Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

5-Inverse-trig-functions/5.4-Inverse-cotangent/154-5.4.1-Inverse-cotangent-functions

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [234]. This is test number [154].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (234)	0.00 (0)
Mathematica	97.44 (228)	2.56 (6)
Maple	97.44 (228)	2.56 (6)
Fricas	71.79 (168)	28.21 (66)
Maxima	61.11 (143)	38.89 (91)
Giac	47.44 (111)	52.56 (123)
Mupad	46.15 (108)	53.85 (126)
Sympy	34.62 (81)	65.38 (153)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

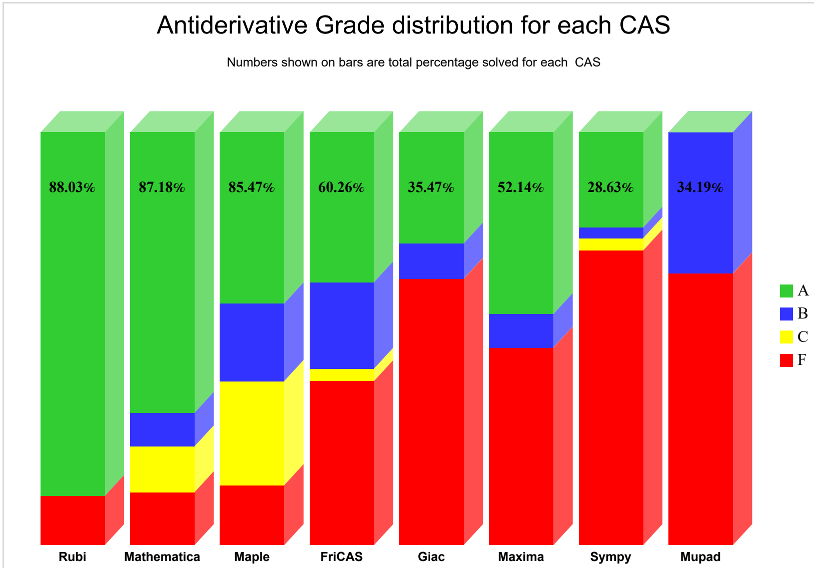
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

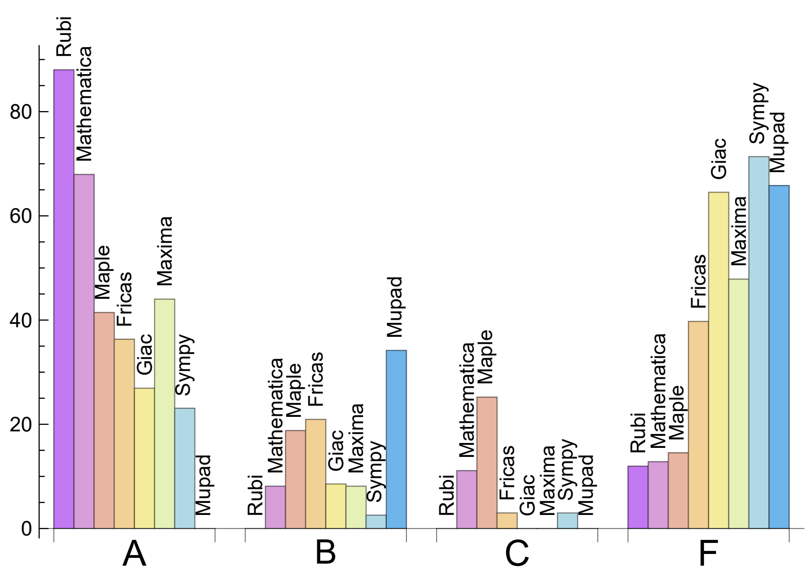
System	% A grade	% B grade	% C grade	% F grade
Rubi	87.607	0.427	0.000	11.966
Mathematica	67.949	8.120	11.111	12.821
Maxima	44.017	8.120	0.000	47.863
Maple	41.453	18.803	25.214	14.530
Fricas	36.325	20.940	2.991	39.744
Giac	26.923	8.547	0.000	64.530
Sympy	23.077	2.564	2.991	71.368
Mupad	0.000	34.188	0.000	65.812

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	6	100.00	0.00	0.00
Maple	6	100.00	0.00	0.00
Fricas	66	98.48	0.00	1.52
Maxima	91	79.12	2.20	18.68
Giac	123	92.68	5.69	1.63
Mupad	126	0.00	100.00	0.00
Sympy	153	52.94	30.72	16.34

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.29
Rubi	0.58
Mathematica	0.70
Mupad	0.96
Maxima	2.14
Giac	2.24
Maple	4.48
Sympy	5.35

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	66.30	1.09	35.00	1.00
Sympy	79.58	1.37	39.00	0.97
Rubi	159.10	1.11	103.50	1.03
Mathematica	173.53	1.39	90.00	1.00
Giac	216.16	2.18	38.00	1.03
Maxima	230.59	2.18	68.00	1.00
Fricas	240.14	1.75	67.50	1.20
Maple	733.20	3.99	128.00	1.16

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

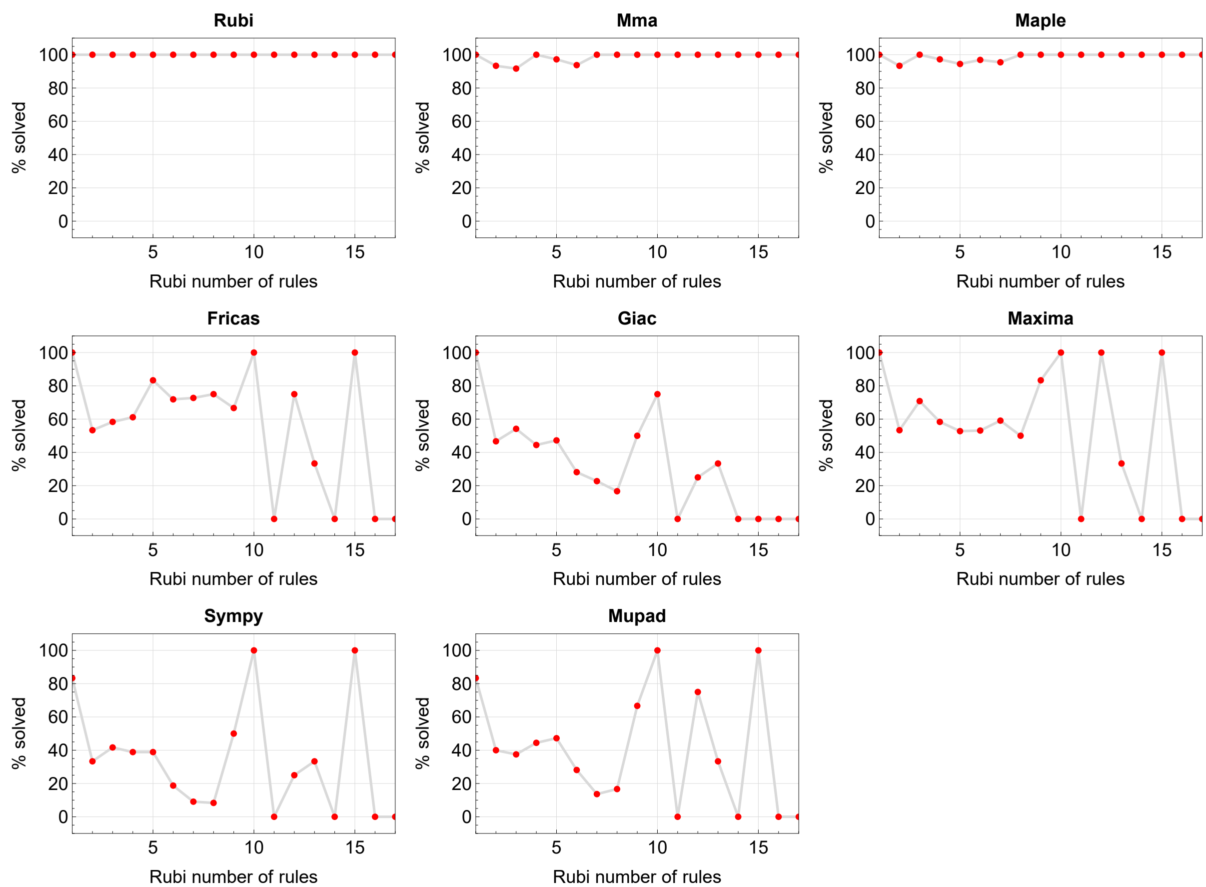


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

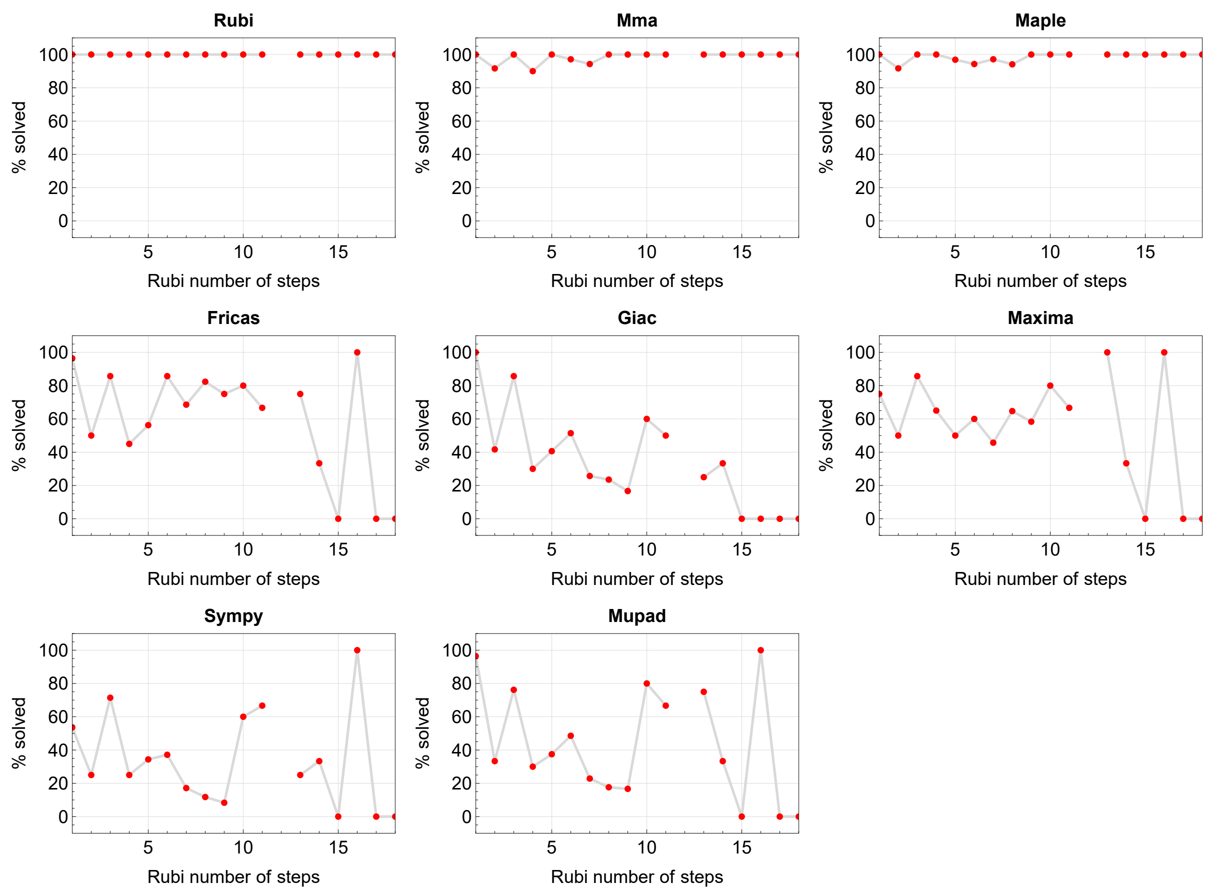


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

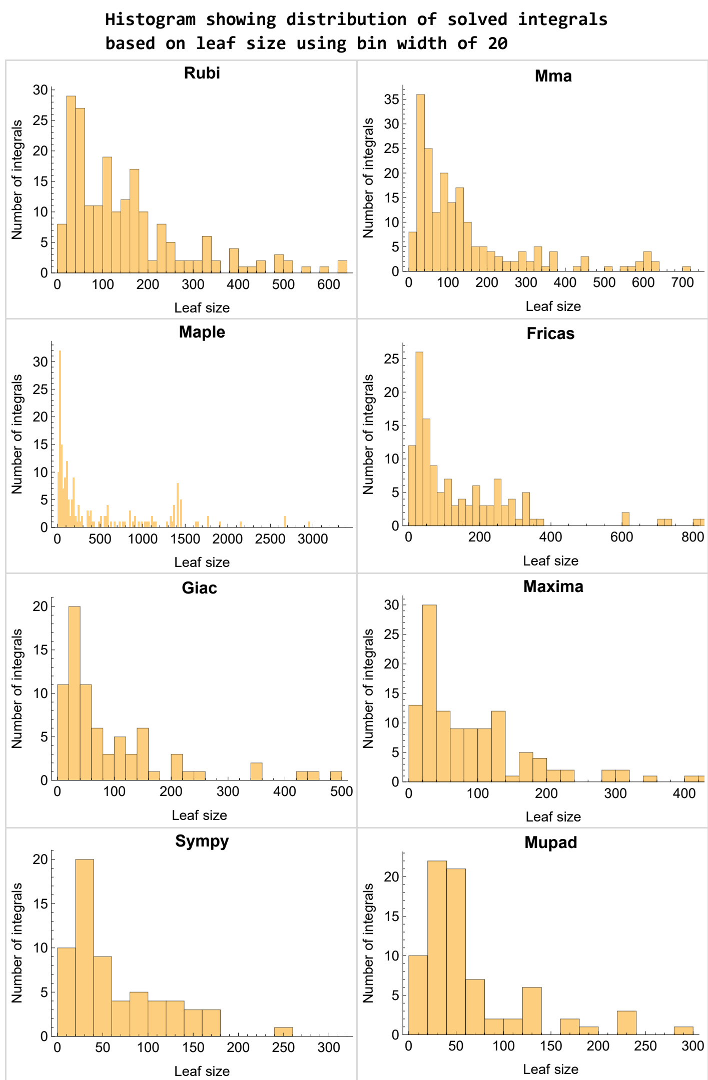


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

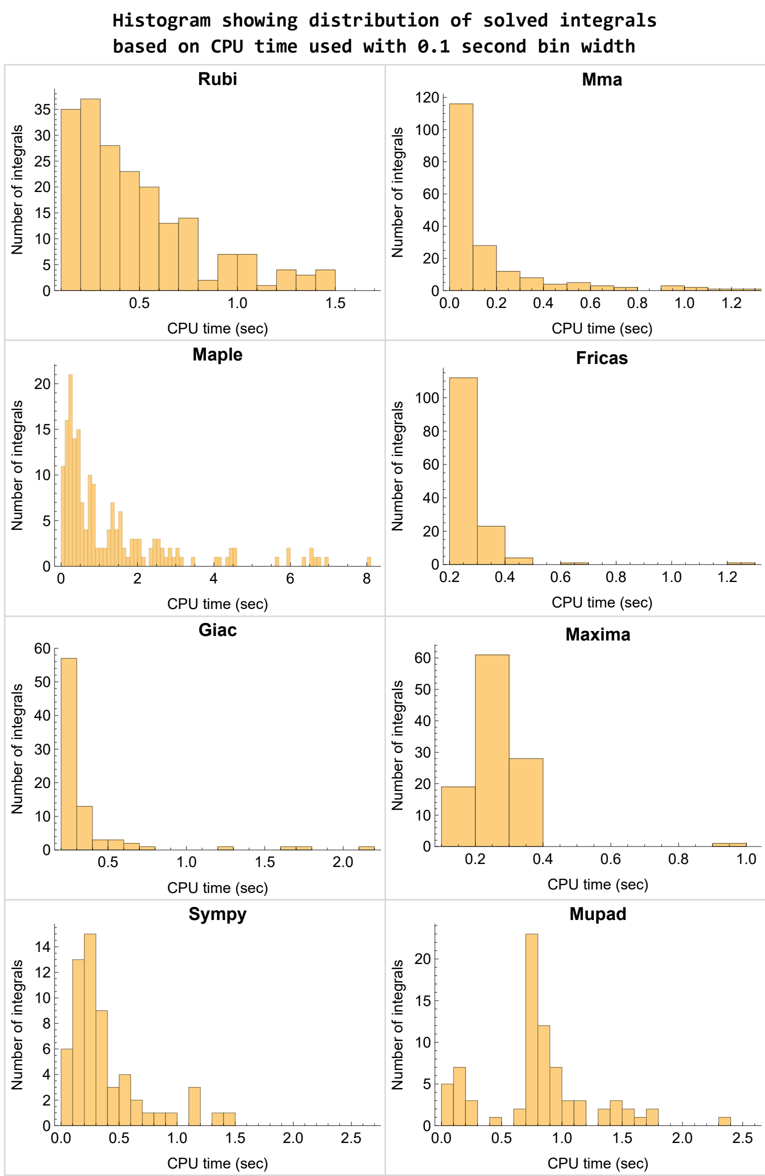


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

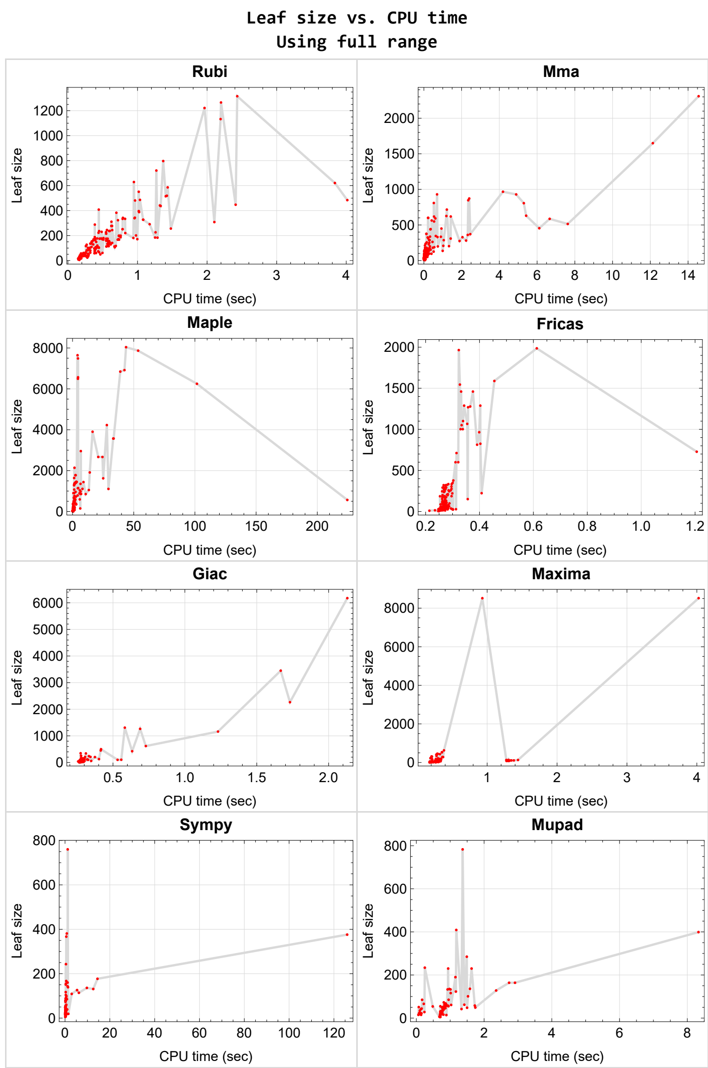


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{34, 35, 59, 60, 116, 117, 120, 121, 128, 147, 148, 151, 155, 156, 161, 165, 169, 174, 178, 182, 187, 191, 195, 199, 204, 208, 212, 216\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{116, 117, 120, 121\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{63, 122, 149, 150\}$

Mathematica $\{58, 141, 160, 164, 168, 173, 177, 181\}$

Maple $\{18, 24, 26, 29, 30, 31, 32, 33, 112, 139, 141, 144, 145, 158, 159, 162, 163, 166, 167, 171, 172, 175, 176, 179, 180, 183, 184, 185, 188, 189, 192, 193, 196, 197, 200, 201, 202, 205, 206, 209, 210, 213, 214, 217, 229, 230, 231, 232, 233, 234\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

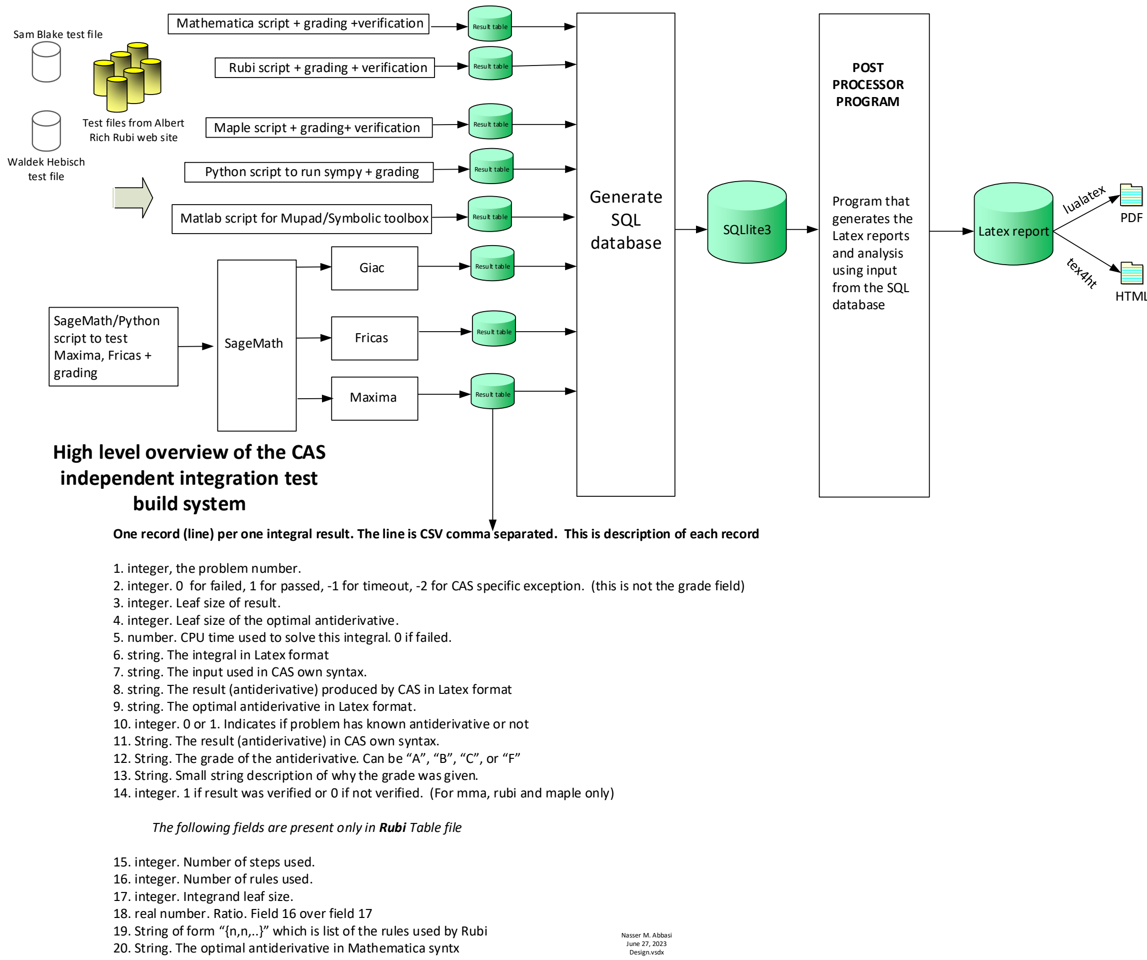
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 118, 119, 122, 123, 124, 125, 126, 127, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 149, 150, 152, 153, 154, 157, 158, 159, 160, 162, 163, 164, 166, 167, 168, 170, 171, 172, 173, 175, 176, 177, 179, 180, 181, 183, 184, 185, 186, 188, 189, 190, 192, 193, 194, 196, 197, 198, 200, 201, 202, 203, 205, 206, 207, 209, 210, 211, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234 }

B grade { 23 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 49, 51, 52, 53, 54, 55, 56, 57, 58, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 91, 92, 93, 94, 95, 96, 97, 98, 102, 107, 109, 110, 111, 112, 114, 115, 118, 119, 122, 124, 125, 126, 127, 132, 136, 137, 138, 140, 142, 143, 146, 149, 150, 154, 157, 158, 159, 162, 163, 166, 167, 170, 171, 172, 175, 176, 179, 180, 184, 185, 186, 188, 189, 190, 192, 193, 194, 196, 197, 198, 201, 202, 203, 205, 206, 207, 209, 210, 211, 213, 214, 215, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 234 }

B grade { 46, 48, 50, 103, 108, 116, 117, 120, 121, 133, 141, 160, 164, 168, 173, 177, 181, 183, 200 }

C grade { 9, 11, 61, 62, 63, 64, 79, 89, 90, 99, 100, 101, 104, 105, 106, 123, 129, 130, 131, 134, 135, 217, 230, 231, 232, 233 }

F normal fail { 113, 139, 144, 145, 152, 153 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 20, 22, 37, 39, 41, 43, 45, 46, 48, 50, 51, 52, 53, 54, 55, 56, 57, 65, 66, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 114, 115, 118, 119, 122, 123, 124, 125, 126, 127, 131, 132, 133, 134, 135, 138, 140, 149, 157, 170, 219, 220, 224, 227, 228 }

B grade { 17, 19, 21, 23, 25, 27, 28, 38, 40, 42, 44, 58, 88, 97, 113, 129, 130, 136, 137, 142, 143, 152, 153, 154, 160, 164, 168, 173, 177, 181, 186, 190, 194, 198, 203, 207, 211, 215, 218, 221, 222, 223, 225, 226 }

C grade { 18, 24, 26, 29, 30, 31, 32, 33, 47, 49, 67, 68, 69, 77, 98, 111, 112, 139, 141, 144, 145, 150, 158, 159, 162, 163, 166, 167, 171, 172, 175, 176, 179, 180, 183, 184, 185, 188, 189, 192, 193, 196, 197, 200, 201, 202, 205, 206, 209, 210, 213, 214, 217, 229, 230, 231, 232, 233, 234 }

F normal fail { 36, 61, 62, 63, 64, 146 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 14, 16, 20, 22, 37, 39, 41, 43, 45, 51, 52, 53, 54, 55, 56, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 102, 104, 105, 106, 122, 123, 125, 129, 130, 131, 132, 134, 149, 150, 157, 170, 175, 176, 177, 179, 180, 181, 217, 219, 220, 223, 224, 225, 226, 227, 228, 234 }

B grade { 61, 62, 63, 64, 135, 158, 159, 160, 162, 163, 164, 166, 167, 168, 171, 172, 173, 183, 184, 185, 186, 188, 189, 190, 192, 193, 194, 196, 197, 198, 200, 201, 202, 203, 205, 206, 207, 209, 210, 211, 213, 214, 215, 218, 221, 222, 229, 230, 233 }

C grade { 80, 81, 82, 83, 84, 231, 232 }

F normal fail { 7, 13, 15, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 36, 38, 40, 42, 44, 46, 47, 48, 49, 50, 57, 58, 65, 66, 77, 88, 98, 103, 107, 108, 109, 110, 111, 112, 113, 114, 115, 118, 119, 124, 126, 127, 133, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 152, 153, 154 }

F(-1) timedout fail { }

F(-2) exception fail { 128 }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 14, 16, 20, 22, 37, 39, 41, 43, 45, 46, 48, 50, 51, 52, 53, 54, 55, 56, 57, 58, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 104, 105, 106, 109, 123, 125, 129, 130, 131, 132, 134, 135, 149, 150, 157, 170, 192, 193, 194, 196, 197, 198, 209, 210, 211, 213, 214, 215, 218, 221, 224, 227, 228, 229, 230, 231, 232, 233, 234 }

B grade { 7, 77, 88, 98, 107, 108, 110, 122, 124, 126, 127, 160, 162, 163, 164, 166, 167, 168, 173 }

C grade { }

F normal fail { 13, 15, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 38, 40, 42, 44, 47, 49, 65, 66, 97, 111, 112, 114, 115, 118, 119, 133, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 152, 153, 154, 158, 159, 171, 172, 183, 184, 185, 186, 188, 189, 190, 200, 201, 202, 203, 205, 206, 207, 217, 219, 220, 222, 223, 225, 226 }

F(-1) timedout fail { 33, 174 }

F(-2) exception fail { 59, 61, 62, 63, 64, 113, 128, 165, 169, 175, 176, 177, 178, 179, 180, 181, 182 }

2.1.6 Giac

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 20, 22, 31, 41, 43, 45, 51, 52, 53, 54, 55, 56, 61, 62, 63, 64, 67, 68, 69, 70, 71, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 126, 150, 157, 170, 227, 228, 229, 230, 233, 234 }

B grade { 6, 99, 100, 101, 102, 104, 105, 106, 122, 123, 124, 125, 127, 129, 130, 131, 132, 134, 135, 149 }

C grade { }

F normal fail { 12, 13, 14, 15, 16, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 36, 37, 38, 39, 40, 42, 44, 46, 47, 48, 49, 50, 57, 58, 65, 66, 72, 73, 77, 97, 98, 103, 108, 109, 114, 115, 118, 119, 133, 136, 137, 138, 139, 141, 142, 143, 146, 152, 153, 154, 158, 159, 160, 162, 163, 164, 166, 167, 168, 171, 172, 173, 175, 176, 177, 179, 180, 181, 183, 184, 185, 186, 188, 189, 190, 192, 193, 194, 196, 197, 198, 201, 202, 203, 205, 206, 207, 209, 210, 211, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 231, 232 }

F(-1) timedout fail { 107, 110, 113, 140, 144, 145, 200 }

F(-2) exception fail { 111, 112 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 14, 16, 17, 20, 22, 37, 39, 41, 43, 45, 51, 52, 53, 54, 55, 56, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 93, 94, 95, 96, 99, 100, 101, 102, 104, 105, 106, 122, 123, 125, 129, 130, 131, 132, 134, 135, 138, 149, 150, 157, 170, 227, 228, 229, 230, 231, 232, 233, 234 }

C grade { }

F normal fail { }

F(-1) timedout fail { 7, 13, 15, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 36, 38, 40, 42, 44, 46, 47, 48, 49, 50, 57, 58, 61, 62, 63, 64, 65, 66, 67, 68, 69, 77, 88, 92, 97, 98, 103, 107, 108, 109, 110, 111, 112, 113, 114, 115, 118, 119, 124, 126, 127, 133, 136, 137, 139, 140, 141, 142, 143, 144, 145, 146, 152, 153, 154, 158, 159, 160, 162, 163, 164, 166, 167, 168, 171, 172, 173, 175, 176, 177, 179, 180, 181, 183, 184, 185, 186, 188, 189, 190, 192, 193, 194, 196, 197, 198, 200, 201, 202, 203, 205, 206, 207, 209, 210, 211, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 14, 16, 20, 22, 37, 39, 41, 43, 45, 51, 52, 53, 54, 55, 56, 70, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 92, 93, 94, 96, 99, 100, 101, 102, 123, 132, 149, 170, 227, 228 }

B grade { 71, 89, 90, 91, 95, 157 }

C grade { 104, 105, 106, 122, 125, 130, 131 }

F normal fail { 7, 13, 15, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 36, 38, 40, 42, 44, 46, 47, 48, 49, 50, 57, 61, 62, 63, 64, 65, 66, 67, 68, 69, 73, 77, 88, 97, 98, 114, 124, 126, 127, 137, 138, 142, 143, 152, 153, 154, 159, 160, 173, 183, 184, 185, 186, 190, 200, 201, 202, 203, 207, 218, 219, 220, 221, 222, 223, 224, 225, 226, 229, 230, 231, 232, 234 }

F(-1) timedout fail { 58, 103, 107, 108, 109, 110, 111, 112, 113, 115, 118, 119, 129, 133, 134, 135, 136, 139, 140, 141, 144, 145, 146, 147, 148, 150, 158, 161, 165, 169, 171, 172, 174, 178, 182, 188, 189, 191, 195, 199, 204, 205, 206, 212, 216, 217, 233 }

F(-2) exception fail { 72, 162, 163, 164, 166, 167, 168, 175, 176, 177, 179, 180, 181, 192, 193, 194, 196, 197, 198, 209, 210, 211, 213, 214, 215 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	52	51	44	47	41	48	59	55
N.S.	1	1.02	1.00	0.86	0.92	0.80	0.94	1.16	1.08
time (sec)	N/A	0.216	0.004	0.170	0.264	0.259	0.292	0.276	0.948

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	50	49	46	46	45	46	74	56
N.S.	1	1.02	1.00	0.94	0.94	0.92	0.94	1.51	1.14
time (sec)	N/A	0.224	0.011	0.124	0.179	0.260	0.257	0.269	0.873

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	42	41	36	38	32	39	51	46
N.S.	1	1.02	1.00	0.88	0.93	0.78	0.95	1.24	1.12
time (sec)	N/A	0.212	0.003	0.164	0.287	0.260	0.228	0.275	0.859

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	40	39	37	36	37	37	64	49
N.S.	1	1.03	1.00	0.95	0.92	0.95	0.95	1.64	1.26
time (sec)	N/A	0.216	0.008	0.108	0.180	0.294	0.193	0.266	0.806

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	32	31	25	28	23	31	36	39
N.S.	1	1.03	1.00	0.81	0.90	0.74	1.00	1.16	1.26
time (sec)	N/A	0.185	0.003	0.146	0.274	0.301	0.165	0.290	0.805

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	24	24	24	45	22
N.S.	1	1.00	1.00	0.96	1.00	1.00	1.00	1.88	0.92
time (sec)	N/A	0.177	0.002	0.103	0.195	0.254	0.087	0.273	0.151

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	33	56	0	0	38	0
N.S.	1	1.00	1.00	0.89	1.51	0.00	0.00	1.03	0.00
time (sec)	N/A	0.219	0.004	0.126	0.305	0.000	0.000	0.277	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	32	30	29	30	31	24	32	28
N.S.	1	1.07	1.00	0.97	1.00	1.03	0.80	1.07	0.93
time (sec)	N/A	0.203	0.003	0.099	0.185	0.256	0.102	0.270	0.238

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	30	36	26	23	24	24	40	44
N.S.	1	0.97	1.16	0.84	0.74	0.77	0.77	1.29	1.42
time (sec)	N/A	0.189	0.004	0.153	0.303	0.253	0.177	0.277	0.815

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	44	42	42	43	39	44	58
N.S.	1	1.00	0.96	0.91	0.91	0.93	0.85	0.96	1.26
time (sec)	N/A	0.223	0.010	0.122	0.186	0.254	0.201	0.271	0.927

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	43	36	35	37	33	32	51	47
N.S.	1	1.05	0.88	0.85	0.90	0.80	0.78	1.24	1.15
time (sec)	N/A	0.204	0.003	0.171	0.271	0.250	0.221	0.275	0.837

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	171	79	90	95	78	104	0	85
N.S.	1	1.64	0.76	0.87	0.91	0.75	1.00	0.00	0.82
time (sec)	N/A	1.103	0.019	0.279	0.308	0.277	0.357	0.000	0.952

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	181	95	205	0	0	0	0	0
N.S.	1	1.34	0.70	1.52	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.021	0.402	0.487	0.000	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	111	61	70	77	60	78	0	66
N.S.	1	1.39	0.76	0.88	0.96	0.75	0.98	0.00	0.82
time (sec)	N/A	0.731	0.016	0.291	0.280	0.260	0.266	0.000	0.219

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	129	76	185	0	0	0	0	0
N.S.	1	1.16	0.68	1.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.679	0.211	0.464	0.000	0.000	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	58	42	44	57	40	54	0	44
N.S.	1	1.09	0.79	0.83	1.08	0.75	1.02	0.00	0.83
time (sec)	N/A	0.420	0.012	0.260	0.282	0.258	0.196	0.000	0.157

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	80	56	130	0	0	0	0	55
N.S.	1	1.19	0.84	1.94	0.00	0.00	0.00	0.00	0.82
time (sec)	N/A	0.410	0.067	0.499	0.000	0.000	0.000	0.000	0.709

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	116	146	132	891	0	0	0	0	0
N.S.	1	1.26	1.14	7.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.651	0.047	5.668	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	76	64	222	0	0	0	0	0
N.S.	1	1.15	0.97	3.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.413	0.033	0.573	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	60	56	64	56	57	53	60	50
N.S.	1	1.02	0.95	1.08	0.95	0.97	0.90	1.02	0.85
time (sec)	N/A	0.439	0.015	0.237	0.275	0.266	0.182	0.282	0.753

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	115	96	251	0	0	0	0	0
N.S.	1	1.02	0.85	2.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.642	0.187	0.718	0.000	0.000	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	113	81	85	95	78	80	91	73
N.S.	1	1.27	0.91	0.96	1.07	0.88	0.90	1.02	0.82
time (sec)	N/A	0.759	0.015	0.287	0.306	0.283	0.232	0.274	0.810

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	447	125	1104	0	0	0	0	0
N.S.	1	2.30	0.64	5.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.662	0.519	29.340	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	205	308	187	1108	0	0	0	0	0
N.S.	1	1.50	0.91	5.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.338	0.553	8.098	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	256	96	857	0	0	0	0	0
N.S.	1	1.73	0.65	5.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.600	0.284	6.969	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	157	184	152	1036	0	0	0	0	0
N.S.	1	1.17	0.97	6.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.358	0.358	5.939	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	117	76	592	0	0	0	0	0
N.S.	1	1.14	0.74	5.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.727	0.068	5.903	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	112	90	187	0	0	0	0	0
N.S.	1	1.17	0.94	1.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.588	0.087	1.154	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	178	220	180	982	0	0	0	0	0
N.S.	1	1.24	1.01	5.52	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.896	0.064	6.607	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	93	116	83	1441	0	0	0	0	0
N.S.	1	1.25	0.89	15.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.620	0.053	8.941	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	105	106	90	2956	0	0	0	29	0
N.S.	1	1.01	0.86	28.15	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.723	0.123	6.764	0.000	0.000	0.000	0.284	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	167	183	151	1622	0	0	0	0	0
N.S.	1	1.10	0.90	9.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.380	0.194	25.043	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	152	226	126	852	0	0	0	0	0
N.S.	1	1.49	0.83	5.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.348	0.221	10.742	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	221	12	10	12	12
N.S.	1	1.00	1.20	1.00	22.10	1.20	1.00	1.20	1.20
time (sec)	N/A	0.170	0.642	0.771	2.291	0.267	1.954	0.291	0.691

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	183	12	10	12	12
N.S.	1	1.00	1.20	1.00	18.30	1.20	1.00	1.20	1.20
time (sec)	N/A	0.172	0.642	0.527	1.569	0.259	0.979	0.297	0.695

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	52	0	0	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.199	0.016	0.000	0.000	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	49	32	34	35	31	34	0	32
N.S.	1	1.22	0.80	0.85	0.88	0.78	0.85	0.00	0.80
time (sec)	N/A	0.508	0.021	0.480	0.278	0.271	0.138	0.000	0.740

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	66	51	126	0	0	0	0	0
N.S.	1	0.99	0.76	1.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.482	0.185	0.738	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	24	19	19	0	19
N.S.	1	1.00	1.00	0.87	1.04	0.83	0.83	0.00	0.83
time (sec)	N/A	0.307	0.014	0.417	0.263	0.256	0.099	0.000	0.703

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	39	98	0	0	0	0	0
N.S.	1	1.00	0.81	2.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.319	0.068	0.353	0.000	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	8	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	1.00	0.75
time (sec)	N/A	0.168	0.004	0.517	0.179	0.250	0.319	0.265	0.681

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	55	43	117	0	0	0	0	0
N.S.	1	1.12	0.88	2.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.314	0.103	0.343	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	33	30	32	29	29	22	26	26
N.S.	1	1.10	1.00	1.07	0.97	0.97	0.73	0.87	0.87
time (sec)	N/A	0.315	0.012	0.354	0.265	0.255	0.140	0.272	0.098

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	74	55	178	0	0	0	0	0
N.S.	1	1.03	0.76	2.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.469	0.168	0.746	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	61	47	43	55	47	42	39	35
N.S.	1	1.30	1.00	0.91	1.17	1.00	0.89	0.83	0.74
time (sec)	N/A	0.542	0.017	0.415	0.280	0.258	0.251	0.268	0.103

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	334	626	261	189	0	0	0	0
N.S.	1	1.62	3.04	1.27	0.92	0.00	0.00	0.00	0.00
time (sec)	N/A	0.895	1.192	0.917	0.298	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	309	134	0	0	0	0	0
N.S.	1	1.00	1.64	0.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.416	1.421	0.764	0.000	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	311	592	183	187	0	0	0	0
N.S.	1	1.70	3.23	1.00	1.02	0.00	0.00	0.00	0.00
time (sec)	N/A	0.703	0.609	1.168	0.301	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	328	197	0	0	0	0	0
N.S.	1	1.00	1.47	0.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.480	2.045	0.816	0.000	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	342	619	257	183	0	0	0	0
N.S.	1	1.61	2.92	1.21	0.86	0.00	0.00	0.00	0.00
time (sec)	N/A	0.863	1.424	0.845	0.317	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	5	5	8	5
N.S.	1	1.00	1.00	1.20	1.00	1.00	1.00	1.60	1.00
time (sec)	N/A	0.173	0.060	0.286	0.177	0.247	0.092	0.268	0.709

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	13	17	15	13
N.S.	1	1.00	1.00	1.08	1.00	1.00	1.31	1.15	1.00
time (sec)	N/A	0.189	0.008	0.645	0.187	0.272	0.688	0.269	0.696

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	212	245	226	237	367	347	234
N.S.	1	1.00	0.87	1.00	0.93	0.97	1.50	1.42	0.96
time (sec)	N/A	0.663	0.057	0.515	0.189	0.268	0.567	0.275	0.250

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	149	167	159	167	243	252	190
N.S.	1	1.00	0.89	0.99	0.95	0.99	1.45	1.50	1.13
time (sec)	N/A	0.553	0.044	0.454	0.215	0.262	0.414	0.278	1.154

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	97	105	103	108	151	171	116
N.S.	1	1.00	0.89	0.96	0.94	0.99	1.39	1.57	1.06
time (sec)	N/A	0.370	0.030	0.432	0.214	0.263	0.310	0.279	1.014

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	67	57	53	57	73	99	62
N.S.	1	1.00	1.16	0.98	0.91	0.98	1.26	1.71	1.07
time (sec)	N/A	0.261	0.010	0.182	0.219	0.256	0.222	0.270	0.830

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	403	629	716	392	528	0	0	0	0
N.S.	1	1.56	1.78	0.97	1.31	0.00	0.00	0.00	0.00
time (sec)	N/A	1.052	1.217	1.316	0.352	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	801	797	806	2141	628	0	0	0	0
N.S.	1	1.00	1.01	2.67	0.78	0.00	0.00	0.00	0.00
time (sec)	N/A	1.517	5.297	1.592	0.382	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	0	16	15	16	16
N.S.	1	1.00	1.12	0.88	0.00	1.00	0.94	1.00	1.00
time (sec)	N/A	0.191	4.298	0.765	0.000	0.260	4.389	0.286	0.725

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	16	15	16	16
N.S.	1	1.00	1.12	0.88	1.00	1.00	0.94	1.00	1.00
time (sec)	N/A	0.195	2.879	0.710	0.919	0.265	1.440	0.286	0.704

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	66	169	0	0	349	0	59	0
N.S.	1	1.00	2.56	0.00	0.00	5.29	0.00	0.89	0.00
time (sec)	N/A	0.281	0.183	0.000	0.000	0.300	0.000	0.302	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	134	136	262	0	0	712	0	126	0
N.S.	1	1.01	1.96	0.00	0.00	5.31	0.00	0.94	0.00
time (sec)	N/A	0.350	0.457	0.000	0.000	0.315	0.000	0.303	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F	A	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	208	201	345	0	0	1278	0	208	0
N.S.	1	0.97	1.66	0.00	0.00	6.14	0.00	1.00	0.00
time (sec)	N/A	1.055	0.632	0.000	0.000	0.366	0.000	0.306	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	293	291	450	0	0	1986	0	340	0
N.S.	1	0.99	1.54	0.00	0.00	6.78	0.00	1.16	0.00
time (sec)	N/A	1.266	0.931	0.000	0.000	0.613	0.000	0.312	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	154	136	117	0	0	0	0	0
N.S.	1	0.79	0.70	0.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.379	0.997	0.870	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	116	89	99	0	0	0	0	0
N.S.	1	0.75	0.57	0.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.288	0.130	0.773	0.000	0.000	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	21	55	31	29	0	33	0
N.S.	1	1.00	0.60	1.57	0.89	0.83	0.00	0.94	0.00
time (sec)	N/A	0.197	0.032	0.568	0.270	0.298	0.000	0.281	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	82	37	101	63	52	0	55	0
N.S.	1	1.04	0.47	1.28	0.80	0.66	0.00	0.70	0.00
time (sec)	N/A	0.290	0.042	1.299	0.274	0.262	0.000	0.301	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	129	47	130	93	70	0	83	0
N.S.	1	1.09	0.40	1.10	0.79	0.59	0.00	0.70	0.00
time (sec)	N/A	0.394	0.046	1.407	0.271	0.270	0.000	0.300	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	37	25	24	26	21	31	32	22
N.S.	1	1.16	0.78	0.75	0.81	0.66	0.97	1.00	0.69
time (sec)	N/A	0.196	0.015	0.335	0.296	0.253	0.185	0.277	0.080

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	54	36	37	39	39	88	40	27
N.S.	1	1.23	0.82	0.84	0.89	0.89	2.00	0.91	0.61
time (sec)	N/A	0.213	0.019	0.359	0.264	0.255	0.257	0.277	0.734

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	28	35	38	26	0	0	22
N.S.	1	1.00	0.82	1.03	1.12	0.76	0.00	0.00	0.65
time (sec)	N/A	0.187	0.012	0.748	0.277	0.282	0.000	0.000	0.734

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	61	46	65	75	40	0	0	51
N.S.	1	1.09	0.82	1.16	1.34	0.71	0.00	0.00	0.91
time (sec)	N/A	0.268	0.020	0.964	0.283	0.257	0.000	0.000	0.072

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	42	41	39	38	39	39	40	35
N.S.	1	1.02	1.00	0.95	0.93	0.95	0.95	0.98	0.85
time (sec)	N/A	0.219	0.012	0.241	0.208	0.279	0.357	0.272	0.769

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	38	37	31	34	27	36	38	31
N.S.	1	1.03	1.00	0.84	0.92	0.73	0.97	1.03	0.84
time (sec)	N/A	0.206	0.006	0.286	0.291	0.263	0.274	0.269	0.709

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	28	28	28	31	47	27
N.S.	1	1.00	1.00	0.90	0.90	0.90	1.00	1.52	0.87
time (sec)	N/A	0.188	0.006	0.162	0.209	0.247	0.139	0.267	0.707

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	41	37	57	68	0	0	0	0
N.S.	1	1.11	1.00	1.54	1.84	0.00	0.00	0.00	0.00
time (sec)	N/A	0.255	0.008	0.276	0.331	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	36	34	31	32	37	29	32	31
N.S.	1	1.06	1.00	0.91	0.94	1.09	0.85	0.94	0.91
time (sec)	N/A	0.193	0.006	0.129	0.214	0.279	0.213	0.276	0.769

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	34	38	31	27	28	29	29	32
N.S.	1	0.97	1.09	0.89	0.77	0.80	0.83	0.83	0.91
time (sec)	N/A	0.202	0.006	0.191	0.261	0.301	0.273	0.280	0.727

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	175	136	112	137	122	136	156	54
N.S.	1	1.15	0.89	0.74	0.90	0.80	0.89	1.03	0.36
time (sec)	N/A	0.421	0.033	0.395	0.290	0.272	9.749	0.286	0.484

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	164	133	109	135	111	126	153	52
N.S.	1	1.09	0.89	0.73	0.90	0.74	0.84	1.02	0.35
time (sec)	N/A	0.393	0.026	0.267	0.285	0.266	5.369	0.288	0.790

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	152	102	97	120	96	109	144	42
N.S.	1	1.15	0.77	0.73	0.91	0.73	0.83	1.09	0.32
time (sec)	N/A	0.367	0.034	0.261	0.282	0.274	2.978	0.278	0.139

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	149	105	98	123	107	114	135	44
N.S.	1	1.10	0.78	0.73	0.91	0.79	0.84	1.00	0.33
time (sec)	N/A	0.370	0.032	0.220	0.262	0.278	6.182	0.285	0.775

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	170	146	106	133	129	131	149	52
N.S.	1	1.13	0.97	0.71	0.89	0.86	0.87	0.99	0.35
time (sec)	N/A	0.400	0.038	0.300	0.270	0.263	12.597	0.282	0.845

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	52	40	32	31	27	39	33	31
N.S.	1	1.02	0.78	0.63	0.61	0.53	0.76	0.65	0.61
time (sec)	N/A	0.194	0.012	0.039	0.288	0.252	0.972	0.280	0.760

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	43	33	27	26	20	32	28	26
N.S.	1	1.02	0.79	0.64	0.62	0.48	0.76	0.67	0.62
time (sec)	N/A	0.193	0.010	0.039	0.264	0.270	0.525	0.285	0.763

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	29	22	17	16	12	19	14	16
N.S.	1	1.32	1.00	0.77	0.73	0.55	0.86	0.64	0.73
time (sec)	N/A	0.175	0.014	0.057	0.269	0.254	0.344	0.268	0.700

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	37	31	61	35	0	0	19	0
N.S.	1	1.19	1.00	1.97	1.13	0.00	0.00	0.61	0.00
time (sec)	N/A	0.254	0.006	0.224	0.299	0.000	0.000	0.284	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	32	29	18	17	19	92	19	17
N.S.	1	1.39	1.26	0.78	0.74	0.83	4.00	0.83	0.74
time (sec)	N/A	0.183	0.009	0.044	0.265	0.266	0.492	0.277	0.742

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	43	34	27	26	27	160	26	24
N.S.	1	1.02	0.81	0.64	0.62	0.64	3.81	0.62	0.57
time (sec)	N/A	0.187	0.009	0.047	0.268	0.252	1.139	0.275	0.760

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	35	29	25	24	24	85	39	24
N.S.	1	0.97	0.81	0.69	0.67	0.67	2.36	1.08	0.67
time (sec)	N/A	0.204	0.013	0.042	0.183	0.270	0.847	0.258	0.729

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	28	25	20	19	19	24	30	0
N.S.	1	0.97	0.86	0.69	0.66	0.66	0.83	1.03	0.00
time (sec)	N/A	0.187	0.010	0.038	0.190	0.265	0.486	0.291	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	17	18	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	0.94	1.00	0.78
time (sec)	N/A	0.168	0.010	0.039	0.209	0.259	0.111	0.284	0.886

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	25	20	16	20
N.S.	1	1.00	1.00	0.86	0.82	1.14	0.91	0.73	0.91
time (sec)	N/A	0.173	0.010	0.040	0.201	0.264	0.331	0.265	0.752

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	32	29	26	25	33	143	23	27
N.S.	1	0.86	0.78	0.70	0.68	0.89	3.86	0.62	0.73
time (sec)	N/A	0.200	0.014	0.049	0.199	0.275	1.138	0.274	0.751

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	14	13	15
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.82	0.76	0.88
time (sec)	N/A	0.173	0.001	0.192	0.203	0.251	0.066	0.264	0.060

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	45	40	83	0	63	0	0	0
N.S.	1	0.96	0.85	1.77	0.00	1.34	0.00	0.00	0.00
time (sec)	N/A	0.260	0.014	0.543	0.000	0.287	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	41	37	57	68	0	0	0	0
N.S.	1	1.11	1.00	1.54	1.84	0.00	0.00	0.00	0.00
time (sec)	N/A	0.256	0.007	0.418	0.334	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	99	95	131	104	92	155	617	133
N.S.	1	0.93	0.90	1.24	0.98	0.87	1.46	5.82	1.25
time (sec)	N/A	0.316	0.061	0.295	0.294	0.263	0.381	0.728	0.922

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	81	114	102	85	73	117	423	101
N.S.	1	1.01	1.42	1.28	1.06	0.91	1.46	5.29	1.26
time (sec)	N/A	0.300	0.032	0.243	0.306	0.272	0.309	0.633	1.525

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	90	63	68	55	78	210	61
N.S.	1	1.00	1.50	1.05	1.13	0.92	1.30	3.50	1.02
time (sec)	N/A	0.275	0.027	0.262	0.269	0.278	0.256	0.344	1.027

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	31	44	30	29	43	46	111	42
N.S.	1	0.94	1.33	0.91	0.88	1.30	1.39	3.36	1.27
time (sec)	N/A	0.205	0.010	0.207	0.181	0.278	0.139	0.321	1.333

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	256	103	133	0	0	0	0
N.S.	1	1.00	2.13	0.86	1.11	0.00	0.00	0.00	0.00
time (sec)	N/A	0.467	0.153	0.389	0.338	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	61	66	61	77	64	167	498	62
N.S.	1	0.98	1.06	0.98	1.24	1.03	2.69	8.03	1.00
time (sec)	N/A	0.241	0.050	0.240	0.289	0.282	0.539	0.417	1.416

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	102	92	83	112	99	381	1309	230
N.S.	1	1.07	0.97	0.87	1.18	1.04	4.01	13.78	2.42
time (sec)	N/A	0.302	0.087	0.310	0.290	0.289	0.798	0.582	1.631

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	136	126	115	165	142	760	3449	285
N.S.	1	1.05	0.98	0.89	1.28	1.10	5.89	26.74	2.21
time (sec)	N/A	0.358	0.108	0.305	0.273	0.272	1.184	1.667	1.490

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	642	721	563	591	8519	0	0	0	0
N.S.	1	1.12	0.88	0.92	13.27	0.00	0.00	0.00	0.00
time (sec)	N/A	1.373	0.413	2.008	4.025	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	170	325	187	283	0	0	0	0
N.S.	1	1.12	2.14	1.23	1.86	0.00	0.00	0.00	0.00
time (sec)	N/A	0.537	0.200	0.788	0.359	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	338	481	514	296	280	0	0	0	0
N.S.	1	1.42	1.52	0.88	0.83	0.00	0.00	0.00	0.00
time (sec)	N/A	1.032	7.625	0.672	0.371	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	735	1222	930	728	8518	0	0	0	0
N.S.	1	1.66	1.27	0.99	11.59	0.00	0.00	0.00	0.00
time (sec)	N/A	2.200	0.699	2.795	0.931	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	693	1133	618	364	0	0	0	0	0
N.S.	1	1.63	0.89	0.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.418	0.538	0.427	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	830	1316	809	388	0	0	0	0	0
N.S.	1	1.59	0.97	0.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.680	0.528	0.471	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F(-2)	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	367	388	0	959	0	0	0	0	0
N.S.	1	1.06	0.00	2.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.125	0.000	2.493	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	127	127	118	0	0	0	0	0
N.S.	1	0.96	0.96	0.89	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.288	0.133	1.284	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	155	138	156	0	0	0	0	0
N.S.	1	0.72	0.64	0.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.370	0.088	1.305	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	177	26	28	28	29	28	28
N.S.	1	1.00	6.32	0.93	1.00	1.00	1.04	1.00	1.00
time (sec)	N/A	0.255	0.302	0.747	0.275	0.265	0.801	0.440	0.820

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	33	33	180	31	33	33	31	33	33
N.S.	1	1.00	5.45	0.94	1.00	1.00	0.94	1.00	1.00
time (sec)	N/A	0.285	0.141	0.783	0.297	0.285	5.943	0.451	0.838

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	177	202	160	0	0	0	0	0
N.S.	1	0.95	1.08	0.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.446	1.332	1.520	0.000	0.000	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	215	207	202	0	0	0	0	0
N.S.	1	0.77	0.74	0.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.594	1.030	1.433	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	35	35	198	33	35	44	36	35	35
N.S.	1	1.00	5.66	0.94	1.00	1.26	1.03	1.00	1.00
time (sec)	N/A	0.343	0.741	0.884	0.357	0.284	3.454	0.496	0.885

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	200	38	40	49	37	40	40
N.S.	1	1.00	5.00	0.95	1.00	1.22	0.92	1.00	1.00
time (sec)	N/A	0.405	0.322	0.853	0.373	0.265	24.399	0.513	0.941

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	43	42	42	93	81	99	203	85
N.S.	1	0.83	0.81	0.81	1.79	1.56	1.90	3.90	1.63
time (sec)	N/A	0.266	0.017	0.433	0.255	0.266	0.620	0.374	0.170

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	141	36	52	33	56	100	49
N.S.	1	1.00	3.62	0.92	1.33	0.85	1.44	2.56	1.26
time (sec)	N/A	0.217	0.044	0.247	0.278	0.261	0.380	0.327	1.741

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	43	38	55	112	0	0	100	0
N.S.	1	0.96	0.84	1.22	2.49	0.00	0.00	2.22	0.00
time (sec)	N/A	0.269	0.007	0.475	0.328	0.000	0.000	0.532	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	45	42	41	53	59	139	238	57
N.S.	1	0.96	0.89	0.87	1.13	1.26	2.96	5.06	1.21
time (sec)	N/A	0.240	0.014	0.467	0.197	0.265	1.333	0.333	0.931

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	39	35	37	64	0	0	26	0
N.S.	1	1.11	1.00	1.06	1.83	0.00	0.00	0.74	0.00
time (sec)	N/A	0.260	0.006	0.564	0.318	0.000	0.000	0.297	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	43	38	55	122	0	0	103	0
N.S.	1	0.96	0.84	1.22	2.71	0.00	0.00	2.29	0.00
time (sec)	N/A	0.272	0.007	0.563	0.320	0.000	0.000	0.558	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	0	0	17	18	18
N.S.	1	1.00	1.11	0.89	0.00	0.00	0.94	1.00	1.00
time (sec)	N/A	0.187	4.040	0.446	0.000	0.000	3.132	0.304	1.148

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	230	157	496	341	325	0	2265	783
N.S.	1	0.99	0.67	2.13	1.46	1.39	0.00	9.72	3.36
time (sec)	N/A	0.513	0.201	0.883	0.276	0.280	0.000	1.731	1.367

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	163	118	294	216	206	376	1161	409
N.S.	1	1.06	0.77	1.91	1.40	1.34	2.44	7.54	2.66
time (sec)	N/A	0.411	0.114	0.676	0.297	0.279	125.905	1.231	1.179

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	103	163	113	113	110	177	451	136
N.S.	1	1.06	1.68	1.16	1.16	1.13	1.82	4.65	1.40
time (sec)	N/A	0.340	0.070	0.353	0.271	0.273	14.479	0.415	1.583

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	49	35	34	52	51	116	48
N.S.	1	1.00	1.29	0.92	0.89	1.37	1.34	3.05	1.26
time (sec)	N/A	0.173	0.010	0.297	0.182	0.268	0.165	0.321	1.498

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	180	336	197	0	0	0	0	0
N.S.	1	1.11	2.07	1.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.545	0.328	0.753	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	149	118	161	177	223	0	1264	128
N.S.	1	0.97	0.77	1.05	1.16	1.46	0.00	8.26	0.84
time (sec)	N/A	0.435	0.162	0.717	0.266	0.408	0.000	0.689	2.356

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	245	180	245	410	728	0	6173	399
N.S.	1	1.07	0.79	1.07	1.80	3.19	0.00	27.07	1.75
time (sec)	N/A	0.593	0.473	1.422	0.273	1.206	0.000	2.130	8.331

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	382	383	586	1072	0	0	0	0	0
N.S.	1	1.00	1.53	2.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.766	6.667	1.340	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	227	286	427	0	0	0	0	0
N.S.	1	1.03	1.30	1.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.589	1.054	0.884	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	118	184	0	0	0	0	123
N.S.	1	1.00	1.16	1.80	0.00	0.00	0.00	0.00	1.21
time (sec)	N/A	0.477	0.150	0.843	0.000	0.000	0.000	0.000	1.167

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	261	288	0	1911	0	0	0	0	0
N.S.	1	1.10	0.00	7.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.419	0.000	14.056	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	567	586	454	784	0	0	0	0	0
N.S.	1	1.03	0.80	1.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.547	6.116	4.178	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	565	551	2309	6248	0	0	0	0	0
N.S.	1	0.98	4.09	11.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.133	14.560	101.639	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	334	630	1051	0	0	0	0	0
N.S.	1	0.99	1.87	3.12	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.840	5.420	13.322	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	141	228	395	0	0	0	0	0
N.S.	1	0.99	1.59	2.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.674	0.256	2.461	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	372	408	0	3903	0	0	0	0	0
N.S.	1	1.10	0.00	10.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.467	0.000	16.365	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	1233	1266	0	4229	0	0	0	0	0
N.S.	1	1.03	0.00	3.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.423	0.000	28.029	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	177	236	162	0	0	0	0	0	0
N.S.	1	1.33	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.476	0.304	0.000	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	618	36	0	22	22
N.S.	1	1.00	1.10	1.00	30.90	1.80	0.00	1.10	1.10
time (sec)	N/A	0.301	4.821	0.469	5.113	0.260	0.000	0.660	0.843

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	880	52	0	22	22
N.S.	1	1.00	1.10	1.00	44.00	2.60	0.00	1.10	1.10
time (sec)	N/A	0.291	0.520	0.551	7.835	0.283	0.000	0.721	0.796

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	34	37	37	35	51	60	127	230
N.S.	1	0.81	0.88	0.88	0.83	1.21	1.43	3.02	5.48
time (sec)	N/A	0.282	0.014	0.340	0.194	0.271	0.568	0.403	0.940

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	36	40	149	38	56	0	60	58
N.S.	1	0.80	0.89	3.31	0.84	1.24	0.00	1.33	1.29
time (sec)	N/A	0.287	0.027	6.300	0.197	0.282	0.000	0.348	1.726

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	42	36	39	38	34	38	39
N.S.	1	1.00	1.05	0.90	0.98	0.95	0.85	0.95	0.98
time (sec)	N/A	0.253	0.151	1.408	0.884	0.274	3.353	0.416	1.195

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	488	486	0	1640	0	0	0	0	0
N.S.	1	1.00	0.00	3.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.121	0.000	1.319	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	321	323	0	903	0	0	0	0	0
N.S.	1	1.01	0.00	2.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.788	0.000	0.701	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	93	93	363	0	0	0	0	0
N.S.	1	0.95	0.95	3.70	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.329	0.029	0.431	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	42	36	39	48	61	38	39
N.S.	1	1.00	1.05	0.90	0.98	1.20	1.52	0.95	0.98
time (sec)	N/A	0.251	0.220	0.767	0.449	0.260	2.832	0.378	0.892

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	42	36	163	91	126	38	39
N.S.	1	1.00	1.05	0.90	4.08	2.28	3.15	0.95	0.98
time (sec)	N/A	0.246	1.029	0.909	0.500	0.275	6.396	0.530	1.693

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	20	17	15	48	30	21
N.S.	1	1.00	1.12	1.25	1.06	0.94	3.00	1.88	1.31
time (sec)	N/A	0.160	0.008	0.304	0.195	0.235	0.069	0.274	0.087

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	403	519	371	8038	0	1965	0	0	0
N.S.	1	1.29	0.92	19.95	0.00	4.88	0.00	0.00	0.00
time (sec)	N/A	1.533	2.459	43.734	0.000	0.323	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	305	396	281	7646	0	1545	0	0	0
N.S.	1	1.30	0.92	25.07	0.00	5.07	0.00	0.00	0.00
time (sec)	N/A	1.105	2.238	4.089	0.000	0.327	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	198	277	549	1138	433	1101	0	0	0
N.S.	1	1.40	2.77	5.75	2.19	5.56	0.00	0.00	0.00
time (sec)	N/A	0.684	0.542	4.354	0.324	0.339	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	0	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.00	1.13	1.13
time (sec)	N/A	0.295	0.332	0.139	227.568	0.258	0.000	1.510	0.900

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	154	198	136	1448	310	321	0	0	0
N.S.	1	1.29	0.88	9.40	2.01	2.08	0.00	0.00	0.00
time (sec)	N/A	0.808	0.181	2.878	0.210	0.274	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	123	155	110	1413	218	270	0	0	0
N.S.	1	1.26	0.89	11.49	1.77	2.20	0.00	0.00	0.00
time (sec)	N/A	0.633	0.081	2.549	0.202	0.273	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	85	107	967	562	455	199	0	0	0
N.S.	1	1.26	11.38	6.61	5.35	2.34	0.00	0.00	0.00
time (sec)	N/A	0.445	4.192	2.062	0.286	0.282	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	0	36	0	21	22
N.S.	1	1.00	1.10	0.90	0.00	1.71	0.00	1.00	1.05
time (sec)	N/A	0.277	0.276	0.205	0.000	0.246	0.000	1.106	1.054

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	155	199	141	1449	312	321	0	0	0
N.S.	1	1.28	0.91	9.35	2.01	2.07	0.00	0.00	0.00
time (sec)	N/A	0.804	0.187	3.056	0.204	0.267	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	124	156	111	1414	220	270	0	0	0
N.S.	1	1.26	0.90	11.40	1.77	2.18	0.00	0.00	0.00
time (sec)	N/A	0.639	0.067	2.635	0.201	0.268	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	86	108	847	595	450	201	0	0	0
N.S.	1	1.26	9.85	6.92	5.23	2.34	0.00	0.00	0.00
time (sec)	N/A	0.444	2.352	2.142	0.294	0.263	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	36	0	22	22
N.S.	1	1.00	1.09	0.91	0.00	1.64	0.00	1.00	1.00
time (sec)	N/A	0.337	0.693	0.228	0.000	0.259	0.000	1.151	1.237

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	17	10	10	19	10	16
N.S.	1	1.00	1.12	1.06	0.62	0.62	1.19	0.62	1.00
time (sec)	N/A	0.161	0.006	0.654	0.175	0.214	0.060	0.274	0.799

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	399	515	360	7868	0	1589	0	0	0
N.S.	1	1.29	0.90	19.72	0.00	3.98	0.00	0.00	0.00
time (sec)	N/A	1.511	2.325	53.611	0.000	0.455	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	303	394	275	7488	0	1289	0	0	0
N.S.	1	1.30	0.91	24.71	0.00	4.25	0.00	0.00	0.00
time (sec)	N/A	1.099	1.892	4.436	0.000	0.403	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	198	277	1649	1146	532	965	0	0	0
N.S.	1	1.40	8.33	5.79	2.69	4.87	0.00	0.00	0.00
time (sec)	N/A	0.660	12.133	4.579	0.355	0.399	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-1)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	0	17	0	17	17
N.S.	1	1.00	1.13	1.00	0.00	1.13	0.00	1.13	1.13
time (sec)	N/A	0.280	0.300	0.194	0.000	0.257	0.000	3.495	0.908

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	154	198	140	1448	0	174	0	0	0
N.S.	1	1.29	0.91	9.40	0.00	1.13	0.00	0.00	0.00
time (sec)	N/A	0.799	0.161	3.115	0.000	0.276	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	123	155	110	1413	0	152	0	0	0
N.S.	1	1.26	0.89	11.49	0.00	1.24	0.00	0.00	0.00
time (sec)	N/A	0.623	0.093	2.616	0.000	0.272	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	A	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	85	107	929	587	0	116	0	0	0
N.S.	1	1.26	10.93	6.91	0.00	1.36	0.00	0.00	0.00
time (sec)	N/A	0.427	4.892	1.566	0.000	0.261	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	26	0	42	0	28	25
N.S.	1	1.00	1.10	1.24	0.00	2.00	0.00	1.33	1.19
time (sec)	N/A	0.282	0.326	0.099	0.000	0.254	0.000	2.381	1.434

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	155	199	136	1449	0	174	0	0	0
N.S.	1	1.28	0.88	9.35	0.00	1.12	0.00	0.00	0.00
time (sec)	N/A	0.819	0.165	3.421	0.000	0.271	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	124	156	110	1414	0	152	0	0	0
N.S.	1	1.26	0.89	11.40	0.00	1.23	0.00	0.00	0.00
time (sec)	N/A	0.636	0.091	2.865	0.000	0.262	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	A	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	86	108	872	630	0	116	0	0	0
N.S.	1	1.26	10.14	7.33	0.00	1.35	0.00	0.00	0.00
time (sec)	N/A	0.447	2.405	1.367	0.000	0.267	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	25	0	42	0	27	25
N.S.	1	1.00	1.09	1.14	0.00	1.91	0.00	1.23	1.14
time (sec)	N/A	0.297	0.365	0.059	0.000	0.272	0.000	2.724	1.544

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	299	327	600	3570	0	1460	0	0	0
N.S.	1	1.09	2.01	11.94	0.00	4.88	0.00	0.00	0.00
time (sec)	N/A	1.200	0.233	33.571	0.000	0.376	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	229	251	375	2668	0	1002	0	0	0
N.S.	1	1.10	1.64	11.65	0.00	4.38	0.00	0.00	0.00
time (sec)	N/A	0.847	0.135	24.430	0.000	0.338	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	159	175	237	1776	0	600	0	0	0
N.S.	1	1.10	1.49	11.17	0.00	3.77	0.00	0.00	0.00
time (sec)	N/A	0.608	0.095	2.572	0.000	0.311	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	79	91	184	0	334	0	0	0
N.S.	1	1.08	1.25	2.52	0.00	4.58	0.00	0.00	0.00
time (sec)	N/A	0.343	0.023	1.737	0.000	0.297	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	14	3	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.93	0.20	1.13
time (sec)	N/A	0.217	0.675	0.240	1.561	0.270	3.143	90.586	0.832

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	355	441	438	6916	0	1289	0	0	0
N.S.	1	1.24	1.23	19.48	0.00	3.63	0.00	0.00	0.00
time (sec)	N/A	1.416	0.347	42.430	0.000	0.343	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	267	343	330	6566	0	1067	0	0	0
N.S.	1	1.28	1.24	24.59	0.00	4.00	0.00	0.00	0.00
time (sec)	N/A	1.022	0.723	4.506	0.000	0.354	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	237	288	352	0	825	0	0	0
N.S.	1	1.36	1.66	2.02	0.00	4.74	0.00	0.00	0.00
time (sec)	N/A	0.659	0.275	3.026	0.000	0.403	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	0	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.00	1.13	1.13
time (sec)	N/A	0.280	3.872	0.155	1.047	0.244	0.000	0.578	0.883

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	142	167	134	1405	129	292	0	0	0
N.S.	1	1.18	0.94	9.89	0.91	2.06	0.00	0.00	0.00
time (sec)	N/A	0.791	0.147	2.531	1.275	0.267	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	113	132	103	1369	107	246	0	0	0
N.S.	1	1.17	0.91	12.12	0.95	2.18	0.00	0.00	0.00
time (sec)	N/A	0.616	0.059	1.962	1.382	0.288	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	93	71	544	80	186	0	0	0
N.S.	1	1.18	0.90	6.89	1.01	2.35	0.00	0.00	0.00
time (sec)	N/A	0.420	0.099	1.833	1.314	0.270	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	17	81	36	0	19	20
N.S.	1	1.00	1.11	0.89	4.26	1.89	0.00	1.00	1.05
time (sec)	N/A	0.282	3.107	0.105	0.642	0.250	0.000	0.370	1.063

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	145	170	134	1409	129	292	0	0	0
N.S.	1	1.17	0.92	9.72	0.89	2.01	0.00	0.00	0.00
time (sec)	N/A	0.784	0.158	2.473	1.441	0.276	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	116	136	103	1373	106	246	0	0	0
N.S.	1	1.17	0.89	11.84	0.91	2.12	0.00	0.00	0.00
time (sec)	N/A	0.620	0.087	2.032	1.347	0.270	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	95	71	517	80	186	0	0	0
N.S.	1	1.16	0.87	6.30	0.98	2.27	0.00	0.00	0.00
time (sec)	N/A	0.424	0.086	1.855	1.276	0.271	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	73	36	0	19	20
N.S.	1	1.00	1.09	0.91	3.32	1.64	0.00	0.86	0.91
time (sec)	N/A	0.295	3.190	0.128	0.628	0.250	0.000	0.375	1.055

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	299	327	600	3570	0	1460	0	0	0
N.S.	1	1.09	2.01	11.94	0.00	4.88	0.00	0.00	0.00
time (sec)	N/A	1.165	0.219	33.246	0.000	0.332	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	229	251	375	2668	0	1002	0	0	0
N.S.	1	1.10	1.64	11.65	0.00	4.38	0.00	0.00	0.00
time (sec)	N/A	0.831	0.134	21.014	0.000	0.330	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	159	175	237	1776	0	600	0	0	0
N.S.	1	1.10	1.49	11.17	0.00	3.77	0.00	0.00	0.00
time (sec)	N/A	0.584	0.100	2.316	0.000	0.321	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	80	91	184	0	334	0	0	0
N.S.	1	1.08	1.23	2.49	0.00	4.51	0.00	0.00	0.00
time (sec)	N/A	0.355	0.024	1.514	0.000	0.284	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	0	3	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.00	0.20	1.13
time (sec)	N/A	0.216	0.702	0.199	1.528	0.285	0.000	105.033	0.818

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	351	437	442	6844	0	1269	0	0	0
N.S.	1	1.25	1.26	19.50	0.00	3.62	0.00	0.00	0.00
time (sec)	N/A	1.432	0.359	39.007	0.000	0.358	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	265	339	334	6494	0	1051	0	0	0
N.S.	1	1.28	1.26	24.51	0.00	3.97	0.00	0.00	0.00
time (sec)	N/A	1.027	0.900	4.400	0.000	0.334	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	237	287	352	0	813	0	0	0
N.S.	1	1.36	1.65	2.02	0.00	4.67	0.00	0.00	0.00
time (sec)	N/A	0.640	0.306	2.918	0.000	0.391	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	14	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.93	1.13	1.13
time (sec)	N/A	0.279	3.839	0.142	1.054	0.261	170.894	0.544	0.859

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	142	167	134	1404	129	292	0	0	0
N.S.	1	1.18	0.94	9.89	0.91	2.06	0.00	0.00	0.00
time (sec)	N/A	0.780	0.155	1.944	1.303	0.263	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	113	133	103	1368	107	246	0	0	0
N.S.	1	1.18	0.91	12.11	0.95	2.18	0.00	0.00	0.00
time (sec)	N/A	0.614	0.073	1.542	1.339	0.261	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	93	71	544	80	186	0	0	0
N.S.	1	1.18	0.90	6.89	1.01	2.35	0.00	0.00	0.00
time (sec)	N/A	0.423	0.105	1.670	1.303	0.262	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	17	73	36	0	19	20
N.S.	1	1.00	1.11	0.89	3.84	1.89	0.00	1.00	1.05
time (sec)	N/A	0.276	3.090	0.123	0.620	0.255	0.000	0.337	1.019

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	145	170	134	1410	129	292	0	0	0
N.S.	1	1.17	0.92	9.72	0.89	2.01	0.00	0.00	0.00
time (sec)	N/A	0.781	0.155	2.308	1.308	0.272	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	116	135	103	1374	106	246	0	0	0
N.S.	1	1.16	0.89	11.84	0.91	2.12	0.00	0.00	0.00
time (sec)	N/A	0.628	0.076	1.911	1.285	0.269	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	95	71	517	80	186	0	0	0
N.S.	1	1.16	0.87	6.30	0.98	2.27	0.00	0.00	0.00
time (sec)	N/A	0.427	0.093	1.872	1.283	0.253	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	81	36	0	19	20
N.S.	1	1.00	1.09	0.91	3.68	1.64	0.00	0.86	0.91
time (sec)	N/A	0.278	3.140	0.135	0.612	0.254	0.000	0.337	1.006

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	187	187	132	566	0	250	0	0	0
N.S.	1	1.00	0.71	3.03	0.00	1.34	0.00	0.00	0.00
time (sec)	N/A	0.838	0.258	224.515	0.000	0.288	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	59	53	34	40	0	0	0
N.S.	1	1.00	1.69	1.51	0.97	1.14	0.00	0.00	0.00
time (sec)	N/A	0.227	0.019	0.820	0.308	0.284	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	63	58	50	0	65	0	0	0
N.S.	1	0.89	0.82	0.70	0.00	0.92	0.00	0.00	0.00
time (sec)	N/A	0.342	0.014	1.042	0.000	0.280	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	105	92	76	0	87	0	0	0
N.S.	1	1.02	0.89	0.74	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.491	0.014	1.694	0.000	0.255	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	49	83	95	63	103	0	0	0
N.S.	1	0.96	1.63	1.86	1.24	2.02	0.00	0.00	0.00
time (sec)	N/A	0.235	0.036	0.353	0.320	0.272	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	97	83	355	0	151	0	0	0
N.S.	1	0.94	0.81	3.45	0.00	1.47	0.00	0.00	0.00
time (sec)	N/A	0.399	0.013	0.802	0.000	0.356	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	159	133	413	0	187	0	0	0
N.S.	1	1.05	0.88	2.74	0.00	1.24	0.00	0.00	0.00
time (sec)	N/A	0.570	0.017	0.803	0.000	0.296	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	177	167	162	189	212	0	0	0
N.S.	1	0.90	0.85	0.83	0.96	1.08	0.00	0.00	0.00
time (sec)	N/A	0.525	0.114	1.250	0.339	0.283	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	484	250	678	0	304	0	0	0
N.S.	1	1.94	1.00	2.71	0.00	1.22	0.00	0.00	0.00
time (sec)	N/A	4.417	0.288	1.044	0.000	0.296	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	621	313	764	0	378	0	0	0
N.S.	1	1.98	1.00	2.44	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	4.231	0.228	1.543	0.000	0.303	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	33	27	25	19	28	19	21	22
N.S.	1	1.22	1.00	0.93	0.70	1.04	0.70	0.78	0.81
time (sec)	N/A	0.217	0.016	0.127	0.235	0.313	1.482	0.280	0.105

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	17	15	12	18	15
N.S.	1	1.00	1.00	0.82	1.00	0.88	0.71	1.06	0.88
time (sec)	N/A	0.206	0.091	0.299	0.230	0.264	0.217	0.279	0.140

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	47	44	61	1281	47	131	0	66	65
N.S.	1	0.94	1.30	27.26	1.00	2.79	0.00	1.40	1.38
time (sec)	N/A	0.320	0.060	1.248	0.310	0.276	0.000	0.293	0.877

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	B	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	103	104	146	1354	131	276	0	154	133
N.S.	1	1.01	1.42	13.15	1.27	2.68	0.00	1.50	1.29
time (sec)	N/A	0.478	0.084	6.524	0.331	0.296	0.000	0.298	1.009

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	180	175	89	1323	167	258	0	0	164
N.S.	1	0.97	0.49	7.35	0.93	1.43	0.00	0.00	0.91
time (sec)	N/A	0.501	0.067	1.479	0.310	0.293	0.000	0.000	2.913

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	180	175	89	1323	167	258	0	0	164
N.S.	1	0.97	0.49	7.35	0.93	1.43	0.00	0.00	0.91
time (sec)	N/A	0.497	0.065	1.320	0.309	0.285	0.000	0.000	2.739

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	103	104	145	855	169	221	0	154	135
N.S.	1	1.01	1.41	8.30	1.64	2.15	0.00	1.50	1.31
time (sec)	N/A	0.444	0.091	6.551	0.342	0.299	0.000	0.285	0.975

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	48	46	59	903	48	75	0	65	68
N.S.	1	0.96	1.23	18.81	1.00	1.56	0.00	1.35	1.42
time (sec)	N/A	0.316	0.053	1.352	0.310	0.284	0.000	0.284	0.863

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [23] had the largest ratio of [1.69999999999999996]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	1.02	8	0.375
2	A	5	4	1.02	8	0.500
3	A	3	3	1.02	8	0.375
4	A	5	4	1.03	8	0.500
5	A	3	3	1.03	6	0.500
6	A	2	2	1.00	4	0.500
7	A	2	2	1.00	8	0.250
8	A	6	5	1.07	8	0.625
9	A	3	3	0.97	8	0.375
10	A	5	4	1.00	8	0.500
11	A	4	4	1.05	8	0.500
12	A	16	15	1.64	10	1.500
13	A	14	13	1.34	10	1.300
14	A	11	10	1.39	10	1.000
15	A	10	9	1.16	10	0.900
16	A	5	5	1.09	8	0.625
17	A	6	5	1.19	6	0.833
18	A	4	4	1.26	10	0.400
19	A	4	4	1.15	10	0.400
20	A	9	8	1.02	10	0.800
21	A	8	8	1.02	10	0.800
22	A	14	13	1.27	10	1.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	B	18	17	2.30	10	1.700
24	A	17	16	1.50	10	1.600
25	A	14	13	1.73	10	1.300
26	A	11	11	1.17	10	1.100
27	A	9	8	1.14	8	1.000
28	A	5	5	1.17	6	0.833
29	A	5	5	1.24	10	0.500
30	A	5	5	1.25	10	0.500
31	A	7	7	1.01	10	0.700
32	A	15	14	1.10	10	1.400
33	A	11	11	1.49	10	1.100
34	N/A	1	0	1.00	10	0.000
35	N/A	1	0	1.00	10	0.000
36	A	2	2	1.00	8	0.250
37	A	10	9	1.22	13	0.692
38	A	9	8	0.99	13	0.615
39	A	4	4	1.00	13	0.308
40	A	5	4	1.00	11	0.364
41	A	1	1	1.00	10	0.100
42	A	3	3	1.12	13	0.231
43	A	8	7	1.10	13	0.538
44	A	7	7	1.03	13	0.538
45	A	13	12	1.30	13	0.923
46	A	9	9	1.62	15	0.600
47	A	2	2	1.00	13	0.154
48	A	6	6	1.70	12	0.500
49	A	2	2	1.00	15	0.133
50	A	13	12	1.61	15	0.800
51	A	1	1	1.00	12	0.083
52	A	1	1	1.00	12	0.083
53	A	6	5	1.00	14	0.357
54	A	6	5	1.00	14	0.357
55	A	6	5	1.00	14	0.357

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	6	5	1.00	12	0.417
57	A	6	6	1.56	14	0.429
58	A	4	4	1.00	14	0.286
59	N/A	1	0	1.00	16	0.000
60	N/A	1	0	1.00	16	0.000
61	A	6	5	1.00	16	0.312
62	A	7	6	1.01	16	0.375
63	A	8	7	0.97	16	0.438
64	A	6	5	0.99	16	0.312
65	A	3	3	0.79	14	0.214
66	A	2	2	0.75	14	0.143
67	A	1	1	1.00	14	0.071
68	A	2	2	1.04	14	0.143
69	A	3	3	1.09	14	0.214
70	A	3	3	1.16	11	0.273
71	A	4	4	1.23	11	0.364
72	A	2	2	1.00	10	0.200
73	A	4	4	1.09	12	0.333
74	A	5	4	1.02	10	0.400
75	A	5	4	1.03	10	0.400
76	A	2	2	1.00	8	0.250
77	A	4	3	1.11	10	0.300
78	A	6	5	1.06	10	0.500
79	A	5	4	0.97	10	0.400
80	A	11	10	1.15	10	1.000
81	A	11	10	1.09	10	1.000
82	A	10	9	1.15	6	1.500
83	A	10	9	1.10	10	0.900
84	A	11	10	1.13	10	1.000
85	A	7	6	1.02	10	0.600
86	A	6	5	1.02	8	0.625
87	A	5	4	1.32	6	0.667
88	A	4	3	1.19	10	0.300

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
89	A	5	4	1.39	10	0.400
90	A	6	5	1.02	10	0.500
91	A	3	3	0.97	12	0.250
92	A	3	3	0.97	12	0.250
93	A	2	2	1.00	12	0.167
94	A	4	4	1.00	12	0.333
95	A	3	3	0.86	12	0.250
96	A	3	3	1.00	4	0.750
97	A	4	3	0.96	10	0.300
98	A	4	3	1.11	10	0.300
99	A	7	6	0.93	10	0.600
100	A	6	5	1.01	10	0.500
101	A	7	6	1.00	8	0.750
102	A	4	3	0.94	6	0.500
103	A	8	7	1.00	10	0.700
104	A	8	7	0.98	10	0.700
105	A	6	5	1.07	10	0.500
106	A	7	6	1.05	10	0.600
107	A	8	7	1.12	16	0.438
108	A	7	6	1.12	14	0.429
109	A	7	7	1.42	16	0.438
110	A	7	7	1.66	16	0.438
111	A	5	4	1.63	18	0.222
112	A	5	4	1.59	18	0.222
113	A	2	2	1.06	19	0.105
114	A	3	2	0.96	28	0.071
115	A	4	3	0.72	33	0.091
116	N/A	3	0	1.00	28	0.000
117	N/A	3	0	1.00	33	0.000
118	A	5	4	0.95	35	0.114
119	A	6	5	0.77	40	0.125
120	N/A	3	0	1.00	35	0.000
121	N/A	3	0	1.00	40	0.000

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
122	A	6	5	0.83	14	0.357
123	A	5	4	1.00	12	0.333
124	A	4	3	0.96	14	0.214
125	A	7	6	0.96	14	0.429
126	A	5	4	1.11	12	0.333
127	A	5	4	0.96	19	0.211
128	N/A	1	0	1.00	18	0.000
129	A	6	5	0.99	18	0.278
130	A	6	5	1.06	18	0.278
131	A	6	5	1.06	16	0.312
132	A	1	1	1.00	10	0.100
133	A	7	6	1.11	18	0.333
134	A	10	9	0.97	18	0.500
135	A	6	6	1.07	18	0.333
136	A	5	4	1.00	20	0.200
137	A	5	4	1.03	18	0.222
138	A	7	6	1.00	12	0.500
139	A	4	3	1.10	20	0.150
140	A	7	6	1.03	20	0.300
141	A	5	4	0.98	20	0.200
142	A	5	4	0.99	18	0.222
143	A	7	6	0.99	12	0.500
144	A	4	3	1.10	20	0.150
145	A	7	6	1.03	20	0.300
146	A	5	4	1.33	18	0.222
147	N/A	3	0	1.00	20	0.000
148	N/A	3	0	1.00	20	0.000
149	A	5	4	0.81	12	0.333
150	A	5	4	0.80	14	0.286
151	N/A	1	0	1.00	40	0.000
152	A	7	6	1.00	40	0.150
153	A	6	5	1.01	40	0.125
154	A	4	3	0.95	38	0.079

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
155	N/A	1	0	1.00	40	0.000
156	N/A	1	0	1.00	40	0.000
157	A	3	2	1.00	7	0.286
158	A	7	6	1.29	15	0.400
159	A	6	5	1.30	13	0.385
160	A	5	4	1.40	11	0.364
161	N/A	1	0	1.00	15	0.000
162	A	8	7	1.29	21	0.333
163	A	7	6	1.26	19	0.316
164	A	6	5	1.26	17	0.294
165	N/A	1	0	1.00	21	0.000
166	A	9	8	1.28	22	0.364
167	A	8	7	1.26	20	0.350
168	A	7	6	1.26	18	0.333
169	N/A	1	0	1.00	22	0.000
170	A	3	2	1.00	7	0.286
171	A	7	6	1.29	15	0.400
172	A	6	5	1.30	13	0.385
173	A	5	4	1.40	11	0.364
174	N/A	1	0	1.00	15	0.000
175	A	9	8	1.29	21	0.381
176	A	8	7	1.26	19	0.368
177	A	7	6	1.26	17	0.353
178	N/A	1	0	1.00	21	0.000
179	A	8	7	1.28	22	0.318
180	A	7	6	1.26	20	0.300
181	A	6	5	1.26	18	0.278
182	N/A	1	0	1.00	22	0.000
183	A	9	8	1.09	15	0.533
184	A	8	7	1.10	15	0.467
185	A	7	6	1.10	13	0.462
186	A	6	5	1.08	7	0.714
187	N/A	1	0	1.00	15	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
188	A	7	6	1.24	15	0.400
189	A	6	5	1.28	13	0.385
190	A	5	4	1.36	11	0.364
191	N/A	1	0	1.00	15	0.000
192	A	9	8	1.18	19	0.421
193	A	8	7	1.17	17	0.412
194	A	7	6	1.18	15	0.400
195	N/A	1	0	1.00	19	0.000
196	A	8	7	1.17	22	0.318
197	A	7	6	1.17	20	0.300
198	A	6	5	1.16	18	0.278
199	N/A	1	0	1.00	22	0.000
200	A	9	8	1.09	15	0.533
201	A	8	7	1.10	15	0.467
202	A	7	6	1.10	13	0.462
203	A	6	5	1.08	7	0.714
204	N/A	1	0	1.00	15	0.000
205	A	7	6	1.25	15	0.400
206	A	6	5	1.28	13	0.385
207	A	5	4	1.36	11	0.364
208	N/A	1	0	1.00	15	0.000
209	A	9	8	1.18	19	0.421
210	A	8	7	1.18	17	0.412
211	A	7	6	1.18	15	0.400
212	N/A	1	0	1.00	19	0.000
213	A	8	7	1.17	22	0.318
214	A	7	6	1.16	20	0.300
215	A	6	5	1.16	18	0.278
216	N/A	1	0	1.00	22	0.000
217	A	2	2	1.00	24	0.083
218	A	4	3	1.00	4	0.750
219	A	5	4	0.89	6	0.667
220	A	6	5	1.02	8	0.625

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
221	A	4	3	0.96	8	0.375
222	A	5	4	0.94	10	0.400
223	A	6	5	1.05	12	0.417
224	A	9	8	0.90	12	0.667
225	A	7	7	1.94	14	0.500
226	A	7	7	1.98	16	0.438
227	A	7	6	1.22	10	0.600
228	A	1	1	1.00	19	0.053
229	A	6	5	0.94	20	0.250
230	A	8	7	1.01	20	0.350
231	A	13	12	0.97	20	0.600
232	A	13	12	0.97	20	0.600
233	A	9	8	1.01	20	0.400
234	A	7	6	0.96	20	0.300

CHAPTER 3

Listing of integrals

3.1 $\int x^5 \cot^{-1}(ax)\,dx$

3.1.1 Optimal result

Integrand size = 8, antiderivative size = 51

$$\int x^5 \cot^{-1}(ax)\,dx = \frac{x}{6a^5} - \frac{x^3}{18a^3} + \frac{x^5}{30a} + \frac{1}{6}x^6 \cot^{-1}(ax) - \frac{\arctan(ax)}{6a^6}$$

output
```
1/6*x/a^5-1/18*x^3/a^3+1/30*x^5/a+1/6*x^6*arccot(a*x)-1/6*arctan(a*x)/a^6
```

3.1.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int x^5 \cot^{-1}(ax)\,dx = \frac{x}{6a^5} - \frac{x^3}{18a^3} + \frac{x^5}{30a} + \frac{1}{6}x^6 \cot^{-1}(ax) - \frac{\arctan(ax)}{6a^6}$$

input
```
Integrate[x^5*ArcCot[a*x],x]
```

output
```
x/(6*a^5) - x^3/(18*a^3) + x^5/(30*a) + (x^6*ArcCot[a*x])/6 - ArcTan[a*x]/
(6*a^6)
```

3.1.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5362, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \cot^{-1}(ax) \, dx$$

$$\Big\downarrow \text{5362}$$

$$\frac{1}{6} a \int \frac{x^6}{a^2 x^2+1} dx + \frac{1}{6} x^6 \cot^{-1}(ax)$$

$$\Big\downarrow \text{254}$$

$$\frac{1}{6} a \int \left(\frac{x^4}{a^2} - \frac{x^2}{a^4} - \frac{1}{a^6 (a^2 x^2+1)} + \frac{1}{a^6}\right) dx + \frac{1}{6} x^6 \cot^{-1}(ax)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6} a \left(-\frac{\arctan(ax)}{a^7} + \frac{x}{a^6} - \frac{x^3}{3a^4} + \frac{x^5}{5a^2}\right) + \frac{1}{6} x^6 \cot^{-1}(ax)$$

input
```
Int[x^5*ArcCot[a*x],x]
```

output
```
(x^6*ArcCot[a*x])/6 + (a*(x/a^6 - x^3/(3*a^4) + x^5/(5*a^2) - ArcTan[a*x]/
a^7))/6
```

3.1.3.1 Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.1.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{\frac{a^6x^6\operatorname{arccot}(ax)}{6}+\frac{a^5x^5}{30}-\frac{a^3x^3}{18}+\frac{ax}{6}-\frac{\arctan(ax)}{6}}{a^6}$	44
default	$\frac{\frac{a^6x^6\operatorname{arccot}(ax)}{6}+\frac{a^5x^5}{30}-\frac{a^3x^3}{18}+\frac{ax}{6}-\frac{\arctan(ax)}{6}}{a^6}$	44
parallelrisch	$\frac{15a^6x^6\operatorname{arccot}(ax)+3a^5x^5-5a^3x^3+15ax+15\operatorname{arccot}(ax)}{90a^6}$	45
parts	$\frac{x^6\operatorname{arccot}(ax)}{6}+\frac{a\left(\frac{\frac{1}{5}a^4x^5-\frac{1}{3}a^2x^3+x}{a^6}-\frac{\arctan(ax)}{a^7}\right)}{6}$	46
risch	$\frac{ix^6\ln(iax+1)}{12}-\frac{ix^6\ln(-iax+1)}{12}+\frac{x^6\pi}{12}+\frac{x^5}{30a}-\frac{x^3}{18a^3}+\frac{x}{6a^5}-\frac{\arctan(ax)}{6a^6}$	67

input
```
int(x^5*arccot(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^6*(1/6*a^6*x^6*arccot(a*x)+1/30*a^5*x^5-1/18*a^3*x^3+1/6*a*x-1/6*arcta
n(a*x))
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int x^5\cot^{-1}(ax)\,dx=\frac{3\,a^5x^5-5\,a^3x^3+15\,ax+15\,(a^6x^6+1)\operatorname{arccot}(ax)}{90\,a^6}$$

input
```
integrate(x^5*arccot(a*x),x, algorithm="fricas")
```

output
```
1/90*(3*a^5*x^5 - 5*a^3*x^3 + 15*a*x + 15*(a^6*x^6 + 1)*arccot(a*x))/a^6
```

3.1.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int x^5 \cot^{-1}(ax)\,dx = \begin{cases} \frac{x^6 \operatorname{acot}(ax)}{6} + \frac{x^5}{30a} - \frac{x^3}{18a^3} + \frac{x}{6a^5} + \frac{\operatorname{acot}(ax)}{6a^6} & \text{for } a \neq 0 \\ \frac{\pi x^6}{12} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*acot(a*x),x)
```

output
```
Piecewise((x**6*acot(a*x)/6 + x**5/(30*a) - x**3/(18*a**3) + x/(6*a**5) +
acot(a*x)/(6*a**6), Ne(a, 0)), (pi*x**6/12, True))
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int x^5 \cot^{-1}(ax)\,dx = \frac{1}{6}\,x^6 \operatorname{arccot}(ax) + \frac{1}{90}\,a\left(\frac{3\,a^4x^5 - 5\,a^2x^3 + 15\,x}{a^6} - \frac{15\,\arctan(ax)}{a^7}\right)$$

input
```
integrate(x^5*arccot(a*x),x, algorithm="maxima")
```

output
```
1/6*x^6*arccot(a*x) + 1/90*a*((3*a^4*x^5 - 5*a^2*x^3 + 15*x)/a^6 - 15*arct
an(a*x)/a^7)
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

$$\int x^5 \cot^{-1}(ax)\,dx = \frac{1}{90}\left(\frac{15\,x^6 \arctan\left(\frac{1}{ax}\right)}{a} - \frac{x^5\left(\frac{5}{a^2x^2} - \frac{15}{a^4x^4} - 3\right)}{a^2} + \frac{15\,\arctan\left(\frac{1}{ax}\right)}{a^7}\right)a$$

input
```
integrate(x^5*arccot(a*x),x, algorithm="giac")
```

output
```
1/90*(15*x^6*arctan(1/(a*x))/a - x^5*(5/(a^2*x^2) - 15/(a^4*x^4) - 3)/a^2
+ 15*arctan(1/(a*x))/a^7)*a
```

3.1.9 Mupad [B] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

$$\int x^5 \cot^{-1}(ax) \, dx = \begin{cases} \frac{\pi x^6}{12} & \text{if } a = 0 \\ \frac{x^6 \operatorname{acot}(a x)}{6} - \frac{\frac{\operatorname{atan}(a x)}{6} - \frac{a x}{6} + \frac{a^3 x^3}{18} - \frac{a^5 x^5}{30}}{a^6} & \text{if } a \neq 0 \end{cases}$$

input
```
int(x^5*acot(a*x),x)
```

output
```
piecewise(a == 0, (x^6*pi)/12, a ~= 0, - (atan(a*x)/6 - (a*x)/6 + (a^3*x^3
)/18 - (a^5*x^5)/30)/a^6 + (x^6*acot(a*x))/6)
```

3.2 $\int x^4 \cot^{-1}(ax)\,dx$

3.2.1 Optimal result

Integrand size = 8, antiderivative size = 49

$$\int x^4 \cot^{-1}(ax)\,dx = -\frac{x^2}{10a^3} + \frac{x^4}{20a} + \frac{1}{5}x^5 \cot^{-1}(ax) + \frac{\log(1 + a^2x^2)}{10a^5}$$

output
```
-1/10*x^2/a^3+1/20*x^4/a+1/5*x^5*arccot(a*x)+1/10*ln(a^2*x^2+1)/a^5
```

3.2.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int x^4 \cot^{-1}(ax)\,dx = -\frac{x^2}{10a^3} + \frac{x^4}{20a} + \frac{1}{5}x^5 \cot^{-1}(ax) + \frac{\log(1 + a^2x^2)}{10a^5}$$

input
```
Integrate[x^4*ArcCot[a*x],x]
```

output
```
-1/10*x^2/a^3 + x^4/(20*a) + (x^5*ArcCot[a*x])/5 + Log[1 + a^2*x^2]/(10*a^
5)
```

3.2.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5362, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \cot^{-1}(ax)\,dx$$

$$\Big\downarrow \text{5362}$$

$$\frac{1}{5}a\int \frac{x^5}{a^2x^2+1}dx+\frac{1}{5}x^5\cot^{-1}(ax)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{10}a\int \frac{x^4}{a^2x^2+1}dx^2+\frac{1}{5}x^5\cot^{-1}(ax)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{10}a\int \left(\frac{x^2}{a^2}+\frac{1}{a^4\left(a^2x^2+1\right)}-\frac{1}{a^4}\right)dx^2+\frac{1}{5}x^5\cot^{-1}(ax)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log\left(a^2x^2+1\right)}{a^6}\right)+\frac{1}{5}x^5\cot^{-1}(ax)$$

input
```
Int[x^4*ArcCot[a*x],x]
```

output
```
(x^5*ArcCot[a*x])/5 + (a*(-(x^2/a^4) + x^4/(2*a^2) + Log[1 + a^2*x^2]/a^6)
)/10
```

3.2.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.2.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{\frac{a^5x^5\,\mathrm{arccot}(ax)}{5}+\frac{a^4x^4}{20}-\frac{a^2x^2}{10}+\frac{\ln\left(a^2x^2+1\right)}{10}}{a^5}$	46
default	$\frac{\frac{a^5x^5\,\mathrm{arccot}(ax)}{5}+\frac{a^4x^4}{20}-\frac{a^2x^2}{10}+\frac{\ln\left(a^2x^2+1\right)}{10}}{a^5}$	46
parallelrisch	$\frac{4a^5x^5\,\mathrm{arccot}(ax)+a^4x^4-2a^2x^2+2+2\ln(a^2x^2+1)}{20a^5}$	47
parts	$\frac{x^5\,\mathrm{arccot}(ax)}{5}+\frac{a\left(\frac{\frac{1}{2}a^2x^4-x^2}{2a^4}+\frac{\ln\left(a^2x^2+1\right)}{2a^6}\right)}{5}$	49
risch	$\frac{ix^5\ln(iax+1)}{10}-\frac{ix^5\ln(-iax+1)}{10}+\frac{x^5\pi}{10}+\frac{x^4}{20a}-\frac{x^2}{10a^3}+\frac{\ln(-a^2x^2-1)}{10a^5}$	68

input
```
int(x^4*arccot(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(1/5*a^5*x^5*arccot(a*x)+1/20*a^4*x^4-1/10*a^2*x^2+1/10*ln(a^2*x^2+1
))
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int x^4 \cot^{-1}(ax)\,dx = \frac{4\,a^5x^5 \operatorname{arccot}(ax) + a^4x^4 - 2\,a^2x^2 + 2\,\log(a^2x^2+1)}{20\,a^5}$$

input
```
integrate(x^4*arccot(a*x),x, algorithm="fricas")
```

output
```
1/20*(4*a^5*x^5*arccot(a*x) + a^4*x^4 - 2*a^2*x^2 + 2*log(a^2*x^2 + 1))/a^
5
```

3.2.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int x^4 \cot^{-1}(ax)\,dx = \begin{cases} \frac{x^5 \operatorname{acot}(ax)}{5} + \frac{x^4}{20a} - \frac{x^2}{10a^3} + \frac{\log(a^2x^2+1)}{10a^5} & \text{for } a \neq 0 \\ \frac{\pi x^5}{10} & \text{otherwise} \end{cases}$$

input
```
integrate(x**4*acot(a*x),x)
```

output
```
Piecewise((x**5*acot(a*x)/5 + x**4/(20*a) - x**2/(10*a**3) + log(a**2*x**2
 + 1)/(10*a**5), Ne(a, 0)), (pi*x**5/10, True))
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int x^4 \cot^{-1}(ax)\,dx = \frac{1}{5}\,x^5 \operatorname{arccot}(ax) + \frac{1}{20}\,a\left(\frac{a^2x^4 - 2\,x^2}{a^4} + \frac{2\,\log(a^2x^2+1)}{a^6}\right)$$

input
```
integrate(x^4*arccot(a*x),x, algorithm="maxima")
```

output
```
1/5*x^5*arccot(a*x) + 1/20*a*((a^2*x^4 - 2*x^2)/a^4 + 2*log(a^2*x^2 + 1)/a
^6)
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.51

$$\int x^4 \cot^{-1}(ax)\,dx$$
$$= \frac{1}{20}\left(\frac{4\,x^5 \arctan\left(\frac{1}{ax}\right)}{a} - \frac{x^4\left(\frac{2}{a^2x^2} - \frac{3}{a^4x^4} - 1\right)}{a^2} + \frac{2\,\log\left(\frac{1}{a^2x^2} + 1\right)}{a^6} - \frac{2\,\log\left(\frac{1}{a^2x^2}\right)}{a^6}\right)a$$

input
```
integrate(x^4*arccot(a*x),x, algorithm="giac")
```

output
```
1/20*(4*x^5*arctan(1/(a*x))/a - x^4*(2/(a^2*x^2) - 3/(a^4*x^4) - 1)/a^2 +
2*log(1/(a^2*x^2) + 1)/a^6 - 2*log(1/(a^2*x^2))/a^6)*a
```

3.2.9 Mupad [B] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.14

$$\int x^4 \cot^{-1}(ax)\,dx = \begin{cases} \frac{\pi\,x^5}{10} & \text{if } a = 0 \\ \frac{2\,\ln\left(a^2\,x^2+1\right)-2\,a^2\,x^2+a^4\,x^4}{20\,a^5} + \frac{x^5\,\mathrm{acot}(a\,x)}{5} & \text{if } a \neq 0 \end{cases}$$

input
```
int(x^4*acot(a*x),x)
```

output
```
piecewise(a == 0, (x^5*pi)/10, a ~= 0, (2*log(a^2*x^2 + 1) - 2*a^2*x^2 + a
^4*x^4)/(20*a^5) + (x^5*acot(a*x))/5)
```

3.3 $\int x^3 \cot^{-1}(ax)\,dx$

3.3.1 Optimal result

Integrand size = 8, antiderivative size = 41

$$\int x^3 \cot^{-1}(ax)\,dx = -\frac{x}{4a^3} + \frac{x^3}{12a} + \frac{1}{4}x^4 \cot^{-1}(ax) + \frac{\arctan(ax)}{4a^4}$$

output
```
-1/4*x/a^3+1/12*x^3/a+1/4*x^4*arccot(a*x)+1/4*arctan(a*x)/a^4
```

3.3.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int x^3 \cot^{-1}(ax)\,dx = -\frac{x}{4a^3} + \frac{x^3}{12a} + \frac{1}{4}x^4 \cot^{-1}(ax) + \frac{\arctan(ax)}{4a^4}$$

input
```
Integrate[x^3*ArcCot[a*x],x]
```

output
```
-1/4*x/a^3 + x^3/(12*a) + (x^4*ArcCot[a*x])/4 + ArcTan[a*x]/(4*a^4)
```

3.3.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5362, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \cot^{-1}(ax) \, dx$$

$$\Big\downarrow \text{ 5362}$$

$$\frac{1}{4}a\int \frac{x^4}{a^2x^2+1}dx+\frac{1}{4}x^4\cot^{-1}(ax)$$

$$\Big\downarrow \text{ 254}$$

$$\frac{1}{4}a\int \left(\frac{x^2}{a^2}+\frac{1}{a^4\left(a^2x^2+1\right)}-\frac{1}{a^4}\right)dx+\frac{1}{4}x^4\cot^{-1}(ax)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)$$

input
```
Int[x^3*ArcCot[a*x],x]
```

output
```
(x^4*ArcCot[a*x])/4 + (a*(-(x/a^4) + x^3/(3*a^2) + ArcTan[a*x]/a^5))/4
```

3.3.3.1 Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.3.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\frac{a^4x^4 \operatorname{arccot}(ax)}{4}+\frac{a^3x^3}{12}-\frac{ax}{4}+\frac{\arctan(ax)}{4}}{a^4}$	36
default	$\frac{\frac{a^4x^4 \operatorname{arccot}(ax)}{4}+\frac{a^3x^3}{12}-\frac{ax}{4}+\frac{\arctan(ax)}{4}}{a^4}$	36
parallelrisch	$\frac{3a^4x^4 \operatorname{arccot}(ax)+a^3x^3-3ax-3 \operatorname{arccot}(ax)}{12a^4}$	36
parts	$\frac{x^4 \operatorname{arccot}(ax)}{4}+\frac{a\left(\frac{\frac{1}{3}a^2x^3-x}{a^4}+\frac{\arctan(ax)}{a^5}\right)}{4}$	39
risch	$\frac{ix^4 \ln(iax+1)}{8}-\frac{ix^4 \ln(-iax+1)}{8}+\frac{\pi x^4}{8}+\frac{x^3}{12a}-\frac{x}{4a^3}+\frac{\arctan(ax)}{4a^4}$	59

input
```
int(x^3*arccot(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/4*a^4*x^4*arccot(a*x)+1/12*a^3*x^3-1/4*a*x+1/4*arctan(a*x))
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int x^3 \cot^{-1}(ax)\,dx = \frac{a^3x^3 - 3\,ax + 3\,(a^4x^4 - 1)\operatorname{arccot}(ax)}{12\,a^4}$$

input
```
integrate(x^3*arccot(a*x),x, algorithm="fricas")
```

output
```
1/12*(a^3*x^3 - 3*a*x + 3*(a^4*x^4 - 1)*arccot(a*x))/a^4
```

3.3.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int x^3 \cot^{-1}(ax)\,dx = \begin{cases} \frac{x^4 \operatorname{acot}(ax)}{4} + \frac{x^3}{12a} - \frac{x}{4a^3} - \frac{\operatorname{acot}(ax)}{4a^4} & \text{for } a \neq 0 \\ \frac{\pi x^4}{8} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*acot(a*x),x)
```

output
```
Piecewise((x**4*acot(a*x)/4 + x**3/(12*a) - x/(4*a**3) - acot(a*x)/(4*a**4
), Ne(a, 0)), (pi*x**4/8, True))
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int x^3 \cot^{-1}(ax)\,dx = \frac{1}{4}\,x^4 \operatorname{arccot}(ax) + \frac{1}{12}\,a\left(\frac{a^2x^3 - 3\,x}{a^4} + \frac{3\,\arctan(ax)}{a^5}\right)$$

input
```
integrate(x^3*arccot(a*x),x, algorithm="maxima")
```

output
```
1/4*x^4*arccot(a*x) + 1/12*a*((a^2*x^3 - 3*x)/a^4 + 3*arctan(a*x)/a^5)
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.24

$$\int x^3 \cot^{-1}(ax)\,dx = \frac{1}{12}\left(\frac{3\,x^4 \arctan\left(\frac{1}{ax}\right)}{a} - \frac{x^3\left(\frac{3}{a^2x^2} - 1\right)}{a^2} - \frac{3\,\arctan\left(\frac{1}{ax}\right)}{a^5}\right)a$$

input
```
integrate(x^3*arccot(a*x),x, algorithm="giac")
```

output
```
1/12*(3*x^4*arctan(1/(a*x))/a - x^3*(3/(a^2*x^2) - 1)/a^2 - 3*arctan(1/(a*
x))/a^5)*a
```

3.3.9 Mupad [B] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int x^3 \cot^{-1}(ax)\,dx = \begin{cases} \frac{\pi x^4}{8} & \text{if } a = 0 \\ \frac{3\,\mathrm{atan}(a\,x)-3\,a\,x+a^3\,x^3}{12\,a^4} + \frac{x^4\,\mathrm{acot}(a\,x)}{4} & \text{if } a \neq 0 \end{cases}$$

input
```
int(x^3*acot(a*x),x)
```

output
```
piecewise(a == 0, (x^4*pi)/8, a ~= 0, (3*atan(a*x) - 3*a*x + a^3*x^3)/(12*
a^4) + (x^4*acot(a*x))/4)
```

3.4 $\int x^2 \cot^{-1}(ax)\,dx$

3.4.1 Optimal result

Integrand size = 8, antiderivative size = 39

$$\int x^2 \cot^{-1}(ax)\,dx = \frac{x^2}{6a} + \frac{1}{3}x^3\cot^{-1}(ax) - \frac{\log(1+a^2x^2)}{6a^3}$$

output
```
1/6*x^2/a+1/3*x^3*arccot(a*x)-1/6*ln(a^2*x^2+1)/a^3
```

3.4.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int x^2 \cot^{-1}(ax)\,dx = \frac{x^2}{6a} + \frac{1}{3}x^3\cot^{-1}(ax) - \frac{\log(1+a^2x^2)}{6a^3}$$

input
```
Integrate[x^2*ArcCot[a*x],x]
```

output
```
x^2/(6*a) + (x^3*ArcCot[a*x])/3 - Log[1 + a^2*x^2]/(6*a^3)
```

3.4.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5362, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cot^{-1}(ax) \, dx$$

$$\Big\downarrow \text{ 5362}$$

$$\frac{1}{3} a \int \frac{x^3}{a^2x^2+1} dx + \frac{1}{3} x^3 \cot^{-1}(ax)$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{6} a \int \frac{x^2}{a^2x^2+1} dx^2 + \frac{1}{3} x^3 \cot^{-1}(ax)$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{6} a \int \left(\frac{1}{a^2} - \frac{1}{a^2\left(a^2x^2+1\right)}\right) dx^2 + \frac{1}{3} x^3 \cot^{-1}(ax)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{6} a \left(\frac{x^2}{a^2} - \frac{\log\left(a^2x^2+1\right)}{a^4}\right) + \frac{1}{3} x^3 \cot^{-1}(ax)$$

input
```
Int[x^2*ArcCot[a*x],x]
```

output
```
(x^3*ArcCot[a*x])/3 + (a*(x^2/a^2 - Log[1 + a^2*x^2]/a^4))/6
```

3.4.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.4.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

method	result	size
parallelrisch	$-\frac{-2a^{3}x^{3}\operatorname{arccot}\left(ax\right)-a^{2}x^{2}+\ln\left(a^{2}x^{2}+1\right)}{6a^{3}}$	37
derivativedivides	$\frac{\frac{a^{3}x^{3}\operatorname{arccot}\left(ax\right)}{3}+\frac{a^{2}x^{2}}{6}-\frac{\ln\left(a^{2}x^{2}+1\right)}{6}}{a^{3}}$	38
default	$\frac{\frac{a^{3}x^{3}\operatorname{arccot}\left(ax\right)}{3}+\frac{a^{2}x^{2}}{6}-\frac{\ln\left(a^{2}x^{2}+1\right)}{6}}{a^{3}}$	38
parts	$\frac{x^{3}\operatorname{arccot}\left(ax\right)}{3}+\frac{a\left(\frac{x^{2}}{2a^{2}}-\frac{\ln\left(a^{2}x^{2}+1\right)}{2a^{4}}\right)}{3}$	38
risch	$\frac{ix^{3}\ln\left(iax+1\right)}{6}-\frac{ix^{3}\ln\left(-iax+1\right)}{6}+\frac{\pi x^{3}}{6}+\frac{x^{2}}{6a}-\frac{\ln\left(-a^{2}x^{2}-1\right)}{6a^{3}}$	60

input
```
int(x^2*arccot(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-2*a^3*x^3*arccot(a*x)-a^2*x^2+ln(a^2*x^2+1))/a^3
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int x^2 \cot^{-1}(ax)\,dx = \frac{2\,a^3x^3\,\mathrm{arccot}\,(ax) + a^2x^2 - \log\left(a^2x^2 + 1\right)}{6\,a^3}$$

input
```
integrate(x^2*arccot(a*x),x, algorithm="fricas")
```

output
```
1/6*(2*a^3*x^3*arccot(a*x) + a^2*x^2 - log(a^2*x^2 + 1))/a^3
```

3.4.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int x^2 \cot^{-1}(ax)\,dx = \begin{cases} \frac{x^3\,\mathrm{acot}\,(ax)}{3} + \frac{x^2}{6a} - \frac{\log\left(a^2x^2+1\right)}{6a^3} & \text{for } a \neq 0 \\ \frac{\pi x^3}{6} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*acot(a*x),x)
```

output
```
Piecewise((x**3*acot(a*x)/3 + x**2/(6*a) - log(a**2*x**2 + 1)/(6*a**3), Ne
(a, 0)), (pi*x**3/6, True))
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int x^2 \cot^{-1}(ax)\,dx = \frac{1}{3}\,x^3\,\mathrm{arccot}\,(ax) + \frac{1}{6}\,a\left(\frac{x^2}{a^2} - \frac{\log\left(a^2x^2 + 1\right)}{a^4}\right)$$

input
```
integrate(x^2*arccot(a*x),x, algorithm="maxima")
```

output
```
1/3*x^3*arccot(a*x) + 1/6*a*(x^2/a^2 - log(a^2*x^2 + 1)/a^4)
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.64

$$\int x^2 \cot^{-1}(ax)\,dx = \frac{1}{6}\left(\frac{2\,x^3 \arctan\left(\frac{1}{ax}\right)}{a} - \frac{x^2\left(\frac{1}{a^2x^2}-1\right)}{a^2} - \frac{\log\left(\frac{1}{a^2x^2}+1\right)}{a^4} + \frac{\log\left(\frac{1}{a^2x^2}\right)}{a^4}\right)a$$

input
```
integrate(x^2*arccot(a*x),x, algorithm="giac")
```

output
```
1/6*(2*x^3*arctan(1/(a*x))/a - x^2*(1/(a^2*x^2) - 1)/a^2 - log(1/(a^2*x^2)
 + 1)/a^4 + log(1/(a^2*x^2))/a^4)*a
```

3.4.9 Mupad [B] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int x^2 \cot^{-1}(ax)\,dx = \begin{cases} \frac{\pi\,x^3}{6} & \text{if } a=0 \\ \frac{\frac{x^2}{2}-\frac{\ln\left(a^2\,x^2+1\right)}{2\,a^2}}{3\,a} + \frac{x^3\,\mathrm{acot}(a\,x)}{3} & \text{if } a\neq 0 \end{cases}$$

input
```
int(x^2*acot(a*x),x)
```

output
```
piecewise(a == 0, (x^3*pi)/6, a ~= 0, (x^2/2 - log(a^2*x^2 + 1)/(2*a^2))/(
3*a) + (x^3*acot(a*x))/3)
```

3.5 $\int x \cot^{-1}(ax) \, dx$

3.5.1 Optimal result

Integrand size = 6, antiderivative size = 31

$$\int x \cot^{-1}(ax) \, dx = \frac{x}{2a} + \frac{1}{2}x^2 \cot^{-1}(ax) - \frac{\arctan(ax)}{2a^2}$$

output
```
1/2*x/a+1/2*x^2*arccot(a*x)-1/2*arctan(a*x)/a^2
```

3.5.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int x \cot^{-1}(ax) \, dx = \frac{x}{2a} + \frac{1}{2}x^2 \cot^{-1}(ax) - \frac{\arctan(ax)}{2a^2}$$

input
```
Integrate[x*ArcCot[a*x],x]
```

output
```
x/(2*a) + (x^2*ArcCot[a*x])/2 - ArcTan[a*x]/(2*a^2)
```

3.5.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5362, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \cot^{-1}(ax) \, dx$$
$$\Big\downarrow \ 5362$$
$$\frac{1}{2} a \int \frac{x^2}{a^2x^2+1} dx + \frac{1}{2} x^2 \cot^{-1}(ax)$$
$$\Big\downarrow \ 262$$
$$\frac{1}{2} a \left(\frac{x}{a^2} - \frac{\int \frac{1}{a^2x^2+1} dx}{a^2}\right) + \frac{1}{2} x^2 \cot^{-1}(ax)$$
$$\Big\downarrow \ 216$$
$$\frac{1}{2} a \left(\frac{x}{a^2} - \frac{\arctan(ax)}{a^3}\right) + \frac{1}{2} x^2 \cot^{-1}(ax)$$

input
```
Int[x*ArcCot[a*x],x]
```

output
```
(x^2*ArcCot[a*x])/2 + (a*(x/a^2 - ArcTan[a*x]/a^3))/2
```

3.5.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.5.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

method	result	size
parallelrisch	$\frac{\operatorname{arccot}(ax)a^2x^2+ax+\operatorname{arccot}(ax)}{2a^2}$	25
derivativedivides	$\frac{\frac{\operatorname{arccot}(ax)a^2x^2}{2}+\frac{ax}{2}-\frac{\arctan(ax)}{2}}{a^2}$	28
default	$\frac{\frac{\operatorname{arccot}(ax)a^2x^2}{2}+\frac{ax}{2}-\frac{\arctan(ax)}{2}}{a^2}$	28
parts	$\frac{x^2\operatorname{arccot}(ax)}{2}+\frac{a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{2}$	29
risch	$\frac{ix^2\ln(iax+1)}{4}-\frac{ix^2\ln(-iax+1)}{4}+\frac{\pi x^2}{4}+\frac{x}{2a}-\frac{\arctan(ax)}{2a^2}$	51

input
```
int(x*arccot(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*(arccot(a*x)*a^2*x^2+a*x+arccot(a*x))/a^2
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int x\cot^{-1}(ax)\,dx=\frac{ax+(a^2x^2+1)\operatorname{arccot}(ax)}{2\,a^2}$$

input
```
integrate(x*arccot(a*x),x, algorithm="fricas")
```

output
```
1/2*(a*x + (a^2*x^2 + 1)*arccot(a*x))/a^2
```

3.5.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int x\cot^{-1}(ax)\,dx=\begin{cases}\frac{x^2\operatorname{acot}(ax)}{2}+\frac{x}{2a}+\frac{\operatorname{acot}(ax)}{2a^2} & \text{for } a\neq 0\\ \frac{\pi x^2}{4} & \text{otherwise}\end{cases}$$

input
```
integrate(x*acot(a*x),x)
```

output
```
Piecewise((x**2*acot(a*x)/2 + x/(2*a) + acot(a*x)/(2*a**2), Ne(a, 0)), (pi
*x**2/4, True))
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int x\cot^{-1}(ax)\,dx=\frac{1}{2}x^2\operatorname{arccot}(ax)+\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)$$

input
```
integrate(x*arccot(a*x),x, algorithm="maxima")
```

output
```
1/2*x^2*arccot(a*x) + 1/2*a*(x/a^2 - arctan(a*x)/a^3)
```

3.5.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int x\cot^{-1}(ax)\,dx=\frac{1}{2}\left(\frac{x^2\arctan\left(\frac{1}{ax}\right)}{a}+\frac{x}{a^2}+\frac{\arctan\left(\frac{1}{ax}\right)}{a^3}\right)a$$

input
```
integrate(x*arccot(a*x),x, algorithm="giac")
```

output
```
1/2*(x^2*arctan(1/(a*x))/a + x/a^2 + arctan(1/(a*x))/a^3)*a
```

3.5.9 Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int x\cot^{-1}(ax)\,dx = \begin{cases} \frac{\pi x^2}{4} & \text{if } a=0 \\ \frac{x-\frac{\mathrm{atan}(a\,x)}{a}}{2\,a}+\frac{x^2\,\mathrm{acot}(a\,x)}{2} & \text{if } a\neq 0 \end{cases}$$

input

```
int(x*acot(a*x),x)
```

output

```
piecewise(a == 0, (x^2*pi)/4, a ~= 0, (x - atan(a*x)/a)/(2*a) + (x^2*acot(
a*x))/2)
```

3.6 $\int \cot^{-1}(ax)\,dx$

3.6.1 Optimal result

Integrand size = 4, antiderivative size = 24

$$\int \cot^{-1}(ax)\,dx = x\cot^{-1}(ax) + \frac{\log(1+a^2x^2)}{2a}$$

output
```
x*arccot(a*x)+1/2*ln(a^2*x^2+1)/a
```

3.6.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \cot^{-1}(ax)\,dx = x\cot^{-1}(ax) + \frac{\log(1+a^2x^2)}{2a}$$

input
```
Integrate[ArcCot[a*x],x]
```

output
```
x*ArcCot[a*x] + Log[1 + a^2*x^2]/(2*a)
```

3.6.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5346, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}(ax)\,dx$$

$$\Big\downarrow \text{5346}$$

$$a\int \frac{x}{a^2x^2+1}dx + x\cot^{-1}(ax)$$

$$\Big\downarrow \text{240}$$

$$\frac{\log(a^2x^2+1)}{2a} + x\cot^{-1}(ax)$$

input
```
Int[ArcCot[a*x],x]
```

output
```
x*ArcCot[a*x] + Log[1 + a^2*x^2]/(2*a)
```

3.6.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

3.6.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
parts	$x \operatorname{arccot}(ax) + \frac{\ln(a^2x^2+1)}{2a}$	23
derivativedivides	$\frac{\operatorname{arccot}(ax)ax+\frac{\ln(a^2x^2+1)}{2}}{a}$	25
default	$\frac{\operatorname{arccot}(ax)ax+\frac{\ln(a^2x^2+1)}{2}}{a}$	25
parallelrisch	$\frac{2\operatorname{arccot}(ax)ax+\ln(a^2x^2+1)}{2a}$	25
risch	$\frac{ix\ln(iax+1)}{2} - \frac{ix\ln(-iax+1)}{2} + \frac{\pi x}{2} + \frac{\ln(-a^2x^2-1)}{2a}$	46

input
```
int(arccot(a*x),x,method=_RETURNVERBOSE)
```

output
```
x*arccot(a*x)+1/2*ln(a^2*x^2+1)/a
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \cot^{-1}(ax)\,dx = \frac{2\,ax \operatorname{arccot}(ax) + \log(a^2x^2+1)}{2\,a}$$

input
```
integrate(arccot(a*x),x, algorithm="fricas")
```

output
```
1/2*(2*a*x*arccot(a*x) + log(a^2*x^2 + 1))/a
```

3.6.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \cot^{-1}(ax)\,dx = \begin{cases} x \operatorname{acot}(ax) + \frac{\log(a^2x^2+1)}{2a} & \text{for } a \neq 0 \\ \frac{\pi x}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(acot(a*x),x)
```

output
```
Piecewise((x*acot(a*x) + log(a**2*x**2 + 1)/(2*a), Ne(a, 0)), (pi*x/2, Tru
e))
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \cot^{-1}(ax)\,dx = \frac{2\,ax\,\mathrm{arccot}\,(ax) + \log\left(a^2x^2 + 1\right)}{2\,a}$$

input
```
integrate(arccot(a*x),x, algorithm="maxima")
```

output
```
1/2*(2*a*x*arccot(a*x) + log(a^2*x^2 + 1))/a
```

3.6.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(22) = 44.

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.88

$$\int \cot^{-1}(ax)\,dx = \frac{1}{2}\,a\left(\frac{2\,x\arctan\left(\frac{1}{ax}\right)}{a} + \frac{\log\left(\frac{1}{a^2x^2} + 1\right)}{a^2} - \frac{\log\left(\frac{1}{a^2x^2}\right)}{a^2}\right)$$

input
```
integrate(arccot(a*x),x, algorithm="giac")
```

output
```
1/2*a*(2*x*arctan(1/(a*x))/a + log(1/(a^2*x^2) + 1)/a^2 - log(1/(a^2*x^2))
/a^2)
```

3.6.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \cot^{-1}(ax)\,dx = x\,\mathrm{acot}(a\,x) + \frac{\ln\left(a^2\,x^2+1\right)}{2\,a}$$

input
```
int(acot(a*x),x)
```

output
```
x*acot(a*x) + log(a^2*x^2 + 1)/(2*a)
```

3.7 $\int \frac{\cot^{-1}(ax)}{x} dx$

3.7.1 Optimal result

Integrand size = 8, antiderivative size = 37

$$\int \frac{\cot^{-1}(ax)}{x} dx = -\frac{1}{2} i \operatorname{PolyLog}\left(2, -\frac{i}{ax}\right) + \frac{1}{2} i \operatorname{PolyLog}\left(2, \frac{i}{ax}\right)$$

output
```
-1/2*I*polylog(2,-I/a/x)+1/2*I*polylog(2,I/a/x)
```

3.7.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(ax)}{x} dx = -\frac{1}{2} i \operatorname{PolyLog}\left(2, -\frac{i}{ax}\right) + \frac{1}{2} i \operatorname{PolyLog}\left(2, \frac{i}{ax}\right)$$

input
```
Integrate[ArcCot[a*x]/x,x]
```

output
```
(-1/2*I)*PolyLog[2, (-I)/(a*x)] + (I/2)*PolyLog[2, I/(a*x)]
```

3.7.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5356, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax)}{x} \, dx$$

$$\Big\downarrow \text{ 5356}$$

$$\frac{1}{2} i \int \frac{\log\left(1-\frac{i}{ax}\right)}{x}dx - \frac{1}{2} i \int \frac{\log\left(1+\frac{i}{ax}\right)}{x}dx$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{1}{2} i \operatorname{PolyLog}\left(2, \frac{i}{ax}\right) - \frac{1}{2} i \operatorname{PolyLog}\left(2, -\frac{i}{ax}\right)$$

input
```
Int[ArcCot[a*x]/x,x]
```

output
```
(-1/2*I)*PolyLog[2, (-I)/(a*x)] + (I/2)*PolyLog[2, I/(a*x)]
```

3.7.3.1 Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5356
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Simp[I*(b/2)   Int[Log[1 + I/(c*x)]/x, x], x] + Simp[I*(b/2)   Int[Log
[1 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

3.7.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

method	result	siz
risch	$\frac{\pi \ln(-iax)}{2}+\frac{i \,\mathrm{dilog}(-iax+1)}{2}-\frac{i \,\mathrm{dilog}(iax+1)}{2}$	33
derivativedivides	$\ln(ax)\,\mathrm{arccot}(ax)-\frac{i\ln(ax)\ln(iax+1)}{2}+\frac{i\ln(ax)\ln(-iax+1)}{2}-\frac{i \,\mathrm{dilog}(iax+1)}{2}+\frac{i \,\mathrm{dilog}(-iax+1)}{2}$	63
default	$\ln(ax)\,\mathrm{arccot}(ax)-\frac{i\ln(ax)\ln(iax+1)}{2}+\frac{i\ln(ax)\ln(-iax+1)}{2}-\frac{i \,\mathrm{dilog}(iax+1)}{2}+\frac{i \,\mathrm{dilog}(-iax+1)}{2}$	63
parts	$\ln(x)\,\mathrm{arccot}(ax)+a\left(-\frac{i\ln(x)(-\ln(-iax+1)+\ln(iax+1))}{2a}-\frac{i(\mathrm{dilog}(iax+1)-\mathrm{dilog}(-iax+1))}{2a}\right)$	64

input
```
int(arccot(a*x)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*Pi*ln(-I*a*x)+1/2*I*dilog(1-I*a*x)-1/2*I*dilog(1+I*a*x)
```

3.7.5 Fricas [F]

$$\int \frac{\cot^{-1}(ax)}{x}\,dx=\int \frac{\mathrm{arccot}(ax)}{x}\,dx$$

input
```
integrate(arccot(a*x)/x,x, algorithm="fricas")
```

output
```
integral(arccot(a*x)/x, x)
```

3.7.6 Sympy [F]

$$\int \frac{\cot^{-1}(ax)}{x}\,dx=\int \frac{\mathrm{acot}(ax)}{x}\,dx$$

input
```
integrate(acot(a*x)/x,x)
```

output
```
Integral(acot(a*x)/x, x)
```

3.7.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(23) = 46.

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.51

$$\int \frac{\cot^{-1}(ax)}{x}\,dx = \frac{1}{4}\pi\log\left(a^2x^2+1\right) - \arctan(ax)\log(ax) + \operatorname{arccot}(ax)\log(x) + \arctan(ax)\log(x) + \frac{1}{2}i\,\mathrm{Li}_2(i\,ax+1) - \frac{1}{2}i\,\mathrm{Li}_2(-i\,ax+1)$$

input
```
integrate(arccot(a*x)/x,x, algorithm="maxima")
```

output
```
1/4*pi*log(a^2*x^2 + 1) - arctan(a*x)*log(a*x) + arccot(a*x)*log(x) + arct
an(a*x)*log(x) + 1/2*I*dilog(I*a*x + 1) - 1/2*I*dilog(-I*a*x + 1)
```

3.7.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int \frac{\cot^{-1}(ax)}{x}\,dx = -\frac{1}{2}\left(\frac{x^2\arctan\left(\frac{1}{ax}\right)}{a} + \frac{x}{a^2} + \frac{\arctan\left(\frac{1}{ax}\right)}{a^3}\right)a^2$$

input
```
integrate(arccot(a*x)/x,x, algorithm="giac")
```

output
```
-1/2*(x^2*arctan(1/(a*x))/a + x/a^2 + arctan(1/(a*x))/a^3)*a^2
```

3.7.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)}{x}\,dx = \int \frac{\operatorname{acot}(a\,x)}{x}\,dx$$

input
```
int(acot(a*x)/x,x)
```

output
```
int(acot(a*x)/x, x)
```

3.8 $\int \frac{\cot^{-1}(ax)}{x^2} dx$

3.8.1 Optimal result

Integrand size = 8, antiderivative size = 30

$$\int \frac{\cot^{-1}(ax)}{x^2} dx = -\frac{\cot^{-1}(ax)}{x} - a\log(x) + \frac{1}{2}a\log\left(1+a^2x^2\right)$$

output
```
-arccot(a*x)/x-a*ln(x)+1/2*a*ln(a^2*x^2+1)
```

3.8.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(ax)}{x^2} dx = -\frac{\cot^{-1}(ax)}{x} - a\log(x) + \frac{1}{2}a\log\left(1+a^2x^2\right)$$

input
```
Integrate[ArcCot[a*x]/x^2,x]
```

output
```
-(ArcCot[a*x]/x) - a*Log[x] + (a*Log[1 + a^2*x^2])/2
```

3.8.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5362, 243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax)}{x^2} \, dx$$

$$\Big\downarrow \text{ 5362}$$

$$-a\int \frac{1}{x\left(a^2x^2+1\right)}dx-\frac{\cot^{-1}(ax)}{x}$$

$$\Big\downarrow \text{ 243}$$

$$-\frac{1}{2}a\int \frac{1}{x^2\left(a^2x^2+1\right)}dx^2-\frac{\cot^{-1}(ax)}{x}$$

$$\Big\downarrow \text{ 47}$$

$$-\frac{1}{2}a\left(\int \frac{1}{x^2}dx^2-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\cot^{-1}(ax)}{x}$$

$$\Big\downarrow \text{ 14}$$

$$-\frac{1}{2}a\left(\log\left(x^2\right)-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\cot^{-1}(ax)}{x}$$

$$\Big\downarrow \text{ 16}$$

$$-\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{\cot^{-1}(ax)}{x}$$

input
```
Int[ArcCot[a*x]/x^2,x]
```

output
```
-(ArcCot[a*x]/x) - (a*(Log[x^2] - Log[1 + a^2*x^2]))/2
```

3.8.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.8.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

method	result	size
parts	$-\frac{\operatorname{arccot}(ax)}{x}-a\left(\ln(x)-\frac{\ln(a^2x^2+1)}{2}\right)$	29
parallelrisch	$-\frac{2a\ln(x)x-a\ln(a^2x^2+1)x+2\operatorname{arccot}(ax)}{2x}$	33
derivativedivides	$a\left(-\frac{\operatorname{arccot}(ax)}{ax}+\frac{\ln(a^2x^2+1)}{2}-\ln(ax)\right)$	34
default	$a\left(-\frac{\operatorname{arccot}(ax)}{ax}+\frac{\ln(a^2x^2+1)}{2}-\ln(ax)\right)$	34
risch	$-\frac{i\ln(iax+1)}{2x}-\frac{2a\ln(x)x-a\ln(a^2x^2+1)x-i\ln(-iax+1)+\pi}{2x}$	54

input
```
int(arccot(a*x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-arccot(a*x)/x-a*(ln(x)-1/2*ln(a^2*x^2+1))
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{\cot^{-1}(ax)}{x^2}\,dx = \frac{ax\log\left(a^2x^2+1\right) - 2\,ax\log\left(x\right) - 2\,\operatorname{arccot}\left(ax\right)}{2\,x}$$

input
```
integrate(arccot(a*x)/x^2,x, algorithm="fricas")
```

output
```
1/2*(a*x*log(a^2*x^2 + 1) - 2*a*x*log(x) - 2*arccot(a*x))/x
```

3.8.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{\cot^{-1}(ax)}{x^2}\,dx = -a\log\left(x\right) + \frac{a\log\left(a^2x^2+1\right)}{2} - \frac{\operatorname{acot}\left(ax\right)}{x}$$

input
```
integrate(acot(a*x)/x**2,x)
```

output
```
-a*log(x) + a*log(a**2*x**2 + 1)/2 - acot(a*x)/x
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(ax)}{x^2}\,dx = \frac{1}{2}\,a\left(\log\left(a^2x^2+1\right) - \log\left(x^2\right)\right) - \frac{\operatorname{arccot}\left(ax\right)}{x}$$

input
```
integrate(arccot(a*x)/x^2,x, algorithm="maxima")
```

output
```
1/2*a*(log(a^2*x^2 + 1) - log(x^2)) - arccot(a*x)/x
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{\cot^{-1}(ax)}{x^2}\,dx = -\frac{1}{2}\,a\left(\frac{2\,\arctan\left(\frac{1}{ax}\right)}{ax} - \log\left(\frac{1}{a^2x^2}+1\right)\right)$$

input
```
integrate(arccot(a*x)/x^2,x, algorithm="giac")
```

output
```
-1/2*a*(2*arctan(1/(a*x))/(a*x) - log(1/(a^2*x^2) + 1))
```

3.8.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{\cot^{-1}(ax)}{x^2}\,dx = \frac{a\,(\ln\left(a^2\,x^2+1\right) - 2\,\ln\left(x\right))}{2} - \frac{\mathrm{acot}(a\,x)}{x}$$

input
```
int(acot(a*x)/x^2,x)
```

output
```
(a*(log(a^2*x^2 + 1) - 2*log(x)))/2 - acot(a*x)/x
```

3.9 $\int \frac{\cot^{-1}(ax)}{x^3} \, dx$

3.9.1 Optimal result

Integrand size = 8, antiderivative size = 31

$$\int \frac{\cot^{-1}(ax)}{x^3} \, dx = \frac{a}{2x} - \frac{\cot^{-1}(ax)}{2x^2} + \frac{1}{2}a^2 \arctan(ax)$$

output
```
1/2*a/x-1/2*arccot(a*x)/x^2+1/2*a^2*arctan(a*x)
```

3.9.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \frac{\cot^{-1}(ax)}{x^3} \, dx = -\frac{\cot^{-1}(ax)}{2x^2} + \frac{a \, \text{Hypergeometric2F1}\left(-\frac{1}{2}, 1, \frac{1}{2}, -a^2x^2\right)}{2x}$$

input
```
Integrate[ArcCot[a*x]/x^3,x]
```

output
```
-1/2*ArcCot[a*x]/x^2 + (a*Hypergeometric2F1[-1/2, 1, 1/2, -(a^2*x^2)])/(2*
x)
```

3.9.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5362, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax)}{x^3} dx$$

$$\Big\downarrow \text{ 5362}$$

$$-\frac{1}{2}a\int \frac{1}{x^2(a^2x^2+1)}dx - \frac{\cot^{-1}(ax)}{2x^2}$$

$$\Big\downarrow \text{ 264}$$

$$-\frac{1}{2}a\left(a^2\left(-\int \frac{1}{a^2x^2+1}dx\right) - \frac{1}{x}\right) - \frac{\cot^{-1}(ax)}{2x^2}$$

$$\Big\downarrow \text{ 216}$$

$$-\frac{1}{2}a\left(-a\arctan(ax) - \frac{1}{x}\right) - \frac{\cot^{-1}(ax)}{2x^2}$$

input
```
Int[ArcCot[a*x]/x^3,x]
```

output
```
-1/2*ArcCot[a*x]/x^2 - (a*(-x^(-1) - a*ArcTan[a*x]))/2
```

3.9.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.9.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
parallelrisch	$-\frac{\operatorname{arccot}(ax) a^2 x^2 - ax + \operatorname{arccot}(ax)}{2x^2}$	26
parts	$-\frac{\operatorname{arccot}(ax)}{2x^2} - \frac{a\left(-\frac{1}{x} - a\arctan(ax)\right)}{2}$	27
derivativedivides	$a^2\left(-\frac{\operatorname{arccot}(ax)}{2a^2x^2} + \frac{\arctan(ax)}{2} + \frac{1}{2ax}\right)$	32
default	$a^2\left(-\frac{\operatorname{arccot}(ax)}{2a^2x^2} + \frac{\arctan(ax)}{2} + \frac{1}{2ax}\right)$	32
risch	$-\frac{i\ln(iax+1)}{4x^2} - \frac{-ia^2\ln(-ax-i)x^2 + ia^2\ln(-ax+i)x^2 - i\ln(-iax+1) - 2ax + \pi}{4x^2}$	72

input
```
int(arccot(a*x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(arccot(a*x)*a^2*x^2-a*x+arccot(a*x))/x^2
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{\cot^{-1}(ax)}{x^3} \, dx = \frac{ax - (a^2x^2 + 1)\operatorname{arccot}(ax)}{2x^2}$$

input
```
integrate(arccot(a*x)/x^3,x, algorithm="fricas")
```

output
```
1/2*(a*x - (a^2*x^2 + 1)*arccot(a*x))/x^2
```

3.9.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{\cot^{-1}(ax)}{x^3}\,dx = -\frac{a^2\operatorname{acot}(ax)}{2} + \frac{a}{2x} - \frac{\operatorname{acot}(ax)}{2x^2}$$

input
```
integrate(acot(a*x)/x**3,x)
```

output
```
-a**2*acot(a*x)/2 + a/(2*x) - acot(a*x)/(2*x**2)
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{\cot^{-1}(ax)}{x^3}\,dx = \frac{1}{2}\left(a\arctan(ax) + \frac{1}{x}\right)a - \frac{\operatorname{arccot}(ax)}{2\,x^2}$$

input
```
integrate(arccot(a*x)/x^3,x, algorithm="maxima")
```

output
```
1/2*(a*arctan(a*x) + 1/x)*a - 1/2*arccot(a*x)/x^2
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \frac{\cot^{-1}(ax)}{x^3}\,dx = \frac{1}{2}\left(a\left(\frac{1}{ax} - \arctan\left(\frac{1}{ax}\right)\right) - \frac{\arctan\left(\frac{1}{ax}\right)}{ax^2}\right)a$$

input
```
integrate(arccot(a*x)/x^3,x, algorithm="giac")
```

output
```
1/2*(a*(1/(a*x) - arctan(1/(a*x))) - arctan(1/(a*x))/(a*x^2))*a
```

3.9.9 Mupad [B] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \frac{\cot^{-1}(ax)}{x^3}\,dx = \begin{cases} -\frac{\pi}{4\,x^2} & \text{if } a=0 \\ \frac{a^3\,\mathrm{atan}(a\,x)+\frac{a^2}{x}}{2\,a}-\frac{\mathrm{acot}(a\,x)}{2\,x^2} & \text{if } a\neq 0 \end{cases}$$

input
```
int(acot(a*x)/x^3,x)
```

output
```
piecewise(a == 0, -pi/(4*x^2), a ~= 0, (a^3*atan(a*x) + a^2/x)/(2*a) - aco
t(a*x)/(2*x^2))
```

3.10 $\int \frac{\cot^{-1}(ax)}{x^4} dx$

3.10.1 Optimal result

Integrand size = 8, antiderivative size = 46

$$\int \frac{\cot^{-1}(ax)}{x^4} dx = \frac{a}{6x^2} - \frac{\cot^{-1}(ax)}{3x^3} + \frac{1}{3}a^3 \log(x) - \frac{1}{6}a^3 \log\left(1 + a^2x^2\right)$$

output
```
1/6*a/x^2-1/3*arccot(a*x)/x^3+1/3*a^3*ln(x)-1/6*a^3*ln(a^2*x^2+1)
```

3.10.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{\cot^{-1}(ax)}{x^4} dx = -\frac{\cot^{-1}(ax)}{3x^3} - \frac{1}{6}a\left(-\frac{1}{x^2} - 2a^2 \log(x) + a^2 \log\left(1 + a^2x^2\right)\right)$$

input
```
Integrate[ArcCot[a*x]/x^4,x]
```

output
```
-1/3*ArcCot[a*x]/x^3 - (a*(-x^(-2) - 2*a^2*Log[x] + a^2*Log[1 + a^2*x^2]))
/6
```

3.10.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5362, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax)}{x^4} \, dx$$

$$\Big\downarrow \text{5362}$$

$$-\frac{1}{3}a\int \frac{1}{x^3\left(a^2x^2+1\right)}dx-\frac{\cot^{-1}(ax)}{3x^3}$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{6}a\int \frac{1}{x^4\left(a^2x^2+1\right)}dx^2-\frac{\cot^{-1}(ax)}{3x^3}$$

$$\Big\downarrow \text{54}$$

$$-\frac{1}{6}a\int \left(\frac{a^4}{a^2x^2+1}-\frac{a^2}{x^2}+\frac{1}{x^4}\right)dx^2-\frac{\cot^{-1}(ax)}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{\cot^{-1}(ax)}{3x^3}$$

input
```
Int[ArcCot[a*x]/x^4,x]
```

output
```
-1/3*ArcCot[a*x]/x^3 - (a*(-x^(-2) - a^2*Log[x^2] + a^2*Log[1 + a^2*x^2]))
/6
```

3.10.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.10.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

method	result	size
parts	$-\frac{\operatorname{arccot}(ax)}{3x^3}-\frac{a\left(-\frac{1}{2x^2}-a^2\ln(x)+\frac{a^2\ln\left(a^2x^2+1\right)}{2}\right)}{3}$	42
derivativedivides	$a^3\left(-\frac{\operatorname{arccot}(ax)}{3a^3x^3}+\frac{1}{6a^2x^2}+\frac{\ln(ax)}{3}-\frac{\ln(a^2x^2+1)}{6}\right)$	44
default	$a^3\left(-\frac{\operatorname{arccot}(ax)}{3a^3x^3}+\frac{1}{6a^2x^2}+\frac{\ln(ax)}{3}-\frac{\ln(a^2x^2+1)}{6}\right)$	44
parallelrisch	$\frac{2a^3\ln(x)x^3-a^3\ln(a^2x^2+1)x^3-a^3x^3+ax-2\operatorname{arccot}(ax)}{6x^3}$	52
risch	$-\frac{i\ln(iax+1)}{6x^3}-\frac{-2a^3\ln(x)x^3+a^3\ln(-a^2x^2-1)x^3-i\ln(-iax+1)-ax+\pi}{6x^3}$	66

input
```
int(arccot(a*x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*arccot(a*x)/x^3-1/3*a*(-1/2/x^2-a^2*ln(x)+1/2*a^2*ln(a^2*x^2+1))
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{\cot^{-1}(ax)}{x^4} \, dx = -\frac{a^3x^3 \log \left(a^2x^2+1\right) - 2\,a^3x^3 \log \left(x\right) - ax + 2\,\operatorname{arccot}\left(ax\right)}{6\,x^3}$$

input
```
integrate(arccot(a*x)/x^4,x, algorithm="fricas")
```

output
```
-1/6*(a^3*x^3*log(a^2*x^2 + 1) - 2*a^3*x^3*log(x) - a*x + 2*arccot(a*x))/x
^3
```

3.10.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{\cot^{-1}(ax)}{x^4} \, dx = \frac{a^3 \log \left(x\right)}{3} - \frac{a^3 \log \left(a^2x^2+1\right)}{6} + \frac{a}{6x^2} - \frac{\operatorname{acot}\left(ax\right)}{3x^3}$$

input
```
integrate(acot(a*x)/x**4,x)
```

output
```
a**3*log(x)/3 - a**3*log(a**2*x**2 + 1)/6 + a/(6*x**2) - acot(a*x)/(3*x**3
)
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{\cot^{-1}(ax)}{x^4} \, dx = -\frac{1}{6}\left(a^2 \log \left(a^2x^2+1\right) - a^2 \log \left(x^2\right) - \frac{1}{x^2}\right)a - \frac{\operatorname{arccot}\left(ax\right)}{3\,x^3}$$

input
```
integrate(arccot(a*x)/x^4,x, algorithm="maxima")
```

output
```
-1/6*(a^2*log(a^2*x^2 + 1) - a^2*log(x^2) - 1/x^2)*a - 1/3*arccot(a*x)/x^3
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{\cot^{-1}(ax)}{x^4}\,dx = \frac{1}{6}\left(a^2\left(\frac{1}{a^2x^2} - \log\left(\frac{1}{a^2x^2}+1\right)\right) - \frac{2\arctan\left(\frac{1}{ax}\right)}{ax^3}\right)a$$

input
```
integrate(arccot(a*x)/x^4,x, algorithm="giac")
```

output
```
1/6*(a^2*(1/(a^2*x^2) - log(1/(a^2*x^2) + 1)) - 2*arctan(1/(a*x))/(a*x^3))
*a
```

3.10.9 Mupad [B] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \frac{\cot^{-1}(ax)}{x^4}\,dx = \begin{cases} -\frac{\pi}{6x^3} & \text{if } a=0 \\ \frac{a^4\,\ln(x) - \frac{a^4\,\ln\left(a^2x^2+1\right)}{2} + \frac{a^2}{2x^2}}{3a} - \frac{\operatorname{acot}(a\,x)}{3x^3} & \text{if } a\neq 0 \end{cases}$$

input
```
int(acot(a*x)/x^4,x)
```

output
```
piecewise(a == 0, -pi/(6*x^3), a ~= 0, (a^4*log(x) - (a^4*log(a^2*x^2 + 1)
)/2 + a^2/(2*x^2))/(3*a) - acot(a*x)/(3*x^3))
```

3.11 $\int \frac{\cot^{-1}(ax)}{x^5} dx$

3.11.1 Optimal result

Integrand size = 8, antiderivative size = 41

$$\int \frac{\cot^{-1}(ax)}{x^5} dx = \frac{a}{12x^3} - \frac{a^3}{4x} - \frac{\cot^{-1}(ax)}{4x^4} - \frac{1}{4}a^4 \arctan(ax)$$

output
```
1/12*a/x^3-1/4*a^3/x-1/4*arccot(a*x)/x^4-1/4*a^4*arctan(a*x)
```

3.11.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{\cot^{-1}(ax)}{x^5} dx = -\frac{\cot^{-1}(ax)}{4x^4} + \frac{a \, \text{Hypergeometric2F1}\left(-\frac{3}{2}, 1, -\frac{1}{2}, -a^2x^2\right)}{12x^3}$$

input
```
Integrate[ArcCot[a*x]/x^5,x]
```

output
```
-1/4*ArcCot[a*x]/x^4 + (a*Hypergeometric2F1[-3/2, 1, -1/2, -(a^2*x^2)])/(1
2*x^3)
```

3.11.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5362, 264, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax)}{x^5} \, dx$$

$$\Big\downarrow \text{5362}$$

$$-\frac{1}{4}a\int \frac{1}{x^4\left(a^2x^2+1\right)}dx-\frac{\cot^{-1}(ax)}{4x^4}$$

$$\Big\downarrow \text{264}$$

$$-\frac{1}{4}a\left(a^2\left(-\int \frac{1}{x^2\left(a^2x^2+1\right)}dx\right)-\frac{1}{3x^3}\right)-\frac{\cot^{-1}(ax)}{4x^4}$$

$$\Big\downarrow \text{264}$$

$$-\frac{1}{4}a\left(-\left(a^2\left(a^2\left(-\int \frac{1}{a^2x^2+1}dx\right)-\frac{1}{x}\right)\right)-\frac{1}{3x^3}\right)-\frac{\cot^{-1}(ax)}{4x^4}$$

$$\Big\downarrow \text{216}$$

$$-\frac{1}{4}a\left(-\left(a^2\left(-a\arctan(ax)-\frac{1}{x}\right)\right)-\frac{1}{3x^3}\right)-\frac{\cot^{-1}(ax)}{4x^4}$$

input
```
Int[ArcCot[a*x]/x^5,x]
```

output
```
-1/4*ArcCot[a*x]/x^4 - (a*(-1/3*1/x^3 - a^2*(-x^(-1) - a*ArcTan[a*x])))/4
```

3.11.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.11.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

method	result	size
parts	$-\frac{\operatorname{arccot}(ax)}{4x^4}-\frac{a\left(-\frac{1}{3x^3}+\frac{a^2}{x}+a^3\arctan(ax)\right)}{4}$	35
parallelrisch	$\frac{3a^4x^4\operatorname{arccot}(ax)-3a^3x^3+ax-3\operatorname{arccot}(ax)}{12x^4}$	36
derivativedivides	$a^4\left(-\frac{\operatorname{arccot}(ax)}{4a^4x^4}-\frac{\arctan(ax)}{4}+\frac{1}{12a^3x^3}-\frac{1}{4ax}\right)$	40
default	$a^4\left(-\frac{\operatorname{arccot}(ax)}{4a^4x^4}-\frac{\arctan(ax)}{4}+\frac{1}{12a^3x^3}-\frac{1}{4ax}\right)$	40
risch	$-\frac{i\ln(iax+1)}{8x^4}-\frac{-3ia^4\ln(-ax+i)x^4+3ia^4\ln(-ax-i)x^4+6a^3x^3-3i\ln(-iax+1)-2ax+3\pi}{24x^4}$	82

input
```
int(arccot(a*x)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*arccot(a*x)/x^4-1/4*a*(-1/3/x^3+a^2/x+a^3*arctan(a*x))
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{\cot^{-1}(ax)}{x^5}\,dx = -\frac{3\,a^3x^3 - ax - 3\,(a^4x^4 - 1)\,\mathrm{arccot}\,(ax)}{12\,x^4}$$

input
```
integrate(arccot(a*x)/x^5,x, algorithm="fricas")
```

output
```
-1/12*(3*a^3*x^3 - a*x - 3*(a^4*x^4 - 1)*arccot(a*x))/x^4
```

3.11.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{\cot^{-1}(ax)}{x^5}\,dx = \frac{a^4\,\mathrm{acot}\,(ax)}{4} - \frac{a^3}{4x} + \frac{a}{12x^3} - \frac{\mathrm{acot}\,(ax)}{4x^4}$$

input
```
integrate(acot(a*x)/x**5,x)
```

output
```
a**4*acot(a*x)/4 - a**3/(4*x) + a/(12*x**3) - acot(a*x)/(4*x**4)
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{\cot^{-1}(ax)}{x^5}\,dx = -\frac{1}{12}\left(3\,a^3\arctan\,(ax) + \frac{3\,a^2x^2 - 1}{x^3}\right)a - \frac{\mathrm{arccot}\,(ax)}{4\,x^4}$$

input
```
integrate(arccot(a*x)/x^5,x, algorithm="maxima")
```

output
```
-1/12*(3*a^3*arctan(a*x) + (3*a^2*x^2 - 1)/x^3)*a - 1/4*arccot(a*x)/x^4
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.24

$$\int \frac{\cot^{-1}(ax)}{x^5}\,dx = -\frac{1}{12}\left(a^3\left(\frac{3}{ax} - \frac{1}{a^3x^3} - 3\arctan\left(\frac{1}{ax}\right)\right) + \frac{3\arctan\left(\frac{1}{ax}\right)}{ax^4}\right)a$$

input
```
integrate(arccot(a*x)/x^5,x, algorithm="giac")
```

output
```
-1/12*(a^3*(3/(a*x) - 1/(a^3*x^3) - 3*arctan(1/(a*x))) + 3*arctan(1/(a*x))
/(a*x^4))*a
```

3.11.9 Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.15

$$\int \frac{\cot^{-1}(ax)}{x^5}\,dx = \begin{cases} -\frac{\pi}{8\,x^4} & \text{if } a = 0 \\ -\frac{a^4\,\mathrm{atan}(a\,x)}{4} - \frac{\frac{\mathrm{acot}(a\,x)}{4} - \frac{a\,x}{12} + \frac{a^3\,x^3}{4}}{x^4} & \text{if } a \neq 0 \end{cases}$$

input
```
int(acot(a*x)/x^5,x)
```

output
```
piecewise(a == 0, -pi/(8*x^4), a ~= 0, - (a^4*atan(a*x))/4 - (acot(a*x)/4
- (a*x)/12 + (a^3*x^3)/4)/x^4)
```

3.12 $\int x^5 \cot^{-1}(ax)^2 \, dx$

3.12.1 Optimal result

Integrand size = 10, antiderivative size = 104

$$\int x^5 \cot^{-1}(ax)^2 \, dx = -\frac{4x^2}{45a^4} + \frac{x^4}{60a^2} + \frac{x\cot^{-1}(ax)}{3a^5} - \frac{x^3\cot^{-1}(ax)}{9a^3} + \frac{x^5\cot^{-1}(ax)}{15a} + \frac{\cot^{-1}(ax)^2}{6a^6} + \frac{1}{6}x^6\cot^{-1}(ax)^2 + \frac{23\log(1+a^2x^2)}{90a^6}$$

output
```
-4/45*x^2/a^4+1/60*x^4/a^2+1/3*x*arccot(a*x)/a^5-1/9*x^3*arccot(a*x)/a^3+1
/15*x^5*arccot(a*x)/a+1/6*arccot(a*x)^2/a^6+1/6*x^6*arccot(a*x)^2+23/90*ln
(a^2*x^2+1)/a^6
```

3.12.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.76

$$\int x^5 \cot^{-1}(ax)^2 \, dx = \frac{-16a^2x^2 + 3a^4x^4 + 4ax(15 - 5a^2x^2 + 3a^4x^4)\cot^{-1}(ax) + 30(1 + a^6x^6)\cot^{-1}(ax)^2 + 46\log(1 + a^2x^2)}{180a^6}$$

input
```
Integrate[x^5*ArcCot[a*x]^2,x]
```

output
```
(-16*a^2*x^2 + 3*a^4*x^4 + 4*a*x*(15 - 5*a^2*x^2 + 3*a^4*x^4)*ArcCot[a*x]
+ 30*(1 + a^6*x^6)*ArcCot[a*x]^2 + 46*Log[1 + a^2*x^2])/(180*a^6)
```

3.12.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.64, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {5362, 5452, 5362, 243, 49, 2009, 5452, 5362, 243, 49, 2009, 5452, 5346, 240, 5420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \cot^{-1}(ax)^2 \, dx$$

$$\Big\downarrow \text{5362}$$

$$\frac{1}{3} a \int \frac{x^6 \cot^{-1}(ax)}{a^2x^2+1} dx + \frac{1}{6} x^6 \cot^{-1}(ax)^2$$

$$\Big\downarrow \text{5452}$$

$$\frac{1}{3} a \left(\frac{\int x^4 \cot^{-1}(ax) dx}{a^2} - \frac{\int \frac{x^4 \cot^{-1}(ax)}{a^2x^2+1} dx}{a^2} \right) + \frac{1}{6} x^6 \cot^{-1}(ax)^2$$

$$\Big\downarrow \text{5362}$$

$$\frac{1}{3} a \left(\frac{\frac{1}{5} a \int \frac{x^5}{a^2x^2+1} dx + \frac{1}{5} x^5 \cot^{-1}(ax)}{a^2} - \frac{\int \frac{x^4 \cot^{-1}(ax)}{a^2x^2+1} dx}{a^2} \right) + \frac{1}{6} x^6 \cot^{-1}(ax)^2$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{3} a \left(\frac{\frac{1}{10} a \int \frac{x^4}{a^2x^2+1} dx^2 + \frac{1}{5} x^5 \cot^{-1}(ax)}{a^2} - \frac{\int \frac{x^4 \cot^{-1}(ax)}{a^2x^2+1} dx}{a^2} \right) + \frac{1}{6} x^6 \cot^{-1}(ax)^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3} a \left(\frac{\frac{1}{10} a \int \left(\frac{x^2}{a^2} + \frac{1}{a^4(a^2x^2+1)} - \frac{1}{a^4} \right) dx^2 + \frac{1}{5} x^5 \cot^{-1}(ax)}{a^2} - \frac{\int \frac{x^4 \cot^{-1}(ax)}{a^2x^2+1} dx}{a^2} \right) + \frac{1}{6} x^6 \cot^{-1}(ax)^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} a \left(\frac{\frac{1}{10} a \left(-\frac{x^2}{a^4} + \frac{x^4}{2a^2} + \frac{\log(a^2x^2+1)}{a^6} \right) + \frac{1}{5} x^5 \cot^{-1}(ax)}{a^2} - \frac{\int \frac{x^4 \cot^{-1}(ax)}{a^2x^2+1} dx}{a^2} \right) + \frac{1}{6} x^6 \cot^{-1}(ax)^2$$

$$\Big\downarrow \text{5452}$$

$$\frac{1}{3}a\left(\frac{\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)+\frac{1}{5}x^5\cot^{-1}(ax)}{a^2}-\frac{\frac{\int x^2\cot^{-1}(ax)dx}{a^2}-\frac{\int\frac{x^2\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{6}x^6\cot^{-1}(ax)^2$$

$$\Big\downarrow 5362$$

$$\frac{1}{3}a\left(\frac{\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)+\frac{1}{5}x^5\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{3}a\int\frac{x^3}{a^2x^2+1}dx+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x^2\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{6}x^6\cot^{-1}(ax)^2$$

$$\Big\downarrow 243$$

$$\frac{1}{3}a\left(\frac{\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)+\frac{1}{5}x^5\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{6}a\int\frac{x^2}{a^2x^2+1}dx^2+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x^2\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{6}x^6\cot^{-1}(ax)^2$$

$$\Big\downarrow 49$$

$$\frac{1}{3}a\left(\frac{\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)+\frac{1}{5}x^5\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{6}a\int\left(\frac{1}{a^2}-\frac{1}{a^2\left(a^2x^2+1\right)}\right)dx^2+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x^2\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{6}x^6\cot^{-1}(ax)^2$$

$$\Big\downarrow 2009$$

$$\frac{1}{3}a\left(\frac{\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)+\frac{1}{5}x^5\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x^2\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{6}x^6\cot^{-1}(ax)^2$$

$$\Big\downarrow 5452$$

3.12. $\int x^5\cot^{-1}(ax)^2\,dx$

$$\frac{1}{3}a\left(\frac{\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)+\frac{1}{5}x^5\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\frac{\int\cot^{-1}(ax)dx}{a^2}-\frac{\int\frac{\cot^{-1}(ax)}{a^2x^2+1}d}{a^2}}{a^2}}{a^2}\right.$$

$$\frac{1}{6}x^6\cot^{-1}(ax)^2$$

$\Big\downarrow$ 5346

$$\frac{1}{3}a\left(\frac{\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)+\frac{1}{5}x^5\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\frac{a\int\frac{x}{a^2x^2+1}dx+x\cot^{-1}(ax)}{a^2}-}{a^2}}{a^2}\right.$$

$$\frac{1}{6}x^6\cot^{-1}(ax)^2$$

$\Big\downarrow$ 240

$$\frac{1}{3}a\left(\frac{\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)+\frac{1}{5}x^5\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{\log(a^2x^2+1)}{2a}+x\cot^{-1}(ax)}{a^2}-}{a^2}}{a^2}\right.$$

$$\frac{1}{6}x^6\cot^{-1}(ax)^2$$

$\Big\downarrow$ 5420

$$\frac{1}{3}a\left(\frac{\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)+\frac{1}{5}x^5\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\frac{\cot^{-1}(ax)^2}{2a^3}+\frac{\frac{\log(a^2x^2+1)}{2a}+x}{a^2}}{a^2}}{a^2}\right.$$

$$\frac{1}{6}x^6\cot^{-1}(ax)^2$$

input
```
Int[x^5*ArcCot[a*x]^2,x]
```

output
```
(x^6*ArcCot[a*x]^2)/6 + (a*((((x^5*ArcCot[a*x])/5 + (a*(-(x^2/a^4) + x^4/(2
*a^2) + Log[1 + a^2*x^2]/a^6))/10)/a^2 - (((x^3*ArcCot[a*x])/3 + (a*(x^2/a
^2 - Log[1 + a^2*x^2]/a^4))/6)/a^2 - (ArcCot[a*x]^2/(2*a^3) + (x*ArcCot[a*
x] + Log[1 + a^2*x^2]/(2*a))/a^2)/a^2)/a^2))/3
```

3.12. $\int x^5\cot^{-1}(ax)^2\,dx$

3.12.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5420
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5452
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

3.12.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

method	result
parallelrisch	$\frac{30a^6x^6 \operatorname{arccot}(ax)^2+12a^5x^5 \operatorname{arccot}(ax)+3a^4x^4-20a^3x^3 \operatorname{arccot}(ax)+16-16a^2x^2+60 \operatorname{arccot}(ax)ax+30 \operatorname{arccot}(ax)^2+4}{180a^6}$
parts	$\frac{x^6 \operatorname{arccot}(ax)^2}{6}+\frac{\frac{a^5x^5 \operatorname{arccot}(ax)}{5}-\frac{a^3x^3 \operatorname{arccot}(ax)}{3}+\operatorname{arccot}(ax)ax-\operatorname{arccot}(ax)\arctan(ax)+\frac{a^4x^4}{20}-\frac{4a^2x^2}{15}+\frac{23\ln(a^2x^2+}{30}}{3a^6}$
derivativedivides	$\frac{\frac{a^6x^6 \operatorname{arccot}(ax)^2}{6}+\frac{a^5x^5 \operatorname{arccot}(ax)}{15}-\frac{a^3x^3 \operatorname{arccot}(ax)}{9}+\frac{\operatorname{arccot}(ax)ax}{3}-\frac{\operatorname{arccot}(ax)\arctan(ax)}{3}+\frac{a^4x^4}{60}-\frac{4a^2x^2}{45}+\frac{23\ln(a^2x^2+1)}{90}-}{a^6}$
default	$\frac{\frac{a^6x^6 \operatorname{arccot}(ax)^2}{6}+\frac{a^5x^5 \operatorname{arccot}(ax)}{15}-\frac{a^3x^3 \operatorname{arccot}(ax)}{9}+\frac{\operatorname{arccot}(ax)ax}{3}-\frac{\operatorname{arccot}(ax)\arctan(ax)}{3}+\frac{a^4x^4}{60}-\frac{4a^2x^2}{45}+\frac{23\ln(a^2x^2+1)}{90}-}{a^6}$
risch	$-\frac{(a^6x^6+1)\ln(iax+1)^2}{24a^6}+\frac{(15i\pi a^6x^6+15x^6\ln(-iax+1)a^6+6ia^5x^5-10ia^3x^3+30iax+15\ln(-iax+1))\ln(iax+1)}{180a^6}+$

input
```
int(x^5*arccot(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/180*(30*a^6*x^6*arccot(a*x)^2+12*a^5*x^5*arccot(a*x)+3*a^4*x^4-20*a^3*x^
3*arccot(a*x)+16-16*a^2*x^2+60*arccot(a*x)*a*x+30*arccot(a*x)^2+46*ln(a^2*
x^2+1))/a^6
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.75

$$\int x^5 \cot^{-1}(ax)^2 \, dx = \frac{3\,a^4x^4-16\,a^2x^2+30\,(a^6x^6+1)\operatorname{arccot}(ax)^2+4\,(3\,a^5x^5-5\,a^3x^3+15\,ax)\operatorname{arccot}(ax)+46\,\log(a^2x^2+1}{180\,a^6}$$

input
```
integrate(x^5*arccot(a*x)^2,x, algorithm="fricas")
```

output
```
1/180*(3*a^4*x^4 - 16*a^2*x^2 + 30*(a^6*x^6 + 1)*arccot(a*x)^2 + 4*(3*a^5*
x^5 - 5*a^3*x^3 + 15*a*x)*arccot(a*x) + 46*log(a^2*x^2 + 1))/a^6
```

3.12.6 Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int x^5 \cot^{-1}(ax)^2 \, dx$$
$$= \begin{cases} \frac{x^6 \operatorname{acot}^2(ax)}{6} + \frac{x^5 \operatorname{acot}(ax)}{15a} + \frac{x^4}{60a^2} - \frac{x^3 \operatorname{acot}(ax)}{9a^3} - \frac{4x^2}{45a^4} + \frac{x \operatorname{acot}(ax)}{3a^5} + \frac{23 \log(a^2x^2+1)}{90a^6} + \frac{\operatorname{acot}^2(ax)}{6a^6} & \text{for } a \neq 0 \\ \frac{\pi^2 x^6}{24} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*acot(a*x)**2,x)
```

output
```
Piecewise((x**6*acot(a*x)**2/6 + x**5*acot(a*x)/(15*a) + x**4/(60*a**2) -
x**3*acot(a*x)/(9*a**3) - 4*x**2/(45*a**4) + x*acot(a*x)/(3*a**5) + 23*log
(a**2*x**2 + 1)/(90*a**6) + acot(a*x)**2/(6*a**6), Ne(a, 0)), (pi**2*x**6/
24, True))
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.91

$$\int x^5 \cot^{-1}(ax)^2 \, dx = \frac{1}{6} x^6 \operatorname{arccot}(ax)^2 + \frac{1}{45} a \left(\frac{3a^4x^5 - 5a^2x^3 + 15x}{a^6} - \frac{15 \arctan(ax)}{a^7} \right) \operatorname{arccot}(ax) + \frac{3a^4x^4 - 16a^2x^2 - 30 \arctan(ax)^2 + 46 \log(a^2x^2 + 1)}{180a^6}$$

input
```
integrate(x^5*arccot(a*x)^2,x, algorithm="maxima")
```

output
```
1/6*x^6*arccot(a*x)^2 + 1/45*a*((3*a^4*x^5 - 5*a^2*x^3 + 15*x)/a^6 - 15*ar
ctan(a*x)/a^7)*arccot(a*x) + 1/180*(3*a^4*x^4 - 16*a^2*x^2 - 30*arctan(a*x
)^2 + 46*log(a^2*x^2 + 1))/a^6
```

3.12.8 Giac [F]

$$\int x^5 \cot^{-1}(ax)^2\,dx = \int x^5 \operatorname{arccot}(ax)^2\,dx$$

input
```
integrate(x^5*arccot(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^5*arccot(a*x)^2, x)
```

3.12.9 Mupad [B] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.82

$$\int x^5 \cot^{-1}(ax)^2\,dx = \frac{x^6 \operatorname{acot}(a\,x)^2}{6} + \frac{\frac{23\,\ln(a^2\,x^2+1)}{90} - \frac{4\,a^2\,x^2}{45} + \frac{a^4\,x^4}{60} + \frac{\operatorname{acot}(a\,x)^2}{6} - \frac{a^3\,x^3\operatorname{acot}(a\,x)}{9} + \frac{a^5\,x^5\operatorname{acot}(a\,x)}{15} + \frac{a\,x\operatorname{acot}(a\,x)}{3}}{a^6}$$

input
```
int(x^5*acot(a*x)^2,x)
```

output
```
(x^6*acot(a*x)^2)/6 + ((23*log(a^2*x^2 + 1))/90 - (4*a^2*x^2)/45 + (a^4*x^
4)/60 + acot(a*x)^2/6 - (a^3*x^3*acot(a*x))/9 + (a^5*x^5*acot(a*x))/15 + (
a*x*acot(a*x))/3)/a^6
```

3.13 $\int x^4 \cot^{-1}(ax)^2\,dx$

3.13.1 Optimal result

Integrand size = 10, antiderivative size = 135

$$\int x^4 \cot^{-1}(ax)^2\,dx = -\frac{3x}{10a^4}+\frac{x^3}{30a^2}-\frac{x^2\cot^{-1}(ax)}{5a^3}+\frac{x^4\cot^{-1}(ax)}{10a}+\frac{i\cot^{-1}(ax)^2}{5a^5}+\frac{1}{5}x^5\cot^{-1}(ax)^2+\frac{3\arctan(ax)}{10a^5}-\frac{2\cot^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{5a^5}+\frac{i\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{5a^5}$$

output
```
-3/10*x/a^4+1/30*x^3/a^2-1/5*x^2*arccot(a*x)/a^3+1/10*x^4*arccot(a*x)/a+1/
5*I*arccot(a*x)^2/a^5+1/5*x^5*arccot(a*x)^2+3/10*arctan(a*x)/a^5-2/5*arcco
t(a*x)*ln(2/(1+I*a*x))/a^5+1/5*I*polylog(2,1-2/(1+I*a*x))/a^5
```

3.13.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.70

$$\int x^4\cot^{-1}(ax)^2\,dx = \frac{ax(-9+a^2x^2)+6(i+a^5x^5)\cot^{-1}(ax)^2+3\cot^{-1}(ax)\left(-3-2a^2x^2+a^4x^4-4\log\left(1-e^{2i\cot^{-1}(ax)}\right)\right)+}{30a^5}$$

input
```
Integrate[x^4*ArcCot[a*x]^2,x]
```

output
```
(a*x*(-9 + a^2*x^2) + 6*(I + a^5*x^5)*ArcCot[a*x]^2 + 3*ArcCot[a*x]*(-3 -
2*a^2*x^2 + a^4*x^4 - 4*Log[1 - E^((2*I)*ArcCot[a*x])]) + (6*I)*PolyLog[2,
 E^((2*I)*ArcCot[a*x])])/(30*a^5)
```

3.13.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.34, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.300$, Rules used = {5362, 5452, 5362, 254, 2009, 5452, 5362, 262, 216, 5456, 5380, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \cot^{-1}(ax)^2 \, dx$$

$$\Big\downarrow \; 5362$$

$$\frac{2}{5}a\int \frac{x^5 \cot^{-1}(ax)}{a^2x^2+1}dx+\frac{1}{5}x^5\cot^{-1}(ax)^2$$

$$\Big\downarrow \; 5452$$

$$\frac{2}{5}a\left(\frac{\int x^3\cot^{-1}(ax)dx}{a^2}-\frac{\int \frac{x^3\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}(ax)^2$$

$$\Big\downarrow \; 5362$$

$$\frac{2}{5}a\left(\frac{\frac{1}{4}a\int \frac{x^4}{a^2x^2+1}dx+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\int \frac{x^3\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}(ax)^2$$

$$\Big\downarrow \; 254$$

$$\frac{2}{5}a\left(\frac{\frac{1}{4}a\int \left(\frac{x^2}{a^2}+\frac{1}{a^4(a^2x^2+1)}-\frac{1}{a^4}\right)dx+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\int \frac{x^3\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}(ax)^2$$

$$\Big\downarrow \; 2009$$

$$\frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\int \frac{x^3\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}(ax)^2$$

$$\Big\downarrow \; 5452$$

$$\frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\frac{\int x\cot^{-1}(ax)dx}{a^2}-\frac{\int\frac{x\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}(ax)^2$$

$\Big\downarrow$ 5362

$$\frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{2}a\int\frac{x^2}{a^2x^2+1}dx+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}(ax)^2$$

$\Big\downarrow$ 262

$$\frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\int\frac{1}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}(ax)^2$$

$\Big\downarrow$ 216

$$\frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}(ax)^2$$

$\Big\downarrow$ 5456

$$\frac{1}{5}x^5\cot^{-1}(ax)^2+\frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\int\frac{\cot^{-1}(ax)}{i-ax}dx}{a}}{a^2}}{a^2}\right)$$

$\Big\downarrow$ 5380

3.13. $\int x^4\cot^{-1}(ax)^2\,dx$

$$\frac{1}{5}x^5\cot^{-1}(ax)^2 + \frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2} - \frac{\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2} - \frac{\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\int\frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx+\frac{\log\left(1-\right.}{a}}{a^2}}{a^2}\right.$$

$\Big\downarrow$ 2849

$$\frac{1}{5}x^5\cot^{-1}(ax)^2 + \frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2} - \frac{\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2} - \frac{\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\frac{\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)}{a}-}{a^2}}{a^2}\right.$$

$\Big\downarrow$ 2752

$$\frac{1}{5}x^5\cot^{-1}(ax)^2 + \frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2} - \frac{\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2} - \frac{\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\frac{\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)}{a}-}{a^2}}{a^2}\right.$$

input
```
Int[x^4*ArcCot[a*x]^2,x]
```

output
```
(x^5*ArcCot[a*x]^2)/5 + (2*a*(((x^4*ArcCot[a*x])/4 + (a*(-(x/a^4) + x^3/(3
*a^2) + ArcTan[a*x]/a^5))/4)/a^2 - (((x^2*ArcCot[a*x])/2 + (a*(x/a^2 - Arc
Tan[a*x]/a^3))/2)/a^2 - (((I/2)*ArcCot[a*x]^2)/a^2 - ((ArcCot[a*x]*Log[2/(
1 + I*a*x)])/a - ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/a)/a^2)/a^2))/5
```

3.13.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5380
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Simp[b*c*(
p/e)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5452
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

3.13. $\int x^4 \cot^{-1}(ax)^2 \, dx$

rule 5456
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Simp[
1/(c*d)   Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.13.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.52

method	result
parts	$\frac{x^5 \operatorname{arccot}(ax)^2}{5} + \frac{\frac{a^4x^4 \operatorname{arccot}(ax)}{10} - \frac{\operatorname{arccot}(ax)a^2x^2}{5} + \frac{\operatorname{arccot}(ax)\ln\left(a^2x^2+1\right)}{5} + \frac{a^3x^3}{30} - \frac{3ax}{10} + \frac{3\arctan(ax)}{10} - \frac{i\left(\ln(ax-i)\ln\left(a^2x\right.\right.}{}}{}$
derivativedivides	$\frac{\frac{a^5x^5 \operatorname{arccot}(ax)^2}{5} + \frac{a^4x^4 \operatorname{arccot}(ax)}{10} - \frac{\operatorname{arccot}(ax)a^2x^2}{5} + \frac{\operatorname{arccot}(ax)\ln\left(a^2x^2+1\right)}{5} + \frac{a^3x^3}{30} - \frac{3ax}{10} + \frac{3\arctan(ax)}{10} - \frac{i\left(\ln(ax-i)\ln\left(a^2x^2\right.\right.}{}}{}$
default	$\frac{\frac{a^5x^5 \operatorname{arccot}(ax)^2}{5} + \frac{a^4x^4 \operatorname{arccot}(ax)}{10} - \frac{\operatorname{arccot}(ax)a^2x^2}{5} + \frac{\operatorname{arccot}(ax)\ln\left(a^2x^2+1\right)}{5} + \frac{a^3x^3}{30} - \frac{3ax}{10} + \frac{3\arctan(ax)}{10} - \frac{i\left(\ln(ax-i)\ln\left(a^2x^2\right.\right.}{}}{}$
risch	$\frac{i\pi\ln(iax+1)x^5}{10} - \frac{i\ln(iax+1)x^2}{10a^3} + \frac{i\ln(iax+1)x^4}{20a} - \frac{\pi x^2}{10a^3} + \frac{\pi x^4}{20a} + \frac{23i\ln\left(a^2x^2+1\right)}{150a^5} + \frac{i\,\mathrm{dilog}\left(\frac{1}{2}-\frac{iax}{2}\right)}{5a^5} - 47i$

input
```
int(x^4*arccot(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5*arccot(a*x)^2+2/5/a^5*(1/4*a^4*x^4*arccot(a*x)-1/2*arccot(a*x)*a^2
*x^2+1/2*arccot(a*x)*ln(a^2*x^2+1)+1/12*a^3*x^3-3/4*a*x+3/4*arctan(a*x)-1/
4*I*(ln(a*x-I)*ln(a^2*x^2+1)-1/2*ln(a*x-I)^2-dilog(-1/2*I*(I+a*x))-ln(a*x-
I)*ln(-1/2*I*(I+a*x)))+1/4*I*(ln(I+a*x)*ln(a^2*x^2+1)-1/2*ln(I+a*x)^2-dilo
g(1/2*I*(a*x-I))-ln(I+a*x)*ln(1/2*I*(a*x-I))))
```

3.13.5 Fricas [F]

$$\int x^4 \cot^{-1}(ax)^2 \, dx = \int x^4 \operatorname{arccot}(ax)^2 \, dx$$

input
```
integrate(x^4*arccot(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x^4*arccot(a*x)^2, x)
```

3.13.6 Sympy [F]

$$\int x^4 \cot^{-1}(ax)^2\, dx = \int x^4 \operatorname{acot}^2(ax)\, dx$$

input
```
integrate(x**4*acot(a*x)**2,x)
```

output
```
Integral(x**4*acot(a*x)**2, x)
```

3.13.7 Maxima [F]

$$\int x^4 \cot^{-1}(ax)^2\, dx = \int x^4 \operatorname{arccot}(ax)^2\, dx$$

input
```
integrate(x^4*arccot(a*x)^2,x, algorithm="maxima")
```

output
```
1/20*x^5*arctan2(1, a*x)^2 - 1/80*x^5*log(a^2*x^2 + 1)^2 + integrate(1/80*
(60*a^2*x^6*arctan2(1, a*x)^2 + 4*a^2*x^6*log(a^2*x^2 + 1) + 8*a*x^5*arcta
n2(1, a*x) + 60*x^4*arctan2(1, a*x)^2 + 5*(a^2*x^6 + x^4)*log(a^2*x^2 + 1)
^2)/(a^2*x^2 + 1), x)
```

3.13.8 Giac [F]

$$\int x^4 \cot^{-1}(ax)^2\, dx = \int x^4 \operatorname{arccot}(ax)^2\, dx$$

input
```
integrate(x^4*arccot(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^4*arccot(a*x)^2, x)
```

3.13.9 Mupad [F(-1)]

Timed out.

$$\int x^4 \cot^{-1}(ax)^2 \, dx = \int x^4 \operatorname{acot}(a\,x)^2 \, dx$$

input
```
int(x^4*acot(a*x)^2,x)
```

output
```
int(x^4*acot(a*x)^2, x)
```

3.14 $\int x^3 \cot^{-1}(ax)^2 \, dx$

3.14.1 Optimal result

Integrand size = 10, antiderivative size = 80

$$\int x^3 \cot^{-1}(ax)^2 \, dx = \frac{x^2}{12a^2} - \frac{x \cot^{-1}(ax)}{2a^3} + \frac{x^3 \cot^{-1}(ax)}{6a} - \frac{\cot^{-1}(ax)^2}{4a^4} + \frac{1}{4}x^4 \cot^{-1}(ax)^2 - \frac{\log(1+a^2x^2)}{3a^4}$$

output
```
1/12*x^2/a^2-1/2*x*arccot(a*x)/a^3+1/6*x^3*arccot(a*x)/a-1/4*arccot(a*x)^2
/a^4+1/4*x^4*arccot(a*x)^2-1/3*ln(a^2*x^2+1)/a^4
```

3.14.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.76

$$\int x^3 \cot^{-1}(ax)^2 \, dx = \frac{a^2x^2 + 2ax(-3+a^2x^2)\cot^{-1}(ax) + 3(-1+a^4x^4)\cot^{-1}(ax)^2 - 4\log(1+a^2x^2)}{12a^4}$$

input
```
Integrate[x^3*ArcCot[a*x]^2,x]
```

output
```
(a^2*x^2 + 2*a*x*(-3 + a^2*x^2)*ArcCot[a*x] + 3*(-1 + a^4*x^4)*ArcCot[a*x]
^2 - 4*Log[1 + a^2*x^2])/(12*a^4)
```

3.14.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.39, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5362, 5452, 5362, 243, 49, 2009, 5452, 5346, 240, 5420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \cot^{-1}(ax)^2 \, dx$$

$$\Big\downarrow \text{5362}$$

$$\frac{1}{2}a\int \frac{x^4 \cot^{-1}(ax)}{a^2x^2+1}dx+\frac{1}{4}x^4\cot^{-1}(ax)^2$$

$$\Big\downarrow \text{5452}$$

$$\frac{1}{2}a\left(\frac{\int x^2\cot^{-1}(ax)dx}{a^2}-\frac{\int \frac{x^2\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^2$$

$$\Big\downarrow \text{5362}$$

$$\frac{1}{2}a\left(\frac{\frac{1}{3}a\int \frac{x^3}{a^2x^2+1}dx+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\int \frac{x^2\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^2$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}a\left(\frac{\frac{1}{6}a\int \frac{x^2}{a^2x^2+1}dx^2+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\int \frac{x^2\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}a\left(\frac{\frac{1}{6}a\int \left(\frac{1}{a^2}-\frac{1}{a^2(a^2x^2+1)}\right)dx^2+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\int \frac{x^2\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}a\left(\frac{\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\int \frac{x^2\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^2$$

$$\Big\downarrow \text{5452}$$

$$\frac{1}{2}a\left(\frac{\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\frac{\int\cot^{-1}(ax)dx}{a^2}-\frac{\int\frac{\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^2$$

$$\Big\downarrow \; 5346$$

$$\frac{1}{2}a\left(\frac{\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\frac{a\int\frac{x}{a^2x^2+1}dx+x\cot^{-1}(ax)}{a^2}-\frac{\int\frac{\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^2$$

$$\Big\downarrow \; 240$$

$$\frac{1}{2}a\left(\frac{\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{\log\left(a^2x^2+1\right)}{2a}+x\cot^{-1}(ax)}{a^2}-\frac{\int\frac{\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^2$$

$$\Big\downarrow \; 5420$$

$$\frac{1}{2}a\left(\frac{\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\frac{\cot^{-1}(ax)^2}{2a^3}+\frac{\frac{\log\left(a^2x^2+1\right)}{2a}+x\cot^{-1}(ax)}{a^2}}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^2$$

input
```
Int[x^3*ArcCot[a*x]^2,x]
```

output
```
(x^4*ArcCot[a*x]^2)/4 + (a*(((x^3*ArcCot[a*x])/3 + (a*(x^2/a^2 - Log[1 + a
^2*x^2]/a^4))/6)/a^2 - (ArcCot[a*x]^2/(2*a^3) + (x*ArcCot[a*x] + Log[1 + a
^2*x^2]/(2*a))/a^2)/a^2))/2
```

3.14.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5420
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5452
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

3.14. $\int x^3 \cot^{-1}(ax)^2 \, dx$

3.14.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

method	result
parallelrisch	$-\frac{-3a^4x^4 \operatorname{arccot}(ax)^2-2a^3x^3 \operatorname{arccot}(ax)-a^2x^2+6 \operatorname{arccot}(ax)ax+1+3 \operatorname{arccot}(ax)^2+4\ln(a^2x^2+1)}{12a^4}$
parts	$\frac{x^4 \operatorname{arccot}(ax)^2}{4}+\frac{\frac{a^3x^3 \operatorname{arccot}(ax)}{3}-\operatorname{arccot}(ax)ax+\operatorname{arccot}(ax)\arctan(ax)+\frac{a^2x^2}{6}-\frac{2\ln(a^2x^2+1)}{3}+\frac{\arctan(ax)^2}{2}}{2a^4}$
derivativedivides	$\frac{\frac{a^4x^4 \operatorname{arccot}(ax)^2}{4}+\frac{a^3x^3 \operatorname{arccot}(ax)}{6}-\frac{\operatorname{arccot}(ax)ax}{2}+\frac{\operatorname{arccot}(ax)\arctan(ax)}{2}+\frac{a^2x^2}{12}-\frac{\ln(a^2x^2+1)}{3}+\frac{\arctan(ax)^2}{4}}{a^4}$
default	$\frac{\frac{a^4x^4 \operatorname{arccot}(ax)^2}{4}+\frac{a^3x^3 \operatorname{arccot}(ax)}{6}-\frac{\operatorname{arccot}(ax)ax}{2}+\frac{\operatorname{arccot}(ax)\arctan(ax)}{2}+\frac{a^2x^2}{12}-\frac{\ln(a^2x^2+1)}{3}+\frac{\arctan(ax)^2}{4}}{a^4}$
risch	$-\frac{(a^4x^4-1)\ln(iax+1)^2}{16a^4}+\frac{(3i\pi a^4x^4+3x^4\ln(-iax+1)a^4+2ia^3x^3-6iax-3\ln(-iax+1))\ln(iax+1)}{24a^4}-\frac{i\pi x^4\ln(-iax}{8}$

input
```
int(x^3*arccot(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/12*(-3*a^4*x^4*arccot(a*x)^2-2*a^3*x^3*arccot(a*x)-a^2*x^2+6*arccot(a*x
)*a*x+1+3*arccot(a*x)^2+4*ln(a^2*x^2+1))/a^4
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.75

$$\int x^3 \cot^{-1}(ax)^2 \, dx = \frac{a^2x^2 + 3\left(a^4x^4 - 1\right) \operatorname{arccot}\left(ax\right)^2 + 2\left(a^3x^3 - 3\,ax\right) \operatorname{arccot}\left(ax\right) - 4\,\log\left(a^2x^2 + 1\right)}{12\,a^4}$$

input
```
integrate(x^3*arccot(a*x)^2,x, algorithm="fricas")
```

output
```
1/12*(a^2*x^2 + 3*(a^4*x^4 - 1)*arccot(a*x)^2 + 2*(a^3*x^3 - 3*a*x)*arccot
(a*x) - 4*log(a^2*x^2 + 1))/a^4
```

3.14.6 Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int x^3 \cot^{-1}(ax)^2 \, dx = \begin{cases} \frac{x^4 \operatorname{acot}^2(ax)}{4} + \frac{x^3 \operatorname{acot}(ax)}{6a} + \frac{x^2}{12a^2} - \frac{x \operatorname{acot}(ax)}{2a^3} - \frac{\log(a^2x^2+1)}{3a^4} - \frac{\operatorname{acot}^2(ax)}{4a^4} & \text{for } a \neq 0 \\ \frac{\pi^2 x^4}{16} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*acot(a*x)**2,x)
```

output
```
Piecewise((x**4*acot(a*x)**2/4 + x**3*acot(a*x)/(6*a) + x**2/(12*a**2) - x
*acot(a*x)/(2*a**3) - log(a**2*x**2 + 1)/(3*a**4) - acot(a*x)**2/(4*a**4),
 Ne(a, 0)), (pi**2*x**4/16, True))
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

$$\int x^3 \cot^{-1}(ax)^2 \, dx = \frac{1}{4} x^4 \operatorname{arccot}(ax)^2 + \frac{1}{6} a \left(\frac{a^2x^3 - 3x}{a^4} + \frac{3 \arctan(ax)}{a^5} \right) \operatorname{arccot}(ax) + \frac{a^2x^2 + 3 \arctan(ax)^2 - 4 \log(a^2x^2 + 1)}{12 a^4}$$

input
```
integrate(x^3*arccot(a*x)^2,x, algorithm="maxima")
```

output
```
1/4*x^4*arccot(a*x)^2 + 1/6*a*((a^2*x^3 - 3*x)/a^4 + 3*arctan(a*x)/a^5)*ar
ccot(a*x) + 1/12*(a^2*x^2 + 3*arctan(a*x)^2 - 4*log(a^2*x^2 + 1))/a^4
```

3.14.8 Giac [F]

$$\int x^3 \cot^{-1}(ax)^2 \, dx = \int x^3 \operatorname{arccot}(ax)^2 \, dx$$

input
```
integrate(x^3*arccot(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^3*arccot(a*x)^2, x)
```

3.14.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.82

$$\int x^3 \cot^{-1}(ax)^2\,dx = \frac{x^4\,\mathrm{acot}(a\,x)^2}{4} - \frac{\frac{\ln(a^2\,x^2+1)}{3} - \frac{a^2\,x^2}{12} + \frac{\mathrm{acot}(a\,x)^2}{4} - \frac{a^3\,x^3\,\mathrm{acot}(a\,x)}{6} + \frac{a\,x\,\mathrm{acot}(a\,x)}{2}}{a^4}$$

input
```
int(x^3*acot(a*x)^2,x)
```

output
```
(x^4*acot(a*x)^2)/4 - (log(a^2*x^2 + 1)/3 - (a^2*x^2)/12 + acot(a*x)^2/4 -
 (a^3*x^3*acot(a*x))/6 + (a*x*acot(a*x))/2)/a^4
```

3.15 $\int x^2 \cot^{-1}(ax)^2 \, dx$

3.15.1 Optimal result

Integrand size = 10, antiderivative size = 111

$$\int x^2 \cot^{-1}(ax)^2 \, dx = \frac{x}{3a^2} + \frac{x^2 \cot^{-1}(ax)}{3a} - \frac{i \cot^{-1}(ax)^2}{3a^3} + \frac{1}{3} x^3 \cot^{-1}(ax)^2 - \frac{\arctan(ax)}{3a^3} + \frac{2 \cot^{-1}(ax) \log \left(\frac{2}{1+iax}\right)}{3a^3} - \frac{i \operatorname{PolyLog}\left(2, 1 - \frac{2}{1+iax}\right)}{3a^3}$$

output
```
1/3*x/a^2+1/3*x^2*arccot(a*x)/a-1/3*I*arccot(a*x)^2/a^3+1/3*x^3*arccot(a*x
)^2-1/3*arctan(a*x)/a^3+2/3*arccot(a*x)*ln(2/(1+I*a*x))/a^3-1/3*I*polylog(
2,1-2/(1+I*a*x))/a^3
```

3.15.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.68

$$\int x^2 \cot^{-1}(ax)^2 \, dx = \frac{ax + (-i + a^3x^3) \cot^{-1}(ax)^2 + \cot^{-1}(ax) \left(1 + a^2x^2 + 2 \log \left(1 - e^{2i \cot^{-1}(ax)}\right)\right) - i \operatorname{PolyLog}\left(2, e^{2i \cot^{-1}(ax}\right.}{3a^3}$$

input
```
Integrate[x^2*ArcCot[a*x]^2,x]
```

output
```
(a*x + (-I + a^3*x^3)*ArcCot[a*x]^2 + ArcCot[a*x]*(1 + a^2*x^2 + 2*Log[1 -
 E^((2*I)*ArcCot[a*x])]) - I*PolyLog[2, E^((2*I)*ArcCot[a*x])])/(3*a^3)
```

3.15.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.16, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {5362, 5452, 5362, 262, 216, 5456, 5380, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cot^{-1}(ax)^2 \, dx$$

$$\Big\downarrow \text{ 5362}$$

$$\frac{2}{3}a\int \frac{x^3 \cot^{-1}(ax)}{a^2x^2+1}dx+\frac{1}{3}x^3\cot^{-1}(ax)^2$$

$$\Big\downarrow \text{ 5452}$$

$$\frac{2}{3}a\left(\frac{\int x\cot^{-1}(ax)dx}{a^2}-\frac{\int \frac{x\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{3}x^3\cot^{-1}(ax)^2$$

$$\Big\downarrow \text{ 5362}$$

$$\frac{2}{3}a\left(\frac{\frac{1}{2}a\int \frac{x^2}{a^2x^2+1}dx+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\int \frac{x\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{3}x^3\cot^{-1}(ax)^2$$

$$\Big\downarrow \text{ 262}$$

$$\frac{2}{3}a\left(\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\int \frac{1}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\int \frac{x\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{3}x^3\cot^{-1}(ax)^2$$

$$\Big\downarrow \text{ 216}$$

$$\frac{2}{3}a\left(\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\int \frac{x\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{3}x^3\cot^{-1}(ax)^2$$

$$\Big\downarrow \text{ 5456}$$

$$\frac{1}{3}x^3\cot^{-1}(ax)^2+\frac{2}{3}a\left(\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\int \frac{\cot^{-1}(ax)}{i-ax}dx}{a}}{a^2}\right)$$

$$\Big\downarrow \text{ 5380}$$

$$\frac{1}{3}x^3\cot^{-1}(ax)^2+\frac{2}{3}a\left(\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\int\frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx+\frac{\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)}{a}}{a}}{a^2}\right)$$

$\downarrow$ 2849

$$\frac{1}{3}x^3\cot^{-1}(ax)^2+\frac{2}{3}a\left(\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\frac{\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)}{a}-\frac{i\int\frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}}{a}}{a^2}\right)$$

$\downarrow$ 2752

$$\frac{1}{3}x^3\cot^{-1}(ax)^2+\frac{2}{3}a\left(\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\frac{\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2}\right)$$

input
```
Int[x^2*ArcCot[a*x]^2,x]
```

output
```
(x^3*ArcCot[a*x]^2)/3 + (2*a*(((x^2*ArcCot[a*x])/2 + (a*(x/a^2 - ArcTan[a*
x]/a^3))/2)/a^2 - (((I/2)*ArcCot[a*x]^2)/a^2 - ((ArcCot[a*x]*Log[2/(1 + I*
a*x)])/a - ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/a)/a^2))/3
```

3.15.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5380
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Simp[b*c*(
p/e)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5452
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5456
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Simp[
1/(c*d)   Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.15. $\int x^2 \cot^{-1}(ax)^2 \, dx$

3.15.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.67

method	result
parts	$\frac{x^3 \operatorname{arccot}(ax)^2}{3} + \frac{\frac{\operatorname{arccot}(ax)a^2x^2}{3} - \frac{\operatorname{arccot}(ax)\ln(a^2x^2+1)}{3} + \frac{ax}{3} - \frac{\arctan(ax)}{3} + \frac{i\left(\ln(ax-i)\ln(a^2x^2+1) - \frac{\ln(ax-i)^2}{2} - \operatorname{dilog}\left(-\right.\right.}{6}}{}$
derivativedivides	$\frac{\frac{a^3x^3\operatorname{arccot}(ax)^2}{3} + \frac{\operatorname{arccot}(ax)a^2x^2}{3} - \frac{\operatorname{arccot}(ax)\ln(a^2x^2+1)}{3} + \frac{ax}{3} - \frac{\arctan(ax)}{3} + \frac{i\left(\ln(ax-i)\ln(a^2x^2+1) - \frac{\ln(ax-i)^2}{2} - \operatorname{dilog}\left(-\frac{i(}{}\right.\right.}{6}}{a^3}$
default	$\frac{\frac{a^3x^3\operatorname{arccot}(ax)^2}{3} + \frac{\operatorname{arccot}(ax)a^2x^2}{3} - \frac{\operatorname{arccot}(ax)\ln(a^2x^2+1)}{3} + \frac{ax}{3} - \frac{\arctan(ax)}{3} + \frac{i\left(\ln(ax-i)\ln(a^2x^2+1) - \frac{\ln(ax-i)^2}{2} - \operatorname{dilog}\left(-\frac{i(}{}\right.\right.}{6}}{a^3}$
risch	$\frac{\pi^2x^3}{12} + \frac{\pi x^2}{6a} + \frac{\ln(iax+1)\ln(-iax+1)x^3}{6} - \frac{i\operatorname{dilog}\left(\frac{1}{2} - \frac{iax}{2}\right)}{3a^3} - \frac{2i\ln(a^2x^2+1)}{9a^3} + \frac{5i\ln(-iax+1)}{36a^3} + \frac{11\pi}{18a^3} - \ln($

input
```
int(x^2*arccot(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*arccot(a*x)^2+2/3/a^3*(1/2*arccot(a*x)*a^2*x^2-1/2*arccot(a*x)*ln(
a^2*x^2+1)+1/2*a*x-1/2*arctan(a*x)+1/4*I*(ln(a*x-I)*ln(a^2*x^2+1)-1/2*ln(a
*x-I)^2-dilog(-1/2*I*(I+a*x))-ln(a*x-I)*ln(-1/2*I*(I+a*x)))-1/4*I*(ln(I+a*
x)*ln(a^2*x^2+1)-1/2*ln(I+a*x)^2-dilog(1/2*I*(a*x-I))-ln(I+a*x)*ln(1/2*I*(
a*x-I))))
```

3.15.5 Fricas [F]

$$\int x^2 \cot^{-1}(ax)^2 \, dx = \int x^2 \operatorname{arccot}(ax)^2 \, dx$$

input
```
integrate(x^2*arccot(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x^2*arccot(a*x)^2, x)
```

3.15.6 Sympy [F]

$$\int x^2 \cot^{-1}(ax)^2 \, dx = \int x^2 \operatorname{acot}^2(ax) \, dx$$

input
```
integrate(x**2*acot(a*x)**2,x)
```

output
```
Integral(x**2*acot(a*x)**2, x)
```

3.15.7 Maxima [F]

$$\int x^2 \cot^{-1}(ax)^2 \, dx = \int x^2 \operatorname{arccot}(ax)^2 \, dx$$

input
```
integrate(x^2*arccot(a*x)^2,x, algorithm="maxima")
```

output
```
1/12*x^3*arctan2(1, a*x)^2 - 1/48*x^3*log(a^2*x^2 + 1)^2 + integrate(1/48*
(36*a^2*x^4*arctan2(1, a*x)^2 + 4*a^2*x^4*log(a^2*x^2 + 1) + 8*a*x^3*arcta
n2(1, a*x) + 36*x^2*arctan2(1, a*x)^2 + 3*(a^2*x^4 + x^2)*log(a^2*x^2 + 1)
^2)/(a^2*x^2 + 1), x)
```

3.15.8 Giac [F]

$$\int x^2 \cot^{-1}(ax)^2 \, dx = \int x^2 \operatorname{arccot}(ax)^2 \, dx$$

input
```
integrate(x^2*arccot(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^2*arccot(a*x)^2, x)
```

3.15.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(ax)^2\,dx = \int x^2\,\mathrm{acot}(a\,x)^2\,dx$$

input
```
int(x^2*acot(a*x)^2,x)
```

output
```
int(x^2*acot(a*x)^2, x)
```

3.16 $\int x \cot^{-1}(ax)^2 \, dx$

3.16.1 Optimal result

Integrand size = 8, antiderivative size = 53

$$\int x \cot^{-1}(ax)^2 \, dx = \frac{x \cot^{-1}(ax)}{a} + \frac{\cot^{-1}(ax)^2}{2a^2} + \frac{1}{2}x^2 \cot^{-1}(ax)^2 + \frac{\log(1+a^2x^2)}{2a^2}$$

output
```
x*arccot(a*x)/a+1/2*arccot(a*x)^2/a^2+1/2*x^2*arccot(a*x)^2+1/2*ln(a^2*x^2
+1)/a^2
```

3.16.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int x \cot^{-1}(ax)^2 \, dx = \frac{2ax \cot^{-1}(ax) + (1+a^2x^2) \cot^{-1}(ax)^2 + \log(1+a^2x^2)}{2a^2}$$

input
```
Integrate[x*ArcCot[a*x]^2,x]
```

output
```
(2*a*x*ArcCot[a*x] + (1 + a^2*x^2)*ArcCot[a*x]^2 + Log[1 + a^2*x^2])/(2*a^
2)
```

3.16.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5362, 5452, 5346, 240, 5420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \cot^{-1}(ax)^2 \, dx$$

$$\Big\downarrow \text{5362}$$

$$a \int \frac{x^2 \cot^{-1}(ax)}{a^2x^2+1} dx + \frac{1}{2}x^2 \cot^{-1}(ax)^2$$

$$\Big\downarrow \text{5452}$$

$$a \left(\frac{\int \cot^{-1}(ax) dx}{a^2} - \frac{\int \frac{\cot^{-1}(ax)}{a^2x^2+1} dx}{a^2} \right) + \frac{1}{2}x^2 \cot^{-1}(ax)^2$$

$$\Big\downarrow \text{5346}$$

$$a \left(\frac{a \int \frac{x}{a^2x^2+1} dx + x \cot^{-1}(ax)}{a^2} - \frac{\int \frac{\cot^{-1}(ax)}{a^2x^2+1} dx}{a^2} \right) + \frac{1}{2}x^2 \cot^{-1}(ax)^2$$

$$\Big\downarrow \text{240}$$

$$a \left(\frac{\frac{\log(a^2x^2+1)}{2a} + x \cot^{-1}(ax)}{a^2} - \frac{\int \frac{\cot^{-1}(ax)}{a^2x^2+1} dx}{a^2} \right) + \frac{1}{2}x^2 \cot^{-1}(ax)^2$$

$$\Big\downarrow \text{5420}$$

$$a \left(\frac{\cot^{-1}(ax)^2}{2a^3} + \frac{\frac{\log(a^2x^2+1)}{2a} + x \cot^{-1}(ax)}{a^2} \right) + \frac{1}{2}x^2 \cot^{-1}(ax)^2$$

input
```
Int[x*ArcCot[a*x]^2,x]
```

output
```
(x^2*ArcCot[a*x]^2)/2 + a*(ArcCot[a*x]^2/(2*a^3) + (x*ArcCot[a*x] + Log[1
+ a^2*x^2]/(2*a))/a^2)
```

3.16.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5420
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5452
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

3.16.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

method	result
parallelrisch	$\frac{a^{2}x^{2}\operatorname{arccot}(ax)^{2}+2\operatorname{arccot}(ax)ax+\operatorname{arccot}(ax)^{2}+\ln(a^{2}x^{2}+1)}{2a^{2}}$
parts	$\frac{x^{2}\operatorname{arccot}(ax)^{2}}{2}+\frac{-\operatorname{arccot}(ax)\arctan(ax)+\operatorname{arccot}(ax)ax+\frac{\ln\left(a^{2}x^{2}+1\right)}{2}-\frac{\arctan(ax)^{2}}{2}}{a^{2}}$
derivativedivides	$\frac{\frac{a^{2}x^{2}\operatorname{arccot}(ax)^{2}}{2}-\operatorname{arccot}(ax)\arctan(ax)+\operatorname{arccot}(ax)ax+\frac{\ln\left(a^{2}x^{2}+1\right)}{2}-\frac{\arctan(ax)^{2}}{2}}{a^{2}}$
default	$\frac{\frac{a^{2}x^{2}\operatorname{arccot}(ax)^{2}}{2}-\operatorname{arccot}(ax)\arctan(ax)+\operatorname{arccot}(ax)ax+\frac{\ln\left(a^{2}x^{2}+1\right)}{2}-\frac{\arctan(ax)^{2}}{2}}{a^{2}}$
risch	$-\frac{(a^{2}x^{2}+1)\ln(iax+1)^{2}}{8a^{2}}+\frac{(i\pi a^{2}x^{2}+x^{2}\ln(-iax+1)a^{2}+2iax+\ln(-iax+1))\ln(iax+1)}{4a^{2}}-\frac{i\pi x^{2}\ln(-iax+1)}{4}+\frac{\pi^{2}x}{8}$

input
```
int(x*arccot(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(a^2*x^2*arccot(a*x)^2+2*arccot(a*x)*a*x+arccot(a*x)^2+ln(a^2*x^2+1))/
a^2
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

$$\int x\cot^{-1}(ax)^{2}\,dx=\frac{2\,ax\operatorname{arccot}\left(ax\right)+\left(a^{2}x^{2}+1\right)\operatorname{arccot}\left(ax\right)^{2}+\log\left(a^{2}x^{2}+1\right)}{2\,a^{2}}$$

input
```
integrate(x*arccot(a*x)^2,x, algorithm="fricas")
```

output
```
1/2*(2*a*x*arccot(a*x) + (a^2*x^2 + 1)*arccot(a*x)^2 + log(a^2*x^2 + 1))/a
^2
```

3.16.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int x\cot^{-1}(ax)^{2}\,dx=\begin{cases}\frac{x^{2}\operatorname{acot}^{2}\left(ax\right)}{2}+\frac{x\operatorname{acot}\left(ax\right)}{a}+\frac{\log\left(a^{2}x^{2}+1\right)}{2a^{2}}+\frac{\operatorname{acot}^{2}\left(ax\right)}{2a^{2}} & \text{for } a\neq 0\\ \frac{\pi^{2}x^{2}}{8} & \text{otherwise}\end{cases}$$

input
```
integrate(x*acot(a*x)**2,x)
```

output
```
Piecewise((x**2*acot(a*x)**2/2 + x*acot(a*x)/a + log(a**2*x**2 + 1)/(2*a**
2) + acot(a*x)**2/(2*a**2), Ne(a, 0)), (pi**2*x**2/8, True))
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08

$$\int x \cot^{-1}(ax)^2 \, dx = \frac{1}{2} x^2 \operatorname{arccot}(ax)^2 + a\left(\frac{x}{a^2} - \frac{\arctan(ax)}{a^3}\right) \operatorname{arccot}(ax) - \frac{\arctan(ax)^2 - \log(a^2x^2 + 1)}{2a^2}$$

input
```
integrate(x*arccot(a*x)^2,x, algorithm="maxima")
```

output
```
1/2*x^2*arccot(a*x)^2 + a*(x/a^2 - arctan(a*x)/a^3)*arccot(a*x) - 1/2*(arc
tan(a*x)^2 - log(a^2*x^2 + 1))/a^2
```

3.16.8 Giac [F]

$$\int x \cot^{-1}(ax)^2 \, dx = \int x \operatorname{arccot}(ax)^2 \, dx$$

input
```
integrate(x*arccot(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x*arccot(a*x)^2, x)
```

3.16.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int x \cot^{-1}(ax)^2 \, dx = \frac{x^2 \operatorname{acot}(a\,x)^2}{2} + \frac{\frac{\operatorname{acot}(a\,x)^2}{2} + a\,x \operatorname{acot}(a\,x) + \frac{\ln(a^2\,x^2+1)}{2}}{a^2}$$

input
```
int(x*acot(a*x)^2,x)
```

output
```
(x^2*acot(a*x)^2)/2 + (log(a^2*x^2 + 1)/2 + acot(a*x)^2/2 + a*x*acot(a*x))
/a^2
```

3.17 $\int \cot^{-1}(ax)^2 \, dx$

3.17.1 Optimal result

Integrand size = 6, antiderivative size = 67

$$\int \cot^{-1}(ax)^2 \, dx = \frac{i \cot^{-1}(ax)^2}{a} + x \cot^{-1}(ax)^2 - \frac{2 \cot^{-1}(ax) \log \left(\frac{2}{1+iax}\right)}{a} + \frac{i \, \text{PolyLog}\left(2, 1 - \frac{2}{1+iax}\right)}{a}$$

output
```
I*arccot(a*x)^2/a+x*arccot(a*x)^2-2*arccot(a*x)*ln(2/(1+I*a*x))/a+I*polylo
g(2,1-2/(1+I*a*x))/a
```

3.17.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

$$\int \cot^{-1}(ax)^2 \, dx = \frac{\cot^{-1}(ax) \left((i + ax) \cot^{-1}(ax) - 2 \log \left(1 - e^{2i \cot^{-1}(ax)}\right)\right) + i \, \text{PolyLog}\left(2, e^{2i \cot^{-1}(ax)}\right)}{a}$$

input
```
Integrate[ArcCot[a*x]^2,x]
```

output
```
(ArcCot[a*x]*((I + a*x)*ArcCot[a*x] - 2*Log[1 - E^((2*I)*ArcCot[a*x])]) +
I*PolyLog[2, E^((2*I)*ArcCot[a*x])])/a
```

3.17.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5346, 5456, 5380, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}(ax)^2\,dx$$

$$\Big\downarrow \text{5346}$$

$$2a\int \frac{x\cot^{-1}(ax)}{a^2x^2+1}dx+x\cot^{-1}(ax)^2$$

$$\Big\downarrow \text{5456}$$

$$x\cot^{-1}(ax)^2+2a\left(\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\int \frac{\cot^{-1}(ax)}{i-ax}dx}{a}\right)$$

$$\Big\downarrow \text{5380}$$

$$x\cot^{-1}(ax)^2+2a\left(\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\int \frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx+\frac{\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)}{a}}{a}\right)$$

$$\Big\downarrow \text{2849}$$

$$x\cot^{-1}(ax)^2+2a\left(\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\frac{\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)}{a}-\frac{i\int \frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}}{a}\right)$$

$$\Big\downarrow \text{2752}$$

$$x\cot^{-1}(ax)^2+2a\left(\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\frac{\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}\right)$$

input
```
Int[ArcCot[a*x]^2,x]
```

output
```
x*ArcCot[a*x]^2 + 2*a*(((I/2)*ArcCot[a*x]^2)/a^2 - ((ArcCot[a*x]*Log[2/(1
+ I*a*x)])/a - ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/a)
```

3.17.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5380
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Simp[b*c*(
p/e)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5456
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Simp[
1/(c*d)   Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.17.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(63) = 126$.

Time = 0.50 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.94

method	result
derivativedivides	$\frac{\operatorname{arccot}(ax)^{2}(ax-i)-2\operatorname{arccot}(ax)\ln\left(1-\frac{ax+i}{\sqrt{a^{2}x^{2}+1}}\right)-2\operatorname{arccot}(ax)\ln\left(1+\frac{ax+i}{\sqrt{a^{2}x^{2}+1}}\right)+2i\operatorname{arccot}(ax)^{2}+2i\operatorname{polylog}\left(2,\frac{}{\sqrt{a}}\right.}{a}$
default	$\frac{\operatorname{arccot}(ax)^{2}(ax-i)-2\operatorname{arccot}(ax)\ln\left(1-\frac{ax+i}{\sqrt{a^{2}x^{2}+1}}\right)-2\operatorname{arccot}(ax)\ln\left(1+\frac{ax+i}{\sqrt{a^{2}x^{2}+1}}\right)+2i\operatorname{arccot}(ax)^{2}+2i\operatorname{polylog}\left(2,\frac{}{\sqrt{a}}\right.}{a}$
risch	$\frac{i\pi^{2}}{4a}-\frac{i\ln(-iax+1)\pi x}{2}+\frac{i\ln\left(\frac{1}{2}+\frac{iax}{2}\right)\ln\left(\frac{1}{2}-\frac{iax}{2}\right)}{a}-\frac{i\ln(-iax+1)^{2}}{4a}+\frac{i\ln(iax+1)^{2}}{4a}+\frac{i\ln(a^{2}x^{2}+1)}{2a}+\frac{\pi\ln(ia}{2a}$

input
```
int(arccot(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(arccot(a*x)^2*(a*x-I)-2*arccot(a*x)*ln(1-(I+a*x)/(a^2*x^2+1)^(1/2))-2
*arccot(a*x)*ln(1+(I+a*x)/(a^2*x^2+1)^(1/2))+2*I*arccot(a*x)^2+2*I*polylog
(2,(I+a*x)/(a^2*x^2+1)^(1/2))+2*I*polylog(2,-(I+a*x)/(a^2*x^2+1)^(1/2)))
```

3.17.5 Fricas [F]

$$\int \cot^{-1}(ax)^2\,dx = \int \operatorname{arccot}\left(ax\right)^2\,dx$$

input
```
integrate(arccot(a*x)^2,x, algorithm="fricas")
```

output
```
integral(arccot(a*x)^2, x)
```

3.17.6 Sympy [F]

$$\int \cot^{-1}(ax)^2\,dx = \int \operatorname{acot}^2\left(ax\right)dx$$

input
```
integrate(acot(a*x)**2,x)
```

output
```
Integral(acot(a*x)**2, x)
```

3.17.7 Maxima [F]

$$\int \cot^{-1}(ax)^2\,dx = \int \operatorname{arccot}(ax)^2\,dx$$

input
```
integrate(arccot(a*x)^2,x, algorithm="maxima")
```

output
```
1/4*x*arctan2(1, a*x)^2 + 12*a^2*integrate(1/16*x^2*arctan(1/(a*x))^2/(a^2
*x^2 + 1), x) + a^2*integrate(1/16*x^2*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x
) + 4*a^2*integrate(1/16*x^2*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) - 1/16*x*l
og(a^2*x^2 + 1)^2 + 1/4*arctan(a*x)^3/a + 3/4*arctan(a*x)^2*arctan(1/(a*x)
)/a + 3/4*arctan(a*x)*arctan(1/(a*x))^2/a + 8*a*integrate(1/16*x*arctan(1/
(a*x))/(a^2*x^2 + 1), x) + integrate(1/16*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1)
, x)
```

3.17.8 Giac [F]

$$\int \cot^{-1}(ax)^2\,dx = \int \operatorname{arccot}(ax)^2\,dx$$

input
```
integrate(arccot(a*x)^2,x, algorithm="giac")
```

output
```
integrate(arccot(a*x)^2, x)
```

3.17.9 Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

$$\int \cot^{-1}(ax)^2\,dx = \frac{-2\,\ln\left(1-\mathrm{e}^{\mathrm{acot}(a\,x)\,2\mathrm{i}}\right)\,\mathrm{acot}(a\,x)+\mathrm{polylog}\left(2,\mathrm{e}^{\mathrm{acot}(a\,x)\,2\mathrm{i}}\right)\,1\mathrm{i}+\mathrm{acot}(a\,x)^2\,1\mathrm{i}}{a} + x\,\mathrm{acot}(a\,x)^2$$

input
```
int(acot(a*x)^2,x)
```

output
```
(polylog(2, exp(acot(a*x)*2i))*1i - 2*log(1 - exp(acot(a*x)*2i))*acot(a*x)
 + acot(a*x)^2*1i)/a + x*acot(a*x)^2
```

3.18 $\int \frac{\cot^{-1}(ax)^2}{x} dx$

3.18.1 Optimal result

Integrand size = 10, antiderivative size = 116

$$\begin{aligned}\int \frac{\cot^{-1}(ax)^2}{x} dx = {} & 2\cot^{-1}(ax)^2 \coth^{-1}\left(1-\frac{2}{1+iax}\right) - i\cot^{-1}(ax)\,\mathrm{PolyLog}\left(2,1-\frac{2i}{i+ax}\right) \\ & + i\cot^{-1}(ax)\,\mathrm{PolyLog}\left(2,1-\frac{2ax}{i+ax}\right) \\ & - \frac{1}{2}\mathrm{PolyLog}\left(3,1-\frac{2i}{i+ax}\right) + \frac{1}{2}\mathrm{PolyLog}\left(3,1-\frac{2ax}{i+ax}\right)\end{aligned}$$

output
```
2*arccot(a*x)^2*arccoth(1-2/(1+I*a*x))-I*arccot(a*x)*polylog(2,1-2*I/(I+a*
x))+I*arccot(a*x)*polylog(2,1-2*a*x/(I+a*x))-1/2*polylog(3,1-2*I/(I+a*x))+
1/2*polylog(3,1-2*a*x/(I+a*x))
```

3.18.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.14

$$\begin{aligned}\int \frac{\cot^{-1}(ax)^2}{x} dx = {} & -\frac{2}{3}i\cot^{-1}(ax)^3 - \cot^{-1}(ax)^2 \log\left(1-e^{-2i\cot^{-1}(ax)}\right) \\ & + \cot^{-1}(ax)^2 \log\left(1+e^{2i\cot^{-1}(ax)}\right) - i\cot^{-1}(ax)\,\mathrm{PolyLog}\left(2,e^{-2i\cot^{-1}(ax)}\right) \\ & - i\cot^{-1}(ax)\,\mathrm{PolyLog}\left(2,-e^{2i\cot^{-1}(ax)}\right) \\ & - \frac{1}{2}\mathrm{PolyLog}\left(3,e^{-2i\cot^{-1}(ax)}\right) + \frac{1}{2}\mathrm{PolyLog}\left(3,-e^{2i\cot^{-1}(ax)}\right)\end{aligned}$$

input
```
Integrate[ArcCot[a*x]^2/x,x]
```

output
```
((-2*I)/3)*ArcCot[a*x]^3 - ArcCot[a*x]^2*Log[1 - E^((-2*I)*ArcCot[a*x])] +
 ArcCot[a*x]^2*Log[1 + E^((2*I)*ArcCot[a*x])] - I*ArcCot[a*x]*PolyLog[2, E
^((-2*I)*ArcCot[a*x])] - I*ArcCot[a*x]*PolyLog[2, -E^((2*I)*ArcCot[a*x])]
- PolyLog[3, E^((-2*I)*ArcCot[a*x])]/2 + PolyLog[3, -E^((2*I)*ArcCot[a*x])
]/2
```

3.18.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.26, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5358, 5524, 5528, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax)^2}{x} \, dx$$

$$\Big\downarrow \text{5358}$$

$$4a\int \frac{\cot^{-1}(ax)\coth^{-1}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx+2\cot^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{1+iax}\right)$$

$$\Big\downarrow \text{5524}$$

$$4a\left(\frac{1}{2}\int \frac{\cot^{-1}(ax)\log\left(\frac{2ax}{ax+i}\right)}{a^2x^2+1}dx-\frac{1}{2}\int \frac{\cot^{-1}(ax)\log\left(\frac{2i}{ax+i}\right)}{a^2x^2+1}dx\right)+ 2\cot^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{1+iax}\right)$$

$$\Big\downarrow \text{5528}$$

$$4a\left(\frac{1}{2}\left(-\frac{1}{2}i\int \frac{\text{PolyLog}\left(2,1-\frac{2i}{ax+i}\right)}{a^2x^2+1}dx-\frac{i\,\text{PolyLog}\left(2,1-\frac{2i}{ax+i}\right)\cot^{-1}(ax)}{2a}\right)+\frac{1}{2}\left(\frac{1}{2}i\int \frac{\text{PolyLog}\left(2,1-\frac{2ax}{ax+i}\right)}{a^2x^2+1}\right.\right.$$
$$2\cot^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{1+iax}\right)$$

$$\Big\downarrow \text{7164}$$

$$4a\left(\frac{1}{2}\left(-\frac{\text{PolyLog}\left(3,1-\frac{2i}{ax+i}\right)}{4a}-\frac{i\,\text{PolyLog}\left(2,1-\frac{2i}{ax+i}\right)\cot^{-1}(ax)}{2a}\right)+\frac{1}{2}\left(\frac{\text{PolyLog}\left(3,1-\frac{2ax}{ax+i}\right)}{4a}+\frac{i\,\text{PolyLo}}{}\right.\right.$$
$$2\cot^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{1+iax}\right)$$

input
```
Int[ArcCot[a*x]^2/x,x]
```

output
```
2*ArcCot[a*x]^2*ArcCoth[1 - 2/(1 + I*a*x)] + 4*a*((((-1/2*I)*ArcCot[a*x]*P
olyLog[2, 1 - (2*I)/(I + a*x)])/a - PolyLog[3, 1 - (2*I)/(I + a*x)]/(4*a))
/2 + (((I/2)*ArcCot[a*x]*PolyLog[2, 1 - (2*a*x)/(I + a*x)])/a + PolyLog[3,
 1 - (2*a*x)/(I + a*x)]/(4*a))/2)
```

3.18.3.1 Defintions of rubi rules used

rule 5358
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcCot[c*x])^p*ArcCoth[1 - 2/(1 + I*c*x)], x] + Simp[2*b*c*p   Int[(a + b
*ArcCot[c*x])^(p - 1)*(ArcCoth[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
 FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 5524
```
Int[(ArcCoth[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[1/2   Int[Log[SimplifyIntegrand[1 + 1/u, x]]*((a +
 b*ArcCot[c*x])^p/(d + e*x^2)), x], x] - Simp[1/2   Int[Log[SimplifyIntegra
nd[1 - 1/u, x]]*((a + b*ArcCot[c*x])^p/(d + e*x^2)), x], x] /; FreeQ[{a, b,
 c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*
x)))^2, 0]
```

rule 5528
```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] + Simp[b*p*(I/2)   Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.18.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.67 (sec) , antiderivative size = 891, normalized size of antiderivative = 7.68

method	result
derivativedivides	$\ln(ax)\operatorname{arccot}(ax)^2 + \frac{i\pi\left(\operatorname{csgn}\left(\frac{i}{\frac{(ax+i)^2}{a^2x^2+1}-1}\right)\operatorname{csgn}\left(i\left(1+\frac{(ax+i)^2}{a^2x^2+1}\right)\right)\operatorname{csgn}\left(\frac{i\left(1+\frac{(ax+i)^2}{a^2x^2+1}\right)}{\frac{(ax+i)^2}{a^2x^2+1}-1}\right)-\operatorname{csgn}\left(\frac{i}{\frac{(ax+i)^2}{a^2x^2+1}}\right.\right.}{}$
default	$\ln(ax)\operatorname{arccot}(ax)^2 + \frac{i\pi\left(\operatorname{csgn}\left(\frac{i}{\frac{(ax+i)^2}{a^2x^2+1}-1}\right)\operatorname{csgn}\left(i\left(1+\frac{(ax+i)^2}{a^2x^2+1}\right)\right)\operatorname{csgn}\left(\frac{i\left(1+\frac{(ax+i)^2}{a^2x^2+1}\right)}{\frac{(ax+i)^2}{a^2x^2+1}-1}\right)-\operatorname{csgn}\left(\frac{i}{\frac{(ax+i)^2}{a^2x^2+1}}\right.\right.}{}$
parts	Expression too large to display

input
```
int(arccot(a*x)^2/x,x,method=_RETURNVERBOSE)
```

output
```
ln(a*x)*arccot(a*x)^2+1/2*I*Pi*(csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*(
1+(I+a*x)^2/(a^2*x^2+1)))*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a
^2*x^2+1)))-csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I/((I+a*x)^2/(a^2*x^2+1
)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^2-csgn(I*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn(I
/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^2+csgn(I/((I+a*x)^2/
(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^3-csgn(I/((I+a*x)^2/(a^2*x^2+1)-
1)*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn(1/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^
2/(a^2*x^2+1)))^2+csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1
)))*csgn(1/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))-csgn(1/((I
+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^3+csgn(1/((I+a*x)^2/(a^2
*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^2-1)*arccot(a*x)^2+arccot(a*x)^2*ln(
(I+a*x)^2/(a^2*x^2+1)-1)-arccot(a*x)^2*ln(1-(I+a*x)/(a^2*x^2+1)^(1/2))+2*I
*arccot(a*x)*polylog(2,(I+a*x)/(a^2*x^2+1)^(1/2))-2*polylog(3,(I+a*x)/(a^2
*x^2+1)^(1/2))-arccot(a*x)^2*ln(1+(I+a*x)/(a^2*x^2+1)^(1/2))+2*I*arccot(a*
x)*polylog(2,-(I+a*x)/(a^2*x^2+1)^(1/2))-2*polylog(3,-(I+a*x)/(a^2*x^2+1)^
(1/2))-I*arccot(a*x)*polylog(2,-(I+a*x)^2/(a^2*x^2+1))+1/2*polylog(3,-(I+a
*x)^2/(a^2*x^2+1))
```

3.18.5 Fricas [F]

$$\int \frac{\cot^{-1}(ax)^2}{x}\,dx = \int \frac{\operatorname{arccot}(ax)^2}{x}\,dx$$

input
```
integrate(arccot(a*x)^2/x,x, algorithm="fricas")
```

output
```
integral(arccot(a*x)^2/x, x)
```

3.18.6 Sympy [F]

$$\int \frac{\cot^{-1}(ax)^2}{x}\,dx = \int \frac{\operatorname{acot}^2(ax)}{x}\,dx$$

input
```
integrate(acot(a*x)**2/x,x)
```

output
```
Integral(acot(a*x)**2/x, x)
```

3.18.7 Maxima [F]

$$\int \frac{\cot^{-1}(ax)^2}{x}\,dx = \int \frac{\operatorname{arccot}(ax)^2}{x}\,dx$$

input
```
integrate(arccot(a*x)^2/x,x, algorithm="maxima")
```

output
```
integrate(arccot(a*x)^2/x, x)
```

3.18.8 Giac [F]

$$\int \frac{\cot^{-1}(ax)^2}{x}\,dx = \int \frac{\operatorname{arccot}\left(ax\right)^2}{x}\,dx$$

input
```
integrate(arccot(a*x)^2/x,x, algorithm="giac")
```

output
```
integrate(arccot(a*x)^2/x, x)
```

3.18.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)^2}{x}\,dx = \int \frac{\operatorname{acot}(a\,x)^2}{x}\,dx$$

input
```
int(acot(a*x)^2/x,x)
```

output
```
int(acot(a*x)^2/x, x)
```

3.19 $\int \frac{\cot^{-1}(ax)^2}{x^2}\,dx$

3.19.1 Optimal result

Integrand size = 10, antiderivative size = 66

$$\int \frac{\cot^{-1}(ax)^2}{x^2}\,dx = -ia\cot^{-1}(ax)^2 - \frac{\cot^{-1}(ax)^2}{x} - 2a\cot^{-1}(ax)\log\left(2 - \frac{2}{1-iax}\right) - ia\,\text{PolyLog}\left(2, -1 + \frac{2}{1-iax}\right)$$

output
```
-I*a*arccot(a*x)^2-arccot(a*x)^2/x-2*a*arccot(a*x)*ln(2-2/(1-I*a*x))-I*a*p
olylog(2,-1+2/(1-I*a*x))
```

3.19.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

$$\int \frac{\cot^{-1}(ax)^2}{x^2}\,dx = a\left(i\cot^{-1}(ax)^2 - \frac{\cot^{-1}(ax)^2}{ax} - 2\cot^{-1}(ax)\log\left(1 + e^{2i\cot^{-1}(ax)}\right) + i\,\text{PolyLog}\left(2, -e^{2i\cot^{-1}(ax)}\right)\right)$$

input
```
Integrate[ArcCot[a*x]^2/x^2,x]
```

output
```
a*(I*ArcCot[a*x]^2 - ArcCot[a*x]^2/(a*x) - 2*ArcCot[a*x]*Log[1 + E^((2*I)*
ArcCot[a*x])] + I*PolyLog[2, -E^((2*I)*ArcCot[a*x])])
```

3.19.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5362, 5460, 5404, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax)^2}{x^2} \, dx$$

$$\Big\downarrow \text{5362}$$

$$-2a \int \frac{\cot^{-1}(ax)}{x\left(a^2x^2+1\right)}dx - \frac{\cot^{-1}(ax)^2}{x}$$

$$\Big\downarrow \text{5460}$$

$$-\frac{\cot^{-1}(ax)^2}{x} - 2a\left(i\int \frac{\cot^{-1}(ax)}{x(ax+i)}dx + \frac{1}{2}i\cot^{-1}(ax)^2\right)$$

$$\Big\downarrow \text{5404}$$

$$-\frac{\cot^{-1}(ax)^2}{x} - 2a\left(i\left(-ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\log\left(2-\frac{2}{1-iax}\right)\cot^{-1}(ax)\right) + \frac{1}{2}i\cot^{-1}(ax)^2\right)$$

$$\Big\downarrow \text{2897}$$

$$-\frac{\cot^{-1}(ax)^2}{x} - 2a\left(i\left(\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right) - i\log\left(2-\frac{2}{1-iax}\right)\cot^{-1}(ax)\right) + \frac{1}{2}i\cot^{-1}(ax)^2\right)$$

input
```
Int[ArcCot[a*x]^2/x^2,x]
```

output
```
-(ArcCot[a*x]^2/x) - 2*a*((I/2)*ArcCot[a*x]^2 + I*((-I)*ArcCot[a*x]*Log[2
- 2/(1 - I*a*x)] + PolyLog[2, -1 + 2/(1 - I*a*x)]/2))
```

3.19.3.1 Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5404
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcCot[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] + Si
mp[b*c*(p/d)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5460
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*d*(p + 1))), x] + Simp[
I/d   Int[(a + b*ArcCot[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.19.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(62) = 124$.

Time = 0.57 (sec) , antiderivative size = 222, normalized size of antiderivative = 3.36

method	result
parts	$-\frac{\operatorname{arccot}(ax)^2}{x} - 2a\left(-\frac{\operatorname{arccot}(ax)\ln(a^2x^2+1)}{2} + \operatorname{arccot}(ax)\ln(ax) - \frac{i\ln(ax)\ln(iax+1)}{2} + \frac{i\ln(ax)\ln(\cdot}{2}\right.$
derivativedivides	$a\left(-\frac{\operatorname{arccot}(ax)^2}{ax} + \operatorname{arccot}(ax)\ln(a^2x^2+1) - 2\operatorname{arccot}(ax)\ln(ax) + i\ln(ax)\ln(iax+1\right.$
default	$a\left(-\frac{\operatorname{arccot}(ax)^2}{ax} + \operatorname{arccot}(ax)\ln(a^2x^2+1) - 2\operatorname{arccot}(ax)\ln(ax) + i\ln(ax)\ln(iax+1\right.$

input
```
int(arccot(a*x)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-arccot(a*x)^2/x-2*a*(-1/2*arccot(a*x)*ln(a^2*x^2+1)+arccot(a*x)*ln(a*x)-1
/2*I*ln(a*x)*ln(1+I*a*x)+1/2*I*ln(a*x)*ln(1-I*a*x)-1/2*I*dilog(1+I*a*x)+1/
2*I*dilog(1-I*a*x)+1/4*I*(ln(a*x-I)*ln(a^2*x^2+1)-1/2*ln(a*x-I)^2-dilog(-1
/2*I*(I+a*x))-ln(a*x-I)*ln(-1/2*I*(I+a*x)))-1/4*I*(ln(I+a*x)*ln(a^2*x^2+1)
-1/2*ln(I+a*x)^2-dilog(1/2*I*(a*x-I))-ln(I+a*x)*ln(1/2*I*(a*x-I))))
```

3.19.5 Fricas [F]

$$\int \frac{\cot^{-1}(ax)^2}{x^2}\,dx = \int \frac{\operatorname{arccot}(ax)^2}{x^2}\,dx$$

input
```
integrate(arccot(a*x)^2/x^2,x, algorithm="fricas")
```

output
```
integral(arccot(a*x)^2/x^2, x)
```

3.19.6 Sympy [F]

$$\int \frac{\cot^{-1}(ax)^2}{x^2}\,dx = \int \frac{\operatorname{acot}^2(ax)}{x^2}\,dx$$

input
```
integrate(acot(a*x)**2/x**2,x)
```

output
```
Integral(acot(a*x)**2/x**2, x)
```

3.19.7 Maxima [F]

$$\int \frac{\cot^{-1}(ax)^2}{x^2}\,dx = \int \frac{\operatorname{arccot}(ax)^2}{x^2}\,dx$$

input
```
integrate(arccot(a*x)^2/x^2,x, algorithm="maxima")
```

output
```
1/16*(4*(3*a*arctan(a*x)*arctan(1/(a*x))^2 + (arctan(a*x)^3/a + 3*arctan(a
*x)^2*arctan(1/(a*x))/a)*a^2 + 4*a^2*integrate(1/16*x^2*log(a^2*x^2 + 1)^2
/(a^2*x^4 + x^2), x) - 16*a^2*integrate(1/16*x^2*log(a^2*x^2 + 1)/(a^2*x^4
 + x^2), x) - 32*a*integrate(1/16*x*arctan(1/(a*x))/(a^2*x^4 + x^2), x) +
48*integrate(1/16*arctan(1/(a*x))^2/(a^2*x^4 + x^2), x) + 4*integrate(1/16
*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x))*x - 4*arctan2(1, a*x)^2 + log(a^2
*x^2 + 1)^2)/x
```

3.19.8 Giac [F]

$$\int \frac{\cot^{-1}(ax)^2}{x^2}\,dx = \int \frac{\operatorname{arccot}(ax)^2}{x^2}\,dx$$

input
```
integrate(arccot(a*x)^2/x^2,x, algorithm="giac")
```

output
```
integrate(arccot(a*x)^2/x^2, x)
```

3.19.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)^2}{x^2}\,dx = \int \frac{\operatorname{acot}(a\,x)^2}{x^2}\,dx$$

input
```
int(acot(a*x)^2/x^2,x)
```

output
```
int(acot(a*x)^2/x^2, x)
```

3.20 $\int \frac{\cot^{-1}(ax)^2}{x^3}\,dx$

3.20.1 Optimal result

Integrand size = 10, antiderivative size = 59

$$\int \frac{\cot^{-1}(ax)^2}{x^3}\,dx=\frac{a\cot^{-1}(ax)}{x}-\frac{1}{2}a^2\cot^{-1}(ax)^2-\frac{\cot^{-1}(ax)^2}{2x^2}+a^2\log(x)-\frac{1}{2}a^2\log\left(1+a^2x^2\right)$$

output
```
a*arccot(a*x)/x-1/2*a^2*arccot(a*x)^2-1/2*arccot(a*x)^2/x^2+a^2*ln(x)-1/2*
a^2*ln(a^2*x^2+1)
```

3.20.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int \frac{\cot^{-1}(ax)^2}{x^3}\,dx=\frac{a\cot^{-1}(ax)}{x}+\frac{(-1-a^2x^2)\cot^{-1}(ax)^2}{2x^2}+a^2\log(x)-\frac{1}{2}a^2\log\left(1+a^2x^2\right)$$

input
```
Integrate[ArcCot[a*x]^2/x^3,x]
```

output
```
(a*ArcCot[a*x])/x + ((-1 - a^2*x^2)*ArcCot[a*x]^2)/(2*x^2) + a^2*Log[x] -
(a^2*Log[1 + a^2*x^2])/2
```

3.20.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {5362, 5454, 5362, 243, 47, 14, 16, 5420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax)^2}{x^3}dx$$

$$\Big\downarrow \text{5362}$$

$$-a\int \frac{\cot^{-1}(ax)}{x^2\left(a^2x^2+1\right)}dx-\frac{\cot^{-1}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{5454}$$

$$-a\left(\int \frac{\cot^{-1}(ax)}{x^2}dx-a^2\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx\right)-\frac{\cot^{-1}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{5362}$$

$$-a\left(-a\int \frac{1}{x\left(a^2x^2+1\right)}dx+a^2\left(-\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx\right)-\frac{\cot^{-1}(ax)}{x}\right)-\frac{\cot^{-1}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{243}$$

$$-a\left(-\frac{1}{2}a\int \frac{1}{x^2\left(a^2x^2+1\right)}dx^2+a^2\left(-\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx\right)-\frac{\cot^{-1}(ax)}{x}\right)-\frac{\cot^{-1}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{47}$$

$$-a\left(-\frac{1}{2}a\left(\int \frac{1}{x^2}dx^2-a^2\int \frac{1}{a^2x^2+1}dx^2\right)+a^2\left(-\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx\right)-\frac{\cot^{-1}(ax)}{x}\right)-\frac{\cot^{-1}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{14}$$

$$-a\left(-\frac{1}{2}a\left(\log\left(x^2\right)-a^2\int \frac{1}{a^2x^2+1}dx^2\right)+a^2\left(-\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx\right)-\frac{\cot^{-1}(ax)}{x}\right)-\frac{\cot^{-1}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{16}$$

$$-a\left(a^2\left(-\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx\right)-\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{\cot^{-1}(ax)}{x}\right)-\frac{\cot^{-1}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{5420}$$

$$-a\left(-\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)+\frac{1}{2}a\cot^{-1}(ax)^2-\frac{\cot^{-1}(ax)}{x}\right)-\frac{\cot^{-1}(ax)^2}{2x^2}$$

input
```
Int[ArcCot[a*x]^2/x^3,x]
```

output
```
-1/2*ArcCot[a*x]^2/x^2 - a*(-(ArcCot[a*x]/x) + (a*ArcCot[a*x]^2)/2 - (a*(L
og[x^2] - Log[1 + a^2*x^2]))/2)
```

3.20.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5420
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5454
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcCot[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcCot[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.20.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08

method	result
derivativedivides	$a^2\left(-\frac{\operatorname{arccot}(ax)^2}{2a^2x^2}+\frac{\operatorname{arccot}(ax)}{ax}+\operatorname{arccot}(ax)\arctan(ax)+\ln(ax)-\frac{\ln(a^2x^2+1)}{2}+\frac{\arctan(ax)^2}{2}\right)$
default	$a^2\left(-\frac{\operatorname{arccot}(ax)^2}{2a^2x^2}+\frac{\operatorname{arccot}(ax)}{ax}+\operatorname{arccot}(ax)\arctan(ax)+\ln(ax)-\frac{\ln(a^2x^2+1)}{2}+\frac{\arctan(ax)^2}{2}\right)$
parallelrisch	$\frac{-a^2x^2\operatorname{arccot}(ax)^2+2a^2\ln(x)x^2-a^2\ln(a^2x^2+1)x^2+2\operatorname{arccot}(ax)ax-\operatorname{arccot}(ax)^2}{2x^2}$
parts	$-\frac{\operatorname{arccot}(ax)^2}{2x^2}-a^2\left(-\frac{\operatorname{arccot}(ax)}{ax}-\operatorname{arccot}(ax)\arctan(ax)-\ln(ax)+\frac{\ln(a^2x^2+1)}{2}-\frac{\arctan(ax)}{2}\right.$
risch	$\frac{(a^2x^2+1)\ln(iax+1)^2}{8x^2}-\frac{i(-ix^2\ln(-iax+1)a^2-2ax+\pi-i\ln(-iax+1))\ln(iax+1)}{4x^2}-\frac{2ia^2\ln((-\pi a+6ia)x+6+i\pi)\pi}{}$

input
```
int(arccot(a*x)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/2/a^2/x^2*arccot(a*x)^2+1/a/x*arccot(a*x)+arccot(a*x)*arctan(a*x)+
ln(a*x)-1/2*ln(a^2*x^2+1)+1/2*arctan(a*x)^2)
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int \frac{\cot^{-1}(ax)^2}{x^3}\,dx = -\frac{a^2x^2\log\left(a^2x^2+1\right)-2\,a^2x^2\log\left(x\right)-2\,ax\operatorname{arccot}\left(ax\right)+\left(a^2x^2+1\right)\operatorname{arccot}\left(ax\right)^2}{2\,x^2}$$

input
```
integrate(arccot(a*x)^2/x^3,x, algorithm="fricas")
```

output
```
-1/2*(a^2*x^2*log(a^2*x^2 + 1) - 2*a^2*x^2*log(x) - 2*a*x*arccot(a*x) + (a
^2*x^2 + 1)*arccot(a*x)^2)/x^2
```

3.20.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{\cot^{-1}(ax)^2}{x^3}\,dx = a^2\log(x) - \frac{a^2\log(a^2x^2+1)}{2} - \frac{a^2\operatorname{acot}^2(ax)}{2} + \frac{a\operatorname{acot}(ax)}{x} - \frac{\operatorname{acot}^2(ax)}{2x^2}$$

input
```
integrate(acot(a*x)**2/x**3,x)
```

output
```
a**2*log(x) - a**2*log(a**2*x**2 + 1)/2 - a**2*acot(a*x)**2/2 + a*acot(a*x
)/x - acot(a*x)**2/(2*x**2)
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int \frac{\cot^{-1}(ax)^2}{x^3}\,dx = \frac{1}{2}\left(\arctan(ax)^2 - \log\left(a^2x^2+1\right) + 2\log(x)\right)a^2 + \left(a\arctan(ax) + \frac{1}{x}\right)a\operatorname{arccot}(ax) - \frac{\operatorname{arccot}(ax)^2}{2x^2}$$

input
```
integrate(arccot(a*x)^2/x^3,x, algorithm="maxima")
```

output
```
1/2*(arctan(a*x)^2 - log(a^2*x^2 + 1) + 2*log(x))*a^2 + (a*arctan(a*x) + 1
/x)*a*arccot(a*x) - 1/2*arccot(a*x)^2/x^2
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

$$\int \frac{\cot^{-1}(ax)^2}{x^3}\,dx = -\frac{1}{2}\left(\left(\arctan\left(\frac{1}{ax}\right)^2 - \frac{2\arctan\left(\frac{1}{ax}\right)}{ax} + \log\left(\frac{1}{a^2x^2}+1\right)\right)a + \frac{\arctan\left(\frac{1}{ax}\right)^2}{ax^2}\right)a$$

input
```
integrate(arccot(a*x)^2/x^3,x, algorithm="giac")
```

output
```
-1/2*((arctan(1/(a*x))^2 - 2*arctan(1/(a*x))/(a*x) + log(1/(a^2*x^2) + 1))
*a + arctan(1/(a*x))^2/(a*x^2))*a
```

3.20.9 Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int \frac{\cot^{-1}(ax)^2}{x^3}\,dx = a^2\,\ln(x) - \operatorname{acot}(a\,x)^2\left(\frac{a^2}{2} + \frac{1}{2\,x^2}\right) - \frac{a^2\,\ln(a^2\,x^2+1)}{2} + \frac{a\operatorname{acot}(a\,x)}{x}$$

input
```
int(acot(a*x)^2/x^3,x)
```

output
```
a^2*log(x) - acot(a*x)^2*(a^2/2 + 1/(2*x^2)) - (a^2*log(a^2*x^2 + 1))/2 +
(a*acot(a*x))/x
```

3.21 $\int \frac{\cot^{-1}(ax)^2}{x^4}\,dx$

3.21.1 Optimal result

Integrand size = 10, antiderivative size = 113

$$\int \frac{\cot^{-1}(ax)^2}{x^4}\,dx = -\frac{a^2}{3x} + \frac{a\cot^{-1}(ax)}{3x^2} + \frac{1}{3}ia^3\cot^{-1}(ax)^2 - \frac{\cot^{-1}(ax)^2}{3x^3} - \frac{1}{3}a^3\arctan(ax) + \frac{2}{3}a^3\cot^{-1}(ax)\log\left(2 - \frac{2}{1-iax}\right) + \frac{1}{3}ia^3\,\text{PolyLog}\left(2, -1 + \frac{2}{1-iax}\right)$$

output
```
-1/3*a^2/x+1/3*a*arccot(a*x)/x^2+1/3*I*a^3*arccot(a*x)^2-1/3*arccot(a*x)^2
/x^3-1/3*a^3*arctan(a*x)+2/3*a^3*arccot(a*x)*ln(2-2/(1-I*a*x))+1/3*I*a^3*p
olylog(2,-1+2/(1-I*a*x))
```

3.21.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.85

$$\int \frac{\cot^{-1}(ax)^2}{x^4}\,dx = \frac{-a^2x^2 + (-1 - ia^3x^3)\cot^{-1}(ax)^2 + ax\cot^{-1}(ax)\left(1 + a^2x^2 + 2a^2x^2\log\left(1 + e^{2i\cot^{-1}(ax)}\right)\right) - ia^3x^3\,\text{PolyL}}{3x^3}$$

input
```
Integrate[ArcCot[a*x]^2/x^4,x]
```

output
```
(-(a^2*x^2) + (-1 - I*a^3*x^3)*ArcCot[a*x]^2 + a*x*ArcCot[a*x]*(1 + a^2*x^
2 + 2*a^2*x^2*Log[1 + E^((2*I)*ArcCot[a*x])]) - I*a^3*x^3*PolyLog[2, -E^((
2*I)*ArcCot[a*x])])/(3*x^3)
```

3.21.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {5362, 5454, 5362, 264, 216, 5460, 5404, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax)^2}{x^4}\,dx$$

$$\Big\downarrow \text{5362}$$

$$-\frac{2}{3}a\int \frac{\cot^{-1}(ax)}{x^3\left(a^2x^2+1\right)}dx-\frac{\cot^{-1}(ax)^2}{3x^3}$$

$$\Big\downarrow \text{5454}$$

$$-\frac{2}{3}a\left(\int \frac{\cot^{-1}(ax)}{x^3}dx-a^2\int \frac{\cot^{-1}(ax)}{x\left(a^2x^2+1\right)}dx\right)-\frac{\cot^{-1}(ax)^2}{3x^3}$$

$$\Big\downarrow \text{5362}$$

$$-\frac{2}{3}a\left(-\frac{1}{2}a\int \frac{1}{x^2\left(a^2x^2+1\right)}dx+a^2\left(-\int \frac{\cot^{-1}(ax)}{x\left(a^2x^2+1\right)}dx\right)-\frac{\cot^{-1}(ax)}{2x^2}\right)-\frac{\cot^{-1}(ax)^2}{3x^3}$$

$$\Big\downarrow \text{264}$$

$$-\frac{2}{3}a\left(-\frac{1}{2}a\left(a^2\left(-\int \frac{1}{a^2x^2+1}dx\right)-\frac{1}{x}\right)+a^2\left(-\int \frac{\cot^{-1}(ax)}{x\left(a^2x^2+1\right)}dx\right)-\frac{\cot^{-1}(ax)}{2x^2}\right)-\frac{\cot^{-1}(ax)^2}{3x^3}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2}{3}a\left(a^2\left(-\int \frac{\cot^{-1}(ax)}{x\left(a^2x^2+1\right)}dx\right)-\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)-\frac{\cot^{-1}(ax)}{2x^2}\right)-\frac{\cot^{-1}(ax)^2}{3x^3}$$

$$\Big\downarrow \text{5460}$$

$$-\frac{\cot^{-1}(ax)^2}{3x^3}-\frac{2}{3}a\left(-\left(a^2\left(i\int \frac{\cot^{-1}(ax)}{x(ax+i)}dx+\frac{1}{2}i\cot^{-1}(ax)^2\right)\right)-\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)-\frac{\cot^{-1}(ax)}{2x^2}\right)$$

$$\Big\downarrow \text{5404}$$

$$-\frac{\cot^{-1}(ax)^2}{3x^3}-$$
$$\frac{2}{3}a\left(-\left(a^2\left(i\left(-ia\int\frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\log\left(2-\frac{2}{1-iax}\right)\cot^{-1}(ax)\right)+\frac{1}{2}i\cot^{-1}(ax)^2\right)\right)-\frac{1}{2}a\left(-a\,\text{arcta}\right.\right.$$

$\Big\downarrow$ 2897

$$-\frac{\cot^{-1}(ax)^2}{3x^3}-$$
$$\frac{2}{3}a\left(-\left(a^2\left(i\left(\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)-i\log\left(2-\frac{2}{1-iax}\right)\cot^{-1}(ax)\right)+\frac{1}{2}i\cot^{-1}(ax)^2\right)\right)-\frac{1}{2}a\left(-a\,\text{arct}\right.\right.$$

input
```
Int[ArcCot[a*x]^2/x^4,x]
```

output
```
-1/3*ArcCot[a*x]^2/x^3 - (2*a*(-1/2*ArcCot[a*x]/x^2 - (a*(-x^(-1) - a*ArcT
an[a*x]))/2 - a^2*((I/2)*ArcCot[a*x]^2 + I*((-I)*ArcCot[a*x]*Log[2 - 2/(1
- I*a*x)] + PolyLog[2, -1 + 2/(1 - I*a*x)]/2))))/3
```

3.21.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5404
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcCot[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] + Si
mp[b*c*(p/d)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5454
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcCot[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcCot[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5460
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*d*(p + 1))), x] + Simp[
I/d   Int[(a + b*ArcCot[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.21.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(95) = 190$.

Time = 0.72 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.22

method	result
parts	$-\frac{\operatorname{arccot}(ax)^2}{3x^3}-\frac{2a^3\left(\frac{\operatorname{arccot}(ax)\ln\left(a^2x^2+1\right)}{2}-\frac{\operatorname{arccot}(ax)}{2a^2x^2}-\operatorname{arccot}(ax)\ln(ax)-\frac{i\left(\ln(ax-i)\ln\left(a^2x^2+1\right)-\frac{\ln(ax-i)^2}{2}-\operatorname{dilog}\right.}{4}\right.}{}$
derivativedivides	$a^3\left(-\frac{\operatorname{arccot}(ax)^2}{3a^3x^3}-\frac{\operatorname{arccot}(ax)\ln(a^2x^2+1)}{3}+\frac{\operatorname{arccot}(ax)}{3a^2x^2}+\frac{2\operatorname{arccot}(ax)\ln(ax)}{3}+\frac{i\left(\ln(ax-i)\ln(a^2x^2+1)-\right.}{}\right.$
default	$a^3\left(-\frac{\operatorname{arccot}(ax)^2}{3a^3x^3}-\frac{\operatorname{arccot}(ax)\ln(a^2x^2+1)}{3}+\frac{\operatorname{arccot}(ax)}{3a^2x^2}+\frac{2\operatorname{arccot}(ax)\ln(ax)}{3}+\frac{i\left(\ln(ax-i)\ln(a^2x^2+1)-\right.}{}\right.$

input
```
int(arccot(a*x)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*arccot(a*x)^2/x^3-2/3*a^3*(1/2*arccot(a*x)*ln(a^2*x^2+1)-1/2*arccot(a
*x)/a^2/x^2-arccot(a*x)*ln(a*x)-1/4*I*(ln(a*x-I)*ln(a^2*x^2+1)-1/2*ln(a*x-
I)^2-dilog(-1/2*I*(I+a*x))-ln(a*x-I)*ln(-1/2*I*(I+a*x)))+1/4*I*(ln(I+a*x)*
ln(a^2*x^2+1)-1/2*ln(I+a*x)^2-dilog(1/2*I*(a*x-I))-ln(I+a*x)*ln(1/2*I*(a*x
-I)))+1/2*arctan(a*x)+1/2/a/x+1/2*I*ln(a*x)*ln(1+I*a*x)-1/2*I*ln(a*x)*ln(1
-I*a*x)+1/2*I*dilog(1+I*a*x)-1/2*I*dilog(1-I*a*x))
```

3.21.5 Fricas [F]

$$\int \frac{\cot^{-1}(ax)^2}{x^4}\,dx=\int \frac{\operatorname{arccot}(ax)^2}{x^4}\,dx$$

input
```
integrate(arccot(a*x)^2/x^4,x, algorithm="fricas")
```

output
```
integral(arccot(a*x)^2/x^4, x)
```

3.21.6 Sympy [F]

$$\int \frac{\cot^{-1}(ax)^2}{x^4}\,dx = \int \frac{\operatorname{acot}^2(ax)}{x^4}\,dx$$

input
```
integrate(acot(a*x)**2/x**4,x)
```

output
```
Integral(acot(a*x)**2/x**4, x)
```

3.21.7 Maxima [F]

$$\int \frac{\cot^{-1}(ax)^2}{x^4}\,dx = \int \frac{\operatorname{arccot}(ax)^2}{x^4}\,dx$$

input
```
integrate(arccot(a*x)^2/x^4,x, algorithm="maxima")
```

output
```
1/48*(48*x^3*integrate(1/48*(36*a^2*x^2*arctan2(1, a*x)^2 - 4*a^2*x^2*log(
a^2*x^2 + 1) - 8*a*x*arctan2(1, a*x) + 3*(a^2*x^2 + 1)*log(a^2*x^2 + 1)^2
+ 36*arctan2(1, a*x)^2)/(a^2*x^6 + x^4), x) - 4*arctan2(1, a*x)^2 + log(a^
2*x^2 + 1)^2)/x^3
```

3.21.8 Giac [F]

$$\int \frac{\cot^{-1}(ax)^2}{x^4}\,dx = \int \frac{\operatorname{arccot}(ax)^2}{x^4}\,dx$$

input
```
integrate(arccot(a*x)^2/x^4,x, algorithm="giac")
```

output
```
integrate(arccot(a*x)^2/x^4, x)
```

3.21.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)^2}{x^4}\,dx = \int \frac{\operatorname{acot}(a\,x)^2}{x^4}\,dx$$

input
```
int(acot(a*x)^2/x^4,x)
```

output
```
int(acot(a*x)^2/x^4, x)
```

3.22 $\int \frac{\cot^{-1}(ax)^2}{x^5} dx$

3.22.1 Optimal result

Integrand size = 10, antiderivative size = 89

$$\int \frac{\cot^{-1}(ax)^2}{x^5} dx = -\frac{a^2}{12x^2} + \frac{a\cot^{-1}(ax)}{6x^3} - \frac{a^3\cot^{-1}(ax)}{2x} + \frac{1}{4}a^4\cot^{-1}(ax)^2 - \frac{\cot^{-1}(ax)^2}{4x^4} - \frac{2}{3}a^4\log(x) + \frac{1}{3}a^4\log\left(1+a^2x^2\right)$$

output
```
-1/12*a^2/x^2+1/6*a*arccot(a*x)/x^3-1/2*a^3*arccot(a*x)/x+1/4*a^4*arccot(a
*x)^2-1/4*arccot(a*x)^2/x^4-2/3*a^4*ln(x)+1/3*a^4*ln(a^2*x^2+1)
```

3.22.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int \frac{\cot^{-1}(ax)^2}{x^5} dx = -\frac{a^2}{12x^2} - \frac{a(-1+3a^2x^2)\cot^{-1}(ax)}{6x^3} + \frac{(-1+a^4x^4)\cot^{-1}(ax)^2}{4x^4} - \frac{2}{3}a^4\log(x) + \frac{1}{3}a^4\log\left(1+a^2x^2\right)$$

input
```
Integrate[ArcCot[a*x]^2/x^5,x]
```

output
```
-1/12*a^2/x^2 - (a*(-1 + 3*a^2*x^2)*ArcCot[a*x])/(6*x^3) + ((-1 + a^4*x^4)
*ArcCot[a*x]^2)/(4*x^4) - (2*a^4*Log[x])/3 + (a^4*Log[1 + a^2*x^2])/3
```

3.22.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.27, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.300, Rules used = {5362, 5454, 5362, 243, 54, 2009, 5454, 5362, 243, 47, 14, 16, 5420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax)^2}{x^5}dx$$

$$\Big\downarrow \text{5362}$$

$$-\frac{1}{2}a\int \frac{\cot^{-1}(ax)}{x^4(a^2x^2+1)}dx-\frac{\cot^{-1}(ax)^2}{4x^4}$$

$$\Big\downarrow \text{5454}$$

$$-\frac{1}{2}a\left(\int \frac{\cot^{-1}(ax)}{x^4}dx-a^2\int \frac{\cot^{-1}(ax)}{x^2(a^2x^2+1)}dx\right)-\frac{\cot^{-1}(ax)^2}{4x^4}$$

$$\Big\downarrow \text{5362}$$

$$-\frac{1}{2}a\left(a^2\left(-\int \frac{\cot^{-1}(ax)}{x^2(a^2x^2+1)}dx\right)-\frac{1}{3}a\int \frac{1}{x^3(a^2x^2+1)}dx-\frac{\cot^{-1}(ax)}{3x^3}\right)-\frac{\cot^{-1}(ax)^2}{4x^4}$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{2}a\left(a^2\left(-\int \frac{\cot^{-1}(ax)}{x^2(a^2x^2+1)}dx\right)-\frac{1}{6}a\int \frac{1}{x^4(a^2x^2+1)}dx^2-\frac{\cot^{-1}(ax)}{3x^3}\right)-\frac{\cot^{-1}(ax)^2}{4x^4}$$

$$\Big\downarrow \text{54}$$

$$-\frac{1}{2}a\left(a^2\left(-\int \frac{\cot^{-1}(ax)}{x^2(a^2x^2+1)}dx\right)-\frac{1}{6}a\int \left(\frac{a^4}{a^2x^2+1}-\frac{a^2}{x^2}+\frac{1}{x^4}\right)dx^2-\frac{\cot^{-1}(ax)}{3x^3}\right)-\frac{\cot^{-1}(ax)^2}{4x^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}a\left(a^2\left(-\int \frac{\cot^{-1}(ax)}{x^2(a^2x^2+1)}dx\right)-\frac{1}{6}a\left(a^2(-\log(x^2))+a^2\log(a^2x^2+1)-\frac{1}{x^2}\right)-\frac{\cot^{-1}(ax)}{3x^3}\right)-\frac{\cot^{-1}(ax)^2}{4x^4}$$

$$\Big\downarrow \text{5454}$$

$$-\frac{1}{2}a\left(-\left(a^2\left(\int \frac{\cot^{-1}(ax)}{x^2}dx-a^2\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx\right)\right)-\frac{1}{6}a\left(a^2(-\log(x^2))+a^2\log(a^2x^2+1)-\frac{1}{x^2}\right)-\frac{\cot^{-1}(}{3x^3}\right.$$
$$\frac{\cot^{-1}(ax)^2}{4x^4}$$

$\big\downarrow$ 5362

$$-\frac{1}{2}a\left(-\left(a^2\left(-a\int\frac{1}{x(a^2x^2+1)}dx+a^2\left(-\int\frac{\cot^{-1}(ax)}{a^2x^2+1}dx\right)-\frac{\cot^{-1}(ax)}{x}\right)\right)-\frac{1}{6}a\left(a^2(-\log(x^2))+a^2\log(a^2\right.\right.$$
$$\frac{\cot^{-1}(ax)^2}{4x^4}$$

$\big\downarrow$ 243

$$-\frac{1}{2}a\left(-\left(a^2\left(-\frac{1}{2}a\int\frac{1}{x^2(a^2x^2+1)}dx^2+a^2\left(-\int\frac{\cot^{-1}(ax)}{a^2x^2+1}dx\right)-\frac{\cot^{-1}(ax)}{x}\right)\right)-\frac{1}{6}a\left(a^2(-\log(x^2))+a^2\log\right.\right.$$
$$\frac{\cot^{-1}(ax)^2}{4x^4}$$

$\big\downarrow$ 47

$$-\frac{1}{2}a\left(-\left(a^2\left(-\frac{1}{2}a\left(\int\frac{1}{x^2}dx^2-a^2\int\frac{1}{a^2x^2+1}dx^2\right)+a^2\left(-\int\frac{\cot^{-1}(ax)}{a^2x^2+1}dx\right)-\frac{\cot^{-1}(ax)}{x}\right)\right)-\frac{1}{6}a\left(a^2(-\log\right.\right.$$
$$\frac{\cot^{-1}(ax)^2}{4x^4}$$

$\big\downarrow$ 14

$$-\frac{1}{2}a\left(-\left(a^2\left(-\frac{1}{2}a\left(\log(x^2)-a^2\int\frac{1}{a^2x^2+1}dx^2\right)+a^2\left(-\int\frac{\cot^{-1}(ax)}{a^2x^2+1}dx\right)-\frac{\cot^{-1}(ax)}{x}\right)\right)-\frac{1}{6}a\left(a^2(-\log(x\right.\right.$$
$$\frac{\cot^{-1}(ax)^2}{4x^4}$$

$\big\downarrow$ 16

$$-\frac{1}{2}a\left(-\left(a^2\left(a^2\left(-\int\frac{\cot^{-1}(ax)}{a^2x^2+1}dx\right)-\frac{1}{2}a(\log(x^2)-\log(a^2x^2+1))-\frac{\cot^{-1}(ax)}{x}\right)\right)-\frac{1}{6}a\left(a^2(-\log(x^2))+a\right.\right.$$
$$\frac{\cot^{-1}(ax)^2}{4x^4}$$

$\big\downarrow$ 5420

$$-\frac{1}{2}a\left(-\frac{1}{6}a\left(a^2(-\log(x^2))+a^2\log(a^2x^2+1)-\frac{1}{x^2}\right)-\left(a^2\left(-\frac{1}{2}a(\log(x^2)-\log(a^2x^2+1))+\frac{1}{2}a\cot^{-1}(ax)^2\right.\right.\right.$$
$$\frac{\cot^{-1}(ax)^2}{4x^4}$$

input
```
Int[ArcCot[a*x]^2/x^5,x]
```

3.22. $\int\frac{\cot^{-1}(ax)^2}{x^5}\,dx$

output
```
-1/4*ArcCot[a*x]^2/x^4 - (a*(-1/3*ArcCot[a*x]/x^3 - a^2*(-(ArcCot[a*x]/x)
+ (a*ArcCot[a*x]^2)/2 - (a*(Log[x^2] - Log[1 + a^2*x^2]))/2) - (a*(-x^(-2)
 - a^2*Log[x^2] + a^2*Log[1 + a^2*x^2]))/6))/2
```

3.22.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5420
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5454
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcCot[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcCot[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.22.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

method	result
parts	$-\frac{\operatorname{arccot}(ax)^2}{4x^4}-\frac{a^4\left(-\frac{\operatorname{arccot}(ax)}{3a^3x^3}+\frac{\operatorname{arccot}(ax)}{ax}+\operatorname{arccot}(ax)\arctan(ax)+\frac{1}{6a^2x^2}+\frac{4\ln(ax)}{3}-\frac{2\ln\left(a^2x^2+1\right)}{3}+\frac{\arctan(ax)^2}{2}\right)}{2}$
derivativedivides	$a^4\left(-\frac{\operatorname{arccot}(ax)^2}{4a^4x^4}+\frac{\operatorname{arccot}(ax)}{6a^3x^3}-\frac{\operatorname{arccot}(ax)}{2ax}-\frac{\operatorname{arccot}(ax)\arctan(ax)}{2}-\frac{1}{12a^2x^2}-\frac{2\ln(ax)}{3}+\frac{\ln(a^2x^2+1)}{3}\right.$
default	$a^4\left(-\frac{\operatorname{arccot}(ax)^2}{4a^4x^4}+\frac{\operatorname{arccot}(ax)}{6a^3x^3}-\frac{\operatorname{arccot}(ax)}{2ax}-\frac{\operatorname{arccot}(ax)\arctan(ax)}{2}-\frac{1}{12a^2x^2}-\frac{2\ln(ax)}{3}+\frac{\ln(a^2x^2+1)}{3}\right.$
parallelrisch	$-\frac{-3a^4x^4\operatorname{arccot}(ax)^2+8a^4\ln(x)x^4-4a^4\ln(a^2x^2+1)x^4-a^4x^4+6a^3x^3\operatorname{arccot}(ax)+a^2x^2-2\operatorname{arccot}(ax)ax+3\operatorname{arccot}(ax}{12x^4}$
risch	$-\frac{(a^4x^4-1)\ln(iax+1)^2}{16x^4}-\frac{i(3ia^4\ln(-iax+1)x^4+6a^3x^3-3i\ln(-iax+1)-2ax+3\pi)\ln(iax+1)}{24x^4}-\frac{-6ia^4\ln((-\pi a+8}{}$

input
```
int(arccot(a*x)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*arccot(a*x)^2/x^4-1/2*a^4*(-1/3/a^3/x^3*arccot(a*x)+1/a/x*arccot(a*x)
+arccot(a*x)*arctan(a*x)+1/6/a^2/x^2+4/3*ln(a*x)-2/3*ln(a^2*x^2+1)+1/2*arc
tan(a*x)^2)
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.88

$$\int\frac{\cot^{-1}(ax)^2}{x^5}\,dx$$
$$=\frac{4\,a^4x^4\log\left(a^2x^2+1\right)-8\,a^4x^4\log\left(x\right)-a^2x^2+3\left(a^4x^4-1\right)\operatorname{arccot}\left(ax\right)^2-2\left(3\,a^3x^3-ax\right)\operatorname{arccot}\left(ax\right)}{12\,x^4}$$

input
```
integrate(arccot(a*x)^2/x^5,x, algorithm="fricas")
```

output
```
1/12*(4*a^4*x^4*log(a^2*x^2 + 1) - 8*a^4*x^4*log(x) - a^2*x^2 + 3*(a^4*x^4
 - 1)*arccot(a*x)^2 - 2*(3*a^3*x^3 - a*x)*arccot(a*x))/x^4
```

3.22.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.90

$$\int \frac{\cot^{-1}(ax)^2}{x^5} dx = -\frac{2a^4 \log(x)}{3} + \frac{a^4 \log(a^2x^2+1)}{3} + \frac{a^4 \operatorname{acot}^2(ax)}{4} - \frac{a^3 \operatorname{acot}(ax)}{2x} - \frac{a^2}{12x^2} + \frac{a \operatorname{acot}(ax)}{6x^3} - \frac{\operatorname{acot}^2(ax)}{4x^4}$$

input
```
integrate(acot(a*x)**2/x**5,x)
```

output
```
-2*a**4*log(x)/3 + a**4*log(a**2*x**2 + 1)/3 + a**4*acot(a*x)**2/4 - a**3*
acot(a*x)/(2*x) - a**2/(12*x**2) + a*acot(a*x)/(6*x**3) - acot(a*x)**2/(4*
x**4)
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07

$$\int \frac{\cot^{-1}(ax)^2}{x^5} dx = -\frac{1}{6}\left(3a^3 \arctan(ax) + \frac{3a^2x^2-1}{x^3}\right)a \operatorname{arccot}(ax) - \frac{(3a^2x^2 \arctan(ax)^2 - 4a^2x^2 \log(a^2x^2+1) + 8a^2x^2 \log(x) + 1)a^2}{12x^2} - \frac{\operatorname{arccot}(ax)^2}{4x^4}$$

input
```
integrate(arccot(a*x)^2/x^5,x, algorithm="maxima")
```

output
```
-1/6*(3*a^3*arctan(a*x) + (3*a^2*x^2 - 1)/x^3)*a*arccot(a*x) - 1/12*(3*a^2
*x^2*arctan(a*x)^2 - 4*a^2*x^2*log(a^2*x^2 + 1) + 8*a^2*x^2*log(x) + 1)*a^
2/x^2 - 1/4*arccot(a*x)^2/x^4
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.02

$$\int \frac{\cot^{-1}(ax)^2}{x^5}\,dx$$
$$= \frac{1}{12}\left(\left(3\arctan\left(\frac{1}{ax}\right)^2 - \frac{6\arctan\left(\frac{1}{ax}\right)}{ax} - \frac{1}{a^2x^2} + \frac{2\arctan\left(\frac{1}{ax}\right)}{a^3x^3} + 4\log\left(\frac{1}{a^2x^2}+1\right)\right)a^3 - \frac{3\arctan\left(\frac{1}{ax}\right)^2}{ax^4}\right)$$

input
```
integrate(arccot(a*x)^2/x^5,x, algorithm="giac")
```

output
```
1/12*((3*arctan(1/(a*x))^2 - 6*arctan(1/(a*x))/(a*x) - 1/(a^2*x^2) + 2*arc
tan(1/(a*x))/(a^3*x^3) + 4*log(1/(a^2*x^2) + 1))*a^3 - 3*arctan(1/(a*x))^2
/(a*x^4))*a
```

3.22.9 Mupad [B] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.82

$$\int \frac{\cot^{-1}(ax)^2}{x^5}\,dx = \mathrm{acot}(a\,x)^2\left(\frac{a^4}{4} - \frac{1}{4\,x^4}\right) - \frac{2\,a^4\,\ln(x)}{3} + \frac{a^4\,\ln(a^2\,x^2+1)}{3} - \frac{a^2}{12\,x^2} - \frac{a^2\,\mathrm{acot}(a\,x)\,\left(\frac{a\,x^2}{2} - \frac{1}{6\,a}\right)}{x^3}$$

input
```
int(acot(a*x)^2/x^5,x)
```

output
```
acot(a*x)^2*(a^4/4 - 1/(4*x^4)) - (2*a^4*log(x))/3 + (a^4*log(a^2*x^2 + 1)
)/3 - a^2/(12*x^2) - (a^2*acot(a*x)*((a*x^2)/2 - 1/(6*a)))/x^3
```

3.23 $\int x^5 \cot^{-1}(ax)^3 \, dx$

3.23.1 Optimal result

Integrand size = 10, antiderivative size = 194

$$\int x^5 \cot^{-1}(ax)^3 \, dx = -\frac{19x}{60a^5} + \frac{x^3}{60a^3} - \frac{4x^2 \cot^{-1}(ax)}{15a^4} + \frac{x^4 \cot^{-1}(ax)}{20a^2} + \frac{23i \cot^{-1}(ax)^2}{30a^6} + \frac{x \cot^{-1}(ax)^2}{2a^5} - \frac{x^3 \cot^{-1}(ax)^2}{6a^3} + \frac{x^5 \cot^{-1}(ax)^2}{10a} + \frac{\cot^{-1}(ax)^3}{6a^6} + \frac{1}{6}x^6 \cot^{-1}(ax)^3 + \frac{19 \arctan(ax)}{60a^6} - \frac{23 \cot^{-1}(ax) \log\left(\frac{2}{1+iax}\right)}{15a^6} + \frac{23i \operatorname{PolyLog}\left(2, 1 - \frac{2}{1+iax}\right)}{30a^6}$$

output
```
-19/60*x/a^5+1/60*x^3/a^3-4/15*x^2*arccot(a*x)/a^4+1/20*x^4*arccot(a*x)/a^
2+23/30*I*arccot(a*x)^2/a^6+1/2*x*arccot(a*x)^2/a^5-1/6*x^3*arccot(a*x)^2/
a^3+1/10*x^5*arccot(a*x)^2/a+1/6*arccot(a*x)^3/a^6+1/6*x^6*arccot(a*x)^3+1
9/60*arctan(a*x)/a^6-23/15*arccot(a*x)*ln(2/(1+I*a*x))/a^6+23/30*I*polylog
(2,1-2/(1+I*a*x))/a^6
```

3.23.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.64

$$\int x^5 \cot^{-1}(ax)^3\,dx = \frac{ax(-19+a^2x^2)+2(23i+15ax-5a^3x^3+3a^5x^5)\cot^{-1}(ax)^2+10(1+a^6x^6)\cot^{-1}(ax)^3+\cot^{-1}(ax)\left(-\right.}{60a^6}$$

input
```
Integrate[x^5*ArcCot[a*x]^3,x]
```

output
```
(a*x*(-19 + a^2*x^2) + 2*(23*I + 15*a*x - 5*a^3*x^3 + 3*a^5*x^5)*ArcCot[a*
x]^2 + 10*(1 + a^6*x^6)*ArcCot[a*x]^3 + ArcCot[a*x]*(-19 - 16*a^2*x^2 + 3*
a^4*x^4 - 92*Log[1 - E^((2*I)*ArcCot[a*x])]) + (46*I)*PolyLog[2, E^((2*I)*
ArcCot[a*x])])/(60*a^6)
```

3.23.3 Rubi [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 447 vs. 2(194) = 388.

Time = 2.66 (sec) , antiderivative size = 447, normalized size of antiderivative = 2.30, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.700, Rules used = {5362, 5452, 5362, 5452, 5362, 254, 2009, 5452, 5346, 5362, 262, 216, 5420, 5456, 5380, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \cot^{-1}(ax)^3\,dx$$

$$\Big\downarrow \; 5362$$

$$\frac{1}{2}a\int \frac{x^6\cot^{-1}(ax)^2}{a^2x^2+1}dx+\frac{1}{6}x^6\cot^{-1}(ax)^3$$

$$\Big\downarrow \; 5452$$

$$\frac{1}{2}a\left(\frac{\int x^4\cot^{-1}(ax)^2dx}{a^2}-\frac{\int \frac{x^4\cot^{-1}(ax)^2}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{6}x^6\cot^{-1}(ax)^3$$

$$\Big\downarrow \; 5362$$

$$\frac{1}{2}a\left(\frac{\frac{2}{5}a\int \frac{x^5\cot^{-1}(ax)}{a^2x^2+1}dx+\frac{1}{5}x^5\cot^{-1}(ax)^2}{a^2}-\frac{\int \frac{x^4\cot^{-1}(ax)^2}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{6}x^6\cot^{-1}(ax)^3$$

$\downarrow$ 5452

$$\frac{1}{2}a\left(\frac{\frac{2}{5}a\left(\frac{\int x^3\cot^{-1}(ax)dx}{a^2}-\frac{\int\frac{x^3\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}(ax)^2}{a^2}-\frac{\frac{\int x^2\cot^{-1}(ax)^2dx}{a^2}-\frac{\int\frac{x^2\cot^{-1}(ax)^2}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{6}x^6\cot^{-1}(ax)^3$$

$\downarrow$ 5362

$$\frac{1}{2}a\left(\frac{\frac{2}{5}a\left(\frac{\frac{1}{4}a\int\frac{x^4}{a^2x^2+1}dx+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x^3\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}(ax)^2}{a^2}-\frac{\frac{\frac{2}{3}a\int\frac{x^3\cot^{-1}(ax)}{a^2x^2+1}dx+\frac{1}{3}x^3\cot^{-1}(ax)^2}{a^2}-\int\frac{x^2\cdots}{a\cdots}}{a^2}\cdots\right.$$

$$\frac{1}{6}x^6\cot^{-1}(ax)^3$$

$\downarrow$ 254

$$\frac{1}{2}a\left(\frac{\frac{2}{5}a\left(\frac{\frac{1}{4}a\int\left(\frac{x^2}{a^2}+\frac{1}{a^4\left(a^2x^2+1\right)}-\frac{1}{a^4}\right)dx+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x^3\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}(ax)^2}{a^2}-\frac{\frac{\frac{2}{3}a\int\frac{x^3\cot^{-1}(ax)}{a^2x^2+1}dx+\frac{1}{3}x^3\cot\cdots}{a^2}}{a\cdots}\cdots\right.$$

$$\frac{1}{6}x^6\cot^{-1}(ax)^3$$

$\downarrow$ 2009

$$\frac{1}{2}a\left(\frac{\frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x^3\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}(ax)^2}{a^2}-\frac{\frac{\frac{2}{3}a\int\frac{x^3\cot^{-1}(ax)}{a^2x^2+1}dx+\frac{1}{3}x^3\cot^{-1}(ax)^2\cdots}{a^2}}{a^2}\cdots\right.$$

$$\frac{1}{6}x^6\cot^{-1}(ax)^3$$

$\downarrow$ 5452

$$\frac{1}{2}a\left(\frac{\frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\frac{\int x\cot^{-1}(ax)dx}{a^2}-\frac{\int\frac{x\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}(ax)^2}{a^2}-\frac{\frac{2}{3}a\left(\frac{\int x\cot^{-1}(ax)dx}{a^2}-\right.}{}\right.$$

$$\frac{1}{6}x^6\cot^{-1}(ax)^3$$

$\downarrow$ 5346

$$\frac{1}{2}a\left(\frac{\frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\frac{\int x\cot^{-1}(ax)dx}{a^2}-\frac{\int\frac{x\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}(ax)^2}{a^2}-\frac{\frac{2}{3}a\left(\frac{\int x\cot^{-1}(ax)dx}{a^2}-\right.}{}\right.$$

$$\frac{1}{6}x^6\cot^{-1}(ax)^3$$

$\downarrow$ 5362

$$\frac{1}{2}a\left(\frac{\frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{2}a\int\frac{x^2}{a^2x^2+1}dx+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}(ax)^2}{a^2}-\frac{\frac{2}{3}a\left(\right.}{}\right.$$

$$\frac{1}{6}x^6\cot^{-1}(ax)^3$$

$\downarrow$ 262

3.23. $\int x^5\cot^{-1}(ax)^3\,dx$

$$\frac{1}{2}a\left(\frac{\frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\int\frac{1}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}(ax)^2}{a^2}-\right.$$

$$\frac{1}{6}x^6\cot^{-1}(ax)^3$$

$\downarrow$ 216

$$\frac{1}{2}a\left(\frac{\frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}(ax)^2}{a^2}-\right.$$

$$\frac{1}{6}x^6\cot^{-1}(ax)^3$$

$\downarrow$ 5420

$$\frac{1}{2}a\left(\frac{\frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}(ax)^2}{a^2}-\right.$$

$$\frac{1}{6}x^6\cot^{-1}(ax)^3$$

$\downarrow$ 5456

3.23. $\int x^5\cot^{-1}(ax)^3\,dx$

$$\frac{1}{6}x^6\cot^{-1}(ax)^3+\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\cot^{-1}(ax)^2+\frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\int\frac{\cot^{-1}(a\cdots}{i-ax}\cdots}{a}\cdots}{a^2}\cdots}{a^2}\cdots\right.}{a^2}\cdots\right.$$

$\downarrow$ 5380

$$\frac{1}{6}x^6\cot^{-1}(ax)^3+\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\cot^{-1}(ax)^2+\frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\int\frac{\log\left(\frac{2}{iax\cdots}\cdots\right)}{a^2x^2+\cdots}\cdots}{\cdots}\cdots}{a^2}\cdots\right.}{a^2}\cdots\right.$$

$\downarrow$ 2849

$$\frac{1}{6}x^6\cot^{-1}(ax)^3+\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\cot^{-1}(ax)^2+\frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\log\left(\frac{2}{1+iax\cdots}\cdots\right)\cdots}{\cdots}\cdots}{a^2}\cdots\right.}{a^2}\cdots\right.$$

$\downarrow$ 2752

3.23. $\int x^5\cot^{-1}(ax)^3\,dx$

$$\frac{1}{6}x^6\cot^{-1}(ax)^3+$$

$$\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\cot^{-1}(ax)^2+\frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\log\left(\frac{2}{1+iax}\right.}{}}{a^2}}{}\right.}{a^2}\right.$$

input
```
Int[x^5*ArcCot[a*x]^3,x]
```

output
```
(x^6*ArcCot[a*x]^3)/6 + (a*(((x^5*ArcCot[a*x]^2)/5 + (2*a*(((x^4*ArcCot[a*
x])/4 + (a*(-(x/a^4) + x^3/(3*a^2) + ArcTan[a*x]/a^5))/4)/a^2 - (((x^2*Arc
Cot[a*x])/2 + (a*(x/a^2 - ArcTan[a*x]/a^3))/2)/a^2 - (((I/2)*ArcCot[a*x]^2
)/a^2 - ((ArcCot[a*x]*Log[2/(1 + I*a*x)])/a - ((I/2)*PolyLog[2, 1 - 2/(1 +
 I*a*x)])/a)/a)/a^2)/a^2))/5)/a^2 - (((x^3*ArcCot[a*x]^2)/3 + (2*a*(((x^2*
ArcCot[a*x])/2 + (a*(x/a^2 - ArcTan[a*x]/a^3))/2)/a^2 - (((I/2)*ArcCot[a*x
]^2)/a^2 - ((ArcCot[a*x]*Log[2/(1 + I*a*x)])/a - ((I/2)*PolyLog[2, 1 - 2/(
1 + I*a*x)])/a)/a)/a^2))/3)/a^2 - (ArcCot[a*x]^3/(3*a^3) + (x*ArcCot[a*x]^
2 + 2*a*(((I/2)*ArcCot[a*x]^2)/a^2 - ((ArcCot[a*x]*Log[2/(1 + I*a*x)])/a -
 ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/a))/a^2)/a^2)/a^2))/2
```

3.23.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5380
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Simp[b*c*(
p/e)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5420
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5452
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

3.23. $\int x^5 \cot^{-1}(ax)^3 \, dx$

rule 5456
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Simp[
1/(c*d)   Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.23.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1103 vs. 2(164) = 328.

Time = 29.34 (sec) , antiderivative size = 1104, normalized size of antiderivative = 5.69

method	result	size
risch	Expression too large to display	1104
parts	Expression too large to display	2453
derivativedivides	Expression too large to display	2455
default	Expression too large to display	2455

input
```
int(x^5*arccot(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
23/120*I/a^6*Pi^2+8929/57600*I/a^6*ln(a^2*x^2+1)-1/4*I/a^5*Pi*ln(1-I*a*x)*
x-61/1920*I/a^2*ln(1-I*a*x)*x^4+151/960*I/a^4*ln(1-I*a*x)*x^2+37/480*I/a^6
*Pi*arctan(a*x)+1/40/a^2*Pi*x^4+1/40/a*Pi^2*x^5+1/12*I/a^3*Pi*ln(1-I*a*x)*
x^3+23/30*I/a^6*dilog(1/2-1/2*I*a*x)-1291/3600*I/a^6*ln(1-I*a*x)-1/8/a^6*P
i^2*arctan(a*x)+331/960/a^6*Pi*ln(a^2*x^2+1)-1/240*(-15*I*x^6*ln(1-I*a*x)*
a^6+15*Pi*a^6*x^6+6*a^5*x^5-10*a^3*x^3-15*I*ln(1-I*a*x)+30*a*x+15*Pi-46*I)
/a^6*ln(1+I*a*x)^2+1/48/a^3*ln(1-I*a*x)^2*x^3+7/1440/a^3*ln(1-I*a*x)*x^3-1
/16/a^5*ln(1-I*a*x)^2*x-1/1200/a*ln(1-I*a*x)*x^5-37/480/a^5*ln(1-I*a*x)*x-
1/80/a*ln(1-I*a*x)^2*x^5-1/3*I/a^6+23/30*I/a^6*ln(1/2-1/2*I*a*x)*ln(1/2+1/
2*I*a*x)-23/30*I/a^6*ln(1-I*a*x)*ln(1/2+1/2*I*a*x)+1/48*x^6*Pi^3+1/48/a^6*
Pi^3-19/120/a^6*Pi+(-1/16*I*(a^6*x^6+1)/a^6*ln(1-I*a*x)^2+1/120*x*(15*Pi*a
^5*x^5+6*a^4*x^4-10*a^2*x^2+30)/a^5*ln(1-I*a*x)-1/240*(-15*I*Pi^2*a^6*x^6-
12*I*Pi*a^5*x^5-6*I*a^4*x^4+20*I*Pi*a^3*x^3+32*I*a^2*x^2-60*I*Pi*a*x-30*ln
(1-I*a*x)*Pi-92*I*ln(1-I*a*x))/a^6)*ln(1+I*a*x)-1/16*I*Pi^2*ln(1-I*a*x)*x^
6-1/20*I/a*Pi*ln(1-I*a*x)*x^5-1/32*I/a^4*ln(1-I*a*x)^2*x^2+1/64*I/a^2*ln(1
-I*a*x)^2*x^4-1/48*I*(a^6*x^6+1)/a^6*ln(1+I*a*x)^3-1/24/a^3*Pi^2*x^3+1/8/a
^5*Pi^2*x-1/16/a^6*Pi*ln(1-I*a*x)^2+37/480/a^6*Pi*ln(1-I*a*x)-1/16*Pi*ln(1
-I*a*x)^2*x^6-2/15/a^4*Pi*x^2+1/48*I/a^6*ln(1-I*a*x)^3-49/320*I/a^6*ln(1-I
*a*x)^2+1/48*I*ln(1-I*a*x)^3*x^6-1/96*I*ln(1-I*a*x)^2*x^6+1/288*I*ln(1-I*a
*x)*x^6-1/50*I/a^6*(1-I*a*x)^5*ln(1-I*a*x)+1/8*I/a^6*(1-I*a*x)^3*ln(1-I...
```

3.23.5 Fricas [F]

$$\int x^5 \cot^{-1}(ax)^3 \, dx = \int x^5 \operatorname{arccot}(ax)^3 \, dx$$

input
```
integrate(x^5*arccot(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^5*arccot(a*x)^3, x)
```

3.23.6 Sympy [F]

$$\int x^5 \cot^{-1}(ax)^3 \, dx = \int x^5 \operatorname{acot}^3(ax) \, dx$$

input
```
integrate(x**5*acot(a*x)**3,x)
```

output
```
Integral(x**5*acot(a*x)**3, x)
```

3.23.7 Maxima [F]

$$\int x^5 \cot^{-1}(ax)^3 \, dx = \int x^5 \operatorname{arccot}(ax)^3 \, dx$$

input
```
integrate(x^5*arccot(a*x)^3,x, algorithm="maxima")
```

output
```
1/480*(40*a^6*x^6*arctan2(1, a*x)^3 + 12*a^5*x^5*arctan2(1, a*x)^2 - 20*a^
3*x^3*arctan2(1, a*x)^2 + 20*(5760*a^7*integrate(1/480*x^7*arctan(1/(a*x))
^3/(a^7*x^2 + a^5), x) + 1440*a^6*integrate(1/480*x^6*arctan(1/(a*x))^2/(a
^7*x^2 + a^5), x) + 360*a^6*integrate(1/480*x^6*log(a^2*x^2 + 1)^2/(a^7*x^
2 + a^5), x) + 288*a^6*integrate(1/480*x^6*log(a^2*x^2 + 1)/(a^7*x^2 + a^5
), x) + 5760*a^5*integrate(1/480*x^5*arctan(1/(a*x))^3/(a^7*x^2 + a^5), x)
 + 576*a^5*integrate(1/480*x^5*arctan(1/(a*x))/(a^7*x^2 + a^5), x) - 480*a
^4*integrate(1/480*x^4*log(a^2*x^2 + 1)/(a^7*x^2 + a^5), x) - 960*a^3*inte
grate(1/480*x^3*arctan(1/(a*x))/(a^7*x^2 + a^5), x) + 1440*a^2*integrate(1
/480*x^2*log(a^2*x^2 + 1)/(a^7*x^2 + a^5), x) + 2880*a*integrate(1/480*x*a
rctan(1/(a*x))/(a^7*x^2 + a^5), x) + arctan(a*x)^3/a^6 + 3*arctan(a*x)^2*a
rctan(1/(a*x))/a^6 + 3*arctan(a*x)*arctan(1/(a*x))^2/a^6 + 360*integrate(1
/480*log(a^2*x^2 + 1)^2/(a^7*x^2 + a^5), x))*a^6 + 60*a*x*arctan2(1, a*x)^
2 + 40*arctan2(1, a*x)^3 - (3*a^5*x^5 - 5*a^3*x^3 + 15*a*x)*log(a^2*x^2 +
1)^2)/a^6
```

3.23.8 Giac [F]

$$\int x^5 \cot^{-1}(ax)^3 \, dx = \int x^5 \operatorname{arccot}(ax)^3 \, dx$$

input
```
integrate(x^5*arccot(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^5*arccot(a*x)^3, x)
```

3.23.9 Mupad [F(-1)]

Timed out.

$$\int x^5 \cot^{-1}(ax)^3 \, dx = \int x^5 \operatorname{acot}(a\,x)^3 \, dx$$

input
```
int(x^5*acot(a*x)^3,x)
```

output
```
int(x^5*acot(a*x)^3, x)
```

3.24 $\int x^4 \cot^{-1}(ax)^3\, dx$

3.24.1 Optimal result

Integrand size = 10, antiderivative size = 205

$$\begin{aligned}\int x^4 \cot^{-1}(ax)^3\, dx = {} & \frac{x^2}{20a^3} - \frac{9x\cot^{-1}(ax)}{10a^4} + \frac{x^3\cot^{-1}(ax)}{10a^2} - \frac{9\cot^{-1}(ax)^2}{20a^5} \\ & - \frac{3x^2\cot^{-1}(ax)^2}{10a^3} + \frac{3x^4\cot^{-1}(ax)^2}{20a} + \frac{i\cot^{-1}(ax)^3}{5a^5} \\ & + \frac{1}{5}x^5\cot^{-1}(ax)^3 - \frac{3\cot^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{5a^5} - \frac{\log(1+a^2x^2)}{2a^5} \\ & + \frac{3i\cot^{-1}(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{5a^5} - \frac{3\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{10a^5}\end{aligned}$$

output
```
1/20*x^2/a^3-9/10*x*arccot(a*x)/a^4+1/10*x^3*arccot(a*x)/a^2-9/20*arccot(a
*x)^2/a^5-3/10*x^2*arccot(a*x)^2/a^3+3/20*x^4*arccot(a*x)^2/a+1/5*I*arccot
(a*x)^3/a^5+1/5*x^5*arccot(a*x)^3-3/5*arccot(a*x)^2*ln(2/(1+I*a*x))/a^5-1/
2*ln(a^2*x^2+1)/a^5+3/5*I*arccot(a*x)*polylog(2,1-2/(1+I*a*x))/a^5-3/10*po
lylog(3,1-2/(1+I*a*x))/a^5
```

3.24.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.91

$$\int x^4 \cot^{-1}(ax)^3 \, dx$$

$$= \frac{2 + i\pi^3 + 2a^2x^2 - 36ax\cot^{-1}(ax) + 4a^3x^3\cot^{-1}(ax) - 18\cot^{-1}(ax)^2 - 12a^2x^2\cot^{-1}(ax)^2 + 6a^4x^4\cot^{-1}(}{}$$

input
```
Integrate[x^4*ArcCot[a*x]^3,x]
```

output
```
(2 + I*Pi^3 + 2*a^2*x^2 - 36*a*x*ArcCot[a*x] + 4*a^3*x^3*ArcCot[a*x] - 18*
ArcCot[a*x]^2 - 12*a^2*x^2*ArcCot[a*x]^2 + 6*a^4*x^4*ArcCot[a*x]^2 - (8*I)
*ArcCot[a*x]^3 + 8*a^5*x^5*ArcCot[a*x]^3 - 24*ArcCot[a*x]^2*Log[1 - E^((-2
*I)*ArcCot[a*x])] + 40*Log[1/Sqrt[1 + 1/(a^2*x^2)]] + 40*Log[1/(a*x)] - (2
4*I)*ArcCot[a*x]*PolyLog[2, E^((-2*I)*ArcCot[a*x])] - 12*PolyLog[3, E^((-2
*I)*ArcCot[a*x])])/(40*a^5)
```

3.24.3 Rubi [A] (verified)

Time = 2.34 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.50, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.600, Rules used = {5362, 5452, 5362, 5452, 5362, 243, 49, 2009, 5452, 5346, 240, 5420, 5456, 5380, 5530, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \cot^{-1}(ax)^3 \, dx$$

$$\Big\downarrow \; 5362$$

$$\frac{3}{5}a\int \frac{x^5\cot^{-1}(ax)^2}{a^2x^2+1}dx + \frac{1}{5}x^5\cot^{-1}(ax)^3$$

$$\Big\downarrow \; 5452$$

$$\frac{3}{5}a\left(\frac{\int x^3\cot^{-1}(ax)^2dx}{a^2} - \frac{\int \frac{x^3\cot^{-1}(ax)^2}{a^2x^2+1}dx}{a^2}\right) + \frac{1}{5}x^5\cot^{-1}(ax)^3$$

$$\Big\downarrow \; 5362$$

$$\frac{3}{5}a\left(\frac{\frac{1}{2}a\int\frac{x^4\cot^{-1}(ax)}{a^2x^2+1}dx+\frac{1}{4}x^4\cot^{-1}(ax)^2}{a^2}-\frac{\int\frac{x^3\cot^{-1}(ax)^2}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}(ax)^3$$

$\big\downarrow$ 5452

$$\frac{3}{5}a\left(\frac{\frac{1}{2}a\left(\frac{\int x^2\cot^{-1}(ax)dx}{a^2}-\frac{\int\frac{x^2\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^2}{a^2}-\frac{\frac{\int x\cot^{-1}(ax)^2dx}{a^2}-\frac{\int\frac{x\cot^{-1}(ax)^2}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}(ax)^3$$

$\big\downarrow$ 5362

$$\frac{3}{5}a\left(\frac{\frac{1}{2}a\left(\frac{\frac{1}{3}a\int\frac{x^3}{a^2x^2+1}dx+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x^2\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^2}{a^2}-\frac{\frac{a\int\frac{x^2\cot^{-1}(ax)}{a^2x^2+1}dx+\frac{1}{2}x^2\cot^{-1}(ax)^2}{a^2}-\int\frac{x\cot^{-1}\cdots}{a^2x\cdots}}{a^2}\cdots\right.$$

$$\frac{1}{5}x^5\cot^{-1}(ax)^3$$

$\big\downarrow$ 243

$$\frac{3}{5}a\left(\frac{\frac{1}{2}a\left(\frac{\frac{1}{6}a\int\frac{x^2}{a^2x^2+1}dx^2+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x^2\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^2}{a^2}-\frac{\frac{a\int\frac{x^2\cot^{-1}(ax)}{a^2x^2+1}dx+\frac{1}{2}x^2\cot^{-1}(ax)^2}{a^2}-\int\frac{x\cot\cdots}{a^2\cdots}}{a^2}\cdots\right.$$

$$\frac{1}{5}x^5\cot^{-1}(ax)^3$$

$\big\downarrow$ 49

$$\frac{3}{5}a\left(\frac{\frac{1}{2}a\left(\frac{\frac{1}{6}a\int\left(\frac{1}{a^2}-\frac{1}{a^2\left(a^2x^2+1\right)}\right)dx^2+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x^2\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^2}{a^2}-\frac{\frac{a\int\frac{x^2\cot^{-1}(ax)}{a^2x^2+1}dx+\frac{1}{2}x^2\cot^{-1}(ax\cdots}{a^2}}{a^2}\cdots\right.$$

$$\frac{1}{5}x^5\cot^{-1}(ax)^3$$

$\big\downarrow$ 2009

3.24. $\int x^4\cot^{-1}(ax)^3\,dx$

$$\frac{3}{5}a\left(\frac{\frac{1}{2}a\left(\frac{\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x^2\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^2}{a^2}-\frac{\frac{a\int\frac{x^2\cot^{-1}(ax)}{a^2x^2+1}dx+\frac{1}{2}x^2\cot^{-1}(ax)^2}{a^2}-\cdots}{a^2}\cdots\right.$$

$$\frac{1}{5}x^5\cot^{-1}(ax)^3$$

$$\Big\downarrow \; 5452$$

$$\frac{3}{5}a\left(\frac{\frac{1}{2}a\left(\frac{\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\frac{\int\cot^{-1}(ax)dx}{a^2}-\frac{\int\frac{\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^2}{a^2}-\frac{a\left(\frac{\int\cot^{-1}(ax)dx}{a^2}-\frac{\int\frac{\cot^{-1}\cdots}{a^2x^2\cdots}}{a^2}\cdots\right.}{a\cdots}\cdots\right.$$

$$\frac{1}{5}x^5\cot^{-1}(ax)^3$$

$$\Big\downarrow \; 5346$$

$$\frac{3}{5}a\left(\frac{\frac{1}{2}a\left(\frac{\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\frac{a\int\frac{x}{a^2x^2+1}dx+x\cot^{-1}(ax)}{a^2}-\frac{\int\frac{\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^2}{a^2}-\frac{a\left(\frac{a\int\frac{x}{a^2x^2+1}\cdots}{\cdots}\right.}{\cdots}\cdots\right.$$

$$\frac{1}{5}x^5\cot^{-1}(ax)^3$$

$$\Big\downarrow \; 240$$

$$\frac{3}{5}a\left(\frac{\frac{1}{2}a\left(\frac{\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{\log\left(a^2x^2+1\right)}{2a}+x\cot^{-1}(ax)}{a^2}-\frac{\int\frac{\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^2}{a^2}-\frac{a\left(\frac{\frac{\log\left(a^2x^2+1\cdots\right.}{2a}\cdots}{\cdots}\right.}{\cdots}\cdots\right.$$

$$\frac{1}{5}x^5\cot^{-1}(ax)^3$$

3.24. $\int x^4\cot^{-1}(ax)^3\,dx$

$\downarrow$ 5420

$$\frac{3}{5}a\left(\frac{\frac{1}{2}a\left(\frac{\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\frac{\cot^{-1}(ax)^2}{2a^3}+\frac{\frac{\log\left(a^2x^2+1\right)}{2a}+x\cot^{-1}(ax)}{a^2}}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^2}{a^2}-\frac{a\left(\frac{\cot^{-1}(ax)^2}{2a^3}+\cdots\right.}{\cdots}\right.$$

$$\frac{1}{5}x^5\cot^{-1}(ax)^3$$

$\downarrow$ 5456

$$\frac{1}{5}x^5\cot^{-1}(ax)^3+\frac{3}{5}a\left(\frac{\frac{1}{2}a\left(\frac{\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\frac{\cot^{-1}(ax)^2}{2a^3}+\frac{\frac{\log\left(a^2x^2+1\right)}{2a}+x\cot^{-1}(ax)}{a^2}}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^2}{a^2}-\frac{a\left(\frac{\cot^{-1}(ax)^2}{2a^3}+\cdots\right.}{\cdots}\right.$$

$\downarrow$ 5380

$$\frac{1}{5}x^5\cot^{-1}(ax)^3+\frac{3}{5}a\left(\frac{\frac{1}{2}a\left(\frac{\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\frac{\cot^{-1}(ax)^2}{2a^3}+\frac{\frac{\log\left(a^2x^2+1\right)}{2a}+x\cot^{-1}(ax)}{a^2}}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^2}{a^2}-\frac{a\left(\frac{\cot^{-1}(ax)^2}{2a^3}+\cdots\right.}{\cdots}\right.$$

$\downarrow$ 5530

$$\frac{1}{5}x^5\cot^{-1}(ax)^3+\frac{3}{5}a\left(\frac{\frac{1}{2}a\left(\frac{\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\frac{\cot^{-1}(ax)^2}{2a^3}+\frac{\frac{\log\left(a^2x^2+1\right)}{2a}+x\cot^{-1}(ax)}{a^2}}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^2}{a^2}-\frac{a\left(\frac{\cot^{-1}(ax)^2}{2a^3}+\cdots\right.}{\cdots}\right.$$

3.24. $\int x^4\cot^{-1}(ax)^3\,dx$

$$\Big\downarrow \; 7164$$

$$\frac{1}{5}x^5\cot^{-1}(ax)^3+$$

$$\frac{3}{5}a\left(\frac{\frac{1}{2}a\left(\frac{\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\frac{\cot^{-1}(ax)^2}{2a^3}+\frac{\frac{\log\left(a^2x^2+1\right)}{2a}+x\cot^{-1}(ax)}{a^2}}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^2}{a^2}-\frac{a\left(\frac{\cot^{-1}(ax)^2}{2a^3}+\cdots\right.}{\cdots}\right.$$

input
```
Int[x^4*ArcCot[a*x]^3,x]
```

output
```
(x^5*ArcCot[a*x]^3)/5 + (3*a*(((x^4*ArcCot[a*x]^2)/4 + (a*(((x^3*ArcCot[a*
x])/3 + (a*(x^2/a^2 - Log[1 + a^2*x^2]/a^4))/6)/a^2 - (ArcCot[a*x]^2/(2*a^
3) + (x*ArcCot[a*x] + Log[1 + a^2*x^2]/(2*a))/a^2)/a^2))/2)/a^2 - (((x^2*A
rcCot[a*x]^2)/2 + a*(ArcCot[a*x]^2/(2*a^3) + (x*ArcCot[a*x] + Log[1 + a^2*
x^2]/(2*a))/a^2))/a^2 - (((I/3)*ArcCot[a*x]^3)/a^2 - ((ArcCot[a*x]^2*Log[2
/(1 + I*a*x)])/a + 2*(((-1/2*I)*ArcCot[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])
/a + PolyLog[3, 1 - 2/(1 + I*a*x)]/(4*a)))/a)/a^2)/a^2))/5
```

3.24.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5346

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5362

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5380

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Simp[b*c*(
p/e)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5420

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5452

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5456

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Simp[
1/(c*d)   Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5530

```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] - Simp[b*p*(I/2)   Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

3.24. $\int x^4 \cot^{-1}(ax)^3 \, dx$

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.24.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 8.10 (sec) , antiderivative size = 1108, normalized size of antiderivative = 5.40

method	result	size
derivativedivides	Expression too large to display	1108
default	Expression too large to display	1108
parts	Expression too large to display	1110

input
```
int(x^4*arccot(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(1/5*a^5*x^5*arccot(a*x)^3+3/20*a^4*x^4*arccot(a*x)^2-3/10*a^2*x^2*a
rccot(a*x)^2+3/10*arccot(a*x)^2*ln(a^2*x^2+1)-3/5*arccot(a*x)^2*ln((I+a*x)
/(a^2*x^2+1)^(1/2))+3/5*arccot(a*x)^2*ln((I+a*x)^2/(a^2*x^2+1)-1)+1/20*I*(
-3*arccot(a*x)^2*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)
^3*Pi-3*arccot(a*x)^2*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-
1)^2)^2*csgn(I*(I+a*x)^2/(a^2*x^2+1))*Pi-3*arccot(a*x)^2*csgn(I*(I+a*x)^2/
(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)^2*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1
)-1)^2)+3*arccot(a*x)^2*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1
)-1)^2)*csgn(I*(I+a*x)^2/(a^2*x^2+1))*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)^
2)+3*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1))^3*arccot(a*x)^2-6*Pi*csgn(I*(I+a*x)/
(a^2*x^2+1)^(1/2))*csgn(I*(I+a*x)^2/(a^2*x^2+1))^2*arccot(a*x)^2+3*Pi*csgn
(I*(I+a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(I+a*x)^2/(a^2*x^2+1))*arccot(a*x)^
2-3*arccot(a*x)^2*Pi*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1))^2*csgn(I*((I+a*x)^2
/(a^2*x^2+1)-1)^2)+6*arccot(a*x)^2*Pi*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1))*cs
gn(I*((I+a*x)^2/(a^2*x^2+1)-1)^2)^2-3*arccot(a*x)^2*Pi*csgn(I*((I+a*x)^2/(
a^2*x^2+1)-1)^2)^3+6*arccot(a*x)^2*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2
/(a^2*x^2+1)-1)^2)^2*Pi+4*arccot(a*x)^3-6*Pi*arccot(a*x)^2-I*a^2*x^2+12*I*
arccot(a*x)^2*ln(2)+18*I*arccot(a*x)*a*x-20*arccot(a*x)+9*I*arccot(a*x)^2-
2*I*arccot(a*x)*a^3*x^3-I)+ln((I+a*x)/(a^2*x^2+1)^(1/2)-1)+ln(1+(I+a*x)/(a
^2*x^2+1)^(1/2))-3/5*arccot(a*x)^2*ln(1-(I+a*x)/(a^2*x^2+1)^(1/2))+6/5*...
```

3.24.5 Fricas [F]

$$\int x^4 \cot^{-1}(ax)^3 \, dx = \int x^4 \operatorname{arccot}(ax)^3 \, dx$$

input
```
integrate(x^4*arccot(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^4*arccot(a*x)^3, x)
```

3.24.6 Sympy [F]

$$\int x^4 \cot^{-1}(ax)^3 \, dx = \int x^4 \operatorname{acot}^3(ax) \, dx$$

input
```
integrate(x**4*acot(a*x)**3,x)
```

output
```
Integral(x**4*acot(a*x)**3, x)
```

3.24.7 Maxima [F]

$$\int x^4 \cot^{-1}(ax)^3 \, dx = \int x^4 \operatorname{arccot}(ax)^3 \, dx$$

input
```
integrate(x^4*arccot(a*x)^3,x, algorithm="maxima")
```

output
```
1/40*x^5*arctan2(1, a*x)^3 - 3/160*x^5*arctan2(1, a*x)*log(a^2*x^2 + 1)^2
+ integrate(1/160*(140*a^2*x^6*arctan2(1, a*x)^3 + 12*a^2*x^6*arctan2(1, a
*x)*log(a^2*x^2 + 1) + 12*a*x^5*arctan2(1, a*x)^2 + 140*x^4*arctan2(1, a*x
)^3 + 3*(5*a^2*x^6*arctan2(1, a*x) - a*x^5 + 5*x^4*arctan2(1, a*x))*log(a^
2*x^2 + 1)^2)/(a^2*x^2 + 1), x)
```

3.24.8 Giac [F]

$$\int x^4 \cot^{-1}(ax)^3 \, dx = \int x^4 \operatorname{arccot}(ax)^3 \, dx$$

input
```
integrate(x^4*arccot(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^4*arccot(a*x)^3, x)
```

3.24.9 Mupad [F(-1)]

Timed out.

$$\int x^4 \cot^{-1}(ax)^3 \, dx = \int x^4 \operatorname{acot}(a\,x)^3 \, dx$$

input
```
int(x^4*acot(a*x)^3,x)
```

output
```
int(x^4*acot(a*x)^3, x)
```

3.25 $\int x^3 \cot^{-1}(ax)^3 \, dx$

3.25.1 Optimal result

Integrand size = 10, antiderivative size = 148

$$\int x^3 \cot^{-1}(ax)^3 \, dx = \frac{x}{4a^3} + \frac{x^2 \cot^{-1}(ax)}{4a^2} - \frac{i \cot^{-1}(ax)^2}{a^4} - \frac{3x \cot^{-1}(ax)^2}{4a^3} + \frac{x^3 \cot^{-1}(ax)^2}{4a} - \frac{\cot^{-1}(ax)^3}{4a^4} + \frac{1}{4}x^4 \cot^{-1}(ax)^3 - \frac{\arctan(ax)}{4a^4} + \frac{2\cot^{-1}(ax) \log\left(\frac{2}{1+iax}\right)}{a^4} - \frac{i\, \text{PolyLog}\left(2, 1 - \frac{2}{1+iax}\right)}{a^4}$$

output
```
1/4*x/a^3+1/4*x^2*arccot(a*x)/a^2-I*arccot(a*x)^2/a^4-3/4*x*arccot(a*x)^2/
a^3+1/4*x^3*arccot(a*x)^2/a-1/4*arccot(a*x)^3/a^4+1/4*x^4*arccot(a*x)^3-1/
4*arctan(a*x)/a^4+2*arccot(a*x)*ln(2/(1+I*a*x))/a^4-I*polylog(2,1-2/(1+I*a
*x))/a^4
```

3.25.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.65

$$\int x^3 \cot^{-1}(ax)^3 \, dx$$

$$= \frac{ax + (-4i - 3ax + a^3x^3)\cot^{-1}(ax)^2 + (-1 + a^4x^4)\cot^{-1}(ax)^3 + \cot^{-1}(ax)\left(1 + a^2x^2 + 8\log\left(1 - e^{2i\cot}\right.\right.}{4a^4}$$

input
```
Integrate[x^3*ArcCot[a*x]^3,x]
```

output
```
(a*x + (-4*I - 3*a*x + a^3*x^3)*ArcCot[a*x]^2 + (-1 + a^4*x^4)*ArcCot[a*x]
^3 + ArcCot[a*x]*(1 + a^2*x^2 + 8*Log[1 - E^((2*I)*ArcCot[a*x])]) - (4*I)*
PolyLog[2, E^((2*I)*ArcCot[a*x])])/(4*a^4)
```

3.25.3 Rubi [A] (verified)

Time = 1.60 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.73, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.300$, Rules used = {5362, 5452, 5362, 5452, 5346, 5362, 262, 216, 5420, 5456, 5380, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \cot^{-1}(ax)^3 \, dx$$

$$\Big\downarrow \text{5362}$$

$$\frac{3}{4} a \int \frac{x^4 \cot^{-1}(ax)^2}{a^2x^2+1} dx + \frac{1}{4} x^4 \cot^{-1}(ax)^3$$

$$\Big\downarrow \text{5452}$$

$$\frac{3}{4} a \left(\frac{\int x^2 \cot^{-1}(ax)^2 dx}{a^2} - \frac{\int \frac{x^2 \cot^{-1}(ax)^2}{a^2x^2+1} dx}{a^2} \right) + \frac{1}{4} x^4 \cot^{-1}(ax)^3$$

$$\Big\downarrow \text{5362}$$

$$\frac{3}{4} a \left(\frac{\frac{2}{3} a \int \frac{x^3 \cot^{-1}(ax)}{a^2x^2+1} dx + \frac{1}{3} x^3 \cot^{-1}(ax)^2}{a^2} - \frac{\int \frac{x^2 \cot^{-1}(ax)^2}{a^2x^2+1} dx}{a^2} \right) + \frac{1}{4} x^4 \cot^{-1}(ax)^3$$

$$\Big\downarrow \text{5452}$$

$$\frac{3}{4} a \left(\frac{\frac{2}{3} a \left(\frac{\int x \cot^{-1}(ax) dx}{a^2} - \frac{\int \frac{x \cot^{-1}(ax)}{a^2x^2+1} dx}{a^2} \right) + \frac{1}{3} x^3 \cot^{-1}(ax)^2}{a^2} - \frac{\frac{\int \cot^{-1}(ax)^2 dx}{a^2} - \frac{\int \frac{\cot^{-1}(ax)^2}{a^2x^2+1} dx}{a^2}}{a^2} \right) + \frac{1}{4} x^4 \cot^{-1}(ax)^3$$

$$\Big\downarrow \text{5346}$$

$$\frac{3}{4}a\left(\frac{\frac{2}{3}a\left(\frac{\int x\cot^{-1}(ax)dx}{a^2}-\frac{\int \frac{x\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{3}x^3\cot^{-1}(ax)^2}{a^2}-\frac{\frac{2a\int \frac{x\cot^{-1}(ax)}{a^2x^2+1}dx+x\cot^{-1}(ax)^2}{a^2}-\frac{\int \frac{\cot^{-1}(ax)^2}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^3$$

$$\Big\downarrow \text{5362}$$

$$\frac{3}{4}a\left(\frac{\frac{2}{3}a\left(\frac{\frac{1}{2}a\int \frac{x^2}{a^2x^2+1}dx+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\int \frac{x\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{3}x^3\cot^{-1}(ax)^2}{a^2}-\frac{\frac{2a\int \frac{x\cot^{-1}(ax)}{a^2x^2+1}dx+x\cot^{-1}(ax)^2}{a^2}-\frac{\int \frac{\cot^{-1}(ax)^2}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^3$$

$$\Big\downarrow \text{262}$$

$$\frac{3}{4}a\left(\frac{\frac{2}{3}a\left(\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\int \frac{1}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\int \frac{x\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{3}x^3\cot^{-1}(ax)^2}{a^2}-\frac{\frac{2a\int \frac{x\cot^{-1}(ax)}{a^2x^2+1}dx+x\cot^{-1}(ax)^2}{a^2}-\frac{\int \frac{\cot^{-1}(ax)^2}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^3$$

$$\Big\downarrow \text{216}$$

$$\frac{3}{4}a\left(\frac{\frac{2}{3}a\left(\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\int \frac{x\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{3}x^3\cot^{-1}(ax)^2}{a^2}-\frac{\frac{2a\int \frac{x\cot^{-1}(ax)}{a^2x^2+1}dx+x\cot^{-1}(ax)^2}{a^2}-\frac{\int \frac{\cot^{-1}(ax)^2}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^3$$

$$\Big\downarrow \text{5420}$$

3.25. $\int x^3\cot^{-1}(ax)^3\,dx$

$$\frac{3}{4}a\left(\frac{\frac{2}{3}a\left(\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{3}x^3\cot^{-1}(ax)^2}{a^2}-\frac{\frac{2a\int\frac{x\cot^{-1}(ax)}{a^2x^2+1}dx+x\cot^{-1}(ax)^2}{a^2}+\frac{\cot^{-1}(}{3a}}{a^2}\right.$$

$$\frac{1}{4}x^4\cot^{-1}(ax)^3$$

$\downarrow$ 5456

$$\frac{1}{4}x^4\cot^{-1}(ax)^3+$$

$$\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\cot^{-1}(ax)^2+\frac{2}{3}a\left(\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\int\frac{\cot^{-1}(ax)}{i-ax}dx}{a}}{a^2}\right)}{a^2}-\frac{\frac{\cot^{-1}(ax)^3}{3a^3}+\frac{x\cot^{-1}(ax)^2+2}{}}{}\right.$$

$\downarrow$ 5380

$$\frac{1}{4}x^4\cot^{-1}(ax)^3+$$

$$\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\cot^{-1}(ax)^2+\frac{2}{3}a\left(\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\int\frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx+\frac{\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)}{a}}{a}}{a^2}\right)}{a^2}-\frac{\frac{\cot^{-1}(}{3a}}{}\right.$$

$\downarrow$ 2849

$$\frac{1}{4}x^4\cot^{-1}(ax)^3+$$

$$\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\cot^{-1}(ax)^2+\frac{2}{3}a\left(\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\frac{\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)}{a}-\frac{i\int\frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}}{a}}{a^2}\right)}{a^2}-\right.$$

$\downarrow$ 2752

3.25. $\int x^3\cot^{-1}(ax)^3\,dx$

$$\frac{1}{4}x^4\cot^{-1}(ax)^3+\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\cot^{-1}(ax)^2+\frac{2}{3}a\left(\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\frac{\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2}\right)}{a^2}-\cdots\right.$$

input
```
Int[x^3*ArcCot[a*x]^3,x]
```

output
```
(x^4*ArcCot[a*x]^3)/4 + (3*a*(((x^3*ArcCot[a*x]^2)/3 + (2*a*(((x^2*ArcCot[
a*x])/2 + (a*(x/a^2 - ArcTan[a*x]/a^3))/2)/a^2 - (((I/2)*ArcCot[a*x]^2)/a^
2 - ((ArcCot[a*x]*Log[2/(1 + I*a*x)])/a - ((I/2)*PolyLog[2, 1 - 2/(1 + I*a
*x)])/a)/a)/a^2))/3)/a^2 - (ArcCot[a*x]^3/(3*a^3) + (x*ArcCot[a*x]^2 + 2*a
*(((I/2)*ArcCot[a*x]^2)/a^2 - ((ArcCot[a*x]*Log[2/(1 + I*a*x)])/a - ((I/2)
*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/a))/a^2)/a^2))/4
```

3.25.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5380
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Simp[b*c*(
p/e)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5420
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5452
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5456
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Simp[
1/(c*d)   Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.25. $\int x^3 \cot^{-1}(ax)^3 \, dx$

3.25.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 856 vs. 2(130) = 260.

Time = 6.97 (sec) , antiderivative size = 857, normalized size of antiderivative = 5.79

method	result
risch	$\frac{\pi x^2}{8a^2}+\frac{\pi^2 x^3}{16a}+\frac{\pi}{8a^4}-\frac{\pi^3}{32a^4}+\frac{3i\ln(-iax+1)^2 x^2}{64a^2}-\frac{i(a^4x^4-1)\ln(iax+1)^3}{32a^4}+\frac{i\ln\left(\frac{1}{2}+\frac{iax}{2}\right)\ln(-iax+1)}{a^4}-\frac{i\ln}{}$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(x^3*arccot(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
3/8*I/a^3*Pi*ln(1-I*a*x)*x+3/64*I/a^2*ln(1-I*a*x)^2*x^2-1/32*I*(a^4*x^4-1)
/a^4*ln(1+I*a*x)^3+1/8/a^2*Pi*x^2+1/16/a*Pi^2*x^3+1/8/a^4*Pi-1/32/a^4*Pi^3
+(-3/32*I*(a^4*x^4-1)/a^4*ln(1-I*a*x)^2+1/16*x*(3*Pi*a^3*x^3+2*a^2*x^2-6)/
a^3*ln(1-I*a*x)+1/32*(3*I*Pi^2*a^4*x^4+4*I*Pi*a^3*x^3+4*I*a^2*x^2-12*I*Pi*
a*x-6*ln(1-I*a*x)*Pi-16*I*ln(1-I*a*x))/a^4)*ln(1+I*a*x)+I/a^4*ln(1/2+1/2*I
*a*x)*ln(1-I*a*x)-I/a^4*ln(1/2-1/2*I*a*x)*ln(1/2+1/2*I*a*x)-3/32*I*Pi^2*ln
(1-I*a*x)*x^4-1/8*I/a*Pi*ln(1-I*a*x)*x^3-1/4*I/a^4*Pi^2-319/1536*I/a^4*ln(
a^2*x^2+1)+1/32*x^4*Pi^3-57/128/a^4*Pi*ln(a^2*x^2+1)-21/128*I/a^2*x^2*ln(1
-I*a*x)-7/64*I/a^4*Pi*arctan(a*x)+23/48*I/a^4*ln(1-I*a*x)-I/a^4*dilog(1/2-
1/2*I*a*x)+3/16/a^4*Pi^2*arctan(a*x)-1/32*(-3*I*x^4*ln(1-I*a*x)*a^4+3*Pi*a
^4*x^4+2*a^3*x^3+3*I*ln(1-I*a*x)-6*a*x-3*Pi+8*I)/a^4*ln(1+I*a*x)^2-1/32/a*
ln(1-I*a*x)^2*x^3-1/192/a*ln(1-I*a*x)*x^3+3/32/a^4*Pi*ln(1-I*a*x)^2-7/64/a
^4*Pi*ln(1-I*a*x)-3/32*Pi*ln(1-I*a*x)^2*x^4+3/32/a^3*ln(1-I*a*x)^2*x+7/64/
a^3*ln(1-I*a*x)*x-3/16/a^3*Pi^2*x+1/4*x/a^3-511/768*arctan(a*x)/a^4+1/32*I
*ln(1-I*a*x)^3*x^4-3/32*I/a^4*(1-I*a*x)^2*ln(1-I*a*x)+3/32*I/a^4*(1-I*a*x)
^2*ln(1-I*a*x)^2+1/24*I/a^4*(1-I*a*x)^3*ln(1-I*a*x)-1/16*I/a^4*(1-I*a*x)^3
*ln(1-I*a*x)^2-3/256*I/a^4*(1-I*a*x)^4*ln(1-I*a*x)+3/128*I/a^4*(1-I*a*x)^4
*ln(1-I*a*x)^2-1/32*I/a^4*ln(1-I*a*x)^3+25/128*I/a^4*ln(1-I*a*x)^2-3/128*I
*ln(1-I*a*x)^2*x^4+3/256*I*ln(1-I*a*x)*x^4+1/4*I/a^4
```

3.25.5 Fricas [F]

$$\int x^3 \cot^{-1}(ax)^3 \, dx = \int x^3 \operatorname{arccot}(ax)^3 \, dx$$

input
```
integrate(x^3*arccot(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^3*arccot(a*x)^3, x)
```

3.25.6 Sympy [F]

$$\int x^3 \cot^{-1}(ax)^3 \, dx = \int x^3 \operatorname{acot}^3(ax) \, dx$$

input
```
integrate(x**3*acot(a*x)**3,x)
```

output
```
Integral(x**3*acot(a*x)**3, x)
```

3.25.7 Maxima [F]

$$\int x^3 \cot^{-1}(ax)^3 \, dx = \int x^3 \operatorname{arccot}(ax)^3 \, dx$$

input
```
integrate(x^3*arccot(a*x)^3,x, algorithm="maxima")
```

output
```
1/64*(8*a^4*x^4*arctan2(1, a*x)^3 + 4*a^3*x^3*arctan2(1, a*x)^2 + 4*(512*a
^5*integrate(1/64*x^5*arctan(1/(a*x))^3/(a^5*x^2 + a^3), x) + 192*a^4*inte
grate(1/64*x^4*arctan(1/(a*x))^2/(a^5*x^2 + a^3), x) + 48*a^4*integrate(1/
64*x^4*log(a^2*x^2 + 1)^2/(a^5*x^2 + a^3), x) + 64*a^4*integrate(1/64*x^4*
log(a^2*x^2 + 1)/(a^5*x^2 + a^3), x) + 512*a^3*integrate(1/64*x^3*arctan(1
/(a*x))^3/(a^5*x^2 + a^3), x) + 128*a^3*integrate(1/64*x^3*arctan(1/(a*x))
/(a^5*x^2 + a^3), x) - 192*a^2*integrate(1/64*x^2*log(a^2*x^2 + 1)/(a^5*x^
2 + a^3), x) - 384*a*integrate(1/64*x*arctan(1/(a*x))/(a^5*x^2 + a^3), x)
- arctan(a*x)^3/a^4 - 3*arctan(a*x)^2*arctan(1/(a*x))/a^4 - 3*arctan(a*x)*
arctan(1/(a*x))^2/a^4 - 48*integrate(1/64*log(a^2*x^2 + 1)^2/(a^5*x^2 + a^
3), x))*a^4 - 12*a*x*arctan2(1, a*x)^2 - 8*arctan2(1, a*x)^3 - (a^3*x^3 -
3*a*x)*log(a^2*x^2 + 1)^2)/a^4
```

3.25.8 Giac [F]

$$\int x^3 \cot^{-1}(ax)^3 \, dx = \int x^3 \operatorname{arccot}(ax)^3 \, dx$$

input
```
integrate(x^3*arccot(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^3*arccot(a*x)^3, x)
```

3.25.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \cot^{-1}(ax)^3 \, dx = \int x^3 \operatorname{acot}(a\,x)^3 \, dx$$

input
```
int(x^3*acot(a*x)^3,x)
```

output
```
int(x^3*acot(a*x)^3, x)
```

3.26 $\int x^2 \cot^{-1}(ax)^3 \, dx$

3.26.1 Optimal result

Integrand size = 10, antiderivative size = 157

$$\int x^2 \cot^{-1}(ax)^3 \, dx = \frac{x\cot^{-1}(ax)}{a^2} + \frac{\cot^{-1}(ax)^2}{2a^3} + \frac{x^2\cot^{-1}(ax)^2}{2a} - \frac{i\cot^{-1}(ax)^3}{3a^3} + \frac{1}{3}x^3\cot^{-1}(ax)^3 + \frac{\cot^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a^3} + \frac{\log(1+a^2x^2)}{2a^3} - \frac{i\cot^{-1}(ax)\,\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a^3} + \frac{\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{2a^3}$$

output
```
x*arccot(a*x)/a^2+1/2*arccot(a*x)^2/a^3+1/2*x^2*arccot(a*x)^2/a-1/3*I*arcc
ot(a*x)^3/a^3+1/3*x^3*arccot(a*x)^3+arccot(a*x)^2*ln(2/(1+I*a*x))/a^3+1/2*
ln(a^2*x^2+1)/a^3-I*arccot(a*x)*polylog(2,1-2/(1+I*a*x))/a^3+1/2*polylog(3
,1-2/(1+I*a*x))/a^3
```

3.26.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.97

$$\int x^2 \cot^{-1}(ax)^3 \, dx$$

$$= \frac{-i\pi^3 + 24ax\cot^{-1}(ax) + 12\cot^{-1}(ax)^2 + 12a^2x^2\cot^{-1}(ax)^2 + 8i\cot^{-1}(ax)^3 + 8a^3x^3\cot^{-1}(ax)^3 + 24\,\mathrm{cot}}{}$$

input
```
Integrate[x^2*ArcCot[a*x]^3,x]
```

output
```
((-I)*Pi^3 + 24*a*x*ArcCot[a*x] + 12*ArcCot[a*x]^2 + 12*a^2*x^2*ArcCot[a*x
]^2 + (8*I)*ArcCot[a*x]^3 + 8*a^3*x^3*ArcCot[a*x]^3 + 24*ArcCot[a*x]^2*Log
[1 - E^((-2*I)*ArcCot[a*x])] - 24*Log[1/Sqrt[1 + 1/(a^2*x^2)]] - 24*Log[1/
(a*x)] + (24*I)*ArcCot[a*x]*PolyLog[2, E^((-2*I)*ArcCot[a*x])] + 12*PolyLo
g[3, E^((-2*I)*ArcCot[a*x])])/(24*a^3)
```

3.26.3 Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.17, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules used = {5362, 5452, 5362, 5452, 5346, 240, 5420, 5456, 5380, 5530, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cot^{-1}(ax)^3 \, dx$$

$$\Big\downarrow \text{5362}$$

$$a \int \frac{x^3 \cot^{-1}(ax)^2}{a^2x^2+1}dx + \frac{1}{3}x^3 \cot^{-1}(ax)^3$$

$$\Big\downarrow \text{5452}$$

$$a \left(\frac{\int x \cot^{-1}(ax)^2dx}{a^2} - \frac{\int \frac{x \cot^{-1}(ax)^2}{a^2x^2+1}dx}{a^2}\right) + \frac{1}{3}x^3 \cot^{-1}(ax)^3$$

$$\Big\downarrow \text{5362}$$

$$a \left(\frac{a \int \frac{x^2 \cot^{-1}(ax)}{a^2x^2+1}dx + \frac{1}{2}x^2 \cot^{-1}(ax)^2}{a^2} - \frac{\int \frac{x \cot^{-1}(ax)^2}{a^2x^2+1}dx}{a^2}\right) + \frac{1}{3}x^3 \cot^{-1}(ax)^3$$

$$\Big\downarrow \text{5452}$$

$$a \left(\frac{a \left(\frac{\int \cot^{-1}(ax)dx}{a^2} - \frac{\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right) + \frac{1}{2}x^2 \cot^{-1}(ax)^2}{a^2} - \frac{\int \frac{x \cot^{-1}(ax)^2}{a^2x^2+1}dx}{a^2}\right) + \frac{1}{3}x^3 \cot^{-1}(ax)^3$$

$$\Big\downarrow \text{5346}$$

$$a\left(\frac{a\left(\frac{a\int \frac{x}{a^2x^2+1}dx+x\cot^{-1}(ax)}{a^2}-\frac{\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{2}x^2\cot^{-1}(ax)^2}{a^2}-\frac{\int \frac{x\cot^{-1}(ax)^2}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{3}x^3\cot^{-1}(ax)^3$$

$$\downarrow \textbf{240}$$

$$a\left(\frac{a\left(\frac{\frac{\log\left(a^2x^2+1\right)}{2a}+x\cot^{-1}(ax)}{a^2}-\frac{\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{2}x^2\cot^{-1}(ax)^2}{a^2}-\frac{\int \frac{x\cot^{-1}(ax)^2}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{3}x^3\cot^{-1}(ax)^3$$

$$\downarrow \textbf{5420}$$

$$a\left(\frac{a\left(\frac{\cot^{-1}(ax)^2}{2a^3}+\frac{\frac{\log\left(a^2x^2+1\right)}{2a}+x\cot^{-1}(ax)}{a^2}\right)+\frac{1}{2}x^2\cot^{-1}(ax)^2}{a^2}-\frac{\int \frac{x\cot^{-1}(ax)^2}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{3}x^3\cot^{-1}(ax)^3$$

$$\downarrow \textbf{5456}$$

$$\frac{1}{3}x^3\cot^{-1}(ax)^3+a\left(\frac{a\left(\frac{\cot^{-1}(ax)^2}{2a^3}+\frac{\frac{\log\left(a^2x^2+1\right)}{2a}+x\cot^{-1}(ax)}{a^2}\right)+\frac{1}{2}x^2\cot^{-1}(ax)^2}{a^2}-\frac{\frac{i\cot^{-1}(ax)^3}{3a^2}-\frac{\int \frac{\cot^{-1}(ax)^2}{i-ax}dx}{a}}{a^2}\right)$$

$$\downarrow \textbf{5380}$$

$$\frac{1}{3}x^3\cot^{-1}(ax)^3+a\left(\frac{a\left(\frac{\cot^{-1}(ax)^2}{2a^3}+\frac{\frac{\log\left(a^2x^2+1\right)}{2a}+x\cot^{-1}(ax)}{a^2}\right)+\frac{1}{2}x^2\cot^{-1}(ax)^2}{a^2}-\frac{\frac{i\cot^{-1}(ax)^3}{3a^2}-\frac{2\int \frac{\cot^{-1}(ax)\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx+\frac{\log\left(\frac{2}{1+iax}\right)\cot^{-1}(}{a}}{a}}{a^2}$$

$$\downarrow \textbf{5530}$$

3.26. $\int x^2\cot^{-1}(ax)^3\,dx$

$$\frac{1}{3}x^3\cot^{-1}(ax)^3+ a\left(\frac{a\left(\frac{\cot^{-1}(ax)^2}{2a^3}+\frac{\frac{\log(a^2x^2+1)}{2a}+x\cot^{-1}(ax)}{a^2}\right)+\frac{1}{2}x^2\cot^{-1}(ax)^2}{a^2}-\frac{i\cot^{-1}(ax)^3}{3a^2}-\frac{2\left(-\frac{1}{2}i\int\frac{\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx-\frac{i\,\text{PolyLog}\left(2,1\right.}{}\right.}{a}}{a^2}\right.$$

$$\Big\downarrow \text{7164}$$

$$\frac{1}{3}x^3\cot^{-1}(ax)^3+ a\left(\frac{a\left(\frac{\cot^{-1}(ax)^2}{2a^3}+\frac{\frac{\log(a^2x^2+1)}{2a}+x\cot^{-1}(ax)}{a^2}\right)+\frac{1}{2}x^2\cot^{-1}(ax)^2}{a^2}-\frac{i\cot^{-1}(ax)^3}{3a^2}-\frac{2\left(\frac{\text{PolyLog}\left(3,1-\frac{2}{iax+1}\right)}{4a}-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}\right.}{a}}{a^2}\right.$$

input
```
Int[x^2*ArcCot[a*x]^3,x]
```

output
```
(x^3*ArcCot[a*x]^3)/3 + a*(((x^2*ArcCot[a*x]^2)/2 + a*(ArcCot[a*x]^2/(2*a^
3) + (x*ArcCot[a*x] + Log[1 + a^2*x^2]/(2*a))/a^2))/a^2 - (((I/3)*ArcCot[a
*x]^3)/a^2 - ((ArcCot[a*x]^2*Log[2/(1 + I*a*x)])/a + 2*(((-1/2*I)*ArcCot[a
*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a + PolyLog[3, 1 - 2/(1 + I*a*x)]/(4*a)
))/a)/a^2)
```

3.26.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5380
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Simp[b*c*(
p/e)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5420
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5452
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5456
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Simp[
1/(c*d)   Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5530
```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] - Simp[b*p*(I/2)   Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.26. $\int x^2 \cot^{-1}(ax)^3 \, dx$

3.26.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.94 (sec) , antiderivative size = 1036, normalized size of antiderivative = 6.60

method	result	size
parts	Expression too large to display	1036
derivativedivides	Expression too large to display	1038
default	Expression too large to display	1038

input
```
int(x^2*arccot(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*arccot(a*x)^3+1/a^3*(1/2*a^2*x^2*arccot(a*x)^2-1/2*arccot(a*x)^2*l
n(a^2*x^2+1)+arccot(a*x)^2*ln((I+a*x)/(a^2*x^2+1)^(1/2))-arccot(a*x)^2*ln(
(I+a*x)^2/(a^2*x^2+1)-1)-1/12*I*arccot(a*x)*(-3*arccot(a*x)*Pi*csgn(I*((I+
a*x)^2/(a^2*x^2+1)-1))^2*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1)^2)+6*arccot(a*x)
*Pi*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1)^2)^
2-3*arccot(a*x)*Pi*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1)^2)^3-3*arccot(a*x)*Pi*
csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)^2)*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^
2/(a^2*x^2+1)-1)^2)^2+3*arccot(a*x)*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)^2)
*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)*csgn(I*(I+a*x)^
2/(a^2*x^2+1))-3*arccot(a*x)*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a
^2*x^2+1)-1)^2)^3-3*arccot(a*x)*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2
/(a^2*x^2+1)-1)^2)^2*csgn(I*(I+a*x)^2/(a^2*x^2+1))+3*arccot(a*x)*Pi*csgn(I
*(I+a*x)^2/(a^2*x^2+1))^3-6*arccot(a*x)*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1))^2
*csgn(I*(I+a*x)/(a^2*x^2+1)^(1/2))+3*arccot(a*x)*Pi*csgn(I*(I+a*x)^2/(a^2*
x^2+1))*csgn(I*(I+a*x)/(a^2*x^2+1)^(1/2))^2+6*arccot(a*x)*Pi*csgn(I*(I+a*x
)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)^2+4*arccot(a*x)^2-6*Pi*arccot
(a*x)+6*I*arccot(a*x)+12*I*arccot(a*x)*ln(2)-12+12*I*a*x)-ln((I+a*x)/(a^2*
x^2+1)^(1/2)-1)-ln(1+(I+a*x)/(a^2*x^2+1)^(1/2))+arccot(a*x)^2*ln(1-(I+a*x)
/(a^2*x^2+1)^(1/2))-2*I*arccot(a*x)*polylog(2,(I+a*x)/(a^2*x^2+1)^(1/2))+2
*polylog(3,(I+a*x)/(a^2*x^2+1)^(1/2))+arccot(a*x)^2*ln(1+(I+a*x)/(a^2*x...
```

3.26.5 Fricas [F]

$$\int x^2 \cot^{-1}(ax)^3 \, dx = \int x^2 \operatorname{arccot}(ax)^3 \, dx$$

input
```
integrate(x^2*arccot(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^2*arccot(a*x)^3, x)
```

3.26.6 Sympy [F]

$$\int x^2 \cot^{-1}(ax)^3 \, dx = \int x^2 \operatorname{acot}^3(ax) \, dx$$

input
```
integrate(x**2*acot(a*x)**3,x)
```

output
```
Integral(x**2*acot(a*x)**3, x)
```

3.26.7 Maxima [F]

$$\int x^2 \cot^{-1}(ax)^3 \, dx = \int x^2 \operatorname{arccot}(ax)^3 \, dx$$

input
```
integrate(x^2*arccot(a*x)^3,x, algorithm="maxima")
```

output
```
1/24*x^3*arctan2(1, a*x)^3 - 1/32*x^3*arctan2(1, a*x)*log(a^2*x^2 + 1)^2 +
 integrate(1/32*(28*a^2*x^4*arctan2(1, a*x)^3 + 4*a^2*x^4*arctan2(1, a*x)*
log(a^2*x^2 + 1) + 4*a*x^3*arctan2(1, a*x)^2 + 28*x^2*arctan2(1, a*x)^3 +
(3*a^2*x^4*arctan2(1, a*x) - a*x^3 + 3*x^2*arctan2(1, a*x))*log(a^2*x^2 +
1)^2)/(a^2*x^2 + 1), x)
```

3.26.8 Giac [F]

$$\int x^2 \cot^{-1}(ax)^3\,dx = \int x^2 \operatorname{arccot}\left(ax\right)^3\,dx$$

input
```
integrate(x^2*arccot(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^2*arccot(a*x)^3, x)
```

3.26.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(ax)^3\,dx = \int x^2\,\mathrm{acot}(a\,x)^3\,dx$$

input
```
int(x^2*acot(a*x)^3,x)
```

output
```
int(x^2*acot(a*x)^3, x)
```

3.27 $\int x \cot^{-1}(ax)^3 \, dx$

3.27.1 Optimal result

Integrand size = 8, antiderivative size = 103

$$\int x \cot^{-1}(ax)^3 \, dx = \frac{3i \cot^{-1}(ax)^2}{2a^2} + \frac{3x \cot^{-1}(ax)^2}{2a} + \frac{\cot^{-1}(ax)^3}{2a^2} + \frac{1}{2}x^2 \cot^{-1}(ax)^3 - \frac{3 \cot^{-1}(ax) \log\left(\frac{2}{1+iax}\right)}{a^2} + \frac{3i \operatorname{PolyLog}\left(2, 1 - \frac{2}{1+iax}\right)}{2a^2}$$

output
```
3/2*I*arccot(a*x)^2/a^2+3/2*x*arccot(a*x)^2/a+1/2*arccot(a*x)^3/a^2+1/2*x^
2*arccot(a*x)^3-3*arccot(a*x)*ln(2/(1+I*a*x))/a^2+3/2*I*polylog(2,1-2/(1+I
*a*x))/a^2
```

3.27.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.74

$$\int x \cot^{-1}(ax)^3 \, dx = \frac{\cot^{-1}(ax) \left(3(i+ax) \cot^{-1}(ax) + (1+a^2x^2) \cot^{-1}(ax)^2 - 6 \log\left(1 - e^{2i \cot^{-1}(ax)}\right)\right) + 3i \operatorname{PolyLog}\left(2, e^{2i \cot^{-1}(ax)}\right)}{2a^2}$$

input
```
Integrate[x*ArcCot[a*x]^3,x]
```

output
```
(ArcCot[a*x]*(3*(I + a*x)*ArcCot[a*x] + (1 + a^2*x^2)*ArcCot[a*x]^2 - 6*Lo
g[1 - E^((2*I)*ArcCot[a*x])]) + (3*I)*PolyLog[2, E^((2*I)*ArcCot[a*x])])/(
2*a^2)
```

3.27.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {5362, 5452, 5346, 5420, 5456, 5380, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \cot^{-1}(ax)^3 \, dx$$

$$\Big\downarrow \text{5362}$$

$$\frac{3}{2}a\int \frac{x^2 \cot^{-1}(ax)^2}{a^2x^2+1}dx+\frac{1}{2}x^2\cot^{-1}(ax)^3$$

$$\Big\downarrow \text{5452}$$

$$\frac{3}{2}a\left(\frac{\int \cot^{-1}(ax)^2dx}{a^2}-\frac{\int \frac{\cot^{-1}(ax)^2}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{2}x^2\cot^{-1}(ax)^3$$

$$\Big\downarrow \text{5346}$$

$$\frac{3}{2}a\left(\frac{2a\int \frac{x\cot^{-1}(ax)}{a^2x^2+1}dx+x\cot^{-1}(ax)^2}{a^2}-\frac{\int \frac{\cot^{-1}(ax)^2}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{2}x^2\cot^{-1}(ax)^3$$

$$\Big\downarrow \text{5420}$$

$$\frac{3}{2}a\left(\frac{2a\int \frac{x\cot^{-1}(ax)}{a^2x^2+1}dx+x\cot^{-1}(ax)^2}{a^2}+\frac{\cot^{-1}(ax)^3}{3a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)^3$$

$$\Big\downarrow \text{5456}$$

$$\frac{1}{2}x^2\cot^{-1}(ax)^3+\frac{3}{2}a\left(\frac{\cot^{-1}(ax)^3}{3a^3}+\frac{x\cot^{-1}(ax)^2+2a\left(\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\int \frac{\cot^{-1}(ax)}{i-ax}dx}{a}\right)}{a^2}\right)$$

$$\Big\downarrow \text{5380}$$

$$\frac{1}{2}x^2\cot^{-1}(ax)^3+\frac{3}{2}a\left(\frac{\cot^{-1}(ax)^3}{3a^3}+\frac{x\cot^{-1}(ax)^2+2a\left(\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\int\frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx+\frac{\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)}{a}}{a}\right)}{a^2}\right)$$

$$\Big\downarrow \text{2849}$$

$$\frac{1}{2}x^2\cot^{-1}(ax)^3+\frac{3}{2}a\left(\frac{\cot^{-1}(ax)^3}{3a^3}+\frac{x\cot^{-1}(ax)^2+2a\left(\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\frac{\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)}{a}-\frac{i\int\frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}}{a}\right)}{a^2}\right)$$

$$\Big\downarrow \text{2752}$$

$$\frac{1}{2}x^2\cot^{-1}(ax)^3+\frac{3}{2}a\left(\frac{\cot^{-1}(ax)^3}{3a^3}+\frac{x\cot^{-1}(ax)^2+2a\left(\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\frac{\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}\right)}{a^2}\right)$$

input
```
Int[x*ArcCot[a*x]^3,x]
```

output
```
(x^2*ArcCot[a*x]^3)/2 + (3*a*(ArcCot[a*x]^3/(3*a^3) + (x*ArcCot[a*x]^2 + 2
*a*(((I/2)*ArcCot[a*x]^2)/a^2 - ((ArcCot[a*x]*Log[2/(1 + I*a*x)])/a - ((I/
2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/a))/a^2))/2
```

3.27. $\int x\cot^{-1}(ax)^3\,dx$

3.27.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5380
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Simp[b*c*(
p/e)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5420
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5452
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5456
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Simp[
1/(c*d)   Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.27.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 591 vs. 2(89) = 178.

Time = 5.90 (sec) , antiderivative size = 592, normalized size of antiderivative = 5.75

method	result
risch	$\frac{3i\pi \arctan(ax)}{16a^2} + \frac{3i \ln\left(\frac{1}{2}-\frac{iax}{2}\right) \ln\left(\frac{1}{2}+\frac{iax}{2}\right)}{2a^2} - \frac{3i \ln\left(\frac{1}{2}+\frac{iax}{2}\right) \ln(-iax+1)}{2a^2} - \frac{i(a^2x^2+1) \ln(iax+1)^3}{16a^2} - \frac{3i\pi^2 \ln(-ia}{16}$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(x*arccot(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
3/16*I/a^2*Pi*arctan(a*x)+(-3/16*I*(a^2*x^2+1)/a^2*ln(1-I*a*x)^2+3/8*x*(Pi
*a*x+2)/a*ln(1-I*a*x)+3/16*(I*Pi^2*a^2*x^2+4*I*Pi*a*x+4*I*ln(1-I*a*x)+2*ln
(1-I*a*x)*Pi)/a^2)*ln(1+I*a*x)+1/16/a^2*Pi^3+3/8*I/a^2*Pi^2+1/16*Pi^3*x^2+
3/8*Pi^2*x/a-3/16*Pi*ln(1-I*a*x)^2*x^2-3/16/a*ln(1-I*a*x)^2*x-3/16/a*ln(1-
I*a*x)*x-3/16/a^2*Pi*ln(1-I*a*x)^2+3/16/a^2*Pi*ln(1-I*a*x)-3/16*(-I*x^2*ln
(1-I*a*x)*a^2+Pi*a^2*x^2-I*ln(1-I*a*x)+2*a*x-2*I+Pi)/a^2*ln(1+I*a*x)^2+21/
32*arctan(a*x)/a^2+3/2*I/a^2*ln(1/2-1/2*I*a*x)*ln(1/2+1/2*I*a*x)-3/2*I/a^2
*ln(1/2+1/2*I*a*x)*ln(1-I*a*x)-1/16*I*(a^2*x^2+1)/a^2*ln(1+I*a*x)^3-3/16*I
*Pi^2*ln(1-I*a*x)*x^2-3/4*I/a*Pi*ln(1-I*a*x)*x+3/32*I/a^2*(1-I*a*x)^2*ln(1
-I*a*x)-3/32*I/a^2*(1-I*a*x)^2*ln(1-I*a*x)^2+1/16*I*ln(1-I*a*x)^3*x^2-9/32
*I/a^2*ln(1-I*a*x)^2+1/16*I/a^2*ln(1-I*a*x)^3-3/32*I*ln(1-I*a*x)^2*x^2+3/3
2*I*ln(1-I*a*x)*x^2-3/4*I/a^2*ln(1-I*a*x)+3/2*I/a^2*dilog(1/2-1/2*I*a*x)+2
1/64*I/a^2*ln(a^2*x^2+1)+21/32/a^2*Pi*ln(a^2*x^2+1)-3/8*Pi^2/a^2*arctan(a*
x)
```

3.27.5 Fricas [F]

$$\int x\cot^{-1}(ax)^3\,dx = \int x\,\mathrm{arccot}\left(ax\right)^3\,dx$$

input
```
integrate(x*arccot(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x*arccot(a*x)^3, x)
```

3.27.6 Sympy [F]

$$\int x\cot^{-1}(ax)^3\,dx = \int x\,\mathrm{acot}^3\left(ax\right)\,dx$$

input
```
integrate(x*acot(a*x)**3,x)
```

output
```
Integral(x*acot(a*x)**3, x)
```

3.27.7 Maxima [F]

$$\int x\cot^{-1}(ax)^3\,dx = \int x\,\mathrm{arccot}\left(ax\right)^3\,dx$$

input
```
integrate(x*arccot(a*x)^3,x, algorithm="maxima")
```

output
```
1/32*(8*a^2*x^2*arctan2(1, a*x)^3 + 12*a*x*arctan2(1, a*x)^2 - 3*a*x*log(a
^2*x^2 + 1)^2 + 4*(128*a^3*integrate(1/32*x^3*arctan(1/(a*x))^3/(a^3*x^2 +
 a), x) + 96*a^2*integrate(1/32*x^2*arctan(1/(a*x))^2/(a^3*x^2 + a), x) +
24*a^2*integrate(1/32*x^2*log(a^2*x^2 + 1)^2/(a^3*x^2 + a), x) + 96*a^2*in
tegrate(1/32*x^2*log(a^2*x^2 + 1)/(a^3*x^2 + a), x) + 128*a*integrate(1/32
*x*arctan(1/(a*x))^3/(a^3*x^2 + a), x) + 192*a*integrate(1/32*x*arctan(1/(
a*x))/(a^3*x^2 + a), x) + arctan(a*x)^3/a^2 + 3*arctan(a*x)^2*arctan(1/(a*
x))/a^2 + 3*arctan(a*x)*arctan(1/(a*x))^2/a^2 + 24*integrate(1/32*log(a^2*
x^2 + 1)^2/(a^3*x^2 + a), x))*a^2 + 8*arctan2(1, a*x)^3)/a^2
```

3.27.8 Giac [F]

$$\int x \cot^{-1}(ax)^3 \, dx = \int x \operatorname{arccot}(ax)^3 \, dx$$

input
```
integrate(x*arccot(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x*arccot(a*x)^3, x)
```

3.27.9 Mupad [F(-1)]

Timed out.

$$\int x \cot^{-1}(ax)^3 \, dx = \int x \operatorname{acot}(a\,x)^3 \, dx$$

input
```
int(x*acot(a*x)^3,x)
```

output
```
int(x*acot(a*x)^3, x)
```

3.28 $\int \cot^{-1}(ax)^3 \, dx$

3.28.1 Optimal result

Integrand size = 6, antiderivative size = 96

$$\int \cot^{-1}(ax)^3 \, dx = \frac{i\cot^{-1}(ax)^3}{a} + x\cot^{-1}(ax)^3 - \frac{3\cot^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a} + \frac{3i\cot^{-1}(ax)\,\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a} - \frac{3\,\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{2a}$$

output
```
I*arccot(a*x)^3/a+x*arccot(a*x)^3-3*arccot(a*x)^2*ln(2/(1+I*a*x))/a+3*I*ar
ccot(a*x)*polylog(2,1-2/(1+I*a*x))/a-3/2*polylog(3,1-2/(1+I*a*x))/a
```

3.28.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.94

$$\int \cot^{-1}(ax)^3 \, dx = -\frac{i\cot^{-1}(ax)^3}{a} + x\cot^{-1}(ax)^3 - \frac{3\cot^{-1}(ax)^2\log\left(1-e^{-2i\cot^{-1}(ax)}\right)}{a} - \frac{3i\cot^{-1}(ax)\,\text{PolyLog}\left(2,e^{-2i\cot^{-1}(ax)}\right)}{a} - \frac{3\,\text{PolyLog}\left(3,e^{-2i\cot^{-1}(ax)}\right)}{2a}$$

input
```
Integrate[ArcCot[a*x]^3,x]
```

output
```
((-I)*ArcCot[a*x]^3)/a + x*ArcCot[a*x]^3 - (3*ArcCot[a*x]^2*Log[1 - E^((-2
*I)*ArcCot[a*x])])/a - ((3*I)*ArcCot[a*x]*PolyLog[2, E^((-2*I)*ArcCot[a*x]
)])/a - (3*PolyLog[3, E^((-2*I)*ArcCot[a*x])])/(2*a)
```

3.28.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5346, 5456, 5380, 5530, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}(ax)^3 \, dx$$

$$\Big\downarrow \text{5346}$$

$$3a\int \frac{x\cot^{-1}(ax)^2}{a^2x^2+1}dx+x\cot^{-1}(ax)^3$$

$$\Big\downarrow \text{5456}$$

$$x\cot^{-1}(ax)^3+3a\left(\frac{i\cot^{-1}(ax)^3}{3a^2}-\frac{\int \frac{\cot^{-1}(ax)^2}{i-ax}dx}{a}\right)$$

$$\Big\downarrow \text{5380}$$

$$x\cot^{-1}(ax)^3+3a\left(\frac{i\cot^{-1}(ax)^3}{3a^2}-\frac{2\int \frac{\cot^{-1}(ax)\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx+\frac{\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)^2}{a}}{a}\right)$$

$$\Big\downarrow \text{5530}$$

$$x\cot^{-1}(ax)^3+3a\left(\frac{i\cot^{-1}(ax)^3}{3a^2}-\frac{2\left(-\frac{1}{2}i\int \frac{\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)\cot^{-1}(ax)}{2a}\right)+\frac{\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)^2}{a}}{a}\right)$$

$$\Big\downarrow \text{7164}$$

$$x\cot^{-1}(ax)^3+3a\left(\frac{i\cot^{-1}(ax)^3}{3a^2}-\frac{2\left(\frac{\text{PolyLog}\left(3,1-\frac{2}{iax+1}\right)}{4a}-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)\cot^{-1}(ax)}{2a}\right)+\frac{\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)^2}{a}}{a}\right)$$

input
```
Int[ArcCot[a*x]^3,x]
```

output
```
x*ArcCot[a*x]^3 + 3*a*(((I/3)*ArcCot[a*x]^3)/a^2 - ((ArcCot[a*x]^2*Log[2/(
1 + I*a*x)])/a + 2*(((-1/2*I)*ArcCot[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a
 + PolyLog[3, 1 - 2/(1 + I*a*x)]/(4*a)))/a)
```

3.28.3.1 Defintions of rubi rules used

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5380
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Simp[b*c*(
p/e)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5456
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Simp[
1/(c*d)   Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5530
```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] - Simp[b*p*(I/2)   Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.28.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(89) = 178$.

Time = 1.15 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.95

method	result
derivativedivides	$\frac{\operatorname{arccot}(ax)^3(ax-i)+2i\operatorname{arccot}(ax)^3-3\operatorname{arccot}(ax)^2\ln\left(1-\frac{ax+i}{\sqrt{a^2x^2+1}}\right)+6i\operatorname{arccot}(ax)\operatorname{polylog}\left(2,\frac{ax+i}{\sqrt{a^2x^2+1}}\right)-6\operatorname{polylog}\left(\right.}{a}$
default	$\frac{\operatorname{arccot}(ax)^3(ax-i)+2i\operatorname{arccot}(ax)^3-3\operatorname{arccot}(ax)^2\ln\left(1-\frac{ax+i}{\sqrt{a^2x^2+1}}\right)+6i\operatorname{arccot}(ax)\operatorname{polylog}\left(2,\frac{ax+i}{\sqrt{a^2x^2+1}}\right)-6\operatorname{polylog}\left(\right.}{a}$

input
```
int(arccot(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a*(arccot(a*x)^3*(a*x-I)+2*I*arccot(a*x)^3-3*arccot(a*x)^2*ln(1-(I+a*x)/
(a^2*x^2+1)^(1/2))+6*I*arccot(a*x)*polylog(2,(I+a*x)/(a^2*x^2+1)^(1/2))-6*
polylog(3,(I+a*x)/(a^2*x^2+1)^(1/2))-3*arccot(a*x)^2*ln(1+(I+a*x)/(a^2*x^2
+1)^(1/2))+6*I*arccot(a*x)*polylog(2,-(I+a*x)/(a^2*x^2+1)^(1/2))-6*polylog
(3,-(I+a*x)/(a^2*x^2+1)^(1/2)))
```

3.28.5 Fricas [F]

$$\int \cot^{-1}(ax)^3 \, dx = \int \operatorname{arccot}(ax)^3 \, dx$$

input
```
integrate(arccot(a*x)^3,x, algorithm="fricas")
```

output
```
integral(arccot(a*x)^3, x)
```

3.28.6 Sympy [F]

$$\int \cot^{-1}(ax)^3 \, dx = \int \operatorname{acot}^3(ax) \, dx$$

input
```
integrate(acot(a*x)**3,x)
```

output
```
Integral(acot(a*x)**3, x)
```

3.28.7 Maxima [F]

$$\int \cot^{-1}(ax)^3 \, dx = \int \operatorname{arccot}(ax)^3 \, dx$$

input
```
integrate(arccot(a*x)^3,x, algorithm="maxima")
```

output
```
1/8*x*arctan2(1, a*x)^3 - 3/32*x*arctan2(1, a*x)*log(a^2*x^2 + 1)^2 + 21/1
6*arctan(a*x)^2*arctan(1/(a*x))^2/a + 7/8*arctan(a*x)*arctan(1/(a*x))^3/a
+ 28*a^2*integrate(1/32*x^2*arctan(1/(a*x))^3/(a^2*x^2 + 1), x) + 3*a^2*in
tegrate(1/32*x^2*arctan(1/(a*x))*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 12
*a^2*integrate(1/32*x^2*arctan(1/(a*x))*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x)
 + 12*a*integrate(1/32*x*arctan(1/(a*x))^2/(a^2*x^2 + 1), x) - 3*a*integra
te(1/32*x*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 7/32*(a*arctan(a*x)^4 + 4
*a*arctan(a*x)^3*arctan(1/(a*x)))/a^2 + 3*integrate(1/32*arctan(1/(a*x))*l
og(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x)
```

3.28.8 Giac [F]

$$\int \cot^{-1}(ax)^3 \, dx = \int \operatorname{arccot}(ax)^3 \, dx$$

input
```
integrate(arccot(a*x)^3,x, algorithm="giac")
```

output
```
integrate(arccot(a*x)^3, x)
```

3.28.9 Mupad [F(-1)]

Timed out.

$$\int \cot^{-1}(ax)^3\,dx = \int \operatorname{acot}(a\,x)^3\,dx$$

input
```
int(acot(a*x)^3,x)
```

output
```
int(acot(a*x)^3, x)
```

3.29 $\int \frac{\cot^{-1}(ax)^3}{x} dx$

3.29.1 Optimal result

Integrand size = 10, antiderivative size = 178

$$\begin{aligned}\int \frac{\cot^{-1}(ax)^3}{x} dx =& 2\cot^{-1}(ax)^3 \coth^{-1}\left(1-\frac{2}{1+iax}\right) \\ &- \frac{3}{2}i\cot^{-1}(ax)^2 \operatorname{PolyLog}\left(2,1-\frac{2i}{i+ax}\right) \\ &+ \frac{3}{2}i\cot^{-1}(ax)^2 \operatorname{PolyLog}\left(2,1-\frac{2ax}{i+ax}\right) \\ &- \frac{3}{2}\cot^{-1}(ax) \operatorname{PolyLog}\left(3,1-\frac{2i}{i+ax}\right) \\ &+ \frac{3}{2}\cot^{-1}(ax) \operatorname{PolyLog}\left(3,1-\frac{2ax}{i+ax}\right) \\ &+ \frac{3}{4}i \operatorname{PolyLog}\left(4,1-\frac{2i}{i+ax}\right) - \frac{3}{4}i \operatorname{PolyLog}\left(4,1-\frac{2ax}{i+ax}\right)\end{aligned}$$

output
```
2*arccot(a*x)^3*arccoth(1-2/(1+I*a*x))-3/2*I*arccot(a*x)^2*polylog(2,1-2*I
/(I+a*x))+3/2*I*arccot(a*x)^2*polylog(2,1-2*a*x/(I+a*x))-3/2*arccot(a*x)*p
olylog(3,1-2*I/(I+a*x))+3/2*arccot(a*x)*polylog(3,1-2*a*x/(I+a*x))+3/4*I*p
olylog(4,1-2*I/(I+a*x))-3/4*I*polylog(4,1-2*a*x/(I+a*x))
```

3.29.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.01

$$\int \frac{\cot^{-1}(ax)^3}{x} \, dx = \frac{1}{64} i \left(\pi^4 - 32 \cot^{-1}(ax)^4 + 64 i \cot^{-1}(ax)^3 \log \left(1 - e^{-2 i \cot^{-1}(ax)}\right) - 64 i \cot^{-1}(ax)^3 \log \left(1 + e^{2 i \cot^{-1}(ax)}\right) - 96 \cot^{-1}(ax)^2 \operatorname{PolyLog}\left(2, e^{-2 i \cot^{-1}(ax)}\right) - 96 \cot^{-1}(ax)^2 \operatorname{PolyLog}\left(2, -e^{2 i \cot^{-1}(ax)}\right) + 96 i \cot^{-1}(ax) \operatorname{PolyLog}\left(3, e^{-2 i \cot^{-1}(ax)}\right) - 96 i \cot^{-1}(ax) \operatorname{PolyLog}\left(3, -e^{2 i \cot^{-1}(ax)}\right) + 48 \operatorname{PolyLog}\left(4, e^{-2 i \cot^{-1}(ax)}\right) + 48 \operatorname{PolyLog}\left(4, -e^{2 i \cot^{-1}(ax)}\right)\right)$$

input
```
Integrate[ArcCot[a*x]^3/x,x]
```

output
```
(I/64)*(Pi^4 - 32*ArcCot[a*x]^4 + (64*I)*ArcCot[a*x]^3*Log[1 - E^((-2*I)*A
rcCot[a*x])] - (64*I)*ArcCot[a*x]^3*Log[1 + E^((2*I)*ArcCot[a*x])] - 96*Ar
cCot[a*x]^2*PolyLog[2, E^((-2*I)*ArcCot[a*x])] - 96*ArcCot[a*x]^2*PolyLog[
2, -E^((2*I)*ArcCot[a*x])] + (96*I)*ArcCot[a*x]*PolyLog[3, E^((-2*I)*ArcCo
t[a*x])] - (96*I)*ArcCot[a*x]*PolyLog[3, -E^((2*I)*ArcCot[a*x])] + 48*Poly
Log[4, E^((-2*I)*ArcCot[a*x])] + 48*PolyLog[4, -E^((2*I)*ArcCot[a*x])])
```

3.29.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5358, 5524, 5528, 5532, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax)^3}{x} \, dx$$

$$\Big\downarrow \; 5358$$

$$6a \int \frac{\cot^{-1}(ax)^2 \coth^{-1}\left(1 - \frac{2}{iax+1}\right)}{a^2x^2+1} dx + 2 \cot^{-1}(ax)^3 \coth^{-1}\left(1 - \frac{2}{1+iax}\right)$$

$$\Big\downarrow \text{5524}$$

$$6a\left(\frac{1}{2}\int \frac{\cot^{-1}(ax)^2\log\left(\frac{2ax}{ax+i}\right)}{a^2x^2+1}dx-\frac{1}{2}\int \frac{\cot^{-1}(ax)^2\log\left(\frac{2i}{ax+i}\right)}{a^2x^2+1}dx\right)+$$
$$2\cot^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1+iax}\right)$$

$$\Big\downarrow \text{5528}$$

$$6a\left(\frac{1}{2}\left(-i\int \frac{\cot^{-1}(ax)\,\text{PolyLog}\left(2,1-\frac{2i}{ax+i}\right)}{a^2x^2+1}dx-\frac{i\,\text{PolyLog}\left(2,1-\frac{2i}{ax+i}\right)\cot^{-1}(ax)^2}{2a}\right)+\frac{1}{2}\left(i\int \frac{\cot^{-1}(ax)\,\text{Po}\ldots}{\ldots}\right.\right.$$
$$2\cot^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1+iax}\right)$$

$$\Big\downarrow \text{5532}$$

$$6a\left(\frac{1}{2}\left(-i\left(-\frac{1}{2}i\int \frac{\text{PolyLog}\left(3,1-\frac{2i}{ax+i}\right)}{a^2x^2+1}dx-\frac{i\,\text{PolyLog}\left(3,1-\frac{2i}{ax+i}\right)\cot^{-1}(ax)}{2a}\right)-\frac{i\,\text{PolyLog}\left(2,1-\frac{2i}{ax+i}\right)\text{c}\ldots}{2a}\right.\right.$$
$$2\cot^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1+iax}\right)$$

$$\Big\downarrow \text{7164}$$

$$6a\left(\frac{1}{2}\left(-\frac{i\,\text{PolyLog}\left(2,1-\frac{2i}{ax+i}\right)\cot^{-1}(ax)^2}{2a}-i\left(-\frac{\text{PolyLog}\left(4,1-\frac{2i}{ax+i}\right)}{4a}-\frac{i\,\text{PolyLog}\left(3,1-\frac{2i}{ax+i}\right)\cot^{-1}(ax)}{2a}\ldots\right.\right.\right.$$
$$2\cot^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1+iax}\right)$$

input
```
Int[ArcCot[a*x]^3/x,x]
```

output
```
2*ArcCot[a*x]^3*ArcCoth[1 - 2/(1 + I*a*x)] + 6*a*((((-1/2*I)*ArcCot[a*x]^2
*PolyLog[2, 1 - (2*I)/(I + a*x)])/a - I*(((-1/2*I)*ArcCot[a*x]*PolyLog[3,
1 - (2*I)/(I + a*x)])/a - PolyLog[4, 1 - (2*I)/(I + a*x)]/(4*a)))/2 + (((I
/2)*ArcCot[a*x]^2*PolyLog[2, 1 - (2*a*x)/(I + a*x)])/a + I*(((-1/2*I)*ArcC
ot[a*x]*PolyLog[3, 1 - (2*a*x)/(I + a*x)])/a - PolyLog[4, 1 - (2*a*x)/(I +
 a*x)]/(4*a)))/2)
```

3.29. $\int \frac{\cot^{-1}(ax)^3}{x}\,dx$

3.29.3.1 Defintions of rubi rules used

rule 5358
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcCot[c*x])^p*ArcCoth[1 - 2/(1 + I*c*x)], x] + Simp[2*b*c*p   Int[(a + b
*ArcCot[c*x])^(p - 1)*(ArcCoth[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
 FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 5524
```
Int[(ArcCoth[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[1/2   Int[Log[SimplifyIntegrand[1 + 1/u, x]]*((a +
 b*ArcCot[c*x])^p/(d + e*x^2)), x], x] - Simp[1/2   Int[Log[SimplifyIntegra
nd[1 - 1/u, x]]*((a + b*ArcCot[c*x])^p/(d + e*x^2)), x], x] /; FreeQ[{a, b,
 c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*
x)))^2, 0]
```

rule 5528
```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] + Simp[b*p*(I/2)   Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 5532
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(-I)*(a + b*ArcCot[c*x])^p*(PolyLog[k + 1, u]/
(2*c*d)), x] - Simp[b*p*(I/2)   Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[k
+ 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0]
&& EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.29.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.61 (sec) , antiderivative size = 982, normalized size of antiderivative = 5.52

method	result	size
derivativedivides	Expression too large to display	982
default	Expression too large to display	982
parts	Expression too large to display	1417

input
```
int(arccot(a*x)^3/x,x,method=_RETURNVERBOSE)
```

output
```
ln(a*x)*arccot(a*x)^3+1/2*I*Pi*(csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*(
1+(I+a*x)^2/(a^2*x^2+1)))*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a
^2*x^2+1)))-csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I/((I+a*x)^2/(a^2*x^2+1
)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^2-csgn(I*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn(I
/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^2+csgn(I/((I+a*x)^2/
(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^3-csgn(I/((I+a*x)^2/(a^2*x^2+1)-
1)*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn(1/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^
2/(a^2*x^2+1)))^2+csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1
)))*csgn(1/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))-csgn(1/((I
+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^3+csgn(1/((I+a*x)^2/(a^2
*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^2-1)*arccot(a*x)^3+arccot(a*x)^3*ln(
(I+a*x)^2/(a^2*x^2+1)-1)-arccot(a*x)^3*ln(1-(I+a*x)/(a^2*x^2+1)^(1/2))+3*I
*arccot(a*x)^2*polylog(2,(I+a*x)/(a^2*x^2+1)^(1/2))-6*arccot(a*x)*polylog(
3,(I+a*x)/(a^2*x^2+1)^(1/2))-6*I*polylog(4,(I+a*x)/(a^2*x^2+1)^(1/2))-arcc
ot(a*x)^3*ln(1+(I+a*x)/(a^2*x^2+1)^(1/2))+3*I*arccot(a*x)^2*polylog(2,-(I+
a*x)/(a^2*x^2+1)^(1/2))-6*arccot(a*x)*polylog(3,-(I+a*x)/(a^2*x^2+1)^(1/2)
)-6*I*polylog(4,-(I+a*x)/(a^2*x^2+1)^(1/2))-3/2*I*arccot(a*x)^2*polylog(2,
-(I+a*x)^2/(a^2*x^2+1))+3/2*arccot(a*x)*polylog(3,-(I+a*x)^2/(a^2*x^2+1))+
3/4*I*polylog(4,-(I+a*x)^2/(a^2*x^2+1))
```

3.29.5 Fricas [F]

$$\int \frac{\cot^{-1}(ax)^3}{x} \, dx = \int \frac{\operatorname{arccot}(ax)^3}{x} \, dx$$

input
```
integrate(arccot(a*x)^3/x,x, algorithm="fricas")
```

output
```
integral(arccot(a*x)^3/x, x)
```

3.29.6 Sympy [F]

$$\int \frac{\cot^{-1}(ax)^3}{x}\,dx = \int \frac{\operatorname{acot}^3(ax)}{x}\,dx$$

input
```
integrate(acot(a*x)**3/x,x)
```

output
```
Integral(acot(a*x)**3/x, x)
```

3.29.7 Maxima [F]

$$\int \frac{\cot^{-1}(ax)^3}{x}\,dx = \int \frac{\operatorname{arccot}(ax)^3}{x}\,dx$$

input
```
integrate(arccot(a*x)^3/x,x, algorithm="maxima")
```

output
```
integrate(arccot(a*x)^3/x, x)
```

3.29.8 Giac [F]

$$\int \frac{\cot^{-1}(ax)^3}{x}\,dx = \int \frac{\operatorname{arccot}(ax)^3}{x}\,dx$$

input
```
integrate(arccot(a*x)^3/x,x, algorithm="giac")
```

output
```
integrate(arccot(a*x)^3/x, x)
```

3.29.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)^3}{x} \, dx = \int \frac{\operatorname{acot}(a\,x)^3}{x} \, dx$$

input
```
int(acot(a*x)^3/x,x)
```

output
```
int(acot(a*x)^3/x, x)
```

3.30 $\int \frac{\cot^{-1}(ax)^3}{x^2} dx$

3.30.1 Optimal result

Integrand size = 10, antiderivative size = 93

$$\int \frac{\cot^{-1}(ax)^3}{x^2} dx = -ia\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{x} - 3a\cot^{-1}(ax)^2\log\left(2 - \frac{2}{1-iax}\right) - 3ia\cot^{-1}(ax)\,\text{PolyLog}\left(2, -1 + \frac{2}{1-iax}\right) - \frac{3}{2}a\,\text{PolyLog}\left(3, -1 + \frac{2}{1-iax}\right)$$

output
```
-I*a*arccot(a*x)^3-arccot(a*x)^3/x-3*a*arccot(a*x)^2*ln(2-2/(1-I*a*x))-3*I
*a*arccot(a*x)*polylog(2,-1+2/(1-I*a*x))-3/2*a*polylog(3,-1+2/(1-I*a*x))
```

3.30.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.89

$$\int \frac{\cot^{-1}(ax)^3}{x^2} dx = \frac{(-1+iax)\cot^{-1}(ax)^3}{x} - 3a\cot^{-1}(ax)^2\log\left(1+e^{2i\cot^{-1}(ax)}\right) + 3ia\cot^{-1}(ax)\,\text{PolyLog}\left(2, -e^{2i\cot^{-1}(ax)}\right) - \frac{3}{2}a\,\text{PolyLog}\left(3, -e^{2i\cot^{-1}(ax)}\right)$$

input
```
Integrate[ArcCot[a*x]^3/x^2,x]
```

output
```
((-1 + I*a*x)*ArcCot[a*x]^3)/x - 3*a*ArcCot[a*x]^2*Log[1 + E^((2*I)*ArcCot
[a*x])] + (3*I)*a*ArcCot[a*x]*PolyLog[2, -E^((2*I)*ArcCot[a*x])] - (3*a*Po
lyLog[3, -E^((2*I)*ArcCot[a*x])])/2
```

3.30.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5362, 5460, 5404, 5528, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax)^3}{x^2}dx$$

$$\Big\downarrow \text{5362}$$

$$-3a\int \frac{\cot^{-1}(ax)^2}{x\left(a^2x^2+1\right)}dx-\frac{\cot^{-1}(ax)^3}{x}$$

$$\Big\downarrow \text{5460}$$

$$-\frac{\cot^{-1}(ax)^3}{x}-3a\left(i\int \frac{\cot^{-1}(ax)^2}{x(ax+i)}dx+\frac{1}{3}i\cot^{-1}(ax)^3\right)$$

$$\Big\downarrow \text{5404}$$

$$-\frac{\cot^{-1}(ax)^3}{x}-$$
$$3a\left(i\left(-2ia\int \frac{\cot^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\log\left(2-\frac{2}{1-iax}\right)\cot^{-1}(ax)^2\right)+\frac{1}{3}i\cot^{-1}(ax)^3\right)$$

$$\Big\downarrow \text{5528}$$

$$-\frac{\cot^{-1}(ax)^3}{x}-$$
$$3a\left(i\left(-2ia\left(\frac{1}{2}i\int \frac{\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx+\frac{i\,\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\cot^{-1}(ax)}{2a}\right)-i\log\left(2-\frac{2}{1-iax}\right)\text{cot}\right.\right.$$

$$\Big\downarrow \text{7164}$$

$$-\frac{\cot^{-1}(ax)^3}{x} - 3a\left(i\left(-2ia\left(\frac{\text{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{4a}+\frac{i\,\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\cot^{-1}(ax)}{2a}\right)-i\log\left(2-\frac{2}{1-iax}\right)\cot^{-1}(ax)^2\right.\right.$$

input
```
Int[ArcCot[a*x]^3/x^2,x]
```

output
```
-(ArcCot[a*x]^3/x) - 3*a*((I/3)*ArcCot[a*x]^3 + I*((-I)*ArcCot[a*x]^2*Log[
2 - 2/(1 - I*a*x)] - (2*I)*a*(((I/2)*ArcCot[a*x]*PolyLog[2, -1 + 2/(1 - I*
a*x)])/a + PolyLog[3, -1 + 2/(1 - I*a*x)]/(4*a))))
```

3.30.3.1 Defintions of rubi rules used

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5404
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcCot[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] + Si
mp[b*c*(p/d)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5460
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*d*(p + 1))), x] + Simp[
I/d   Int[(a + b*ArcCot[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5528
```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] + Simp[b*p*(I/2)   Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.30.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 8.94 (sec) , antiderivative size = 1441, normalized size of antiderivative = 15.49

method	result	size
parts	Expression too large to display	1441
derivativedivides	Expression too large to display	1444
default	Expression too large to display	1444

input
```
int(arccot(a*x)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
-arccot(a*x)^3/x-3*a*(-1/2*arccot(a*x)^2*ln(a^2*x^2+1)+arccot(a*x)^2*ln(a*
x)+arccot(a*x)^2*ln((I+a*x)/(a^2*x^2+1)^(1/2))-1/3*I*arccot(a*x)^3+1/4*(-2
*I*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(
1+(I+a*x)^2/(a^2*x^2+1)))^2+I*Pi*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1))^2*csgn(
I*((I+a*x)^2/(a^2*x^2+1)-1)^2)-I*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1))^3+2*I*Pi
*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn(1/((I+a*
x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))-2*I*Pi*csgn(I*(1+(I+a*x)^2/
(a^2*x^2+1)))*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^
2+2*I*Pi*csgn(I*(I+a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(I+a*x)^2/(a^2*x^2+1))^2
+I*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)^2)*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I
+a*x)^2/(a^2*x^2+1)-1)^2)^2-2*I*Pi*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1))*csgn(
I*((I+a*x)^2/(a^2*x^2+1)-1)^2)^2-2*I*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(
1+(I+a*x)^2/(a^2*x^2+1)))*csgn(1/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a
^2*x^2+1)))^2+2*I*Pi*csgn(1/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^
2+1)))^2-I*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)^2)*csgn(I*(I+a*x)^2/(a^2*x^
2+1))*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)+I*Pi*csgn(
I*(I+a*x)^2/(a^2*x^2+1))*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+
1)-1)^2)^2+2*I*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1
)))^3+I*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)^3+2*I
*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*(1+(I+a*x)^2/(a^2*x^2+1)))...
```

3.30.5 Fricas [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^2}\,dx = \int \frac{\operatorname{arccot}(ax)^3}{x^2}\,dx$$

input
```
integrate(arccot(a*x)^3/x^2,x, algorithm="fricas")
```

output
```
integral(arccot(a*x)^3/x^2, x)
```

3.30.6 Sympy [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^2}\,dx = \int \frac{\operatorname{acot}^3(ax)}{x^2}\,dx$$

input
```
integrate(acot(a*x)**3/x**2,x)
```

output
```
Integral(acot(a*x)**3/x**2, x)
```

3.30.7 Maxima [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^2}\,dx = \int \frac{\operatorname{arccot}(ax)^3}{x^2}\,dx$$

input
```
integrate(arccot(a*x)^3/x^2,x, algorithm="maxima")
```

output
```
-1/32*(4*arctan2(1, a*x)^3 - 3*arctan2(1, a*x)*log(a^2*x^2 + 1)^2 - (28*a*
arctan(a*x)*arctan(1/(a*x))^3 + 7*(6*arctan(a*x)^2*arctan(1/(a*x))^2/a + (
a*arctan(a*x)^4 + 4*a*arctan(a*x)^3*arctan(1/(a*x)))/a^2)*a^2 + 96*a^2*int
egrate(1/32*x^2*arctan(1/(a*x))*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) - 3
84*a^2*integrate(1/32*x^2*arctan(1/(a*x))*log(a^2*x^2 + 1)/(a^2*x^4 + x^2)
, x) - 384*a*integrate(1/32*x*arctan(1/(a*x))^2/(a^2*x^4 + x^2), x) + 96*a
*integrate(1/32*x*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) + 896*integrate(1
/32*arctan(1/(a*x))^3/(a^2*x^4 + x^2), x) + 96*integrate(1/32*arctan(1/(a*
x))*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x))*x)/x
```

3.30.8 Giac [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^2}\,dx = \int \frac{\operatorname{arccot}\left(ax\right)^3}{x^2}\,dx$$

input
```
integrate(arccot(a*x)^3/x^2,x, algorithm="giac")
```

output
```
integrate(arccot(a*x)^3/x^2, x)
```

3.30.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)^3}{x^2}\,dx = \int \frac{\operatorname{acot}(a\,x)^3}{x^2}\,dx$$

input
```
int(acot(a*x)^3/x^2,x)
```

output
```
int(acot(a*x)^3/x^2, x)
```

3.31 $\int \frac{\cot^{-1}(ax)^3}{x^3} dx$

3.31.1 Optimal result

Integrand size = 10, antiderivative size = 105

$$\int \frac{\cot^{-1}(ax)^3}{x^3} dx = \frac{3}{2}ia^2\cot^{-1}(ax)^2 + \frac{3a\cot^{-1}(ax)^2}{2x} - \frac{1}{2}a^2\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{2x^2} + 3a^2\cot^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right) + \frac{3}{2}ia^2\,\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)$$

output
```
3/2*I*a^2*arccot(a*x)^2+3/2*a*arccot(a*x)^2/x-1/2*a^2*arccot(a*x)^3-1/2*ar
ccot(a*x)^3/x^2+3*a^2*arccot(a*x)*ln(2-2/(1-I*a*x))+3/2*I*a^2*polylog(2,-1
+2/(1-I*a*x))
```

3.31.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.86

$$\int \frac{\cot^{-1}(ax)^3}{x^3} dx = -\frac{\cot^{-1}(ax)\left(3iax(i+ax)\cot^{-1}(ax) + (1+a^2x^2)\cot^{-1}(ax)^2 - 6a^2x^2\log\left(1+e^{2i\cot^{-1}(ax)}\right)\right)}{2x^2} - \frac{3}{2}ia^2\,\text{PolyLog}\left(2,-e^{2i\cot^{-1}(ax)}\right)$$

input
```
Integrate[ArcCot[a*x]^3/x^3,x]
```

output
```
-1/2*(ArcCot[a*x]*((3*I)*a*x*(I + a*x)*ArcCot[a*x] + (1 + a^2*x^2)*ArcCot[
a*x]^2 - 6*a^2*x^2*Log[1 + E^((2*I)*ArcCot[a*x])]))/x^2 - ((3*I)/2)*a^2*Po
lyLog[2, -E^((2*I)*ArcCot[a*x])]
```

3.31.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5362, 5454, 5362, 5420, 5460, 5404, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax)^3}{x^3}dx$$

$$\Big\downarrow \text{ 5362}$$

$$-\frac{3}{2}a\int \frac{\cot^{-1}(ax)^2}{x^2(a^2x^2+1)}dx-\frac{\cot^{-1}(ax)^3}{2x^2}$$

$$\Big\downarrow \text{ 5454}$$

$$-\frac{3}{2}a\left(\int \frac{\cot^{-1}(ax)^2}{x^2}dx-a^2\int \frac{\cot^{-1}(ax)^2}{a^2x^2+1}dx\right)-\frac{\cot^{-1}(ax)^3}{2x^2}$$

$$\Big\downarrow \text{ 5362}$$

$$-\frac{3}{2}a\left(a^2\left(-\int \frac{\cot^{-1}(ax)^2}{a^2x^2+1}dx\right)-2a\int \frac{\cot^{-1}(ax)}{x(a^2x^2+1)}dx-\frac{\cot^{-1}(ax)^2}{x}\right)-\frac{\cot^{-1}(ax)^3}{2x^2}$$

$$\Big\downarrow \text{ 5420}$$

$$-\frac{3}{2}a\left(-2a\int \frac{\cot^{-1}(ax)}{x(a^2x^2+1)}dx+\frac{1}{3}a\cot^{-1}(ax)^3-\frac{\cot^{-1}(ax)^2}{x}\right)-\frac{\cot^{-1}(ax)^3}{2x^2}$$

$$\Big\downarrow \text{ 5460}$$

$$-\frac{\cot^{-1}(ax)^3}{2x^2}-\frac{3}{2}a\left(-2a\left(i\int \frac{\cot^{-1}(ax)}{x(ax+i)}dx+\frac{1}{2}i\cot^{-1}(ax)^2\right)+\frac{1}{3}a\cot^{-1}(ax)^3-\frac{\cot^{-1}(ax)^2}{x}\right)$$

$$\Big\downarrow \text{ 5404}$$

$$-\frac{\cot^{-1}(ax)^3}{2x^2}-$$

$$\frac{3}{2}a\left(-2a\left(i\left(-ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\log\left(2-\frac{2}{1-iax}\right)\cot^{-1}(ax)\right)+\frac{1}{2}i\cot^{-1}(ax)^2\right)+\frac{1}{3}a\cot^{-1}(ax)^3-\right.$$

$$\Big\downarrow \text{2897}$$

$$-\frac{\cot^{-1}(ax)^3}{2x^2} - \frac{3}{2}a\left(-2a\left(i\left(\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)-i\log\left(2-\frac{2}{1-iax}\right)\cot^{-1}(ax)\right)+\frac{1}{2}i\cot^{-1}(ax)^2\right)+\frac{1}{3}a\cot^{-1}(ax)^3\right.$$

input
```
Int[ArcCot[a*x]^3/x^3,x]
```

output
```
-1/2*ArcCot[a*x]^3/x^2 - (3*a*(-(ArcCot[a*x]^2/x) + (a*ArcCot[a*x]^3)/3 -
2*a*((I/2)*ArcCot[a*x]^2 + I*((-I)*ArcCot[a*x]*Log[2 - 2/(1 - I*a*x)] + Po
lyLog[2, -1 + 2/(1 - I*a*x)]/2))))/2
```

3.31.3.1 Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5404
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcCot[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] + Si
mp[b*c*(p/d)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5420
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5454
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcCot[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcCot[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5460
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*d*(p + 1))), x] + Simp[
I/d   Int[(a + b*ArcCot[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.31.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.76 (sec) , antiderivative size = 2956, normalized size of antiderivative = 28.15

method	result	size
parts	Expression too large to display	2956
derivativedivides	Expression too large to display	2957
default	Expression too large to display	2957

input
```
int(arccot(a*x)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*arccot(a*x)^3/x^2-3/2*a^2*(1/2*Pi*arccot(a*x)^2-1/2*I*Pi*arccot(a*x)*
ln(1+I*(I+a*x)/(a^2*x^2+1)^(1/2))-1/2*I*Pi*arccot(a*x)*ln(1-I*(I+a*x)/(a^2
*x^2+1)^(1/2))+1/2*I*Pi*arccot(a*x)*ln(1+(I+a*x)^2/(a^2*x^2+1))+1/8*Pi*csg
n(I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*(I+a*x)^2/(a^2*x^2+1))*csgn(I*(I+a*x
)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))*(2*I*arccot(a*x)*ln(1+(I+a*x)^2
/(a^2*x^2+1))+2*arccot(a*x)^2+polylog(2,-(I+a*x)^2/(a^2*x^2+1)))-1/4*Pi*cs
gn(I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*(I+a*x)^2/(a^2*x^2+1))*csgn(I*(I+a*
x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))*(I*arccot(a*x)*ln(1+I*(I+a*x)/
(a^2*x^2+1)^(1/2))+I*arccot(a*x)*ln(1-I*(I+a*x)/(a^2*x^2+1)^(1/2))+dilog(1
+I*(I+a*x)/(a^2*x^2+1)^(1/2))+dilog(1-I*(I+a*x)/(a^2*x^2+1)^(1/2)))+1/8*Pi
*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))^3*(2*I*arccot(a*x
)*ln(1+(I+a*x)^2/(a^2*x^2+1))+2*arccot(a*x)^2+polylog(2,-(I+a*x)^2/(a^2*x^
2+1)))-1/4*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))^3*(I
*arccot(a*x)*ln(1+I*(I+a*x)/(a^2*x^2+1)^(1/2))+I*arccot(a*x)*ln(1-I*(I+a*x
)/(a^2*x^2+1)^(1/2))+dilog(1+I*(I+a*x)/(a^2*x^2+1)^(1/2))+dilog(1-I*(I+a*x
)/(a^2*x^2+1)^(1/2)))+1/2*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))^2*(I*arccot
(a*x)*ln(1+I*(I+a*x)/(a^2*x^2+1)^(1/2))+I*arccot(a*x)*ln(1-I*(I+a*x)/(a^2*
x^2+1)^(1/2))+dilog(1+I*(I+a*x)/(a^2*x^2+1)^(1/2))+dilog(1-I*(I+a*x)/(a^2*
x^2+1)^(1/2)))+1/4*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))^3*(2*I*arccot(a*x)
*ln(1+(I+a*x)^2/(a^2*x^2+1))+2*arccot(a*x)^2+polylog(2,-(I+a*x)^2/(a^2*...
```

3.31.5 Fricas [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^3}\, dx = \int \frac{\operatorname{arccot}(ax)^3}{x^3}\, dx$$

input
```
integrate(arccot(a*x)^3/x^3,x, algorithm="fricas")
```

output
```
integral(arccot(a*x)^3/x^3, x)
```

3.31.6 Sympy [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^3} \, dx = \int \frac{\operatorname{acot}^3(ax)}{x^3} \, dx$$

input
```
integrate(acot(a*x)**3/x**3,x)
```

output
```
Integral(acot(a*x)**3/x**3, x)
```

3.31.7 Maxima [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^3} \, dx = \int \frac{\operatorname{arccot}(ax)^3}{x^3} \, dx$$

input
```
integrate(arccot(a*x)^3/x^3,x, algorithm="maxima")
```

output
```
-1/32*(8*a^2*x^2*arctan2(1, a*x)^3 - 12*a*x*arctan2(1, a*x)^2 + 3*a*x*log(
a^2*x^2 + 1)^2 + 4*(3*a^2*arctan(a*x)*arctan(1/(a*x))^2 + (arctan(a*x)^3/a
 + 3*arctan(a*x)^2*arctan(1/(a*x))/a)*a^3 + 24*a^3*integrate(1/32*x^3*log(
a^2*x^2 + 1)^2/(a^2*x^5 + x^3), x) - 96*a^3*integrate(1/32*x^3*log(a^2*x^2
 + 1)/(a^2*x^5 + x^3), x) - 128*a^2*integrate(1/32*x^2*arctan(1/(a*x))^3/(
a^2*x^5 + x^3), x) - 192*a^2*integrate(1/32*x^2*arctan(1/(a*x))/(a^2*x^5 +
 x^3), x) + 96*a*integrate(1/32*x*arctan(1/(a*x))^2/(a^2*x^5 + x^3), x) +
24*a*integrate(1/32*x*log(a^2*x^2 + 1)^2/(a^2*x^5 + x^3), x) - 128*integra
te(1/32*arctan(1/(a*x))^3/(a^2*x^5 + x^3), x))*x^2 + 8*arctan2(1, a*x)^3)/
x^2
```

3.31.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.28

$$\int \frac{\cot^{-1}(ax)^3}{x^3} \, dx = -\frac{1}{2} \, a \arctan\left(\frac{1}{ax}\right)^3 - \frac{\arctan\left(\frac{1}{ax}\right)^3}{2 \, x^2}$$

input
```
integrate(arccot(a*x)^3/x^3,x, algorithm="giac")
```

output
```
-1/2*a*arctan(1/(a*x))^3 - 1/2*arctan(1/(a*x))^3/x^2
```

3.31.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)^3}{x^3}\,dx = \int \frac{\mathrm{acot}(a\,x)^3}{x^3}\,dx$$

input
```
int(acot(a*x)^3/x^3,x)
```

output
```
int(acot(a*x)^3/x^3, x)
```

3.32 $\int \frac{\cot^{-1}(ax)^3}{x^4} dx$

3.32.1 Optimal result

Integrand size = 10, antiderivative size = 167

$$\int \frac{\cot^{-1}(ax)^3}{x^4} dx = -\frac{a^2 \cot^{-1}(ax)}{x} + \frac{1}{2}a^3 \cot^{-1}(ax)^2 + \frac{a \cot^{-1}(ax)^2}{2x^2} + \frac{1}{3}ia^3 \cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{3x^3} - a^3 \log(x) + \frac{1}{2}a^3 \log\left(1+a^2x^2\right) + a^3 \cot^{-1}(ax)^2 \log\left(2 - \frac{2}{1-iax}\right) + ia^3 \cot^{-1}(ax) \operatorname{PolyLog}\left(2, -1 + \frac{2}{1-iax}\right) + \frac{1}{2}a^3 \operatorname{PolyLog}\left(3, -1 + \frac{2}{1-iax}\right)$$

output
```
-a^2*arccot(a*x)/x+1/2*a^3*arccot(a*x)^2+1/2*a*arccot(a*x)^2/x^2+1/3*I*a^3
*arccot(a*x)^3-1/3*arccot(a*x)^3/x^3-a^3*ln(x)+1/2*a^3*ln(a^2*x^2+1)+a^3*a
rccot(a*x)^2*ln(2-2/(1-I*a*x))+I*a^3*arccot(a*x)*polylog(2,-1+2/(1-I*a*x))
+1/2*a^3*polylog(3,-1+2/(1-I*a*x))
```

3.32.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.90

$$\int \frac{\cot^{-1}(ax)^3}{x^4}\,dx = \frac{1}{6}\left(-\frac{6a^2\cot^{-1}(ax)}{x}+3a^3\cot^{-1}(ax)^2+\frac{3a\cot^{-1}(ax)^2}{x^2}-2ia^3\cot^{-1}(ax)^3 - \frac{2\cot^{-1}(ax)^3}{x^3}+6a^3\cot^{-1}(ax)^2\log\left(1+e^{2i\cot^{-1}(ax)}\right) - 6a^3\log\left(\frac{1}{\sqrt{1+\frac{1}{a^2x^2}}}\right)-6ia^3\cot^{-1}(ax)\,\text{PolyLog}\left(2,-e^{2i\cot^{-1}(ax)}\right) + 3a^3\,\text{PolyLog}\left(3,-e^{2i\cot^{-1}(ax)}\right)\right)$$

input
```
Integrate[ArcCot[a*x]^3/x^4,x]
```

output
```
((-6*a^2*ArcCot[a*x])/x + 3*a^3*ArcCot[a*x]^2 + (3*a*ArcCot[a*x]^2)/x^2 -
(2*I)*a^3*ArcCot[a*x]^3 - (2*ArcCot[a*x]^3)/x^3 + 6*a^3*ArcCot[a*x]^2*Log[
1 + E^((2*I)*ArcCot[a*x])] - 6*a^3*Log[1/Sqrt[1 + 1/(a^2*x^2)]] - (6*I)*a^
3*ArcCot[a*x]*PolyLog[2, -E^((2*I)*ArcCot[a*x])] + 3*a^3*PolyLog[3, -E^((2
*I)*ArcCot[a*x])])/6
```

3.32.3 Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.10, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.400$, Rules used = {5362, 5454, 5362, 5454, 5362, 243, 47, 14, 16, 5420, 5460, 5404, 5528, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax)^3}{x^4}\,dx$$

$$\Big\downarrow \text{5362}$$

$$-a\int \frac{\cot^{-1}(ax)^2}{x^3\left(a^2x^2+1\right)}dx-\frac{\cot^{-1}(ax)^3}{3x^3}$$

$$\Big\downarrow \text{5454}$$

$$-a\left(\int \frac{\cot^{-1}(ax)^2}{x^3}dx - a^2\int \frac{\cot^{-1}(ax)^2}{x\left(a^2x^2+1\right)}dx\right) - \frac{\cot^{-1}(ax)^3}{3x^3}$$

$$\Big\downarrow \; 5362$$

$$-a\left(a^2\left(-\int \frac{\cot^{-1}(ax)^2}{x\left(a^2x^2+1\right)}dx\right) - a\int \frac{\cot^{-1}(ax)}{x^2\left(a^2x^2+1\right)}dx - \frac{\cot^{-1}(ax)^2}{2x^2}\right) - \frac{\cot^{-1}(ax)^3}{3x^3}$$

$$\Big\downarrow \; 5454$$

$$-a\left(a^2\left(-\int \frac{\cot^{-1}(ax)^2}{x\left(a^2x^2+1\right)}dx\right) - a\left(\int \frac{\cot^{-1}(ax)}{x^2}dx - a^2\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx\right) - \frac{\cot^{-1}(ax)^2}{2x^2}\right) - \frac{\cot^{-1}(ax)^3}{3x^3}$$

$$\Big\downarrow \; 5362$$

$$-a\left(a^2\left(-\int \frac{\cot^{-1}(ax)^2}{x\left(a^2x^2+1\right)}dx\right) - a\left(-a\int \frac{1}{x\left(a^2x^2+1\right)}dx + a^2\left(-\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx\right) - \frac{\cot^{-1}(ax)}{x}\right) - \frac{\cot^{-1}(ax)^2}{2x^2}\right. \frac{\cot^{-1}(ax)^3}{3x^3}$$

$$\Big\downarrow \; 243$$

$$-a\left(a^2\left(-\int \frac{\cot^{-1}(ax)^2}{x\left(a^2x^2+1\right)}dx\right) - a\left(-\frac{1}{2}a\int \frac{1}{x^2\left(a^2x^2+1\right)}dx^2 + a^2\left(-\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx\right) - \frac{\cot^{-1}(ax)}{x}\right) - \frac{\cot^{-1}(\right. \frac{\cot^{-1}(ax)^3}{3x^3}$$

$$\Big\downarrow \; 47$$

$$-a\left(a^2\left(-\int \frac{\cot^{-1}(ax)^2}{x\left(a^2x^2+1\right)}dx\right) - a\left(-\frac{1}{2}a\left(\int \frac{1}{x^2}dx^2 - a^2\int \frac{1}{a^2x^2+1}dx^2\right) + a^2\left(-\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx\right) - \frac{\cot^{-1}(ax}{x}\right.\right. \frac{\cot^{-1}(ax)^3}{3x^3}$$

$$\Big\downarrow \; 14$$

$$-a\left(a^2\left(-\int \frac{\cot^{-1}(ax)^2}{x\left(a^2x^2+1\right)}dx\right) - a\left(-\frac{1}{2}a\left(\log\left(x^2\right) - a^2\int \frac{1}{a^2x^2+1}dx^2\right) + a^2\left(-\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx\right) - \frac{\cot^{-1}(ax)}{x}\right.\right. \frac{\cot^{-1}(ax)^3}{3x^3}$$

$$\Big\downarrow \; 16$$

3.32. $\int \frac{\cot^{-1}(ax)^3}{x^4}\,dx$

$$-a\left(a^2\left(-\int \frac{\cot^{-1}(ax)^2}{x\left(a^2x^2+1\right)}dx\right)-a\left(a^2\left(-\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx\right)-\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{\cot^{-1}(ax)}{x}\right)-\text{co}\right.$$
$$\frac{\cot^{-1}(ax)^3}{3x^3}$$

$\big\downarrow$ 5420

$$-a\left(a^2\left(-\int \frac{\cot^{-1}(ax)^2}{x\left(a^2x^2+1\right)}dx\right)-a\left(-\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)+\frac{1}{2}a\cot^{-1}(ax)^2-\frac{\cot^{-1}(ax)}{x}\right)-\frac{\cot^{-1}(ax}{2x^2}\right.$$
$$\frac{\cot^{-1}(ax)^3}{3x^3}$$

$\big\downarrow$ 5460

$$-\frac{\cot^{-1}(ax)^3}{3x^3}-$$
$$a\left(-\left(a^2\left(i\int \frac{\cot^{-1}(ax)^2}{x(ax+i)}dx+\frac{1}{3}i\cot^{-1}(ax)^3\right)\right)-a\left(-\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)+\frac{1}{2}a\cot^{-1}(ax)^2-\frac{\cot^{-1}(}{x}\right.\right.$$

$\big\downarrow$ 5404

$$-\frac{\cot^{-1}(ax)^3}{3x^3}-$$
$$a\left(-\left(a^2\left(i\left(-2ia\int \frac{\cot^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\log\left(2-\frac{2}{1-iax}\right)\cot^{-1}(ax)^2\right)+\frac{1}{3}i\cot^{-1}(ax)^3\right)\right)-a\right.$$

$\big\downarrow$ 5528

$$-\frac{\cot^{-1}(ax)^3}{3x^3}-$$
$$a\left(-\left(a^2\left(i\left(-2ia\left(\frac{1}{2}i\int \frac{\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx+\frac{i\,\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\cot^{-1}(ax)}{2a}\right)-i\log\left(2-\frac{2}{1-ia}\right.\right.\right.\right.\right.$$

$\big\downarrow$ 7164

$$-\frac{\cot^{-1}(ax)^3}{3x^3}-$$
$$a\left(-\left(a^2\left(i\left(-2ia\left(\frac{\text{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{4a}+\frac{i\,\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\cot^{-1}(ax)}{2a}\right)-i\log\left(2-\frac{2}{1-iax}\right)\cot^{-}\right.\right.\right.\right.$$

input
```
Int[ArcCot[a*x]^3/x^4,x]
```

3.32. $\int \frac{\cot^{-1}(ax)^3}{x^4}\,dx$

output
```
-1/3*ArcCot[a*x]^3/x^3 - a*(-1/2*ArcCot[a*x]^2/x^2 - a*(-(ArcCot[a*x]/x) +
 (a*ArcCot[a*x]^2)/2 - (a*(Log[x^2] - Log[1 + a^2*x^2]))/2) - a^2*((I/3)*A
rcCot[a*x]^3 + I*((-I)*ArcCot[a*x]^2*Log[2 - 2/(1 - I*a*x)] - (2*I)*a*(((I
/2)*ArcCot[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/a + PolyLog[3, -1 + 2/(1 -
 I*a*x)]/(4*a)))))
```

3.32.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5404
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcCot[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] + Si
mp[b*c*(p/d)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5420
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5454
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcCot[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcCot[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5460
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*d*(p + 1))), x] + Simp[
I/d   Int[(a + b*ArcCot[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5528
```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] + Simp[b*p*(I/2)   Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.32.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 25.04 (sec) , antiderivative size = 1622, normalized size of antiderivative = 9.71

method	result	size
parts	Expression too large to display	1622
derivativedivides	Expression too large to display	1623
default	Expression too large to display	1623

input
```
int(arccot(a*x)^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*arccot(a*x)^3/x^3-a^3*(-1/2/a^2/x^2*arccot(a*x)^2-arccot(a*x)^2*ln(a*
x)+1/2*arccot(a*x)^2*ln(a^2*x^2+1)-arccot(a*x)^2*ln((I+a*x)/(a^2*x^2+1)^(1
/2))+I*arccot(a*x)*polylog(2,-(I+a*x)^2/(a^2*x^2+1))-1/2*polylog(3,-(I+a*x
)^2/(a^2*x^2+1))+1/12*arccot(a*x)*(4*I*arccot(a*x)^2*a*x-3*I*arccot(a*x)*P
i*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1))^2*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1)^2)*
a*x+6*I*arccot(a*x)*Pi*csgn(I*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn(I/((I+a*x)^2
/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^2*a*x+3*I*arccot(a*x)*Pi*csgn(I
*(I+a*x)^2/(a^2*x^2+1))^3*a*x-6*I*arccot(a*x)*Pi*csgn(I/((I+a*x)^2/(a^2*x^
2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn(1/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+
a*x)^2/(a^2*x^2+1)))*a*x-3*I*arccot(a*x)*Pi*csgn(I*((I+a*x)^2/(a^2*x^2+1)-
1)^2)^3*a*x-6*I*arccot(a*x)*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1))^2*csgn(I*(I+a
*x)/(a^2*x^2+1)^(1/2))*a*x+6*I*arccot(a*x)*Pi*csgn(I*((I+a*x)^2/(a^2*x^2+1
)-1))*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1)^2)^2*a*x+6*I*arccot(a*x)*Pi*csgn(1/
((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^3*a*x-12*I*a*x-6*I*ar
ccot(a*x)*Pi*csgn(I*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn(I/((I+a*x)^2/(a^2*x^2+
1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*a*x+6*I
*arccot(a*x)*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1))
)^2*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*a*x-3*I*arccot(a*x)*Pi*csgn(I*(I+a*x
)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)^3*a*x-6*I*arccot(a*x)*Pi*csgn
(1/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^2*a*x+6*I*arcco...
```

3.32.5 Fricas [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^4} \, dx = \int \frac{\operatorname{arccot}(ax)^3}{x^4} \, dx$$

input
```
integrate(arccot(a*x)^3/x^4,x, algorithm="fricas")
```

output
```
integral(arccot(a*x)^3/x^4, x)
```

3.32.6 Sympy [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^4}\,dx = \int \frac{\operatorname{acot}^3(ax)}{x^4}\,dx$$

input
```
integrate(acot(a*x)**3/x**4,x)
```

output
```
Integral(acot(a*x)**3/x**4, x)
```

3.32.7 Maxima [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^4}\,dx = \int \frac{\operatorname{arccot}(ax)^3}{x^4}\,dx$$

input
```
integrate(arccot(a*x)^3/x^4,x, algorithm="maxima")
```

output
```
1/96*(96*x^3*integrate(1/32*(28*a^2*x^2*arctan2(1, a*x)^3 - 4*a^2*x^2*arct
an2(1, a*x)*log(a^2*x^2 + 1) - 4*a*x*arctan2(1, a*x)^2 + 28*arctan2(1, a*x
)^3 + (3*a^2*x^2*arctan2(1, a*x) + a*x + 3*arctan2(1, a*x))*log(a^2*x^2 +
1)^2)/(a^2*x^6 + x^4), x) - 4*arctan2(1, a*x)^3 + 3*arctan2(1, a*x)*log(a^
2*x^2 + 1)^2)/x^3
```

3.32.8 Giac [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^4}\,dx = \int \frac{\operatorname{arccot}(ax)^3}{x^4}\,dx$$

input
```
integrate(arccot(a*x)^3/x^4,x, algorithm="giac")
```

output
```
integrate(arccot(a*x)^3/x^4, x)
```

3.32.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)^3}{x^4}\,dx = \int \frac{\mathrm{acot}(a\,x)^3}{x^4}\,dx$$

input
```
int(acot(a*x)^3/x^4,x)
```

output
```
int(acot(a*x)^3/x^4, x)
```

3.33 $\int \frac{\cot^{-1}(ax)^3}{x^5} dx$

3.33.1 Optimal result

Integrand size = 10, antiderivative size = 152

$$\begin{aligned}\int \frac{\cot^{-1}(ax)^3}{x^5} dx = {} & \frac{a^3}{4x} - \frac{a^2\cot^{-1}(ax)}{4x^2} - ia^4\cot^{-1}(ax)^2 + \frac{a\cot^{-1}(ax)^2}{4x^3} \\ & - \frac{3a^3\cot^{-1}(ax)^2}{4x} + \frac{1}{4}a^4\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{4x^4} + \frac{1}{4}a^4\arctan(ax) \\ & - 2a^4\cot^{-1}(ax)\log\left(2 - \frac{2}{1-iax}\right) - ia^4\,\mathrm{PolyLog}\left(2, -1 + \frac{2}{1-iax}\right)\end{aligned}$$

output
```
1/4*a^3/x-1/4*a^2*arccot(a*x)/x^2-I*a^4*arccot(a*x)^2+1/4*a*arccot(a*x)^2/
x^3-3/4*a^3*arccot(a*x)^2/x+1/4*a^4*arccot(a*x)^3-1/4*arccot(a*x)^3/x^4+1/
4*a^4*arctan(a*x)-2*a^4*arccot(a*x)*ln(2-2/(1-I*a*x))-I*a^4*polylog(2,-1+2
/(1-I*a*x))
```

3.33.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.83

$$\int \frac{\cot^{-1}(ax)^3}{x^5} dx$$
$$= \frac{a^3x^3 + (ax - 3a^3x^3 + 4ia^4x^4)\cot^{-1}(ax)^2 + (-1 + a^4x^4)\cot^{-1}(ax)^3 - a^2x^2\cot^{-1}(ax)\left(1 + a^2x^2 + 8a^2x^2 \right.}{4x^4}$$

input
```
Integrate[ArcCot[a*x]^3/x^5,x]
```

output
```
(a^3*x^3 + (a*x - 3*a^3*x^3 + (4*I)*a^4*x^4)*ArcCot[a*x]^2 + (-1 + a^4*x^4
)*ArcCot[a*x]^3 - a^2*x^2*ArcCot[a*x]*(1 + a^2*x^2 + 8*a^2*x^2*Log[1 + E^(
(2*I)*ArcCot[a*x])]) + (4*I)*a^4*x^4*PolyLog[2, -E^((2*I)*ArcCot[a*x])])/(
4*x^4)
```

3.33.3 Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.49, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules used = {5362, 5454, 5362, 5454, 5362, 264, 216, 5420, 5460, 5404, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax)^3}{x^5}dx$$

$$\Big\downarrow \text{5362}$$

$$-\frac{3}{4}a\int \frac{\cot^{-1}(ax)^2}{x^4(a^2x^2+1)}dx-\frac{\cot^{-1}(ax)^3}{4x^4}$$

$$\Big\downarrow \text{5454}$$

$$-\frac{3}{4}a\left(\int \frac{\cot^{-1}(ax)^2}{x^4}dx-a^2\int \frac{\cot^{-1}(ax)^2}{x^2(a^2x^2+1)}dx\right)-\frac{\cot^{-1}(ax)^3}{4x^4}$$

$$\Big\downarrow \text{5362}$$

$$-\frac{3}{4}a\left(a^2\left(-\int \frac{\cot^{-1}(ax)^2}{x^2(a^2x^2+1)}dx\right)-\frac{2}{3}a\int \frac{\cot^{-1}(ax)}{x^3(a^2x^2+1)}dx-\frac{\cot^{-1}(ax)^2}{3x^3}\right)-\frac{\cot^{-1}(ax)^3}{4x^4}$$

$$\Big\downarrow \text{5454}$$

$$-\frac{3}{4}a\left(-\left(a^2\left(\int \frac{\cot^{-1}(ax)^2}{x^2}dx-a^2\int \frac{\cot^{-1}(ax)^2}{a^2x^2+1}dx\right)\right)-\frac{2}{3}a\left(\int \frac{\cot^{-1}(ax)}{x^3}dx-a^2\int \frac{\cot^{-1}(ax)}{x(a^2x^2+1)}dx\right)-\frac{\cot^{-1}}{3}\right.$$
$$\frac{\cot^{-1}(ax)^3}{4x^4}$$

$$\Big\downarrow \text{5362}$$

$$-\frac{3}{4}a\left(-\left(a^2\left(a^2\left(-\int \frac{\cot^{-1}(ax)^2}{a^2x^2+1}dx\right)-2a\int \frac{\cot^{-1}(ax)}{x\left(a^2x^2+1\right)}dx-\frac{\cot^{-1}(ax)^2}{x}\right)\right)-\frac{2}{3}a\left(-\frac{1}{2}a\int \frac{1}{x^2\left(a^2x^2+1\right)}dx -\right.\right.$$
$$\frac{\cot^{-1}(ax)^3}{4x^4}$$

$$\Big\downarrow \; 264$$

$$-\frac{3}{4}a\left(-\left(a^2\left(a^2\left(-\int \frac{\cot^{-1}(ax)^2}{a^2x^2+1}dx\right)-2a\int \frac{\cot^{-1}(ax)}{x\left(a^2x^2+1\right)}dx-\frac{\cot^{-1}(ax)^2}{x}\right)\right)-\frac{2}{3}a\left(-\frac{1}{2}a\left(a^2\left(-\int \frac{1}{a^2x^2+1}d\right.\right.\right.\right.$$
$$\frac{\cot^{-1}(ax)^3}{4x^4}$$

$$\Big\downarrow \; 216$$

$$-\frac{3}{4}a\left(-\frac{2}{3}a\left(a^2\left(-\int \frac{\cot^{-1}(ax)}{x\left(a^2x^2+1\right)}dx\right)-\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)-\frac{\cot^{-1}(ax)}{2x^2}\right)-\left(a^2\left(a^2\left(-\int \frac{\cot^{-1}(ax)^2}{a^2x^2+1}dx\right.\right.\right.\right.$$
$$\frac{\cot^{-1}(ax)^3}{4x^4}$$

$$\Big\downarrow \; 5420$$

$$-\frac{3}{4}a\left(-\frac{2}{3}a\left(a^2\left(-\int \frac{\cot^{-1}(ax)}{x\left(a^2x^2+1\right)}dx\right)-\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)-\frac{\cot^{-1}(ax)}{2x^2}\right)-\left(a^2\left(-2a\int \frac{\cot^{-1}(ax)}{x\left(a^2x^2+1\right)}dx\right.\right.\right.$$
$$\frac{\cot^{-1}(ax)^3}{4x^4}$$

$$\Big\downarrow \; 5460$$

$$-\frac{\cot^{-1}(ax)^3}{4x^4}-$$
$$\frac{3}{4}a\left(-\frac{2}{3}a\left(-\left(a^2\left(i\int \frac{\cot^{-1}(ax)}{x(ax+i)}dx+\frac{1}{2}i\cot^{-1}(ax)^2\right)\right)-\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)-\frac{\cot^{-1}(ax)}{2x^2}\right)-\left(a^2\left(-2a\left(\right.\right.\right.\right.$$

$$\Big\downarrow \; 5404$$

$$-\frac{\cot^{-1}(ax)^3}{4x^4}-$$
$$\frac{3}{4}a\left(-\frac{2}{3}a\left(-\left(a^2\left(i\left(-ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\log\left(2-\frac{2}{1-iax}\right)\cot^{-1}(ax)\right)+\frac{1}{2}i\cot^{-1}(ax)^2\right)\right)-\frac{1}{2}a\left(-\right.\right.\right.$$

$$\Big\downarrow \; 2897$$

$$-\frac{\cot^{-1}(ax)^3}{4x^4}-$$
$$\frac{3}{4}a\left(-\frac{2}{3}a\left(-\left(a^2\left(i\left(\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)-i\log\left(2-\frac{2}{1-iax}\right)\cot^{-1}(ax)\right)+\frac{1}{2}i\cot^{-1}(ax)^2\right)\right)-\frac{1}{2}a\left(-\right.\right.\right.$$

3.33. $\int \frac{\cot^{-1}(ax)^3}{x^5}\,dx$

input
```
Int[ArcCot[a*x]^3/x^5,x]
```

output
```
-1/4*ArcCot[a*x]^3/x^4 - (3*a*(-1/3*ArcCot[a*x]^2/x^3 - a^2*(-(ArcCot[a*x]
^2/x) + (a*ArcCot[a*x]^3)/3 - 2*a*((I/2)*ArcCot[a*x]^2 + I*((-I)*ArcCot[a*
x]*Log[2 - 2/(1 - I*a*x)] + PolyLog[2, -1 + 2/(1 - I*a*x)]/2))) - (2*a*(-1
/2*ArcCot[a*x]/x^2 - (a*(-x^(-1) - a*ArcTan[a*x]))/2 - a^2*((I/2)*ArcCot[a
*x]^2 + I*((-I)*ArcCot[a*x]*Log[2 - 2/(1 - I*a*x)] + PolyLog[2, -1 + 2/(1
- I*a*x)]/2))))/3))/4
```

3.33.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5404
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcCot[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] + Si
mp[b*c*(p/d)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5420
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5454
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcCot[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcCot[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5460
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*d*(p + 1))), x] + Simp[
I/d   Int[(a + b*ArcCot[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.33.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 10.74 (sec) , antiderivative size = 852, normalized size of antiderivative = 5.61

method	result	size
parts	Expression too large to display	852
derivativedivides	Expression too large to display	855
default	Expression too large to display	855

input
```
int(arccot(a*x)^3/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*arccot(a*x)^3/x^4-3/4*a^4*(-1/3/a^3/x^3*arccot(a*x)^2+1/x*arccot(a*x)
^2/a+arccot(a*x)^2*arctan(a*x)-1/3*arccot(a*x)^3-1/2*Pi*arccot(a*x)^2+8/3*
arccot(a*x)*ln(1+I*(I+a*x)/(a^2*x^2+1)^(1/2))+8/3*arccot(a*x)*ln(1-I*(I+a*
x)/(a^2*x^2+1)^(1/2))-1/3*I/a/x*(a*x-I)-1/4*Pi*csgn(I*(I+a*x)/(a^2*x^2+1)^
(1/2))^2*csgn(I*(I+a*x)^2/(a^2*x^2+1))*arccot(a*x)^2+1/4*Pi*csgn(I/((I+a*x
)^2/(a^2*x^2+1)-1))*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)
)^2*arccot(a*x)^2+1/2*Pi*csgn(I*(I+a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(I+a*x)^
2/(a^2*x^2+1))^2*arccot(a*x)^2+1/4*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1))*csgn(I
*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))^2*arccot(a*x)^2-1/4*Pi*c
sgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))^3*arccot(a*x)^2-I*a
rccot(a*x)^2*ln((I+a*x)/(a^2*x^2+1)^(1/2))-1/2*Pi*csgn(I/((I+a*x)^2/(a^2*x
^2+1)-1))^3*arccot(a*x)^2-1/4*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*
(I+a*x)^2/(a^2*x^2+1))*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)
-1))*arccot(a*x)^2+1/2*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))^2*arccot(a*x)^
2-1/4*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1))^3*arccot(a*x)^2-8/3*I*dilog(1+I*(I+
a*x)/(a^2*x^2+1)^(1/2))-2/3*arccot(a*x)*(I+a*x)/a/x+1/3*arccot(a*x)*(I+a*x
)^2/a^2/x^2+2/3*arccot(a*x)*(I+a*x)*(a*x-I)/a^2/x^2-8/3*I*dilog(1-I*(I+a*x
)/(a^2*x^2+1)^(1/2))-4/3*I*arccot(a*x)^2)
```

3.33.5 Fricas [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^5} \, dx = \int \frac{\operatorname{arccot}(ax)^3}{x^5} \, dx$$

input
```
integrate(arccot(a*x)^3/x^5,x, algorithm="fricas")
```

output
```
integral(arccot(a*x)^3/x^5, x)
```

3.33.6 Sympy [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^5} \, dx = \int \frac{\operatorname{acot}^3(ax)}{x^5} \, dx$$

input
```
integrate(acot(a*x)**3/x**5,x)
```

output
```
Integral(acot(a*x)**3/x**5, x)
```

3.33.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)^3}{x^5} \, dx = \text{Timed out}$$

input
```
integrate(arccot(a*x)^3/x^5,x, algorithm="maxima")
```

output
```
Timed out
```

3.33.8 Giac [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^5} \, dx = \int \frac{\operatorname{arccot}(ax)^3}{x^5} \, dx$$

input
```
integrate(arccot(a*x)^3/x^5,x, algorithm="giac")
```

output
```
integrate(arccot(a*x)^3/x^5, x)
```

3.33.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)^3}{x^5} \, dx = \int \frac{\operatorname{acot}(a\,x)^3}{x^5} \, dx$$

input
```
int(acot(a*x)^3/x^5,x)
```

output
```
int(acot(a*x)^3/x^5, x)
```

3.34 $\int x^m \cot^{-1}(ax)^3 \, dx$

3.34.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int x^m \cot^{-1}(ax)^3 \, dx = \text{Int}\left(x^m \cot^{-1}(ax)^3, x\right)$$

output
```
Unintegrable(x^m*arccot(a*x)^3,x)
```

3.34.2 Mathematica [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \cot^{-1}(ax)^3 \, dx = \int x^m \cot^{-1}(ax)^3 \, dx$$

input
```
Integrate[x^m*ArcCot[a*x]^3,x]
```

output
```
Integrate[x^m*ArcCot[a*x]^3, x]
```

3.34.3 Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5378}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \cot^{-1}(ax)^3 \, dx$$

$$\Big\downarrow \text{5378}$$

$$\int x^m \cot^{-1}(ax)^3 dx$$

input
```
Int[x^m*ArcCot[a*x]^3,x]
```

output
```
$Aborted
```

3.34.3.1 Defintions of rubi rules used

rule 5378
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcCot[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

3.34.4 Maple [N/A] (verified)

Not integrable

Time = 0.77 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x^m \operatorname{arccot}(ax)^3 \, dx$$

input
```
int(x^m*arccot(a*x)^3,x)
```

output
```
int(x^m*arccot(a*x)^3,x)
```

3.34.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \cot^{-1}(ax)^3 \, dx = \int x^m \operatorname{arccot}(ax)^3 \, dx$$

input
```
integrate(x^m*arccot(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^m*arccot(a*x)^3, x)
```

3.34.6 Sympy [N/A]

Not integrable

Time = 1.95 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x^m \cot^{-1}(ax)^3 \, dx = \int x^m \operatorname{acot}^3(ax) \, dx$$

input
```
integrate(x**m*acot(a*x)**3,x)
```

output
```
Integral(x**m*acot(a*x)**3, x)
```

3.34.7 Maxima [N/A]

Not integrable

Time = 2.29 (sec) , antiderivative size = 221, normalized size of antiderivative = 22.10

$$\int x^m \cot^{-1}(ax)^3 \, dx = \int x^m \operatorname{arccot}(ax)^3 \, dx$$

input
```
integrate(x^m*arccot(a*x)^3,x, algorithm="maxima")
```

output
```
1/32*(4*x*x^m*arctan2(1, a*x)^3 - 3*x*x^m*arctan2(1, a*x)*log(a^2*x^2 + 1)
^2 + 32*(m + 1)*integrate(1/32*(12*a^2*x^2*x^m*arctan2(1, a*x)*log(a^2*x^2
 + 1) + 3*((a^2*m*arctan2(1, a*x) + a^2*arctan2(1, a*x))*x^2 - a*x + m*arc
tan2(1, a*x) + arctan2(1, a*x))*x^m*log(a^2*x^2 + 1)^2 + 4*(3*a*x*arctan2(
1, a*x)^2 + 7*m*arctan2(1, a*x)^3 + 7*(a^2*m*arctan2(1, a*x)^3 + a^2*arcta
n2(1, a*x)^3)*x^2 + 7*arctan2(1, a*x)^3)*x^m)/((a^2*m + a^2)*x^2 + m + 1),
 x))/(m + 1)
```

3.34.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \cot^{-1}(ax)^3 \, dx = \int x^m \operatorname{arccot}(ax)^3 \, dx$$

input
```
integrate(x^m*arccot(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^m*arccot(a*x)^3, x)
```

3.34.9 Mupad [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \cot^{-1}(ax)^3 \, dx = \int x^m \operatorname{acot}(a\,x)^3 \, dx$$

input
```
int(x^m*acot(a*x)^3,x)
```

output
```
int(x^m*acot(a*x)^3, x)
```

3.35 $\int x^m \cot^{-1}(ax)^2 \, dx$

3.35.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int x^m \cot^{-1}(ax)^2 \, dx = \operatorname{Int}\left(x^m \cot^{-1}(ax)^2, x\right)$$

output
```
Unintegrable(x^m*arccot(a*x)^2,x)
```

3.35.2 Mathematica [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \cot^{-1}(ax)^2 \, dx = \int x^m \cot^{-1}(ax)^2 \, dx$$

input
```
Integrate[x^m*ArcCot[a*x]^2,x]
```

output
```
Integrate[x^m*ArcCot[a*x]^2, x]
```

3.35.3 Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5378}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \cot^{-1}(ax)^2 \, dx$$

$$\Big\downarrow \text{ 5378}$$

$$\int x^m \cot^{-1}(ax)^2 dx$$

input
```
Int[x^m*ArcCot[a*x]^2,x]
```

output
```
$Aborted
```

3.35.3.1 Defintions of rubi rules used

rule 5378
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcCot[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

3.35.4 Maple [N/A] (verified)

Not integrable

Time = 0.53 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x^m \operatorname{arccot}(ax)^2 \, dx$$

input
```
int(x^m*arccot(a*x)^2,x)
```

output
```
int(x^m*arccot(a*x)^2,x)
```

3.35.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \cot^{-1}(ax)^2\,dx = \int x^m \operatorname{arccot}(ax)^2\,dx$$

input
```
integrate(x^m*arccot(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x^m*arccot(a*x)^2, x)
```

3.35.6 Sympy [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x^m \cot^{-1}(ax)^2\,dx = \int x^m \operatorname{acot}^2(ax)\,dx$$

input
```
integrate(x**m*acot(a*x)**2,x)
```

output
```
Integral(x**m*acot(a*x)**2, x)
```

3.35.7 Maxima [N/A]

Not integrable

Time = 1.57 (sec) , antiderivative size = 183, normalized size of antiderivative = 18.30

$$\int x^m \cot^{-1}(ax)^2\,dx = \int x^m \operatorname{arccot}(ax)^2\,dx$$

input
```
integrate(x^m*arccot(a*x)^2,x, algorithm="maxima")
```

output
```
1/16*(4*x*x^m*arctan2(1, a*x)^2 - x*x^m*log(a^2*x^2 + 1)^2 + 16*(m + 1)*in
tegrate(1/16*(4*a^2*x^2*x^m*log(a^2*x^2 + 1) + ((a^2*m + a^2)*x^2 + m + 1)
*x^m*log(a^2*x^2 + 1)^2 + 4*(3*(a^2*m*arctan2(1, a*x)^2 + a^2*arctan2(1, a
*x)^2)*x^2 + 2*a*x*arctan2(1, a*x) + 3*m*arctan2(1, a*x)^2 + 3*arctan2(1,
a*x)^2)*x^m)/((a^2*m + a^2)*x^2 + m + 1), x))/(m + 1)
```

3.35.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \cot^{-1}(ax)^2 \, dx = \int x^m \operatorname{arccot}(ax)^2 \, dx$$

input
```
integrate(x^m*arccot(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^m*arccot(a*x)^2, x)
```

3.35.9 Mupad [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \cot^{-1}(ax)^2 \, dx = \int x^m \operatorname{acot}(a\,x)^2 \, dx$$

input
```
int(x^m*acot(a*x)^2,x)
```

output
```
int(x^m*acot(a*x)^2, x)
```

3.36 $\int x^m \cot^{-1}(ax) \, dx$

3.36.1 Optimal result

Integrand size = 8, antiderivative size = 57

$$\int x^m \cot^{-1}(ax) \, dx = \frac{x^{1+m} \cot^{-1}(ax)}{1+m} + \frac{ax^{2+m} \text{Hypergeometric2F1}\left(1, \frac{2+m}{2}, \frac{4+m}{2}, -a^2x^2\right)}{2+3m+m^2}$$

output
```
x^(1+m)*arccot(a*x)/(1+m)+a*x^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],-a^2*
x^2)/(m^2+3*m+2)
```

3.36.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int x^m \cot^{-1}(ax) \, dx = \frac{x^{1+m}\left((2+m)\cot^{-1}(ax) + ax \, \text{Hypergeometric2F1}\left(1, 1+\frac{m}{2}, 2+\frac{m}{2}, -a^2x^2\right)\right)}{(1+m)(2+m)}$$

input
```
Integrate[x^m*ArcCot[a*x],x]
```

output
```
(x^(1 + m)*((2 + m)*ArcCot[a*x] + a*x*Hypergeometric2F1[1, 1 + m/2, 2 + m/
2, -(a^2*x^2)]))/((1 + m)*(2 + m))
```

3.36.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5362, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \cot^{-1}(ax)\,dx$$

$$\Big\downarrow \text{5362}$$

$$\frac{a\int \frac{x^{m+1}}{a^2x^2+1}dx}{m+1}+\frac{x^{m+1}\cot^{-1}(ax)}{m+1}$$

$$\Big\downarrow \text{278}$$

$$\frac{ax^{m+2}\,\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{(m+1)(m+2)}+\frac{x^{m+1}\cot^{-1}(ax)}{m+1}$$

input
```
Int[x^m*ArcCot[a*x],x]
```

output
```
(x^(1 + m)*ArcCot[a*x])/(1 + m) + (a*x^(2 + m)*Hypergeometric2F1[1, (2 + m
)/2, (4 + m)/2, -(a^2*x^2)])/((1 + m)*(2 + m))
```

3.36.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.36.4 Maple [F]

$$\int x^m \operatorname{arccot}(ax)\, dx$$

input
```
int(x^m*arccot(a*x),x)
```

output
```
int(x^m*arccot(a*x),x)
```

3.36.5 Fricas [F]

$$\int x^m \cot^{-1}(ax)\, dx = \int x^m \operatorname{arccot}(ax)\, dx$$

input
```
integrate(x^m*arccot(a*x),x, algorithm="fricas")
```

output
```
integral(x^m*arccot(a*x), x)
```

3.36.6 Sympy [F]

$$\int x^m \cot^{-1}(ax)\, dx = \int x^m \operatorname{acot}(ax)\, dx$$

input
```
integrate(x**m*acot(a*x),x)
```

output
```
Integral(x**m*acot(a*x), x)
```

3.36.7 Maxima [F]

$$\int x^m \cot^{-1}(ax)\,dx = \int x^m \operatorname{arccot}(ax)\,dx$$

input
```
integrate(x^m*arccot(a*x),x, algorithm="maxima")
```

output
```
(x*x^m*arctan2(1, a*x) + (a*m + a)*integrate(x*x^m/((a^2*m + a^2)*x^2 + m
+ 1), x))/(m + 1)
```

3.36.8 Giac [F]

$$\int x^m \cot^{-1}(ax)\,dx = \int x^m \operatorname{arccot}(ax)\,dx$$

input
```
integrate(x^m*arccot(a*x),x, algorithm="giac")
```

output
```
integrate(x^m*arccot(a*x), x)
```

3.36.9 Mupad [F(-1)]

Timed out.

$$\int x^m \cot^{-1}(ax)\,dx = \int x^m \operatorname{acot}(a\,x)\,dx$$

input
```
int(x^m*acot(a*x),x)
```

output
```
int(x^m*acot(a*x), x)
```

3.37 $\int \frac{x^4 \cot^{-1}(x)}{1+x^2} dx$

3.37.1 Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{x^4 \cot^{-1}(x)}{1+x^2} \, dx = \frac{x^2}{6} - x\cot^{-1}(x) + \frac{1}{3}x^3\cot^{-1}(x) - \frac{1}{2}\cot^{-1}(x)^2 - \frac{2}{3}\log\left(1+x^2\right)$$

output
```
1/6*x^2-x*arccot(x)+1/3*x^3*arccot(x)-1/2*arccot(x)^2-2/3*ln(x^2+1)
```

3.37.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{x^4 \cot^{-1}(x)}{1+x^2} \, dx = \frac{1}{6}\left(x^2 + 2x\left(-3+x^2\right)\cot^{-1}(x) - 3\cot^{-1}(x)^2 - 4\log\left(1+x^2\right)\right)$$

input
```
Integrate[(x^4*ArcCot[x])/(1 + x^2),x]
```

output
```
(x^2 + 2*x*(-3 + x^2)*ArcCot[x] - 3*ArcCot[x]^2 - 4*Log[1 + x^2])/6
```

3.37.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {5452, 5362, 243, 49, 2009, 5452, 5346, 240, 5420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \cot^{-1}(x)}{x^2+1} \, dx$$

$$\Big\downarrow \ 5452$$

$$\int x^2 \cot^{-1}(x)dx-\int \frac{x^2 \cot^{-1}(x)}{x^2+1}dx$$

$$\Big\downarrow \ 5362$$

$$-\int \frac{x^2 \cot^{-1}(x)}{x^2+1}dx+\frac{1}{3}\int \frac{x^3}{x^2+1}dx+\frac{1}{3}x^3 \cot^{-1}(x)$$

$$\Big\downarrow \ 243$$

$$\frac{1}{6}\int \frac{x^2}{x^2+1}dx^2-\int \frac{x^2 \cot^{-1}(x)}{x^2+1}dx+\frac{1}{3}x^3 \cot^{-1}(x)$$

$$\Big\downarrow \ 49$$

$$\frac{1}{6}\int \left(1+\frac{1}{-x^2-1}\right)dx^2-\int \frac{x^2 \cot^{-1}(x)}{x^2+1}dx+\frac{1}{3}x^3 \cot^{-1}(x)$$

$$\Big\downarrow \ 2009$$

$$-\int \frac{x^2 \cot^{-1}(x)}{x^2+1}dx+\frac{1}{3}x^3 \cot^{-1}(x)+\frac{1}{6}\left(x^2-\log\left(x^2+1\right)\right)$$

$$\Big\downarrow \ 5452$$

$$\int \frac{\cot^{-1}(x)}{x^2+1}dx-\int \cot^{-1}(x)dx+\frac{1}{3}x^3 \cot^{-1}(x)+\frac{1}{6}\left(x^2-\log\left(x^2+1\right)\right)$$

$$\Big\downarrow \ 5346$$

$$-\int \frac{x}{x^2+1}dx+\int \frac{\cot^{-1}(x)}{x^2+1}dx+\frac{1}{3}x^3 \cot^{-1}(x)+\frac{1}{6}\left(x^2-\log\left(x^2+1\right)\right)-x\cot^{-1}(x)$$

$$\Big\downarrow \ 240$$

$$\int \frac{\cot^{-1}(x)}{x^2+1}dx+\frac{1}{3}x^3 \cot^{-1}(x)+\frac{1}{6}\left(x^2-\log\left(x^2+1\right)\right)-\frac{1}{2}\log\left(x^2+1\right)-x\cot^{-1}(x)$$

$$\Big\downarrow \ 5420$$

$$\frac{1}{3}x^3\cot^{-1}(x)+\frac{1}{6}\left(x^2-\log\left(x^2+1\right)\right)-\frac{1}{2}\log\left(x^2+1\right)-x\cot^{-1}(x)-\frac{1}{2}\cot^{-1}(x)^2$$

input
```
Int[(x^4*ArcCot[x])/(1 + x^2),x]
```

output
```
-(x*ArcCot[x]) + (x^3*ArcCot[x])/3 - ArcCot[x]^2/2 + (x^2 - Log[1 + x^2])/
6 - Log[1 + x^2]/2
```

3.37.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5420
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5452
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

3.37.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

method	result
parallelrisch	$\frac{x^3 \operatorname{arccot}(x)}{3}+\frac{x^2}{6}-x \operatorname{arccot}(x)-\frac{\operatorname{arccot}(x)^2}{2}-\frac{2\ln(x^2+1)}{3}-\frac{1}{3}$
default	$\frac{x^3 \operatorname{arccot}(x)}{3}-x \operatorname{arccot}(x)+\operatorname{arccot}(x)\arctan(x)+\frac{x^2}{6}-\frac{2\ln(x^2+1)}{3}+\frac{\arctan(x)^2}{2}$
parts	$\frac{x^3 \operatorname{arccot}(x)}{3}-x \operatorname{arccot}(x)+\operatorname{arccot}(x)\arctan(x)+\frac{x^2}{6}-\frac{2\ln(x^2+1)}{3}+\frac{\arctan(x)^2}{2}$
risch	$\frac{\ln(ix+1)^2}{8}+\left(\frac{ix^3}{6}-\frac{ix}{2}-\frac{\ln(-ix+1)}{4}\right)\ln(ix+1)+\frac{\ln(-ix+1)^2}{8}-\frac{ix^3\ln(-ix+1)}{6}+\frac{i\ln(-ix+1)x}{2}+\frac{\pi x^3}{6}-$

input
```
int(x^4*arccot(x)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*arccot(x)+1/6*x^2-x*arccot(x)-1/2*arccot(x)^2-2/3*ln(x^2+1)-1/3
```

3.37.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{x^4\cot^{-1}(x)}{1+x^2}\,dx=\frac{1}{6}\,x^2+\frac{1}{3}\left(x^3-3\,x\right)\operatorname{arccot}(x)-\frac{1}{2}\operatorname{arccot}(x)^2-\frac{2}{3}\log\left(x^2+1\right)$$

input
```
integrate(x^4*arccot(x)/(x^2+1),x, algorithm="fricas")
```

output
```
1/6*x^2 + 1/3*(x^3 - 3*x)*arccot(x) - 1/2*arccot(x)^2 - 2/3*log(x^2 + 1)
```

3.37.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{x^4 \cot^{-1}(x)}{1+x^2} \, dx = \frac{x^3 \operatorname{acot}(x)}{3} + \frac{x^2}{6} - x \operatorname{acot}(x) - \frac{2 \log(x^2+1)}{3} - \frac{\operatorname{acot}^2(x)}{2}$$

input
```
integrate(x**4*acot(x)/(x**2+1),x)
```

output
```
x**3*acot(x)/3 + x**2/6 - x*acot(x) - 2*log(x**2 + 1)/3 - acot(x)**2/2
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int \frac{x^4 \cot^{-1}(x)}{1+x^2} \, dx = \frac{1}{6} x^2 + \frac{1}{3} (x^3 - 3x + 3 \arctan(x)) \operatorname{arccot}(x) + \frac{1}{2} \arctan(x)^2 - \frac{2}{3} \log(x^2+1)$$

input
```
integrate(x^4*arccot(x)/(x^2+1),x, algorithm="maxima")
```

output
```
1/6*x^2 + 1/3*(x^3 - 3*x + 3*arctan(x))*arccot(x) + 1/2*arctan(x)^2 - 2/3*
log(x^2 + 1)
```

3.37.8 Giac [F]

$$\int \frac{x^4 \cot^{-1}(x)}{1+x^2} \, dx = \int \frac{x^4 \operatorname{arccot}(x)}{x^2+1} \, dx$$

input
```
integrate(x^4*arccot(x)/(x^2+1),x, algorithm="giac")
```

output
```
integrate(x^4*arccot(x)/(x^2 + 1), x)
```

3.37.9 Mupad [B] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{x^4 \cot^{-1}(x)}{1+x^2} \, dx = \frac{x^3 \operatorname{acot}(x)}{3} - \frac{2 \ln(x^2+1)}{3} - \frac{\operatorname{acot}(x)^2}{2} - x \operatorname{acot}(x) + \frac{x^2}{6}$$

input

```
int((x^4*acot(x))/(x^2 + 1),x)
```

output

```
(x^3*acot(x))/3 - (2*log(x^2 + 1))/3 - acot(x)^2/2 - x*acot(x) + x^2/6
```

3.38 $\int \frac{x^3 \cot^{-1}(x)}{1+x^2} \, dx$

3.38.1 Optimal result

Integrand size = 13, antiderivative size = 67

$$\int \frac{x^3 \cot^{-1}(x)}{1+x^2} \, dx = \frac{x}{2} + \frac{1}{2}x^2 \cot^{-1}(x) - \frac{1}{2}i \cot^{-1}(x)^2 - \frac{\arctan(x)}{2} + \cot^{-1}(x) \log\left(\frac{2}{1+ix}\right) - \frac{1}{2}i \operatorname{PolyLog}\left(2, 1 - \frac{2}{1+ix}\right)$$

output
```
1/2*x+1/2*x^2*arccot(x)-1/2*I*arccot(x)^2-1/2*arctan(x)+arccot(x)*ln(2/(1+
I*x))-1/2*I*polylog(2,1-2/(1+I*x))
```

3.38.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.76

$$\int \frac{x^3 \cot^{-1}(x)}{1+x^2} \, dx = \frac{1}{2}\left(x - i \cot^{-1}(x)^2 + \cot^{-1}(x) \left(1 + x^2 + 2 \log\left(1 - e^{2i \cot^{-1}(x)}\right)\right) - i \operatorname{PolyLog}\left(2, e^{2i \cot^{-1}(x)}\right)\right)$$

input
```
Integrate[(x^3*ArcCot[x])/(1 + x^2),x]
```

output
```
(x - I*ArcCot[x]^2 + ArcCot[x]*(1 + x^2 + 2*Log[1 - E^((2*I)*ArcCot[x])])
- I*PolyLog[2, E^((2*I)*ArcCot[x])])/2
```

3.38.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {5452, 5362, 262, 216, 5456, 5380, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \cot^{-1}(x)}{x^2+1} dx$$

$$\Big\downarrow \text{5452}$$

$$\int x \cot^{-1}(x)dx - \int \frac{x \cot^{-1}(x)}{x^2+1}dx$$

$$\Big\downarrow \text{5362}$$

$$\frac{1}{2}\int \frac{x^2}{x^2+1}dx - \int \frac{x \cot^{-1}(x)}{x^2+1}dx + \frac{1}{2}x^2 \cot^{-1}(x)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(x - \int \frac{1}{x^2+1}dx\right) - \int \frac{x \cot^{-1}(x)}{x^2+1}dx + \frac{1}{2}x^2 \cot^{-1}(x)$$

$$\Big\downarrow \text{216}$$

$$-\int \frac{x \cot^{-1}(x)}{x^2+1}dx + \frac{1}{2}(x - \arctan(x)) + \frac{1}{2}x^2 \cot^{-1}(x)$$

$$\Big\downarrow \text{5456}$$

$$\int \frac{\cot^{-1}(x)}{i-x}dx + \frac{1}{2}(x - \arctan(x)) + \frac{1}{2}x^2 \cot^{-1}(x) - \frac{1}{2}i \cot^{-1}(x)^2$$

$$\Big\downarrow \text{5380}$$

$$\int \frac{\log\left(\frac{2}{ix+1}\right)}{x^2+1}dx + \frac{1}{2}(x - \arctan(x)) + \frac{1}{2}x^2 \cot^{-1}(x) - \frac{1}{2}i \cot^{-1}(x)^2 + \log\left(\frac{2}{1+ix}\right)\cot^{-1}(x)$$

$$\Big\downarrow \text{2849}$$

$$-i\int \frac{\log\left(\frac{2}{ix+1}\right)}{1-\frac{2}{ix+1}}d\frac{1}{ix+1} + \frac{1}{2}(x - \arctan(x)) + \frac{1}{2}x^2 \cot^{-1}(x) - \frac{1}{2}i \cot^{-1}(x)^2 + \log\left(\frac{2}{1+ix}\right)\cot^{-1}(x)$$

$$\Big\downarrow \text{2752}$$

$$\frac{1}{2}(x - \arctan(x)) - \frac{1}{2}i \operatorname{PolyLog}\left(2, 1 - \frac{2}{ix + 1}\right) + \frac{1}{2}x^2 \cot^{-1}(x) - \frac{1}{2}i \cot^{-1}(x)^2 + \log\left(\frac{2}{1 + ix}\right) \cot^{-1}(x)$$

input
```
Int[(x^3*ArcCot[x])/(1 + x^2),x]
```

output
```
(x^2*ArcCot[x])/2 - (I/2)*ArcCot[x]^2 + (x - ArcTan[x])/2 + ArcCot[x]*Log[
2/(1 + I*x)] - (I/2)*PolyLog[2, 1 - 2/(1 + I*x)]
```

3.38.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5380
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Simp[b*c*(
p/e)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5452
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5456
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Simp[
1/(c*d)   Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.38.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(53) = 106.

Time = 0.74 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.88

method	result
default	$\frac{x^2 \operatorname{arccot}(x)}{2} - \frac{\operatorname{arccot}(x)\ln(x^2+1)}{2} + \frac{x}{2} - \frac{\arctan(x)}{2} + \frac{i\left(\ln(x-i)\ln(x^2+1) - \frac{\ln(x-i)^2}{2} - \operatorname{dilog}\left(-\frac{i(i+x)}{2}\right) - \ln(x-i)\ln\left(-\frac{i(i+x)}{2}\right.\right.}{4}$
parts	$\frac{x^2 \operatorname{arccot}(x)}{2} - \frac{\operatorname{arccot}(x)\ln(x^2+1)}{2} + \frac{x}{2} - \frac{\arctan(x)}{2} + \frac{i\left(\ln(x-i)\ln(x^2+1) - \frac{\ln(x-i)^2}{2} - \operatorname{dilog}\left(-\frac{i(i+x)}{2}\right) - \ln(x-i)\ln\left(-\frac{i(i+x)}{2}\right.\right.}{4}$
risch	$\frac{\pi x^2}{4} + \frac{\pi}{4} - \frac{\pi\ln(x^2+1)}{4} - \frac{i\ln(-ix+1)x^2}{4} + \frac{i\ln\left(\frac{1}{2}+\frac{ix}{2}\right)\ln(-ix+1)}{4} - \frac{i\operatorname{dilog}\left(\frac{1}{2}-\frac{ix}{2}\right)}{4} + \frac{x}{2} + \frac{i\operatorname{dilog}\left(\frac{1}{2}+\frac{ix}{2}\right)}{4} + \frac{i\ln(-ix+}{8}$

input
```
int(x^3*arccot(x)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*arccot(x)-1/2*arccot(x)*ln(x^2+1)+1/2*x-1/2*arctan(x)+1/4*I*(ln(x-
I)*ln(x^2+1)-1/2*ln(x-I)^2-dilog(-1/2*I*(I+x))-ln(x-I)*ln(-1/2*I*(I+x)))-1
/4*I*(ln(I+x)*ln(x^2+1)-1/2*ln(I+x)^2-dilog(1/2*I*(x-I))-ln(I+x)*ln(1/2*I*
(x-I)))
```

3.38.5 Fricas [F]

$$\int \frac{x^3 \cot^{-1}(x)}{1+x^2}\,dx = \int \frac{x^3 \operatorname{arccot}(x)}{x^2+1}\,dx$$

input
```
integrate(x^3*arccot(x)/(x^2+1),x, algorithm="fricas")
```

output
```
integral(x^3*arccot(x)/(x^2 + 1), x)
```

3.38.6 Sympy [F]

$$\int \frac{x^3 \cot^{-1}(x)}{1+x^2}\,dx = \int \frac{x^3 \operatorname{acot}(x)}{x^2+1}\,dx$$

input
```
integrate(x**3*acot(x)/(x**2+1),x)
```

output
```
Integral(x**3*acot(x)/(x**2 + 1), x)
```

3.38.7 Maxima [F]

$$\int \frac{x^3 \cot^{-1}(x)}{1+x^2}\,dx = \int \frac{x^3 \operatorname{arccot}(x)}{x^2+1}\,dx$$

input
```
integrate(x^3*arccot(x)/(x^2+1),x, algorithm="maxima")
```

output
```
integrate(x^3*arccot(x)/(x^2 + 1), x)
```

3.38.8 Giac [F]

$$\int \frac{x^3 \cot^{-1}(x)}{1+x^2} \, dx = \int \frac{x^3 \operatorname{arccot}(x)}{x^2+1} \, dx$$

input
```
integrate(x^3*arccot(x)/(x^2+1),x, algorithm="giac")
```

output
```
integrate(x^3*arccot(x)/(x^2 + 1), x)
```

3.38.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \cot^{-1}(x)}{1+x^2} \, dx = \int \frac{x^3 \operatorname{acot}(x)}{x^2+1} \, dx$$

input
```
int((x^3*acot(x))/(x^2 + 1),x)
```

output
```
int((x^3*acot(x))/(x^2 + 1), x)
```

3.39 $\int \frac{x^2 \cot^{-1}(x)}{1+x^2} dx$

3.39.1 Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{x^2 \cot^{-1}(x)}{1+x^2} dx = x \cot^{-1}(x) + \frac{1}{2} \cot^{-1}(x)^2 + \frac{1}{2} \log \left(1+x^2\right)$$

output
```
x*arccot(x)+1/2*arccot(x)^2+1/2*ln(x^2+1)
```

3.39.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2 \cot^{-1}(x)}{1+x^2} dx = x \cot^{-1}(x) + \frac{1}{2} \cot^{-1}(x)^2 + \frac{1}{2} \log \left(1+x^2\right)$$

input
```
Integrate[(x^2*ArcCot[x])/(1 + x^2),x]
```

output
```
x*ArcCot[x] + ArcCot[x]^2/2 + Log[1 + x^2]/2
```

3.39.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5452, 5346, 240, 5420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \cot^{-1}(x)}{x^2+1} \, dx$$

$$\Big\downarrow \text{5452}$$

$$\int \cot^{-1}(x)dx - \int \frac{\cot^{-1}(x)}{x^2+1}dx$$

$$\Big\downarrow \text{5346}$$

$$\int \frac{x}{x^2+1}dx - \int \frac{\cot^{-1}(x)}{x^2+1}dx + x\cot^{-1}(x)$$

$$\Big\downarrow \text{240}$$

$$-\int \frac{\cot^{-1}(x)}{x^2+1}dx + \frac{1}{2}\log\left(x^2+1\right) + x\cot^{-1}(x)$$

$$\Big\downarrow \text{5420}$$

$$\frac{1}{2}\log\left(x^2+1\right) + \frac{1}{2}\cot^{-1}(x)^2 + x\cot^{-1}(x)$$

input
```
Int[(x^2*ArcCot[x])/(1 + x^2),x]
```

output
```
x*ArcCot[x] + ArcCot[x]^2/2 + Log[1 + x^2]/2
```

3.39.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5420
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5452
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

3.39.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result
parallelrisch	$x \operatorname{arccot}(x)+\frac{\operatorname{arccot}(x)^2}{2}+\frac{\ln(x^2+1)}{2}$
default	$-\operatorname{arccot}(x)\arctan(x)+x\operatorname{arccot}(x)+\frac{\ln(x^2+1)}{2}-\frac{\arctan(x)^2}{2}$
parts	$-\operatorname{arccot}(x)\arctan(x)+x\operatorname{arccot}(x)+\frac{\ln(x^2+1)}{2}-\frac{\arctan(x)^2}{2}$
risch	$-\frac{\ln(ix+1)^2}{8}+\left(\frac{ix}{2}+\frac{\ln(-ix+1)}{4}\right)\ln(ix+1)-\frac{\ln(-ix+1)^2}{8}-\frac{i\ln(-ix+1)x}{2}+\frac{\pi x}{2}+\frac{\ln(x^2+1)}{2}-\frac{\pi\arctan(x}{2}$

input
```
int(x^2*arccot(x)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
x*arccot(x)+1/2*arccot(x)^2+1/2*ln(x^2+1)
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x^2\cot^{-1}(x)}{1+x^2}\,dx = x\operatorname{arccot}(x)+\frac{1}{2}\operatorname{arccot}(x)^2+\frac{1}{2}\log\left(x^2+1\right)$$

input
```
integrate(x^2*arccot(x)/(x^2+1),x, algorithm="fricas")
```

output
```
x*arccot(x) + 1/2*arccot(x)^2 + 1/2*log(x^2 + 1)
```

3.39.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x^2 \cot^{-1}(x)}{1+x^2} \, dx = x \operatorname{acot}(x) + \frac{\log(x^2+1)}{2} + \frac{\operatorname{acot}^2(x)}{2}$$

input
```
integrate(x**2*acot(x)/(x**2+1),x)
```

output
```
x*acot(x) + log(x**2 + 1)/2 + acot(x)**2/2
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{x^2 \cot^{-1}(x)}{1+x^2} \, dx = (x - \arctan(x)) \operatorname{arccot}(x) - \frac{1}{2} \arctan(x)^2 + \frac{1}{2} \log(x^2+1)$$

input
```
integrate(x^2*arccot(x)/(x^2+1),x, algorithm="maxima")
```

output
```
(x - arctan(x))*arccot(x) - 1/2*arctan(x)^2 + 1/2*log(x^2 + 1)
```

3.39.8 Giac [F]

$$\int \frac{x^2 \cot^{-1}(x)}{1+x^2} \, dx = \int \frac{x^2 \operatorname{arccot}(x)}{x^2+1} \, dx$$

input
```
integrate(x^2*arccot(x)/(x^2+1),x, algorithm="giac")
```

output
```
integrate(x^2*arccot(x)/(x^2 + 1), x)
```

3.39.9 Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x^2 \cot^{-1}(x)}{1+x^2} \, dx = \frac{\text{acot}(x)^2}{2} + x \, \text{acot}(x) + \frac{\ln(x^2+1)}{2}$$

input
```
int((x^2*acot(x))/(x^2 + 1),x)
```

output
```
log(x^2 + 1)/2 + acot(x)^2/2 + x*acot(x)
```

3.40 $\int \frac{x\cot^{-1}(x)}{1+x^2}\,dx$

3.40.1 Optimal result

Integrand size = 11, antiderivative size = 48

$$\int \frac{x\cot^{-1}(x)}{1+x^2}\,dx=\frac{1}{2}i\cot^{-1}(x)^2-\cot^{-1}(x)\log\left(\frac{2}{1+ix}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{1+ix}\right)$$

output
```
1/2*I*arccot(x)^2-arccot(x)*ln(2/(1+I*x))+1/2*I*polylog(2,1-2/(1+I*x))
```

3.40.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \frac{x\cot^{-1}(x)}{1+x^2}\,dx=-\cot^{-1}(x)\log\left(1-e^{2i\cot^{-1}(x)}\right)+\frac{1}{2}i\left(\cot^{-1}(x)^2+\text{PolyLog}\left(2,e^{2i\cot^{-1}(x)}\right)\right)$$

input
```
Integrate[(x*ArcCot[x])/(1 + x^2),x]
```

output
```
-(ArcCot[x]*Log[1 - E^((2*I)*ArcCot[x])]) + (I/2)*(ArcCot[x]^2 + PolyLog[2
, E^((2*I)*ArcCot[x])])
```

3.40.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5456, 5380, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \cot^{-1}(x)}{x^2+1} \, dx$$

$$\Big\downarrow \text{ 5456}$$

$$\frac{1}{2} i \cot^{-1}(x)^2-\int \frac{\cot^{-1}(x)}{i-x}dx$$

$$\Big\downarrow \text{ 5380}$$

$$-\int \frac{\log\left(\frac{2}{ix+1}\right)}{x^2+1}dx+\frac{1}{2} i \cot^{-1}(x)^2-\log\left(\frac{2}{1+ix}\right)\cot^{-1}(x)$$

$$\Big\downarrow \text{ 2849}$$

$$i \int \frac{\log\left(\frac{2}{ix+1}\right)}{1-\frac{2}{ix+1}}d\frac{1}{ix+1}+\frac{1}{2} i \cot^{-1}(x)^2-\log\left(\frac{2}{1+ix}\right)\cot^{-1}(x)$$

$$\Big\downarrow \text{ 2752}$$

$$\frac{1}{2} i \, \text{PolyLog}\left(2,1-\frac{2}{ix+1}\right)+\frac{1}{2} i \cot^{-1}(x)^2-\log\left(\frac{2}{1+ix}\right)\cot^{-1}(x)$$

input
```
Int[(x*ArcCot[x])/(1 + x^2),x]
```

output
```
(I/2)*ArcCot[x]^2 - ArcCot[x]*Log[2/(1 + I*x)] + (I/2)*PolyLog[2, 1 - 2/(1
 + I*x)]
```

3.40.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5380
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Simp[b*c*(
p/e)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5456
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Simp[
1/(c*d)   Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.40.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(40) = 80$.

Time = 0.35 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.04

method	result
risch	$\frac{\pi \ln\left(-2+(-ix+1)^2+2ix\right)}{4}-\frac{i\ln(-ix+1)^2}{8}-\frac{i\ln\left(\frac{1}{2}+\frac{ix}{2}\right)\ln(-ix+1)}{4}+\frac{i\,\text{dilog}\left(\frac{1}{2}-\frac{ix}{2}\right)}{4}+\frac{i\ln(ix+1)^2}{8}+\frac{i\ln\left(\frac{1}{2}-\frac{ix}{2}\right)\ln(ix+1)}{4}$
default	$\frac{\text{arccot}(x)\ln(x^2+1)}{2}-\frac{i\left(\ln(x-i)\ln(x^2+1)-\frac{\ln(x-i)^2}{2}-\text{dilog}\left(-\frac{i(i+x)}{2}\right)-\ln(x-i)\ln\left(-\frac{i(i+x)}{2}\right)\right)}{4}+\frac{i\left(\ln(i+x)\ln(x^2+1)-\frac{\ln(i+x)^2}{2}\right.}{}$
parts	$\frac{\text{arccot}(x)\ln(x^2+1)}{2}-\frac{i\left(\ln(x-i)\ln(x^2+1)-\frac{\ln(x-i)^2}{2}-\text{dilog}\left(-\frac{i(i+x)}{2}\right)-\ln(x-i)\ln\left(-\frac{i(i+x)}{2}\right)\right)}{4}+\frac{i\left(\ln(i+x)\ln(x^2+1)-\frac{\ln(i+x)^2}{2}\right.}{}$

input
```
int(x*arccot(x)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*Pi*ln(-2+(1-I*x)^2+2*I*x)-1/8*I*ln(1-I*x)^2-1/4*I*ln(1/2+1/2*I*x)*ln(1
-I*x)+1/4*I*dilog(1/2-1/2*I*x)+1/8*I*ln(1+I*x)^2+1/4*I*ln(1/2-1/2*I*x)*ln(
1+I*x)-1/4*I*dilog(1/2+1/2*I*x)
```

3.40.5 Fricas [F]

$$\int \frac{x\cot^{-1}(x)}{1+x^2}\,dx = \int \frac{x\,\mathrm{arccot}\,(x)}{x^2+1}\,dx$$

input
```
integrate(x*arccot(x)/(x^2+1),x, algorithm="fricas")
```

output
```
integral(x*arccot(x)/(x^2 + 1), x)
```

3.40.6 Sympy [F]

$$\int \frac{x\cot^{-1}(x)}{1+x^2}\,dx = \int \frac{x\,\mathrm{acot}\,(x)}{x^2+1}\,dx$$

input
```
integrate(x*acot(x)/(x**2+1),x)
```

output
```
Integral(x*acot(x)/(x**2 + 1), x)
```

3.40.7 Maxima [F]

$$\int \frac{x\cot^{-1}(x)}{1+x^2}\,dx = \int \frac{x\,\mathrm{arccot}\,(x)}{x^2+1}\,dx$$

input
```
integrate(x*arccot(x)/(x^2+1),x, algorithm="maxima")
```

output
```
integrate(x*arccot(x)/(x^2 + 1), x)
```

3.40.8 Giac [F]

$$\int \frac{x \cot^{-1}(x)}{1+x^2} \, dx = \int \frac{x \operatorname{arccot}(x)}{x^2+1} \, dx$$

input
```
integrate(x*arccot(x)/(x^2+1),x, algorithm="giac")
```

output
```
integrate(x*arccot(x)/(x^2 + 1), x)
```

3.40.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x \cot^{-1}(x)}{1+x^2} \, dx = \int \frac{x \operatorname{acot}(x)}{x^2+1} \, dx$$

input
```
int((x*acot(x))/(x^2 + 1),x)
```

output
```
int((x*acot(x))/(x^2 + 1), x)
```

3.41 $\int \frac{\cot^{-1}(x)}{1+x^2} \, dx$

3.41.1 Optimal result

Integrand size = 10, antiderivative size = 8

$$\int \frac{\cot^{-1}(x)}{1+x^2} \, dx = -\frac{1}{2}\cot^{-1}(x)^2$$

output
```
-1/2*arccot(x)^2
```

3.41.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(x)}{1+x^2} \, dx = -\frac{1}{2}\cot^{-1}(x)^2$$

input
```
Integrate[ArcCot[x]/(1 + x^2),x]
```

output
```
-1/2*ArcCot[x]^2
```

3.41.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(x)}{x^2+1}\,dx$$

$$\Big\downarrow \; 5420$$

$$-\frac{1}{2}\cot^{-1}(x)^2$$

input
```
Int[ArcCot[x]/(1 + x^2),x]
```

output
```
-1/2*ArcCot[x]^2
```

3.41.3.1 Defintions of rubi rules used

rule 5420
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

3.41.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-\frac{\operatorname{arccot}(x)^2}{2}$	7
default	$-\frac{\operatorname{arccot}(x)^2}{2}$	7
parts	$\operatorname{arccot}(x)\arctan(x)+\frac{\arctan(x)^2}{2}$	13
risch	$\frac{\ln(ix+1)^2}{8}-\frac{\ln(-ix+1)\ln(ix+1)}{4}+\frac{\ln(-ix+1)^2}{8}+\frac{\pi\arctan(x)}{2}$	45

input
```
int(arccot(x)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arccot(x)^2
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\cot^{-1}(x)}{1+x^2}\,dx = -\frac{1}{2}\,\mathrm{arccot}\,(x)^2$$

input
```
integrate(arccot(x)/(x^2+1),x, algorithm="fricas")
```

output
```
-1/2*arccot(x)^2
```

3.41.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{\cot^{-1}(x)}{1+x^2}\,dx = -\frac{\mathrm{acot}^2\,(x)}{2}$$

input
```
integrate(acot(x)/(x**2+1),x)
```

output
```
-acot(x)**2/2
```

3.41.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\cot^{-1}(x)}{1+x^2}\,dx = -\frac{1}{2}\,\mathrm{arccot}\,(x)^2$$

input
```
integrate(arccot(x)/(x^2+1),x, algorithm="maxima")
```

output
```
-1/2*arccot(x)^2
```

3.41.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(x)}{1+x^2}\,dx = -\frac{1}{2}\arctan\left(\frac{1}{x}\right)^2$$

input
```
integrate(arccot(x)/(x^2+1),x, algorithm="giac")
```

output
```
-1/2*arctan(1/x)^2
```

3.41.9 Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\cot^{-1}(x)}{1+x^2}\,dx = -\frac{\mathrm{acot}(x)^2}{2}$$

input
```
int(acot(x)/(x^2 + 1),x)
```

output
```
-acot(x)^2/2
```

3.42 $\int \frac{\cot^{-1}(x)}{x(1+x^2)}\,dx$

3.42.1 Optimal result

Integrand size = 13, antiderivative size = 49

$$\int \frac{\cot^{-1}(x)}{x(1+x^2)}\,dx = \frac{1}{2}i\cot^{-1}(x)^2+\cot^{-1}(x)\log\left(2-\frac{2}{1-ix}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,-1+\frac{2}{1-ix}\right)$$

output
```
1/2*I*arccot(x)^2+arccot(x)*ln(2-2/(1-I*x))+1/2*I*polylog(2,-1+2/(1-I*x))
```

3.42.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{\cot^{-1}(x)}{x(1+x^2)}\,dx = -\frac{1}{2}i\cot^{-1}(x)^2+\cot^{-1}(x)\log\left(1+e^{2i\cot^{-1}(x)}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\cot^{-1}(x)}\right)$$

input
```
Integrate[ArcCot[x]/(x*(1 + x^2)),x]
```

output
```
(-1/2*I)*ArcCot[x]^2 + ArcCot[x]*Log[1 + E^((2*I)*ArcCot[x])] - (I/2)*Poly
Log[2, -E^((2*I)*ArcCot[x])]
```

3.42.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5460, 5404, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(x)}{x(x^2+1)}dx$$

$$\Big\downarrow \text{5460}$$

$$i\int \frac{\cot^{-1}(x)}{x(x+i)}dx+\frac{1}{2}i\cot^{-1}(x)^2$$

$$\Big\downarrow \text{5404}$$

$$i\left(-i\int \frac{\log\left(2-\frac{2}{1-ix}\right)}{x^2+1}dx-i\log\left(2-\frac{2}{1-ix}\right)\cot^{-1}(x)\right)+\frac{1}{2}i\cot^{-1}(x)^2$$

$$\Big\downarrow \text{2897}$$

$$i\left(\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-ix}-1\right)-i\log\left(2-\frac{2}{1-ix}\right)\cot^{-1}(x)\right)+\frac{1}{2}i\cot^{-1}(x)^2$$

input
```
Int[ArcCot[x]/(x*(1 + x^2)),x]
```

output
```
(I/2)*ArcCot[x]^2 + I*((-I)*ArcCot[x]*Log[2 - 2/(1 - I*x)] + PolyLog[2, -1
 + 2/(1 - I*x)]/2)
```

3.42.3.1 Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5404
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcCot[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] + Si
mp[b*c*(p/d)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5460
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*d*(p + 1))), x] + Simp[
I/d   Int[(a + b*ArcCot[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.42.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(41) = 82$.

Time = 0.34 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.39

method	result
risch	$-\frac{\pi \ln(x^2+1)}{4}+\frac{\pi \ln(-ix)}{2}+\frac{i \ln(-ix+1)^2}{8}+\frac{i \ln\left(\frac{1}{2}+\frac{ix}{2}\right) \ln(-ix+1)}{4}-\frac{i\,\mathrm{dilog}\left(\frac{1}{2}-\frac{ix}{2}\right)}{4}+\frac{i\,\mathrm{dilog}(-ix+1)}{2}-\frac{i \ln(ix+1)^2}{8}-$
default	$\operatorname{arccot}(x)\ln(x)-\frac{\operatorname{arccot}(x)\ln(x^2+1)}{2}-\frac{i\ln(x)\ln(ix+1)}{2}+\frac{i\ln(x)\ln(-ix+1)}{2}-\frac{i\,\mathrm{dilog}(ix+1)}{2}+\frac{i\,\mathrm{dilog}(-ix+1)}{2}+$
parts	$\operatorname{arccot}(x)\ln(x)-\frac{\operatorname{arccot}(x)\ln(x^2+1)}{2}-\frac{i\ln(x)\ln(ix+1)}{2}+\frac{i\ln(x)\ln(-ix+1)}{2}-\frac{i\,\mathrm{dilog}(ix+1)}{2}+\frac{i\,\mathrm{dilog}(-ix+1)}{2}+$

input
```
int(arccot(x)/x/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*Pi*ln(x^2+1)+1/2*Pi*ln(-I*x)+1/8*I*ln(1-I*x)^2+1/4*I*ln(1/2+1/2*I*x)*
ln(1-I*x)-1/4*I*dilog(1/2-1/2*I*x)+1/2*I*dilog(1-I*x)-1/8*I*ln(1+I*x)^2-1/
4*I*ln(1/2-1/2*I*x)*ln(1+I*x)+1/4*I*dilog(1/2+1/2*I*x)-1/2*I*dilog(1+I*x)
```

3.42.5 Fricas [F]

$$\int \frac{\cot^{-1}(x)}{x(1+x^2)}\,dx = \int \frac{\operatorname{arccot}(x)}{(x^2+1)x}\,dx$$

input
```
integrate(arccot(x)/x/(x^2+1),x, algorithm="fricas")
```

output
```
integral(arccot(x)/(x^3 + x), x)
```

3.42.6 Sympy [F]

$$\int \frac{\cot^{-1}(x)}{x(1+x^2)}\,dx = \int \frac{\operatorname{acot}(x)}{x(x^2+1)}\,dx$$

input
```
integrate(acot(x)/x/(x**2+1),x)
```

output
```
Integral(acot(x)/(x*(x**2 + 1)), x)
```

3.42.7 Maxima [F]

$$\int \frac{\cot^{-1}(x)}{x(1+x^2)}\,dx = \int \frac{\operatorname{arccot}(x)}{(x^2+1)x}\,dx$$

input
```
integrate(arccot(x)/x/(x^2+1),x, algorithm="maxima")
```

output
```
integrate(arccot(x)/((x^2 + 1)*x), x)
```

3.42.8 Giac [F]

$$\int \frac{\cot^{-1}(x)}{x(1+x^2)}\,dx = \int \frac{\operatorname{arccot}(x)}{(x^2+1)x}\,dx$$

input
```
integrate(arccot(x)/x/(x^2+1),x, algorithm="giac")
```

output
```
integrate(arccot(x)/((x^2 + 1)*x), x)
```

3.42.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(x)}{x(1+x^2)}\,dx = \int \frac{\operatorname{acot}(x)}{x\,(x^2+1)}\,dx$$

input
```
int(acot(x)/(x*(x^2 + 1)),x)
```

output
```
int(acot(x)/(x*(x^2 + 1)), x)
```

3.43 $\int \frac{\cot^{-1}(x)}{x^2(1+x^2)}\,dx$

3.43.1 Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{\cot^{-1}(x)}{x^2(1+x^2)}\,dx = -\frac{\cot^{-1}(x)}{x} + \frac{1}{2}\cot^{-1}(x)^2 - \log(x) + \frac{1}{2}\log\left(1+x^2\right)$$

output
```
-arccot(x)/x+1/2*arccot(x)^2-ln(x)+1/2*ln(x^2+1)
```

3.43.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(x)}{x^2(1+x^2)}\,dx = -\frac{\cot^{-1}(x)}{x} + \frac{1}{2}\cot^{-1}(x)^2 - \log(x) + \frac{1}{2}\log\left(1+x^2\right)$$

input
```
Integrate[ArcCot[x]/(x^2*(1 + x^2)),x]
```

output
```
-(ArcCot[x]/x) + ArcCot[x]^2/2 - Log[x] + Log[1 + x^2]/2
```

3.43.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {5454, 5362, 243, 47, 14, 16, 5420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(x)}{x^2(x^2+1)}dx$$

$$\Big\downarrow \text{5454}$$

$$\int \frac{\cot^{-1}(x)}{x^2}dx-\int \frac{\cot^{-1}(x)}{x^2+1}dx$$

$$\Big\downarrow \text{5362}$$

$$-\int \frac{1}{x(x^2+1)}dx-\int \frac{\cot^{-1}(x)}{x^2+1}dx-\frac{\cot^{-1}(x)}{x}$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{2}\int \frac{1}{x^2(x^2+1)}dx^2-\int \frac{\cot^{-1}(x)}{x^2+1}dx-\frac{\cot^{-1}(x)}{x}$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2}\left(\int \frac{1}{x^2+1}dx^2-\int \frac{1}{x^2}dx^2\right)-\int \frac{\cot^{-1}(x)}{x^2+1}dx-\frac{\cot^{-1}(x)}{x}$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}\left(\int \frac{1}{x^2+1}dx^2-\log\left(x^2\right)\right)-\int \frac{\cot^{-1}(x)}{x^2+1}dx-\frac{\cot^{-1}(x)}{x}$$

$$\Big\downarrow \text{16}$$

$$-\int \frac{\cot^{-1}(x)}{x^2+1}dx+\frac{1}{2}\left(\log\left(x^2+1\right)-\log\left(x^2\right)\right)-\frac{\cot^{-1}(x)}{x}$$

$$\Big\downarrow \text{5420}$$

$$\frac{1}{2}\left(\log\left(x^2+1\right)-\log\left(x^2\right)\right)+\frac{1}{2}\cot^{-1}(x)^2-\frac{\cot^{-1}(x)}{x}$$

input
```
Int[ArcCot[x]/(x^2*(1 + x^2)),x]
```

output
```
-(ArcCot[x]/x) + ArcCot[x]^2/2 + (-Log[x^2] + Log[1 + x^2])/2
```

3.43.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5420
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5454
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcCot[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcCot[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.43.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

method	result
parallelrisch	$-\frac{-\operatorname{arccot}(x)^{2} x+2 \ln(x) x-\ln(x^{2}+1) x+2 \operatorname{arccot}(x)}{2 x}$
default	$-\frac{\operatorname{arccot}(x)}{x}-\operatorname{arccot}(x) \arctan(x)-\ln(x)+\frac{\ln(x^{2}+1)}{2}-\frac{\arctan(x)^{2}}{2}$
parts	$-\frac{\operatorname{arccot}(x)}{x}-\operatorname{arccot}(x) \arctan(x)-\ln(x)+\frac{\ln(x^{2}+1)}{2}-\frac{\arctan(x)^{2}}{2}$
risch	$-\frac{\ln(ix+1)^{2}}{8}+\frac{(\ln(-ix+1) x-2i) \ln(ix+1)}{4x}-\frac{-2i \ln((-\pi+6i) x+6+i\pi) \pi x+2i \ln((-\pi-6i) x+6-i\pi) \pi x+\ln(-ix+1)^{2} x-4i \ln}{8}$

input
```
int(arccot(x)/x^2/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-arccot(x)^2*x+2*ln(x)*x-ln(x^2+1)*x+2*arccot(x))/x
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{\cot^{-1}(x)}{x^2(1+x^2)}\,dx = \frac{x \operatorname{arccot}(x)^2 + x \log(x^2+1) - 2\,x \log(x) - 2\,\operatorname{arccot}(x)}{2\,x}$$

input
```
integrate(arccot(x)/x^2/(x^2+1),x, algorithm="fricas")
```

output
```
1/2*(x*arccot(x)^2 + x*log(x^2 + 1) - 2*x*log(x) - 2*arccot(x))/x
```

3.43.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{\cot^{-1}(x)}{x^2(1+x^2)}\,dx = -\log(x) + \frac{\log(x^2+1)}{2} + \frac{\operatorname{acot}^2(x)}{2} - \frac{\operatorname{acot}(x)}{x}$$

input
```
integrate(acot(x)/x**2/(x**2+1),x)
```

output
```
-log(x) + log(x**2 + 1)/2 + acot(x)**2/2 - acot(x)/x
```

3.43.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{\cot^{-1}(x)}{x^2(1+x^2)}\,dx = -\left(\frac{1}{x}+\arctan(x)\right)\operatorname{arccot}(x)-\frac{1}{2}\arctan(x)^2+\frac{1}{2}\log\left(x^2+1\right)-\log(x)$$

input
```
integrate(arccot(x)/x^2/(x^2+1),x, algorithm="maxima")
```

output
```
-(1/x + arctan(x))*arccot(x) - 1/2*arctan(x)^2 + 1/2*log(x^2 + 1) - log(x)
```

3.43.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\cot^{-1}(x)}{x^2(1+x^2)}\,dx = \frac{1}{2}\arctan\left(\frac{1}{x}\right)^2-\frac{\arctan\left(\frac{1}{x}\right)}{x}+\frac{1}{2}\log\left(\frac{1}{x^2}+1\right)$$

input
```
integrate(arccot(x)/x^2/(x^2+1),x, algorithm="giac")
```

output
```
1/2*arctan(1/x)^2 - arctan(1/x)/x + 1/2*log(1/x^2 + 1)
```

3.43.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\cot^{-1}(x)}{x^2(1+x^2)}\,dx = \frac{\ln\left(x^2+1\right)}{2}-\ln(x)-\frac{\operatorname{acot}(x)}{x}+\frac{\operatorname{acot}(x)^2}{2}$$

input
```
int(acot(x)/(x^2*(x^2 + 1)),x)
```

output
```
log(x^2 + 1)/2 - log(x) - acot(x)/x + acot(x)^2/2
```

3.44 $\int \frac{\cot^{-1}(x)}{x^3(1+x^2)}\,dx$

3.44.1 Optimal result

Integrand size = 13, antiderivative size = 72

$$\int \frac{\cot^{-1}(x)}{x^3(1+x^2)}\,dx = \frac{1}{2x}-\frac{\cot^{-1}(x)}{2x^2}-\frac{1}{2}i\cot^{-1}(x)^2+\frac{\arctan(x)}{2}-\cot^{-1}(x)\log\left(2-\frac{2}{1-ix}\right)-\frac{1}{2}i\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-ix}\right)$$

output
```
1/2/x-1/2*arccot(x)/x^2-1/2*I*arccot(x)^2+1/2*arctan(x)-arccot(x)*ln(2-2/(
1-I*x))-1/2*I*polylog(2,-1+2/(1-I*x))
```

3.44.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.76

$$\int \frac{\cot^{-1}(x)}{x^3(1+x^2)}\,dx = \frac{1}{2}\left(\frac{1}{x}+i\cot^{-1}(x)^2+\cot^{-1}(x)\left(-1-\frac{1}{x^2}-2\log\left(1+e^{2i\cot^{-1}(x)}\right)\right)+i\,\mathrm{PolyLog}\left(2,-e^{2i\cot^{-1}(x)}\right)\right)$$

input
```
Integrate[ArcCot[x]/(x^3*(1 + x^2)),x]
```

output
```
(x^(-1) + I*ArcCot[x]^2 + ArcCot[x]*(-1 - x^(-2) - 2*Log[1 + E^((2*I)*ArcC
ot[x])]) + I*PolyLog[2, -E^((2*I)*ArcCot[x])])/2
```

3.44.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {5454, 5362, 264, 216, 5460, 5404, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(x)}{x^3\left(x^2+1\right)}dx$$

$$\Big\downarrow \text{5454}$$

$$\int \frac{\cot^{-1}(x)}{x^3}dx-\int \frac{\cot^{-1}(x)}{x\left(x^2+1\right)}dx$$

$$\Big\downarrow \text{5362}$$

$$-\frac{1}{2}\int \frac{1}{x^2\left(x^2+1\right)}dx-\int \frac{\cot^{-1}(x)}{x\left(x^2+1\right)}dx-\frac{\cot^{-1}(x)}{2x^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2}\left(\int \frac{1}{x^2+1}dx+\frac{1}{x}\right)-\int \frac{\cot^{-1}(x)}{x\left(x^2+1\right)}dx-\frac{\cot^{-1}(x)}{2x^2}$$

$$\Big\downarrow \text{216}$$

$$-\int \frac{\cot^{-1}(x)}{x\left(x^2+1\right)}dx+\frac{1}{2}\left(\arctan(x)+\frac{1}{x}\right)-\frac{\cot^{-1}(x)}{2x^2}$$

$$\Big\downarrow \text{5460}$$

$$-i\int \frac{\cot^{-1}(x)}{x(x+i)}dx+\frac{1}{2}\left(\arctan(x)+\frac{1}{x}\right)-\frac{\cot^{-1}(x)}{2x^2}-\frac{1}{2}i\cot^{-1}(x)^2$$

$$\Big\downarrow \text{5404}$$

$$-i\left(-i\int \frac{\log\left(2-\frac{2}{1-ix}\right)}{x^2+1}dx-i\log\left(2-\frac{2}{1-ix}\right)\cot^{-1}(x)\right)+\frac{1}{2}\left(\arctan(x)+\frac{1}{x}\right)-\frac{\cot^{-1}(x)}{2x^2}-\frac{1}{2}i\cot^{-1}(x)^2$$

$$\Big\downarrow \text{2897}$$

$$\frac{1}{2}\left(\arctan(x)+\frac{1}{x}\right)-i\left(\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-ix}-1\right)-i\log\left(2-\frac{2}{1-ix}\right)\cot^{-1}(x)\right)-\frac{\cot^{-1}(x)}{2x^2}-\frac{1}{2}i\cot^{-1}(x)^2$$

input
```
Int[ArcCot[x]/(x^3*(1 + x^2)),x]
```

output
```
-1/2*ArcCot[x]/x^2 - (I/2)*ArcCot[x]^2 + (x^(-1) + ArcTan[x])/2 - I*((-I)*
ArcCot[x]*Log[2 - 2/(1 - I*x)] + PolyLog[2, -1 + 2/(1 - I*x)]/2)
```

3.44.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5404
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcCot[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] + Si
mp[b*c*(p/d)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5454
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcCot[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcCot[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5460
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*d*(p + 1))), x] + Simp[
I/d   Int[(a + b*ArcCot[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.44.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(58) = 116$.

Time $= 0.75$ (sec) , antiderivative size $= 178$, normalized size of antiderivative $= 2.47$

method	result
default	$-\frac{\operatorname{arccot}(x)}{2x^2} - \operatorname{arccot}(x)\ln(x) + \frac{\operatorname{arccot}(x)\ln(x^2+1)}{2} - \frac{i\left(\ln(x-i)\ln(x^2+1) - \frac{\ln(x-i)^2}{2} - \operatorname{dilog}\left(-\frac{i(i+x)}{2}\right) - \ln(x-i)\ln\left(-\frac{i}{\right.}\right.}{4}$
parts	$-\frac{\operatorname{arccot}(x)}{2x^2} - \operatorname{arccot}(x)\ln(x) + \frac{\operatorname{arccot}(x)\ln(x^2+1)}{2} - \frac{i\left(\ln(x-i)\ln(x^2+1) - \frac{\ln(x-i)^2}{2} - \operatorname{dilog}\left(-\frac{i(i+x)}{2}\right) - \ln(x-i)\ln\left(-\frac{i}{\right.}\right.}{4}$
risch	$\frac{\pi\ln(x^2+1)}{4} - \frac{\pi}{4x^2} - \frac{\pi\ln(-ix)}{2} + \frac{i\operatorname{dilog}\left(\frac{1}{2}-\frac{ix}{2}\right)}{4} - \frac{i\operatorname{dilog}\left(\frac{1}{2}+\frac{ix}{2}\right)}{4} + \frac{1}{2x} + \frac{i\operatorname{dilog}(ix+1)}{2} - \frac{i\ln(-ix+1)^2}{8} - \frac{i\operatorname{dilog}(-ix-}{2}$

input
```
int(arccot(x)/x^3/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arccot(x)/x^2-arccot(x)*ln(x)+1/2*arccot(x)*ln(x^2+1)-1/4*I*(ln(x-I)*
ln(x^2+1)-1/2*ln(x-I)^2-dilog(-1/2*I*(I+x))-ln(x-I)*ln(-1/2*I*(I+x)))+1/4*
I*(ln(I+x)*ln(x^2+1)-1/2*ln(I+x)^2-dilog(1/2*I*(x-I))-ln(I+x)*ln(1/2*I*(x-
I)))+1/2/x+1/2*arctan(x)+1/2*I*ln(x)*ln(1+I*x)-1/2*I*ln(x)*ln(1-I*x)+1/2*I
*dilog(1+I*x)-1/2*I*dilog(1-I*x)
```

3.44.5 Fricas [F]

$$\int \frac{\cot^{-1}(x)}{x^3\left(1+x^2\right)}\,dx=\int \frac{\operatorname{arccot}(x)}{(x^2+1)x^3}\,dx$$

input
```
integrate(arccot(x)/x^3/(x^2+1),x, algorithm="fricas")
```

output
```
integral(arccot(x)/(x^5 + x^3), x)
```

3.44.6 Sympy [F]

$$\int \frac{\cot^{-1}(x)}{x^3\left(1+x^2\right)}\,dx=\int \frac{\operatorname{acot}(x)}{x^3\left(x^2+1\right)}\,dx$$

input
```
integrate(acot(x)/x**3/(x**2+1),x)
```

output
```
Integral(acot(x)/(x**3*(x**2 + 1)), x)
```

3.44.7 Maxima [F]

$$\int \frac{\cot^{-1}(x)}{x^3\left(1+x^2\right)}\,dx=\int \frac{\operatorname{arccot}(x)}{(x^2+1)x^3}\,dx$$

input
```
integrate(arccot(x)/x^3/(x^2+1),x, algorithm="maxima")
```

output
```
integrate(arccot(x)/((x^2 + 1)*x^3), x)
```

3.44.8 Giac [F]

$$\int \frac{\cot^{-1}(x)}{x^3(1+x^2)}\,dx = \int \frac{\operatorname{arccot}(x)}{(x^2+1)x^3}\,dx$$

input
```
integrate(arccot(x)/x^3/(x^2+1),x, algorithm="giac")
```

output
```
integrate(arccot(x)/((x^2 + 1)*x^3), x)
```

3.44.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(x)}{x^3(1+x^2)}\,dx = \int \frac{\operatorname{acot}(x)}{x^3\,(x^2+1)}\,dx$$

input
```
int(acot(x)/(x^3*(x^2 + 1)),x)
```

output
```
int(acot(x)/(x^3*(x^2 + 1)), x)
```

3.45 $\int \frac{\cot^{-1}(x)}{x^4(1+x^2)} \, dx$

3.45.1 Optimal result

Integrand size = 13, antiderivative size = 47

$$\int \frac{\cot^{-1}(x)}{x^4(1+x^2)} \, dx = \frac{1}{6x^2} - \frac{\cot^{-1}(x)}{3x^3} + \frac{\cot^{-1}(x)}{x} - \frac{1}{2}\cot^{-1}(x)^2 + \frac{4\log(x)}{3} - \frac{2}{3}\log\left(1+x^2\right)$$

output
```
1/6/x^2-1/3*arccot(x)/x^3+arccot(x)/x-1/2*arccot(x)^2+4/3*ln(x)-2/3*ln(x^2
+1)
```

3.45.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(x)}{x^4(1+x^2)} \, dx = \frac{1}{6x^2} - \frac{\cot^{-1}(x)}{3x^3} + \frac{\cot^{-1}(x)}{x} - \frac{1}{2}\cot^{-1}(x)^2 + \frac{4\log(x)}{3} - \frac{2}{3}\log\left(1+x^2\right)$$

input
```
Integrate[ArcCot[x]/(x^4*(1 + x^2)),x]
```

output
```
1/(6*x^2) - ArcCot[x]/(3*x^3) + ArcCot[x]/x - ArcCot[x]^2/2 + (4*Log[x])/3
 - (2*Log[1 + x^2])/3
```

3.45.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.30, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.923, Rules used = {5454, 5362, 243, 54, 2009, 5454, 5362, 243, 47, 14, 16, 5420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(x)}{x^4(x^2+1)}dx$$

$$\Big\downarrow \text{5454}$$

$$\int \frac{\cot^{-1}(x)}{x^4}dx-\int \frac{\cot^{-1}(x)}{x^2(x^2+1)}dx$$

$$\Big\downarrow \text{5362}$$

$$-\int \frac{\cot^{-1}(x)}{x^2(x^2+1)}dx-\frac{1}{3}\int \frac{1}{x^3(x^2+1)}dx-\frac{\cot^{-1}(x)}{3x^3}$$

$$\Big\downarrow \text{243}$$

$$-\int \frac{\cot^{-1}(x)}{x^2(x^2+1)}dx-\frac{1}{6}\int \frac{1}{x^4(x^2+1)}dx^2-\frac{\cot^{-1}(x)}{3x^3}$$

$$\Big\downarrow \text{54}$$

$$-\int \frac{\cot^{-1}(x)}{x^2(x^2+1)}dx-\frac{1}{6}\int \left(-\frac{1}{x^2}+\frac{1}{x^4}+\frac{1}{x^2+1}\right)dx^2-\frac{\cot^{-1}(x)}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$-\int \frac{\cot^{-1}(x)}{x^2(x^2+1)}dx-\frac{\cot^{-1}(x)}{3x^3}+\frac{1}{6}\left(\frac{1}{x^2}+\log(x^2)-\log(x^2+1)\right)$$

$$\Big\downarrow \text{5454}$$

$$-\int \frac{\cot^{-1}(x)}{x^2}dx+\int \frac{\cot^{-1}(x)}{x^2+1}dx-\frac{\cot^{-1}(x)}{3x^3}+\frac{1}{6}\left(\frac{1}{x^2}+\log(x^2)-\log(x^2+1)\right)$$

$$\Big\downarrow \text{5362}$$

$$\int \frac{1}{x(x^2+1)}dx+\int \frac{\cot^{-1}(x)}{x^2+1}dx-\frac{\cot^{-1}(x)}{3x^3}+\frac{1}{6}\left(\frac{1}{x^2}+\log(x^2)-\log(x^2+1)\right)+\frac{\cot^{-1}(x)}{x}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^2(x^2+1)}dx^2+\int \frac{\cot^{-1}(x)}{x^2+1}dx-\frac{\cot^{-1}(x)}{3x^3}+\frac{1}{6}\left(\frac{1}{x^2}+\log(x^2)-\log(x^2+1)\right)+\frac{\cot^{-1}(x)}{x}$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2}\left(\int \frac{1}{x^2}dx^2 - \int \frac{1}{x^2+1}dx^2\right) + \int \frac{\cot^{-1}(x)}{x^2+1}dx - \frac{\cot^{-1}(x)}{3x^3} + \frac{1}{6}\left(\frac{1}{x^2} + \log\left(x^2\right) - \log\left(x^2+1\right)\right) + \frac{\cot^{-1}(x)}{x}$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}\left(\log\left(x^2\right) - \int \frac{1}{x^2+1}dx^2\right) + \int \frac{\cot^{-1}(x)}{x^2+1}dx - \frac{\cot^{-1}(x)}{3x^3} + \frac{1}{6}\left(\frac{1}{x^2} + \log\left(x^2\right) - \log\left(x^2+1\right)\right) + \frac{\cot^{-1}(x)}{x}$$

$$\Big\downarrow \text{16}$$

$$\int \frac{\cot^{-1}(x)}{x^2+1}dx - \frac{\cot^{-1}(x)}{3x^3} + \frac{1}{2}\left(\log\left(x^2\right) - \log\left(x^2+1\right)\right) + \frac{1}{6}\left(\frac{1}{x^2} + \log\left(x^2\right) - \log\left(x^2+1\right)\right) + \frac{\cot^{-1}(x)}{x}$$

$$\Big\downarrow \text{5420}$$

$$-\frac{\cot^{-1}(x)}{3x^3} + \frac{1}{2}\left(\log\left(x^2\right) - \log\left(x^2+1\right)\right) + \frac{1}{6}\left(\frac{1}{x^2} + \log\left(x^2\right) - \log\left(x^2+1\right)\right) - \frac{1}{2}\cot^{-1}(x)^2 + \frac{\cot^{-1}(x)}{x}$$

input
```
Int[ArcCot[x]/(x^4*(1 + x^2)),x]
```

output
```
-1/3*ArcCot[x]/x^3 + ArcCot[x]/x - ArcCot[x]^2/2 + (Log[x^2] - Log[1 + x^2
])/2 + (x^(-2) + Log[x^2] - Log[1 + x^2])/6
```

3.45.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5420
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5454
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcCot[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcCot[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.45.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

method	result
default	$-\frac{\operatorname{arccot}(x)}{3x^3}+\frac{\operatorname{arccot}(x)}{x}+\operatorname{arccot}(x)\arctan(x)+\frac{1}{6x^2}+\frac{4\ln(x)}{3}-\frac{2\ln(x^2+1)}{3}+\frac{\arctan(x)^2}{2}$
parts	$-\frac{\operatorname{arccot}(x)}{3x^3}+\frac{\operatorname{arccot}(x)}{x}+\operatorname{arccot}(x)\arctan(x)+\frac{1}{6x^2}+\frac{4\ln(x)}{3}-\frac{2\ln(x^2+1)}{3}+\frac{\arctan(x)^2}{2}$
parallelrisch	$\frac{-3\operatorname{arccot}(x)^2x^3+8\ln(x)x^3-4\ln(x^2+1)x^3+6x^2\operatorname{arccot}(x)+x-2\operatorname{arccot}(x)}{6x^3}$
risch	$\frac{\ln(ix+1)^2}{8}-\frac{(3\ln(-ix+1)x^3-6ix^2+2i)\ln(ix+1)}{12x^3}+\frac{-6i\ln((-\pi+8i)x+8+i\pi)\pi x^3+6i\ln((-\pi-8i)x+8-i\pi)\pi x^3+3\ln(-ix}{}$

3.45. $\int \frac{\cot^{-1}(x)}{x^4(1+x^2)}\,dx$

input
```
int(arccot(x)/x^4/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3*arccot(x)/x^3+arccot(x)/x+arccot(x)*arctan(x)+1/6/x^2+4/3*ln(x)-2/3*l
n(x^2+1)+1/2*arctan(x)^2
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(x)}{x^4(1+x^2)}\,dx = -\frac{3\,x^3\,\mathrm{arccot}\left(x\right)^2+4\,x^3\log\left(x^2+1\right)-8\,x^3\log\left(x\right)-2\left(3\,x^2-1\right)\mathrm{arccot}\left(x\right)-x}{6\,x^3}$$

input
```
integrate(arccot(x)/x^4/(x^2+1),x, algorithm="fricas")
```

output
```
-1/6*(3*x^3*arccot(x)^2 + 4*x^3*log(x^2 + 1) - 8*x^3*log(x) - 2*(3*x^2 - 1
)*arccot(x) - x)/x^3
```

3.45.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{\cot^{-1}(x)}{x^4(1+x^2)}\,dx = \frac{4\log\left(x\right)}{3}-\frac{2\log\left(x^2+1\right)}{3}-\frac{\mathrm{acot}^2\left(x\right)}{2}+\frac{\mathrm{acot}\left(x\right)}{x}+\frac{1}{6x^2}-\frac{\mathrm{acot}\left(x\right)}{3x^3}$$

input
```
integrate(acot(x)/x**4/(x**2+1),x)
```

output
```
4*log(x)/3 - 2*log(x**2 + 1)/3 - acot(x)**2/2 + acot(x)/x + 1/(6*x**2) - a
cot(x)/(3*x**3)
```

3.45.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.17

$$\int \frac{\cot^{-1}(x)}{x^4(1+x^2)}\,dx = \frac{1}{3}\left(\frac{3\,x^2-1}{x^3}+3\,\arctan(x)\right)\operatorname{arccot}(x) + \frac{3\,x^2\arctan(x)^2-4\,x^2\log(x^2+1)+8\,x^2\log(x)+1}{6\,x^2}$$

input
```
integrate(arccot(x)/x^4/(x^2+1),x, algorithm="maxima")
```

output
```
1/3*((3*x^2 - 1)/x^3 + 3*arctan(x))*arccot(x) + 1/6*(3*x^2*arctan(x)^2 - 4
*x^2*log(x^2 + 1) + 8*x^2*log(x) + 1)/x^2
```

3.45.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{\cot^{-1}(x)}{x^4(1+x^2)}\,dx = -\frac{1}{2}\arctan\left(\frac{1}{x}\right)^2+\frac{\arctan\left(\frac{1}{x}\right)}{x}+\frac{1}{6\,x^2}-\frac{\arctan\left(\frac{1}{x}\right)}{3\,x^3}-\frac{2}{3}\log\left(\frac{1}{x^2}+1\right)$$

input
```
integrate(arccot(x)/x^4/(x^2+1),x, algorithm="giac")
```

output
```
-1/2*arctan(1/x)^2 + arctan(1/x)/x + 1/6/x^2 - 1/3*arctan(1/x)/x^3 - 2/3*l
og(1/x^2 + 1)
```

3.45.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{\cot^{-1}(x)}{x^4(1+x^2)}\,dx = \frac{4\,\ln(x)}{3}-\frac{2\,\ln(x^2+1)}{3}-\frac{\operatorname{acot}(x)^2}{2}+\frac{1}{6\,x^2}+\frac{\operatorname{acot}(x)\,\left(x^2-\frac{1}{3}\right)}{x^3}$$

input
```
int(acot(x)/(x^4*(x^2 + 1)),x)
```

output
```
(4*log(x))/3 - (2*log(x^2 + 1))/3 - acot(x)^2/2 + 1/(6*x^2) + (acot(x)*(x^
2 - 1/3))/x^3
```

3.46 $\int \frac{x^2 \cot^{-1}(cx)}{1+x^2} \, dx$

3.46.1 Optimal result

Integrand size = 15, antiderivative size = 206

$$\begin{aligned}\int \frac{x^2 \cot^{-1}(cx)}{1+x^2} \, dx = {} & x\cot^{-1}(cx) - \frac{1}{2} i \arctan(x) \log\left(1-\frac{i}{cx}\right) \\ & + \frac{1}{2} i \arctan(x) \log\left(1+\frac{i}{cx}\right) + \frac{1}{2} i \arctan(x) \log\left(-\frac{2i(i-cx)}{(1-c)(1-ix)}\right) \\ & - \frac{1}{2} i \arctan(x) \log\left(-\frac{2i(i+cx)}{(1+c)(1-ix)}\right) \\ & + \frac{\log(1+c^2x^2)}{2c} + \frac{1}{4} \operatorname{PolyLog}\left(2, 1+\frac{2i(i-cx)}{(1-c)(1-ix)}\right) \\ & - \frac{1}{4} \operatorname{PolyLog}\left(2, 1+\frac{2i(i+cx)}{(1+c)(1-ix)}\right)\end{aligned}$$

output
```
x*arccot(c*x)-1/2*I*arctan(x)*ln(1-I/c/x)+1/2*I*arctan(x)*ln(1+I/c/x)+1/2*
I*arctan(x)*ln(-2*I*(I-c*x)/(1-c)/(1-I*x))-1/2*I*arctan(x)*ln(-2*I*(c*x+I)
/(1+c)/(1-I*x))+1/2*ln(c^2*x^2+1)/c+1/4*polylog(2,1+2*I*(I-c*x)/(1-c)/(1-I
*x))-1/4*polylog(2,1+2*I*(c*x+I)/(1+c)/(1-I*x))
```

3.46.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 626 vs. 2(206) = 412.

Time = 1.19 (sec) , antiderivative size = 626, normalized size of antiderivative = 3.04

$$\int \frac{x^2 \cot^{-1}(cx)}{1+x^2}\,dx$$
$$= \frac{cx\cot^{-1}(cx) - \log\left(\frac{1}{c\sqrt{1+\frac{1}{c^2x^2}}\,x}\right) + \frac{1}{4}\sqrt{-c^2}\left(2\arccos\left(\frac{1+c^2}{-1+c^2}\right)\operatorname{arctanh}\left(\frac{\sqrt{-c^2}}{cx}\right) - 4\cot^{-1}(cx)\operatorname{arctanh}\left(\frac{cx}{\sqrt{-c^2}}\right.\right.}{}$$

input
```
Integrate[(x^2*ArcCot[c*x])/(1 + x^2),x]
```

output
```
(c*x*ArcCot[c*x] - Log[1/(c*Sqrt[1 + 1/(c^2*x^2)]*x)] + (Sqrt[-c^2]*(2*Arc
Cos[(1 + c^2)/(-1 + c^2)]*ArcTanh[Sqrt[-c^2]/(c*x)] - 4*ArcCot[c*x]*ArcTan
h[(c*x)/Sqrt[-c^2]] - (ArcCos[(1 + c^2)/(-1 + c^2)] - (2*I)*ArcTanh[Sqrt[-
c^2]/(c*x)])*Log[(-2*(c^2 + I*Sqrt[-c^2])*(-I + c*x))/((-1 + c^2)*(Sqrt[-c
^2] - c*x))] - (ArcCos[(1 + c^2)/(-1 + c^2)] + (2*I)*ArcTanh[Sqrt[-c^2]/(c
*x)])*Log[((2*I)*(I*c^2 + Sqrt[-c^2])*(I + c*x))/((-1 + c^2)*(Sqrt[-c^2] -
 c*x))] + (ArcCos[(1 + c^2)/(-1 + c^2)] - (2*I)*ArcTanh[Sqrt[-c^2]/(c*x)]
+ (2*I)*ArcTanh[(c*x)/Sqrt[-c^2]])*Log[(Sqrt[2]*Sqrt[-c^2])/(Sqrt[-1 + c^2
]*E^(I*ArcCot[c*x])*Sqrt[-1 - c^2 + (-1 + c^2)*Cos[2*ArcCot[c*x]]])] + (Ar
cCos[(1 + c^2)/(-1 + c^2)] + (2*I)*ArcTanh[Sqrt[-c^2]/(c*x)] - (2*I)*ArcTa
nh[(c*x)/Sqrt[-c^2]])*Log[(Sqrt[2]*Sqrt[-c^2]*E^(I*ArcCot[c*x]))/(Sqrt[-1
+ c^2]*Sqrt[-1 - c^2 + (-1 + c^2)*Cos[2*ArcCot[c*x]]])] + I*(-PolyLog[2, (
(1 + c^2 - (2*I)*Sqrt[-c^2])*(Sqrt[-c^2] + c*x))/((-1 + c^2)*(Sqrt[-c^2] -
 c*x))] + PolyLog[2, ((1 + c^2 + (2*I)*Sqrt[-c^2])*(Sqrt[-c^2] + c*x))/((-
1 + c^2)*(Sqrt[-c^2] - c*x))])))/4)/c
```

3.46.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.62, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5452, 5346, 240, 5444, 2920, 27, 2005, 5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \cot^{-1}(cx)}{x^2+1}\,dx$$

$$\Big\downarrow \; 5452$$

$$\int \cot^{-1}(cx)dx - \int \frac{\cot^{-1}(cx)}{x^2+1}dx$$

$$\Big\downarrow \; 5346$$

$$c\int \frac{x}{c^2x^2+1}dx - \int \frac{\cot^{-1}(cx)}{x^2+1}dx + x\cot^{-1}(cx)$$

$$\Big\downarrow \; 240$$

$$-\int \frac{\cot^{-1}(cx)}{x^2+1}dx + \frac{\log\left(c^2x^2+1\right)}{2c} + x\cot^{-1}(cx)$$

$$\Big\downarrow \; 5444$$

$$-\frac{1}{2}i\int \frac{\log\left(1-\frac{i}{cx}\right)}{x^2+1}dx + \frac{1}{2}i\int \frac{\log\left(1+\frac{i}{cx}\right)}{x^2+1}dx + \frac{\log\left(c^2x^2+1\right)}{2c} + x\cot^{-1}(cx)$$

$$\Big\downarrow \; 2920$$

$$-\frac{1}{2}i\left(\arctan(x)\log\left(1-\frac{i}{cx}\right) - \frac{i\int \frac{c\arctan(x)}{\left(c-\frac{i}{x}\right)x^2}dx}{c}\right) + \frac{1}{2}i\left(\frac{i\int \frac{c\arctan(x)}{\left(c+\frac{i}{x}\right)x^2}dx}{c} + \arctan(x)\log\left(1+\frac{i}{cx}\right)\right) + \frac{\log\left(c^2x^2+1\right)}{2c} + x\cot^{-1}(cx)$$

$$\Big\downarrow \; 27$$

$$-\frac{1}{2}i\left(\arctan(x)\log\left(1-\frac{i}{cx}\right) - i\int \frac{\arctan(x)}{\left(c-\frac{i}{x}\right)x^2}dx\right) + \frac{1}{2}i\left(i\int \frac{\arctan(x)}{\left(c+\frac{i}{x}\right)x^2}dx + \arctan(x)\log\left(1+\frac{i}{cx}\right)\right) + \frac{\log\left(c^2x^2+1\right)}{2c} + x\cot^{-1}(cx)$$

$$\Big\downarrow \; 2005$$

$$-\frac{1}{2}i\left(\arctan(x)\log\left(1-\frac{i}{cx}\right) - i\int \frac{\arctan(x)}{x(cx-i)}dx\right) + \frac{1}{2}i\left(i\int \frac{\arctan(x)}{x(cx+i)}dx + \arctan(x)\log\left(1+\frac{i}{cx}\right)\right) + \frac{\log\left(c^2x^2+1\right)}{2c} + x\cot^{-1}(cx)$$

$$\Big\downarrow \; 5411$$

3.46. $\int \frac{x^2\cot^{-1}(cx)}{1+x^2}\,dx$

$$-\frac{1}{2}i\left(\arctan(x)\log\left(1-\frac{i}{cx}\right)-i\int\left(\frac{i\arctan(x)}{x}-\frac{ic\arctan(x)}{cx-i}\right)dx\right)+\frac{1}{2}i\left(i\int\left(\frac{ic\arctan(x)}{cx+i}-\frac{i\arctan(x)}{x}\right)dx+\arctan(x)\log\left(1+\frac{i}{cx}\right)\right)+\frac{\log\left(c^2x^2+1\right)}{2c}+x\cot^{-1}(cx)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}i\left(\arctan(x)\log\left(1-\frac{i}{cx}\right)-i\left(-i\arctan(x)\log\left(-\frac{2i(-cx+i)}{(1-c)(1-ix)}\right)+i\arctan(x)\log\left(\frac{2}{1-ix}\right)-\frac{1}{2}\text{PolyLog}\right.\right.$$
$$\frac{1}{2}i\left(i\left(i\arctan(x)\log\left(-\frac{2i(cx+i)}{(c+1)(1-ix)}\right)-i\arctan(x)\log\left(\frac{2}{1-ix}\right)+\frac{1}{2}\text{PolyLog}\left(2,\frac{2i(cx+i)}{(c+1)(1-ix)}+1\right)-\frac{1}{2}\right.\right.$$
$$\frac{\log\left(c^2x^2+1\right)}{2c}+x\cot^{-1}(cx)$$

input
```
Int[(x^2*ArcCot[c*x])/(1 + x^2),x]
```

output
```
x*ArcCot[c*x] + Log[1 + c^2*x^2]/(2*c) - (I/2)*(ArcTan[x]*Log[1 - I/(c*x)]
 - I*(I*ArcTan[x]*Log[2/(1 - I*x)] - I*ArcTan[x]*Log[((-2*I)*(I - c*x))/((
1 - c)*(1 - I*x))] + PolyLog[2, 1 - 2/(1 - I*x)]/2 - PolyLog[2, (-I)*x]/2
+ PolyLog[2, I*x]/2 - PolyLog[2, 1 + ((2*I)*(I - c*x))/((1 - c)*(1 - I*x))
]/2)) + (I/2)*(ArcTan[x]*Log[1 + I/(c*x)] + I*((-I)*ArcTan[x]*Log[2/(1 - I
*x)] + I*ArcTan[x]*Log[((-2*I)*(I + c*x))/((1 + c)*(1 - I*x))] - PolyLog[2
, 1 - 2/(1 - I*x)]/2 + PolyLog[2, (-I)*x]/2 - PolyLog[2, I*x]/2 + PolyLog[
2, 1 + ((2*I)*(I + c*x))/((1 + c)*(1 - I*x))]/2))
```

3.46.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2920
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Simp[b*e*n*p   Int[u*(x^(n - 1)/(d + e*x^n)), x
], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

rule 5444
```
Int[ArcCot[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Simp[I/2   Int[
Log[1 - I/(c*x)]/(d + e*x^2), x], x] - Simp[I/2   Int[Log[1 + I/(c*x)]/(d +
 e*x^2), x], x] /; FreeQ[{c, d, e}, x]
```

rule 5452
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

3.46. $\int \frac{x^2 \cot^{-1}(cx)}{1+x^2} \, dx$

3.46.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.27

method	result
risch	$\frac{i\ln(icx+1)x}{2}+\frac{\pi x}{2}-\frac{i\ln(-icx+1)x}{2}-\frac{\pi\arctan(x)}{2}+\frac{i\pi}{2c}-\frac{i\arctan(cx)}{2c}+\frac{\ln(c^2x^2+1)}{4c}-\frac{1}{c}+\frac{\ln(-icx+1)\ln}{4}$
derivativedivides	$\frac{-\operatorname{arccot}(cx)\arctan(x)c^3+\operatorname{arccot}(cx)c^3x+c^3\left(\frac{\ln(c^2x^2+1)}{2c}-\frac{i\arctan(x)\ln\left(1-\frac{(c-1)(ix+1)^2}{(x^2+1)(1+c)}\right)}{2}-\frac{\arctan(x)^2}{2}-\frac{\operatorname{polylog}\left(2,\frac{(c-}{(x^2}\right.}{4}\right.}{}$
default	$\frac{-\operatorname{arccot}(cx)\arctan(x)c^3+\operatorname{arccot}(cx)c^3x+c^3\left(\frac{\ln(c^2x^2+1)}{2c}-\frac{i\arctan(x)\ln\left(1-\frac{(c-1)(ix+1)^2}{(x^2+1)(1+c)}\right)}{2}-\frac{\arctan(x)^2}{2}-\frac{\operatorname{polylog}\left(2,\frac{(c-}{(x^2}\right.}{4}\right.}{}$
parts	$-\operatorname{arccot}(cx)\arctan(x)+x\operatorname{arccot}(cx)+c\left(\frac{\ln(c^2x^2+1)}{2c^2}-\frac{i\arctan(x)\ln\left(1-\frac{(c-1)(ix+1)^2}{(x^2+1)(1+c)}\right)}{2c}-\operatorname{ar}\right.$

input
```
int(x^2*arccot(c*x)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*ln(1+I*c*x)*x+1/2*Pi*x-1/2*I*ln(1-I*c*x)*x-1/2*Pi*arctan(x)+1/2*I/c*
Pi-1/2*I/c*arctan(c*x)+1/4*ln(c^2*x^2+1)/c-1/c+1/4*ln(1-I*c*x)*ln((-c-I*c*
x)/(-c-1))+1/4*dilog((-c-I*c*x)/(-c-1))-1/4*ln(1-I*c*x)*ln((c-I*c*x)/(c-1)
)-1/4*dilog((c-I*c*x)/(c-1))+1/2/c*ln(1+I*c*x)+1/4*ln(1+I*c*x)*ln((-c+I*c*
x)/(-c-1))+1/4*dilog((-c+I*c*x)/(-c-1))-1/4*ln(1+I*c*x)*ln((c+I*c*x)/(c-1)
)-1/4*dilog((c+I*c*x)/(c-1))
```

3.46.5 Fricas [F]

$$\int \frac{x^2\cot^{-1}(cx)}{1+x^2}\,dx=\int \frac{x^2\operatorname{arccot}(cx)}{x^2+1}\,dx$$

input
```
integrate(x^2*arccot(c*x)/(x^2+1),x, algorithm="fricas")
```

output
```
integral(x^2*arccot(c*x)/(x^2 + 1), x)
```

3.46.6 Sympy [F]

$$\int \frac{x^2 \cot^{-1}(cx)}{1+x^2}\,dx = \int \frac{x^2 \operatorname{acot}(cx)}{x^2+1}\,dx$$

input
```
integrate(x**2*acot(c*x)/(x**2+1),x)
```

output
```
Integral(x**2*acot(c*x)/(x**2 + 1), x)
```

3.46.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.92

$$\int \frac{x^2 \cot^{-1}(cx)}{1+x^2}\,dx = (x - \arctan(x))\operatorname{arccot}(cx) - \frac{4\,c\arctan(cx)\arctan(x) - 4\,c\arctan(x)\arctan\left(\frac{cx}{c-1}, -\frac{1}{c-1}\right) + c\log(x^2+1)\log\left(\frac{c^2x^2+1}{c^2+2\,c+1}\right) - c\log(x^2}{8}$$

input
```
integrate(x^2*arccot(c*x)/(x^2+1),x, algorithm="maxima")
```

output
```
(x - arctan(x))*arccot(c*x) - 1/8*(4*c*arctan(c*x)*arctan(x) - 4*c*arctan(
x)*arctan2(c*x/(c - 1), -1/(c - 1)) + c*log(x^2 + 1)*log((c^2*x^2 + 1)/(c^
2 + 2*c + 1)) - c*log(x^2 + 1)*log((c^2*x^2 + 1)/(c^2 - 2*c + 1)) + 2*c*di
log((I*c*x + c)/(c + 1)) + 2*c*dilog(-(I*c*x - c)/(c + 1)) - 2*c*dilog((I*
c*x + c)/(c - 1)) - 2*c*dilog(-(I*c*x - c)/(c - 1)) - 4*log(c^2*x^2 + 1))/
c
```

3.46.8 Giac [F]

$$\int \frac{x^2 \cot^{-1}(cx)}{1+x^2}\,dx = \int \frac{x^2 \operatorname{arccot}(cx)}{x^2+1}\,dx$$

input
```
integrate(x^2*arccot(c*x)/(x^2+1),x, algorithm="giac")
```

output
```
integrate(x^2*arccot(c*x)/(x^2 + 1), x)
```

3.46.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \cot^{-1}(cx)}{1+x^2} \, dx = \int \frac{x^2 \, \text{acot}(c\,x)}{x^2+1} \, dx$$

input
```
int((x^2*acot(c*x))/(x^2 + 1),x)
```

output
```
int((x^2*acot(c*x))/(x^2 + 1), x)
```

3.47 $\int \frac{x\cot^{-1}(cx)}{1+x^2}\,dx$

3.47.1 Optimal result

Integrand size = 13, antiderivative size = 188

$$\begin{aligned}\int \frac{x\cot^{-1}(cx)}{1+x^2}\,dx &= -\cot^{-1}(cx)\log\left(\frac{2}{1-icx}\right)+\frac{1}{2}\cot^{-1}(cx)\log\left(\frac{2ic(i-x)}{(1-c)(1-icx)}\right)\\ &+\frac{1}{2}\cot^{-1}(cx)\log\left(-\frac{2ic(i+x)}{(1+c)(1-icx)}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)\\ &+\frac{1}{4}i\,\text{PolyLog}\left(2,1-\frac{2ic(i-x)}{(1-c)(1-icx)}\right)\\ &+\frac{1}{4}i\,\text{PolyLog}\left(2,1+\frac{2ic(i+x)}{(1+c)(1-icx)}\right)\end{aligned}$$

output
```
-arccot(c*x)*ln(2/(1-I*c*x))+1/2*arccot(c*x)*ln(2*I*c*(I-x)/(1-c)/(1-I*c*x
))+1/2*arccot(c*x)*ln(-2*I*c*(I+x)/(1+c)/(1-I*c*x))-1/2*I*polylog(2,1-2/(1
-I*c*x))+1/4*I*polylog(2,1-2*I*c*(I-x)/(1-c)/(1-I*c*x))+1/4*I*polylog(2,1+
2*I*c*(I+x)/(1+c)/(1-I*c*x))
```

3.47.2 Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.64

$$\int \frac{x\cot^{-1}(cx)}{1+x^2}\,dx = \frac{1}{2}\left(-i\cot^{-1}(cx)^2-2i\arcsin\left(\sqrt{\frac{1}{1-c^2}}\right)\arctan\left(\frac{\sqrt{c^2}}{cx}\right)-2\cot^{-1}(cx)\log\left(1-e^{2i\cot^{-1}(cx)}\right)+\left(\cot^{-1}(cx)-\arcsin\left(\sqrt{\frac{1}{1-c^2}}\right)\right)\log\left(\frac{-1+\left(-1+2\sqrt{c^2}\right)e^{2i\cot^{-1}(cx)}-c^2\left(-1+e^{2i\cot^{-1}(cx)}\right)}{-1+c^2}\right)+\left(\cot^{-1}(cx)+\arcsin\left(\sqrt{\frac{1}{1-c^2}}\right)\right)\log\left(-\frac{1+\left(1+2\sqrt{c^2}\right)e^{2i\cot^{-1}(cx)}+c^2\left(-1+e^{2i\cot^{-1}(cx)}\right)}{-1+c^2}\right)+i\left(\cot^{-1}(cx)^2+\text{PolyLog}\left(2,e^{2i\cot^{-1}(cx)}\right)\right)-\frac{1}{2}i\left(\text{PolyLog}\left(2,\frac{\left(1+c^2-2\sqrt{c^2}\right)e^{2i\cot^{-1}(cx)}}{-1+c^2}\right)+\text{PolyLog}\left(2,\frac{\left(1+c^2+2\sqrt{c^2}\right)e^{2i\cot^{-1}(cx)}}{-1+c^2}\right)\right)\right)$$

input
```
Integrate[(x*ArcCot[c*x])/(1 + x^2),x]
```

output
```
((-I)*ArcCot[c*x]^2 - (2*I)*ArcSin[Sqrt[(1 - c^2)^(-1)]]*ArcTan[Sqrt[c^2]/
(c*x)] - 2*ArcCot[c*x]*Log[1 - E^((2*I)*ArcCot[c*x])] + (ArcCot[c*x] - Arc
Sin[Sqrt[(1 - c^2)^(-1)]])*Log[(-1 + (-1 + 2*Sqrt[c^2])*E^((2*I)*ArcCot[c*
x]) - c^2*(-1 + E^((2*I)*ArcCot[c*x])))/(-1 + c^2)] + (ArcCot[c*x] + ArcSi
n[Sqrt[(1 - c^2)^(-1)]])*Log[-((1 + (1 + 2*Sqrt[c^2])*E^((2*I)*ArcCot[c*x]
) + c^2*(-1 + E^((2*I)*ArcCot[c*x])))/(-1 + c^2))] + I*(ArcCot[c*x]^2 + Po
lyLog[2, E^((2*I)*ArcCot[c*x])]) - (I/2)*(PolyLog[2, ((1 + c^2 - 2*Sqrt[c^
2])*E^((2*I)*ArcCot[c*x]))/(-1 + c^2)] + PolyLog[2, ((1 + c^2 + 2*Sqrt[c^2
])*E^((2*I)*ArcCot[c*x]))/(-1 + c^2)]))/2
```

3.47.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5464, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \cot^{-1}(cx)}{x^2+1} \, dx$$

$$\Big\downarrow \text{5464}$$

$$\int \left(\frac{\cot^{-1}(cx)}{2(x+i)}-\frac{\cot^{-1}(cx)}{2(-x+i)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}i \operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)+\frac{1}{4}i \operatorname{PolyLog}\left(2,1-\frac{2ic(i-x)}{(1-c)(1-icx)}\right)+\frac{1}{4}i \operatorname{PolyLog}\left(2,\frac{2ic(x+i)}{(c+1)(1-icx)}+1\right)+\log\left(\frac{2}{1-icx}\right)\left(-\cot^{-1}(cx)\right)+\frac{1}{2}\log\left(\frac{2ic(-x+i)}{(1-c)(1-icx)}\right)\cot^{-1}(cx)+\frac{1}{2}\log\left(-\frac{2ic(x+i)}{(c+1)(1-icx)}\right)\cot^{-1}(cx)$$

input
```
Int[(x*ArcCot[c*x])/(1 + x^2),x]
```

output
```
-(ArcCot[c*x]*Log[2/(1 - I*c*x)]) + (ArcCot[c*x]*Log[((2*I)*c*(I - x))/((1
 - c)*(1 - I*c*x))])/2 + (ArcCot[c*x]*Log[((-2*I)*c*(I + x))/((1 + c)*(1 -
 I*c*x))])/2 - (I/2)*PolyLog[2, 1 - 2/(1 - I*c*x)] + (I/4)*PolyLog[2, 1 -
((2*I)*c*(I - x))/((1 - c)*(1 - I*c*x))] + (I/4)*PolyLog[2, 1 + ((2*I)*c*(
I + x))/((1 + c)*(1 - I*c*x))]
```

3.47.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5464
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcCot[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

3.47.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.76 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.71

method	result
parts	$\frac{\ln(x^2+1)\operatorname{arccot}(cx)}{2}+\frac{\sum_{\_\alpha=\operatorname{RootOf}(\_Z^2c^2+1)}\frac{\ln(x-\_\alpha)\ln(x^2+1)-\ln(x-\_\alpha)\ln\left(\frac{c\_\alpha+x}{\_\alpha(1+c)}\right)-\ln(x-\_\alpha)\ln\left(\frac{c\_\alpha-x}{\_\alpha(c-1)}\right)-\text{d}}{\_\alpha}}{4c}$
risch	$\frac{\pi\ln\left(-c^2+(-icx+1)^2-1+2icx\right)}{4}-\frac{i\operatorname{dilog}\left(\frac{-icx-c}{-c-1}\right)}{4}-\frac{i\ln(-icx+1)\ln\left(\frac{-icx-c}{-c-1}\right)}{4}-\frac{i\operatorname{dilog}\left(\frac{-icx+c}{c-1}\right)}{4}-\frac{i\ln(-icx}{}$
derivativedivides	$\frac{\frac{c^2\ln(c^2x^2+c^2)\operatorname{arccot}(cx)}{2}+\frac{c^2\left(-\frac{i\left(\ln(cx-i)\ln(c^2x^2+c^2)-i\left(-i\operatorname{dilog}\left(\frac{i(cx-i)+c-1}{c-1}\right)-i\ln(cx-i)\ln\left(\frac{i(cx-i)+c-1}{c-1}\right)\right)-i\left(-i\operatorname{dil}\right.\right.}{2}\right.}{}}{}$
default	$\frac{\frac{c^2\ln(c^2x^2+c^2)\operatorname{arccot}(cx)}{2}+\frac{c^2\left(-\frac{i\left(\ln(cx-i)\ln(c^2x^2+c^2)-i\left(-i\operatorname{dilog}\left(\frac{i(cx-i)+c-1}{c-1}\right)-i\ln(cx-i)\ln\left(\frac{i(cx-i)+c-1}{c-1}\right)\right)-i\left(-i\operatorname{dil}\right.\right.}{2}\right.}{}}{}$

input
```
int(x*arccot(c*x)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x^2+1)*arccot(c*x)+1/4/c*sum(1/_alpha*(ln(x-_alpha)*ln(x^2+1)-ln(x-
_alpha)*ln((_alpha*c+x)/_alpha/(1+c))-ln(x-_alpha)*ln((_alpha*c-x)/_alpha/
(c-1))-dilog((_alpha*c+x)/_alpha/(1+c))-dilog((_alpha*c-x)/_alpha/(c-1))),
_alpha=RootOf(_Z^2*c^2+1))
```

3.47.5 Fricas [F]

$$\int\frac{x\cot^{-1}(cx)}{1+x^2}\,dx=\int\frac{x\operatorname{arccot}(cx)}{x^2+1}\,dx$$

input
```
integrate(x*arccot(c*x)/(x^2+1),x, algorithm="fricas")
```

output
```
integral(x*arccot(c*x)/(x^2 + 1), x)
```

3.47.6 Sympy [F]

$$\int \frac{x \cot^{-1}(cx)}{1+x^2} \, dx = \int \frac{x \operatorname{acot}(cx)}{x^2+1} \, dx$$

input
```
integrate(x*acot(c*x)/(x**2+1),x)
```

output
```
Integral(x*acot(c*x)/(x**2 + 1), x)
```

3.47.7 Maxima [F]

$$\int \frac{x \cot^{-1}(cx)}{1+x^2} \, dx = \int \frac{x \operatorname{arccot}(cx)}{x^2+1} \, dx$$

input
```
integrate(x*arccot(c*x)/(x^2+1),x, algorithm="maxima")
```

output
```
integrate(x*arccot(c*x)/(x^2 + 1), x)
```

3.47.8 Giac [F]

$$\int \frac{x \cot^{-1}(cx)}{1+x^2} \, dx = \int \frac{x \operatorname{arccot}(cx)}{x^2+1} \, dx$$

input
```
integrate(x*arccot(c*x)/(x^2+1),x, algorithm="giac")
```

output
```
integrate(x*arccot(c*x)/(x^2 + 1), x)
```

3.47.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x \cot^{-1}(cx)}{1+x^2}\,dx = \int \frac{x\,\mathrm{acot}(c\,x)}{x^2+1}\,dx$$

input
```
int((x*acot(c*x))/(x^2 + 1),x)
```

output
```
int((x*acot(c*x))/(x^2 + 1), x)
```

3.48 $\int \frac{\cot^{-1}(cx)}{1+x^2} \, dx$

3.48.1 Optimal result

Integrand size = 12, antiderivative size = 183

$$\begin{aligned}\int \frac{\cot^{-1}(cx)}{1+x^2} \, dx &= \frac{1}{2} i \arctan(x) \log\left(1-\frac{i}{cx}\right) - \frac{1}{2} i \arctan(x) \log\left(1+\frac{i}{cx}\right) \\ &\quad - \frac{1}{2} i \arctan(x) \log\left(-\frac{2i(i-cx)}{(1-c)(1-ix)}\right) \\ &\quad + \frac{1}{2} i \arctan(x) \log\left(-\frac{2i(i+cx)}{(1+c)(1-ix)}\right) \\ &\quad - \frac{1}{4} \operatorname{PolyLog}\left(2, 1+\frac{2i(i-cx)}{(1-c)(1-ix)}\right) \\ &\quad + \frac{1}{4} \operatorname{PolyLog}\left(2, 1+\frac{2i(i+cx)}{(1+c)(1-ix)}\right)\end{aligned}$$

output
```
1/2*I*arctan(x)*ln(1-I/c/x)-1/2*I*arctan(x)*ln(1+I/c/x)-1/2*I*arctan(x)*ln
(-2*I*(I-c*x)/(1-c)/(1-I*x))+1/2*I*arctan(x)*ln(-2*I*(c*x+I)/(1+c)/(1-I*x)
)-1/4*polylog(2,1+2*I*(I-c*x)/(1-c)/(1-I*x))+1/4*polylog(2,1+2*I*(c*x+I)/(
1+c)/(1-I*x))
```

3.48.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 592 vs. 2(183) = 366.

Time = 0.61 (sec) , antiderivative size = 592, normalized size of antiderivative = 3.23

$$\int \frac{\cot^{-1}(cx)}{1+x^2}\,dx$$
$$= \frac{c\left(2\arccos\left(\frac{1+c^2}{-1+c^2}\right)\operatorname{arctanh}\left(\frac{\sqrt{-c^2}}{cx}\right) - 4\cot^{-1}(cx)\operatorname{arctanh}\left(\frac{cx}{\sqrt{-c^2}}\right) - \left(\arccos\left(\frac{1+c^2}{-1+c^2}\right) - 2i\operatorname{arctanh}\left(\frac{\sqrt{-c^2}}{cx}\right)\right.\right.}{}$$

input
```
Integrate[ArcCot[c*x]/(1 + x^2),x]
```

output
```
(c*(2*ArcCos[(1 + c^2)/(-1 + c^2)]*ArcTanh[Sqrt[-c^2]/(c*x)] - 4*ArcCot[c*
x]*ArcTanh[(c*x)/Sqrt[-c^2]] - (ArcCos[(1 + c^2)/(-1 + c^2)] - (2*I)*ArcTa
nh[Sqrt[-c^2]/(c*x)])*Log[(-2*(c^2 + I*Sqrt[-c^2])*(-I + c*x))/((-1 + c^2)
*(Sqrt[-c^2] - c*x))] - (ArcCos[(1 + c^2)/(-1 + c^2)] + (2*I)*ArcTanh[Sqrt
[-c^2]/(c*x)])*Log[((2*I)*(I*c^2 + Sqrt[-c^2])*(I + c*x))/((-1 + c^2)*(Sqr
t[-c^2] - c*x))] + (ArcCos[(1 + c^2)/(-1 + c^2)] - (2*I)*ArcTanh[Sqrt[-c^2
]/(c*x)] + (2*I)*ArcTanh[(c*x)/Sqrt[-c^2]])*Log[(Sqrt[2]*Sqrt[-c^2])/(Sqrt
[-1 + c^2]*E^(I*ArcCot[c*x])*Sqrt[-1 - c^2 + (-1 + c^2)*Cos[2*ArcCot[c*x]]
])] + (ArcCos[(1 + c^2)/(-1 + c^2)] + (2*I)*ArcTanh[Sqrt[-c^2]/(c*x)] - (2
*I)*ArcTanh[(c*x)/Sqrt[-c^2]])*Log[(Sqrt[2]*Sqrt[-c^2]*E^(I*ArcCot[c*x]))/
(Sqrt[-1 + c^2]*Sqrt[-1 - c^2 + (-1 + c^2)*Cos[2*ArcCot[c*x]]])] + I*(-Pol
yLog[2, ((1 + c^2 - (2*I)*Sqrt[-c^2])*(Sqrt[-c^2] + c*x))/((-1 + c^2)*(Sqr
t[-c^2] - c*x))] + PolyLog[2, ((1 + c^2 + (2*I)*Sqrt[-c^2])*(Sqrt[-c^2] +
c*x))/((-1 + c^2)*(Sqrt[-c^2] - c*x))])))/(4*Sqrt[-c^2])
```

3.48.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.70, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5444, 2920, 27, 2005, 5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(cx)}{x^2+1}\,dx$$

$$\Big\downarrow \text{ 5444}$$

$$\frac{1}{2}i\int \frac{\log\left(1-\frac{i}{cx}\right)}{x^2+1}dx-\frac{1}{2}i\int \frac{\log\left(1+\frac{i}{cx}\right)}{x^2+1}dx$$

$$\Big\downarrow \text{ 2920}$$

$$\frac{1}{2}i\left(\arctan(x)\log\left(1-\frac{i}{cx}\right)-\frac{i\int \frac{c\arctan(x)}{\left(c-\frac{i}{x}\right)x^2}dx}{c}\right)-\frac{1}{2}i\left(\frac{i\int \frac{c\arctan(x)}{\left(c+\frac{i}{x}\right)x^2}dx}{c}+\arctan(x)\log\left(1+\frac{i}{cx}\right)\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}i\left(\arctan(x)\log\left(1-\frac{i}{cx}\right)-i\int \frac{\arctan(x)}{\left(c-\frac{i}{x}\right)x^2}dx\right)-\frac{1}{2}i\left(i\int \frac{\arctan(x)}{\left(c+\frac{i}{x}\right)x^2}dx+\arctan(x)\log\left(1+\frac{i}{cx}\right)\right)$$

$$\Big\downarrow \text{ 2005}$$

$$\frac{1}{2}i\left(\arctan(x)\log\left(1-\frac{i}{cx}\right)-i\int \frac{\arctan(x)}{x(cx-i)}dx\right)-\frac{1}{2}i\left(i\int \frac{\arctan(x)}{x(cx+i)}dx+\arctan(x)\log\left(1+\frac{i}{cx}\right)\right)$$

$$\Big\downarrow \text{ 5411}$$

$$\frac{1}{2}i\left(\arctan(x)\log\left(1-\frac{i}{cx}\right)-i\int \left(\frac{i\arctan(x)}{x}-\frac{ic\arctan(x)}{cx-i}\right)dx\right)-\frac{1}{2}i\left(i\int \left(\frac{ic\arctan(x)}{cx+i}-\frac{i\arctan(x)}{x}\right)dx+\arctan(x)\log\left(1+\frac{i}{cx}\right)\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}i\left(\arctan(x)\log\left(1-\frac{i}{cx}\right)-i\left(-i\arctan(x)\log\left(-\frac{2i(-cx+i)}{(1-c)(1-ix)}\right)+i\arctan(x)\log\left(\frac{2}{1-ix}\right)-\frac{1}{2}\text{PolyLog}\left(\right.\right.\right.$$
$$\frac{1}{2}i\left(i\left(i\arctan(x)\log\left(-\frac{2i(cx+i)}{(c+1)(1-ix)}\right)-i\arctan(x)\log\left(\frac{2}{1-ix}\right)+\frac{1}{2}\text{PolyLog}\left(2,\frac{2i(cx+i)}{(c+1)(1-ix)}+1\right)-\frac{1}{2}\right.\right.$$

input
```
Int[ArcCot[c*x]/(1 + x^2),x]
```

3.48. $\int \frac{\cot^{-1}(cx)}{1+x^2}\,dx$

output
```
(I/2)*(ArcTan[x]*Log[1 - I/(c*x)] - I*(I*ArcTan[x]*Log[2/(1 - I*x)] - I*Ar
cTan[x]*Log[((-2*I)*(I - c*x))/((1 - c)*(1 - I*x))] + PolyLog[2, 1 - 2/(1
- I*x)]/2 - PolyLog[2, (-I)*x]/2 + PolyLog[2, I*x]/2 - PolyLog[2, 1 + ((2*
I)*(I - c*x))/((1 - c)*(1 - I*x))]/2)) - (I/2)*(ArcTan[x]*Log[1 + I/(c*x)]
 + I*((-I)*ArcTan[x]*Log[2/(1 - I*x)] + I*ArcTan[x]*Log[((-2*I)*(I + c*x))
/((1 + c)*(1 - I*x))] - PolyLog[2, 1 - 2/(1 - I*x)]/2 + PolyLog[2, (-I)*x]
/2 - PolyLog[2, I*x]/2 + PolyLog[2, 1 + ((2*I)*(I + c*x))/((1 + c)*(1 - I*
x))]/2))
```

3.48.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2920
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Simp[b*e*n*p   Int[u*(x^(n - 1)/(d + e*x^n)), x
], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

rule 5444
```
Int[ArcCot[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Simp[I/2   Int[
Log[1 - I/(c*x)]/(d + e*x^2), x], x] - Simp[I/2   Int[Log[1 + I/(c*x)]/(d +
 e*x^2), x], x] /; FreeQ[{c, d, e}, x]
```

3.48.4 Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00

method	result
risch	$\frac{\pi \arctan(x)}{2} - \frac{\ln(-icx+1)\ln\left(\frac{-icx-c}{-c-1}\right)}{4} - \frac{\text{dilog}\left(\frac{-icx-c}{-c-1}\right)}{4} + \frac{\ln(-icx+1)\ln\left(\frac{-icx+c}{c-1}\right)}{4} + \frac{\text{dilog}\left(\frac{-icx+c}{c-1}\right)}{4} - \frac{\ln(i$
parts	$\text{arccot}(cx)\arctan(x) + c\left(\frac{\arctan(cx)\arctan(x)}{c} - \frac{ic^2\ln\left(1-\frac{(c-1)(icx+1)^2}{(c^2x^2+1)(-c-1)}\right)\arctan(cx)}{2+2c} + \frac{ic\arctan(cx)\ln\left(\right.}{}\right.$
derivativedivides	$\frac{c\arctan(x)\,\text{arccot}(cx)+c^2\left(\frac{\arctan(cx)\arctan(x)}{c} - \frac{ic^2\ln\left(1-\frac{(c-1)(icx+1)^2}{(c^2x^2+1)(-c-1)}\right)\arctan(cx)}{2+2c} + \frac{ic\arctan(cx)\ln\left(1-\frac{(c-1)(icx+1}{(c^2x^2+1)(-c}\right.}{2+2c}\right.}{}$
default	$\frac{c\arctan(x)\,\text{arccot}(cx)+c^2\left(\frac{\arctan(cx)\arctan(x)}{c} - \frac{ic^2\ln\left(1-\frac{(c-1)(icx+1)^2}{(c^2x^2+1)(-c-1)}\right)\arctan(cx)}{2+2c} + \frac{ic\arctan(cx)\ln\left(1-\frac{(c-1)(icx+1}{(c^2x^2+1)(-c}\right.}{2+2c}\right.}{}$

input
```
int(arccot(c*x)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*Pi*arctan(x)-1/4*ln(1-I*c*x)*ln((-c-I*c*x)/(-c-1))-1/4*dilog((-c-I*c*x
)/(-c-1))+1/4*ln(1-I*c*x)*ln((c-I*c*x)/(c-1))+1/4*dilog((c-I*c*x)/(c-1))-1
/4*ln(1+I*c*x)*ln((-c+I*c*x)/(-c-1))-1/4*dilog((-c+I*c*x)/(-c-1))+1/4*ln(1
+I*c*x)*ln((c+I*c*x)/(c-1))+1/4*dilog((c+I*c*x)/(c-1))
```

3.48.5 Fricas [F]

$$\int \frac{\cot^{-1}(cx)}{1+x^2}\,dx = \int \frac{\text{arccot}(cx)}{x^2+1}\,dx$$

input
```
integrate(arccot(c*x)/(x^2+1),x, algorithm="fricas")
```

output
```
integral(arccot(c*x)/(x^2 + 1), x)
```

3.48.6 Sympy [F]

$$\int \frac{\cot^{-1}(cx)}{1+x^2}\,dx = \int \frac{\operatorname{acot}(cx)}{x^2+1}\,dx$$

input
```
integrate(acot(c*x)/(x**2+1),x)
```

output
```
Integral(acot(c*x)/(x**2 + 1), x)
```

3.48.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.02

$$\int \frac{\cot^{-1}(cx)}{1+x^2}\,dx =$$
$$-\frac{1}{8}c\left(\frac{8\arctan(cx)\arctan(x)}{c} - \frac{4\arctan(cx)\arctan(x) - 4\arctan(x)\arctan\left(\frac{cx}{c-1}, -\frac{1}{c-1}\right) + \log(x^2+\right.$$
$$+\operatorname{arccot}(cx)\arctan(x) + \arctan(cx)\arctan(x)$$

input
```
integrate(arccot(c*x)/(x^2+1),x, algorithm="maxima")
```

output
```
-1/8*c*(8*arctan(c*x)*arctan(x)/c - (4*arctan(c*x)*arctan(x) - 4*arctan(x)
*arctan2(c*x/(c - 1), -1/(c - 1)) + log(x^2 + 1)*log((c^2*x^2 + 1)/(c^2 +
2*c + 1)) - log(x^2 + 1)*log((c^2*x^2 + 1)/(c^2 - 2*c + 1)) + 2*dilog((I*c
*x + c)/(c + 1)) + 2*dilog(-(I*c*x - c)/(c + 1)) - 2*dilog((I*c*x + c)/(c
- 1)) - 2*dilog(-(I*c*x - c)/(c - 1)))/c) + arccot(c*x)*arctan(x) + arctan
(c*x)*arctan(x)
```

3.48.8 Giac [F]

$$\int \frac{\cot^{-1}(cx)}{1+x^2}\,dx = \int \frac{\operatorname{arccot}(cx)}{x^2+1}\,dx$$

input
```
integrate(arccot(c*x)/(x^2+1),x, algorithm="giac")
```

output
```
integrate(arccot(c*x)/(x^2 + 1), x)
```

3.48.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(cx)}{1+x^2}\,dx = \int \frac{\operatorname{acot}(c\,x)}{x^2+1}\,dx$$

input
```
int(acot(c*x)/(x^2 + 1),x)
```

output
```
int(acot(c*x)/(x^2 + 1), x)
```

3.49 $\int \frac{\cot^{-1}(cx)}{x(1+x^2)}\,dx$

3.49.1 Optimal result

Integrand size = 15, antiderivative size = 223

$$\begin{aligned}\int \frac{\cot^{-1}(cx)}{x(1+x^2)}\,dx &= \cot^{-1}(cx)\log\left(\frac{2}{1-icx}\right)-\frac{1}{2}\cot^{-1}(cx)\log\left(\frac{2ic(i-x)}{(1-c)(1-icx)}\right)\\&\quad-\frac{1}{2}\cot^{-1}(cx)\log\left(-\frac{2ic(i+x)}{(1+c)(1-icx)}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,-\frac{i}{cx}\right)\\&\quad+\frac{1}{2}i\,\text{PolyLog}\left(2,\frac{i}{cx}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)\\&\quad-\frac{1}{4}i\,\text{PolyLog}\left(2,1-\frac{2ic(i-x)}{(1-c)(1-icx)}\right)\\&\quad-\frac{1}{4}i\,\text{PolyLog}\left(2,1+\frac{2ic(i+x)}{(1+c)(1-icx)}\right)\end{aligned}$$

output
```
arccot(c*x)*ln(2/(1-I*c*x))-1/2*arccot(c*x)*ln(2*I*c*(I-x)/(1-c)/(1-I*c*x)
)-1/2*arccot(c*x)*ln(-2*I*c*(I+x)/(1+c)/(1-I*c*x))-1/2*I*polylog(2,-I/c/x)
+1/2*I*polylog(2,I/c/x)+1/2*I*polylog(2,1-2/(1-I*c*x))-1/4*I*polylog(2,1-2
*I*c*(I-x)/(1-c)/(1-I*c*x))-1/4*I*polylog(2,1+2*I*c*(I+x)/(1+c)/(1-I*c*x))
```

3.49.2 Mathematica [A] (verified)

Time = 2.04 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.47

$$\int \frac{\cot^{-1}(cx)}{x(1+x^2)}\,dx$$
$$= \frac{1}{2}\Bigg(-i\left(\cot^{-1}(cx)\left(\cot^{-1}(cx)+2i\log\left(1+e^{2i\cot^{-1}(cx)}\right)\right)+\text{PolyLog}\left(2,-e^{2i\cot^{-1}(cx)}\right)\right)$$
$$+\frac{(-1+c)(1+c)\left(i\cot^{-1}(cx)^2+2i\arcsin\left(\sqrt{\frac{1}{1-c^2}}\right)\arctan\left(\frac{\sqrt{c^2}}{cx}\right)-\left(\cot^{-1}(cx)-\arcsin\left(\sqrt{\frac{1}{1-c^2}}\right)\right)\log$$

input
```
Integrate[ArcCot[c*x]/(x*(1 + x^2)),x]
```

output
```
((-I)*(ArcCot[c*x]*(ArcCot[c*x] + (2*I)*Log[1 + E^((2*I)*ArcCot[c*x])]) +
PolyLog[2, -E^((2*I)*ArcCot[c*x])]) + ((-1 + c)*(1 + c)*(I*ArcCot[c*x]^2 +
 (2*I)*ArcSin[Sqrt[(1 - c^2)^(-1)]]*ArcTan[Sqrt[c^2]/(c*x)] - (ArcCot[c*x]
 - ArcSin[Sqrt[(1 - c^2)^(-1)]])*Log[(-1 + (-1 + 2*Sqrt[c^2])*E^((2*I)*Arc
Cot[c*x]) - c^2*(-1 + E^((2*I)*ArcCot[c*x])))/(-1 + c^2)] - (ArcCot[c*x] +
 ArcSin[Sqrt[(1 - c^2)^(-1)]])*Log[-((1 + (1 + 2*Sqrt[c^2])*E^((2*I)*ArcCo
t[c*x]) + c^2*(-1 + E^((2*I)*ArcCot[c*x])))/(-1 + c^2))] + (I/2)*(PolyLog[
2, ((1 + c^2 - 2*Sqrt[c^2])*E^((2*I)*ArcCot[c*x]))/(-1 + c^2)] + PolyLog[2
, ((1 + c^2 + 2*Sqrt[c^2])*E^((2*I)*ArcCot[c*x]))/(-1 + c^2)])))/(-1 + c^2
))/2
```

3.49.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5464, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(cx)}{x(x^2+1)}\,dx$$
$$\Big\downarrow \; 5464$$

$$\int \left(\frac{\cot^{-1}(cx)}{x} - \frac{x\cot^{-1}(cx)}{x^2+1}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{2}i\,\text{PolyLog}\left(2,-\frac{i}{cx}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,\frac{i}{cx}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)-\frac{1}{4}i\,\text{PolyLog}\left(2,1-\frac{2ic(i-x)}{(1-c)(1-icx)}\right)-\frac{1}{4}i\,\text{PolyLog}\left(2,\frac{2ic(x+i)}{(c+1)(1-icx)}+1\right)+\log\left(\frac{2}{1-icx}\right)\cot^{-1}(cx)-\frac{1}{2}\log\left(\frac{2ic(-x+i)}{(1-c)(1-icx)}\right)\cot^{-1}(cx)-\frac{1}{2}\log\left(-\frac{2ic(x+i)}{(c+1)(1-icx)}\right)\cot^{-1}(cx)$$

input
```
Int[ArcCot[c*x]/(x*(1 + x^2)),x]
```

output
```
ArcCot[c*x]*Log[2/(1 - I*c*x)] - (ArcCot[c*x]*Log[((2*I)*c*(I - x))/((1 -
c)*(1 - I*c*x))])/2 - (ArcCot[c*x]*Log[((-2*I)*c*(I + x))/((1 + c)*(1 - I*
c*x))])/2 - (I/2)*PolyLog[2, (-I)/(c*x)] + (I/2)*PolyLog[2, I/(c*x)] + (I/
2)*PolyLog[2, 1 - 2/(1 - I*c*x)] - (I/4)*PolyLog[2, 1 - ((2*I)*c*(I - x))/
((1 - c)*(1 - I*c*x))] - (I/4)*PolyLog[2, 1 + ((2*I)*c*(I + x))/((1 + c)*(
1 - I*c*x))]
```

3.49.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5464
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcCot[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

3.49.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.82 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.88

method	result
parts	$\operatorname{arccot}(cx)\ln(x) - \frac{\ln(x^2+1)\operatorname{arccot}(cx)}{2} + \frac{c\left(-\frac{i\ln(x)(\ln(icx+1)-\ln(-icx+1))}{c} - \frac{i(\operatorname{dilog}(icx+1)-\operatorname{dilog}(-icx+1))}{c} - \right.}{}$
risch	$-\frac{\pi\ln(c^2x^2+c^2)}{4} + \frac{\pi\ln(-icx)}{2} + \frac{i\ln(-icx+1)\ln\left(\frac{-icx-c}{-c-1}\right)}{4} + \frac{i\operatorname{dilog}\left(\frac{-icx-c}{-c-1}\right)}{4} + \frac{i\ln(-icx+1)\ln\left(\frac{-icx+c}{c-1}\right)}{4} +$
derivativedivides	$-\frac{\operatorname{arccot}(cx)\ln(c^2x^2+c^2)}{2} + \operatorname{arccot}(cx)\ln(cx) + \frac{c^2\left(-\frac{i\ln(cx)\ln(icx+1)}{c^2} + \frac{i\ln(cx)\ln(-icx+1)}{c^2} - \frac{i\operatorname{dilog}(icx+1)}{c^2} + \right.}{}$
default	$-\frac{\operatorname{arccot}(cx)\ln(c^2x^2+c^2)}{2} + \operatorname{arccot}(cx)\ln(cx) + \frac{c^2\left(-\frac{i\ln(cx)\ln(icx+1)}{c^2} + \frac{i\ln(cx)\ln(-icx+1)}{c^2} - \frac{i\operatorname{dilog}(icx+1)}{c^2} + \right.}{}$

input
```
int(arccot(c*x)/x/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
arccot(c*x)*ln(x)-1/2*ln(x^2+1)*arccot(c*x)+1/2*c*(-I*ln(x)*(ln(1+I*c*x)-l
n(1-I*c*x))/c-I*(dilog(1+I*c*x)-dilog(1-I*c*x))/c-1/2/c^2*sum(1/_alpha*(ln
(x-_alpha)*ln(x^2+1)-ln(x-_alpha)*ln((_alpha*c+x)/_alpha/(1+c))-ln(x-_alph
a)*ln((_alpha*c-x)/_alpha/(c-1))-dilog((_alpha*c+x)/_alpha/(1+c))-dilog((_
alpha*c-x)/_alpha/(c-1))),_alpha=RootOf(_Z^2*c^2+1)))
```

3.49.5 Fricas [F]

$$\int \frac{\cot^{-1}(cx)}{x(1+x^2)}\,dx = \int \frac{\operatorname{arccot}(cx)}{(x^2+1)x}\,dx$$

input
```
integrate(arccot(c*x)/x/(x^2+1),x, algorithm="fricas")
```

output
```
integral(arccot(c*x)/(x^3 + x), x)
```

3.49.6 Sympy [F]

$$\int \frac{\cot^{-1}(cx)}{x(1+x^2)} dx = \int \frac{\operatorname{acot}(cx)}{x(x^2+1)} dx$$

input
```
integrate(acot(c*x)/x/(x**2+1),x)
```

output
```
Integral(acot(c*x)/(x*(x**2 + 1)), x)
```

3.49.7 Maxima [F]

$$\int \frac{\cot^{-1}(cx)}{x(1+x^2)} dx = \int \frac{\operatorname{arccot}(cx)}{(x^2+1)x} dx$$

input
```
integrate(arccot(c*x)/x/(x^2+1),x, algorithm="maxima")
```

output
```
integrate(arccot(c*x)/((x^2 + 1)*x), x)
```

3.49.8 Giac [F]

$$\int \frac{\cot^{-1}(cx)}{x(1+x^2)} dx = \int \frac{\operatorname{arccot}(cx)}{(x^2+1)x} dx$$

input
```
integrate(arccot(c*x)/x/(x^2+1),x, algorithm="giac")
```

output
```
integrate(arccot(c*x)/((x^2 + 1)*x), x)
```

3.49.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(cx)}{x\,(1+x^2)}\,dx = \int \frac{\mathrm{acot}(c\,x)}{x\,\left(x^2+1\right)}\,dx$$

input
```
int(acot(c*x)/(x*(x^2 + 1)),x)
```

output
```
int(acot(c*x)/(x*(x^2 + 1)), x)
```

3.50 $\int \frac{\cot^{-1}(cx)}{x^2(1+x^2)}\,dx$

3.50.1 Optimal result

Integrand size = 15, antiderivative size = 212

$$\begin{aligned}\int \frac{\cot^{-1}(cx)}{x^2(1+x^2)}\,dx =& -\frac{\cot^{-1}(cx)}{x}-\frac{1}{2}i\arctan(x)\log\left(1-\frac{i}{cx}\right)+\frac{1}{2}i\arctan(x)\log\left(1+\frac{i}{cx}\right)\\&-c\log(x)+\frac{1}{2}i\arctan(x)\log\left(-\frac{2i(i-cx)}{(1-c)(1-ix)}\right)\\&-\frac{1}{2}i\arctan(x)\log\left(-\frac{2i(i+cx)}{(1+c)(1-ix)}\right)\\&+\frac{1}{2}c\log\left(1+c^2x^2\right)+\frac{1}{4}\operatorname{PolyLog}\left(2,1+\frac{2i(i-cx)}{(1-c)(1-ix)}\right)\\&-\frac{1}{4}\operatorname{PolyLog}\left(2,1+\frac{2i(i+cx)}{(1+c)(1-ix)}\right)\end{aligned}$$

output
```
-arccot(c*x)/x-1/2*I*arctan(x)*ln(1-I/c/x)+1/2*I*arctan(x)*ln(1+I/c/x)-c*l
n(x)+1/2*I*arctan(x)*ln(-2*I*(I-c*x)/(1-c)/(1-I*x))-1/2*I*arctan(x)*ln(-2*
I*(c*x+I)/(1+c)/(1-I*x))+1/2*c*ln(c^2*x^2+1)+1/4*polylog(2,1+2*I*(I-c*x)/(
1-c)/(1-I*x))-1/4*polylog(2,1+2*I*(c*x+I)/(1+c)/(1-I*x))
```

3.50.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 619 vs. 2(212) = 424.

Time = 1.42 (sec) , antiderivative size = 619, normalized size of antiderivative = 2.92

$$\int \frac{\cot^{-1}(cx)}{x^2(1+x^2)}\,dx = -\frac{\cot^{-1}(cx)}{x} - c\log\left(\frac{1}{\sqrt{1+\frac{1}{c^2x^2}}}\right)$$
$$-\frac{c\left(2\arccos\left(\frac{1+c^2}{-1+c^2}\right)\operatorname{arctanh}\left(\frac{\sqrt{-c^2}}{cx}\right) - 4\cot^{-1}(cx)\operatorname{arctanh}\left(\frac{cx}{\sqrt{-c^2}}\right) - \left(\arccos\left(\frac{1+c^2}{-1+c^2}\right) - 2i\operatorname{arctanh}\left(\frac{\sqrt{-c^2}}{cx}\right.\right.\right.}{}$$

input
```
Integrate[ArcCot[c*x]/(x^2*(1 + x^2)),x]
```

output
```
-(ArcCot[c*x]/x) - c*Log[1/Sqrt[1 + 1/(c^2*x^2)]] - (c*(2*ArcCos[(1 + c^2)
/(-1 + c^2)]*ArcTanh[Sqrt[-c^2]/(c*x)] - 4*ArcCot[c*x]*ArcTanh[(c*x)/Sqrt[
-c^2]] - (ArcCos[(1 + c^2)/(-1 + c^2)] - (2*I)*ArcTanh[Sqrt[-c^2]/(c*x)])*
Log[(-2*(c^2 + I*Sqrt[-c^2])*(-I + c*x))/((-1 + c^2)*(Sqrt[-c^2] - c*x))]
- (ArcCos[(1 + c^2)/(-1 + c^2)] + (2*I)*ArcTanh[Sqrt[-c^2]/(c*x)])*Log[((2
*I)*(I*c^2 + Sqrt[-c^2])*(I + c*x))/((-1 + c^2)*(Sqrt[-c^2] - c*x))] + (Ar
cCos[(1 + c^2)/(-1 + c^2)] - (2*I)*ArcTanh[Sqrt[-c^2]/(c*x)] + (2*I)*ArcTa
nh[(c*x)/Sqrt[-c^2]])*Log[(Sqrt[2]*Sqrt[-c^2])/(Sqrt[-1 + c^2]*E^(I*ArcCot
[c*x])*Sqrt[-1 - c^2 + (-1 + c^2)*Cos[2*ArcCot[c*x]]])] + (ArcCos[(1 + c^2
)/(-1 + c^2)] + (2*I)*ArcTanh[Sqrt[-c^2]/(c*x)] - (2*I)*ArcTanh[(c*x)/Sqrt
[-c^2]])*Log[(Sqrt[2]*Sqrt[-c^2]*E^(I*ArcCot[c*x]))/(Sqrt[-1 + c^2]*Sqrt[-
1 - c^2 + (-1 + c^2)*Cos[2*ArcCot[c*x]]])] + I*(-PolyLog[2, ((1 + c^2 - (2
*I)*Sqrt[-c^2])*(Sqrt[-c^2] + c*x))/((-1 + c^2)*(Sqrt[-c^2] - c*x))] + Pol
yLog[2, ((1 + c^2 + (2*I)*Sqrt[-c^2])*(Sqrt[-c^2] + c*x))/((-1 + c^2)*(Sqr
t[-c^2] - c*x))])))/(4*Sqrt[-c^2])
```

3.50.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.61, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {5454, 5362, 243, 47, 14, 16, 5444, 2920, 27, 2005, 5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(cx)}{x^2\left(x^2+1\right)}dx$$

$\big\downarrow$ 5454

$$\int \frac{\cot^{-1}(cx)}{x^2}dx-\int \frac{\cot^{-1}(cx)}{x^2+1}dx$$

$\big\downarrow$ 5362

$$-c\int \frac{1}{x\left(c^2x^2+1\right)}dx-\int \frac{\cot^{-1}(cx)}{x^2+1}dx-\frac{\cot^{-1}(cx)}{x}$$

$\big\downarrow$ 243

$$-\frac{1}{2}c\int \frac{1}{x^2\left(c^2x^2+1\right)}dx^2-\int \frac{\cot^{-1}(cx)}{x^2+1}dx-\frac{\cot^{-1}(cx)}{x}$$

$\big\downarrow$ 47

$$-\frac{1}{2}c\left(\int \frac{1}{x^2}dx^2-c^2\int \frac{1}{c^2x^2+1}dx^2\right)-\int \frac{\cot^{-1}(cx)}{x^2+1}dx-\frac{\cot^{-1}(cx)}{x}$$

$\big\downarrow$ 14

$$-\frac{1}{2}c\left(\log\left(x^2\right)-c^2\int \frac{1}{c^2x^2+1}dx^2\right)-\int \frac{\cot^{-1}(cx)}{x^2+1}dx-\frac{\cot^{-1}(cx)}{x}$$

$\big\downarrow$ 16

$$-\int \frac{\cot^{-1}(cx)}{x^2+1}dx-\frac{1}{2}c\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)-\frac{\cot^{-1}(cx)}{x}$$

$\big\downarrow$ 5444

$$-\frac{1}{2}i\int \frac{\log\left(1-\frac{i}{cx}\right)}{x^2+1}dx+\frac{1}{2}i\int \frac{\log\left(1+\frac{i}{cx}\right)}{x^2+1}dx-\frac{1}{2}c\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)-\frac{\cot^{-1}(cx)}{x}$$

$\big\downarrow$ 2920

$$-\frac{1}{2}i\left(\arctan(x)\log\left(1-\frac{i}{cx}\right)-\frac{i\int \frac{c\arctan(x)}{\left(c-\frac{i}{x}\right)x^2}dx}{c}\right)+$$
$$\frac{1}{2}i\left(\frac{i\int \frac{c\arctan(x)}{\left(c+\frac{i}{x}\right)x^2}dx}{c}+\arctan(x)\log\left(1+\frac{i}{cx}\right)\right)-\frac{1}{2}c\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)-\frac{\cot^{-1}(cx)}{x}$$

$\big\downarrow$ 27

3.50. $\int \frac{\cot^{-1}(cx)}{x^2(1+x^2)}\,dx$

$$-\frac{1}{2}i\left(\arctan(x)\log\left(1-\frac{i}{cx}\right)-i\int\frac{\arctan(x)}{\left(c-\frac{i}{x}\right)x^2}dx\right)+$$
$$\frac{1}{2}i\left(i\int\frac{\arctan(x)}{\left(c+\frac{i}{x}\right)x^2}dx+\arctan(x)\log\left(1+\frac{i}{cx}\right)\right)-\frac{1}{2}c\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)-\frac{\cot^{-1}(cx)}{x}$$

$\big\downarrow$ 2005

$$-\frac{1}{2}i\left(\arctan(x)\log\left(1-\frac{i}{cx}\right)-i\int\frac{\arctan(x)}{x(cx-i)}dx\right)+$$
$$\frac{1}{2}i\left(i\int\frac{\arctan(x)}{x(cx+i)}dx+\arctan(x)\log\left(1+\frac{i}{cx}\right)\right)-\frac{1}{2}c\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)-\frac{\cot^{-1}(cx)}{x}$$

$\big\downarrow$ 5411

$$-\frac{1}{2}i\left(\arctan(x)\log\left(1-\frac{i}{cx}\right)-i\int\left(\frac{i\arctan(x)}{x}-\frac{ic\arctan(x)}{cx-i}\right)dx\right)+$$
$$\frac{1}{2}i\left(i\int\left(\frac{ic\arctan(x)}{cx+i}-\frac{i\arctan(x)}{x}\right)dx+\arctan(x)\log\left(1+\frac{i}{cx}\right)\right)-$$
$$\frac{1}{2}c\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)-\frac{\cot^{-1}(cx)}{x}$$

$\big\downarrow$ 2009

$$-\frac{1}{2}i\left(\arctan(x)\log\left(1-\frac{i}{cx}\right)-i\left(-i\arctan(x)\log\left(-\frac{2i(-cx+i)}{(1-c)(1-ix)}\right)+i\arctan(x)\log\left(\frac{2}{1-ix}\right)-\frac{1}{2}\text{PolyLog}\right.\right.$$
$$\frac{1}{2}i\left(i\left(i\arctan(x)\log\left(-\frac{2i(cx+i)}{(c+1)(1-ix)}\right)-i\arctan(x)\log\left(\frac{2}{1-ix}\right)+\frac{1}{2}\text{PolyLog}\left(2,\frac{2i(cx+i)}{(c+1)(1-ix)}+1\right)-\frac{1}{2}\right.\right.$$
$$\frac{1}{2}c\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)-\frac{\cot^{-1}(cx)}{x}$$

input
```
Int[ArcCot[c*x]/(x^2*(1 + x^2)),x]
```

output
```
-(ArcCot[c*x]/x) - (c*(Log[x^2] - Log[1 + c^2*x^2]))/2 - (I/2)*(ArcTan[x]*
Log[1 - I/(c*x)] - I*(I*ArcTan[x]*Log[2/(1 - I*x)] - I*ArcTan[x]*Log[((-2*
I)*(I - c*x))/((1 - c)*(1 - I*x))] + PolyLog[2, 1 - 2/(1 - I*x)]/2 - PolyL
og[2, (-I)*x]/2 + PolyLog[2, I*x]/2 - PolyLog[2, 1 + ((2*I)*(I - c*x))/((1
 - c)*(1 - I*x))]/2)) + (I/2)*(ArcTan[x]*Log[1 + I/(c*x)] + I*((-I)*ArcTan
[x]*Log[2/(1 - I*x)] + I*ArcTan[x]*Log[((-2*I)*(I + c*x))/((1 + c)*(1 - I*
x))] - PolyLog[2, 1 - 2/(1 - I*x)]/2 + PolyLog[2, (-I)*x]/2 - PolyLog[2, I
*x]/2 + PolyLog[2, 1 + ((2*I)*(I + c*x))/((1 + c)*(1 - I*x))]/2))
```

3.50. $\int \frac{\cot^{-1}(cx)}{x^2(1+x^2)}\,dx$

3.50.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2920
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Simp[b*e*n*p   Int[u*(x^(n - 1)/(d + e*x^n)), x
], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.50. $\int \frac{\cot^{-1}(cx)}{x^2(1+x^2)}\,dx$

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

rule 5444
```
Int[ArcCot[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Simp[I/2   Int[
Log[1 - I/(c*x)]/(d + e*x^2), x], x] - Simp[I/2   Int[Log[1 + I/(c*x)]/(d +
 e*x^2), x], x] /; FreeQ[{c, d, e}, x]
```

rule 5454
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcCot[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcCot[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.50.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.21

method	result
risch	$-\frac{\pi \arctan(x)}{2}-\frac{\pi}{2x}+\frac{\ln(-icx+1)\ln\left(\frac{-icx-c}{-c-1}\right)}{4}+\frac{\operatorname{dilog}\left(\frac{-icx-c}{-c-1}\right)}{4}-\frac{\ln(-icx+1)\ln\left(\frac{-icx+c}{c-1}\right)}{4}-\frac{\operatorname{dilog}\left(\frac{-icx+c}{c-1}\right)}{4}$
parts	$-\frac{\operatorname{arccot}(cx)}{x}-\operatorname{arccot}(cx)\arctan(x)+c\left(-\ln(x)+\frac{\ln(c^2x^2+1)}{2}-\frac{i\arctan(x)\ln\left(1-\frac{(c-1)(ix+1)^2}{(x^2+1)(1+c)}\right)}{2c}\right.$
derivativedivides	$c\left(-\frac{\operatorname{arccot}(cx)\arctan(x)}{c}-\frac{\operatorname{arccot}(cx)}{cx}+c^3\left(-\frac{\ln(x)-\frac{\ln(c^2x^2+1)}{2}}{c^3}-\frac{i\arctan(x)\ln\left(1-\frac{(c-1)(ix+1)^2}{(x^2+1)(1+c)}\right)}{2c^4}-\text{ar}\right.\right.$
default	$c\left(-\frac{\operatorname{arccot}(cx)\arctan(x)}{c}-\frac{\operatorname{arccot}(cx)}{cx}+c^3\left(-\frac{\ln(x)-\frac{\ln(c^2x^2+1)}{2}}{c^3}-\frac{i\arctan(x)\ln\left(1-\frac{(c-1)(ix+1)^2}{(x^2+1)(1+c)}\right)}{2c^4}-\text{ar}\right.\right.$

input
```
int(arccot(c*x)/x^2/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*Pi*arctan(x)-1/2*Pi/x+1/4*ln(1-I*c*x)*ln((-c-I*c*x)/(-c-1))+1/4*dilog
((-c-I*c*x)/(-c-1))-1/4*ln(1-I*c*x)*ln((c-I*c*x)/(c-1))-1/4*dilog((c-I*c*x
)/(c-1))-1/2*c*ln(-I*c*x)+1/2*c*ln(1-I*c*x)+1/2*I*ln(1-I*c*x)/x+1/4*ln(1+I
*c*x)*ln((-c+I*c*x)/(-c-1))+1/4*dilog((-c+I*c*x)/(-c-1))-1/4*ln(1+I*c*x)*l
n((c+I*c*x)/(c-1))-1/4*dilog((c+I*c*x)/(c-1))-1/2*c*ln(I*c*x)+1/2*c*ln(1+I
*c*x)-1/2*I*ln(1+I*c*x)/x
```

3.50.5 Fricas [F]

$$\int \frac{\cot^{-1}(cx)}{x^2(1+x^2)}\,dx = \int \frac{\operatorname{arccot}(cx)}{(x^2+1)x^2}\,dx$$

input
```
integrate(arccot(c*x)/x^2/(x^2+1),x, algorithm="fricas")
```

output
```
integral(arccot(c*x)/(x^4 + x^2), x)
```

3.50.6 Sympy [F]

$$\int \frac{\cot^{-1}(cx)}{x^2(1+x^2)}\,dx = \int \frac{\operatorname{acot}(cx)}{x^2(x^2+1)}\,dx$$

input
```
integrate(acot(c*x)/x**2/(x**2+1),x)
```

output
```
Integral(acot(c*x)/(x**2*(x**2 + 1)), x)
```

3.50.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.86

$$\int \frac{\cot^{-1}(cx)}{x^2(1+x^2)}\,dx = -\left(\frac{1}{x}+\arctan(x)\right)\operatorname{arccot}(cx) - \frac{1}{2}\arctan(cx)\arctan(x) + \frac{1}{2}\arctan(x)\arctan\left(\frac{cx}{c-1}, -\frac{1}{c-1}\right) + \frac{1}{2}c\log\left(c^2x^2+1\right) - c\log(x) - \frac{1}{8}\log\left(x^2+1\right)\log\left(\frac{c^2x^2+1}{c^2+2c+1}\right) + \frac{1}{8}\log\left(x^2+1\right)\log\left(\frac{c^2x^2+1}{c^2-2c+1}\right) - \frac{1}{4}\operatorname{Li}_2\left(\frac{icx+c}{c+1}\right) - \frac{1}{4}\operatorname{Li}_2\left(-\frac{icx-c}{c+1}\right) + \frac{1}{4}\operatorname{Li}_2\left(\frac{icx+c}{c-1}\right) + \frac{1}{4}\operatorname{Li}_2\left(-\frac{icx-c}{c-1}\right)$$

input
```
integrate(arccot(c*x)/x^2/(x^2+1),x, algorithm="maxima")
```

output
```
-(1/x + arctan(x))*arccot(c*x) - 1/2*arctan(c*x)*arctan(x) + 1/2*arctan(x)
*arctan2(c*x/(c - 1), -1/(c - 1)) + 1/2*c*log(c^2*x^2 + 1) - c*log(x) - 1/
8*log(x^2 + 1)*log((c^2*x^2 + 1)/(c^2 + 2*c + 1)) + 1/8*log(x^2 + 1)*log((
c^2*x^2 + 1)/(c^2 - 2*c + 1)) - 1/4*dilog((I*c*x + c)/(c + 1)) - 1/4*dilog
(-(I*c*x - c)/(c + 1)) + 1/4*dilog((I*c*x + c)/(c - 1)) + 1/4*dilog(-(I*c*
x - c)/(c - 1))
```

3.50.8 Giac [F]

$$\int \frac{\cot^{-1}(cx)}{x^2(1+x^2)}\,dx = \int \frac{\operatorname{arccot}(cx)}{(x^2+1)x^2}\,dx$$

input
```
integrate(arccot(c*x)/x^2/(x^2+1),x, algorithm="giac")
```

output
```
integrate(arccot(c*x)/((x^2 + 1)*x^2), x)
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(cx)}{x^2 (1+x^2)} \, dx = \int \frac{\operatorname{acot}(c\,x)}{x^2\,(x^2+1)} \, dx$$

input
```
int(acot(c*x)/(x^2*(x^2 + 1)),x)
```

output
```
int(acot(c*x)/(x^2*(x^2 + 1)), x)
```

3.51 $\int \frac{1}{(1+x^2)\cot^{-1}(x)}\,dx$

3.51.1 Optimal result

Integrand size = 12, antiderivative size = 5

$$\int \frac{1}{(1+x^2)\cot^{-1}(x)}\,dx = -\log\left(\cot^{-1}(x)\right)$$

output
```
-ln(arccot(x))
```

3.51.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x^2)\cot^{-1}(x)}\,dx = -\log\left(\cot^{-1}(x)\right)$$

input
```
Integrate[1/((1 + x^2)*ArcCot[x]),x]
```

output
```
-Log[ArcCot[x]]
```

3.51.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2+1)\cot^{-1}(x)} \, dx$$

$$\Big\downarrow \; 5418$$

$$-\log\left(\cot^{-1}(x)\right)$$

input
```
Int[1/((1 + x^2)*ArcCot[x]),x]
```

output
```
-Log[ArcCot[x]]
```

3.51.3.1 Defintions of rubi rules used

rule 5418
```
Int[1/(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)), x_Symbol]
 :> Simp[-Log[RemoveContent[a + b*ArcCot[c*x], x]]/(b*c*d), x] /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[e, c^2*d]
```

3.51.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$-\ln\left(\operatorname{arccot}\left(x\right)\right)$	6
default	$-\ln\left(\operatorname{arccot}\left(x\right)\right)$	6
parallelrisch	$-\ln\left(\operatorname{arccot}\left(x\right)\right)$	6
risch	$-\ln\left(\ln\left(ix+1\right)+i(i\ln\left(-ix+1\right)-\pi)\right)$	29

input
```
int(1/(x^2+1)/arccot(x),x,method=_RETURNVERBOSE)
```

output
```
-ln(arccot(x))
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x^2)\cot^{-1}(x)}\,dx = -\log\left(\operatorname{arccot}(x)\right)$$

input
```
integrate(1/(x^2+1)/arccot(x),x, algorithm="fricas")
```

output
```
-log(arccot(x))
```

3.51.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x^2)\cot^{-1}(x)}\,dx = -\log\left(\operatorname{acot}(x)\right)$$

input
```
integrate(1/(x**2+1)/acot(x),x)
```

output
```
-log(acot(x))
```

3.51.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x^2)\cot^{-1}(x)}\,dx = -\log\left(\operatorname{arccot}(x)\right)$$

input
```
integrate(1/(x^2+1)/arccot(x),x, algorithm="maxima")
```

output
```
-log(arccot(x))
```

3.51.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.60

$$\int \frac{1}{(1+x^2)\cot^{-1}(x)}\,dx = -\log\left(\left|\arctan\left(\frac{1}{x}\right)\right|\right)$$

input
```
integrate(1/(x^2+1)/arccot(x),x, algorithm="giac")
```

output
```
-log(abs(arctan(1/x)))
```

3.51.9 Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x^2)\cot^{-1}(x)}\,dx = -\ln\left(\operatorname{acot}(x)\right)$$

input
```
int(1/(acot(x)*(x^2 + 1)),x)
```

output
```
-log(acot(x))
```

3.52 $\int \frac{\cot^{-1}(x)^n}{1+x^2} dx$

3.52.1 Optimal result

Integrand size = 12, antiderivative size = 13

$$\int \frac{\cot^{-1}(x)^n}{1+x^2} dx = -\frac{\cot^{-1}(x)^{1+n}}{1+n}$$

output
```
-arccot(x)^(1+n)/(1+n)
```

3.52.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(x)^n}{1+x^2} dx = -\frac{\cot^{-1}(x)^{1+n}}{1+n}$$

input
```
Integrate[ArcCot[x]^n/(1 + x^2),x]
```

output
```
-(ArcCot[x]^(1 + n)/(1 + n))
```

3.52.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(x)^n}{x^2+1} \, dx$$
$$\Big\downarrow \text{ 5420}$$
$$-\frac{\cot^{-1}(x)^{n+1}}{n+1}$$

input
```
Int[ArcCot[x]^n/(1 + x^2),x]
```

output
```
-(ArcCot[x]^(1 + n)/(1 + n))
```

3.52.3.1 Defintions of rubi rules used

rule 5420
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

3.52.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$-\frac{\operatorname{arccot}(x)^{n+1}}{n+1}$	14
default	$-\frac{\operatorname{arccot}(x)^{n+1}}{n+1}$	14
risch	$-\frac{(\pi-i\ln(-i(i+x))+i\ln(-i(i-x)))(\pi-i\ln(-i(i+x))+i\ln(-i(i-x)))^n\left(\frac{1}{2}\right)^n}{2(n+1)}$	65

input
```
int(arccot(x)^n/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-arccot(x)^(n+1)/(n+1)
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(x)^n}{1+x^2}\,dx = -\frac{\operatorname{arccot}(x)^n\operatorname{arccot}(x)}{n+1}$$

input
```
integrate(arccot(x)^n/(x^2+1),x, algorithm="fricas")
```

output
```
-arccot(x)^n*arccot(x)/(n + 1)
```

3.52.6 Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{\cot^{-1}(x)^n}{1+x^2}\,dx = -\begin{cases} \frac{\operatorname{acot}^{n+1}(x)}{n+1} & \text{for } n \neq -1 \\ \log(\operatorname{acot}(x)) & \text{otherwise} \end{cases}$$

input
```
integrate(acot(x)**n/(x**2+1),x)
```

output
```
-Piecewise((acot(x)**(n + 1)/(n + 1), Ne(n, -1)), (log(acot(x)), True))
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(x)^n}{1+x^2}\,dx = -\frac{\operatorname{arccot}(x)^{n+1}}{n+1}$$

input
```
integrate(arccot(x)^n/(x^2+1),x, algorithm="maxima")
```

output
```
-arccot(x)^(n + 1)/(n + 1)
```

3.52.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{\cot^{-1}(x)^n}{1+x^2}\,dx = -\frac{\arctan\left(\frac{1}{x}\right)^{n+1}}{n+1}$$

input
```
integrate(arccot(x)^n/(x^2+1),x, algorithm="giac")
```

output
```
-arctan(1/x)^(n + 1)/(n + 1)
```

3.52.9 Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(x)^n}{1+x^2}\,dx = -\frac{\operatorname{acot}(x)^{n+1}}{n+1}$$

input
```
int(acot(x)^n/(x^2 + 1),x)
```

output
```
-acot(x)^(n + 1)/(n + 1)
```

3.53 $\int (c+dx^2)^4 \cot^{-1}(ax)\,dx$

3.53.1 Optimal result

Integrand size = 14, antiderivative size = 244

$$\begin{aligned}\int (c+dx^2)^4 \cot^{-1}(ax)\,dx &= \frac{d(420a^6c^3-378a^4c^2d+180a^2cd^2-35d^3)\,x^2}{630a^7}+\frac{d^2(378a^4c^2-180a^2cd+35d^2)\,x^4}{1260a^5}\\&+\frac{(36a^2c-7d)\,d^3x^6}{378a^3}+\frac{d^4x^8}{72a}+c^4x\cot^{-1}(ax)+\frac{4}{3}c^3dx^3\cot^{-1}(ax)\\&+\frac{6}{5}c^2d^2x^5\cot^{-1}(ax)+\frac{4}{7}cd^3x^7\cot^{-1}(ax)+\frac{1}{9}d^4x^9\cot^{-1}(ax)\\&+\frac{(315a^8c^4-420a^6c^3d+378a^4c^2d^2-180a^2cd^3+35d^4)\log(1+a^2x^2)}{630a^9}\end{aligned}$$

output
```
1/630*d*(420*a^6*c^3-378*a^4*c^2*d+180*a^2*c*d^2-35*d^3)*x^2/a^7+1/1260*d^
2*(378*a^4*c^2-180*a^2*c*d+35*d^2)*x^4/a^5+1/378*(36*a^2*c-7*d)*d^3*x^6/a^
3+1/72*d^4*x^8/a+c^4*x*arccot(a*x)+4/3*c^3*d*x^3*arccot(a*x)+6/5*c^2*d^2*x
^5*arccot(a*x)+4/7*c*d^3*x^7*arccot(a*x)+1/9*d^4*x^9*arccot(a*x)+1/630*(31
5*a^8*c^4-420*a^6*c^3*d+378*a^4*c^2*d^2-180*a^2*c*d^3+35*d^4)*ln(a^2*x^2+1
)/a^9
```

3.53.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.87

$$\int \left(c+dx^2\right)^4 \cot^{-1}(ax)\,dx$$
$$=\frac{a^2dx^2(-420d^3+30a^2d^2(72c+7dx^2)-4a^4d(1134c^2+270cdx^2+35d^2x^4)+3a^6(1680c^3+756c^2dx^2+240 \ldots}{}$$

input
```
Integrate[(c + d*x^2)^4*ArcCot[a*x],x]
```

output
```
(a^2*d*x^2*(-420*d^3 + 30*a^2*d^2*(72*c + 7*d*x^2) - 4*a^4*d*(1134*c^2 + 2
70*c*d*x^2 + 35*d^2*x^4) + 3*a^6*(1680*c^3 + 756*c^2*d*x^2 + 240*c*d^2*x^4
 + 35*d^3*x^6)) + 24*a^9*x*(315*c^4 + 420*c^3*d*x^2 + 378*c^2*d^2*x^4 + 18
0*c*d^3*x^6 + 35*d^4*x^8)*ArcCot[a*x] + 12*(315*a^8*c^4 - 420*a^6*c^3*d +
378*a^4*c^2*d^2 - 180*a^2*c*d^3 + 35*d^4)*Log[1 + a^2*x^2])/(7560*a^9)
```

3.53.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5448, 27, 2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}(ax)\left(c+dx^2\right)^4dx$$

$$\Big\downarrow \text{5448}$$

$$a\int \frac{x\left(35d^4x^8+180cd^3x^6+378c^2d^2x^4+420c^3dx^2+315c^4\right)}{315\left(a^2x^2+1\right)}dx+c^4x\cot^{-1}(ax)+\frac{4}{3}c^3dx^3\cot^{-1}(ax)+\frac{6}{5}c^2d^2x^5\cot^{-1}(ax)+\frac{4}{7}cd^3x^7\cot^{-1}(ax)+\frac{1}{9}d^4x^9\cot^{-1}(ax)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{315}a\int \frac{x\left(35d^4x^8+180cd^3x^6+378c^2d^2x^4+420c^3dx^2+315c^4\right)}{a^2x^2+1}dx+c^4x\cot^{-1}(ax)+\frac{4}{3}c^3dx^3\cot^{-1}(ax)+\frac{6}{5}c^2d^2x^5\cot^{-1}(ax)+\frac{4}{7}cd^3x^7\cot^{-1}(ax)+\frac{1}{9}d^4x^9\cot^{-1}(ax)$$

$$\Big\downarrow \text{2331}$$

$$\frac{1}{630}a\int \frac{35d^4x^8+180cd^3x^6+378c^2d^2x^4+420c^3dx^2+315c^4}{a^2x^2+1}dx^2+c^4x\cot^{-1}(ax)+\frac{4}{3}c^3dx^3\cot^{-1}(ax)+\frac{6}{5}c^2d^2x^5\cot^{-1}(ax)+\frac{4}{7}cd^3x^7\cot^{-1}(ax)+\frac{1}{9}d^4x^9\cot^{-1}(ax)$$

$$\Big\downarrow \text{2389}$$

$$\frac{1}{630}a\int \left(\frac{35d^4x^6}{a^2}+\frac{5(36a^2c-7d)d^3x^4}{a^4}+\frac{d^2(378c^2a^4-180cda^2+35d^2)x^2}{a^6}+\frac{d(420c^3a^6-378c^2da^4+180cd^2a^2}{a^8}\right.$$
$$c^4x\cot^{-1}(ax)+\frac{4}{3}c^3dx^3\cot^{-1}(ax)+\frac{6}{5}c^2d^2x^5\cot^{-1}(ax)+\frac{4}{7}cd^3x^7\cot^{-1}(ax)+\frac{1}{9}d^4x^9\cot^{-1}(ax)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{630}a\left(\frac{35d^4x^8}{4a^2}+\frac{5d^3x^6(36a^2c-7d)}{3a^4}+\frac{d^2x^4(378a^4c^2-180a^2cd+35d^2)}{2a^6}+\frac{dx^2(420a^6c^3-378a^4c^2d+180a^2cd^2}{a^8}\right.$$
$$c^4x\cot^{-1}(ax)+\frac{4}{3}c^3dx^3\cot^{-1}(ax)+\frac{6}{5}c^2d^2x^5\cot^{-1}(ax)+\frac{4}{7}cd^3x^7\cot^{-1}(ax)+\frac{1}{9}d^4x^9\cot^{-1}(ax)$$

input
```
Int[(c + d*x^2)^4*ArcCot[a*x],x]
```

output
```
c^4*x*ArcCot[a*x] + (4*c^3*d*x^3*ArcCot[a*x])/3 + (6*c^2*d^2*x^5*ArcCot[a*
x])/5 + (4*c*d^3*x^7*ArcCot[a*x])/7 + (d^4*x^9*ArcCot[a*x])/9 + (a*((d*(42
0*a^6*c^3 - 378*a^4*c^2*d + 180*a^2*c*d^2 - 35*d^3)*x^2)/a^8 + (d^2*(378*a
^4*c^2 - 180*a^2*c*d + 35*d^2)*x^4)/(2*a^6) + (5*(36*a^2*c - 7*d)*d^3*x^6)
/(3*a^4) + (35*d^4*x^8)/(4*a^2) + ((315*a^8*c^4 - 420*a^6*c^3*d + 378*a^4*
c^2*d^2 - 180*a^2*c*d^3 + 35*d^4)*Log[1 + a^2*x^2])/a^10))/630
```

3.53.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

rule 5448
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcCot[c*x])   u,
x] + Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2), x], x], x]] /; FreeQ
[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

3.53.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00

method	result
parts	$\frac{d^4 x^9 \operatorname{arccot}(ax)}{9} + \frac{4c\,d^3 x^7 \operatorname{arccot}(ax)}{7} + \frac{6c^2 d^2 x^5 \operatorname{arccot}(ax)}{5} + \frac{4c^3 d\,x^3 \operatorname{arccot}(ax)}{3} + c^4 x \operatorname{arccot}(ax) + \frac{a(\ldots}{\ldots}$
derivativedivides	$\frac{\operatorname{arccot}(ax) c^4 a x + \frac{4a \operatorname{arccot}(ax) c^3 d\,x^3}{3} + \frac{6a \operatorname{arccot}(ax) c^2 d^2 x^5}{5} + \frac{4a \operatorname{arccot}(ax) c\,d^3 x^7}{7} + \frac{a \operatorname{arccot}(ax) d^4 x^9}{9} + \frac{210c^3 a^8 d\,x^2 + \frac{189c^2 a^8 \ldots}{2}}{\ldots}}{\ldots}$
default	$\frac{\operatorname{arccot}(ax) c^4 a x + \frac{4a \operatorname{arccot}(ax) c^3 d\,x^3}{3} + \frac{6a \operatorname{arccot}(ax) c^2 d^2 x^5}{5} + \frac{4a \operatorname{arccot}(ax) c\,d^3 x^7}{7} + \frac{a \operatorname{arccot}(ax) d^4 x^9}{9} + \frac{210c^3 a^8 d\,x^2 + \frac{189c^2 a^8 \ldots}{2}}{\ldots}}{\ldots}$
parallelrisch	$\frac{840x^9 \operatorname{arccot}(ax) a^9 d^4 + 4320x^7 \operatorname{arccot}(ax) a^9 c\,d^3 + 105d^4 a^8 x^8 + 9072x^5 \operatorname{arccot}(ax) a^9 c^2 d^2 + 720c\,a^8 d^3 x^6 + 10080x^3 \operatorname{arccot} \ldots}{\ldots}$
risch	$-\frac{d^4 x^6}{54a^3} + \frac{d^4 x^4}{36a^5} - \frac{d^4 x^2}{18a^7} + \frac{\ln(-a^2x^2-1) c^4}{2a} + \frac{\ln(-a^2x^2-1) d^4}{18a^9} + \frac{\pi\,d^4 x^9}{18} + \frac{\pi\,c^4 x}{2} + \frac{2c\,d^3 x^6}{21a} + \frac{3c^2 d^2 x^4}{10a} + \ldots$

input
```
int((d*x^2+c)^4*arccot(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/9*d^4*x^9*arccot(a*x)+4/7*c*d^3*x^7*arccot(a*x)+6/5*c^2*d^2*x^5*arccot(a
*x)+4/3*c^3*d*x^3*arccot(a*x)+c^4*x*arccot(a*x)+1/315*a*(1/2*d/a^8*(35/4*a
^6*d^3*x^8+60*a^6*c*d^2*x^6+189*a^6*c^2*d*x^4+420*a^6*c^3*x^2-35/3*a^4*d^3
*x^6-90*a^4*c*d^2*x^4-378*a^4*c^2*d*x^2+35/2*a^2*d^3*x^4+180*a^2*c*d^2*x^2
-35*d^3*x^2)+1/2*(315*a^8*c^4-420*a^6*c^3*d+378*a^4*c^2*d^2-180*a^2*c*d^3+
35*d^4)/a^10*ln(a^2*x^2+1))
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.97

$$\int (c+dx^2)^4 \cot^{-1}(ax)\, dx$$
$$= \frac{105\, a^8 d^4 x^8 + 20\,(36\, a^8 c d^3 - 7\, a^6 d^4)x^6 + 6\,(378\, a^8 c^2 d^2 - 180\, a^6 c d^3 + 35\, a^4 d^4)x^4 + 12\,(420\, a^8 c^3 d - 378\, a^6 c^2 \cdots}{\cdots}$$

input
```
integrate((d*x^2+c)^4*arccot(a*x),x, algorithm="fricas")
```

output
```
1/7560*(105*a^8*d^4*x^8 + 20*(36*a^8*c*d^3 - 7*a^6*d^4)*x^6 + 6*(378*a^8*c
^2*d^2 - 180*a^6*c*d^3 + 35*a^4*d^4)*x^4 + 12*(420*a^8*c^3*d - 378*a^6*c^2
*d^2 + 180*a^4*c*d^3 - 35*a^2*d^4)*x^2 + 24*(35*a^9*d^4*x^9 + 180*a^9*c*d^
3*x^7 + 378*a^9*c^2*d^2*x^5 + 420*a^9*c^3*d*x^3 + 315*a^9*c^4*x)*arccot(a*
x) + 12*(315*a^8*c^4 - 420*a^6*c^3*d + 378*a^4*c^2*d^2 - 180*a^2*c*d^3 + 3
5*d^4)*log(a^2*x^2 + 1))/a^9
```

3.53.6 Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.50

$$\int (c+dx^2)^4 \cot^{-1}(ax)\, dx$$
$$= \begin{cases} c^4 x \operatorname{acot}(ax) + \frac{4c^3 d x^3 \operatorname{acot}(ax)}{3} + \frac{6c^2 d^2 x^5 \operatorname{acot}(ax)}{5} + \frac{4cd^3 x^7 \operatorname{acot}(ax)}{7} + \frac{d^4 x^9 \operatorname{acot}(ax)}{9} + \frac{c^4 \log\left(x^2+\frac{1}{a^2}\right)}{2a} + \frac{2c^3 dx^2}{3a} + \frac{3c^2 \cdots}{10 \cdots} \\ \frac{\pi\left(c^4 x + \frac{4c^3 d x^3}{3} + \frac{6c^2 d^2 x^5}{5} + \frac{4cd^3 x^7}{7} + \frac{d^4 x^9}{9}\right)}{2} \end{cases}$$

input
```
integrate((d*x**2+c)**4*acot(a*x),x)
```

output
```
Piecewise((c**4*x*acot(a*x) + 4*c**3*d*x**3*acot(a*x)/3 + 6*c**2*d**2*x**5
*acot(a*x)/5 + 4*c*d**3*x**7*acot(a*x)/7 + d**4*x**9*acot(a*x)/9 + c**4*lo
g(x**2 + a**(-2))/(2*a) + 2*c**3*d*x**2/(3*a) + 3*c**2*d**2*x**4/(10*a) +
2*c*d**3*x**6/(21*a) + d**4*x**8/(72*a) - 2*c**3*d*log(x**2 + a**(-2))/(3*
a**3) - 3*c**2*d**2*x**2/(5*a**3) - c*d**3*x**4/(7*a**3) - d**4*x**6/(54*a
**3) + 3*c**2*d**2*log(x**2 + a**(-2))/(5*a**5) + 2*c*d**3*x**2/(7*a**5) +
 d**4*x**4/(36*a**5) - 2*c*d**3*log(x**2 + a**(-2))/(7*a**7) - d**4*x**2/(
18*a**7) + d**4*log(x**2 + a**(-2))/(18*a**9), Ne(a, 0)), (pi*(c**4*x + 4*
c**3*d*x**3/3 + 6*c**2*d**2*x**5/5 + 4*c*d**3*x**7/7 + d**4*x**9/9)/2, Tru
e))
```

3.53.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.93

$$\int \left(c+dx^2\right)^4 \cot^{-1}(ax)\,dx$$
$$= \frac{1}{7560}a\left(\frac{105\,a^6d^4x^8+20\,(36\,a^6cd^3-7\,a^4d^4)x^6+6\,(378\,a^6c^2d^2-180\,a^4cd^3+35\,a^2d^4)x^4+12\,(420\,a^6c^3d-\right.}{a^8}$$
$$+\frac{1}{315}\left(35\,d^4x^9+180\,cd^3x^7+378\,c^2d^2x^5+420\,c^3dx^3+315\,c^4x\right)\operatorname{arccot}(ax)$$

input
```
integrate((d*x^2+c)^4*arccot(a*x),x, algorithm="maxima")
```

output
```
1/7560*a*((105*a^6*d^4*x^8 + 20*(36*a^6*c*d^3 - 7*a^4*d^4)*x^6 + 6*(378*a^
6*c^2*d^2 - 180*a^4*c*d^3 + 35*a^2*d^4)*x^4 + 12*(420*a^6*c^3*d - 378*a^4*
c^2*d^2 + 180*a^2*c*d^3 - 35*d^4)*x^2)/a^8 + 12*(315*a^8*c^4 - 420*a^6*c^3
*d + 378*a^4*c^2*d^2 - 180*a^2*c*d^3 + 35*d^4)*log(a^2*x^2 + 1)/a^10) + 1/
315*(35*d^4*x^9 + 180*c*d^3*x^7 + 378*c^2*d^2*x^5 + 420*c^3*d*x^3 + 315*c^
4*x)*arccot(a*x)
```

3.53.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.42

$$\int \left(c+dx^2\right)^4 \cot^{-1}(ax)\,dx$$
$$= \frac{1}{7560}\left(\frac{24\left(35\,d^4+\frac{180\,cd^3}{x^2}+\frac{378\,c^2d^2}{x^4}+\frac{420\,c^3d}{x^6}+\frac{315\,c^4}{x^8}\right)x^9\arctan\left(\frac{1}{ax}\right)}{a}+\frac{\left(105\,d^4+\frac{720\,cd^3}{x^2}+\frac{2268\,c^2d^2}{x^4}-\frac{140\,d^4}{a^2x^2}\right.}{}\right.$$

input
```
integrate((d*x^2+c)^4*arccot(a*x),x, algorithm="giac")
```

output
```
1/7560*(24*(35*d^4 + 180*c*d^3/x^2 + 378*c^2*d^2/x^4 + 420*c^3*d/x^6 + 315
*c^4/x^8)*x^9*arctan(1/(a*x))/a + (105*d^4 + 720*c*d^3/x^2 + 2268*c^2*d^2/
x^4 - 140*d^4/(a^2*x^2) + 5040*c^3*d/x^6 - 1080*c*d^3/(a^2*x^4) + 7875*c^4
/x^8 - 4536*c^2*d^2/(a^2*x^6) + 210*d^4/(a^4*x^4) - 10500*c^3*d/(a^2*x^8)
+ 2160*c*d^3/(a^4*x^6) + 9450*c^2*d^2/(a^4*x^8) - 420*d^4/(a^6*x^6) - 4500
*c*d^3/(a^6*x^8) + 875*d^4/(a^8*x^8))*x^8/a^2 + 12*(315*a^8*c^4 - 420*a^6*
c^3*d + 378*a^4*c^2*d^2 - 180*a^2*c*d^3 + 35*d^4)*log(1/(a^2*x^2) + 1)/a^1
0 - 12*(315*a^8*c^4 - 420*a^6*c^3*d + 378*a^4*c^2*d^2 - 180*a^2*c*d^3 + 35
*d^4)*log(1/(a^2*x^2))/a^10)*a
```

3.53.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.96

$$\int (c+dx^2)^4 \cot^{-1}(ax)\,dx$$
$$= \operatorname{acot}(a\,x)\left(c^4\,x+\frac{4\,c^3\,d\,x^3}{3}+\frac{6\,c^2\,d^2\,x^5}{5}+\frac{4\,c\,d^3\,x^7}{7}+\frac{d^4\,x^9}{9}\right)$$
$$-x^2\left(\frac{\frac{\frac{d^4}{9\,a^3}-\frac{4\,c\,d^3}{7\,a}}{a^2}+\frac{6\,c^2\,d^2}{5\,a}}{2\,a^2}-\frac{2\,c^3\,d}{3\,a}\right)-x^6\left(\frac{d^4}{54\,a^3}-\frac{2\,c\,d^3}{21\,a}\right)+x^4\left(\frac{\frac{d^4}{9\,a^3}-\frac{4\,c\,d^3}{7\,a}}{4\,a^2}+\frac{3\,c^2\,d^2}{10\,a}\right)$$
$$+\frac{\ln\left(a^2\,x^2+1\right)\,\left(315\,a^8\,c^4-420\,a^6\,c^3\,d+378\,a^4\,c^2\,d^2-180\,a^2\,c\,d^3+35\,d^4\right)}{630\,a^9}+\frac{d^4\,x^8}{72\,a}$$

input
```
int(acot(a*x)*(c + d*x^2)^4,x)
```

output
```
acot(a*x)*(c^4*x + (d^4*x^9)/9 + (4*c^3*d*x^3)/3 + (4*c*d^3*x^7)/7 + (6*c^
2*d^2*x^5)/5) - x^2*(((d^4/(9*a^3) - (4*c*d^3)/(7*a))/a^2 + (6*c^2*d^2)/(5
*a))/(2*a^2) - (2*c^3*d)/(3*a)) - x^6*(d^4/(54*a^3) - (2*c*d^3)/(21*a)) +
x^4*((d^4/(9*a^3) - (4*c*d^3)/(7*a))/(4*a^2) + (3*c^2*d^2)/(10*a)) + (log(
a^2*x^2 + 1)*(35*d^4 + 315*a^8*c^4 - 180*a^2*c*d^3 - 420*a^6*c^3*d + 378*a
^4*c^2*d^2))/(630*a^9) + (d^4*x^8)/(72*a)
```

3.54 $\int (c+dx^2)^3 \cot^{-1}(ax) \, dx$

3.54.1 Optimal result

Integrand size = 14, antiderivative size = 168

$$\int (c+dx^2)^3 \cot^{-1}(ax) \, dx = \frac{d(35a^4c^2-21a^2cd+5d^2)\,x^2}{70a^5} + \frac{(21a^2c-5d)\,d^2x^4}{140a^3} + \frac{d^3x^6}{42a} + c^3x\cot^{-1}(ax) + c^2dx^3\cot^{-1}(ax) + \frac{3}{5}cd^2x^5\cot^{-1}(ax) + \frac{1}{7}d^3x^7\cot^{-1}(ax) + \frac{(35a^6c^3-35a^4c^2d+21a^2cd^2-5d^3)\log(1+a^2x^2)}{70a^7}$$

output
```
1/70*d*(35*a^4*c^2-21*a^2*c*d+5*d^2)*x^2/a^5+1/140*(21*a^2*c-5*d)*d^2*x^4/
a^3+1/42*d^3*x^6/a+c^3*x*arccot(a*x)+c^2*d*x^3*arccot(a*x)+3/5*c*d^2*x^5*a
rccot(a*x)+1/7*d^3*x^7*arccot(a*x)+1/70*(35*a^6*c^3-35*a^4*c^2*d+21*a^2*c*
d^2-5*d^3)*ln(a^2*x^2+1)/a^7
```

3.54.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.89

$$\int (c+dx^2)^3 \cot^{-1}(ax) \, dx = \frac{a^2dx^2(30d^2-3a^2d(42c+5dx^2)+a^4(210c^2+63cdx^2+10d^2x^4))+12a^7x(35c^3+35c^2dx^2+21cd^2x^4+5a}{420a^7}$$

input
```
Integrate[(c + d*x^2)^3*ArcCot[a*x],x]
```

output
```
(a^2*d*x^2*(30*d^2 - 3*a^2*d*(42*c + 5*d*x^2) + a^4*(210*c^2 + 63*c*d*x^2
+ 10*d^2*x^4)) + 12*a^7*x*(35*c^3 + 35*c^2*d*x^2 + 21*c*d^2*x^4 + 5*d^3*x^
6)*ArcCot[a*x] + 6*(35*a^6*c^3 - 35*a^4*c^2*d + 21*a^2*c*d^2 - 5*d^3)*Log[
1 + a^2*x^2])/(420*a^7)
```

3.54.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5448, 27, 2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}(ax)\left(c+dx^2\right)^3dx$$

$$\Big\downarrow \text{5448}$$

$$a\int \frac{x\left(5d^3x^6+21cd^2x^4+35c^2dx^2+35c^3\right)}{35\left(a^2x^2+1\right)}dx+c^3x\cot^{-1}(ax)+c^2dx^3\cot^{-1}(ax)+\frac{3}{5}cd^2x^5\cot^{-1}(ax)+\frac{1}{7}d^3x^7\cot^{-1}(ax)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{35}a\int \frac{x\left(5d^3x^6+21cd^2x^4+35c^2dx^2+35c^3\right)}{a^2x^2+1}dx+c^3x\cot^{-1}(ax)+c^2dx^3\cot^{-1}(ax)+\frac{3}{5}cd^2x^5\cot^{-1}(ax)+\frac{1}{7}d^3x^7\cot^{-1}(ax)$$

$$\Big\downarrow \text{2331}$$

$$\frac{1}{70}a\int \frac{5d^3x^6+21cd^2x^4+35c^2dx^2+35c^3}{a^2x^2+1}dx^2+c^3x\cot^{-1}(ax)+c^2dx^3\cot^{-1}(ax)+\frac{3}{5}cd^2x^5\cot^{-1}(ax)+\frac{1}{7}d^3x^7\cot^{-1}(ax)$$

$$\Big\downarrow \text{2389}$$

$$\frac{1}{70}a\int \left(\frac{5d^3x^4}{a^2}+\frac{\left(21a^2c-5d\right)d^2x^2}{a^4}+\frac{d\left(35c^2a^4-21cda^2+5d^2\right)}{a^6}+\frac{35c^3a^6-35c^2da^4+21cd^2a^2-5d^3}{a^6\left(a^2x^2+1\right)}\right)dx^2+c^3x\cot^{-1}(ax)+c^2dx^3\cot^{-1}(ax)+\frac{3}{5}cd^2x^5\cot^{-1}(ax)+\frac{1}{7}d^3x^7\cot^{-1}(ax)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{70}a\left(\frac{5d^3x^6}{3a^2}+\frac{d^2x^4(21a^2c-5d)}{2a^4}+\frac{dx^2(35a^4c^2-21a^2cd+5d^2)}{a^6}+\frac{(35a^6c^3-35a^4c^2d+21a^2cd^2-5d^3)\log(a^2x^2\right.$$
$$c^3x\cot^{-1}(ax)+c^2dx^3\cot^{-1}(ax)+\frac{3}{5}cd^2x^5\cot^{-1}(ax)+\frac{1}{7}d^3x^7\cot^{-1}(ax)$$

input
```
Int[(c + d*x^2)^3*ArcCot[a*x],x]
```

output
```
c^3*x*ArcCot[a*x] + c^2*d*x^3*ArcCot[a*x] + (3*c*d^2*x^5*ArcCot[a*x])/5 +
(d^3*x^7*ArcCot[a*x])/7 + (a*((d*(35*a^4*c^2 - 21*a^2*c*d + 5*d^2)*x^2)/a^
6 + ((21*a^2*c - 5*d)*d^2*x^4)/(2*a^4) + (5*d^3*x^6)/(3*a^2) + ((35*a^6*c^
3 - 35*a^4*c^2*d + 21*a^2*c*d^2 - 5*d^3)*Log[1 + a^2*x^2])/a^8))/70
```

3.54.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

rule 5448
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcCot[c*x])   u,
x] + Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2), x], x], x]] /; FreeQ
[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

3.54.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.99

method	result
parts	$\frac{d^3x^7 \operatorname{arccot}(ax)}{7}+\frac{3c\,d^2x^5 \operatorname{arccot}(ax)}{5}+c^2d\,x^3 \operatorname{arccot}(ax)+c^3x \operatorname{arccot}(ax)+\frac{a\left(\frac{d\left(\frac{5}{3}a^4d^2x^6+\frac{21}{2}a^4c\cdots\right.}{\cdots}\right.}{\cdots}$
derivativedivides	$\frac{\operatorname{arccot}(ax)c^3ax+a \operatorname{arccot}(ax)c^2d\,x^3+\frac{3a \operatorname{arccot}(ax)c\,d^2x^5}{5}+\frac{a \operatorname{arccot}(ax)d^3x^7}{7}+\frac{\frac{35c^2a^6d\,x^2}{2}+\frac{21c\,a^6d^2x^4}{4}-\frac{21c\,a^4d^2x^2}{2}+\frac{5d^3a\cdots}{6}\cdots}{\cdots}}{a}$
default	$\frac{\operatorname{arccot}(ax)c^3ax+a \operatorname{arccot}(ax)c^2d\,x^3+\frac{3a \operatorname{arccot}(ax)c\,d^2x^5}{5}+\frac{a \operatorname{arccot}(ax)d^3x^7}{7}+\frac{\frac{35c^2a^6d\,x^2}{2}+\frac{21c\,a^6d^2x^4}{4}-\frac{21c\,a^4d^2x^2}{2}+\frac{5d^3a\cdots}{6}\cdots}{\cdots}}{a}$
parallelrisch	$\frac{60x^7 \operatorname{arccot}(ax)a^7d^3+252x^5 \operatorname{arccot}(ax)a^7c\,d^2+10d^3a^6x^6+420x^3 \operatorname{arccot}(ax)a^7c^2d+63c\,a^6d^2x^4+420x \operatorname{arccot}(ax)a^7c^3-\cdots}{\cdots}$
risch	$-\frac{ic^2d\,x^3\ln(-iax+1)}{2}-\frac{ic^3x\ln(-iax+1)}{2}+\frac{\pi\,d^3x^7}{14}-\frac{3ic\,d^2x^5\ln(-iax+1)}{10}+\frac{3\pi c\,d^2x^5}{10}+\frac{i(5d^3x^7+21c\,d^2x^5+\cdots}{\cdots}$

input
```
int((d*x^2+c)^3*arccot(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/7*d^3*x^7*arccot(a*x)+3/5*c*d^2*x^5*arccot(a*x)+c^2*d*x^3*arccot(a*x)+c^
3*x*arccot(a*x)+1/35*a*(1/2*d/a^6*(5/3*a^4*d^2*x^6+21/2*a^4*c*d*x^4+35*a^4
*c^2*x^2-5/2*a^2*d^2*x^4-21*a^2*c*d*x^2+5*d^2*x^2)+1/2*(35*a^6*c^3-35*a^4*
c^2*d+21*a^2*c*d^2-5*d^3)/a^8*ln(a^2*x^2+1))
```

3.54.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.99

$$\int \left(c+dx^2\right)^3 \cot^{-1}(ax)\,dx$$
$$=\frac{10\,a^6d^3x^6+3\,(21\,a^6cd^2-5\,a^4d^3)x^4+6\,(35\,a^6c^2d-21\,a^4cd^2+5\,a^2d^3)x^2+12\,(5\,a^7d^3x^7+21\,a^7cd^2x^5+35\cdots}{420\,a^7}$$

input
```
integrate((d*x^2+c)^3*arccot(a*x),x, algorithm="fricas")
```

output
```
1/420*(10*a^6*d^3*x^6 + 3*(21*a^6*c*d^2 - 5*a^4*d^3)*x^4 + 6*(35*a^6*c^2*d
 - 21*a^4*c*d^2 + 5*a^2*d^3)*x^2 + 12*(5*a^7*d^3*x^7 + 21*a^7*c*d^2*x^5 +
35*a^7*c^2*d*x^3 + 35*a^7*c^3*x)*arccot(a*x) + 6*(35*a^6*c^3 - 35*a^4*c^2*
d + 21*a^2*c*d^2 - 5*d^3)*log(a^2*x^2 + 1))/a^7
```

3.54.6 Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.45

$$\int (c+dx^2)^3 \cot^{-1}(ax)\,dx$$
$$= \begin{cases} c^3x\,\mathrm{acot}\,(ax) + c^2dx^3\,\mathrm{acot}\,(ax) + \frac{3cd^2x^5\,\mathrm{acot}\,(ax)}{5} + \frac{d^3x^7\,\mathrm{acot}\,(ax)}{7} + \frac{c^3\log\left(x^2+\frac{1}{a^2}\right)}{2a} + \frac{c^2dx^2}{2a} + \frac{3cd^2x^4}{20a} + \frac{d^3x^6}{42a} - \frac{c^2 \cdots}{\cdots} \\ \frac{\pi\left(c^3x+c^2dx^3+\frac{3cd^2x^5}{5}+\frac{d^3x^7}{7}\right)}{2} \end{cases}$$

input
```
integrate((d*x**2+c)**3*acot(a*x),x)
```

output
```
Piecewise((c**3*x*acot(a*x) + c**2*d*x**3*acot(a*x) + 3*c*d**2*x**5*acot(a
*x)/5 + d**3*x**7*acot(a*x)/7 + c**3*log(x**2 + a**(-2))/(2*a) + c**2*d*x*
*2/(2*a) + 3*c*d**2*x**4/(20*a) + d**3*x**6/(42*a) - c**2*d*log(x**2 + a**
(-2))/(2*a**3) - 3*c*d**2*x**2/(10*a**3) - d**3*x**4/(28*a**3) + 3*c*d**2*
log(x**2 + a**(-2))/(10*a**5) + d**3*x**2/(14*a**5) - d**3*log(x**2 + a**(
-2))/(14*a**7), Ne(a, 0)), (pi*(c**3*x + c**2*d*x**3 + 3*c*d**2*x**5/5 + d
**3*x**7/7)/2, True))
```

3.54.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.95

$$\int (c+dx^2)^3 \cot^{-1}(ax)\,dx$$
$$= \frac{1}{420}a\left(\frac{10\,a^4d^3x^6 + 3\,(21\,a^4cd^2 - 5\,a^2d^3)x^4 + 6\,(35\,a^4c^2d - 21\,a^2cd^2 + 5\,d^3)x^2}{a^6} + \frac{6\,(35\,a^6c^3 - 35\,a^4c^2d + 2\cdots}{\cdots}\right.$$
$$+ \frac{1}{35}\left(5\,d^3x^7 + 21\,cd^2x^5 + 35\,c^2dx^3 + 35\,c^3x\right)\mathrm{arccot}\,(ax)$$

input
```
integrate((d*x^2+c)^3*arccot(a*x),x, algorithm="maxima")
```

output
```
1/420*a*((10*a^4*d^3*x^6 + 3*(21*a^4*c*d^2 - 5*a^2*d^3)*x^4 + 6*(35*a^4*c^
2*d - 21*a^2*c*d^2 + 5*d^3)*x^2)/a^6 + 6*(35*a^6*c^3 - 35*a^4*c^2*d + 21*a
^2*c*d^2 - 5*d^3)*log(a^2*x^2 + 1)/a^8) + 1/35*(5*d^3*x^7 + 21*c*d^2*x^5 +
 35*c^2*d*x^3 + 35*c^3*x)*arccot(a*x)
```

3.54.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.50

$$\int (c+dx^2)^3 \cot^{-1}(ax)\,dx$$
$$= \frac{1}{420}\left(\frac{12\left(5\,d^3+\frac{21\,cd^2}{x^2}+\frac{35\,c^2d}{x^4}+\frac{35\,c^3}{x^6}\right)x^7\arctan\left(\frac{1}{ax}\right)}{a}+\frac{\left(10\,d^3+\frac{63\,cd^2}{x^2}+\frac{210\,c^2d}{x^4}-\frac{15\,d^3}{a^2x^2}+\frac{385\,c^3}{x^6}-\frac{126\,cd^2}{a^2x^4}-\right.}{a^2}\right.$$

input
```
integrate((d*x^2+c)^3*arccot(a*x),x, algorithm="giac")
```

output
```
1/420*(12*(5*d^3 + 21*c*d^2/x^2 + 35*c^2*d/x^4 + 35*c^3/x^6)*x^7*arctan(1/
(a*x))/a + (10*d^3 + 63*c*d^2/x^2 + 210*c^2*d/x^4 - 15*d^3/(a^2*x^2) + 385
*c^3/x^6 - 126*c*d^2/(a^2*x^4) - 385*c^2*d/(a^2*x^6) + 30*d^3/(a^4*x^4) +
231*c*d^2/(a^4*x^6) - 55*d^3/(a^6*x^6))*x^6/a^2 + 6*(35*a^6*c^3 - 35*a^4*c
^2*d + 21*a^2*c*d^2 - 5*d^3)*log(1/(a^2*x^2) + 1)/a^8 - 6*(35*a^6*c^3 - 35
*a^4*c^2*d + 21*a^2*c*d^2 - 5*d^3)*log(1/(a^2*x^2))/a^8)*a
```

3.54.9 Mupad [B] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.13

$$\begin{aligned}\int (c+dx^2)^3 \cot^{-1}(ax)\,dx &= c^3\,x\,\mathrm{acot}(a\,x)+\frac{d^3\,x^7\,\mathrm{acot}(a\,x)}{7}+\frac{c^3\,\ln\left(a^2\,x^2+1\right)}{2\,a}\\ &\quad-\frac{d^3\,\ln\left(a^2\,x^2+1\right)}{14\,a^7}+\frac{d^3\,x^6}{42\,a}-\frac{d^3\,x^4}{28\,a^3}+\frac{d^3\,x^2}{14\,a^5}\\ &\quad-\frac{c^2\,d\,\ln\left(a^2\,x^2+1\right)}{2\,a^3}+\frac{3\,c\,d^2\,\ln\left(a^2\,x^2+1\right)}{10\,a^5}+\frac{c^2\,d\,x^2}{2\,a}\\ &\quad+\frac{3\,c\,d^2\,x^4}{20\,a}-\frac{3\,c\,d^2\,x^2}{10\,a^3}+c^2\,d\,x^3\,\mathrm{acot}(a\,x)+\frac{3\,c\,d^2\,x^5\,\mathrm{acot}(a\,x)}{5}\end{aligned}$$

input
```
int(acot(a*x)*(c + d*x^2)^3,x)
```

output
```
c^3*x*acot(a*x) + (d^3*x^7*acot(a*x))/7 + (c^3*log(a^2*x^2 + 1))/(2*a) - (
d^3*log(a^2*x^2 + 1))/(14*a^7) + (d^3*x^6)/(42*a) - (d^3*x^4)/(28*a^3) + (
d^3*x^2)/(14*a^5) - (c^2*d*log(a^2*x^2 + 1))/(2*a^3) + (3*c*d^2*log(a^2*x^
2 + 1))/(10*a^5) + (c^2*d*x^2)/(2*a) + (3*c*d^2*x^4)/(20*a) - (3*c*d^2*x^2
)/(10*a^3) + c^2*d*x^3*acot(a*x) + (3*c*d^2*x^5*acot(a*x))/5
```

3.55 $\int (c+dx^2)^2 \cot^{-1}(ax)\,dx$

3.55.1 Optimal result

Integrand size = 14, antiderivative size = 109

$$\int (c+dx^2)^2 \cot^{-1}(ax)\,dx = \frac{(10a^2c-3d)\,dx^2}{30a^3}+\frac{d^2x^4}{20a}+c^2x\cot^{-1}(ax)+\frac{2}{3}cdx^3\cot^{-1}(ax)+\frac{1}{5}d^2x^5\cot^{-1}(ax)+\frac{(15a^4c^2-10a^2cd+3d^2)\log(1+a^2x^2)}{30a^5}$$

output
```
1/30*(10*a^2*c-3*d)*d*x^2/a^3+1/20*d^2*x^4/a+c^2*x*arccot(a*x)+2/3*c*d*x^3
*arccot(a*x)+1/5*d^2*x^5*arccot(a*x)+1/30*(15*a^4*c^2-10*a^2*c*d+3*d^2)*ln
(a^2*x^2+1)/a^5
```

3.55.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.89

$$\int (c+dx^2)^2 \cot^{-1}(ax)\,dx = \frac{a^2dx^2(-6d+a^2(20c+3dx^2))+4a^5x(15c^2+10cdx^2+3d^2x^4)\cot^{-1}(ax)+(30a^4c^2-20a^2cd+6d^2)\log(1}{60a^5}$$

input
```
Integrate[(c + d*x^2)^2*ArcCot[a*x],x]
```

output
```
(a^2*d*x^2*(-6*d + a^2*(20*c + 3*d*x^2)) + 4*a^5*x*(15*c^2 + 10*c*d*x^2 +
3*d^2*x^4)*ArcCot[a*x] + (30*a^4*c^2 - 20*a^2*c*d + 6*d^2)*Log[1 + a^2*x^2
])/(60*a^5)
```

3.55.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5448, 27, 1576, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}(ax)\left(c+dx^2\right)^2 dx$$

$$\Big\downarrow \text{5448}$$

$$a\int \frac{x\left(3d^2x^4+10cdx^2+15c^2\right)}{15\left(a^2x^2+1\right)}dx+c^2x\cot^{-1}(ax)+\frac{2}{3}cdx^3\cot^{-1}(ax)+\frac{1}{5}d^2x^5\cot^{-1}(ax)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{15}a\int \frac{x\left(3d^2x^4+10cdx^2+15c^2\right)}{a^2x^2+1}dx+c^2x\cot^{-1}(ax)+\frac{2}{3}cdx^3\cot^{-1}(ax)+\frac{1}{5}d^2x^5\cot^{-1}(ax)$$

$$\Big\downarrow \text{1576}$$

$$\frac{1}{30}a\int \frac{3d^2x^4+10cdx^2+15c^2}{a^2x^2+1}dx^2+c^2x\cot^{-1}(ax)+\frac{2}{3}cdx^3\cot^{-1}(ax)+\frac{1}{5}d^2x^5\cot^{-1}(ax)$$

$$\Big\downarrow \text{1140}$$

$$\frac{1}{30}a\int \left(\frac{3d^2x^2}{a^2}+\frac{\left(10a^2c-3d\right)d}{a^4}+\frac{15c^2a^4-10cda^2+3d^2}{a^4\left(a^2x^2+1\right)}\right)dx^2+c^2x\cot^{-1}(ax)+\frac{2}{3}cdx^3\cot^{-1}(ax)+\frac{1}{5}d^2x^5\cot^{-1}(ax)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{30}a\left(\frac{3d^2x^4}{2a^2}+\frac{dx^2\left(10a^2c-3d\right)}{a^4}+\frac{\left(15a^4c^2-10a^2cd+3d^2\right)\log\left(a^2x^2+1\right)}{a^6}\right)+c^2x\cot^{-1}(ax)+\frac{2}{3}cdx^3\cot^{-1}(ax)+\frac{1}{5}d^2x^5\cot^{-1}(ax)$$

input
```
Int[(c + d*x^2)^2*ArcCot[a*x],x]
```

output
```
c^2*x*ArcCot[a*x] + (2*c*d*x^3*ArcCot[a*x])/3 + (d^2*x^5*ArcCot[a*x])/5 +
(a*(((10*a^2*c - 3*d)*d*x^2)/a^4 + (3*d^2*x^4)/(2*a^2) + ((15*a^4*c^2 - 10
*a^2*c*d + 3*d^2)*Log[1 + a^2*x^2])/a^6))/30
```

3.55.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5448
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcCot[c*x])   u,
x] + Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2), x], x], x]] /; FreeQ
[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

3.55.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.96

method	result
parts	$\frac{d^2x^5\operatorname{arccot}(ax)}{5}+\frac{2cd\,x^3\operatorname{arccot}(ax)}{3}+c^2x\operatorname{arccot}(ax)+\frac{a\left(\frac{d\left(\frac{3}{2}a^2d\,x^4+10a^2c\,x^2-3d\,x^2\right)}{2a^4}+\frac{\left(15a^4c^2-10a^2cd+3d\right.}{2a^6}\right.}{15}$
derivativedivides	$\frac{\operatorname{arccot}(ax)c^2ax+\frac{2a\operatorname{arccot}(ax)cd\,x^3}{3}+\frac{a\operatorname{arccot}(ax)d^2x^5}{5}+\frac{5c\,a^4d\,x^2+\frac{3d^2a^4x^4}{4}-\frac{3a^2d^2x^2}{2}+\frac{\left(15a^4c^2-10a^2cd+3d^2\right)\ln\left(a^2x^2+1\right)}{2}}{15a^4}}{a}$
default	$\frac{\operatorname{arccot}(ax)c^2ax+\frac{2a\operatorname{arccot}(ax)cd\,x^3}{3}+\frac{a\operatorname{arccot}(ax)d^2x^5}{5}+\frac{5c\,a^4d\,x^2+\frac{3d^2a^4x^4}{4}-\frac{3a^2d^2x^2}{2}+\frac{\left(15a^4c^2-10a^2cd+3d^2\right)\ln\left(a^2x^2+1\right)}{2}}{15a^4}}{a}$
parallelrisch	$\frac{12x^5\operatorname{arccot}(ax)a^5d^2+40x^3\operatorname{arccot}(ax)a^5cd+3d^2a^4x^4+60c^2\operatorname{arccot}(ax)x\,a^5+20c\,a^4d\,x^2+30\ln\left(a^2x^2+1\right)a^4c^2-6a^2d^2x^2}{60a^5}$
risch	$\frac{i\left(3d^2x^5+10cd\,x^3+15c^2x\right)\ln(iax+1)}{30}-\frac{id^2x^5\ln(-iax+1)}{10}+\frac{\pi\,d^2x^5}{10}-\frac{icd\,x^3\ln(-iax+1)}{3}+\frac{\pi cd\,x^3}{3}-\frac{ic^2x\ln(-}{2}$

input
```
int((d*x^2+c)^2*arccot(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/5*d^2*x^5*arccot(a*x)+2/3*c*d*x^3*arccot(a*x)+c^2*x*arccot(a*x)+1/15*a*(
1/2*d/a^4*(3/2*a^2*d*x^4+10*a^2*c*x^2-3*d*x^2)+1/2*(15*a^4*c^2-10*a^2*c*d+
3*d^2)/a^6*ln(a^2*x^2+1))
```

3.55.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.99

$$\int (c+dx^2)^2 \cot^{-1}(ax)\,dx$$
$$= \frac{3\,a^4d^2x^4 + 2\,(10\,a^4cd - 3\,a^2d^2)x^2 + 4\,(3\,a^5d^2x^5 + 10\,a^5cdx^3 + 15\,a^5c^2x)\operatorname{arccot}(ax) + 2\,(15\,a^4c^2 - 10\,a^2cd}{60\,a^5}$$

input
```
integrate((d*x^2+c)^2*arccot(a*x),x, algorithm="fricas")
```

output
```
1/60*(3*a^4*d^2*x^4 + 2*(10*a^4*c*d - 3*a^2*d^2)*x^2 + 4*(3*a^5*d^2*x^5 +
10*a^5*c*d*x^3 + 15*a^5*c^2*x)*arccot(a*x) + 2*(15*a^4*c^2 - 10*a^2*c*d +
3*d^2)*log(a^2*x^2 + 1))/a^5
```

3.55.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.39

$$\int (c+dx^2)^2 \cot^{-1}(ax)\,dx$$
$$= \begin{cases} c^2x\operatorname{acot}(ax) + \frac{2cdx^3\operatorname{acot}(ax)}{3} + \frac{d^2x^5\operatorname{acot}(ax)}{5} + \frac{c^2\log\left(x^2+\frac{1}{a^2}\right)}{2a} + \frac{cdx^2}{3a} + \frac{d^2x^4}{20a} - \frac{cd\log\left(x^2+\frac{1}{a^2}\right)}{3a^3} - \frac{d^2x^2}{10a^3} + \frac{d^2\log\left(x^2+\right.}{10a^5} \\ \frac{\pi\left(c^2x+\frac{2cdx^3}{3}+\frac{d^2x^5}{5}\right)}{2} \end{cases}$$

input
```
integrate((d*x**2+c)**2*acot(a*x),x)
```

output
```
Piecewise(((c**2*x*acot(a*x) + 2*c*d*x**3*acot(a*x)/3 + d**2*x**5*acot(a*x)
/5 + c**2*log(x**2 + a**(-2))/(2*a) + c*d*x**2/(3*a) + d**2*x**4/(20*a) -
c*d*log(x**2 + a**(-2))/(3*a**3) - d**2*x**2/(10*a**3) + d**2*log(x**2 + a
**(-2))/(10*a**5), Ne(a, 0)), (pi*(c**2*x + 2*c*d*x**3/3 + d**2*x**5/5)/2,
 True))
```

3.55.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

$$\int \left(c+dx^2\right)^2 \cot^{-1}(ax)\,dx$$
$$=\frac{1}{60}a\left(\frac{3\,a^2d^2x^4+2\,(10\,a^2cd-3\,d^2)x^2}{a^4}+\frac{2\,(15\,a^4c^2-10\,a^2cd+3\,d^2)\log\left(a^2x^2+1\right)}{a^6}\right)$$
$$+\frac{1}{15}\left(3\,d^2x^5+10\,cdx^3+15\,c^2x\right)\operatorname{arccot}\left(ax\right)$$

input
```
integrate((d*x^2+c)^2*arccot(a*x),x, algorithm="maxima")
```

output
```
1/60*a*((3*a^2*d^2*x^4 + 2*(10*a^2*c*d - 3*d^2)*x^2)/a^4 + 2*(15*a^4*c^2 -
 10*a^2*c*d + 3*d^2)*log(a^2*x^2 + 1)/a^6) + 1/15*(3*d^2*x^5 + 10*c*d*x^3
+ 15*c^2*x)*arccot(a*x)
```

3.55.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.57

$$\int \left(c+dx^2\right)^2 \cot^{-1}(ax)\,dx$$
$$=\frac{1}{60}\left(\frac{4\left(3\,d^2+\frac{10\,cd}{x^2}+\frac{15\,c^2}{x^4}\right)x^5\arctan\left(\frac{1}{ax}\right)}{a}+\frac{\left(3\,d^2+\frac{20\,cd}{x^2}+\frac{45\,c^2}{x^4}-\frac{6\,d^2}{a^2x^2}-\frac{30\,cd}{a^2x^4}+\frac{9\,d^2}{a^4x^4}\right)x^4}{a^2}+\frac{2\,(15\,a^4c^2-\right.$$

input
```
integrate((d*x^2+c)^2*arccot(a*x),x, algorithm="giac")
```

output
```
1/60*(4*(3*d^2 + 10*c*d/x^2 + 15*c^2/x^4)*x^5*arctan(1/(a*x))/a + (3*d^2 +
 20*c*d/x^2 + 45*c^2/x^4 - 6*d^2/(a^2*x^2) - 30*c*d/(a^2*x^4) + 9*d^2/(a^4
*x^4))*x^4/a^2 + 2*(15*a^4*c^2 - 10*a^2*c*d + 3*d^2)*log(1/(a^2*x^2) + 1)/
a^6 - 2*(15*a^4*c^2 - 10*a^2*c*d + 3*d^2)*log(1/(a^2*x^2))/a^6)*a
```

3.55.9 Mupad [B] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.06

$$\int \left(c+dx^2\right)^2 \cot^{-1}(ax)\,dx$$
$$= \frac{a^4\left(\frac{c^2 \ln\left(a^2 x^2+1\right)}{2}+\frac{d^2 x^4}{20}+\frac{c\,d\,x^2}{3}\right)-a^2\left(\frac{d^2 x^2}{10}+\frac{c\,d\,\ln\left(a^2 x^2+1\right)}{3}\right)+\frac{d^2 \ln\left(a^2 x^2+1\right)}{10}}{a^5} + c^2\,x\,\mathrm{acot}(a\,x)+\frac{d^2\,x^5\,\mathrm{acot}(a\,x)}{5}+\frac{2\,c\,d\,x^3\,\mathrm{acot}(a\,x)}{3}$$

input
```
int(acot(a*x)*(c + d*x^2)^2,x)
```

output
```
(a^4*((c^2*log(a^2*x^2 + 1))/2 + (d^2*x^4)/20 + (c*d*x^2)/3) - a^2*((d^2*x
^2)/10 + (c*d*log(a^2*x^2 + 1))/3) + (d^2*log(a^2*x^2 + 1))/10)/a^5 + c^2*
x*acot(a*x) + (d^2*x^5*acot(a*x))/5 + (2*c*d*x^3*acot(a*x))/3
```

3.56 $\int (c + dx^2) \cot^{-1}(ax) \, dx$

3.56.1 Optimal result

Integrand size = 12, antiderivative size = 58

$$\int (c+dx^2) \cot^{-1}(ax) \, dx = \frac{dx^2}{6a} + cx \cot^{-1}(ax) + \frac{1}{3} dx^3 \cot^{-1}(ax) + \frac{(3a^2c - d) \log(1 + a^2x^2)}{6a^3}$$

output
```
1/6*d*x^2/a+c*x*arccot(a*x)+1/3*d*x^3*arccot(a*x)+1/6*(3*a^2*c-d)*ln(a^2*x
^2+1)/a^3
```

3.56.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.16

$$\int (c+dx^2) \cot^{-1}(ax) \, dx = \frac{dx^2}{6a} + cx \cot^{-1}(ax) + \frac{1}{3} dx^3 \cot^{-1}(ax) + \frac{c \log(1 + a^2x^2)}{2a} - \frac{d \log(1 + a^2x^2)}{6a^3}$$

input
```
Integrate[(c + d*x^2)*ArcCot[a*x],x]
```

output
```
(d*x^2)/(6*a) + c*x*ArcCot[a*x] + (d*x^3*ArcCot[a*x])/3 + (c*Log[1 + a^2*x
^2])/(2*a) - (d*Log[1 + a^2*x^2])/(6*a^3)
```

3.56.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5448, 27, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}(ax)\left(c+dx^2\right)\,dx$$

$$\Big\downarrow \text{5448}$$

$$a\int \frac{x\left(dx^2+3c\right)}{3\left(a^2x^2+1\right)}dx+cx\cot^{-1}(ax)+\frac{1}{3}dx^3\cot^{-1}(ax)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}a\int \frac{x\left(dx^2+3c\right)}{a^2x^2+1}dx+cx\cot^{-1}(ax)+\frac{1}{3}dx^3\cot^{-1}(ax)$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{6}a\int \frac{dx^2+3c}{a^2x^2+1}dx^2+cx\cot^{-1}(ax)+\frac{1}{3}dx^3\cot^{-1}(ax)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{6}a\int \left(\frac{3a^2c-d}{a^2\left(a^2x^2+1\right)}+\frac{d}{a^2}\right)dx^2+cx\cot^{-1}(ax)+\frac{1}{3}dx^3\cot^{-1}(ax)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}a\left(\frac{dx^2}{a^2}+\frac{\left(3a^2c-d\right)\log\left(a^2x^2+1\right)}{a^4}\right)+cx\cot^{-1}(ax)+\frac{1}{3}dx^3\cot^{-1}(ax)$$

input
```
Int[(c + d*x^2)*ArcCot[a*x],x]
```

output
```
c*x*ArcCot[a*x] + (d*x^3*ArcCot[a*x])/3 + (a*((d*x^2)/a^2 + ((3*a^2*c - d)
*Log[1 + a^2*x^2])/a^4))/6
```

3.56.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5448
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcCot[c*x])   u,
x] + Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2), x], x], x]] /; FreeQ
[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

3.56.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

method	result
parts	$\frac{d\,x^{3} \operatorname{arccot}(ax)}{3}+cx \operatorname{arccot}\left(ax\right)+\frac{a\left(\frac{d\,x^{2}}{2a^{2}}+\frac{\left(3a^{2}c-d\right)\ln\left(a^{2}x^{2}+1\right)}{2a^{4}}\right)}{3}$
derivativedivides	$\frac{\operatorname{arccot}(ax) c a x+\frac{a \operatorname{arccot}(ax) d\,x^{3}}{3}+\frac{\frac{a^{2} d\,x^{2}}{2}+\frac{\left(3a^{2}c-d\right)\ln\left(a^{2}x^{2}+1\right)}{2}}{3a^{2}}}{a}$
default	$\frac{\operatorname{arccot}(ax) c a x+\frac{a \operatorname{arccot}(ax) d\,x^{3}}{3}+\frac{\frac{a^{2} d\,x^{2}}{2}+\frac{\left(3a^{2}c-d\right)\ln\left(a^{2}x^{2}+1\right)}{2}}{3a^{2}}}{a}$
parallelrisch	$\frac{2x^{3} \operatorname{arccot}(ax) a^{3} d+6x \operatorname{arccot}(ax) a^{3} c+a^{2} d\,x^{2}+3\ln\left(a^{2}x^{2}+1\right)a^{2}c-\ln\left(a^{2}x^{2}+1\right)d}{6a^{3}}$
risch	$\frac{i\left(d\,x^{3}+3cx\right)\ln\left(iax+1\right)}{6}-\frac{id\,x^{3}\ln\left(-iax+1\right)}{6}+\frac{\pi d\,x^{3}}{6}-\frac{icx\ln\left(-iax+1\right)}{2}+\frac{\pi cx}{2}+\frac{d\,x^{2}}{6a}+\frac{\ln\left(-a^{2}x^{2}-1\right)c}{2a}-\ln$

input
```
int((d*x^2+c)*arccot(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*d*x^3*arccot(a*x)+c*x*arccot(a*x)+1/3*a*(1/2*d/a^2*x^2+1/2*(3*a^2*c-d)
/a^4*ln(a^2*x^2+1))
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int (c+dx^2)\cot^{-1}(ax)\,dx = \frac{a^2dx^2 + 2\,(a^3dx^3 + 3\,a^3cx)\,\text{arccot}\,(ax) + (3\,a^2c - d)\log\,(a^2x^2 + 1)}{6\,a^3}$$

input
```
integrate((d*x^2+c)*arccot(a*x),x, algorithm="fricas")
```

output
```
1/6*(a^2*d*x^2 + 2*(a^3*d*x^3 + 3*a^3*c*x)*arccot(a*x) + (3*a^2*c - d)*log
(a^2*x^2 + 1))/a^3
```

3.56.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.26

$$\int (c+dx^2)\cot^{-1}(ax)\,dx = \begin{cases} cx\,\text{acot}\,(ax) + \frac{dx^3\,\text{acot}\,(ax)}{3} + \frac{c\log\left(x^2+\frac{1}{a^2}\right)}{2a} + \frac{dx^2}{6a} - \frac{d\log\left(x^2+\frac{1}{a^2}\right)}{6a^3} & \text{for } a \neq 0 \\ \frac{\pi\left(cx+\frac{dx^3}{3}\right)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x**2+c)*acot(a*x),x)
```

output
```
Piecewise((c*x*acot(a*x) + d*x**3*acot(a*x)/3 + c*log(x**2 + a**(-2))/(2*a
) + d*x**2/(6*a) - d*log(x**2 + a**(-2))/(6*a**3), Ne(a, 0)), (pi*(c*x + d
*x**3/3)/2, True))
```

3.56.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

$$\int (c+dx^2)\cot^{-1}(ax)\,dx$$
$$=\frac{1}{6}\,a\left(\frac{dx^2}{a^2}+\frac{(3\,a^2c-d)\log\left(a^2x^2+1\right)}{a^4}\right)+\frac{1}{3}\left(dx^3+3\,cx\right)\operatorname{arccot}\left(ax\right)$$

input
```
integrate((d*x^2+c)*arccot(a*x),x, algorithm="maxima")
```

output
```
1/6*a*(d*x^2/a^2 + (3*a^2*c - d)*log(a^2*x^2 + 1)/a^4) + 1/3*(d*x^3 + 3*c*
x)*arccot(a*x)
```

3.56.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.71

$$\int (c+dx^2)\cot^{-1}(ax)\,dx$$
$$=\frac{1}{6}\left(\frac{2\left(d+\frac{3\,c}{x^2}\right)x^3\arctan\left(\frac{1}{ax}\right)}{a}+\frac{\left(d+\frac{3\,c}{x^2}-\frac{d}{a^2x^2}\right)x^2}{a^2}+\frac{(3\,a^2c-d)\log\left(\frac{1}{a^2x^2}+1\right)}{a^4}-\frac{(3\,a^2c-d)\log\left(\frac{1}{a^2x^2}\right)}{a^4}\right)a$$

input
```
integrate((d*x^2+c)*arccot(a*x),x, algorithm="giac")
```

output
```
1/6*(2*(d + 3*c/x^2)*x^3*arctan(1/(a*x))/a + (d + 3*c/x^2 - d/(a^2*x^2))*x
^2/a^2 + (3*a^2*c - d)*log(1/(a^2*x^2) + 1)/a^4 - (3*a^2*c - d)*log(1/(a^2
*x^2))/a^4)*a
```

3.56.9 Mupad [B] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07

$$\int (c+dx^2)\cot^{-1}(ax)\,dx=\frac{d\,x^3\,\mathrm{acot}(a\,x)}{3}+c\,x\,\mathrm{acot}(a\,x)-\frac{\frac{d\,\ln(a^2\,x^2+1)}{6}-a^2\left(\frac{c\,\ln(a^2\,x^2+1)}{2}+\frac{d\,x^2}{6}\right)}{a^3}$$

input
```
int(acot(a*x)*(c + d*x^2),x)
```

output
```
(d*x^3*acot(a*x))/3 - ((d*log(a^2*x^2 + 1))/6 - a^2*((c*log(a^2*x^2 + 1))/
2 + (d*x^2)/6))/a^3 + c*x*acot(a*x)
```

3.56. $\int (c+dx^2)\cot^{-1}(ax)\,dx$

3.57 $\int \frac{\cot^{-1}(ax)}{c+dx^2}\,dx$

3.57.1 Optimal result

Integrand size = 14, antiderivative size = 403

$$\begin{aligned}\int \frac{\cot^{-1}(ax)}{c+dx^2}\,dx &= \frac{i\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(1-\frac{i}{ax}\right)}{2\sqrt{c}\sqrt{d}} - \frac{i\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(1+\frac{i}{ax}\right)}{2\sqrt{c}\sqrt{d}} \\ &\quad - \frac{i\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(\frac{2i\sqrt{c}\sqrt{d}(i-ax)}{\left(a\sqrt{c}-\sqrt{d}\right)\left(\sqrt{c}-i\sqrt{d}x\right)}\right)}{2\sqrt{c}\sqrt{d}} \\ &\quad + \frac{i\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(-\frac{2i\sqrt{c}\sqrt{d}(i+ax)}{\left(a\sqrt{c}+\sqrt{d}\right)\left(\sqrt{c}-i\sqrt{d}x\right)}\right)}{2\sqrt{c}\sqrt{d}} \\ &\quad - \frac{\operatorname{PolyLog}\left(2,1-\frac{2i\sqrt{c}\sqrt{d}(i-ax)}{\left(a\sqrt{c}-\sqrt{d}\right)\left(\sqrt{c}-i\sqrt{d}x\right)}\right)}{4\sqrt{c}\sqrt{d}} \\ &\quad + \frac{\operatorname{PolyLog}\left(2,1+\frac{2i\sqrt{c}\sqrt{d}(i+ax)}{\left(a\sqrt{c}+\sqrt{d}\right)\left(\sqrt{c}-i\sqrt{d}x\right)}\right)}{4\sqrt{c}\sqrt{d}}\end{aligned}$$

output
```
1/2*I*arctan(x*d^(1/2)/c^(1/2))*ln(1-I/a/x)/c^(1/2)/d^(1/2)-1/2*I*arctan(x
*d^(1/2)/c^(1/2))*ln(1+I/a/x)/c^(1/2)/d^(1/2)-1/2*I*arctan(x*d^(1/2)/c^(1/
2))*ln(2*I*(I-a*x)*c^(1/2)*d^(1/2)/(a*c^(1/2)-d^(1/2))/(c^(1/2)-I*x*d^(1/2
)))/c^(1/2)/d^(1/2)+1/2*I*arctan(x*d^(1/2)/c^(1/2))*ln(-2*I*(I+a*x)*c^(1/2
)*d^(1/2)/(a*c^(1/2)+d^(1/2))/(c^(1/2)-I*x*d^(1/2)))/c^(1/2)/d^(1/2)-1/4*p
olylog(2,1-2*I*(I-a*x)*c^(1/2)*d^(1/2)/(a*c^(1/2)-d^(1/2))/(c^(1/2)-I*x*d^
(1/2)))/c^(1/2)/d^(1/2)+1/4*polylog(2,1+2*I*(I+a*x)*c^(1/2)*d^(1/2)/(a*c^(
1/2)+d^(1/2))/(c^(1/2)-I*x*d^(1/2)))/c^(1/2)/d^(1/2)
```

3.57.2 Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 716, normalized size of antiderivative = 1.78

$$\int \frac{\cot^{-1}(ax)}{c+dx^2}\,dx = \frac{a\left(-2\arccos\left(\frac{a^2c+d}{a^2c-d}\right)\operatorname{arctanh}\left(\frac{ac}{\sqrt{-a^2cdx}}\right)-4\cot^{-1}(ax)\operatorname{arctanh}\left(\frac{adx}{\sqrt{-a^2cd}}\right)-\left(\arccos\left(\frac{a^2c+d}{a^2c-d}\right)-2i\operatorname{arctanh}\left(\right.\right.\right.}{}$$

input
```
Integrate[ArcCot[a*x]/(c + d*x^2),x]
```

output
```
(a*(-2*ArcCos[(a^2*c + d)/(a^2*c - d)]*ArcTanh[(a*c)/(Sqrt[-(a^2*c*d)]*x)]
 - 4*ArcCot[a*x]*ArcTanh[(a*d*x)/Sqrt[-(a^2*c*d)]] - (ArcCos[(a^2*c + d)/(
a^2*c - d)] - (2*I)*ArcTanh[(a*c)/(Sqrt[-(a^2*c*d)]*x)])*Log[((2*I)*d*(I*a
^2*c + Sqrt[-(a^2*c*d)])*(I + a*x))/((a^2*c - d)*(Sqrt[-(a^2*c*d)] - a*d*x
))] - (ArcCos[(a^2*c + d)/(a^2*c - d)] + (2*I)*ArcTanh[(a*c)/(Sqrt[-(a^2*c
*d)]*x)])*Log[(2*d*(a^2*c + I*Sqrt[-(a^2*c*d)])*(-I + a*x))/((a^2*c - d)*(
-Sqrt[-(a^2*c*d)] + a*d*x))] + (ArcCos[(a^2*c + d)/(a^2*c - d)] + (2*I)*Ar
cTanh[(a*c)/(Sqrt[-(a^2*c*d)]*x)] + (2*I)*ArcTanh[(a*d*x)/Sqrt[-(a^2*c*d)]
])*Log[(Sqrt[2]*Sqrt[-(a^2*c*d)])/(Sqrt[a^2*c - d]*E^(I*ArcCot[a*x])*Sqrt[
-(a^2*c) - d + (a^2*c - d)*Cos[2*ArcCot[a*x]]])] + (ArcCos[(a^2*c + d)/(a^
2*c - d)] - (2*I)*ArcTanh[(a*c)/(Sqrt[-(a^2*c*d)]*x)] - (2*I)*ArcTanh[(a*d
*x)/Sqrt[-(a^2*c*d)]])*Log[(Sqrt[2]*Sqrt[-(a^2*c*d)]*E^(I*ArcCot[a*x]))/(S
qrt[a^2*c - d]*Sqrt[-(a^2*c) - d + (a^2*c - d)*Cos[2*ArcCot[a*x]]])] + I*(
-PolyLog[2, ((a^2*c + d - (2*I)*Sqrt[-(a^2*c*d)])*(Sqrt[-(a^2*c*d)] + a*d*
x))/((a^2*c - d)*(Sqrt[-(a^2*c*d)] - a*d*x))] + PolyLog[2, ((a^2*c + d + (
2*I)*Sqrt[-(a^2*c*d)])*(Sqrt[-(a^2*c*d)] + a*d*x))/((a^2*c - d)*(Sqrt[-(a^
2*c*d)] - a*d*x))])))/(4*Sqrt[-(a^2*c*d)])
```

3.57.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 629, normalized size of antiderivative = 1.56, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5444, 2920, 27, 2005, 5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax)}{c+dx^2}dx$$

$$\Big\downarrow \text{5444}$$

$$\frac{1}{2}i\int \frac{\log\left(1-\frac{i}{ax}\right)}{dx^2+c}dx-\frac{1}{2}i\int \frac{\log\left(1+\frac{i}{ax}\right)}{dx^2+c}dx$$

$$\Big\downarrow \text{2920}$$

$$\frac{1}{2}i\left(\frac{\log\left(1-\frac{i}{ax}\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}}-\frac{i\int \frac{a\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}\left(a-\frac{i}{x}\right)x^2}dx}{a}\right)-\frac{1}{2}i\left(\frac{i\int \frac{a\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}\left(a+\frac{i}{x}\right)x^2}dx}{a}+\frac{\log\left(1+\frac{i}{ax}\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}i\left(\frac{\log\left(1-\frac{i}{ax}\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}}-\frac{i\int \frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\left(a-\frac{i}{x}\right)x^2}dx}{\sqrt{c}\sqrt{d}}\right)-\frac{1}{2}i\left(\frac{i\int \frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\left(a+\frac{i}{x}\right)x^2}dx}{\sqrt{c}\sqrt{d}}+\frac{\log\left(1+\frac{i}{ax}\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}}\right)$$

$$\Big\downarrow \text{2005}$$

$$\frac{1}{2}i\left(\frac{\log\left(1-\frac{i}{ax}\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}}-\frac{i\int \frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{x(ax-i)}dx}{\sqrt{c}\sqrt{d}}\right)-\frac{1}{2}i\left(\frac{i\int \frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{x(ax+i)}dx}{\sqrt{c}\sqrt{d}}+\frac{\log\left(1+\frac{i}{ax}\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}}\right)$$

$$\Big\downarrow \text{5411}$$

$$\frac{1}{2}i\left(\frac{\log\left(1-\frac{i}{ax}\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}}-\frac{i\int\left(\frac{i\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{x}-\frac{ia\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{ax-i}\right)dx}{\sqrt{c}\sqrt{d}}\right)-\frac{1}{2}i\left(\frac{i\int\left(\frac{ia\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{ax+i}-\frac{i\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{x}\right)dx}{\sqrt{c}\sqrt{d}}+\frac{\log\left(1+\frac{i}{ax}\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}i\left(\frac{\log\left(1-\frac{i}{ax}\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}}-\frac{i\left(-i\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(\frac{2i\sqrt{c}\sqrt{d}(-ax+i)}{\left(a\sqrt{c}-\sqrt{d}\right)\left(\sqrt{c}-i\sqrt{d}x\right)}\right)-\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2i\sqrt{c}\sqrt{d}}{\left(a\sqrt{c}-\sqrt{d}\right)(}\right.\right.}{}\right.$$

$$\frac{1}{2}i\left(\frac{i\left(i\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(-\frac{2i\sqrt{c}\sqrt{d}(ax+i)}{\left(a\sqrt{c}+\sqrt{d}\right)\left(\sqrt{c}-i\sqrt{d}x\right)}\right)+\frac{1}{2}\text{PolyLog}\left(2,\frac{2i\sqrt{c}\sqrt{d}(ax+i)}{\left(\sqrt{c}a+\sqrt{d}\right)\left(\sqrt{c}-i\sqrt{d}x\right)}+1\right)-i\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right.}{\sqrt{c}\sqrt{d}}\right.$$

input
```
Int[ArcCot[a*x]/(c + d*x^2),x]
```

output
```
(I/2)*((ArcTan[(Sqrt[d]*x)/Sqrt[c]]*Log[1 - I/(a*x)])/(Sqrt[c]*Sqrt[d]) -
(I*(I*ArcTan[(Sqrt[d]*x)/Sqrt[c]]*Log[(2*Sqrt[c])/(Sqrt[c] - I*Sqrt[d]*x)]
 - I*ArcTan[(Sqrt[d]*x)/Sqrt[c]]*Log[((2*I)*Sqrt[c]*Sqrt[d]*(I - a*x))/((a
*Sqrt[c] - Sqrt[d])*(Sqrt[c] - I*Sqrt[d]*x))] - PolyLog[2, ((-I)*Sqrt[d]*x
)/Sqrt[c]]/2 + PolyLog[2, (I*Sqrt[d]*x)/Sqrt[c]]/2 + PolyLog[2, 1 - (2*Sqr
t[c])/(Sqrt[c] - I*Sqrt[d]*x)]/2 - PolyLog[2, 1 - ((2*I)*Sqrt[c]*Sqrt[d]*(
I - a*x))/((a*Sqrt[c] - Sqrt[d])*(Sqrt[c] - I*Sqrt[d]*x))]/2))/(Sqrt[c]*Sq
rt[d])) - (I/2)*((ArcTan[(Sqrt[d]*x)/Sqrt[c]]*Log[1 + I/(a*x)])/(Sqrt[c]*S
qrt[d]) + (I*((-I)*ArcTan[(Sqrt[d]*x)/Sqrt[c]]*Log[(2*Sqrt[c])/(Sqrt[c] -
I*Sqrt[d]*x)] + I*ArcTan[(Sqrt[d]*x)/Sqrt[c]]*Log[((-2*I)*Sqrt[c]*Sqrt[d]*
(I + a*x))/((a*Sqrt[c] + Sqrt[d])*(Sqrt[c] - I*Sqrt[d]*x))] + PolyLog[2, (
(-I)*Sqrt[d]*x)/Sqrt[c]]/2 - PolyLog[2, (I*Sqrt[d]*x)/Sqrt[c]]/2 - PolyLog
[2, 1 - (2*Sqrt[c])/(Sqrt[c] - I*Sqrt[d]*x)]/2 + PolyLog[2, 1 + ((2*I)*Sqr
t[c]*Sqrt[d]*(I + a*x))/((a*Sqrt[c] + Sqrt[d])*(Sqrt[c] - I*Sqrt[d]*x))]/2
))/(Sqrt[c]*Sqrt[d]))
```

3.57. $\int \frac{\cot^{-1}(ax)}{c+dx^2}\,dx$

3.57.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2920
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Simp[b*e*n*p   Int[u*(x^(n - 1)/(d + e*x^n)), x
], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

rule 5444
```
Int[ArcCot[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Simp[I/2   Int[
Log[1 - I/(c*x)]/(d + e*x^2), x], x] - Simp[I/2   Int[Log[1 + I/(c*x)]/(d +
 e*x^2), x], x] /; FreeQ[{c, d, e}, x]
```

3.57.4 Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.97

method	result
risch	$\frac{i\pi \operatorname{arctanh}\left(\frac{2(-iax+1)d-2d}{2a\sqrt{cd}}\right)}{2\sqrt{cd}}-\frac{\ln(-iax+1)\ln\left(\frac{a\sqrt{cd}-(-iax+1)d+d}{a\sqrt{cd}+d}\right)}{4\sqrt{cd}}+\frac{\ln(-iax+1)\ln\left(\frac{a\sqrt{cd}+(-iax+1)d-d}{a\sqrt{cd}-d}\right)}{4\sqrt{cd}}-$ dil
derivativedivides	$-\frac{i\sqrt{a^2cd}\,\operatorname{arccot}(ax)\ln\left(1-\frac{\left(a^2c-d\right)(ax+i)^2}{\left(a^2x^2+1\right)\left(a^2c+2\sqrt{a^2cd}+d\right)}\right)}{2cd}-\frac{\sqrt{a^2cd}\,\operatorname{arccot}(ax)^2}{2cd}-\frac{\sqrt{a^2cd}\,\operatorname{polylog}\left(2,\frac{\left(a^2c-d\right)(ax+i)^2}{\left(a^2x^2+1\right)\left(a^2c+2\sqrt{a^2cd}+d\right)}\right)}{4cd}$
default	$-\frac{i\sqrt{a^2cd}\,\operatorname{arccot}(ax)\ln\left(1-\frac{\left(a^2c-d\right)(ax+i)^2}{\left(a^2x^2+1\right)\left(a^2c+2\sqrt{a^2cd}+d\right)}\right)}{2cd}-\frac{\sqrt{a^2cd}\,\operatorname{arccot}(ax)^2}{2cd}-\frac{\sqrt{a^2cd}\,\operatorname{polylog}\left(2,\frac{\left(a^2c-d\right)(ax+i)^2}{\left(a^2x^2+1\right)\left(a^2c+2\sqrt{a^2cd}+d\right)}\right)}{4cd}$

input
```
int(arccot(a*x)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*Pi/(c*d)^(1/2)*arctanh(1/2*(2*(1-I*a*x)*d-2*d)/a/(c*d)^(1/2))-1/4*ln
(1-I*a*x)/(c*d)^(1/2)*ln((a*(c*d)^(1/2)-(1-I*a*x)*d+d)/(a*(c*d)^(1/2)+d))+
1/4*ln(1-I*a*x)/(c*d)^(1/2)*ln((a*(c*d)^(1/2)+(1-I*a*x)*d-d)/(a*(c*d)^(1/2
)-d))-1/4/(c*d)^(1/2)*dilog((a*(c*d)^(1/2)-(1-I*a*x)*d+d)/(a*(c*d)^(1/2)+d
))+1/4/(c*d)^(1/2)*dilog((a*(c*d)^(1/2)+(1-I*a*x)*d-d)/(a*(c*d)^(1/2)-d))-
1/4*ln(1+I*a*x)/(c*d)^(1/2)*ln((a*(c*d)^(1/2)-(1+I*a*x)*d+d)/(a*(c*d)^(1/2
)+d))+1/4*ln(1+I*a*x)/(c*d)^(1/2)*ln((a*(c*d)^(1/2)+(1+I*a*x)*d-d)/(a*(c*d
)^(1/2)-d))-1/4/(c*d)^(1/2)*dilog((a*(c*d)^(1/2)-(1+I*a*x)*d+d)/(a*(c*d)^(
1/2)+d))+1/4/(c*d)^(1/2)*dilog((a*(c*d)^(1/2)+(1+I*a*x)*d-d)/(a*(c*d)^(1/2
)-d))
```

3.57.5 Fricas [F]

$$\int \frac{\cot^{-1}(ax)}{c+dx^2}\,dx=\int \frac{\operatorname{arccot}(ax)}{dx^2+c}\,dx$$

input
```
integrate(arccot(a*x)/(d*x^2+c),x, algorithm="fricas")
```

output
```
integral(arccot(a*x)/(d*x^2 + c), x)
```

3.57.6 Sympy [F]

$$\int \frac{\cot^{-1}(ax)}{c+dx^2}\,dx = \int \frac{\operatorname{acot}(ax)}{c+dx^2}\,dx$$

input
```
integrate(acot(a*x)/(d*x**2+c),x)
```

output
```
Integral(acot(a*x)/(c + d*x**2), x)
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 528, normalized size of antiderivative = 1.31

$$\int \frac{\cot^{-1}(ax)}{c+dx^2}\,dx =$$

$$-\frac{a\left(\frac{8\arctan(ax)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{a} - \frac{4\arctan(ax)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)+4\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\arctan\left(-\frac{a\sqrt{d}x}{a\sqrt{c}-\sqrt{d}}, -\frac{\sqrt{d}}{a\sqrt{c}-\sqrt{d}}\right)+\log(dx^2+c)\log\left(\frac{a^2cd+(a^4}{a^4c}\right.}{}\right.}{}$$

$$+\frac{\operatorname{arccot}(ax)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}}+\frac{\arctan(ax)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}}$$

input
```
integrate(arccot(a*x)/(d*x^2+c),x, algorithm="maxima")
```

output
```
-1/8*a*(8*arctan(a*x)*arctan(d*x/sqrt(c*d))/a - (4*arctan(a*x)*arctan(sqrt
(d)*x/sqrt(c)) + 4*arctan(sqrt(d)*x/sqrt(c))*arctan2(-a*sqrt(d)*x/(a*sqrt(
c) - sqrt(d)), -sqrt(d)/(a*sqrt(c) - sqrt(d))) + log(d*x^2 + c)*log((a^2*c
*d + (a^4*c*d + a^2*d^2)*x^2 + 2*(a^3*d*x^2 + a*d)*sqrt(c)*sqrt(d) + d^2)/
(a^4*c^2 + 6*a^2*c*d + 4*(a^3*c + a*d)*sqrt(c)*sqrt(d) + d^2)) - log(d*x^2
 + c)*log((a^2*c*d + (a^4*c*d + a^2*d^2)*x^2 - 2*(a^3*d*x^2 + a*d)*sqrt(c)
*sqrt(d) + d^2)/(a^4*c^2 + 6*a^2*c*d - 4*(a^3*c + a*d)*sqrt(c)*sqrt(d) + d
^2)) + 2*dilog((a^2*c + I*a*d*x + (I*a^2*x + a)*sqrt(c)*sqrt(d))/(a^2*c +
2*a*sqrt(c)*sqrt(d) + d)) + 2*dilog((a^2*c - I*a*d*x - (I*a^2*x - a)*sqrt(
c)*sqrt(d))/(a^2*c + 2*a*sqrt(c)*sqrt(d) + d)) - 2*dilog((a^2*c + I*a*d*x
- (I*a^2*x + a)*sqrt(c)*sqrt(d))/(a^2*c - 2*a*sqrt(c)*sqrt(d) + d)) - 2*di
log((a^2*c - I*a*d*x + (I*a^2*x - a)*sqrt(c)*sqrt(d))/(a^2*c - 2*a*sqrt(c)
*sqrt(d) + d)))/a)/sqrt(c*d) + arccot(a*x)*arctan(d*x/sqrt(c*d))/sqrt(c*d)
 + arctan(a*x)*arctan(d*x/sqrt(c*d))/sqrt(c*d)
```

3.57.8 Giac [F]

$$\int \frac{\cot^{-1}(ax)}{c+dx^2}\,dx = \int \frac{\operatorname{arccot}(ax)}{dx^2+c}\,dx$$

input
```
integrate(arccot(a*x)/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate(arccot(a*x)/(d*x^2 + c), x)
```

3.57.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)}{c+dx^2}\,dx = \int \frac{\operatorname{acot}(a\,x)}{d\,x^2+c}\,dx$$

input
```
int(acot(a*x)/(c + d*x^2),x)
```

output
```
int(acot(a*x)/(c + d*x^2), x)
```

3.58 $\int \frac{\cot^{-1}(ax)}{(c+dx^2)^2} \, dx$

3.58.1 Optimal result

Integrand size = 14, antiderivative size = 801

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^2}\,dx = \frac{x\cot^{-1}(ax)}{2c(c+dx^2)} + \frac{\cot^{-1}(ax)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} - \frac{ia\log\left(\frac{\sqrt{d}\left(1-\sqrt{-a^2}x\right)}{i\sqrt{-a^2}\sqrt{c}+\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}} + \frac{ia\log\left(-\frac{\sqrt{d}\left(1+\sqrt{-a^2}x\right)}{i\sqrt{-a^2}\sqrt{c}-\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}} + \frac{ia\log\left(-\frac{\sqrt{d}\left(1-\sqrt{-a^2}x\right)}{i\sqrt{-a^2}\sqrt{c}-\sqrt{d}}\right)\log\left(1+\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}} - \frac{ia\log\left(\frac{\sqrt{d}\left(1+\sqrt{-a^2}x\right)}{i\sqrt{-a^2}\sqrt{c}+\sqrt{d}}\right)\log\left(1+\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}} + \frac{a\log(1+a^2x^2)}{4c(a^2c-d)} - \frac{a\log(c+dx^2)}{4c(a^2c-d)} - \frac{ia\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-a^2}\left(\sqrt{c}-i\sqrt{d}x\right)}{\sqrt{-a^2}\sqrt{c}-i\sqrt{d}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}} + \frac{ia\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-a^2}\left(\sqrt{c}-i\sqrt{d}x\right)}{\sqrt{-a^2}\sqrt{c}+i\sqrt{d}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}} - \frac{ia\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-a^2}\left(\sqrt{c}+i\sqrt{d}x\right)}{\sqrt{-a^2}\sqrt{c}-i\sqrt{d}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}} + \frac{ia\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-a^2}\left(\sqrt{c}+i\sqrt{d}x\right)}{\sqrt{-a^2}\sqrt{c}+i\sqrt{d}}\right)}{8\sqrt{-a^2}c^{3/2}\sqrt{d}}$$

output
```
1/2*x*arccot(a*x)/c/(d*x^2+c)+1/4*a*ln(a^2*x^2+1)/c/(a^2*c-d)-1/4*a*ln(d*x
^2+c)/c/(a^2*c-d)+1/2*arccot(a*x)*arctan(x*d^(1/2)/c^(1/2))/c^(3/2)/d^(1/2
)+1/8*I*a*ln(-(1+x*(-a^2)^(1/2))*d^(1/2)/(I*(-a^2)^(1/2)*c^(1/2)-d^(1/2)))
*ln(1-I*x*d^(1/2)/c^(1/2))/c^(3/2)/(-a^2)^(1/2)/d^(1/2)-1/8*I*a*ln((1-x*(-
a^2)^(1/2))*d^(1/2)/(I*(-a^2)^(1/2)*c^(1/2)+d^(1/2)))*ln(1-I*x*d^(1/2)/c^(
1/2))/c^(3/2)/(-a^2)^(1/2)/d^(1/2)+1/8*I*a*ln(-(1-x*(-a^2)^(1/2))*d^(1/2)/
(I*(-a^2)^(1/2)*c^(1/2)-d^(1/2)))*ln(1+I*x*d^(1/2)/c^(1/2))/c^(3/2)/(-a^2)
^(1/2)/d^(1/2)-1/8*I*a*ln((1+x*(-a^2)^(1/2))*d^(1/2)/(I*(-a^2)^(1/2)*c^(1/
2)+d^(1/2)))*ln(1+I*x*d^(1/2)/c^(1/2))/c^(3/2)/(-a^2)^(1/2)/d^(1/2)-1/8*I*
a*polylog(2,(-a^2)^(1/2)*(c^(1/2)-I*x*d^(1/2))/((-a^2)^(1/2)*c^(1/2)-I*d^(
1/2)))/c^(3/2)/(-a^2)^(1/2)/d^(1/2)+1/8*I*a*polylog(2,(-a^2)^(1/2)*(c^(1/2
)-I*x*d^(1/2))/((-a^2)^(1/2)*c^(1/2)+I*d^(1/2)))/c^(3/2)/(-a^2)^(1/2)/d^(1
/2)-1/8*I*a*polylog(2,(-a^2)^(1/2)*(c^(1/2)+I*x*d^(1/2))/((-a^2)^(1/2)*c^(
1/2)-I*d^(1/2)))/c^(3/2)/(-a^2)^(1/2)/d^(1/2)+1/8*I*a*polylog(2,(-a^2)^(1/
2)*(c^(1/2)+I*x*d^(1/2))/((-a^2)^(1/2)*c^(1/2)+I*d^(1/2)))/c^(3/2)/(-a^2)^
(1/2)/d^(1/2)
```

3.58.2 Mathematica [A] (warning: unable to verify)

Time = 5.30 (sec) , antiderivative size = 806, normalized size of antiderivative = 1.01

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^2}\,dx =$$

$$-\frac{a\left(\frac{2\log\left(\frac{a^2c+d+\left(-a^2c+d\right)\cos\left(2\cot^{-1}(ax)\right)}{a^2c+d}\right)}{a^2c-d}+\frac{2\arccos\left(\frac{a^2c+d}{a^2c-d}\right)\text{arctanh}\left(\frac{ac}{\sqrt{-a^2cd}x}\right)+4\cot^{-1}(ax)\text{arctanh}\left(\frac{adx}{\sqrt{-a^2cd}}\right)+\left(\arccos\left(\cdots\right.\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[ArcCot[a*x]/(c + d*x^2)^2,x]
```

output
```
-1/8*(a*((2*Log[(a^2*c + d + (-(a^2*c) + d)*Cos[2*ArcCot[a*x]])/(a^2*c + d
)])/(a^2*c - d) + (2*ArcCos[(a^2*c + d)/(a^2*c - d)]*ArcTanh[(a*c)/(Sqrt[-
(a^2*c*d)]*x)] + 4*ArcCot[a*x]*ArcTanh[(a*d*x)/Sqrt[-(a^2*c*d)]] + (ArcCos
[(a^2*c + d)/(a^2*c - d)] - (2*I)*ArcTanh[(a*c)/(Sqrt[-(a^2*c*d)]*x)])*Log
[((2*I)*d*(I*a^2*c + Sqrt[-(a^2*c*d)])*(I + a*x))/((a^2*c - d)*(Sqrt[-(a^2
*c*d)] - a*d*x))] + (ArcCos[(a^2*c + d)/(a^2*c - d)] + (2*I)*ArcTanh[(a*c)
/(Sqrt[-(a^2*c*d)]*x)])*Log[(2*d*(a^2*c + I*Sqrt[-(a^2*c*d)])*(-I + a*x))/
((a^2*c - d)*(-Sqrt[-(a^2*c*d)] + a*d*x))] - (ArcCos[(a^2*c + d)/(a^2*c -
d)] + (2*I)*ArcTanh[(a*c)/(Sqrt[-(a^2*c*d)]*x)] + (2*I)*ArcTanh[(a*d*x)/Sq
rt[-(a^2*c*d)]])*Log[(Sqrt[2]*Sqrt[-(a^2*c*d)])/(Sqrt[a^2*c - d]*E^(I*ArcC
ot[a*x])*Sqrt[-(a^2*c) - d + (a^2*c - d)*Cos[2*ArcCot[a*x]]])] - (ArcCos[(
a^2*c + d)/(a^2*c - d)] - (2*I)*ArcTanh[(a*c)/(Sqrt[-(a^2*c*d)]*x)] - (2*I
)*ArcTanh[(a*d*x)/Sqrt[-(a^2*c*d)]])*Log[(Sqrt[2]*Sqrt[-(a^2*c*d)]*E^(I*Ar
cCot[a*x]))/(Sqrt[a^2*c - d]*Sqrt[-(a^2*c) - d + (a^2*c - d)*Cos[2*ArcCot[
a*x]]])] + I*(PolyLog[2, ((a^2*c + d - (2*I)*Sqrt[-(a^2*c*d)])*(Sqrt[-(a^2
*c*d)] + a*d*x))/((a^2*c - d)*(Sqrt[-(a^2*c*d)] - a*d*x))] - PolyLog[2, ((
a^2*c + d + (2*I)*Sqrt[-(a^2*c*d)])*(Sqrt[-(a^2*c*d)] + a*d*x))/((a^2*c -
d)*(Sqrt[-(a^2*c*d)] - a*d*x))]))/Sqrt[-(a^2*c*d)] - (4*ArcCot[a*x]*Sin[2*
ArcCot[a*x]])/(a^2*c + d + (-(a^2*c) + d)*Cos[2*ArcCot[a*x]])))/c
```

3.58.3 Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 797, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5448, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^2}dx$$

$$\Big\downarrow \text{5448}$$

$$a\int \frac{\frac{x}{c(dx^2+c)}+\frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}\sqrt{d}}}{2(a^2x^2+1)}dx+\frac{\cot^{-1}(ax)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}}+\frac{x\cot^{-1}(ax)}{2c(c+dx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}a\int \frac{\frac{x}{c(dx^2+c)}+\frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}\sqrt{d}}}{a^2x^2+1}dx+\frac{\cot^{-1}(ax)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}}+\frac{x\cot^{-1}(ax)}{2c(c+dx^2)}$$

$$\Big\downarrow \ 7276$$

$$\frac{1}{2}a\int \left(\frac{x}{c\left(a^2x^2+1\right)\left(dx^2+c\right)}+\frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}\sqrt{d}\left(a^2x^2+1\right)}\right)dx+\frac{\cot^{-1}(ax)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}}+\frac{x\cot^{-1}(ax)}{2c\left(c+dx^2\right)}$$

$$\Big\downarrow \ 2009$$

$$\frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\cot^{-1}(ax)}{2c^{3/2}\sqrt{d}}+\frac{x\cot^{-1}(ax)}{2c\left(dx^2+c\right)}+$$
$$\frac{1}{2}a\left(-\frac{i\log\left(\frac{\sqrt{d}\left(1-\sqrt{-a^2}x\right)}{i\sqrt{-a^2}\sqrt{c}+\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{4\sqrt{-a^2}c^{3/2}\sqrt{d}}+\frac{i\log\left(-\frac{\sqrt{d}\left(\sqrt{-a^2}x+1\right)}{i\sqrt{-a^2}\sqrt{c}-\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{4\sqrt{-a^2}c^{3/2}\sqrt{d}}+\frac{i\log\left(-\frac{\sqrt{d}\left(1-\sqrt{-a^2}x\right)}{i\sqrt{-a^2}\sqrt{c}-\sqrt{d}}\right)}{4\sqrt{-a^2}c^{3/2}}\right.$$

input
```
Int[ArcCot[a*x]/(c + d*x^2)^2,x]
```

output
```
(x*ArcCot[a*x])/(2*c*(c + d*x^2)) + (ArcCot[a*x]*ArcTan[(Sqrt[d]*x)/Sqrt[c
]])/(2*c^(3/2)*Sqrt[d]) + (a*(((-1/4*I)*Log[(Sqrt[d]*(1 - Sqrt[-a^2]*x))/(
I*Sqrt[-a^2]*Sqrt[c] + Sqrt[d])]*Log[1 - (I*Sqrt[d]*x)/Sqrt[c]])/(Sqrt[-a^
2]*c^(3/2)*Sqrt[d]) + ((I/4)*Log[-((Sqrt[d]*(1 + Sqrt[-a^2]*x))/(I*Sqrt[-a
^2]*Sqrt[c] - Sqrt[d]))]*Log[1 - (I*Sqrt[d]*x)/Sqrt[c]])/(Sqrt[-a^2]*c^(3/
2)*Sqrt[d]) + ((I/4)*Log[-((Sqrt[d]*(1 - Sqrt[-a^2]*x))/(I*Sqrt[-a^2]*Sqrt
[c] - Sqrt[d]))]*Log[1 + (I*Sqrt[d]*x)/Sqrt[c]])/(Sqrt[-a^2]*c^(3/2)*Sqrt[
d]) - ((I/4)*Log[(Sqrt[d]*(1 + Sqrt[-a^2]*x))/(I*Sqrt[-a^2]*Sqrt[c] + Sqrt
[d])]*Log[1 + (I*Sqrt[d]*x)/Sqrt[c]])/(Sqrt[-a^2]*c^(3/2)*Sqrt[d]) + Log[1
 + a^2*x^2]/(2*c*(a^2*c - d)) - Log[c + d*x^2]/(2*c*(a^2*c - d)) - ((I/4)*
PolyLog[2, (Sqrt[-a^2]*(Sqrt[c] - I*Sqrt[d]*x))/(Sqrt[-a^2]*Sqrt[c] - I*Sq
rt[d])]])/(Sqrt[-a^2]*c^(3/2)*Sqrt[d]) + ((I/4)*PolyLog[2, (Sqrt[-a^2]*(Sqr
t[c] - I*Sqrt[d]*x))/(Sqrt[-a^2]*Sqrt[c] + I*Sqrt[d])]])/(Sqrt[-a^2]*c^(3/2
)*Sqrt[d]) - ((I/4)*PolyLog[2, (Sqrt[-a^2]*(Sqrt[c] + I*Sqrt[d]*x))/(Sqrt[
-a^2]*Sqrt[c] - I*Sqrt[d])]])/(Sqrt[-a^2]*c^(3/2)*Sqrt[d]) + ((I/4)*PolyLog
[2, (Sqrt[-a^2]*(Sqrt[c] + I*Sqrt[d]*x))/(Sqrt[-a^2]*Sqrt[c] + I*Sqrt[d])]
)/(Sqrt[-a^2]*c^(3/2)*Sqrt[d])))/2
```

3.58. $\int \frac{\cot^{-1}(ax)}{\left(c+dx^2\right)^2}\,dx$

3.58.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5448
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcCot[c*x])   u,
x] + Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2), x], x], x]] /; FreeQ
[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.58.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2140 vs. 2(593) = 1186.

Time = 1.59 (sec) , antiderivative size = 2141, normalized size of antiderivative = 2.67

method	result	size
risch	Expression too large to display	2141
derivativedivides	Expression too large to display	2275
default	Expression too large to display	2275

input
```
int(arccot(a*x)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*I*a^4*ln(1-I*a*x)/(a^2*c-d)/(-a^2*d*x^2-a^2*c)*x-1/4*I*a^4*ln(1+I*a*x)
/(a^2*c-d)/(-a^2*d*x^2-a^2*c)*x-1/8*a^4*ln(1-I*a*x)*c/(a^2*c-d)/(-a^2*d*x^
2-a^2*c)/(c*d)^(1/2)*ln((a*(c*d)^(1/2)+(1-I*a*x)*d-d)/(a*(c*d)^(1/2)-d))+1
/8*a^4*ln(1-I*a*x)*c/(a^2*c-d)/(-a^2*d*x^2-a^2*c)/(c*d)^(1/2)*ln((a*(c*d)^
(1/2)-(1-I*a*x)*d+d)/(a*(c*d)^(1/2)+d))+1/8*a^2*ln(1-I*a*x)/(a^2*c-d)/(-a^
2*d*x^2-a^2*c)/(c*d)^(1/2)*ln((a*(c*d)^(1/2)+(1-I*a*x)*d-d)/(a*(c*d)^(1/2)
-d))*d-1/4*a^3*ln(1-I*a*x)/c/(a^2*c-d)/(-a^2*d*x^2-a^2*c)*d*x^2-1/8*a^2*ln
(1-I*a*x)/(a^2*c-d)/(-a^2*d*x^2-a^2*c)/(c*d)^(1/2)*ln((a*(c*d)^(1/2)-(1-I*
a*x)*d+d)/(a*(c*d)^(1/2)+d))*d-1/4*a^3*ln(1+I*a*x)/c/(a^2*c-d)/(-a^2*d*x^2
-a^2*c)*d*x^2+1/8*a^4*ln(1+I*a*x)*c/(a^2*c-d)/(-a^2*d*x^2-a^2*c)/(c*d)^(1/
2)*ln((a*(c*d)^(1/2)-(1+I*a*x)*d+d)/(a*(c*d)^(1/2)+d))-1/8*a^4*ln(1+I*a*x)
*c/(a^2*c-d)/(-a^2*d*x^2-a^2*c)/(c*d)^(1/2)*ln((a*(c*d)^(1/2)+(1+I*a*x)*d-
d)/(a*(c*d)^(1/2)-d))-1/8*a^2*ln(1+I*a*x)/(a^2*c-d)/(-a^2*d*x^2-a^2*c)/(c*
d)^(1/2)*ln((a*(c*d)^(1/2)-(1+I*a*x)*d+d)/(a*(c*d)^(1/2)+d))*d+1/8*a^2*ln(
1+I*a*x)/(a^2*c-d)/(-a^2*d*x^2-a^2*c)/(c*d)^(1/2)*ln((a*(c*d)^(1/2)+(1+I*a
*x)*d-d)/(a*(c*d)^(1/2)-d))*d+1/8*a^4*ln(1-I*a*x)/(a^2*c-d)/(-a^2*d*x^2-a^
2*c)/(c*d)^(1/2)*ln((a*(c*d)^(1/2)-(1-I*a*x)*d+d)/(a*(c*d)^(1/2)+d))*d*x^2
-1/8/c/(c*d)^(1/2)*dilog((a*(c*d)^(1/2)-(1-I*a*x)*d+d)/(a*(c*d)^(1/2)+d))+
1/8/c/(c*d)^(1/2)*dilog((a*(c*d)^(1/2)+(1-I*a*x)*d-d)/(a*(c*d)^(1/2)-d))-1
/8/c/(c*d)^(1/2)*dilog((a*(c*d)^(1/2)-(1+I*a*x)*d+d)/(a*(c*d)^(1/2)+d))...
```

3.58.5 Fricas [F]

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^2}\,dx = \int \frac{\operatorname{arccot}(ax)}{(dx^2+c)^2}\,dx$$

input
```
integrate(arccot(a*x)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(arccot(a*x)/(d^2*x^4 + 2*c*d*x^2 + c^2), x)
```

3.58.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(acot(a*x)/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.58.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 628, normalized size of antiderivative = 0.78

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^2}\,dx = \frac{1}{2}\left(\frac{x}{cdx^2+c^2} + \frac{\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}c}\right)\operatorname{arccot}(ax)$$
$$+ \frac{\left(4\,acd\log\left(a^2x^2+1\right) - 4\,acd\log\left(dx^2+c\right) + \left(4\left(a^2c-d\right)\arctan\left(ax\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right) + 4\left(a^2c-d\right)\operatorname{arctar}\right.\right.}{}$$

input
```
integrate(arccot(a*x)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/2*(x/(c*d*x^2 + c^2) + arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c))*arccot(a*x)
+ 1/16*(4*a*c*d*log(a^2*x^2 + 1) - 4*a*c*d*log(d*x^2 + c) + (4*(a^2*c - d)
*arctan(a*x)*arctan(sqrt(d)*x/sqrt(c)) + 4*(a^2*c - d)*arctan(sqrt(d)*x/sq
rt(c))*arctan2(-a*sqrt(d)*x/(a*sqrt(c) - sqrt(d)), -sqrt(d)/(a*sqrt(c) - s
qrt(d))) + (a^2*c - d)*log(d*x^2 + c)*log((a^2*c*d + (a^4*c*d + a^2*d^2)*x
^2 + 2*(a^3*d*x^2 + a*d)*sqrt(c)*sqrt(d) + d^2)/(a^4*c^2 + 6*a^2*c*d + 4*(
a^3*c + a*d)*sqrt(c)*sqrt(d) + d^2)) - (a^2*c - d)*log(d*x^2 + c)*log((a^2
*c*d + (a^4*c*d + a^2*d^2)*x^2 - 2*(a^3*d*x^2 + a*d)*sqrt(c)*sqrt(d) + d^2
)/(a^4*c^2 + 6*a^2*c*d - 4*(a^3*c + a*d)*sqrt(c)*sqrt(d) + d^2)) + 2*(a^2*
c - d)*dilog((a^2*c + I*a*d*x + (I*a^2*x + a)*sqrt(c)*sqrt(d))/(a^2*c + 2*
a*sqrt(c)*sqrt(d) + d)) + 2*(a^2*c - d)*dilog((a^2*c - I*a*d*x - (I*a^2*x
- a)*sqrt(c)*sqrt(d))/(a^2*c + 2*a*sqrt(c)*sqrt(d) + d)) - 2*(a^2*c - d)*d
ilog((a^2*c + I*a*d*x - (I*a^2*x + a)*sqrt(c)*sqrt(d))/(a^2*c - 2*a*sqrt(c
)*sqrt(d) + d)) - 2*(a^2*c - d)*dilog((a^2*c - I*a*d*x + (I*a^2*x - a)*sqr
t(c)*sqrt(d))/(a^2*c - 2*a*sqrt(c)*sqrt(d) + d)))*sqrt(c)*sqrt(d))*a/(a^3*
c^3*d - a*c^2*d^2)
```

3.58.8 Giac [F]

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^2} \, dx = \int \frac{\operatorname{arccot}(ax)}{(dx^2+c)^2} \, dx$$

input
```
integrate(arccot(a*x)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(arccot(a*x)/(d*x^2 + c)^2, x)
```

3.58.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^2} \, dx = \int \frac{\operatorname{acot}(a\,x)}{(d\,x^2+c)^2} \, dx$$

input
```
int(acot(a*x)/(c + d*x^2)^2,x)
```

output
```
int(acot(a*x)/(c + d*x^2)^2, x)
```

3.59 $\int \sqrt{c+dx^2} \cot^{-1}(ax) \, dx$

3.59.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \sqrt{c+dx^2} \cot^{-1}(ax) \, dx = \text{Int}\left(\sqrt{c+dx^2} \cot^{-1}(ax), x\right)$$

output
```
Unintegrable((d*x^2+c)^(1/2)*arccot(a*x),x)
```

3.59.2 Mathematica [N/A]

Not integrable

Time = 4.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \sqrt{c+dx^2} \cot^{-1}(ax) \, dx = \int \sqrt{c+dx^2} \cot^{-1}(ax) \, dx$$

input
```
Integrate[Sqrt[c + d*x^2]*ArcCot[a*x],x]
```

output
```
Integrate[Sqrt[c + d*x^2]*ArcCot[a*x], x]
```

3.59.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5561}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}(ax)\sqrt{c+dx^2}\,dx$$

$$\Big\downarrow \text{ 5561}$$

$$\int \cot^{-1}(ax)\sqrt{c+dx^2}dx$$

input
```
Int[Sqrt[c + d*x^2]*ArcCot[a*x],x]
```

output
```
$Aborted
```

3.59.3.1 Defintions of rubi rules used

rule 5561
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCot[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.59.4 Maple [N/A] (verified)

Not integrable

Time = 0.76 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \sqrt{d\,x^2+c}\,\operatorname{arccot}(ax)\,dx$$

input
```
int((d*x^2+c)^(1/2)*arccot(a*x),x)
```

output
```
int((d*x^2+c)^(1/2)*arccot(a*x),x)
```

3.59.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{c+dx^2}\cot^{-1}(ax)\,dx=\int \sqrt{dx^2+c}\,\mathrm{arccot}\left(ax\right)\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*arccot(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^2 + c)*arccot(a*x), x)
```

3.59.6 Sympy [N/A]

Not integrable

Time = 4.39 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \sqrt{c+dx^2}\cot^{-1}(ax)\,dx=\int \sqrt{c+dx^2}\,\mathrm{acot}\left(ax\right)dx$$

input
```
integrate((d*x**2+c)**(1/2)*acot(a*x),x)
```

output
```
Integral(sqrt(c + d*x**2)*acot(a*x), x)
```

3.59.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{c+dx^2}\cot^{-1}(ax)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((d*x^2+c)^(1/2)*arccot(a*x),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(d-a^2*c>0)', see `assume?` for m
ore detail
```

3.59.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{c+dx^2}\cot^{-1}(ax)\,dx=\int \sqrt{dx^2+c}\,\mathrm{arccot}\left(ax\right)\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*arccot(a*x),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^2 + c)*arccot(a*x), x)
```

3.59.9 Mupad [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{c+dx^2}\cot^{-1}(ax)\,dx=\int \mathrm{acot}(a\,x)\,\sqrt{d\,x^2+c}\,dx$$

input
```
int(acot(a*x)*(c + d*x^2)^(1/2),x)
```

output
```
int(acot(a*x)*(c + d*x^2)^(1/2), x)
```

3.60 $\int \frac{\cot^{-1}(ax)}{\sqrt{c+dx^2}}\,dx$

3.60.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\cot^{-1}(ax)}{\sqrt{c+dx^2}}\,dx = \text{Int}\left(\frac{\cot^{-1}(ax)}{\sqrt{c+dx^2}},x\right)$$

output
```
Unintegrable(arccot(a*x)/(d*x^2+c)^(1/2),x)
```

3.60.2 Mathematica [N/A]

Not integrable

Time = 2.88 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cot^{-1}(ax)}{\sqrt{c+dx^2}}\,dx = \int \frac{\cot^{-1}(ax)}{\sqrt{c+dx^2}}\,dx$$

input
```
Integrate[ArcCot[a*x]/Sqrt[c + d*x^2],x]
```

output
```
Integrate[ArcCot[a*x]/Sqrt[c + d*x^2], x]
```

3.60.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5561}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax)}{\sqrt{c+dx^2}} \, dx$$

$$\Big\downarrow \text{ 5561}$$

$$\int \frac{\cot^{-1}(ax)}{\sqrt{c+dx^2}}dx$$

input
```
Int[ArcCot[a*x]/Sqrt[c + d*x^2],x]
```

output
```
$Aborted
```

3.60.3.1 Defintions of rubi rules used

rule 5561
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCot[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.60.4 Maple [N/A] (verified)

Not integrable

Time = 0.71 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\operatorname{arccot}\left(a x \right)}{\sqrt{d \,x^{2}+c}}d x$$

input
```
int(arccot(a*x)/(d*x^2+c)^(1/2),x)
```

output
```
int(arccot(a*x)/(d*x^2+c)^(1/2),x)
```

3.60.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(ax)}{\sqrt{c+dx^2}}\,dx = \int \frac{\operatorname{arccot}(ax)}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate(arccot(a*x)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(arccot(a*x)/sqrt(d*x^2 + c), x)
```

3.60.6 Sympy [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\cot^{-1}(ax)}{\sqrt{c+dx^2}}\,dx = \int \frac{\operatorname{acot}(ax)}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate(acot(a*x)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(acot(a*x)/sqrt(c + d*x**2), x)
```

3.60.7 Maxima [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(ax)}{\sqrt{c+dx^2}}\,dx = \int \frac{\operatorname{arccot}(ax)}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate(arccot(a*x)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arccot(a*x)/sqrt(d*x^2 + c), x)
```

3.60.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(ax)}{\sqrt{c+dx^2}}\,dx = \int \frac{\operatorname{arccot}(ax)}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate(arccot(a*x)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(arccot(a*x)/sqrt(d*x^2 + c), x)
```

3.60.9 Mupad [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(ax)}{\sqrt{c+dx^2}}\,dx = \int \frac{\operatorname{acot}(a\,x)}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int(acot(a*x)/(c + d*x^2)^(1/2),x)
```

output
```
int(acot(a*x)/(c + d*x^2)^(1/2), x)
```

3.61 $\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{3/2}}\,dx$

3.61.1 Optimal result

Integrand size = 16, antiderivative size = 66

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{3/2}}\,dx = \frac{x\cot^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{\operatorname{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{c\sqrt{a^2c-d}}$$

output
```
-arctanh(a*(d*x^2+c)^(1/2)/(a^2*c-d)^(1/2))/c/(a^2*c-d)^(1/2)+x*arccot(a*x
)/c/(d*x^2+c)^(1/2)
```

3.61.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.56

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{3/2}}\,dx = \frac{\frac{2x\cot^{-1}(ax)}{\sqrt{c+dx^2}} + \frac{-\log\left(\frac{4ac\left(ac-idx+\sqrt{a^2c-d}\sqrt{c+dx^2}\right)}{\sqrt{a^2c-d}(i+ax)}\right)-\log\left(\frac{4ac\left(ac+idx+\sqrt{a^2c-d}\sqrt{c+dx^2}\right)}{\sqrt{a^2c-d}(-i+ax)}\right)}{\sqrt{a^2c-d}}}{2c}$$

input
```
Integrate[ArcCot[a*x]/(c + d*x^2)^(3/2),x]
```

output
```
((2*x*ArcCot[a*x])/Sqrt[c + d*x^2] + (-Log[(4*a*c*(a*c - I*d*x + Sqrt[a^2*
c - d]*Sqrt[c + d*x^2]))/(Sqrt[a^2*c - d]*(I + a*x))] - Log[(4*a*c*(a*c +
I*d*x + Sqrt[a^2*c - d]*Sqrt[c + d*x^2]))/(Sqrt[a^2*c - d]*(-I + a*x))])/S
qrt[a^2*c - d])/(2*c)
```

3.61.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5448, 27, 353, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{3/2}}dx$$

$$\Big\downarrow \text{5448}$$

$$a\int \frac{x}{c\left(a^2x^2+1\right)\sqrt{dx^2+c}}dx+\frac{x\cot^{-1}(ax)}{c\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{x}{(a^2x^2+1)\sqrt{dx^2+c}}dx}{c}+\frac{x\cot^{-1}(ax)}{c\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{353}$$

$$\frac{a\int \frac{1}{(a^2x^2+1)\sqrt{dx^2+c}}dx^2}{2c}+\frac{x\cot^{-1}(ax)}{c\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{a\int \frac{1}{\frac{a^2x^4}{d}-\frac{a^2c}{d}+1}d\sqrt{dx^2+c}}{cd}+\frac{x\cot^{-1}(ax)}{c\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{x\cot^{-1}(ax)}{c\sqrt{c+dx^2}}-\frac{\text{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{c\sqrt{a^2c-d}}$$

input
```
Int[ArcCot[a*x]/(c + d*x^2)^(3/2),x]
```

output
```
(x*ArcCot[a*x])/(c*Sqrt[c + d*x^2]) - ArcTanh[(a*Sqrt[c + d*x^2])/Sqrt[a^2
*c - d]]/(c*Sqrt[a^2*c - d])
```

3.61.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 5448
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcCot[c*x])   u,
x] + Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2), x], x], x]] /; FreeQ
[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

3.61.4 Maple [F]

$$\int \frac{\operatorname{arccot}(ax)}{(d\,x^{2}+c)^{\frac{3}{2}}}dx$$

input
```
int(arccot(a*x)/(d*x^2+c)^(3/2),x)
```

output
```
int(arccot(a*x)/(d*x^2+c)^(3/2),x)
```

3.61.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(58) = 116$.

Time = 0.30 (sec) , antiderivative size = 349, normalized size of antiderivative = 5.29

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{3/2}}\,dx = \left[\frac{4\,(a^2c-d)\sqrt{dx^2+c}x\,\mathrm{arccot}\,(ax)+\sqrt{a^2c-d}(dx^2+c)\log\left(\frac{a^4d^2x^4+8\,a^4c^2-8\,a^2cd+2\,(4\,a^4cd}{}\right.}{4\,(a^2c^3-c^2d+(a^2c^2d-cd^2)x^2)}\right.$$

input
```
integrate(arccot(a*x)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(4*(a^2*c - d)*sqrt(d*x^2 + c)*x*arccot(a*x) + sqrt(a^2*c - d)*(d*x^2
 + c)*log((a^4*d^2*x^4 + 8*a^4*c^2 - 8*a^2*c*d + 2*(4*a^4*c*d - 3*a^2*d^2)
*x^2 - 4*(a^3*d*x^2 + 2*a^3*c - a*d)*sqrt(a^2*c - d)*sqrt(d*x^2 + c) + d^2
)/(a^4*x^4 + 2*a^2*x^2 + 1)))/(a^2*c^3 - c^2*d + (a^2*c^2*d - c*d^2)*x^2),
 1/2*(2*(a^2*c - d)*sqrt(d*x^2 + c)*x*arccot(a*x) - sqrt(-a^2*c + d)*(d*x^
2 + c)*arctan(-1/2*(a^2*d*x^2 + 2*a^2*c - d)*sqrt(-a^2*c + d)*sqrt(d*x^2 +
 c)/(a^3*c^2 - a*c*d + (a^3*c*d - a*d^2)*x^2)))/(a^2*c^3 - c^2*d + (a^2*c^
2*d - c*d^2)*x^2)]
```

3.61.6 Sympy [F]

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{3/2}}\,dx = \int \frac{\operatorname{acot}\,(ax)}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(acot(a*x)/(d*x**2+c)**(3/2),x)
```

output
```
Integral(acot(a*x)/(c + d*x**2)**(3/2), x)
```

3.61.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(arccot(a*x)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(d-a^2*c>0)', see `assume?` for m
ore detail
```

3.61.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.89

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{3/2}}\,dx = \frac{x\arctan\left(\frac{1}{ax}\right)}{\sqrt{dx^2+c}c} + \frac{\arctan\left(\frac{\sqrt{dx^2+c}a}{\sqrt{-a^2c+d}}\right)}{\sqrt{-a^2c+d}c}$$

input
```
integrate(arccot(a*x)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
x*arctan(1/(a*x))/(sqrt(d*x^2 + c)*c) + arctan(sqrt(d*x^2 + c)*a/sqrt(-a^2
*c + d))/(sqrt(-a^2*c + d)*c)
```

3.61.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{3/2}}\,dx = \int \frac{\mathrm{acot}(a\,x)}{(d\,x^2+c)^{3/2}}\,dx$$

input
```
int(acot(a*x)/(c + d*x^2)^(3/2),x)
```

output
```
int(acot(a*x)/(c + d*x^2)^(3/2), x)
```

3.62 $\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{5/2}}\,dx$

3.62.1 Optimal result

Integrand size = 16, antiderivative size = 134

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{5/2}}\,dx = \frac{a}{3c(a^2c-d)\sqrt{c+dx^2}} + \frac{x\cot^{-1}(ax)}{3c(c+dx^2)^{3/2}} + \frac{2x\cot^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{(3a^2c-2d)\operatorname{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{3c^2(a^2c-d)^{3/2}}$$

output
```
1/3*x*arccot(a*x)/c/(d*x^2+c)^(3/2)-1/3*(3*a^2*c-2*d)*arctanh(a*(d*x^2+c)^
(1/2)/(a^2*c-d)^(1/2))/c^2/(a^2*c-d)^(3/2)+1/3*a/c/(a^2*c-d)/(d*x^2+c)^(1/
2)+2/3*x*arccot(a*x)/c^2/(d*x^2+c)^(1/2)
```

3.62.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.96

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{5/2}}\,dx =$$

$$-\frac{-\frac{2ac}{(a^2c-d)\sqrt{c+dx^2}} - \frac{2x(3c+2dx^2)\cot^{-1}(ax)}{(c+dx^2)^{3/2}} + \frac{(3a^2c-2d)\log\left(\frac{12ac^2\sqrt{a^2c-d}\left(ac-idx+\sqrt{a^2c-d}\sqrt{c+dx^2}\right)}{(3a^2c-2d)(i+ax)}\right)}{(a^2c-d)^{3/2}} + \cdots}{6c^2}$$

input
```
Integrate[ArcCot[a*x]/(c + d*x^2)^(5/2),x]
```

output
```
-1/6*((-2*a*c)/((a^2*c - d)*Sqrt[c + d*x^2]) - (2*x*(3*c + 2*d*x^2)*ArcCot
[a*x])/(c + d*x^2)^(3/2) + ((3*a^2*c - 2*d)*Log[(12*a*c^2*Sqrt[a^2*c - d]*
(a*c - I*d*x + Sqrt[a^2*c - d]*Sqrt[c + d*x^2]))/((3*a^2*c - 2*d)*(I + a*x
))]])/(a^2*c - d)^(3/2) + ((3*a^2*c - 2*d)*Log[(12*a*c^2*Sqrt[a^2*c - d]*(a
*c + I*d*x + Sqrt[a^2*c - d]*Sqrt[c + d*x^2]))/((3*a^2*c - 2*d)*(-I + a*x)
)]])/(a^2*c - d)^(3/2))/c^2
```

3.62.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5448, 27, 435, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{5/2}}dx$$

$$\Big\downarrow \text{5448}$$

$$a\int \frac{x(2dx^2+3c)}{3c^2(a^2x^2+1)(dx^2+c)^{3/2}}dx+\frac{2x\cot^{-1}(ax)}{3c^2\sqrt{c+dx^2}}+\frac{x\cot^{-1}(ax)}{3c(c+dx^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{x(2dx^2+3c)}{(a^2x^2+1)(dx^2+c)^{3/2}}dx}{3c^2}+\frac{2x\cot^{-1}(ax)}{3c^2\sqrt{c+dx^2}}+\frac{x\cot^{-1}(ax)}{3c(c+dx^2)^{3/2}}$$

$$\Big\downarrow \text{435}$$

$$\frac{a\int \frac{2dx^2+3c}{(a^2x^2+1)(dx^2+c)^{3/2}}dx^2}{6c^2}+\frac{2x\cot^{-1}(ax)}{3c^2\sqrt{c+dx^2}}+\frac{x\cot^{-1}(ax)}{3c(c+dx^2)^{3/2}}$$

$$\Big\downarrow \text{87}$$

$$\frac{a\left(\frac{(3a^2c-2d)\int \frac{1}{(a^2x^2+1)\sqrt{dx^2+c}}dx^2}{a^2c-d}+\frac{2c}{(a^2c-d)\sqrt{c+dx^2}}\right)}{6c^2}+\frac{2x\cot^{-1}(ax)}{3c^2\sqrt{c+dx^2}}+\frac{x\cot^{-1}(ax)}{3c(c+dx^2)^{3/2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{a\left(\frac{2(3a^2c-2d)\int \frac{1}{\frac{a^2x^4}{d}-\frac{a^2c}{d}+1}d\sqrt{dx^2+c}}{d(a^2c-d)}+\frac{2c}{(a^2c-d)\sqrt{c+dx^2}}\right)}{6c^2}+\frac{2x\cot^{-1}(ax)}{3c^2\sqrt{c+dx^2}}+\frac{x\cot^{-1}(ax)}{3c\,(c+dx^2)^{3/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{a\left(\frac{2c}{(a^2c-d)\sqrt{c+dx^2}}-\frac{2(3a^2c-2d)\text{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{a(a^2c-d)^{3/2}}\right)}{6c^2}+\frac{2x\cot^{-1}(ax)}{3c^2\sqrt{c+dx^2}}+\frac{x\cot^{-1}(ax)}{3c\,(c+dx^2)^{3/2}}$$

input
```
Int[ArcCot[a*x]/(c + d*x^2)^(5/2),x]
```

output
```
(x*ArcCot[a*x])/(3*c*(c + d*x^2)^(3/2)) + (2*x*ArcCot[a*x])/(3*c^2*Sqrt[c
+ d*x^2]) + (a*((2*c)/((a^2*c - d)*Sqrt[c + d*x^2]) - (2*(3*a^2*c - 2*d)*A
rcTanh[(a*Sqrt[c + d*x^2])/Sqrt[a^2*c - d]])/(a*(a^2*c - d)^(3/2))))/(6*c^
2)
```

3.62.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 5448
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcCot[c*x])   u,
x] + Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2), x], x], x]] /; FreeQ
[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

3.62.4 Maple [F]

$$\int \frac{\operatorname{arccot}(ax)}{(d\,x^2+c)^{\frac{5}{2}}}dx$$

input
```
int(arccot(a*x)/(d*x^2+c)^(5/2),x)
```

output
```
int(arccot(a*x)/(d*x^2+c)^(5/2),x)
```

3.62.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. 2(114) = 228.

Time = 0.31 (sec) , antiderivative size = 712, normalized size of antiderivative = 5.31

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{5/2}}\,dx = \left[\frac{(3\,a^2c^3+(3\,a^2cd^2-2\,d^3)x^4-2\,c^2d+2\,(3\,a^2c^2d-2\,cd^2)x^2)\sqrt{a^2c-d}\log\left(\frac{a^4d^2x^4+8\,a^4c^2\cdots}{\cdots}\right.}{\cdots}\right.$$
$$-\frac{(3\,a^2c^3+(3\,a^2cd^2-2\,d^3)x^4-2\,c^2d+2\,(3\,a^2c^2d-2\,cd^2)x^2)\sqrt{-a^2c+d}\arctan\left(-\frac{(a^2dx^2+2\,a^2c-d)\sqrt{-a^2c+d}\sqrt{dx\cdots}}{2\,(a^3c^2-acd+(a^3cd-ad^2)x^2)\cdots}\right.}{6\,(a^4c^6-2\,a^2c^5d+c^4d^2+(a^4c^4d^2-\cdots}$$

input
```
integrate(arccot(a*x)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*((3*a^2*c^3 + (3*a^2*c*d^2 - 2*d^3)*x^4 - 2*c^2*d + 2*(3*a^2*c^2*d -
 2*c*d^2)*x^2)*sqrt(a^2*c - d)*log((a^4*d^2*x^4 + 8*a^4*c^2 - 8*a^2*c*d +
2*(4*a^4*c*d - 3*a^2*d^2)*x^2 - 4*(a^3*d*x^2 + 2*a^3*c - a*d)*sqrt(a^2*c -
 d)*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 + 2*a^2*x^2 + 1)) + 4*(a^3*c^3 - a*c^2
*d + (a^3*c^2*d - a*c*d^2)*x^2 + (2*(a^4*c^2*d - 2*a^2*c*d^2 + d^3)*x^3 +
3*(a^4*c^3 - 2*a^2*c^2*d + c*d^2)*x)*arccot(a*x))*sqrt(d*x^2 + c))/(a^4*c^
6 - 2*a^2*c^5*d + c^4*d^2 + (a^4*c^4*d^2 - 2*a^2*c^3*d^3 + c^2*d^4)*x^4 +
2*(a^4*c^5*d - 2*a^2*c^4*d^2 + c^3*d^3)*x^2), -1/6*((3*a^2*c^3 + (3*a^2*c*
d^2 - 2*d^3)*x^4 - 2*c^2*d + 2*(3*a^2*c^2*d - 2*c*d^2)*x^2)*sqrt(-a^2*c +
d)*arctan(-1/2*(a^2*d*x^2 + 2*a^2*c - d)*sqrt(-a^2*c + d)*sqrt(d*x^2 + c)/
(a^3*c^2 - a*c*d + (a^3*c*d - a*d^2)*x^2)) - 2*(a^3*c^3 - a*c^2*d + (a^3*c
^2*d - a*c*d^2)*x^2 + (2*(a^4*c^2*d - 2*a^2*c*d^2 + d^3)*x^3 + 3*(a^4*c^3
- 2*a^2*c^2*d + c*d^2)*x)*arccot(a*x))*sqrt(d*x^2 + c))/(a^4*c^6 - 2*a^2*c
^5*d + c^4*d^2 + (a^4*c^4*d^2 - 2*a^2*c^3*d^3 + c^2*d^4)*x^4 + 2*(a^4*c^5*
d - 2*a^2*c^4*d^2 + c^3*d^3)*x^2)]
```

3.62.6 Sympy [F]

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{5/2}}\,dx = \int \frac{\operatorname{acot}(ax)}{(c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(acot(a*x)/(d*x**2+c)**(5/2),x)
```

output
```
Integral(acot(a*x)/(c + d*x**2)**(5/2), x)
```

3.62.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(arccot(a*x)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(d-a^2*c>0)', see `assume?` for m
ore detail
```

3.62.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.94

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{5/2}}\,dx = \frac{1}{3}\,a\left(\frac{(3\,a^2c-2\,d)\arctan\left(\frac{\sqrt{dx^2+c}a}{\sqrt{-a^2c+d}}\right)}{(a^2c^3-c^2d)\sqrt{-a^2c+da}}+\frac{1}{(a^2c^2-cd)\sqrt{dx^2+c}}\right) + \frac{x\left(\frac{2\,dx^2}{c^2}+\frac{3}{c}\right)\arctan\left(\frac{1}{ax}\right)}{3\,(dx^2+c)^{\frac{3}{2}}}$$

input
```
integrate(arccot(a*x)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
1/3*a*((3*a^2*c - 2*d)*arctan(sqrt(d*x^2 + c)*a/sqrt(-a^2*c + d))/((a^2*c^
3 - c^2*d)*sqrt(-a^2*c + d)*a) + 1/((a^2*c^2 - c*d)*sqrt(d*x^2 + c))) + 1/
3*x*(2*d*x^2/c^2 + 3/c)*arctan(1/(a*x))/(d*x^2 + c)^(3/2)
```

3.62.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{5/2}}\,dx = \int \frac{\mathrm{acot}(a\,x)}{(d\,x^2+c)^{5/2}}\,dx$$

input
```
int(acot(a*x)/(c + d*x^2)^(5/2),x)
```

output
```
int(acot(a*x)/(c + d*x^2)^(5/2), x)
```

3.63 $\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx$

3.63.1 Optimal result

Integrand size = 16, antiderivative size = 208

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx = \frac{a}{15c\,(a^2c-d)\,(c+dx^2)^{3/2}} + \frac{a(7a^2c-4d)}{15c^2\,(a^2c-d)^2\sqrt{c+dx^2}} + \frac{x\cot^{-1}(ax)}{5c\,(c+dx^2)^{5/2}} + \frac{4x\cot^{-1}(ax)}{15c^2\,(c+dx^2)^{3/2}} + \frac{8x\cot^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{(15a^4c^2-20a^2cd+8d^2)\,\text{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{15c^3\,(a^2c-d)^{5/2}}$$

output
```
1/15*a/c/(a^2*c-d)/(d*x^2+c)^(3/2)+1/5*x*arccot(a*x)/c/(d*x^2+c)^(5/2)+4/1
5*x*arccot(a*x)/c^2/(d*x^2+c)^(3/2)-1/15*(15*a^4*c^2-20*a^2*c*d+8*d^2)*arc
tanh(a*(d*x^2+c)^(1/2)/(a^2*c-d)^(1/2))/c^3/(a^2*c-d)^(5/2)+1/15*a*(7*a^2*
c-4*d)/c^2/(a^2*c-d)^2/(d*x^2+c)^(1/2)+8/15*x*arccot(a*x)/c^3/(d*x^2+c)^(1
/2)
```

3.63.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.66

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx =$$

$$-\frac{-\frac{2ac(-d(5c+4dx^2)+a^2c(8c+7dx^2))}{(-a^2c+d)^2(c+dx^2)^{3/2}}-\frac{2x(15c^2+20cdx^2+8d^2x^4)\cot^{-1}(ax)}{(c+dx^2)^{5/2}}+\frac{(15a^4c^2-20a^2cd+8d^2)\log\left(\frac{60ac^3(a^2c-d)^{3/2}(ac-idx+\sqrt{a^2c}}{(15a^4c^2-20a^2cd+8d^2)(i-}\right.}{(a^2c-d)^{5/2}}}{30c^3}$$

input
```
Integrate[ArcCot[a*x]/(c + d*x^2)^(7/2),x]
```

output
```
-1/30*((-2*a*c*(-(d*(5*c + 4*d*x^2)) + a^2*c*(8*c + 7*d*x^2)))/((-(a^2*c)
+ d)^2*(c + d*x^2)^(3/2)) - (2*x*(15*c^2 + 20*c*d*x^2 + 8*d^2*x^4)*ArcCot[
a*x])/(c + d*x^2)^(5/2) + ((15*a^4*c^2 - 20*a^2*c*d + 8*d^2)*Log[(60*a*c^3
*(a^2*c - d)^(3/2)*(a*c - I*d*x + Sqrt[a^2*c - d]*Sqrt[c + d*x^2]))/((15*a
^4*c^2 - 20*a^2*c*d + 8*d^2)*(I + a*x))])/(a^2*c - d)^(5/2) + ((15*a^4*c^2
 - 20*a^2*c*d + 8*d^2)*Log[(60*a*c^3*(a^2*c - d)^(3/2)*(a*c + I*d*x + Sqrt
[a^2*c - d]*Sqrt[c + d*x^2]))/((15*a^4*c^2 - 20*a^2*c*d + 8*d^2)*(-I + a*x
))])/(a^2*c - d)^(5/2))/c^3
```

3.63.3 Rubi [A] (warning: unable to verify)

Time = 1.06 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5448, 27, 7266, 1192, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx$$

$$\Big\downarrow \; 5448$$

$$a\int \frac{x(8d^2x^4+20cdx^2+15c^2)}{15c^3(a^2x^2+1)(dx^2+c)^{5/2}}dx+\frac{8x\cot^{-1}(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4x\cot^{-1}(ax)}{15c^2(c+dx^2)^{3/2}}+\frac{x\cot^{-1}(ax)}{5c(c+dx^2)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{a\int \frac{x(8d^2x^4+20cdx^2+15c^2)}{(a^2x^2+1)(dx^2+c)^{5/2}}dx}{15c^3}+\frac{8x\cot^{-1}(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4x\cot^{-1}(ax)}{15c^2(c+dx^2)^{3/2}}+\frac{x\cot^{-1}(ax)}{5c(c+dx^2)^{5/2}}$$

$$\Big\downarrow \text{7266}$$

$$\frac{a\int \frac{8d^2x^4+20cdx^2+15c^2}{(a^2x^2+1)(dx^2+c)^{5/2}}dx^2}{30c^3}+\frac{8x\cot^{-1}(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4x\cot^{-1}(ax)}{15c^2(c+dx^2)^{3/2}}+\frac{x\cot^{-1}(ax)}{5c(c+dx^2)^{5/2}}$$

$$\Big\downarrow \text{1192}$$

$$\frac{a\int -\frac{8d^2x^8+4cd^2x^4+3c^2d^2}{x^8(-a^2x^4+a^2c-d)}d\sqrt{dx^2+c}}{15c^3d^2}+\frac{8x\cot^{-1}(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4x\cot^{-1}(ax)}{15c^2(c+dx^2)^{3/2}}+\frac{x\cot^{-1}(ax)}{5c(c+dx^2)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a\int \frac{8d^2x^8+4cd^2x^4+3c^2d^2}{x^8(-a^2x^4+a^2c-d)}d\sqrt{dx^2+c}}{15c^3d^2}+\frac{8x\cot^{-1}(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4x\cot^{-1}(ax)}{15c^2(c+dx^2)^{3/2}}+\frac{x\cot^{-1}(ax)}{5c(c+dx^2)^{5/2}}$$

$$\Big\downarrow \text{1584}$$

$$-\frac{a\int \left(-\frac{(15c^2a^4-20cda^2+8d^2)d^2}{(d-a^2c)^2(a^2x^4-a^2c+d)}+\frac{c(7a^2c-4d)d^2}{(a^2c-d)^2x^4}+\frac{3c^2d^2}{(a^2c-d)x^8}\right)d\sqrt{dx^2+c}}{15c^3d^2}+\frac{8x\cot^{-1}(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4x\cot^{-1}(ax)}{15c^2(c+dx^2)^{3/2}}+\frac{x\cot^{-1}(ax)}{5c(c+dx^2)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\left(\frac{c^2d^2}{x^6(a^2c-d)}+\frac{cd^2(7a^2c-4d)}{x^2(a^2c-d)^2}-\frac{d^2(15a^4c^2-20a^2cd+8d^2)\text{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{a(a^2c-d)^{5/2}}\right)}{15c^3d^2}+\frac{8x\cot^{-1}(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4x\cot^{-1}(ax)}{15c^2(c+dx^2)^{3/2}}+\frac{x\cot^{-1}(ax)}{5c(c+dx^2)^{5/2}}$$

input
```
Int[ArcCot[a*x]/(c + d*x^2)^(7/2),x]
```

output
```
(x*ArcCot[a*x])/(5*c*(c + d*x^2)^(5/2)) + (4*x*ArcCot[a*x])/(15*c^2*(c + d
*x^2)^(3/2)) + (8*x*ArcCot[a*x])/(15*c^3*Sqrt[c + d*x^2]) + (a*((c^2*d^2)/
((a^2*c - d)*x^6) + (c*(7*a^2*c - 4*d)*d^2)/((a^2*c - d)^2*x^2) - (d^2*(15
*a^4*c^2 - 20*a^2*c*d + 8*d^2)*ArcTanh[(a*Sqrt[c + d*x^2])/Sqrt[a^2*c - d]
])/(a*(a^2*c - d)^(5/2))))/(15*c^3*d^2)
```

3.63. $\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx$

3.63.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5448
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcCot[c*x])   u,
x] + Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2), x], x], x]] /; FreeQ
[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.63.4 Maple [F]

$$\int \frac{\operatorname{arccot}(ax)}{(d\,x^2+c)^{\frac{7}{2}}}dx$$

input
```
int(arccot(a*x)/(d*x^2+c)^(7/2),x)
```

output
```
int(arccot(a*x)/(d*x^2+c)^(7/2),x)
```

3.63.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 618 vs. $2(180) = 360$.

Time = 0.37 (sec) , antiderivative size = 1278, normalized size of antiderivative = 6.14

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(arccot(a*x)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

output
```
[1/60*((15*a^4*c^5 - 20*a^2*c^4*d + (15*a^4*c^2*d^3 - 20*a^2*c*d^4 + 8*d^5
)*x^6 + 8*c^3*d^2 + 3*(15*a^4*c^3*d^2 - 20*a^2*c^2*d^3 + 8*c*d^4)*x^4 + 3*
(15*a^4*c^4*d - 20*a^2*c^3*d^2 + 8*c^2*d^3)*x^2)*sqrt(a^2*c - d)*log((a^4*
d^2*x^4 + 8*a^4*c^2 - 8*a^2*c*d + 2*(4*a^4*c*d - 3*a^2*d^2)*x^2 - 4*(a^3*d
*x^2 + 2*a^3*c - a*d)*sqrt(a^2*c - d)*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 + 2*
a^2*x^2 + 1)) + 4*(8*a^5*c^5 - 13*a^3*c^4*d + 5*a*c^3*d^2 + (7*a^5*c^3*d^2
 - 11*a^3*c^2*d^3 + 4*a*c*d^4)*x^4 + 3*(5*a^5*c^4*d - 8*a^3*c^3*d^2 + 3*a*
c^2*d^3)*x^2 + (8*(a^6*c^3*d^2 - 3*a^4*c^2*d^3 + 3*a^2*c*d^4 - d^5)*x^5 +
20*(a^6*c^4*d - 3*a^4*c^3*d^2 + 3*a^2*c^2*d^3 - c*d^4)*x^3 + 15*(a^6*c^5 -
 3*a^4*c^4*d + 3*a^2*c^3*d^2 - c^2*d^3)*x)*arccot(a*x))*sqrt(d*x^2 + c))/(
a^6*c^9 - 3*a^4*c^8*d + 3*a^2*c^7*d^2 - c^6*d^3 + (a^6*c^6*d^3 - 3*a^4*c^5
*d^4 + 3*a^2*c^4*d^5 - c^3*d^6)*x^6 + 3*(a^6*c^7*d^2 - 3*a^4*c^6*d^3 + 3*a
^2*c^5*d^4 - c^4*d^5)*x^4 + 3*(a^6*c^8*d - 3*a^4*c^7*d^2 + 3*a^2*c^6*d^3 -
 c^5*d^4)*x^2), -1/30*((15*a^4*c^5 - 20*a^2*c^4*d + (15*a^4*c^2*d^3 - 20*a
^2*c*d^4 + 8*d^5)*x^6 + 8*c^3*d^2 + 3*(15*a^4*c^3*d^2 - 20*a^2*c^2*d^3 + 8
*c*d^4)*x^4 + 3*(15*a^4*c^4*d - 20*a^2*c^3*d^2 + 8*c^2*d^3)*x^2)*sqrt(-a^2
*c + d)*arctan(-1/2*(a^2*d*x^2 + 2*a^2*c - d)*sqrt(-a^2*c + d)*sqrt(d*x^2
+ c)/(a^3*c^2 - a*c*d + (a^3*c*d - a*d^2)*x^2)) - 2*(8*a^5*c^5 - 13*a^3*c^
4*d + 5*a*c^3*d^2 + (7*a^5*c^3*d^2 - 11*a^3*c^2*d^3 + 4*a*c*d^4)*x^4 + 3*(
5*a^5*c^4*d - 8*a^3*c^3*d^2 + 3*a*c^2*d^3)*x^2 + (8*(a^6*c^3*d^2 - 3*a^...
```

3.63.6 Sympy [F]

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx = \int \frac{\operatorname{acot}(ax)}{(c+dx^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate(acot(a*x)/(d*x**2+c)**(7/2),x)
```

output
```
Integral(acot(a*x)/(c + d*x**2)**(7/2), x)
```

3.63.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(arccot(a*x)/(d*x^2+c)^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(d-a^2*c>0)', see `assume?` for m
ore detail
```

3.63.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx = \frac{1}{15}\,a\left(\frac{(15\,a^4c^2-20\,a^2cd+8\,d^2)\arctan\left(\frac{\sqrt{dx^2+ca}}{\sqrt{-a^2c+d}}\right)}{(a^4c^5-2\,a^2c^4d+c^3d^2)\sqrt{-a^2c+d}a}+\frac{7\,(dx^2+c)a^2c+a^2c^2-4\,(dx^2+}{(a^4c^4-2\,a^2c^3d+c^2d^2)(dx^2-}\right.$$
$$+\frac{\left(4\,x^2\left(\frac{2\,d^2x^2}{c^3}+\frac{5\,d}{c^2}\right)+\frac{15}{c}\right)x\arctan\left(\frac{1}{ax}\right)}{15\,(dx^2+c)^{\frac{5}{2}}}$$

input
```
integrate(arccot(a*x)/(d*x^2+c)^(7/2),x, algorithm="giac")
```

output
```
1/15*a*((15*a^4*c^2 - 20*a^2*c*d + 8*d^2)*arctan(sqrt(d*x^2 + c)*a/sqrt(-a
^2*c + d))/((a^4*c^5 - 2*a^2*c^4*d + c^3*d^2)*sqrt(-a^2*c + d)*a) + (7*(d*
x^2 + c)*a^2*c + a^2*c^2 - 4*(d*x^2 + c)*d - c*d)/((a^4*c^4 - 2*a^2*c^3*d
+ c^2*d^2)*(d*x^2 + c)^(3/2))) + 1/15*(4*x^2*(2*d^2*x^2/c^3 + 5*d/c^2) + 1
5/c)*x*arctan(1/(a*x))/(d*x^2 + c)^(5/2)
```

3.63.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx = \int \frac{\mathrm{acot}(a\,x)}{(d\,x^2+c)^{7/2}}\,dx$$

input
```
int(acot(a*x)/(c + d*x^2)^(7/2),x)
```

output
```
int(acot(a*x)/(c + d*x^2)^(7/2), x)
```

3.64 $\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{9/2}} \, dx$

3.64.1 Optimal result

Integrand size = 16, antiderivative size = 293

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{9/2}} \, dx = \frac{a}{35c(a^2c-d)(c+dx^2)^{5/2}} + \frac{a(11a^2c-6d)}{105c^2(a^2c-d)^2(c+dx^2)^{3/2}} + \frac{a(19a^4c^2-22a^2cd+8d^2)}{35c^3(a^2c-d)^3\sqrt{c+dx^2}} + \frac{x\cot^{-1}(ax)}{7c(c+dx^2)^{7/2}} + \frac{6x\cot^{-1}(ax)}{35c^2(c+dx^2)^{5/2}} + \frac{8x\cot^{-1}(ax)}{35c^3(c+dx^2)^{3/2}} + \frac{16x\cot^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{(35a^6c^3-70a^4c^2d+56a^2cd^2-16d^3)\operatorname{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{35c^4(a^2c-d)^{7/2}}$$

output
```
1/35*a/c/(a^2*c-d)/(d*x^2+c)^(5/2)+1/105*a*(11*a^2*c-6*d)/c^2/(a^2*c-d)^2/
(d*x^2+c)^(3/2)+1/7*x*arccot(a*x)/c/(d*x^2+c)^(7/2)+6/35*x*arccot(a*x)/c^2
/(d*x^2+c)^(5/2)+8/35*x*arccot(a*x)/c^3/(d*x^2+c)^(3/2)-1/35*(35*a^6*c^3-7
0*a^4*c^2*d+56*a^2*c*d^2-16*d^3)*arctanh(a*(d*x^2+c)^(1/2)/(a^2*c-d)^(1/2)
)/c^4/(a^2*c-d)^(7/2)+1/35*a*(19*a^4*c^2-22*a^2*c*d+8*d^2)/c^3/(a^2*c-d)^3
/(d*x^2+c)^(1/2)+16/35*x*arccot(a*x)/c^4/(d*x^2+c)^(1/2)
```

3.64.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.54

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx = \frac{\frac{2ac\left(3c^2(-a^2c+d)^2+c(11a^2c-6d)(a^2c-d)(c+dx^2)+3(19a^4c^2-22a^2cd+8d^2)(c+dx^2)^2\right)}{(a^2c-d)^3(c+dx^2)^{5/2}} + \frac{6x(35c^3+70c^2dx^2+56cd}{(c+dx}}{}$$

input
```
Integrate[ArcCot[a*x]/(c + d*x^2)^(9/2),x]
```

output
```
((2*a*c*(3*c^2*(-(a^2*c) + d)^2 + c*(11*a^2*c - 6*d)*(a^2*c - d)*(c + d*x^
2) + 3*(19*a^4*c^2 - 22*a^2*c*d + 8*d^2)*(c + d*x^2)^2))/((a^2*c - d)^3*(c
 + d*x^2)^(5/2)) + (6*x*(35*c^3 + 70*c^2*d*x^2 + 56*c*d^2*x^4 + 16*d^3*x^6
)*ArcCot[a*x])/(c + d*x^2)^(7/2) - (3*(35*a^6*c^3 - 70*a^4*c^2*d + 56*a^2*
c*d^2 - 16*d^3)*Log[(140*a*c^4*(a^2*c - d)^(5/2)*(a*c - I*d*x + Sqrt[a^2*c
 - d]*Sqrt[c + d*x^2]))/((35*a^6*c^3 - 70*a^4*c^2*d + 56*a^2*c*d^2 - 16*d^
3)*(I + a*x))])/(a^2*c - d)^(7/2) - (3*(35*a^6*c^3 - 70*a^4*c^2*d + 56*a^2
*c*d^2 - 16*d^3)*Log[(140*a*c^4*(a^2*c - d)^(5/2)*(a*c + I*d*x + Sqrt[a^2*
c - d]*Sqrt[c + d*x^2]))/((35*a^6*c^3 - 70*a^4*c^2*d + 56*a^2*c*d^2 - 16*d
^3)*(-I + a*x))])/(a^2*c - d)^(7/2))/(210*c^4)
```

3.64.3 Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5448, 27, 7266, 2122, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx$$

$$\Big\downarrow \; 5448$$

$$a\int \frac{x\left(16d^3x^6+56cd^2x^4+70c^2dx^2+35c^3\right)}{35c^4(a^2x^2+1)(dx^2+c)^{7/2}}dx+\frac{16x\cot^{-1}(ax)}{35c^4\sqrt{c+dx^2}}+\frac{8x\cot^{-1}(ax)}{35c^3(c+dx^2)^{3/2}}+\frac{6x\cot^{-1}(ax)}{35c^2(c+dx^2)^{5/2}}+\frac{x\cot^{-1}(ax)}{7c(c+dx^2)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{x(16d^3x^6+56cd^2x^4+70c^2dx^2+35c^3)}{(a^2x^2+1)(dx^2+c)^{7/2}}dx}{35c^4}+\frac{16x\cot^{-1}(ax)}{35c^4\sqrt{c+dx^2}}+\frac{8x\cot^{-1}(ax)}{35c^3(c+dx^2)^{3/2}}+\frac{6x\cot^{-1}(ax)}{35c^2(c+dx^2)^{5/2}}+\frac{x\cot^{-1}(ax)}{7c(c+dx^2)^{7/2}}$$

$$\Big\downarrow \text{7266}$$

$$\frac{a\int \frac{16d^3x^6+56cd^2x^4+70c^2dx^2+35c^3}{(a^2x^2+1)(dx^2+c)^{7/2}}dx^2}{70c^4}+\frac{16x\cot^{-1}(ax)}{35c^4\sqrt{c+dx^2}}+\frac{8x\cot^{-1}(ax)}{35c^3(c+dx^2)^{3/2}}+\frac{6x\cot^{-1}(ax)}{35c^2(c+dx^2)^{5/2}}+\frac{x\cot^{-1}(ax)}{7c(c+dx^2)^{7/2}}$$

$$\Big\downarrow \text{2122}$$

$$\frac{a\int \left(-\frac{5dc^3}{(a^2c-d)(dx^2+c)^{7/2}}-\frac{(11a^2c-6d)dc^2}{(d-a^2c)^2(dx^2+c)^{5/2}}+\frac{d(19c^2a^4-22cda^2+8d^2)c}{(d-a^2c)^3(dx^2+c)^{3/2}}+\frac{35c^3a^6-70c^2da^4+56cd^2a^2-16d^3}{(a^2c-d)^3(a^2x^2+1)\sqrt{dx^2+c}}\right)dx^2}{70c^4}+\frac{16x\cot^{-1}(ax)}{35c^4\sqrt{c+dx^2}}+\frac{8x\cot^{-1}(ax)}{35c^3(c+dx^2)^{3/2}}+\frac{6x\cot^{-1}(ax)}{35c^2(c+dx^2)^{5/2}}+\frac{x\cot^{-1}(ax)}{7c(c+dx^2)^{7/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\left(\frac{2c^3}{(a^2c-d)(c+dx^2)^{5/2}}+\frac{2c^2(11a^2c-6d)}{3(a^2c-d)^2(c+dx^2)^{3/2}}+\frac{2c(19a^4c^2-22a^2cd+8d^2)}{(a^2c-d)^3\sqrt{c+dx^2}}-\frac{2(35a^6c^3-70a^4c^2d+56a^2cd^2-16d^3)\text{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{a(a^2c-d)^{7/2}}\right)}{70c^4}+\frac{16x\cot^{-1}(ax)}{35c^4\sqrt{c+dx^2}}+\frac{8x\cot^{-1}(ax)}{35c^3(c+dx^2)^{3/2}}+\frac{6x\cot^{-1}(ax)}{35c^2(c+dx^2)^{5/2}}+\frac{x\cot^{-1}(ax)}{7c(c+dx^2)^{7/2}}$$

input
```
Int[ArcCot[a*x]/(c + d*x^2)^(9/2),x]
```

output
```
(x*ArcCot[a*x])/(7*c*(c + d*x^2)^(7/2)) + (6*x*ArcCot[a*x])/(35*c^2*(c + d
*x^2)^(5/2)) + (8*x*ArcCot[a*x])/(35*c^3*(c + d*x^2)^(3/2)) + (16*x*ArcCot
[a*x])/(35*c^4*Sqrt[c + d*x^2]) + (a*((2*c^3)/((a^2*c - d)*(c + d*x^2)^(5/
2)) + (2*c^2*(11*a^2*c - 6*d))/(3*(a^2*c - d)^2*(c + d*x^2)^(3/2)) + (2*c*
(19*a^4*c^2 - 22*a^2*c*d + 8*d^2))/((a^2*c - d)^3*Sqrt[c + d*x^2]) - (2*(3
5*a^6*c^3 - 70*a^4*c^2*d + 56*a^2*c*d^2 - 16*d^3)*ArcTanh[(a*Sqrt[c + d*x^
2])/Sqrt[a^2*c - d]])/(a*(a^2*c - d)^(7/2))))/(70*c^4)
```

3.64. $\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx$

3.64.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2122
```
Int[((Px_)*((c_.) + (d_.)*(x_))^(n_))/((a_.) + (b_.)*(x_)), x_Symbol] :> In
t[ExpandIntegrand[1/Sqrt[c + d*x], Px*((c + d*x)^(n + 1/2)/(a + b*x)), x],
x] /; FreeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[n + 1/2, 0]
```

rule 5448
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcCot[c*x])   u,
x] + Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2), x], x], x]] /; FreeQ
[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.64.4 Maple [F]

$$\int \frac{\operatorname{arccot}(ax)}{(d\,x^2+c)^{\frac{9}{2}}}dx$$

input
```
int(arccot(a*x)/(d*x^2+c)^(9/2),x)
```

output
```
int(arccot(a*x)/(d*x^2+c)^(9/2),x)
```

3.64.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 972 vs. $2(257) = 514$.

Time = 0.61 (sec) , antiderivative size = 1986, normalized size of antiderivative = 6.78

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate(arccot(a*x)/(d*x^2+c)^(9/2),x, algorithm="fricas")
```

output
```
[1/420*(3*(35*a^6*c^7 - 70*a^4*c^6*d + 56*a^2*c^5*d^2 + (35*a^6*c^3*d^4 -
70*a^4*c^2*d^5 + 56*a^2*c*d^6 - 16*d^7)*x^8 - 16*c^4*d^3 + 4*(35*a^6*c^4*d
^3 - 70*a^4*c^3*d^4 + 56*a^2*c^2*d^5 - 16*c*d^6)*x^6 + 6*(35*a^6*c^5*d^2 -
 70*a^4*c^4*d^3 + 56*a^2*c^3*d^4 - 16*c^2*d^5)*x^4 + 4*(35*a^6*c^6*d - 70*
a^4*c^5*d^2 + 56*a^2*c^4*d^3 - 16*c^3*d^4)*x^2)*sqrt(a^2*c - d)*log((a^4*d
^2*x^4 + 8*a^4*c^2 - 8*a^2*c*d + 2*(4*a^4*c*d - 3*a^2*d^2)*x^2 - 4*(a^3*d*
x^2 + 2*a^3*c - a*d)*sqrt(a^2*c - d)*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 + 2*a
^2*x^2 + 1)) + 4*(71*a^7*c^7 - 160*a^5*c^6*d + 122*a^3*c^5*d^2 - 33*a*c^4*
d^3 + 3*(19*a^7*c^4*d^3 - 41*a^5*c^3*d^4 + 30*a^3*c^2*d^5 - 8*a*c*d^6)*x^6
 + (182*a^7*c^5*d^2 - 397*a^5*c^4*d^3 + 293*a^3*c^3*d^4 - 78*a*c^2*d^5)*x^
4 + (196*a^7*c^6*d - 434*a^5*c^5*d^2 + 325*a^3*c^4*d^3 - 87*a*c^3*d^4)*x^2
 + 3*(16*(a^8*c^4*d^3 - 4*a^6*c^3*d^4 + 6*a^4*c^2*d^5 - 4*a^2*c*d^6 + d^7)
*x^7 + 56*(a^8*c^5*d^2 - 4*a^6*c^4*d^3 + 6*a^4*c^3*d^4 - 4*a^2*c^2*d^5 + c
*d^6)*x^5 + 70*(a^8*c^6*d - 4*a^6*c^5*d^2 + 6*a^4*c^4*d^3 - 4*a^2*c^3*d^4
+ c^2*d^5)*x^3 + 35*(a^8*c^7 - 4*a^6*c^6*d + 6*a^4*c^5*d^2 - 4*a^2*c^4*d^3
 + c^3*d^4)*x)*arccot(a*x))*sqrt(d*x^2 + c))/(a^8*c^12 - 4*a^6*c^11*d + 6*
a^4*c^10*d^2 - 4*a^2*c^9*d^3 + c^8*d^4 + (a^8*c^8*d^4 - 4*a^6*c^7*d^5 + 6*
a^4*c^6*d^6 - 4*a^2*c^5*d^7 + c^4*d^8)*x^8 + 4*(a^8*c^9*d^3 - 4*a^6*c^8*d^
4 + 6*a^4*c^7*d^5 - 4*a^2*c^6*d^6 + c^5*d^7)*x^6 + 6*(a^8*c^10*d^2 - 4*a^6
*c^9*d^3 + 6*a^4*c^8*d^4 - 4*a^2*c^7*d^5 + c^6*d^6)*x^4 + 4*(a^8*c^11*d...
```

3.64.6 Sympy [F]

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx = \int \frac{\operatorname{acot}(ax)}{(c+dx^2)^{\frac{9}{2}}}\,dx$$

input
```
integrate(acot(a*x)/(d*x**2+c)**(9/2),x)
```

output
```
Integral(acot(a*x)/(c + d*x**2)**(9/2), x)
```

3.64.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(arccot(a*x)/(d*x^2+c)^(9/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(d-a^2*c>0)', see `assume?` for m
ore detail
```

3.64.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.16

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx = \frac{1}{105}\,a\left(\frac{3\,(35\,a^6c^3 - 70\,a^4c^2d + 56\,a^2cd^2 - 16\,d^3)\arctan\left(\frac{\sqrt{dx^2+c}a}{\sqrt{-a^2c+d}}\right)}{(a^6c^7 - 3\,a^4c^6d + 3\,a^2c^5d^2 - c^4d^3)\sqrt{-a^2c+d}a} + \frac{57\,(dx^2+c)^2a^4c^2 \cdots}{\cdots}\right)$$
$$+ \frac{\left(2\left(4\,x^2\left(\frac{2\,d^3x^2}{c^4} + \frac{7\,d^2}{c^3}\right) + \frac{35\,d}{c^2}\right)x^2 + \frac{35}{c}\right)x\arctan\left(\frac{1}{ax}\right)}{35\,(dx^2+c)^{\frac{7}{2}}}$$

input
```
integrate(arccot(a*x)/(d*x^2+c)^(9/2),x, algorithm="giac")
```

output
```
1/105*a*(3*(35*a^6*c^3 - 70*a^4*c^2*d + 56*a^2*c*d^2 - 16*d^3)*arctan(sqrt
(d*x^2 + c)*a/sqrt(-a^2*c + d))/((a^6*c^7 - 3*a^4*c^6*d + 3*a^2*c^5*d^2 -
c^4*d^3)*sqrt(-a^2*c + d)*a) + (57*(d*x^2 + c)^2*a^4*c^2 + 11*(d*x^2 + c)*
a^4*c^3 + 3*a^4*c^4 - 66*(d*x^2 + c)^2*a^2*c*d - 17*(d*x^2 + c)*a^2*c^2*d
- 6*a^2*c^3*d + 24*(d*x^2 + c)^2*d^2 + 6*(d*x^2 + c)*c*d^2 + 3*c^2*d^2)/((
a^6*c^6 - 3*a^4*c^5*d + 3*a^2*c^4*d^2 - c^3*d^3)*(d*x^2 + c)^(5/2))) + 1/3
5*(2*(4*x^2*(2*d^3*x^2/c^4 + 7*d^2/c^3) + 35*d/c^2)*x^2 + 35/c)*x*arctan(1
/(a*x))/(d*x^2 + c)^(7/2)
```

3.64.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx = \int \frac{\mathrm{acot}(a\,x)}{(d\,x^2+c)^{9/2}}\,dx$$

input
```
int(acot(a*x)/(c + d*x^2)^(9/2),x)
```

output
```
int(acot(a*x)/(c + d*x^2)^(9/2), x)
```

3.65 $\int \sqrt{a+ax^2} \cot^{-1}(x)\,dx$

3.65.1 Optimal result

Integrand size = 14, antiderivative size = 195

$$\int \sqrt{a+ax^2} \cot^{-1}(x)\,dx = \frac{1}{2}\sqrt{a+ax^2} + \frac{1}{2}x\sqrt{a+ax^2}\cot^{-1}(x) - \frac{ia\sqrt{1+x^2}\cot^{-1}(x)\arctan\left(\frac{\sqrt{1+ix}}{\sqrt{1-ix}}\right)}{\sqrt{a+ax^2}} - \frac{ia\sqrt{1+x^2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1+ix}}{\sqrt{1-ix}}\right)}{2\sqrt{a+ax^2}} + \frac{ia\sqrt{1+x^2}\,\text{PolyLog}\left(2,\frac{i\sqrt{1+ix}}{\sqrt{1-ix}}\right)}{2\sqrt{a+ax^2}}$$

output
```
-I*a*arccot(x)*arctan((1+I*x)^(1/2)/(1-I*x)^(1/2))*(x^2+1)^(1/2)/(a*x^2+a)
^(1/2)-1/2*I*a*polylog(2,-I*(1+I*x)^(1/2)/(1-I*x)^(1/2))*(x^2+1)^(1/2)/(a*
x^2+a)^(1/2)+1/2*I*a*polylog(2,I*(1+I*x)^(1/2)/(1-I*x)^(1/2))*(x^2+1)^(1/2
)/(a*x^2+a)^(1/2)+1/2*(a*x^2+a)^(1/2)+1/2*x*arccot(x)*(a*x^2+a)^(1/2)
```

3.65.2 Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.70

$$\int \sqrt{a+ax^2}\cot^{-1}(x)\,dx =$$

$$-\frac{(a(1+x^2))^{3/2}\left(-2\cot\left(\frac{1}{2}\cot^{-1}(x)\right)-\cot^{-1}(x)\csc^2\left(\frac{1}{2}\cot^{-1}(x)\right)+4\cot^{-1}(x)\log\left(1-e^{i\cot^{-1}(x)}\right)-4\,\mathrm{c}}{}$$

input
```
Integrate[Sqrt[a + a*x^2]*ArcCot[x],x]
```

output
```
-1/8*((a*(1 + x^2))^(3/2)*(-2*Cot[ArcCot[x]/2] - ArcCot[x]*Csc[ArcCot[x]/2
]^2 + 4*ArcCot[x]*Log[1 - E^(I*ArcCot[x])] - 4*ArcCot[x]*Log[1 + E^(I*ArcC
ot[x])] + (4*I)*PolyLog[2, -E^(I*ArcCot[x])] - (4*I)*PolyLog[2, E^(I*ArcCo
t[x])] + ArcCot[x]*Sec[ArcCot[x]/2]^2 - 2*Tan[ArcCot[x]/2]))/(a*(1 + x^(-2
))^(3/2)*x^3)
```

3.65.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.79, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5414, 5426, 5422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ax^2+a}\cot^{-1}(x)\,dx$$

$$\Big\downarrow \text{5414}$$

$$\frac{1}{2}a\int \frac{\cot^{-1}(x)}{\sqrt{ax^2+a}}dx+\frac{1}{2}\sqrt{ax^2+a}+\frac{1}{2}x\sqrt{ax^2+a}\cot^{-1}(x)$$

$$\Big\downarrow \text{5426}$$

$$\frac{a\sqrt{x^2+1}\int \frac{\cot^{-1}(x)}{\sqrt{x^2+1}}dx}{2\sqrt{ax^2+a}}+\frac{1}{2}\sqrt{ax^2+a}+\frac{1}{2}x\sqrt{ax^2+a}\cot^{-1}(x)$$

$$\Big\downarrow \text{5422}$$

$$\frac{a\sqrt{x^2+1}\left(-2i\arctan\left(\frac{\sqrt{1+ix}}{\sqrt{1-ix}}\right)\cot^{-1}(x)-i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{ix+1}}{\sqrt{1-ix}}\right)+i\operatorname{PolyLog}\left(2,\frac{i\sqrt{ix+1}}{\sqrt{1-ix}}\right)\right)}{2\sqrt{ax^2+a}}+\frac{1}{2}\sqrt{ax^2+a}+\frac{1}{2}x\sqrt{ax^2+a}\cot^{-1}(x)$$

input
```
Int[Sqrt[a + a*x^2]*ArcCot[x],x]
```

output
```
Sqrt[a + a*x^2]/2 + (x*Sqrt[a + a*x^2]*ArcCot[x])/2 + (a*Sqrt[1 + x^2]*((-
2*I)*ArcCot[x]*ArcTan[Sqrt[1 + I*x]/Sqrt[1 - I*x]] - I*PolyLog[2, ((-I)*Sq
rt[1 + I*x])/Sqrt[1 - I*x]] + I*PolyLog[2, (I*Sqrt[1 + I*x])/Sqrt[1 - I*x]
]))/(2*Sqrt[a + a*x^2])
```

3.65.3.1 Defintions of rubi rules used

rule 5414
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)^q*
((a + b*ArcCot[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x
^2)^(q - 1)*(a + b*ArcCot[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] && Eq
Q[e, c^2*d] && GtQ[q, 0]
```

rule 5422
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcCot[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x])]/(c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 -
I*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 GtQ[d, 0]
```

rule 5426
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcCot[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

3.65.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.60

method	result
default	$\frac{\sqrt{a(i+x)(x-i)}\,(x\,\mathrm{arccot}(x)+1)}{2}-\frac{i\sqrt{a(i+x)(x-i)}\,\left(i\,\mathrm{arccot}(x)\ln\left(\frac{i+x}{\sqrt{x^2+1}}+1\right)-i\,\mathrm{arccot}(x)\ln\left(1-\frac{i+x}{\sqrt{x^2+1}}\right)+\mathrm{polylog}\left(2,-\frac{i+x}{\sqrt{x^2+1}}\right.\right.}{2\sqrt{x^2+1}}$

input
```
int((a*x^2+a)^(1/2)*arccot(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*(I+x)*(x-I))^(1/2)*(x*arccot(x)+1)-1/2*I*(a*(I+x)*(x-I))^(1/2)*(I*a
rccot(x)*ln((I+x)/(x^2+1)^(1/2)+1)-I*arccot(x)*ln(1-(I+x)/(x^2+1)^(1/2))+p
olylog(2,-(I+x)/(x^2+1)^(1/2))-polylog(2,(I+x)/(x^2+1)^(1/2)))/(x^2+1)^(1/
2)
```

3.65.5 Fricas [F]

$$\int \sqrt{a+ax^2}\cot^{-1}(x)\,dx=\int \sqrt{ax^2+a}\,\mathrm{arccot}\,(x)\;dx$$

input
```
integrate((a*x^2+a)^(1/2)*arccot(x),x, algorithm="fricas")
```

output
```
integral(sqrt(a*x^2 + a)*arccot(x), x)
```

3.65.6 Sympy [F]

$$\int \sqrt{a+ax^2}\cot^{-1}(x)\,dx=\int \sqrt{a\,(x^2+1)}\,\mathrm{acot}\,(x)\,dx$$

input
```
integrate((a*x**2+a)**(1/2)*acot(x),x)
```

output
```
Integral(sqrt(a*(x**2 + 1))*acot(x), x)
```

3.65.7 Maxima [F]

$$\int \sqrt{a+ax^2}\cot^{-1}(x)\,dx = \int \sqrt{ax^2+a}\,\mathrm{arccot}\,(x)\,dx$$

input
```
integrate((a*x^2+a)^(1/2)*arccot(x),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x^2 + a)*arccot(x), x)
```

3.65.8 Giac [F]

$$\int \sqrt{a+ax^2}\cot^{-1}(x)\,dx = \int \sqrt{ax^2+a}\,\mathrm{arccot}\,(x)\,dx$$

input
```
integrate((a*x^2+a)^(1/2)*arccot(x),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x^2 + a)*arccot(x), x)
```

3.65.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+ax^2}\cot^{-1}(x)\,dx = \int \mathrm{acot}(x)\,\sqrt{a\,x^2+a}\,dx$$

input
```
int(acot(x)*(a + a*x^2)^(1/2),x)
```

output
```
int(acot(x)*(a + a*x^2)^(1/2), x)
```

3.66 $\int \frac{\cot^{-1}(x)}{\sqrt{a+ax^2}} dx$

3.66.1 Optimal result

Integrand size = 14, antiderivative size = 155

$$\int \frac{\cot^{-1}(x)}{\sqrt{a+ax^2}} dx = -\frac{2i\sqrt{1+x^2}\cot^{-1}(x)\arctan\left(\frac{\sqrt{1+ix}}{\sqrt{1-ix}}\right)}{\sqrt{a+ax^2}} - \frac{i\sqrt{1+x^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+ix}}{\sqrt{1-ix}}\right)}{\sqrt{a+ax^2}} + \frac{i\sqrt{1+x^2}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1+ix}}{\sqrt{1-ix}}\right)}{\sqrt{a+ax^2}}$$

output
```
-2*I*arccot(x)*arctan((1+I*x)^(1/2)/(1-I*x)^(1/2))*(x^2+1)^(1/2)/(a*x^2+a)
^(1/2)-I*polylog(2,-I*(1+I*x)^(1/2)/(1-I*x)^(1/2))*(x^2+1)^(1/2)/(a*x^2+a)
^(1/2)+I*polylog(2,I*(1+I*x)^(1/2)/(1-I*x)^(1/2))*(x^2+1)^(1/2)/(a*x^2+a)^
(1/2)
```

3.66.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.57

$$\int \frac{\cot^{-1}(x)}{\sqrt{a+ax^2}} dx = -\frac{\sqrt{a(1+x^2)}\left(\cot^{-1}(x)\left(\log\left(1-e^{i\cot^{-1}(x)}\right)-\log\left(1+e^{i\cot^{-1}(x)}\right)\right)+i\,\mathrm{PolyLog}\left(2,-e^{i\cot^{-1}(x)}\right)-i\,\mathrm{Pol}\right.}{a\sqrt{1+\frac{1}{x^2}}x}$$

input
```
Integrate[ArcCot[x]/Sqrt[a + a*x^2],x]
```

output
```
-((Sqrt[a*(1 + x^2)]*(ArcCot[x]*(Log[1 - E^(I*ArcCot[x])] - Log[1 + E^(I*A
rcCot[x])]) + I*PolyLog[2, -E^(I*ArcCot[x])] - I*PolyLog[2, E^(I*ArcCot[x]
)]))/(a*Sqrt[1 + x^(-2)]*x))
```

3.66.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.75, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5426, 5422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(x)}{\sqrt{ax^2+a}}dx$$
$$\Big\downarrow \text{5426}$$
$$\frac{\sqrt{x^2+1}\int \frac{\cot^{-1}(x)}{\sqrt{x^2+1}}dx}{\sqrt{ax^2+a}}$$
$$\Big\downarrow \text{5422}$$
$$\frac{\sqrt{x^2+1}\left(-2i\arctan\left(\frac{\sqrt{1+ix}}{\sqrt{1-ix}}\right)\cot^{-1}(x)-i\,\text{PolyLog}\left(2,-\frac{i\sqrt{ix+1}}{\sqrt{1-ix}}\right)+i\,\text{PolyLog}\left(2,\frac{i\sqrt{ix+1}}{\sqrt{1-ix}}\right)\right)}{\sqrt{ax^2+a}}$$

input
```
Int[ArcCot[x]/Sqrt[a + a*x^2],x]
```

output
```
(Sqrt[1 + x^2]*((-2*I)*ArcCot[x]*ArcTan[Sqrt[1 + I*x]/Sqrt[1 - I*x]] - I*P
olyLog[2, ((-I)*Sqrt[1 + I*x])/Sqrt[1 - I*x]] + I*PolyLog[2, (I*Sqrt[1 + I
*x])/Sqrt[1 - I*x]]))/Sqrt[a + a*x^2]
```

3.66.3.1 Defintions of rubi rules used

rule 5422
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcCot[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x])]/(c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 -
I*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 GtQ[d, 0]
```

rule 5426
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcCot[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

3.66.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.64

method	result	size
default	$-\frac{i\left(i \operatorname{arccot}(x)\ln\left(\frac{i+x}{\sqrt{x^2+1}}+1\right)-i\operatorname{arccot}(x)\ln\left(1-\frac{i+x}{\sqrt{x^2+1}}\right)+\operatorname{polylog}\left(2,-\frac{i+x}{\sqrt{x^2+1}}\right)-\operatorname{polylog}\left(2,\frac{i+x}{\sqrt{x^2+1}}\right)\right)\sqrt{a(i+x)(x-i)}}{\sqrt{x^2+1}\,a}$	99

input
```
int(arccot(x)/(a*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*(I*arccot(x)*ln((I+x)/(x^2+1)^(1/2)+1)-I*arccot(x)*ln(1-(I+x)/(x^2+1)^(
1/2))+polylog(2,-(I+x)/(x^2+1)^(1/2))-polylog(2,(I+x)/(x^2+1)^(1/2)))*(a*(
I+x)*(x-I))^(1/2)/(x^2+1)^(1/2)/a
```

3.66.5 Fricas [F]

$$\int \frac{\cot^{-1}(x)}{\sqrt{a+ax^2}}\,dx = \int \frac{\operatorname{arccot}(x)}{\sqrt{ax^2+a}}\,dx$$

input
```
integrate(arccot(x)/(a*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(arccot(x)/sqrt(a*x^2 + a), x)
```

3.66.6 Sympy [F]

$$\int \frac{\cot^{-1}(x)}{\sqrt{a+ax^2}}\,dx = \int \frac{\operatorname{acot}(x)}{\sqrt{a(x^2+1)}}\,dx$$

input
```
integrate(acot(x)/(a*x**2+a)**(1/2),x)
```

output
```
Integral(acot(x)/sqrt(a*(x**2 + 1)), x)
```

3.66.7 Maxima [F]

$$\int \frac{\cot^{-1}(x)}{\sqrt{a+ax^2}}\,dx = \int \frac{\operatorname{arccot}(x)}{\sqrt{ax^2+a}}\,dx$$

input
```
integrate(arccot(x)/(a*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arccot(x)/sqrt(a*x^2 + a), x)
```

3.66.8 Giac [F]

$$\int \frac{\cot^{-1}(x)}{\sqrt{a+ax^2}}\,dx = \int \frac{\operatorname{arccot}(x)}{\sqrt{ax^2+a}}\,dx$$

input
```
integrate(arccot(x)/(a*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(arccot(x)/sqrt(a*x^2 + a), x)
```

3.66.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(x)}{\sqrt{a+ax^2}}\,dx = \int \frac{\mathrm{acot}(x)}{\sqrt{a\,x^2+a}}\,dx$$

input
```
int(acot(x)/(a + a*x^2)^(1/2),x)
```

output
```
int(acot(x)/(a + a*x^2)^(1/2), x)
```

3.67 $\int \frac{\cot^{-1}(x)}{(a+ax^2)^{3/2}}\,dx$

3.67.1 Optimal result

Integrand size = 14, antiderivative size = 35

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{3/2}}\,dx = -\frac{1}{a\sqrt{a+ax^2}} + \frac{x\cot^{-1}(x)}{a\sqrt{a+ax^2}}$$

output
```
-1/a/(a*x^2+a)^(1/2)+x*arccot(x)/a/(a*x^2+a)^(1/2)
```

3.67.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.60

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{3/2}}\,dx = \frac{-1+x\cot^{-1}(x)}{a\sqrt{a(1+x^2)}}$$

input
```
Integrate[ArcCot[x]/(a + a*x^2)^(3/2),x]
```

output
```
(-1 + x*ArcCot[x])/(a*Sqrt[a*(1 + x^2)])
```

3.67.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5430}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(x)}{(ax^2+a)^{3/2}}\,dx$$

$$\Big\downarrow \ 5430$$

$$\frac{x\cot^{-1}(x)}{a\sqrt{ax^2+a}}-\frac{1}{a\sqrt{ax^2+a}}$$

input
```
Int[ArcCot[x]/(a + a*x^2)^(3/2),x]
```

output
```
-(1/(a*Sqrt[a + a*x^2])) + (x*ArcCot[x])/(a*Sqrt[a + a*x^2])
```

3.67.3.1 Defintions of rubi rules used

rule 5430
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcCot[c*x])/(d*Sq
rt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

3.67.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.57

method	result	size
risch	$\frac{ix\ln(ix+1)}{2a\sqrt{a(x^2+1)}}+\frac{-ix\ln(-ix+1)+\pi x-2}{2a\sqrt{a(x^2+1)}}$	55
default	$\frac{(\text{arccot}(x)+i)(i+x)\sqrt{a(i+x)(x-i)}}{2(x^2+1)a^2}+\frac{\sqrt{a(i+x)(x-i)}\,(x-i)(\text{arccot}(x)-i)}{2(x^2+1)a^2}$	68

input
```
int(arccot(x)/(a*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*I/a*x/(a*(x^2+1))^(1/2)*ln(1+I*x)+1/2/a*(-I*x*ln(1-I*x)+Pi*x-2)/(a*(x^
2+1))^(1/2)
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{3/2}}\,dx = \frac{\sqrt{ax^2+a}(x\operatorname{arccot}(x)-1)}{a^2x^2+a^2}$$

input
```
integrate(arccot(x)/(a*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(a*x^2 + a)*(x*arccot(x) - 1)/(a^2*x^2 + a^2)
```

3.67.6 Sympy [F]

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{3/2}}\,dx = \int \frac{\operatorname{acot}(x)}{(a(x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(acot(x)/(a*x**2+a)**(3/2),x)
```

output
```
Integral(acot(x)/(a*(x**2 + 1))**(3/2), x)
```

3.67.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{3/2}}\,dx = \frac{x\operatorname{arccot}(x)}{\sqrt{ax^2+a}a} - \frac{1}{\sqrt{ax^2+a}a}$$

input
```
integrate(arccot(x)/(a*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
x*arccot(x)/(sqrt(a*x^2 + a)*a) - 1/(sqrt(a*x^2 + a)*a)
```

3.67.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{3/2}}\,dx = \frac{x\arctan\left(\frac{1}{x}\right)}{\sqrt{ax^2+a}a} - \frac{1}{\sqrt{ax^2+a}a}$$

input
```
integrate(arccot(x)/(a*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
x*arctan(1/x)/(sqrt(a*x^2 + a)*a) - 1/(sqrt(a*x^2 + a)*a)
```

3.67.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{3/2}}\,dx = \int \frac{\mathrm{acot}(x)}{(a\,x^2+a)^{3/2}}\,dx$$

input
```
int(acot(x)/(a + a*x^2)^(3/2),x)
```

output
```
int(acot(x)/(a + a*x^2)^(3/2), x)
```

3.68 $\int \frac{\cot^{-1}(x)}{(a+ax^2)^{5/2}} dx$

3.68.1 Optimal result

Integrand size = 14, antiderivative size = 79

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{5/2}} dx = -\frac{1}{9a(a+ax^2)^{3/2}} - \frac{2}{3a^2\sqrt{a+ax^2}} + \frac{x\cot^{-1}(x)}{3a(a+ax^2)^{3/2}} + \frac{2x\cot^{-1}(x)}{3a^2\sqrt{a+ax^2}}$$

output
```
-1/9/a/(a*x^2+a)^(3/2)+1/3*x*arccot(x)/a/(a*x^2+a)^(3/2)-2/3/a^2/(a*x^2+a)
^(1/2)+2/3*x*arccot(x)/a^2/(a*x^2+a)^(1/2)
```

3.68.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.47

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{5/2}} dx = \frac{-7-6x^2+(9x+6x^3)\cot^{-1}(x)}{9a(a(1+x^2))^{3/2}}$$

input
```
Integrate[ArcCot[x]/(a + a*x^2)^(5/2),x]
```

output
```
(-7 - 6*x^2 + (9*x + 6*x^3)*ArcCot[x])/(9*a*(a*(1 + x^2))^(3/2))
```

3.68.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5432, 5430}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(x)}{(ax^2+a)^{5/2}}dx$$

$$\Big\downarrow \text{5432}$$

$$\frac{2\int \frac{\cot^{-1}(x)}{(ax^2+a)^{3/2}}dx}{3a}-\frac{1}{9a(ax^2+a)^{3/2}}+\frac{x\cot^{-1}(x)}{3a(ax^2+a)^{3/2}}$$

$$\Big\downarrow \text{5430}$$

$$-\frac{1}{9a(ax^2+a)^{3/2}}+\frac{x\cot^{-1}(x)}{3a(ax^2+a)^{3/2}}+\frac{2\left(\frac{x\cot^{-1}(x)}{a\sqrt{ax^2+a}}-\frac{1}{a\sqrt{ax^2+a}}\right)}{3a}$$

input
```
Int[ArcCot[x]/(a + a*x^2)^(5/2),x]
```

output
```
-1/9*1/(a*(a + a*x^2)^(3/2)) + (x*ArcCot[x])/(3*a*(a + a*x^2)^(3/2)) + (2*
(-(1/(a*Sqrt[a + a*x^2])) + (x*ArcCot[x])/(a*Sqrt[a + a*x^2])))/(3*a)
```

3.68.3.1 Defintions of rubi rules used

rule 5430
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcCot[c*x])/(d*Sq
rt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 5432
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d +
e*x^2)^(q + 1)*((a + b*ArcCot[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*
d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcCot[c*x]), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

3.68.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.28

method	result
risch	$\frac{ix(2x^2+3)\ln(ix+1)}{6a^2(x^2+1)\sqrt{a(x^2+1)}}+\frac{-6ix^3\ln(-ix+1)+6\pi x^3-9ix\ln(-ix+1)+9\pi x-12x^2-14}{18a^2(x^2+1)\sqrt{a(x^2+1)}}$
default	$-\frac{(i+3\,\mathrm{arccot}(x))(x^3+3ix^2-3x-i)\sqrt{a(i+x)(x-i)}}{72(x^2+1)^2a^3}+\frac{3(\mathrm{arccot}(x)+i)(i+x)\sqrt{a(i+x)(x-i)}}{8a^3(x^2+1)}+\frac{3\sqrt{a(i+x)(x-i)}\,(x-i)(\mathrm{arccot}(x)-}{8a^3(x^2+1)}$

input
```
int(arccot(x)/(a*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*I/a^2*x*(2*x^2+3)/(x^2+1)/(a*(x^2+1))^(1/2)*ln(1+I*x)+1/18/a^2*(-6*I*x
^3*ln(1-I*x)+6*Pi*x^3-9*I*x*ln(1-I*x)+9*Pi*x-12*x^2-14)/(x^2+1)/(a*(x^2+1)
)^(1/2)
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.66

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{5/2}}\,dx = -\frac{\sqrt{ax^2+a}(6\,x^2-3\,(2\,x^3+3\,x)\,\mathrm{arccot}\,(x)+7)}{9\,(a^3x^4+2\,a^3x^2+a^3)}$$

input
```
integrate(arccot(x)/(a*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/9*sqrt(a*x^2 + a)*(6*x^2 - 3*(2*x^3 + 3*x)*arccot(x) + 7)/(a^3*x^4 + 2*
a^3*x^2 + a^3)
```

3.68.6 Sympy [F]

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{5/2}}\,dx = \int \frac{\mathrm{acot}\,(x)}{(a\,(x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(acot(x)/(a*x**2+a)**(5/2),x)
```

output
```
Integral(acot(x)/(a*(x**2 + 1))**(5/2), x)
```

3.68.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.80

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{5/2}}\,dx = \frac{1}{3}\left(\frac{2x}{\sqrt{ax^2+a}a^2} + \frac{x}{(ax^2+a)^{\frac{3}{2}}a}\right)\mathrm{arccot}\,(x) - \frac{2}{3\sqrt{ax^2+a}a^2} - \frac{1}{9\,(ax^2+a)^{\frac{3}{2}}a}$$

input
```
integrate(arccot(x)/(a*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/3*(2*x/(sqrt(a*x^2 + a)*a^2) + x/((a*x^2 + a)^(3/2)*a))*arccot(x) - 2/3/
(sqrt(a*x^2 + a)*a^2) - 1/9/((a*x^2 + a)^(3/2)*a)
```

3.68.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.70

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{5/2}}\,dx = \frac{x\left(\frac{2x^2}{a}+\frac{3}{a}\right)\arctan\left(\frac{1}{x}\right)}{3\,(ax^2+a)^{\frac{3}{2}}} - \frac{6\,ax^2+7\,a}{9\,(ax^2+a)^{\frac{3}{2}}a^2}$$

input
```
integrate(arccot(x)/(a*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*x*(2*x^2/a + 3/a)*arctan(1/x)/(a*x^2 + a)^(3/2) - 1/9*(6*a*x^2 + 7*a)/
((a*x^2 + a)^(3/2)*a^2)
```

3.68.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{5/2}}\,dx = \int \frac{\mathrm{acot}(x)}{(a\,x^2+a)^{5/2}}\,dx$$

input
```
int(acot(x)/(a + a*x^2)^(5/2),x)
```

output
```
int(acot(x)/(a + a*x^2)^(5/2), x)
```

3.69 $\int \frac{\cot^{-1}(x)}{(a+ax^2)^{7/2}}\,dx$

3.69.1 Optimal result

Integrand size = 14, antiderivative size = 118

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{7/2}}\,dx = -\frac{1}{25a(a+ax^2)^{5/2}} - \frac{4}{45a^2(a+ax^2)^{3/2}} - \frac{8}{15a^3\sqrt{a+ax^2}} + \frac{x\cot^{-1}(x)}{5a(a+ax^2)^{5/2}} + \frac{4x\cot^{-1}(x)}{15a^2(a+ax^2)^{3/2}} + \frac{8x\cot^{-1}(x)}{15a^3\sqrt{a+ax^2}}$$

output
```
-1/25/a/(a*x^2+a)^(5/2)-4/45/a^2/(a*x^2+a)^(3/2)+1/5*x*arccot(x)/a/(a*x^2+
a)^(5/2)+4/15*x*arccot(x)/a^2/(a*x^2+a)^(3/2)-8/15/a^3/(a*x^2+a)^(1/2)+8/1
5*x*arccot(x)/a^3/(a*x^2+a)^(1/2)
```

3.69.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.40

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{7/2}}\,dx = \frac{-149-260x^2-120x^4+15x(15+20x^2+8x^4)\cot^{-1}(x)}{225a(a(1+x^2))^{5/2}}$$

input
```
Integrate[ArcCot[x]/(a + a*x^2)^(7/2),x]
```

output
```
(-149 - 260*x^2 - 120*x^4 + 15*x*(15 + 20*x^2 + 8*x^4)*ArcCot[x])/(225*a*(
a*(1 + x^2))^(5/2))
```

3.69.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5432, 5432, 5430}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(x)}{(ax^2+a)^{7/2}}dx$$

$$\Big\downarrow \text{5432}$$

$$\frac{4\int \frac{\cot^{-1}(x)}{(ax^2+a)^{5/2}}dx}{5a}-\frac{1}{25a(ax^2+a)^{5/2}}+\frac{x\cot^{-1}(x)}{5a(ax^2+a)^{5/2}}$$

$$\Big\downarrow \text{5432}$$

$$\frac{4\left(\frac{2\int \frac{\cot^{-1}(x)}{(ax^2+a)^{3/2}}dx}{3a}-\frac{1}{9a(ax^2+a)^{3/2}}+\frac{x\cot^{-1}(x)}{3a(ax^2+a)^{3/2}}\right)}{5a}-\frac{1}{25a(ax^2+a)^{5/2}}+\frac{x\cot^{-1}(x)}{5a(ax^2+a)^{5/2}}$$

$$\Big\downarrow \text{5430}$$

$$-\frac{1}{25a(ax^2+a)^{5/2}}+\frac{x\cot^{-1}(x)}{5a(ax^2+a)^{5/2}}+\frac{4\left(-\frac{1}{9a(ax^2+a)^{3/2}}+\frac{x\cot^{-1}(x)}{3a(ax^2+a)^{3/2}}+\frac{2\left(\frac{x\cot^{-1}(x)}{a\sqrt{ax^2+a}}-\frac{1}{a\sqrt{ax^2+a}}\right)}{3a}\right)}{5a}$$

input
```
Int[ArcCot[x]/(a + a*x^2)^(7/2),x]
```

output
```
-1/25*1/(a*(a + a*x^2)^(5/2)) + (x*ArcCot[x])/(5*a*(a + a*x^2)^(5/2)) + (4
*(-1/9*1/(a*(a + a*x^2)^(3/2)) + (x*ArcCot[x])/(3*a*(a + a*x^2)^(3/2)) + (
2*(-(1/(a*Sqrt[a + a*x^2])) + (x*ArcCot[x])/(a*Sqrt[a + a*x^2])))/(3*a)))/
(5*a)
```

3.69.3.1 Defintions of rubi rules used

rule 5430
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcCot[c*x])/(d*Sq
rt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 5432
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d +
e*x^2)^(q + 1)*((a + b*ArcCot[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*
d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcCot[c*x]), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

3.69.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.41 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.10

method	result
risch	$\frac{ix(8x^4+20x^2+15)\ln(ix+1)}{30a^3(x^2+1)^2\sqrt{a(x^2+1)}}+\frac{-120ix^5\ln(-ix+1)+120\pi x^5-300ix^3\ln(-ix+1)+300\pi x^3-240x^4-225ix\ln(-ix+1)+225\pi x-520}{450a^3(x^2+1)^2\sqrt{a(x^2+1)}}$
default	$\frac{(i+5\operatorname{arccot}(x))(x^5+5ix^4-10x^3-10ix^2+5x+i)\sqrt{a(i+x)(x-i)}}{800(x^2+1)^3a^4}+\frac{5(\operatorname{arccot}(x)+i)(i+x)\sqrt{a(i+x)(x-i)}}{16(x^2+1)a^4}+\frac{5\sqrt{a(i+x)(x-i)}(x-i)(}{16(x^2+1)a}$

input
```
int(arccot(x)/(a*x^2+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/30*I/a^3*x*(8*x^4+20*x^2+15)/(x^2+1)^2/(a*(x^2+1))^(1/2)*ln(1+I*x)+1/450
/a^3*(-120*I*x^5*ln(1-I*x)+120*Pi*x^5-300*I*x^3*ln(1-I*x)+300*Pi*x^3-240*x
^4-225*I*x*ln(1-I*x)+225*Pi*x-520*x^2-298)/(x^2+1)^2/(a*(x^2+1))^(1/2)
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.59

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{7/2}}\,dx=-\frac{(120\,x^4+260\,x^2-15\,(8\,x^5+20\,x^3+15\,x)\operatorname{arccot}(x)+149)\sqrt{ax^2+a}}{225\,(a^4x^6+3\,a^4x^4+3\,a^4x^2+a^4)}$$

input
```
integrate(arccot(x)/(a*x^2+a)^(7/2),x, algorithm="fricas")
```

output
```
-1/225*(120*x^4 + 260*x^2 - 15*(8*x^5 + 20*x^3 + 15*x)*arccot(x) + 149)*sq
rt(a*x^2 + a)/(a^4*x^6 + 3*a^4*x^4 + 3*a^4*x^2 + a^4)
```

3.69.6 Sympy [F]

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{7/2}}\,dx = \int \frac{\operatorname{acot}(x)}{(a(x^2+1))^{\frac{7}{2}}}\,dx$$

input
```
integrate(acot(x)/(a*x**2+a)**(7/2),x)
```

output
```
Integral(acot(x)/(a*(x**2 + 1))**(7/2), x)
```

3.69.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.79

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{7/2}}\,dx = \frac{1}{15}\left(\frac{8\,x}{\sqrt{ax^2+a}a^3} + \frac{4\,x}{(ax^2+a)^{\frac{3}{2}}a^2} + \frac{3\,x}{(ax^2+a)^{\frac{5}{2}}a}\right)\operatorname{arccot}(x) - \frac{8}{15\sqrt{ax^2+a}a^3} - \frac{4}{45\,(ax^2+a)^{\frac{3}{2}}a^2} - \frac{1}{25\,(ax^2+a)^{\frac{5}{2}}a}$$

input
```
integrate(arccot(x)/(a*x^2+a)^(7/2),x, algorithm="maxima")
```

output
```
1/15*(8*x/(sqrt(a*x^2 + a)*a^3) + 4*x/((a*x^2 + a)^(3/2)*a^2) + 3*x/((a*x^
2 + a)^(5/2)*a))*arccot(x) - 8/15/(sqrt(a*x^2 + a)*a^3) - 4/45/((a*x^2 + a
)^(3/2)*a^2) - 1/25/((a*x^2 + a)^(5/2)*a)
```

3.69.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.70

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{7/2}}\,dx = \frac{\left(4\,x^2\left(\frac{2\,x^2}{a}+\frac{5}{a}\right)+\frac{15}{a}\right)x\arctan\left(\frac{1}{x}\right)}{15\,(ax^2+a)^{\frac{5}{2}}} - \frac{120\,(ax^2+a)^2+20\,(ax^2+a)a+9\,a^2}{225\,(ax^2+a)^{\frac{5}{2}}a^3}$$

input
```
integrate(arccot(x)/(a*x^2+a)^(7/2),x, algorithm="giac")
```

output
```
1/15*(4*x^2*(2*x^2/a + 5/a) + 15/a)*x*arctan(1/x)/(a*x^2 + a)^(5/2) - 1/22
5*(120*(a*x^2 + a)^2 + 20*(a*x^2 + a)*a + 9*a^2)/((a*x^2 + a)^(5/2)*a^3)
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{7/2}}\,dx = \int \frac{\mathrm{acot}(x)}{(a\,x^2+a)^{7/2}}\,dx$$

input
```
int(acot(x)/(a + a*x^2)^(7/2),x)
```

output
```
int(acot(x)/(a + a*x^2)^(7/2), x)
```

3.70 $\int \frac{x \cot^{-1}(x)}{(1+x^2)^2} \, dx$

3.70.1 Optimal result

Integrand size = 11, antiderivative size = 32

$$\int \frac{x \cot^{-1}(x)}{(1+x^2)^2} \, dx = -\frac{x}{4(1+x^2)} - \frac{\cot^{-1}(x)}{2(1+x^2)} - \frac{\arctan(x)}{4}$$

output
```
-1/4*x/(x^2+1)-1/2*arccot(x)/(x^2+1)-1/4*arctan(x)
```

3.70.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int \frac{x \cot^{-1}(x)}{(1+x^2)^2} \, dx = -\frac{x + 2\cot^{-1}(x) + \arctan(x) + x^2 \arctan(x)}{4 + 4x^2}$$

input
```
Integrate[(x*ArcCot[x])/(1 + x^2)^2,x]
```

output
```
-((x + 2*ArcCot[x] + ArcTan[x] + x^2*ArcTan[x])/(4 + 4*x^2))
```

3.70.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {5466, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cot^{-1}(x)}{(x^2+1)^2}dx$$

$$\Big\downarrow \text{5466}$$

$$-\frac{1}{2}\int \frac{1}{(x^2+1)^2}dx-\frac{\cot^{-1}(x)}{2(x^2+1)}$$

$$\Big\downarrow \text{215}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^2+1}dx-\frac{x}{2(x^2+1)}\right)-\frac{\cot^{-1}(x)}{2(x^2+1)}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(-\frac{\arctan(x)}{2}-\frac{x}{2(x^2+1)}\right)-\frac{\cot^{-1}(x)}{2(x^2+1)}$$

input
```
Int[(x*ArcCot[x])/(1 + x^2)^2,x]
```

output
```
-1/2*ArcCot[x]/(1 + x^2) + (-1/2*x/(1 + x^2) - ArcTan[x]/2)/2
```

3.70.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5466
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcCot[c*x])^p/(2*e*(q +
1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcCot[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.70.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

method	result	size
parallelrisch	$\frac{x^2 \operatorname{arccot}(x)-x-\operatorname{arccot}(x)}{4x^2+4}$	24
default	$-\frac{x}{4(x^2+1)}-\frac{\operatorname{arccot}(x)}{2(x^2+1)}-\frac{\arctan(x)}{4}$	27
parts	$-\frac{x}{4(x^2+1)}-\frac{\operatorname{arccot}(x)}{2(x^2+1)}-\frac{\arctan(x)}{4}$	27
risch	$-\frac{i\ln(ix+1)}{4(x^2+1)}-\frac{-2i\ln(-ix+1)+i\ln(i+x)+i\ln(i+x)x^2-i\ln(x-i)-i\ln(x-i)x^2+2\pi+2x}{8(i+x)(x-i)}$	88

input
```
int(x*arccot(x)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(x^2*arccot(x)-x-arccot(x))/(x^2+1)
```

3.70.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int \frac{x\cot^{-1}(x)}{(1+x^2)^2}\,dx = \frac{(x^2-1)\operatorname{arccot}(x)-x}{4(x^2+1)}$$

input
```
integrate(x*arccot(x)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
1/4*((x^2 - 1)*arccot(x) - x)/(x^2 + 1)
```

3.70.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{x\cot^{-1}(x)}{(1+x^2)^2}\,dx = \frac{x^2\operatorname{acot}(x)}{4x^2+4} - \frac{x}{4x^2+4} - \frac{\operatorname{acot}(x)}{4x^2+4}$$

input
```
integrate(x*acot(x)/(x**2+1)**2,x)
```

output
```
x**2*acot(x)/(4*x**2 + 4) - x/(4*x**2 + 4) - acot(x)/(4*x**2 + 4)
```

3.70.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{x\cot^{-1}(x)}{(1+x^2)^2}\,dx = -\frac{x}{4(x^2+1)} - \frac{\operatorname{arccot}(x)}{2(x^2+1)} - \frac{1}{4}\arctan(x)$$

input
```
integrate(x*arccot(x)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
-1/4*x/(x^2 + 1) - 1/2*arccot(x)/(x^2 + 1) - 1/4*arctan(x)
```

3.70.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{x\cot^{-1}(x)}{(1+x^2)^2}\,dx = -\frac{\arctan\left(\frac{1}{x}\right)}{2(x^2+1)} - \frac{1}{4x\left(\frac{1}{x^2}+1\right)} + \frac{1}{4}\arctan\left(\frac{1}{x}\right)$$

input
```
integrate(x*arccot(x)/(x^2+1)^2,x, algorithm="giac")
```

output
```
-1/2*arctan(1/x)/(x^2 + 1) - 1/4/(x*(1/x^2 + 1)) + 1/4*arctan(1/x)
```

3.70.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.69

$$\int \frac{x\cot^{-1}(x)}{(1+x^2)^2}\,dx = \frac{\operatorname{acot}(x)}{4} - \frac{\frac{x}{4} + \frac{\operatorname{acot}(x)}{2}}{x^2+1}$$

input
```
int((x*acot(x))/(x^2 + 1)^2,x)
```

output
```
acot(x)/4 - (x/4 + acot(x)/2)/(x^2 + 1)
```

3.71 $\int \frac{x\cot^{-1}(x)}{(1+x^2)^3}\,dx$

3.71.1 Optimal result

Integrand size = 11, antiderivative size = 44

$$\int \frac{x\cot^{-1}(x)}{(1+x^2)^3}\,dx = -\frac{x}{16(1+x^2)^2} - \frac{3x}{32(1+x^2)} - \frac{\cot^{-1}(x)}{4(1+x^2)^2} - \frac{3\arctan(x)}{32}$$

output
```
-1/16*x/(x^2+1)^2-3/32*x/(x^2+1)-1/4*arccot(x)/(x^2+1)^2-3/32*arctan(x)
```

3.71.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \frac{x\cot^{-1}(x)}{(1+x^2)^3}\,dx = -\frac{x(5+3x^2)+8\cot^{-1}(x)+3(1+x^2)^2\arctan(x)}{32(1+x^2)^2}$$

input
```
Integrate[(x*ArcCot[x])/(1 + x^2)^3,x]
```

output
```
-1/32*(x*(5 + 3*x^2) + 8*ArcCot[x] + 3*(1 + x^2)^2*ArcTan[x])/(1 + x^2)^2
```

3.71.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.23, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5466, 215, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cot^{-1}(x)}{(x^2+1)^3}dx$$

$$\Big\downarrow \text{5466}$$

$$-\frac{1}{4}\int \frac{1}{(x^2+1)^3}dx-\frac{\cot^{-1}(x)}{4(x^2+1)^2}$$

$$\Big\downarrow \text{215}$$

$$\frac{1}{4}\left(-\frac{3}{4}\int \frac{1}{(x^2+1)^2}dx-\frac{x}{4(x^2+1)^2}\right)-\frac{\cot^{-1}(x)}{4(x^2+1)^2}$$

$$\Big\downarrow \text{215}$$

$$\frac{1}{4}\left(-\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{x^2+1}dx+\frac{x}{2(x^2+1)}\right)-\frac{x}{4(x^2+1)^2}\right)-\frac{\cot^{-1}(x)}{4(x^2+1)^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(-\frac{3}{4}\left(\frac{\arctan(x)}{2}+\frac{x}{2(x^2+1)}\right)-\frac{x}{4(x^2+1)^2}\right)-\frac{\cot^{-1}(x)}{4(x^2+1)^2}$$

input
```
Int[(x*ArcCot[x])/(1 + x^2)^3,x]
```

output
```
-1/4*ArcCot[x]/(1 + x^2)^2 + (-1/4*x/(1 + x^2)^2 - (3*(x/(2*(1 + x^2)) + A
rcTan[x]/2))/4)/4
```

3.71.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5466
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcCot[c*x])^p/(2*e*(q +
1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcCot[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.71.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

method	result
default	$-\frac{x}{16(x^2+1)^2}-\frac{3x}{32(x^2+1)}-\frac{\operatorname{arccot}(x)}{4(x^2+1)^2}-\frac{3\arctan(x)}{32}$
parallelrisch	$\frac{3\ \operatorname{arccot}(x)x^4-3x^3+6x^2\operatorname{arccot}(x)-5x-5\ \operatorname{arccot}(x)}{32(x^2+1)^2}$
parts	$-\frac{x}{16(x^2+1)^2}-\frac{3x}{32(x^2+1)}-\frac{\operatorname{arccot}(x)}{4(x^2+1)^2}-\frac{3\arctan(x)}{32}$
risch	$-\frac{i\ln(ix+1)}{8(x^2+1)^2}-\frac{-8i\ln(-ix+1)-6i\ln(x-i)x^2-3i\ln(x-i)-3i\ln(x-i)x^4+6i\ln(i+x)x^2+3i\ln(i+x)+3i\ln(i+x)x^4+6x^3+8\pi+}{64(i+x)(x^2+1)(x-i)}$

input
```
int(x*arccot(x)/(x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*x/(x^2+1)^2-3/32*x/(x^2+1)-1/4*arccot(x)/(x^2+1)^2-3/32*arctan(x)
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \frac{x \cot^{-1}(x)}{(1+x^2)^3} \, dx = -\frac{3\,x^3 - (3\,x^4 + 6\,x^2 - 5)\operatorname{arccot}(x) + 5\,x}{32\,(x^4 + 2\,x^2 + 1)}$$

input
```
integrate(x*arccot(x)/(x^2+1)^3,x, algorithm="fricas")
```

output
```
-1/32*(3*x^3 - (3*x^4 + 6*x^2 - 5)*arccot(x) + 5*x)/(x^4 + 2*x^2 + 1)
```

3.71.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(39) = 78.

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.00

$$\int \frac{x \cot^{-1}(x)}{(1+x^2)^3} \, dx = \frac{3x^4 \operatorname{acot}(x)}{32x^4 + 64x^2 + 32} - \frac{3x^3}{32x^4 + 64x^2 + 32} + \frac{6x^2 \operatorname{acot}(x)}{32x^4 + 64x^2 + 32} - \frac{5x}{32x^4 + 64x^2 + 32} - \frac{5 \operatorname{acot}(x)}{32x^4 + 64x^2 + 32}$$

input
```
integrate(x*acot(x)/(x**2+1)**3,x)
```

output
```
3*x**4*acot(x)/(32*x**4 + 64*x**2 + 32) - 3*x**3/(32*x**4 + 64*x**2 + 32)
+ 6*x**2*acot(x)/(32*x**4 + 64*x**2 + 32) - 5*x/(32*x**4 + 64*x**2 + 32) -
 5*acot(x)/(32*x**4 + 64*x**2 + 32)
```

3.71.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \frac{x \cot^{-1}(x)}{(1+x^2)^3} \, dx = -\frac{3\,x^3 + 5\,x}{32\,(x^4 + 2\,x^2 + 1)} - \frac{\operatorname{arccot}(x)}{4\,(x^2 + 1)^2} - \frac{3}{32}\arctan(x)$$

input
```
integrate(x*arccot(x)/(x^2+1)^3,x, algorithm="maxima")
```

output
```
-1/32*(3*x^3 + 5*x)/(x^4 + 2*x^2 + 1) - 1/4*arccot(x)/(x^2 + 1)^2 - 3/32*a
rctan(x)
```

3.71.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int \frac{x\cot^{-1}(x)}{(1+x^2)^3}\,dx = -\frac{\frac{3}{x}+\frac{5}{x^3}}{32\left(\frac{1}{x^2}+1\right)^2} - \frac{\arctan\left(\frac{1}{x}\right)}{4\left(x^2+1\right)^2} + \frac{3}{32}\arctan\left(\frac{1}{x}\right)$$

input
```
integrate(x*arccot(x)/(x^2+1)^3,x, algorithm="giac")
```

output
```
-1/32*(3/x + 5/x^3)/(1/x^2 + 1)^2 - 1/4*arctan(1/x)/(x^2 + 1)^2 + 3/32*arc
tan(1/x)
```

3.71.9 Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int \frac{x\cot^{-1}(x)}{(1+x^2)^3}\,dx = -\frac{3\,\mathrm{atan}(x)}{32} - \frac{\frac{5\,x}{32}+\frac{\mathrm{acot}(x)}{4}+\frac{3\,x^3}{32}}{\left(x^2+1\right)^2}$$

input
```
int((x*acot(x))/(x^2 + 1)^3,x)
```

output
```
- (3*atan(x))/32 - ((5*x)/32 + acot(x)/4 + (3*x^3)/32)/(x^2 + 1)^2
```

3.72 $\int \frac{\cot^{-1}(x)}{(1+x^2)^2} dx$

3.72.1 Optimal result

Integrand size = 10, antiderivative size = 34

$$\int \frac{\cot^{-1}(x)}{(1+x^2)^2} dx = -\frac{1}{4(1+x^2)} + \frac{x\cot^{-1}(x)}{2(1+x^2)} - \frac{1}{4}\cot^{-1}(x)^2$$

output
```
-1/4/(x^2+1)+1/2*x*arccot(x)/(x^2+1)-1/4*arccot(x)^2
```

3.72.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{\cot^{-1}(x)}{(1+x^2)^2} dx = -\frac{1-2x\cot^{-1}(x)+(1+x^2)\cot^{-1}(x)^2}{4(1+x^2)}$$

input
```
Integrate[ArcCot[x]/(1 + x^2)^2,x]
```

output
```
-1/4*(1 - 2*x*ArcCot[x] + (1 + x^2)*ArcCot[x]^2)/(1 + x^2)
```

3.72.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5428, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(x)}{(x^2+1)^2} dx$$

$$\Big\downarrow \text{ 5428}$$

$$\frac{1}{2}\int \frac{x}{(x^2+1)^2} dx + \frac{x\cot^{-1}(x)}{2(x^2+1)} - \frac{1}{4}\cot^{-1}(x)^2$$

$$\Big\downarrow \text{ 241}$$

$$-\frac{1}{4(x^2+1)} + \frac{x\cot^{-1}(x)}{2(x^2+1)} - \frac{1}{4}\cot^{-1}(x)^2$$

input
```
Int[ArcCot[x]/(1 + x^2)^2,x]
```

output
```
-1/4*1/(1 + x^2) + (x*ArcCot[x])/(2*(1 + x^2)) - ArcCot[x]^2/4
```

3.72.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5428
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcCot[c*x])^p/(2*d*(d + e*x^2))), x] + (-Simp[(a +
b*ArcCot[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] + Simp[b*c*(p/2)   Int[x*((a
 + b*ArcCot[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.72.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

method	result
default	$\frac{x \operatorname{arccot}(x)}{2x^2+2}+\frac{\operatorname{arccot}(x)\arctan(x)}{2}-\frac{1}{4(x^2+1)}+\frac{\arctan(x)^2}{4}$
parts	$\frac{x \operatorname{arccot}(x)}{2x^2+2}+\frac{\operatorname{arccot}(x)\arctan(x)}{2}-\frac{1}{4(x^2+1)}+\frac{\arctan(x)^2}{4}$
risch	$\frac{\ln(ix+1)^2}{16}-\frac{\left(x^2\ln(-ix+1)-2ix+\ln(-ix+1)\right)\ln(ix+1)}{8(x^2+1)}+\frac{x^2\ln(-ix+1)^2+\ln(-ix+1)^2+2i\pi\ln(i+x)+2i\pi\ln(i+x)x^2-2i\pi\ln(x-}{16(i+x)(x-i)}$

input
```
int(arccot(x)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x*arccot(x)/(x^2+1)+1/2*arccot(x)*arctan(x)-1/4/(x^2+1)+1/4*arctan(x)^
2
```

3.72.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{\cot^{-1}(x)}{(1+x^2)^2}\,dx=-\frac{(x^2+1)\operatorname{arccot}(x)^2-2\,x\operatorname{arccot}(x)+1}{4\,(x^2+1)}$$

input
```
integrate(arccot(x)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
-1/4*((x^2 + 1)*arccot(x)^2 - 2*x*arccot(x) + 1)/(x^2 + 1)
```

3.72.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\cot^{-1}(x)}{(1+x^2)^2}\,dx=\text{Exception raised: RecursionError}$$

input
```
integrate(acot(x)/(x**2+1)**2,x)
```

output
```
Exception raised: RecursionError >> maximum recursion depth exceeded in co
mparison
```

3.72.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{\cot^{-1}(x)}{(1+x^2)^2}\,dx = \frac{1}{2}\left(\frac{x}{x^2+1} + \arctan(x)\right)\operatorname{arccot}(x) + \frac{(x^2+1)\arctan(x)^2 - 1}{4\,(x^2+1)}$$

input
```
integrate(arccot(x)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
1/2*(x/(x^2 + 1) + arctan(x))*arccot(x) + 1/4*((x^2 + 1)*arctan(x)^2 - 1)/
(x^2 + 1)
```

3.72.8 Giac [F]

$$\int \frac{\cot^{-1}(x)}{(1+x^2)^2}\,dx = \int \frac{\operatorname{arccot}(x)}{(x^2+1)^2}\,dx$$

input
```
integrate(arccot(x)/(x^2+1)^2,x, algorithm="giac")
```

output
```
integrate(arccot(x)/(x^2 + 1)^2, x)
```

3.72.9 Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \frac{\cot^{-1}(x)}{(1+x^2)^2}\,dx = \frac{\frac{x\,\mathrm{acot}(x)}{2} - \frac{1}{4}}{x^2+1} - \frac{\mathrm{acot}(x)^2}{4}$$

input
```
int(acot(x)/(x^2 + 1)^2,x)
```

output
```
((x*acot(x))/2 - 1/4)/(x^2 + 1) - acot(x)^2/4
```

3.73 $\int \frac{\cot^{-1}(x)^2}{(1+x^2)^2}\,dx$

3.73.1 Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \frac{\cot^{-1}(x)^2}{(1+x^2)^2}\,dx = -\frac{x}{4(1+x^2)} - \frac{\cot^{-1}(x)}{2(1+x^2)} + \frac{x\cot^{-1}(x)^2}{2(1+x^2)} - \frac{1}{6}\cot^{-1}(x)^3 - \frac{\arctan(x)}{4}$$

output
```
-1/4*x/(x^2+1)-1/2*arccot(x)/(x^2+1)+1/2*x*arccot(x)^2/(x^2+1)-1/6*arccot(
x)^3-1/4*arctan(x)
```

3.73.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \frac{\cot^{-1}(x)^2}{(1+x^2)^2}\,dx = -\frac{6\cot^{-1}(x) - 6x\cot^{-1}(x)^2 + 2(1+x^2)\cot^{-1}(x)^3 + 3(x+(1+x^2)\arctan(x))}{12(1+x^2)}$$

input
```
Integrate[ArcCot[x]^2/(1 + x^2)^2,x]
```

output
```
-1/12*(6*ArcCot[x] - 6*x*ArcCot[x]^2 + 2*(1 + x^2)*ArcCot[x]^3 + 3*(x + (1
 + x^2)*ArcTan[x]))/(1 + x^2)
```

3.73.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5428, 5466, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(x)^2}{(x^2+1)^2}dx$$

$$\Big\downarrow \text{5428}$$

$$\int \frac{x\cot^{-1}(x)}{(x^2+1)^2}dx+\frac{x\cot^{-1}(x)^2}{2(x^2+1)}-\frac{1}{6}\cot^{-1}(x)^3$$

$$\Big\downarrow \text{5466}$$

$$-\frac{1}{2}\int \frac{1}{(x^2+1)^2}dx+\frac{x\cot^{-1}(x)^2}{2(x^2+1)}-\frac{\cot^{-1}(x)}{2(x^2+1)}-\frac{1}{6}\cot^{-1}(x)^3$$

$$\Big\downarrow \text{215}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^2+1}dx-\frac{x}{2(x^2+1)}\right)+\frac{x\cot^{-1}(x)^2}{2(x^2+1)}-\frac{\cot^{-1}(x)}{2(x^2+1)}-\frac{1}{6}\cot^{-1}(x)^3$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(-\frac{\arctan(x)}{2}-\frac{x}{2(x^2+1)}\right)+\frac{x\cot^{-1}(x)^2}{2(x^2+1)}-\frac{\cot^{-1}(x)}{2(x^2+1)}-\frac{1}{6}\cot^{-1}(x)^3$$

input
```
Int[ArcCot[x]^2/(1 + x^2)^2,x]
```

output
```
-1/2*ArcCot[x]/(1 + x^2) + (x*ArcCot[x]^2)/(2*(1 + x^2)) - ArcCot[x]^3/6 +
 (-1/2*x/(1 + x^2) - ArcTan[x]/2)/2
```

3.73.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5428
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcCot[c*x])^p/(2*d*(d + e*x^2))), x] + (-Simp[(a +
b*ArcCot[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] + Simp[b*c*(p/2)   Int[x*((a
 + b*ArcCot[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5466
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcCot[c*x])^p/(2*e*(q +
1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcCot[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.73.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.16

method	result
default	$\frac{\operatorname{arccot}(x)^2 x}{2x^2+2}+\frac{\operatorname{arccot}(x)^2 \arctan(x)}{2}-\frac{\pi \operatorname{arccot}(x)^2}{4}+\frac{\operatorname{arccot}(x)^3}{3}+\frac{x^2 \operatorname{arccot}(x)}{2x^2+2}-\frac{x}{4(x^2+1)}-\frac{\operatorname{arccot}(x)}{4}$
parts	$\frac{\operatorname{arccot}(x)^2 x}{2x^2+2}+\frac{\operatorname{arccot}(x)^2 \arctan(x)}{2}-\frac{\pi \operatorname{arccot}(x)^2}{4}+\frac{\operatorname{arccot}(x)^3}{3}+\frac{x^2 \operatorname{arccot}(x)}{2x^2+2}-\frac{x}{4(x^2+1)}-\frac{\operatorname{arccot}(x)}{4}$
risch	$\frac{i\ln(ix+1)^3}{48}+\frac{(-ix^2\ln(-ix+1)+\pi x^2-i\ln(-ix+1)+\pi-2x)\ln(ix+1)^2}{16x^2+16}-\frac{\left(-ix^2\ln(-ix+1)^2-i\ln(-ix+1)^2-4\ln(-ix+1)x+2\ln\right.}{16(i+x)(}$

input
```
int(arccot(x)^2/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x*arccot(x)^2/(x^2+1)+1/2*arccot(x)^2*arctan(x)-1/4*Pi*arccot(x)^2+1/3
*arccot(x)^3+1/2*x^2*arccot(x)/(x^2+1)-1/4*x/(x^2+1)-1/4*arccot(x)
```

3.73.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.71

$$\int \frac{\cot^{-1}(x)^2}{(1+x^2)^2}\,dx = -\frac{2\,(x^2+1)\operatorname{arccot}(x)^3 - 6\,x\operatorname{arccot}(x)^2 - 3\,(x^2-1)\operatorname{arccot}(x) + 3\,x}{12\,(x^2+1)}$$

input
```
integrate(arccot(x)^2/(x^2+1)^2,x, algorithm="fricas")
```

output
```
-1/12*(2*(x^2 + 1)*arccot(x)^3 - 6*x*arccot(x)^2 - 3*(x^2 - 1)*arccot(x) +
 3*x)/(x^2 + 1)
```

3.73.6 Sympy [F]

$$\int \frac{\cot^{-1}(x)^2}{(1+x^2)^2}\,dx = \int \frac{\operatorname{acot}^2(x)}{(x^2+1)^2}\,dx$$

input
```
integrate(acot(x)**2/(x**2+1)**2,x)
```

output
```
Integral(acot(x)**2/(x**2 + 1)**2, x)
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.34

$$\int \frac{\cot^{-1}(x)^2}{(1+x^2)^2}\,dx = \frac{1}{2}\left(\frac{x}{x^2+1} + \arctan(x)\right)\operatorname{arccot}(x)^2 + \frac{\left((x^2+1)\arctan(x)^2 - 1\right)\operatorname{arccot}(x)}{2\,(x^2+1)} + \frac{2\,(x^2+1)\arctan(x)^3 - 3\,(x^2+1)\arctan(x) - 3\,x}{12\,(x^2+1)}$$

input
```
integrate(arccot(x)^2/(x^2+1)^2,x, algorithm="maxima")
```

output
```
1/2*(x/(x^2 + 1) + arctan(x))*arccot(x)^2 + 1/2*((x^2 + 1)*arctan(x)^2 - 1
)*arccot(x)/(x^2 + 1) + 1/12*(2*(x^2 + 1)*arctan(x)^3 - 3*(x^2 + 1)*arctan
(x) - 3*x)/(x^2 + 1)
```

3.73.8 Giac [F]

$$\int \frac{\cot^{-1}(x)^2}{(1+x^2)^2}\,dx = \int \frac{\operatorname{arccot}(x)^2}{(x^2+1)^2}\,dx$$

input
```
integrate(arccot(x)^2/(x^2+1)^2,x, algorithm="giac")
```

output
```
integrate(arccot(x)^2/(x^2 + 1)^2, x)
```

3.73.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int \frac{\cot^{-1}(x)^2}{(1+x^2)^2}\,dx = \frac{x\,\mathrm{acot}(x)^2}{2\,(x^2+1)} - \frac{\mathrm{acot}(x)^3}{6} - \frac{x}{4\,(x^2+1)} - \frac{\mathrm{acot}(x)}{2\,(x^2+1)} - \frac{\mathrm{atan}(x)}{4}$$

input
```
int(acot(x)^2/(x^2 + 1)^2,x)
```

output
```
(x*acot(x)^2)/(2*(x^2 + 1)) - acot(x)^3/6 - x/(4*(x^2 + 1)) - acot(x)/(2*(
x^2 + 1)) - atan(x)/4
```

3.74 $\int x^5 \cot^{-1}(ax^2)\, dx$

3.74.1 Optimal result

Integrand size = 10, antiderivative size = 41

$$\int x^5 \cot^{-1}(ax^2)\, dx = \frac{x^4}{12a} + \frac{1}{6}x^6 \cot^{-1}(ax^2) - \frac{\log(1 + a^2x^4)}{12a^3}$$

output
```
1/12*x^4/a+1/6*x^6*arccot(a*x^2)-1/12*ln(a^2*x^4+1)/a^3
```

3.74.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int x^5 \cot^{-1}(ax^2)\, dx = \frac{x^4}{12a} + \frac{1}{6}x^6 \cot^{-1}(ax^2) - \frac{\log(1 + a^2x^4)}{12a^3}$$

input
```
Integrate[x^5*ArcCot[a*x^2],x]
```

output
```
x^4/(12*a) + (x^6*ArcCot[a*x^2])/6 - Log[1 + a^2*x^4]/(12*a^3)
```

3.74.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5362, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \cot^{-1}\left(ax^2\right) dx$$

$$\Big\downarrow \text{5362}$$

$$\frac{1}{3}a\int \frac{x^7}{a^2x^4+1}dx+\frac{1}{6}x^6\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{12}a\int \frac{x^4}{a^2x^4+1}dx^4+\frac{1}{6}x^6\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{12}a\int \left(\frac{1}{a^2}-\frac{1}{a^2\left(a^2x^4+1\right)}\right)dx^4+\frac{1}{6}x^6\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{12}a\left(\frac{x^4}{a^2}-\frac{\log\left(a^2x^4+1\right)}{a^4}\right)+\frac{1}{6}x^6\cot^{-1}\left(ax^2\right)$$

input
```
Int[x^5*ArcCot[a*x^2],x]
```

output
```
(x^6*ArcCot[a*x^2])/6 + (a*(x^4/a^2 - Log[1 + a^2*x^4]/a^4))/12
```

3.74.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.74.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

method	result	size
parallelrisch	$-\frac{-2x^{6} \operatorname{arccot}\left(a\,x^{2}\right) a^{3}-a^{2} x^{4}+\ln\left(a^{2} x^{4}+1\right)}{12 a^{3}}$	39
default	$\frac{x^{6} \operatorname{arccot}\left(a\,x^{2}\right)}{6}+\frac{a \left(\frac{x^{4}}{4 a^{2}}-\frac{\ln\left(a^{2} x^{4}+1\right)}{4 a^{4}}\right)}{3}$	40
parts	$\frac{x^{6} \operatorname{arccot}\left(a\,x^{2}\right)}{6}+\frac{a \left(\frac{x^{4}}{4 a^{2}}-\frac{\ln\left(a^{2} x^{4}+1\right)}{4 a^{4}}\right)}{3}$	40
risch	$\frac{i x^{6} \ln\left(i a\,x^{2}+1\right)}{12}-\frac{i x^{6} \ln\left(-i a\,x^{2}+1\right)}{12}+\frac{\pi\,x^{6}}{12}+\frac{x^{4}}{12 a}-\frac{\ln\left(-a^{2} x^{4}-1\right)}{12 a^{3}}$	64

input
```
int(x^5*arccot(a*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(-2*x^6*arccot(a*x^2)*a^3-a^2*x^4+ln(a^2*x^4+1))/a^3
```

3.74.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int x^5 \cot^{-1}\left(ax^2\right)\, dx = \frac{2\, a^3 x^6 \operatorname{arccot}\left(ax^2\right) + a^2 x^4 - \log\left(a^2 x^4 + 1\right)}{12\, a^3}$$

input
```
integrate(x^5*arccot(a*x^2),x, algorithm="fricas")
```

output
```
1/12*(2*a^3*x^6*arccot(a*x^2) + a^2*x^4 - log(a^2*x^4 + 1))/a^3
```

3.74.6 Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int x^5 \cot^{-1}\left(ax^2\right)\, dx = \begin{cases} \frac{x^6 \operatorname{acot}\left(ax^2\right)}{6} + \frac{x^4}{12a} - \frac{\log\left(a^2x^4+1\right)}{12a^3} & \text{for } a \neq 0 \\ \frac{\pi x^6}{12} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*acot(a*x**2),x)
```

output
```
Piecewise((x**6*acot(a*x**2)/6 + x**4/(12*a) - log(a**2*x**4 + 1)/(12*a**3
), Ne(a, 0)), (pi*x**6/12, True))
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int x^5 \cot^{-1}\left(ax^2\right)\, dx = \frac{1}{6}\, x^6 \operatorname{arccot}\left(ax^2\right) + \frac{1}{12}\left(\frac{x^4}{a^2} - \frac{\log\left(a^2x^4 + 1\right)}{a^4}\right)a$$

input
```
integrate(x^5*arccot(a*x^2),x, algorithm="maxima")
```

output
```
1/6*x^6*arccot(a*x^2) + 1/12*(x^4/a^2 - log(a^2*x^4 + 1)/a^4)*a
```

3.74.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int x^5 \cot^{-1}\left(ax^2\right) dx = \frac{1}{6}x^6 \arctan\left(\frac{1}{ax^2}\right) + \frac{1}{12}\left(\frac{x^4}{a^2} - \frac{\log\left(a^2x^4+1\right)}{a^4}\right)a$$

input
```
integrate(x^5*arccot(a*x^2),x, algorithm="giac")
```

output
```
1/6*x^6*arctan(1/(a*x^2)) + 1/12*(x^4/a^2 - log(a^2*x^4 + 1)/a^4)*a
```

3.74.9 Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int x^5 \cot^{-1}\left(ax^2\right) dx = \frac{x^6 \operatorname{acot}(a\,x^2)}{6} - \frac{\ln\left(a^2\,x^4+1\right)}{12\,a^3} + \frac{x^4}{12\,a}$$

input
```
int(x^5*acot(a*x^2),x)
```

output
```
(x^6*acot(a*x^2))/6 - log(a^2*x^4 + 1)/(12*a^3) + x^4/(12*a)
```

3.75 $\int x^3 \cot^{-1}(ax^2)\, dx$

3.75.1 Optimal result

Integrand size = 10, antiderivative size = 37

$$\int x^3 \cot^{-1}(ax^2)\, dx = \frac{x^2}{4a} + \frac{1}{4}x^4 \cot^{-1}(ax^2) - \frac{\arctan(ax^2)}{4a^2}$$

output
```
1/4*x^2/a+1/4*x^4*arccot(a*x^2)-1/4*arctan(a*x^2)/a^2
```

3.75.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int x^3 \cot^{-1}(ax^2)\, dx = \frac{x^2}{4a} + \frac{1}{4}x^4 \cot^{-1}(ax^2) - \frac{\arctan(ax^2)}{4a^2}$$

input
```
Integrate[x^3*ArcCot[a*x^2],x]
```

output
```
x^2/(4*a) + (x^4*ArcCot[a*x^2])/4 - ArcTan[a*x^2]/(4*a^2)
```

3.75.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5362, 807, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \cot^{-1}\left(ax^2\right) dx$$

$$\Big\downarrow \text{5362}$$

$$\frac{1}{2}a\int \frac{x^5}{a^2x^4+1}dx+\frac{1}{4}x^4\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{4}a\int \frac{x^4}{a^2x^4+1}dx^2+\frac{1}{4}x^4\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{4}a\left(\frac{x^2}{a^2}-\frac{\int \frac{1}{a^2x^4+1}dx^2}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}a\left(\frac{x^2}{a^2}-\frac{\arctan\left(ax^2\right)}{a^3}\right)+\frac{1}{4}x^4\cot^{-1}\left(ax^2\right)$$

input
```
Int[x^3*ArcCot[a*x^2],x]
```

output
```
(x^4*ArcCot[a*x^2])/4 + (a*(x^2/a^2 - ArcTan[a*x^2]/a^3))/4
```

3.75.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.75.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

method	result	size
parallelrisch	$\frac{\operatorname{arccot}\left(a\,x^{2}\right) a^{2} x^{4}+a\,x^{2}+\operatorname{arccot}\left(a\,x^{2}\right)}{4 a^{2}}$	31
default	$\frac{x^{4} \operatorname{arccot}\left(a\,x^{2}\right)}{4}+\frac{a \left(\frac{x^{2}}{2 a^{2}}-\frac{\arctan\left(a\,x^{2}\right)}{2 a^{3}}\right)}{2}$	36
parts	$\frac{x^{4} \operatorname{arccot}\left(a\,x^{2}\right)}{4}+\frac{a \left(\frac{x^{2}}{2 a^{2}}-\frac{\arctan\left(a\,x^{2}\right)}{2 a^{3}}\right)}{2}$	36
risch	$\frac{i x^{4} \ln\left(i a\,x^{2}+1\right)}{8}-\frac{i x^{4} \ln\left(-i a\,x^{2}+1\right)}{8}+\frac{\pi\,x^{4}}{8}+\frac{x^{2}}{4 a}-\frac{\arctan\left(a\,x^{2}\right)}{4 a^{2}}+\frac{1}{8 \pi\,a^{2}}$	67

input
```
int(x^3*arccot(a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(arccot(a*x^2)*a^2*x^4+a*x^2+arccot(a*x^2))/a^2
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int x^3 \cot^{-1}\left(ax^2\right) dx = \frac{ax^2 + (a^2x^4 + 1) \operatorname{arccot}\left(ax^2\right)}{4\,a^2}$$

input
```
integrate(x^3*arccot(a*x^2),x, algorithm="fricas")
```

output
```
1/4*(a*x^2 + (a^2*x^4 + 1)*arccot(a*x^2))/a^2
```

3.75.6 Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int x^3 \cot^{-1}\left(ax^2\right) dx = \begin{cases} \frac{x^4 \operatorname{acot}\left(ax^2\right)}{4} + \frac{x^2}{4a} + \frac{\operatorname{acot}\left(ax^2\right)}{4a^2} & \text{for } a \neq 0 \\ \frac{\pi x^4}{8} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*acot(a*x**2),x)
```

output
```
Piecewise((x**4*acot(a*x**2)/4 + x**2/(4*a) + acot(a*x**2)/(4*a**2), Ne(a,
 0)), (pi*x**4/8, True))
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int x^3 \cot^{-1}\left(ax^2\right) dx = \frac{1}{4}\,x^4 \operatorname{arccot}\left(ax^2\right) + \frac{1}{4}\,a\left(\frac{x^2}{a^2} - \frac{\arctan\left(ax^2\right)}{a^3}\right)$$

input
```
integrate(x^3*arccot(a*x^2),x, algorithm="maxima")
```

output
```
1/4*x^4*arccot(a*x^2) + 1/4*a*(x^2/a^2 - arctan(a*x^2)/a^3)
```

3.75.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int x^3 \cot^{-1}\left(ax^2\right)\,dx = \frac{1}{4}\left(\frac{x^4 \arctan\left(\frac{1}{ax^2}\right)}{a} + \frac{x^2}{a^2} + \frac{\arctan\left(\frac{1}{ax^2}\right)}{a^3}\right)a$$

input
```
integrate(x^3*arccot(a*x^2),x, algorithm="giac")
```

output
```
1/4*(x^4*arctan(1/(a*x^2))/a + x^2/a^2 + arctan(1/(a*x^2))/a^3)*a
```

3.75.9 Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int x^3 \cot^{-1}\left(ax^2\right)\,dx = \frac{x^4\,\mathrm{acot}(a\,x^2)}{4} - \frac{\mathrm{atan}(a\,x^2)}{4\,a^2} + \frac{x^2}{4\,a}$$

input
```
int(x^3*acot(a*x^2),x)
```

output
```
(x^4*acot(a*x^2))/4 - atan(a*x^2)/(4*a^2) + x^2/(4*a)
```

3.76 $\int x \cot^{-1}(ax^2) \, dx$

3.76.1 Optimal result

Integrand size = 8, antiderivative size = 31

$$\int x \cot^{-1}(ax^2) \, dx = \frac{1}{2}x^2 \cot^{-1}(ax^2) + \frac{\log(1 + a^2x^4)}{4a}$$

output
```
1/2*x^2*arccot(a*x^2)+1/4*ln(a^2*x^4+1)/a
```

3.76.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int x \cot^{-1}(ax^2) \, dx = \frac{1}{2}x^2 \cot^{-1}(ax^2) + \frac{\log(1 + a^2x^4)}{4a}$$

input
```
Integrate[x*ArcCot[a*x^2],x]
```

output
```
(x^2*ArcCot[a*x^2])/2 + Log[1 + a^2*x^4]/(4*a)
```

3.76.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5362, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cot^{-1}\left(ax^2\right)\,dx$$

$$\Big\downarrow \text{5362}$$

$$a\int \frac{x^3}{a^2x^4+1}dx+\frac{1}{2}x^2\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \text{792}$$

$$\frac{\log\left(a^2x^4+1\right)}{4a}+\frac{1}{2}x^2\cot^{-1}\left(ax^2\right)$$

input
```
Int[x*ArcCot[a*x^2],x]
```

output
```
(x^2*ArcCot[a*x^2])/2 + Log[1 + a^2*x^4]/(4*a)
```

3.76.3.1 Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.76.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

method	result	size
parts	$\frac{x^2 \operatorname{arccot}(a\,x^2)}{2}+\frac{\ln(a^2x^4+1)}{4a}$	28
parallelrisch	$\frac{2\operatorname{arccot}(a\,x^2)a\,x^2+\ln(a^2x^4+1)}{4a}$	29
derivativedivides	$\frac{\operatorname{arccot}(a\,x^2)a\,x^2+\frac{\ln\left(a^2x^4+1\right)}{2}}{2a}$	30
default	$\frac{\operatorname{arccot}(a\,x^2)a\,x^2+\frac{\ln\left(a^2x^4+1\right)}{2}}{2a}$	30
risch	$\frac{ix^2\ln(ia\,x^2+1)}{4}-\frac{ix^2\ln(-ia\,x^2+1)}{4}+\frac{\pi\,x^2}{4}+\frac{\ln(-a^2x^4-1)}{4a}$	56

input
```
int(x*arccot(a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*arccot(a*x^2)+1/4*ln(a^2*x^4+1)/a
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int x\cot^{-1}\left(ax^2\right)\,dx=\frac{2\,ax^2\operatorname{arccot}\left(ax^2\right)+\log\left(a^2x^4+1\right)}{4\,a}$$

input
```
integrate(x*arccot(a*x^2),x, algorithm="fricas")
```

output
```
1/4*(2*a*x^2*arccot(a*x^2) + log(a^2*x^4 + 1))/a
```

3.76.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int x\cot^{-1}\left(ax^2\right)\,dx=\begin{cases}\frac{x^2\operatorname{acot}\left(ax^2\right)}{2}+\frac{\log\left(a^2x^4+1\right)}{4a} & \text{for } a\neq 0\\ \frac{\pi x^2}{4} & \text{otherwise}\end{cases}$$

input
```
integrate(x*acot(a*x**2),x)
```

output
```
Piecewise((x**2*acot(a*x**2)/2 + log(a**2*x**4 + 1)/(4*a), Ne(a, 0)), (pi*
x**2/4, True))
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int x\cot^{-1}\left(ax^2\right)\,dx=\frac{2\,ax^2\operatorname{arccot}\left(ax^2\right)+\log\left(a^2x^4+1\right)}{4\,a}$$

input
```
integrate(x*arccot(a*x^2),x, algorithm="maxima")
```

output
```
1/4*(2*a*x^2*arccot(a*x^2) + log(a^2*x^4 + 1))/a
```

3.76.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.52

$$\int x\cot^{-1}\left(ax^2\right)\,dx=\frac{1}{4}\left(\frac{2\,x^2\arctan\left(\frac{1}{ax^2}\right)}{a}+\frac{\log\left(\frac{1}{a^2x^4}+1\right)}{a^2}-\frac{\log\left(\frac{1}{a^2x^4}\right)}{a^2}\right)a$$

input
```
integrate(x*arccot(a*x^2),x, algorithm="giac")
```

output
```
1/4*(2*x^2*arctan(1/(a*x^2))/a + log(1/(a^2*x^4) + 1)/a^2 - log(1/(a^2*x^4
))/a^2)*a
```

3.76.9 Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int x \cot^{-1}\left(ax^2\right)\,dx = \frac{x^2\,\mathrm{acot}(a\,x^2)}{2} + \frac{\ln\left(a^2\,x^4+1\right)}{4\,a}$$

input
```
int(x*acot(a*x^2),x)
```

output
```
(x^2*acot(a*x^2))/2 + log(a^2*x^4 + 1)/(4*a)
```

3.77 $\int \frac{\cot^{-1}(ax^2)}{x} \, dx$

3.77.1 Optimal result

Integrand size = 10, antiderivative size = 37

$$\int \frac{\cot^{-1}(ax^2)}{x} \, dx = -\frac{1}{4} i \,\text{PolyLog}\left(2, -\frac{i}{ax^2}\right) + \frac{1}{4} i \,\text{PolyLog}\left(2, \frac{i}{ax^2}\right)$$

output
```
-1/4*I*polylog(2,-I/a/x^2)+1/4*I*polylog(2,I/a/x^2)
```

3.77.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(ax^2)}{x} \, dx = -\frac{1}{4} i \,\text{PolyLog}\left(2, -\frac{i}{ax^2}\right) + \frac{1}{4} i \,\text{PolyLog}\left(2, \frac{i}{ax^2}\right)$$

input
```
Integrate[ArcCot[a*x^2]/x,x]
```

output
```
(-1/4*I)*PolyLog[2, (-I)/(a*x^2)] + (I/4)*PolyLog[2, I/(a*x^2)]
```

3.77.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5360, 5356, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax^2)}{x} dx$$

$$\Big\downarrow \text{5360}$$

$$\frac{1}{2}\int \frac{\cot^{-1}(ax^2)}{x^2}dx^2$$

$$\Big\downarrow \text{5356}$$

$$\frac{1}{2}\left(\frac{1}{2}i\int \frac{\log\left(1-\frac{i}{ax^2}\right)}{x^2}dx^2-\frac{1}{2}i\int \frac{\log\left(1+\frac{i}{x^2a}\right)}{x^2}dx^2\right)$$

$$\Big\downarrow \text{2838}$$

$$\frac{1}{2}\left(\frac{1}{2}i\operatorname{PolyLog}\left(2,\frac{i}{ax^2}\right)-\frac{1}{2}i\operatorname{PolyLog}\left(2,-\frac{i}{ax^2}\right)\right)$$

input
```
Int[ArcCot[a*x^2]/x,x]
```

output
```
((-1/2*I)*PolyLog[2, (-I)/(a*x^2)] + (I/2)*PolyLog[2, I/(a*x^2)])/2
```

3.77.3.1 Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5356
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Simp[I*(b/2)   Int[Log[1 + I/(c*x)]/x, x], x] + Simp[I*(b/2)   Int[Log
[1 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5360
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1
/n   Subst[Int[(a + b*ArcCot[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
n}, x] && IGtQ[p, 0]
```

3.77.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.54

method	result
default	$\ln\left(x\right)\operatorname{arccot}\left(a\,x^{2}\right)+\frac{\sum_{\_R1=\operatorname{RootOf}\left(a^{2}\_Z^{4}+1\right)}\frac{\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1-x}{\_R1}\right)}{\_R1^{2}}}{2a}$
parts	$\ln\left(x\right)\operatorname{arccot}\left(a\,x^{2}\right)+\frac{\sum_{\_R1=\operatorname{RootOf}\left(a^{2}\_Z^{4}+1\right)}\frac{\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1-x}{\_R1}\right)}{\_R1^{2}}}{2a}$
risch	$\frac{\pi\ln(x)}{2}+\frac{i\ln(x)\ln(1-ix\sqrt{-ia})}{2}+\frac{i\ln(x)\ln(1+ix\sqrt{-ia})}{2}-\frac{i\ln(x)\ln(-ia\,x^{2}+1)}{2}+\frac{i\operatorname{dilog}(1-ix\sqrt{-ia})}{2}+\frac{i\operatorname{dilog}(1+ix\sqrt{-ia}}{2}$

input
```
int(arccot(a*x^2)/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*arccot(a*x^2)+1/2/a*sum(1/_R1^2*(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)
/_R1)),_R1=RootOf(_Z^4*a^2+1))
```

3.77.5 Fricas [F]

$$\int \frac{\cot^{-1}\left(ax^{2}\right)}{x}\,dx=\int \frac{\operatorname{arccot}\left(ax^{2}\right)}{x}\,dx$$

input
```
integrate(arccot(a*x^2)/x,x, algorithm="fricas")
```

output
```
integral(arccot(a*x^2)/x, x)
```

3.77.6 Sympy [F]

$$\int \frac{\cot^{-1}(ax^2)}{x}\,dx = \int \frac{\operatorname{acot}(ax^2)}{x}\,dx$$

input
```
integrate(acot(a*x**2)/x,x)
```

output
```
Integral(acot(a*x**2)/x, x)
```

3.77.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(23) = 46$.

Time = 0.33 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.84

$$\int \frac{\cot^{-1}(ax^2)}{x}\,dx = \frac{1}{8}\pi\log\left(a^2x^4+1\right) - \frac{1}{2}\arctan\left(ax^2\right)\log\left(ax^2\right) + \operatorname{arccot}\left(ax^2\right)\log(x) + \arctan\left(ax^2\right)\log(x) + \frac{1}{4}i\operatorname{Li}_2\left(i\,ax^2+1\right) - \frac{1}{4}i\operatorname{Li}_2\left(-i\,ax^2+1\right)$$

input
```
integrate(arccot(a*x^2)/x,x, algorithm="maxima")
```

output
```
1/8*pi*log(a^2*x^4 + 1) - 1/2*arctan(a*x^2)*log(a*x^2) + arccot(a*x^2)*log
(x) + arctan(a*x^2)*log(x) + 1/4*I*dilog(I*a*x^2 + 1) - 1/4*I*dilog(-I*a*x
^2 + 1)
```

3.77.8 Giac [F]

$$\int \frac{\cot^{-1}(ax^2)}{x}\,dx = \int \frac{\operatorname{arccot}(ax^2)}{x}\,dx$$

input
```
integrate(arccot(a*x^2)/x,x, algorithm="giac")
```

output
```
integrate(arccot(a*x^2)/x, x)
```

3.77.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax^2)}{x} \, dx = \int \frac{\operatorname{acot}(a\,x^2)}{x} \, dx$$

input
```
int(acot(a*x^2)/x,x)
```

output
```
int(acot(a*x^2)/x, x)
```

3.78 $\int \frac{\cot^{-1}(ax^2)}{x^3}\,dx$

3.78.1 Optimal result

Integrand size = 10, antiderivative size = 34

$$\int \frac{\cot^{-1}(ax^2)}{x^3}\,dx = -\frac{\cot^{-1}(ax^2)}{2x^2} - a\log(x) + \frac{1}{4}a\log\left(1+a^2x^4\right)$$

output
```
-1/2*arccot(a*x^2)/x^2-a*ln(x)+1/4*a*ln(a^2*x^4+1)
```

3.78.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(ax^2)}{x^3}\,dx = -\frac{\cot^{-1}(ax^2)}{2x^2} - a\log(x) + \frac{1}{4}a\log\left(1+a^2x^4\right)$$

input
```
Integrate[ArcCot[a*x^2]/x^3,x]
```

output
```
-1/2*ArcCot[a*x^2]/x^2 - a*Log[x] + (a*Log[1 + a^2*x^4])/4
```

3.78.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5362, 798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax^2)}{x^3}dx$$

$$\Big\downarrow \text{5362}$$

$$-a\int \frac{1}{x(a^2x^4+1)}dx-\frac{\cot^{-1}(ax^2)}{2x^2}$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{4}a\int \frac{1}{x^4(a^2x^4+1)}dx^4-\frac{\cot^{-1}(ax^2)}{2x^2}$$

$$\Big\downarrow \text{47}$$

$$-\frac{1}{4}a\left(\int \frac{1}{x^4}dx^4-a^2\int \frac{1}{a^2x^4+1}dx^4\right)-\frac{\cot^{-1}(ax^2)}{2x^2}$$

$$\Big\downarrow \text{14}$$

$$-\frac{1}{4}a\left(\log(x^4)-a^2\int \frac{1}{a^2x^4+1}dx^4\right)-\frac{\cot^{-1}(ax^2)}{2x^2}$$

$$\Big\downarrow \text{16}$$

$$-\frac{1}{4}a(\log(x^4)-\log(a^2x^4+1))-\frac{\cot^{-1}(ax^2)}{2x^2}$$

input
```
Int[ArcCot[a*x^2]/x^3,x]
```

output
```
-1/2*ArcCot[a*x^2]/x^2 - (a*(Log[x^4] - Log[1 + a^2*x^4]))/4
```

3.78.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.78.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{\operatorname{arccot}\left(a\,x^{2}\right)}{2x^{2}}-a\left(\ln\left(x\right)-\frac{\ln\left(a^{2}x^{4}+1\right)}{4}\right)$	31
parts	$-\frac{\operatorname{arccot}\left(a\,x^{2}\right)}{2x^{2}}-a\left(\ln\left(x\right)-\frac{\ln\left(a^{2}x^{4}+1\right)}{4}\right)$	31
parallelrisch	$-\frac{4a\ln\left(x\right)x^{2}-a\ln\left(a^{2}x^{4}+1\right)x^{2}+2\ \operatorname{arccot}\left(a\,x^{2}\right)}{4x^{2}}$	39
risch	$-\frac{i\ln\left(ia\,x^{2}+1\right)}{4x^{2}}-\frac{4a\ln\left(x\right)x^{2}-a\ln\left(a^{2}x^{4}+1\right)x^{2}-i\ln\left(-ia\,x^{2}+1\right)+\pi}{4x^{2}}$	62

input
```
int(arccot(a*x^2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*arccot(a*x^2)/x^2-a*(ln(x)-1/4*ln(a^2*x^4+1))
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \frac{\cot^{-1}(ax^2)}{x^3}\,dx = \frac{ax^2\log(a^2x^4+1) - 4\,ax^2\log(x) - 2\,\operatorname{arccot}(ax^2)}{4\,x^2}$$

input
```
integrate(arccot(a*x^2)/x^3,x, algorithm="fricas")
```

output
```
1/4*(a*x^2*log(a^2*x^4 + 1) - 4*a*x^2*log(x) - 2*arccot(a*x^2))/x^2
```

3.78.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{\cot^{-1}(ax^2)}{x^3}\,dx = -a\log(x) + \frac{a\log(a^2x^4+1)}{4} - \frac{\operatorname{acot}(ax^2)}{2x^2}$$

input
```
integrate(acot(a*x**2)/x**3,x)
```

output
```
-a*log(x) + a*log(a**2*x**4 + 1)/4 - acot(a*x**2)/(2*x**2)
```

3.78.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{\cot^{-1}(ax^2)}{x^3}\,dx = \frac{1}{4}\,a\big(\log\left(a^2x^4+1\right) - \log\left(x^4\right)\big) - \frac{\operatorname{arccot}(ax^2)}{2\,x^2}$$

input
```
integrate(arccot(a*x^2)/x^3,x, algorithm="maxima")
```

output
```
1/4*a*(log(a^2*x^4 + 1) - log(x^4)) - 1/2*arccot(a*x^2)/x^2
```

3.78.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{\cot^{-1}(ax^2)}{x^3}\,dx = -\frac{1}{4}\,a\left(\frac{2\,\arctan\left(\frac{1}{ax^2}\right)}{ax^2} - \log\left(\frac{1}{a^2x^4}+1\right)\right)$$

input
```
integrate(arccot(a*x^2)/x^3,x, algorithm="giac")
```

output
```
-1/4*a*(2*arctan(1/(a*x^2))/(a*x^2) - log(1/(a^2*x^4) + 1))
```

3.78.9 Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{\cot^{-1}(ax^2)}{x^3}\,dx = \frac{a\,\ln\left(-a^2\,x^4-1\right)}{4} - \frac{\mathrm{acot}(a\,x^2)}{2\,x^2} - a\,\ln(x)$$

input
```
int(acot(a*x^2)/x^3,x)
```

output
```
(a*log(- a^2*x^4 - 1))/4 - acot(a*x^2)/(2*x^2) - a*log(x)
```

3.79 $\int \frac{\cot^{-1}(ax^2)}{x^5}\,dx$

3.79.1 Optimal result

Integrand size = 10, antiderivative size = 35

$$\int \frac{\cot^{-1}(ax^2)}{x^5}\,dx = \frac{a}{4x^2} - \frac{\cot^{-1}(ax^2)}{4x^4} + \frac{1}{4}a^2 \arctan(ax^2)$$

output
```
1/4*a/x^2-1/4*arccot(a*x^2)/x^4+1/4*a^2*arctan(a*x^2)
```

3.79.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \frac{\cot^{-1}(ax^2)}{x^5}\,dx = -\frac{\cot^{-1}(ax^2)}{4x^4} + \frac{a\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-a^2x^4\right)}{4x^2}$$

input
```
Integrate[ArcCot[a*x^2]/x^5,x]
```

output
```
-1/4*ArcCot[a*x^2]/x^4 + (a*Hypergeometric2F1[-1/2, 1, 1/2, -(a^2*x^4)])/(
4*x^2)
```

3.79.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5362, 807, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}\left(ax^2\right)}{x^5} \, dx$$

$$\Big\downarrow \text{ 5362}$$

$$-\frac{1}{2}a\int \frac{1}{x^3\left(a^2x^4+1\right)}dx-\frac{\cot^{-1}\left(ax^2\right)}{4x^4}$$

$$\Big\downarrow \text{ 807}$$

$$-\frac{1}{4}a\int \frac{1}{x^4\left(a^2x^4+1\right)}dx^2-\frac{\cot^{-1}\left(ax^2\right)}{4x^4}$$

$$\Big\downarrow \text{ 264}$$

$$-\frac{1}{4}a\left(a^2\left(-\int \frac{1}{a^2x^4+1}dx^2\right)-\frac{1}{x^2}\right)-\frac{\cot^{-1}\left(ax^2\right)}{4x^4}$$

$$\Big\downarrow \text{ 216}$$

$$-\frac{1}{4}a\left(-a\arctan\left(ax^2\right)-\frac{1}{x^2}\right)-\frac{\cot^{-1}\left(ax^2\right)}{4x^4}$$

input
```
Int[ArcCot[a*x^2]/x^5,x]
```

output
```
-1/4*ArcCot[a*x^2]/x^4 - (a*(-x^(-2) - a*ArcTan[a*x^2]))/4
```

3.79.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.79.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{\operatorname{arccot}\left(a\,x^{2}\right)}{4x^{4}}-\frac{a\left(-\frac{a\arctan\left(a\,x^{2}\right)}{2}-\frac{1}{2x^{2}}\right)}{2}$	31
parts	$-\frac{\operatorname{arccot}\left(a\,x^{2}\right)}{4x^{4}}-\frac{a\left(-\frac{a\arctan\left(a\,x^{2}\right)}{2}-\frac{1}{2x^{2}}\right)}{2}$	31
parallelrisch	$-\frac{\operatorname{arccot}\left(a\,x^{2}\right)a^{2}x^{4}-a\,x^{2}+\operatorname{arccot}\left(a\,x^{2}\right)}{4x^{4}}$	32
risch	$-\frac{i\ln\left(ia\,x^{2}+1\right)}{8x^{4}}-\frac{ia^{2}\ln\left(-a\,x^{2}+i\right)x^{4}-ia^{2}\ln\left(-a\,x^{2}-i\right)x^{4}-2a\,x^{2}-i\ln\left(-ia\,x^{2}+1\right)+\pi}{8x^{4}}$	82

input
```
int(arccot(a*x^2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*arccot(a*x^2)/x^4-1/2*a*(-1/2*a*arctan(a*x^2)-1/2/x^2)
```

3.79.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \frac{\cot^{-1}(ax^2)}{x^5}\,dx = \frac{ax^2 - (a^2x^4 + 1)\,\mathrm{arccot}\,(ax^2)}{4\,x^4}$$

input
```
integrate(arccot(a*x^2)/x^5,x, algorithm="fricas")
```

output
```
1/4*(a*x^2 - (a^2*x^4 + 1)*arccot(a*x^2))/x^4
```

3.79.6 Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{\cot^{-1}(ax^2)}{x^5}\,dx = -\frac{a^2\,\mathrm{acot}\,(ax^2)}{4} + \frac{a}{4x^2} - \frac{\mathrm{acot}\,(ax^2)}{4x^4}$$

input
```
integrate(acot(a*x**2)/x**5,x)
```

output
```
-a**2*acot(a*x**2)/4 + a/(4*x**2) - acot(a*x**2)/(4*x**4)
```

3.79.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{\cot^{-1}(ax^2)}{x^5}\,dx = \frac{1}{4}\left(a\arctan\left(ax^2\right) + \frac{1}{x^2}\right)a - \frac{\mathrm{arccot}\,(ax^2)}{4\,x^4}$$

input
```
integrate(arccot(a*x^2)/x^5,x, algorithm="maxima")
```

output
```
1/4*(a*arctan(a*x^2) + 1/x^2)*a - 1/4*arccot(a*x^2)/x^4
```

3.79.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{\cot^{-1}(ax^2)}{x^5}\,dx = \frac{1}{4}\left(a\arctan\left(ax^2\right) + \frac{1}{x^2}\right)a - \frac{\arctan\left(\frac{1}{ax^2}\right)}{4\,x^4}$$

input
```
integrate(arccot(a*x^2)/x^5,x, algorithm="giac")
```

output
```
1/4*(a*arctan(a*x^2) + 1/x^2)*a - 1/4*arctan(1/(a*x^2))/x^4
```

3.79.9 Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{\cot^{-1}(ax^2)}{x^5}\,dx = \frac{a\,x^2 - \mathrm{acot}(a\,x^2) + a^2\,x^4\,\mathrm{atan}(a\,x^2)}{4\,x^4}$$

input
```
int(acot(a*x^2)/x^5,x)
```

output
```
(a*x^2 - acot(a*x^2) + a^2*x^4*atan(a*x^2))/(4*x^4)
```

3.80 $\int x^4 \cot^{-1}(ax^2)\, dx$

3.80.1 Optimal result

Integrand size = 10, antiderivative size = 152

$$\int x^4 \cot^{-1}(ax^2)\, dx = \frac{2x^3}{15a} + \frac{1}{5}x^5 \cot^{-1}(ax^2) + \frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{5\sqrt{2}a^{5/2}} - \frac{\arctan\left(1+\sqrt{2}\sqrt{a}x\right)}{5\sqrt{2}a^{5/2}} - \frac{\log\left(1-\sqrt{2}\sqrt{a}x+ax^2\right)}{10\sqrt{2}a^{5/2}} + \frac{\log\left(1+\sqrt{2}\sqrt{a}x+ax^2\right)}{10\sqrt{2}a^{5/2}}$$

output
```
2/15*x^3/a+1/5*x^5*arccot(a*x^2)-1/10*arctan(-1+x*2^(1/2)*a^(1/2))/a^(5/2)
*2^(1/2)-1/10*arctan(1+x*2^(1/2)*a^(1/2))/a^(5/2)*2^(1/2)-1/20*ln(1+a*x^2-
x*2^(1/2)*a^(1/2))/a^(5/2)*2^(1/2)+1/20*ln(1+a*x^2+x*2^(1/2)*a^(1/2))/a^(5
/2)*2^(1/2)
```

3.80.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.89

$$\int x^4 \cot^{-1}(ax^2)\, dx = \frac{8a^{3/2}x^3 + 12a^{5/2}x^5 \cot^{-1}(ax^2) + 6\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{a}x\right) - 6\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{a}x\right) - 3\sqrt{2}\log\left(1-\right.}{60a^{5/2}}$$

input
```
Integrate[x^4*ArcCot[a*x^2],x]
```

output
```
(8*a^(3/2)*x^3 + 12*a^(5/2)*x^5*ArcCot[a*x^2] + 6*Sqrt[2]*ArcTan[1 - Sqrt[
2]*Sqrt[a]*x] - 6*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[a]*x] - 3*Sqrt[2]*Log[1
- Sqrt[2]*Sqrt[a]*x + a*x^2] + 3*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[a]*x + a*x^2
])/(60*a^(5/2))
```

3.80.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.15, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5362, 843, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \cot^{-1}\left(ax^2\right) dx$$

$$\Big\downarrow \text{5362}$$

$$\frac{2}{5}a\int \frac{x^6}{a^2x^4+1}dx+\frac{1}{5}x^5\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \text{843}$$

$$\frac{2}{5}a\left(\frac{x^3}{3a^2}-\frac{\int \frac{x^2}{a^2x^4+1}dx}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \text{826}$$

$$\frac{2}{5}a\left(\frac{x^3}{3a^2}-\frac{\frac{\int \frac{ax^2+1}{a^2x^4+1}dx}{2a}-\frac{\int \frac{1-ax^2}{a^2x^4+1}dx}{2a}}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \text{1476}$$

$$\frac{2}{5}a\left(\frac{x^3}{3a^2}-\frac{\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}}dx}{2a}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}}dx}{2a}}{2a}-\frac{\int \frac{1-ax^2}{a^2x^4+1}dx}{2a}}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{2}{5}a\left(\frac{x^3}{3a^2}-\frac{\frac{\frac{\int \frac{1}{-\left(1-\sqrt{2}\sqrt{a}x\right)^2-1}d\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}-\frac{\int \frac{1}{-\left(\sqrt{2}\sqrt{a}x+1\right)^2-1}d\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}}{2a}-\frac{\int \frac{1-ax^2}{a^2x^4+1}dx}{2a}}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \ 217$$

$$\frac{2}{5}a\left(\frac{x^3}{3a^2}-\frac{\frac{\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}}{2a}-\frac{\int \frac{1-ax^2}{a^2x^4+1}dx}{2a}}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \ 1479$$

$$\frac{2}{5}a\left(\frac{x^3}{3a^2}-\frac{\frac{\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}}{2a}-\frac{-\frac{\int -\frac{\sqrt{2}-2\sqrt{a}x}{\sqrt{a}\left(x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{a}\left(x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}}{2a}}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \ 25$$

$$\frac{2}{5}a\left(\frac{x^3}{3a^2}-\frac{\frac{\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}}{2a}-\frac{\frac{\int \frac{\sqrt{2}-2\sqrt{a}x}{\sqrt{a}\left(x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{a}\left(x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}}{2a}}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \ 27$$

$$\frac{2}{5}a\left(\frac{x^3}{3a^2}-\frac{\frac{\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}}{2a}-\frac{\frac{\int \frac{\sqrt{2}-2\sqrt{a}x}{x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}}dx}{2\sqrt{2}a}+\frac{\int \frac{\sqrt{2}\sqrt{a}x+1}{x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}}dx}{2a}}{2a}}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \ 1103$$

$$\frac{2}{5}a\left(\frac{x^3}{3a^2}-\frac{\frac{\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}}{2a}-\frac{\frac{\log\left(ax^2+\sqrt{2}\sqrt{a}x+1\right)}{2\sqrt{2}\sqrt{a}}-\frac{\log\left(ax^2-\sqrt{2}\sqrt{a}x+1\right)}{2\sqrt{2}\sqrt{a}}}{2a}}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}\left(ax^2\right)$$

3.80. $\int x^4\cot^{-1}\left(ax^2\right)\,dx$

input
```
Int[x^4*ArcCot[a*x^2],x]
```

output
```
(x^5*ArcCot[a*x^2])/5 + (2*a*(x^3/(3*a^2) - ((-(ArcTan[1 - Sqrt[2]*Sqrt[a]
*x]/(Sqrt[2]*Sqrt[a])) + ArcTan[1 + Sqrt[2]*Sqrt[a]*x]/(Sqrt[2]*Sqrt[a]))/
(2*a) - (-1/2*Log[1 - Sqrt[2]*Sqrt[a]*x + a*x^2]/(Sqrt[2]*Sqrt[a]) + Log[1
 + Sqrt[2]*Sqrt[a]*x + a*x^2]/(2*Sqrt[2]*Sqrt[a]))/(2*a))/a^2))/5
```

3.80.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.80.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{x^5 \operatorname{arccot}(a x^2)}{5}+\frac{2a\left(\frac{x^3}{3a^2}-\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{1}{a^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^2}}}{x^2+\left(\frac{1}{a^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}-1\right)\right)}{8a^4\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}\right)}{5}$	112
parts	$\frac{x^5 \operatorname{arccot}(a x^2)}{5}+\frac{2a\left(\frac{x^3}{3a^2}-\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{1}{a^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^2}}}{x^2+\left(\frac{1}{a^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}-1\right)\right)}{8a^4\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}\right)}{5}$	112

input
```
int(x^4*arccot(a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5*arccot(a*x^2)+2/5*a*(1/3*x^3/a^2-1/8/a^4/(1/a^2)^(1/4)*2^(1/2)*(ln
((x^2-(1/a^2)^(1/4)*x*2^(1/2)+(1/a^2)^(1/2))/(x^2+(1/a^2)^(1/4)*x*2^(1/2)+
(1/a^2)^(1/2)))+2*arctan(2^(1/2)/(1/a^2)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/a^
2)^(1/4)*x-1)))
```

3.80.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.80

$$\int x^4 \cot^{-1}\left(ax^2\right) dx$$
$$= \frac{6\,ax^5 \operatorname{arccot}\left(ax^2\right) + 4\,x^3 - 3\,a\left(-\frac{1}{a^{10}}\right)^{\frac{1}{4}} \log\left(a^7\left(-\frac{1}{a^{10}}\right)^{\frac{3}{4}} + x\right) + 3i\,a\left(-\frac{1}{a^{10}}\right)^{\frac{1}{4}} \log\left(i\,a^7\left(-\frac{1}{a^{10}}\right)^{\frac{3}{4}} + x\right) - 3i\,a(}{30\,a}$$

input
```
integrate(x^4*arccot(a*x^2),x, algorithm="fricas")
```

output
```
1/30*(6*a*x^5*arccot(a*x^2) + 4*x^3 - 3*a*(-1/a^10)^(1/4)*log(a^7*(-1/a^10
)^(3/4) + x) + 3*I*a*(-1/a^10)^(1/4)*log(I*a^7*(-1/a^10)^(3/4) + x) - 3*I*
a*(-1/a^10)^(1/4)*log(-I*a^7*(-1/a^10)^(3/4) + x) + 3*a*(-1/a^10)^(1/4)*lo
g(-a^7*(-1/a^10)^(3/4) + x))/a
```

3.80.6 Sympy [A] (verification not implemented)

Time = 9.75 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.89

$$\int x^4 \cot^{-1}\left(ax^2\right) dx$$
$$= \begin{cases} \frac{x^5 \operatorname{acot}\left(ax^2\right)}{5} + \frac{2x^3}{15a} - \frac{\sqrt[4]{-\frac{1}{a^2}}\operatorname{acot}\left(ax^2\right)}{5a^2} - \frac{\log\left(x - \sqrt[4]{-\frac{1}{a^2}}\right)}{5a^3\sqrt[4]{-\frac{1}{a^2}}} + \frac{\log\left(x^2 + \sqrt{-\frac{1}{a^2}}\right)}{10a^3\sqrt[4]{-\frac{1}{a^2}}} - \frac{\operatorname{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{a^2}}}\right)}{5a^3\sqrt[4]{-\frac{1}{a^2}}} & \text{for}\ a \neq 0 \\ \frac{\pi x^5}{10} & \text{otherwise} \end{cases}$$

input
```
integrate(x**4*acot(a*x**2),x)
```

output
```
Piecewise((x**5*acot(a*x**2)/5 + 2*x**3/(15*a) - (-1/a**2)**(1/4)*acot(a*x
**2)/(5*a**2) - log(x - (-1/a**2)**(1/4))/(5*a**3*(-1/a**2)**(1/4)) + log(
x**2 + sqrt(-1/a**2))/(10*a**3*(-1/a**2)**(1/4)) - atan(x/(-1/a**2)**(1/4)
)/(5*a**3*(-1/a**2)**(1/4)), Ne(a, 0)), (pi*x**5/10, True))
```

3.80.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.90

$$\int x^4 \cot^{-1}\left(ax^2\right)\,dx = \frac{1}{5}\,x^5\,\mathrm{arccot}\left(ax^2\right) + \frac{1}{60}\,a\left(\frac{8\,x^3}{a^2} - \frac{3\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,ax+\sqrt{2}\sqrt{a}\right)}{2\,\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,ax-\sqrt{2}\sqrt{a}\right)}{2\,\sqrt{a}}\right)}{a^{\frac{3}{2}}} - \frac{\sqrt{2}\log\left(ax^2+\sqrt{2}\sqrt{a}x+1\right)}{a^{\frac{3}{2}}} + \frac{\sqrt{2}\log\left(ax^2-\right.}{a^{\frac{3}{2}}}\right.}{a^2}\right.$$

input
```
integrate(x^4*arccot(a*x^2),x, algorithm="maxima")
```

output
```
1/5*x^5*arccot(a*x^2) + 1/60*a*(8*x^3/a^2 - 3*(2*sqrt(2)*arctan(1/2*sqrt(2
)*(2*a*x + sqrt(2)*sqrt(a))/sqrt(a))/a^(3/2) + 2*sqrt(2)*arctan(1/2*sqrt(2
)*(2*a*x - sqrt(2)*sqrt(a))/sqrt(a))/a^(3/2) - sqrt(2)*log(a*x^2 + sqrt(2)
*sqrt(a)*x + 1)/a^(3/2) + sqrt(2)*log(a*x^2 - sqrt(2)*sqrt(a)*x + 1)/a^(3/
2))/a^2)
```

3.80.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.03

$$\int x^4 \cot^{-1}\left(ax^2\right)\,dx = \frac{1}{5}\,x^5\arctan\left(\frac{1}{ax^2}\right) + \frac{1}{60}\,a\left(\frac{8\,x^3}{a^2} - \frac{6\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\frac{\sqrt{2}}{\sqrt{|a|}}\right)\sqrt{|a|}\right)}{a^2|a|^{\frac{3}{2}}} - \frac{6\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\frac{\sqrt{2}}{\sqrt{|a|}}\right)\sqrt{|a|}\right)}{a^2|a|^{\frac{3}{2}}} + \frac{3\,\sqrt{2}\sqrt{}}{}\right.$$

input
```
integrate(x^4*arccot(a*x^2),x, algorithm="giac")
```

output
```
1/5*x^5*arctan(1/(a*x^2)) + 1/60*a*(8*x^3/a^2 - 6*sqrt(2)*arctan(1/2*sqrt(
2)*(2*x + sqrt(2)/sqrt(abs(a)))*sqrt(abs(a)))/(a^2*abs(a)^(3/2)) - 6*sqrt(
2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)/sqrt(abs(a)))*sqrt(abs(a)))/(a^2*abs(
a)^(3/2)) + 3*sqrt(2)*sqrt(abs(a))*log(x^2 + sqrt(2)*x/sqrt(abs(a)) + 1/ab
s(a))/a^4 - 3*sqrt(2)*log(x^2 - sqrt(2)*x/sqrt(abs(a)) + 1/abs(a))/(a^2*ab
s(a)^(3/2)))
```

3.80.9 Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.36

$$\int x^4 \cot^{-1}\left(ax^2\right) \, dx = \frac{x^5 \operatorname{acot}(a\,x^2)}{5} + \frac{2\,x^3}{15\,a} - \frac{(-1)^{1/4} \operatorname{atan}\left((-1)^{1/4}\,\sqrt{a}\,x\right)}{5\,a^{5/2}} - \frac{(-1)^{1/4} \operatorname{atan}\left((-1)^{1/4}\,\sqrt{a}\,x\,1\mathrm{i}\right)\,1\mathrm{i}}{5\,a^{5/2}}$$

input
```
int(x^4*acot(a*x^2),x)
```

output
```
(x^5*acot(a*x^2))/5 + (2*x^3)/(15*a) - ((-1)^(1/4)*atan((-1)^(1/4)*a^(1/2)
*x))/(5*a^(5/2)) - ((-1)^(1/4)*atan((-1)^(1/4)*a^(1/2)*x*1i)*1i)/(5*a^(5/2
))
```

3.81 $\int x^2 \cot^{-1}(ax^2)\,dx$

3.81.1 Optimal result

Integrand size = 10, antiderivative size = 150

$$\int x^2 \cot^{-1}(ax^2)\,dx = \frac{2x}{3a}+\frac{1}{3}x^3\cot^{-1}(ax^2)+\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{3\sqrt{2}a^{3/2}}-\frac{\arctan\left(1+\sqrt{2}\sqrt{a}x\right)}{3\sqrt{2}a^{3/2}}+\frac{\log\left(1-\sqrt{2}\sqrt{a}x+ax^2\right)}{6\sqrt{2}a^{3/2}}-\frac{\log\left(1+\sqrt{2}\sqrt{a}x+ax^2\right)}{6\sqrt{2}a^{3/2}}$$

output
```
2/3*x/a+1/3*x^3*arccot(a*x^2)-1/6*arctan(-1+x*2^(1/2)*a^(1/2))/a^(3/2)*2^(
1/2)-1/6*arctan(1+x*2^(1/2)*a^(1/2))/a^(3/2)*2^(1/2)+1/12*ln(1+a*x^2-x*2^(
1/2)*a^(1/2))/a^(3/2)*2^(1/2)-1/12*ln(1+a*x^2+x*2^(1/2)*a^(1/2))/a^(3/2)*2
^(1/2)
```

3.81.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.89

$$\int x^2 \cot^{-1}(ax^2)\,dx = \frac{8\sqrt{a}x+4a^{3/2}x^3\cot^{-1}(ax^2)+2\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{a}x\right)-2\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{a}x\right)+\sqrt{2}\log\left(1-\sqrt{2}\sqrt{}\right.}{12a^{3/2}}$$

input
```
Integrate[x^2*ArcCot[a*x^2],x]
```

output
```
(8*Sqrt[a]*x + 4*a^(3/2)*x^3*ArcCot[a*x^2] + 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*
Sqrt[a]*x] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[a]*x] + Sqrt[2]*Log[1 - Sqr
t[2]*Sqrt[a]*x + a*x^2] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[a]*x + a*x^2])/(12*
a^(3/2))
```

3.81.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5362, 843, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cot^{-1}\left(ax^2\right) dx$$

$$\Big\downarrow \text{5362}$$

$$\frac{2}{3}a\int \frac{x^4}{a^2x^4+1}dx+\frac{1}{3}x^3\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \text{843}$$

$$\frac{2}{3}a\left(\frac{x}{a^2}-\frac{\int \frac{1}{a^2x^4+1}dx}{a^2}\right)+\frac{1}{3}x^3\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \text{755}$$

$$\frac{2}{3}a\left(\frac{x}{a^2}-\frac{\frac{1}{2}\int \frac{1-ax^2}{a^2x^4+1}dx+\frac{1}{2}\int \frac{ax^2+1}{a^2x^4+1}dx}{a^2}\right)+\frac{1}{3}x^3\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \text{1476}$$

$$\frac{2}{3}a\left(\frac{x}{a^2}-\frac{\frac{1}{2}\int \frac{1-ax^2}{a^2x^4+1}dx+\frac{1}{2}\left(\frac{\int \frac{1}{x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}}dx}{2a}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}}dx}{2a}\right)}{a^2}\right)+\frac{1}{3}x^3\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{2}{3}a\left(\frac{x}{a^2}-\frac{\frac{1}{2}\int\frac{1-ax^2}{a^2x^4+1}dx+\frac{1}{2}\left(\frac{\int\frac{1}{-\left(1-\sqrt{2}\sqrt{a}x\right)^2-1}d\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}-\frac{\int\frac{1}{-\left(\sqrt{2}\sqrt{a}x+1\right)^2-1}d\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}\right)}{a^2}\right)+\frac{1}{3}x^3\cot^{-1}\left(ax^2\right)$$

$\downarrow$ 217

$$\frac{2}{3}a\left(\frac{x}{a^2}-\frac{\frac{1}{2}\int\frac{1-ax^2}{a^2x^4+1}dx+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}\right)}{a^2}\right)+\frac{1}{3}x^3\cot^{-1}\left(ax^2\right)$$

$\downarrow$ 1479

$$\frac{2}{3}a\left(\frac{x}{a^2}-\frac{\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{a}x}{\sqrt{a}\left(x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{a}\left(x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}\right)}{a^2}\right)+\frac{1}{3}x^3\cot^{-1}\left(ax^2\right)$$

$\downarrow$ 25

$$\frac{2}{3}a\left(\frac{x}{a^2}-\frac{\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{a}x}{\sqrt{a}\left(x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{a}\left(x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}\right)}{a^2}\right)+\frac{1}{3}x^3\cot^{-1}\left(ax^2\right)$$

$\downarrow$ 27

3.81. $\int x^2\cot^{-1}\left(ax^2\right)\,dx$

$$\frac{2}{3}a\left(\frac{x}{a^2}-\frac{\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{a}x}{x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}}dx}{2\sqrt{2}a}+\frac{\int \frac{\sqrt{2}\sqrt{a}x+1}{x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}}dx}{2a}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}\right)}{a^2}\right)+\frac{1}{3}x^3\cot^{-1}\left(ax^2\right)$$

$\big\downarrow$ 1103

$$\frac{2}{3}a\left(\frac{x}{a^2}-\frac{\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}\right)+\frac{1}{2}\left(\frac{\log\left(ax^2+\sqrt{2}\sqrt{a}x+1\right)}{2\sqrt{2}\sqrt{a}}-\frac{\log\left(ax^2-\sqrt{2}\sqrt{a}x+1\right)}{2\sqrt{2}\sqrt{a}}\right)}{a^2}\right)+\frac{1}{3}x^3\cot^{-1}\left(ax^2\right)$$

input
```
Int[x^2*ArcCot[a*x^2],x]
```

output
```
(x^3*ArcCot[a*x^2])/3 + (2*a*(x/a^2 - ((-(ArcTan[1 - Sqrt[2]*Sqrt[a]*x]/(S
qrt[2]*Sqrt[a])) + ArcTan[1 + Sqrt[2]*Sqrt[a]*x]/(Sqrt[2]*Sqrt[a]))/2 + (-
1/2*Log[1 - Sqrt[2]*Sqrt[a]*x + a*x^2]/(Sqrt[2]*Sqrt[a]) + Log[1 + Sqrt[2]
*Sqrt[a]*x + a*x^2]/(2*Sqrt[2]*Sqrt[a]))/2)/a^2))/3
```

3.81.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.81. $\int x^2 \cot^{-1}(ax^2)\,dx$

3.81.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{x^3 \operatorname{arccot}(a\,x^2)}{3}+\frac{2a\left(\frac{x}{a^2}-\frac{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{1}{a^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^2}}}{x^2-\left(\frac{1}{a^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}-1\right)\right)}{8a^2}\right)}{3}$	109
parts	$\frac{x^3 \operatorname{arccot}(a\,x^2)}{3}+\frac{2a\left(\frac{x}{a^2}-\frac{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{1}{a^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^2}}}{x^2-\left(\frac{1}{a^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}-1\right)\right)}{8a^2}\right)}{3}$	109

input
```
int(x^2*arccot(a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*arccot(a*x^2)+2/3*a*(x/a^2-1/8/a^2*(1/a^2)^(1/4)*2^(1/2)*(ln((x^2+
(1/a^2)^(1/4)*x*2^(1/2)+(1/a^2)^(1/2))/(x^2-(1/a^2)^(1/4)*x*2^(1/2)+(1/a^2
)^(1/2)))+2*arctan(2^(1/2)/(1/a^2)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/a^2)^(1/
4)*x-1)))
```

3.81.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.74

$$\int x^2 \cot^{-1}\left(ax^2\right)\,dx$$
$$= \frac{2\,ax^3\operatorname{arccot}\left(ax^2\right)-a\left(-\frac{1}{a^6}\right)^{\frac{1}{4}}\log\left(a\left(-\frac{1}{a^6}\right)^{\frac{1}{4}}+x\right)-i\,a\left(-\frac{1}{a^6}\right)^{\frac{1}{4}}\log\left(i\,a\left(-\frac{1}{a^6}\right)^{\frac{1}{4}}+x\right)+i\,a\left(-\frac{1}{a^6}\right)^{\frac{1}{4}}\log\left(-i\,\right.}{6\,a}$$

input
```
integrate(x^2*arccot(a*x^2),x, algorithm="fricas")
```

output
```
1/6*(2*a*x^3*arccot(a*x^2) - a*(-1/a^6)^(1/4)*log(a*(-1/a^6)^(1/4) + x) -
I*a*(-1/a^6)^(1/4)*log(I*a*(-1/a^6)^(1/4) + x) + I*a*(-1/a^6)^(1/4)*log(-I
*a*(-1/a^6)^(1/4) + x) + a*(-1/a^6)^(1/4)*log(-a*(-1/a^6)^(1/4) + x) + 4*x
)/a
```

3.81.6 Sympy [A] (verification not implemented)

Time = 5.37 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.84

$$\int x^2 \cot^{-1}(ax^2)\, dx$$
$$= \begin{cases} \frac{x^3 \operatorname{acot}(ax^2)}{3} + \frac{\left(-\frac{1}{a^2}\right)^{\frac{3}{4}} \operatorname{acot}(ax^2)}{3} + \frac{2x}{3a} + \frac{\sqrt[4]{-\frac{1}{a^2}} \log\left(x - \sqrt[4]{-\frac{1}{a^2}}\right)}{3a} - \frac{\sqrt[4]{-\frac{1}{a^2}} \log\left(x^2 + \sqrt{-\frac{1}{a^2}}\right)}{6a} - \frac{\sqrt[4]{-\frac{1}{a^2}} \operatorname{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{a^2}}}\right)}{3a} \\ \frac{\pi x^3}{6} \end{cases}$$

input
```
integrate(x**2*acot(a*x**2),x)
```

output
```
Piecewise((x**3*acot(a*x**2)/3 + (-1/a**2)**(3/4)*acot(a*x**2)/3 + 2*x/(3*
a) + (-1/a**2)**(1/4)*log(x - (-1/a**2)**(1/4))/(3*a) - (-1/a**2)**(1/4)*l
og(x**2 + sqrt(-1/a**2))/(6*a) - (-1/a**2)**(1/4)*atan(x/(-1/a**2)**(1/4))
/(3*a), Ne(a, 0)), (pi*x**3/6, True))
```

3.81.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.90

$$\int x^2 \cot^{-1}(ax^2)\, dx = \frac{1}{3} x^3 \operatorname{arccot}(ax^2) + \frac{1}{12} a \left(\frac{8x}{a^2} - \frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}(2ax+\sqrt{2}\sqrt{a})}{2\sqrt{a}}\right)}{\sqrt{a}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}(2ax-\sqrt{2}\sqrt{a})}{2\sqrt{a}}\right)}{\sqrt{a}} + \frac{\sqrt{2}\log\left(ax^2+\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{a}} - \frac{\sqrt{2}\log\left(ax^2-\sqrt{2}\sqrt{a}x\right.}{\sqrt{a}}}{a^2} \right.$$

input
```
integrate(x^2*arccot(a*x^2),x, algorithm="maxima")
```

output
```
1/3*x^3*arccot(a*x^2) + 1/12*a*(8*x/a^2 - (2*sqrt(2)*arctan(1/2*sqrt(2)*(2
*a*x + sqrt(2)*sqrt(a))/sqrt(a))/sqrt(a) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2
*a*x - sqrt(2)*sqrt(a))/sqrt(a))/sqrt(a) + sqrt(2)*log(a*x^2 + sqrt(2)*sqr
t(a)*x + 1)/sqrt(a) - sqrt(2)*log(a*x^2 - sqrt(2)*sqrt(a)*x + 1)/sqrt(a))/
a^2)
```

3.81.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.02

$$\int x^2 \cot^{-1}\left(ax^2\right) dx = \frac{1}{3}x^3 \arctan\left(\frac{1}{ax^2}\right) + \frac{1}{12}a\left(\frac{8x}{a^2} - \frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\frac{\sqrt{2}}{\sqrt{|a|}}\right)\sqrt{|a|}\right)}{a^2\sqrt{|a|}} - \frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\frac{\sqrt{2}}{\sqrt{|a|}}\right)\sqrt{|a|}\right)}{a^2\sqrt{|a|}} - \frac{\sqrt{2}\log}{}\right.$$

input
```
integrate(x^2*arccot(a*x^2),x, algorithm="giac")
```

output
```
1/3*x^3*arctan(1/(a*x^2)) + 1/12*a*(8*x/a^2 - 2*sqrt(2)*arctan(1/2*sqrt(2)
*(2*x + sqrt(2)/sqrt(abs(a)))*sqrt(abs(a)))/(a^2*sqrt(abs(a))) - 2*sqrt(2)
*arctan(1/2*sqrt(2)*(2*x - sqrt(2)/sqrt(abs(a)))*sqrt(abs(a)))/(a^2*sqrt(a
bs(a))) - sqrt(2)*log(x^2 + sqrt(2)*x/sqrt(abs(a)) + 1/abs(a))/(a^2*sqrt(a
bs(a))) + sqrt(2)*log(x^2 - sqrt(2)*x/sqrt(abs(a)) + 1/abs(a))/(a^2*sqrt(a
bs(a))))
```

3.81.9 Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.35

$$\int x^2 \cot^{-1}\left(ax^2\right) dx = \frac{x^3\,\mathrm{acot}(a\,x^2)}{3} + \frac{2\,x}{3\,a} + \frac{(-1)^{1/4}\,\mathrm{atan}\left((-1)^{1/4}\sqrt{a}\,x\right)\,1\mathrm{i}}{3\,a^{3/2}} + \frac{(-1)^{1/4}\,\mathrm{atan}\left((-1)^{1/4}\sqrt{a}\,x\,1\mathrm{i}\right)}{3\,a^{3/2}}$$

input
```
int(x^2*acot(a*x^2),x)
```

output
```
(x^3*acot(a*x^2))/3 + (2*x)/(3*a) + ((-1)^(1/4)*atan((-1)^(1/4)*a^(1/2)*x)
*1i)/(3*a^(3/2)) + ((-1)^(1/4)*atan((-1)^(1/4)*a^(1/2)*x*1i))/(3*a^(3/2))
```

3.82 $\int \cot^{-1}(ax^2)\,dx$

3.82.1 Optimal result

Integrand size = 6, antiderivative size = 132

$$\int \cot^{-1}(ax^2)\,dx = x\cot^{-1}(ax^2) - \frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}} + \frac{\arctan\left(1+\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}} + \frac{\log\left(1-\sqrt{2}\sqrt{a}x+ax^2\right)}{2\sqrt{2}\sqrt{a}} - \frac{\log\left(1+\sqrt{2}\sqrt{a}x+ax^2\right)}{2\sqrt{2}\sqrt{a}}$$

output
```
x*arccot(a*x^2)+1/2*arctan(-1+x*2^(1/2)*a^(1/2))*2^(1/2)/a^(1/2)+1/2*arcta
n(1+x*2^(1/2)*a^(1/2))*2^(1/2)/a^(1/2)+1/4*ln(1+a*x^2-x*2^(1/2)*a^(1/2))*2
^(1/2)/a^(1/2)-1/4*ln(1+a*x^2+x*2^(1/2)*a^(1/2))*2^(1/2)/a^(1/2)
```

3.82.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.77

$$\int \cot^{-1}(ax^2)\,dx = x\cot^{-1}(ax^2) + \frac{-2\arctan\left(1-\sqrt{2}\sqrt{a}x\right)+2\arctan\left(1+\sqrt{2}\sqrt{a}x\right)+\log\left(1-\sqrt{2}\sqrt{a}x+ax^2\right)-\log\left(1+\sqrt{2}\sqrt{a}x+ax^2\right)}{2\sqrt{2}\sqrt{a}}$$

input
```
Integrate[ArcCot[a*x^2],x]
```

output
```
x*ArcCot[a*x^2] + (-2*ArcTan[1 - Sqrt[2]*Sqrt[a]*x] + 2*ArcTan[1 + Sqrt[2]
*Sqrt[a]*x] + Log[1 - Sqrt[2]*Sqrt[a]*x + a*x^2] - Log[1 + Sqrt[2]*Sqrt[a]
*x + a*x^2])/(2*Sqrt[2]*Sqrt[a])
```

3.82.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {5346, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}\left(ax^2\right) dx$$

$$\Big\downarrow \text{5346}$$

$$2a\int \frac{x^2}{a^2x^4+1}dx+x\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \text{826}$$

$$2a\left(\frac{\int \frac{ax^2+1}{a^2x^4+1}dx}{2a}-\frac{\int \frac{1-ax^2}{a^2x^4+1}dx}{2a}\right)+x\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \text{1476}$$

$$2a\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}}dx}{2a}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}}dx}{2a}}{2a}-\frac{\int \frac{1-ax^2}{a^2x^4+1}dx}{2a}\right)+x\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \text{1082}$$

$$2a\left(\frac{\frac{\int \frac{1}{-\left(1-\sqrt{2}\sqrt{a}x\right)^2-1}d\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}-\frac{\int \frac{1}{-\left(\sqrt{2}\sqrt{a}x+1\right)^2-1}d\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}}{2a}-\frac{\int \frac{1-ax^2}{a^2x^4+1}dx}{2a}\right)+x\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \text{217}$$

$$2a\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}}{2a}-\frac{\int \frac{1-ax^2}{a^2x^4+1}dx}{2a}\right)+x\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \ 1479$$

$$2a\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}}{2a}-\frac{-\frac{\int -\frac{\sqrt{2}-2\sqrt{a}x}{\sqrt{a}\left(x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{a}\left(x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}}{2a}\right)+ x\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \ 25$$

$$2a\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}}{2a}-\frac{\frac{\int \frac{\sqrt{2}-2\sqrt{a}x}{\sqrt{a}\left(x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{a}\left(x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}}{2a}\right)+ x\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \ 27$$

$$2a\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}}{2a}-\frac{\frac{\int \frac{\sqrt{2}-2\sqrt{a}x}{x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}}dx}{2\sqrt{2}a}+\frac{\int \frac{\sqrt{2}\sqrt{a}x+1}{x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}}dx}{2a}}{2a}\right)+x\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \ 1103$$

$$2a\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}}{2a}-\frac{\frac{\log\left(ax^2+\sqrt{2}\sqrt{a}x+1\right)}{2\sqrt{2}\sqrt{a}}-\frac{\log\left(ax^2-\sqrt{2}\sqrt{a}x+1\right)}{2\sqrt{2}\sqrt{a}}}{2a}\right)+x\cot^{-1}\left(ax^2\right)$$

input
```
Int[ArcCot[a*x^2],x]
```

output
```
x*ArcCot[a*x^2] + 2*a*((-(ArcTan[1 - Sqrt[2]*Sqrt[a]*x]/(Sqrt[2]*Sqrt[a]))
 + ArcTan[1 + Sqrt[2]*Sqrt[a]*x]/(Sqrt[2]*Sqrt[a]))/(2*a) - (-1/2*Log[1 -
Sqrt[2]*Sqrt[a]*x + a*x^2]/(Sqrt[2]*Sqrt[a]) + Log[1 + Sqrt[2]*Sqrt[a]*x +
 a*x^2]/(2*Sqrt[2]*Sqrt[a]))/(2*a))
```

3.82. $\int \cot^{-1}\left(ax^2\right)\,dx$

3.82.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

3.82.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.73

method	result	size
default	$x \,\mathrm{arccot}\left(a \,x^{2}\right)+\frac{\sqrt{2}\, \left(\ln \left(\frac{x^{2}-\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{1}{a^{2}}}}{x^{2}+\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{1}{a^{2}}}}\right)+2 \arctan \left(\frac{\sqrt{2}\, x}{\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}}+1\right)+2 \arctan \left(\frac{\sqrt{2}\, x}{\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}}-1\right)\right)}{4 a \left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}}$	97
parts	$x \,\mathrm{arccot}\left(a \,x^{2}\right)+\frac{\sqrt{2}\, \left(\ln \left(\frac{x^{2}-\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{1}{a^{2}}}}{x^{2}+\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{1}{a^{2}}}}\right)+2 \arctan \left(\frac{\sqrt{2}\, x}{\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}}+1\right)+2 \arctan \left(\frac{\sqrt{2}\, x}{\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}}-1\right)\right)}{4 a \left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}}$	97

input
```
int(arccot(a*x^2),x,method=_RETURNVERBOSE)
```

output
```
x*arccot(a*x^2)+1/4/a/(1/a^2)^(1/4)*2^(1/2)*(ln((x^2-(1/a^2)^(1/4)*x*2^(1/
2)+(1/a^2)^(1/2))/(x^2+(1/a^2)^(1/4)*x*2^(1/2)+(1/a^2)^(1/2)))+2*arctan(2^
(1/2)/(1/a^2)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/a^2)^(1/4)*x-1))
```

3.82.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.73

$$\int \cot^{-1}\left(ax^2\right) dx = x \operatorname{arccot}\left(ax^2\right) + \frac{1}{2}\left(-\frac{1}{a^2}\right)^{\frac{1}{4}} \log\left(a\left(-\frac{1}{a^2}\right)^{\frac{3}{4}} + x\right) - \frac{1}{2}i\left(-\frac{1}{a^2}\right)^{\frac{1}{4}} \log\left(i\,a\left(-\frac{1}{a^2}\right)^{\frac{3}{4}} + x\right) + \frac{1}{2}i\left(-\frac{1}{a^2}\right)^{\frac{1}{4}} \log\left(-i\,a\left(-\frac{1}{a^2}\right)^{\frac{3}{4}} + x\right) - \frac{1}{2}\left(-\frac{1}{a^2}\right)^{\frac{1}{4}} \log\left(-a\left(-\frac{1}{a^2}\right)^{\frac{3}{4}} + x\right)$$

input
```
integrate(arccot(a*x^2),x, algorithm="fricas")
```

output
```
x*arccot(a*x^2) + 1/2*(-1/a^2)^(1/4)*log(a*(-1/a^2)^(3/4) + x) - 1/2*I*(-1
/a^2)^(1/4)*log(I*a*(-1/a^2)^(3/4) + x) + 1/2*I*(-1/a^2)^(1/4)*log(-I*a*(-
1/a^2)^(3/4) + x) - 1/2*(-1/a^2)^(1/4)*log(-a*(-1/a^2)^(3/4) + x)
```

3.82.6 Sympy [A] (verification not implemented)

Time = 2.98 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.83

$$\int \cot^{-1}\left(ax^2\right) dx = \begin{cases} x \operatorname{acot}\left(ax^2\right) + \sqrt[4]{-\frac{1}{a^2}} \operatorname{acot}\left(ax^2\right) + \frac{\log\left(x - \sqrt[4]{-\frac{1}{a^2}}\right)}{a\sqrt[4]{-\frac{1}{a^2}}} - \frac{\log\left(x^2 + \sqrt{-\frac{1}{a^2}}\right)}{2a\sqrt[4]{-\frac{1}{a^2}}} + \frac{\operatorname{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{a^2}}}\right)}{a\sqrt[4]{-\frac{1}{a^2}}} & \text{for } a \neq 0 \\ \frac{\pi x}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(acot(a*x**2),x)
```

output
```
Piecewise((x*acot(a*x**2) + (-1/a**2)**(1/4)*acot(a*x**2) + log(x - (-1/a*
*2)**(1/4))/(a*(-1/a**2)**(1/4)) - log(x**2 + sqrt(-1/a**2))/(2*a*(-1/a**2
)**(1/4)) + atan(x/(-1/a**2)**(1/4))/(a*(-1/a**2)**(1/4)), Ne(a, 0)), (pi*
x/2, True))
```

3.82.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.91

$$\int \cot^{-1}\left(ax^2\right)\,dx$$
$$=\frac{1}{4}a\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2ax+\sqrt{2}\sqrt{a}\right)}{2\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2ax-\sqrt{2}\sqrt{a}\right)}{2\sqrt{a}}\right)}{a^{\frac{3}{2}}}-\frac{\sqrt{2}\log\left(ax^2+\sqrt{2}\sqrt{a}x+1\right)}{a^{\frac{3}{2}}}+\sqrt{2}\right.$$
$$+x\,\mathrm{arccot}\left(ax^2\right)$$

input
```
integrate(arccot(a*x^2),x, algorithm="maxima")
```

output
```
1/4*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*a*x + sqrt(2)*sqrt(a))/sqrt(a))/a^(
3/2) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*a*x - sqrt(2)*sqrt(a))/sqrt(a))/a^(
3/2) - sqrt(2)*log(a*x^2 + sqrt(2)*sqrt(a)*x + 1)/a^(3/2) + sqrt(2)*log(a*
x^2 - sqrt(2)*sqrt(a)*x + 1)/a^(3/2)) + x*arccot(a*x^2)
```

3.82.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.09

$$\int \cot^{-1}\left(ax^2\right)\,dx$$
$$=\frac{1}{4}a\left(\frac{2\sqrt{2}\sqrt{|a|}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\frac{\sqrt{2}}{\sqrt{|a|}}\right)\sqrt{|a|}\right)}{a^2}+\frac{2\sqrt{2}\sqrt{|a|}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\frac{\sqrt{2}}{\sqrt{|a|}}\right)\sqrt{|a|}\right)}{a^2}-\sqrt{2}\sqrt{|a}\right.$$
$$+x\arctan\left(\frac{1}{ax^2}\right)$$

input
```
integrate(arccot(a*x^2),x, algorithm="giac")
```

output
```
1/4*a*(2*sqrt(2)*sqrt(abs(a))*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(a
)))*sqrt(abs(a)))/a^2 + 2*sqrt(2)*sqrt(abs(a))*arctan(1/2*sqrt(2)*(2*x - s
qrt(2)/sqrt(abs(a)))*sqrt(abs(a)))/a^2 - sqrt(2)*sqrt(abs(a))*log(x^2 + sq
rt(2)*x/sqrt(abs(a)) + 1/abs(a))/a^2 + sqrt(2)*sqrt(abs(a))*log(x^2 - sqrt
(2)*x/sqrt(abs(a)) + 1/abs(a))/a^2) + x*arctan(1/(a*x^2))
```

3.82.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.32

$$\int \cot^{-1}\left(ax^2\right)\,dx = x\,\mathrm{acot}\left(a\,x^2\right) + \frac{(-1)^{1/4}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{a}\,x\right)}{\sqrt{a}} - \frac{(-1)^{1/4}\,\mathrm{atanh}\left((-1)^{1/4}\,\sqrt{a}\,x\right)}{\sqrt{a}}$$

input
```
int(acot(a*x^2),x)
```

output
```
x*acot(a*x^2) + ((-1)^(1/4)*atan((-1)^(1/4)*a^(1/2)*x))/a^(1/2) - ((-1)^(1
/4)*atanh((-1)^(1/4)*a^(1/2)*x))/a^(1/2)
```

3.83 $\int \frac{\cot^{-1}(ax^2)}{x^2} dx$

3.83.1 Optimal result

Integrand size = 10, antiderivative size = 135

$$\int \frac{\cot^{-1}(ax^2)}{x^2} dx = -\frac{\cot^{-1}(ax^2)}{x} + \frac{\sqrt{a}\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}} - \frac{\sqrt{a}\arctan\left(1+\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}} + \frac{\sqrt{a}\log\left(1-\sqrt{2}\sqrt{a}x+ax^2\right)}{2\sqrt{2}} - \frac{\sqrt{a}\log\left(1+\sqrt{2}\sqrt{a}x+ax^2\right)}{2\sqrt{2}}$$

output
```
-arccot(a*x^2)/x-1/2*arctan(-1+x*2^(1/2)*a^(1/2))*a^(1/2)*2^(1/2)-1/2*arct
an(1+x*2^(1/2)*a^(1/2))*a^(1/2)*2^(1/2)+1/4*ln(1+a*x^2-x*2^(1/2)*a^(1/2))*
a^(1/2)*2^(1/2)-1/4*ln(1+a*x^2+x*2^(1/2)*a^(1/2))*a^(1/2)*2^(1/2)
```

3.83.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.78

$$\int \frac{\cot^{-1}(ax^2)}{x^2} dx = -\frac{\cot^{-1}(ax^2)}{x} + \frac{\sqrt{a}\left(2\arctan\left(1-\sqrt{2}\sqrt{a}x\right) - 2\arctan\left(1+\sqrt{2}\sqrt{a}x\right) + \log\left(1-\sqrt{2}\sqrt{a}x+ax^2\right) - \log\left(1+\sqrt{2}\sqrt{a}x+a\right.}{2\sqrt{2}}$$

input
```
Integrate[ArcCot[a*x^2]/x^2,x]
```

output
```
-(ArcCot[a*x^2]/x) + (Sqrt[a]*(2*ArcTan[1 - Sqrt[2]*Sqrt[a]*x] - 2*ArcTan[
1 + Sqrt[2]*Sqrt[a]*x] + Log[1 - Sqrt[2]*Sqrt[a]*x + a*x^2] - Log[1 + Sqrt
[2]*Sqrt[a]*x + a*x^2]))/(2*Sqrt[2])
```

3.83.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {5362, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax^2)}{x^2}dx$$

$$\Big\downarrow \text{5362}$$

$$-2a\int \frac{1}{a^2x^4+1}dx-\frac{\cot^{-1}(ax^2)}{x}$$

$$\Big\downarrow \text{755}$$

$$-2a\left(\frac{1}{2}\int \frac{1-ax^2}{a^2x^4+1}dx+\frac{1}{2}\int \frac{ax^2+1}{a^2x^4+1}dx\right)-\frac{\cot^{-1}(ax^2)}{x}$$

$$\Big\downarrow \text{1476}$$

$$-2a\left(\frac{1}{2}\int \frac{1-ax^2}{a^2x^4+1}dx+\frac{1}{2}\left(\frac{\int \frac{1}{x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}}dx}{2a}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}}dx}{2a}\right)\right)-\frac{\cot^{-1}(ax^2)}{x}$$

$$\Big\downarrow \text{1082}$$

$$-2a\left(\frac{1}{2}\int \frac{1-ax^2}{a^2x^4+1}dx+\frac{1}{2}\left(\frac{\int \frac{1}{-\left(1-\sqrt{2}\sqrt{a}x\right)^2-1}d\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}-\frac{\int \frac{1}{-\left(\sqrt{2}\sqrt{a}x+1\right)^2-1}d\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}\right)\right)-\frac{\cot^{-1}(ax^2)}{x}$$

$$\Big\downarrow \text{217}$$

$$-2a\left(\frac{1}{2}\int \frac{1-ax^2}{a^2x^4+1}dx+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}\right)\right)-\frac{\cot^{-1}(ax^2)}{x}$$

$$\Big\downarrow \text{1479}$$

$$-2a\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{a}x}{\sqrt{a}\left(x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{a}\left(x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}\right)\right.$$
$$\frac{\cot^{-1}\left(ax^2\right)}{x}$$

$$\Big\downarrow \textcolor{blue}{25}$$

$$-2a\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{a}x}{\sqrt{a}\left(x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{a}\left(x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}\right)\right)-$$
$$\frac{\cot^{-1}\left(ax^2\right)}{x}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$-2a\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{a}x}{x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}}dx}{2\sqrt{2}a}+\frac{\int \frac{\sqrt{2}\sqrt{a}x+1}{x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}}dx}{2a}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}\right)\right)-$$
$$\frac{\cot^{-1}\left(ax^2\right)}{x}$$

$$\Big\downarrow \textcolor{blue}{1103}$$

$$-2a\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}\right)+\frac{1}{2}\left(\frac{\log\left(ax^2+\sqrt{2}\sqrt{a}x+1\right)}{2\sqrt{2}\sqrt{a}}-\frac{\log\left(ax^2-\sqrt{2}\sqrt{a}x+1\right.}{2\sqrt{2}\sqrt{a}}\right.\right.$$
$$\frac{\cot^{-1}\left(ax^2\right)}{x}$$

input
```
Int[ArcCot[a*x^2]/x^2,x]
```

output
```
-(ArcCot[a*x^2]/x) - 2*a*((-(ArcTan[1 - Sqrt[2]*Sqrt[a]*x]/(Sqrt[2]*Sqrt[a
])) + ArcTan[1 + Sqrt[2]*Sqrt[a]*x]/(Sqrt[2]*Sqrt[a]))/2 + (-1/2*Log[1 - S
qrt[2]*Sqrt[a]*x + a*x^2]/(Sqrt[2]*Sqrt[a]) + Log[1 + Sqrt[2]*Sqrt[a]*x +
a*x^2]/(2*Sqrt[2]*Sqrt[a]))/2)
```

3.83. $\int \frac{\cot^{-1}\left(ax^2\right)}{x^2}\,dx$

3.83.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.83. $\int \frac{\cot^{-1}(ax^2)}{x^2} dx$

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.83.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.73

method	result	size
default	$-\frac{\operatorname{arccot}\left(a\,x^{2}\right)}{x}-\frac{a\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^{2}}}}{x^{2}-\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^{2}}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}}-1\right)\right)}{4}$	98
parts	$-\frac{\operatorname{arccot}\left(a\,x^{2}\right)}{x}-\frac{a\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^{2}}}}{x^{2}-\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^{2}}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}}-1\right)\right)}{4}$	98

input
```
int(arccot(a*x^2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-arccot(a*x^2)/x-1/4*a*(1/a^2)^(1/4)*2^(1/2)*(ln((x^2+(1/a^2)^(1/4)*x*2^(1
/2)+(1/a^2)^(1/2))/(x^2-(1/a^2)^(1/4)*x*2^(1/2)+(1/a^2)^(1/2)))+2*arctan(2
^(1/2)/(1/a^2)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/a^2)^(1/4)*x-1))
```

3.83.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.79

$$\int \frac{\cot^{-1}(ax^2)}{x^2}\,dx =$$
$$-\frac{(-a^2)^{\frac{1}{4}}\,x\log\left(ax+(-a^2)^{\frac{1}{4}}\right)+i\,(-a^2)^{\frac{1}{4}}\,x\log\left(ax+i\,(-a^2)^{\frac{1}{4}}\right)-i\,(-a^2)^{\frac{1}{4}}\,x\log\left(ax-i\,(-a^2)^{\frac{1}{4}}\right)-(-a}{2\,x}$$

input
```
integrate(arccot(a*x^2)/x^2,x, algorithm="fricas")
```

output
```
-1/2*((-a^2)^(1/4)*x*log(a*x + (-a^2)^(1/4)) + I*(-a^2)^(1/4)*x*log(a*x +
I*(-a^2)^(1/4)) - I*(-a^2)^(1/4)*x*log(a*x - I*(-a^2)^(1/4)) - (-a^2)^(1/4
)*x*log(a*x - (-a^2)^(1/4)) + 2*arccot(a*x^2))/x
```

3.83.6 Sympy [A] (verification not implemented)

Time = 6.18 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.84

$$\int \frac{\cot^{-1}(ax^2)}{x^2}\,dx = \begin{cases} a^2\left(-\frac{1}{a^2}\right)^{\frac{3}{4}}\operatorname{acot}(ax^2) + a\sqrt[4]{-\frac{1}{a^2}}\log\left(x - \sqrt[4]{-\frac{1}{a^2}}\right) - \frac{a\sqrt[4]{-\frac{1}{a^2}}\log\left(x^2+\sqrt{-\frac{1}{a^2}}\right)}{2} - a\sqrt[4]{-\frac{1}{a^2}}\operatorname{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{a^2}}}\right) - \\ -\frac{\pi}{2x} \end{cases}$$

input
```
integrate(acot(a*x**2)/x**2,x)
```

output
```
Piecewise((a**2*(-1/a**2)**(3/4)*acot(a*x**2) + a*(-1/a**2)**(1/4)*log(x -
 (-1/a**2)**(1/4)) - a*(-1/a**2)**(1/4)*log(x**2 + sqrt(-1/a**2))/2 - a*(-
1/a**2)**(1/4)*atan(x/(-1/a**2)**(1/4)) - acot(a*x**2)/x, Ne(a, 0)), (-pi/
(2*x), True))
```

3.83.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.91

$$\int \frac{\cot^{-1}(ax^2)}{x^2}\,dx = -\frac{1}{4}a\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,ax+\sqrt{2}\sqrt{a}\right)}{2\sqrt{a}}\right)}{\sqrt{a}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,ax-\sqrt{2}\sqrt{a}\right)}{2\sqrt{a}}\right)}{\sqrt{a}} + \frac{\sqrt{2}\log\left(ax^2+\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{a}} - \right. - \frac{\operatorname{arccot}(ax^2)}{x}$$

input
```
integrate(arccot(a*x^2)/x^2,x, algorithm="maxima")
```

output
```
-1/4*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*a*x + sqrt(2)*sqrt(a))/sqrt(a))/sq
rt(a) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*a*x - sqrt(2)*sqrt(a))/sqrt(a))/sq
rt(a) + sqrt(2)*log(a*x^2 + sqrt(2)*sqrt(a)*x + 1)/sqrt(a) - sqrt(2)*log(a
*x^2 - sqrt(2)*sqrt(a)*x + 1)/sqrt(a)) - arccot(a*x^2)/x
```

3.83.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(ax^2)}{x^2}\,dx = -\frac{1}{4}\,a\left(\frac{2\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\frac{\sqrt{2}}{\sqrt{|a|}}\right)\sqrt{|a|}\right)}{\sqrt{|a|}}+\frac{2\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\frac{\sqrt{2}}{\sqrt{|a|}}\right)\sqrt{|a|}\right)}{\sqrt{|a|}}+\frac{\sqrt{2}\log\left(x^2+\frac{\sqrt{2}}{\sqrt{|a|}}\right.}{\sqrt{|a|}}\right. - \frac{\arctan\left(\frac{1}{ax^2}\right)}{x}$$

input
```
integrate(arccot(a*x^2)/x^2,x, algorithm="giac")
```

output
```
-1/4*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(a)))*sqrt(abs
(a)))/sqrt(abs(a)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)/sqrt(abs(
a)))*sqrt(abs(a)))/sqrt(abs(a)) + sqrt(2)*log(x^2 + sqrt(2)*x/sqrt(abs(a))
 + 1/abs(a))/sqrt(abs(a)) - sqrt(2)*log(x^2 - sqrt(2)*x/sqrt(abs(a)) + 1/a
bs(a))/sqrt(abs(a))) - arctan(1/(a*x^2))/x
```

3.83.9 Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.33

$$\int \frac{\cot^{-1}(ax^2)}{x^2}\,dx = -\frac{\mathrm{acot}(a\,x^2)}{x}+(-1)^{1/4}\,\sqrt{a}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{a}\,x\right)\,1\mathrm{i} + (-1)^{1/4}\,\sqrt{a}\,\mathrm{atanh}\left((-1)^{1/4}\,\sqrt{a}\,x\right)\,1\mathrm{i}$$

input
```
int(acot(a*x^2)/x^2,x)
```

output
```
(-1)^(1/4)*a^(1/2)*atan((-1)^(1/4)*a^(1/2)*x)*1i - acot(a*x^2)/x + (-1)^(1
/4)*a^(1/2)*atanh((-1)^(1/4)*a^(1/2)*x)*1i
```

3.84 $\int \frac{\cot^{-1}(ax^2)}{x^4} \, dx$

3.84.1 Optimal result

Integrand size = 10, antiderivative size = 150

$$\int \frac{\cot^{-1}(ax^2)}{x^4} \, dx = \frac{2a}{3x} - \frac{\cot^{-1}(ax^2)}{3x^3} - \frac{a^{3/2}\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{3\sqrt{2}} + \frac{a^{3/2}\arctan\left(1+\sqrt{2}\sqrt{a}x\right)}{3\sqrt{2}} + \frac{a^{3/2}\log\left(1-\sqrt{2}\sqrt{a}x+ax^2\right)}{6\sqrt{2}} - \frac{a^{3/2}\log\left(1+\sqrt{2}\sqrt{a}x+ax^2\right)}{6\sqrt{2}}$$

output
```
2/3*a/x-1/3*arccot(a*x^2)/x^3+1/6*a^(3/2)*arctan(-1+x*2^(1/2)*a^(1/2))*2^(
1/2)+1/6*a^(3/2)*arctan(1+x*2^(1/2)*a^(1/2))*2^(1/2)+1/12*a^(3/2)*ln(1+a*x
^2-x*2^(1/2)*a^(1/2))*2^(1/2)-1/12*a^(3/2)*ln(1+a*x^2+x*2^(1/2)*a^(1/2))*2
^(1/2)
```

3.84.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.97

$$\int \frac{\cot^{-1}(ax^2)}{x^4} \, dx = \frac{-4\cot^{-1}(ax^2) + ax^2\left(8 - 2\sqrt{2}\sqrt{a}x\arctan\left(1-\sqrt{2}\sqrt{a}x\right) + 2\sqrt{2}\sqrt{a}x\arctan\left(1+\sqrt{2}\sqrt{a}x\right) + \sqrt{2}\sqrt{a}x\log\right.}{12x^3}$$

input
```
Integrate[ArcCot[a*x^2]/x^4,x]
```

output
```
(-4*ArcCot[a*x^2] + a*x^2*(8 - 2*Sqrt[2]*Sqrt[a]*x*ArcTan[1 - Sqrt[2]*Sqrt
[a]*x] + 2*Sqrt[2]*Sqrt[a]*x*ArcTan[1 + Sqrt[2]*Sqrt[a]*x] + Sqrt[2]*Sqrt[
a]*x*Log[1 - Sqrt[2]*Sqrt[a]*x + a*x^2] - Sqrt[2]*Sqrt[a]*x*Log[1 + Sqrt[2
]*Sqrt[a]*x + a*x^2]))/(12*x^3)
```

3.84.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {5362, 847, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax^2)}{x^4}dx$$

$$\Big\downarrow \text{5362}$$

$$-\frac{2}{3}a\int \frac{1}{x^2(a^2x^4+1)}dx-\frac{\cot^{-1}(ax^2)}{3x^3}$$

$$\Big\downarrow \text{847}$$

$$-\frac{2}{3}a\left(a^2\left(-\int \frac{x^2}{a^2x^4+1}dx\right)-\frac{1}{x}\right)-\frac{\cot^{-1}(ax^2)}{3x^3}$$

$$\Big\downarrow \text{826}$$

$$-\frac{2}{3}a\left(-\left(a^2\left(\frac{\int \frac{ax^2+1}{a^2x^4+1}dx}{2a}-\frac{\int \frac{1-ax^2}{a^2x^4+1}dx}{2a}\right)\right)-\frac{1}{x}\right)-\frac{\cot^{-1}(ax^2)}{3x^3}$$

$$\Big\downarrow \text{1476}$$

$$-\frac{2}{3}a\left(-\left(a^2\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}}dx}{2a}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}}dx}{2a}}{2a}-\frac{\int \frac{1-ax^2}{a^2x^4+1}dx}{2a}\right)\right)-\frac{1}{x}\right)-\frac{\cot^{-1}(ax^2)}{3x^3}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{2}{3}a\left(-\left(a^2\left(\frac{\frac{\int \frac{1}{-\left(1-\sqrt{2}\sqrt{a}x\right)^2-1}d\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}-\frac{\int \frac{1}{-\left(\sqrt{2}\sqrt{a}x+1\right)^2-1}d\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}}{2a}-\frac{\int \frac{1-ax^2}{a^2x^4+1}dx}{2a}\right)\right)-\frac{1}{x}\right)-\frac{\cot^{-1}\left(ax^2\right)}{3x^3}$$

$$\Big\downarrow \; 217$$

$$-\frac{2}{3}a\left(-\left(a^2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}}{2a}-\frac{\int \frac{1-ax^2}{a^2x^4+1}dx}{2a}\right)\right)-\frac{1}{x}\right)-\frac{\cot^{-1}\left(ax^2\right)}{3x^3}$$

$$\Big\downarrow \; 1479$$

$$-\frac{2}{3}a\left(-\left(a^2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}}{2a}-\frac{-\frac{\int -\frac{\sqrt{2}-2\sqrt{a}x}{\sqrt{a}\left(x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{a}\left(x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}}{2a}\right)\right)-\frac{1}{x}\right)-\frac{\cot^{-1}\left(ax^2\right)}{3x^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{2}{3}a\left(-\left(a^2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}}{2a}-\frac{\frac{\int \frac{\sqrt{2}-2\sqrt{a}x}{\sqrt{a}\left(x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{a}\left(x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}}{2a}\right)\right)-\frac{1}{x}\right)-\frac{\cot^{-1}\left(ax^2\right)}{3x^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{2}{3}a\left(-\left(a^2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}}{2a}-\frac{\frac{\int \frac{\sqrt{2}-2\sqrt{a}x}{x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}}dx}{2\sqrt{2}a}+\frac{\int \frac{\sqrt{2}\sqrt{a}x+1}{x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}}dx}{2a}}{2a}\right)\right)-\frac{1}{x}\right)-\frac{\cot^{-1}\left(ax^2\right)}{3x^3}$$

$$\Big\downarrow \; 1103$$

3.84. $\int \frac{\cot^{-1}\left(ax^2\right)}{x^4}\,dx$

$$-\frac{2}{3}a\left(-\left(a^2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}}{2a}-\frac{\frac{\log\left(ax^2+\sqrt{2}\sqrt{a}x+1\right)}{2\sqrt{2}\sqrt{a}}-\frac{\log\left(ax^2-\sqrt{2}\sqrt{a}x+1\right)}{2\sqrt{2}\sqrt{a}}}{2a}\right)\right)-\frac{1}{x}\right)-\frac{\cot^{-1}\left(ax^2\right)}{3x^3}$$

input
```
Int[ArcCot[a*x^2]/x^4,x]
```

output
```
-1/3*ArcCot[a*x^2]/x^3 - (2*a*(-x^(-1) - a^2*((-(ArcTan[1 - Sqrt[2]*Sqrt[a
]*x]/(Sqrt[2]*Sqrt[a])) + ArcTan[1 + Sqrt[2]*Sqrt[a]*x]/(Sqrt[2]*Sqrt[a]))
/(2*a) - (-1/2*Log[1 - Sqrt[2]*Sqrt[a]*x + a*x^2]/(Sqrt[2]*Sqrt[a]) + Log[
1 + Sqrt[2]*Sqrt[a]*x + a*x^2]/(2*Sqrt[2]*Sqrt[a]))/(2*a))))/3
```

3.84.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.84.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.71

method	result	size
default	$-\frac{\operatorname{arccot}(a x^2)}{3x^3} - \frac{2a\left(-\frac{1}{x} - \frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{1}{a^2}\right)^{\frac{1}{4}} x\sqrt{2}+\sqrt{\frac{1}{a^2}}}{x^2+\left(\frac{1}{a^2}\right)^{\frac{1}{4}} x\sqrt{2}+\sqrt{\frac{1}{a^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}-1\right)\right)}{8\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}\right)}{3}$	106
parts	$-\frac{\operatorname{arccot}(a x^2)}{3x^3} - \frac{2a\left(-\frac{1}{x} - \frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{1}{a^2}\right)^{\frac{1}{4}} x\sqrt{2}+\sqrt{\frac{1}{a^2}}}{x^2+\left(\frac{1}{a^2}\right)^{\frac{1}{4}} x\sqrt{2}+\sqrt{\frac{1}{a^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}-1\right)\right)}{8\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}\right)}{3}$	106

input
```
int(arccot(a*x^2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*arccot(a*x^2)/x^3-2/3*a*(-1/x-1/8/(1/a^2)^(1/4)*2^(1/2)*(ln((x^2-(1/a
^2)^(1/4)*x*2^(1/2)+(1/a^2)^(1/2))/(x^2+(1/a^2)^(1/4)*x*2^(1/2)+(1/a^2)^(1
/2)))+2*arctan(2^(1/2)/(1/a^2)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/a^2)^(1/4)*x
-1)))
```

3.84.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.86

$$\int \frac{\cot^{-1}(ax^2)}{x^4}\,dx = \frac{(-a^6)^{\frac{1}{4}}\,x^3\log\left(a^5x+(-a^6)^{\frac{3}{4}}\right) - i\,(-a^6)^{\frac{1}{4}}\,x^3\log\left(a^5x+i\,(-a^6)^{\frac{3}{4}}\right) + i\,(-a^6)^{\frac{1}{4}}\,x^3\log\left(a^5x-i\,(-a^6)^{\frac{3}{4}}\right) - \cdots}{6\,x^3}$$

input
```
integrate(arccot(a*x^2)/x^4,x, algorithm="fricas")
```

output
```
1/6*((-a^6)^(1/4)*x^3*log(a^5*x + (-a^6)^(3/4)) - I*(-a^6)^(1/4)*x^3*log(a
^5*x + I*(-a^6)^(3/4)) + I*(-a^6)^(1/4)*x^3*log(a^5*x - I*(-a^6)^(3/4)) -
(-a^6)^(1/4)*x^3*log(a^5*x - (-a^6)^(3/4)) + 4*a*x^2 - 2*arccot(a*x^2))/x^
3
```

3.84.6 Sympy [A] (verification not implemented)

Time = 12.60 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.87

$$\int \frac{\cot^{-1}(ax^2)}{x^4}\,dx$$
$$= \begin{cases} \frac{a^2\sqrt[4]{-\frac{1}{a^2}}\operatorname{acot}(ax^2)}{3} + \frac{a\log\left(x-\sqrt[4]{-\frac{1}{a^2}}\right)}{3\sqrt[4]{-\frac{1}{a^2}}} - \frac{a\log\left(x^2+\sqrt{-\frac{1}{a^2}}\right)}{6\sqrt[4]{-\frac{1}{a^2}}} + \frac{a\operatorname{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{a^2}}}\right)}{3\sqrt[4]{-\frac{1}{a^2}}} + \frac{2a}{3x} - \frac{\operatorname{acot}(ax^2)}{3x^3} & \text{for } a \neq 0 \\ -\frac{\pi}{6x^3} & \text{otherwise} \end{cases}$$

input
```
integrate(acot(a*x**2)/x**4,x)
```

output
```
Piecewise((a**2*(-1/a**2)**(1/4)*acot(a*x**2)/3 + a*log(x - (-1/a**2)**(1/
4))/(3*(-1/a**2)**(1/4)) - a*log(x**2 + sqrt(-1/a**2))/(6*(-1/a**2)**(1/4)
) + a*atan(x/(-1/a**2)**(1/4))/(3*(-1/a**2)**(1/4)) + 2*a/(3*x) - acot(a*x
**2)/(3*x**3), Ne(a, 0)), (-pi/(6*x**3), True))
```

3.84.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.89

$$\int \frac{\cot^{-1}(ax^2)}{x^4}\,dx$$
$$= \frac{1}{12}\left(a^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}(2ax+\sqrt{2}\sqrt{a})}{2\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}(2ax-\sqrt{2}\sqrt{a})}{2\sqrt{a}}\right)}{a^{\frac{3}{2}}} - \frac{\sqrt{2}\log\left(ax^2+\sqrt{2}\sqrt{a}x+1\right)}{a^{\frac{3}{2}}} + \right.\right.$$
$$- \frac{\operatorname{arccot}(ax^2)}{3x^3}$$

input
```
integrate(arccot(a*x^2)/x^4,x, algorithm="maxima")
```

output
```
1/12*(a^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*a*x + sqrt(2)*sqrt(a))/sqrt(a))
/a^(3/2) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*a*x - sqrt(2)*sqrt(a))/sqrt(a))
/a^(3/2) - sqrt(2)*log(a*x^2 + sqrt(2)*sqrt(a)*x + 1)/a^(3/2) + sqrt(2)*lo
g(a*x^2 - sqrt(2)*sqrt(a)*x + 1)/a^(3/2)) + 8/x)*a - 1/3*arccot(a*x^2)/x^3
```

3.84.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.99

$$\int \frac{\cot^{-1}(ax^2)}{x^4}\,dx = \frac{1}{12}\left(\frac{2\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\frac{\sqrt{2}}{\sqrt{|a|}}\right)\sqrt{|a|}\right)}{|a|^{\frac{3}{2}}} + \frac{2\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\frac{\sqrt{2}}{\sqrt{|a|}}\right)\sqrt{|a|}\right)}{|a|^{\frac{3}{2}}} - \sqrt{2}\sqrt{|a|}\log\left(\right. - \frac{\arctan\left(\frac{1}{ax^2}\right)}{3x^3}$$

input
```
integrate(arccot(a*x^2)/x^4,x, algorithm="giac")
```

output
```
1/12*(2*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(a)))*sqrt(a
bs(a)))/abs(a)^(3/2) + 2*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(2*x - sqrt(2)/sqr
t(abs(a)))*sqrt(abs(a)))/abs(a)^(3/2) - sqrt(2)*sqrt(abs(a))*log(x^2 + sqr
t(2)*x/sqrt(abs(a)) + 1/abs(a)) + sqrt(2)*a^2*log(x^2 - sqrt(2)*x/sqrt(abs
(a)) + 1/abs(a))/abs(a)^(3/2) + 8/x)*a - 1/3*arctan(1/(a*x^2))/x^3
```

3.84.9 Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.35

$$\int \frac{\cot^{-1}(ax^2)}{x^4}\,dx = \frac{2a}{3x} - \frac{\operatorname{acot}(a\,x^2)}{3\,x^3} + \frac{(-1)^{1/4}\,a^{3/2}\operatorname{atan}\left((-1)^{1/4}\sqrt{a}\,x\right)}{3} + \frac{(-1)^{1/4}\,a^{3/2}\operatorname{atan}\left((-1)^{1/4}\sqrt{a}\,x\,1\mathrm{i}\right)\,1\mathrm{i}}{3}$$

input
```
int(acot(a*x^2)/x^4,x)
```

output
```
(2*a)/(3*x) - acot(a*x^2)/(3*x^3) + ((-1)^(1/4)*a^(3/2)*atan((-1)^(1/4)*a^
(1/2)*x))/3 + ((-1)^(1/4)*a^(3/2)*atan((-1)^(1/4)*a^(1/2)*x*1i)*1i)/3
```

3.85 $\int x^2 \cot^{-1}(\sqrt{x})\,dx$

3.85.1 Optimal result

Integrand size = 10, antiderivative size = 51

$$\int x^2 \cot^{-1}(\sqrt{x})\,dx = \frac{\sqrt{x}}{3} - \frac{x^{3/2}}{9} + \frac{x^{5/2}}{15} + \frac{1}{3}x^3 \cot^{-1}(\sqrt{x}) - \frac{\arctan(\sqrt{x})}{3}$$

output
```
-1/9*x^(3/2)+1/15*x^(5/2)+1/3*x^3*arccot(x^(1/2))-1/3*arctan(x^(1/2))+1/3*
x^(1/2)
```

3.85.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int x^2 \cot^{-1}(\sqrt{x})\,dx = \frac{1}{45}\left(\sqrt{x}(15-5x+3x^2) + 15x^3 \cot^{-1}(\sqrt{x}) - 15\arctan(\sqrt{x})\right)$$

input
```
Integrate[x^2*ArcCot[Sqrt[x]],x]
```

output
```
(Sqrt[x]*(15 - 5*x + 3*x^2) + 15*x^3*ArcCot[Sqrt[x]] - 15*ArcTan[Sqrt[x]])
/45
```

3.85.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5362, 60, 60, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cot^{-1}\left(\sqrt{x}\right) dx$$

$$\Big\downarrow \text{5362}$$

$$\frac{1}{6}\int \frac{x^{5/2}}{x+1}dx+\frac{1}{3}x^3\cot^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{6}\left(\frac{2x^{5/2}}{5}-\int \frac{x^{3/2}}{x+1}dx\right)+\frac{1}{3}x^3\cot^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{6}\left(\int \frac{\sqrt{x}}{x+1}dx+\frac{2x^{5/2}}{5}-\frac{2x^{3/2}}{3}\right)+\frac{1}{3}x^3\cot^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{6}\left(-\int \frac{1}{\sqrt{x}(x+1)}dx+\frac{2x^{5/2}}{5}-\frac{2x^{3/2}}{3}+2\sqrt{x}\right)+\frac{1}{3}x^3\cot^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{6}\left(-2\int \frac{1}{x+1}d\sqrt{x}+\frac{2x^{5/2}}{5}-\frac{2x^{3/2}}{3}+2\sqrt{x}\right)+\frac{1}{3}x^3\cot^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{6}\left(-2\arctan\left(\sqrt{x}\right)+\frac{2x^{5/2}}{5}-\frac{2x^{3/2}}{3}+2\sqrt{x}\right)+\frac{1}{3}x^3\cot^{-1}\left(\sqrt{x}\right)$$

input
```
Int[x^2*ArcCot[Sqrt[x]],x]
```

output
```
(x^3*ArcCot[Sqrt[x]])/3 + (2*Sqrt[x] - (2*x^(3/2))/3 + (2*x^(5/2))/5 - 2*A
rcTan[Sqrt[x]])/6
```

3.85.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.85.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.63

method	result	size
derivativedivides	$-\frac{x^{\frac{3}{2}}}{9}+\frac{x^{\frac{5}{2}}}{15}+\frac{x^{3}\operatorname{arccot}\left(\sqrt{x}\right)}{3}-\frac{\arctan\left(\sqrt{x}\right)}{3}+\frac{\sqrt{x}}{3}$	32
default	$-\frac{x^{\frac{3}{2}}}{9}+\frac{x^{\frac{5}{2}}}{15}+\frac{x^{3}\operatorname{arccot}\left(\sqrt{x}\right)}{3}-\frac{\arctan\left(\sqrt{x}\right)}{3}+\frac{\sqrt{x}}{3}$	32
parts	$-\frac{x^{\frac{3}{2}}}{9}+\frac{x^{\frac{5}{2}}}{15}+\frac{x^{3}\operatorname{arccot}\left(\sqrt{x}\right)}{3}-\frac{\arctan\left(\sqrt{x}\right)}{3}+\frac{\sqrt{x}}{3}$	32

input
```
int(x^2*arccot(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-1/9*x^(3/2)+1/15*x^(5/2)+1/3*x^3*arccot(x^(1/2))-1/3*arctan(x^(1/2))+1/3*
x^(1/2)
```

3.85.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.53

$$\int x^2 \cot^{-1}\left(\sqrt{x}\right) dx = \frac{1}{3}\left(x^3+1\right)\operatorname{arccot}\left(\sqrt{x}\right)+\frac{1}{45}\left(3x^2-5x+15\right)\sqrt{x}$$

input
```
integrate(x^2*arccot(x^(1/2)),x, algorithm="fricas")
```

output
```
1/3*(x^3 + 1)*arccot(sqrt(x)) + 1/45*(3*x^2 - 5*x + 15)*sqrt(x)
```

3.85.6 Sympy [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int x^2 \cot^{-1}\left(\sqrt{x}\right) dx = \frac{x^{\frac{5}{2}}}{15}-\frac{x^{\frac{3}{2}}}{9}+\frac{\sqrt{x}}{3}+\frac{x^3 \operatorname{acot}\left(\sqrt{x}\right)}{3}-\frac{\operatorname{atan}\left(\sqrt{x}\right)}{3}$$

input
```
integrate(x**2*acot(x**(1/2)),x)
```

output
```
x**(5/2)/15 - x**(3/2)/9 + sqrt(x)/3 + x**3*acot(sqrt(x))/3 - atan(sqrt(x)
)/3
```

3.85.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.61

$$\int x^2 \cot^{-1}\left(\sqrt{x}\right) dx = \frac{1}{3}x^3 \operatorname{arccot}\left(\sqrt{x}\right)+\frac{1}{15}x^{\frac{5}{2}}-\frac{1}{9}x^{\frac{3}{2}}+\frac{1}{3}\sqrt{x}-\frac{1}{3}\arctan\left(\sqrt{x}\right)$$

input
```
integrate(x^2*arccot(x^(1/2)),x, algorithm="maxima")
```

output
```
1/3*x^3*arccot(sqrt(x)) + 1/15*x^(5/2) - 1/9*x^(3/2) + 1/3*sqrt(x) - 1/3*a
rctan(sqrt(x))
```

3.85.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.65

$$\int x^2 \cot^{-1}\left(\sqrt{x}\right) dx = \frac{1}{3} x^3 \arctan\left(\frac{1}{\sqrt{x}}\right) - \frac{1}{45} x^{\frac{5}{2}} \left(\frac{5}{x} - \frac{15}{x^2} - 3\right) + \frac{1}{3} \arctan\left(\frac{1}{\sqrt{x}}\right)$$

input
```
integrate(x^2*arccot(x^(1/2)),x, algorithm="giac")
```

output
```
1/3*x^3*arctan(1/sqrt(x)) - 1/45*x^(5/2)*(5/x - 15/x^2 - 3) + 1/3*arctan(1
/sqrt(x))
```

3.85.9 Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.61

$$\int x^2 \cot^{-1}\left(\sqrt{x}\right) dx = \frac{x^3 \operatorname{acot}\left(\sqrt{x}\right)}{3} - \frac{\operatorname{atan}\left(\sqrt{x}\right)}{3} + \frac{\sqrt{x}}{3} - \frac{x^{3/2}}{9} + \frac{x^{5/2}}{15}$$

input
```
int(x^2*acot(x^(1/2)),x)
```

output
```
(x^3*acot(x^(1/2)))/3 - atan(x^(1/2))/3 + x^(1/2)/3 - x^(3/2)/9 + x^(5/2)/
15
```

3.86 $\int x \cot^{-1}\left(\sqrt{x}\right)\,dx$

3.86.1 Optimal result

Integrand size = 8, antiderivative size = 42

$$\int x \cot^{-1}\left(\sqrt{x}\right)\,dx = -\frac{\sqrt{x}}{2} + \frac{x^{3/2}}{6} + \frac{1}{2}x^2\cot^{-1}\left(\sqrt{x}\right) + \frac{\arctan\left(\sqrt{x}\right)}{2}$$

output
```
1/6*x^(3/2)+1/2*x^2*arccot(x^(1/2))+1/2*arctan(x^(1/2))-1/2*x^(1/2)
```

3.86.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int x \cot^{-1}\left(\sqrt{x}\right)\,dx = \frac{1}{6}\left((-3+x)\sqrt{x} + 3x^2\cot^{-1}\left(\sqrt{x}\right) + 3\arctan\left(\sqrt{x}\right)\right)$$

input
```
Integrate[x*ArcCot[Sqrt[x]],x]
```

output
```
((-3 + x)*Sqrt[x] + 3*x^2*ArcCot[Sqrt[x]] + 3*ArcTan[Sqrt[x]])/6
```

3.86.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5362, 60, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cot^{-1}\left(\sqrt{x}\right)dx$$

$$\Big\downarrow \text{5362}$$

$$\frac{1}{4}\int \frac{x^{3/2}}{x+1}dx+\frac{1}{2}x^2\cot^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(\frac{2x^{3/2}}{3}-\int \frac{\sqrt{x}}{x+1}dx\right)+\frac{1}{2}x^2\cot^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(\int \frac{1}{\sqrt{x}(x+1)}dx+\frac{2x^{3/2}}{3}-2\sqrt{x}\right)+\frac{1}{2}x^2\cot^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(2\int \frac{1}{x+1}d\sqrt{x}+\frac{2x^{3/2}}{3}-2\sqrt{x}\right)+\frac{1}{2}x^2\cot^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(2\arctan\left(\sqrt{x}\right)+\frac{2x^{3/2}}{3}-2\sqrt{x}\right)+\frac{1}{2}x^2\cot^{-1}\left(\sqrt{x}\right)$$

input
```
Int[x*ArcCot[Sqrt[x]],x]
```

output
```
(x^2*ArcCot[Sqrt[x]])/2 + (-2*Sqrt[x] + (2*x^(3/2))/3 + 2*ArcTan[Sqrt[x]])
/4
```

3.86.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.86.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.64

method	result	size
derivativedivides	$\frac{x^{\frac{3}{2}}}{6}+\frac{x^{2}\operatorname{arccot}\left(\sqrt{x}\right)}{2}+\frac{\arctan\left(\sqrt{x}\right)}{2}-\frac{\sqrt{x}}{2}$	27
default	$\frac{x^{\frac{3}{2}}}{6}+\frac{x^{2}\operatorname{arccot}\left(\sqrt{x}\right)}{2}+\frac{\arctan\left(\sqrt{x}\right)}{2}-\frac{\sqrt{x}}{2}$	27
parts	$\frac{x^{\frac{3}{2}}}{6}+\frac{x^{2}\operatorname{arccot}\left(\sqrt{x}\right)}{2}+\frac{\arctan\left(\sqrt{x}\right)}{2}-\frac{\sqrt{x}}{2}$	27

input
```
int(x*arccot(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/6*x^(3/2)+1/2*x^2*arccot(x^(1/2))+1/2*arctan(x^(1/2))-1/2*x^(1/2)
```

3.86.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.48

$$\int x\cot^{-1}\left(\sqrt{x}\right)\,dx=\frac{1}{2}\left(x^{2}-1\right)\operatorname{arccot}\left(\sqrt{x}\right)+\frac{1}{6}\,(x-3)\sqrt{x}$$

input
```
integrate(x*arccot(x^(1/2)),x, algorithm="fricas")
```

output
```
1/2*(x^2 - 1)*arccot(sqrt(x)) + 1/6*(x - 3)*sqrt(x)
```

3.86.6 Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int x\cot^{-1}\left(\sqrt{x}\right)\,dx=\frac{x^{\frac{3}{2}}}{6}-\frac{\sqrt{x}}{2}+\frac{x^{2}\operatorname{acot}\left(\sqrt{x}\right)}{2}+\frac{\operatorname{atan}\left(\sqrt{x}\right)}{2}$$

input
```
integrate(x*acot(x**(1/2)),x)
```

output
```
x**(3/2)/6 - sqrt(x)/2 + x**2*acot(sqrt(x))/2 + atan(sqrt(x))/2
```

3.86.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int x\cot^{-1}\left(\sqrt{x}\right)\,dx=\frac{1}{2}\,x^{2}\operatorname{arccot}\left(\sqrt{x}\right)+\frac{1}{6}\,x^{\frac{3}{2}}-\frac{1}{2}\,\sqrt{x}+\frac{1}{2}\,\arctan\left(\sqrt{x}\right)$$

input
```
integrate(x*arccot(x^(1/2)),x, algorithm="maxima")
```

output
```
1/2*x^2*arccot(sqrt(x)) + 1/6*x^(3/2) - 1/2*sqrt(x) + 1/2*arctan(sqrt(x))
```

3.86.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.67

$$\int x\cot^{-1}\left(\sqrt{x}\right)\,dx=\frac{1}{2}\,x^2\arctan\left(\frac{1}{\sqrt{x}}\right)-\frac{1}{6}\,x^{\frac{3}{2}}\left(\frac{3}{x}-1\right)-\frac{1}{2}\,\arctan\left(\frac{1}{\sqrt{x}}\right)$$

input
```
integrate(x*arccot(x^(1/2)),x, algorithm="giac")
```

output
```
1/2*x^2*arctan(1/sqrt(x)) - 1/6*x^(3/2)*(3/x - 1) - 1/2*arctan(1/sqrt(x))
```

3.86.9 Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int x\cot^{-1}\left(\sqrt{x}\right)\,dx=\frac{\mathrm{atan}\left(\sqrt{x}\right)}{2}+\frac{x^2\,\mathrm{acot}\left(\sqrt{x}\right)}{2}-\frac{\sqrt{x}}{2}+\frac{x^{3/2}}{6}$$

input
```
int(x*acot(x^(1/2)),x)
```

output
```
atan(x^(1/2))/2 + (x^2*acot(x^(1/2)))/2 - x^(1/2)/2 + x^(3/2)/6
```

3.87 $\int \cot^{-1}\left(\sqrt{x}\right)\,dx$

3.87.1 Optimal result

Integrand size = 6, antiderivative size = 22

$$\int \cot^{-1}\left(\sqrt{x}\right)\,dx=\sqrt{x}+x\cot^{-1}\left(\sqrt{x}\right)-\arctan\left(\sqrt{x}\right)$$

output
```
x*arccot(x^(1/2))-arctan(x^(1/2))+x^(1/2)
```

3.87.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \cot^{-1}\left(\sqrt{x}\right)\,dx=\sqrt{x}+x\cot^{-1}\left(\sqrt{x}\right)-\arctan\left(\sqrt{x}\right)$$

input
```
Integrate[ArcCot[Sqrt[x]],x]
```

output
```
Sqrt[x] + x*ArcCot[Sqrt[x]] - ArcTan[Sqrt[x]]
```

3.87.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5346, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}\left(\sqrt{x}\right) dx$$

$$\Big\downarrow \text{5346}$$

$$\frac{1}{2}\int \frac{\sqrt{x}}{x+1}dx + x\cot^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(2\sqrt{x} - \int \frac{1}{\sqrt{x}(x+1)}dx\right) + x\cot^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(2\sqrt{x} - 2\int \frac{1}{x+1}d\sqrt{x}\right) + x\cot^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(2\sqrt{x} - 2\arctan\left(\sqrt{x}\right)\right) + x\cot^{-1}\left(\sqrt{x}\right)$$

input
```
Int[ArcCot[Sqrt[x]],x]
```

output
```
x*ArcCot[Sqrt[x]] + (2*Sqrt[x] - 2*ArcTan[Sqrt[x]])/2
```

3.87.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

3.87.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$x \operatorname{arccot}\left(\sqrt{x}\right)-\arctan\left(\sqrt{x}\right)+\sqrt{x}$	17
default	$x \operatorname{arccot}\left(\sqrt{x}\right)-\arctan\left(\sqrt{x}\right)+\sqrt{x}$	17
parts	$x \operatorname{arccot}\left(\sqrt{x}\right)-\arctan\left(\sqrt{x}\right)+\sqrt{x}$	17

input
```
int(arccot(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
x*arccot(x^(1/2))-arctan(x^(1/2))+x^(1/2)
```

3.87.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.55

$$\int \cot^{-1}\left(\sqrt{x}\right)\,dx = (x+1)\,\mathrm{arccot}\left(\sqrt{x}\right) + \sqrt{x}$$

input
```
integrate(arccot(x^(1/2)),x, algorithm="fricas")
```

output
```
(x + 1)*arccot(sqrt(x)) + sqrt(x)
```

3.87.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \cot^{-1}\left(\sqrt{x}\right)\,dx = \sqrt{x} + x\,\mathrm{acot}\left(\sqrt{x}\right) - \mathrm{atan}\left(\sqrt{x}\right)$$

input
```
integrate(acot(x**(1/2)),x)
```

output
```
sqrt(x) + x*acot(sqrt(x)) - atan(sqrt(x))
```

3.87.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \cot^{-1}\left(\sqrt{x}\right)\,dx = x\,\mathrm{arccot}\left(\sqrt{x}\right) + \sqrt{x} - \arctan\left(\sqrt{x}\right)$$

input
```
integrate(arccot(x^(1/2)),x, algorithm="maxima")
```

output
```
x*arccot(sqrt(x)) + sqrt(x) - arctan(sqrt(x))
```

3.87.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.64

$$\int \cot^{-1}\left(\sqrt{x}\right)\,dx = x\arctan\left(\frac{1}{\sqrt{x}}\right) + \sqrt{x} + \arctan\left(\frac{1}{\sqrt{x}}\right)$$

input
```
integrate(arccot(x^(1/2)),x, algorithm="giac")
```

output
```
x*arctan(1/sqrt(x)) + sqrt(x) + arctan(1/sqrt(x))
```

3.87.9 Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \cot^{-1}\left(\sqrt{x}\right)\,dx = x\,\mathrm{acot}\left(\sqrt{x}\right) - \mathrm{atan}\left(\sqrt{x}\right) + \sqrt{x}$$

input
```
int(acot(x^(1/2)),x)
```

output
```
x*acot(x^(1/2)) - atan(x^(1/2)) + x^(1/2)
```

3.88 $\int \frac{\cot^{-1}(\sqrt{x})}{x} \, dx$

3.88.1 Optimal result

Integrand size = 10, antiderivative size = 31

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x} \, dx = -i \operatorname{PolyLog}\left(2, -\frac{i}{\sqrt{x}}\right) + i \operatorname{PolyLog}\left(2, \frac{i}{\sqrt{x}}\right)$$

output
```
-I*polylog(2,-I/x^(1/2))+I*polylog(2,I/x^(1/2))
```

3.88.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x} \, dx = -i \operatorname{PolyLog}\left(2, -\frac{i}{\sqrt{x}}\right) + i \operatorname{PolyLog}\left(2, \frac{i}{\sqrt{x}}\right)$$

input
```
Integrate[ArcCot[Sqrt[x]]/x,x]
```

output
```
(-I)*PolyLog[2, (-I)/Sqrt[x]] + I*PolyLog[2, I/Sqrt[x]]
```

3.88.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5360, 5356, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x}dx$$

$$\Big\downarrow \text{ 5360}$$

$$2\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{ 5356}$$

$$2\left(\frac{1}{2}i\int \frac{\log\left(1-\frac{i}{\sqrt{x}}\right)}{\sqrt{x}}d\sqrt{x}-\frac{1}{2}i\int \frac{\log\left(1+\frac{i}{\sqrt{x}}\right)}{\sqrt{x}}d\sqrt{x}\right)$$

$$\Big\downarrow \text{ 2838}$$

$$2\left(\frac{1}{2}i\,\text{PolyLog}\left(2,\frac{i}{\sqrt{x}}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,-\frac{i}{\sqrt{x}}\right)\right)$$

input
```
Int[ArcCot[Sqrt[x]]/x,x]
```

output
```
2*((-1/2*I)*PolyLog[2, (-I)/Sqrt[x]] + (I/2)*PolyLog[2, I/Sqrt[x]])
```

3.88.3.1 Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5356
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Simp[I*(b/2)   Int[Log[1 + I/(c*x)]/x, x], x] + Simp[I*(b/2)   Int[Log
[1 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5360
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1
/n   Subst[Int[(a + b*ArcCot[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
n}, x] && IGtQ[p, 0]
```

3.88.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(23) = 46$.

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.97

method	result
derivativedivides	$\ln(x)\operatorname{arccot}\left(\sqrt{x}\right)-\frac{i\ln(x)\ln\left(1+i\sqrt{x}\right)}{2}+\frac{i\ln(x)\ln\left(1-i\sqrt{x}\right)}{2}-i\operatorname{dilog}\left(1+i\sqrt{x}\right)+i\operatorname{dilog}\left(1-i\right.$
default	$\ln(x)\operatorname{arccot}\left(\sqrt{x}\right)-\frac{i\ln(x)\ln\left(1+i\sqrt{x}\right)}{2}+\frac{i\ln(x)\ln\left(1-i\sqrt{x}\right)}{2}-i\operatorname{dilog}\left(1+i\sqrt{x}\right)+i\operatorname{dilog}\left(1-i\right.$
parts	$\ln(x)\operatorname{arccot}\left(\sqrt{x}\right)-\frac{i\ln(x)\ln\left(1+i\sqrt{x}\right)}{2}+\frac{i\ln(x)\ln\left(1-i\sqrt{x}\right)}{2}-i\operatorname{dilog}\left(1+i\sqrt{x}\right)+i\operatorname{dilog}\left(1-i\right.$

input
```
int(arccot(x^(1/2))/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*arccot(x^(1/2))-1/2*I*ln(x)*ln(1+I*x^(1/2))+1/2*I*ln(x)*ln(1-I*x^(1/
2))-I*dilog(1+I*x^(1/2))+I*dilog(1-I*x^(1/2))
```

3.88.5 Fricas [F]

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x}\,dx=\int \frac{\operatorname{arccot}\left(\sqrt{x}\right)}{x}\,dx$$

input
```
integrate(arccot(x^(1/2))/x,x, algorithm="fricas")
```

output
```
integral(arccot(sqrt(x))/x, x)
```

3.88.6 Sympy [F]

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x}\,dx = \int \frac{\operatorname{acot}\left(\sqrt{x}\right)}{x}\,dx$$

input
```
integrate(acot(x**(1/2))/x,x)
```

output
```
Integral(acot(sqrt(x))/x, x)
```

3.88.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x}\,dx = \frac{1}{2}\pi\log\left(x+1\right) + \operatorname{arccot}\left(\sqrt{x}\right)\log\left(x\right) + i\operatorname{Li}_2\left(i\sqrt{x}+1\right) - i\operatorname{Li}_2\left(-i\sqrt{x}+1\right)$$

input
```
integrate(arccot(x^(1/2))/x,x, algorithm="maxima")
```

output
```
1/2*pi*log(x + 1) + arccot(sqrt(x))*log(x) + I*dilog(I*sqrt(x) + 1) - I*di
log(-I*sqrt(x) + 1)
```

3.88.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x}\,dx = -x\arctan\left(\frac{1}{\sqrt{x}}\right) - \sqrt{x} - \arctan\left(\frac{1}{\sqrt{x}}\right)$$

input
```
integrate(arccot(x^(1/2))/x,x, algorithm="giac")
```

output
```
-x*arctan(1/sqrt(x)) - sqrt(x) - arctan(1/sqrt(x))
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x}\,dx = \int \frac{\mathrm{acot}\left(\sqrt{x}\right)}{x}\,dx$$

input
```
int(acot(x^(1/2))/x,x)
```

output
```
int(acot(x^(1/2))/x, x)
```

3.89 $\int \frac{\cot^{-1}(\sqrt{x})}{x^2}\,dx$

3.89.1 Optimal result

Integrand size = 10, antiderivative size = 23

$$\int \frac{\cot^{-1}(\sqrt{x})}{x^2}\,dx = \frac{1}{\sqrt{x}} - \frac{\cot^{-1}(\sqrt{x})}{x} + \arctan(\sqrt{x})$$

output
```
-arccot(x^(1/2))/x+arctan(x^(1/2))+1/x^(1/2)
```

3.89.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{\cot^{-1}(\sqrt{x})}{x^2}\,dx = -\frac{\cot^{-1}(\sqrt{x})}{x} + \frac{\text{Hypergeometric2F1}\left(-\frac{1}{2}, 1, \frac{1}{2}, -x\right)}{\sqrt{x}}$$

input
```
Integrate[ArcCot[Sqrt[x]]/x^2,x]
```

output
```
-(ArcCot[Sqrt[x]]/x) + Hypergeometric2F1[-1/2, 1, 1/2, -x]/Sqrt[x]
```

3.89.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5362, 61, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^2}dx$$

$$\Big\downarrow \text{5362}$$

$$-\frac{1}{2}\int \frac{1}{x^{3/2}(x+1)}dx-\frac{\cot^{-1}\left(\sqrt{x}\right)}{x}$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(\int \frac{1}{\sqrt{x}(x+1)}dx+\frac{2}{\sqrt{x}}\right)-\frac{\cot^{-1}\left(\sqrt{x}\right)}{x}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(2\int \frac{1}{x+1}d\sqrt{x}+\frac{2}{\sqrt{x}}\right)-\frac{\cot^{-1}\left(\sqrt{x}\right)}{x}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(2\arctan\left(\sqrt{x}\right)+\frac{2}{\sqrt{x}}\right)-\frac{\cot^{-1}\left(\sqrt{x}\right)}{x}$$

input
```
Int[ArcCot[Sqrt[x]]/x^2,x]
```

output
```
-(ArcCot[Sqrt[x]]/x) + (2/Sqrt[x] + 2*ArcTan[Sqrt[x]])/2
```

3.89.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.89.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$-\frac{\operatorname{arccot}\left(\sqrt{x}\right)}{x}+\arctan\left(\sqrt{x}\right)+\frac{1}{\sqrt{x}}$	18
default	$-\frac{\operatorname{arccot}\left(\sqrt{x}\right)}{x}+\arctan\left(\sqrt{x}\right)+\frac{1}{\sqrt{x}}$	18
parts	$-\frac{\operatorname{arccot}\left(\sqrt{x}\right)}{x}+\arctan\left(\sqrt{x}\right)+\frac{1}{\sqrt{x}}$	18

input
```
int(arccot(x^(1/2))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-arccot(x^(1/2))/x+arctan(x^(1/2))+1/x^(1/2)
```

3.89.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^2}\, dx = -\frac{(x+1)\operatorname{arccot}\left(\sqrt{x}\right) - \sqrt{x}}{x}$$

input
```
integrate(arccot(x^(1/2))/x^2,x, algorithm="fricas")
```

output
```
-((x + 1)*arccot(sqrt(x)) - sqrt(x))/x
```

3.89.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(20) = 40.

Time = 0.49 (sec) , antiderivative size = 92, normalized size of antiderivative = 4.00

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^2}\, dx = -\frac{x^{\frac{5}{2}}\operatorname{acot}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}+x^{\frac{3}{2}}} - \frac{2x^{\frac{3}{2}}\operatorname{acot}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}+x^{\frac{3}{2}}} - \frac{\sqrt{x}\operatorname{acot}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}+x^{\frac{3}{2}}} + \frac{x^2}{x^{\frac{5}{2}}+x^{\frac{3}{2}}} + \frac{x}{x^{\frac{5}{2}}+x^{\frac{3}{2}}}$$

input
```
integrate(acot(x**(1/2))/x**2,x)
```

output
```
-x**(5/2)*acot(sqrt(x))/(x**(5/2) + x**(3/2)) - 2*x**(3/2)*acot(sqrt(x))/(
x**(5/2) + x**(3/2)) - sqrt(x)*acot(sqrt(x))/(x**(5/2) + x**(3/2)) + x**2/
(x**(5/2) + x**(3/2)) + x/(x**(5/2) + x**(3/2))
```

3.89.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^2}\, dx = -\frac{\operatorname{arccot}\left(\sqrt{x}\right)}{x} + \frac{1}{\sqrt{x}} + \arctan\left(\sqrt{x}\right)$$

input
```
integrate(arccot(x^(1/2))/x^2,x, algorithm="maxima")
```

output
```
-arccot(sqrt(x))/x + 1/sqrt(x) + arctan(sqrt(x))
```

3.89.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^2}\,dx = -\frac{\arctan\left(\frac{1}{\sqrt{x}}\right)}{x} + \frac{1}{\sqrt{x}} - \arctan\left(\frac{1}{\sqrt{x}}\right)$$

input
```
integrate(arccot(x^(1/2))/x^2,x, algorithm="giac")
```

output
```
-arctan(1/sqrt(x))/x + 1/sqrt(x) - arctan(1/sqrt(x))
```

3.89.9 Mupad [B] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^2}\,dx = \mathrm{atan}\left(\sqrt{x}\right) - \frac{\mathrm{acot}\left(\sqrt{x}\right)}{x} + \frac{1}{\sqrt{x}}$$

input
```
int(acot(x^(1/2))/x^2,x)
```

output
```
atan(x^(1/2)) - acot(x^(1/2))/x + 1/x^(1/2)
```

3.90 $\int \frac{\cot^{-1}(\sqrt{x})}{x^3}\,dx$

3.90.1 Optimal result

Integrand size = 10, antiderivative size = 42

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = \frac{1}{6x^{3/2}} - \frac{1}{2\sqrt{x}} - \frac{\cot^{-1}\left(\sqrt{x}\right)}{2x^2} - \frac{\arctan\left(\sqrt{x}\right)}{2}$$

output
```
1/6/x^(3/2)-1/2*arccot(x^(1/2))/x^2-1/2*arctan(x^(1/2))-1/2/x^(1/2)
```

3.90.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = -\frac{\cot^{-1}\left(\sqrt{x}\right)}{2x^2} + \frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-x\right)}{6x^{3/2}}$$

input
```
Integrate[ArcCot[Sqrt[x]]/x^3,x]
```

output
```
-1/2*ArcCot[Sqrt[x]]/x^2 + Hypergeometric2F1[-3/2, 1, -1/2, -x]/(6*x^(3/2)
)
```

3.90.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5362, 61, 61, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^3}dx$$

$$\Big\downarrow \text{5362}$$

$$-\frac{1}{4}\int \frac{1}{x^{5/2}(x+1)}dx-\frac{\cot^{-1}\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{4}\left(\int \frac{1}{x^{3/2}(x+1)}dx+\frac{2}{3x^{3/2}}\right)-\frac{\cot^{-1}\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{4}\left(-\int \frac{1}{\sqrt{x}(x+1)}dx+\frac{2}{3x^{3/2}}-\frac{2}{\sqrt{x}}\right)-\frac{\cot^{-1}\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(-2\int \frac{1}{x+1}d\sqrt{x}+\frac{2}{3x^{3/2}}-\frac{2}{\sqrt{x}}\right)-\frac{\cot^{-1}\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(-2\arctan\left(\sqrt{x}\right)+\frac{2}{3x^{3/2}}-\frac{2}{\sqrt{x}}\right)-\frac{\cot^{-1}\left(\sqrt{x}\right)}{2x^2}$$

input
```
Int[ArcCot[Sqrt[x]]/x^3,x]
```

output
```
-1/2*ArcCot[Sqrt[x]]/x^2 + (2/(3*x^(3/2)) - 2/Sqrt[x] - 2*ArcTan[Sqrt[x]])
/4
```

3.90.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.90.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.64

method	result	size
derivativedivides	$\frac{1}{6x^{\frac{3}{2}}}-\frac{\operatorname{arccot}\left(\sqrt{x}\right)}{2x^{2}}-\frac{\arctan\left(\sqrt{x}\right)}{2}-\frac{1}{2\sqrt{x}}$	27
default	$\frac{1}{6x^{\frac{3}{2}}}-\frac{\operatorname{arccot}\left(\sqrt{x}\right)}{2x^{2}}-\frac{\arctan\left(\sqrt{x}\right)}{2}-\frac{1}{2\sqrt{x}}$	27
parts	$\frac{1}{6x^{\frac{3}{2}}}-\frac{\operatorname{arccot}\left(\sqrt{x}\right)}{2x^{2}}-\frac{\arctan\left(\sqrt{x}\right)}{2}-\frac{1}{2\sqrt{x}}$	27

input
```
int(arccot(x^(1/2))/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/6/x^(3/2)-1/2*arccot(x^(1/2))/x^2-1/2*arctan(x^(1/2))-1/2/x^(1/2)
```

3.90.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.64

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = \frac{3\,(x^2-1)\operatorname{arccot}\left(\sqrt{x}\right)-(3\,x-1)\sqrt{x}}{6\,x^2}$$

input
```
integrate(arccot(x^(1/2))/x^3,x, algorithm="fricas")
```

output
```
1/6*(3*(x^2 - 1)*arccot(sqrt(x)) - (3*x - 1)*sqrt(x))/x^2
```

3.90.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(36) = 72.

Time = 1.14 (sec) , antiderivative size = 160, normalized size of antiderivative = 3.81

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = \frac{3x^{\frac{7}{2}}\operatorname{acot}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}} + \frac{3x^{\frac{5}{2}}\operatorname{acot}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}} - \frac{3x^{\frac{3}{2}}\operatorname{acot}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}} - \frac{3\sqrt{x}\operatorname{acot}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}} - \frac{3x^3}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}} - \frac{2x^2}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}} + \frac{x}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}$$

input
```
integrate(acot(x**(1/2))/x**3,x)
```

output
```
3*x**(7/2)*acot(sqrt(x))/(6*x**(7/2) + 6*x**(5/2)) + 3*x**(5/2)*acot(sqrt(
x))/(6*x**(7/2) + 6*x**(5/2)) - 3*x**(3/2)*acot(sqrt(x))/(6*x**(7/2) + 6*x
**(5/2)) - 3*sqrt(x)*acot(sqrt(x))/(6*x**(7/2) + 6*x**(5/2)) - 3*x**3/(6*x
**(7/2) + 6*x**(5/2)) - 2*x**2/(6*x**(7/2) + 6*x**(5/2)) + x/(6*x**(7/2) +
 6*x**(5/2))
```

3.90.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = -\frac{3\,x-1}{6\,x^{\frac{3}{2}}} - \frac{\operatorname{arccot}\left(\sqrt{x}\right)}{2\,x^2} - \frac{1}{2}\,\arctan\left(\sqrt{x}\right)$$

input
```
integrate(arccot(x^(1/2))/x^3,x, algorithm="maxima")
```

output
```
-1/6*(3*x - 1)/x^(3/2) - 1/2*arccot(sqrt(x))/x^2 - 1/2*arctan(sqrt(x))
```

3.90.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = -\frac{1}{2\,\sqrt{x}} - \frac{\arctan\left(\frac{1}{\sqrt{x}}\right)}{2\,x^2} + \frac{1}{6\,x^{\frac{3}{2}}} + \frac{1}{2}\,\arctan\left(\frac{1}{\sqrt{x}}\right)$$

input
```
integrate(arccot(x^(1/2))/x^3,x, algorithm="giac")
```

output
```
-1/2/sqrt(x) - 1/2*arctan(1/sqrt(x))/x^2 + 1/6/x^(3/2) + 1/2*arctan(1/sqrt
(x))
```

3.90.9 Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.57

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = -\frac{\operatorname{atan}\left(\sqrt{x}\right)}{2} - \frac{x-\frac{1}{3}}{2\,x^{3/2}} - \frac{\operatorname{acot}\left(\sqrt{x}\right)}{2\,x^2}$$

input
```
int(acot(x^(1/2))/x^3,x)
```

output
```
- atan(x^(1/2))/2 - (x - 1/3)/(2*x^(3/2)) - acot(x^(1/2))/(2*x^2)
```

3.91 $\int x^{3/2} \cot^{-1}(\sqrt{x})\, dx$

3.91.1 Optimal result

Integrand size = 12, antiderivative size = 36

$$\int x^{3/2} \cot^{-1}(\sqrt{x})\, dx = -\frac{x}{5} + \frac{x^2}{10} + \frac{2}{5} x^{5/2} \cot^{-1}(\sqrt{x}) + \frac{1}{5} \log(1+x)$$

output
```
-1/5*x+1/10*x^2+2/5*x^(5/2)*arccot(x^(1/2))+1/5*ln(1+x)
```

3.91.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int x^{3/2} \cot^{-1}(\sqrt{x})\, dx = \frac{1}{10}\left((-2+x)x + 4x^{5/2} \cot^{-1}(\sqrt{x}) + 2\log(1+x)\right)$$

input
```
Integrate[x^(3/2)*ArcCot[Sqrt[x]],x]
```

output
```
((-2 + x)*x + 4*x^(5/2)*ArcCot[Sqrt[x]] + 2*Log[1 + x])/10
```

3.91.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5362, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2} \cot^{-1}\left(\sqrt{x}\right) \, dx$$

$$\Big\downarrow \text{ 5362}$$

$$\frac{1}{5} \int \frac{x^2}{x+1} dx + \frac{2}{5} x^{5/2} \cot^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{5} \int \left(x + \frac{1}{x+1} - 1\right) dx + \frac{2}{5} x^{5/2} \cot^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{5} x^{5/2} \cot^{-1}\left(\sqrt{x}\right) + \frac{1}{5}\left(\frac{x^2}{2} - x + \log(x+1)\right)$$

input
```
Int[x^(3/2)*ArcCot[Sqrt[x]],x]
```

output
```
(2*x^(5/2)*ArcCot[Sqrt[x]])/5 + (-x + x^2/2 + Log[1 + x])/5
```

3.91.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.91.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$-\frac{x}{5}+\frac{x^{2}}{10}+\frac{2x^{\frac{5}{2}}\operatorname{arccot}\left(\sqrt{x}\right)}{5}+\frac{\ln\left(1+x\right)}{5}$	25
default	$-\frac{x}{5}+\frac{x^{2}}{10}+\frac{2x^{\frac{5}{2}}\operatorname{arccot}\left(\sqrt{x}\right)}{5}+\frac{\ln\left(1+x\right)}{5}$	25

input
```
int(x^(3/2)*arccot(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-1/5*x+1/10*x^2+2/5*x^(5/2)*arccot(x^(1/2))+1/5*ln(1+x)
```

3.91.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{3/2}\cot^{-1}\left(\sqrt{x}\right)\,dx=\frac{2}{5}\,x^{\frac{5}{2}}\operatorname{arccot}\left(\sqrt{x}\right)+\frac{1}{10}\,x^{2}-\frac{1}{5}\,x+\frac{1}{5}\,\log\left(x+1\right)$$

input
```
integrate(x^(3/2)*arccot(x^(1/2)),x, algorithm="fricas")
```

output
```
2/5*x^(5/2)*arccot(sqrt(x)) + 1/10*x^2 - 1/5*x + 1/5*log(x + 1)
```

3.91.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(29) = 58.

Time = 0.85 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.36

$$\int x^{3/2}\cot^{-1}\left(\sqrt{x}\right)\,dx=\frac{4x^{\frac{7}{2}}\operatorname{acot}\left(\sqrt{x}\right)}{10x+10}+\frac{4x^{\frac{5}{2}}\operatorname{acot}\left(\sqrt{x}\right)}{10x+10}+\frac{x^3}{10x+10}-\frac{x^2}{10x+10}+\frac{2x\log\left(x+1\right)}{10x+10}+\frac{2\log\left(x+1\right)}{10x+10}+\frac{2}{10x+10}$$

input
```
integrate(x**(3/2)*acot(x**(1/2)),x)
```

output
```
4*x**(7/2)*acot(sqrt(x))/(10*x + 10) + 4*x**(5/2)*acot(sqrt(x))/(10*x + 10
) + x**3/(10*x + 10) - x**2/(10*x + 10) + 2*x*log(x + 1)/(10*x + 10) + 2*l
og(x + 1)/(10*x + 10) + 2/(10*x + 10)
```

3.91.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{3/2}\cot^{-1}\left(\sqrt{x}\right)\,dx=\frac{2}{5}\,x^{\frac{5}{2}}\operatorname{arccot}\left(\sqrt{x}\right)+\frac{1}{10}\,x^2-\frac{1}{5}\,x+\frac{1}{5}\,\log\left(x+1\right)$$

input
```
integrate(x^(3/2)*arccot(x^(1/2)),x, algorithm="maxima")
```

output
```
2/5*x^(5/2)*arccot(sqrt(x)) + 1/10*x^2 - 1/5*x + 1/5*log(x + 1)
```

3.91.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int x^{3/2}\cot^{-1}\left(\sqrt{x}\right)\,dx=\frac{2}{5}\,x^{\frac{5}{2}}\arctan\left(\frac{1}{\sqrt{x}}\right)-\frac{1}{10}\,x^2\left(\frac{2}{x}-\frac{3}{x^2}-1\right)+\frac{1}{5}\,\log\left(x\right)+\frac{1}{5}\,\log\left(\frac{1}{x}+1\right)$$

input
```
integrate(x^(3/2)*arccot(x^(1/2)),x, algorithm="giac")
```

output
```
2/5*x^(5/2)*arctan(1/sqrt(x)) - 1/10*x^2*(2/x - 3/x^2 - 1) + 1/5*log(x) +
1/5*log(1/x + 1)
```

3.91.9 Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{3/2}\cot^{-1}\left(\sqrt{x}\right)\,dx=\frac{\ln\left(x+1\right)}{5}-\frac{x}{5}+\frac{2\,x^{5/2}\,\mathrm{acot}\left(\sqrt{x}\right)}{5}+\frac{x^2}{10}$$

input
```
int(x^(3/2)*acot(x^(1/2)),x)
```

output
```
log(x + 1)/5 - x/5 + (2*x^(5/2)*acot(x^(1/2)))/5 + x^2/10
```

3.92 $\int \sqrt{x}\cot^{-1}\left(\sqrt{x}\right)\,dx$

3.92.1 Optimal result

Integrand size = 12, antiderivative size = 29

$$\int \sqrt{x}\cot^{-1}\left(\sqrt{x}\right)\,dx=\frac{x}{3}+\frac{2}{3}x^{3/2}\cot^{-1}\left(\sqrt{x}\right)-\frac{1}{3}\log(1+x)$$

output
```
1/3*x+2/3*x^(3/2)*arccot(x^(1/2))-1/3*ln(1+x)
```

3.92.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \sqrt{x}\cot^{-1}\left(\sqrt{x}\right)\,dx=\frac{1}{3}\left(x+2x^{3/2}\cot^{-1}\left(\sqrt{x}\right)-\log(1+x)\right)$$

input
```
Integrate[Sqrt[x]*ArcCot[Sqrt[x]],x]
```

output
```
(x + 2*x^(3/2)*ArcCot[Sqrt[x]] - Log[1 + x])/3
```

3.92.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5362, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\cot^{-1}\left(\sqrt{x}\right)\,dx$$

$$\Big\downarrow \text{ 5362}$$

$$\frac{1}{3}\int \frac{x}{x+1}dx+\frac{2}{3}x^{3/2}\cot^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{3}\int \left(1+\frac{1}{-x-1}\right)dx+\frac{2}{3}x^{3/2}\cot^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{3}x^{3/2}\cot^{-1}\left(\sqrt{x}\right)+\frac{1}{3}(x-\log(x+1))$$

input
```
Int[Sqrt[x]*ArcCot[Sqrt[x]],x]
```

output
```
(2*x^(3/2)*ArcCot[Sqrt[x]])/3 + (x - Log[1 + x])/3
```

3.92.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.92.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$\frac{x}{3}+\frac{2x^{\frac{3}{2}}\operatorname{arccot}\left(\sqrt{x}\right)}{3}-\frac{\ln\left(1+x\right)}{3}$	20
default	$\frac{x}{3}+\frac{2x^{\frac{3}{2}}\operatorname{arccot}\left(\sqrt{x}\right)}{3}-\frac{\ln\left(1+x\right)}{3}$	20

input
```
int(x^(1/2)*arccot(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/3*x+2/3*x^(3/2)*arccot(x^(1/2))-1/3*ln(1+x)
```

3.92.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \sqrt{x}\cot^{-1}\left(\sqrt{x}\right)\,dx=\frac{2}{3}\,x^{\frac{3}{2}}\operatorname{arccot}\left(\sqrt{x}\right)+\frac{1}{3}\,x-\frac{1}{3}\,\log\left(x+1\right)$$

input
```
integrate(x^(1/2)*arccot(x^(1/2)),x, algorithm="fricas")
```

output
```
2/3*x^(3/2)*arccot(sqrt(x)) + 1/3*x - 1/3*log(x + 1)
```

3.92.6 Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \sqrt{x}\cot^{-1}\left(\sqrt{x}\right)\,dx = \frac{2x^{\frac{3}{2}}\operatorname{acot}\left(\sqrt{x}\right)}{3} + \frac{x}{3} - \frac{\log\left(x+1\right)}{3}$$

input
```
integrate(x**(1/2)*acot(x**(1/2)),x)
```

output
```
2*x**(3/2)*acot(sqrt(x))/3 + x/3 - log(x + 1)/3
```

3.92.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \sqrt{x}\cot^{-1}\left(\sqrt{x}\right)\,dx = \frac{2}{3}\,x^{\frac{3}{2}}\operatorname{arccot}\left(\sqrt{x}\right) + \frac{1}{3}\,x - \frac{1}{3}\,\log\left(x+1\right)$$

input
```
integrate(x^(1/2)*arccot(x^(1/2)),x, algorithm="maxima")
```

output
```
2/3*x^(3/2)*arccot(sqrt(x)) + 1/3*x - 1/3*log(x + 1)
```

3.92.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \sqrt{x}\cot^{-1}\left(\sqrt{x}\right)\,dx = \frac{2}{3}\,x^{\frac{3}{2}}\arctan\left(\frac{1}{\sqrt{x}}\right) - \frac{1}{3}\,x\left(\frac{1}{x}-1\right) - \frac{1}{3}\,\log\left(x\right) - \frac{1}{3}\,\log\left(\frac{1}{x}+1\right)$$

input
```
integrate(x^(1/2)*arccot(x^(1/2)),x, algorithm="giac")
```

output
```
2/3*x^(3/2)*arctan(1/sqrt(x)) - 1/3*x*(1/x - 1) - 1/3*log(x) - 1/3*log(1/x
 + 1)
```

3.92.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}\cot^{-1}\left(\sqrt{x}\right)\,dx = \int \sqrt{x}\,\mathrm{acot}\left(\sqrt{x}\right)\,dx$$

input
```
int(x^(1/2)*acot(x^(1/2)),x)
```

output
```
int(x^(1/2)*acot(x^(1/2)), x)
```

3.93 $\int \frac{\cot^{-1}(\sqrt{x})}{\sqrt{x}}\,dx$

3.93.1 Optimal result

Integrand size = 12, antiderivative size = 18

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\cot^{-1}\left(\sqrt{x}\right) + \log(1+x)$$

output
```
ln(1+x)+2*x^(1/2)*arccot(x^(1/2))
```

3.93.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\cot^{-1}\left(\sqrt{x}\right) + \log(1+x)$$

input
```
Integrate[ArcCot[Sqrt[x]]/Sqrt[x],x]
```

output
```
2*Sqrt[x]*ArcCot[Sqrt[x]] + Log[1 + x]
```

3.93.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5362, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}dx$$
$$\Big\downarrow \text{ 5362}$$
$$\int \frac{1}{x+1}dx+2\sqrt{x}\cot^{-1}\left(\sqrt{x}\right)$$
$$\Big\downarrow \text{ 16}$$
$$\log(x+1)+2\sqrt{x}\cot^{-1}\left(\sqrt{x}\right)$$

input
```
Int[ArcCot[Sqrt[x]]/Sqrt[x],x]
```

output
```
2*Sqrt[x]*ArcCot[Sqrt[x]] + Log[1 + x]
```

3.93.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.93.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\ln(1+x)+2\sqrt{x}\,\operatorname{arccot}\left(\sqrt{x}\right)$	15
default	$\ln(1+x)+2\sqrt{x}\,\operatorname{arccot}\left(\sqrt{x}\right)$	15

input
```
int(arccot(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(1+x)+2*x^(1/2)*arccot(x^(1/2))
```

3.93.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\operatorname{arccot}\left(\sqrt{x}\right)+\log(x+1)$$

input
```
integrate(arccot(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(x)*arccot(sqrt(x)) + log(x + 1)
```

3.93.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\operatorname{acot}\left(\sqrt{x}\right)+\log(x+1)$$

input
```
integrate(acot(x**(1/2))/x**(1/2),x)
```

output
```
2*sqrt(x)*acot(sqrt(x)) + log(x + 1)
```

3.93.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\,\mathrm{arccot}\left(\sqrt{x}\right) + \log\left(x+1\right)$$

input
```
integrate(arccot(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(x)*arccot(sqrt(x)) + log(x + 1)
```

3.93.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\arctan\left(\frac{1}{\sqrt{x}}\right) + \log\left(x\right) + \log\left(\frac{1}{x}+1\right)$$

input
```
integrate(arccot(x^(1/2))/x^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(x)*arctan(1/sqrt(x)) + log(x) + log(1/x + 1)
```

3.93.9 Mupad [B] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \ln\left(x+1\right) + 2\sqrt{x}\,\mathrm{acot}\left(\sqrt{x}\right)$$

input
```
int(acot(x^(1/2))/x^(1/2),x)
```

output
```
log(x + 1) + 2*x^(1/2)*acot(x^(1/2))
```

3.94 $\int \frac{\cot^{-1}(\sqrt{x})}{x^{3/2}}\,dx$

3.94.1 Optimal result

Integrand size = 12, antiderivative size = 22

$$\int \frac{\cot^{-1}(\sqrt{x})}{x^{3/2}}\,dx = -\frac{2\cot^{-1}(\sqrt{x})}{\sqrt{x}} - \log(x) + \log(1+x)$$

output
```
-ln(x)+ln(1+x)-2*arccot(x^(1/2))/x^(1/2)
```

3.94.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(\sqrt{x})}{x^{3/2}}\,dx = -\frac{2\cot^{-1}(\sqrt{x})}{\sqrt{x}} - \log(x) + \log(1+x)$$

input
```
Integrate[ArcCot[Sqrt[x]]/x^(3/2),x]
```

output
```
(-2*ArcCot[Sqrt[x]])/Sqrt[x] - Log[x] + Log[1 + x]
```

3.94.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5362, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{3/2}}dx$$

$$\Big\downarrow \text{5362}$$

$$-\int \frac{1}{x(x+1)}dx-\frac{2\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}$$

$$\Big\downarrow \text{47}$$

$$-\int \frac{1}{x}dx+\int \frac{1}{x+1}dx-\frac{2\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}$$

$$\Big\downarrow \text{14}$$

$$\int \frac{1}{x+1}dx-\log(x)-\frac{2\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}$$

$$\Big\downarrow \text{16}$$

$$-\log(x)+\log(x+1)-\frac{2\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}$$

input
```
Int[ArcCot[Sqrt[x]]/x^(3/2),x]
```

output
```
(-2*ArcCot[Sqrt[x]])/Sqrt[x] - Log[x] + Log[1 + x]
```

3.94.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.94.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$-\ln(x)+\ln(1+x)-\frac{2\,\mathrm{arccot}(\sqrt{x})}{\sqrt{x}}$	19
default	$-\ln(x)+\ln(1+x)-\frac{2\,\mathrm{arccot}(\sqrt{x})}{\sqrt{x}}$	19

input
```
int(arccot(x^(1/2))/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-ln(x)+ln(1+x)-2*arccot(x^(1/2))/x^(1/2)
```

3.94.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{\cot^{-1}(\sqrt{x})}{x^{3/2}}\,dx = \frac{x\log(x+1)-x\log(x)-2\sqrt{x}\,\mathrm{arccot}(\sqrt{x})}{x}$$

input
```
integrate(arccot(x^(1/2))/x^(3/2),x, algorithm="fricas")
```

output
```
(x*log(x + 1) - x*log(x) - 2*sqrt(x)*arccot(sqrt(x)))/x
```

3.94.6 Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\,dx = -\log\left(x\right) + \log\left(x+1\right) - \frac{2\,\mathrm{acot}\left(\sqrt{x}\right)}{\sqrt{x}}$$

input
```
integrate(acot(x**(1/2))/x**(3/2),x)
```

output
```
-log(x) + log(x + 1) - 2*acot(sqrt(x))/sqrt(x)
```

3.94.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\,dx = -\frac{2\,\mathrm{arccot}\left(\sqrt{x}\right)}{\sqrt{x}} + \log\left(x+1\right) - \log\left(x\right)$$

input
```
integrate(arccot(x^(1/2))/x^(3/2),x, algorithm="maxima")
```

output
```
-2*arccot(sqrt(x))/sqrt(x) + log(x + 1) - log(x)
```

3.94.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\,dx = -\frac{2\,\arctan\left(\frac{1}{\sqrt{x}}\right)}{\sqrt{x}} + \log\left(\frac{1}{x}+1\right)$$

input
```
integrate(arccot(x^(1/2))/x^(3/2),x, algorithm="giac")
```

output
```
-2*arctan(1/sqrt(x))/sqrt(x) + log(1/x + 1)
```

3.94.9 Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\,dx = \ln\left(x+1\right) - 2\,\ln\left(\sqrt{x}\right) - \frac{2\,\mathrm{acot}\left(\sqrt{x}\right)}{\sqrt{x}}$$

input
```
int(acot(x^(1/2))/x^(3/2),x)
```

output
```
log(x + 1) - 2*log(x^(1/2)) - (2*acot(x^(1/2)))/x^(1/2)
```

3.95 $\int \frac{\cot^{-1}(\sqrt{x})}{x^{5/2}}\,dx$

3.95.1 Optimal result

Integrand size = 12, antiderivative size = 37

$$\int \frac{\cot^{-1}(\sqrt{x})}{x^{5/2}}\,dx=\frac{1}{3x}-\frac{2\cot^{-1}(\sqrt{x})}{3x^{3/2}}+\frac{\log(x)}{3}-\frac{1}{3}\log(1+x)$$

output
```
1/3/x-2/3*arccot(x^(1/2))/x^(3/2)+1/3*ln(x)-1/3*ln(1+x)
```

3.95.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{\cot^{-1}(\sqrt{x})}{x^{5/2}}\,dx=\frac{1}{3}\left(\frac{1}{x}-\frac{2\cot^{-1}(\sqrt{x})}{x^{3/2}}+\log(x)-\log(1+x)\right)$$

input
```
Integrate[ArcCot[Sqrt[x]]/x^(5/2),x]
```

output
```
(x^(-1) - (2*ArcCot[Sqrt[x]])/x^(3/2) + Log[x] - Log[1 + x])/3
```

3.95.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5362, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{5/2}}dx$$

$$\Big\downarrow \text{5362}$$

$$-\frac{1}{3}\int \frac{1}{x^2(x+1)}dx-\frac{2\cot^{-1}\left(\sqrt{x}\right)}{3x^{3/2}}$$

$$\Big\downarrow \text{54}$$

$$-\frac{1}{3}\int \left(\frac{1}{x+1}-\frac{1}{x}+\frac{1}{x^2}\right)dx-\frac{2\cot^{-1}\left(\sqrt{x}\right)}{3x^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{1}{x}+\log(x)-\log(x+1)\right)-\frac{2\cot^{-1}\left(\sqrt{x}\right)}{3x^{3/2}}$$

input
```
Int[ArcCot[Sqrt[x]]/x^(5/2),x]
```

output
```
(-2*ArcCot[Sqrt[x]])/(3*x^(3/2)) + (x^(-1) + Log[x] - Log[1 + x])/3
```

3.95.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.95.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{1}{3x}-\frac{2\,\mathrm{arccot}\left(\sqrt{x}\right)}{3x^{\frac{3}{2}}}+\frac{\ln\left(x\right)}{3}-\frac{\ln\left(1+x\right)}{3}$	26
default	$\frac{1}{3x}-\frac{2\,\mathrm{arccot}\left(\sqrt{x}\right)}{3x^{\frac{3}{2}}}+\frac{\ln\left(x\right)}{3}-\frac{\ln\left(1+x\right)}{3}$	26

input
```
int(arccot(x^(1/2))/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/x-2/3*arccot(x^(1/2))/x^(3/2)+1/3*ln(x)-1/3*ln(1+x)
```

3.95.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{5/2}}\,dx=-\frac{x^2\log\left(x+1\right)-x^2\log\left(x\right)+2\sqrt{x}\,\mathrm{arccot}\left(\sqrt{x}\right)-x}{3\,x^2}$$

input
```
integrate(arccot(x^(1/2))/x^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(x^2*log(x + 1) - x^2*log(x) + 2*sqrt(x)*arccot(sqrt(x)) - x)/x^2
```

3.95.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(31) = 62$.

Time = 1.14 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.86

$$\int \frac{\cot^{-1}(\sqrt{x})}{x^{5/2}}\,dx = -\frac{2x^{\frac{3}{2}}\operatorname{acot}(\sqrt{x})}{3x^3+3x^2} - \frac{2\sqrt{x}\operatorname{acot}(\sqrt{x})}{3x^3+3x^2} + \frac{x^3\log(x)}{3x^3+3x^2} - \frac{x^3\log(x+1)}{3x^3+3x^2} + \frac{x^2\log(x)}{3x^3+3x^2} - \frac{x^2\log(x+1)}{3x^3+3x^2} + \frac{x^2}{3x^3+3x^2} + \frac{x}{3x^3+3x^2}$$

input
```
integrate(acot(x**(1/2))/x**(5/2),x)
```

output
```
-2*x**(3/2)*acot(sqrt(x))/(3*x**3 + 3*x**2) - 2*sqrt(x)*acot(sqrt(x))/(3*x
**3 + 3*x**2) + x**3*log(x)/(3*x**3 + 3*x**2) - x**3*log(x + 1)/(3*x**3 +
3*x**2) + x**2*log(x)/(3*x**3 + 3*x**2) - x**2*log(x + 1)/(3*x**3 + 3*x**2
) + x**2/(3*x**3 + 3*x**2) + x/(3*x**3 + 3*x**2)
```

3.95.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int \frac{\cot^{-1}(\sqrt{x})}{x^{5/2}}\,dx = -\frac{2\operatorname{arccot}(\sqrt{x})}{3x^{\frac{3}{2}}} + \frac{1}{3x} - \frac{1}{3}\log(x+1) + \frac{1}{3}\log(x)$$

input
```
integrate(arccot(x^(1/2))/x^(5/2),x, algorithm="maxima")
```

output
```
-2/3*arccot(sqrt(x))/x^(3/2) + 1/3/x - 1/3*log(x + 1) + 1/3*log(x)
```

3.95.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.62

$$\int \frac{\cot^{-1}(\sqrt{x})}{x^{5/2}}\,dx = -\frac{2\arctan\left(\frac{1}{\sqrt{x}}\right)}{3x^{\frac{3}{2}}} + \frac{1}{3x} - \frac{1}{3}\log\left(\frac{1}{x}+1\right)$$

input
```
integrate(arccot(x^(1/2))/x^(5/2),x, algorithm="giac")
```

output
```
-2/3*arctan(1/sqrt(x))/x^(3/2) + 1/3/x - 1/3*log(1/x + 1)
```

3.95.9 Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{5/2}}\,dx = \frac{2\,\ln\left(\sqrt{x}\right)}{3} - \frac{\ln\left(x+1\right)}{3} - \frac{2\,\mathrm{acot}\left(\sqrt{x}\right)}{3\,x^{3/2}} + \frac{1}{3\,x}$$

input
```
int(acot(x^(1/2))/x^(5/2),x)
```

output
```
(2*log(x^(1/2)))/3 - log(x + 1)/3 - (2*acot(x^(1/2)))/(3*x^(3/2)) + 1/(3*x
)
```

3.96 $\int \cot^{-1}\left(\frac{1}{x}\right) dx$

3.96.1 Optimal result

Integrand size = 4, antiderivative size = 17

$$\int \cot^{-1}\left(\frac{1}{x}\right) dx = x\cot^{-1}\left(\frac{1}{x}\right) - \frac{1}{2}\log\left(1+x^2\right)$$

output
```
x*arccot(1/x)-1/2*ln(x^2+1)
```

3.96.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cot^{-1}\left(\frac{1}{x}\right) dx = x\cot^{-1}\left(\frac{1}{x}\right) - \frac{1}{2}\log\left(1+x^2\right)$$

input
```
Integrate[ArcCot[x^(-1)],x]
```

output
```
x*ArcCot[x^(-1)] - Log[1 + x^2]/2
```

3.96.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5346, 795, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}\left(\frac{1}{x}\right)dx$$

$$\Big\downarrow \text{5346}$$

$$x\cot^{-1}\left(\frac{1}{x}\right)-\int \frac{1}{\left(1+\frac{1}{x^2}\right)x}dx$$

$$\Big\downarrow \text{795}$$

$$x\cot^{-1}\left(\frac{1}{x}\right)-\int \frac{x}{x^2+1}dx$$

$$\Big\downarrow \text{240}$$

$$x\cot^{-1}\left(\frac{1}{x}\right)-\frac{1}{2}\log\left(x^2+1\right)$$

input
```
Int[ArcCot[x^(-1)],x]
```

output
```
x*ArcCot[x^(-1)] - Log[1 + x^2]/2
```

3.96.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

3.96.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result
parallelrisch	$x\,\mathrm{arccot}\left(\frac{1}{x}\right)-\frac{\ln\left(x^{2}+1\right)}{2}$
parts	$x\,\mathrm{arccot}\left(\frac{1}{x}\right)-\frac{\ln\left(x^{2}+1\right)}{2}$
derivativedivides	$x\,\mathrm{arccot}\left(\frac{1}{x}\right)+\ln\left(\frac{1}{x}\right)-\frac{\ln\left(\frac{1}{x^{2}}+1\right)}{2}$
default	$x\,\mathrm{arccot}\left(\frac{1}{x}\right)+\ln\left(\frac{1}{x}\right)-\frac{\ln\left(\frac{1}{x^{2}}+1\right)}{2}$
risch	$\frac{ix\ln(i+x)}{2}-\frac{i\ln(x-i)x}{2}-\frac{\pi\,\mathrm{csgn}\left(\frac{i}{x}\right)\mathrm{csgn}\left(i\left(-\mathrm{RootOf}\left(\_Z^{2}+1,\mathrm{index}=1\right)+x\right)\right)\mathrm{csgn}\left(\frac{i\left(-\mathrm{RootOf}\left(\_Z^{2}+1,\mathrm{index}\right.\right.}{x}\right.}{4}$

input
```
int(arccot(1/x),x,method=_RETURNVERBOSE)
```

output
```
x*arccot(1/x)-1/2*ln(x^2+1)
```

3.96.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \cot^{-1}\left(\frac{1}{x}\right)\,dx = x\,\mathrm{arccot}\left(\frac{1}{x}\right)-\frac{1}{2}\log\left(x^{2}+1\right)$$

input
```
integrate(arccot(1/x),x, algorithm="fricas")
```

output
```
x*arccot(1/x) - 1/2*log(x^2 + 1)
```

3.96.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \cot^{-1}\left(\frac{1}{x}\right)\,dx = x\,\mathrm{acot}\left(\frac{1}{x}\right) - \frac{\log\left(x^2+1\right)}{2}$$

input
```
integrate(acot(1/x),x)
```

output
```
x*acot(1/x) - log(x**2 + 1)/2
```

3.96.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \cot^{-1}\left(\frac{1}{x}\right)\,dx = x\,\mathrm{arccot}\left(\frac{1}{x}\right) - \frac{1}{2}\log\left(x^2+1\right)$$

input
```
integrate(arccot(1/x),x, algorithm="maxima")
```

output
```
x*arccot(1/x) - 1/2*log(x^2 + 1)
```

3.96.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cot^{-1}\left(\frac{1}{x}\right)\,dx = x\arctan\left(x\right) - \frac{1}{2}\log\left(x^2+1\right)$$

input
```
integrate(arccot(1/x),x, algorithm="giac")
```

output
```
x*arctan(x) - 1/2*log(x^2 + 1)
```

3.96.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \cot^{-1}\left(\frac{1}{x}\right)\,dx = x\,\mathrm{acot}\left(\frac{1}{x}\right) - \frac{\ln\left(x^2+1\right)}{2}$$

input
```
int(acot(1/x),x)
```

output
```
x*acot(1/x) - log(x^2 + 1)/2
```

3.97 $\int \frac{\cot^{-1}(ax^n)}{x} dx$

3.97.1 Optimal result

Integrand size = 10, antiderivative size = 47

$$\int \frac{\cot^{-1}(ax^n)}{x} dx = -\frac{i \operatorname{PolyLog}\left(2, -\frac{ix^{-n}}{a}\right)}{2n} + \frac{i \operatorname{PolyLog}\left(2, \frac{ix^{-n}}{a}\right)}{2n}$$

output
```
-1/2*I*polylog(2,-I/a/(x^n))/n+1/2*I*polylog(2,I/a/(x^n))/n
```

3.97.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{\cot^{-1}(ax^n)}{x} dx = -\frac{i\left(\operatorname{PolyLog}\left(2, -\frac{ix^{-n}}{a}\right) - \operatorname{PolyLog}\left(2, \frac{ix^{-n}}{a}\right)\right)}{2n}$$

input
```
Integrate[ArcCot[a*x^n]/x,x]
```

output
```
((-1/2*I)*(PolyLog[2, (-I)/(a*x^n)] - PolyLog[2, I/(a*x^n)]))/n
```

3.97.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5360, 5356, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax^n)}{x} \, dx$$
$$\Big\downarrow \ 5360$$
$$\frac{\int x^{-n}\cot^{-1}(ax^n)\,dx^n}{n}$$
$$\Big\downarrow \ 5356$$
$$\frac{\frac{1}{2}i\int x^{-n}\log\left(1-\frac{ix^{-n}}{a}\right)dx^n-\frac{1}{2}i\int x^{-n}\log\left(\frac{ix^{-n}}{a}+1\right)dx^n}{n}$$
$$\Big\downarrow \ 2838$$
$$\frac{\frac{1}{2}i\,\text{PolyLog}\left(2,\frac{ix^{-n}}{a}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,-\frac{ix^{-n}}{a}\right)}{n}$$

input
```
Int[ArcCot[a*x^n]/x,x]
```

output
```
((-1/2*I)*PolyLog[2, (-I)/(a*x^n)] + (I/2)*PolyLog[2, I/(a*x^n)])/n
```

3.97.3.1 Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5356
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Simp[I*(b/2)   Int[Log[1 + I/(c*x)]/x, x], x] + Simp[I*(b/2)   Int[Log
[1 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5360
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1
/n   Subst[Int[(a + b*ArcCot[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
n}, x] && IGtQ[p, 0]
```

3.97.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(39) = 78.

Time = 0.54 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.77

method	result
derivativedivides	$\frac{\ln(a\,x^{n})\operatorname{arccot}(a\,x^{n})-\frac{i\ln(a\,x^{n})\ln(1+ix^{n}a)}{2}+\frac{i\ln(a\,x^{n})\ln(1-ix^{n}a)}{2}-\frac{i\,\mathrm{dilog}(1+ix^{n}a)}{2}+\frac{i\,\mathrm{dilog}(1-ix^{n}a)}{2}}{n}$
default	$\frac{\ln(a\,x^{n})\operatorname{arccot}(a\,x^{n})-\frac{i\ln(a\,x^{n})\ln(1+ix^{n}a)}{2}+\frac{i\ln(a\,x^{n})\ln(1-ix^{n}a)}{2}-\frac{i\,\mathrm{dilog}(1+ix^{n}a)}{2}+\frac{i\,\mathrm{dilog}(1-ix^{n}a)}{2}}{n}$
risch	$\frac{i\ln(x)\ln(1+ix^{n}a)}{2}+\frac{\pi\ln(x)}{2}+\frac{i\,\mathrm{dilog}(1-ix^{n}a)}{2n}-\frac{i\ln(-i(-a\,x^{n}+i))\ln(x)}{2}+\frac{i\ln(-i(-a\,x^{n}+i))\ln(-ix^{n}a)}{2n}+$

input
```
int(arccot(a*x^n)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(ln(a*x^n)*arccot(a*x^n)-1/2*I*ln(a*x^n)*ln(1+I*x^n*a)+1/2*I*ln(a*x^n)
*ln(1-I*x^n*a)-1/2*I*dilog(1+I*x^n*a)+1/2*I*dilog(1-I*x^n*a))
```

3.97.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.34

$$\int \frac{\cot^{-1}(ax^n)}{x}\,dx = \frac{2\,n\operatorname{arccot}(ax^n)\log(x)-i\,n\log(i\,ax^n+1)\log(x)+i\,n\log(-i\,ax^n+1)\log(x)+i\,\mathrm{Li}_2(i\,ax^n)-i\,\mathrm{Li}_2(-i\,a}{2\,n}$$

input
```
integrate(arccot(a*x^n)/x,x, algorithm="fricas")
```

output
```
1/2*(2*n*arccot(a*x^n)*log(x) - I*n*log(I*a*x^n + 1)*log(x) + I*n*log(-I*a
*x^n + 1)*log(x) + I*dilog(I*a*x^n) - I*dilog(-I*a*x^n))/n
```

3.97.6 Sympy [F]

$$\int \frac{\cot^{-1}(ax^n)}{x} \, dx = \int \frac{\operatorname{acot}(ax^n)}{x} \, dx$$

input
```
integrate(acot(a*x**n)/x,x)
```

output
```
Integral(acot(a*x**n)/x, x)
```

3.97.7 Maxima [F]

$$\int \frac{\cot^{-1}(ax^n)}{x} \, dx = \int \frac{\operatorname{arccot}(ax^n)}{x} \, dx$$

input
```
integrate(arccot(a*x^n)/x,x, algorithm="maxima")
```

output
```
a*n*integrate(x^n*log(x)/(a^2*x*x^(2*n) + x), x) + arctan(1/(a*x^n))*log(x
)
```

3.97.8 Giac [F]

$$\int \frac{\cot^{-1}(ax^n)}{x} \, dx = \int \frac{\operatorname{arccot}(ax^n)}{x} \, dx$$

input
```
integrate(arccot(a*x^n)/x,x, algorithm="giac")
```

output
```
integrate(arccot(a*x^n)/x, x)
```

3.97.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax^n)}{x}\,dx = \int \frac{\operatorname{acot}(a\,x^n)}{x}\,dx$$

input
```
int(acot(a*x^n)/x,x)
```

output
```
int(acot(a*x^n)/x, x)
```

3.98 $\int \frac{\cot^{-1}(ax^5)}{x} \, dx$

3.98.1 Optimal result

Integrand size = 10, antiderivative size = 37

$$\int \frac{\cot^{-1}(ax^5)}{x} \, dx = -\frac{1}{10} i \operatorname{PolyLog}\left(2,-\frac{i}{ax^5}\right)+\frac{1}{10} i \operatorname{PolyLog}\left(2,\frac{i}{ax^5}\right)$$

output
```
-1/10*I*polylog(2,-I/a/x^5)+1/10*I*polylog(2,I/a/x^5)
```

3.98.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(ax^5)}{x} \, dx = -\frac{1}{10} i \operatorname{PolyLog}\left(2,-\frac{i}{ax^5}\right)+\frac{1}{10} i \operatorname{PolyLog}\left(2,\frac{i}{ax^5}\right)$$

input
```
Integrate[ArcCot[a*x^5]/x,x]
```

output
```
(-1/10*I)*PolyLog[2, (-I)/(a*x^5)] + (I/10)*PolyLog[2, I/(a*x^5)]
```

3.98.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5360, 5356, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax^5)}{x}\,dx$$

$$\Big\downarrow \; 5360$$

$$\frac{1}{5}\int \frac{\cot^{-1}(ax^5)}{x^5}dx^5$$

$$\Big\downarrow \; 5356$$

$$\frac{1}{5}\left(\frac{1}{2}i\int \frac{\log\left(1-\frac{i}{ax^5}\right)}{x^5}dx^5-\frac{1}{2}i\int \frac{\log\left(1+\frac{i}{x^5a}\right)}{x^5}dx^5\right)$$

$$\Big\downarrow \; 2838$$

$$\frac{1}{5}\left(\frac{1}{2}i\,\text{PolyLog}\left(2,\frac{i}{ax^5}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,-\frac{i}{ax^5}\right)\right)$$

input
```
Int[ArcCot[a*x^5]/x,x]
```

output
```
((-1/2*I)*PolyLog[2, (-I)/(a*x^5)] + (I/2)*PolyLog[2, I/(a*x^5)])/5
```

3.98.3.1 Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5356
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Simp[I*(b/2)   Int[Log[1 + I/(c*x)]/x, x], x] + Simp[I*(b/2)   Int[Log
[1 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5360
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1
/n   Subst[Int[(a + b*ArcCot[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
n}, x] && IGtQ[p, 0]
```

3.98.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.42 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.54

method	result
default	$\ln(x)\operatorname{arccot}(a\,x^5)+\frac{\sum_{\_R1=\mathrm{RootOf}(a^2\_Z^{10}+1)}\frac{\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-x}{\_R1}\right)}{\_R1^5}}{2a}$
parts	$\ln(x)\operatorname{arccot}(a\,x^5)+\frac{\sum_{\_R1=\mathrm{RootOf}(a^2\_Z^{10}+1)}\frac{\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-x}{\_R1}\right)}{\_R1^5}}{2a}$
risch	$\frac{\pi\ln(x)}{2}+\frac{i\left(\sum_{\_R1=\mathrm{RootOf}(\_Z^5 a+\mathrm{RootOf}(\_Z^2+1,\mathrm{index}=1))}\left(\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-x}{\_R1}\right)\right)\right)}{2}-\frac{i\ln(x)\ln(-ia\,x^5+1)}{2}$

input
```
int(arccot(a*x^5)/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*arccot(a*x^5)+1/2/a*sum(1/_R1^5*(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)
/_R1)),_R1=RootOf(_Z^10*a^2+1))
```

3.98.5 Fricas [F]

$$\int \frac{\cot^{-1}(ax^5)}{x}\,dx=\int \frac{\operatorname{arccot}(ax^5)}{x}\,dx$$

input
```
integrate(arccot(a*x^5)/x,x, algorithm="fricas")
```

output
```
integral(arccot(a*x^5)/x, x)
```

3.98.6 Sympy [F]

$$\int \frac{\cot^{-1}(ax^5)}{x}\,dx = \int \frac{\operatorname{acot}(ax^5)}{x}\,dx$$

input
```
integrate(acot(a*x**5)/x,x)
```

output
```
Integral(acot(a*x**5)/x, x)
```

3.98.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(23) = 46$.

Time = 0.33 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.84

$$\int \frac{\cot^{-1}(ax^5)}{x}\,dx = \frac{1}{20}\pi\log\left(a^2x^{10}+1\right) - \frac{1}{5}\arctan\left(ax^5\right)\log\left(ax^5\right) + \operatorname{arccot}\left(ax^5\right)\log\left(x\right) + \arctan\left(ax^5\right)\log\left(x\right) + \frac{1}{10}i\operatorname{Li}_2\left(i\,ax^5+1\right) - \frac{1}{10}i\operatorname{Li}_2\left(-i\,ax^5+1\right)$$

input
```
integrate(arccot(a*x^5)/x,x, algorithm="maxima")
```

output
```
1/20*pi*log(a^2*x^10 + 1) - 1/5*arctan(a*x^5)*log(a*x^5) + arccot(a*x^5)*l
og(x) + arctan(a*x^5)*log(x) + 1/10*I*dilog(I*a*x^5 + 1) - 1/10*I*dilog(-I
*a*x^5 + 1)
```

3.98.8 Giac [F]

$$\int \frac{\cot^{-1}(ax^5)}{x}\,dx = \int \frac{\operatorname{arccot}(ax^5)}{x}\,dx$$

input
```
integrate(arccot(a*x^5)/x,x, algorithm="giac")
```

output
```
integrate(arccot(a*x^5)/x, x)
```

3.98.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax^5)}{x}\,dx = \int \frac{\mathrm{acot}(a\,x^5)}{x}\,dx$$

input
```
int(acot(a*x^5)/x,x)
```

output
```
int(acot(a*x^5)/x, x)
```

3.99 $\int x^3 \cot^{-1}(a+bx)\,dx$

3.99.1 Optimal result

Integrand size = 10, antiderivative size = 106

$$\int x^3 \cot^{-1}(a+bx)\,dx = -\frac{(1-6a^2)\,x}{4b^3} - \frac{a(a+bx)^2}{2b^4} + \frac{(a+bx)^3}{12b^4} + \frac{1}{4}x^4\cot^{-1}(a+bx) + \frac{(1-6a^2+a^4)\arctan(a+bx)}{4b^4} + \frac{a(1-a^2)\log\left(1+(a+bx)^2\right)}{2b^4}$$

output
```
-1/4*(-6*a^2+1)*x/b^3-1/2*a*(b*x+a)^2/b^4+1/12*(b*x+a)^3/b^4+1/4*x^4*arcco
t(b*x+a)+1/4*(a^4-6*a^2+1)*arctan(b*x+a)/b^4+1/2*a*(-a^2+1)*ln(1+(b*x+a)^2
)/b^4
```

3.99.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.90

$$\int x^3 \cot^{-1}(a+bx)\,dx = \frac{6(-1+6a^2)\,bx - 12a(a+bx)^2 + 2(a+bx)^3 + 6b^4x^4\cot^{-1}(a+bx) - 3i(-i+a)^4\log(i-a-bx) + 3i(i+}{24b^4}$$

input
```
Integrate[x^3*ArcCot[a + b*x],x]
```

output
```
(6*(-1 + 6*a^2)*b*x - 12*a*(a + b*x)^2 + 2*(a + b*x)^3 + 6*b^4*x^4*ArcCot[
a + b*x] - (3*I)*(-I + a)^4*Log[I - a - b*x] + (3*I)*(I + a)^4*Log[I + a +
 b*x])/(24*b^4)
```

3.99.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5571, 25, 27, 5388, 478, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \cot^{-1}(a+bx)\,dx$$

$$\Big\downarrow \text{5571}$$

$$\frac{\int x^3 \cot^{-1}(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -x^3 \cot^{-1}(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int -b^3x^3 \cot^{-1}(a+bx)d(a+bx)}{b^4}$$

$$\Big\downarrow \text{5388}$$

$$-\frac{-\frac{1}{4}\int \frac{b^4x^4}{(a+bx)^2+1}d(a+bx)-\frac{1}{4}b^4x^4\cot^{-1}(a+bx)}{b^4}$$

$$\Big\downarrow \text{478}$$

$$-\frac{-\frac{1}{4}\int \left(6a^2-4(a+bx)a+(a+bx)^2+\frac{a^4-6a^2+4(1-a^2)(a+bx)a+1}{(a+bx)^2+1}-1\right)d(a+bx)-\frac{1}{4}b^4x^4\cot^{-1}(a+bx)}{b^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{4}\left(\left(1-6a^2\right)(a+bx)-2a\left(1-a^2\right)\log\left((a+bx)^2+1\right)-\left(a^4-6a^2+1\right)\arctan(a+bx)-\frac{1}{3}(a+bx)^3+2a(a+b\right.}{b^4}$$

input
```
Int[x^3*ArcCot[a + b*x],x]
```

output
```
-((-1/4*(b^4*x^4*ArcCot[a + b*x]) + ((1 - 6*a^2)*(a + b*x) + 2*a*(a + b*x)
^2 - (a + b*x)^3/3 - (1 - 6*a^2 + a^4)*ArcTan[a + b*x] - 2*a*(1 - a^2)*Log
[1 + (a + b*x)^2])/4)/b^4)
```

3.99.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 478
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ
[n, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5388
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])/(e*(q + 1))), x] + Simp[b*(
c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b
, c, d, e, q}, x] && NeQ[q, -1]
```

rule 5571
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.99.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.24

method	result
parallelrisch	$-\frac{-3 \operatorname{arccot}(bx+a)x^4b^4-b^3x^3+3a\,b^2x^2+3 \operatorname{arccot}(bx+a)a^4+6a^3\ln\left(b^2x^2+2abx+a^2+1\right)-9a^2bx-18 \operatorname{arccot}(bx+a)a^2+}{12b^4}$
parts	$\frac{x^4 \operatorname{arccot}(bx+a)}{4}+\frac{b\left(\frac{\frac{1}{3}b^2x^3-ab\,x^2+3a^2x-x}{b^4}+\frac{\left(-4a^3b+4ab\right)\ln\left(b^2x^2+2abx+a^2+1\right)}{2b^2}+\frac{\left(-3a^4-2a^2+1-\frac{\left(-4a^3b+4ab\right)a}{b}\right)\text{arc}}{b^4}\right.}{4}$
derivativedivides	$\frac{\frac{\operatorname{arccot}(bx+a)a^4}{4}-\operatorname{arccot}(bx+a)a^3(bx+a)+\frac{3 \operatorname{arccot}(bx+a)a^2(bx+a)^2}{2}-\operatorname{arccot}(bx+a)a(bx+a)^3+\frac{\operatorname{arccot}(bx+a)(bx+a)^4}{4}+\frac{3a^2(}{}}{b^4}$
default	$\frac{\frac{\operatorname{arccot}(bx+a)a^4}{4}-\operatorname{arccot}(bx+a)a^3(bx+a)+\frac{3 \operatorname{arccot}(bx+a)a^2(bx+a)^2}{2}-\operatorname{arccot}(bx+a)a(bx+a)^3+\frac{\operatorname{arccot}(bx+a)(bx+a)^4}{4}+\frac{3a^2(}{}}{b^4}$
risch	$\frac{ix^4\ln(1+i(bx+a))}{8}-\frac{ix^4\ln(1-i(bx+a))}{8}+\frac{\pi x^4}{8}+\frac{x^3}{12b}+\frac{a^4\arctan(bx+a)}{4b^4}-\frac{a\,x^2}{4b^2}-\frac{a^3\ln\left(b^2x^2+2abx+a^2+1\right)}{2b^4}$

input
```
int(x^3*arccot(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(-3*arccot(b*x+a)*x^4*b^4-b^3*x^3+3*a*b^2*x^2+3*arccot(b*x+a)*a^4+6*
a^3*ln(b^2*x^2+2*a*b*x+a^2+1)-9*a^2*b*x-18*arccot(b*x+a)*a^2+15*a^3-6*a*ln
(b^2*x^2+2*a*b*x+a^2+1)+3*b*x+3*arccot(b*x+a)-9*a)/b^4
```

3.99.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

$$\int x^3\cot^{-1}(a+bx)\,dx$$
$$=\frac{3\,b^4x^4\operatorname{arccot}(bx+a)+b^3x^3-3\,ab^2x^2+3\,(3\,a^2-1)bx+3\,(a^4-6\,a^2+1)\arctan(bx+a)-6\,(a^3-a)\,\text{lo}}{12\,b^4}$$

input
```
integrate(x^3*arccot(b*x+a),x, algorithm="fricas")
```

output
```
1/12*(3*b^4*x^4*arccot(b*x + a) + b^3*x^3 - 3*a*b^2*x^2 + 3*(3*a^2 - 1)*b*
x + 3*(a^4 - 6*a^2 + 1)*arctan(b*x + a) - 6*(a^3 - a)*log(b^2*x^2 + 2*a*b*
x + a^2 + 1))/b^4
```

3.99.6 Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.46

$$\int x^3 \cot^{-1}(a+bx)\,dx$$
$$= \begin{cases} -\frac{a^4 \operatorname{acot}(a+bx)}{4b^4} - \frac{a^3 \log(a^2+2abx+b^2x^2+1)}{2b^4} + \frac{3a^2x}{4b^3} + \frac{3a^2 \operatorname{acot}(a+bx)}{2b^4} - \frac{ax^2}{4b^2} + \frac{a \log(a^2+2abx+b^2x^2+1)}{2b^4} + \frac{x^4 \operatorname{acot}(a+bx)}{4} + \\ \frac{x^4 \operatorname{acot}(a)}{4} \end{cases}$$

input
```
integrate(x**3*acot(b*x+a),x)
```

output
```
Piecewise((-a**4*acot(a + b*x)/(4*b**4) - a**3*log(a**2 + 2*a*b*x + b**2*x
**2 + 1)/(2*b**4) + 3*a**2*x/(4*b**3) + 3*a**2*acot(a + b*x)/(2*b**4) - a*
x**2/(4*b**2) + a*log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(2*b**4) + x**4*acot
(a + b*x)/4 + x**3/(12*b) - x/(4*b**3) - acot(a + b*x)/(4*b**4), Ne(b, 0))
, (x**4*acot(a)/4, True))
```

3.99.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.98

$$\int x^3 \cot^{-1}(a+bx)\,dx = \frac{1}{4}x^4 \operatorname{arccot}(bx+a)$$
$$+ \frac{1}{12}b\left(\frac{b^2x^3 - 3\,abx^2 + 3\,(3\,a^2-1)x}{b^4} + \frac{3\,(a^4 - 6\,a^2 + 1)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^5} - \frac{6\,(a^3-a)\log(b^2x^2 + 2\,abx +}{b^5}\right.$$

input
```
integrate(x^3*arccot(b*x+a),x, algorithm="maxima")
```

output
```
1/4*x^4*arccot(b*x + a) + 1/12*b*((b^2*x^3 - 3*a*b*x^2 + 3*(3*a^2 - 1)*x)/
b^4 + 3*(a^4 - 6*a^2 + 1)*arctan((b^2*x + a*b)/b)/b^5 - 6*(a^3 - a)*log(b^
2*x^2 + 2*a*b*x + a^2 + 1)/b^5)
```

3.99.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(92) = 184$.

Time = 0.73 (sec) , antiderivative size = 617, normalized size of antiderivative = 5.82

$$\int x^3 \cot^{-1}(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x^3*arccot(b*x+a),x, algorithm="giac")
```

output
```
1/192*(96*a^3*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^5 + 72*a^2*
arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^6 + 24*a*arctan(1/(b*x +
a))*tan(1/2*arctan(1/(b*x + a)))^7 + 3*arctan(1/(b*x + a))*tan(1/2*arctan(
1/(b*x + a)))^8 + 96*a^3*log(16*tan(1/2*arctan(1/(b*x + a)))^2/(tan(1/2*ar
ctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arct
an(1/(b*x + a)))^4 - 96*a^3*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)
))^3 + 144*a^2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^4 - 144*a^
2*tan(1/2*arctan(1/(b*x + a)))^5 - 72*a*arctan(1/(b*x + a))*tan(1/2*arctan
(1/(b*x + a)))^5 - 24*a*tan(1/2*arctan(1/(b*x + a)))^6 - 12*arctan(1/(b*x
+ a))*tan(1/2*arctan(1/(b*x + a)))^6 - 2*tan(1/2*arctan(1/(b*x + a)))^7 -
96*a*log(16*tan(1/2*arctan(1/(b*x + a)))^2/(tan(1/2*arctan(1/(b*x + a)))^4
 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^4 +
 72*a^2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^2 + 144*a^2*tan(1
/2*arctan(1/(b*x + a)))^3 + 72*a*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x
 + a)))^3 - 48*a*tan(1/2*arctan(1/(b*x + a)))^4 - 30*arctan(1/(b*x + a))*t
an(1/2*arctan(1/(b*x + a)))^4 + 30*tan(1/2*arctan(1/(b*x + a)))^5 - 24*a*a
rctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a))) - 24*a*tan(1/2*arctan(1/(b
*x + a)))^2 - 12*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^2 - 30*t
an(1/2*arctan(1/(b*x + a)))^3 + 3*arctan(1/(b*x + a)) + 2*tan(1/2*arctan(1
/(b*x + a))))/(b^4*tan(1/2*arctan(1/(b*x + a)))^4)
```

3.99.9 Mupad [B] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.25

$$\begin{aligned}\int x^3 \cot^{-1}(a+bx)\,dx &= \frac{\mathrm{atan}(a+b\,x)}{4\,b^4} + \frac{x^4\,\mathrm{acot}(a+b\,x)}{4} - \frac{x}{4\,b^3} + \frac{x^3}{12\,b} \\ &\quad - \frac{a^3\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2\,b^4} - \frac{3\,a^2\,\mathrm{atan}(a+b\,x)}{2\,b^4} \\ &\quad + \frac{a^4\,\mathrm{atan}(a+b\,x)}{4\,b^4} - \frac{a\,x^2}{4\,b^2} + \frac{3\,a^2\,x}{4\,b^3} + \frac{a\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2\,b^4}\end{aligned}$$

input
```
int(x^3*acot(a + b*x),x)
```

output
```
atan(a + b*x)/(4*b^4) + (x^4*acot(a + b*x))/4 - x/(4*b^3) + x^3/(12*b) - (
a^3*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/(2*b^4) - (3*a^2*atan(a + b*x))/(2*b
^4) + (a^4*atan(a + b*x))/(4*b^4) - (a*x^2)/(4*b^2) + (3*a^2*x)/(4*b^3) +
(a*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/(2*b^4)
```

3.99. $\int x^3 \cot^{-1}(a+bx)\,dx$

3.100 $\int x^2 \cot^{-1}(a+bx)\,dx$

3.100.1 Optimal result

Integrand size = 10, antiderivative size = 80

$$\int x^2 \cot^{-1}(a+bx)\,dx = -\frac{ax}{b^2}+\frac{(a+bx)^2}{6b^3}+\frac{1}{3}x^3\cot^{-1}(a+bx)+\frac{a(3-a^2)\arctan(a+bx)}{3b^3}-\frac{(1-3a^2)\log\left(1+(a+bx)^2\right)}{6b^3}$$

output
```
-a*x/b^2+1/6*(b*x+a)^2/b^3+1/3*x^3*arccot(b*x+a)+1/3*a*(-a^2+3)*arctan(b*x
+a)/b^3-1/6*(-3*a^2+1)*ln(1+(b*x+a)^2)/b^3
```

3.100.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.42

$$\int x^2 \cot^{-1}(a+bx)\,dx = \frac{\frac{1}{3}b\left(-\frac{a}{b}+\frac{a+bx}{b}\right)^3\cot^{-1}(a+bx)+\frac{1}{3}b\left(-\frac{3ax}{b^2}+\frac{(a+bx)^2}{2b^3}-\frac{(1+ia)^3\log(i-a-bx)}{2b^3}-\frac{(1-ia)^3\log(i+a+bx)}{2b^3}\right)}{b}$$

input
```
Integrate[x^2*ArcCot[a + b*x],x]
```

output
```
((b*(-(a/b) + (a + b*x)/b)^3*ArcCot[a + b*x])/3 + (b*((-3*a*x)/b^2 + (a +
b*x)^2/(2*b^3) - ((1 + I*a)^3*Log[I - a - b*x])/(2*b^3) - ((1 - I*a)^3*Log
[I + a + b*x])/(2*b^3)))/3)/b
```

3.100.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5571, 27, 5388, 478, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cot^{-1}(a+bx)\,dx$$

$$\Big\downarrow \text{5571}$$

$$\frac{\int x^2 \cot^{-1}(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int b^2x^2 \cot^{-1}(a+bx)d(a+bx)}{b^3}$$

$$\Big\downarrow \text{5388}$$

$$\frac{\frac{1}{3}b^3x^3 \cot^{-1}(a+bx)-\frac{1}{3}\int -\frac{b^3x^3}{(a+bx)^2+1}d(a+bx)}{b^3}$$

$$\Big\downarrow \text{478}$$

$$\frac{\frac{1}{3}b^3x^3 \cot^{-1}(a+bx)-\frac{1}{3}\int \left(2a-bx-\frac{a(3-a^2)-(1-3a^2)(a+bx)}{(a+bx)^2+1}\right)d(a+bx)}{b^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{3}\left(a(3-a^2)\arctan(a+bx)-\frac{1}{2}(1-3a^2)\log\left((a+bx)^2+1\right)+\frac{1}{2}(a+bx)^2-3a(a+bx)\right)+\frac{1}{3}b^3x^3\cot^{-1}(a+bx)}{b^3}$$

input

```
Int[x^2*ArcCot[a + b*x],x]
```

output

```
((b^3*x^3*ArcCot[a + b*x])/3 + (-3*a*(a + b*x) + (a + b*x)^2/2 + a*(3 - a^
2)*ArcTan[a + b*x] - ((1 - 3*a^2)*Log[1 + (a + b*x)^2])/2)/3)/b^3
```

3.100.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 478
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ
[n, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5388
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])/(e*(q + 1))), x] + Simp[b*(
c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b
, c, d, e, q}, x] && NeQ[q, -1]
```

rule 5571
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.100.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.28

method	result
parallelrisch	$\frac{2\operatorname{arccot}(bx+a)x^3b^3+b^2x^2+2\operatorname{arccot}(bx+a)a^3+3a^2\ln(b^2x^2+2abx+a^2+1)-4abx-6a\operatorname{arccot}(bx+a)+7a^2-1-\ln(b^2x^2+}{6b^3}$
derivativedivides	$\frac{-\frac{\operatorname{arccot}(bx+a)a^3}{3}+\operatorname{arccot}(bx+a)a^2(bx+a)-\operatorname{arccot}(bx+a)a(bx+a)^2+\frac{\operatorname{arccot}(bx+a)(bx+a)^3}{3}-(bx+a)a+\frac{(bx+a)^2}{6}-\frac{(-3a^2+}{}}{b^3}$
default	$\frac{-\frac{\operatorname{arccot}(bx+a)a^3}{3}+\operatorname{arccot}(bx+a)a^2(bx+a)-\operatorname{arccot}(bx+a)a(bx+a)^2+\frac{\operatorname{arccot}(bx+a)(bx+a)^3}{3}-(bx+a)a+\frac{(bx+a)^2}{6}-\frac{(-3a^2+}{}}{b^3}$
parts	$\frac{x^3\operatorname{arccot}(bx+a)}{3}+\frac{b\left(-\frac{-\frac{1}{2}x^2b+2ax}{b^3}+\frac{\frac{(3a^2b-b)\ln(b^2x^2+2abx+a^2+1)}{2b^2}+\frac{\left(2a^3+2a-\frac{(3a^2b-b)a}{b}\right)\arctan\left(\frac{2b^2x+2ab}{2b}\right)}{b}}{b^3}\right)}{3}$
risch	$\frac{ix^3\ln(1+i(bx+a))}{6}-\frac{ix^3\ln(1-i(bx+a))}{6}+\frac{\pi x^3}{6}-\frac{a^3\arctan(bx+a)}{3b^3}+\frac{x^2}{6b}+\frac{a^2\ln(b^2x^2+2abx+a^2+1)}{2b^3}-\frac{2ax}{3b^2}$

input
```
int(x^2*arccot(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*arccot(b*x+a)*x^3*b^3+b^2*x^2+2*arccot(b*x+a)*a^3+3*a^2*ln(b^2*x^2+
2*a*b*x+a^2+1)-4*a*b*x-6*a*arccot(b*x+a)+7*a^2-1-ln(b^2*x^2+2*a*b*x+a^2+1)
)/b^3
```

3.100.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.91

$$\int x^2\cot^{-1}(a+bx)\,dx$$
$$=\frac{2\,b^3x^3\operatorname{arccot}(bx+a)+b^2x^2-4\,abx-2\,(a^3-3\,a)\arctan(bx+a)+(3\,a^2-1)\log(b^2x^2+2\,abx+a^2+1}{6\,b^3}$$

input
```
integrate(x^2*arccot(b*x+a),x, algorithm="fricas")
```

output
```
1/6*(2*b^3*x^3*arccot(b*x + a) + b^2*x^2 - 4*a*b*x - 2*(a^3 - 3*a)*arctan(
b*x + a) + (3*a^2 - 1)*log(b^2*x^2 + 2*a*b*x + a^2 + 1))/b^3
```

3.100.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.46

$$\int x^2 \cot^{-1}(a+bx)\,dx$$
$$= \begin{cases} \frac{a^3 \operatorname{acot}(a+bx)}{3b^3} + \frac{a^2 \log(a^2+2abx+b^2x^2+1)}{2b^3} - \frac{2ax}{3b^2} - \frac{a \operatorname{acot}(a+bx)}{b^3} + \frac{x^3 \operatorname{acot}(a+bx)}{3} + \frac{x^2}{6b} - \frac{\log(a^2+2abx+b^2x^2+1)}{6b^3} & \text{for } b \neq \\ \frac{x^3 \operatorname{acot}(a)}{3} & \text{otherw} \end{cases}$$

input
```
integrate(x**2*acot(b*x+a),x)
```

output
```
Piecewise(((a**3*acot(a + b*x)/(3*b**3) + a**2*log(a**2 + 2*a*b*x + b**2*x*
*2 + 1)/(2*b**3) - 2*a*x/(3*b**2) - a*acot(a + b*x)/b**3 + x**3*acot(a + b
*x)/3 + x**2/(6*b) - log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(6*b**3), Ne(b, 0
)), (x**3*acot(a)/3, True))
```

3.100.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.06

$$\int x^2 \cot^{-1}(a+bx)\,dx = \frac{1}{3}\,x^3 \operatorname{arccot}(bx+a)$$
$$+\frac{1}{6}\,b\left(\frac{bx^2-4\,ax}{b^3} - \frac{2\,(a^3-3\,a)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^4} + \frac{(3\,a^2-1)\log(b^2x^2+2\,abx+a^2+1)}{b^4}\right)$$

input
```
integrate(x^2*arccot(b*x+a),x, algorithm="maxima")
```

output
```
1/3*x^3*arccot(b*x + a) + 1/6*b*((b*x^2 - 4*a*x)/b^3 - 2*(a^3 - 3*a)*arcta
n((b^2*x + a*b)/b)/b^4 + (3*a^2 - 1)*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^4)
```

3.100.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(70) = 140$.

Time = 0.63 (sec) , antiderivative size = 423, normalized size of antiderivative = 5.29

$$\int x^2 \cot^{-1}(a+bx)\,dx =$$

$$-\frac{12\,a^2 \arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^4 + 6\,a \arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^5 + \arctan\left(\frac{1}{bx+a}\right)\mathrm{t}}{}$$

input
```
integrate(x^2*arccot(b*x+a),x, algorithm="giac")
```

output
```
-1/24*(12*a^2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^4 + 6*a*arc
tan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^5 + arctan(1/(b*x + a))*tan(
1/2*arctan(1/(b*x + a)))^6 + 12*a^2*log(16*tan(1/2*arctan(1/(b*x + a)))^2/
(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1))*t
an(1/2*arctan(1/(b*x + a)))^3 - 12*a^2*arctan(1/(b*x + a))*tan(1/2*arctan(
1/(b*x + a)))^2 + 12*a*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^3
- 12*a*tan(1/2*arctan(1/(b*x + a)))^4 - 3*arctan(1/(b*x + a))*tan(1/2*arct
an(1/(b*x + a)))^4 - tan(1/2*arctan(1/(b*x + a)))^5 - 4*log(16*tan(1/2*arc
tan(1/(b*x + a)))^2/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/(
b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^3 + 6*a*arctan(1/(b*x + a)
)*tan(1/2*arctan(1/(b*x + a))) + 12*a*tan(1/2*arctan(1/(b*x + a)))^2 + 3*a
rctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^2 - 2*tan(1/2*arctan(1/(b*
x + a)))^3 - arctan(1/(b*x + a)) - tan(1/2*arctan(1/(b*x + a))))/(b^3*tan(
1/2*arctan(1/(b*x + a)))^3)
```

3.100.9 Mupad [B] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.26

$$\begin{aligned}\int x^2 \cot^{-1}(a+bx)\,dx = {} & \frac{x^3\,\mathrm{acot}(a+b\,x)}{3} - \frac{\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{6\,b^3} \\ & + \frac{x^2}{6\,b} + \frac{a^2\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2\,b^3} \\ & - \frac{a^3\,\mathrm{atan}(a+b\,x)}{3\,b^3} + \frac{a\,\mathrm{atan}(a+b\,x)}{b^3} - \frac{2\,a\,x}{3\,b^2}\end{aligned}$$

input
```
int(x^2*acot(a + b*x),x)
```

output
```
(x^3*acot(a + b*x))/3 - log(a^2 + b^2*x^2 + 2*a*b*x + 1)/(6*b^3) + x^2/(6*
b) + (a^2*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/(2*b^3) - (a^3*atan(a + b*x))/
(3*b^3) + (a*atan(a + b*x))/b^3 - (2*a*x)/(3*b^2)
```

3.100. $\int x^2 \cot^{-1}(a+bx)\, dx$

3.101 $\int x \cot^{-1}(a+bx)\,dx$

3.101.1 Optimal result

Integrand size = 8, antiderivative size = 60

$$\int x \cot^{-1}(a+bx)\,dx = \frac{x}{2b}+\frac{1}{2}x^2\cot^{-1}(a+bx)-\frac{(1-a^2)\arctan(a+bx)}{2b^2}-\frac{a\log\left(1+(a+bx)^2\right)}{2b^2}$$

output
```
1/2*x/b+1/2*x^2*arccot(b*x+a)-1/2*(-a^2+1)*arctan(b*x+a)/b^2-1/2*a*ln(1+(b
*x+a)^2)/b^2
```

3.101.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.50

$$\int x\cot^{-1}(a+bx)\,dx = \frac{2bx+2b^2x^2\cot^{-1}(a+bx)-i(-i+a)^2\log(i-a-bx)-i\log(i+a+bx)-2a\log(i+a+bx)+ia^2\log(}{4b^2}$$

input
```
Integrate[x*ArcCot[a + b*x],x]
```

output
```
(2*b*x + 2*b^2*x^2*ArcCot[a + b*x] - I*(-I + a)^2*Log[I - a - b*x] - I*Log
[I + a + b*x] - 2*a*Log[I + a + b*x] + I*a^2*Log[I + a + b*x])/(4*b^2)
```

3.101.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5571, 25, 27, 5388, 478, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cot^{-1}(a+bx)\,dx$$

$$\Big\downarrow \text{5571}$$

$$\frac{\int x\cot^{-1}(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -x\cot^{-1}(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int -bx\cot^{-1}(a+bx)d(a+bx)}{b^2}$$

$$\Big\downarrow \text{5388}$$

$$-\frac{-\frac{1}{2}\int \frac{b^2x^2}{(a+bx)^2+1}d(a+bx)-\frac{1}{2}b^2x^2\cot^{-1}(a+bx)}{b^2}$$

$$\Big\downarrow \text{478}$$

$$-\frac{-\frac{1}{2}\int \left(1-\frac{-a^2+2(a+bx)a+1}{(a+bx)^2+1}\right)d(a+bx)-\frac{1}{2}b^2x^2\cot^{-1}(a+bx)}{b^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{2}\left(\left(1-a^2\right)\arctan(a+bx)+a\log\left((a+bx)^2+1\right)-a-bx\right)-\frac{1}{2}b^2x^2\cot^{-1}(a+bx)}{b^2}$$

input
```
Int[x*ArcCot[a + b*x],x]
```

output
```
-((-1/2*(b^2*x^2*ArcCot[a + b*x]) + (-a - b*x + (1 - a^2)*ArcTan[a + b*x]
+ a*Log[1 + (a + b*x)^2])/2)/b^2)
```

3.101.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 478
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ
[n, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5388
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])/(e*(q + 1))), x] + Simp[b*(
c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b
, c, d, e, q}, x] && NeQ[q, -1]
```

rule 5571
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.101.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{\frac{(bx+a)^2\operatorname{arccot}(bx+a)}{2}-\operatorname{arccot}(bx+a)a(bx+a)+\frac{bx}{2}+\frac{a}{2}-\frac{a\ln\left(1+(bx+a)^2\right)}{2}-\frac{\arctan(bx+a)}{2}}{b^2}$
default	$\frac{\frac{(bx+a)^2\operatorname{arccot}(bx+a)}{2}-\operatorname{arccot}(bx+a)a(bx+a)+\frac{bx}{2}+\frac{a}{2}-\frac{a\ln\left(1+(bx+a)^2\right)}{2}-\frac{\arctan(bx+a)}{2}}{b^2}$
parallelrisch	$-\frac{-\operatorname{arccot}(bx+a)b^2x^2+\operatorname{arccot}(bx+a)a^2+a\ln(b^2x^2+2abx+a^2+1)-bx-\operatorname{arccot}(bx+a)+2a}{2b^2}$
parts	$\frac{x^2\operatorname{arccot}(bx+a)}{2}+\frac{b\left(\frac{x}{b^2}+\frac{-\frac{a\ln\left(b^2x^2+2abx+a^2+1\right)}{b}+\frac{\left(a^2-1\right)\arctan\left(\frac{2b^2x+2ab}{2b}\right)}{b}}{b^2}\right)}{2}$
risch	$\frac{ix^2\ln(1+i(bx+a))}{4}-\frac{ix^2\ln(1-i(bx+a))}{4}+\frac{\pi x^2}{4}+\frac{a^2\arctan(bx+a)}{2b^2}-\frac{a\ln(b^2x^2+2abx+a^2+1)}{2b^2}+\frac{x}{2b}-\frac{\text{arctan}}{2}$

input
```
int(x*arccot(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(1/2*(b*x+a)^2*arccot(b*x+a)-arccot(b*x+a)*a*(b*x+a)+1/2*b*x+1/2*a-1
/2*a*ln(1+(b*x+a)^2)-1/2*arctan(b*x+a))
```

3.101.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

$$\int x\cot^{-1}(a+bx)\,dx$$
$$=\frac{b^2x^2\operatorname{arccot}(bx+a)+bx+(a^2-1)\arctan(bx+a)-a\log(b^2x^2+2abx+a^2+1)}{2b^2}$$

input
```
integrate(x*arccot(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(b^2*x^2*arccot(b*x + a) + b*x + (a^2 - 1)*arctan(b*x + a) - a*log(b^2
*x^2 + 2*a*b*x + a^2 + 1))/b^2
```

3.101.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.30

$$\int x\cot^{-1}(a+bx)\,dx = \begin{cases} -\frac{a^2\operatorname{acot}(a+bx)}{2b^2} - \frac{a\log(a^2+2abx+b^2x^2+1)}{2b^2} + \frac{x^2\operatorname{acot}(a+bx)}{2} + \frac{x}{2b} + \frac{\operatorname{acot}(a+bx)}{2b^2} & \text{for } b \neq 0 \\ \frac{x^2\operatorname{acot}(a)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*acot(b*x+a),x)
```

output
```
Piecewise((-a**2*acot(a + b*x)/(2*b**2) - a*log(a**2 + 2*a*b*x + b**2*x**2
 + 1)/(2*b**2) + x**2*acot(a + b*x)/2 + x/(2*b) + acot(a + b*x)/(2*b**2),
Ne(b, 0)), (x**2*acot(a)/2, True))
```

3.101.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13

$$\int x\cot^{-1}(a+bx)\,dx = \frac{1}{2}x^2\operatorname{arccot}(bx+a) + \frac{1}{2}b\left(\frac{x}{b^2} + \frac{(a^2-1)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^3} - \frac{a\log(b^2x^2+2abx+a^2+1)}{b^3}\right)$$

input
```
integrate(x*arccot(b*x+a),x, algorithm="maxima")
```

output
```
1/2*x^2*arccot(b*x + a) + 1/2*b*(x/b^2 + (a^2 - 1)*arctan((b^2*x + a*b)/b)
/b^3 - a*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^3)
```

3.101.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(50) = 100.

Time = 0.34 (sec) , antiderivative size = 210, normalized size of antiderivative = 3.50

$$\int x\cot^{-1}(a+bx)\,dx$$

$$= \frac{4\,a\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^3+\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^4+4\,a\log\left(\frac{16\tan}{\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+}\right.\right.}\right.}{}$$

input
```
integrate(x*arccot(b*x+a),x, algorithm="giac")
```

output
```
1/8*(4*a*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^3 + arctan(1/(b*
x + a))*tan(1/2*arctan(1/(b*x + a)))^4 + 4*a*log(16*tan(1/2*arctan(1/(b*x
+ a)))^2/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^
2 + 1))*tan(1/2*arctan(1/(b*x + a)))^2 - 4*a*arctan(1/(b*x + a))*tan(1/2*a
rctan(1/(b*x + a))) + 2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^2
 - 2*tan(1/2*arctan(1/(b*x + a)))^3 + arctan(1/(b*x + a)) + 2*tan(1/2*arct
an(1/(b*x + a))))/(b^2*tan(1/2*arctan(1/(b*x + a)))^2)
```

3.101.9 Mupad [B] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

$$\int x\cot^{-1}(a+bx)\,dx = \frac{x^2\,\mathrm{acot}(a+b\,x)}{2}+\frac{\frac{\mathrm{acot}(a+b\,x)}{2}+\frac{b\,x}{2}-\frac{a^2\,\mathrm{acot}(a+b\,x)}{2}-\frac{a\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2}}{b^2}$$

input
```
int(x*acot(a + b*x),x)
```

output
```
(x^2*acot(a + b*x))/2 + (acot(a + b*x)/2 + (b*x)/2 - (a^2*acot(a + b*x))/2
 - (a*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/2)/b^2
```

3.102 $\int \cot^{-1}(a+bx)\,dx$

3.102.1 Optimal result

Integrand size = 6, antiderivative size = 33

$$\int \cot^{-1}(a+bx)\,dx=\frac{(a+bx)\cot^{-1}(a+bx)}{b}+\frac{\log\left(1+(a+bx)^2\right)}{2b}$$

output
```
(b*x+a)*arccot(b*x+a)/b+1/2*ln(1+(b*x+a)^2)/b
```

3.102.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \cot^{-1}(a+bx)\,dx=x\cot^{-1}(a+bx)+\frac{-2a\arctan(a+bx)+\log\left(1+a^2+2abx+b^2x^2\right)}{2b}$$

input
```
Integrate[ArcCot[a + b*x],x]
```

output
```
x*ArcCot[a + b*x] + (-2*a*ArcTan[a + b*x] + Log[1 + a^2 + 2*a*b*x + b^2*x^
2])/(2*b)
```

3.102.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5563, 5346, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}(a+bx)\,dx$$

$$\Big\downarrow \text{5563}$$

$$\frac{\int \cot^{-1}(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{5346}$$

$$\frac{\int \frac{a+bx}{(a+bx)^2+1}d(a+bx)+(a+bx)\cot^{-1}(a+bx)}{b}$$

$$\Big\downarrow \text{240}$$

$$\frac{\frac{1}{2}\log\left((a+bx)^2+1\right)+(a+bx)\cot^{-1}(a+bx)}{b}$$

input
```
Int[ArcCot[a + b*x],x]
```

output
```
((a + b*x)*ArcCot[a + b*x] + Log[1 + (a + b*x)^2]/2)/b
```

3.102.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5563
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[1/d
  Subst[Int[(a + b*ArcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
 x] && IGtQ[p, 0]
```

3.102.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{(bx+a)\,\text{arccot}(bx+a)+\frac{\ln\left(1+(bx+a)^2\right)}{2}}{b}$	30
default	$\frac{(bx+a)\,\text{arccot}(bx+a)+\frac{\ln\left(1+(bx+a)^2\right)}{2}}{b}$	30
parallelrisch	$\frac{2x\ \text{arccot}(bx+a)b^2+2\ \text{arccot}(bx+a)ab+\ln(b^2x^2+2abx+a^2+1)b}{2b^2}$	49
parts	$x\ \text{arccot}\left(bx+a\right)+b\left(\frac{\ln(b^2x^2+2abx+a^2+1)}{2b^2}-\frac{a\arctan\left(\frac{2b^2x+2ab}{2b}\right)}{b^2}\right)$	59
risch	$\frac{ix\ln(1+i(bx+a))}{2}-\frac{ix\ln(1-i(bx+a))}{2}+\frac{\pi x}{2}-\frac{a\arctan(bx+a)}{b}+\frac{\ln(b^2x^2+2abx+a^2+1)}{2b}$	71

input
```
int(arccot(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*((b*x+a)*arccot(b*x+a)+1/2*ln(1+(b*x+a)^2))
```

3.102.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.30

$$\int \cot^{-1}(a+bx)\,dx=\frac{2\,bx\,\text{arccot}\left(bx+a\right)-2\,a\arctan\left(bx+a\right)+\log\left(b^2x^2+2\,abx+a^2+1\right)}{2\,b}$$

input
```
integrate(arccot(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(2*b*x*arccot(b*x + a) - 2*a*arctan(b*x + a) + log(b^2*x^2 + 2*a*b*x +
 a^2 + 1))/b
```

3.102.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39

$$\int \cot^{-1}(a+bx)\,dx = \begin{cases} \frac{a \operatorname{acot}(a+bx)}{b} + x \operatorname{acot}(a+bx) + \frac{\log(a^2+2abx+b^2x^2+1)}{2b} & \text{for } b \neq 0 \\ x \operatorname{acot}(a) & \text{otherwise} \end{cases}$$

input
```
integrate(acot(b*x+a),x)
```

output
```
Piecewise((a*acot(a + b*x)/b + x*acot(a + b*x) + log(a**2 + 2*a*b*x + b**2
*x**2 + 1)/(2*b), Ne(b, 0)), (x*acot(a), True))
```

3.102.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \cot^{-1}(a+bx)\,dx = \frac{2(bx+a)\operatorname{arccot}(bx+a) + \log\left((bx+a)^2+1\right)}{2b}$$

input
```
integrate(arccot(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(2*(b*x + a)*arccot(b*x + a) + log((b*x + a)^2 + 1))/b
```

3.102.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(31) = 62.

Time = 0.32 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.36

$$\int \cot^{-1}(a+bx)\,dx =$$

$$-\frac{\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2 + \log\left(\frac{16\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2}{\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^4+2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2+1}\right)\tan\left(\frac{1}{2}\arctan\right.}{2b\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)}$$

input
```
integrate(arccot(b*x+a),x, algorithm="giac")
```

output
```
-1/2*(arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^2 + log(16*tan(1/2*
arctan(1/(b*x + a)))^2/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(
1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a))) - arctan(1/(b*x + a)))/
(b*tan(1/2*arctan(1/(b*x + a))))
```

3.102.9 Mupad [B] (verification not implemented)

Time = 1.33 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

$$\int \cot^{-1}(a+bx)\,dx = \frac{\frac{\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2}+a\,\mathrm{acot}(a+b\,x)}{b}+x\,\mathrm{acot}(a+b\,x)$$

input
```
int(acot(a + b*x),x)
```

output
```
(log(a^2 + b^2*x^2 + 2*a*b*x + 1)/2 + a*acot(a + b*x))/b + x*acot(a + b*x)
```

3.103 $\int \frac{\cot^{-1}(a+bx)}{x} dx$

3.103.1 Optimal result

Integrand size = 10, antiderivative size = 120

$$\begin{aligned}\int \frac{\cot^{-1}(a+bx)}{x} dx = & -\cot^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right) \\ & +\cot^{-1}(a+bx)\log\left(\frac{2bx}{(i-a)(1-i(a+bx))}\right) \\ & -\frac{1}{2}i\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right) \\ & +\frac{1}{2}i\,\mathrm{PolyLog}\left(2,1-\frac{2bx}{(i-a)(1-i(a+bx))}\right)\end{aligned}$$

output
```
-arccot(b*x+a)*ln(2/(1-I*(b*x+a)))+arccot(b*x+a)*ln(2*b*x/(I-a)/(1-I*(b*x+
a)))-1/2*I*polylog(2,1-2/(1-I*(b*x+a)))+1/2*I*polylog(2,1-2*b*x/(I-a)/(1-I
*(b*x+a)))
```

3.103.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 256 vs. 2(120) = 240.

Time = 0.15 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.13

$$\int \frac{\cot^{-1}(a+bx)}{x} \, dx = \left(\cot^{-1}(a+bx) + \arctan(a+bx)\right)\log(x) + \arctan(a+bx)\left(\log\left(\frac{1}{\sqrt{1+(a+bx)^2}}\right) - \log(-\sin(\arctan(a) - \arctan(a+bx)))\right) + \frac{1}{2}\left(\frac{1}{4}i(\pi - 2\arctan(a+bx))^2 + i(\arctan(a) - \arctan(a+bx))^2 - (\pi - 2\arctan(a+bx))\log\left(1 + e^{-2i\arctan(a+bx)}\right) + 2(\arctan(a) - \arctan(a+bx))\log\left(1 - e^{2i(-\arctan(a)+\arctan(a+bx))}\right) + (\pi - 2\arctan(a+bx))\log\left(\frac{2}{\sqrt{1+(a+bx)^2}}\right) + 2(-\arctan(a) + \arctan(a+bx))\log(-2\sin(\arctan(a) - \arctan(a+bx))) + i\,\mathrm{PolyLog}\left(2, -e^{-2i\arctan(a+bx)}\right) + i\,\mathrm{PolyLog}\left(2, e^{2i(-\arctan(a)+\arctan(a+bx))}\right)\right)$$

input
```
Integrate[ArcCot[a + b*x]/x,x]
```

output
```
(ArcCot[a + b*x] + ArcTan[a + b*x])*Log[x] + ArcTan[a + b*x]*(Log[1/Sqrt[1
 + (a + b*x)^2]] - Log[-Sin[ArcTan[a] - ArcTan[a + b*x]]]) + ((I/4)*(Pi -
2*ArcTan[a + b*x])^2 + I*(ArcTan[a] - ArcTan[a + b*x])^2 - (Pi - 2*ArcTan[
a + b*x])*Log[1 + E^((-2*I)*ArcTan[a + b*x])] + 2*(ArcTan[a] - ArcTan[a +
b*x])*Log[1 - E^((2*I)*(-ArcTan[a] + ArcTan[a + b*x]))] + (Pi - 2*ArcTan[a
 + b*x])*Log[2/Sqrt[1 + (a + b*x)^2]] + 2*(-ArcTan[a] + ArcTan[a + b*x])*L
og[-2*Sin[ArcTan[a] - ArcTan[a + b*x]]] + I*PolyLog[2, -E^((-2*I)*ArcTan[a
 + b*x])] + I*PolyLog[2, E^((2*I)*(-ArcTan[a] + ArcTan[a + b*x]))])/2
```

3.103. $\int \frac{\cot^{-1}(a+bx)}{x} \, dx$

3.103.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5571, 25, 27, 5382, 2849, 2752, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(a+bx)}{x}\,dx$$

$$\Big\downarrow \text{ 5571}$$

$$\frac{\int \frac{\cot^{-1}(a+bx)}{x}d(a+bx)}{b}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int -\frac{\cot^{-1}(a+bx)}{x}d(a+bx)}{b}$$

$$\Big\downarrow \text{ 27}$$

$$-\int -\frac{\cot^{-1}(a+bx)}{bx}d(a+bx)$$

$$\Big\downarrow \text{ 5382}$$

$$-\int \frac{\log\left(\frac{2}{1-i(a+bx)}\right)}{(a+bx)^2+1}d(a+bx)+\int \frac{\log\left(\frac{2bx}{(i-a)(1-i(a+bx))}\right)}{(a+bx)^2+1}d(a+bx)+\log\left(\frac{2}{1-i(a+bx)}\right)\left(-\cot^{-1}(a+bx)\right)+\log\left(\frac{2bx}{(-a+i)(1-i(a+bx))}\right)\cot^{-1}(a+bx)$$

$$\Big\downarrow \text{ 2849}$$

$$-i\int \frac{\log\left(\frac{2}{1-i(a+bx)}\right)}{1-\frac{2}{1-i(a+bx)}}d\frac{1}{1-i(a+bx)}+\int \frac{\log\left(\frac{2bx}{(i-a)(1-i(a+bx))}\right)}{(a+bx)^2+1}d(a+bx)+\log\left(\frac{2}{1-i(a+bx)}\right)\left(-\cot^{-1}(a+bx)\right)+\log\left(\frac{2bx}{(-a+i)(1-i(a+bx))}\right)\cot^{-1}(a+bx)$$

$$\Big\downarrow \text{ 2752}$$

$$\int \frac{\log\left(\frac{2bx}{(i-a)(1-i(a+bx))}\right)}{(a+bx)^2+1}d(a+bx)-\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right)+\log\left(\frac{2}{1-i(a+bx)}\right)\left(-\cot^{-1}(a+bx)\right)+\log\left(\frac{2bx}{(-a+i)(1-i(a+bx))}\right)\cot^{-1}(a+bx)$$

$$\Big\downarrow \text{ 2897}$$

$$-\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2bx}{(i-a)(1-i(a+bx))}\right)+\log\left(\frac{2}{1-i(a+bx)}\right)\left(-\cot^{-1}(a+bx)\right)+\log\left(\frac{2bx}{(-a+i)(1-i(a+bx))}\right)\cot^{-1}(a+bx)$$

input
```
Int[ArcCot[a + b*x]/x,x]
```

output
```
-(ArcCot[a + b*x]*Log[2/(1 - I*(a + b*x))]) + ArcCot[a + b*x]*Log[(2*b*x)/
((I - a)*(1 - I*(a + b*x)))] - (I/2)*PolyLog[2, 1 - 2/(1 - I*(a + b*x))] +
 (I/2)*PolyLog[2, 1 - (2*b*x)/((I - a)*(1 - I*(a + b*x)))]
```

3.103.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5382
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcCot[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*ArcCot[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] - Simp[b*(c/e)
   Int[Log[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] + Simp[b*(c/e)   Int[Log[2*
c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x]) /; FreeQ[{a
, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

rule 5571
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.103.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.86

method	result
risch	$\frac{\pi \ln(-ibx)}{2}-\frac{i\ln(-ibx-ia+1)\ln\left(-\frac{ixb}{ia-1}\right)}{2}-\frac{i\,\mathrm{dilog}\left(-\frac{ixb}{ia-1}\right)}{2}+\frac{i\ln(ibx+ia+1)\ln\left(\frac{ixb}{-ia-1}\right)}{2}+\frac{i\,\mathrm{dilog}\left(\frac{ixb}{-ia-1}\right)}{2}$
parts	$\ln(x)\,\mathrm{arccot}(bx+a)+b\Bigg(-\frac{i\ln(x)\left(\ln\left(\frac{-bx-a+i}{i-a}\right)-\ln\left(\frac{bx+a+i}{i+a}\right)\right)}{2b}-\frac{i\left(\mathrm{dilog}\left(\frac{-bx-a+i}{i-a}\right)-\mathrm{dilog}\left(\frac{bx+a+i}{i+a}\right)\right.}{2b}$
derivativedivides	$\ln(-bx)\,\mathrm{arccot}(bx+a)+\frac{i\ln(-bx)\ln\left(\frac{bx+a+i}{i+a}\right)}{2}-\frac{i\ln(-bx)\ln\left(\frac{-bx-a+i}{i-a}\right)}{2}+\frac{i\,\mathrm{dilog}\left(\frac{bx+a+i}{i+a}\right)}{2}-\frac{i\,\mathrm{dilog}}{}$
default	$\ln(-bx)\,\mathrm{arccot}(bx+a)+\frac{i\ln(-bx)\ln\left(\frac{bx+a+i}{i+a}\right)}{2}-\frac{i\ln(-bx)\ln\left(\frac{-bx-a+i}{i-a}\right)}{2}+\frac{i\,\mathrm{dilog}\left(\frac{bx+a+i}{i+a}\right)}{2}-\frac{i\,\mathrm{dilog}}{}$

input
```
int(arccot(b*x+a)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*Pi*ln(-I*b*x)-1/2*I*ln(1-I*a-I*b*x)*ln(-I*x*b/(I*a-1))-1/2*I*dilog(-I*
x*b/(I*a-1))+1/2*I*ln(1+I*a+I*b*x)*ln(I*x*b/(-I*a-1))+1/2*I*dilog(I*x*b/(-
I*a-1))
```

3.103.5 Fricas [F]

$$\int \frac{\cot^{-1}(a+bx)}{x}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{x}\,dx$$

input
```
integrate(arccot(b*x+a)/x,x, algorithm="fricas")
```

output
```
integral(arccot(b*x + a)/x, x)
```

3.103.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{x}\,dx = \text{Timed out}$$

input
```
integrate(acot(b*x+a)/x,x)
```

output
```
Timed out
```

3.103.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.11

$$\begin{aligned}\int \frac{\cot^{-1}(a+bx)}{x}\,dx =& \frac{1}{2}\arctan\left(\frac{bx}{a^2+1}, -\frac{abx}{a^2+1}\right)\log\left(b^2x^2+2\,abx+a^2+1\right) \\ &- \frac{1}{2}\arctan(bx+a)\log\left(\frac{b^2x^2}{a^2+1}\right) \\ &+ \operatorname{arccot}(bx+a)\log(x) + \arctan\left(\frac{b^2x+ab}{b}\right)\log(x) \\ &+ \frac{1}{2}i\,\mathrm{Li}_2\left(\frac{i\,bx+i\,a+1}{i\,a+1}\right) - \frac{1}{2}i\,\mathrm{Li}_2\left(\frac{i\,bx+i\,a-1}{i\,a-1}\right)\end{aligned}$$

input
```
integrate(arccot(b*x+a)/x,x, algorithm="maxima")
```

output
```
1/2*arctan2(b*x/(a^2 + 1), -a*b*x/(a^2 + 1))*log(b^2*x^2 + 2*a*b*x + a^2 +
 1) - 1/2*arctan(b*x + a)*log(b^2*x^2/(a^2 + 1)) + arccot(b*x + a)*log(x)
+ arctan((b^2*x + a*b)/b)*log(x) + 1/2*I*dilog((I*b*x + I*a + 1)/(I*a + 1)
) - 1/2*I*dilog((I*b*x + I*a - 1)/(I*a - 1))
```

3.103.8 Giac [F]

$$\int \frac{\cot^{-1}(a+bx)}{x}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{x}\,dx$$

input
```
integrate(arccot(b*x+a)/x,x, algorithm="giac")
```

output
```
integrate(arccot(b*x + a)/x, x)
```

3.103.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{x}\,dx = \int \frac{\operatorname{acot}(a+b\,x)}{x}\,dx$$

input
```
int(acot(a + b*x)/x,x)
```

output
```
int(acot(a + b*x)/x, x)
```

3.104 $\int \frac{\cot^{-1}(a+bx)}{x^2} dx$

3.104.1 Optimal result

Integrand size = 10, antiderivative size = 62

$$\int \frac{\cot^{-1}(a+bx)}{x^2} dx = -\frac{\cot^{-1}(a+bx)}{x} + \frac{ab\arctan(a+bx)}{1+a^2} - \frac{b\log(x)}{1+a^2} + \frac{b\log\left(1+(a+bx)^2\right)}{2(1+a^2)}$$

output
```
-arccot(b*x+a)/x+a*b*arctan(b*x+a)/(a^2+1)-b*ln(x)/(a^2+1)+1/2*b*ln(1+(b*x
+a)^2)/(a^2+1)
```

3.104.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06

$$\int \frac{\cot^{-1}(a+bx)}{x^2} dx = -\frac{\cot^{-1}(a+bx)}{x} + \frac{b(-2\log(x) + (1-ia)\log(i-a-bx) + (1+ia)\log(i+a+bx))}{2(1+a^2)}$$

input
```
Integrate[ArcCot[a + b*x]/x^2,x]
```

output
```
-(ArcCot[a + b*x]/x) + (b*(-2*Log[x] + (1 - I*a)*Log[I - a - b*x] + (1 + I
*a)*Log[I + a + b*x]))/(2*(1 + a^2))
```

3.104.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5569, 896, 25, 479, 452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(a+bx)}{x^2}\,dx$$

$$\Big\downarrow \text{ 5569}$$

$$-b\int \frac{1}{x\left((a+bx)^2+1\right)}dx-\frac{\cot^{-1}(a+bx)}{x}$$

$$\Big\downarrow \text{ 896}$$

$$-b\int \frac{1}{bx\left((a+bx)^2+1\right)}d(a+bx)-\frac{\cot^{-1}(a+bx)}{x}$$

$$\Big\downarrow \text{ 25}$$

$$b\int -\frac{1}{bx\left((a+bx)^2+1\right)}d(a+bx)-\frac{\cot^{-1}(a+bx)}{x}$$

$$\Big\downarrow \text{ 479}$$

$$-b\left(\frac{\log(-bx)}{a^2+1}-\frac{\int \frac{2a+bx}{(a+bx)^2+1}d(a+bx)}{a^2+1}\right)-\frac{\cot^{-1}(a+bx)}{x}$$

$$\Big\downarrow \text{ 452}$$

$$-b\left(\frac{\log(-bx)}{a^2+1}-\frac{a\int \frac{1}{(a+bx)^2+1}d(a+bx)+\int \frac{a+bx}{(a+bx)^2+1}d(a+bx)}{a^2+1}\right)-\frac{\cot^{-1}(a+bx)}{x}$$

$$\Big\downarrow \text{ 216}$$

$$-b\left(\frac{\log(-bx)}{a^2+1}-\frac{\int \frac{a+bx}{(a+bx)^2+1}d(a+bx)+a\arctan(a+bx)}{a^2+1}\right)-\frac{\cot^{-1}(a+bx)}{x}$$

$$\Big\downarrow \text{ 240}$$

$$-b\left(\frac{\log(-bx)}{a^2+1}-\frac{a\arctan(a+bx)+\frac{1}{2}\log\left((a+bx)^2+1\right)}{a^2+1}\right)-\frac{\cot^{-1}(a+bx)}{x}$$

input
```
Int[ArcCot[a + b*x]/x^2,x]
```

output
```
-(ArcCot[a + b*x]/x) - b*(Log[-(b*x)]/(1 + a^2) - (a*ArcTan[a + b*x] + Log
[1 + (a + b*x)^2]/2)/(1 + a^2))
```

3.104.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 479
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[d*(Log
[RemoveContent[c + d*x, x]]/(b*c^2 + a*d^2)), x] + Simp[b/(b*c^2 + a*d^2)
 Int[(c - d*x)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 5569
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCot[c + d*x])^p/(f*(m +
1))), x] + Simp[b*d*(p/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*((a + b*ArcCot[
c + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[p, 0] && ILtQ[m, -1]
```

3.104.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

method	result
derivativedivides	$b\left(-\frac{\operatorname{arccot}(bx+a)}{bx}-\frac{\ln(-bx)}{a^2+1}+\frac{\frac{\ln\left(1+(bx+a)^2\right)}{2}+a\arctan(bx+a)}{a^2+1}\right)$
default	$b\left(-\frac{\operatorname{arccot}(bx+a)}{bx}-\frac{\ln(-bx)}{a^2+1}+\frac{\frac{\ln\left(1+(bx+a)^2\right)}{2}+a\arctan(bx+a)}{a^2+1}\right)$
parts	$-\frac{\operatorname{arccot}(bx+a)}{x}-b\left(\frac{\ln(x)}{a^2+1}-\frac{b\left(\frac{\ln(b^2x^2+2abx+a^2+1)}{2b}+\frac{a\arctan\left(\frac{2b^2x+2ab}{2b}\right)}{b}\right)}{a^2+1}\right)$
parallelrisch	$-\frac{2x\ \operatorname{arccot}(bx+a)a^2b^2+2b^2\ln(x)ax-b^2\ln(b^2x^2+2abx+a^2+1)ax+2\ \operatorname{arccot}(bx+a)a^3b+2\ \operatorname{arccot}(bx+a)ab}{2xab(a^2+1)}$
risch	$-\frac{i\ln(1+i(bx+a))}{2x}-\frac{-ia^2\ln(1-i(bx+a))-i\ln(1-i(bx+a))+\pi a^2+\pi+2b\ln(x)x-xb\ln((iab+3b)x+ia^2+3i+2a)-ixb}{2x(i+a}$

input
```
int(arccot(b*x+a)/x^2,x,method=_RETURNVERBOSE)
```

output
```
b*(-1/b/x*arccot(b*x+a)-1/(a^2+1)*ln(-b*x)+1/(a^2+1)*(1/2*ln(1+(b*x+a)^2)+
a*arctan(b*x+a)))
```

3.104.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.03

$$\int\frac{\cot^{-1}(a+bx)}{x^2}\,dx$$
$$=\frac{2\,abx\arctan\left(bx+a\right)+bx\log\left(b^2x^2+2\,abx+a^2+1\right)-2\,bx\log\left(x\right)-2\left(a^2+1\right)\operatorname{arccot}\left(bx+a\right)}{2\left(a^2+1\right)x}$$

input
```
integrate(arccot(b*x+a)/x^2,x, algorithm="fricas")
```

output
```
1/2*(2*a*b*x*arctan(b*x + a) + b*x*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*b*
x*log(x) - 2*(a^2 + 1)*arccot(b*x + a))/((a^2 + 1)*x)
```

3.104.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.69

$$\int \frac{\cot^{-1}(a+bx)}{x^2} dx$$
$$= \begin{cases} -\frac{ib \operatorname{acot}(bx-i)}{2} - \frac{\operatorname{acot}(bx-i)}{x} + \frac{i}{2x} & \text{for } a = -i \\ \frac{ib \operatorname{acot}(bx+i)}{2} - \frac{\operatorname{acot}(bx+i)}{x} - \frac{i}{2x} & \text{for } a = i \\ -\frac{2a^2 \operatorname{acot}(a+bx)}{2a^2x+2x} - \frac{2abx \operatorname{acot}(a+bx)}{2a^2x+2x} - \frac{2bx \log(x)}{2a^2x+2x} + \frac{bx \log(a^2+2abx+b^2x^2+1)}{2a^2x+2x} - \frac{2 \operatorname{acot}(a+bx)}{2a^2x+2x} & \text{otherwise} \end{cases}$$

input
```
integrate(acot(b*x+a)/x**2,x)
```

output
```
Piecewise(((-I*b*acot(b*x - I)/2 - acot(b*x - I)/x + I/(2*x), Eq(a, -I)), (
I*b*acot(b*x + I)/2 - acot(b*x + I)/x - I/(2*x), Eq(a, I)), (-2*a**2*acot(
a + b*x)/(2*a**2*x + 2*x) - 2*a*b*x*acot(a + b*x)/(2*a**2*x + 2*x) - 2*b*x
*log(x)/(2*a**2*x + 2*x) + b*x*log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(2*a**2
*x + 2*x) - 2*acot(a + b*x)/(2*a**2*x + 2*x), True))
```

3.104.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.24

$$\int \frac{\cot^{-1}(a+bx)}{x^2} dx = \frac{1}{2} b \left(\frac{2 a \arctan\left(\frac{b^2x+ab}{b}\right)}{a^2+1} + \frac{\log(b^2x^2+2abx+a^2+1)}{a^2+1} - \frac{2 \log(x)}{a^2+1} \right) - \frac{\operatorname{arccot}(bx+a)}{x}$$

input
```
integrate(arccot(b*x+a)/x^2,x, algorithm="maxima")
```

output
```
1/2*b*(2*a*arctan((b^2*x + a*b)/b)/(a^2 + 1) + log(b^2*x^2 + 2*a*b*x + a^2
 + 1)/(a^2 + 1) - 2*log(x)/(a^2 + 1)) - arccot(b*x + a)/x
```

3.104.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(60) = 120$.

Time = 0.42 (sec) , antiderivative size = 498, normalized size of antiderivative = 8.03

$$\int \frac{\cot^{-1}(a+bx)}{x^2}\,dx =$$

$$-\frac{\left(2a\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2+2a\log\left(\frac{4\left(4a^2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2+4a\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^3+\tan\cdots\right.}{\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\cdots\right.\right.}\right.\right.}{\cdots}$$

input
```
integrate(arccot(b*x+a)/x^2,x, algorithm="giac")
```

output
```
-1/2*(2*a*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^2 + 2*a*log(4*(
4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3
+ tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*ta
n(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(
1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a))) + log(4*(4*a
^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3 + t
an(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*tan(1
/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2
*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^2 - 2*a*arctan(
1/(b*x + a)) - 4*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a))) - log(4*
(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3
 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*t
an(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan
(1/2*arctan(1/(b*x + a)))^2 + 1)))*b/(2*a^3*tan(1/2*arctan(1/(b*x + a))) +
 a^2*tan(1/2*arctan(1/(b*x + a)))^2 - a^2 + 2*a*tan(1/2*arctan(1/(b*x + a)
)) + tan(1/2*arctan(1/(b*x + a)))^2 - 1)
```

3.104.9 Mupad [B] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(a+bx)}{x^2}\,dx = -\frac{\operatorname{acot}(a+b\,x)}{x}-\frac{b\,x\,\ln(x)-\frac{b\,x\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2}+a\,b\,x\operatorname{acot}(a+b\,x)}{x\,\left(a^2+1\right)}$$

input
```
int(acot(a + b*x)/x^2,x)
```

output
```
- acot(a + b*x)/x - (b*x*log(x) - (b*x*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/2
 + a*b*x*acot(a + b*x))/(x*(a^2 + 1))
```

3.104. $\int \frac{\cot^{-1}(a+bx)}{x^2} dx$

3.105 $\int \frac{\cot^{-1}(a+bx)}{x^3} dx$

3.105.1 Optimal result

Integrand size = 10, antiderivative size = 95

$$\int \frac{\cot^{-1}(a+bx)}{x^3} dx = \frac{b}{2(1+a^2)x} - \frac{\cot^{-1}(a+bx)}{2x^2} + \frac{(1-a^2)b^2\arctan(a+bx)}{2(1+a^2)^2} + \frac{ab^2\log(x)}{(1+a^2)^2} - \frac{ab^2\log(1+(a+bx)^2)}{2(1+a^2)^2}$$

output
```
1/2*b/(a^2+1)/x-1/2*arccot(b*x+a)/x^2+1/2*(-a^2+1)*b^2*arctan(b*x+a)/(a^2+
1)^2+a*b^2*ln(x)/(a^2+1)^2-1/2*a*b^2*ln(1+(b*x+a)^2)/(a^2+1)^2
```

3.105.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.97

$$\int \frac{\cot^{-1}(a+bx)}{x^3} dx = \frac{-2\cot^{-1}(a+bx) + \frac{bx(4abx\log(x)+i(i+a)^2bx\log(i-a-bx)+(-i+a)(2(i+a)+(-1-ia)bx\log(i+a+bx)))}{(1+a^2)^2}}{4x^2}$$

input
```
Integrate[ArcCot[a + b*x]/x^3,x]
```

output
```
(-2*ArcCot[a + b*x] + (b*x*(4*a*b*x*Log[x] + I*(I + a)^2*b*x*Log[I - a - b
*x] + (-I + a)*(2*(I + a) + (-1 - I*a)*b*x*Log[I + a + b*x])))/(1 + a^2)^2
)/(4*x^2)
```

3.105.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5569, 896, 480, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(a+bx)}{x^3} \, dx$$

$$\Big\downarrow \text{ 5569}$$

$$-\frac{1}{2} b \int \frac{1}{x^2 \left((a+bx)^2+1\right)} dx - \frac{\cot^{-1}(a+bx)}{2x^2}$$

$$\Big\downarrow \text{ 896}$$

$$-\frac{1}{2} b^2 \int \frac{1}{b^2 x^2 \left((a+bx)^2+1\right)} d(a+bx) - \frac{\cot^{-1}(a+bx)}{2x^2}$$

$$\Big\downarrow \text{ 480}$$

$$-\frac{1}{2} b^2 \left(\frac{\int -\frac{2a+bx}{bx\left((a+bx)^2+1\right)} d(a+bx)}{a^2+1} - \frac{1}{\left(a^2+1\right) bx}\right) - \frac{\cot^{-1}(a+bx)}{2x^2}$$

$$\Big\downarrow \text{ 657}$$

$$-\frac{1}{2} b^2 \left(\frac{\int \left(\frac{a^2+2(a+bx)a-1}{\left(a^2+1\right)\left((a+bx)^2+1\right)} - \frac{2a}{\left(a^2+1\right) bx}\right) d(a+bx)}{a^2+1} - \frac{1}{\left(a^2+1\right) bx}\right) - \frac{\cot^{-1}(a+bx)}{2x^2}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{2} b^2 \left(\frac{-\frac{\left(1-a^2\right) \arctan(a+bx)}{a^2+1} - \frac{2a \log(-bx)}{a^2+1} + \frac{a \log\left((a+bx)^2+1\right)}{a^2+1}}{a^2+1} - \frac{1}{\left(a^2+1\right) bx}\right) - \frac{\cot^{-1}(a+bx)}{2x^2}$$

input
```
Int[ArcCot[a + b*x]/x^3,x]
```

output
```
-1/2*ArcCot[a + b*x]/x^2 - (b^2*(-(1/((1 + a^2)*b*x)) + (-(((1 - a^2)*ArcT
an[a + b*x])/(1 + a^2)) - (2*a*Log[-(b*x)])/(1 + a^2) + (a*Log[1 + (a + b*
x)^2])/(1 + a^2))/(1 + a^2)))/2
```

3.105.3.1 Defintions of rubi rules used

rule 480
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[d*((c
 + d*x)^(n + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + Simp[b/(b*c^2 + a*d^2)   I
nt[(c + d*x)^(n + 1)*((c - d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[n, -1]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5569
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCot[c + d*x])^p/(f*(m +
1))), x] + Simp[b*d*(p/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*((a + b*ArcCot[
c + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[p, 0] && ILtQ[m, -1]
```

3.105.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.87

method	result
derivativedivides	$b^{2}\left(-\frac{\operatorname{arccot}(bx+a)}{2b^{2}x^{2}}-\frac{a\ln\left(1+(bx+a)^{2}\right)+(a^{2}-1)\arctan(bx+a)}{2(a^{2}+1)^{2}}+\frac{1}{2(a^{2}+1)bx}+\frac{a\ln(-bx)}{(a^{2}+1)^{2}}\right)$
default	$b^{2}\left(-\frac{\operatorname{arccot}(bx+a)}{2b^{2}x^{2}}-\frac{a\ln\left(1+(bx+a)^{2}\right)+(a^{2}-1)\arctan(bx+a)}{2(a^{2}+1)^{2}}+\frac{1}{2(a^{2}+1)bx}+\frac{a\ln(-bx)}{(a^{2}+1)^{2}}\right)$
parts	$-\frac{\operatorname{arccot}(bx+a)}{2x^{2}}-\frac{b\left(-\frac{1}{(a^{2}+1)x}-\frac{2ab\ln(x)}{(a^{2}+1)^{2}}+\frac{b^{2}\left(\frac{a\ln(b^{2}x^{2}+2abx+a^{2}+1)}{b}+\frac{(a^{2}-1)\arctan\left(\frac{2b^{2}x+2ab}{2b}\right)}{b}\right)}{(a^{2}+1)^{2}}\right)}{2}$
parallelrisch	$\frac{x^{2}\operatorname{arccot}(bx+a)a^{2}b^{2}+2b^{2}a\ln(x)x^{2}-b^{2}a\ln(b^{2}x^{2}+2abx+a^{2}+1)x^{2}-\operatorname{arccot}(bx+a)b^{2}x^{2}-2a\,b^{2}x^{2}-\operatorname{arccot}(bx+a)a^{4}+a^{2}b}{2x^{2}(a^{4}+2a^{2}+1)}$
risch	$-\frac{i\ln(1+i(bx+a))}{4x^{2}}+\frac{ia^{4}\ln(1-i(bx+a))+2ia^{2}\ln(1-i(bx+a))+i\ln(1-i(bx+a))+4b^{2}a\ln(-x)x^{2}-\pi\,a^{4}+2a^{2}bx-2\pi\,a^{2}}{}$

input
```
int(arccot(b*x+a)/x^3,x,method=_RETURNVERBOSE)
```

output
```
b^2*(-1/2/b^2/x^2*arccot(b*x+a)-1/2/(a^2+1)^2*(a*ln(1+(b*x+a)^2)+(a^2-1)*a
rctan(b*x+a))+1/2/(a^2+1)/b/x+1/(a^2+1)^2*a*ln(-b*x))
```

3.105.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04

$$\int \frac{\cot^{-1}(a+bx)}{x^{3}}\,dx = -\frac{(a^{2}-1)b^{2}x^{2}\arctan(bx+a)+ab^{2}x^{2}\log(b^{2}x^{2}+2\,abx+a^{2}+1)-2\,ab^{2}x^{2}\log(x)-(a^{2}+1)bx+(a^{4}+}{2\,(a^{4}+2\,a^{2}+1)x^{2}}$$

input
```
integrate(arccot(b*x+a)/x^3,x, algorithm="fricas")
```

output
```
-1/2*((a^2 - 1)*b^2*x^2*arctan(b*x + a) + a*b^2*x^2*log(b^2*x^2 + 2*a*b*x
+ a^2 + 1) - 2*a*b^2*x^2*log(x) - (a^2 + 1)*b*x + (a^4 + 2*a^2 + 1)*arccot
(b*x + a))/((a^4 + 2*a^2 + 1)*x^2)
```

3.105.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 381, normalized size of antiderivative = 4.01

$$\int \frac{\cot^{-1}(a+bx)}{x^3}\,dx$$
$$= \begin{cases} -\frac{b^2 \operatorname{acot}(bx-i)}{8} + \frac{b}{8x} - \frac{\operatorname{acot}(bx-i)}{2x^2} + \frac{i}{8x^2} \\ -\frac{b^2 \operatorname{acot}(bx+i)}{8} + \frac{b}{8x} - \frac{\operatorname{acot}(bx+i)}{2x^2} - \frac{i}{8x^2} \\ -\frac{a^4 \operatorname{acot}(a+bx)}{2a^4x^2+4a^2x^2+2x^2} + \frac{a^2b^2x^2 \operatorname{acot}(a+bx)}{2a^4x^2+4a^2x^2+2x^2} + \frac{a^2bx}{2a^4x^2+4a^2x^2+2x^2} - \frac{2a^2 \operatorname{acot}(a+bx)}{2a^4x^2+4a^2x^2+2x^2} + \frac{2ab^2x^2 \log(x)}{2a^4x^2+4a^2x^2+2x^2} - \frac{ab^2x^2 \log(a^2+2ab}{2a^4x^2+4a^2x} \end{cases}$$

input
```
integrate(acot(b*x+a)/x**3,x)
```

output
```
Piecewise((-b**2*acot(b*x - I)/8 + b/(8*x) - acot(b*x - I)/(2*x**2) + I/(8
*x**2), Eq(a, -I)), (-b**2*acot(b*x + I)/8 + b/(8*x) - acot(b*x + I)/(2*x*
*2) - I/(8*x**2), Eq(a, I)), (-a**4*acot(a + b*x)/(2*a**4*x**2 + 4*a**2*x*
*2 + 2*x**2) + a**2*b**2*x**2*acot(a + b*x)/(2*a**4*x**2 + 4*a**2*x**2 + 2
*x**2) + a**2*b*x/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**2) - 2*a**2*acot(a + b
*x)/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**2) + 2*a*b**2*x**2*log(x)/(2*a**4*x*
*2 + 4*a**2*x**2 + 2*x**2) - a*b**2*x**2*log(a**2 + 2*a*b*x + b**2*x**2 +
1)/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**2) - b**2*x**2*acot(a + b*x)/(2*a**4*
x**2 + 4*a**2*x**2 + 2*x**2) + b*x/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**2) -
acot(a + b*x)/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**2), True))
```

3.105.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.18

$$\int \frac{\cot^{-1}(a+bx)}{x^3}\,dx = -\frac{1}{2}\left(\frac{(a^2-1)b\arctan\left(\frac{b^2x+ab}{b}\right)}{a^4+2a^2+1} + \frac{ab\log(b^2x^2+2abx+a^2+1)}{a^4+2a^2+1} - \frac{2ab\log(x)}{a^4+2a^2+1} - \frac{1}{(a^2+1)x}\right)b - \frac{\operatorname{arccot}(bx+a)}{2x^2}$$

input
```
integrate(arccot(b*x+a)/x^3,x, algorithm="maxima")
```

output
```
-1/2*((a^2 - 1)*b*arctan((b^2*x + a*b)/b)/(a^4 + 2*a^2 + 1) + a*b*log(b^2*
x^2 + 2*a*b*x + a^2 + 1)/(a^4 + 2*a^2 + 1) - 2*a*b*log(x)/(a^4 + 2*a^2 + 1
) - 1/((a^2 + 1)*x))*b - 1/2*arccot(b*x + a)/x^2
```

3.105.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1309 vs. $2(85) = 170$.

Time = 0.58 (sec) , antiderivative size = 1309, normalized size of antiderivative = 13.78

$$\int \frac{\cot^{-1}(a+bx)}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate(arccot(b*x+a)/x^3,x, algorithm="giac")
```

output
```
1/2*(4*a^3*b*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^3 + a^2*b*ar
ctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^4 + 4*a^3*b*log(4*(4*a^2*ta
n(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3 + tan(1/
2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*tan(1/2*ar
ctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arct
an(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a^2*b*log(4*(4
*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3 +
 tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*tan
(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1
/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^3 + a*b*log(4
*(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^
3 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*
tan(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*ta
n(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^4 - 4*a^3*
b*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a))) - 14*a^2*b*arctan(1/(b*
x + a))*tan(1/2*arctan(1/(b*x + a)))^2 + 2*a^2*b*tan(1/2*arctan(1/(b*x + a
)))^3 - 4*a*b*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^3 + a*b*tan
(1/2*arctan(1/(b*x + a)))^4 - b*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x
+ a)))^4 - 4*a^2*b*log(4*(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1
/2*arctan(1/(b*x + a)))^3 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/...
```

3.105.9 Mupad [B] (verification not implemented)

Time = 1.63 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.42

$$\int \frac{\cot^{-1}(a+bx)}{x^3}\,dx = \frac{\operatorname{atan}\left(\frac{2\,x\,b^2+2\,a\,b}{2\,\sqrt{b^2\,(a^2+1)-a^2\,b^2}}\right)\,(b^3-a^2\,b^3)}{\sqrt{b^2}\,(2\,a^4+4\,a^2+2)} - \frac{a\,b^2\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2\,(a^2+1)^2} - \frac{\operatorname{acot}(a+b\,x)\,\left(\frac{a^2}{2}+\frac{1}{2}\right)-\frac{b\,x}{2}+\frac{b^2\,x^2\,\operatorname{acot}(a+b\,x)}{2}-\frac{x^3\,(b^3-3\,a^2\,b^3)}{2\,(a^4+2\,a^2+1)}+\frac{a\,b^4\,x^4}{(a^2+1)^2}+a\,b\,x\,\operatorname{acot}(a+b\,x)}{a^2\,x^2+2\,a\,b\,x^3+b^2\,x^4+x^2} + \frac{a\,b^2\,\ln(x)}{(a^2+1)^2}$$

input

```
int(acot(a + b*x)/x^3,x)
```

output

```
(atan((2*a*b + 2*b^2*x)/(2*(b^2*(a^2 + 1) - a^2*b^2)^(1/2)))*(b^3 - a^2*b^
3))/((b^2)^(1/2)*(4*a^2 + 2*a^4 + 2)) - (a*b^2*log(a^2 + b^2*x^2 + 2*a*b*x
 + 1))/(2*(a^2 + 1)^2) - (acot(a + b*x)*(a^2/2 + 1/2) - (b*x)/2 + (b^2*x^2
*acot(a + b*x))/2 - (x^3*(b^3 - 3*a^2*b^3))/(2*(2*a^2 + a^4 + 1)) + (a*b^4
*x^4)/(a^2 + 1)^2 + a*b*x*acot(a + b*x))/(x^2 + a^2*x^2 + b^2*x^4 + 2*a*b*
x^3) + (a*b^2*log(x))/(a^2 + 1)^2
```

3.106 $\int \frac{\cot^{-1}(a+bx)}{x^4}\,dx$

3.106.1 Optimal result

Integrand size = 10, antiderivative size = 129

$$\int \frac{\cot^{-1}(a+bx)}{x^4}\,dx = \frac{b}{6(1+a^2)\,x^2} - \frac{2ab^2}{3(1+a^2)^2\,x} - \frac{\cot^{-1}(a+bx)}{3x^3} - \frac{a(3-a^2)\,b^3\arctan(a+bx)}{3(1+a^2)^3} + \frac{(1-3a^2)\,b^3\log(x)}{3(1+a^2)^3} - \frac{(1-3a^2)\,b^3\log\left(1+(a+bx)^2\right)}{6(1+a^2)^3}$$

output
```
1/6*b/(a^2+1)/x^2-2/3*a*b^2/(a^2+1)^2/x-1/3*arccot(b*x+a)/x^3-1/3*a*(-a^2+
3)*b^3*arctan(b*x+a)/(a^2+1)^3+1/3*(-3*a^2+1)*b^3*ln(x)/(a^2+1)^3-1/6*(-3*
a^2+1)*b^3*ln(1+(b*x+a)^2)/(a^2+1)^3
```

3.106.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.98

$$\int \frac{\cot^{-1}(a+bx)}{x^4}\,dx = \frac{-2(1+a^2)^3\cot^{-1}(a+bx) + 2(1-3a^2)\,b^3x^3\log(x) + (-1+ia)^3b^3x^3\log(i-a-bx) + (-i+a)bx((i+a$$

$$6(1+a^2)^3\,x^3$$

input
```
Integrate[ArcCot[a + b*x]/x^4,x]
```

output
```
(-2*(1 + a^2)^3*ArcCot[a + b*x] + 2*(1 - 3*a^2)*b^3*x^3*Log[x] + (-1 + I*a
)^3*b^3*x^3*Log[I - a - b*x] + (-I + a)*b*x*((I + a)*(1 + a^2 - 4*a*b*x) +
 I*(-I + a)^2*b^2*x^2*Log[I + a + b*x]))/(6*(1 + a^2)^3*x^3)
```

3.106.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5569, 896, 25, 480, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(a+bx)}{x^4}dx$$

$$\Big\downarrow \text{5569}$$

$$-\frac{1}{3}b\int \frac{1}{x^3\left((a+bx)^2+1\right)}dx-\frac{\cot^{-1}(a+bx)}{3x^3}$$

$$\Big\downarrow \text{896}$$

$$-\frac{1}{3}b^3\int \frac{1}{b^3x^3\left((a+bx)^2+1\right)}d(a+bx)-\frac{\cot^{-1}(a+bx)}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}b^3\int -\frac{1}{b^3x^3\left((a+bx)^2+1\right)}d(a+bx)-\frac{\cot^{-1}(a+bx)}{3x^3}$$

$$\Big\downarrow \text{480}$$

$$-\frac{1}{3}b^3\left(-\frac{\int \frac{2a+bx}{b^2x^2((a+bx)^2+1)}d(a+bx)}{a^2+1}-\frac{1}{2\left(a^2+1\right)b^2x^2}\right)-\frac{\cot^{-1}(a+bx)}{3x^3}$$

$$\Big\downarrow \text{657}$$

$$-\frac{1}{3}b^3\left(-\frac{\int \left(\frac{2a}{(a^2+1)b^2x^2}-\frac{3a^2-1}{(a^2+1)^2bx}+\frac{-a(3-a^2)-(1-3a^2)(a+bx)}{(a^2+1)^2((a+bx)^2+1)}\right)d(a+bx)}{a^2+1}-\frac{1}{2\left(a^2+1\right)b^2x^2}\right)-\frac{\cot^{-1}(a+bx)}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{3}b^3\left(-\frac{-\frac{(3-a^2)a\arctan(a+bx)}{(a^2+1)^2}-\frac{2a}{(a^2+1)bx}+\frac{(1-3a^2)\log(-bx)}{(a^2+1)^2}-\frac{(1-3a^2)\log((a+bx)^2+1)}{2(a^2+1)^2}}{a^2+1}-\frac{1}{2(a^2+1)b^2x^2}\right)-\frac{\cot^{-1}(a+bx)}{3x^3}$$

input
```
Int[ArcCot[a + b*x]/x^4,x]
```

output
```
-1/3*ArcCot[a + b*x]/x^3 - (b^3*(-1/2*1/((1 + a^2)*b^2*x^2) - ((-2*a)/((1
+ a^2)*b*x) - (a*(3 - a^2)*ArcTan[a + b*x])/(1 + a^2)^2 + ((1 - 3*a^2)*Log
[-(b*x)])/(1 + a^2)^2 - ((1 - 3*a^2)*Log[1 + (a + b*x)^2])/(2*(1 + a^2)^2)
)/(1 + a^2)))/3
```

3.106.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 480
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[d*((c
 + d*x)^(n + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + Simp[b/(b*c^2 + a*d^2)   I
nt[(c + d*x)^(n + 1)*((c - d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[n, -1]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5569
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCot[c + d*x])^p/(f*(m +
1))), x] + Simp[b*d*(p/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*((a + b*ArcCot[
c + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[p, 0] && ILtQ[m, -1]
```

3.106.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.89

method	result
derivativedivides	$b^{3} \left(-\frac{\operatorname{arccot}(bx+a)}{3b^{3}x^{3}}+\frac{(-3a^{2}+1)\ln(-bx)}{3(a^{2}+1)^{3}}+\frac{1}{6(a^{2}+1)b^{2}x^{2}}-\frac{2a}{3(a^{2}+1)^{2}bx}+\frac{\frac{(3a^{2}-1)\ln\left(1+(bx+a)^{2}\right)}{2}+(a^{3}-3a)\ a}{3(a^{2}+1)^{3}}\right.$
default	$b^{3} \left(-\frac{\operatorname{arccot}(bx+a)}{3b^{3}x^{3}}+\frac{(-3a^{2}+1)\ln(-bx)}{3(a^{2}+1)^{3}}+\frac{1}{6(a^{2}+1)b^{2}x^{2}}-\frac{2a}{3(a^{2}+1)^{2}bx}+\frac{\frac{(3a^{2}-1)\ln\left(1+(bx+a)^{2}\right)}{2}+(a^{3}-3a)\ a}{3(a^{2}+1)^{3}}\right.$
parts	$-\frac{\operatorname{arccot}(bx+a)}{3x^{3}}-\frac{b\left(-\frac{1}{2\left(a^{2}+1\right)x^{2}}+\frac{b^{2}\left(3a^{2}-1\right)\ln(x)}{\left(a^{2}+1\right)^{3}}+\frac{2ba}{\left(a^{2}+1\right)^{2}x}-\frac{b^{3}\left(\frac{\left(3a^{2}b-b\right)\ln\left(b^{2}x^{2}+2abx+a^{2}+1\right)}{2b^{2}}+\frac{\left(4a^{3}-4a-\frac{(3a}{}\right.}{}\right.}{\left(a^{2}+1\right)^{3}}\right.}{3}$
parallelrisch	$-\frac{2x^{3}\operatorname{arccot}(bx+a)a^{3}b^{3}+6\ln(x)x^{3}a^{2}b^{3}-3\ln\left(b^{2}x^{2}+2abx+a^{2}+1\right)x^{3}a^{2}b^{3}-6x^{3}\operatorname{arccot}(bx+a)a\,b^{3}-7a^{2}b^{3}x^{3}-2b^{3}\ln(x)x^{}}{}$
risch	$-\frac{i\ln(1+i(bx+a))}{6x^{3}}-\frac{ix^{3}\ln\left(\left(-a^{7}b-5ia^{6}b-27a^{5}b+41ia^{4}b+29a^{3}b-15ia^{2}b-9ab+3ib\right)x-a^{8}-32a^{6}-4ia^{7}+70a^{4}+68ia^{5}}{}$

input
```
int(arccot(b*x+a)/x^4,x,method=_RETURNVERBOSE)
```

output
```
b^3*(-1/3/b^3/x^3*arccot(b*x+a)+1/3*(-3*a^2+1)/(a^2+1)^3*ln(-b*x)+1/6/(a^2
+1)/b^2/x^2-2/3/(a^2+1)^2*a/b/x+1/3/(a^2+1)^3*(1/2*(3*a^2-1)*ln(1+(b*x+a)^
2)+(a^3-3*a)*arctan(b*x+a)))
```

3.106.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.10

$$\int \frac{\cot^{-1}(a+bx)}{x^4}\,dx$$
$$= \frac{2\,(a^3-3\,a)b^3x^3\arctan\left(bx+a\right)+(3\,a^2-1)b^3x^3\log\left(b^2x^2+2\,abx+a^2+1\right)-2\,(3\,a^2-1)b^3x^3\log\left(x\right)-4}{6\,(a^6+3\,a^4+3\,a^2+1)x^3}$$

input
```
integrate(arccot(b*x+a)/x^4,x, algorithm="fricas")
```

output
```
1/6*(2*(a^3 - 3*a)*b^3*x^3*arctan(b*x + a) + (3*a^2 - 1)*b^3*x^3*log(b^2*x
^2 + 2*a*b*x + a^2 + 1) - 2*(3*a^2 - 1)*b^3*x^3*log(x) - 4*(a^3 + a)*b^2*x
^2 + (a^4 + 2*a^2 + 1)*b*x - 2*(a^6 + 3*a^4 + 3*a^2 + 1)*arccot(b*x + a))/
((a^6 + 3*a^4 + 3*a^2 + 1)*x^3)
```

3.106.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.18 (sec) , antiderivative size = 760, normalized size of antiderivative = 5.89

$$\int \frac{\cot^{-1}(a+bx)}{x^4}\,dx$$
$$= \begin{cases} \frac{ib^3\operatorname{acot}(bx-i)}{24}-\frac{ib^2}{24x}+\frac{b}{24x^2}-\frac{\operatorname{acot}(bx-i)}{3x^3}+\frac{i}{18x^3} \\ -\frac{ib^3\operatorname{acot}(bx+i)}{24}+\frac{ib^2}{24x}+\frac{b}{24x^2}-\frac{\operatorname{acot}(bx+i)}{3x^3}-\frac{i}{18x^3} \\ -\frac{2a^6\operatorname{acot}(a+bx)}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}+\frac{a^4bx}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}-\frac{6a^4\operatorname{acot}(a+bx)}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}-\frac{2a^3b^3x^3\operatorname{acot}(a+bx)}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}-\frac{}{6a} \end{cases}$$

input
```
integrate(acot(b*x+a)/x**4,x)
```

output
```
Piecewise((I*b**3*acot(b*x - I)/24 - I*b**2/(24*x) + b/(24*x**2) - acot(b*
x - I)/(3*x**3) + I/(18*x**3), Eq(a, -I)), (-I*b**3*acot(b*x + I)/24 + I*b
**2/(24*x) + b/(24*x**2) - acot(b*x + I)/(3*x**3) - I/(18*x**3), Eq(a, I))
, (-2*a**6*acot(a + b*x)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x*
*3) + a**4*b*x/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 6*a*
*4*acot(a + b*x)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 2*
a**3*b**3*x**3*acot(a + b*x)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 +
6*x**3) - 4*a**3*b**2*x**2/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*
x**3) - 6*a**2*b**3*x**3*log(x)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3
 + 6*x**3) + 3*a**2*b**3*x**3*log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(6*a**6*
x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) + 2*a**2*b*x/(6*a**6*x**3 + 1
8*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 6*a**2*acot(a + b*x)/(6*a**6*x**3 +
 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) + 6*a*b**3*x**3*acot(a + b*x)/(6*a*
*6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 4*a*b**2*x**2/(6*a**6*x*
*3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) + 2*b**3*x**3*log(x)/(6*a**6*x*
*3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - b**3*x**3*log(a**2 + 2*a*b*x
+ b**2*x**2 + 1)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) + b*
x/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 2*acot(a + b*x)/(
6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3), True))
```

3.106.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.28

$$\int \frac{\cot^{-1}(a+bx)}{x^4}\,dx$$
$$= \frac{1}{6}\left(\frac{2\,(a^3-3\,a)b^2\arctan\left(\frac{b^2x+ab}{b}\right)}{a^6+3\,a^4+3\,a^2+1}+\frac{(3\,a^2-1)b^2\log\left(b^2x^2+2\,abx+a^2+1\right)}{a^6+3\,a^4+3\,a^2+1}-\frac{2\,(3\,a^2-1)b^2\log\left(x\right)}{a^6+3\,a^4+3\,a^2+1}-\frac{4}{(a}\right.$$
$$-\frac{\operatorname{arccot}\left(bx+a\right)}{3\,x^3}$$

input
```
integrate(arccot(b*x+a)/x^4,x, algorithm="maxima")
```

output
```
1/6*(2*(a^3 - 3*a)*b^2*arctan((b^2*x + a*b)/b)/(a^6 + 3*a^4 + 3*a^2 + 1) +
 (3*a^2 - 1)*b^2*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/(a^6 + 3*a^4 + 3*a^2 + 1
) - 2*(3*a^2 - 1)*b^2*log(x)/(a^6 + 3*a^4 + 3*a^2 + 1) - (4*a*b*x - a^2 -
1)/((a^4 + 2*a^2 + 1)*x^2))*b - 1/3*arccot(b*x + a)/x^3
```

3.106.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3449 vs. $2(115) = 230$.

Time = 1.67 (sec) , antiderivative size = 3449, normalized size of antiderivative = 26.74

$$\int \frac{\cot^{-1}(a+bx)}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate(arccot(b*x+a)/x^4,x, algorithm="giac")
```

output
```
-1/6*(24*a^5*b^2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^4 + 12*a
^4*b^2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^5 + 2*a^3*b^2*arct
an(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^6 + 24*a^5*b^2*log(4*(4*a^2*t
an(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3 + tan(1
/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*tan(1/2*a
rctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arc
tan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^3 + 36*a^4*b^2*log(
4*(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))
^3 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2
*tan(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*t
an(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^4 + 18*a^
3*b^2*log(4*(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(
b*x + a)))^3 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x
+ a))) - 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)
))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))
^5 + 3*a^2*b^2*log(4*(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*a
rctan(1/(b*x + a)))^3 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arcta
n(1/(b*x + a))) - 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/
(b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b
*x + a)))^6 - 24*a^5*b^2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)...
```

3.106.9 Mupad [B] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.21

$$\int \frac{\cot^{-1}(a+bx)}{x^4}\,dx = \frac{\ln(x)\left(\frac{b^3}{3}-a^2\,b^3\right)}{a^6+3\,a^4+3\,a^2+1} - \frac{\mathrm{acot}(a+b\,x)\left(\frac{a^2}{3}+\frac{1}{3}\right)-\frac{b\,x}{6}+\frac{b^2\,x^2\,\mathrm{acot}(a+b\,x)}{3}-\frac{x^3\,\left(b^3-7\,a^2\,b^3\right)}{6\,\left(a^4+2\,a^2+1\right)}+\frac{a\,b^2\,x^2}{3\,\left(a^2+1\right)}+\frac{2\,a\,b^4\,x^4}{3\,{\left(a^2+1\right)}^2}+\frac{2\,a\,b\,x\,\mathrm{acot}(a+b\,x)}{3}}{a^2\,x^3+2\,a\,b\,x^4+b^2\,x^5+x^3} + \frac{b^3\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)\,\left(3\,a^2-1\right)}{6\,\left(a^6+3\,a^4+3\,a^2+1\right)} + \frac{a\,\mathrm{atan}\left(\frac{2\,x\,b^2+2\,a\,b}{2\,\sqrt{b^2\,\left(a^2+1\right)-a^2\,b^2}}\right)\,\left(a^2-3\right)\,{\left(b^2\right)}^{3/2}}{3\,\left(a^6+3\,a^4+3\,a^2+1\right)}$$

input
```
int(acot(a + b*x)/x^4,x)
```

output
```
(log(x)*(b^3/3 - a^2*b^3))/(3*a^2 + 3*a^4 + a^6 + 1) - (acot(a + b*x)*(a^2
/3 + 1/3) - (b*x)/6 + (b^2*x^2*acot(a + b*x))/3 - (x^3*(b^3 - 7*a^2*b^3))/
(6*(2*a^2 + a^4 + 1)) + (a*b^2*x^2)/(3*(a^2 + 1)) + (2*a*b^4*x^4)/(3*(a^2
+ 1)^2) + (2*a*b*x*acot(a + b*x))/3)/(x^3 + a^2*x^3 + b^2*x^5 + 2*a*b*x^4)
 + (b^3*log(a^2 + b^2*x^2 + 2*a*b*x + 1)*(3*a^2 - 1))/(6*(3*a^2 + 3*a^4 +
a^6 + 1)) + (a*atan((2*a*b + 2*b^2*x)/(2*(b^2*(a^2 + 1) - a^2*b^2)^(1/2)))
*(a^2 - 3)*(b^2)^(3/2))/(3*(3*a^2 + 3*a^4 + a^6 + 1))
```

3.107 $\int \frac{\cot^{-1}(a+bx)}{c+dx^2} \, dx$

3.107.1 Optimal result

Integrand size = 16, antiderivative size = 642

$$\int \frac{\cot^{-1}(a+bx)}{c+dx^2}\,dx = -\frac{\log\left(\frac{i+a+bx}{a+bx}\right)\log\left(-\frac{b\left(i\sqrt{c}-\sqrt{d}x\right)}{\left(b\sqrt{c}+(1-ia)\sqrt{d}\right)(a+bx)}\right)}{4\sqrt{c}\sqrt{d}} + \frac{\log\left(-\frac{i-a-bx}{a+bx}\right)\log\left(\frac{ib\left(\sqrt{c}+i\sqrt{d}x\right)}{\left(b\sqrt{c}-(1+ia)\sqrt{d}\right)(a+bx)}\right)}{4\sqrt{c}\sqrt{d}} - \frac{\log\left(-\frac{i-a-bx}{a+bx}\right)\log\left(\frac{b\left(i\sqrt{c}+\sqrt{d}x\right)}{\left(b\sqrt{c}+(1+ia)\sqrt{d}\right)(a+bx)}\right)}{4\sqrt{c}\sqrt{d}} + \frac{\log\left(\frac{i+a+bx}{a+bx}\right)\log\left(-\frac{b\left(i\sqrt{c}+\sqrt{d}x\right)}{\left(b\sqrt{c}+i(i+a)\sqrt{d}\right)(a+bx)}\right)}{4\sqrt{c}\sqrt{d}} + \frac{\text{PolyLog}\left(2,-\frac{\left(b\sqrt{c}-ia\sqrt{d}\right)(i-a-bx)}{\left(b\sqrt{c}-(1+ia)\sqrt{d}\right)(a+bx)}\right)}{4\sqrt{c}\sqrt{d}} - \frac{\text{PolyLog}\left(2,-\frac{\left(b\sqrt{c}+ia\sqrt{d}\right)(i-a-bx)}{\left(b\sqrt{c}+(1+ia)\sqrt{d}\right)(a+bx)}\right)}{4\sqrt{c}\sqrt{d}} - \frac{\text{PolyLog}\left(2,\frac{\left(b\sqrt{c}-ia\sqrt{d}\right)(i+a+bx)}{\left(b\sqrt{c}+(1-ia)\sqrt{d}\right)(a+bx)}\right)}{4\sqrt{c}\sqrt{d}} + \frac{\text{PolyLog}\left(2,\frac{\left(b\sqrt{c}+ia\sqrt{d}\right)(i+a+bx)}{\left(b\sqrt{c}+i(i+a)\sqrt{d}\right)(a+bx)}\right)}{4\sqrt{c}\sqrt{d}}$$

output
```
-1/4*ln((I+a+b*x)/(b*x+a))*ln(-b*(I*c^(1/2)-x*d^(1/2))/(b*x+a)/(b*c^(1/2)+
(1-I*a)*d^(1/2)))/c^(1/2)/d^(1/2)+1/4*ln((-I+a+b*x)/(b*x+a))*ln(I*b*(c^(1/
2)+I*x*d^(1/2))/(b*x+a)/(b*c^(1/2)-(1+I*a)*d^(1/2)))/c^(1/2)/d^(1/2)-1/4*l
n((-I+a+b*x)/(b*x+a))*ln(b*(I*c^(1/2)+x*d^(1/2))/(b*x+a)/(b*c^(1/2)+(1+I*a
)*d^(1/2)))/c^(1/2)/d^(1/2)+1/4*ln((I+a+b*x)/(b*x+a))*ln(-b*(I*c^(1/2)+x*d
^(1/2))/(b*x+a)/(b*c^(1/2)+I*(I+a)*d^(1/2)))/c^(1/2)/d^(1/2)-1/4*polylog(2
,(I+a+b*x)*(b*c^(1/2)-I*a*d^(1/2))/(b*x+a)/(b*c^(1/2)+(1-I*a)*d^(1/2)))/c^
(1/2)/d^(1/2)+1/4*polylog(2,-(I-a-b*x)*(b*c^(1/2)-I*a*d^(1/2))/(b*x+a)/(b*
c^(1/2)-(1+I*a)*d^(1/2)))/c^(1/2)/d^(1/2)-1/4*polylog(2,-(I-a-b*x)*(b*c^(1
/2)+I*a*d^(1/2))/(b*x+a)/(b*c^(1/2)+(1+I*a)*d^(1/2)))/c^(1/2)/d^(1/2)+1/4*
polylog(2,(I+a+b*x)*(b*c^(1/2)+I*a*d^(1/2))/(b*x+a)/(b*c^(1/2)+I*(I+a)*d^(
1/2)))/c^(1/2)/d^(1/2)
```

3.107.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 563, normalized size of antiderivative = 0.88

$$\int \frac{\cot^{-1}(a+bx)}{c+dx^2}\,dx =$$

$$-\frac{i\left(\log\left(\frac{\sqrt{d}(-i+a+bx)}{b\sqrt{-c}+(-i+a)\sqrt{d}}\right)\log\left(\sqrt{-c}-\sqrt{d}x\right)-\log\left(\frac{-i+a+bx}{a+bx}\right)\log\left(\sqrt{-c}-\sqrt{d}x\right)-\log\left(\frac{\sqrt{d}(i+a+bx)}{b\sqrt{-c}+(i+a)\sqrt{d}}\right)\log}{}$$

input
```
Integrate[ArcCot[a + b*x]/(c + d*x^2),x]
```

output
```
((-1/4*I)*(Log[(Sqrt[d]*(-I + a + b*x))/(b*Sqrt[-c] + (-I + a)*Sqrt[d])]*L
og[Sqrt[-c] - Sqrt[d]*x] - Log[(-I + a + b*x)/(a + b*x)]*Log[Sqrt[-c] - Sq
rt[d]*x] - Log[(Sqrt[d]*(I + a + b*x))/(b*Sqrt[-c] + (I + a)*Sqrt[d])]*Log
[Sqrt[-c] - Sqrt[d]*x] + Log[(I + a + b*x)/(a + b*x)]*Log[Sqrt[-c] - Sqrt[
d]*x] - Log[-((Sqrt[d]*(-I + a + b*x))/(b*Sqrt[-c] - (-I + a)*Sqrt[d]))]*L
og[Sqrt[-c] + Sqrt[d]*x] + Log[(-I + a + b*x)/(a + b*x)]*Log[Sqrt[-c] + Sq
rt[d]*x] + Log[-((Sqrt[d]*(I + a + b*x))/(b*Sqrt[-c] - (I + a)*Sqrt[d]))]*
Log[Sqrt[-c] + Sqrt[d]*x] - Log[(I + a + b*x)/(a + b*x)]*Log[Sqrt[-c] + Sq
rt[d]*x] + PolyLog[2, (b*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c] + (-I + a)*Sq
rt[d])] - PolyLog[2, (b*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c] + (I + a)*Sqrt
[d])] - PolyLog[2, (b*(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[-c] - (-I + a)*Sqrt[
d])] + PolyLog[2, (b*(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[-c] - (I + a)*Sqrt[d]
)]))/(Sqrt[-c]*Sqrt[d])
```

3.107.3 Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 721, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5575, 2976, 2804, 2009, 2977, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(a+bx)}{c+dx^2}dx$$

$$\Big\downarrow \text{5575}$$

$$\frac{1}{2}i\int \frac{\log\left(-\frac{-a-bx+i}{a+bx}\right)}{dx^2+c}dx-\frac{1}{2}i\int \frac{\log\left(\frac{a+bx+i}{a+bx}\right)}{dx^2+c}dx$$

$$\Big\downarrow \text{2976}$$

$$\frac{1}{2}i\int \frac{\log\left(-\frac{-a-bx+i}{a+bx}\right)}{dx^2+c}dx-\frac{1}{2}b\int \frac{\log\left(\frac{a+bx+i}{a+bx}\right)}{d(a+i)^2+\frac{(da^2+b^2c)(a+bx+i)^2}{(a+bx)^2}+b^2c-\frac{2(cb^2+a(a+i)d)(a+bx+i)}{a+bx}}d\frac{a+bx+i}{a+bx}$$

$$\Big\downarrow \text{2804}$$

$$\frac{1}{2}i\int \frac{\log\left(-\frac{-a-bx+i}{a+bx}\right)}{dx^2+c}dx-\frac{1}{2}b\int \left(\frac{(da^2+b^2c)\log\left(\frac{a+bx+i}{a+bx}\right)}{b\sqrt{c}\sqrt{d}\left(2da^2+2ida+2b^2c-\frac{2(da^2+b^2c)(a+bx+i)}{a+bx}-2b\sqrt{c}\sqrt{d}\right)}+\frac{(da^2+b^2c)\log\left(\frac{a+bx+}{a+bx}\right.}{b\sqrt{c}\sqrt{d}\left(-2da^2-2ida-2b^2c+\frac{2(da^2+b^2c}{a+}\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}i\int \frac{\log\left(-\frac{-a-bx+i}{a+bx}\right)}{dx^2+c}dx-\frac{1}{2}b\left(\frac{\text{PolyLog}\left(2,\frac{(b\sqrt{c}-ia\sqrt{d})(a+bx+i)}{(\sqrt{d}(1-ia)+b\sqrt{c})(a+bx)}\right)}{2b\sqrt{c}\sqrt{d}}-\frac{\text{PolyLog}\left(2,\frac{(i\sqrt{d}a+b\sqrt{c})(a+bx+i)}{(i\sqrt{d}(a+i)+b\sqrt{c})(a+bx)}\right)}{2b\sqrt{c}\sqrt{d}}+\frac{\log\left(\frac{a+bx+i}{a+bx}\right)\log\left(1-\frac{(a+bx+i)}{(a+bx)(b\surd}\right.}{2b\sqrt{c}\sqrt{d}}\right.$$

$$\Big\downarrow \text{2977}$$

$$\frac{1}{2}b\int \frac{\log\left(-\frac{-a-bx+i}{a+bx}\right)}{d(i-a)^2+b^2c+\frac{2(b^2c-(i-a)ad)(-a-bx+i)}{a+bx}+\frac{(da^2+b^2c)(-a-bx+i)^2}{(a+bx)^2}}d\frac{-a-bx+i}{a+bx}-$$

$$\frac{1}{2}b\left(\frac{\text{PolyLog}\left(2,\frac{\left(b\sqrt{c}-ia\sqrt{d}\right)(a+bx+i)}{\left(\sqrt{d}(1-ia)+b\sqrt{c}\right)(a+bx)}\right)}{2b\sqrt{c}\sqrt{d}}-\frac{\text{PolyLog}\left(2,\frac{\left(i\sqrt{d}a+b\sqrt{c}\right)(a+bx+i)}{\left(i\sqrt{d}(a+i)+b\sqrt{c}\right)(a+bx)}\right)}{2b\sqrt{c}\sqrt{d}}+\frac{\log\left(\frac{a+bx+i}{a+bx}\right)\log\left(1-\frac{(a+bx+i)}{(a+bx)\left(b\sqrt{\right.}}\right.}{2b\sqrt{c}\sqrt{d}}\right.$$

$\Big\downarrow$ 2804

$$\frac{1}{2}b\int\left(\frac{(da^2+b^2c)\log\left(-\frac{-a-bx+i}{a+bx}\right)}{b\sqrt{c}\sqrt{d}\left(-2da^2+2ida-2b^2c-2b\sqrt{c}\sqrt{d}-\frac{2(da^2+b^2c)(-a-bx+i)}{a+bx}\right)}+\frac{(da^2+b^2c)\log\left(-\frac{-a}{a}\right.}{b\sqrt{c}\sqrt{d}\left(2da^2-2ida+2b^2c-2b\sqrt{c}\sqrt{\right.}}\right.$$

$$\frac{1}{2}b\left(\frac{\text{PolyLog}\left(2,\frac{\left(b\sqrt{c}-ia\sqrt{d}\right)(a+bx+i)}{\left(\sqrt{d}(1-ia)+b\sqrt{c}\right)(a+bx)}\right)}{2b\sqrt{c}\sqrt{d}}-\frac{\text{PolyLog}\left(2,\frac{\left(i\sqrt{d}a+b\sqrt{c}\right)(a+bx+i)}{\left(i\sqrt{d}(a+i)+b\sqrt{c}\right)(a+bx)}\right)}{2b\sqrt{c}\sqrt{d}}+\frac{\log\left(\frac{a+bx+i}{a+bx}\right)\log\left(1-\frac{(a+bx+i)}{(a+bx)\left(b\sqrt{\right.}}\right.}{2b\sqrt{c}\sqrt{d}}\right.$$

$\Big\downarrow$ 2009

$$\frac{1}{2}b\left(\frac{\text{PolyLog}\left(2,-\frac{\left(b\sqrt{c}-ia\sqrt{d}\right)(-a-bx+i)}{\left(b\sqrt{c}-(ia+1)\sqrt{d}\right)(a+bx)}\right)}{2b\sqrt{c}\sqrt{d}}-\frac{\text{PolyLog}\left(2,-\frac{\left(i\sqrt{d}a+b\sqrt{c}\right)(-a-bx+i)}{\left(\sqrt{d}(ia+1)+b\sqrt{c}\right)(a+bx)}\right)}{2b\sqrt{c}\sqrt{d}}+\frac{\log\left(-\frac{-a-bx+i}{a+bx}\right)\log\left(1+\frac{(}{(}\right.}{2b\sqrt{c}\sqrt{}}\right.$$

$$\frac{1}{2}b\left(\frac{\text{PolyLog}\left(2,\frac{\left(b\sqrt{c}-ia\sqrt{d}\right)(a+bx+i)}{\left(\sqrt{d}(1-ia)+b\sqrt{c}\right)(a+bx)}\right)}{2b\sqrt{c}\sqrt{d}}-\frac{\text{PolyLog}\left(2,\frac{\left(i\sqrt{d}a+b\sqrt{c}\right)(a+bx+i)}{\left(i\sqrt{d}(a+i)+b\sqrt{c}\right)(a+bx)}\right)}{2b\sqrt{c}\sqrt{d}}+\frac{\log\left(\frac{a+bx+i}{a+bx}\right)\log\left(1-\frac{(a+bx+i)}{(a+bx)\left(b\sqrt{\right.}}\right.}{2b\sqrt{c}\sqrt{d}}\right.$$

input
```
Int[ArcCot[a + b*x]/(c + d*x^2),x]
```

3.107. $\int \frac{\cot^{-1}(a+bx)}{c+dx^2}\,dx$

output
```
(b*((Log[-((I - a - b*x)/(a + b*x))]*Log[1 + ((b*Sqrt[c] - I*a*Sqrt[d])*(I
 - a - b*x))/((b*Sqrt[c] - (1 + I*a)*Sqrt[d])*(a + b*x))]])/(2*b*Sqrt[c]*Sq
rt[d]) - (Log[-((I - a - b*x)/(a + b*x))]*Log[1 + ((b*Sqrt[c] + I*a*Sqrt[d
])*(I - a - b*x))/((b*Sqrt[c] + (1 + I*a)*Sqrt[d])*(a + b*x))]])/(2*b*Sqrt[
c]*Sqrt[d]) + PolyLog[2, -(((b*Sqrt[c] - I*a*Sqrt[d])*(I - a - b*x))/((b*S
qrt[c] - (1 + I*a)*Sqrt[d])*(a + b*x)))]/(2*b*Sqrt[c]*Sqrt[d]) - PolyLog[2
, -(((b*Sqrt[c] + I*a*Sqrt[d])*(I - a - b*x))/((b*Sqrt[c] + (1 + I*a)*Sqrt
[d])*(a + b*x)))]/(2*b*Sqrt[c]*Sqrt[d])))/2 - (b*((Log[(I + a + b*x)/(a +
b*x)]*Log[1 - ((b*Sqrt[c] - I*a*Sqrt[d])*(I + a + b*x))/((b*Sqrt[c] + (1 -
 I*a)*Sqrt[d])*(a + b*x))]])/(2*b*Sqrt[c]*Sqrt[d]) - (Log[(I + a + b*x)/(a
+ b*x)]*Log[1 - ((b*Sqrt[c] + I*a*Sqrt[d])*(I + a + b*x))/((b*Sqrt[c] + I*
(I + a)*Sqrt[d])*(a + b*x))]])/(2*b*Sqrt[c]*Sqrt[d]) + PolyLog[2, ((b*Sqrt[
c] - I*a*Sqrt[d])*(I + a + b*x))/((b*Sqrt[c] + (1 - I*a)*Sqrt[d])*(a + b*x
))]/(2*b*Sqrt[c]*Sqrt[d]) - PolyLog[2, ((b*Sqrt[c] + I*a*Sqrt[d])*(I + a +
 b*x))/((b*Sqrt[c] + I*(I + a)*Sqrt[d])*(a + b*x))]/(2*b*Sqrt[c]*Sqrt[d]))
)/2
```

3.107.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2976
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*(P2x_)^(m_.), x_Symbol] :> With[{f = Coeff[P2x, x, 0], g = Coef
f[P2x, x, 1], h = Coeff[P2x, x, 2]}, Simp[(b*c - a*d)   Subst[Int[(b^2*f -
a*b*g + a^2*h - (2*b*d*f - b*c*g - a*d*g + 2*a*c*h)*x + (d^2*f - c*d*g + c^
2*h)*x^2)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(2*(m + 1))), x], x, (a + b*x)/
(c + d*x)], x]] /; FreeQ[{a, b, c, d, e, A, B, n}, x] && PolyQ[P2x, x, 2] &
& NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

rule 2977
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*(P2x_)^(m_.), x_Symbol] :> With[{f = Coeff[P2x, x, 0], g =
Coeff[P2x, x, 1], h = Coeff[P2x, x, 2]}, Simp[(b*c - a*d)   Subst[Int[(b^2*
f - a*b*g + a^2*h - (2*b*d*f - b*c*g - a*d*g + 2*a*c*h)*x + (d^2*f - c*d*g
+ c^2*h)*x^2)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(2*(m + 1))), x], x, (a + b
*x)/(c + d*x)], x]] /; FreeQ[{a, b, c, d, e, A, B, n}, x] && PolyQ[P2x, x,
2] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] && I
GtQ[p, 0]
```

rule 5575
```
Int[ArcCot[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Simp[
I/2   Int[Log[(-I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] - Simp[I/2   In
t[Log[(I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x
] && RationalQ[n]
```

3.107.4 Maple [A] (verified)

Time = 2.01 (sec) , antiderivative size = 591, normalized size of antiderivative = 0.92

method	result
risch	$-\frac{ib\pi \arctan\left(\frac{2iad+2(-ibx-ia+1)d-2d}{2\sqrt{-b^2cd}}\right)}{2\sqrt{-b^2cd}}-\frac{\ln(-ibx-ia+1)\ln\left(\frac{iad-b\sqrt{cd}+(-ibx-ia+1)d-d}{iad-b\sqrt{cd}-d}\right)\sqrt{cd}}{4cd}+\frac{\ln(-ibx-ia+1)\ln}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(arccot(b*x+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*b*Pi/(-b^2*c*d)^(1/2)*arctan(1/2*(2*I*a*d+2*(1-I*a-I*b*x)*d-2*d)/(-
b^2*c*d)^(1/2))-1/4*ln(1-I*a-I*b*x)/c/d*ln((I*a*d-b*(c*d)^(1/2)+(1-I*a-I*b
*x)*d-d)/(I*a*d-b*(c*d)^(1/2)-d))*(c*d)^(1/2)+1/4*ln(1-I*a-I*b*x)/c/d*ln((
I*a*d+b*(c*d)^(1/2)+(1-I*a-I*b*x)*d-d)/(I*a*d+b*(c*d)^(1/2)-d))*(c*d)^(1/2
)-1/4/c/d*dilog((I*a*d-b*(c*d)^(1/2)+(1-I*a-I*b*x)*d-d)/(I*a*d-b*(c*d)^(1/
2)-d))*(c*d)^(1/2)+1/4/c/d*dilog((I*a*d+b*(c*d)^(1/2)+(1-I*a-I*b*x)*d-d)/(
I*a*d+b*(c*d)^(1/2)-d))*(c*d)^(1/2)-1/4*ln(1+I*a+I*b*x)/c/d*ln((I*a*d+b*(c
*d)^(1/2)-(1+I*a+I*b*x)*d+d)/(I*a*d+b*(c*d)^(1/2)+d))*(c*d)^(1/2)+1/4*ln(1
+I*a+I*b*x)/c/d*ln((I*a*d-b*(c*d)^(1/2)-(1+I*a+I*b*x)*d+d)/(I*a*d-b*(c*d)^
(1/2)+d))*(c*d)^(1/2)-1/4/c/d*(c*d)^(1/2)*dilog((I*a*d+b*(c*d)^(1/2)-(1+I*
a+I*b*x)*d+d)/(I*a*d+b*(c*d)^(1/2)+d))+1/4/c/d*(c*d)^(1/2)*dilog((I*a*d-b*
(c*d)^(1/2)-(1+I*a+I*b*x)*d+d)/(I*a*d-b*(c*d)^(1/2)+d))
```

3.107.5 Fricas [F]

$$\int \frac{\cot^{-1}(a+bx)}{c+dx^2}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{dx^2+c}\,dx$$

input
```
integrate(arccot(b*x+a)/(d*x^2+c),x, algorithm="fricas")
```

output
```
integral(arccot(b*x + a)/(d*x^2 + c), x)
```

3.107.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+dx^2}\,dx = \text{Timed out}$$

input
```
integrate(acot(b*x+a)/(d*x**2+c),x)
```

output
```
Timed out
```

3.107.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 8519 vs. $2(456) = 912$.

Time = 4.03 (sec) , antiderivative size = 8519, normalized size of antiderivative = 13.27

$$\int \frac{\cot^{-1}(a+bx)}{c+dx^2}\,dx = \text{Too large to display}$$

input
```
integrate(arccot(b*x+a)/(d*x^2+c),x, algorithm="maxima")
```

output
```
-1/8*b*(8*arctan(d*x/sqrt(c*d))*arctan((b^2*x + a*b)/b)/b - (4*arctan(sqrt
(d)*x/sqrt(c))*arctan2((2*a*b^2*c*d + (a*b^3*c + (a^3 + a)*b*d + (b^4*c +
(a^2 + 3)*b^2*d)*x)*sqrt(c)*sqrt(d) + (3*b^3*c*d + (a^2 + 1)*b*d^2)*x)/(b^
4*c^2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 + 4*(b^3*c + (a^2 + 1)
*b*d)*sqrt(c)*sqrt(d)), ((a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 + (2*a*
b^2*d*x + b^3*c + 3*(a^2 + 1)*b*d)*sqrt(c)*sqrt(d) + (a*b^3*c*d + (a^3 + a
)*b*d^2)*x)/(b^4*c^2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 + 4*(b^
3*c + (a^2 + 1)*b*d)*sqrt(c)*sqrt(d))) + 4*arctan(sqrt(d)*x/sqrt(c))*arcta
n2((2*a*b^2*c*d - (a*b^3*c + (a^3 + a)*b*d + (b^4*c + (a^2 + 3)*b^2*d)*x)*
sqrt(c)*sqrt(d) + (3*b^3*c*d + (a^2 + 1)*b*d^2)*x)/(b^4*c^2 + 2*(a^2 + 3)*
b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 - 4*(b^3*c + (a^2 + 1)*b*d)*sqrt(c)*sqrt(d
)), ((a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 - (2*a*b^2*d*x + b^3*c + 3*
(a^2 + 1)*b*d)*sqrt(c)*sqrt(d) + (a*b^3*c*d + (a^3 + a)*b*d^2)*x)/(b^4*c^2
 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 - 4*(b^3*c + (a^2 + 1)*b*d)
*sqrt(c)*sqrt(d))) + log(d*x^2 + c)*log(((a^2 + 1)*b^22*c^11*d + 11*(a^4 +
 22*a^2 + 21)*b^20*c^10*d^2 + 55*(a^6 + 39*a^4 + 171*a^2 + 133)*b^18*c^9*d
^3 + 33*(5*a^8 + 260*a^6 + 1870*a^4 + 3876*a^2 + 2261)*b^16*c^8*d^4 + 330*
(a^10 + 61*a^8 + 570*a^6 + 1802*a^4 + 2261*a^2 + 969)*b^14*c^7*d^5 + 22*(2
1*a^12 + 1386*a^10 + 15015*a^8 + 60060*a^6 + 109395*a^4 + 92378*a^2 + 2939
3)*b^12*c^6*d^6 + 22*(21*a^14 + 1407*a^12 + 16401*a^10 + 75075*a^8 + 16...
```

3.107.8 Giac [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+dx^2}\,dx = \text{Timed out}$$

input
```
integrate(arccot(b*x+a)/(d*x^2+c),x, algorithm="giac")
```

output
```
Timed out
```

3.107.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+dx^2}\,dx=\int \frac{\operatorname{acot}(a+b\,x)}{d\,x^2+c}\,dx$$

input
```
int(acot(a + b*x)/(c + d*x^2),x)
```

output
```
int(acot(a + b*x)/(c + d*x^2), x)
```

3.108 $\int \frac{\cot^{-1}(a+bx)}{c+dx}\,dx$

3.108.1 Optimal result

Integrand size = 14, antiderivative size = 152

$$\int \frac{\cot^{-1}(a+bx)}{c+dx}\,dx = -\frac{\cot^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)}{d} + \frac{\cot^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(bc+id-ad)(1-i(a+bx))}\right)}{d} - \frac{i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right)}{2d} + \frac{i\,\text{PolyLog}\left(2,1-\frac{2b(c+dx)}{(bc+id-ad)(1-i(a+bx))}\right)}{2d}$$

output
```
-arccot(b*x+a)*ln(2/(1-I*(b*x+a)))/d+arccot(b*x+a)*ln(2*b*(d*x+c)/(b*c+I*d
-a*d)/(1-I*(b*x+a)))/d-1/2*I*polylog(2,1-2/(1-I*(b*x+a)))/d+1/2*I*polylog(
2,1-2*b*(d*x+c)/(b*c+I*d-a*d)/(1-I*(b*x+a)))/d
```

3.108.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 325 vs. 2(152) = 304.

Time = 0.20 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.14

$$\int \frac{\cot^{-1}(a+bx)}{c+dx}\,dx$$
$$= \frac{(\cot^{-1}(a+bx)+\arctan(a+bx))\log(c+dx)+\arctan(a+bx)\left(\log\left(\frac{1}{\sqrt{1+(a+bx)^2}}\right)-\log\left(\sin\left(\arctan\left(\frac{bc-}{d}\right.\right.\right.\right.}{}$$

input
```
Integrate[ArcCot[a + b*x]/(c + d*x),x]
```

output
```
((ArcCot[a + b*x] + ArcTan[a + b*x])*Log[c + d*x] + ArcTan[a + b*x]*(Log[1
/Sqrt[1 + (a + b*x)^2]] - Log[Sin[ArcTan[(b*c - a*d)/d] + ArcTan[a + b*x]]
]) + ((I/4)*(Pi - 2*ArcTan[a + b*x])^2 + I*(ArcTan[(b*c - a*d)/d] + ArcTan
[a + b*x])^2 - (Pi - 2*ArcTan[a + b*x])*Log[1 + E^((-2*I)*ArcTan[a + b*x])
] - 2*(ArcTan[(b*c - a*d)/d] + ArcTan[a + b*x])*Log[1 - E^((2*I)*(ArcTan[(
b*c - a*d)/d] + ArcTan[a + b*x]))] + (Pi - 2*ArcTan[a + b*x])*Log[2/Sqrt[1
 + (a + b*x)^2]] + 2*(ArcTan[(b*c - a*d)/d] + ArcTan[a + b*x])*Log[2*Sin[A
rcTan[(b*c - a*d)/d] + ArcTan[a + b*x]]] + I*PolyLog[2, -E^((-2*I)*ArcTan[
a + b*x])] + I*PolyLog[2, E^((2*I)*(ArcTan[(b*c - a*d)/d] + ArcTan[a + b*x
]))])/2)/d
```

3.108.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5571, 27, 5382, 2849, 2752, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(a+bx)}{c+dx}\,dx$$
$$\Big\downarrow \text{5571}$$
$$\frac{\int \frac{b\cot^{-1}(a+bx)}{b\left(c-\frac{ad}{b}\right)+d(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \; 27$$

$$\int \frac{\cot^{-1}(a+bx)}{d(a+bx)-ad+bc}d(a+bx)$$

$$\Big\downarrow \; 5382$$

$$\frac{\int \frac{\log\left(\frac{2(bc-ad+d(a+bx))}{(bc-ad+id)(1-i(a+bx))}\right)}{(a+bx)^2+1}d(a+bx)}{d}-\frac{\int \frac{\log\left(\frac{2}{1-i(a+bx)}\right)}{(a+bx)^2+1}d(a+bx)}{d}+\frac{\cot^{-1}(a+bx)\log\left(\frac{2(d(a+bx)-ad+bc)}{(1-i(a+bx))(-ad+bc+id)}\right)}{d}-\frac{\log\left(\frac{2}{1-i(a+bx)}\right)\cot^{-1}(a+bx)}{d}$$

$$\Big\downarrow \; 2849$$

$$\frac{\int \frac{\log\left(\frac{2(bc-ad+d(a+bx))}{(bc-ad+id)(1-i(a+bx))}\right)}{(a+bx)^2+1}d(a+bx)}{d}-\frac{i\int \frac{\log\left(\frac{2}{1-i(a+bx)}\right)}{1-\frac{2}{1-i(a+bx)}}d\frac{1}{1-i(a+bx)}}{d}+\frac{\cot^{-1}(a+bx)\log\left(\frac{2(d(a+bx)-ad+bc)}{(1-i(a+bx))(-ad+bc+id)}\right)}{d}-\frac{\log\left(\frac{2}{1-i(a+bx)}\right)\cot^{-1}(a+bx)}{d}$$

$$\Big\downarrow \; 2752$$

$$\frac{\int \frac{\log\left(\frac{2(bc-ad+d(a+bx))}{(bc-ad+id)(1-i(a+bx))}\right)}{(a+bx)^2+1}d(a+bx)}{d}+\frac{\cot^{-1}(a+bx)\log\left(\frac{2(d(a+bx)-ad+bc)}{(1-i(a+bx))(-ad+bc+id)}\right)}{d}-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right)}{2d}-\frac{\log\left(\frac{2}{1-i(a+bx)}\right)\cot^{-1}(a+bx)}{d}$$

$$\Big\downarrow \; 2897$$

$$\frac{i\,\text{PolyLog}\left(2,1-\frac{2(bc-ad+d(a+bx))}{(bc-ad+id)(1-i(a+bx))}\right)}{2d}+\frac{\cot^{-1}(a+bx)\log\left(\frac{2(d(a+bx)-ad+bc)}{(1-i(a+bx))(-ad+bc+id)}\right)}{d}-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right)}{2d}-\frac{\log\left(\frac{2}{1-i(a+bx)}\right)\cot^{-1}(a+bx)}{d}$$

input
```
Int[ArcCot[a + b*x]/(c + d*x),x]
```

output
```
-((ArcCot[a + b*x]*Log[2/(1 - I*(a + b*x))])/d) + (ArcCot[a + b*x]*Log[(2*
(b*c - a*d + d*(a + b*x)))/((b*c + I*d - a*d)*(1 - I*(a + b*x)))])/d - ((I
/2)*PolyLog[2, 1 - 2/(1 - I*(a + b*x))])/d + ((I/2)*PolyLog[2, 1 - (2*(b*c
 - a*d + d*(a + b*x)))/((b*c + I*d - a*d)*(1 - I*(a + b*x)))])/d
```

3.108. $\int \frac{\cot^{-1}(a+bx)}{c+dx}\,dx$

3.108.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5382
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcCot[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*ArcCot[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] - Simp[b*(c/e)
   Int[Log[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] + Simp[b*(c/e)   Int[Log[2*
c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x]) /; FreeQ[{a
, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

rule 5571
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.108.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{\frac{b\ln(ad-bc-d(bx+a))\operatorname{arccot}(bx+a)}{d}-b\left(-\frac{i\ln(ad-bc-d(bx+a))\left(\ln\left(\frac{id+d(bx+a)}{ad-bc+id}\right)-\ln\left(\frac{id-d(bx+a)}{-ad+bc+id}\right)\right)}{2d}-\frac{i\left(\operatorname{dilog}\left(\frac{id+d(bx+a)}{ad-bc+id}\right)-\right.}{2d}\right.}{b}$
default	$\frac{\frac{b\ln(ad-bc-d(bx+a))\operatorname{arccot}(bx+a)}{d}-b\left(-\frac{i\ln(ad-bc-d(bx+a))\left(\ln\left(\frac{id+d(bx+a)}{ad-bc+id}\right)-\ln\left(\frac{id-d(bx+a)}{-ad+bc+id}\right)\right)}{2d}-\frac{i\left(\operatorname{dilog}\left(\frac{id+d(bx+a)}{ad-bc+id}\right)-\right.}{2d}\right.}{b}$
parts	$\frac{\ln(dx+c)\operatorname{arccot}(bx+a)}{d}+b\left(-\frac{i\ln(dx+c)\left(\ln\left(\frac{id-ad+bc-b(dx+c)}{-ad+bc+id}\right)-\ln\left(\frac{id+ad-bc+b(dx+c)}{ad-bc+id}\right)\right)}{2db}-\frac{i\left(\operatorname{dilog}\left(\frac{id-ad+bc-}{-ad+b}\right.\right.}{}\right.$
risch	$-\frac{i\operatorname{dilog}\left(\frac{iad-ibc+(-ibx-ia+1)d-d}{iad-ibc-d}\right)}{2d}-\frac{i\ln(-ibx-ia+1)\ln\left(\frac{iad-ibc+(-ibx-ia+1)d-d}{iad-ibc-d}\right)}{2d}+\frac{\pi\ln(iad-ibc+(-ibx-ia+}{2d}$

input
```
int(arccot(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/b*(b*ln(a*d-b*c-d*(b*x+a))/d*arccot(b*x+a)-b*(-1/2*I*ln(a*d-b*c-d*(b*x+a
))*(ln((I*d+d*(b*x+a))/(a*d-b*c+I*d))-ln((I*d-d*(b*x+a))/(b*c+I*d-a*d)))/d
-1/2*I*(dilog((I*d+d*(b*x+a))/(a*d-b*c+I*d))-dilog((I*d-d*(b*x+a))/(b*c+I*
d-a*d)))/d))
```

3.108.5 Fricas [F]

$$\int \frac{\cot^{-1}(a+bx)}{c+dx}\,dx=\int \frac{\operatorname{arccot}(bx+a)}{dx+c}\,dx$$

input
```
integrate(arccot(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(arccot(b*x + a)/(d*x + c), x)
```

3.108.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+dx} \, dx = \text{Timed out}$$

input
```
integrate(acot(b*x+a)/(d*x+c),x)
```

output
```
Timed out
```

3.108.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(130) = 260.

Time = 0.36 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.86

$$\int \frac{\cot^{-1}(a+bx)}{c+dx} \, dx = \frac{\operatorname{arccot}(bx+a)\log(dx+c)}{d} + \frac{\arctan\left(\frac{b^2x+ab}{b}\right)\log(dx+c)}{d} + \frac{\arctan\left(\frac{bd^2x+bcd}{b^2c^2-2\,abcd+(a^2+1)d^2}, \frac{b^2c^2-abcd+(b^2cd-abd^2)x}{b^2c^2-2\,abcd+(a^2+1)d^2}\right)\log\left(b^2x^2+2\,abx+a^2+1\right) - \arctan(bx+a)\log\left(\frac{b^2d^2}{b^2c^2}\right.}{2\,d}$$

input
```
integrate(arccot(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
arccot(b*x + a)*log(d*x + c)/d + arctan((b^2*x + a*b)/b)*log(d*x + c)/d +
1/2*(arctan2((b*d^2*x + b*c*d)/(b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2), (b^2
*c^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x)/(b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d
^2))*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - arctan(b*x + a)*log((b^2*d^2*x^2 +
 2*b^2*c*d*x + b^2*c^2)/(b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)) + I*dilog((
I*b*d*x + (I*a + 1)*d)/(-I*b*c + (I*a + 1)*d)) - I*dilog((I*b*d*x + (I*a -
 1)*d)/(-I*b*c + (I*a - 1)*d)))/d
```

3.108.8 Giac [F]

$$\int \frac{\cot^{-1}(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{dx+c}\,dx$$

input
```
integrate(arccot(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(arccot(b*x + a)/(d*x + c), x)
```

3.108.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{acot}(a+b\,x)}{c+d\,x}\,dx$$

input
```
int(acot(a + b*x)/(c + d*x),x)
```

output
```
int(acot(a + b*x)/(c + d*x), x)
```

3.109 $\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x}}\,dx$

3.109.1 Optimal result

Integrand size = 16, antiderivative size = 338

$$\begin{aligned}\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x}}\,dx &= \frac{\log(i-a-bx)}{2bc}+\frac{i(a+bx)\log\left(-\frac{i-a-bx}{a+bx}\right)}{2bc}\\ &+\frac{\log(i+a+bx)}{2bc}-\frac{i(a+bx)\log\left(\frac{i+a+bx}{a+bx}\right)}{2bc}\\ &+\frac{id\log\left(\frac{c(i-a-bx)}{ic-ac+bd}\right)\log(d+cx)}{2c^2}-\frac{id\log\left(-\frac{i-a-bx}{a+bx}\right)\log(d+cx)}{2c^2}\\ &-\frac{id\log\left(\frac{c(i+a+bx)}{(i+a)c-bd}\right)\log(d+cx)}{2c^2}+\frac{id\log\left(\frac{i+a+bx}{a+bx}\right)\log(d+cx)}{2c^2}\\ &-\frac{id\,\mathrm{PolyLog}\left(2,-\frac{b(d+cx)}{(i+a)c-bd}\right)}{2c^2}+\frac{id\,\mathrm{PolyLog}\left(2,\frac{b(d+cx)}{ic-ac+bd}\right)}{2c^2}\end{aligned}$$

output
```
1/2*ln(I-a-b*x)/b/c+1/2*I*(b*x+a)*ln((-I+a+b*x)/(b*x+a))/b/c+1/2*ln(I+a+b*
x)/b/c-1/2*I*(b*x+a)*ln((I+a+b*x)/(b*x+a))/b/c+1/2*I*d*ln(c*(I-a-b*x)/(I*c
-a*c+b*d))*ln(c*x+d)/c^2-1/2*I*d*ln((-I+a+b*x)/(b*x+a))*ln(c*x+d)/c^2-1/2*
I*d*ln(c*(I+a+b*x)/((I+a)*c-b*d))*ln(c*x+d)/c^2+1/2*I*d*ln((I+a+b*x)/(b*x+
a))*ln(c*x+d)/c^2-1/2*I*d*polylog(2,-b*(c*x+d)/((I+a)*c-b*d))/c^2+1/2*I*d*
polylog(2,b*(c*x+d)/(I*c-a*c+b*d))/c^2
```

3.109.2 Mathematica [A] (verified)

Time = 7.62 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.52

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x}}\,dx$$

$$= \frac{2ac^2\cot^{-1}(a+bx) - ibcd\pi\cot^{-1}(a+bx) + 2bc^2x\cot^{-1}(a+bx) - ibcd\cot^{-1}(a+bx)^2 + abcd\cot^{-1}(a+b}{}$$

input
```
Integrate[ArcCot[a + b*x]/(c + d/x),x]
```

output
```
(2*a*c^2*ArcCot[a + b*x] - I*b*c*d*Pi*ArcCot[a + b*x] + 2*b*c^2*x*ArcCot[a
 + b*x] - I*b*c*d*ArcCot[a + b*x]^2 + a*b*c*d*ArcCot[a + b*x]^2 - b^2*d^2*
ArcCot[a + b*x]^2 - a*b*c*d*Sqrt[1 + c^2/(a*c - b*d)^2]*E^(I*ArcTan[c/(-(a
*c) + b*d)])*ArcCot[a + b*x]^2 + b^2*d^2*Sqrt[1 + c^2/(a*c - b*d)^2]*E^(I*
ArcTan[c/(-(a*c) + b*d)])*ArcCot[a + b*x]^2 + (2*I)*b*c*d*ArcCot[a + b*x]*
ArcTan[c/(-(a*c) + b*d)] - b*c*d*Pi*Log[1 + E^((-2*I)*ArcCot[a + b*x])] +
2*b*c*d*ArcCot[a + b*x]*Log[1 - E^((2*I)*ArcCot[a + b*x])] - 2*b*c*d*ArcCo
t[a + b*x]*Log[1 - E^((2*I)*(ArcCot[a + b*x] + ArcTan[c/(-(a*c) + b*d)]))]
 - 2*b*c*d*ArcTan[c/(-(a*c) + b*d)]*Log[1 - E^((2*I)*(ArcCot[a + b*x] + Ar
cTan[c/(-(a*c) + b*d)]))] - 2*c^2*Log[(a + b*x)^(-1)] - 2*c^2*Log[1/Sqrt[1
 + (a + b*x)^(-2)]] + b*c*d*Pi*Log[1/Sqrt[1 + (a + b*x)^(-2)]] + 2*b*c*d*A
rcTan[c/(-(a*c) + b*d)]*Log[Sin[ArcCot[a + b*x] + ArcTan[c/(-(a*c) + b*d)]
]] - I*b*c*d*PolyLog[2, E^((2*I)*ArcCot[a + b*x])] + I*b*c*d*PolyLog[2, E^
((2*I)*(ArcCot[a + b*x] + ArcTan[c/(-(a*c) + b*d)]))])/(2*b*c^3)
```

3.109.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.42, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5575, 2993, 772, 49, 2009, 2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x}}\,dx$$

$$\Big\downarrow \ 5575$$

$$\frac{1}{2}i\int \frac{\log\left(-\frac{-a-bx+i}{a+bx}\right)}{c+\frac{d}{x}}dx-\frac{1}{2}i\int \frac{\log\left(\frac{a+bx+i}{a+bx}\right)}{c+\frac{d}{x}}dx$$

$\Big\downarrow$ 2993

$$\frac{1}{2}i\left(-\left(\left(\log(-a-bx+i)-\log\left(-\frac{-a-bx+i}{a+bx}\right)-\log(a+bx)\right)\int \frac{1}{c+\frac{d}{x}}dx\right)+\int \frac{\log(-a-bx+i)}{c+\frac{d}{x}}dx-\int \frac{\text{lo}}{}\right.$$
$$\frac{1}{2}i\left(\left(\log(a+bx)-\log(a+bx+i)+\log\left(\frac{a+bx+i}{a+bx}\right)\right)\int \frac{1}{c+\frac{d}{x}}dx-\int \frac{\log(a+bx)}{c+\frac{d}{x}}dx+\int \frac{\log(a+bx+i)}{c+\frac{d}{x}}dx\right)$$

$\Big\downarrow$ 772

$$\frac{1}{2}i\left(-\left(\left(\log(-a-bx+i)-\log\left(-\frac{-a-bx+i}{a+bx}\right)-\log(a+bx)\right)\int \frac{x}{d+cx}dx\right)+\int \frac{\log(-a-bx+i)}{c+\frac{d}{x}}dx-\int \frac{\text{lo}}{}\right.$$
$$\frac{1}{2}i\left(\left(\log(a+bx)-\log(a+bx+i)+\log\left(\frac{a+bx+i}{a+bx}\right)\right)\int \frac{x}{d+cx}dx-\int \frac{\log(a+bx)}{c+\frac{d}{x}}dx+\int \frac{\log(a+bx+i)}{c+\frac{d}{x}}dx\right.$$

$\Big\downarrow$ 49

$$\frac{1}{2}i\left(-\left(\left(\log(-a-bx+i)-\log\left(-\frac{-a-bx+i}{a+bx}\right)-\log(a+bx)\right)\int \left(\frac{1}{c}-\frac{d}{c(d+cx)}\right)dx\right)+\int \frac{\log(-a-bx+}{c+\frac{d}{x}}\right.$$
$$\frac{1}{2}i\left(\left(\log(a+bx)-\log(a+bx+i)+\log\left(\frac{a+bx+i}{a+bx}\right)\right)\int \left(\frac{1}{c}-\frac{d}{c(d+cx)}\right)dx-\int \frac{\log(a+bx)}{c+\frac{d}{x}}dx+\int \frac{\log(a-}{c}\right.$$

$\Big\downarrow$ 2009

$$\frac{1}{2}i\left(\int \frac{\log(-a-bx+i)}{c+\frac{d}{x}}dx-\int \frac{\log(a+bx)}{c+\frac{d}{x}}dx-\left(\left(\log(-a-bx+i)-\log\left(-\frac{-a-bx+i}{a+bx}\right)-\log(a+bx)\right)\left(\frac{x}{c}\right.\right.\right.$$
$$\frac{1}{2}i\left(-\int \frac{\log(a+bx)}{c+\frac{d}{x}}dx+\int \frac{\log(a+bx+i)}{c+\frac{d}{x}}dx+\left(\log(a+bx)-\log(a+bx+i)+\log\left(\frac{a+bx+i}{a+bx}\right)\right)\left(\frac{x}{c}-\frac{d\,\text{lo}}{}\right.\right.$$

$\Big\downarrow$ 2856

$$\frac{1}{2}i\left(\int \left(\frac{\log(-a-bx+i)}{c}-\frac{d\log(-a-bx+i)}{c(d+cx)}\right)dx-\int \left(\frac{\log(a+bx)}{c}-\frac{d\log(a+bx)}{c(d+cx)}\right)dx-\left(\left(\log(-a-bx+\right.\right.\right.$$
$$\frac{1}{2}i\left(-\int \left(\frac{\log(a+bx)}{c}-\frac{d\log(a+bx)}{c(d+cx)}\right)dx+\int \left(\frac{\log(a+bx+i)}{c}-\frac{d\log(a+bx+i)}{c(d+cx)}\right)dx+\left(\log(a+bx)-\log(\right.\right.$$

$\Big\downarrow$ 2009

3.109. $\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x}}\,dx$

$$\frac{1}{2}i\left(-\frac{d\,\text{PolyLog}\left(2,\frac{c(-a-bx+i)}{-ac+ic+bd}\right)}{c^2}+\frac{d\,\text{PolyLog}\left(2,\frac{c(a+bx)}{ac-bd}\right)}{c^2}-\frac{d\log(-a-bx+i)\log\left(\frac{b(cx+d)}{-ac+bd+ic}\right)}{c^2}-\left(\log(-a-bx\right.\right.$$

$$\frac{1}{2}i\left(\frac{d\,\text{PolyLog}\left(2,\frac{c(a+bx)}{ac-bd}\right)}{c^2}-\frac{d\,\text{PolyLog}\left(2,\frac{c(a+bx+i)}{(a+i)c-bd}\right)}{c^2}+\frac{d\log(a+bx)\log\left(-\frac{b(cx+d)}{ac-bd}\right)}{c^2}+\left(\log(a+bx)-\log(a+\right.\right.$$

input
```
Int[ArcCot[a + b*x]/(c + d/x),x]
```

output
```
(I/2)*(-(((I - a - b*x)*Log[I - a - b*x])/(b*c)) - ((a + b*x)*Log[a + b*x]
)/(b*c) - (Log[I - a - b*x] - Log[-((I - a - b*x)/(a + b*x))] - Log[a + b*
x])*(x/c - (d*Log[d + c*x])/c^2) + (d*Log[a + b*x]*Log[-((b*(d + c*x))/(a*
c - b*d))])/c^2 - (d*Log[I - a - b*x]*Log[(b*(d + c*x))/(I*c - a*c + b*d)]
)/c^2 - (d*PolyLog[2, (c*(I - a - b*x))/(I*c - a*c + b*d)])/c^2 + (d*PolyL
og[2, (c*(a + b*x))/(a*c - b*d)])/c^2) - (I/2)*(-(((a + b*x)*Log[a + b*x])
/(b*c)) + ((I + a + b*x)*Log[I + a + b*x])/(b*c) + (Log[a + b*x] - Log[I +
 a + b*x] + Log[(I + a + b*x)/(a + b*x)])*(x/c - (d*Log[d + c*x])/c^2) + (
d*Log[a + b*x]*Log[-((b*(d + c*x))/(a*c - b*d))])/c^2 - (d*Log[I + a + b*x
]*Log[-((b*(d + c*x))/((I + a)*c - b*d))])/c^2 + (d*PolyLog[2, (c*(a + b*x
))/(a*c - b*d)])/c^2 - (d*PolyLog[2, (c*(I + a + b*x))/((I + a)*c - b*d)])
/c^2)
```

3.109.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

rule 2993
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Simp[p*r   Int[RFx*Log[a + b*x], x], x] + (Si
mp[q*r   Int[RFx*Log[c + d*x], x], x] - Simp[(p*r*Log[a + b*x] + q*r*Log[c
+ d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])   Int[RFx, x], x]) /; FreeQ[
{a, b, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a
*d, 0] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[
m, n]]
```

rule 5575
```
Int[ArcCot[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Simp[
I/2   Int[Log[(-I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] - Simp[I/2   In
t[Log[(I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x
] && RationalQ[n]
```

3.109.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{\frac{\operatorname{arccot}(bx+a)(bx+a)}{c}-\frac{\operatorname{arccot}(bx+a)db\ln(ac-bd-c(bx+a))}{c^2}-\frac{-\frac{\ln\left(a^2c^2-2abcd+b^2d^2-2ac(ac-bd-c(bx+a))+2bd(ac-bd-c(bx+a))\right.}{2}}{}}{}$
default	$\frac{\frac{\operatorname{arccot}(bx+a)(bx+a)}{c}-\frac{\operatorname{arccot}(bx+a)db\ln(ac-bd-c(bx+a))}{c^2}-\frac{-\frac{\ln\left(a^2c^2-2abcd+b^2d^2-2ac(ac-bd-c(bx+a))+2bd(ac-bd-c(bx+a))\right.}{2}}{}}{}$
parts	$\frac{\operatorname{arccot}(bx+a)x}{c}-\frac{\operatorname{arccot}(bx+a)d\ln(cx+d)}{c^2}+\frac{b\left(\frac{\ln\left(a^2c^2-2abcd+2abc(cx+d)+b^2d^2-2b^2d(cx+d)+b^2(cx+d)^2+c^2\right)}{2b^2}-\frac{a\operatorname{arct}}{}\right.}{}$
risch	$\frac{id\operatorname{dilog}\left(\frac{iac-ibd+(-ibx-ia+1)c-c}{iac-ibd-c}\right)}{2c^2}-\frac{id\operatorname{dilog}\left(\frac{-iac+ibd+(ibx+ia+1)c-c}{-iac+ibd-c}\right)}{2c^2}-\frac{id\ln(ibx+ia+1)\ln\left(\frac{-iac+ibd+(ibx+ia+}{-iac+ibd-c}\right.}{2c^2}$

input
```
int(arccot(b*x+a)/(c+d/x),x,method=_RETURNVERBOSE)
```

output
```
1/b*(arccot(b*x+a)/c*(b*x+a)-arccot(b*x+a)*d*b/c^2*ln(a*c-b*d-c*(b*x+a))-1
/c*(-1/2*ln(a^2*c^2-2*a*b*c*d+b^2*d^2-2*a*c*(a*c-b*d-c*(b*x+a))+2*b*d*(a*c
-b*d-c*(b*x+a))+c^2+(a*c-b*d-c*(b*x+a))^2)-b*d*(-1/2*I*ln(a*c-b*d-c*(b*x+a
))*(ln((I*c+c*(b*x+a))/(a*c-b*d+I*c))-ln((I*c-c*(b*x+a))/(I*c-a*c+b*d)))/c
-1/2*I*(dilog((I*c+c*(b*x+a))/(a*c-b*d+I*c))-dilog((I*c-c*(b*x+a))/(I*c-a*
c+b*d)))/c)))
```

3.109.5 Fricas [F]

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x}}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{c+\frac{d}{x}}\,dx$$

input
```
integrate(arccot(b*x+a)/(c+d/x),x, algorithm="fricas")
```

output
```
integral(x*arccot(b*x + a)/(c*x + d), x)
```

3.109.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x}}\,dx = \text{Timed out}$$

input
```
integrate(acot(b*x+a)/(c+d/x),x)
```

output
```
Timed out
```

3.109.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.83

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x}}\,dx$$
$$= \frac{2\,bcx\arctan(1,bx+a) - bd\arctan(1,bx+a)\log\left(-\frac{b^2c^2x^2+2\,b^2cdx+b^2d^2}{2\,abcd-b^2d^2-(a^2+1)c^2}\right) - 2\,ac\arctan(bx+a) + i\,bd\mathrm{Li}_2(\cdots}{\cdots}$$

input
```
integrate(arccot(b*x+a)/(c+d/x),x, algorithm="maxima")
```

output
```
1/2*(2*b*c*x*arctan2(1, b*x + a) - b*d*arctan2(1, b*x + a)*log(-(b^2*c^2*x
^2 + 2*b^2*c*d*x + b^2*d^2)/(2*a*b*c*d - b^2*d^2 - (a^2 + 1)*c^2)) - 2*a*c
*arctan(b*x + a) + I*b*d*dilog((b*c*x + (a + I)*c)/((a + I)*c - b*d)) - I*
b*d*dilog((b*c*x + (a - I)*c)/((a - I)*c - b*d)) - (b*d*arctan2(-(b*c^2*x
+ b*c*d)/(2*a*b*c*d - b^2*d^2 - (a^2 + 1)*c^2), (a*b*c*d - b^2*d^2 + (a*b*
c^2 - b^2*c*d)*x)/(2*a*b*c*d - b^2*d^2 - (a^2 + 1)*c^2)) - c)*log(b^2*x^2
+ 2*a*b*x + a^2 + 1))/(b*c^2)
```

3.109.8 Giac [F]

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x}}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{c+\frac{d}{x}}\,dx$$

input
```
integrate(arccot(b*x+a)/(c+d/x),x, algorithm="giac")
```

output
```
integrate(arccot(b*x + a)/(c + d/x), x)
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x}}\,dx = \int \frac{\operatorname{acot}(a+b\,x)}{c+\frac{d}{x}}\,dx$$

input
```
int(acot(a + b*x)/(c + d/x),x)
```

output
```
int(acot(a + b*x)/(c + d/x), x)
```

3.110 $\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx$

3.110.1 Optimal result

Integrand size = 16, antiderivative size = 735

$$\begin{aligned}\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx &= \frac{\log(i-a-bx)}{2bc}+\frac{i(a+bx)\log\left(-\frac{i-a-bx}{a+bx}\right)}{2bc} \\ &\quad -\frac{i\sqrt{d}\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\log\left(-\frac{i-a-bx}{a+bx}\right)}{2c^{3/2}}+\frac{\log(i+a+bx)}{2bc} \\ &\quad -\frac{i(a+bx)\log\left(\frac{i+a+bx}{a+bx}\right)}{2bc}+\frac{i\sqrt{d}\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\log\left(\frac{i+a+bx}{a+bx}\right)}{2c^{3/2}} \\ &\quad -\frac{\sqrt{d}\log\left(\frac{\sqrt{c}(i-a-bx)}{(i-a)\sqrt{c}+ib\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{c}x}{\sqrt{d}}\right)}{4c^{3/2}} \\ &\quad +\frac{\sqrt{d}\log\left(\frac{\sqrt{c}(i+a+bx)}{(i+a)\sqrt{c}-ib\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{c}x}{\sqrt{d}}\right)}{4c^{3/2}} \\ &\quad +\frac{\sqrt{d}\log\left(\frac{\sqrt{c}(i-a-bx)}{(i-a)\sqrt{c}-ib\sqrt{d}}\right)\log\left(1+\frac{i\sqrt{c}x}{\sqrt{d}}\right)}{4c^{3/2}} \\ &\quad -\frac{\sqrt{d}\log\left(\frac{\sqrt{c}(i+a+bx)}{(i+a)\sqrt{c}+ib\sqrt{d}}\right)\log\left(1+\frac{i\sqrt{c}x}{\sqrt{d}}\right)}{4c^{3/2}} \\ &\quad -\frac{\sqrt{d}\,\mathrm{PolyLog}\left(2,\frac{b(\sqrt{d}-i\sqrt{c}x)}{(1+ia)\sqrt{c}+b\sqrt{d}}\right)}{4c^{3/2}}+\frac{\sqrt{d}\,\mathrm{PolyLog}\left(2,\frac{b(\sqrt{d}-i\sqrt{c}x)}{i(i+a)\sqrt{c}+b\sqrt{d}}\right)}{4c^{3/2}} \\ &\quad +\frac{\sqrt{d}\,\mathrm{PolyLog}\left(2,-\frac{b(\sqrt{d}+i\sqrt{c}x)}{(1+ia)\sqrt{c}-b\sqrt{d}}\right)}{4c^{3/2}}-\frac{\sqrt{d}\,\mathrm{PolyLog}\left(2,\frac{b(\sqrt{d}+i\sqrt{c}x)}{(1-ia)\sqrt{c}+b\sqrt{d}}\right)}{4c^{3/2}}\end{aligned}$$

output
```
1/2*ln(I-a-b*x)/b/c+1/2*I*(b*x+a)*ln((-I+a+b*x)/(b*x+a))/b/c+1/2*ln(I+a+b*
x)/b/c-1/2*I*(b*x+a)*ln((I+a+b*x)/(b*x+a))/b/c-1/2*I*arctan(x*c^(1/2)/d^(1
/2))*ln((-I+a+b*x)/(b*x+a))*d^(1/2)/c^(3/2)+1/2*I*arctan(x*c^(1/2)/d^(1/2)
)*ln((I+a+b*x)/(b*x+a))*d^(1/2)/c^(3/2)+1/4*ln(1+I*x*c^(1/2)/d^(1/2))*ln((
I-a-b*x)*c^(1/2)/((I-a)*c^(1/2)-I*b*d^(1/2)))*d^(1/2)/c^(3/2)+1/4*ln(1-I*x
*c^(1/2)/d^(1/2))*ln((I+a+b*x)*c^(1/2)/((I+a)*c^(1/2)-I*b*d^(1/2)))*d^(1/2
)/c^(3/2)-1/4*ln(1-I*x*c^(1/2)/d^(1/2))*ln((I-a-b*x)*c^(1/2)/((I-a)*c^(1/2
)+I*b*d^(1/2)))*d^(1/2)/c^(3/2)-1/4*ln(1+I*x*c^(1/2)/d^(1/2))*ln((I+a+b*x)
*c^(1/2)/((I+a)*c^(1/2)+I*b*d^(1/2)))*d^(1/2)/c^(3/2)+1/4*polylog(2,-b*(I*
x*c^(1/2)+d^(1/2))/((1+I*a)*c^(1/2)-b*d^(1/2)))*d^(1/2)/c^(3/2)-1/4*polylo
g(2,b*(I*x*c^(1/2)+d^(1/2))/((1-I*a)*c^(1/2)+b*d^(1/2)))*d^(1/2)/c^(3/2)-1
/4*polylog(2,b*(-I*x*c^(1/2)+d^(1/2))/((1+I*a)*c^(1/2)+b*d^(1/2)))*d^(1/2)
/c^(3/2)+1/4*polylog(2,b*(-I*x*c^(1/2)+d^(1/2))/(I*(I+a)*c^(1/2)+b*d^(1/2)
))*d^(1/2)/c^(3/2)
```

3.110.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 930, normalized size of antiderivative = 1.27

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx$$

$$= \frac{4c\log(a+bx)+2c\log\left(\frac{-i+a+bx}{a+bx}\right)+2iac\log\left(\frac{-i+a+bx}{a+bx}\right)+2ibcx\log\left(\frac{-i+a+bx}{a+bx}\right)+2c\log\left(\frac{i+a+bx}{a+bx}\right)-2iac\log\left(\right.}{}$$

input
```
Integrate[ArcCot[a + b*x]/(c + d/x^2),x]
```

output
```
(4*c*Log[a + b*x] + 2*c*Log[(-I + a + b*x)/(a + b*x)] + (2*I)*a*c*Log[(-I
+ a + b*x)/(a + b*x)] + (2*I)*b*c*x*Log[(-I + a + b*x)/(a + b*x)] + 2*c*Lo
g[(I + a + b*x)/(a + b*x)] - (2*I)*a*c*Log[(I + a + b*x)/(a + b*x)] - (2*I
)*b*c*x*Log[(I + a + b*x)/(a + b*x)] + I*b*Sqrt[-c]*Sqrt[d]*Log[(Sqrt[-c]*
(-I + a + b*x))/((-I)*Sqrt[-c] + a*Sqrt[-c] + b*Sqrt[d])]*Log[Sqrt[d] - Sq
rt[-c]*x] - I*b*Sqrt[-c]*Sqrt[d]*Log[(-I + a + b*x)/(a + b*x)]*Log[Sqrt[d]
 - Sqrt[-c]*x] - I*b*Sqrt[-c]*Sqrt[d]*Log[(Sqrt[-c]*(I + a + b*x))/((I + a
)*Sqrt[-c] + b*Sqrt[d])]*Log[Sqrt[d] - Sqrt[-c]*x] + I*b*Sqrt[-c]*Sqrt[d]*
Log[(I + a + b*x)/(a + b*x)]*Log[Sqrt[d] - Sqrt[-c]*x] - I*b*Sqrt[-c]*Sqrt
[d]*Log[(Sqrt[-c]*(I - a - b*x))/(-((-I + a)*Sqrt[-c]) + b*Sqrt[d])]*Log[S
qrt[d] + Sqrt[-c]*x] + I*b*Sqrt[-c]*Sqrt[d]*Log[(-I + a + b*x)/(a + b*x)]*
Log[Sqrt[d] + Sqrt[-c]*x] + I*b*Sqrt[-c]*Sqrt[d]*Log[(Sqrt[-c]*(I + a + b*
x))/((I + a)*Sqrt[-c] - b*Sqrt[d])]*Log[Sqrt[d] + Sqrt[-c]*x] - I*b*Sqrt[-
c]*Sqrt[d]*Log[(I + a + b*x)/(a + b*x)]*Log[Sqrt[d] + Sqrt[-c]*x] + I*b*Sq
rt[-c]*Sqrt[d]*PolyLog[2, (b*(Sqrt[d] - Sqrt[-c]*x))/((-I)*Sqrt[-c] + a*Sq
rt[-c] + b*Sqrt[d])] - I*b*Sqrt[-c]*Sqrt[d]*PolyLog[2, (b*(Sqrt[d] - Sqrt[
-c]*x))/(I*Sqrt[-c] + a*Sqrt[-c] + b*Sqrt[d])] + I*b*Sqrt[-c]*Sqrt[d]*Poly
Log[2, (b*(Sqrt[d] + Sqrt[-c]*x))/((-I)*Sqrt[-c] - a*Sqrt[-c] + b*Sqrt[d])
] - I*b*Sqrt[-c]*Sqrt[d]*PolyLog[2, (b*(Sqrt[d] + Sqrt[-c]*x))/(I*Sqrt[-c]
 - a*Sqrt[-c] + b*Sqrt[d])])/(4*b*c^2)
```

3.110.3 Rubi [A] (verified)

Time = 2.20 (sec) , antiderivative size = 1222, normalized size of antiderivative = 1.66, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5575, 2993, 772, 262, 218, 2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x^2}}dx$$

$$\Big\downarrow \text{5575}$$

$$\frac{1}{2}i\int \frac{\log\left(-\frac{-a-bx+i}{a+bx}\right)}{c+\frac{d}{x^2}}dx-\frac{1}{2}i\int \frac{\log\left(\frac{a+bx+i}{a+bx}\right)}{c+\frac{d}{x^2}}dx$$

$$\Big\downarrow \text{2993}$$

$$\frac{1}{2}i\left(-\left(\left(\log(-a-bx+i)-\log\left(-\frac{-a-bx+i}{a+bx}\right)-\log(a+bx)\right)\int\frac{1}{c+\frac{d}{x^2}}dx\right)+\int\frac{\log(-a-bx+i)}{c+\frac{d}{x^2}}dx-\int\frac{\text{lo}}{}\right.$$
$$\frac{1}{2}i\left(\left(\log(a+bx)-\log(a+bx+i)+\log\left(\frac{a+bx+i}{a+bx}\right)\right)\int\frac{1}{c+\frac{d}{x^2}}dx-\int\frac{\log(a+bx)}{c+\frac{d}{x^2}}dx+\int\frac{\log(a+bx+i)}{c+\frac{d}{x^2}}dx\right.$$

$\downarrow$ 772

$$\frac{1}{2}i\left(-\left(\left(\log(-a-bx+i)-\log\left(-\frac{-a-bx+i}{a+bx}\right)-\log(a+bx)\right)\int\frac{x^2}{cx^2+d}dx\right)+\int\frac{\log(-a-bx+i)}{c+\frac{d}{x^2}}dx-\int\right.$$
$$\frac{1}{2}i\left(\left(\log(a+bx)-\log(a+bx+i)+\log\left(\frac{a+bx+i}{a+bx}\right)\right)\int\frac{x^2}{cx^2+d}dx-\int\frac{\log(a+bx)}{c+\frac{d}{x^2}}dx+\int\frac{\log(a+bx+i)}{c+\frac{d}{x^2}}d\right.$$

$\downarrow$ 262

$$\frac{1}{2}i\left(-\left(\left(\log(-a-bx+i)-\log\left(-\frac{-a-bx+i}{a+bx}\right)-\log(a+bx)\right)\left(\frac{x}{c}-\frac{d\int\frac{1}{cx^2+d}dx}{c}\right)\right)+\int\frac{\log(-a-bx+i)}{c+\frac{d}{x^2}}d\right.$$
$$\frac{1}{2}i\left(\left(\log(a+bx)-\log(a+bx+i)+\log\left(\frac{a+bx+i}{a+bx}\right)\right)\left(\frac{x}{c}-\frac{d\int\frac{1}{cx^2+d}dx}{c}\right)-\int\frac{\log(a+bx)}{c+\frac{d}{x^2}}dx+\int\frac{\log(a+b}{c+}\right.$$

$\downarrow$ 218

$$\frac{1}{2}i\left(\int\frac{\log(-a-bx+i)}{c+\frac{d}{x^2}}dx-\int\frac{\log(a+bx)}{c+\frac{d}{x^2}}dx-\left(\left(\log(-a-bx+i)-\log\left(-\frac{-a-bx+i}{a+bx}\right)-\log(a+bx)\right)\right(\right.$$
$$\frac{1}{2}i\left(-\int\frac{\log(a+bx)}{c+\frac{d}{x^2}}dx+\int\frac{\log(a+bx+i)}{c+\frac{d}{x^2}}dx+\left(\log(a+bx)-\log(a+bx+i)+\log\left(\frac{a+bx+i}{a+bx}\right)\right)\left(\frac{x}{c}-\frac{\sqrt{}}{}\right.\right.$$

$\downarrow$ 2856

$$\frac{1}{2}i\left(\int\left(\frac{\log(-a-bx+i)}{c}-\frac{d\log(-a-bx+i)}{c(cx^2+d)}\right)dx-\int\left(\frac{\log(a+bx)}{c}-\frac{d\log(a+bx)}{c(cx^2+d)}\right)dx-\left(\left(\log(-a-bx+\right.\right.\right.$$
$$\frac{1}{2}i\left(-\int\left(\frac{\log(a+bx)}{c}-\frac{d\log(a+bx)}{c(cx^2+d)}\right)dx+\int\left(\frac{\log(a+bx+i)}{c}-\frac{d\log(a+bx+i)}{c(cx^2+d)}\right)dx+\left(\log(a+bx)-\log\right.\right.$$

$\downarrow$ 2009

3.110. $\int\frac{\cot^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx$

$$\frac{1}{2}i\left(-\frac{(-a-bx+i)\log(-a-bx+i)}{bc}-\frac{\sqrt{d}\log\left(-\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{(i-a)\sqrt{-c}-b\sqrt{d}}\right)\log(-a-bx+i)}{2(-c)^{3/2}}+\frac{\sqrt{d}\log\left(\frac{b\left(\sqrt{-c}x+\sqrt{d}\right)}{\sqrt{-c}(i-a)+b\sqrt{d}}\right)\mathrm{lo}}{2(-c)^{3/2}}\right.$$

$$\frac{1}{2}i\left(-\frac{(a+bx)\log(a+bx)}{bc}+\frac{\sqrt{d}\log\left(\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{\sqrt{-c}a+b\sqrt{d}}\right)\log(a+bx)}{2(-c)^{3/2}}-\frac{\sqrt{d}\log\left(-\frac{b\left(\sqrt{-c}x+\sqrt{d}\right)}{a\sqrt{-c}-b\sqrt{d}}\right)\log(a+bx)}{2(-c)^{3/2}}+\frac{(a+b}{}\right.$$

input
```
Int[ArcCot[a + b*x]/(c + d/x^2),x]
```

output
```
(I/2)*(-(((I - a - b*x)*Log[I - a - b*x])/(b*c)) - (x/c - (Sqrt[d]*ArcTan[
(Sqrt[c]*x)/Sqrt[d]])/c^(3/2))*(Log[I - a - b*x] - Log[-((I - a - b*x)/(a
+ b*x))] - Log[a + b*x]) - ((a + b*x)*Log[a + b*x])/(b*c) - (Sqrt[d]*Log[I
 - a - b*x]*Log[-((b*(Sqrt[d] - Sqrt[-c]*x))/((I - a)*Sqrt[-c] - b*Sqrt[d]
))])/(2*(-c)^(3/2)) + (Sqrt[d]*Log[a + b*x]*Log[(b*(Sqrt[d] - Sqrt[-c]*x))
/(a*Sqrt[-c] + b*Sqrt[d])])/(2*(-c)^(3/2)) - (Sqrt[d]*Log[a + b*x]*Log[-((
b*(Sqrt[d] + Sqrt[-c]*x))/(a*Sqrt[-c] - b*Sqrt[d]))])/(2*(-c)^(3/2)) + (Sq
rt[d]*Log[I - a - b*x]*Log[(b*(Sqrt[d] + Sqrt[-c]*x))/((I - a)*Sqrt[-c] +
b*Sqrt[d])])/(2*(-c)^(3/2)) - (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(I - a - b*x))
/(I*Sqrt[-c] - a*Sqrt[-c] - b*Sqrt[d])])/(2*(-c)^(3/2)) + (Sqrt[d]*PolyLog
[2, (Sqrt[-c]*(I - a - b*x))/((I - a)*Sqrt[-c] + b*Sqrt[d])])/(2*(-c)^(3/2
)) - (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(a + b*x))/(a*Sqrt[-c] - b*Sqrt[d])])/(
2*(-c)^(3/2)) + (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(a + b*x))/(a*Sqrt[-c] + b*S
qrt[d])])/(2*(-c)^(3/2))) - (I/2)*(-(((a + b*x)*Log[a + b*x])/(b*c)) + ((I
 + a + b*x)*Log[I + a + b*x])/(b*c) + (x/c - (Sqrt[d]*ArcTan[(Sqrt[c]*x)/S
qrt[d]])/c^(3/2))*(Log[a + b*x] - Log[I + a + b*x] + Log[(I + a + b*x)/(a
+ b*x)]) + (Sqrt[d]*Log[a + b*x]*Log[(b*(Sqrt[d] - Sqrt[-c]*x))/(a*Sqrt[-c
] + b*Sqrt[d])])/(2*(-c)^(3/2)) - (Sqrt[d]*Log[I + a + b*x]*Log[(b*(Sqrt[d
] - Sqrt[-c]*x))/((I + a)*Sqrt[-c] + b*Sqrt[d])])/(2*(-c)^(3/2)) - (Sqrt[d
]*Log[a + b*x]*Log[-((b*(Sqrt[d] + Sqrt[-c]*x))/(a*Sqrt[-c] - b*Sqrt[d]...
```

3.110. $\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx$

3.110.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

rule 2993
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Simp[p*r   Int[RFx*Log[a + b*x], x], x] + (Si
mp[q*r   Int[RFx*Log[c + d*x], x], x] - Simp[(p*r*Log[a + b*x] + q*r*Log[c
+ d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])   Int[RFx, x], x]) /; FreeQ[
{a, b, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a
*d, 0] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[
m, n]]
```

rule 5575
```
Int[ArcCot[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Simp[
I/2   Int[Log[(-I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] - Simp[I/2   In
t[Log[(I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x
] && RationalQ[n]
```

3.110.4 Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 728, normalized size of antiderivative = 0.99

method	result
risch	$\frac{\pi x}{2c}+\frac{\pi a}{2bc}+\frac{i\ln(ibx+ia+1)a}{2bc}-\frac{i\ln(-ibx-ia+1)a}{2bc}+\frac{i\ln(ibx+ia+1)x}{2c}-\frac{i\ln(-ibx-ia+1)x}{2c}+\frac{i\pi}{2bc}+\frac{ib\pi d\,\mathrm{arc}}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(arccot(b*x+a)/(c+d/x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*Pi/c*x+1/2/b*Pi/c*a+1/2*I/b/c*ln(1+I*a+I*b*x)*a-1/2*I/b/c*ln(1-I*a-I*b
*x)*a+1/2*I/c*ln(1+I*a+I*b*x)*x-1/2*I/c*ln(1-I*a-I*b*x)*x+1/2*I/b*Pi/c+1/2
*I*b*Pi*d/c/(-b^2*c*d)^(1/2)*arctan(1/2*(2*I*a*c+2*(1-I*a-I*b*x)*c-2*c)/(-
b^2*c*d)^(1/2))+1/2/b/c*ln(1-I*a-I*b*x)-1/b/c+1/4/c^2*ln(1-I*a-I*b*x)*(c*d
)^(1/2)*ln((I*a*c-b*(c*d)^(1/2)+(1-I*a-I*b*x)*c-c)/(I*a*c-b*(c*d)^(1/2)-c)
)-1/4/c^2*ln(1-I*a-I*b*x)*(c*d)^(1/2)*ln((I*a*c+b*(c*d)^(1/2)+(1-I*a-I*b*x
)*c-c)/(I*a*c+b*(c*d)^(1/2)-c))+1/4/c^2*dilog((I*a*c-b*(c*d)^(1/2)+(1-I*a-
I*b*x)*c-c)/(I*a*c-b*(c*d)^(1/2)-c))*(c*d)^(1/2)-1/4/c^2*dilog((I*a*c+b*(c
*d)^(1/2)+(1-I*a-I*b*x)*c-c)/(I*a*c+b*(c*d)^(1/2)-c))*(c*d)^(1/2)+1/2/b/c*
ln(1+I*a+I*b*x)+1/4/c^2*ln(1+I*a+I*b*x)*(c*d)^(1/2)*ln((I*a*c+b*(c*d)^(1/2
)-(1+I*a+I*b*x)*c+c)/(I*a*c+b*(c*d)^(1/2)+c))-1/4/c^2*ln(1+I*a+I*b*x)*(c*d
)^(1/2)*ln((I*a*c-b*(c*d)^(1/2)-(1+I*a+I*b*x)*c+c)/(I*a*c-b*(c*d)^(1/2)+c)
)-1/4/c^2*(c*d)^(1/2)*dilog((I*a*c-b*(c*d)^(1/2)-(1+I*a+I*b*x)*c+c)/(I*a*c
-b*(c*d)^(1/2)+c))+1/4/c^2*(c*d)^(1/2)*dilog((I*a*c+b*(c*d)^(1/2)-(1+I*a+I
*b*x)*c+c)/(I*a*c+b*(c*d)^(1/2)+c))
```

3.110.5 Fricas [F]

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx=\int \frac{\operatorname{arccot}(bx+a)}{c+\frac{d}{x^2}}\,dx$$

input
```
integrate(arccot(b*x+a)/(c+d/x^2),x, algorithm="fricas")
```

output
```
integral(x^2*arccot(b*x + a)/(c*x^2 + d), x)
```

3.110.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx = \text{Timed out}$$

input
```
integrate(acot(b*x+a)/(c+d/x**2),x)
```

output
```
Timed out
```

3.110.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 8518 vs. $2(502) = 1004$.

Time $= 0.93$ (sec) , antiderivative size $= 8518$, normalized size of antiderivative $= 11.59$

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx = \text{Too large to display}$$

input
```
integrate(arccot(b*x+a)/(c+d/x^2),x, algorithm="maxima")
```

output
```
-(d*arctan(c*x/sqrt(c*d))/(sqrt(c*d)*c) - x/c)*arccot(b*x + a) - 1/8*(8*a*
c*arctan(b*x + a) + (4*b*arctan(sqrt(c)*x/sqrt(d))*arctan2((2*a*b^2*c*d +
(a*b^3*d + (a^3 + a)*b*c + (b^4*d + (a^2 + 3)*b^2*c)*x)*sqrt(c)*sqrt(d) +
(3*b^3*c*d + (a^2 + 1)*b*c^2)*x)/(b^4*d^2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2
*a^2 + 1)*c^2 + 4*(b^3*d + (a^2 + 1)*b*c)*sqrt(c)*sqrt(d)), ((a^2 + 3)*b^2
*c*d + (a^4 + 2*a^2 + 1)*c^2 + (2*a*b^2*c*x + b^3*d + 3*(a^2 + 1)*b*c)*sqr
t(c)*sqrt(d) + (a*b^3*c*d + (a^3 + a)*b*c^2)*x)/(b^4*d^2 + 2*(a^2 + 3)*b^2
*c*d + (a^4 + 2*a^2 + 1)*c^2 + 4*(b^3*d + (a^2 + 1)*b*c)*sqrt(c)*sqrt(d)))
 + 4*b*arctan(sqrt(c)*x/sqrt(d))*arctan2((2*a*b^2*c*d - (a*b^3*d + (a^3 +
a)*b*c + (b^4*d + (a^2 + 3)*b^2*c)*x)*sqrt(c)*sqrt(d) + (3*b^3*c*d + (a^2
+ 1)*b*c^2)*x)/(b^4*d^2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*c^2 - 4*
(b^3*d + (a^2 + 1)*b*c)*sqrt(c)*sqrt(d)), ((a^2 + 3)*b^2*c*d + (a^4 + 2*a^
2 + 1)*c^2 - (2*a*b^2*c*x + b^3*d + 3*(a^2 + 1)*b*c)*sqrt(c)*sqrt(d) + (a*
b^3*c*d + (a^3 + a)*b*c^2)*x)/(b^4*d^2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^
2 + 1)*c^2 - 4*(b^3*d + (a^2 + 1)*b*c)*sqrt(c)*sqrt(d))) + b*log(c*x^2 + d
)*log(((a^2 + 1)*b^22*c*d^11 + 11*(a^4 + 22*a^2 + 21)*b^20*c^2*d^10 + 55*(
a^6 + 39*a^4 + 171*a^2 + 133)*b^18*c^3*d^9 + 33*(5*a^8 + 260*a^6 + 1870*a^
4 + 3876*a^2 + 2261)*b^16*c^4*d^8 + 330*(a^10 + 61*a^8 + 570*a^6 + 1802*a^
4 + 2261*a^2 + 969)*b^14*c^5*d^7 + 22*(21*a^12 + 1386*a^10 + 15015*a^8 + 6
0060*a^6 + 109395*a^4 + 92378*a^2 + 29393)*b^12*c^6*d^6 + 22*(21*a^14 +...
```

3.110.8 Giac [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx = \text{Timed out}$$

input
```
integrate(arccot(b*x+a)/(c+d/x^2),x, algorithm="giac")
```

output
```
Timed out
```

3.110.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx = \int \frac{\operatorname{acot}(a+b\,x)}{c+\frac{d}{x^2}}\,dx$$

input
```
int(acot(a + b*x)/(c + d/x^2),x)
```

output
```
int(acot(a + b*x)/(c + d/x^2), x)
```

3.111 $\int \frac{\cot^{-1}(a+bx)}{c+d\sqrt{x}}\,dx$

3.111.1 Optimal result

Integrand size = 18, antiderivative size = 693

$$\begin{aligned}\int \frac{\cot^{-1}(a+bx)}{c+d\sqrt{x}}\,dx = &-\frac{2i\sqrt{i+a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}d}+\frac{2i\sqrt{i-a}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}d}\\
&-\frac{ic\log\left(\frac{d\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{-i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}\\
&+\frac{ic\log\left(\frac{d\left(\sqrt{i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}\\
&-\frac{ic\log\left(-\frac{d\left(\sqrt{-i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c-\sqrt{-i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}\\
&+\frac{ic\log\left(-\frac{d\left(\sqrt{i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c-\sqrt{i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}\\
&+\frac{i\sqrt{x}\log\left(-\frac{i-a-bx}{a+bx}\right)}{d}-\frac{ic\log\left(c+d\sqrt{x}\right)\log\left(-\frac{i-a-bx}{a+bx}\right)}{d^2}\\
&-\frac{i\sqrt{x}\log\left(\frac{i+a+bx}{a+bx}\right)}{d}+\frac{ic\log\left(c+d\sqrt{x}\right)\log\left(\frac{i+a+bx}{a+bx}\right)}{d^2}\\
&-\frac{ic\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{-i-a}d}\right)}{d^2}-\frac{ic\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c+\sqrt{-i-a}d}\right)}{d^2}\\
&+\frac{ic\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{i-a}d}\right)}{d^2}+\frac{ic\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c+\sqrt{i-a}d}\right)}{d^2}\end{aligned}$$

output
```
-I*c*ln((-I+a+b*x)/(b*x+a))*ln(c+d*x^(1/2))/d^2+I*c*ln((I+a+b*x)/(b*x+a))*
ln(c+d*x^(1/2))/d^2-I*c*ln(c+d*x^(1/2))*ln(d*((-I-a)^(1/2)-b^(1/2)*x^(1/2)
)/(d*(-I-a)^(1/2)+c*b^(1/2)))/d^2+I*c*ln(c+d*x^(1/2))*ln(d*((I-a)^(1/2)-b^
(1/2)*x^(1/2))/(d*(I-a)^(1/2)+c*b^(1/2)))/d^2-I*c*ln(c+d*x^(1/2))*ln(-d*((
-I-a)^(1/2)+b^(1/2)*x^(1/2))/(-d*(-I-a)^(1/2)+c*b^(1/2)))/d^2+I*c*ln(c+d*x
^(1/2))*ln(-d*((I-a)^(1/2)+b^(1/2)*x^(1/2))/(-d*(I-a)^(1/2)+c*b^(1/2)))/d^
2-I*c*polylog(2,b^(1/2)*(c+d*x^(1/2))/(-d*(-I-a)^(1/2)+c*b^(1/2)))/d^2-I*c
*polylog(2,b^(1/2)*(c+d*x^(1/2))/(d*(-I-a)^(1/2)+c*b^(1/2)))/d^2+I*c*polyl
og(2,b^(1/2)*(c+d*x^(1/2))/(-d*(I-a)^(1/2)+c*b^(1/2)))/d^2+I*c*polylog(2,b
^(1/2)*(c+d*x^(1/2))/(d*(I-a)^(1/2)+c*b^(1/2)))/d^2+2*I*arctanh(b^(1/2)*x^
(1/2)/(I-a)^(1/2))*(I-a)^(1/2)/d/b^(1/2)-2*I*arctan(b^(1/2)*x^(1/2)/(I+a)^
(1/2))*(I+a)^(1/2)/d/b^(1/2)+I*ln((-I+a+b*x)/(b*x+a))*x^(1/2)/d-I*ln((I+a+
b*x)/(b*x+a))*x^(1/2)/d
```

3.111.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 618, normalized size of antiderivative = 0.89

$$\int \frac{\cot^{-1}(a+bx)}{c+d\sqrt{x}}\,dx =$$

$$-\frac{i\left(\frac{2\sqrt{i+a}d\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}}-\frac{2\sqrt{i-a}d\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}}+c\log\left(\frac{d\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{-i-a}d}\right)\log\left(c+d\sqrt{x}\right)-c\log\left(\frac{d\left(\sqrt{i-a}-\right.}{\sqrt{b}c+\sqrt{}}\right.\right.}{}$$

input
```
Integrate[ArcCot[a + b*x]/(c + d*Sqrt[x]),x]
```

output
```
((-I)*((2*Sqrt[I + a]*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]])/Sqrt[b] - (
2*Sqrt[I - a]*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[I - a]])/Sqrt[b] + c*Log[(d
*(Sqrt[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-I - a]*d)]*Log[c + d
*Sqrt[x]] - c*Log[(d*(Sqrt[I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[I
- a]*d)]*Log[c + d*Sqrt[x]] + c*Log[(d*(Sqrt[-I - a] + Sqrt[b]*Sqrt[x]))/(
-(Sqrt[b]*c) + Sqrt[-I - a]*d)]*Log[c + d*Sqrt[x]] - c*Log[(d*(Sqrt[I - a]
 + Sqrt[b]*Sqrt[x]))/(-(Sqrt[b]*c) + Sqrt[I - a]*d)]*Log[c + d*Sqrt[x]] -
d*Sqrt[x]*Log[(-I + a + b*x)/(a + b*x)] + c*Log[c + d*Sqrt[x]]*Log[(-I + a
 + b*x)/(a + b*x)] + d*Sqrt[x]*Log[(I + a + b*x)/(a + b*x)] - c*Log[c + d*
Sqrt[x]]*Log[(I + a + b*x)/(a + b*x)] + c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[
x]))/(Sqrt[b]*c - Sqrt[-I - a]*d)] + c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x])
)/(Sqrt[b]*c + Sqrt[-I - a]*d)] - c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(
Sqrt[b]*c - Sqrt[I - a]*d)] - c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt
[b]*c + Sqrt[I - a]*d)]))/d^2
```

3.111.3 Rubi [A] (verified)

Time = 2.42 (sec) , antiderivative size = 1133, normalized size of antiderivative = 1.63, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5575, 7267, 3008, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(a+bx)}{c+d\sqrt{x}}dx$$

$$\Big\downarrow \text{ 5575}$$

$$\frac{1}{2}i\int \frac{\log\left(-\frac{-a-bx+i}{a+bx}\right)}{c+d\sqrt{x}}dx-\frac{1}{2}i\int \frac{\log\left(\frac{a+bx+i}{a+bx}\right)}{c+d\sqrt{x}}dx$$

$$\Big\downarrow \text{ 7267}$$

$$i\int \frac{\sqrt{x}\log\left(-\frac{-a-bx+i}{a+bx}\right)}{c+d\sqrt{x}}d\sqrt{x}-i\int \frac{\sqrt{x}\log\left(\frac{a+bx+i}{a+bx}\right)}{c+d\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{ 3008}$$

$$i\int \left(\frac{\log\left(-\frac{-a-bx+i}{a+bx}\right)}{d}-\frac{c\log\left(-\frac{-a-bx+i}{a+bx}\right)}{d\left(c+d\sqrt{x}\right)}\right)d\sqrt{x}-i\int \left(\frac{\log\left(\frac{a+bx+i}{a+bx}\right)}{d}-\frac{c\log\left(\frac{a+bx+i}{a+bx}\right)}{d\left(c+d\sqrt{x}\right)}\right)d\sqrt{x}$$

$$\Big\downarrow \text{ 2009}$$

$$i\left(-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}d}+\frac{2\sqrt{i-a}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}d}+\frac{c\log\left(\frac{d\left(\sqrt{i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}-\frac{c\log\left(\frac{d\left(\sqrt{-a}-\sqrt{b}\right.}{\sqrt{b}c+\sqrt{-a}\right.}\right.}{}\right.$$

$$i\left(-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}d}+\frac{2\sqrt{a+i}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+i}}\right)}{\sqrt{b}d}+\frac{c\log\left(\frac{d\left(\sqrt{-a-i}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{-a-i}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}-\frac{c\log\left(\frac{d\left(\sqrt{-a}-\sqrt{\right.}}{\sqrt{b}c+\sqrt{-\right.}}\right.}{}\right.$$

input
```
Int[ArcCot[a + b*x]/(c + d*Sqrt[x]),x]
```

output
```
(-I)*((-2*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[b]*d) + (2*Sqrt
[I + a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]])/(Sqrt[b]*d) + (c*Log[(d*(Sq
rt[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-I - a]*d)]*Log[c + d*Sqr
t[x]])/d^2 - (c*Log[(d*(Sqrt[-a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-a]
*d)]*Log[c + d*Sqrt[x]])/d^2 + (c*Log[-((d*(Sqrt[-I - a] + Sqrt[b]*Sqrt[x]
))/(Sqrt[b]*c - Sqrt[-I - a]*d))]*Log[c + d*Sqrt[x]])/d^2 - (c*Log[-((d*(S
qrt[-a] + Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c - Sqrt[-a]*d))]*Log[c + d*Sqrt[x]])
/d^2 + (Sqrt[x]*Log[(I + a + b*x)/(a + b*x)])/d - (c*Log[c + d*Sqrt[x]]*Lo
g[(I + a + b*x)/(a + b*x)])/d^2 + (c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/
(Sqrt[b]*c - Sqrt[-I - a]*d)])/d^2 + (c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]
))/(Sqrt[b]*c + Sqrt[-I - a]*d)])/d^2 - (c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt
[x]))/(Sqrt[b]*c - Sqrt[-a]*d)])/d^2 - (c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[
x]))/(Sqrt[b]*c + Sqrt[-a]*d)])/d^2) + I*((-2*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt
[x])/Sqrt[a]])/(Sqrt[b]*d) + (2*Sqrt[I - a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt
[I - a]])/(Sqrt[b]*d) + (c*Log[(d*(Sqrt[I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b
]*c + Sqrt[I - a]*d)]*Log[c + d*Sqrt[x]])/d^2 - (c*Log[(d*(Sqrt[-a] - Sqrt
[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-a]*d)]*Log[c + d*Sqrt[x]])/d^2 + (c*Log[-
((d*(Sqrt[I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c - Sqrt[I - a]*d))]*Log[c +
 d*Sqrt[x]])/d^2 - (c*Log[-((d*(Sqrt[-a] + Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c -
Sqrt[-a]*d))]*Log[c + d*Sqrt[x]])/d^2 + (Sqrt[x]*Log[-((I - a - b*x)/(a...
```

3.111.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3008
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

rule 5575
```
Int[ArcCot[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Simp[
I/2   Int[Log[(-I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] - Simp[I/2   In
t[Log[(I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x
] && RationalQ[n]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.111.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.43 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.53

method	result
derivativedivides	$\frac{2\,\mathrm{arccot}(bx+a)\sqrt{x}}{d}-\frac{2\,\mathrm{arccot}(bx+a)c\ln(c+d\sqrt{x})}{d^2}+\frac{4b\left(d^2\left(\sum_{\_R=\mathrm{RootOf}\left(b^2\_Z^4-4b^2c\_Z^3+\left(2ab\,d^2+6b^2c^2\right)\_Z^2+\left(-\right.\right.}\right.\right.}{}$
default	$\frac{2\,\mathrm{arccot}(bx+a)\sqrt{x}}{d}-\frac{2\,\mathrm{arccot}(bx+a)c\ln(c+d\sqrt{x})}{d^2}+\frac{4b\left(d^2\left(\sum_{\_R=\mathrm{RootOf}\left(b^2\_Z^4-4b^2c\_Z^3+\left(2ab\,d^2+6b^2c^2\right)\_Z^2+\left(-\right.\right.}\right.\right.}{}$

input
```
int(arccot(b*x+a)/(c+d*x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2*arccot(b*x+a)/d*x^(1/2)-2*arccot(b*x+a)*c/d^2*ln(c+d*x^(1/2))+4*b/d^2*(1
/4*d^2/b*sum((_R^2-2*_R*c+c^2)/(_R^3*b-3*_R^2*b*c+_R*a*d^2+3*_R*b*c^2-a*c*
d^2-b*c^3)*ln(d*x^(1/2)-_R+c),_R=RootOf(b^2*_Z^4-4*b^2*c*_Z^3+(2*a*b*d^2+6
*b^2*c^2)*_Z^2+(-4*a*b*c*d^2-4*b^2*c^3)*_Z+a^2*d^4+2*a*b*c^2*d^2+b^2*c^4+d
^4))-1/4*c*d^2/b*sum(1/(_R1^2*b-2*_R1*b*c+a*d^2+b*c^2)*(ln(c+d*x^(1/2))*ln
((-d*x^(1/2)+_R1-c)/_R1)+dilog((-d*x^(1/2)+_R1-c)/_R1)),_R1=RootOf(b^2*_Z^
4-4*b^2*c*_Z^3+(2*a*b*d^2+6*b^2*c^2)*_Z^2+(-4*a*b*c*d^2-4*b^2*c^3)*_Z+a^2*
d^4+2*a*b*c^2*d^2+b^2*c^4+d^4))
```

3.111.5 Fricas [F]

$$\int \frac{\cot^{-1}(a+bx)}{c+d\sqrt{x}}\,dx=\int \frac{\mathrm{arccot}\,(bx+a)}{d\sqrt{x}+c}\,dx$$

input
```
integrate(arccot(b*x+a)/(c+d*x^(1/2)),x, algorithm="fricas")
```

output
```
integral((d*sqrt(x)*arccot(b*x + a) - c*arccot(b*x + a))/(d^2*x - c^2), x)
```

3.111.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+d\sqrt{x}}\,dx = \text{Timed out}$$

input
```
integrate(acot(b*x+a)/(c+d*x**(1/2)),x)
```

output
```
Timed out
```

3.111.7 Maxima [F]

$$\int \frac{\cot^{-1}(a+bx)}{c+d\sqrt{x}}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{d\sqrt{x}+c}\,dx$$

input
```
integrate(arccot(b*x+a)/(c+d*x^(1/2)),x, algorithm="maxima")
```

output
```
integrate(arccot(b*x + a)/(d*sqrt(x) + c), x)
```

3.111.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\cot^{-1}(a+bx)}{c+d\sqrt{x}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arccot(b*x+a)/(c+d*x^(1/2)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0]W
arning, replacing 0 by -24, a substitution variable should perhaps be purg
ed.Warnin
```

3.111.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+d\sqrt{x}}\,dx = \int \frac{\mathrm{acot}(a+b\,x)}{c+d\,\sqrt{x}}\,dx$$

input
```
int(acot(a + b*x)/(c + d*x^(1/2)),x)
```

output
```
int(acot(a + b*x)/(c + d*x^(1/2)), x)
```

3.112 $\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx$

3.112.1 Optimal result

Integrand size = 18, antiderivative size = 830

$$
\begin{aligned}
\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx =& \frac{2i\sqrt{i+a}d\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}c^2} - \frac{2i\sqrt{i-a}d\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}c^2} \\
&+ \frac{id^2\log\left(\frac{c\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}c+\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3} \\
&- \frac{id^2\log\left(\frac{c\left(\sqrt{i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{i-a}c+\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3} \\
&+ \frac{id^2\log\left(\frac{c\left(\sqrt{-i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}c-\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3} \\
&- \frac{id^2\log\left(\frac{c\left(\sqrt{i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{i-a}c-\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3} \\
&+ \frac{(1+ia)\log(i-a-bx)}{2bc} - \frac{id\sqrt{x}\log\left(-\frac{i-a-bx}{a+bx}\right)}{c^2} \\
&+ \frac{ix\log\left(-\frac{i-a-bx}{a+bx}\right)}{2c} + \frac{id^2\log\left(d+c\sqrt{x}\right)\log\left(-\frac{i-a-bx}{a+bx}\right)}{c^3} \\
&+ \frac{(1-ia)\log(i+a+bx)}{2bc} + \frac{id\sqrt{x}\log\left(\frac{i+a+bx}{a+bx}\right)}{c^2} \\
&- \frac{ix\log\left(\frac{i+a+bx}{a+bx}\right)}{2c} - \frac{id^2\log\left(d+c\sqrt{x}\right)\log\left(\frac{i+a+bx}{a+bx}\right)}{c^3} \\
&+ \frac{id^2\,\text{PolyLog}\left(2,-\frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{-i-a}c-\sqrt{b}d}\right)}{c^3} - \frac{id^2\,\text{PolyLog}\left(2,-\frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{i-a}c-\sqrt{b}d}\right)}{c^3} \\
&+ \frac{id^2\,\text{PolyLog}\left(2,\frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{-i-a}c+\sqrt{b}d}\right)}{c^3} - \frac{id^2\,\text{PolyLog}\left(2,\frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{i-a}c+\sqrt{b}d}\right)}{c^3}
\end{aligned}
$$

output
```
1/2*(1+I*a)*ln(I-a-b*x)/b/c+I*d^2*polylog(2,-b^(1/2)*(d+c*x^(1/2))/(c*(-I-
a)^(1/2)-d*b^(1/2)))/c^3+1/2*(1-I*a)*ln(I+a+b*x)/b/c-I*d*ln((-I+a+b*x)/(b*
x+a))*x^(1/2)/c^2+I*d^2*ln(d+c*x^(1/2))*ln(c*((-I-a)^(1/2)-b^(1/2)*x^(1/2)
)/(c*(-I-a)^(1/2)+d*b^(1/2)))/c^3+2*I*d*arctan(b^(1/2)*x^(1/2)/(I+a)^(1/2)
)*(I+a)^(1/2)/c^2/b^(1/2)-2*I*d*arctanh(b^(1/2)*x^(1/2)/(I-a)^(1/2))*(I-a)
^(1/2)/c^2/b^(1/2)-I*d^2*polylog(2,b^(1/2)*(d+c*x^(1/2))/(c*(I-a)^(1/2)+d*
b^(1/2)))/c^3+I*d^2*polylog(2,b^(1/2)*(d+c*x^(1/2))/(c*(-I-a)^(1/2)+d*b^(1
/2)))/c^3-I*d^2*ln(d+c*x^(1/2))*ln(c*((I-a)^(1/2)+b^(1/2)*x^(1/2))/(c*(I-a
)^(1/2)-d*b^(1/2)))/c^3-I*d^2*ln((I+a+b*x)/(b*x+a))*ln(d+c*x^(1/2))/c^3-I*
d^2*polylog(2,-b^(1/2)*(d+c*x^(1/2))/(c*(I-a)^(1/2)-d*b^(1/2)))/c^3+I*d^2*
ln(d+c*x^(1/2))*ln(c*((-I-a)^(1/2)+b^(1/2)*x^(1/2))/(c*(-I-a)^(1/2)-d*b^(1
/2)))/c^3+1/2*I*x*ln((-I+a+b*x)/(b*x+a))/c+I*d^2*ln((-I+a+b*x)/(b*x+a))*ln
(d+c*x^(1/2))/c^3-1/2*I*x*ln((I+a+b*x)/(b*x+a))/c+I*d*ln((I+a+b*x)/(b*x+a)
)*x^(1/2)/c^2-I*d^2*ln(d+c*x^(1/2))*ln(c*((I-a)^(1/2)-b^(1/2)*x^(1/2))/(c*
(I-a)^(1/2)+d*b^(1/2)))/c^3
```

3.112.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 809, normalized size of antiderivative = 0.97

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx$$

$$= \frac{4i\sqrt{i+a}\sqrt{b}cd\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)-4i\sqrt{i-a}\sqrt{b}cd\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)+2ibd^2\log\left(\frac{c\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}c+\sqrt{b}d}\right)\log\left(d+c\sqrt{\right.}}{}$$

input
```
Integrate[ArcCot[a + b*x]/(c + d/Sqrt[x]),x]
```

output
```
((4*I)*Sqrt[I + a]*Sqrt[b]*c*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]] - (4*
I)*Sqrt[I - a]*Sqrt[b]*c*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[I - a]] + (2*I)*
b*d^2*Log[(c*(Sqrt[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[-I - a]*c + Sqrt[b]*d
)]*Log[d + c*Sqrt[x]] - (2*I)*b*d^2*Log[(c*(Sqrt[I - a] - Sqrt[b]*Sqrt[x])
)/(Sqrt[I - a]*c + Sqrt[b]*d)]*Log[d + c*Sqrt[x]] + (2*I)*b*d^2*Log[(c*(Sq
rt[-I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[-I - a]*c - Sqrt[b]*d)]*Log[d + c*Sqr
t[x]] - (2*I)*b*d^2*Log[(c*(Sqrt[I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[I - a]*c
 - Sqrt[b]*d)]*Log[d + c*Sqrt[x]] + c^2*Log[I - a - b*x] + I*a*c^2*Log[I -
 a - b*x] - (2*I)*b*c*d*Sqrt[x]*Log[(-I + a + b*x)/(a + b*x)] + I*b*c^2*x*
Log[(-I + a + b*x)/(a + b*x)] + (2*I)*b*d^2*Log[d + c*Sqrt[x]]*Log[(-I + a
 + b*x)/(a + b*x)] + c^2*Log[I + a + b*x] - I*a*c^2*Log[I + a + b*x] + (2*
I)*b*c*d*Sqrt[x]*Log[(I + a + b*x)/(a + b*x)] - I*b*c^2*x*Log[(I + a + b*x
)/(a + b*x)] - (2*I)*b*d^2*Log[d + c*Sqrt[x]]*Log[(I + a + b*x)/(a + b*x)]
 + (2*I)*b*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(-(Sqrt[-I - a]*c) + S
qrt[b]*d)] + (2*I)*b*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[-I - a
]*c + Sqrt[b]*d)] - (2*I)*b*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(-(Sq
rt[I - a]*c) + Sqrt[b]*d)] - (2*I)*b*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x
]))/(Sqrt[I - a]*c + Sqrt[b]*d)])/(2*b*c^3)
```

3.112.3 Rubi [A] (verified)

Time = 2.68 (sec) , antiderivative size = 1316, normalized size of antiderivative = 1.59, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5575, 7267, 3008, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}dx$$

$$\Big\downarrow \text{5575}$$

$$\frac{1}{2}i\int \frac{\log\left(-\frac{-a-bx+i}{a+bx}\right)}{c+\frac{d}{\sqrt{x}}}dx-\frac{1}{2}i\int \frac{\log\left(\frac{a+bx+i}{a+bx}\right)}{c+\frac{d}{\sqrt{x}}}dx$$

$$\Big\downarrow \text{7267}$$

$$i\int \frac{x\log\left(-\frac{-a-bx+i}{a+bx}\right)}{\sqrt{x}c+d}d\sqrt{x}-i\int \frac{x\log\left(\frac{a+bx+i}{a+bx}\right)}{\sqrt{x}c+d}d\sqrt{x}$$

$$\Big\downarrow \text{3008}$$

$$i\int\left(\frac{\log\left(-\frac{-a-bx+i}{a+bx}\right)d^2}{c^2\left(\sqrt{x}c+d\right)}-\frac{\log\left(-\frac{-a-bx+i}{a+bx}\right)d}{c^2}+\frac{\sqrt{x}\log\left(-\frac{-a-bx+i}{a+bx}\right)}{c}\right)d\sqrt{x}-$$
$$i\int\left(\frac{\log\left(\frac{a+bx+i}{a+bx}\right)d^2}{c^2\left(\sqrt{x}c+d\right)}-\frac{\log\left(\frac{a+bx+i}{a+bx}\right)d}{c^2}+\frac{\sqrt{x}\log\left(\frac{a+bx+i}{a+bx}\right)}{c}\right)d\sqrt{x}$$

$\downarrow$ 2009

$$i\left(-\frac{\log\left(\frac{c\left(\sqrt{i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{i-a}c+\sqrt{b}d}\right)\log\left(\sqrt{x}c+d\right)d^2}{c^3}+\frac{\log\left(\frac{c\left(\sqrt{-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-a}c+\sqrt{b}d}\right)\log\left(\sqrt{x}c+d\right)d^2}{c^3}-\frac{\log\left(\frac{c\left(\sqrt{i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{i-a}c-\sqrt{b}d}\right)\log}{c^3}\right.$$
$$i\left(-\frac{\log\left(\frac{c\left(\sqrt{-a-i}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-a-i}c+\sqrt{b}d}\right)\log\left(\sqrt{x}c+d\right)d^2}{c^3}+\frac{\log\left(\frac{c\left(\sqrt{-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-a}c+\sqrt{b}d}\right)\log\left(\sqrt{x}c+d\right)d^2}{c^3}-\frac{\log\left(\frac{c\left(\sqrt{-a-i}+\sqrt{b}\sqrt{x}\right)}{\sqrt{-a-i}c-\sqrt{b}d}\right)\text{l}}{c^3}\right.$$

input

```
Int[ArcCot[a + b*x]/(c + d/Sqrt[x]),x]
```

output

```
(-I)*((2*Sqrt[a]*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[b]*c^2) - (2*S
qrt[I + a]*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]])/(Sqrt[b]*c^2) - (d^2*L
og[(c*(Sqrt[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[-I - a]*c + Sqrt[b]*d)]*Log[
d + c*Sqrt[x]])/c^3 + (d^2*Log[(c*(Sqrt[-a] - Sqrt[b]*Sqrt[x]))/(Sqrt[-a]*
c + Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3 - (d^2*Log[(c*(Sqrt[-I - a] + Sqrt
[b]*Sqrt[x]))/(Sqrt[-I - a]*c - Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3 + (d^2
*Log[(c*(Sqrt[-a] + Sqrt[b]*Sqrt[x]))/(Sqrt[-a]*c - Sqrt[b]*d)]*Log[d + c*
Sqrt[x]])/c^3 - (a*Log[a + b*x])/(2*b*c) + ((I + a)*Log[I + a + b*x])/(2*b
*c) - (d*Sqrt[x]*Log[(I + a + b*x)/(a + b*x)])/c^2 + (x*Log[(I + a + b*x)/
(a + b*x)])/(2*c) + (d^2*Log[d + c*Sqrt[x]]*Log[(I + a + b*x)/(a + b*x)])/
c^3 - (d^2*PolyLog[2, -((Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[-I - a]*c - Sqrt[b
]*d))])/c^3 + (d^2*PolyLog[2, -((Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[-a]*c - Sq
rt[b]*d))])/c^3 - (d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[-I - a]*
c + Sqrt[b]*d)])/c^3 + (d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[-a]
*c + Sqrt[b]*d)])/c^3) + I*((2*Sqrt[a]*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]
)/(Sqrt[b]*c^2) - (2*Sqrt[I - a]*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[I - a]])
/(Sqrt[b]*c^2) - (d^2*Log[(c*(Sqrt[I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[I - a]
*c + Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3 + (d^2*Log[(c*(Sqrt[-a] - Sqrt[b]
*Sqrt[x]))/(Sqrt[-a]*c + Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3 - (d^2*Log[(c
*(Sqrt[I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[I - a]*c - Sqrt[b]*d)]*Log[d + ...
```

3.112. $\int\frac{\cot^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx$

3.112.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3008
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

rule 5575
```
Int[ArcCot[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Simp[
I/2   Int[Log[(-I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] - Simp[I/2   In
t[Log[(I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x
] && RationalQ[n]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.112.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.47 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.47

method	result
derivativedivides	$\frac{\operatorname{arccot}(bx+a)x}{c}-\frac{2\operatorname{arccot}(bx+a)d\sqrt{x}}{c^2}+\frac{2\operatorname{arccot}(bx+a)d^2\ln(d+c\sqrt{x})}{c^3}+\frac{4b\left(-\frac{c\left(\sum_{\_R=\operatorname{RootOf}\left(b^2\_Z^4-4b^2d\_Z^3+\right.}\right.}{}\right.}{}$
default	$\frac{\operatorname{arccot}(bx+a)x}{c}-\frac{2\operatorname{arccot}(bx+a)d\sqrt{x}}{c^2}+\frac{2\operatorname{arccot}(bx+a)d^2\ln(d+c\sqrt{x})}{c^3}+\frac{4b\left(-\frac{c\left(\sum_{\_R=\operatorname{RootOf}\left(b^2\_Z^4-4b^2d\_Z^3+\right.}\right.}{}\right.}{}$

input
```
int(arccot(b*x+a)/(c+d/x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
arccot(b*x+a)/c*x-2*arccot(b*x+a)/c^2*d*x^(1/2)+2*arccot(b*x+a)*d^2/c^3*ln
(d+c*x^(1/2))+4*b/c^2*(-1/8*c/b*sum((-_R^3+5*_R^2*d-7*_R*d^2+3*d^3)/(_R^3*
b-3*_R^2*b*d+_R*a*c^2+3*_R*b*d^2-a*c^2*d-b*d^3)*ln(c*x^(1/2)-_R+d),_R=Root
Of(b^2*_Z^4-4*b^2*d*_Z^3+(2*a*b*c^2+6*b^2*d^2)*_Z^2+(-4*a*b*c^2*d-4*b^2*d^
3)*_Z+a^2*c^4+2*a*b*c^2*d^2+b^2*d^4+c^4))+1/4*c*d^2/b*sum(1/(_R1^2*b-2*_R1
*b*d+a*c^2+b*d^2)*(ln(d+c*x^(1/2))*ln((-c*x^(1/2)+_R1-d)/_R1)+dilog((-c*x^
(1/2)+_R1-d)/_R1)),_R1=RootOf(b^2*_Z^4-4*b^2*d*_Z^3+(2*a*b*c^2+6*b^2*d^2)*
_Z^2+(-4*a*b*c^2*d-4*b^2*d^3)*_Z+a^2*c^4+2*a*b*c^2*d^2+b^2*d^4+c^4))
```

3.112.5 Fricas [F]

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{c+\frac{d}{\sqrt{x}}}\,dx$$

input
```
integrate(arccot(b*x+a)/(c+d/x^(1/2)),x, algorithm="fricas")
```

output
```
integral((c*x*arccot(b*x + a) - d*sqrt(x)*arccot(b*x + a))/(c^2*x - d^2),
x)
```

3.112.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = \text{Timed out}$$

input
```
integrate(acot(b*x+a)/(c+d/x**(1/2)),x)
```

output
```
Timed out
```

3.112.7 Maxima [F]

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{c+\frac{d}{\sqrt{x}}}\,dx$$

input
```
integrate(arccot(b*x+a)/(c+d/x^(1/2)),x, algorithm="maxima")
```

output
```
integrate(arccot(b*x + a)/(c + d/sqrt(x)), x)
```

3.112.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arccot(b*x+a)/(c+d/x^(1/2)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0]W
arning, replacing 0 by -24, a substitution variable should perhaps be purg
ed.Warnin
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = \int \frac{\operatorname{acot}(a+b\,x)}{c+\frac{d}{\sqrt{x}}}\,dx$$

input
```
int(acot(a + b*x)/(c + d/x^(1/2)),x)
```

output
```
int(acot(a + b*x)/(c + d/x^(1/2)), x)
```

3.113 $\int \frac{\cot^{-1}(d+ex)}{a+bx+cx^2}\,dx$

3.113.1 Optimal result

Integrand size = 19, antiderivative size = 367

$$\int \frac{\cot^{-1}(d+ex)}{a+bx+cx^2}\,dx = \frac{\cot^{-1}(d+ex)\log\left(\frac{2e\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(2c(i-d)+\left(b-\sqrt{b^2-4ac}\right)e\right)(1-i(d+ex))}\right)}{\sqrt{b^2-4ac}} - \frac{\cot^{-1}(d+ex)\log\left(\frac{2e\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(2c(i-d)+\left(b+\sqrt{b^2-4ac}\right)e\right)(1-i(d+ex))}\right)}{\sqrt{b^2-4ac}} + \frac{i\,\mathrm{PolyLog}\left(2,1+\frac{2\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e-2c(d+ex)\right)}{\left(2ic-2cd+be-\sqrt{b^2-4ac}e\right)(1-i(d+ex))}\right)}{2\sqrt{b^2-4ac}} - \frac{i\,\mathrm{PolyLog}\left(2,1+\frac{2\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e-2c(d+ex)\right)}{\left(2c(i-d)+\left(b+\sqrt{b^2-4ac}\right)e\right)(1-i(d+ex))}\right)}{2\sqrt{b^2-4ac}}$$

output
```
arccot(e*x+d)*ln(2*e*(b+2*c*x-(-4*a*c+b^2)^(1/2))/(1-I*(e*x+d))/(2*c*(I-d)
+e*(b-(-4*a*c+b^2)^(1/2))))/(-4*a*c+b^2)^(1/2)-arccot(e*x+d)*ln(2*e*(b+2*c
*x+(-4*a*c+b^2)^(1/2))/(1-I*(e*x+d))/(2*c*(I-d)+e*(b+(-4*a*c+b^2)^(1/2))))
/(-4*a*c+b^2)^(1/2)+1/2*I*polylog(2,1+2*(2*c*d-2*c*(e*x+d)-e*(b-(-4*a*c+b^
2)^(1/2)))/(1-I*(e*x+d))/(2*I*c-2*c*d+b*e-e*(-4*a*c+b^2)^(1/2)))/(-4*a*c+b
^2)^(1/2)-1/2*I*polylog(2,1+2*(2*c*d-2*c*(e*x+d)-e*(b+(-4*a*c+b^2)^(1/2)))
/(1-I*(e*x+d))/(2*c*(I-d)+e*(b+(-4*a*c+b^2)^(1/2))))/(-4*a*c+b^2)^(1/2)
```

3.113.2 Mathematica [F]

$$\int \frac{\cot^{-1}(d+ex)}{a+bx+cx^2}\,dx = \int \frac{\cot^{-1}(d+ex)}{a+bx+cx^2}\,dx$$

input
```
Integrate[ArcCot[d + e*x]/(a + b*x + c*x^2),x]
```

output
```
Integrate[ArcCot[d + e*x]/(a + b*x + c*x^2), x]
```

3.113.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(d+ex)}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \; 7279$$

$$\int \left(\frac{2c\cot^{-1}(d+ex)}{\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}+b+2cx\right)}-\frac{2c\cot^{-1}(d+ex)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b+2cx\right)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{i\,\text{PolyLog}\left(2,\frac{2\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e-2c(d+ex)\right)}{\left(-2dc+2ic+be-\sqrt{b^2-4ac}e\right)(1-i(d+ex))}+1\right)}{2\sqrt{b^2-4ac}}-\frac{i\,\text{PolyLog}\left(2,\frac{2\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e-2c(d+ex)\right)}{\left(2c(i-d)+\left(b+\sqrt{b^2-4ac}\right)e\right)(1-i(d+ex))}+1\right)}{2\sqrt{b^2-4ac}}+\frac{\cot^{-1}(d+ex)\log\left(-\frac{2\left(-e\left(b-\sqrt{b^2-4ac}\right)-2c(d+ex)+2cd\right)}{(1-i(d+ex))\left(-e\sqrt{b^2-4ac}+be-2cd+2ic\right)}\right)}{\sqrt{b^2-4ac}}-\frac{\cot^{-1}(d+ex)\log\left(-\frac{2\left(-e\left(\sqrt{b^2-4ac}+b\right)-2c(d+ex)+2cd\right)}{(1-i(d+ex))\left(e\left(\sqrt{b^2-4ac}+b\right)+2c(-d+i)\right)}\right)}{\sqrt{b^2-4ac}}$$

input
```
Int[ArcCot[d + e*x]/(a + b*x + c*x^2),x]
```

output
```
(ArcCot[d + e*x]*Log[(-2*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e - 2*c*(d + e*x
)))/(((2*I)*c - 2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e)*(1 - I*(d + e*x)))])/Sq
rt[b^2 - 4*a*c] - (ArcCot[d + e*x]*Log[(-2*(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e - 2*c*(d + e*x)))/((2*c*(I - d) + (b + Sqrt[b^2 - 4*a*c])*e)*(1 - I*(d
 + e*x)))])/Sqrt[b^2 - 4*a*c] + ((I/2)*PolyLog[2, 1 + (2*(2*c*d - (b - Sqr
t[b^2 - 4*a*c])*e - 2*c*(d + e*x)))/(((2*I)*c - 2*c*d + b*e - Sqrt[b^2 - 4
*a*c]*e)*(1 - I*(d + e*x)))])/Sqrt[b^2 - 4*a*c] - ((I/2)*PolyLog[2, 1 + (2
*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e - 2*c*(d + e*x)))/((2*c*(I - d) + (b +
 Sqrt[b^2 - 4*a*c])*e)*(1 - I*(d + e*x)))])/Sqrt[b^2 - 4*a*c]
```

3.113.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.113.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 958 vs. $2(329) = 658$.

Time = 2.49 (sec) , antiderivative size = 959, normalized size of antiderivative = 2.61

method	result
risch	$\frac{ie\pi \arctan\left(\frac{ibe-2icd-2(-iex-id+1)c+2c}{\sqrt{-4ac\,e^2+b^2e^2}}\right)}{\sqrt{-4ac\,e^2+b^2e^2}} - \frac{e\ln(-iex-id+1)\ln\left(\frac{ibe-2icd-2(-iex-id+1)c+\sqrt{4ac\,e^2-b^2e^2}+2c}{ibe-2icd+2c+\sqrt{4ac\,e^2-b^2e^2}}\right)}{2\sqrt{4ac\,e^2-b^2e^2}} + -\frac{e\ln}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(arccot(e*x+d)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
I*e*Pi/(-4*a*c*e^2+b^2*e^2)^(1/2)*arctan((I*b*e-2*I*c*d-2*(1-I*d-I*e*x)*c+
2*c)/(-4*a*c*e^2+b^2*e^2)^(1/2))-1/2*e*ln(1-I*d-I*e*x)/(4*a*c*e^2-b^2*e^2)
^(1/2)*ln((I*b*e-2*I*c*d-2*(1-I*d-I*e*x)*c+(4*a*c*e^2-b^2*e^2)^(1/2)+2*c)/
(I*b*e-2*I*c*d+2*c+(4*a*c*e^2-b^2*e^2)^(1/2)))+1/2*e*ln(1-I*d-I*e*x)/(4*a*
c*e^2-b^2*e^2)^(1/2)*ln((I*b*e-2*I*c*d-2*(1-I*d-I*e*x)*c-(4*a*c*e^2-b^2*e^
2)^(1/2)+2*c)/(I*b*e-2*I*c*d+2*c-(4*a*c*e^2-b^2*e^2)^(1/2)))-1/2*e/(4*a*c*
e^2-b^2*e^2)^(1/2)*dilog((I*b*e-2*I*c*d-2*(1-I*d-I*e*x)*c+(4*a*c*e^2-b^2*e
^2)^(1/2)+2*c)/(I*b*e-2*I*c*d+2*c+(4*a*c*e^2-b^2*e^2)^(1/2)))+1/2*e/(4*a*c
*e^2-b^2*e^2)^(1/2)*dilog((I*b*e-2*I*c*d-2*(1-I*d-I*e*x)*c-(4*a*c*e^2-b^2*
e^2)^(1/2)+2*c)/(I*b*e-2*I*c*d+2*c-(4*a*c*e^2-b^2*e^2)^(1/2)))-1/2*e*ln(1+
I*d+I*e*x)/(4*a*c*e^2-b^2*e^2)^(1/2)*ln((I*b*e-2*I*c*d+2*(1+I*d+I*e*x)*c-(
4*a*c*e^2-b^2*e^2)^(1/2)-2*c)/(I*b*e-2*I*c*d-(4*a*c*e^2-b^2*e^2)^(1/2)-2*c
))+1/2*e*ln(1+I*d+I*e*x)/(4*a*c*e^2-b^2*e^2)^(1/2)*ln((I*b*e-2*I*c*d+2*(1+
I*d+I*e*x)*c+(4*a*c*e^2-b^2*e^2)^(1/2)-2*c)/(I*b*e-2*I*c*d+(4*a*c*e^2-b^2*
e^2)^(1/2)-2*c))-1/2*e/(4*a*c*e^2-b^2*e^2)^(1/2)*dilog((I*b*e-2*I*c*d+2*(1
+I*d+I*e*x)*c-(4*a*c*e^2-b^2*e^2)^(1/2)-2*c)/(I*b*e-2*I*c*d-(4*a*c*e^2-b^2
*e^2)^(1/2)-2*c))+1/2*e/(4*a*c*e^2-b^2*e^2)^(1/2)*dilog((I*b*e-2*I*c*d+2*(
1+I*d+I*e*x)*c+(4*a*c*e^2-b^2*e^2)^(1/2)-2*c)/(I*b*e-2*I*c*d+(4*a*c*e^2-b^
2*e^2)^(1/2)-2*c))
```

3.113.5 Fricas [F]

$$\int \frac{\cot^{-1}(d+ex)}{a+bx+cx^2}\,dx = \int \frac{\operatorname{arccot}(ex+d)}{cx^2+bx+a}\,dx$$

input
```
integrate(arccot(e*x+d)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
integral(arccot(e*x + d)/(c*x^2 + b*x + a), x)
```

3.113.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(d+ex)}{a+bx+cx^2}\,dx = \text{Timed out}$$

input
```
integrate(acot(e*x+d)/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

3.113.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^{-1}(d+ex)}{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(arccot(e*x+d)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.113.8 Giac [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(d+ex)}{a+bx+cx^2}\,dx = \text{Timed out}$$

input
```
integrate(arccot(e*x+d)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
Timed out
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(d+ex)}{a+bx+cx^2}\,dx = \int \frac{\mathrm{acot}(d+e\,x)}{c\,x^2+b\,x+a}\,dx$$

input
```
int(acot(d + e*x)/(a + b*x + c*x^2),x)
```

output
```
int(acot(d + e*x)/(a + b*x + c*x^2), x)
```

3.114 $\int \frac{\cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx$

3.114.1 Optimal result

Integrand size = 28, antiderivative size = 132

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx=-\frac{2i\cot^{-1}(a+bx)\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}$$

output
```
-2*I*arccot(b*x+a)*arctan((1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b-I*pol
ylog(2,-I*(1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b+I*polylog(2,I*(1+I*(b
*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b
```

3.114.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.96

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx=-\frac{\sqrt{1+a^2+2abx+b^2x^2}\left(\cot^{-1}(a+bx)\left(\log\left(1-e^{i\cot^{-1}(a+bx)}\right)-\log\left(1+e^{i\cot^{-1}(a+bx)}\right)\right)+i\,\text{PolyLog}\left(\right.\right.}{b(a+bx)\sqrt{1+\frac{1}{(a+bx)^2}}}$$

input
```
Integrate[ArcCot[a + b*x]/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
-((Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(ArcCot[a + b*x]*(Log[1 - E^(I*ArcCot
[a + b*x])] - Log[1 + E^(I*ArcCot[a + b*x])]) + I*PolyLog[2, -E^(I*ArcCot[
a + b*x])] - I*PolyLog[2, E^(I*ArcCot[a + b*x])]))/(b*(a + b*x)*Sqrt[1 + (
a + b*x)^(-2)]))
```

3.114.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5579, 5422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{a^2+2abx+b^2x^2+1}}dx$$

$$\Big\downarrow \text{5579}$$

$$\frac{\int \frac{\cot^{-1}(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)}{b}$$

$$\Big\downarrow \text{5422}$$

$$\frac{-2i\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)\cot^{-1}(a+bx)-i\,\text{PolyLog}\left(2,-\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)+i\,\text{PolyLog}\left(2,\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{b}$$

input
```
Int[ArcCot[a + b*x]/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
((-2*I)*ArcCot[a + b*x]*ArcTan[Sqrt[1 + I*(a + b*x)]/Sqrt[1 - I*(a + b*x)]
] - I*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]] + I*P
olyLog[2, (I*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]])/b
```

3.114.3.1 Defintions of rubi rules used

rule 5422
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcCot[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x])]/(c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 -
I*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 GtQ[d, 0]
```

rule 5579
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((A_.) + (B_.)*(x_) + (
C_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/d   Subst[Int[(C/d^2 + (C/d^2)*x^2)
^q*(a + b*ArcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
p, q}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

3.114.4 Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.89

method	result
default	$-\frac{\operatorname{arccot}(bx+a)\ln\left(1-\frac{bx+a+i}{\sqrt{1+(bx+a)^2}}\right)-\operatorname{arccot}(bx+a)\ln\left(\frac{bx+a+i}{\sqrt{1+(bx+a)^2}}+1\right)+i\operatorname{dilog}\left(\frac{bx+a+i}{\sqrt{1+(bx+a)^2}}+1\right)-i\operatorname{dilog}\left(1-\frac{bx+a+i}{\sqrt{1+(bx+a)^2}}\right)}{b}$

input
```
int(arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/b*(arccot(b*x+a)*ln(1-(I+a+b*x)/(1+(b*x+a)^2)^(1/2))-arccot(b*x+a)*ln((
I+a+b*x)/(1+(b*x+a)^2)^(1/2)+1)+I*dilog((I+a+b*x)/(1+(b*x+a)^2)^(1/2)+1)-I
*dilog(1-(I+a+b*x)/(1+(b*x+a)^2)^(1/2)))
```

3.114.5 Fricas [F]

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{\sqrt{b^2x^2+2\,abx+a^2+1}}\,dx$$

input
```
integrate(arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fricas
")
```

output
```
integral(arccot(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

3.114.6 Sympy [F]

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\operatorname{acot}(a+bx)}{\sqrt{a^2+2abx+b^2x^2+1}}\,dx$$

input
```
integrate(acot(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

output
```
Integral(acot(a + b*x)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1), x)
```

3.114.7 Maxima [F]

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{\sqrt{b^2x^2+2\,abx+a^2+1}}\,dx$$

input
```
integrate(arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(arccot(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

3.114.8 Giac [F]

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{\sqrt{b^2x^2+2\,abx+a^2+1}}\,dx$$

input
```
integrate(arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arccot(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

3.114.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\operatorname{acot}(a+b\,x)}{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}\,dx$$

input
```
int(acot(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
```

output
```
int(acot(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2), x)
```

3.115 $\int \frac{\cot^{-1}(a+bx)}{\sqrt{(1+a^2)c+2abcx+b^2cx^2}}\,dx$

3.115.1 Optimal result

Integrand size = 33, antiderivative size = 216

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = -\frac{2i\sqrt{1+(a+bx)^2}\cot^{-1}(a+bx)\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c+c(a+bx)^2}} - \frac{i\sqrt{1+(a+bx)^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c+c(a+bx)^2}} + \frac{i\sqrt{1+(a+bx)^2}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c+c(a+bx)^2}}$$

output
```
-2*I*arccot(b*x+a)*arctan((1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))*(1+(b*x
+a)^2)^(1/2)/b/(c+c*(b*x+a)^2)^(1/2)-I*polylog(2,-I*(1+I*(b*x+a))^(1/2)/(1
-I*(b*x+a))^(1/2))*(1+(b*x+a)^2)^(1/2)/b/(c+c*(b*x+a)^2)^(1/2)+I*polylog(2
,I*(1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))*(1+(b*x+a)^2)^(1/2)/b/(c+c*(b*
x+a)^2)^(1/2)
```

3.115.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.64

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{(1+a^2)c+2abcx+b^2cx^2}}\,dx = -\frac{(1+(a+bx)^2)\left(\cot^{-1}(a+bx)\left(\log\left(1-e^{i\cot^{-1}(a+bx)}\right)-\log\left(1+e^{i\cot^{-1}(a+bx)}\right)\right)+i\,\text{PolyLog}\left(2,-e^{i\cot^{-1}(a+bx)}\right)\right.}{b(a+bx)\sqrt{c(1+a^2+2abx+b^2x^2)}\sqrt{1+\frac{1}{(a+bx)^2}}}$$

input
```
Integrate[ArcCot[a + b*x]/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2],x]
```

output
```
-(((1 + (a + b*x)^2)*(ArcCot[a + b*x]*(Log[1 - E^(I*ArcCot[a + b*x])] - Lo
g[1 + E^(I*ArcCot[a + b*x])]) + I*PolyLog[2, -E^(I*ArcCot[a + b*x])] - I*P
olyLog[2, E^(I*ArcCot[a + b*x])]))/(b*(a + b*x)*Sqrt[c*(1 + a^2 + 2*a*b*x
+ b^2*x^2)]*Sqrt[1 + (a + b*x)^(-2)]))
```

3.115.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5579, 5426, 5422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{(a^2+1)c+2abcx+b^2cx^2}}\,dx$$

$$\Big\downarrow \ 5579$$

$$\frac{\int \frac{\cot^{-1}(a+bx)}{\sqrt{c(a+bx)^2+c}}d(a+bx)}{b}$$

$$\Big\downarrow \ 5426$$

$$\frac{\sqrt{(a+bx)^2+1}\int \frac{\cot^{-1}(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)}{b\sqrt{c(a+bx)^2+c}}$$

$$\Big\downarrow \ 5422$$

$$\frac{\sqrt{(a+bx)^2+1}\left(-2i\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)\cot^{-1}(a+bx)-i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)+i\operatorname{PolyLog}\left(2,\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+b}}\right.\right.}{b\sqrt{c(a+bx)^2+c}}$$

input
```
Int[ArcCot[a + b*x]/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2],x]
```

output
```
(Sqrt[1 + (a + b*x)^2]*((-2*I)*ArcCot[a + b*x]*ArcTan[Sqrt[1 + I*(a + b*x)
]/Sqrt[1 - I*(a + b*x)]] - I*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)])/Sqrt[
1 - I*(a + b*x)]] + I*PolyLog[2, (I*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a +
 b*x)]]))/(b*Sqrt[c + c*(a + b*x)^2])
```

3.115.3.1 Defintions of rubi rules used

rule 5422
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcCot[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x])]/(c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 -
I*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 GtQ[d, 0]
```

rule 5426
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcCot[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5579
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((A_.) + (B_.)*(x_) + (
C_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/d   Subst[Int[(C/d^2 + (C/d^2)*x^2)
^q*(a + b*ArcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
p, q}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

3.115.4 Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.72

method	result
default	$\frac{i\left(i\,\mathrm{arccot}(bx+a)\ln\left(1-\frac{bx+a+i}{\sqrt{1+(bx+a)^2}}\right)-i\,\mathrm{arccot}(bx+a)\ln\left(\frac{bx+a+i}{\sqrt{1+(bx+a)^2}}+1\right)+\mathrm{polylog}\left(2,\frac{bx+a+i}{\sqrt{1+(bx+a)^2}}\right)-\mathrm{polylog}\left(2,-\frac{bx+a+i}{\sqrt{1+(bx+a)^2}}\right.\right.}{\sqrt{b^2x^2+2abx+a^2+1}\,bc}$

input
```
int(arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
I*(I*arccot(b*x+a)*ln(1-(I+a+b*x)/(1+(b*x+a)^2)^(1/2))-I*arccot(b*x+a)*ln(
(I+a+b*x)/(1+(b*x+a)^2)^(1/2)+1)+polylog(2,(I+a+b*x)/(1+(b*x+a)^2)^(1/2))-
polylog(2,-(I+a+b*x)/(1+(b*x+a)^2)^(1/2)))*(c*(-I+a+b*x)*(I+a+b*x))^(1/2)/
(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b/c
```

3.115.5 Fricas [F]

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{\mathrm{arccot}\,(bx+a)}{\sqrt{b^2cx^2+2\,abcx+(a^2+1)c}}\,dx$$

input
```
integrate(arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x, algorithm
="fricas")
```

output
```
integral(arccot(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c), x)
```

3.115.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\text{Timed out}$$

input
```
integrate(acot(b*x+a)/((a**2+1)*c+2*a*b*c*x+b**2*c*x**2)**(1/2),x)
```

output
```
Timed out
```

3.115.7 Maxima [F]

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{\operatorname{arccot}\,(bx+a)}{\sqrt{b^2cx^2+2\,abcx+(a^2+1)c}}\,dx$$

input
```
integrate(arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x, algorithm
="maxima")
```

output
```
integrate(arccot(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c), x)
```

3.115.8 Giac [F]

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{\operatorname{arccot}\,(bx+a)}{\sqrt{b^2cx^2+2\,abcx+(a^2+1)c}}\,dx$$

input
```
integrate(arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x, algorithm
="giac")
```

output
```
integrate(arccot(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c), x)
```

3.115.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{\operatorname{acot}(a+b\,x)}{\sqrt{c\,b^2\,x^2+2\,a\,c\,b\,x+c\,\left(a^2+1\right)}}\,dx$$

input
```
int(acot(a + b*x)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/2),x)
```

output
```
int(acot(a + b*x)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/2), x)
```

3.116 $\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx$

3.116.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \text{Int}\left(\frac{\cot^{-1}(a+bx)}{\sqrt[3]{1+(a+bx)^2}},x\right)$$

output
```
Unintegrable(arccot(b*x+a)/(1+(b*x+a)^2)^(1/3),x)
```

3.116.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 177 vs. 2(23) = 46.

Time = 0.30 (sec) , antiderivative size = 177, normalized size of antiderivative = 6.32

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \frac{6\,\text{Gamma}\left(\frac{11}{6}\right)\text{Gamma}\left(\frac{7}{3}\right)\left(5(1+a^2+2abx+b^2x^2)\left(-3+2(a+bx)\cot^{-1}(a+bx)\right)+4(a+bx)\cot^{-1}(a\right.}{20b\left(1+a^2+2abx+b^2x^2\right.}$$

input
```
Integrate[ArcCot[a + b*x]/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3),x]
```

output
```
(6*Gamma[11/6]*Gamma[7/3]*(5*(1 + a^2 + 2*a*b*x + b^2*x^2)*(-3 + 2*(a + b*
x)*ArcCot[a + b*x]) + 4*(a + b*x)*ArcCot[a + b*x]*Hypergeometric2F1[1, 4/3
, 11/6, (1 + a^2 + 2*a*b*x + b^2*x^2)^(-1)]) - 5*2^(1/3)*Sqrt[Pi]*Gamma[5/
3]*HypergeometricPFQ[{1, 4/3, 4/3}, {11/6, 7/3}, (1 + a^2 + 2*a*b*x + b^2*
x^2)^(-1)])/(20*b*(1 + a^2 + 2*a*b*x + b^2*x^2)^(4/3)*Gamma[11/6]*Gamma[7/
3])
```

3.116.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5579, 5561}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{a^2+2abx+b^2x^2+1}}dx$$

$$\Big\downarrow \text{5579}$$

$$\frac{\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{(a+bx)^2+1}}d(a+bx)}{b}$$

$$\Big\downarrow \text{5561}$$

$$\frac{\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{(a+bx)^2+1}}d(a+bx)}{b}$$

input
```
Int[ArcCot[a + b*x]/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3),x]
```

output
```
$Aborted
```

3.116.3.1 Defintions of rubi rules used

rule 5561
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCot[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

rule 5579
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((A_.) + (B_.)*(x_) + (
C_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/d   Subst[Int[(C/d^2 + (C/d^2)*x^2)
^q*(a + b*ArcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
p, q}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

3.116.4 Maple [N/A] (verified)

Not integrable

Time = 0.75 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\operatorname{arccot}(bx+a)}{(b^2x^2+2abx+a^2+1)^{\frac{1}{3}}}dx$$

input
```
int(arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)
```

output
```
int(arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)
```

3.116.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\operatorname{arccot}(bx+a)}{(b^2x^2+2\,abx+a^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorithm="fricas
")
```

output
```
integral(arccot(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3), x)
```

3.116.6 Sympy [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx=\int \frac{\operatorname{acot}(a+bx)}{\sqrt[3]{a^2+2abx+b^2x^2+1}}\,dx$$

input
```
integrate(acot(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/3),x)
```

output
```
Integral(acot(a + b*x)/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(1/3), x)
```

3.116.7 Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx=\int \frac{\operatorname{arccot}(bx+a)}{(b^2x^2+2\,abx+a^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorithm="maxima
")
```

output
```
integrate(arccot(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3), x)
```

3.116.8 Giac [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx=\int \frac{\operatorname{arccot}(bx+a)}{(b^2x^2+2\,abx+a^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorithm="giac")
```

output
```
integrate(arccot(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3), x)
```

3.116.9 Mupad [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\mathrm{acot}(a+b\,x)}{\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)^{1/3}}\,dx$$

input
```
int(acot(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/3),x)
```

output
```
int(acot(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/3), x)
```

3.117 $\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx$

3.117.1 Optimal result

Integrand size = 33, antiderivative size = 33

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\operatorname{Int}\left(\frac{\cot^{-1}(a+bx)}{\sqrt[3]{c+c(a+bx)^2}},x\right)$$

output
```
Unintegrable(arccot(b*x+a)/(c+c*(b*x+a)^2)^(1/3),x)
```

3.117.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 180 vs. 2(25) = 50.

Time = 0.14 (sec) , antiderivative size = 180, normalized size of antiderivative = 5.45

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx$$
$$=\frac{c\Big(6\,\text{Gamma}\left(\frac{11}{6}\right)\text{Gamma}\left(\frac{7}{3}\right)\left(5(1+a^2+2abx+b^2x^2)\left(-3+2(a+bx)\cot^{-1}(a+bx)\right)+4(a+bx)\cot^{-1}\right.\cdots}{20b\left(c\left(1+a^2+2abx+b^2\cdots\right.\right.}$$

input
```
Integrate[ArcCot[a + b*x]/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2)^(1/3),x]
```

output
```
(c*(6*Gamma[11/6]*Gamma[7/3]*(5*(1 + a^2 + 2*a*b*x + b^2*x^2)*(-3 + 2*(a +
 b*x)*ArcCot[a + b*x]) + 4*(a + b*x)*ArcCot[a + b*x]*Hypergeometric2F1[1,
4/3, 11/6, (1 + a^2 + 2*a*b*x + b^2*x^2)^(-1)]) - 5*2^(1/3)*Sqrt[Pi]*Gamma
[5/3]*HypergeometricPFQ[{1, 4/3, 4/3}, {11/6, 7/3}, (1 + a^2 + 2*a*b*x + b
^2*x^2)^(-1)]))/(20*b*(c*(1 + a^2 + 2*a*b*x + b^2*x^2))^(4/3)*Gamma[11/6]*
Gamma[7/3])
```

3.117.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5579, 5561}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{(a^2+1)c+2abcx+b^2cx^2}}dx$$

$$\Big\downarrow \text{ 5579}$$

$$\frac{\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{c(a+bx)^2+c}}d(a+bx)}{b}$$

$$\Big\downarrow \text{ 5561}$$

$$\frac{\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{c(a+bx)^2+c}}d(a+bx)}{b}$$

input
```
Int[ArcCot[a + b*x]/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2)^(1/3),x]
```

output
```
$Aborted
```

3.117.3.1 Defintions of rubi rules used

rule 5561
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCot[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

rule 5579
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((A_.) + (B_.)*(x_) + (
C_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/d   Subst[Int[(C/d^2 + (C/d^2)*x^2)
^q*(a + b*ArcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
p, q}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

3.117.4 Maple [N/A] (verified)

Not integrable

Time = 0.78 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{arccot}(bx+a)}{\left((a^2+1)\,c+2abcx+b^2c\,x^2\right)^{\frac{1}{3}}}dx$$

input
```
int(arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)
```

output
```
int(arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)
```

3.117.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{\operatorname{arccot}(bx+a)}{\left(b^2cx^2+2\,abcx+(a^2+1)c\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x, algorithm
="fricas")
```

output
```
integral(arccot(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c)^(1/3), x)
```

3.117.6 Sympy [N/A]

Not integrable

Time = 5.94 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{\operatorname{acot}\left(a+bx\right)}{\sqrt[3]{c\left(a^2+2abx+b^2x^2+1\right)}}\,dx$$

input
```
integrate(acot(b*x+a)/((a**2+1)*c+2*a*b*c*x+b**2*c*x**2)**(1/3),x)
```

output
```
Integral(acot(a + b*x)/(c*(a**2 + 2*a*b*x + b**2*x**2 + 1))**(1/3), x)
```

3.117.7 Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{\operatorname{arccot}\left(bx+a\right)}{\left(b^2cx^2+2\,abcx+(a^2+1)c\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x, algorithm
="maxima")
```

output
```
integrate(arccot(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c)^(1/3), x)
```

3.117.8 Giac [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{\operatorname{arccot}\left(bx+a\right)}{\left(b^2cx^2+2\,abcx+(a^2+1)c\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x, algorithm
="giac")
```

output
```
integrate(arccot(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c)^(1/3), x)
```

3.117.9 Mupad [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{\mathrm{acot}(a+b\,x)}{{\left(c\,b^2\,x^2+2\,a\,c\,b\,x+c\,\left(a^2+1\right)\right)}^{1/3}}\,dx$$

input
```
int(acot(a + b*x)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/3),x)
```

output
```
int(acot(a + b*x)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/3), x)
```

3.118 $\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}} \, dx$

3.118.1 Optimal result

Integrand size = 35, antiderivative size = 187

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}} \, dx = \frac{\sqrt{1+(a+bx)^2}}{2b} + \frac{(a+bx)\sqrt{1+(a+bx)^2}\cot^{-1}(a+bx)}{2b} + \frac{i\cot^{-1}(a+bx)\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b} + \frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b} - \frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b}$$

output
```
I*arccot(b*x+a)*arctan((1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b+1/2*I*po
lylog(2,-I*(1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b-1/2*I*polylog(2,I*(1
+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b+1/2*(1+(b*x+a)^2)^(1/2)/b+1/2*(b*
x+a)*arccot(b*x+a)*(1+(b*x+a)^2)^(1/2)/b
```

3.118.2 Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}} \, dx = -\frac{\sqrt{(a+bx)^2\left(1+\frac{1}{(a+bx)^2}\right)}\left(-2\cot\left(\frac{1}{2}\cot^{-1}(a+bx)\right)-\cot^{-1}(a+bx)\csc^2\left(\frac{1}{2}\cot^{-1}(a+bx)\right)-4\cot^{-1}(a$$

input
```
Integrate[((a + b*x)^2*ArcCot[a + b*x])/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],
x]
```

output
```
-1/8*(Sqrt[(a + b*x)^2*(1 + (a + b*x)^(-2))]*(-2*Cot[ArcCot[a + b*x]/2] -
ArcCot[a + b*x]*Csc[ArcCot[a + b*x]/2]^2 - 4*ArcCot[a + b*x]*Log[1 - E^(I*
ArcCot[a + b*x])] + 4*ArcCot[a + b*x]*Log[1 + E^(I*ArcCot[a + b*x])] - (4*
I)*PolyLog[2, -E^(I*ArcCot[a + b*x])] + (4*I)*PolyLog[2, E^(I*ArcCot[a + b
*x])] + ArcCot[a + b*x]*Sec[ArcCot[a + b*x]/2]^2 - 2*Tan[ArcCot[a + b*x]/2
]))/(b*(a + b*x)*Sqrt[1 + (a + b*x)^(-2)])
```

3.118.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {5581, 5488, 241, 5422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{a^2+2abx+b^2x^2+1}}dx$$

$$\Big\downarrow \text{ 5581}$$

$$\frac{\int \frac{(a+bx)^2\cot^{-1}(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)}{b}$$

$$\Big\downarrow \text{ 5488}$$

$$\frac{\frac{1}{2}\int \frac{a+bx}{\sqrt{(a+bx)^2+1}}d(a+bx)-\frac{1}{2}\int \frac{\cot^{-1}(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)+\frac{1}{2}(a+bx)\sqrt{(a+bx)^2+1}\cot^{-1}(a+bx)}{b}$$

$$\Big\downarrow \text{ 241}$$

$$\frac{-\frac{1}{2}\int \frac{\cot^{-1}(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)+\frac{1}{2}\sqrt{(a+bx)^2+1}+\frac{1}{2}(a+bx)\sqrt{(a+bx)^2+1}\cot^{-1}(a+bx)}{b}$$

$$\Big\downarrow \text{ 5422}$$

$$\frac{\frac{1}{2}\left(2i\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)\cot^{-1}(a+bx)+i\,\text{PolyLog}\left(2,-\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)-i\,\text{PolyLog}\left(2,\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)\right)+\frac{1}{2}\sqrt{(a+}{b}$$

input
```
Int[((a + b*x)^2*ArcCot[a + b*x])/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
(Sqrt[1 + (a + b*x)^2]/2 + ((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcCot[a + b*x
])/2 + ((2*I)*ArcCot[a + b*x]*ArcTan[Sqrt[1 + I*(a + b*x)]/Sqrt[1 - I*(a +
 b*x)]] + I*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]]
 - I*PolyLog[2, (I*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]])/2)/b
```

3.118.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5422
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcCot[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x])]/(c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 -
I*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 GtQ[d, 0]
```

rule 5488
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcCot[c*x])^p/(c^2*d*m)), x] + (Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((a
 + b*ArcCot[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^2
*m))   Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/Sqrt[d + e*x^2]), x], x]) /
; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5581
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/d   Subs
t[Int[((d*e - c*f)/d + f*(x/d))^m*(C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcCot[x])
^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &
& EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

3.118.4 Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.86

method	result
default	$\frac{(\operatorname{arccot}(bx+a)bx+a\ \operatorname{arccot}(bx+a)+1)\sqrt{b^2x^2+2abx+a^2+1}}{2b} - \frac{i\left(i\operatorname{arccot}(bx+a)\ln\left(1-\frac{bx+a+i}{\sqrt{1+(bx+a)^2}}\right)-i\operatorname{arccot}(bx+a)\ln\left(\frac{bx+a+}{\sqrt{1+(bx+}}\right.\right.}{2b}$

input
```
int((b*x+a)^2*arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURN
VERBOSE)
```

output
```
1/2*(arccot(b*x+a)*b*x+a*arccot(b*x+a)+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b-
1/2*I*(I*arccot(b*x+a)*ln(1-(I+a+b*x)/(1+(b*x+a)^2)^(1/2))-I*arccot(b*x+a)
*ln((I+a+b*x)/(1+(b*x+a)^2)^(1/2)+1)+polylog(2,(I+a+b*x)/(1+(b*x+a)^2)^(1/
2))-polylog(2,-(I+a+b*x)/(1+(b*x+a)^2)^(1/2)))/b
```

3.118.5 Fricas [F]

$$\int \frac{(a+bx)^2\cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{(bx+a)^2\operatorname{arccot}(bx+a)}{\sqrt{b^2x^2+2\,abx+a^2+1}}\,dx$$

input
```
integrate((b*x+a)^2*arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorit
hm="fricas")
```

output
```
integral((b^2*x^2 + 2*a*b*x + a^2)*arccot(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1), x)
```

3.118.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2\cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**2*acot(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

output
```
Timed out
```

3.118.7 Maxima [F]

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{\sqrt{b^2x^2+2\,abx+a^2+1}}\,dx$$

input
```
integrate((b*x+a)^2*arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorit
hm="maxima")
```

output
```
integrate((b*x + a)^2*arccot(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x
)
```

3.118.8 Giac [F]

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{\sqrt{b^2x^2+2\,abx+a^2+1}}\,dx$$

input
```
integrate((b*x+a)^2*arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorit
hm="giac")
```

output
```
integrate((b*x + a)^2*arccot(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x
)
```

3.118.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\operatorname{acot}(a+b\,x)\,\left(a+b\,x\right)^2}{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}\,dx$$

input
```
int((acot(a + b*x)*(a + b*x)^2)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
```

output
```
int((acot(a + b*x)*(a + b*x)^2)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2), x)
```

3.119 $\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{(1+a^2)c+2abcx+b^2cx^2}}\,dx$

3.119.1 Optimal result

Integrand size = 40, antiderivative size = 281

$$\begin{aligned}\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx &= \frac{\sqrt{c+c(a+bx)^2}}{2bc} \\ &+ \frac{(a+bx)\sqrt{c+c(a+bx)^2}\cot^{-1}(a+bx)}{2bc} \\ &+ \frac{i\sqrt{1+(a+bx)^2}\cot^{-1}(a+bx)\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c+c(a+bx)^2}} \\ &+ \frac{i\sqrt{1+(a+bx)^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b\sqrt{c+c(a+bx)^2}} \\ &- \frac{i\sqrt{1+(a+bx)^2}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b\sqrt{c+c(a+bx)^2}}\end{aligned}$$

output
```
I*arccot(b*x+a)*arctan((1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))*(1+(b*x+a)
^2)^(1/2)/b/(c+c*(b*x+a)^2)^(1/2)+1/2*I*polylog(2,-I*(1+I*(b*x+a))^(1/2)/(
1-I*(b*x+a))^(1/2))*(1+(b*x+a)^2)^(1/2)/b/(c+c*(b*x+a)^2)^(1/2)-1/2*I*poly
log(2,I*(1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))*(1+(b*x+a)^2)^(1/2)/b/(c+
c*(b*x+a)^2)^(1/2)+1/2*(c+c*(b*x+a)^2)^(1/2)/b/c+1/2*(b*x+a)*arccot(b*x+a)
*(c+c*(b*x+a)^2)^(1/2)/b/c
```

3.119.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{(1+a^2)c+2abcx+b^2cx^2}}\,dx = -\frac{\sqrt{c(1+a^2+2abx+b^2x^2)}\left(-2\cot\left(\frac{1}{2}\cot^{-1}(a+bx)\right)-\cot^{-1}(a+bx)\csc^2\left(\frac{1}{2}\cot^{-1}(a+bx)\right)-4\cot^{-1}(\right.}{}$$

input
```
Integrate[((a + b*x)^2*ArcCot[a + b*x])/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2
*c*x^2],x]
```

output
```
-1/8*(Sqrt[c*(1 + a^2 + 2*a*b*x + b^2*x^2)]*(-2*Cot[ArcCot[a + b*x]/2] - A
rcCot[a + b*x]*Csc[ArcCot[a + b*x]/2]^2 - 4*ArcCot[a + b*x]*Log[1 - E^(I*A
rcCot[a + b*x])] + 4*ArcCot[a + b*x]*Log[1 + E^(I*ArcCot[a + b*x])] - (4*I
)*PolyLog[2, -E^(I*ArcCot[a + b*x])] + (4*I)*PolyLog[2, E^(I*ArcCot[a + b*
x])] + ArcCot[a + b*x]*Sec[ArcCot[a + b*x]/2]^2 - 2*Tan[ArcCot[a + b*x]/2]
))/(b*c*(a + b*x)*Sqrt[1 + (a + b*x)^(-2)])
```

3.119.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.77, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5581, 5488, 241, 5426, 5422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{(a^2+1)c+2abcx+b^2cx^2}}\,dx$$

$$\Big\downarrow \ 5581$$

$$\frac{\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{c(a+bx)^2+c}}\,d(a+bx)}{b}$$

$$\Big\downarrow \ 5488$$

$$\frac{\frac{1}{2}\int \frac{a+bx}{\sqrt{c(a+bx)^2+c}}\,d(a+bx)-\frac{1}{2}\int \frac{\cot^{-1}(a+bx)}{\sqrt{c(a+bx)^2+c}}\,d(a+bx)+\frac{(a+bx)\sqrt{c(a+bx)^2+c}\cot^{-1}(a+bx)}{2c}}{b}$$

$$\Big\downarrow \; 241$$

$$\frac{-\frac{1}{2}\int \frac{\cot^{-1}(a+bx)}{\sqrt{c(a+bx)^2+c}}d(a+bx)+\frac{\sqrt{c(a+bx)^2+c}}{2c}+\frac{(a+bx)\sqrt{c(a+bx)^2+c}\cot^{-1}(a+bx)}{2c}}{b}$$

$$\Big\downarrow \; 5426$$

$$\frac{-\frac{\sqrt{(a+bx)^2+1}\int \frac{\cot^{-1}(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)}{2\sqrt{c(a+bx)^2+c}}+\frac{\sqrt{c(a+bx)^2+c}}{2c}+\frac{(a+bx)\sqrt{c(a+bx)^2+c}\cot^{-1}(a+bx)}{2c}}{b}$$

$$\Big\downarrow \; 5422$$

$$\frac{-\frac{\sqrt{(a+bx)^2+1}\left(-2i\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)\cot^{-1}(a+bx)-i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)+i\operatorname{PolyLog}\left(2,\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)\right)}{2\sqrt{c(a+bx)^2+c}}+\frac{\sqrt{c(a+bx)^2+c}}{2c}+\frac{(a}{}}{b}$$

input
```
Int[((a + b*x)^2*ArcCot[a + b*x])/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2
],x]
```

output
```
(Sqrt[c + c*(a + b*x)^2]/(2*c) + ((a + b*x)*Sqrt[c + c*(a + b*x)^2]*ArcCot
[a + b*x])/(2*c) - (Sqrt[1 + (a + b*x)^2]*((-2*I)*ArcCot[a + b*x]*ArcTan[S
qrt[1 + I*(a + b*x)]/Sqrt[1 - I*(a + b*x)]] - I*PolyLog[2, ((-I)*Sqrt[1 +
I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]] + I*PolyLog[2, (I*Sqrt[1 + I*(a + b*x
)])/Sqrt[1 - I*(a + b*x)]]))/(2*Sqrt[c + c*(a + b*x)^2]))/b
```

3.119.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5422
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcCot[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x])]/(c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 -
I*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 GtQ[d, 0]
```

3.119. $\int \frac{(a+bx)^2\cot^{-1}(a+bx)}{\sqrt{(1+a^2)c+2abcx+b^2cx^2}}\,dx$

rule 5426
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcCot[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5488
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcCot[c*x])^p/(c^2*d*m)), x] + (Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((a
 + b*ArcCot[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^2
*m))   Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/Sqrt[d + e*x^2]), x], x]) /
; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5581
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/d   Subs
t[Int[((d*e - c*f)/d + f*(x/d))^m*(C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcCot[x])
^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &
& EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

3.119.4 Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.72

method	result
default	$\frac{(\operatorname{arccot}(bx+a)bx+a\ \operatorname{arccot}(bx+a)+1)\sqrt{c(bx+a-i)(bx+a+i)}}{2bc}-\frac{i\left(i\operatorname{arccot}(bx+a)\ln\left(1-\frac{bx+a+i}{\sqrt{1+(bx+a)^2}}\right)-i\operatorname{arccot}(bx+a)\ln\left(\frac{bx+}{\sqrt{1+(}}\right.\right.}{}$

input
```
int((b*x+a)^2*arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x,method
=_RETURNVERBOSE)
```

output
```
1/2*(arccot(b*x+a)*b*x+a*arccot(b*x+a)+1)*(c*(-I+a+b*x)*(I+a+b*x))^(1/2)/b
/c-1/2*I*(I*arccot(b*x+a)*ln(1-(I+a+b*x)/(1+(b*x+a)^2)^(1/2))-I*arccot(b*x
+a)*ln((I+a+b*x)/(1+(b*x+a)^2)^(1/2)+1)+polylog(2,(I+a+b*x)/(1+(b*x+a)^2)^
(1/2))-polylog(2,-(I+a+b*x)/(1+(b*x+a)^2)^(1/2)))*(c*(-I+a+b*x)*(I+a+b*x))
^(1/2)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b/c
```

3.119.5 Fricas [F]

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{\sqrt{b^2cx^2+2\,abcx+(a^2+1)c}}\,dx$$

input
```
integrate((b*x+a)^2*arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x,
 algorithm="fricas")
```

output
```
integral((b^2*x^2 + 2*a*b*x + a^2)*arccot(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*
c*x + (a^2 + 1)*c), x)
```

3.119.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**2*acot(b*x+a)/((a**2+1)*c+2*a*b*c*x+b**2*c*x**2)**(1/2)
,x)
```

output
```
Timed out
```

3.119.7 Maxima [F]

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{\sqrt{b^2cx^2+2\,abcx+(a^2+1)c}}\,dx$$

input
```
integrate((b*x+a)^2*arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x,
 algorithm="maxima")
```

output
```
integrate((b*x + a)^2*arccot(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c*x + (a^2 +
1)*c), x)
```

3.119.8 Giac [F]

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{(1+a^2)c+2abcx+b^2cx^2}}\,dx = \int \frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{\sqrt{b^2cx^2+2\,abcx+(a^2+1)c}}\,dx$$

input
```
integrate((b*x+a)^2*arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x,
 algorithm="giac")
```

output
```
integrate((b*x + a)^2*arccot(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c*x + (a^2 +
1)*c), x)
```

3.119.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{(1+a^2)c+2abcx+b^2cx^2}}\,dx = \int \frac{\operatorname{acot}(a+b\,x)\,(a+b\,x)^2}{\sqrt{c\,b^2\,x^2+2\,a\,c\,b\,x+c\,(a^2+1)}}\,dx$$

input
```
int((acot(a + b*x)*(a + b*x)^2)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/2
),x)
```

output
```
int((acot(a + b*x)*(a + b*x)^2)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/2
), x)
```

3.120 $\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}} dx$

3.120.1 Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}} dx = \text{Int}\left(\frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{1+(a+bx)^2}}, x\right)$$

output
```
Unintegrable((b*x+a)^2*arccot(b*x+a)/(1+(b*x+a)^2)^(1/3),x)
```

3.120.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 198 vs. 2(30) = 60.

Time = 0.74 (sec) , antiderivative size = 198, normalized size of antiderivative = 5.66

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}} dx$$
$$= \frac{3\Big(\text{Gamma}\left(\frac{11}{6}\right) \text{Gamma}\left(\frac{7}{3}\right) \left(5(1+(a+bx)^2)\left(3(7+(a+bx)^2)+4(a+bx)\left(-2+(a+bx)^2\right)\cot^{-1}(a+b\right.\right.}{140b\sqrt[3]{1+a^2+2ab}}$$

input
```
Integrate[((a + b*x)^2*ArcCot[a + b*x])/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3
),x]
```

output
```
(3*(Gamma[11/6]*Gamma[7/3]*(5*(1 + (a + b*x)^2)*(3*(7 + (a + b*x)^2) + 4*(
a + b*x)*(-2 + (a + b*x)^2)*ArcCot[a + b*x]) - 24*(a + b*x)*ArcCot[a + b*x
]*Hypergeometric2F1[1, 4/3, 11/6, (1 + a^2 + 2*a*b*x + b^2*x^2)^(-1)]) + 5
*2^(1/3)*Sqrt[Pi]*Gamma[5/3]*HypergeometricPFQ[{1, 4/3, 4/3}, {11/6, 7/3},
 (1 + a^2 + 2*a*b*x + b^2*x^2)^(-1)]))/(140*b*(1 + a^2 + 2*a*b*x + b^2*x^2
)^(1/3)*(1 + (a + b*x)^2)*Gamma[11/6]*Gamma[7/3])
```

3.120.3 Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5581, 5561}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{a^2+2abx+b^2x^2+1}}dx$$

$$\Big\downarrow \text{5581}$$

$$\frac{\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{(a+bx)^2+1}}d(a+bx)}{b}$$

$$\Big\downarrow \text{5561}$$

$$\frac{\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{(a+bx)^2+1}}d(a+bx)}{b}$$

input
```
Int[((a + b*x)^2*ArcCot[a + b*x])/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3),x]
```

output
```
$Aborted
```

3.120.3.1 Defintions of rubi rules used

rule 5561
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCot[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

rule 5581
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/d   Subs
t[Int[((d*e - c*f)/d + f*(x/d))^m*(C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcCot[x])
^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &
& EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

3.120.4 Maple [N/A] (verified)

Not integrable

Time = 0.88 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{(b^2x^2+2abx+a^2+1)^{\frac{1}{3}}}dx$$

input
```
int((b*x+a)^2*arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)
```

output
```
int((b*x+a)^2*arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)
```

3.120.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}} dx = \int \frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{(b^2x^2+2\,abx+a^2+1)^{\frac{1}{3}}} dx$$

input
```
integrate((b*x+a)^2*arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorit
hm="fricas")
```

output
```
integral((b^2*x^2 + 2*a*b*x + a^2)*arccot(b*x + a)/(b^2*x^2 + 2*a*b*x + a^
2 + 1)^(1/3), x)
```

3.120.6 Sympy [N/A]

Not integrable

Time = 3.45 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{(a+bx)^2 \operatorname{acot}(a+bx)}{\sqrt[3]{a^2+2abx+b^2x^2+1}}\,dx$$

input
```
integrate((b*x+a)**2*acot(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/3),x)
```

output
```
Integral((a + b*x)**2*acot(a + b*x)/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(1/3
), x)
```

3.120.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{(b^2x^2+2\,abx+a^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x+a)^2*arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorit
hm="maxima")
```

output
```
integrate((b*x + a)^2*arccot(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3),
 x)
```

3.120.8 Giac [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{(b^2x^2+2\,abx+a^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x+a)^2*arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorit
hm="giac")
```

output
```
integrate((b*x + a)^2*arccot(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3),
 x)
```

3.120.9 Mupad [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\operatorname{acot}(a+b\,x)\,(a+b\,x)^2}{(a^2+2\,a\,b\,x+b^2\,x^2+1)^{1/3}}\,dx$$

input
```
int((acot(a + b*x)*(a + b*x)^2)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/3),x)
```

output
```
int((acot(a + b*x)*(a + b*x)^2)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/3), x)
```

3.121 $\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)c+2abcx+b^2cx^2}}\,dx$

3.121.1 Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)c+2abcx+b^2cx^2}}\,dx = \text{Int}\left(\frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{c+c(a+bx)^2}},x\right)$$

output
```
Unintegrable((b*x+a)^2*arccot(b*x+a)/(c+c*(b*x+a)^2)^(1/3),x)
```

3.121.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 200 vs. $2(32) = 64$.

Time = 0.32 (sec) , antiderivative size = 200, normalized size of antiderivative = 5.00

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)c+2abcx+b^2cx^2}}\,dx = \frac{3\Big(\text{Gamma}\left(\frac{11}{6}\right)\text{Gamma}\left(\frac{7}{3}\right)\left(5(1+(a+bx)^2)\right)\left(3(7+(a+bx)^2)+4(a+bx)\left(-2+(a+bx)^2\right)\cot^{-1}(a+b}{140b\sqrt[3]{c\left(1+a^2+2a\right.}}$$

input
```
Integrate[((a + b*x)^2*ArcCot[a + b*x])/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x
^2)^(1/3),x]
```

output
```
(3*(Gamma[11/6]*Gamma[7/3]*(5*(1 + (a + b*x)^2)*(3*(7 + (a + b*x)^2) + 4*(
a + b*x)*(-2 + (a + b*x)^2)*ArcCot[a + b*x]) - 24*(a + b*x)*ArcCot[a + b*x
]*Hypergeometric2F1[1, 4/3, 11/6, (1 + a^2 + 2*a*b*x + b^2*x^2)^(-1)]) + 5
*2^(1/3)*Sqrt[Pi]*Gamma[5/3]*HypergeometricPFQ[{1, 4/3, 4/3}, {11/6, 7/3},
 (1 + a^2 + 2*a*b*x + b^2*x^2)^(-1)]))/(140*b*(c*(1 + a^2 + 2*a*b*x + b^2*
x^2))^(1/3)*(1 + (a + b*x)^2)*Gamma[11/6]*Gamma[7/3])
```

3.121.3 Rubi [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5581, 5561}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2\cot^{-1}(a+bx)}{\sqrt[3]{(a^2+1)c+2abcx+b^2cx^2}}dx$$

$$\Big\downarrow \text{5581}$$

$$\frac{\int \frac{(a+bx)^2\cot^{-1}(a+bx)}{\sqrt[3]{c(a+bx)^2+c}}d(a+bx)}{b}$$

$$\Big\downarrow \text{5561}$$

$$\frac{\int \frac{(a+bx)^2\cot^{-1}(a+bx)}{\sqrt[3]{c(a+bx)^2+c}}d(a+bx)}{b}$$

input
```
Int[((a + b*x)^2*ArcCot[a + b*x])/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2)^(1
/3),x]
```

output
```
$Aborted
```

3.121.3.1 Defintions of rubi rules used

rule 5561
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCot[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

rule 5581
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/d   Subs
t[Int[((d*e - c*f)/d + f*(x/d))^m*(C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcCot[x])
^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &
& EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

3.121.4 Maple [N/A] (verified)

Not integrable

Time = 0.85 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{((a^2+1)c+2abcx+b^2c\,x^2)^{\frac{1}{3}}}dx$$

input
```
int((b*x+a)^2*arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)
```

output
```
int((b*x+a)^2*arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)
```

3.121.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{(b^2cx^2+2\,abcx+(a^2+1)c)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x+a)^2*arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x,
 algorithm="fricas")
```

output
```
integral((b^2*x^2 + 2*a*b*x + a^2)*arccot(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x
+ (a^2 + 1)*c)^(1/3), x)
```

3.121.6 Sympy [N/A]

Not integrable

Time = 24.40 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{(a+bx)^2\operatorname{acot}(a+bx)}{\sqrt[3]{c\,(a^2+2abx+b^2x^2+1)}}\,dx$$

input
```
integrate((b*x+a)**2*acot(b*x+a)/((a**2+1)*c+2*a*b*c*x+b**2*c*x**2)**(1/3)
,x)
```

output
```
Integral((a + b*x)**2*acot(a + b*x)/(c*(a**2 + 2*a*b*x + b**2*x**2 + 1))**
(1/3), x)
```

3.121.7 Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{(bx+a)^2\operatorname{arccot}(bx+a)}{(b^2cx^2+2\,abcx+(a^2+1)c)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x+a)^2*arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x,
 algorithm="maxima")
```

output
```
integrate((b*x + a)^2*arccot(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c
)^(1/3), x)
```

3.121.8 Giac [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{(b^2cx^2+2\,abcx+(a^2+1)c)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x+a)^2*arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x,
 algorithm="giac")
```

output
```
integrate((b*x + a)^2*arccot(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c
)^(1/3), x)
```

3.121.9 Mupad [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{\operatorname{acot}(a+b\,x)\,\left(a+b\,x\right)^2}{\left(c\,b^2\,x^2+2\,a\,c\,b\,x+c\,\left(a^2+1\right)\right)^{1/3}}\,dx$$

input
```
int((acot(a + b*x)*(a + b*x)^2)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/3
),x)
```

output
```
int((acot(a + b*x)*(a + b*x)^2)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/3
), x)
```

3.122 $\int (a+bx)^2 \cot^{-1}(a+bx)\,dx$

3.122.1 Optimal result

Integrand size = 14, antiderivative size = 52

$$\int (a+bx)^2 \cot^{-1}(a+bx)\,dx=\frac{(a+bx)^2}{6b}+\frac{(a+bx)^3\cot^{-1}(a+bx)}{3b}-\frac{\log\left(1+(a+bx)^2\right)}{6b}$$

output
```
1/6*(b*x+a)^2/b+1/3*(b*x+a)^3*arccot(b*x+a)/b-1/6*ln(1+(b*x+a)^2)/b
```

3.122.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int (a+bx)^2 \cot^{-1}(a+bx)\,dx=\frac{(a+bx)^2+2(a+bx)^3\cot^{-1}(a+bx)-\log\left(1+(a+bx)^2\right)}{6b}$$

input
```
Integrate[(a + b*x)^2*ArcCot[a + b*x],x]
```

output
```
((a + b*x)^2 + 2*(a + b*x)^3*ArcCot[a + b*x] - Log[1 + (a + b*x)^2])/(6*b)
```

3.122.3 Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5567, 5362, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2\cot^{-1}(a+bx)\,dx$$

$$\Big\downarrow \text{ 5567}$$

$$\frac{\int (a+bx)^2\cot^{-1}(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{ 5362}$$

$$\frac{\frac{1}{3}\int \frac{(a+bx)^3}{(a+bx)^2+1}d(a+bx)+\frac{1}{3}(a+bx)^3\cot^{-1}(a+bx)}{b}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{\frac{1}{6}\int \frac{(a+bx)^2}{(a+bx)^2+1}d(a+bx)^2+\frac{1}{3}(a+bx)^3\cot^{-1}(a+bx)}{b}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\frac{1}{6}\int \left(1+\frac{1}{-a-bx-1}\right)d(a+bx)^2+\frac{1}{3}(a+bx)^3\cot^{-1}(a+bx)}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{6}\left((a+bx)^2-\log(a+bx+1)\right)+\frac{1}{3}(a+bx)^3\cot^{-1}(a+bx)}{b}$$

input
```
Int[(a + b*x)^2*ArcCot[a + b*x],x]
```

output
```
(((a + b*x)^3*ArcCot[a + b*x])/3 + ((a + b*x)^2 - Log[1 + a + b*x])/6)/b
```

3.122.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5567
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcCot[x])^p, x],
 x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
 IGtQ[p, 0]
```

3.122.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{\frac{\operatorname{arccot}(bx+a)(bx+a)^3}{3}+\frac{(bx+a)^2}{6}-\frac{\ln\left(1+(bx+a)^2\right)}{6}}{b}$
default	$\frac{\frac{\operatorname{arccot}(bx+a)(bx+a)^3}{3}+\frac{(bx+a)^2}{6}-\frac{\ln\left(1+(bx+a)^2\right)}{6}}{b}$
parts	$\frac{\operatorname{arccot}(bx+a)b^2x^3}{3}+\operatorname{arccot}(bx+a)\,ba\,x^2+\operatorname{arccot}(bx+a)\,a^2x+\frac{\operatorname{arccot}(bx+a)a^3}{3b}+\frac{x^2b}{6}+\frac{ax}{3}-$
parallelrisch	$-\frac{-2\operatorname{arccot}(bx+a)x^3b^4-6a\,b^3\operatorname{arccot}(bx+a)x^2-6x\operatorname{arccot}(bx+a)a^2b^2-b^3x^2-2\operatorname{arccot}(bx+a)a^3b-2a\,b^2x+5a^2b+\ln(b}{6b^2}$
risch	$\frac{i(bx+a)^3\ln(1+i(bx+a))}{6b}-\frac{ib^2x^3\ln(1-i(bx+a))}{6}-\frac{iba\,x^2\ln(1-i(bx+a))}{2}+\frac{\pi\,b^2x^3}{6}-\frac{ia^2x\ln(1-i(bx+a))}{2}+$

input
```
int((b*x+a)^2*arccot(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/3*arccot(b*x+a)*(b*x+a)^3+1/6*(b*x+a)^2-1/6*ln(1+(b*x+a)^2))
```

3.122.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.56

$$\int (a+bx)^2 \cot^{-1}(a+bx)\,dx$$
$$= \frac{b^2x^2 - 2\,a^3 \arctan\left(bx+a\right) + 2\,abx + 2\left(b^3x^3 + 3\,ab^2x^2 + 3\,a^2bx\right) \operatorname{arccot}\left(bx+a\right) - \log\left(b^2x^2 + 2\,abx + a^2\right.}{6\,b}$$

input
```
integrate((b*x+a)^2*arccot(b*x+a),x, algorithm="fricas")
```

output
```
1/6*(b^2*x^2 - 2*a^3*arctan(b*x + a) + 2*a*b*x + 2*(b^3*x^3 + 3*a*b^2*x^2
+ 3*a^2*b*x)*arccot(b*x + a) - log(b^2*x^2 + 2*a*b*x + a^2 + 1))/b
```

3.122.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.90

$$\int (a+bx)^2 \cot^{-1}(a+bx)\,dx$$
$$= \begin{cases} \frac{a^3 \operatorname{acot}(a+bx)}{3b} + a^2x \operatorname{acot}(a+bx) + abx^2 \operatorname{acot}(a+bx) + \frac{ax}{3} + \frac{b^2x^3 \operatorname{acot}(a+bx)}{3} + \frac{bx^2}{6} - \frac{\log\left(\frac{a}{b}+x-\frac{i}{b}\right)}{3b} - \frac{i \operatorname{acot}(a+b}{3b} \\ a^2x \operatorname{acot}(a) \end{cases}$$

input
```
integrate((b*x+a)**2*acot(b*x+a),x)
```

output
```
Piecewise((a**3*acot(a + b*x)/(3*b) + a**2*x*acot(a + b*x) + a*b*x**2*acot
(a + b*x) + a*x/3 + b**2*x**3*acot(a + b*x)/3 + b*x**2/6 - log(a/b + x - I
/b)/(3*b) - I*acot(a + b*x)/(3*b), Ne(b, 0)), (a**2*x*acot(a), True))
```

3.122.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(46) = 92$.

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.79

$$\int (a+bx)^2 \cot^{-1}(a+bx)\,dx$$
$$= -\frac{1}{6}\left(\frac{2\,a^3 \arctan\left(\frac{b^2x+ab}{b}\right)}{b^2} - \frac{bx^2+2\,ax}{b} + \frac{\log\left(b^2x^2+2\,abx+a^2+1\right)}{b^2}\right)b$$
$$+ \frac{1}{3}\left(b^2x^3+3\,abx^2+3\,a^2x\right)\text{arccot}\left(bx+a\right)$$

input
```
integrate((b*x+a)^2*arccot(b*x+a),x, algorithm="maxima")
```

output
```
-1/6*(2*a^3*arctan((b^2*x + a*b)/b)/b^2 - (b*x^2 + 2*a*x)/b + log(b^2*x^2
+ 2*a*b*x + a^2 + 1)/b^2)*b + 1/3*(b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)*arccot(b
*x + a)
```

3.122.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(46) = 92$.

Time = 0.37 (sec) , antiderivative size = 203, normalized size of antiderivative = 3.90

$$\int (a+bx)^2 \cot^{-1}(a+bx)\,dx =$$
$$-\frac{\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^6 - 3\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^4 - \tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)}{}$$

input
```
integrate((b*x+a)^2*arccot(b*x+a),x, algorithm="giac")
```

output
```
-1/24*(arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^6 - 3*arctan(1/(b*
x + a))*tan(1/2*arctan(1/(b*x + a)))^4 - tan(1/2*arctan(1/(b*x + a)))^5 -
4*log(16*tan(1/2*arctan(1/(b*x + a)))^2/(tan(1/2*arctan(1/(b*x + a)))^4 +
2*tan(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^3 + 3*
arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^2 - 2*tan(1/2*arctan(1/(b
*x + a)))^3 - arctan(1/(b*x + a)) - tan(1/2*arctan(1/(b*x + a))))/(b*tan(1
/2*arctan(1/(b*x + a)))^3)
```

3.122.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.63

$$\int (a+bx)^2 \cot^{-1}(a+bx)\,dx = \frac{a\,x}{3} - \frac{\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{6\,b} + \frac{b\,x^2}{6} - \frac{a^3\,\mathrm{atan}(a+b\,x)}{3\,b} + \frac{b^2\,x^3\,\mathrm{acot}(a+b\,x)}{3} + a^2\,x\,\mathrm{acot}(a+b\,x) + a\,b\,x^2\,\mathrm{acot}(a+b\,x)$$

input
```
int(acot(a + b*x)*(a + b*x)^2,x)
```

output
```
(a*x)/3 - log(a^2 + b^2*x^2 + 2*a*b*x + 1)/(6*b) + (b*x^2)/6 - (a^3*atan(a
 + b*x))/(3*b) + (b^2*x^3*acot(a + b*x))/3 + a^2*x*acot(a + b*x) + a*b*x^2
*acot(a + b*x)
```

3.123 $\int (a+bx)\cot^{-1}(a+bx)\,dx$

3.123.1 Optimal result

Integrand size = 12, antiderivative size = 39

$$\int (a+bx)\cot^{-1}(a+bx)\,dx=\frac{x}{2}+\frac{(a+bx)^2\cot^{-1}(a+bx)}{2b}-\frac{\arctan(a+bx)}{2b}$$

output
```
1/2*x+1/2*(b*x+a)^2*arccot(b*x+a)/b-1/2*arctan(b*x+a)/b
```

3.123.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 141, normalized size of antiderivative = 3.62

$$\int (a+bx)\cot^{-1}(a+bx)\,dx=ax\cot^{-1}(a+bx)+\frac{1}{2}b\left(-\frac{a}{b}+\frac{a+bx}{b}\right)^2\cot^{-1}(a+bx)+\frac{1}{2}b\left(\frac{x}{b}-\frac{i(i-a)^2\log(i-a-bx)}{2b^2}+\frac{i(i+a)^2\log(i+a+bx)}{2b^2}\right)+\frac{a(-2a\arctan(a+bx)+\log(1+a^2+2abx+b^2x^2))}{2b}$$

input
```
Integrate[(a + b*x)*ArcCot[a + b*x],x]
```

output
```
a*x*ArcCot[a + b*x] + (b*(-(a/b) + (a + b*x)/b)^2*ArcCot[a + b*x])/2 + (b*
(x/b - ((I/2)*(I - a)^2*Log[I - a - b*x])/b^2 + ((I/2)*(I + a)^2*Log[I + a
 + b*x])/b^2))/2 + (a*(-2*a*ArcTan[a + b*x] + Log[1 + a^2 + 2*a*b*x + b^2*
x^2]))/(2*b)
```

3.123.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5567, 5362, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)\cot^{-1}(a+bx)\,dx$$

$$\Big\downarrow \text{5567}$$

$$\frac{\int (a+bx)\cot^{-1}(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{5362}$$

$$\frac{\frac{1}{2}\int \frac{(a+bx)^2}{(a+bx)^2+1}d(a+bx)+\frac{1}{2}(a+bx)^2\cot^{-1}(a+bx)}{b}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{1}{2}\left(-\int \frac{1}{(a+bx)^2+1}d(a+bx)+a+bx\right)+\frac{1}{2}(a+bx)^2\cot^{-1}(a+bx)}{b}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{2}(-\arctan(a+bx)+a+bx)+\frac{1}{2}(a+bx)^2\cot^{-1}(a+bx)}{b}$$

input
```
Int[(a + b*x)*ArcCot[a + b*x],x]
```

output
```
(((a + b*x)^2*ArcCot[a + b*x])/2 + (a + b*x - ArcTan[a + b*x])/2)/b
```

3.123.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5567
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcCot[x])^p, x],
 x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
 IGtQ[p, 0]
```

3.123.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{\frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}-\frac{\arctan(bx+a)}{2}}{b}$
default	$\frac{\frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}-\frac{\arctan(bx+a)}{2}}{b}$
parts	$\frac{\operatorname{arccot}(bx+a)x^2 b}{2}+\operatorname{arccot}(bx+a)\,ax+\frac{b\left(\frac{x}{b}+\frac{(-a^2-1)\arctan(bx+a)}{b^2}\right)}{2}$
parallelrisch	$\frac{\operatorname{arccot}(bx+a)x^2b^3+2a\operatorname{arccot}(bx+a)x\,b^2+\operatorname{arccot}(bx+a)a^2b+b^2x+\operatorname{arccot}(bx+a)b-2ab}{2b^2}$
risch	$\frac{i(x^2b+2ax)\ln(1+i(bx+a))}{4}-\frac{ib\,x^2\ln(1-i(bx+a))}{4}-\frac{iax\ln(1-i(bx+a))}{2}+\frac{\pi b\,x^2}{4}+\frac{\pi ax}{2}-\frac{a^2\arctan(bx+a)}{2b}+$

input
```
int((b*x+a)*arccot(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*(b*x+a)^2*arccot(b*x+a)+1/2*b*x+1/2*a-1/2*arctan(b*x+a))
```

3.123.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int (a+bx)\cot^{-1}(a+bx)\,dx = \frac{bx + (b^2x^2 + 2\,abx + a^2 + 1)\,\mathrm{arccot}\,(bx+a)}{2\,b}$$

input
```
integrate((b*x+a)*arccot(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(b*x + (b^2*x^2 + 2*a*b*x + a^2 + 1)*arccot(b*x + a))/b
```

3.123.6 Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.44

$$\int (a+bx)\cot^{-1}(a+bx)\,dx$$
$$= \begin{cases} \frac{a^2\,\mathrm{acot}\,(a+bx)}{2b} + ax\,\mathrm{acot}\,(a+bx) + \frac{bx^2\,\mathrm{acot}\,(a+bx)}{2} + \frac{x}{2} + \frac{\mathrm{acot}\,(a+bx)}{2b} & \text{for } b \neq 0 \\ ax\,\mathrm{acot}\,(a) & \text{otherwise} \end{cases}$$

input
```
integrate((b*x+a)*acot(b*x+a),x)
```

output
```
Piecewise((a**2*acot(a + b*x)/(2*b) + a*x*acot(a + b*x) + b*x**2*acot(a +
b*x)/2 + x/2 + acot(a + b*x)/(2*b), Ne(b, 0)), (a*x*acot(a), True))
```

3.123.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.33

$$\int (a+bx)\cot^{-1}(a+bx)\,dx = \frac{1}{2}\,b\left(\frac{x}{b} - \frac{(a^2+1)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^2}\right) + \frac{1}{2}\left(bx^2 + 2\,ax\right)\mathrm{arccot}\,(bx+a)$$

input
```
integrate((b*x+a)*arccot(b*x+a),x, algorithm="maxima")
```

output
```
1/2*b*(x/b - (a^2 + 1)*arctan((b^2*x + a*b)/b)/b^2) + 1/2*(b*x^2 + 2*a*x)*
arccot(b*x + a)
```

3.123.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(33) = 66.

Time = 0.33 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.56

$$\int (a+bx)\cot^{-1}(a+bx)\,dx = \frac{\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^4 + 2\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2 - 2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)}{8\,b\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2}$$

input
```
integrate((b*x+a)*arccot(b*x+a),x, algorithm="giac")
```

output
```
1/8*(arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^4 + 2*arctan(1/(b*x
+ a))*tan(1/2*arctan(1/(b*x + a)))^2 - 2*tan(1/2*arctan(1/(b*x + a)))^3 +
arctan(1/(b*x + a)) + 2*tan(1/2*arctan(1/(b*x + a))))/(b*tan(1/2*arctan(1/
(b*x + a)))^2)
```

3.123.9 Mupad [B] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int (a+bx)\cot^{-1}(a+bx)\,dx = \frac{x}{2} + \frac{\frac{\mathrm{acot}(a+b\,x)}{2} + \frac{a^2\,\mathrm{acot}(a+b\,x)}{2}}{b} + a\,x\,\mathrm{acot}(a+b\,x) + \frac{b\,x^2\,\mathrm{acot}(a+b\,x)}{2}$$

input
```
int(acot(a + b*x)*(a + b*x),x)
```

output
```
x/2 + (acot(a + b*x)/2 + (a^2*acot(a + b*x))/2)/b + a*x*acot(a + b*x) + (b
*x^2*acot(a + b*x))/2
```

3.124 $\int \frac{\cot^{-1}(a+bx)}{a+bx}\,dx$

3.124.1 Optimal result

Integrand size = 14, antiderivative size = 45

$$\int \frac{\cot^{-1}(a+bx)}{a+bx}\,dx=-\frac{i\,\text{PolyLog}\left(2,-\frac{i}{a+bx}\right)}{2b}+\frac{i\,\text{PolyLog}\left(2,\frac{i}{a+bx}\right)}{2b}$$

output
```
-1/2*I*polylog(2,-I/(b*x+a))/b+1/2*I*polylog(2,I/(b*x+a))/b
```

3.124.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \frac{\cot^{-1}(a+bx)}{a+bx}\,dx=-\frac{i\left(\text{PolyLog}\left(2,-\frac{i}{a+bx}\right)-\text{PolyLog}\left(2,\frac{i}{a+bx}\right)\right)}{2b}$$

input
```
Integrate[ArcCot[a + b*x]/(a + b*x),x]
```

output
```
((-1/2*I)*(PolyLog[2, (-I)/(a + b*x)] - PolyLog[2, I/(a + b*x)]))/b
```

3.124.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5567, 5356, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(a+bx)}{a+bx} \, dx$$

$$\Big\downarrow \text{5567}$$

$$\frac{\int \frac{\cot^{-1}(a+bx)}{a+bx}d(a+bx)}{b}$$

$$\Big\downarrow \text{5356}$$

$$\frac{\frac{1}{2}i\int \frac{\log\left(1-\frac{i}{a+bx}\right)}{a+bx}d(a+bx)-\frac{1}{2}i\int \frac{\log\left(1+\frac{i}{a+bx}\right)}{a+bx}d(a+bx)}{b}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\frac{1}{2}i\,\text{PolyLog}\left(2,\frac{i}{a+bx}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,-\frac{i}{a+bx}\right)}{b}$$

input
```
Int[ArcCot[a + b*x]/(a + b*x),x]
```

output
```
((-1/2*I)*PolyLog[2, (-I)/(a + b*x)] + (I/2)*PolyLog[2, I/(a + b*x)])/b
```

3.124.3.1 Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5356
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Simp[I*(b/2)   Int[Log[1 + I/(c*x)]/x, x], x] + Simp[I*(b/2)   Int[Log
[1 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5567
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcCot[x])^p, x],
 x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
 IGtQ[p, 0]
```

3.124.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

method	result
risch	$\frac{\pi\ln(-ibx-ia)}{2b}+\frac{i\,\mathrm{dilog}(-ibx-ia+1)}{2b}-\frac{i\,\mathrm{dilog}(ibx+ia+1)}{2b}$
derivativedivides	$\frac{\ln(bx+a)\,\mathrm{arccot}(bx+a)-\frac{i\ln(bx+a)\ln(1+i(bx+a))}{2}+\frac{i\ln(bx+a)\ln(1-i(bx+a))}{2}-\frac{i\,\mathrm{dilog}(1+i(bx+a))}{2}+\frac{i\,\mathrm{dilog}(1-i(bx+a))}{2}}{b}$
default	$\frac{\ln(bx+a)\,\mathrm{arccot}(bx+a)-\frac{i\ln(bx+a)\ln(1+i(bx+a))}{2}+\frac{i\ln(bx+a)\ln(1-i(bx+a))}{2}-\frac{i\,\mathrm{dilog}(1+i(bx+a))}{2}+\frac{i\,\mathrm{dilog}(1-i(bx+a))}{2}}{b}$
parts	$\frac{\ln(bx+a)\,\mathrm{arccot}(bx+a)}{b}+\frac{-\frac{i\ln(bx+a)\ln(1+i(bx+a))}{2}+\frac{i\ln(bx+a)\ln(1-i(bx+a))}{2}-\frac{i\,\mathrm{dilog}(1+i(bx+a))}{2}+\frac{i\,\mathrm{dilog}(1-i(bx+a))}{2}}{b}$

input
```
int(arccot(b*x+a)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*Pi*ln(-I*a-I*b*x)+1/2*I/b*dilog(1-I*a-I*b*x)-1/2*I/b*dilog(1+I*a+I*b
*x)
```

3.124.5 Fricas [F]

$$\int \frac{\cot^{-1}(a+bx)}{a+bx}\,dx=\int \frac{\mathrm{arccot}\,(bx+a)}{bx+a}\,dx$$

input
```
integrate(arccot(b*x+a)/(b*x+a),x, algorithm="fricas")
```

output
```
integral(arccot(b*x + a)/(b*x + a), x)
```

3.124.6 Sympy [F]

$$\int \frac{\cot^{-1}(a+bx)}{a+bx}\,dx=\int \frac{\operatorname{acot}(a+bx)}{a+bx}\,dx$$

input
```
integrate(acot(b*x+a)/(b*x+a),x)
```

output
```
Integral(acot(a + b*x)/(a + b*x), x)
```

3.124.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(31) = 62$.

Time = 0.33 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.49

$$\int \frac{\cot^{-1}(a+bx)}{a+bx}\,dx=\frac{\operatorname{arccot}(bx+a)\log(bx+a)}{b}+\frac{\arctan\left(\frac{b^2x+ab}{b}\right)\log(bx+a)}{b}+\frac{\arctan(bx+a,0)\log(b^2x^2+2\,abx+a^2+1)-2\,\arctan(bx+a)\log(|bx+a|)+i\,\mathrm{Li}_2(i\,bx+i\,a+1)-}{2\,b}$$

input
```
integrate(arccot(b*x+a)/(b*x+a),x, algorithm="maxima")
```

output
```
arccot(b*x + a)*log(b*x + a)/b + arctan((b^2*x + a*b)/b)*log(b*x + a)/b +
1/2*(arctan2(b*x + a, 0)*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*arctan(b*x +
 a)*log(abs(b*x + a)) + I*dilog(I*b*x + I*a + 1) - I*dilog(-I*b*x - I*a +
1))/b
```

3.124.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(31) = 62$.

Time = 0.53 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.22

$$\int \frac{\cot^{-1}(a+bx)}{a+bx}\,dx=-\frac{\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^4+2\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2-2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right.\right.}{8\,b^2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2}$$

input
```
integrate(arccot(b*x+a)/(b*x+a),x, algorithm="giac")
```

output
```
-1/8*(arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^4 + 2*arctan(1/(b*x
 + a))*tan(1/2*arctan(1/(b*x + a)))^2 - 2*tan(1/2*arctan(1/(b*x + a)))^3 +
 arctan(1/(b*x + a)) + 2*tan(1/2*arctan(1/(b*x + a))))/(b^2*tan(1/2*arctan
(1/(b*x + a)))^2)
```

3.124.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{a+bx}\,dx=\int \frac{\mathrm{acot}(a+b\,x)}{a+b\,x}\,dx$$

input
```
int(acot(a + b*x)/(a + b*x),x)
```

output
```
int(acot(a + b*x)/(a + b*x), x)
```

3.125 $\int \frac{\cot^{-1}(a+bx)}{(a+bx)^2}\,dx$

3.125.1 Optimal result

Integrand size = 14, antiderivative size = 47

$$\int \frac{\cot^{-1}(a+bx)}{(a+bx)^2}\,dx = -\frac{\cot^{-1}(a+bx)}{b(a+bx)} - \frac{\log(a+bx)}{b} + \frac{\log\left(1+(a+bx)^2\right)}{2b}$$

output
```
-arccot(b*x+a)/b/(b*x+a)-ln(b*x+a)/b+1/2*ln(1+(b*x+a)^2)/b
```

3.125.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{\cot^{-1}(a+bx)}{(a+bx)^2}\,dx = \frac{-\frac{\cot^{-1}(a+bx)}{a+bx} - \log(a+bx) + \frac{1}{2}\log\left(1+(a+bx)^2\right)}{b}$$

input
```
Integrate[ArcCot[a + b*x]/(a + b*x)^2,x]
```

output
```
(-(ArcCot[a + b*x]/(a + b*x)) - Log[a + b*x] + Log[1 + (a + b*x)^2]/2)/b
```

3.125.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5567, 5362, 243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(a+bx)}{(a+bx)^2} \, dx$$

$$\Big\downarrow \text{ 5567}$$

$$\frac{\int \frac{\cot^{-1}(a+bx)}{(a+bx)^2}d(a+bx)}{b}$$

$$\Big\downarrow \text{ 5362}$$

$$\frac{-\int \frac{1}{(a+bx)((a+bx)^2+1)}d(a+bx)-\frac{\cot^{-1}(a+bx)}{a+bx}}{b}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{-\frac{1}{2}\int \frac{1}{(a+bx)^2((a+bx)^2+1)}d(a+bx)^2-\frac{\cot^{-1}(a+bx)}{a+bx}}{b}$$

$$\Big\downarrow \text{ 47}$$

$$\frac{\frac{1}{2}\left(\int \frac{1}{(a+bx)^2+1}d(a+bx)^2-\int \frac{1}{(a+bx)^2}d(a+bx)^2\right)-\frac{\cot^{-1}(a+bx)}{a+bx}}{b}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{\frac{1}{2}\left(\int \frac{1}{(a+bx)^2+1}d(a+bx)^2-\log\left((a+bx)^2\right)\right)-\frac{\cot^{-1}(a+bx)}{a+bx}}{b}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{\frac{1}{2}\left(\log\left((a+bx)^2+1\right)-\log\left((a+bx)^2\right)\right)-\frac{\cot^{-1}(a+bx)}{a+bx}}{b}$$

input
```
Int[ArcCot[a + b*x]/(a + b*x)^2,x]
```

output
```
(-(ArcCot[a + b*x]/(a + b*x)) + (-Log[(a + b*x)^2] + Log[1 + (a + b*x)^2])
/2)/b
```

3.125.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5567
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcCot[x])^p, x],
 x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
 IGtQ[p, 0]
```

3.125.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{-\frac{\operatorname{arccot}(bx+a)}{bx+a}-\ln(bx+a)+\frac{\ln\left(1+(bx+a)^2\right)}{2}}{b}$
default	$\frac{-\frac{\operatorname{arccot}(bx+a)}{bx+a}-\ln(bx+a)+\frac{\ln\left(1+(bx+a)^2\right)}{2}}{b}$
parts	$-\frac{\operatorname{arccot}(bx+a)}{b(bx+a)}+\frac{\ln\left(b^2x^2+2abx+a^2+1\right)}{2b}-\frac{\ln(bx+a)}{b}$
parallelrisch	$-\frac{6\ln(bx+a)xa\,b^2-3b^2\ln\left(b^2x^2+2abx+a^2+1\right)ax+6\ln(bx+a)a^2b-3\ln\left(b^2x^2+2abx+a^2+1\right)a^2b+6\ \operatorname{arccot}(bx+a)ab}{6(bx+a)a\,b^2}$
risch	$-\frac{i\ln(1+i(bx+a))}{2b(bx+a)}-\frac{2\ln(-bx-a)bx-\ln\left(b^2x^2+2abx+a^2+1\right)bx+2\ln(-bx-a)a-a\ln\left(b^2x^2+2abx+a^2+1\right)-i\ln(1-i(b}{2(bx+a)b}$

input
```
int(arccot(b*x+a)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-arccot(b*x+a)/(b*x+a)-ln(b*x+a)+1/2*ln(1+(b*x+a)^2))
```

3.125.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.26

$$\int \frac{\cot^{-1}(a+bx)}{(a+bx)^2}\,dx = \frac{(bx+a)\log\left(b^2x^2+2\,abx+a^2+1\right)-2\,(bx+a)\log\left(bx+a\right)-2\ \operatorname{arccot}\left(bx+a\right)}{2\,(b^2x+ab)}$$

input
```
integrate(arccot(b*x+a)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*((b*x + a)*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*(b*x + a)*log(b*x + a)
 - 2*arccot(b*x + a))/(b^2*x + a*b)
```

3.125.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.33 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.96

$$\int \frac{\cot^{-1}(a+bx)}{(a+bx)^2}\,dx = \begin{cases} -\frac{a\log\left(\frac{a}{b}+x\right)}{ab+b^2x} + \frac{a\log\left(\frac{a}{b}+x-\frac{i}{b}\right)}{ab+b^2x} + \frac{ia\,\mathrm{acot}\,(a+bx)}{ab+b^2x} - \frac{bx\log\left(\frac{a}{b}+x\right)}{ab+b^2x} + \frac{bx\log\left(\frac{a}{b}+x-\frac{i}{b}\right)}{ab+b^2x} + \frac{ibx\,\mathrm{acot}\,(a+bx)}{ab+b^2x} - \frac{\mathrm{acot}\,(a+bx)}{ab+b^2x} & \text{for } b \neq 0 \\ \frac{x\,\mathrm{acot}\,(a)}{a^2} & \text{otherwise} \end{cases}$$

input
```
integrate(acot(b*x+a)/(b*x+a)**2,x)
```

output
```
Piecewise((-a*log(a/b + x)/(a*b + b**2*x) + a*log(a/b + x - I/b)/(a*b + b*
*2*x) + I*a*acot(a + b*x)/(a*b + b**2*x) - b*x*log(a/b + x)/(a*b + b**2*x)
 + b*x*log(a/b + x - I/b)/(a*b + b**2*x) + I*b*x*acot(a + b*x)/(a*b + b**2
*x) - acot(a + b*x)/(a*b + b**2*x), Ne(b, 0)), (x*acot(a)/a**2, True))
```

3.125.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(a+bx)}{(a+bx)^2}\,dx = \frac{\log\left(b^2x^2+2\,abx+a^2+1\right)}{2\,b} - \frac{\log\left(bx+a\right)}{b} - \frac{\mathrm{arccot}\,(bx+a)}{(bx+a)b}$$

input
```
integrate(arccot(b*x+a)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b - log(b*x + a)/b - arccot(b*x + a)/
((b*x + a)*b)
```

3.125.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(45) = 90$.

Time = 0.33 (sec) , antiderivative size = 238, normalized size of antiderivative = 5.06

$$\int \frac{\cot^{-1}(a+bx)}{(a+bx)^2}\,dx =$$

$$-\frac{\arctan\left(\frac{1}{bx+a}\right)^2 - \frac{\arctan\left(\frac{1}{bx+a}\right)^2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2 - \log\left(\frac{4\left(\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^4 - 2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2+1\right)}{\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^4 + 2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2+1}\right)\tan\left(\frac{1}{2}\text{arcta}\right.}{\tan}}{2b}$$

input
```
integrate(arccot(b*x+a)/(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/2*(arctan(1/(b*x + a))^2 - (arctan(1/(b*x + a))^2*tan(1/2*arctan(1/(b*x
 + a)))^2 - log(4*(tan(1/2*arctan(1/(b*x + a)))^4 - 2*tan(1/2*arctan(1/(b*
x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x
+ a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^2 - arctan(1/(b*x + a))^2 + 4*
arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a))) + log(4*(tan(1/2*arctan(1
/(b*x + a)))^4 - 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(
b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)))/(tan(1/2*arctan(1/(
b*x + a)))^2 - 1))/b
```

3.125.9 Mupad [B] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

$$\int \frac{\cot^{-1}(a+bx)}{(a+bx)^2}\,dx = \frac{\ln\left(-a^2-2\,a\,b\,x-b^2\,x^2-1\right)}{2\,b} - \frac{\ln\left(a+b\,x\right)}{b} - \frac{\mathrm{acot}(a+b\,x)}{x\,b^2+a\,b}$$

input
```
int(acot(a + b*x)/(a + b*x)^2,x)
```

output
```
log(- a^2 - b^2*x^2 - 2*a*b*x - 1)/(2*b) - log(a + b*x)/b - acot(a + b*x)/
(a*b + b^2*x)
```

3.126 $\int \frac{\cot^{-1}(1+x)}{2+2x} dx$

3.126.1 Optimal result

Integrand size = 12, antiderivative size = 35

$$\int \frac{\cot^{-1}(1+x)}{2+2x}\,dx=-\frac{1}{4}i\,\text{PolyLog}\left(2,-\frac{i}{1+x}\right)+\frac{1}{4}i\,\text{PolyLog}\left(2,\frac{i}{1+x}\right)$$

output
```
-1/4*I*polylog(2,-I/(1+x))+1/4*I*polylog(2,I/(1+x))
```

3.126.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(1+x)}{2+2x}\,dx=-\frac{1}{4}i\,\text{PolyLog}\left(2,-\frac{i}{1+x}\right)+\frac{1}{4}i\,\text{PolyLog}\left(2,\frac{i}{1+x}\right)$$

input
```
Integrate[ArcCot[1 + x]/(2 + 2*x),x]
```

output
```
(-1/4*I)*PolyLog[2, (-I)/(1 + x)] + (I/4)*PolyLog[2, I/(1 + x)]
```

3.126.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5567, 27, 5356, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(x+1)}{2x+2}dx$$

$$\Big\downarrow \text{5567}$$

$$\int \frac{\cot^{-1}(x+1)}{2(x+1)}d(x+1)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{\cot^{-1}(x+1)}{x+1}d(x+1)$$

$$\Big\downarrow \text{5356}$$

$$\frac{1}{2}\left(\frac{1}{2}i\int \frac{\log\left(1-\frac{i}{x+1}\right)}{x+1}d(x+1)-\frac{1}{2}i\int \frac{\log\left(1+\frac{i}{x+1}\right)}{x+1}d(x+1)\right)$$

$$\Big\downarrow \text{2838}$$

$$\frac{1}{2}\left(\frac{1}{2}i\,\text{PolyLog}\left(2,\frac{i}{x+1}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,-\frac{i}{x+1}\right)\right)$$

input
```
Int[ArcCot[1 + x]/(2 + 2*x),x]
```

output
```
((-1/2*I)*PolyLog[2, (-I)/(1 + x)] + (I/2)*PolyLog[2, I/(1 + x)])/2
```

3.126.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5356
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Simp[I*(b/2)   Int[Log[1 + I/(c*x)]/x, x], x] + Simp[I*(b/2)   Int[Log
[1 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5567
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcCot[x])^p, x],
 x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
 IGtQ[p, 0]
```

3.126.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

method	result
risch	$\frac{i\,\mathrm{dilog}(-ix-i+1)}{4}+\frac{\pi\ln(-ix-i)}{4}-\frac{i\,\mathrm{dilog}(ix+i+1)}{4}$
derivativedivides	$\frac{\ln(1+x)\,\mathrm{arccot}(1+x)}{2}-\frac{i\ln(1+x)\ln(1+i(1+x))}{4}+\frac{i\ln(1+x)\ln(1-i(1+x))}{4}-\frac{i\,\mathrm{dilog}(1+i(1+x))}{4}+\frac{i\,\mathrm{dilog}(1-i(1+$
default	$\frac{\ln(1+x)\,\mathrm{arccot}(1+x)}{2}-\frac{i\ln(1+x)\ln(1+i(1+x))}{4}+\frac{i\ln(1+x)\ln(1-i(1+x))}{4}-\frac{i\,\mathrm{dilog}(1+i(1+x))}{4}+\frac{i\,\mathrm{dilog}(1-i(1+$
parts	$\frac{\ln(1+x)\,\mathrm{arccot}(1+x)}{2}-\frac{i\ln(1+x)\ln(1+i(1+x))}{4}+\frac{i\ln(1+x)\ln(1-i(1+x))}{4}-\frac{i\,\mathrm{dilog}(1+i(1+x))}{4}+\frac{i\,\mathrm{dilog}(1-i(1+$

input
```
int(arccot(1+x)/(2+2*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*I*dilog(-I*x+1-I)+1/4*Pi*ln(-I-I*x)-1/4*I*dilog(I*x+1+I)
```

3.126.5 Fricas [F]

$$\int \frac{\cot^{-1}(1+x)}{2+2x}\,dx=\int \frac{\mathrm{arccot}\,(x+1)}{2\,(x+1)}\,dx$$

input
```
integrate(arccot(1+x)/(2+2*x),x, algorithm="fricas")
```

output
```
integral(1/2*arccot(x + 1)/(x + 1), x)
```

3.126.6 Sympy [F]

$$\int \frac{\cot^{-1}(1+x)}{2+2x}\,dx = \frac{\int \frac{\operatorname{acot}(x+1)}{x+1}\,dx}{2}$$

input
```
integrate(acot(1+x)/(2+2*x),x)
```

output
```
Integral(acot(x + 1)/(x + 1), x)/2
```

3.126.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(21) = 42.

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.83

$$\int \frac{\cot^{-1}(1+x)}{2+2x}\,dx = \frac{1}{4}\arctan(x+1,0)\log\left(x^2+2x+2\right) + \frac{1}{2}\operatorname{arccot}(x+1)\log(x+1) + \frac{1}{2}\arctan(x+1)\log(x+1) - \frac{1}{2}\arctan(x+1)\log(|x+1|) + \frac{1}{4}i\operatorname{Li}_2(ix+i+1) - \frac{1}{4}i\operatorname{Li}_2(-ix-i+1)$$

input
```
integrate(arccot(1+x)/(2+2*x),x, algorithm="maxima")
```

output
```
1/4*arctan2(x + 1, 0)*log(x^2 + 2*x + 2) + 1/2*arccot(x + 1)*log(x + 1) +
1/2*arctan(x + 1)*log(x + 1) - 1/2*arctan(x + 1)*log(abs(x + 1)) + 1/4*I*d
ilog(I*x + I + 1) - 1/4*I*dilog(-I*x - I + 1)
```

3.126.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \frac{\cot^{-1}(1+x)}{2+2x}\,dx = -\frac{1}{4}(x+1)^2\arctan\left(\frac{1}{x+1}\right) - \frac{1}{4}x - \frac{1}{4}\arctan\left(\frac{1}{x+1}\right) - \frac{1}{4}$$

input
```
integrate(arccot(1+x)/(2+2*x),x, algorithm="giac")
```

output
```
-1/4*(x + 1)^2*arctan(1/(x + 1)) - 1/4*x - 1/4*arctan(1/(x + 1)) - 1/4
```

3.126.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(1+x)}{2+2x}\,dx = \int \frac{\operatorname{acot}(x+1)}{2\,x+2}\,dx$$

input
```
int(acot(x + 1)/(2*x + 2),x)
```

output
```
int(acot(x + 1)/(2*x + 2), x)
```

3.127 $\int \frac{\cot^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx$

3.127.1 Optimal result

Integrand size = 19, antiderivative size = 45

$$\int \frac{\cot^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx=-\frac{i\,\text{PolyLog}\left(2,-\frac{i}{a+bx}\right)}{2d}+\frac{i\,\text{PolyLog}\left(2,\frac{i}{a+bx}\right)}{2d}$$

output
```
-1/2*I*polylog(2,-I/(b*x+a))/d+1/2*I*polylog(2,I/(b*x+a))/d
```

3.127.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \frac{\cot^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx=-\frac{i\left(\text{PolyLog}\left(2,-\frac{i}{a+bx}\right)-\text{PolyLog}\left(2,\frac{i}{a+bx}\right)\right)}{2d}$$

input
```
Integrate[ArcCot[a + b*x]/((a*d)/b + d*x),x]
```

output
```
((-1/2*I)*(PolyLog[2, (-I)/(a + b*x)] - PolyLog[2, I/(a + b*x)]))/d
```

3.127.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {5567, 27, 5356, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(a+bx)}{\frac{ad}{b}+dx}dx$$

$$\Big\downarrow \text{ 5567}$$

$$\frac{\int \frac{b\cot^{-1}(a+bx)}{d(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\cot^{-1}(a+bx)}{a+bx}d(a+bx)}{d}$$

$$\Big\downarrow \text{ 5356}$$

$$\frac{\frac{1}{2}i\int \frac{\log\left(1-\frac{i}{a+bx}\right)}{a+bx}d(a+bx)-\frac{1}{2}i\int \frac{\log\left(1+\frac{i}{a+bx}\right)}{a+bx}d(a+bx)}{d}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{\frac{1}{2}i\,\text{PolyLog}\left(2,\frac{i}{a+bx}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,-\frac{i}{a+bx}\right)}{d}$$

input
```
Int[ArcCot[a + b*x]/((a*d)/b + d*x),x]
```

output
```
((-1/2*I)*PolyLog[2, (-I)/(a + b*x)] + (I/2)*PolyLog[2, I/(a + b*x)])/d
```

3.127.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5356
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Simp[I*(b/2)   Int[Log[1 + I/(c*x)]/x, x], x] + Simp[I*(b/2)   Int[Log
[1 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5567
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcCot[x])^p, x],
 x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
 IGtQ[p, 0]
```

3.127.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

method	result
risch	$\frac{\pi\ln(-ibx-ia)}{2d}+\frac{i\,\mathrm{dilog}(-ibx-ia+1)}{2d}-\frac{i\,\mathrm{dilog}(ibx+ia+1)}{2d}$
parts	$\frac{\ln(bx+a)\,\mathrm{arccot}(bx+a)}{d}+\frac{-\frac{i\ln(bx+a)\ln(1+i(bx+a))}{2}+\frac{i\ln(bx+a)\ln(1-i(bx+a))}{2}-\frac{i\,\mathrm{dilog}(1+i(bx+a))}{2}+\frac{i\,\mathrm{dilog}(1-i(bx+a))}{2}}{d}$
derivativedivides	$\frac{\frac{b\ln(bx+a)\,\mathrm{arccot}(bx+a)}{d}+\frac{b\left(-\frac{i\ln(bx+a)\ln(1+i(bx+a))}{2}+\frac{i\ln(bx+a)\ln(1-i(bx+a))}{2}-\frac{i\,\mathrm{dilog}(1+i(bx+a))}{2}+\frac{i\,\mathrm{dilog}(1-i(bx+a))}{2}\right)}{d}}{b}$
default	$\frac{\frac{b\ln(bx+a)\,\mathrm{arccot}(bx+a)}{d}+\frac{b\left(-\frac{i\ln(bx+a)\ln(1+i(bx+a))}{2}+\frac{i\ln(bx+a)\ln(1-i(bx+a))}{2}-\frac{i\,\mathrm{dilog}(1+i(bx+a))}{2}+\frac{i\,\mathrm{dilog}(1-i(bx+a))}{2}\right)}{d}}{b}$

input
```
int(arccot(b*x+a)/(a*d/b+d*x),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*Pi*ln(-I*a-I*b*x)+1/2*I/d*dilog(1-I*a-I*b*x)-1/2*I/d*dilog(1+I*a+I*b
*x)
```

3.127.5 Fricas [F]

$$\int \frac{\cot^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx=\int \frac{\operatorname{arccot}(bx+a)}{dx+\frac{ad}{b}}\,dx$$

input
```
integrate(arccot(b*x+a)/(a*d/b+d*x),x, algorithm="fricas")
```

output
```
integral(b*arccot(b*x + a)/(b*d*x + a*d), x)
```

3.127.6 Sympy [F]

$$\int \frac{\cot^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx=\frac{b\int \frac{\operatorname{acot}(a+bx)}{a+bx}\,dx}{d}$$

input
```
integrate(acot(b*x+a)/(a*d/b+d*x),x)
```

output
```
b*Integral(acot(a + b*x)/(a + b*x), x)/d
```

3.127.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(31) = 62$.

Time = 0.32 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.71

$$\int \frac{\cot^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx=\frac{\operatorname{arccot}(bx+a)\log\left(dx+\frac{ad}{b}\right)}{d}+\frac{\arctan\left(\frac{b^2x+ab}{b}\right)\log\left(dx+\frac{ad}{b}\right)}{d}+\frac{\arctan(bx+a,0)\log\left(b^2x^2+2\,abx+a^2+1\right)-2\,\arctan(bx+a)\log\left(|bx+a|\right)+i\,\mathrm{Li}_2(i\,bx+i\,a+1)-}{2\,d}$$

input
```
integrate(arccot(b*x+a)/(a*d/b+d*x),x, algorithm="maxima")
```

output
```
arccot(b*x + a)*log(d*x + a*d/b)/d + arctan((b^2*x + a*b)/b)*log(d*x + a*d
/b)/d + 1/2*(arctan2(b*x + a, 0)*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*arct
an(b*x + a)*log(abs(b*x + a)) + I*dilog(I*b*x + I*a + 1) - I*dilog(-I*b*x
- I*a + 1))/d
```

3.127.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(31) = 62$.

Time = 0.56 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.29

$$\int \frac{\cot^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx =$$

$$-\frac{\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^4 + 2\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2 - 2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right.\right.}{8\,bd\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2}$$

input
```
integrate(arccot(b*x+a)/(a*d/b+d*x),x, algorithm="giac")
```

output
```
-1/8*(arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^4 + 2*arctan(1/(b*x
 + a))*tan(1/2*arctan(1/(b*x + a)))^2 - 2*tan(1/2*arctan(1/(b*x + a)))^3 +
 arctan(1/(b*x + a)) + 2*tan(1/2*arctan(1/(b*x + a))))/(b*d*tan(1/2*arctan
(1/(b*x + a)))^2)
```

3.127.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx = \int \frac{\mathrm{acot}(a+b\,x)}{d\,x+\frac{a\,d}{b}}\,dx$$

input
```
int(acot(a + b*x)/(d*x + (a*d)/b),x)
```

output
```
int(acot(a + b*x)/(d*x + (a*d)/b), x)
```

3.128 $\int (a+bx)^2 \sqrt{\cot^{-1}(a+bx)}\,dx$

3.128.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (a+bx)^2 \sqrt{\cot^{-1}(a+bx)}\,dx = \text{Int}\left((a+bx)^2 \sqrt{\cot^{-1}(a+bx)},x\right)$$

output
```
Unintegrable((b*x+a)^2*arccot(b*x+a)^(1/2),x)
```

3.128.2 Mathematica [N/A]

Not integrable

Time = 4.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (a+bx)^2 \sqrt{\cot^{-1}(a+bx)}\,dx = \int (a+bx)^2 \sqrt{\cot^{-1}(a+bx)}\,dx$$

input
```
Integrate[(a + b*x)^2*Sqrt[ArcCot[a + b*x]],x]
```

output
```
Integrate[(a + b*x)^2*Sqrt[ArcCot[a + b*x]], x]
```

3.128.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5573}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2\sqrt{\cot^{-1}(a+bx)}\,dx$$

$$\Big\downarrow \; 5573$$

$$\int (a+bx)^2\sqrt{\cot^{-1}(a+bx)}dx$$

input
```
Int[(a + b*x)^2*Sqrt[ArcCot[a + b*x]],x]
```

output
```
$Aborted
```

3.128.3.1 Defintions of rubi rules used

rule 5573
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_)*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Unintegrable[(e + f*x)^m*(a + b*ArcCot[c + d*x])^p, x] /;
FreeQ[{a, b, c, d, e, f, m, p}, x] &&  !IGtQ[p, 0]
```

3.128.4 Maple [N/A] (verified)

Not integrable

Time = 0.45 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int (bx+a)^2\sqrt{\operatorname{arccot}(bx+a)}dx$$

input
```
int((b*x+a)^2*arccot(b*x+a)^(1/2),x)
```

output
```
int((b*x+a)^2*arccot(b*x+a)^(1/2),x)
```

3.128.5 Fricas [F(-2)]

Exception generated.

$$\int (a+bx)^2\sqrt{\cot^{-1}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)^2*arccot(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.128.6 Sympy [N/A]

Not integrable

Time = 3.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int (a+bx)^2\sqrt{\cot^{-1}(a+bx)}\,dx = \int (a+bx)^2\,\sqrt{\operatorname{acot}(a+bx)}\,dx$$

input
```
integrate((b*x+a)**2*acot(b*x+a)**(1/2),x)
```

output
```
Integral((a + b*x)**2*sqrt(acot(a + b*x)), x)
```

3.128.7 Maxima [F(-2)]

Exception generated.

$$\int (a+bx)^2\sqrt{\cot^{-1}(a+bx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((b*x+a)^2*arccot(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.128.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (a+bx)^2\sqrt{\cot^{-1}(a+bx)}\,dx = \int (bx+a)^2\sqrt{\operatorname{arccot}(bx+a)}\,dx$$

input
```
integrate((b*x+a)^2*arccot(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x + a)^2*sqrt(arccot(b*x + a)), x)
```

3.128.9 Mupad [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (a+bx)^2\sqrt{\cot^{-1}(a+bx)}\,dx = \int \sqrt{\operatorname{acot}(a+b\,x)}\,(a+b\,x)^2\,dx$$

input
```
int(acot(a + b*x)^(1/2)*(a + b*x)^2,x)
```

output
```
int(acot(a + b*x)^(1/2)*(a + b*x)^2, x)
```

3.129 $\int (e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right) \, dx$

3.129.1 Optimal result

Integrand size = 18, antiderivative size = 233

$$\int (e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right) \, dx = \frac{bf\left(6d^2e^2-12cdef-\left(1-6c^2\right)f^2\right)x}{4d^3} + \frac{bf^2(de-cf)(c+dx)^2}{2d^4} + \frac{bf^3(c+dx)^3}{12d^4} + \frac{(e+fx)^4 \left(a+b\cot^{-1}(c+dx)\right)}{4f} + \frac{b\left(d^4e^4-4cd^3e^3f-6\left(1-c^2\right)d^2e^2f^2+4c\left(3-c^2\right)def^3+\left(1-6c^2+c^4\right)f^4\right)\arctan(c+dx)}{4d^4f} + \frac{b(de-cf)(de+f-cf)(de-(1+c)f)\log\left(1+(c+dx)^2\right)}{2d^4}$$

output
```
1/4*b*f*(6*d^2*e^2-12*c*d*e*f-(-6*c^2+1)*f^2)*x/d^3+1/2*b*f^2*(-c*f+d*e)*(
d*x+c)^2/d^4+1/12*b*f^3*(d*x+c)^3/d^4+1/4*(f*x+e)^4*(a+b*arccot(d*x+c))/f+
1/4*b*(d^4*e^4-4*c*d^3*e^3*f-6*(-c^2+1)*d^2*e^2*f^2+4*c*(-c^2+3)*d*e*f^3+(
c^4-6*c^2+1)*f^4)*arctan(d*x+c)/d^4/f+1/2*b*(-c*f+d*e)*(-c*f+d*e+f)*(d*e-(
1+c)*f)*ln(1+(d*x+c)^2)/d^4
```

3.129.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.67

$$\int (e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right) \, dx$$
$$= \frac{(e+fx)^4 \left(a+b\cot^{-1}(c+dx)\right)+\frac{b\left(6df^2\left(6d^2e^2-12cdef+(-1+6c^2)f^2\right)x+12f^3(de-cf)(c+dx)^2+2f^4(c+dx)^3-3i(de-(-i+c)f)^4 \mathrm{l}\right.}{6d^4}}{4f}$$

input
```
Integrate[(e + f*x)^3*(a + b*ArcCot[c + d*x]),x]
```

output
```
((e + f*x)^4*(a + b*ArcCot[c + d*x]) + (b*(6*d*f^2*(6*d^2*e^2 - 12*c*d*e*f
 + (-1 + 6*c^2)*f^2)*x + 12*f^3*(d*e - c*f)*(c + d*x)^2 + 2*f^4*(c + d*x)^
3 - (3*I)*(d*e - (-I + c)*f)^4*Log[I - c - d*x] + (3*I)*(d*e - (I + c)*f)^
4*Log[I + c + d*x]))/(6*d^4))/(4*f)
```

3.129.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5571, 27, 5388, 478, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right) \, dx$$
$$\Big\downarrow \text{ 5571}$$
$$\frac{\int \frac{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)^3 (a+b\cot^{-1}(c+dx))}{d^3}d(c+dx)}{d}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{\int (de-cf+f(c+dx))^3 \left(a+b\cot^{-1}(c+dx)\right)d(c+dx)}{d^4}$$
$$\Big\downarrow \text{ 5388}$$
$$\frac{\frac{b\int \frac{(de-cf+f(c+dx))^4}{(c+dx)^2+1}d(c+dx)}{4f}+\frac{(f(c+dx)-cf+de)^4(a+b\cot^{-1}(c+dx))}{4f}}{d^4}$$

$\Big\downarrow$ 478

$$\frac{\frac{b\int \left((c+dx)^2f^4+4(de-cf)(c+dx)f^3+\left(6d^2e^2-12cdfe-\left(1-6c^2\right)f^2\right)f^2+\frac{d^4e^4-4cd^3fe^3-6\left(1-c^2\right)d^2f^2e^2+4c\left(3-c^2\right)df^3e+\left(c^4-6c^2+1\right)f^4+4f(de-cf)(d}{(c+dx)^2+1}}{4f}}{d^4}$$

$\Big\downarrow$ 2009

$$\frac{\frac{(f(c+dx)-cf+de)^4\left(a+b\cot^{-1}(c+dx)\right)}{4f}+\frac{b\left(\arctan(c+dx)\left(-6\left(1-c^2\right)d^2e^2f^2+4c\left(3-c^2\right)def^3+\left(c^4-6c^2+1\right)f^4-4cd^3e^3f+d^4e^4\right)+f^2(c+dx)\left(-(1\right.\right.}{}}{d^4}$$

input
```
Int[(e + f*x)^3*(a + b*ArcCot[c + d*x]),x]
```

output
```
(((d*e - c*f + f*(c + d*x))^4*(a + b*ArcCot[c + d*x]))/(4*f) + (b*(f^2*(6*
d^2*e^2 - 12*c*d*e*f - (1 - 6*c^2)*f^2)*(c + d*x) + 2*f^3*(d*e - c*f)*(c +
 d*x)^2 + (f^4*(c + d*x)^3)/3 + (d^4*e^4 - 4*c*d^3*e^3*f - 6*(1 - c^2)*d^2
*e^2*f^2 + 4*c*(3 - c^2)*d*e*f^3 + (1 - 6*c^2 + c^4)*f^4)*ArcTan[c + d*x]
+ 2*f*(d*e - c*f)*(d*e + f - c*f)*(d*e - (1 + c)*f)*Log[1 + (c + d*x)^2]))
/(4*f))/d^4
```

3.129.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 478
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ
[n, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5388
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])/(e*(q + 1))), x] + Simp[b*(
c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b
, c, d, e, q}, x] && NeQ[q, -1]
```

rule 5571
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.129.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(221) = 442$.

Time = 0.88 (sec) , antiderivative size = 496, normalized size of antiderivative = 2.13

method	result
parts	$\frac{a(fx+e)^4}{4f} - \frac{f^3bc\,x^2}{4d^2} + \frac{f^2be\,x^2}{2d} + \frac{3f^3b\,c^2x}{4d^3} + \frac{3fb\,e^2x}{2d} + \frac{b\,f^3\operatorname{arccot}(dx+c)x^4}{4} + b\operatorname{arccot}(dx+c)\,x\,e^3$
derivativedivides	$\frac{\frac{a(cf-de-f(dx+c))^4}{4d^3f} - \frac{b\left(-\frac{f^3\operatorname{arccot}(dx+c)c^4}{4} + f^2\operatorname{arccot}(dx+c)c^3de + f^3\operatorname{arccot}(dx+c)c^3(dx+c) - \frac{3f\operatorname{arccot}(dx+c)c^2d^2e^2}{2} - 3f^2\operatorname{arc}\right.}{}}{}$
default	$\frac{\frac{a(cf-de-f(dx+c))^4}{4d^3f} - \frac{b\left(-\frac{f^3\operatorname{arccot}(dx+c)c^4}{4} + f^2\operatorname{arccot}(dx+c)c^3de + f^3\operatorname{arccot}(dx+c)c^3(dx+c) - \frac{3f\operatorname{arccot}(dx+c)c^2d^2e^2}{2} - 3f^2\operatorname{arc}\right.}{}}{}$
parallelrisch	$-\frac{-42b\,c^2de\,f^2+24ac\,d^3e^3+6\ln\left(d^2x^2+2cdx+c^2+1\right)b\,c^3f^3-6\ln\left(d^2x^2+2cdx+c^2+1\right)b\,d^3e^3-6\ln\left(d^2x^2+2cdx+c^2+1\right)bc}{}$
risch	$\frac{x^4f^3a}{4} + x\,e^3a - \frac{f^3bc\,x^2}{4d^2} + \frac{f^2be\,x^2}{2d} + \frac{3f^3b\,c^2x}{4d^3} + \frac{3fb\,e^2x}{2d} - \frac{3fb\,e^2\arctan(dx+c)}{2d^2} - \frac{f^2be\ln\left(d^2x^2+2cdx+c\right.}{2d^3}$

input
```
int((f*x+e)^3*(a+b*arccot(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*(f*x+e)^4/f-1/4/d^2*f^3*b*c*x^2+1/2/d*f^2*b*e*x^2+3/4/d^3*f^3*b*c^2*
x+3/2/d*f*b*e^2*x+1/4*b*f^3*arccot(d*x+c)*x^4+b*arccot(d*x+c)*x*e^3+1/4*b/
f*arccot(d*x+c)*e^4-3/2/d^2*f*b*e^2*arctan(d*x+c)+1/4/d^4*f^3*b*c^4*arctan
(d*x+c)-3/2/d^4*f^3*b*c^2*arctan(d*x+c)-1/d*b*c*e^3*arctan(d*x+c)-1/4*b/d^
4*f^3*c+13/12*b/d^4*f^3*c^3+1/2*b/d*ln(1+(d*x+c)^2)*e^3+1/4/d^4*f^3*b*arct
an(d*x+c)+1/12/d*f^3*b*x^3-1/4/d^3*f^3*b*x+1/4/f*b*e^4*arctan(d*x+c)+3/2*b
/d^3*f^2*ln(1+(d*x+c)^2)*c^2*e-1/2*b/d^4*f^3*ln(1+(d*x+c)^2)*c^3+3/2*b*f*a
rccot(d*x+c)*e^2*x^2+b*f^2*arccot(d*x+c)*e*x^3+1/2*b/d^4*f^3*ln(1+(d*x+c)^
2)*c-1/2*b/d^3*f^2*ln(1+(d*x+c)^2)*e-2/d^2*f^2*b*c*e*x-1/d^3*f^2*b*c^3*e*a
rctan(d*x+c)+3/2/d^2*f*b*c^2*e^2*arctan(d*x+c)+3/d^3*f^2*b*c*e*arctan(d*x+
c)-5/2*b/d^3*f^2*c^2*e+3/2*b/d^2*f*c*e^2-3/2*b/d^2*f*ln(1+(d*x+c)^2)*c*e^2
```

3.129.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.39

$$\int (e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right) dx$$
$$= \frac{3\,ad^4f^3x^4 + (12\,ad^4ef^2 + bd^3f^3)x^3 + 3\,(6\,ad^4e^2f + 2\,bd^3ef^2 - bcd^2f^3)x^2 + 3\,(4\,ad^4e^3 + 6\,bd^3e^2f - 8\,bcd^2}{}$$

input
```
integrate((f*x+e)^3*(a+b*arccot(d*x+c)),x, algorithm="fricas")
```

output
```
1/12*(3*a*d^4*f^3*x^4 + (12*a*d^4*e*f^2 + b*d^3*f^3)*x^3 + 3*(6*a*d^4*e^2*
f + 2*b*d^3*e*f^2 - b*c*d^2*f^3)*x^2 + 3*(4*a*d^4*e^3 + 6*b*d^3*e^2*f - 8*
b*c*d^2*e*f^2 + (3*b*c^2 - b)*d*f^3)*x + 3*(b*d^4*f^3*x^4 + 4*b*d^4*e*f^2*
x^3 + 6*b*d^4*e^2*f*x^2 + 4*b*d^4*e^3*x)*arccot(d*x + c) - 3*(4*b*c*d^3*e^
3 - 6*(b*c^2 - b)*d^2*e^2*f + 4*(b*c^3 - 3*b*c)*d*e*f^2 - (b*c^4 - 6*b*c^2
 + b)*f^3)*arctan(d*x + c) + 6*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + (3*b*c^2 - b
)*d*e*f^2 - (b*c^3 - b*c)*f^3)*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/d^4
```

3.129.6 Sympy [F(-1)]

Timed out.

$$\int (e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right) dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*(a+b*acot(d*x+c)),x)
```

output
```
Timed out
```

3.129.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.46

$$\int (e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right)\,dx = \frac{1}{4}\,af^3x^4+aef^2x^3+\frac{3}{2}\,ae^2fx^2$$
$$+\frac{3}{2}\left(x^2\operatorname{arccot}\left(dx+c\right)+d\left(\frac{x}{d^2}+\frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3}-\frac{c\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^3}\right)\right)be^2f$$
$$+\frac{1}{2}\left(2\,x^3\operatorname{arccot}\left(dx+c\right)+d\left(\frac{dx^2-4\,cx}{d^3}-\frac{2\,(c^3-3\,c)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^4}+\frac{(3\,c^2-1)\log\left(d^2x^2+2\,cdx+\right.}{d^4}\right.\right.$$
$$+\frac{1}{12}\left(3\,x^4\operatorname{arccot}\left(dx+c\right)+d\left(\frac{d^2x^3-3\,cdx^2+3\,(3\,c^2-1)x}{d^4}+\frac{3\,(c^4-6\,c^2+1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^5}-\frac{6\,(c}{}\right.\right.$$
$$+ae^3x+\frac{\left(2\,(dx+c)\operatorname{arccot}\left(dx+c\right)+\log\left((dx+c)^2+1\right)\right)be^3}{2\,d}$$

input
```
integrate((f*x+e)^3*(a+b*arccot(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*a*f^3*x^4 + a*e*f^2*x^3 + 3/2*a*e^2*f*x^2 + 3/2*(x^2*arccot(d*x + c) +
 d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*
x + c^2 + 1)/d^3))*b*e^2*f + 1/2*(2*x^3*arccot(d*x + c) + d*((d*x^2 - 4*c*
x)/d^3 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d^2*x
^2 + 2*c*d*x + c^2 + 1)/d^4))*b*e*f^2 + 1/12*(3*x^4*arccot(d*x + c) + d*((
d^2*x^3 - 3*c*d*x^2 + 3*(3*c^2 - 1)*x)/d^4 + 3*(c^4 - 6*c^2 + 1)*arctan((d
^2*x + c*d)/d)/d^5 - 6*(c^3 - c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^5))*b*
f^3 + a*e^3*x + 1/2*(2*(d*x + c)*arccot(d*x + c) + log((d*x + c)^2 + 1))*b
*e^3/d
```

3.129.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2265 vs. $2(216) = 432$.

Time = 1.73 (sec) , antiderivative size = 2265, normalized size of antiderivative = 9.72

$$\int (e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right)\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*(a+b*arccot(d*x+c)),x, algorithm="giac")
```

output
```
-1/192*(96*b*d^3*e^3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^5 -
288*b*c*d^2*e^2*f*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^5 + 288
*b*c^2*d*e*f^2*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^5 - 96*b*c
^3*f^3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^5 - 72*b*d^2*e^2*f
*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^6 + 144*b*c*d*e*f^2*arct
an(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^6 - 72*b*c^2*f^3*arctan(1/(d*
x + c))*tan(1/2*arctan(1/(d*x + c)))^6 + 24*b*d*e*f^2*arctan(1/(d*x + c))*
tan(1/2*arctan(1/(d*x + c)))^7 - 24*b*c*f^3*arctan(1/(d*x + c))*tan(1/2*ar
ctan(1/(d*x + c)))^7 - 3*b*f^3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x +
 c)))^8 + 96*b*d^3*e^3*log(16*tan(1/2*arctan(1/(d*x + c)))^2/(tan(1/2*arct
an(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/2*arctan
(1/(d*x + c)))^4 - 288*b*c*d^2*e^2*f*log(16*tan(1/2*arctan(1/(d*x + c)))^2
/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*
tan(1/2*arctan(1/(d*x + c)))^4 + 288*b*c^2*d*e*f^2*log(16*tan(1/2*arctan(1
/(d*x + c)))^2/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x +
 c)))^2 + 1))*tan(1/2*arctan(1/(d*x + c)))^4 - 96*b*c^3*f^3*log(16*tan(1/2
*arctan(1/(d*x + c)))^2/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan
(1/(d*x + c)))^2 + 1))*tan(1/2*arctan(1/(d*x + c)))^4 + 96*a*d^3*e^3*tan(1
/2*arctan(1/(d*x + c)))^5 - 288*a*c*d^2*e^2*f*tan(1/2*arctan(1/(d*x + c)))
^5 + 288*a*c^2*d*e*f^2*tan(1/2*arctan(1/(d*x + c)))^5 - 96*a*c^3*f^3*ta...
```

3.129. $\int (e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right) \, dx$

3.129.9 Mupad [B] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 783, normalized size of antiderivative = 3.36

$$\int (e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right)\,dx = \mathrm{acot}(c+d\,x)\,\left(b\,e^3\,x+\frac{3\,b\,e^2\,f\,x^2}{2}+b\,e\,f^2\,x^3+\frac{b\,f^3\,x^4}{4}\right)$$
$$+x\left(\frac{e\,(6\,a\,c^2\,f^2+12\,a\,c\,d\,e\,f+2\,a\,d^2\,e^2+3\,b\,d\,e\,f+6\,a\,f^2)}{2\,d^2}-\frac{(4\,c^2+4)\,\left(\frac{f^2\,(b\,f+8\,a\,c\,f+12\,a\,d\,e)}{4\,d}-\frac{2\,a\,c\,f^3}{d}\right)}{4\,d^2}+\frac{2\,c\,\left(\frac{2\,c\,\left(\frac{f^2\,(b\,f+8\,a\,c\,f+12\,a\,d\,e)}{4\,d}-\frac{2\,a\,c\,f^3}{d}\right)}{d}-\frac{4\,a\,c^2\,f^3+24\,a\,c\,d\,e\,f^2+12\,a\,d^2\,e^2\,f+4\,b\,d\,e\,f^2+4\,a\,f^3}{4\,d^2}+\frac{a\,f^3\,(4\,c^2+4)}{4\,d^2}\right)}{d}\right)$$
$$-x^2\left(\frac{c\,\left(\frac{f^2\,(b\,f+8\,a\,c\,f+12\,a\,d\,e)}{4\,d}-\frac{2\,a\,c\,f^3}{d}\right)}{d}-\frac{4\,a\,c^2\,f^3+24\,a\,c\,d\,e\,f^2+12\,a\,d^2\,e^2\,f+4\,b\,d\,e\,f^2+4\,a\,f^3}{8\,d^2}+\frac{a\,f^3\,(4\,c^2+4)}{8\,d^2}\right)$$
$$+x^3\left(\frac{f^2\,(b\,f+8\,a\,c\,f+12\,a\,d\,e)}{12\,d}-\frac{2\,a\,c\,f^3}{3\,d}\right)+\frac{a\,f^3\,x^4}{4}$$
$$+\frac{\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)\,\left(-64\,b\,c^3\,d^4\,f^3+192\,b\,c^2\,d^5\,e\,f^2-192\,b\,c\,d^6\,e^2\,f+64\,b\,c\,d^4\,f^3+64\,b\,d^7\,e^3-\cdots\right.}{128\,d^8}$$
$$+\frac{b\,\mathrm{atan}\left(\frac{4\,d^3\,\left(\frac{c\,\left(c^4\,f^3-4\,c^3\,d\,e\,f^2+6\,c^2\,d^2\,e^2\,f-6\,c^2\,f^3-4\,c\,d^3\,e^3+12\,c\,d\,e\,f^2-6\,d^2\,e^2\,f+f^3\right)}{4\,d^3}+\frac{x\,\left(c^4\,f^3-4\,c^3\,d\,e\,f^2+6\,c^2\,d^2\,e^2\,f-6\,c^2\,f^3-4\,c\,d^3\,e^3+12\cdots\right.}{4\,d^2}\cdots\right.}{c^4\,f^3-4\,c^3\,d\,e\,f^2+6\,c^2\,d^2\,e^2\,f-6\,c^2\,f^3-4\,c\,d^3\,e^3+12\,c\,d\,e\,f^2-6\,d^2\,e^2\,f+f^3\cdots}\right.}{4\,d\cdots}$$

input

```
int((e + f*x)^3*(a + b*acot(c + d*x)),x)
```

output
```
acot(c + d*x)*((b*f^3*x^4)/4 + b*e^3*x + (3*b*e^2*f*x^2)/2 + b*e*f^2*x^3)
+ x*((e*(6*a*f^2 + 6*a*c^2*f^2 + 2*a*d^2*e^2 + 3*b*d*e*f + 12*a*c*d*e*f))/
(2*d^2) - ((4*c^2 + 4)*((f^2*(b*f + 8*a*c*f + 12*a*d*e))/(4*d) - (2*a*c*f^
3)/d))/(4*d^2) + (2*c*((2*c*((f^2*(b*f + 8*a*c*f + 12*a*d*e))/(4*d) - (2*a
*c*f^3)/d))/d - (4*a*f^3 + 4*a*c^2*f^3 + 4*b*d*e*f^2 + 12*a*d^2*e^2*f + 24
*a*c*d*e*f^2)/(4*d^2) + (a*f^3*(4*c^2 + 4))/(4*d^2)))/d) - x^2*((c*((f^2*(
b*f + 8*a*c*f + 12*a*d*e))/(4*d) - (2*a*c*f^3)/d))/d - (4*a*f^3 + 4*a*c^2*
f^3 + 4*b*d*e*f^2 + 12*a*d^2*e^2*f + 24*a*c*d*e*f^2)/(8*d^2) + (a*f^3*(4*c
^2 + 4))/(8*d^2)) + x^3*((f^2*(b*f + 8*a*c*f + 12*a*d*e))/(12*d) - (2*a*c*
f^3)/(3*d)) + (a*f^3*x^4)/4 + (log(c^2 + d^2*x^2 + 2*c*d*x + 1)*(64*b*d^7*
e^3 - 64*b*c^3*d^4*f^3 + 64*b*c*d^4*f^3 - 64*b*d^5*e*f^2 - 192*b*c*d^6*e^2
*f + 192*b*c^2*d^5*e*f^2))/(128*d^8) + (b*atan((4*d^3*((c*(f^3 - 6*c^2*f^3
 + c^4*f^3 - 4*c*d^3*e^3 - 6*d^2*e^2*f + 6*c^2*d^2*e^2*f + 12*c*d*e*f^2 -
4*c^3*d*e*f^2))/(4*d^3) + (x*(f^3 - 6*c^2*f^3 + c^4*f^3 - 4*c*d^3*e^3 - 6*
d^2*e^2*f + 6*c^2*d^2*e^2*f + 12*c*d*e*f^2 - 4*c^3*d*e*f^2))/(4*d^2)))/(f^
3 - 6*c^2*f^3 + c^4*f^3 - 4*c*d^3*e^3 - 6*d^2*e^2*f + 6*c^2*d^2*e^2*f + 12
*c*d*e*f^2 - 4*c^3*d*e*f^2))*(f^3 - 6*c^2*f^3 + c^4*f^3 - 4*c*d^3*e^3 - 6*
d^2*e^2*f + 6*c^2*d^2*e^2*f + 12*c*d*e*f^2 - 4*c^3*d*e*f^2))/(4*d^4)
```

3.129. $\int (e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right)\,dx$

3.130 $\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right) \, dx$

3.130.1 Optimal result

Integrand size = 18, antiderivative size = 154

$$\begin{aligned}\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right) \, dx &= \frac{bf(de-cf)x}{d^2}+\frac{bf^2(c+dx)^2}{6d^3}+\frac{(e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right)}{3f} \\ &\quad+\frac{b(de-cf)\left(d^2e^2-2cdef-(3-c^2)\,f^2\right)\arctan(c+dx)}{3d^3f} \\ &\quad+\frac{b(3d^2e^2-6cdef-(1-3c^2)\,f^2)\log\left(1+(c+dx)^2\right)}{6d^3}\end{aligned}$$

output
```
b*f*(-c*f+d*e)*x/d^2+1/6*b*f^2*(d*x+c)^2/d^3+1/3*(f*x+e)^3*(a+b*arccot(d*x
+c))/f+1/3*b*(-c*f+d*e)*(d^2*e^2-2*c*d*e*f-(-c^2+3)*f^2)*arctan(d*x+c)/d^3
/f+1/6*b*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*ln(1+(d*x+c)^2)/d^3
```

3.130.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.77

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right) \, dx = \frac{(e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right)+\frac{b\left(6df^2(de-cf)x+f^3(c+dx)^2-i(de-(-i+c)f)^3\log(i-c-dx)+i(de-(i+c)f)^3\log(i+c+dx)\right)}{2d^3}}{3f}$$

input
```
Integrate[(e + f*x)^2*(a + b*ArcCot[c + d*x]),x]
```

output
```
((e + f*x)^3*(a + b*ArcCot[c + d*x]) + (b*(6*d*f^2*(d*e - c*f)*x + f^3*(c
+ d*x)^2 - I*(d*e - (-I + c)*f)^3*Log[I - c - d*x] + I*(d*e - (I + c)*f)^3
*Log[I + c + d*x]))/(2*d^3))/(3*f)
```

3.130.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5571, 27, 5388, 478, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right) dx$$

$$\Big\downarrow \; 5571$$

$$\frac{\int \frac{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)^2 \left(a+b\cot^{-1}(c+dx)\right)}{d^2}d(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\int (de-cf+f(c+dx))^2 \left(a+b\cot^{-1}(c+dx)\right)d(c+dx)}{d^3}$$

$$\Big\downarrow \; 5388$$

$$\frac{\frac{b\int \frac{(de-cf+f(c+dx))^3}{(c+dx)^2+1}d(c+dx)}{3f}+\frac{(f(c+dx)-cf+de)^3\left(a+b\cot^{-1}(c+dx)\right)}{3f}}{d^3}$$

$$\Big\downarrow \; 478$$

$$\frac{\frac{b\int \left((c+dx)f^3+3(de-cf)f^2+\frac{(de-cf)\left(d^2e^2-2cdfe+c^2f^2-3f^2\right)+f\left(3d^2e^2-6cdfe-\left(1-3c^2\right)f^2\right)(c+dx)}{(c+dx)^2+1}\right)d(c+dx)}{3f}+\frac{(f(c+dx)-cf+de)^3 (a+b\cot^{-1}(c \ldots}{3f}}{d^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{(f(c+dx)-cf+de)^3\left(a+b\cot^{-1}(c+dx)\right)}{3f}+\frac{b(\arctan(c+dx)(de-cf)\left(-(3-c^2)f^2-2cdef+d^2e^2\right)+\frac{1}{2}f\left(-(1-3c^2)f^2-6cdef+3d^2e^2\right)\log((c+dx)^2+ \ldots}{3f}}{d^3}$$

input
```
Int[(e + f*x)^2*(a + b*ArcCot[c + d*x]),x]
```

output
```
(((d*e - c*f + f*(c + d*x))^3*(a + b*ArcCot[c + d*x]))/(3*f) + (b*(3*f^2*(
d*e - c*f)*(c + d*x) + (f^3*(c + d*x)^2)/2 + (d*e - c*f)*(d^2*e^2 - 2*c*d*
e*f - (3 - c^2)*f^2)*ArcTan[c + d*x] + (f*(3*d^2*e^2 - 6*c*d*e*f - (1 - 3*
c^2)*f^2)*Log[1 + (c + d*x)^2])/2))/(3*f))/d^3
```

3.130.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 478
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ
[n, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5388
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])/(e*(q + 1))), x] + Simp[b*(
c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b
, c, d, e, q}, x] && NeQ[q, -1]
```

rule 5571
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.130.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(146) = 292.

Time = 0.68 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.91

method	result
parts	$\frac{a(fx+e)^3}{3f}-\frac{2f^2bcx}{3d^2}+\frac{fbex}{d}+\frac{b\,f^2\ln\left(1+(dx+c)^2\right)c^2}{2d^3}+\frac{b\,e^2\ln\left(1+(dx+c)^2\right)}{2d}-\frac{f^2b\,c^3\arctan(dx+c)}{3d^3}+\frac{b\,e^3\,\text{ar}}{}$
parallelrisch	$\frac{-f^2b-\ln(d^2x^2+2cdx+c^2+1)b\,f^2+7b\,c^2f^2-6fead-6a\,c^2efd+2x^3\operatorname{arccot}(dx+c)b\,d^3f^2+6x\operatorname{arccot}(dx+c)b\,d^3e^2+6\operatorname{arcc}}{}$
derivativedivides	$\frac{-\frac{a(cf-de-f(dx+c))^3}{3d^2f}+\frac{b\left(-\frac{f^2\operatorname{arccot}(dx+c)c^3}{3}+f\operatorname{arccot}(dx+c)c^2de+f^2\operatorname{arccot}(dx+c)c^2(dx+c)-\operatorname{arccot}(dx+c)c\,d^2e^2-2f\operatorname{arccot}(}{}}{}$
default	$\frac{-\frac{a(cf-de-f(dx+c))^3}{3d^2f}+\frac{b\left(-\frac{f^2\operatorname{arccot}(dx+c)c^3}{3}+f\operatorname{arccot}(dx+c)c^2de+f^2\operatorname{arccot}(dx+c)c^2(dx+c)-\operatorname{arccot}(dx+c)c\,d^2e^2-2f\operatorname{arccot}(}{}}{}$
risch	$-\frac{ifbe\,x^2\ln(1-i(dx+c))}{2}-\frac{ib\,e^3\ln(d^2x^2+2cdx+c^2+1)}{12f}-\frac{f^2b\ln(d^2x^2+2cdx+c^2+1)}{6d^3}+\frac{i(fx+e)^3b\ln(1+i(dx+c))}{6f}$

input
```
int((f*x+e)^2*(a+b*arccot(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*(f*x+e)^3/f-2/3*f^2/d^2*b*c*x+f/d*b*e*x+1/2*b/d^3*f^2*ln(1+(d*x+c)^2
)*c^2+1/2*b*e^2*ln(1+(d*x+c)^2)/d-1/3*f^2/d^3*b*c^3*arctan(d*x+c)+1/3/f*b*
e^3*arctan(d*x+c)+f^2/d^3*b*c*arctan(d*x+c)-f/d^2*b*e*arctan(d*x+c)+1/3*b*
f^2*arccot(d*x+c)*x^3+b*arccot(d*x+c)*x*e^2-5/6*b/d^3*f^2*c^2+b/d^2*f*c*e+
1/3*b/f*arccot(d*x+c)*e^3+1/6*f^2/d*b*x^2-1/6*b/d^3*f^2*ln(1+(d*x+c)^2)-b/
d^2*f*ln(1+(d*x+c)^2)*c*e+f/d^2*b*c^2*e*arctan(d*x+c)-1/d*b*c*e^2*arctan(d
*x+c)+b*f*arccot(d*x+c)*e*x^2
```

3.130.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.34

$$\int (e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)\,dx$$
$$=\frac{2\,ad^3f^2x^3+(6\,ad^3ef+bd^2f^2)x^2+2\,(3\,ad^3e^2+3\,bd^2ef-2\,bcdf^2)x+2\,(bd^3f^2x^3+3\,bd^3efx^2+3\,bd^3e^2x)}{}$$

input
```
integrate((f*x+e)^2*(a+b*arccot(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(2*a*d^3*f^2*x^3 + (6*a*d^3*e*f + b*d^2*f^2)*x^2 + 2*(3*a*d^3*e^2 + 3*
b*d^2*e*f - 2*b*c*d*f^2)*x + 2*(b*d^3*f^2*x^3 + 3*b*d^3*e*f*x^2 + 3*b*d^3*
e^2*x)*arccot(d*x + c) - 2*(3*b*c*d^2*e^2 - 3*(b*c^2 - b)*d*e*f + (b*c^3 -
 3*b*c)*f^2)*arctan(d*x + c) + (3*b*d^2*e^2 - 6*b*c*d*e*f + (3*b*c^2 - b)*
f^2)*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/d^3
```

3.130.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 125.91 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.44

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)\,dx$$
$$= \begin{cases} ae^2x + aefx^2 + \frac{af^2x^3}{3} + \frac{bc^3f^2\,\mathrm{acot}\,(c+dx)}{3d^3} - \frac{bc^2ef\,\mathrm{acot}\,(c+dx)}{d^2} + \frac{bc^2f^2\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{d^3} + \frac{ibc^2f^2\,\mathrm{acot}\,(c+dx)}{d^3} + \frac{bce^2\,\mathrm{acot}\,(c+d}{d} \\ (a+b\,\mathrm{acot}\,(c))\left(e^2x + efx^2 + \frac{f^2x^3}{3}\right) \end{cases}$$

input
```
integrate((f*x+e)**2*(a+b*acot(d*x+c)),x)
```

output
```
Piecewise((a*e**2*x + a*e*f*x**2 + a*f**2*x**3/3 + b*c**3*f**2*acot(c + d*
x)/(3*d**3) - b*c**2*e*f*acot(c + d*x)/d**2 + b*c**2*f**2*log(c/d + x - I/
d)/d**3 + I*b*c**2*f**2*acot(c + d*x)/d**3 + b*c*e**2*acot(c + d*x)/d - 2*
b*c*e*f*log(c/d + x - I/d)/d**2 - 2*I*b*c*e*f*acot(c + d*x)/d**2 - 2*b*c*f
**2*x/(3*d**2) - b*c*f**2*acot(c + d*x)/d**3 + b*e**2*x*acot(c + d*x) + b*
e*f*x**2*acot(c + d*x) + b*f**2*x**3*acot(c + d*x)/3 + b*e**2*log(c/d + x
- I/d)/d + I*b*e**2*acot(c + d*x)/d + b*e*f*x/d + b*f**2*x**2/(6*d) + b*e*
f*acot(c + d*x)/d**2 - b*f**2*log(c/d + x - I/d)/(3*d**3) - I*b*f**2*acot(
c + d*x)/(3*d**3), Ne(d, 0)), ((a + b*acot(c))*(e**2*x + e*f*x**2 + f**2*x
**3/3), True))
```

3.130.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.40

$$\int (e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)\,dx = \frac{1}{3}\,af^2x^3 + aefx^2$$
$$+\left(x^2\,\mathrm{arccot}\left(dx+c\right)+d\left(\frac{x}{d^2}+\frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3}-\frac{c\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^3}\right)\right)bef$$
$$+\frac{1}{6}\left(2\,x^3\,\mathrm{arccot}\left(dx+c\right)+d\left(\frac{dx^2-4\,cx}{d^3}-\frac{2\,(c^3-3\,c)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^4}+\frac{(3\,c^2-1)\log\left(d^2x^2+2\,cdx+\right.}{d^4}\right.\right.$$
$$+ae^2x+\frac{\left(2\,(dx+c)\,\mathrm{arccot}\left(dx+c\right)+\log\left((dx+c)^2+1\right)\right)be^2}{2\,d}$$

input
```
integrate((f*x+e)^2*(a+b*arccot(d*x+c)),x, algorithm="maxima")
```

output
```
1/3*a*f^2*x^3 + a*e*f*x^2 + (x^2*arccot(d*x + c) + d*(x/d^2 + (c^2 - 1)*ar
ctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*b*e*f
 + 1/6*(2*x^3*arccot(d*x + c) + d*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arc
tan((d^2*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^
4))*b*f^2 + a*e^2*x + 1/2*(2*(d*x + c)*arccot(d*x + c) + log((d*x + c)^2 +
 1))*b*e^2/d
```

3.130.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1161 vs. 2(142) = 284.

Time = 1.23 (sec) , antiderivative size = 1161, normalized size of antiderivative = 7.54

$$\int (e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*(a+b*arccot(d*x+c)),x, algorithm="giac")
```

output
```
-1/24*(12*b*d^2*e^2*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^4 - 2
4*b*c*d*e*f*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^4 + 12*b*c^2*
f^2*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^4 - 6*b*d*e*f*arctan(
1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^5 + 6*b*c*f^2*arctan(1/(d*x + c)
)*tan(1/2*arctan(1/(d*x + c)))^5 + b*f^2*arctan(1/(d*x + c))*tan(1/2*arcta
n(1/(d*x + c)))^6 + 12*b*d^2*e^2*log(16*tan(1/2*arctan(1/(d*x + c)))^2/(ta
n(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(
1/2*arctan(1/(d*x + c)))^3 - 24*b*c*d*e*f*log(16*tan(1/2*arctan(1/(d*x + c
)))^2/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 +
 1))*tan(1/2*arctan(1/(d*x + c)))^3 + 12*b*c^2*f^2*log(16*tan(1/2*arctan(1
/(d*x + c)))^2/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x +
 c)))^2 + 1))*tan(1/2*arctan(1/(d*x + c)))^3 + 12*a*d^2*e^2*tan(1/2*arctan
(1/(d*x + c)))^4 - 24*a*c*d*e*f*tan(1/2*arctan(1/(d*x + c)))^4 + 12*a*c^2*
f^2*tan(1/2*arctan(1/(d*x + c)))^4 - 6*a*d*e*f*tan(1/2*arctan(1/(d*x + c))
)^5 + 6*a*c*f^2*tan(1/2*arctan(1/(d*x + c)))^5 + a*f^2*tan(1/2*arctan(1/(d
*x + c)))^6 - 12*b*d^2*e^2*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c))
)^2 + 24*b*c*d*e*f*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^2 - 12
*b*c^2*f^2*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^2 - 12*b*d*e*f
*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^3 + 12*b*c*f^2*arctan(1/
(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^3 + 12*b*d*e*f*tan(1/2*arctan(1...
```

3.130.9 Mupad [B] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.66

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right) dx = x^2 \left(\frac{f\,(b\,f+6\,a\,c\,f+6\,a\,d\,e)}{6\,d}-\frac{a\,c\,f^2}{d}\right) - x \left(\frac{2\,c\left(\frac{f\,(b\,f+6\,a\,c\,f+6\,a\,d\,e)}{3\,d}-\frac{2\,a\,c\,f^2}{d}\right)}{d} - \frac{3\,a\,c^2\,f^2+12\,a\,c\,d\,e\,f+3\,a\,d^2\,e^2+3\,b\,d\,e\,f+3\,a\,f^2}{3\,d^2}+\frac{a\,f^2\,(3\,c^2+3)}{3\,d^2}\right) + \mathrm{acot}(c+d\,x)\left(b\,e^2\,x+b\,e\,f\,x^2+\frac{b\,f^2\,x^3}{3}\right)+\frac{a\,f^2\,x^3}{3} + \frac{\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)\,\left(36\,b\,c^2\,d^3\,f^2-72\,b\,c\,d^4\,e\,f+36\,b\,d^5\,e^2-12\,b\,d^3\,f^2\right)}{72\,d^6} - \frac{b\,\mathrm{atan}\left(\frac{3\,d^2\left(\frac{c\left(c^3\,f^2-3\,c^2\,d\,e\,f+3\,c\,d^2\,e^2-3\,c\,f^2+3\,d\,e\,f\right)}{3\,d^2}+\frac{x\left(c^3\,f^2-3\,c^2\,d\,e\,f+3\,c\,d^2\,e^2-3\,c\,f^2+3\,d\,e\,f\right)}{3\,d}\right)}{c^3\,f^2-3\,c^2\,d\,e\,f+3\,c\,d^2\,e^2-3\,c\,f^2+3\,d\,e\,f}\right)\left(c^3\,f^2-3\,c^2\,d\,e\,f+3\,c\,d^2 \cdots\right.}{3\,d^3}$$

input
```
int((e + f*x)^2*(a + b*acot(c + d*x)),x)
```

output
```
x^2*((f*(b*f + 6*a*c*f + 6*a*d*e))/(6*d) - (a*c*f^2)/d) - x*((2*c*((f*(b*f
 + 6*a*c*f + 6*a*d*e))/(3*d) - (2*a*c*f^2)/d))/d - (3*a*f^2 + 3*a*c^2*f^2
+ 3*a*d^2*e^2 + 3*b*d*e*f + 12*a*c*d*e*f)/(3*d^2) + (a*f^2*(3*c^2 + 3))/(3
*d^2)) + acot(c + d*x)*((b*f^2*x^3)/3 + b*e^2*x + b*e*f*x^2) + (a*f^2*x^3)
/3 + (log(c^2 + d^2*x^2 + 2*c*d*x + 1)*(36*b*d^5*e^2 - 12*b*d^3*f^2 + 36*b
*c^2*d^3*f^2 - 72*b*c*d^4*e*f))/(72*d^6) - (b*atan((3*d^2*((c*(c^3*f^2 - 3
*c*f^2 + 3*c*d^2*e^2 + 3*d*e*f - 3*c^2*d*e*f))/(3*d^2) + (x*(c^3*f^2 - 3*c
*f^2 + 3*c*d^2*e^2 + 3*d*e*f - 3*c^2*d*e*f))/(3*d)))/(c^3*f^2 - 3*c*f^2 +
3*c*d^2*e^2 + 3*d*e*f - 3*c^2*d*e*f))*(c^3*f^2 - 3*c*f^2 + 3*c*d^2*e^2 + 3
*d*e*f - 3*c^2*d*e*f))/(3*d^3)
```

3.130. $\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)\,dx$

3.131 $\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)\,dx$

3.131.1 Optimal result

Integrand size = 16, antiderivative size = 97

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)\,dx=\frac{bfx}{2d}+\frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)}{2f}+\frac{b(de+f-cf)(de-(1+c)f)\arctan(c+dx)}{2d^2f}+\frac{b(de-cf)\log\left(1+(c+dx)^2\right)}{2d^2}$$

output
```
1/2*b*f*x/d+1/2*(f*x+e)^2*(a+b*arccot(d*x+c))/f+1/2*b*(-c*f+d*e+f)*(d*e-(1
+c)*f)*arctan(d*x+c)/d^2/f+1/2*b*(-c*f+d*e)*ln(1+(d*x+c)^2)/d^2
```

3.131.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.68

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)\,dx$$
$$=aex+\frac{1}{2}afx^2+bex\cot^{-1}(c+dx)+\frac{bf\left(\frac{1}{2}d\left(-\frac{c}{d}+\frac{c+dx}{d}\right)^2\cot^{-1}(c+dx)+\frac{1}{2}d\left(\frac{x}{d}-\frac{i(i-c)^2\log(i-c-dx)}{2d^2}+\frac{i(i+c)^2\log(i+c+dx)}{2d^2}\right)\right)}{d}+\frac{be\left(-2c\arctan(c+dx)+\log\left(1+c^2+2cdx+d^2x^2\right)\right)}{2d}$$

input
```
Integrate[(e + f*x)*(a + b*ArcCot[c + d*x]),x]
```

output
```
a*e*x + (a*f*x^2)/2 + b*e*x*ArcCot[c + d*x] + (b*f*((d*(-(c/d) + (c + d*x)
/d)^2*ArcCot[c + d*x])/2 + (d*(x/d - ((I/2)*(I - c)^2*Log[I - c - d*x])/d^
2 + ((I/2)*(I + c)^2*Log[I + c + d*x])/d^2))/2))/d + (b*e*(-2*c*ArcTan[c +
 d*x] + Log[1 + c^2 + 2*c*d*x + d^2*x^2]))/(2*d)
```

3.131.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5571, 27, 5388, 478, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)dx$$

$$\Big\downarrow \text{5571}$$

$$\frac{\int \frac{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)\left(a+b\cot^{-1}(c+dx)\right)}{d}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (de-cf+f(c+dx))\left(a+b\cot^{-1}(c+dx)\right)d(c+dx)}{d^2}$$

$$\Big\downarrow \text{5388}$$

$$\frac{\frac{b\int \frac{(de-cf+f(c+dx))^2}{(c+dx)^2+1}d(c+dx)}{2f}+\frac{(f(c+dx)-cf+de)^2\left(a+b\cot^{-1}(c+dx)\right)}{2f}}{d^2}$$

$$\Big\downarrow \text{478}$$

$$\frac{\frac{b\int \left(f^2+\frac{(de-cf-f)(de-cf+f)+2f(de-cf)(c+dx)}{(c+dx)^2+1}\right)d(c+dx)}{2f}+\frac{(f(c+dx)-cf+de)^2\left(a+b\cot^{-1}(c+dx)\right)}{2f}}{d^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(f(c+dx)-cf+de)^2\left(a+b\cot^{-1}(c+dx)\right)}{2f}+\frac{b\left(\arctan(c+dx)(-cf+de+f)(de-(c+1)f)+f(de-cf)\log\left((c+dx)^2+1\right)+f^2(c+dx)\right)}{2f}}{d^2}$$

input
```
Int[(e + f*x)*(a + b*ArcCot[c + d*x]),x]
```

output
```
(((d*e - c*f + f*(c + d*x))^2*(a + b*ArcCot[c + d*x]))/(2*f) + (b*(f^2*(c
+ d*x) + (d*e + f - c*f)*(d*e - (1 + c)*f)*ArcTan[c + d*x] + f*(d*e - c*f)
*Log[1 + (c + d*x)^2]))/(2*f))/d^2
```

3.131.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 478
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ
[n, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5388
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])/(e*(q + 1))), x] + Simp[b*(
c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b
, c, d, e, q}, x] && NeQ[q, -1]
```

rule 5571
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.131.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.16

method	result
parts	$a\left(\frac{1}{2}f\,x^{2}+ex\right)+\frac{b\left(\frac{\operatorname{arccot}(dx+c)(dx+c)^{2}f}{2d}-\frac{\operatorname{arccot}(dx+c)cf(dx+c)}{d}+\operatorname{arccot}(dx+c)e(dx+c)+\frac{f(dx+c)+\frac{(-2cf+2de)\ln}{2}}{}\right.}{d}$
derivativedivides	$\frac{-\frac{a\left(fc(dx+c)-ed(dx+c)-\frac{f(dx+c)^{2}}{2}\right)}{d}-\frac{b\left(\operatorname{arccot}(dx+c)fc(dx+c)-\operatorname{arccot}(dx+c)ed(dx+c)-\frac{\operatorname{arccot}(dx+c)f(dx+c)^{2}}{2}-\frac{f(dx+c)}{2}+\frac{(2}{}\right.}{d}}{d}$
default	$\frac{-\frac{a\left(fc(dx+c)-ed(dx+c)-\frac{f(dx+c)^{2}}{2}\right)}{d}-\frac{b\left(\operatorname{arccot}(dx+c)fc(dx+c)-\operatorname{arccot}(dx+c)ed(dx+c)-\frac{\operatorname{arccot}(dx+c)f(dx+c)^{2}}{2}-\frac{f(dx+c)}{2}+\frac{(2}{}\right.}{d}}{d}$
parallelrisch	$-\frac{-\operatorname{arccot}(dx+c)b\,d^{2}f\,x^{2}-a\,d^{2}f\,x^{2}-2x\operatorname{arccot}(dx+c)b\,d^{2}e-2a\,d^{2}ex+\operatorname{arccot}(dx+c)b\,c^{2}f-2\operatorname{arccot}(dx+c)bcde+bcf\ln}{2d^{2}}$
risch	$\frac{ib(f\,x^{2}+2ex)\ln(1+i(dx+c))}{4}-\frac{ibf\,x^{2}\ln(1-i(dx+c))}{4}-\frac{ibex\ln(1-i(dx+c))}{2}+\frac{\pi bf\,x^{2}}{4}+\frac{\pi bex}{2}+\frac{af\,x^{2}}{2}+\operatorname{arct}$

input
```
int((f*x+e)*(a+b*arccot(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/2*f*x^2+e*x)+b/d*(1/2/d*arccot(d*x+c)*(d*x+c)^2*f-1/d*arccot(d*x+c)*c
*f*(d*x+c)+arccot(d*x+c)*e*(d*x+c)+1/2/d*(f*(d*x+c)+1/2*(-2*c*f+2*d*e)*ln(
1+(d*x+c)^2)-f*arctan(d*x+c)))
```

3.131.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.13

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)\,dx$$
$$=\frac{ad^{2}fx^{2}+(2\,ad^{2}e+bdf)x+(bd^{2}fx^{2}+2\,bd^{2}ex)\operatorname{arccot}(dx+c)-(2\,bcde-(bc^{2}-b)f)\arctan(dx+c)+}{2\,d^{2}}$$

input
```
integrate((f*x+e)*(a+b*arccot(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(a*d^2*f*x^2 + (2*a*d^2*e + b*d*f)*x + (b*d^2*f*x^2 + 2*b*d^2*e*x)*arc
cot(d*x + c) - (2*b*c*d*e - (b*c^2 - b)*f)*arctan(d*x + c) + (b*d*e - b*c*
f)*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/d^2
```

3.131.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 14.48 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.82

$$\int (e+fx) \left(a+b\cot^{-1}(c+dx)\right) dx$$
$$= \begin{cases} aex + \frac{afx^2}{2} - \frac{bc^2 f \operatorname{acot}(c+dx)}{2d^2} + \frac{bce \operatorname{acot}(c+dx)}{d} - \frac{bcf \log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{d^2} - \frac{ibcf \operatorname{acot}(c+dx)}{d^2} + bex \operatorname{acot}(c+dx) + \frac{bfx^2 \operatorname{acot}}{2} \\ (a+b\operatorname{acot}(c))\left(ex+\frac{fx^2}{2}\right) \end{cases}$$

input
```
integrate((f*x+e)*(a+b*acot(d*x+c)),x)
```

output
```
Piecewise((a*e*x + a*f*x**2/2 - b*c**2*f*acot(c + d*x)/(2*d**2) + b*c*e*ac
ot(c + d*x)/d - b*c*f*log(c/d + x - I/d)/d**2 - I*b*c*f*acot(c + d*x)/d**2
 + b*e*x*acot(c + d*x) + b*f*x**2*acot(c + d*x)/2 + b*e*log(c/d + x - I/d)
/d + I*b*e*acot(c + d*x)/d + b*f*x/(2*d) + b*f*acot(c + d*x)/(2*d**2), Ne(
d, 0)), ((a + b*acot(c))*(e*x + f*x**2/2), True))
```

3.131.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.16

$$\int (e+fx) \left(a+b\cot^{-1}(c+dx)\right) dx = \frac{1}{2}afx^2 + \frac{1}{2}\left(x^2 \operatorname{arccot}(dx+c) + d\left(\frac{x}{d^2} + \frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3} - \frac{c\log(d^2x^2+2cdx+c^2+1)}{d^3}\right)\right)bf + aex + \frac{\left(2(dx+c)\operatorname{arccot}(dx+c) + \log\left((dx+c)^2+1\right)\right)be}{2d}$$

input
```
integrate((f*x+e)*(a+b*arccot(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*a*f*x^2 + 1/2*(x^2*arccot(d*x + c) + d*(x/d^2 + (c^2 - 1)*arctan((d^2*
x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*b*f + a*e*x + 1
/2*(2*(d*x + c)*arccot(d*x + c) + log((d*x + c)^2 + 1))*b*e/d
```

3.131.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(89) = 178$.

Time = 0.41 (sec) , antiderivative size = 451, normalized size of antiderivative = 4.65

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)\,dx =$$

$$-\frac{4\,bde\arctan\left(\frac{1}{dx+c}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{dx+c}\right)\right)^3-4\,bcf\arctan\left(\frac{1}{dx+c}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{dx+c}\right)\right)^3-bf\arctan\left(\frac{1}{dx}\right.}{}$$

input
```
integrate((f*x+e)*(a+b*arccot(d*x+c)),x, algorithm="giac")
```

output
```
-1/8*(4*b*d*e*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^3 - 4*b*c*f
*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^3 - b*f*arctan(1/(d*x +
c))*tan(1/2*arctan(1/(d*x + c)))^4 + 4*b*d*e*log(16*tan(1/2*arctan(1/(d*x
+ c)))^2/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^
2 + 1))*tan(1/2*arctan(1/(d*x + c)))^2 - 4*b*c*f*log(16*tan(1/2*arctan(1/(
d*x + c)))^2/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c
)))^2 + 1))*tan(1/2*arctan(1/(d*x + c)))^2 + 4*a*d*e*tan(1/2*arctan(1/(d*x
 + c)))^3 - 4*a*c*f*tan(1/2*arctan(1/(d*x + c)))^3 - a*f*tan(1/2*arctan(1/
(d*x + c)))^4 - 4*b*d*e*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c))) +
 4*b*c*f*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c))) - 2*b*f*arctan(1
/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^2 + 2*b*f*tan(1/2*arctan(1/(d*x +
 c)))^3 - 4*a*d*e*tan(1/2*arctan(1/(d*x + c))) + 4*a*c*f*tan(1/2*arctan(1/
(d*x + c))) - 2*a*f*tan(1/2*arctan(1/(d*x + c)))^2 - b*f*arctan(1/(d*x + c
)) - 2*b*f*tan(1/2*arctan(1/(d*x + c))) - a*f)/(d^2*tan(1/2*arctan(1/(d*x
+ c)))^2)
```

3.131.9 Mupad [B] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.40

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)\,dx = a\,e\,x+\frac{a\,f\,x^2}{2}+\frac{b\,e\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)}{2\,d}+\frac{b\,f\,\mathrm{acot}(c+d\,x)}{2\,d^2}+\frac{b\,f\,x^2\,\mathrm{acot}(c+d\,x)}{2}+\frac{b\,f\,x}{2\,d}+b\,e\,x\,\mathrm{acot}(c+d\,x)-\frac{b\,c^2\,f\,\mathrm{acot}(c+d\,x)}{2\,d^2}-\frac{b\,c\,f\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)}{2\,d^2}+\frac{b\,c\,e\,\mathrm{acot}(c+d\,x)}{d}$$

input

```
int((e + f*x)*(a + b*acot(c + d*x)),x)
```

output

```
a*e*x + (a*f*x^2)/2 + (b*e*log(c^2 + d^2*x^2 + 2*c*d*x + 1))/(2*d) + (b*f*
acot(c + d*x))/(2*d^2) + (b*f*x^2*acot(c + d*x))/2 + (b*f*x)/(2*d) + b*e*x
*acot(c + d*x) - (b*c^2*f*acot(c + d*x))/(2*d^2) - (b*c*f*log(c^2 + d^2*x^
2 + 2*c*d*x + 1))/(2*d^2) + (b*c*e*acot(c + d*x))/d
```

3.132 $\int (a+b\cot^{-1}(c+dx))\,dx$

3.132.1 Optimal result

Integrand size = 10, antiderivative size = 38

$$\int (a+b\cot^{-1}(c+dx))\,dx = ax+\frac{b(c+dx)\cot^{-1}(c+dx)}{d}+\frac{b\log(1+(c+dx)^2)}{2d}$$

output
```
a*x+b*(d*x+c)*arccot(d*x+c)/d+1/2*b*ln(1+(d*x+c)^2)/d
```

3.132.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int (a+b\cot^{-1}(c+dx))\,dx = ax+bx\cot^{-1}(c+dx) + \frac{b(-2c\arctan(c+dx)+\log(1+c^2+2cdx+d^2x^2))}{2d}$$

input
```
Integrate[a + b*ArcCot[c + d*x],x]
```

output
```
a*x + b*x*ArcCot[c + d*x] + (b*(-2*c*ArcTan[c + d*x] + Log[1 + c^2 + 2*c*d
*x + d^2*x^2]))/(2*d)
```

3.132.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\cot^{-1}(c+dx)\right) \, dx$$

$$\Big\downarrow \; 2009$$

$$ax+\frac{b\log\left((c+dx)^2+1\right)}{2d}+\frac{b(c+dx)\cot^{-1}(c+dx)}{d}$$

input
```
Int[a + b*ArcCot[c + d*x],x]
```

output
```
a*x + (b*(c + d*x)*ArcCot[c + d*x])/d + (b*Log[1 + (c + d*x)^2])/(2*d)
```

3.132.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.132.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

method	result	size
default	$ax+\frac{b\left(\operatorname{arccot}(dx+c)(dx+c)+\frac{\ln\left(1+(dx+c)^2\right)}{2}\right)}{d}$	35
parts	$ax+\frac{b\left(\operatorname{arccot}(dx+c)(dx+c)+\frac{\ln\left(1+(dx+c)^2\right)}{2}\right)}{d}$	35
derivativedivides	$\frac{(dx+c)a+b\left(\operatorname{arccot}(dx+c)(dx+c)+\frac{\ln\left(1+(dx+c)^2\right)}{2}\right)}{d}$	40
parallelrisch	$\frac{b\left(2x\ \operatorname{arccot}(dx+c)d^2+2c\ \operatorname{arccot}(dx+c)d+\ln\left(d^2x^2+2cdx+c^2+1\right)d\right)}{2d^2}+ax$	54
risch	$ax+\frac{ibx\ln(1+i(dx+c))}{2}-\frac{ibx\ln(1-i(dx+c))}{2}+\frac{\pi bx}{2}-\frac{bc\arctan(dx+c)}{d}+\frac{b\ln\left(d^2x^2+2cdx+c^2+1\right)}{2d}$	79

input
```
int(a+b*arccot(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
a*x+b/d*(arccot(d*x+c)*(d*x+c)+1/2*ln(1+(d*x+c)^2))
```

3.132.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

$$\int \left(a+b\cot^{-1}(c+dx)\right)\,dx = \frac{2\,bdx\,\mathrm{arccot}\,(dx+c)+2\,adx-2\,bc\,\mathrm{arctan}\,(dx+c)+b\log\left(d^2x^2+2\,cdx+c^2+1\right)}{2\,d}$$

input
```
integrate(a+b*arccot(d*x+c),x, algorithm="fricas")
```

output
```
1/2*(2*b*d*x*arccot(d*x + c) + 2*a*d*x - 2*b*c*arctan(d*x + c) + b*log(d^2
*x^2 + 2*c*d*x + c^2 + 1))/d
```

3.132.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

$$\int \left(a+b\cot^{-1}(c+dx)\right)\,dx = ax+b\left(\begin{cases}\frac{c\,\mathrm{acot}\,(c+dx)}{d}+x\,\mathrm{acot}\,(c+dx)+\frac{\log\left(c^2+2cdx+d^2x^2+1\right)}{2d} & \text{for } d\neq 0\\ x\,\mathrm{acot}\,(c) & \text{otherwise}\end{cases}\right)$$

input
```
integrate(a+b*acot(d*x+c),x)
```

output
```
a*x + b*Piecewise((c*acot(c + d*x)/d + x*acot(c + d*x) + log(c**2 + 2*c*d*
x + d**2*x**2 + 1)/(2*d), Ne(d, 0)), (x*acot(c), True))
```

3.132.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \left(a + b\cot^{-1}(c + dx)\right) \, dx = ax + \frac{\left(2\left(dx + c\right)\operatorname{arccot}\left(dx + c\right) + \log\left(\left(dx + c\right)^2 + 1\right)\right)b}{2\,d}$$

input
```
integrate(a+b*arccot(d*x+c),x, algorithm="maxima")
```

output
```
a*x + 1/2*(2*(d*x + c)*arccot(d*x + c) + log((d*x + c)^2 + 1))*b/d
```

3.132.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(36) = 72.

Time = 0.32 (sec) , antiderivative size = 116, normalized size of antiderivative = 3.05

$$\int \left(a + b\cot^{-1}(c + dx)\right) \, dx = ax - \frac{\left(\arctan\left(\frac{1}{dx+c}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{dx+c}\right)\right)^2 + \log\left(\frac{16\tan\left(\frac{1}{2}\arctan\left(\frac{1}{dx+c}\right)\right)^2}{\tan\left(\frac{1}{2}\arctan\left(\frac{1}{dx+c}\right)\right)^4 + 2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{dx+c}\right)\right)^2 + 1}\right)\tan\left(\frac{1}{2}\operatorname{arcta}\right.}{2\,d\tan\left(\frac{1}{2}\arctan\left(\frac{1}{dx+c}\right)\right)}$$

input
```
integrate(a+b*arccot(d*x+c),x, algorithm="giac")
```

output
```
a*x - 1/2*(arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^2 + log(16*tan
(1/2*arctan(1/(d*x + c)))^2/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*ar
ctan(1/(d*x + c)))^2 + 1))*tan(1/2*arctan(1/(d*x + c))) - arctan(1/(d*x +
c)))*b/(d*tan(1/2*arctan(1/(d*x + c))))
```

3.132.9 Mupad [B] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int \left(a + b\cot^{-1}(c + dx)\right) \, dx = a\,x + \frac{\frac{b\,\ln\left(c^2 + 2\,c\,d\,x + d^2\,x^2 + 1\right)}{2} + b\,c\,\mathrm{acot}(c + d\,x)}{d} + b\,x\,\mathrm{acot}(c + d\,x)$$

input
```
int(a + b*acot(c + d*x),x)
```

output
```
a*x + ((b*log(c^2 + d^2*x^2 + 2*c*d*x + 1))/2 + b*c*acot(c + d*x))/d + b*x
*acot(c + d*x)
```

3.132. $\int \left(a+b\cot^{-1}(c+dx)\right)\,dx$

3.133 $\int \frac{a+b\cot^{-1}(c+dx)}{e+fx}\,dx$

3.133.1 Optimal result

Integrand size = 18, antiderivative size = 162

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+fx}\,dx = -\frac{(a+b\cot^{-1}(c+dx))\log\left(\frac{2}{1-i(c+dx)}\right)}{f} + \frac{(a+b\cot^{-1}(c+dx))\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f} - \frac{ib\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{2f} + \frac{ib\,\mathrm{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2f}$$

output
```
-(a+b*arccot(d*x+c))*ln(2/(1-I*(d*x+c)))/f+(a+b*arccot(d*x+c))*ln(2*d*(f*x
+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f-1/2*I*b*polylog(2,1-2/(1-I*(d*x+c)))/f+
1/2*I*b*polylog(2,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f
```

3.133.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 336 vs. 2(162) = 324.

Time = 0.33 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.07

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+fx}\,dx$$
$$= \frac{a\log(e+fx)+b\Big((\cot^{-1}(c+dx)+\arctan(c+dx))\log(e+fx)+\arctan(c+dx)\left(\log\left(\frac{1}{\sqrt{1+(c+dx)^2}}\right)-\text{lo}\right.}{}$$

input
```
Integrate[(a + b*ArcCot[c + d*x])/(e + f*x),x]
```

output
```
(a*Log[e + f*x] + b*((ArcCot[c + d*x] + ArcTan[c + d*x])*Log[e + f*x] + Ar
cTan[c + d*x]*(Log[1/Sqrt[1 + (c + d*x)^2]] - Log[Sin[ArcTan[(d*e - c*f)/f
] + ArcTan[c + d*x]]]) + ((I/4)*(Pi - 2*ArcTan[c + d*x])^2 + I*(ArcTan[(d*
e - c*f)/f] + ArcTan[c + d*x])^2 - (Pi - 2*ArcTan[c + d*x])*Log[1 + E^((-2
*I)*ArcTan[c + d*x])] - 2*(ArcTan[(d*e - c*f)/f] + ArcTan[c + d*x])*Log[1
- E^((2*I)*(ArcTan[(d*e - c*f)/f] + ArcTan[c + d*x]))] + (Pi - 2*ArcTan[c
+ d*x])*Log[2/Sqrt[1 + (c + d*x)^2]] + 2*(ArcTan[(d*e - c*f)/f] + ArcTan[c
 + d*x])*Log[2*Sin[ArcTan[(d*e - c*f)/f] + ArcTan[c + d*x]]] + I*PolyLog[2
, -E^((-2*I)*ArcTan[c + d*x])] + I*PolyLog[2, E^((2*I)*(ArcTan[(d*e - c*f)
/f] + ArcTan[c + d*x]))])/2))/f
```

3.133.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5571, 27, 5382, 2849, 2752, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+fx}\,dx$$
$$\Big\downarrow \; 5571$$
$$\frac{\int \frac{d\left(a+b\cot^{-1}(c+dx)\right)}{d\left(e-\frac{cf}{d}\right)+f(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \color{blue}{27}$$

$$\int \frac{a+b\cot^{-1}(c+dx)}{f(c+dx)-cf+de}d(c+dx)$$

$$\Big\downarrow \color{blue}{5382}$$

$$\frac{b\int \frac{\log\left(\frac{2(de-cf+f(c+dx))}{(de-cf+if)(1-i(c+dx))}\right)}{(c+dx)^2+1}d(c+dx)}{f}-\frac{b\int \frac{\log\left(\frac{2}{1-i(c+dx)}\right)}{(c+dx)^2+1}d(c+dx)}{f}+\frac{(a+b\cot^{-1}(c+dx))\log\left(\frac{2(f(c+dx)-cf+de)}{(1-i(c+dx))(-cf+de+if)}\right)}{f}-\frac{\log\left(\frac{2}{1-i(c+dx)}\right)(a+b\cot^{-1}(c+dx))}{f}$$

$$\Big\downarrow \color{blue}{2849}$$

$$\frac{b\int \frac{\log\left(\frac{2(de-cf+f(c+dx))}{(de-cf+if)(1-i(c+dx))}\right)}{(c+dx)^2+1}d(c+dx)}{f}-\frac{ib\int \frac{\log\left(\frac{2}{1-i(c+dx)}\right)}{1-\frac{2}{1-i(c+dx)}}d\frac{1}{1-i(c+dx)}}{f}+\frac{(a+b\cot^{-1}(c+dx))\log\left(\frac{2(f(c+dx)-cf+de)}{(1-i(c+dx))(-cf+de+if)}\right)}{f}-\frac{\log\left(\frac{2}{1-i(c+dx)}\right)(a+b\cot^{-1}(c+dx))}{f}$$

$$\Big\downarrow \color{blue}{2752}$$

$$\frac{b\int \frac{\log\left(\frac{2(de-cf+f(c+dx))}{(de-cf+if)(1-i(c+dx))}\right)}{(c+dx)^2+1}d(c+dx)}{f}+\frac{(a+b\cot^{-1}(c+dx))\log\left(\frac{2(f(c+dx)-cf+de)}{(1-i(c+dx))(-cf+de+if)}\right)}{f}-\frac{\log\left(\frac{2}{1-i(c+dx)}\right)(a+b\cot^{-1}(c+dx))}{f}-\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{2f}$$

$$\Big\downarrow \color{blue}{2897}$$

$$\frac{(a+b\cot^{-1}(c+dx))\log\left(\frac{2(f(c+dx)-cf+de)}{(1-i(c+dx))(-cf+de+if)}\right)}{f}-\frac{\log\left(\frac{2}{1-i(c+dx)}\right)(a+b\cot^{-1}(c+dx))}{f}+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2(de-cf+f(c+dx))}{(de-cf+if)(1-i(c+dx))}\right)}{2f}-\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{2f}$$

input
```
Int[(a + b*ArcCot[c + d*x])/(e + f*x),x]
```

output
```
-(((a + b*ArcCot[c + d*x])*Log[2/(1 - I*(c + d*x))])/f) + ((a + b*ArcCot[c
 + d*x])*Log[(2*(d*e - c*f + f*(c + d*x)))/((d*e + I*f - c*f)*(1 - I*(c +
d*x)))])/f - ((I/2)*b*PolyLog[2, 1 - 2/(1 - I*(c + d*x))])/f + ((I/2)*b*Po
lyLog[2, 1 - (2*(d*e - c*f + f*(c + d*x)))/((d*e + I*f - c*f)*(1 - I*(c +
d*x)))])/f
```

3.133. $\int \frac{a+b\cot^{-1}(c+dx)}{e+fx}dx$

3.133.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5382
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcCot[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*ArcCot[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] - Simp[b*(c/e)
   Int[Log[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] + Simp[b*(c/e)   Int[Log[2*
c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x]) /; FreeQ[{a
, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

rule 5571
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.133.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.22

method	result
parts	$\frac{a\ln(fx+e)}{f}+\frac{b\left(\frac{d\ln(f(dx+c)-cf+de)\operatorname{arccot}(dx+c)}{f}+d\left(-\frac{i\ln(f(dx+c)-cf+de)\left(\ln\left(\frac{if-f(dx+c)}{-cf+de+if}\right)-\ln\left(\frac{if+f(dx+c)}{cf-de+if}\right)\right)}{2f}-\frac{i(}{}\right.\right.}{d}$
derivativedivides	$\frac{\frac{ad\ln(cf-de-f(dx+c))}{f}-bd\left(-\frac{\ln(cf-de-f(dx+c))\operatorname{arccot}(dx+c)}{f}-\frac{i\ln(cf-de-f(dx+c))\left(\ln\left(\frac{if+f(dx+c)}{cf-de+if}\right)-\ln\left(\frac{if-f(dx+c)}{-cf+de+if}\right)\right)}{2f}\right.}{d}$
default	$\frac{\frac{ad\ln(cf-de-f(dx+c))}{f}-bd\left(-\frac{\ln(cf-de-f(dx+c))\operatorname{arccot}(dx+c)}{f}-\frac{i\ln(cf-de-f(dx+c))\left(\ln\left(\frac{if+f(dx+c)}{cf-de+if}\right)-\ln\left(\frac{if-f(dx+c)}{-cf+de+if}\right)\right)}{2f}\right.}{d}$
risch	$-\frac{ib\operatorname{dilog}\left(\frac{icf-ide+(-idx-ic+1)f-f}{icf-ide-f}\right)}{2f}-\frac{ib\ln(-idx-ic+1)\ln\left(\frac{icf-ide+(-idx-ic+1)f-f}{icf-ide-f}\right)}{2f}+\frac{\ln(icf-ide+(-idx-ic}{2f}$

input
```
int((a+b*arccot(d*x+c))/(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
a*ln(f*x+e)/f+b/d*(d*ln(f*(d*x+c)-c*f+d*e)/f*arccot(d*x+c)+d*(-1/2*I*ln(f*
(d*x+c)-c*f+d*e)*(ln((I*f-f*(d*x+c))/(d*e+I*f-c*f))-ln((I*f+f*(d*x+c))/(c*
f-d*e+I*f)))/f-1/2*I*(dilog((I*f-f*(d*x+c))/(d*e+I*f-c*f))-dilog((I*f+f*(d
*x+c))/(c*f-d*e+I*f)))/f))
```

3.133.5 Fricas [F]

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+fx}\,dx=\int \frac{b\operatorname{arccot}(dx+c)+a}{fx+e}\,dx$$

input
```
integrate((a+b*arccot(d*x+c))/(f*x+e),x, algorithm="fricas")
```

output
```
integral((b*arccot(d*x + c) + a)/(f*x + e), x)
```

3.133.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+fx}\,dx=\text{Timed out}$$

input
```
integrate((a+b*acot(d*x+c))/(f*x+e),x)
```

output
```
Timed out
```

3.133.7 Maxima [F]

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+fx}\,dx=\int \frac{b\,\text{arccot}\,(dx+c)+a}{fx+e}\,dx$$

input
```
integrate((a+b*arccot(d*x+c))/(f*x+e),x, algorithm="maxima")
```

output
```
2*b*integrate(1/2*arctan2(1, d*x + c)/(f*x + e), x) + a*log(f*x + e)/f
```

3.133.8 Giac [F]

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+fx}\,dx=\int \frac{b\,\text{arccot}\,(dx+c)+a}{fx+e}\,dx$$

input
```
integrate((a+b*arccot(d*x+c))/(f*x+e),x, algorithm="giac")
```

output
```
integrate((b*arccot(d*x + c) + a)/(f*x + e), x)
```

3.133.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+fx}\,dx=\int \frac{a+b\,\mathrm{acot}(c+d\,x)}{e+f\,x}\,dx$$

input
```
int((a + b*acot(c + d*x))/(e + f*x),x)
```

output
```
int((a + b*acot(c + d*x))/(e + f*x), x)
```

3.134 $\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^2}\,dx$

3.134.1 Optimal result

Integrand size = 18, antiderivative size = 153

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^2}\,dx=-\frac{a+b\cot^{-1}(c+dx)}{f(e+fx)}-\frac{bd(de-cf)\arctan(c+dx)}{f\left(d^2e^2-2cdef+(1+c^2)f^2\right)}-\frac{bd\log(e+fx)}{d^2e^2-2cdef+(1+c^2)f^2}+\frac{bd\log\left(1+c^2+2cdx+d^2x^2\right)}{2\left(d^2e^2-2cdef+(1+c^2)f^2\right)}$$

output
```
(-a-b*arccot(d*x+c))/f/(f*x+e)-b*d*(-c*f+d*e)*arctan(d*x+c)/f/(d^2*e^2-2*c
*d*e*f+(c^2+1)*f^2)-b*d*ln(f*x+e)/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+1/2*b*d*
ln(d^2*x^2+2*c*d*x+c^2+1)/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)
```

3.134.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.77

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^2}\,dx=\frac{-\frac{a+b\cot^{-1}(c+dx)}{e+fx}+\frac{bd((ide+f-icf)\log(i-c-dx)+(-ide+f+icf)\log(i+c+dx)-2f\log(d(e+fx)))}{2(d^2e^2-2cdef+(1+c^2)f^2)}}{f}$$

input
```
Integrate[(a + b*ArcCot[c + d*x])/(e + f*x)^2,x]
```

output
```
(-((a + b*ArcCot[c + d*x])/(e + f*x)) + (b*d*((I*d*e + f - I*c*f)*Log[I -
c - d*x] + ((-I)*d*e + f + I*c*f)*Log[I + c + d*x] - 2*f*Log[d*(e + f*x)])
)/(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)))/f
```

3.134.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5569, 2081, 1144, 27, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^2}dx$$

$$\Big\downarrow \text{5569}$$

$$-\frac{bd\int \frac{1}{(e+fx)((c+dx)^2+1)}dx}{f}-\frac{a+b\cot^{-1}(c+dx)}{f(e+fx)}$$

$$\Big\downarrow \text{2081}$$

$$-\frac{bd\int \frac{1}{(e+fx)(c^2+2dxc+d^2x^2+1)}dx}{f}-\frac{a+b\cot^{-1}(c+dx)}{f(e+fx)}$$

$$\Big\downarrow \text{1144}$$

$$-\frac{bd\left(\frac{\int \frac{d(de-2cf-dfx)}{c^2+2dxc+d^2x^2+1}dx}{(c^2+1)f^2-2cdef+d^2e^2}+\frac{f\log(e+fx)}{(c^2+1)f^2-2cdef+d^2e^2}\right)}{f}-\frac{a+b\cot^{-1}(c+dx)}{f(e+fx)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bd\left(\frac{d\int \frac{de-2cf-dfx}{c^2+2dxc+d^2x^2+1}dx}{(c^2+1)f^2-2cdef+d^2e^2}+\frac{f\log(e+fx)}{(c^2+1)f^2-2cdef+d^2e^2}\right)}{f}-\frac{a+b\cot^{-1}(c+dx)}{f(e+fx)}$$

$$\Big\downarrow \text{1142}$$

$$-\frac{bd\left(\frac{d\left((de-cf)\int \frac{1}{c^2+2dxc+d^2x^2+1}dx-\frac{f\int \frac{2d(c+dx)}{c^2+2dxc+d^2x^2+1}dx}{2d}\right)}{(c^2+1)f^2-2cdef+d^2e^2}+\frac{f\log(e+fx)}{(c^2+1)f^2-2cdef+d^2e^2}\right)}{f}-\frac{a+b\cot^{-1}(c+dx)}{f(e+fx)}$$

$$\Big\downarrow \ 27$$

$$-\frac{bd\left(\frac{d\left((de-cf)\int \frac{1}{c^2+2dxc+d^2x^2+1}dx-f\int \frac{c+dx}{c^2+2dxc+d^2x^2+1}dx\right)}{(c^2+1)f^2-2cdef+d^2e^2}+\frac{f\log(e+fx)}{(c^2+1)f^2-2cdef+d^2e^2}\right)}{f}-\frac{a+b\cot^{-1}(c+dx)}{f(e+fx)}$$

$$\Big\downarrow \ 1083$$

$$-\frac{bd\left(\frac{d\left(-f\int \frac{c+dx}{c^2+2dxc+d^2x^2+1}dx-2(de-cf)\int \frac{1}{-4d^2-\left(2xd^2+2cd\right)^2}d(2xd^2+2cd)\right)}{(c^2+1)f^2-2cdef+d^2e^2}+\frac{f\log(e+fx)}{(c^2+1)f^2-2cdef+d^2e^2}\right)}{f}-\frac{a+b\cot^{-1}(c+dx)}{f(e+fx)}$$

$$\Big\downarrow \ 217$$

$$-\frac{bd\left(\frac{d\left(\frac{\arctan\left(\frac{2cd+2d^2x}{2d}\right)(de-cf)}{d}-f\int \frac{c+dx}{c^2+2dxc+d^2x^2+1}dx\right)}{(c^2+1)f^2-2cdef+d^2e^2}+\frac{f\log(e+fx)}{(c^2+1)f^2-2cdef+d^2e^2}\right)}{f}-\frac{a+b\cot^{-1}(c+dx)}{f(e+fx)}$$

$$\Big\downarrow \ 1103$$

$$-\frac{a+b\cot^{-1}(c+dx)}{f(e+fx)}-\frac{bd\left(\frac{d\left(\frac{\arctan\left(\frac{2cd+2d^2x}{2d}\right)(de-cf)}{d}-\frac{f\log\left(c^2+2cdx+d^2x^2+1\right)}{2d}\right)}{(c^2+1)f^2-2cdef+d^2e^2}+\frac{f\log(e+fx)}{(c^2+1)f^2-2cdef+d^2e^2}\right)}{f}$$

input
```
Int[(a + b*ArcCot[c + d*x])/(e + f*x)^2,x]
```

output
```
-((a + b*ArcCot[c + d*x])/(f*(e + f*x))) - (b*d*((f*Log[e + f*x])/(d^2*e^2
 - 2*c*d*e*f + (1 + c^2)*f^2) + (d*(((d*e - c*f)*ArcTan[(2*c*d + 2*d^2*x)/
(2*d)])/d - (f*Log[1 + c^2 + 2*c*d*x + d^2*x^2])/(2*d)))/(d^2*e^2 - 2*c*d*
e*f + (1 + c^2)*f^2)))/f
```

3.134. $\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^2}\,dx$

3.134.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2081
```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

rule 5569
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCot[c + d*x])^p/(f*(m +
1))), x] + Simp[b*d*(p/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*((a + b*ArcCot[
c + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[p, 0] && ILtQ[m, -1]
```

3.134. $\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^2}\,dx$

3.134.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.05

method	result
parts	$-\frac{a}{(fx+e)f}+\frac{b\left(-\frac{d^2 \operatorname{arccot}(dx+c)}{(f(dx+c)-cf+de)f}-\frac{d^2\left(\frac{f\ln(f(dx+c)-cf+de)}{c^2f^2-2cdef+d^2e^2+f^2}+\frac{-\frac{f\ln\left(1+(dx+c)^2\right)}{2}+(-cf+de)\arctan(dx+c)}{c^2f^2-2cdef+d^2e^2+f^2}\right)}{f}\right)}{d}$
derivativedivides	$\frac{\frac{a d^2}{(cf-de-f(dx+c))f}+b d^2\left(\frac{\operatorname{arccot}(dx+c)}{(cf-de-f(dx+c))f}+\frac{\frac{\frac{f\ln\left(1+(dx+c)^2\right)}{2}+(cf-de)\arctan(dx+c)}{c^2f^2-2cdef+d^2e^2+f^2}-\frac{f\ln(cf-de-f(dx+c))}{c^2f^2-2cdef+d^2e^2+f^2}}{f}\right)}{d}$
default	$\frac{\frac{a d^2}{(cf-de-f(dx+c))f}+b d^2\left(\frac{\operatorname{arccot}(dx+c)}{(cf-de-f(dx+c))f}+\frac{\frac{\frac{f\ln\left(1+(dx+c)^2\right)}{2}+(cf-de)\arctan(dx+c)}{c^2f^2-2cdef+d^2e^2+f^2}-\frac{f\ln(cf-de-f(dx+c))}{c^2f^2-2cdef+d^2e^2+f^2}}{f}\right)}{d}$
parallelrisch	$-\frac{2x\operatorname{arccot}(dx+c)bc d^3f^2-2x\operatorname{arccot}(dx+c)b d^4ef+2\ln(fx+e)xb d^3f^2-\ln(d^2x^2+2cdx+c^2+1)xb d^3f^2+2\operatorname{arccot}(dx+}{2(}$
risch	$-\frac{ib\ln(1+i(dx+c))}{2f(fx+e)}-\frac{2ac^2f^2+\pi bc^2f^2+\pi bd^2e^2+2f^2a+\pi bf^2-ibc^2f^2\ln(1-i(dx+c))-ibd^2e^2\ln(1-i(dx+c))-i\ln(}{}$

input
```
int((a+b*arccot(d*x+c))/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-a/(f*x+e)/f+b/d*(-d^2/(f*(d*x+c)-c*f+d*e)/f*arccot(d*x+c)-d^2/f*(1/(c^2*f
^2-2*c*d*e*f+d^2*e^2+f^2)*f*ln(f*(d*x+c)-c*f+d*e)+1/(c^2*f^2-2*c*d*e*f+d^2
*e^2+f^2)*(-1/2*f*ln(1+(d*x+c)^2)+(-c*f+d*e)*arctan(d*x+c))))
```

3.134.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.46

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^2}\,dx =$$
$$-\frac{2\,ad^2e^2-4\,acdef+2\,(ac^2+a)f^2+2\,(bd^2e^2-2\,bcdef+(bc^2+b)f^2)\operatorname{arccot}(dx+c)+2\,(bd^2e^2-bcde}{2\,(d^2e^3f-2\,cde^2f^2+(c^2+1}$$

input
```
integrate((a+b*arccot(d*x+c))/(f*x+e)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a*d^2*e^2 - 4*a*c*d*e*f + 2*(a*c^2 + a)*f^2 + 2*(b*d^2*e^2 - 2*b*c
*d*e*f + (b*c^2 + b)*f^2)*arccot(d*x + c) + 2*(b*d^2*e^2 - b*c*d*e*f + (b*
d^2*e*f - b*c*d*f^2)*x)*arctan(d*x + c) - (b*d*f^2*x + b*d*e*f)*log(d^2*x^
2 + 2*c*d*x + c^2 + 1) + 2*(b*d*f^2*x + b*d*e*f)*log(f*x + e))/(d^2*e^3*f
- 2*c*d*e^2*f^2 + (c^2 + 1)*e*f^3 + (d^2*e^2*f^2 - 2*c*d*e*f^3 + (c^2 + 1)
*f^4)*x)
```

3.134.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acot(d*x+c))/(f*x+e)**2,x)
```

output
```
Timed out
```

3.134.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.16

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^2}\,dx = -\frac{1}{2}\left(d\left(\frac{2\,(d^2e-cdf)\arctan\left(\frac{d^2x+cd}{d}\right)}{(d^2e^2f-2\,cdef^2+(c^2+1)f^3)d}-\frac{\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^2e^2-2\,cdef+(c^2+1)f^2}+\frac{2\,\log\left(fx+e\right)}{d^2e^2-2\,cdef+(c^2+1)f^2}\right)\right. \\ \left. -\frac{a}{f^2x+ef}\right.$$

input
```
integrate((a+b*arccot(d*x+c))/(f*x+e)^2,x, algorithm="maxima")
```

output
```
-1/2*(d*(2*(d^2*e - c*d*f)*arctan((d^2*x + c*d)/d)/((d^2*e^2*f - 2*c*d*e*f
^2 + (c^2 + 1)*f^3)*d) - log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*e^2 - 2*c*d
*e*f + (c^2 + 1)*f^2) + 2*log(f*x + e)/(d^2*e^2 - 2*c*d*e*f + (c^2 + 1)*f^
2)) + 2*arccot(d*x + c)/(f^2*x + e*f))*b - a/(f^2*x + e*f)
```

3.134.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1264 vs. $2(151) = 302$.

Time = 0.69 (sec) , antiderivative size = 1264, normalized size of antiderivative = 8.26

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*arccot(d*x+c))/(f*x+e)^2,x, algorithm="giac")
```

output
```
-1/2*(2*b*d*e*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^2 - 2*b*c*f
*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^2 + 2*b*d*e*log(4*(4*d^2
*e^2*tan(1/2*arctan(1/(d*x + c)))^2 - 8*c*d*e*f*tan(1/2*arctan(1/(d*x + c)
))^2 + 4*c^2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 - 4*d*e*f*tan(1/2*arctan(1
/(d*x + c)))^3 + 4*c*f^2*tan(1/2*arctan(1/(d*x + c)))^3 + f^2*tan(1/2*arct
an(1/(d*x + c)))^4 + 4*d*e*f*tan(1/2*arctan(1/(d*x + c))) - 4*c*f^2*tan(1/
2*arctan(1/(d*x + c))) - 2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 + f^2)/(tan(
1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/
2*arctan(1/(d*x + c))) - 2*b*c*f*log(4*(4*d^2*e^2*tan(1/2*arctan(1/(d*x +
c)))^2 - 8*c*d*e*f*tan(1/2*arctan(1/(d*x + c)))^2 + 4*c^2*f^2*tan(1/2*arct
an(1/(d*x + c)))^2 - 4*d*e*f*tan(1/2*arctan(1/(d*x + c)))^3 + 4*c*f^2*tan(
1/2*arctan(1/(d*x + c)))^3 + f^2*tan(1/2*arctan(1/(d*x + c)))^4 + 4*d*e*f*
tan(1/2*arctan(1/(d*x + c))) - 4*c*f^2*tan(1/2*arctan(1/(d*x + c))) - 2*f^
2*tan(1/2*arctan(1/(d*x + c)))^2 + f^2)/(tan(1/2*arctan(1/(d*x + c)))^4 +
2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/2*arctan(1/(d*x + c))) + 2*a*
d*e*tan(1/2*arctan(1/(d*x + c)))^2 - 2*a*c*f*tan(1/2*arctan(1/(d*x + c)))^
2 - b*f*log(4*(4*d^2*e^2*tan(1/2*arctan(1/(d*x + c)))^2 - 8*c*d*e*f*tan(1/
2*arctan(1/(d*x + c)))^2 + 4*c^2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 - 4*d*
e*f*tan(1/2*arctan(1/(d*x + c)))^3 + 4*c*f^2*tan(1/2*arctan(1/(d*x + c)))^
3 + f^2*tan(1/2*arctan(1/(d*x + c)))^4 + 4*d*e*f*tan(1/2*arctan(1/(d*x ...
```

3.134.9 Mupad [B] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.84

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^2}\,dx = -\frac{a}{x\,f^2+e\,f} - \frac{b\,\mathrm{acot}(c+d\,x)}{f\,\left(e+f\,x\right)} - \frac{b\,d\,\ln\left(e+f\,x\right)}{d^2\,e^2-2\,c\,d\,e\,f+\left(c^2+1\right)\,f^2} + \frac{b\,d\,\ln\left(c+d\,x-\mathrm{i}\right)\,1\mathrm{i}}{2\,f\,\left(d\,e-c\,f+f\,1\mathrm{i}\right)} + \frac{b\,d\,\ln\left(c+d\,x+1\mathrm{i}\right)}{2\,f\,\left(f-c\,f\,1\mathrm{i}+d\,e\,1\mathrm{i}\right)}$$

input
```
int((a + b*acot(c + d*x))/(e + f*x)^2,x)
```

output
```
(b*d*log(c + d*x - 1i)*1i)/(2*f*(f*1i - c*f + d*e)) - (b*acot(c + d*x))/(f
*(e + f*x)) - (b*d*log(e + f*x))/(f^2*(c^2 + 1) + d^2*e^2 - 2*c*d*e*f) - a
/(e*f + f^2*x) + (b*d*log(c + d*x + 1i))/(2*f*(f - c*f*1i + d*e*1i))
```

3.134. $\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^2}\,dx$

3.135 $\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^3}\,dx$

3.135.1 Optimal result

Integrand size = 18, antiderivative size = 228

$$\begin{aligned}\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^3}\,dx = {} & \frac{bd}{2\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)(e+fx)} - \frac{a+b\cot^{-1}(c+dx)}{2f(e+fx)^2} \\ & - \frac{bd^2(de+f-cf)(de-(1+c)f)\arctan(c+dx)}{2f\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)^2} \\ & - \frac{bd^2(de-cf)\log(e+fx)}{\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)^2} \\ & + \frac{bd^2(de-cf)\log\left(1+c^2+2cdx+d^2x^2\right)}{2\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)^2}\end{aligned}$$

output
```
1/2*b*d/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)/(f*x+e)+1/2*(-a-b*arccot(d*x+c))/f
/(f*x+e)^2-1/2*b*d^2*(-c*f+d*e+f)*(d*e-(1+c)*f)*arctan(d*x+c)/f/(d^2*e^2-2
*c*d*e*f+(c^2+1)*f^2)^2-b*d^2*(-c*f+d*e)*ln(f*x+e)/(d^2*e^2-2*c*d*e*f+(c^2
+1)*f^2)^2+1/2*b*d^2*(-c*f+d*e)*ln(d^2*x^2+2*c*d*x+c^2+1)/(d^2*e^2-2*c*d*e
*f+(c^2+1)*f^2)^2
```

3.135.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.79

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^3}\,dx = \frac{\frac{bdf}{(d^2e^2-2cdef+(1+c^2)f^2)(e+fx)} - \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^2} + \frac{ibd^2\log(i-c-dx)}{2(de-(-i+c)f)^2} - \frac{ibd^2\log(i+c+dx)}{2(de-(i+c)f)^2} - \frac{2bd^2f(de-cf)\log(d(e+fx))}{(d^2e^2-2cdef+(1+c^2)f^2)^2}}{2f}$$

input
```
Integrate[(a + b*ArcCot[c + d*x])/(e + f*x)^3,x]
```

output
```
((b*d*f)/((d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)*(e + f*x)) - (a + b*ArcCot
[c + d*x])/(e + f*x)^2 + ((I/2)*b*d^2*Log[I - c - d*x])/(d*e - (-I + c)*f)
^2 - ((I/2)*b*d^2*Log[I + c + d*x])/(d*e - (I + c)*f)^2 - (2*b*d^2*f*(d*e
- c*f)*Log[d*(e + f*x)])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)^2)/(2*f)
```

3.135.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5569, 2081, 1145, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^3}\,dx$$

$$\Big\downarrow \text{5569}$$

$$-\frac{bd\int \frac{1}{(e+fx)^2((c+dx)^2+1)}dx}{2f} - \frac{a+b\cot^{-1}(c+dx)}{2f(e+fx)^2}$$

$$\Big\downarrow \text{2081}$$

$$-\frac{bd\int \frac{1}{(e+fx)^2(c^2+2dxc+d^2x^2+1)}dx}{2f} - \frac{a+b\cot^{-1}(c+dx)}{2f(e+fx)^2}$$

$$\Big\downarrow \text{1145}$$

$$-\frac{bd\left(\frac{\int \frac{d(de-2cf-dfx)}{(e+fx)\left(c^2+2dxc+d^2x^2+1\right)}dx}{(c^2+1)f^2-2cdef+d^2e^2}-\frac{f}{(e+fx)((c^2+1)f^2-2cdef+d^2e^2)}\right)}{2f}-\frac{a+b\cot^{-1}(c+dx)}{2f(e+fx)^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{bd\left(\frac{d\int \frac{de-2cf-dfx}{(e+fx)\left(c^2+2dxc+d^2x^2+1\right)}dx}{(c^2+1)f^2-2cdef+d^2e^2}-\frac{f}{(e+fx)((c^2+1)f^2-2cdef+d^2e^2)}\right)}{2f}-\frac{a+b\cot^{-1}(c+dx)}{2f(e+fx)^2}$$

$$\Big\downarrow \text{ 1200}$$

$$-\frac{bd\left(\frac{d\int \left(\frac{2(de-cf)f^2}{\left(d^2e^2-2cdfe+\left(c^2+1\right)f^2\right)(e+fx)}+\frac{d\left(d^2e^2-4cdfe-\left(1-3c^2\right)f^2-2df(de-cf)x\right)}{\left(d^2e^2-2cdfe+\left(c^2+1\right)f^2\right)\left(c^2+2dxc+d^2x^2+1\right)}\right)dx}{(c^2+1)f^2-2cdef+d^2e^2}-\frac{f}{(e+fx)((c^2+1)f^2-2cdef+d^2e^2)}\right)}{2f}-\frac{a+b\cot^{-1}(c+dx)}{2f(e+fx)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a+b\cot^{-1}(c+dx)}{2f(e+fx)^2}-\frac{bd\left(\frac{d\left(\frac{\arctan(c+dx)(-cf+de+f)(de-(c+1)f)}{\left(c^2+1\right)f^2-2cdef+d^2e^2}-\frac{f(de-cf)\log\left(c^2+2cdx+d^2x^2+1\right)}{\left(c^2+1\right)f^2-2cdef+d^2e^2}+\frac{2f(de-cf)\log(e+fx)}{\left(c^2+1\right)f^2-2cdef+d^2e^2}\right)}{(c^2+1)f^2-2cdef+d^2e^2}-\frac{f}{(e+fx)((c^2+1)f^2-2cdef+d^2e^2)}\right)}{2f}$$

input
```
Int[(a + b*ArcCot[c + d*x])/(e + f*x)^3,x]
```

output
```
-1/2*(a + b*ArcCot[c + d*x])/(f*(e + f*x)^2) - (b*d*(-(f/((d^2*e^2 - 2*c*d
*e*f + (1 + c^2)*f^2)*(e + f*x))) + (d*(((d*e + f - c*f)*(d*e - (1 + c)*f)
*ArcTan[c + d*x])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (2*f*(d*e - c*f)
*Log[e + f*x])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - (f*(d*e - c*f)*Log[
1 + c^2 + 2*c*d*x + d^2*x^2])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)))/(d^2
*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)))/(2*f)
```

3.135. $\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^3}\,dx$

3.135.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2081
```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

rule 5569
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCot[c + d*x])^p/(f*(m +
1))), x] + Simp[b*d*(p/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*((a + b*ArcCot[
c + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[p, 0] && ILtQ[m, -1]
```

3.135.4 Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.07

method	result
parts	$-\frac{a}{2(fx+e)^2 f}+\frac{b\Big(-\frac{d^3 \operatorname{arccot}(dx+c)}{2(f(dx+c)-cf+de)^2 f}-\frac{d^3\big(-\frac{f}{(c^2f^2-2cdef+d^2e^2+f^2)(f(dx+c)-cf+de)}-\frac{2(cf-de)f\ln(f(dx+c)-cf+\cdots}{(c^2f^2-2cdef+d^2e^2+f^2)\cdots}}{2\cdots}}{d}$
derivativedivides	$\frac{-\frac{a\,d^3}{2(cf-de-f(dx+c))^2 f}-b\,d^3\Big(\frac{\operatorname{arccot}(dx+c)}{2(cf-de-f(dx+c))^2 f}+\frac{\frac{f}{(c^2f^2-2cdef+d^2e^2+f^2)(cf-de-f(dx+c))}-\frac{2f(cf-de)\ln(cf-de-f(dx\cdots}{(c^2f^2-2cdef+d^2e^2+f^2\cdots}}{\cdots}}{d}$
default	$\frac{-\frac{a\,d^3}{2(cf-de-f(dx+c))^2 f}-b\,d^3\Big(\frac{\operatorname{arccot}(dx+c)}{2(cf-de-f(dx+c))^2 f}+\frac{\frac{f}{(c^2f^2-2cdef+d^2e^2+f^2)(cf-de-f(dx+c))}-\frac{2f(cf-de)\ln(cf-de-f(dx\cdots}{(c^2f^2-2cdef+d^2e^2+f^2\cdots}}{\cdots}}{d}$
parallelrisch	$\frac{-a\,d^2f^5-2x^2\operatorname{arccot}(dx+c)bc\,d^5e\,f^4+2x\operatorname{arccot}(dx+c)b\,c^2d^4e\,f^4-4x\operatorname{arccot}(dx+c)bc\,d^5e^2f^3+4\ln(fx+e)xbc\,d^4e\,f^4-2\cdots}{\cdots}$
risch	Expression too large to display

input
```
int((a+b*arccot(d*x+c))/(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/(f*x+e)^2/f+b/d*(-1/2*d^3/(f*(d*x+c)-c*f+d*e)^2/f*arccot(d*x+c)-1/2
*d^3/f*(-f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)/(f*(d*x+c)-c*f+d*e)-2*(c*f-d*e)
*f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)^2*ln(f*(d*x+c)-c*f+d*e)+1/(c^2*f^2-2*c*
d*e*f+d^2*e^2+f^2)^2*(1/2*(2*c*f^2-2*d*e*f)*ln(1+(d*x+c)^2)+(c^2*f^2-2*c*d
*e*f+d^2*e^2-f^2)*arctan(d*x+c))))
```

3.135.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(220) = 440$.

Time = 1.21 (sec) , antiderivative size = 728, normalized size of antiderivative = 3.19

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^3}\,dx =$$
$$-\frac{ad^4e^4-(4\,ac+b)d^3e^3f+2\,(3\,ac^2+bc+a)d^2e^2f^2-(4\,ac^3+bc^2+4\,ac+b)def^3+(ac^4+2\,ac^2+a)f\cdots}{\cdots}$$

input
```
integrate((a+b*arccot(d*x+c))/(f*x+e)^3,x, algorithm="fricas")
```

output
```
-1/2*(a*d^4*e^4 - (4*a*c + b)*d^3*e^3*f + 2*(3*a*c^2 + b*c + a)*d^2*e^2*f^
2 - (4*a*c^3 + b*c^2 + 4*a*c + b)*d*e*f^3 + (a*c^4 + 2*a*c^2 + a)*f^4 - (b
*d^3*e^2*f^2 - 2*b*c*d^2*e*f^3 + (b*c^2 + b)*d*f^4)*x + (b*d^4*e^4 - 4*b*c
*d^3*e^3*f + 2*(3*b*c^2 + b)*d^2*e^2*f^2 - 4*(b*c^3 + b*c)*d*e*f^3 + (b*c^
4 + 2*b*c^2 + b)*f^4)*arccot(d*x + c) + (b*d^4*e^4 - 2*b*c*d^3*e^3*f + (b*
c^2 - b)*d^2*e^2*f^2 + (b*d^4*e^2*f^2 - 2*b*c*d^3*e*f^3 + (b*c^2 - b)*d^2*
f^4)*x^2 + 2*(b*d^4*e^3*f - 2*b*c*d^3*e^2*f^2 + (b*c^2 - b)*d^2*e*f^3)*x)*
arctan(d*x + c) - (b*d^3*e^3*f - b*c*d^2*e^2*f^2 + (b*d^3*e*f^3 - b*c*d^2*
f^4)*x^2 + 2*(b*d^3*e^2*f^2 - b*c*d^2*e*f^3)*x)*log(d^2*x^2 + 2*c*d*x + c^
2 + 1) + 2*(b*d^3*e^3*f - b*c*d^2*e^2*f^2 + (b*d^3*e*f^3 - b*c*d^2*f^4)*x^
2 + 2*(b*d^3*e^2*f^2 - b*c*d^2*e*f^3)*x)*log(f*x + e))/(d^4*e^6*f - 4*c*d^
3*e^5*f^2 + 2*(3*c^2 + 1)*d^2*e^4*f^3 - 4*(c^3 + c)*d*e^3*f^4 + (c^4 + 2*c
^2 + 1)*e^2*f^5 + (d^4*e^4*f^3 - 4*c*d^3*e^3*f^4 + 2*(3*c^2 + 1)*d^2*e^2*f
^5 - 4*(c^3 + c)*d*e*f^6 + (c^4 + 2*c^2 + 1)*f^7)*x^2 + 2*(d^4*e^5*f^2 - 4
*c*d^3*e^4*f^3 + 2*(3*c^2 + 1)*d^2*e^3*f^4 - 4*(c^3 + c)*d*e^2*f^5 + (c^4
+ 2*c^2 + 1)*e*f^6)*x)
```

3.135.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acot(d*x+c))/(f*x+e)**3,x)
```

output
```
Timed out
```

3.135.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.80

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^3}\,dx$$
$$= \frac{1}{2}\left(d\left(\frac{(d^2e-cdf)\log(d^2x^2+2\,cdx+c^2+1)}{d^4e^4-4\,cd^3e^3f+2\,(3\,c^2+1)d^2e^2f^2-4\,(c^3+c)def^3+(c^4+2\,c^2+1)f^4} - \frac{}{d^4e^4-4\,cd^3e^3f+2\,(3\right.\right.$$
$$- \frac{a}{2\,(f^3x^2+2\,ef^2x+e^2f)}$$

input
```
integrate((a+b*arccot(d*x+c))/(f*x+e)^3,x, algorithm="maxima")
```

output
```
1/2*(d*((d^2*e - c*d*f)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^4*e^4 - 4*c*d^
3*e^3*f + 2*(3*c^2 + 1)*d^2*e^2*f^2 - 4*(c^3 + c)*d*e*f^3 + (c^4 + 2*c^2 +
 1)*f^4) - 2*(d^2*e - c*d*f)*log(f*x + e)/(d^4*e^4 - 4*c*d^3*e^3*f + 2*(3*
c^2 + 1)*d^2*e^2*f^2 - 4*(c^3 + c)*d*e*f^3 + (c^4 + 2*c^2 + 1)*f^4) - (d^4
*e^2 - 2*c*d^3*e*f + (c^2 - 1)*d^2*f^2)*arctan((d^2*x + c*d)/d)/((d^4*e^4*
f - 4*c*d^3*e^3*f^2 + 2*(3*c^2 + 1)*d^2*e^2*f^3 - 4*(c^3 + c)*d*e*f^4 + (c
^4 + 2*c^2 + 1)*f^5)*d) + 1/(d^2*e^3 - 2*c*d*e^2*f + (c^2 + 1)*e*f^2 + (d^
2*e^2*f - 2*c*d*e*f^2 + (c^2 + 1)*f^3)*x)) - arccot(d*x + c)/(f^3*x^2 + 2*
e*f^2*x + e^2*f))*b - 1/2*a/(f^3*x^2 + 2*e*f^2*x + e^2*f)
```

3.135.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6173 vs. $2(220) = 440$.

Time = 2.13 (sec) , antiderivative size = 6173, normalized size of antiderivative = 27.07

$$\int \frac{a + b\cot^{-1}(c + dx)}{(e + fx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*arccot(d*x+c))/(f*x+e)^3,x, algorithm="giac")
```

output

```
-1/2*(4*b*d^4*e^3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^3 - 12*
b*c*d^3*e^2*f*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^3 + 12*b*c^
2*d^2*e*f^2*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^3 - 4*b*c^3*d
*f^3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^3 - b*d^3*e^2*f*arct
an(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^4 + 2*b*c*d^2*e*f^2*arctan(1/
(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^4 - b*c^2*d*f^3*arctan(1/(d*x + c)
)*tan(1/2*arctan(1/(d*x + c)))^4 + 4*b*d^4*e^3*log(4*(4*d^2*e^2*tan(1/2*ar
ctan(1/(d*x + c)))^2 - 8*c*d*e*f*tan(1/2*arctan(1/(d*x + c)))^2 + 4*c^2*f^
2*tan(1/2*arctan(1/(d*x + c)))^2 - 4*d*e*f*tan(1/2*arctan(1/(d*x + c)))^3
+ 4*c*f^2*tan(1/2*arctan(1/(d*x + c)))^3 + f^2*tan(1/2*arctan(1/(d*x + c))
)^4 + 4*d*e*f*tan(1/2*arctan(1/(d*x + c))) - 4*c*f^2*tan(1/2*arctan(1/(d*x
 + c))) - 2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 + f^2)/(tan(1/2*arctan(1/(d
*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/2*arctan(1/(d*x
 + c)))^2 - 12*b*c*d^3*e^2*f*log(4*(4*d^2*e^2*tan(1/2*arctan(1/(d*x + c)))
^2 - 8*c*d*e*f*tan(1/2*arctan(1/(d*x + c)))^2 + 4*c^2*f^2*tan(1/2*arctan(1
/(d*x + c)))^2 - 4*d*e*f*tan(1/2*arctan(1/(d*x + c)))^3 + 4*c*f^2*tan(1/2*
arctan(1/(d*x + c)))^3 + f^2*tan(1/2*arctan(1/(d*x + c)))^4 + 4*d*e*f*tan(
1/2*arctan(1/(d*x + c))) - 4*c*f^2*tan(1/2*arctan(1/(d*x + c))) - 2*f^2*ta
n(1/2*arctan(1/(d*x + c)))^2 + f^2)/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*ta
n(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/2*arctan(1/(d*x + c)))^2 + 12*...
```

3.135. $\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^3}\,dx$

3.135.9 Mupad [B] (verification not implemented)

Time = 8.33 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.75

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^3}\,dx = \frac{b\,d\,e}{2\,(e+f\,x)^2\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)} - \frac{a\,f}{2\,(e+f\,x)^2\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)} - \frac{b\,\mathrm{acot}(c+d\,x)}{2\,f\,(e+f\,x)^2} - \frac{a\,c^2\,f}{2\,(e+f\,x)^2\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)} - \frac{b\,d^3\,e\,\ln\left(e+f\,x\right)}{\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)^2} + \frac{b\,c\,d^2\,f\,\ln\left(e+f\,x\right)}{\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)^2} + \frac{a\,c\,d\,e}{(e+f\,x)^2\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)} + \frac{b\,d\,f\,x}{2\,(e+f\,x)^2\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)} - \frac{a\,d^2\,e^2}{2\,f\,(e+f\,x)^2\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)} + \frac{b\,d^2\,\ln\left(c+d\,x-\mathrm{i}\right)\,1\mathrm{i}}{4\,f\,\left(d\,e-c\,f+f\,1\mathrm{i}\right)^2} - \frac{b\,d^2\,\ln\left(c+d\,x+1\mathrm{i}\right)\,1\mathrm{i}}{4\,f\,\left(c\,f-d\,e+f\,1\mathrm{i}\right)^2}$$

input
```
int((a + b*acot(c + d*x))/(e + f*x)^3,x)
```

output
```
(b*d*e)/(2*(e + f*x)^2*(f^2 + c^2*f^2 + d^2*e^2 - 2*c*d*e*f)) - (a*f)/(2*(
e + f*x)^2*(f^2 + c^2*f^2 + d^2*e^2 - 2*c*d*e*f)) - (b*acot(c + d*x))/(2*f
*(e + f*x)^2) + (b*d^2*log(c + d*x - 1i)*1i)/(4*f*(f*1i - c*f + d*e)^2) -
(b*d^2*log(c + d*x + 1i)*1i)/(4*f*(f*1i + c*f - d*e)^2) - (a*c^2*f)/(2*(e
+ f*x)^2*(f^2 + c^2*f^2 + d^2*e^2 - 2*c*d*e*f)) - (b*d^3*e*log(e + f*x))/(
f^2 + c^2*f^2 + d^2*e^2 - 2*c*d*e*f)^2 + (b*c*d^2*f*log(e + f*x))/(f^2 + c
^2*f^2 + d^2*e^2 - 2*c*d*e*f)^2 + (a*c*d*e)/((e + f*x)^2*(f^2 + c^2*f^2 +
d^2*e^2 - 2*c*d*e*f)) + (b*d*f*x)/(2*(e + f*x)^2*(f^2 + c^2*f^2 + d^2*e^2
- 2*c*d*e*f)) - (a*d^2*e^2)/(2*f*(e + f*x)^2*(f^2 + c^2*f^2 + d^2*e^2 - 2*
c*d*e*f))
```

3.136 $\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx$

3.136.1 Optimal result

Integrand size = 20, antiderivative size = 382

$$\begin{aligned}
&\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx \\
&= \frac{b^2f^2x}{3d^2} + \frac{2abf(de-cf)x}{d^2} + \frac{2b^2f(de-cf)(c+dx)\cot^{-1}(c+dx)}{d^3} \\
&\quad + \frac{bf^2(c+dx)^2\left(a+b\cot^{-1}(c+dx)\right)}{3d^3} \\
&\quad + \frac{i(3d^2e^2-6cdef-(1-3c^2)f^2)\left(a+b\cot^{-1}(c+dx)\right)^2}{3d^3} \\
&\quad - \frac{(de-cf)\left(d^2e^2-2cdef-(3-c^2)f^2\right)\left(a+b\cot^{-1}(c+dx)\right)^2}{3d^3f} \\
&\quad + \frac{(e+fx)^3\left(a+b\cot^{-1}(c+dx)\right)^2}{3f} - \frac{b^2f^2\arctan(c+dx)}{3d^3} \\
&\quad - \frac{2b(3d^2e^2-6cdef-(1-3c^2)f^2)\left(a+b\cot^{-1}(c+dx)\right)\log\left(\frac{2}{1+i(c+dx)}\right)}{3d^3} \\
&\quad + \frac{b^2f(de-cf)\log\left(1+(c+dx)^2\right)}{d^3} \\
&\quad + \frac{ib^2(3d^2e^2-6cdef-(1-3c^2)f^2)\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{3d^3}
\end{aligned}$$

output
```
1/3*b^2*f^2*x/d^2+2*a*b*f*(-c*f+d*e)*x/d^2+2*b^2*f*(-c*f+d*e)*(d*x+c)*arcc
ot(d*x+c)/d^3+1/3*b*f^2*(d*x+c)^2*(a+b*arccot(d*x+c))/d^3+1/3*I*(3*d^2*e^2
-6*c*d*e*f-(-3*c^2+1)*f^2)*(a+b*arccot(d*x+c))^2/d^3-1/3*(-c*f+d*e)*(d^2*e
^2-2*c*d*e*f-(-c^2+3)*f^2)*(a+b*arccot(d*x+c))^2/d^3/f+1/3*(f*x+e)^3*(a+b*
arccot(d*x+c))^2/f-1/3*b^2*f^2*arctan(d*x+c)/d^3-2/3*b*(3*d^2*e^2-6*c*d*e*
f-(-3*c^2+1)*f^2)*(a+b*arccot(d*x+c))*ln(2/(1+I*(d*x+c)))/d^3+b^2*f*(-c*f+
d*e)*ln(1+(d*x+c)^2)/d^3+1/3*I*b^2*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*po
lylog(2,1-2/(1+I*(d*x+c)))/d^3
```

3.136.2 Mathematica [A] (verified)

Time = 6.67 (sec) , antiderivative size = 586, normalized size of antiderivative = 1.53

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx$$

$$= \frac{b^2cf^2+3a^2d^3e^2x+6abd^2efx+b^2df^2x-4abcdf^2x+3a^2d^3efx^2+abd^2f^2x^2+a^2d^3f^2x^3+b^2(i+c+dx)}{}$$

input
```
Integrate[(e + f*x)^2*(a + b*ArcCot[c + d*x])^2,x]
```

output
```
(b^2*c*f^2 + 3*a^2*d^3*e^2*x + 6*a*b*d^2*e*f*x + b^2*d*f^2*x - 4*a*b*c*d*f
^2*x + 3*a^2*d^3*e*f*x^2 + a*b*d^2*f^2*x^2 + a^2*d^3*f^2*x^3 + b^2*(I + c
+ d*x)*((I + c)^2*f^2 - (I + c)*d*f*(3*e + f*x) + d^2*(3*e^2 + 3*e*f*x + f
^2*x^2))*ArcCot[c + d*x]^2 - 6*a*b*c*d^2*e^2*ArcTan[c + d*x] - 6*a*b*d*e*f
*ArcTan[c + d*x] + 6*a*b*c^2*d*e*f*ArcTan[c + d*x] + 6*a*b*c*f^2*ArcTan[c
+ d*x] - 2*a*b*c^3*f^2*ArcTan[c + d*x] - b*ArcCot[c + d*x]*(-2*a*d^3*x*(3*
e^2 + 3*e*f*x + f^2*x^2) + b*f*(5*c^2*f - 6*d^2*e*x + c*d*(-6*e + 4*f*x) -
 f*(1 + d^2*x^2)) + 2*b*(3*d^2*e^2 - 6*c*d*e*f + (-1 + 3*c^2)*f^2)*Log[1 -
 E^((2*I)*ArcCot[c + d*x])]) + 6*b^2*c*f^2*Log[(c + d*x)^(-1)] + 3*a*b*d^2
*e^2*Log[1 + c^2 + 2*c*d*x + d^2*x^2] - 6*a*b*c*d*e*f*Log[1 + c^2 + 2*c*d*
x + d^2*x^2] - a*b*f^2*Log[1 + c^2 + 2*c*d*x + d^2*x^2] + 3*a*b*c^2*f^2*Lo
g[1 + c^2 + 2*c*d*x + d^2*x^2] + 6*b^2*c*f^2*Log[1/Sqrt[1 + (c + d*x)^(-2)
]] - 6*b^2*d*e*f*Log[1/((c + d*x)*Sqrt[1 + (c + d*x)^(-2)])] + I*b^2*(3*d^
2*e^2 - 6*c*d*e*f + (-1 + 3*c^2)*f^2)*PolyLog[2, E^((2*I)*ArcCot[c + d*x])
])/(3*d^3)
```

3.136.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5571, 27, 5390, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{5571}$$

$$\frac{\int \frac{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)^2\left(a+b\cot^{-1}(c+dx)\right)^2}{d^2}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (de-cf+f(c+dx))^2\left(a+b\cot^{-1}(c+dx)\right)^2d(c+dx)}{d^3}$$

$$\Big\downarrow \text{5390}$$

$$\frac{\frac{2b\int\left((c+dx)\left(a+b\cot^{-1}(c+dx)\right)f^3+3(de-cf)\left(a+b\cot^{-1}(c+dx)\right)f^2+\frac{\left((de-cf)\left(d^2e^2-2cdfe-\left(3-c^2\right)f^2\right)+f\left(3d^2e^2-6cdfe-\left(1-3c^2\right)f^2\right)(c+dx)\right)\left(a+b\,\mathrm{c}\right.}{(c+dx)^2+1}}{3f}}{d^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(f(c+dx)-cf+de)^3\left(a+b\cot^{-1}(c+dx)\right)^2}{3f}+\frac{2b\left(\frac{if\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)\left(a+b\cot^{-1}(c+dx)\right)^2}{2b}-\frac{(de-cf)\left(-\left(3-c^2\right)f^2-2cdef+d^2e^2\right)\left(a+b\cot^{-1}(\right.}{2b}\right.}{}}{}$$

input
```
Int[(e + f*x)^2*(a + b*ArcCot[c + d*x])^2,x]
```

output
```
(((d*e - c*f + f*(c + d*x))^3*(a + b*ArcCot[c + d*x])^2)/(3*f) + (2*b*((b*
f^3*(c + d*x))/2 + 3*a*f^2*(d*e - c*f)*(c + d*x) + 3*b*f^2*(d*e - c*f)*(c
+ d*x)*ArcCot[c + d*x] + (f^3*(c + d*x)^2*(a + b*ArcCot[c + d*x]))/2 + ((I
/2)*f*(3*d^2*e^2 - 6*c*d*e*f - (1 - 3*c^2)*f^2)*(a + b*ArcCot[c + d*x])^2)
/b - ((d*e - c*f)*(d^2*e^2 - 2*c*d*e*f - (3 - c^2)*f^2)*(a + b*ArcCot[c +
d*x])^2)/(2*b) - (b*f^3*ArcTan[c + d*x])/2 - f*(3*d^2*e^2 - 6*c*d*e*f - (1
 - 3*c^2)*f^2)*(a + b*ArcCot[c + d*x])*Log[2/(1 + I*(c + d*x))] + (3*b*f^2
*(d*e - c*f)*Log[1 + (c + d*x)^2])/2 + (I/2)*b*f*(3*d^2*e^2 - 6*c*d*e*f -
(1 - 3*c^2)*f^2)*PolyLog[2, 1 - 2/(1 + I*(c + d*x))]))/(3*f))/d^3
```

3.136.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5390
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])^p/(e*(q + 1))), x] + S
imp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcCot[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

rule 5571
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.136.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1071 vs. 2(362) = 724.

Time = 1.34 (sec) , antiderivative size = 1072, normalized size of antiderivative = 2.81

method	result	size
parts	Expression too large to display	1072
derivativedivides	Expression too large to display	1087
default	Expression too large to display	1087
risch	Expression too large to display	3165

input
```
int((f*x+e)^2*(a+b*arccot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2*(f*x+e)^3/f+b^2/d*(1/3/d^2*f^2*arccot(d*x+c)^2*(d*x+c)^3-1/d^2*f^2
*arccot(d*x+c)^2*(d*x+c)^2*c+1/d*f*arccot(d*x+c)^2*(d*x+c)^2*e+1/d^2*f^2*a
rccot(d*x+c)^2*(d*x+c)*c^2-2/d*f*arccot(d*x+c)^2*(d*x+c)*c*e+arccot(d*x+c)
^2*(d*x+c)*e^2-1/3/d^2*f^2*arccot(d*x+c)^2*c^3+1/d*f*arccot(d*x+c)^2*c^2*e
-arccot(d*x+c)^2*c*e^2+1/3*d/f*arccot(d*x+c)^2*e^3+2/3/d^2/f*(1/2*arccot(d
*x+c)*f^3*(d*x+c)^2-3*arccot(d*x+c)*c*f^3*(d*x+c)+3*arccot(d*x+c)*d*e*f^2*
(d*x+c)+3/2*arccot(d*x+c)*ln(1+(d*x+c)^2)*c^2*f^3-3*arccot(d*x+c)*ln(1+(d*
x+c)^2)*c*d*e*f^2+3/2*arccot(d*x+c)*ln(1+(d*x+c)^2)*d^2*e^2*f-1/2*arccot(d
*x+c)*ln(1+(d*x+c)^2)*f^3-arccot(d*x+c)*arctan(d*x+c)*c^3*f^3+3*arccot(d*x
+c)*arctan(d*x+c)*c^2*d*e*f^2-3*arccot(d*x+c)*arctan(d*x+c)*c*d^2*e^2*f+ar
ccot(d*x+c)*arctan(d*x+c)*d^3*e^3+3*arccot(d*x+c)*arctan(d*x+c)*c*f^3-3*ar
ccot(d*x+c)*arctan(d*x+c)*d*e*f^2+1/2*f^2*(f*(d*x+c)+1/2*(-6*c*f+6*d*e)*ln
(1+(d*x+c)^2)-f*arctan(d*x+c))+1/2*f*(3*c^2*f^2-6*c*d*e*f+3*d^2*e^2-f^2)*(
-1/2*I*(ln(d*x+c-I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c-I)^2-dilog(-1/2*I*(d*x+c+
I))-ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I)))+1/2*I*(ln(d*x+c+I)*ln(1+(d*x+c)^2)-1
/2*ln(d*x+c+I)^2-dilog(1/2*I*(d*x+c-I))-ln(d*x+c+I)*ln(1/2*I*(d*x+c-I))))+
1/4*(-2*c^3*f^3+6*c^2*d*e*f^2-6*c*d^2*e^2*f+2*d^3*e^3+6*c*f^3-6*d*e*f^2)*a
rctan(d*x+c)^2))+2/3*a*b/f*arccot(d*x+c)*e^3-5/3/d^3*c^2*f^2*b*a+1/3/d*f^2
*b*a*x^2-1/3*a*b/d^3*f^2*ln(1+(d*x+c)^2)-2*a*b/d^2*f*ln(1+(d*x+c)^2)*c*e+2
*b/d^2*arctan(d*x+c)*a*c^2*e*f-2*b/d*arctan(d*x+c)*a*c*e^2+2*a*b*f*arcc...
```

3.136.5 Fricas [F]

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx = \int (fx+e)^2 (b\,\mathrm{arccot}\,(dx+c)+a)^2 \, dx$$

input
```
integrate((f*x+e)^2*(a+b*arccot(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral(a^2*f^2*x^2 + 2*a^2*e*f*x + a^2*e^2 + (b^2*f^2*x^2 + 2*b^2*e*f*x
+ b^2*e^2)*arccot(d*x + c)^2 + 2*(a*b*f^2*x^2 + 2*a*b*e*f*x + a*b*e^2)*arc
cot(d*x + c), x)
```

3.136.6 Sympy [F(-1)]

Timed out.

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*(a+b*acot(d*x+c))**2,x)
```

output
```
Timed out
```

3.136.7 Maxima [F]

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx = \int (fx+e)^2 (b\,\mathrm{arccot}\,(dx+c)+a)^2 \, dx$$

input
```
integrate((f*x+e)^2*(a+b*arccot(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/12*b^2*f^2*x^3*arctan2(1, d*x + c)^2 + 1/4*b^2*e*f*x^2*arctan2(1, d*x +
c)^2 + 1/3*a^2*f^2*x^3 + 1/4*b^2*e^2*x*arctan2(1, d*x + c)^2 + a^2*e*f*x^2
 + 2*(x^2*arccot(d*x + c) + d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d
^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a*b*e*f + 1/3*(2*x^3*arccot(
d*x + c) + d*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)/d)/
d^4 + (3*c^2 - 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*a*b*f^2 + a^2*e^2
*x + (2*(d*x + c)*arccot(d*x + c) + log((d*x + c)^2 + 1))*a*b*e^2/d - 1/48
*(b^2*f^2*x^3 + 3*b^2*e*f*x^2 + 3*b^2*e^2*x)*log(d^2*x^2 + 2*c*d*x + c^2 +
 1)^2 + integrate(1/48*(36*b^2*d^2*f^2*x^4*arctan2(1, d*x + c)^2 + 8*(9*b^
2*d^2*e*f*arctan2(1, d*x + c)^2 + (9*b^2*c*arctan2(1, d*x + c)^2 + b^2*arc
tan2(1, d*x + c))*d*f^2)*x^3 + 36*(b^2*c^2*arctan2(1, d*x + c)^2 + b^2*arc
tan2(1, d*x + c)^2)*e^2 + 12*(3*b^2*d^2*e^2*arctan2(1, d*x + c)^2 + 2*(6*b
^2*c*arctan2(1, d*x + c)^2 + b^2*arctan2(1, d*x + c))*d*e*f + 3*(b^2*c^2*a
rctan2(1, d*x + c)^2 + b^2*arctan2(1, d*x + c)^2)*f^2)*x^2 + 3*(b^2*d^2*f^
2*x^4 + 2*(b^2*d^2*e*f + b^2*c*d*f^2)*x^3 + (b^2*c^2 + b^2)*e^2 + (b^2*d^2
*e^2 + 4*b^2*c*d*e*f + (b^2*c^2 + b^2)*f^2)*x^2 + 2*(b^2*c*d*e^2 + (b^2*c^
2 + b^2)*e*f)*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 24*((3*b^2*c*arctan2
(1, d*x + c)^2 + b^2*arctan2(1, d*x + c))*d*e^2 + 3*(b^2*c^2*arctan2(1, d*
x + c)^2 + b^2*arctan2(1, d*x + c)^2)*e*f)*x + 4*(b^2*d^2*f^2*x^4 + 3*b^2*
c*d*e^2*x + (3*b^2*d^2*e*f + b^2*c*d*f^2)*x^3 + 3*(b^2*d^2*e^2 + b^2*c*...
```

3.136.8 Giac [F]

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx = \int (fx+e)^2 (b \operatorname{arccot}(dx+c)+a)^2 \, dx$$

input
```
integrate((f*x+e)^2*(a+b*arccot(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*(b*arccot(d*x + c) + a)^2, x)
```

3.136.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx = \int (e+f\,x)^2 \,(a+b\,\mathrm{acot}(c+d\,x))^2 \, dx$$

input
```
int((e + f*x)^2*(a + b*acot(c + d*x))^2,x)
```

output
```
int((e + f*x)^2*(a + b*acot(c + d*x))^2, x)
```

3.137 $\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^2\,dx$

3.137.1 Optimal result

Integrand size = 18, antiderivative size = 220

$$\begin{aligned}
&\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^2\,dx \\
&= \frac{abfx}{d}+\frac{b^2f(c+dx)\cot^{-1}(c+dx)}{d^2}+\frac{i(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^2}{d^2} \\
&\quad -\frac{(de+f-cf)(de-(1+c)f)\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2f} \\
&\quad +\frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2f}-\frac{2b(de-cf)\left(a+b\cot^{-1}(c+dx)\right)\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2} \\
&\quad +\frac{b^2f\log\left(1+(c+dx)^2\right)}{2d^2}+\frac{ib^2(de-cf)\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2}
\end{aligned}$$

output
```
a*b*f*x/d+b^2*f*(d*x+c)*arccot(d*x+c)/d^2+I*(-c*f+d*e)*(a+b*arccot(d*x+c))
^2/d^2-1/2*(-c*f+d*e+f)*(d*e-(1+c)*f)*(a+b*arccot(d*x+c))^2/d^2/f+1/2*(f*x
+e)^2*(a+b*arccot(d*x+c))^2/f-2*b*(-c*f+d*e)*(a+b*arccot(d*x+c))*ln(2/(1+I
*(d*x+c)))/d^2+1/2*b^2*f*ln(1+(d*x+c)^2)/d^2+I*b^2*(-c*f+d*e)*polylog(2,1-
2/(1+I*(d*x+c)))/d^2
```

3.137.2 Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.30

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^2\,dx$$

$$= \frac{2a^2cde+2abcf-a^2c^2f+2a^2d^2ex+2abdfx+a^2d^2fx^2+b^2(i+c+dx)(-((i+c)f)+d(2e+fx))\cot^{-}}{}$$

input
```
Integrate[(e + f*x)*(a + b*ArcCot[c + d*x])^2,x]
```

output
```
(2*a^2*c*d*e + 2*a*b*c*f - a^2*c^2*f + 2*a^2*d^2*e*x + 2*a*b*d*f*x + a^2*d
^2*f*x^2 + b^2*(I + c + d*x)*(-((I + c)*f) + d*(2*e + f*x))*ArcCot[c + d*x
]^2 - 2*a*b*f*ArcTan[c + d*x] + 2*b*ArcCot[c + d*x]*(-((c + d*x)*(-(b*f) +
 a*c*f - a*d*(2*e + f*x))) - 2*b*(d*e - c*f)*Log[1 - E^((2*I)*ArcCot[c + d
*x])]) - 4*a*b*d*e*Log[1/((c + d*x)*Sqrt[1 + (c + d*x)^(-2)])] - 2*b^2*f*L
og[1/((c + d*x)*Sqrt[1 + (c + d*x)^(-2)])] + 4*a*b*c*f*Log[1/((c + d*x)*Sq
rt[1 + (c + d*x)^(-2)])] + (2*I)*b^2*(d*e - c*f)*PolyLog[2, E^((2*I)*ArcCo
t[c + d*x])])/(2*d^2)
```

3.137.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5571, 27, 5390, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^2\,dx$$

$$\Big\downarrow \; 5571$$

$$\frac{\int \frac{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)\left(a+b\cot^{-1}(c+dx)\right)^2}{d}d(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\int (de-cf+f(c+dx))\left(a+b\cot^{-1}(c+dx)\right)^2d(c+dx)}{d^2}$$

$$\Big\downarrow \text{5390}$$

$$\frac{\frac{b\int\left(\left(a+b\cot^{-1}(c+dx)\right)f^2+\frac{((de-cf+f)(de-(c+1)f)+2f(de-cf)(c+dx))\left(a+b\cot^{-1}(c+dx)\right)}{(c+dx)^2+1}\right)d(c+dx)}{f}+\frac{(f(c+dx)-cf+de)^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2f}}{d^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(f(c+dx)-cf+de)^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2f}+\frac{b\left(\frac{if(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^2}{b}-\frac{(-cf+de+f)(de-(c+1)f)\left(a+b\cot^{-1}(c+dx)\right)^2}{2b}-2f(de-cf)\log\left(\frac{2}{1+i(c+\ldots}\right.\right.}{\ldots}}{d^2}$$

input
```
Int[(e + f*x)*(a + b*ArcCot[c + d*x])^2,x]
```

output
```
(((d*e - c*f + f*(c + d*x))^2*(a + b*ArcCot[c + d*x])^2)/(2*f) + (b*(a*f^2
*(c + d*x) + b*f^2*(c + d*x)*ArcCot[c + d*x] + (I*f*(d*e - c*f)*(a + b*Arc
Cot[c + d*x])^2)/b - ((d*e + f - c*f)*(d*e - (1 + c)*f)*(a + b*ArcCot[c +
d*x])^2)/(2*b) - 2*f*(d*e - c*f)*(a + b*ArcCot[c + d*x])*Log[2/(1 + I*(c +
 d*x))] + (b*f^2*Log[1 + (c + d*x)^2])/2 + I*b*f*(d*e - c*f)*PolyLog[2, 1
- 2/(1 + I*(c + d*x))]))/f)/d^2
```

3.137.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5390
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])^p/(e*(q + 1))), x] + S
imp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcCot[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

rule 5571
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.137.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(210) = 420$.

Time = 0.88 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.94

method	result
parts	$a^2\left(\frac{1}{2}f\,x^2+ex\right)+\frac{b^2\left(\frac{\operatorname{arccot}(dx+c)^2(dx+c)^2f}{2d}-\frac{\operatorname{arccot}(dx+c)^2cf(dx+c)}{d}+\operatorname{arccot}(dx+c)^2e(dx+c)+\frac{-\operatorname{arccot}(dx+c)\ln\left(\right.}{}\right.}{}$
derivativedivides	$-\frac{a^2\left(fc(dx+c)-ed(dx+c)-\frac{f(dx+c)^2}{2}\right)}{d}-\frac{b^2\left(\operatorname{arccot}(dx+c)^2fc(dx+c)-\operatorname{arccot}(dx+c)^2ed(dx+c)-\frac{\operatorname{arccot}(dx+c)^2f(dx+c)^2}{2}+\operatorname{arccot}(\right.}{}$
default	$-\frac{a^2\left(fc(dx+c)-ed(dx+c)-\frac{f(dx+c)^2}{2}\right)}{d}-\frac{b^2\left(\operatorname{arccot}(dx+c)^2fc(dx+c)-\operatorname{arccot}(dx+c)^2ed(dx+c)-\frac{\operatorname{arccot}(dx+c)^2f(dx+c)^2}{2}+\operatorname{arccot}(\right.}{}$
risch	Expression too large to display

input
```
int((f*x+e)*(a+b*arccot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/2*f*x^2+e*x)+b^2/d*(1/2/d*arccot(d*x+c)^2*(d*x+c)^2*f-1/d*arccot(d*
x+c)^2*c*f*(d*x+c)+arccot(d*x+c)^2*e*(d*x+c)+1/d*(-arccot(d*x+c)*ln(1+(d*x
+c)^2)*c*f+arccot(d*x+c)*ln(1+(d*x+c)^2)*d*e-arccot(d*x+c)*arctan(d*x+c)*f
+arccot(d*x+c)*(d*x+c)*f+1/2*f*ln(1+(d*x+c)^2)-1/2*arctan(d*x+c)^2*f+1/2*(
-2*c*f+2*d*e)*(-1/2*I*(ln(d*x+c-I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c-I)^2-dilog
(-1/2*I*(d*x+c+I))-ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I)))+1/2*I*(ln(d*x+c+I)*ln
(1+(d*x+c)^2)-1/2*ln(d*x+c+I)^2-dilog(1/2*I*(d*x+c-I))-ln(d*x+c+I)*ln(1/2*
I*(d*x+c-I))))))+2*a*b/d*(1/2/d*arccot(d*x+c)*(d*x+c)^2*f-1/d*arccot(d*x+c
)*c*f*(d*x+c)+arccot(d*x+c)*e*(d*x+c)+1/2/d*(f*(d*x+c)+1/2*(-2*c*f+2*d*e)*
ln(1+(d*x+c)^2)-f*arctan(d*x+c)))
```

3.137.5 Fricas [F]

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^2\,dx=\int (fx+e)(b\,\mathrm{arccot}\,(dx+c)+a)^2\,dx$$

input
```
integrate((f*x+e)*(a+b*arccot(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral(a^2*f*x + a^2*e + (b^2*f*x + b^2*e)*arccot(d*x + c)^2 + 2*(a*b*f*
x + a*b*e)*arccot(d*x + c), x)
```

3.137.6 Sympy [F]

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^2\,dx=\int (a+b\,\mathrm{acot}\,(c+dx))^2\,(e+fx)\,dx$$

input
```
integrate((f*x+e)*(a+b*acot(d*x+c))**2,x)
```

output
```
Integral((a + b*acot(c + d*x))**2*(e + f*x), x)
```

3.137.7 Maxima [F]

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^2\,dx=\int (fx+e)(b\,\mathrm{arccot}\,(dx+c)+a)^2\,dx$$

input
```
integrate((f*x+e)*(a+b*arccot(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/8*b^2*f*x^2*arctan2(1, d*x + c)^2 + 1/4*b^2*e*x*arctan2(1, d*x + c)^2 +
1/2*a^2*f*x^2 + (x^2*arccot(d*x + c) + d*(x/d^2 + (c^2 - 1)*arctan((d^2*x
+ c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a*b*f + a^2*e*x +
 (2*(d*x + c)*arccot(d*x + c) + log((d*x + c)^2 + 1))*a*b*e/d - 1/32*(b^2*
f*x^2 + 2*b^2*e*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + integrate(1/16*(12
*b^2*d^2*f*x^3*arctan2(1, d*x + c)^2 + 4*(3*b^2*d^2*e*arctan2(1, d*x + c)^
2 + (6*b^2*c*arctan2(1, d*x + c)^2 + b^2*arctan2(1, d*x + c))*d*f)*x^2 + (
b^2*d^2*f*x^3 + (b^2*d^2*e + 2*b^2*c*d*f)*x^2 + (b^2*c^2 + b^2)*e + (2*b^2
*c*d*e + (b^2*c^2 + b^2)*f)*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 12*(b^
2*c^2*arctan2(1, d*x + c)^2 + b^2*arctan2(1, d*x + c)^2)*e + 4*(2*(3*b^2*c
*arctan2(1, d*x + c)^2 + b^2*arctan2(1, d*x + c))*d*e + 3*(b^2*c^2*arctan2
(1, d*x + c)^2 + b^2*arctan2(1, d*x + c)^2)*f)*x + 2*(b^2*d^2*f*x^3 + 2*b^
2*c*d*e*x + (2*b^2*d^2*e + b^2*c*d*f)*x^2)*log(d^2*x^2 + 2*c*d*x + c^2 + 1
))/(d^2*x^2 + 2*c*d*x + c^2 + 1), x)
```

3.137.8 Giac [F]

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^2\,dx=\int (fx+e)(b\,\mathrm{arccot}\,(dx+c)+a)^2\,dx$$

input
```
integrate((f*x+e)*(a+b*arccot(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((f*x + e)*(b*arccot(d*x + c) + a)^2, x)
```

3.137.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^2\,dx=\int (e+f\,x)\,\left(a+b\,\mathrm{acot}(c+d\,x)\right)^2\,dx$$

input
```
int((e + f*x)*(a + b*acot(c + d*x))^2,x)
```

output
```
int((e + f*x)*(a + b*acot(c + d*x))^2, x)
```

3.138 $\int \left(a+b\cot^{-1}(c+dx)\right)^2\,dx$

3.138.1 Optimal result

Integrand size = 12, antiderivative size = 102

$$\int \left(a+b\cot^{-1}(c+dx)\right)^2\,dx=\frac{i\left(a+b\cot^{-1}(c+dx)\right)^2}{d}+\frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{d}-\frac{2b\left(a+b\cot^{-1}(c+dx)\right)\log\left(\frac{2}{1+i(c+dx)}\right)}{d}+\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d}$$

output
```
I*(a+b*arccot(d*x+c))^2/d+(d*x+c)*(a+b*arccot(d*x+c))^2/d-2*b*(a+b*arccot(
d*x+c))*ln(2/(1+I*(d*x+c)))/d+I*b^2*polylog(2,1-2/(1+I*(d*x+c)))/d
```

3.138.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.16

$$\int \left(a+b\cot^{-1}(c+dx)\right)^2\,dx$$

$$=\frac{b^2(i+c+dx)\cot^{-1}(c+dx)^2+2b\cot^{-1}(c+dx)\left(ac+adx-b\log\left(1-e^{2i\cot^{-1}(c+dx)}\right)\right)+a\Big(ac+adx-}{d}$$

input
```
Integrate[(a + b*ArcCot[c + d*x])^2,x]
```

output
```
(b^2*(I + c + d*x)*ArcCot[c + d*x]^2 + 2*b*ArcCot[c + d*x]*(a*c + a*d*x -
b*Log[1 - E^((2*I)*ArcCot[c + d*x])]) + a*(a*c + a*d*x - 2*b*Log[1/((c + d
*x)*Sqrt[1 + (c + d*x)^(-2)])]) + I*b^2*PolyLog[2, E^((2*I)*ArcCot[c + d*x
])])/d
```

3.138.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5563, 5346, 5456, 5380, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\cot^{-1}(c+dx)\right)^2dx$$

$$\Big\downarrow \text{5563}$$

$$\frac{\int \left(a+b\cot^{-1}(c+dx)\right)^2d(c+dx)}{d}$$

$$\Big\downarrow \text{5346}$$

$$\frac{2b\int \frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)}{(c+dx)^2+1}d(c+dx)+(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{d}$$

$$\Big\downarrow \text{5456}$$

$$\frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2+2b\left(\frac{i\left(a+b\cot^{-1}(c+dx)\right)^2}{2b}-\int \frac{a+b\cot^{-1}(c+dx)}{-c-dx+i}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{5380}$$

$$\frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2+2b\left(-b\int \frac{\log\left(\frac{2}{i(c+dx)+1}\right)}{(c+dx)^2+1}d(c+dx)+\frac{i\left(a+b\cot^{-1}(c+dx)\right)^2}{2b}-\log\left(\frac{2}{1+i(c+dx)}\right)\left(a+b\text{co}\right.\right.}{d}$$

$$\Big\downarrow \text{2849}$$

$$\frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2+2b\left(ib\int \frac{\log\left(\frac{2}{i(c+dx)+1}\right)}{1-\frac{2}{i(c+dx)+1}}d\frac{1}{i(c+dx)+1}+\frac{i\left(a+b\cot^{-1}(c+dx)\right)^2}{2b}-\log\left(\frac{2}{1+i(c+dx)}\right)\left(a+b\text{co}\right.\right.}{d}$$

$$\Big\downarrow \text{2752}$$

$$\frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2+2b\left(\frac{i\left(a+b\cot^{-1}(c+dx)\right)^2}{2b}-\log\left(\frac{2}{1+i(c+dx)}\right)\left(a+b\cot^{-1}(c+dx)\right)+\frac{1}{2}ib\,\mathrm{PolyLog}\left(2\right.\right.}{d}$$

input
```
Int[(a + b*ArcCot[c + d*x])^2,x]
```

output
```
((c + d*x)*(a + b*ArcCot[c + d*x])^2 + 2*b*(((I/2)*(a + b*ArcCot[c + d*x])
^2)/b - (a + b*ArcCot[c + d*x])*Log[2/(1 + I*(c + d*x))] + (I/2)*b*PolyLog
[2, 1 - 2/(1 + I*(c + d*x))]))/d
```

3.138.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5380
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Simp[b*c*(
p/e)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5456
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Simp[
1/(c*d)   Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5563
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[1/d
  Subst[Int[(a + b*ArcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
 x] && IGtQ[p, 0]
```

3.138.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.80

method	result
parts	$a^2x+\frac{b^2\left(\operatorname{arccot}(dx+c)^2(dx+c-i)-2\operatorname{arccot}(dx+c)\ln\left(1+\frac{dx+c+i}{\sqrt{1+(dx+c)^2}}\right)-2\operatorname{arccot}(dx+c)\ln\left(1-\frac{dx+c+i}{\sqrt{1+(dx+c)^2}}\right)+2i\right.}{d}$
derivativedivides	$\frac{(dx+c)a^2+b^2\left(\operatorname{arccot}(dx+c)^2(dx+c-i)-2\operatorname{arccot}(dx+c)\ln\left(1+\frac{dx+c+i}{\sqrt{1+(dx+c)^2}}\right)-2\operatorname{arccot}(dx+c)\ln\left(1-\frac{dx+c+i}{\sqrt{1+(dx+c)^2}}\right)+\right.}{}$
default	$\frac{(dx+c)a^2+b^2\left(\operatorname{arccot}(dx+c)^2(dx+c-i)-2\operatorname{arccot}(dx+c)\ln\left(1+\frac{dx+c+i}{\sqrt{1+(dx+c)^2}}\right)-2\operatorname{arccot}(dx+c)\ln\left(1-\frac{dx+c+i}{\sqrt{1+(dx+c)^2}}\right)+\right.}{}$
risch	$-\frac{\ln(-idx-ic+1)^2b^2x}{4}+\frac{ia^2}{d}-\frac{b^2\arctan(dx+c)\pi c}{2d}-\frac{b\arctan(dx+c)ac}{d}+\frac{\pi abc}{d}+\frac{\pi^2b^2x}{4}+\frac{b^2\ln(d^2x^2+2cdx-}{4d}$

input
```
int((a+b*arccot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+b^2/d*(arccot(d*x+c)^2*(d*x+c-I)-2*arccot(d*x+c)*ln(1+(d*x+c+I)/(1+(
d*x+c)^2)^(1/2))-2*arccot(d*x+c)*ln(1-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+2*I*a
rccot(d*x+c)^2+2*I*polylog(2,-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+2*I*polylog(2
,(d*x+c+I)/(1+(d*x+c)^2)^(1/2)))+2*a*b/d*(arccot(d*x+c)*(d*x+c)+1/2*ln(1+(
d*x+c)^2))
```

3.138.5 Fricas [F]

$$\int \left(a+b\cot^{-1}(c+dx)\right)^2\,dx=\int \left(b\operatorname{arccot}\left(dx+c\right)+a\right)^2dx$$

input
```
integrate((a+b*arccot(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral(b^2*arccot(d*x + c)^2 + 2*a*b*arccot(d*x + c) + a^2, x)
```

3.138.6 Sympy [F]

$$\int \left(a+b\cot^{-1}(c+dx)\right)^2\,dx=\int \left(a+b\,\mathrm{acot}\,(c+dx)\right)^2\,dx$$

input
```
integrate((a+b*acot(d*x+c))**2,x)
```

output
```
Integral((a + b*acot(c + d*x))**2, x)
```

3.138.7 Maxima [F]

$$\int \left(a+b\cot^{-1}(c+dx)\right)^2\,dx=\int \left(b\,\mathrm{arccot}\,(dx+c)+a\right)^2\,dx$$

input
```
integrate((a+b*arccot(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/16*(4*x*arctan2(1, d*x + c)^2 - x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 1
6*integrate(1/16*(12*d^2*x^2*arctan2(1, d*x + c)^2 + 12*c^2*arctan2(1, d*x
 + c)^2 + 8*(3*c*arctan2(1, d*x + c)^2 + arctan2(1, d*x + c))*d*x + (d^2*x
^2 + 2*c*d*x + c^2 + 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 12*arctan2(1,
 d*x + c)^2 + 4*(d^2*x^2 + c*d*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^2*x
^2 + 2*c*d*x + c^2 + 1), x))*b^2 + a^2*x + (2*(d*x + c)*arccot(d*x + c) +
log((d*x + c)^2 + 1))*a*b/d
```

3.138.8 Giac [F]

$$\int \left(a+b\cot^{-1}(c+dx)\right)^2\,dx=\int \left(b\,\mathrm{arccot}\,(dx+c)+a\right)^2\,dx$$

input
```
integrate((a+b*arccot(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((b*arccot(d*x + c) + a)^2, x)
```

3.138.9 Mupad [B] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.21

$$\int \left(a+b\cot^{-1}(c+dx)\right)^2\,dx = a^2\,x+\frac{a\,b\,\left(\ln\left({\left(c+d\,x\right)}^2+1\right)+2\,\mathrm{acot}\left(c+d\,x\right)\,\left(c+d\,x\right)\right)}{d} - \frac{2\,b^2\,\ln\left(1-{\mathrm{e}}^{\mathrm{acot}\left(c+d\,x\right)\,2{}\mathrm{i}}\right)\,\mathrm{acot}\left(c+d\,x\right)}{d} + \frac{b^2\,{\mathrm{acot}\left(c+d\,x\right)}^2\,\left(c+d\,x\right)}{d} + \frac{b^2\,\mathrm{polylog}\left(2,{\mathrm{e}}^{\mathrm{acot}\left(c+d\,x\right)\,2{}\mathrm{i}}\right)\,1{}\mathrm{i}}{d}+\frac{b^2\,{\mathrm{acot}\left(c+d\,x\right)}^2\,1{}\mathrm{i}}{d}$$

input
```
int((a + b*acot(c + d*x))^2,x)
```

output
```
a^2*x + (b^2*polylog(2, exp(acot(c + d*x)*2i))*1i)/d + (b^2*acot(c + d*x)^
2*1i)/d + (a*b*(log((c + d*x)^2 + 1) + 2*acot(c + d*x)*(c + d*x)))/d - (2*
b^2*log(1 - exp(acot(c + d*x)*2i))*acot(c + d*x))/d + (b^2*acot(c + d*x)^2
*(c + d*x))/d
```

3.139 $\int \frac{(a+b\cot^{-1}(c+dx))^2}{e+fx}\,dx$

3.139.1 Optimal result

Integrand size = 20, antiderivative size = 261

$$\begin{aligned}\int \frac{(a+b\cot^{-1}(c+dx))^2}{e+fx}\,dx &= -\frac{(a+b\cot^{-1}(c+dx))^2\log\left(\frac{2}{1-i(c+dx)}\right)}{f} \\ &+\frac{(a+b\cot^{-1}(c+dx))^2\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f} \\ &-\frac{ib(a+b\cot^{-1}(c+dx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{f} \\ &+\frac{ib(a+b\cot^{-1}(c+dx))\operatorname{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f} \\ &-\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)}{2f}+\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2f}\end{aligned}$$

output
```
-(a+b*arccot(d*x+c))^2*ln(2/(1-I*(d*x+c)))/f+(a+b*arccot(d*x+c))^2*ln(2*d*
(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f-I*b*(a+b*arccot(d*x+c))*polylog(2,1
-2/(1-I*(d*x+c)))/f+I*b*(a+b*arccot(d*x+c))*polylog(2,1-2*d*(f*x+e)/(d*e+I
*f-c*f)/(1-I*(d*x+c)))/f-1/2*b^2*polylog(3,1-2/(1-I*(d*x+c)))/f+1/2*b^2*po
lylog(3,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f
```

3.139.2 Mathematica [F]

$$\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{e+fx}\,dx=\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{e+fx}\,dx$$

input
```
Integrate[(a + b*ArcCot[c + d*x])^2/(e + f*x),x]
```

output
```
Integrate[(a + b*ArcCot[c + d*x])^2/(e + f*x), x]
```

3.139.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5571, 27, 5384}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{e+fx}\,dx$$

$$\Big\downarrow \; 5571$$

$$\frac{\int \frac{d\left(a+b\cot^{-1}(c+dx)\right)^2}{d\left(e-\frac{cf}{d}\right)+f(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(c+dx)-cf+de}d(c+dx)$$

$$\Big\downarrow \; 5384$$

$$\frac{ib(a+b\cot^{-1}(c+dx))\operatorname{PolyLog}\left(2,1-\frac{2(de-cf+f(c+dx))}{(de-cf+if)(1-i(c+dx))}\right)}{f}+\frac{(a+b\cot^{-1}(c+dx))^2\log\left(\frac{2(f(c+dx)-cf+de)}{(1-i(c+dx))(-cf+de+if)}\right)}{f}-\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)(a+b\cot^{-1}(c+dx))}{f}-\frac{\log\left(\frac{2}{1-i(c+dx)}\right)(a+b\cot^{-1}(c+dx))^2}{f}+\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2(de-cf+f(c+dx))}{(de-cf+if)(1-i(c+dx))}\right)}{2f}-\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)}{2f}$$

input
```
Int[(a + b*ArcCot[c + d*x])^2/(e + f*x),x]
```

output
```
-(((a + b*ArcCot[c + d*x])^2*Log[2/(1 - I*(c + d*x))])/f) + ((a + b*ArcCot
[c + d*x])^2*Log[(2*(d*e - c*f + f*(c + d*x)))/((d*e + I*f - c*f)*(1 - I*(
c + d*x)))])/f - (I*b*(a + b*ArcCot[c + d*x])*PolyLog[2, 1 - 2/(1 - I*(c +
 d*x))])/f + (I*b*(a + b*ArcCot[c + d*x])*PolyLog[2, 1 - (2*(d*e - c*f + f
*(c + d*x)))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f - (b^2*PolyLog[3, 1
 - 2/(1 - I*(c + d*x))])/(2*f) + (b^2*PolyLog[3, 1 - (2*(d*e - c*f + f*(c
+ d*x)))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/(2*f)
```

3.139.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5384
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcCot[c*x])^2)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Cot[c*x])^2*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] - Simp[I
*b*(a + b*ArcCot[c*x])*(PolyLog[2, 1 - 2/(1 - I*c*x)]/e), x] + Simp[I*b*(a
+ b*ArcCot[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]
/e), x] - Simp[b^2*(PolyLog[3, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp[b^2*(Po
lyLog[3, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x]) /; Free
Q[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

rule 5571
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.139.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 14.06 (sec) , antiderivative size = 1911, normalized size of antiderivative = 7.32

method	result	size
derivativedivides	Expression too large to display	1911
default	Expression too large to display	1911
parts	Expression too large to display	2022

input
```
int((a+b*arccot(d*x+c))^2/(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*d*ln(c*f-d*e-f*(d*x+c))/f-b^2*d*(-ln(c*f-d*e-f*(d*x+c))/f*arccot(
d*x+c)^2-2/f*(-1/2*arccot(d*x+c)^2*ln(-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)+c*f*(
d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+I)^2/(1+(d*x+c)^2)-I*f-c*f+d*e)-1/4*I*
f/(-I*f+c*f-d*e)*polylog(3,(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*
x+c)^2))+1/2*arccot(d*x+c)^2*ln((d*x+c+I)^2/(1+(d*x+c)^2)-1)-1/2*arccot(d*
x+c)^2*ln(1+(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+I*d*e*arccot(d*x+c)*polylog(2,(
d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x+c)^2))/(-2*I*f+2*c*f-2*d*e
)-polylog(3,-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))-1/2*arccot(d*x+c)^2*ln(1-(d*x+
c+I)/(1+(d*x+c)^2)^(1/2))+I*arccot(d*x+c)*polylog(2,-(d*x+c+I)/(1+(d*x+c)^
2)^(1/2))-polylog(3,(d*x+c+I)/(1+(d*x+c)^2)^(1/2))-1/2*I*f/(-I*f+c*f-d*e)*
arccot(d*x+c)^2*ln(1-(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x+c)^2
))+1/2*c*f/(-I*f+c*f-d*e)*arccot(d*x+c)^2*ln(1-(d*e+I*f-c*f)/(-c*f+d*e-I*f
)*(d*x+c+I)^2/(1+(d*x+c)^2))+1/4*c*f/(-I*f+c*f-d*e)*polylog(3,(d*e+I*f-c*f
)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x+c)^2))-1/2*f/(-I*f+c*f-d*e)*arccot(d*
x+c)*polylog(2,(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x+c)^2))-1/2
*I*c*f/(-I*f+c*f-d*e)*arccot(d*x+c)*polylog(2,(d*e+I*f-c*f)/(-c*f+d*e-I*f)
*(d*x+c+I)^2/(1+(d*x+c)^2))+I*arccot(d*x+c)*polylog(2,(d*x+c+I)/(1+(d*x+c)
^2)^(1/2))+1/4*I*Pi*csgn(I*(-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)+c*f*(d*x+c+I)^2
/(1+(d*x+c)^2)-d*e*(d*x+c+I)^2/(1+(d*x+c)^2)-I*f-c*f+d*e)/((d*x+c+I)^2/(1+
(d*x+c)^2)-1))*(csgn(I*(-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)+c*f*(d*x+c+I)^2/...
```

3.139.5 Fricas [F]

$$\int \frac{(a+b\cot^{-1}(c+dx))^2}{e+fx}\,dx=\int \frac{(b\operatorname{arccot}(dx+c)+a)^2}{fx+e}\,dx$$

input
```
integrate((a+b*arccot(d*x+c))^2/(f*x+e),x, algorithm="fricas")
```

output
```
integral((b^2*arccot(d*x + c)^2 + 2*a*b*arccot(d*x + c) + a^2)/(f*x + e),
x)
```

3.139.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cot^{-1}(c+dx))^2}{e+fx}\,dx=\text{Timed out}$$

input
```
integrate((a+b*acot(d*x+c))**2/(f*x+e),x)
```

output
```
Timed out
```

3.139.7 Maxima [F]

$$\int \frac{(a+b\cot^{-1}(c+dx))^2}{e+fx}\,dx=\int \frac{(b\operatorname{arccot}(dx+c)+a)^2}{fx+e}\,dx$$

input
```
integrate((a+b*arccot(d*x+c))^2/(f*x+e),x, algorithm="maxima")
```

output
```
a^2*log(f*x + e)/f + integrate(1/16*(12*b^2*arctan2(1, d*x + c)^2 + b^2*lo
g(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 32*a*b*arctan2(1, d*x + c))/(f*x + e),
x)
```

3.139.8 Giac [F]

$$\int \frac{(a+b\cot^{-1}(c+dx))^2}{e+fx}\,dx=\int \frac{(b\operatorname{arccot}(dx+c)+a)^2}{fx+e}\,dx$$

input
```
integrate((a+b*arccot(d*x+c))^2/(f*x+e),x, algorithm="giac")
```

output
```
integrate((b*arccot(d*x + c) + a)^2/(f*x + e), x)
```

3.139.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cot^{-1}(c+dx))^2}{e+fx}\,dx=\int \frac{(a+b\operatorname{acot}(c+d\,x))^2}{e+f\,x}\,dx$$

input
```
int((a + b*acot(c + d*x))^2/(e + f*x),x)
```

output
```
int((a + b*acot(c + d*x))^2/(e + f*x), x)
```

3.140 $\int \frac{(a+b\cot^{-1}(c+dx))^2}{(e+fx)^2}\,dx$

3.140.1 Optimal result

Integrand size = 20, antiderivative size = 567

$$\begin{aligned}\int \frac{(a+b\cot^{-1}(c+dx))^2}{(e+fx)^2}\,dx &= \frac{ib^2d\cot^{-1}(c+dx)^2}{d^2e^2-2cdef+(1+c^2)f^2}+\frac{b^2d(de-cf)\cot^{-1}(c+dx)^2}{f(d^2e^2-2cdef+(1+c^2)f^2)}\\ &\quad-\frac{(a+b\cot^{-1}(c+dx))^2}{f(e+fx)}-\frac{2abd(de-cf)\arctan(c+dx)}{f(f^2+(de-cf)^2)}\\ &\quad-\frac{2abd\log(e+fx)}{f^2+(de-cf)^2}+\frac{2b^2d\cot^{-1}(c+dx)\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}\\ &\quad-\frac{2b^2d\cot^{-1}(c+dx)\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2}\\ &\quad-\frac{2b^2d\cot^{-1}(c+dx)\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}\\ &\quad+\frac{abd\log(1+(c+dx)^2)}{f^2+(de-cf)^2}+\frac{ib^2d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}\\ &\quad-\frac{ib^2d\,\mathrm{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2}\\ &\quad+\frac{ib^2d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}\end{aligned}$$

output
```
I*b^2*d*arccot(d*x+c)^2/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+b^2*d*(-c*f+d*e)*a
rccot(d*x+c)^2/f/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-(a+b*arccot(d*x+c))^2/f/(
f*x+e)-2*a*b*d*(-c*f+d*e)*arctan(d*x+c)/f/(f^2+(-c*f+d*e)^2)-2*a*b*d*ln(f*
x+e)/(f^2+(-c*f+d*e)^2)+2*b^2*d*arccot(d*x+c)*ln(2/(1-I*(d*x+c)))/(d^2*e^2
-2*c*d*e*f+(c^2+1)*f^2)-2*b^2*d*arccot(d*x+c)*ln(2*d*(f*x+e)/(d*e+I*f-c*f)
/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-2*b^2*d*arccot(d*x+c)*ln(2
/(1+I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+a*b*d*ln(1+(d*x+c)^2)/(f^2
+(-c*f+d*e)^2)+I*b^2*d*polylog(2,1-2/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^
2+1)*f^2)-I*b^2*d*polylog(2,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/(d^
2*e^2-2*c*d*e*f+(c^2+1)*f^2)+I*b^2*d*polylog(2,1-2/(1+I*(d*x+c)))/(d^2*e^2
-2*c*d*e*f+(c^2+1)*f^2)
```

3.140.2 Mathematica [A] (verified)

Time = 6.12 (sec) , antiderivative size = 454, normalized size of antiderivative = 0.80

$$\int \frac{(a+b\cot^{-1}(c+dx))^2}{(e+fx)^2}\,dx =$$

$$-\frac{a^2+\frac{2abf\left((-cde+f+c^2f-d^2ex+cdfx)\cot^{-1}(c+dx)+d(e+fx)\log\left(-\frac{d(e+fx)}{(c+dx)\sqrt{1+\frac{1}{(c+dx)^2}}}\right)\right)}{d^2e^2-2cdef+(1+c^2)f^2}+\frac{b^2d(e+fx)(1+(c+dx)^2)\left(\frac{e^{i\arctan\left(\frac{f}{de-}\right.}}{(-de+cf}\right.}{}}{}$$

input
```
Integrate[(a + b*ArcCot[c + d*x])^2/(e + f*x)^2,x]
```

output
```
-((a^2 + (2*a*b*f*((-(c*d*e) + f + c^2*f - d^2*e*x + c*d*f*x)*ArcCot[c + d
*x] + d*(e + f*x)*Log[-((d*(e + f*x))/((c + d*x)*Sqrt[1 + (c + d*x)^(-2)])
)]))/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (b^2*d*(e + f*x)*(1 + (c + d*
x)^2)*((E^(I*ArcTan[f/(d*e - c*f)])*ArcCot[c + d*x]^2)/((-(d*e) + c*f)*Sqr
t[1 + f^2/(d*e - c*f)^2]) + ArcCot[c + d*x]^2/(d*e + d*f*x) + (f*(I*Pi*Arc
Cot[c + d*x] + Pi*Log[1 + E^((-2*I)*ArcCot[c + d*x])] + 2*ArcCot[c + d*x]*
Log[1 - E^((2*I)*(ArcCot[c + d*x] + ArcTan[f/(d*e - c*f)]))] - Pi*Log[1/Sq
rt[1 + (c + d*x)^(-2)]] + 2*ArcTan[f/(-(d*e) + c*f)]*(I*ArcCot[c + d*x] -
Log[1 - E^((2*I)*(ArcCot[c + d*x] + ArcTan[f/(d*e - c*f)]))] + Log[Sin[Arc
Cot[c + d*x] + ArcTan[f/(d*e - c*f)]]]) - I*PolyLog[2, E^((2*I)*(ArcCot[c
+ d*x] + ArcTan[f/(d*e - c*f)]))]))/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2))
)/((c + d*x)^2*(1 + (c + d*x)^(-2))))/(f*(e + f*x)))
```

3.140.3 Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 586, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5569, 7292, 5581, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{(e+fx)^2}dx$$

$$\Big\downarrow \text{5569}$$

$$-\frac{2bd\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)((c+dx)^2+1)}dx}{f}-\frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(e+fx)}$$

$$\Big\downarrow \text{7292}$$

$$-\frac{2bd\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)(c^2+2dxc+d^2x^2+1)}dx}{f}-\frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(e+fx)}$$

$$\Big\downarrow \text{5581}$$

$$-\frac{2b\int \frac{d(a+b\cot^{-1}(c+dx))}{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)((c+dx)^2+1)}d(c+dx)}{f}-\frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(e+fx)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2bd\int \frac{a+b\cot^{-1}(c+dx)}{(de-cf+f(c+dx))((c+dx)^2+1)}d(c+dx)}{f}-\frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(e+fx)}$$

$$\Big\downarrow \text{7276}$$

$$-\frac{2bd\int \left(\frac{a}{(de-cf+f(c+dx))((c+dx)^2+1)}+\frac{b\cot^{-1}(c+dx)}{(de-cf+f(c+dx))((c+dx)^2+1)}\right)d(c+dx)}{f}-\frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(e+fx)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(e+fx)}-$$

$$2bd\left(\frac{a\arctan(c+dx)(de-cf)}{(de-cf)^2+f^2}+\frac{af\log(f(c+dx)-cf+de)}{(de-cf)^2+f^2}-\frac{af\log((c+dx)^2+1)}{2((de-cf)^2+f^2)}-\frac{ibf\operatorname{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{2((c^2+1)f^2-2cdef+d^2e^2)}-\frac{ibf\operatorname{PolyLog}\left(2,1-\frac{2}{i(c+d}\right.}{2((c^2+1)f^2-2cdef+d}\right.$$

input
```
Int[(a + b*ArcCot[c + d*x])^2/(e + f*x)^2,x]
```

3.140. $\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{(e+fx)^2}dx$

output
```
-((a + b*ArcCot[c + d*x])^2/(f*(e + f*x))) - (2*b*d*(((-1/2*I)*b*f*ArcCot[
c + d*x]^2)/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - (b*(d*e - c*f)*ArcCot[
c + d*x]^2)/(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)) + (a*(d*e - c*f)*Arc
Tan[c + d*x])/(f^2 + (d*e - c*f)^2) - (b*f*ArcCot[c + d*x]*Log[2/(1 - I*(c
 + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (b*f*ArcCot[c + d*x]*Lo
g[2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (a*f*Log[d
*e - c*f + f*(c + d*x)])/(f^2 + (d*e - c*f)^2) + (b*f*ArcCot[c + d*x]*Log[
(2*(d*e - c*f + f*(c + d*x)))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/(d^2
*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - (a*f*Log[1 + (c + d*x)^2])/(2*(f^2 + (
d*e - c*f)^2)) - ((I/2)*b*f*PolyLog[2, 1 - 2/(1 - I*(c + d*x))])/(d^2*e^2
- 2*c*d*e*f + (1 + c^2)*f^2) - ((I/2)*b*f*PolyLog[2, 1 - 2/(1 + I*(c + d*x
))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + ((I/2)*b*f*PolyLog[2, 1 - (2*
(d*e - c*f + f*(c + d*x)))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/(d^2*e^
2 - 2*c*d*e*f + (1 + c^2)*f^2)))/f
```

3.140.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5569
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCot[c + d*x])^p/(f*(m +
1))), x] + Simp[b*d*(p/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*((a + b*ArcCot[
c + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[p, 0] && ILtQ[m, -1]
```

rule 5581
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/d   Subs
t[Int[((d*e - c*f)/d + f*(x/d))^m*(C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcCot[x])
^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &
& EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.140. $\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{(e+fx)^2}\,dx$

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

3.140.4 Maple [A] (verified)

Time = 4.18 (sec) , antiderivative size = 784, normalized size of antiderivative = 1.38

method	result
parts	$-\frac{a^2}{(fx+e)f}+\frac{b^2\left(-\frac{d^2\operatorname{arccot}(dx+c)^2}{(f(dx+c)-cf+de)f}-\frac{2d^2\left(\frac{\operatorname{arccot}(dx+c)f\ln(f(dx+c)-cf+de)}{c^2f^2-2cdef+d^2e^2+f^2}-\frac{\operatorname{arccot}(dx+c)f\ln\left(1+(dx+c)^2\right)}{2\left(c^2f^2-2cdef+d^2e^2+f^2\right)}-\frac{\operatorname{arccot}(d\ldots}{c^2f^2-\ldots}\ldots\right.}{\ldots}\right)}{\ldots}$
derivativedivides	$\frac{\frac{a^2d^2}{(cf-de-f(dx+c))f}+b^2d^2\left(\frac{\operatorname{arccot}(dx+c)^2}{(cf-de-f(dx+c))f}+\frac{-\frac{2\operatorname{arccot}(dx+c)f\ln(cf-de-f(dx+c))}{c^2f^2-2cdef+d^2e^2+f^2}+\frac{2\operatorname{arccot}(dx+c)f\ln\left(1+(dx+c)^2\right)}{2c^2f^2-4cdef+2d^2e^2+2f^2}+\frac{2\operatorname{ar}\ldots}{\ldots}}{\ldots}\right.}{\ldots}$
default	$\frac{\frac{a^2d^2}{(cf-de-f(dx+c))f}+b^2d^2\left(\frac{\operatorname{arccot}(dx+c)^2}{(cf-de-f(dx+c))f}+\frac{-\frac{2\operatorname{arccot}(dx+c)f\ln(cf-de-f(dx+c))}{c^2f^2-2cdef+d^2e^2+f^2}+\frac{2\operatorname{arccot}(dx+c)f\ln\left(1+(dx+c)^2\right)}{2c^2f^2-4cdef+2d^2e^2+2f^2}+\frac{2\operatorname{ar}\ldots}{\ldots}}{\ldots}\right.}{\ldots}$

input
```
int((a+b*arccot(d*x+c))^2/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2/(f*x+e)/f+b^2/d*(-d^2/(f*(d*x+c)-c*f+d*e)/f*arccot(d*x+c)^2-2*d^2/f*(
arccot(d*x+c)/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f*ln(f*(d*x+c)-c*f+d*e)-1/2*
arccot(d*x+c)/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f*ln(1+(d*x+c)^2)-arccot(d*x
+c)/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)*c*f+arccot(d*x+c)/(c^2*f
^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)*d*e+f^2/(c^2*f^2-2*c*d*e*f+d^2*e^2
+f^2)*(-1/2*I*ln(f*(d*x+c)-c*f+d*e)*(ln((I*f-f*(d*x+c))/(d*e+I*f-c*f))-ln(
(I*f+f*(d*x+c))/(c*f-d*e+I*f)))/f-1/2*I*(dilog((I*f-f*(d*x+c))/(d*e+I*f-c*
f))-dilog((I*f+f*(d*x+c))/(c*f-d*e+I*f)))/f)-1/2*f/(c^2*f^2-2*c*d*e*f+d^2*
e^2+f^2)*(-1/2*I*(ln(d*x+c-I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c-I)^2-dilog(-1/2
*I*(d*x+c+I))-ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I)))+1/2*I*(ln(d*x+c+I)*ln(1+(d
*x+c)^2)-1/2*ln(d*x+c+I)^2-dilog(1/2*I*(d*x+c-I))-ln(d*x+c+I)*ln(1/2*I*(d*
x+c-I))))-1/2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*(c*f-d*e)*arctan(d*x+c)^2))+
2*a*b/d*(-d^2/(f*(d*x+c)-c*f+d*e)/f*arccot(d*x+c)-d^2/f*(1/(c^2*f^2-2*c*d*
e*f+d^2*e^2+f^2)*f*ln(f*(d*x+c)-c*f+d*e)+1/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)
*(-1/2*f*ln(1+(d*x+c)^2)+(-c*f+d*e)*arctan(d*x+c))))
```

3.140.5 Fricas [F]

$$\int \frac{(a+b\cot^{-1}(c+dx))^2}{(e+fx)^2}\,dx = \int \frac{(b\,\mathrm{arccot}\,(dx+c)+a)^2}{(fx+e)^2}\,dx$$

input
```
integrate((a+b*arccot(d*x+c))^2/(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral((b^2*arccot(d*x + c)^2 + 2*a*b*arccot(d*x + c) + a^2)/(f^2*x^2 +
2*e*f*x + e^2), x)
```

3.140.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cot^{-1}(c+dx))^2}{(e+fx)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acot(d*x+c))**2/(f*x+e)**2,x)
```

output
```
Timed out
```

3.140.7 Maxima [F]

$$\int \frac{(a + b \cot^{-1}(c + dx))^2}{(e + fx)^2} \, dx = \int \frac{(b \operatorname{arccot}(dx + c) + a)^2}{(fx + e)^2} \, dx$$

input
```
integrate((a+b*arccot(d*x+c))^2/(f*x+e)^2,x, algorithm="maxima")
```

output
```
-(d*(2*(d^2*e - c*d*f)*arctan((d^2*x + c*d)/d)/((d^2*e^2*f - 2*c*d*e*f^2 +
 (c^2 + 1)*f^3)*d) - log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*e^2 - 2*c*d*e*f
 + (c^2 + 1)*f^2) + 2*log(f*x + e)/(d^2*e^2 - 2*c*d*e*f + (c^2 + 1)*f^2))
+ 2*arccot(d*x + c)/(f^2*x + e*f))*a*b - 1/16*(4*arctan2(1, d*x + c)^2 - 1
6*(f^2*x + e*f)*integrate(1/16*(12*d^2*f*x^2*arctan2(1, d*x + c)^2 + 8*(3*
c*arctan2(1, d*x + c)^2 - arctan2(1, d*x + c))*d*f*x - 8*d*e*arctan2(1, d*
x + c) + (d^2*f*x^2 + 2*c*d*f*x + (c^2 + 1)*f)*log(d^2*x^2 + 2*c*d*x + c^2
 + 1)^2 + 12*(c^2*arctan2(1, d*x + c)^2 + arctan2(1, d*x + c)^2)*f - 4*(d^
2*f*x^2 + c*d*e + (d^2*e + c*d*f)*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^
2*f^3*x^4 + (c^2 + 1)*e^2*f + 2*(d^2*e*f^2 + c*d*f^3)*x^3 + (d^2*e^2*f + 4
*c*d*e*f^2 + (c^2 + 1)*f^3)*x^2 + 2*(c*d*e^2*f + (c^2 + 1)*e*f^2)*x), x) -
 log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2)*b^2/(f^2*x + e*f) - a^2/(f^2*x + e*f)
```

3.140.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a + b \cot^{-1}(c + dx))^2}{(e + fx)^2} \, dx = \text{Timed out}$$

input
```
integrate((a+b*arccot(d*x+c))^2/(f*x+e)^2,x, algorithm="giac")
```

output
```
Timed out
```

3.140.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{(e+fx)^2}\,dx=\int \frac{\left(a+b\,\mathrm{acot}(c+d\,x)\right)^2}{\left(e+f\,x\right)^2}\,dx$$

input
```
int((a + b*acot(c + d*x))^2/(e + f*x)^2,x)
```

output
```
int((a + b*acot(c + d*x))^2/(e + f*x)^2, x)
```

3.141 $\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx$

3.141.1 Optimal result

Integrand size = 20, antiderivative size = 565

$$\begin{aligned}
&\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx \\
&= \frac{ab^2f^2x}{d^2} + \frac{b^3f^2(c+dx)\cot^{-1}(c+dx)}{d^3} + \frac{bf^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^3} \\
&+ \frac{3ibf(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^2}{d^3} \\
&+ \frac{3bf(de-cf)(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{d^3} + \frac{bf^2(c+dx)^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^3} \\
&+ \frac{i\left(3d^2e^2-6cdef-(1-3c^2)f^2\right)\left(a+b\cot^{-1}(c+dx)\right)^3}{3d^3} \\
&- \frac{(de-cf)\left(d^2e^2-2cdef-(3-c^2)f^2\right)\left(a+b\cot^{-1}(c+dx)\right)^3}{3d^3f} \\
&+ \frac{(e+fx)^3\left(a+b\cot^{-1}(c+dx)\right)^3}{3f} - \frac{6b^2f(de-cf)\left(a+b\cot^{-1}(c+dx)\right)\log\left(\frac{2}{1+i(c+dx)}\right)}{d^3} \\
&- \frac{b\left(3d^2e^2-6cdef-(1-3c^2)f^2\right)\left(a+b\cot^{-1}(c+dx)\right)^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d^3} \\
&+ \frac{b^3f^2\log\left(1+(c+dx)^2\right)}{2d^3} + \frac{3ib^3f(de-cf)\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^3} \\
&+ \frac{ib^2\left(3d^2e^2-6cdef-(1-3c^2)f^2\right)\left(a+b\cot^{-1}(c+dx)\right)\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^3} \\
&- \frac{b^3\left(3d^2e^2-6cdef-(1-3c^2)f^2\right)\operatorname{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2d^3}
\end{aligned}$$

output
```
a*b^2*f^2*x/d^2+b^3*f^2*(d*x+c)*arccot(d*x+c)/d^3+1/2*b*f^2*(a+b*arccot(d*
x+c))^2/d^3+3*I*b*f*(-c*f+d*e)*(a+b*arccot(d*x+c))^2/d^3+3*b*f*(-c*f+d*e)*
(d*x+c)*(a+b*arccot(d*x+c))^2/d^3+1/2*b*f^2*(d*x+c)^2*(a+b*arccot(d*x+c))^
2/d^3+1/3*I*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*(a+b*arccot(d*x+c))^3/d^3
-1/3*(-c*f+d*e)*(d^2*e^2-2*c*d*e*f-(-c^2+3)*f^2)*(a+b*arccot(d*x+c))^3/d^3
/f+1/3*(f*x+e)^3*(a+b*arccot(d*x+c))^3/f-6*b^2*f*(-c*f+d*e)*(a+b*arccot(d*
x+c))*ln(2/(1+I*(d*x+c)))/d^3-b*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*(a+b*
arccot(d*x+c))^2*ln(2/(1+I*(d*x+c)))/d^3+1/2*b^3*f^2*ln(1+(d*x+c)^2)/d^3+3
*I*b^3*f*(-c*f+d*e)*polylog(2,1-2/(1+I*(d*x+c)))/d^3+I*b^2*(3*d^2*e^2-6*c*
d*e*f-(-3*c^2+1)*f^2)*(a+b*arccot(d*x+c))*polylog(2,1-2/(1+I*(d*x+c)))/d^3
-1/2*b^3*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*polylog(3,1-2/(1+I*(d*x+c)))
/d^3
```

3.141.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2309 vs. 2(565) = 1130.

Time = 14.56 (sec) , antiderivative size = 2309, normalized size of antiderivative = 4.09

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx = \text{Result too large to show}$$

input
```
Integrate[(e + f*x)^2*(a + b*ArcCot[c + d*x])^3,x]
```

output
```
(a^2*(a*d^2*e^2 + 3*b*d*e*f - 2*b*c*f^2)*x)/d^2 + (a^2*f*(2*a*d*e + b*f)*x
^2)/(2*d) + (a^3*f^2*x^3)/3 + a^2*b*x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcCot[c
 + d*x] + ((-3*a^2*b*c*d^2*e^2 - 3*a^2*b*d*e*f + 3*a^2*b*c^2*d*e*f + 3*a^2
*b*c*f^2 - a^2*b*c^3*f^2)*ArcTan[c + d*x])/d^3 + ((3*a^2*b*d^2*e^2 - 6*a^2
*b*c*d*e*f - a^2*b*f^2 + 3*a^2*b*c^2*f^2)*Log[1 + c^2 + 2*c*d*x + d^2*x^2]
)/(2*d^3) - (3*a*b^2*e^2*(1 + (c + d*x)^2)*(-((c + d*x)*ArcCot[c + d*x]^2)
 + 2*ArcCot[c + d*x]*Log[1 - E^((2*I)*ArcCot[c + d*x])] - I*(ArcCot[c + d*
x]^2 + PolyLog[2, E^((2*I)*ArcCot[c + d*x])])))/(d*(c + d*x)^2*(1 + (c + d
*x)^(-2))) + (6*a*b^2*e*f*(1 + (c + d*x)^2)*(((c + d*x)*ArcCot[c + d*x])/d
^2 - (c*(c + d*x)*ArcCot[c + d*x]^2)/d^2 + ((c + d*x)^2*(1 + (c + d*x)^(-2
))*ArcCot[c + d*x]^2)/(2*d^2) - Log[1/((c + d*x)*Sqrt[1 + (c + d*x)^(-2)])
]/d^2 + (2*c*(ArcCot[c + d*x]*Log[1 - E^((2*I)*ArcCot[c + d*x])] - (I/2)*(
ArcCot[c + d*x]^2 + PolyLog[2, E^((2*I)*ArcCot[c + d*x])])))/d^2))/((c + d
*x)^2*(1 + (c + d*x)^(-2))) - (a*b^2*f^2*x^2*(1 + (c + d*x)^2)*(-((c + d*x
)*ArcCot[c + d*x]^2) + ArcCot[c + d*x]*(-1 + 3*c*ArcCot[c + d*x]) - (1 - 6
*c*ArcCot[c + d*x] - ArcCot[c + d*x]^2 + 3*c^2*ArcCot[c + d*x]^2)/((c + d*
x)*(1 + (c + d*x)^(-2))) - (6*c*(Log[(c + d*x)^(-1)] + Log[1/Sqrt[1 + (c +
 d*x)^(-2)]]))/((c + d*x)^2*(1 + (c + d*x)^(-2))) + (I*(ArcCot[c + d*x]*(A
rcCot[c + d*x] + (2*I)*Log[1 - E^((2*I)*ArcCot[c + d*x])]) + PolyLog[2, E^
((2*I)*ArcCot[c + d*x])]))/((c + d*x)^2*(1 + (c + d*x)^(-2))) + (6*c^2*...
```

3.141.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 551, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5571, 27, 5390, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^3 dx$$

$$\Big\downarrow \text{5571}$$

$$\frac{\int \frac{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)^2 \left(a+b\cot^{-1}(c+dx)\right)^3}{d^2}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (de-cf+f(c+dx))^2 \left(a+b\cot^{-1}(c+dx)\right)^3 d(c+dx)}{d^3}$$

$$\Big\downarrow \text{5390}$$

$$\frac{\frac{b\int \left(\left(c+dx\right)\left(a+b\cot^{-1}(c+dx)\right)^2 f^3+3(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^2 f^2+\frac{\left((de-cf)\left(d^2e^2-2cdfe-\left(3-c^2\right)f^2\right)+f\left(3d^2e^2-6cdfe-\left(1-3c^2\right)f^2\right)(c+dx)\right)\left(a+b\right.}{(c+dx)^2+1}\right.}{f}}{d^3}$$

$\downarrow$ 2009

$$\frac{\frac{(f(c+dx)-cf+de)^3\left(a+b\cot^{-1}(c+dx)\right)^3}{3f}+\frac{b\left(ibf\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)\operatorname{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)\left(a+b\cot^{-1}(c+dx)\right)+\frac{if\left(-\left(1-3c^2\right)f^2\right.}{}\right.}{}}{}$$

input
```
Int[(e + f*x)^2*(a + b*ArcCot[c + d*x])^3,x]
```

output
```
(((d*e - c*f + f*(c + d*x))^3*(a + b*ArcCot[c + d*x])^3)/(3*f) + (b*(a*b*f
^3*(c + d*x) + b^2*f^3*(c + d*x)*ArcCot[c + d*x] + (f^3*(a + b*ArcCot[c +
d*x])^2)/2 + (3*I)*f^2*(d*e - c*f)*(a + b*ArcCot[c + d*x])^2 + 3*f^2*(d*e
- c*f)*(c + d*x)*(a + b*ArcCot[c + d*x])^2 + (f^3*(c + d*x)^2*(a + b*ArcCo
t[c + d*x])^2)/2 + ((I/3)*f*(3*d^2*e^2 - 6*c*d*e*f - (1 - 3*c^2)*f^2)*(a +
 b*ArcCot[c + d*x])^3)/b - ((d*e - c*f)*(d^2*e^2 - 2*c*d*e*f - (3 - c^2)*f
^2)*(a + b*ArcCot[c + d*x])^3)/(3*b) - 6*b*f^2*(d*e - c*f)*(a + b*ArcCot[c
 + d*x])*Log[2/(1 + I*(c + d*x))] - f*(3*d^2*e^2 - 6*c*d*e*f - (1 - 3*c^2)
*f^2)*(a + b*ArcCot[c + d*x])^2*Log[2/(1 + I*(c + d*x))] + (b^2*f^3*Log[1
+ (c + d*x)^2])/2 + (3*I)*b^2*f^2*(d*e - c*f)*PolyLog[2, 1 - 2/(1 + I*(c +
 d*x))] + I*b*f*(3*d^2*e^2 - 6*c*d*e*f - (1 - 3*c^2)*f^2)*(a + b*ArcCot[c
+ d*x])*PolyLog[2, 1 - 2/(1 + I*(c + d*x))] - (b^2*f*(3*d^2*e^2 - 6*c*d*e*
f - (1 - 3*c^2)*f^2)*PolyLog[3, 1 - 2/(1 + I*(c + d*x))])/2))/f)/d^3
```

3.141.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5390
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])^p/(e*(q + 1))), x] + S
imp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcCot[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

rule 5571
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.141.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 101.64 (sec) , antiderivative size = 6248, normalized size of antiderivative = 11.06

method	result	size
parts	Expression too large to display	6248
derivativedivides	Expression too large to display	10834
default	Expression too large to display	10834

input
```
int((f*x+e)^2*(a+b*arccot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.141.5 Fricas [F]

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^3 dx = \int (fx+e)^2(b\,\mathrm{arccot}\,(dx+c)+a)^3\,dx$$

input
```
integrate((f*x+e)^2*(a+b*arccot(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral(a^3*f^2*x^2 + 2*a^3*e*f*x + a^3*e^2 + (b^3*f^2*x^2 + 2*b^3*e*f*x
+ b^3*e^2)*arccot(d*x + c)^3 + 3*(a*b^2*f^2*x^2 + 2*a*b^2*e*f*x + a*b^2*e^
2)*arccot(d*x + c)^2 + 3*(a^2*b*f^2*x^2 + 2*a^2*b*e*f*x + a^2*b*e^2)*arcco
t(d*x + c), x)
```

3.141.6 Sympy [F(-1)]

Timed out.

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*(a+b*acot(d*x+c))**3,x)
```

output
```
Timed out
```

3.141.7 Maxima [F]

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx = \int (fx+e)^2 (b\,\text{arccot}\,(dx+c)+a)^3 \, dx$$

input
```
integrate((f*x+e)^2*(a+b*arccot(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/24*b^3*f^2*x^3*arctan2(1, d*x + c)^3 + 1/8*b^3*e*f*x^2*arctan2(1, d*x +
c)^3 + 1/8*b^3*e^2*x*arctan2(1, d*x + c)^3 + 1/3*a^3*f^2*x^3 + a^3*e*f*x^2
 + 3*(x^2*arccot(d*x + c) + d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d
^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a^2*b*e*f + 1/2*(2*x^3*arcco
t(d*x + c) + d*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)/d
)/d^4 + (3*c^2 - 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*a^2*b*f^2 + a^3
*e^2*x + 3/2*(2*(d*x + c)*arccot(d*x + c) + log((d*x + c)^2 + 1))*a^2*b*e^
2/d - 1/32*(b^3*f^2*x^3*arctan2(1, d*x + c) + 3*b^3*e*f*x^2*arctan2(1, d*x
 + c) + 3*b^3*e^2*x*arctan2(1, d*x + c))*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^
2 + integrate(1/32*(4*(7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d
*x + c)^2)*d^2*f^2*x^4 + 4*(2*(7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arct
an2(1, d*x + c)^2)*d^2*e*f + (b^3*arctan2(1, d*x + c)^2 + 2*(7*b^3*arctan2
(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*c)*d*f^2)*x^3 + 4*(7*b^3*
arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2 + (7*b^3*arctan2(1,
 d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*c^2)*e^2 + 4*((7*b^3*arctan2
(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*d^2*e^2 + (3*b^3*arctan2(
1, d*x + c)^2 + 4*(7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x +
 c)^2)*c)*d*e*f + (7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x +
 c)^2 + (7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*c^2
)*f^2)*x^2 + (3*b^3*d^2*f^2*x^4*arctan2(1, d*x + c) + (6*b^3*d^2*e*f*ar...
```

3.141.8 Giac [F]

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx = \int (fx+e)^2 (b\,\mathrm{arccot}\,(dx+c)+a)^3 \, dx$$

input
```
integrate((f*x+e)^2*(a+b*arccot(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*(b*arccot(d*x + c) + a)^3, x)
```

3.141.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx = \int (e+f\,x)^2 \left(a+b\,\mathrm{acot}(c+d\,x)\right)^3 \, dx$$

input
```
int((e + f*x)^2*(a + b*acot(c + d*x))^3,x)
```

output
```
int((e + f*x)^2*(a + b*acot(c + d*x))^3, x)
```

3.142 $\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^3\,dx$

3.142.1 Optimal result

Integrand size = 18, antiderivative size = 337

$$
\begin{aligned}
&\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^3\,dx \\
&= \frac{3ibf\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2}+\frac{3bf(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2} \\
&\quad+\frac{i(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^3}{d^2} \\
&\quad-\frac{(de+f-cf)(de-(1+c)f)\left(a+b\cot^{-1}(c+dx)\right)^3}{2d^2f} \\
&\quad+\frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)^3}{2f}-\frac{3b^2f\left(a+b\cot^{-1}(c+dx)\right)\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2} \\
&\quad-\frac{3b(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2}+\frac{3ib^3f\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{2d^2} \\
&\quad+\frac{3ib^2(de-cf)\left(a+b\cot^{-1}(c+dx)\right)\mathrm{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2} \\
&\quad-\frac{3b^3(de-cf)\,\mathrm{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2d^2}
\end{aligned}
$$

output
```
3/2*I*b*f*(a+b*arccot(d*x+c))^2/d^2+3/2*b*f*(d*x+c)*(a+b*arccot(d*x+c))^2/
d^2+I*(-c*f+d*e)*(a+b*arccot(d*x+c))^3/d^2-1/2*(-c*f+d*e+f)*(d*e-(1+c)*f)*
(a+b*arccot(d*x+c))^3/d^2/f+1/2*(f*x+e)^2*(a+b*arccot(d*x+c))^3/f-3*b^2*f*
(a+b*arccot(d*x+c))*ln(2/(1+I*(d*x+c)))/d^2-3*b*(-c*f+d*e)*(a+b*arccot(d*x
+c))^2*ln(2/(1+I*(d*x+c)))/d^2+3/2*I*b^3*f*polylog(2,1-2/(1+I*(d*x+c)))/d^
2+3*I*b^2*(-c*f+d*e)*(a+b*arccot(d*x+c))*polylog(2,1-2/(1+I*(d*x+c)))/d^2-
3/2*b^3*(-c*f+d*e)*polylog(3,1-2/(1+I*(d*x+c)))/d^2
```

3.142.2 Mathematica [A] (verified)

Time = 5.42 (sec) , antiderivative size = 630, normalized size of antiderivative = 1.87

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^3\,dx$$

$$= \frac{a^2(2ade+3bf-2acf)(c+dx)+a^3f(c+dx)^2-3a^2b(c+dx)(cf-d(2e+fx))\cot^{-1}(c+dx)-3a^2bf\,a}{}$$

input
```
Integrate[(e + f*x)*(a + b*ArcCot[c + d*x])^3,x]
```

output
```
(a^2*(2*a*d*e + 3*b*f - 2*a*c*f)*(c + d*x) + a^3*f*(c + d*x)^2 - 3*a^2*b*(
c + d*x)*(c*f - d*(2*e + f*x))*ArcCot[c + d*x] - 3*a^2*b*f*ArcTan[c + d*x]
 + 6*a*b^2*f*((c + d*x)*ArcCot[c + d*x] + ((1 + (c + d*x)^2)*ArcCot[c + d*
x]^2)/2 - Log[1/((c + d*x)*Sqrt[1 + (c + d*x)^(-2)])]) + 3*a^2*b*(d*e - c*
f)*Log[1 + (c + d*x)^2] + 6*a*b^2*d*e*(ArcCot[c + d*x]*((I + c + d*x)*ArcC
ot[c + d*x] - 2*Log[1 - E^((2*I)*ArcCot[c + d*x])]) + I*PolyLog[2, E^((2*I
)*ArcCot[c + d*x])]) - 6*a*b^2*c*f*(ArcCot[c + d*x]*((I + c + d*x)*ArcCot[
c + d*x] - 2*Log[1 - E^((2*I)*ArcCot[c + d*x])]) + I*PolyLog[2, E^((2*I)*A
rcCot[c + d*x])]) + b^3*f*(3*(c + d*x)*ArcCot[c + d*x]^2 + (1 + (c + d*x)^
2)*ArcCot[c + d*x]^3 - 6*ArcCot[c + d*x]*Log[1 - E^((2*I)*ArcCot[c + d*x])
] + (3*I)*(ArcCot[c + d*x]^2 + PolyLog[2, E^((2*I)*ArcCot[c + d*x])])) + 2
*b^3*d*e*((I/8)*Pi^3 - I*ArcCot[c + d*x]^3 + (c + d*x)*ArcCot[c + d*x]^3 -
 3*ArcCot[c + d*x]^2*Log[1 - E^((-2*I)*ArcCot[c + d*x])] - (3*I)*ArcCot[c
+ d*x]*PolyLog[2, E^((-2*I)*ArcCot[c + d*x])] - (3*PolyLog[3, E^((-2*I)*Ar
cCot[c + d*x])])/2) - 2*b^3*c*f*((I/8)*Pi^3 - I*ArcCot[c + d*x]^3 + (c + d
*x)*ArcCot[c + d*x]^3 - 3*ArcCot[c + d*x]^2*Log[1 - E^((-2*I)*ArcCot[c + d
*x])] - (3*I)*ArcCot[c + d*x]*PolyLog[2, E^((-2*I)*ArcCot[c + d*x])] - (3*
PolyLog[3, E^((-2*I)*ArcCot[c + d*x])])/2))/(2*d^2)
```

3.142.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5571, 27, 5390, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^3\,dx$$

$$\Big\downarrow \; 5571$$

$$\frac{\int \frac{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)\left(a+b\cot^{-1}(c+dx)\right)^3}{d}d(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\int (de-cf+f(c+dx))\left(a+b\cot^{-1}(c+dx)\right)^3d(c+dx)}{d^2}$$

$$\Big\downarrow \; 5390$$

$$\frac{\frac{3b\int \left(f^2\left(a+b\cot^{-1}(c+dx)\right)^2+\frac{((de-cf+f)(de-(c+1)f)+2f(de-cf)(c+dx))\left(a+b\cot^{-1}(c+dx)\right)^2}{(c+dx)^2+1}\right)d(c+dx)}{2f}+\frac{(f(c+dx)-cf+de)^2\left(a+b\cot^{-1}(c+dx)\right)}{2f}}{d^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{(f(c+dx)-cf+de)^2\left(a+b\cot^{-1}(c+dx)\right)^3}{2f}+\frac{3b\left(2ibf(de-cf)\operatorname{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)\left(a+b\cot^{-1}(c+dx)\right)+\frac{2if(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^3}{3b}-\frac{(-c}{}\right.}{}}{}$$

input
```
Int[(e + f*x)*(a + b*ArcCot[c + d*x])^3,x]
```

output
```
(((d*e - c*f + f*(c + d*x))^2*(a + b*ArcCot[c + d*x])^3)/(2*f) + (3*b*(I*f
^2*(a + b*ArcCot[c + d*x])^2 + f^2*(c + d*x)*(a + b*ArcCot[c + d*x])^2 + (
((2*I)/3)*f*(d*e - c*f)*(a + b*ArcCot[c + d*x])^3)/b - ((d*e + f - c*f)*(d
*e - (1 + c)*f)*(a + b*ArcCot[c + d*x])^3)/(3*b) - 2*b*f^2*(a + b*ArcCot[c
 + d*x])*Log[2/(1 + I*(c + d*x))] - 2*f*(d*e - c*f)*(a + b*ArcCot[c + d*x]
)^2*Log[2/(1 + I*(c + d*x))] + I*b^2*f^2*PolyLog[2, 1 - 2/(1 + I*(c + d*x)
)] + (2*I)*b*f*(d*e - c*f)*(a + b*ArcCot[c + d*x])*PolyLog[2, 1 - 2/(1 + I
*(c + d*x))] - b^2*f*(d*e - c*f)*PolyLog[3, 1 - 2/(1 + I*(c + d*x))]))/(2*
f))/d^2
```

3.142.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5390
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])^p/(e*(q + 1))), x] + S
imp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcCot[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

rule 5571
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.142.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1050 vs. $2(316) = 632$.

Time = 13.32 (sec) , antiderivative size = 1051, normalized size of antiderivative = 3.12

method	result	size
parts	Expression too large to display	1051
derivativedivides	Expression too large to display	17316
default	Expression too large to display	17316

input
```
int((f*x+e)*(a+b*arccot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*(1/2*f*x^2+e*x)+b^3/d*(1/2/d*arccot(d*x+c)^3*(d*x+c)^2*f-1/d*arccot(d*
x+c)^3*c*f*(d*x+c)+arccot(d*x+c)^3*e*(d*x+c)+3/2/d*(1/3*f*arccot(d*x+c)^3+
2*I*f*polylog(2,-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))-2/3*I*arccot(d*x+c)^3*c*f+
2*I*f*arccot(d*x+c)^2+arccot(d*x+c)^2*f*(d*x+c-I)-2*f*arccot(d*x+c)*ln(1-(
d*x+c+I)/(1+(d*x+c)^2)^(1/2))+2/3*I*arccot(d*x+c)^3*d*e-2*f*arccot(d*x+c)*
ln(1+(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+4*polylog(3,-(d*x+c+I)/(1+(d*x+c)^2)^(
1/2))*c*f+4*I*d*e*arccot(d*x+c)*polylog(2,(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+2
*ln(1-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))*c*f*arccot(d*x+c)^2+2*I*f*polylog(2,(
d*x+c+I)/(1+(d*x+c)^2)^(1/2))-2*ln(1-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))*d*e*ar
ccot(d*x+c)^2-4*I*c*f*arccot(d*x+c)*polylog(2,(d*x+c+I)/(1+(d*x+c)^2)^(1/2
))-4*polylog(3,-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))*d*e+4*I*d*e*arccot(d*x+c)*p
olylog(2,-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))-4*I*c*f*arccot(d*x+c)*polylog(2,-
(d*x+c+I)/(1+(d*x+c)^2)^(1/2))-4*polylog(3,(d*x+c+I)/(1+(d*x+c)^2)^(1/2))*
d*e+4*polylog(3,(d*x+c+I)/(1+(d*x+c)^2)^(1/2))*c*f+2*ln(1+(d*x+c+I)/(1+(d*
x+c)^2)^(1/2))*c*f*arccot(d*x+c)^2-2*ln(1+(d*x+c+I)/(1+(d*x+c)^2)^(1/2))*d
*e*arccot(d*x+c)^2))+3*a*b^2/d*(1/2/d*arccot(d*x+c)^2*(d*x+c)^2*f-1/d*arcc
ot(d*x+c)^2*c*f*(d*x+c)+arccot(d*x+c)^2*e*(d*x+c)+1/d*(-arccot(d*x+c)*ln(1
+(d*x+c)^2)*c*f+arccot(d*x+c)*ln(1+(d*x+c)^2)*d*e-arccot(d*x+c)*arctan(d*x
+c)*f+arccot(d*x+c)*(d*x+c)*f+1/2*f*ln(1+(d*x+c)^2)-1/2*arctan(d*x+c)^2*f+
1/2*(-2*c*f+2*d*e)*(-1/2*I*(ln(d*x+c-I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c-I)...
```

3.142.5 Fricas [F]

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^3\,dx=\int (fx+e)(b\,\text{arccot}\,(dx+c)+a)^3\,dx$$

input
```
integrate((f*x+e)*(a+b*arccot(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral(a^3*f*x + a^3*e + (b^3*f*x + b^3*e)*arccot(d*x + c)^3 + 3*(a*b^2*
f*x + a*b^2*e)*arccot(d*x + c)^2 + 3*(a^2*b*f*x + a^2*b*e)*arccot(d*x + c)
, x)
```

3.142.6 Sympy [F]

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^3\,dx=\int (a+b\operatorname{acot}(c+dx))^3(e+fx)\,dx$$

input
```
integrate((f*x+e)*(a+b*acot(d*x+c))**3,x)
```

output
```
Integral((a + b*acot(c + d*x))**3*(e + f*x), x)
```

3.142.7 Maxima [F]

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^3\,dx=\int (fx+e)(b\operatorname{arccot}(dx+c)+a)^3\,dx$$

input
```
integrate((f*x+e)*(a+b*arccot(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/16*b^3*f*x^2*arctan2(1, d*x + c)^3 + 1/8*b^3*e*x*arctan2(1, d*x + c)^3 +
 1/2*a^3*f*x^2 + 3/2*(x^2*arccot(d*x + c) + d*(x/d^2 + (c^2 - 1)*arctan((d
^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a^2*b*f + a^
3*e*x + 3/2*(2*(d*x + c)*arccot(d*x + c) + log((d*x + c)^2 + 1))*a^2*b*e/d
 - 3/64*(b^3*f*x^2*arctan2(1, d*x + c) + 2*b^3*e*x*arctan2(1, d*x + c))*lo
g(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + integrate(1/64*(8*(7*b^3*arctan2(1, d*x
 + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*d^2*f*x^3 + 4*(2*(7*b^3*arctan2(
1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*d^2*e + (3*b^3*arctan2(1,
d*x + c)^2 + 4*(7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)
^2)*c)*d*f)*x^2 + 3*(2*b^3*d^2*f*x^3*arctan2(1, d*x + c) + (2*b^3*d^2*e*ar
ctan2(1, d*x + c) + (4*b^3*c*arctan2(1, d*x + c) - b^3)*d*f)*x^2 + 2*(b^3*
c^2*arctan2(1, d*x + c) + b^3*arctan2(1, d*x + c))*e + 2*((2*b^3*c*arctan2
(1, d*x + c) - b^3)*d*e + (b^3*c^2*arctan2(1, d*x + c) + b^3*arctan2(1, d*
x + c))*f)*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 8*(7*b^3*arctan2(1, d*x
 + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2 + (7*b^3*arctan2(1, d*x + c)^3 +
24*a*b^2*arctan2(1, d*x + c)^2)*c^2)*e + 8*((3*b^3*arctan2(1, d*x + c)^2 +
 2*(7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*c)*d*e +
 (7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2 + (7*b^3*ar
ctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*c^2)*f)*x + 12*(b^3*
d^2*f*x^3*arctan2(1, d*x + c) + 2*b^3*c*d*e*x*arctan2(1, d*x + c) + (2*...
```

3.142.8 Giac [F]

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^3\,dx=\int (fx+e)(b\,\mathrm{arccot}\,(dx+c)+a)^3\,dx$$

input
```
integrate((f*x+e)*(a+b*arccot(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((f*x + e)*(b*arccot(d*x + c) + a)^3, x)
```

3.142.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^3\,dx=\int (e+f\,x)\,\left(a+b\,\mathrm{acot}(c+d\,x)\right)^3\,dx$$

input
```
int((e + f*x)*(a + b*acot(c + d*x))^3,x)
```

output
```
int((e + f*x)*(a + b*acot(c + d*x))^3, x)
```

3.143 $\int (a+b\cot^{-1}(c+dx))^3 \, dx$

3.143.1 Optimal result

Integrand size = 12, antiderivative size = 143

$$\int (a+b\cot^{-1}(c+dx))^3 \, dx = \frac{i(a+b\cot^{-1}(c+dx))^3}{d} + \frac{(c+dx)(a+b\cot^{-1}(c+dx))^3}{d} - \frac{3b(a+b\cot^{-1}(c+dx))^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d} + \frac{3ib^2(a+b\cot^{-1}(c+dx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d} - \frac{3b^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2d}$$

output
```
I*(a+b*arccot(d*x+c))^3/d+(d*x+c)*(a+b*arccot(d*x+c))^3/d-3*b*(a+b*arccot(
d*x+c))^2*ln(2/(1+I*(d*x+c)))/d+3*I*b^2*(a+b*arccot(d*x+c))*polylog(2,1-2/
(1+I*(d*x+c)))/d-3/2*b^3*polylog(3,1-2/(1+I*(d*x+c)))/d
```

3.143.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.59

$$\int \left(a+b\cot^{-1}(c+dx)\right)^3\,dx$$
$$=\frac{2a^3(c+dx)+6a^2b(c+dx)\cot^{-1}(c+dx)+3a^2b\log\left(1+(c+dx)^2\right)+6ab^2\left(\cot^{-1}(c+dx)\left((i+c+dx)\right.\right.}{}$$

input
```
Integrate[(a + b*ArcCot[c + d*x])^3,x]
```

output
```
(2*a^3*(c + d*x) + 6*a^2*b*(c + d*x)*ArcCot[c + d*x] + 3*a^2*b*Log[1 + (c
+ d*x)^2] + 6*a*b^2*(ArcCot[c + d*x]*((I + c + d*x)*ArcCot[c + d*x] - 2*Lo
g[1 - E^((2*I)*ArcCot[c + d*x])]) + I*PolyLog[2, E^((2*I)*ArcCot[c + d*x])
]) + 2*b^3*((I/8)*Pi^3 - I*ArcCot[c + d*x]^3 + (c + d*x)*ArcCot[c + d*x]^3
 - 3*ArcCot[c + d*x]^2*Log[1 - E^((-2*I)*ArcCot[c + d*x])] - (3*I)*ArcCot[
c + d*x]*PolyLog[2, E^((-2*I)*ArcCot[c + d*x])] - (3*PolyLog[3, E^((-2*I)*
ArcCot[c + d*x])])/2))/(2*d)
```

3.143.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5563, 5346, 5456, 5380, 5530, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\cot^{-1}(c+dx)\right)^3\,dx$$

$$\Big\downarrow \; 5563$$

$$\frac{\int \left(a+b\cot^{-1}(c+dx)\right)^3\,d(c+dx)}{d}$$

$$\Big\downarrow \; 5346$$

$$\frac{3b\int \frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{(c+dx)^2+1}d(c+dx)+(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^3}{d}$$

$$\Big\downarrow \; 5456$$

$$\frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^3+3b\left(\frac{i\left(a+b\cot^{-1}(c+dx)\right)^3}{3b}-\int\frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{-c-dx+i}d(c+dx)\right)}{d}$$

$\downarrow$ 5380

$$\frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^3+3b\left(-2b\int\frac{\left(a+b\cot^{-1}(c+dx)\right)\log\left(\frac{2}{i(c+dx)+1}\right)}{(c+dx)^2+1}d(c+dx)+\frac{i\left(a+b\cot^{-1}(c+dx)\right)^3}{3b}-\log\left(\frac{}{1+}\right.\right.}{d}$$

$\downarrow$ 5530

$$\frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^3+3b\left(-2b\left(-\frac{1}{2}ib\int\frac{\text{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)}{(c+dx)^2+1}d(c+dx)-\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right.\right.\right.}{d}$$

$\downarrow$ 7164

$$\frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^3+3b\left(-2b\left(\frac{1}{4}b\,\text{PolyLog}\left(3,1-\frac{2}{i(c+dx)+1}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)(a+b\,c\right.\right.}{d}$$

input
```
Int[(a + b*ArcCot[c + d*x])^3,x]
```

output
```
((c + d*x)*(a + b*ArcCot[c + d*x])^3 + 3*b*(((I/3)*(a + b*ArcCot[c + d*x])
^3)/b - (a + b*ArcCot[c + d*x])^2*Log[2/(1 + I*(c + d*x))] - 2*b*((-1/2*I)
*(a + b*ArcCot[c + d*x])*PolyLog[2, 1 - 2/(1 + I*(c + d*x))] + (b*PolyLog[
3, 1 - 2/(1 + I*(c + d*x))])/4)))/d
```

3.143.3.1 Defintions of rubi rules used

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5380
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Simp[b*c*(
p/e)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5456
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Simp[
1/(c*d)   Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5530
```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] - Simp[b*p*(I/2)   Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5563
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[1/d
  Subst[Int[(a + b*ArcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
 x] && IGtQ[p, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.143.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(136) = 272$.

Time = 2.46 (sec) , antiderivative size = 395, normalized size of antiderivative = 2.76

method	result
derivativedivides	$\frac{(dx+c)a^3+b^3\left(\operatorname{arccot}(dx+c)^3(dx+c-i)+2i\operatorname{arccot}(dx+c)^3-3\operatorname{arccot}(dx+c)^2\ln\left(1+\frac{dx+c+i}{\sqrt{1+(dx+c)^2}}\right)+6i\operatorname{arccot}(dx+c)\operatorname{po}\right.}{}$
default	$\frac{(dx+c)a^3+b^3\left(\operatorname{arccot}(dx+c)^3(dx+c-i)+2i\operatorname{arccot}(dx+c)^3-3\operatorname{arccot}(dx+c)^2\ln\left(1+\frac{dx+c+i}{\sqrt{1+(dx+c)^2}}\right)+6i\operatorname{arccot}(dx+c)\operatorname{po}\right.}{}$
parts	$a^3x+\frac{b^3\left(\operatorname{arccot}(dx+c)^3(dx+c-i)+2i\operatorname{arccot}(dx+c)^3-3\operatorname{arccot}(dx+c)^2\ln\left(1+\frac{dx+c+i}{\sqrt{1+(dx+c)^2}}\right)+6i\operatorname{arccot}(dx+c)\operatorname{polyl}\right.}{}$

input
```
int((a+b*arccot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*((d*x+c)*a^3+b^3*(arccot(d*x+c)^3*(d*x+c-I)+2*I*arccot(d*x+c)^3-3*arcc
ot(d*x+c)^2*ln(1+(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+6*I*arccot(d*x+c)*polylog(
2,-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))-6*polylog(3,-(d*x+c+I)/(1+(d*x+c)^2)^(1/
2))-3*arccot(d*x+c)^2*ln(1-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+6*I*arccot(d*x+c
)*polylog(2,(d*x+c+I)/(1+(d*x+c)^2)^(1/2))-6*polylog(3,(d*x+c+I)/(1+(d*x+c
)^2)^(1/2)))+3*a*b^2*(arccot(d*x+c)^2*(d*x+c-I)-2*arccot(d*x+c)*ln(1+(d*x+
c+I)/(1+(d*x+c)^2)^(1/2))-2*arccot(d*x+c)*ln(1-(d*x+c+I)/(1+(d*x+c)^2)^(1/
2))+2*I*arccot(d*x+c)^2+2*I*polylog(2,-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+2*I*
polylog(2,(d*x+c+I)/(1+(d*x+c)^2)^(1/2)))+3*a^2*b*(arccot(d*x+c)*(d*x+c)+1
/2*ln(1+(d*x+c)^2)))
```

3.143.5 Fricas [F]

$$\int \left(a+b\cot^{-1}(c+dx)\right)^3\,dx=\int \left(b\operatorname{arccot}(dx+c)+a\right)^3\,dx$$

input
```
integrate((a+b*arccot(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral(b^3*arccot(d*x + c)^3 + 3*a*b^2*arccot(d*x + c)^2 + 3*a^2*b*arcco
t(d*x + c) + a^3, x)
```

3.143.6 Sympy [F]

$$\int \left(a+b\cot^{-1}(c+dx)\right)^3\,dx=\int \left(a+b\operatorname{acot}\left(c+dx\right)\right)^3\,dx$$

input
```
integrate((a+b*acot(d*x+c))**3,x)
```

output
```
Integral((a + b*acot(c + d*x))**3, x)
```

3.143.7 Maxima [F]

$$\int \left(a+b\cot^{-1}(c+dx)\right)^3\,dx=\int \left(b\operatorname{arccot}\left(dx+c\right)+a\right)^3\,dx$$

input
```
integrate((a+b*arccot(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/8*b^3*x*arctan2(1, d*x + c)^3 - 3/32*b^3*x*arctan2(1, d*x + c)*log(d^2*x
^2 + 2*c*d*x + c^2 + 1)^2 + a^3*x + 3/2*(2*(d*x + c)*arccot(d*x + c) + log
((d*x + c)^2 + 1))*a^2*b/d + integrate(1/32*(28*b^3*arctan2(1, d*x + c)^3
+ 4*(7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*d^2*x^2
 + 96*a*b^2*arctan2(1, d*x + c)^2 + 4*(7*b^3*arctan2(1, d*x + c)^3 + 24*a*
b^2*arctan2(1, d*x + c)^2)*c^2 + 4*(3*b^3*arctan2(1, d*x + c)^2 + 2*(7*b^3
*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*c)*d*x + 3*(b^3*d
^2*x^2*arctan2(1, d*x + c) + b^3*c^2*arctan2(1, d*x + c) + b^3*arctan2(1,
d*x + c) + (2*b^3*c*arctan2(1, d*x + c) - b^3)*d*x)*log(d^2*x^2 + 2*c*d*x
+ c^2 + 1)^2 + 12*(b^3*d^2*x^2*arctan2(1, d*x + c) + b^3*c*d*x*arctan2(1,
d*x + c))*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^2*x^2 + 2*c*d*x + c^2 + 1),
 x)
```

3.143.8 Giac [F]

$$\int \left(a+b\cot^{-1}(c+dx)\right)^3\,dx=\int (b\operatorname{arccot}(dx+c)+a)^3\,dx$$

input
```
integrate((a+b*arccot(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((b*arccot(d*x + c) + a)^3, x)
```

3.143.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\cot^{-1}(c+dx)\right)^3\,dx=\int (a+b\,\mathrm{acot}(c+d\,x))^3\,dx$$

input
```
int((a + b*acot(c + d*x))^3,x)
```

output
```
int((a + b*acot(c + d*x))^3, x)
```

3.144 $\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{e+fx}\,dx$

3.144.1 Optimal result

Integrand size = 20, antiderivative size = 372

$$\begin{aligned}\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{e+fx}\,dx &= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^3\log\left(\frac{2}{1-i(c+dx)}\right)}{f} \\ &+ \frac{\left(a+b\cot^{-1}(c+dx)\right)^3\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f} \\ &- \frac{3ib\left(a+b\cot^{-1}(c+dx)\right)^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{2f} \\ &+ \frac{3ib\left(a+b\cot^{-1}(c+dx)\right)^2\operatorname{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2f} \\ &- \frac{3b^2\left(a+b\cot^{-1}(c+dx)\right)\operatorname{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)}{2f} \\ &+ \frac{3b^2\left(a+b\cot^{-1}(c+dx)\right)\operatorname{PolyLog}\left(3,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2f} \\ &+ \frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2}{1-i(c+dx)}\right)}{4f} - \frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{4f}\end{aligned}$$

output
```
-(a+b*arccot(d*x+c))^3*ln(2/(1-I*(d*x+c)))/f+(a+b*arccot(d*x+c))^3*ln(2*d*
(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f-3/2*I*b*(a+b*arccot(d*x+c))^2*polyl
og(2,1-2/(1-I*(d*x+c)))/f+3/2*I*b*(a+b*arccot(d*x+c))^2*polylog(2,1-2*d*(f
*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f-3/2*b^2*(a+b*arccot(d*x+c))*polylog(3
,1-2/(1-I*(d*x+c)))/f+3/2*b^2*(a+b*arccot(d*x+c))*polylog(3,1-2*d*(f*x+e)/
(d*e+I*f-c*f)/(1-I*(d*x+c)))/f+3/4*I*b^3*polylog(4,1-2/(1-I*(d*x+c)))/f-3/
4*I*b^3*polylog(4,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f
```

3.144.2 Mathematica [F]

$$\int \frac{(a+b\cot^{-1}(c+dx))^3}{e+fx}\,dx=\int \frac{(a+b\cot^{-1}(c+dx))^3}{e+fx}\,dx$$

input
```
Integrate[(a + b*ArcCot[c + d*x])^3/(e + f*x),x]
```

output
```
Integrate[(a + b*ArcCot[c + d*x])^3/(e + f*x), x]
```

3.144.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5571, 27, 5386}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cot^{-1}(c+dx))^3}{e+fx}\,dx$$

$$\Big\downarrow \text{5571}$$

$$\frac{\int \frac{d(a+b\cot^{-1}(c+dx))^3}{d\left(e-\frac{cf}{d}\right)+f(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+b\cot^{-1}(c+dx))^3}{f(c+dx)-cf+de}d(c+dx)$$

$$\Big\downarrow \text{5386}$$

$$\frac{3b^2(a+b\cot^{-1}(c+dx))\operatorname{PolyLog}\left(3,1-\frac{2(de-cf+f(c+dx))}{(de-cf+if)(1-i(c+dx))}\right)}{2f}-\frac{3b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)(a+b\cot^{-1}(c+dx))}{2f}+\frac{3ib(a+b\cot^{-1}(c+dx))^2\operatorname{PolyLog}\left(2,1-\frac{2(de-cf+f(c+dx))}{(de-cf+if)(1-i(c+dx))}\right)}{2f}+\frac{(a+b\cot^{-1}(c+dx))^3\log\left(\frac{2(f(c+dx)-cf+de)}{(1-i(c+dx))(-cf+de+if)}\right)}{f}-\frac{3ib\operatorname{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)(a+b\cot^{-1}(c+dx))^2}{2f}-\frac{\log\left(\frac{2}{1-i(c+dx)}\right)(a+b\cot^{-1}(c+dx))^3}{f}-\frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2(de-cf+f(c+dx))}{(de-cf+if)(1-i(c+dx))}\right)}{4f}+\frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2}{1-i(c+dx)}\right)}{4f}$$

input
```
Int[(a + b*ArcCot[c + d*x])^3/(e + f*x),x]
```

output
```
-(((a + b*ArcCot[c + d*x])^3*Log[2/(1 - I*(c + d*x))])/f) + ((a + b*ArcCot
[c + d*x])^3*Log[(2*(d*e - c*f + f*(c + d*x)))/((d*e + I*f - c*f)*(1 - I*(
c + d*x)))])/f - (((3*I)/2)*b*(a + b*ArcCot[c + d*x])^2*PolyLog[2, 1 - 2/(
1 - I*(c + d*x))])/f + (((3*I)/2)*b*(a + b*ArcCot[c + d*x])^2*PolyLog[2, 1
 - (2*(d*e - c*f + f*(c + d*x)))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f
 - (3*b^2*(a + b*ArcCot[c + d*x])*PolyLog[3, 1 - 2/(1 - I*(c + d*x))])/(2*
f) + (3*b^2*(a + b*ArcCot[c + d*x])*PolyLog[3, 1 - (2*(d*e - c*f + f*(c +
d*x)))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/(2*f) + (((3*I)/4)*b^3*Poly
Log[4, 1 - 2/(1 - I*(c + d*x))])/f - (((3*I)/4)*b^3*PolyLog[4, 1 - (2*(d*e
 - c*f + f*(c + d*x)))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f
```

3.144. $\int \frac{(a+b\cot^{-1}(c+dx))^3}{e+fx}\,dx$

3.144.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5386
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^3/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcCot[c*x])^3)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Cot[c*x])^3*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] - Simp[3
*I*b*(a + b*ArcCot[c*x])^2*(PolyLog[2, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp
[3*I*b*(a + b*ArcCot[c*x])^2*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1
 - I*c*x)))]/(2*e)), x] - Simp[3*b^2*(a + b*ArcCot[c*x])*(PolyLog[3, 1 - 2/
(1 - I*c*x)]/(2*e)), x] + Simp[3*b^2*(a + b*ArcCot[c*x])*(PolyLog[3, 1 - 2*
c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x] + Simp[3*I*b^3*(PolyLog
[4, 1 - 2/(1 - I*c*x)]/(4*e)), x] - Simp[3*I*b^3*(PolyLog[4, 1 - 2*c*((d +
e*x)/((c*d + I*e)*(1 - I*c*x)))]/(4*e)), x]) /; FreeQ[{a, b, c, d, e}, x] &
& NeQ[c^2*d^2 + e^2, 0]
```

rule 5571
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.144.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 16.36 (sec) , antiderivative size = 3903, normalized size of antiderivative = 10.49

method	result	size
derivativedivides	Expression too large to display	3903
default	Expression too large to display	3903
parts	Expression too large to display	4139

input
```
int((a+b*arccot(d*x+c))^3/(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*d*ln(c*f-d*e-f*(d*x+c))/f-b^3*d*(-ln(c*f-d*e-f*(d*x+c))/f*arccot(
d*x+c)^3-3/f*(-1/3*arccot(d*x+c)^3*ln(-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)+c*f*(
d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+I)^2/(1+(d*x+c)^2)-I*f-c*f+d*e)-2*I*po
lylog(4,(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+1/3*arccot(d*x+c)^3*ln((d*x+c+I)^2/
(1+(d*x+c)^2)-1)-1/3*arccot(d*x+c)^3*ln(1-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+I
*arccot(d*x+c)^2*polylog(2,(d*x+c+I)/(1+(d*x+c)^2)^(1/2))-2*arccot(d*x+c)*
polylog(3,(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+I*d*e*arccot(d*x+c)^2*polylog(2,(
d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x+c)^2))/(-2*I*f+2*c*f-2*d*e
)-1/3*arccot(d*x+c)^3*ln(1+(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+I*arccot(d*x+c)^
2*polylog(2,-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))-2*arccot(d*x+c)*polylog(3,-(d*
x+c+I)/(1+(d*x+c)^2)^(1/2))+1/6*I*Pi*csgn(I*(-I*f*(d*x+c+I)^2/(1+(d*x+c)^2
)+c*f*(d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+I)^2/(1+(d*x+c)^2)-I*f-c*f+d*e)
/((d*x+c+I)^2/(1+(d*x+c)^2)-1))*(csgn(I*(-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)+c*
f*(d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+I)^2/(1+(d*x+c)^2)-I*f-c*f+d*e))*cs
gn(I/((d*x+c+I)^2/(1+(d*x+c)^2)-1))-csgn(I*(-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)
+c*f*(d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+I)^2/(1+(d*x+c)^2)-I*f-c*f+d*e)/
((d*x+c+I)^2/(1+(d*x+c)^2)-1))*csgn(I/((d*x+c+I)^2/(1+(d*x+c)^2)-1))-csgn(
I*(-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)+c*f*(d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c
+I)^2/(1+(d*x+c)^2)-I*f-c*f+d*e))*csgn(I*(-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)+c
*f*(d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+I)^2/(1+(d*x+c)^2)-I*f-c*f+d*e)...
```

3.144.5 Fricas [F]

$$\int \frac{(a + b\cot^{-1}(c + dx))^3}{e + fx}\,dx = \int \frac{(b\,\mathrm{arccot}\,(dx + c) + a)^3}{fx + e}\,dx$$

input
```
integrate((a+b*arccot(d*x+c))^3/(f*x+e),x, algorithm="fricas")
```

output
```
integral((b^3*arccot(d*x + c)^3 + 3*a*b^2*arccot(d*x + c)^2 + 3*a^2*b*arcc
ot(d*x + c) + a^3)/(f*x + e), x)
```

3.144.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cot^{-1}(c+dx))^3}{e+fx}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acot(d*x+c))**3/(f*x+e),x)
```

output
```
Timed out
```

3.144.7 Maxima [F]

$$\int \frac{(a+b\cot^{-1}(c+dx))^3}{e+fx}\,dx = \int \frac{(b\,\mathrm{arccot}\,(dx+c)+a)^3}{fx+e}\,dx$$

input
```
integrate((a+b*arccot(d*x+c))^3/(f*x+e),x, algorithm="maxima")
```

output
```
a^3*log(f*x + e)/f + integrate(1/32*(28*b^3*arctan2(1, d*x + c)^3 + 3*b^3*
arctan2(1, d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 96*a*b^2*arctan2(
1, d*x + c)^2 + 96*a^2*b*arctan2(1, d*x + c))/(f*x + e), x)
```

3.144.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\cot^{-1}(c+dx))^3}{e+fx}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arccot(d*x+c))^3/(f*x+e),x, algorithm="giac")
```

output
```
Timed out
```

3.144.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{e+fx}\,dx=\int \frac{\left(a+b\,\mathrm{acot}(c+d\,x)\right)^3}{e+f\,x}\,dx$$

input
```
int((a + b*acot(c + d*x))^3/(e + f*x),x)
```

output
```
int((a + b*acot(c + d*x))^3/(e + f*x), x)
```

3.145 $\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{(e+fx)^2}\,dx$

3.145.1 Optimal result

Integrand size = 20, antiderivative size = 1233

$$
\begin{aligned}
\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{(e+fx)^2}\,dx = {} & \frac{3iab^2d\cot^{-1}(c+dx)^2}{d^2e^2-2cdef+(1+c^2)\,f^2}+\frac{3ab^2d(de-cf)\cot^{-1}(c+dx)^2}{f\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)} \\
& + \frac{ib^3d\cot^{-1}(c+dx)^3}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
& + \frac{b^3d(de-cf)\cot^{-1}(c+dx)^3}{f\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)}-\frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{f(e+fx)} \\
& - \frac{3a^2bd(de-cf)\arctan(c+dx)}{f\left(f^2+(de-cf)^2\right)}-\frac{3a^2bd\log(e+fx)}{f^2+(de-cf)^2} \\
& + \frac{6ab^2d\cot^{-1}(c+dx)\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
& + \frac{3b^3d\cot^{-1}(c+dx)^2\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
& - \frac{6ab^2d\cot^{-1}(c+dx)\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
& - \frac{3b^3d\cot^{-1}(c+dx)^2\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
& - \frac{6ab^2d\cot^{-1}(c+dx)\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
& - \frac{3b^3d\cot^{-1}(c+dx)^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
& + \frac{3a^2bd\log\left(1+(c+dx)^2\right)}{2\left(f^2+(de-cf)^2\right)} \\
& + \frac{3iab^2d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
& + \frac{3ib^3d\cot^{-1}(c+dx)\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
& - \frac{3iab^2d\,\mathrm{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
& - \frac{3ib^3d\cot^{-1}(c+dx)\,\mathrm{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
& + \frac{3iab^2d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
& + \frac{3ib^3d\cot^{-1}(c+dx)\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2}
\end{aligned}
$$

output
```
I*b^3*d*arccot(d*x+c)^3/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3*a*b^2*d*(-c*f+d*
e)*arccot(d*x+c)^2/f/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3*I*a*b^2*d*polylog(2
,1-2/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+b^3*d*(-c*f+d*e)*arcco
t(d*x+c)^3/f/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-(a+b*arccot(d*x+c))^3/f/(f*x+
e)-3*a^2*b*d*(-c*f+d*e)*arctan(d*x+c)/f/(f^2+(-c*f+d*e)^2)-3*a^2*b*d*ln(f*
x+e)/(f^2+(-c*f+d*e)^2)+6*a*b^2*d*arccot(d*x+c)*ln(2/(1-I*(d*x+c)))/(d^2*e
^2-2*c*d*e*f+(c^2+1)*f^2)+3*b^3*d*arccot(d*x+c)^2*ln(2/(1-I*(d*x+c)))/(d^2
*e^2-2*c*d*e*f+(c^2+1)*f^2)-6*a*b^2*d*arccot(d*x+c)*ln(2*d*(f*x+e)/(d*e+I*
f-c*f)/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-3*b^3*d*arccot(d*x+c
)^2*ln(2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)
*f^2)-6*a*b^2*d*arccot(d*x+c)*ln(2/(1+I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+
1)*f^2)-3*b^3*d*arccot(d*x+c)^2*ln(2/(1+I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^
2+1)*f^2)+3/2*a^2*b*d*ln(1+(d*x+c)^2)/(f^2+(-c*f+d*e)^2)+3*I*a*b^2*d*arcco
t(d*x+c)^2/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3*I*b^3*d*arccot(d*x+c)*polylog
(2,1-2/(1+I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3*I*b^3*d*arccot(d*x
+c)*polylog(2,1-2/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-3*I*b^3*d
*arccot(d*x+c)*polylog(2,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/(d^2*e
^2-2*c*d*e*f+(c^2+1)*f^2)+3*I*a*b^2*d*polylog(2,1-2/(1+I*(d*x+c)))/(d^2*e^
2-2*c*d*e*f+(c^2+1)*f^2)-3*I*a*b^2*d*polylog(2,1-2*d*(f*x+e)/(d*e+I*f-c*f)
/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3/2*b^3*d*polylog(3,1-2...
```

3.145.2 Mathematica [F]

$$\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{(e+fx)^2}\,dx=\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{(e+fx)^2}\,dx$$

input
```
Integrate[(a + b*ArcCot[c + d*x])^3/(e + f*x)^2,x]
```

output
```
Integrate[(a + b*ArcCot[c + d*x])^3/(e + f*x)^2, x]
```

3.145.3 Rubi [A] (verified)

Time = 2.42 (sec) , antiderivative size = 1266, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5569, 7292, 5581, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{(e+fx)^2}dx$$

$$\Big\downarrow \text{5569}$$

$$-\frac{3bd\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{(e+fx)\left((c+dx)^2+1\right)}dx}{f}-\frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{f(e+fx)}$$

$$\Big\downarrow \text{7292}$$

$$-\frac{3bd\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{(e+fx)(c^2+2dxc+d^2x^2+1)}dx}{f}-\frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{f(e+fx)}$$

$$\Big\downarrow \text{5581}$$

$$-\frac{3b\int \frac{d\left(a+b\cot^{-1}(c+dx)\right)^2}{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)\left((c+dx)^2+1\right)}d(c+dx)}{f}-\frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{f(e+fx)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3bd\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{(de-cf+f(c+dx))\left((c+dx)^2+1\right)}d(c+dx)}{f}-\frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{f(e+fx)}$$

$$\Big\downarrow \text{7276}$$

$$-\frac{3bd\int \left(\frac{a^2}{(de-cf+f(c+dx))\left((c+dx)^2+1\right)}+\frac{2b\cot^{-1}(c+dx)a}{(de-cf+f(c+dx))\left((c+dx)^2+1\right)}+\frac{b^2\cot^{-1}(c+dx)^2}{(de-cf+f(c+dx))\left((c+dx)^2+1\right)}\right)d(c+dx)}{f}-\frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{f(e+fx)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{f(e+fx)}-$$

$$\frac{3bd\left(-\frac{ib^2f\cot^{-1}(c+dx)^3}{3(d^2e^2-2cdfe+(c^2+1)f^2)}-\frac{b^2(de-cf)\cot^{-1}(c+dx)^3}{3(d^2e^2-2cdfe+(c^2+1)f^2)}-\frac{b^2f\log\left(\frac{2}{1-i(c+dx)}\right)\cot^{-1}(c+dx)^2}{d^2e^2-2cdfe+(c^2+1)f^2}+\frac{b^2f\log\left(\frac{2}{i(c+dx)+1}\right)\cot^{-1}(c+dx)^2}{d^2e^2-2cdfe+(c^2+1)f^2}\cdots\right.}{}$$

input
```
Int[(a + b*ArcCot[c + d*x])^3/(e + f*x)^2,x]
```

output
```
-((a + b*ArcCot[c + d*x])^3/(f*(e + f*x))) - (3*b*d*(((-I)*a*b*f*ArcCot[c
+ d*x]^2)/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - (a*b*(d*e - c*f)*ArcCot[
c + d*x]^2)/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - ((I/3)*b^2*f*ArcCot[c
+ d*x]^3)/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - (b^2*(d*e - c*f)*ArcCot[
c + d*x]^3)/(3*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)) + (a^2*(d*e - c*f)*A
rcTan[c + d*x])/(f^2 + (d*e - c*f)^2) - (2*a*b*f*ArcCot[c + d*x]*Log[2/(1
- I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - (b^2*f*ArcCot[c +
 d*x]^2*Log[2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) +
(2*a*b*f*ArcCot[c + d*x]*Log[2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f +
(1 + c^2)*f^2) + (b^2*f*ArcCot[c + d*x]^2*Log[2/(1 + I*(c + d*x))])/(d^2*e
^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (a^2*f*Log[d*e - c*f + f*(c + d*x)])/(f^
2 + (d*e - c*f)^2) + (2*a*b*f*ArcCot[c + d*x]*Log[(2*(d*e - c*f + f*(c + d
*x)))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/(d^2*e^2 - 2*c*d*e*f + (1 +
c^2)*f^2) + (b^2*f*ArcCot[c + d*x]^2*Log[(2*(d*e - c*f + f*(c + d*x)))/((d
*e + I*f - c*f)*(1 - I*(c + d*x)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)
 - (a^2*f*Log[1 + (c + d*x)^2])/(2*(f^2 + (d*e - c*f)^2)) - (I*a*b*f*PolyL
og[2, 1 - 2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - (I
*b^2*f*ArcCot[c + d*x]*PolyLog[2, 1 - 2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c
*d*e*f + (1 + c^2)*f^2) - (I*a*b*f*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/(d
^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - (I*b^2*f*ArcCot[c + d*x]*PolyLog[...
```

3.145.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5569
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCot[c + d*x])^p/(f*(m +
1))), x] + Simp[b*d*(p/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*((a + b*ArcCot[
c + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[p, 0] && ILtQ[m, -1]
```

rule 5581
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/d   Subs
t[Int[((d*e - c*f)/d + f*(x/d))^m*(C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcCot[x])
^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &
& EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

3.145.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 28.03 (sec) , antiderivative size = 4229, normalized size of antiderivative = 3.43

method	result	size
parts	Expression too large to display	4229
derivativedivides	Expression too large to display	4722
default	Expression too large to display	4722

input
```
int((a+b*arccot(d*x+c))^3/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-a^3/(f*x+e)/f+b^3/d*(-d^2/(f*(d*x+c)-c*f+d*e)/f*arccot(d*x+c)^3-3*d^2/f*(
arccot(d*x+c)^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f*ln(f*(d*x+c)-c*f+d*e)-1/
2*arccot(d*x+c)^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f*ln(1+(d*x+c)^2)-arccot
(d*x+c)^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)*c*f+arccot(d*x+c)^
2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)*d*e+1/(c^2*f^2-2*c*d*e*f+d
^2*e^2+f^2)*f*arctan(d*x+c)^2*ln((1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))-1/3*I/
(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f*arctan(d*x+c)^3+1/4/(c^2*f^2-2*c*d*e*f+d
^2*e^2+f^2)*(-2*Pi*c*f+2*Pi*d*e+4*f*ln(2)+I*f*Pi*csgn(I*(1+I*(d*x+c))^2/(1
+(d*x+c)^2))*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(1+(1+I*(d*x+c))^2/(1+(d
*x+c)^2))^2)^2-2*I*f*Pi*csgn(I*(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(I*(
1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)^2+4*I*f*Pi*csgn(I*(I*f*(1+I*(d*x+c))^2
/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d
*x+c)^2)-I*f+c*f-d*e)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2+2*I*f*Pi*csgn(I
*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*
(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e))*csgn(I*(I*f*(1+I*(d*x+c))^2/(1
+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+
c)^2)-I*f+c*f-d*e)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(I/(1+(1+I*(d*x+
c))^2/(1+(d*x+c)^2)))-I*f*Pi*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(1+(1+I*
(d*x+c))^2/(1+(d*x+c)^2))^2)^3+2*I*f*Pi*csgn(I*(1+I*(d*x+c))/(1+(d*x+c)^2)
^(1/2))*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2+I*f*Pi*csgn(I*(1+(1+I*(...
```

3.145.5 Fricas [F]

$$\int \frac{(a+b\cot^{-1}(c+dx))^3}{(e+fx)^2}\,dx=\int \frac{(b\operatorname{arccot}(dx+c)+a)^3}{(fx+e)^2}\,dx$$

input
```
integrate((a+b*arccot(d*x+c))^3/(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral((b^3*arccot(d*x + c)^3 + 3*a*b^2*arccot(d*x + c)^2 + 3*a^2*b*arcc
ot(d*x + c) + a^3)/(f^2*x^2 + 2*e*f*x + e^2), x)
```

3.145.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{(e+fx)^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*acot(d*x+c))**3/(f*x+e)**2,x)
```

output
```
Timed out
```

3.145.7 Maxima [F]

$$\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{(e+fx)^2}\,dx=\int \frac{\left(b\,\mathrm{arccot}\left(dx+c\right)+a\right)^3}{\left(fx+e\right)^2}\,dx$$

input
```
integrate((a+b*arccot(d*x+c))^3/(f*x+e)^2,x, algorithm="maxima")
```

output
```
-3/2*(d*(2*(d^2*e - c*d*f)*arctan((d^2*x + c*d)/d)/((d^2*e^2*f - 2*c*d*e*f
^2 + (c^2 + 1)*f^3)*d) - log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*e^2 - 2*c*d
*e*f + (c^2 + 1)*f^2) + 2*log(f*x + e)/(d^2*e^2 - 2*c*d*e*f + (c^2 + 1)*f^
2)) + 2*arccot(d*x + c)/(f^2*x + e*f))*a^2*b - a^3/(f^2*x + e*f) - 1/32*(4
*b^3*arctan2(1, d*x + c)^3 - 3*b^3*arctan2(1, d*x + c)*log(d^2*x^2 + 2*c*d
*x + c^2 + 1)^2 - 32*(f^2*x + e*f)*integrate(-1/32*(12*b^3*d*e*arctan2(1,
d*x + c)^2 - 4*(7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)
^2)*d^2*f*x^2 + 4*(3*b^3*arctan2(1, d*x + c)^2 - 2*(7*b^3*arctan2(1, d*x +
 c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*c)*d*f*x - 3*(b^3*d^2*f*x^2*arctan
2(1, d*x + c) + b^3*d*e + (2*b^3*c*arctan2(1, d*x + c) + b^3)*d*f*x + (b^3
*c^2*arctan2(1, d*x + c) + b^3*arctan2(1, d*x + c))*f)*log(d^2*x^2 + 2*c*d
*x + c^2 + 1)^2 - 4*(7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x
 + c)^2 + (7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*c
^2)*f + 12*(b^3*d^2*f*x^2*arctan2(1, d*x + c) + b^3*c*d*e*arctan2(1, d*x +
 c) + (b^3*d^2*e*arctan2(1, d*x + c) + b^3*c*d*f*arctan2(1, d*x + c))*x)*l
og(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^2*f^3*x^4 + (c^2 + 1)*e^2*f + 2*(d^2*e
*f^2 + c*d*f^3)*x^3 + (d^2*e^2*f + 4*c*d*e*f^2 + (c^2 + 1)*f^3)*x^2 + 2*(c
*d*e^2*f + (c^2 + 1)*e*f^2)*x), x))/(f^2*x + e*f)
```

3.145.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a + b\cot^{-1}(c + dx))^3}{(e + fx)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arccot(d*x+c))^3/(f*x+e)^2,x, algorithm="giac")
```

output
```
Timed out
```

3.145.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\cot^{-1}(c + dx))^3}{(e + fx)^2}\,dx = \int \frac{(a + b\,\mathrm{acot}(c + d\,x))^3}{(e + f\,x)^2}\,dx$$

input
```
int((a + b*acot(c + d*x))^3/(e + f*x)^2,x)
```

output
```
int((a + b*acot(c + d*x))^3/(e + f*x)^2, x)
```

3.146 $\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)\,dx$

3.146.1 Optimal result

Integrand size = 18, antiderivative size = 177

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)\,dx = \frac{(e+fx)^{1+m}\left(a+b\cot^{-1}(c+dx)\right)}{f(1+m)} + \frac{ibd(e+fx)^{2+m}\,\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{d(e+fx)}{de+if-cf}\right)}{2f(de+(i-c)f)(1+m)(2+m)} - \frac{ibd(e+fx)^{2+m}\,\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{d(e+fx)}{de-(i+c)f}\right)}{2f(de-(i+c)f)(1+m)(2+m)}$$

output
```
(f*x+e)^(1+m)*(a+b*arccot(d*x+c))/f/(1+m)+1/2*I*b*d*(f*x+e)^(2+m)*hypergeo
m([1, 2+m],[3+m],d*(f*x+e)/(d*e+I*f-c*f))/f/(d*e+(I-c)*f)/(1+m)/(2+m)-1/2*
I*b*d*(f*x+e)^(2+m)*hypergeom([1, 2+m],[3+m],d*(f*x+e)/(d*e-(I+c)*f))/f/(d
*e-(I+c)*f)/(1+m)/(2+m)
```

3.146.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.92

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)\,dx$$
$$=\frac{(e+fx)^{1+m}\left(2(a+b\cot^{-1}(c+dx))+\frac{bd(e+fx)\left((de-(i+c)f)\text{ Hypergeometric2F1}\left(1,2+m,3+m,\frac{d(e+fx)}{de-(-i+c)f}\right)+(-de+(-i+c)f\right.}{(-ide+f+icf)(de-(i+c)f)(2+m)}\right.}{2f(1+m)}$$

input
```
Integrate[(e + f*x)^m*(a + b*ArcCot[c + d*x]),x]
```

output
```
((e + f*x)^(1 + m)*(2*(a + b*ArcCot[c + d*x]) + (b*d*(e + f*x)*((d*e - (I
+ c)*f)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e - (-I + c)*f
)] + (-(d*e) + (-I + c)*f)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x)
)/(d*e - (I + c)*f)]))/(((-I)*d*e + f + I*c*f)*(d*e - (I + c)*f)*(2 + m)))
)/(2*f*(1 + m))
```

3.146.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.33, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5571, 5388, 485, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)\,dx$$
$$\Big\downarrow \text{5571}$$
$$\frac{\int \left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^m \left(a+b\cot^{-1}(c+dx)\right)d(c+dx)}{d}$$
$$\Big\downarrow \text{5388}$$
$$\frac{\frac{bd\int \frac{\left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^{m+1}}{(c+dx)^2+1}d(c+dx)}{f(m+1)}+\frac{d\left(a+b\cot^{-1}(c+dx)\right)\left(\frac{f(c+dx)}{d}-\frac{cf}{d}+e\right)^{m+1}}{f(m+1)}}{d}$$
$$\Big\downarrow \text{485}$$

$$\frac{\frac{d(a+b\cot^{-1}(c+dx))\left(\frac{f(c+dx)}{d}-\frac{cf}{d}+e\right)^{m+1}}{f(m+1)}+\frac{bd\int\left(\frac{i\left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^{m+1}}{2(-c-dx+i)}+\frac{i\left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^{m+1}}{2(c+dx+i)}\right)d(c+dx)}{f(m+1)}}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{d(a+b\cot^{-1}(c+dx))\left(\frac{f(c+dx)}{d}-\frac{cf}{d}+e\right)^{m+1}}{f(m+1)}+\frac{bd\left(\frac{id\left(\frac{f(c+dx)}{d}-\frac{cf}{d}+e\right)^{m+2}\text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{de-cf+f(c+dx)}{de-cf+if}\right)}{2(m+2)(de+(-c+i)f)}-\frac{id\left(\frac{f(c+dx)}{d}-\frac{cf}{d}+e\right.}{}\right.}{f(m+1)}}{d}$$

input
```
Int[(e + f*x)^m*(a + b*ArcCot[c + d*x]),x]
```

output
```
((d*(e - (c*f)/d + (f*(c + d*x))/d)^(1 + m)*(a + b*ArcCot[c + d*x]))/(f*(1
 + m)) + (b*d*(((I/2)*d*(e - (c*f)/d + (f*(c + d*x))/d)^(2 + m)*Hypergeome
tric2F1[1, 2 + m, 3 + m, (d*e - c*f + f*(c + d*x))/(d*e + I*f - c*f)])/((d
*e + (I - c)*f)*(2 + m)) - ((I/2)*d*(e - (c*f)/d + (f*(c + d*x))/d)^(2 + m
)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*e - c*f + f*(c + d*x))/(d*e - (I +
 c)*f)])/((d*e - (I + c)*f)*(2 + m))))/(f*(1 + m)))/d
```

3.146.3.1 Defintions of rubi rules used

rule 485
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n, 1/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d, n}, x] &
&  !IntegerQ[2*n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5388
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])/(e*(q + 1))), x] + Simp[b*(
c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b
, c, d, e, q}, x] && NeQ[q, -1]
```

rule 5571
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.146.4 Maple [F]

$$\int (fx+e)^m (a+b \operatorname{arccot}(dx+c))\, dx$$

input
```
int((f*x+e)^m*(a+b*arccot(d*x+c)),x)
```

output
```
int((f*x+e)^m*(a+b*arccot(d*x+c)),x)
```

3.146.5 Fricas [F]

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)\, dx = \int (b \operatorname{arccot}(dx+c)+a)(fx+e)^m\, dx$$

input
```
integrate((f*x+e)^m*(a+b*arccot(d*x+c)),x, algorithm="fricas")
```

output
```
integral((b*arccot(d*x + c) + a)*(f*x + e)^m, x)
```

3.146.6 Sympy [F(-1)]

Timed out.

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)\, dx = \text{Timed out}$$

input
```
integrate((f*x+e)**m*(a+b*acot(d*x+c)),x)
```

output
```
Timed out
```

3.146.7 Maxima [F]

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right) dx = \int (b\,\mathrm{arccot}\,(dx+c)+a)(fx+e)^m\,dx$$

input
```
integrate((f*x+e)^m*(a+b*arccot(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*((f*x*arctan2(1, d*x + c) + e*arctan2(1, d*x + c))*(f*x + e)^m + 2*(f*
m + f)*integrate(1/2*((c^2*arctan2(1, d*x + c) + arctan2(1, d*x + c))*f*m
+ (d^2*f*m*arctan2(1, d*x + c) + d^2*f*arctan2(1, d*x + c))*x^2 + d*e + (c
^2*arctan2(1, d*x + c) + arctan2(1, d*x + c))*f + (2*c*d*f*m*arctan2(1, d*
x + c) + (2*c*arctan2(1, d*x + c) + 1)*d*f)*x)*(f*x + e)^m/((c^2 + 1)*f*m
+ (d^2*f*m + d^2*f)*x^2 + (c^2 + 1)*f + 2*(c*d*f*m + c*d*f)*x), x))*b/(f*m
 + f) + (f*x + e)^(m + 1)*a/(f*(m + 1))
```

3.146.8 Giac [F]

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right) dx = \int (b\,\mathrm{arccot}\,(dx+c)+a)(fx+e)^m\,dx$$

input
```
integrate((f*x+e)^m*(a+b*arccot(d*x+c)),x, algorithm="giac")
```

output
```
integrate((b*arccot(d*x + c) + a)*(f*x + e)^m, x)
```

3.146.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right) dx = \int (e+f\,x)^m\,(a+b\,\mathrm{acot}(c+d\,x))\,dx$$

input
```
int((e + f*x)^m*(a + b*acot(c + d*x)),x)
```

output
```
int((e + f*x)^m*(a + b*acot(c + d*x)), x)
```

3.147 $\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx$

3.147.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx = \text{Int}\left((e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2, x\right)$$

output
```
Unintegrable((f*x+e)^m*(a+b*arccot(d*x+c))^2,x)
```

3.147.2 Mathematica [N/A]

Not integrable

Time = 4.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx = \int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx$$

input
```
Integrate[(e + f*x)^m*(a + b*ArcCot[c + d*x])^2,x]
```

output
```
Integrate[(e + f*x)^m*(a + b*ArcCot[c + d*x])^2, x]
```

3.147.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5571, 5561}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{5571}$$

$$\frac{\int \left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^m \left(a+b\cot^{-1}(c+dx)\right)^2 d(c+dx)}{d}$$

$$\Big\downarrow \text{5561}$$

$$\frac{\int \left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^m \left(a+b\cot^{-1}(c+dx)\right)^2 d(c+dx)}{d}$$

input
```
Int[(e + f*x)^m*(a + b*ArcCot[c + d*x])^2,x]
```

output
```
$Aborted
```

3.147.3.1 Defintions of rubi rules used

rule 5561
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCot[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

rule 5571
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.147.4 Maple [N/A] (verified)

Not integrable

Time = 0.47 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (fx+e)^m (a+b\,\mathrm{arccot}\,(dx+c))^2\,dx$$

input
```
int((f*x+e)^m*(a+b*arccot(d*x+c))^2,x)
```

output
```
int((f*x+e)^m*(a+b*arccot(d*x+c))^2,x)
```

3.147.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2\,dx = \int (b\,\mathrm{arccot}\,(dx+c)+a)^2(fx+e)^m\,dx$$

input
```
integrate((f*x+e)^m*(a+b*arccot(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral((b^2*arccot(d*x + c)^2 + 2*a*b*arccot(d*x + c) + a^2)*(f*x + e)^m
, x)
```

3.147.6 Sympy [F(-1)]

Timed out.

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**m*(a+b*acot(d*x+c))**2,x)
```

output
```
Timed out
```

3.147.7 Maxima [N/A]

Not integrable

Time = 5.11 (sec) , antiderivative size = 618, normalized size of antiderivative = 30.90

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2 dx = \int (b \operatorname{arccot}(dx+c)+a)^2(fx+e)^m dx$$

input
```
integrate((f*x+e)^m*(a+b*arccot(d*x+c))^2,x, algorithm="maxima")
```

output
```
(f*x + e)^(m + 1)*a^2/(f*(m + 1)) - 1/16*((b^2*f*x + b^2*e)*(f*x + e)^m*lo
g(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 - 4*(b^2*f*x*arctan2(1, d*x + c)^2 + b^2*
e*arctan2(1, d*x + c)^2)*(f*x + e)^m - 16*(f*m + f)*integrate(1/16*(((b^2*
c^2 + b^2)*f*m + (b^2*d^2*f*m + b^2*d^2*f)*x^2 + (b^2*c^2 + b^2)*f + 2*(b^
2*c*d*f*m + b^2*c*d*f)*x)*(f*x + e)^m*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 +
 4*(b^2*d^2*f*x^2 + b^2*c*d*e + (b^2*d^2*e + b^2*c*d*f)*x)*(f*x + e)^m*log
(d^2*x^2 + 2*c*d*x + c^2 + 1) + 4*(2*b^2*d*e*arctan2(1, d*x + c) + (3*b^2*
arctan2(1, d*x + c)^2 + (3*b^2*arctan2(1, d*x + c)^2 + 8*a*b*arctan2(1, d*
x + c))*c^2 + 8*a*b*arctan2(1, d*x + c))*f*m + ((3*b^2*arctan2(1, d*x + c)
^2 + 8*a*b*arctan2(1, d*x + c))*d^2*f*m + (3*b^2*arctan2(1, d*x + c)^2 + 8
*a*b*arctan2(1, d*x + c))*d^2*f)*x^2 + (3*b^2*arctan2(1, d*x + c)^2 + (3*b
^2*arctan2(1, d*x + c)^2 + 8*a*b*arctan2(1, d*x + c))*c^2 + 8*a*b*arctan2(
1, d*x + c))*f + 2*((3*b^2*arctan2(1, d*x + c)^2 + 8*a*b*arctan2(1, d*x +
c))*c*d*f*m + (b^2*arctan2(1, d*x + c) + (3*b^2*arctan2(1, d*x + c)^2 + 8*
a*b*arctan2(1, d*x + c))*c)*d*f)*x)*(f*x + e)^m)/((c^2 + 1)*f*m + (d^2*f*m
 + d^2*f)*x^2 + (c^2 + 1)*f + 2*(c*d*f*m + c*d*f)*x), x))/(f*m + f)
```

3.147.8 Giac [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2 dx = \int (b \operatorname{arccot}(dx+c)+a)^2(fx+e)^m dx$$

input
```
integrate((f*x+e)^m*(a+b*arccot(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((b*arccot(d*x + c) + a)^2*(f*x + e)^m, x)
```

3.147.9 Mupad [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx = \int (e+f\,x)^m\,\left(a+b\,\mathrm{acot}(c+d\,x)\right)^2 \,dx$$

input
```
int((e + f*x)^m*(a + b*acot(c + d*x))^2,x)
```

output
```
int((e + f*x)^m*(a + b*acot(c + d*x))^2, x)
```

3.148 $\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx$

3.148.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx = \text{Int}\left((e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3, x\right)$$

output
```
Unintegrable((f*x+e)^m*(a+b*arccot(d*x+c))^3,x)
```

3.148.2 Mathematica [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx = \int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx$$

input
```
Integrate[(e + f*x)^m*(a + b*ArcCot[c + d*x])^3,x]
```

output
```
Integrate[(e + f*x)^m*(a + b*ArcCot[c + d*x])^3, x]
```

3.148.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5571, 5561}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3 dx$$

$$\Big\downarrow \text{ 5571}$$

$$\frac{\int \left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^m \left(a+b\cot^{-1}(c+dx)\right)^3 d(c+dx)}{d}$$

$$\Big\downarrow \text{ 5561}$$

$$\frac{\int \left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^m \left(a+b\cot^{-1}(c+dx)\right)^3 d(c+dx)}{d}$$

input
```
Int[(e + f*x)^m*(a + b*ArcCot[c + d*x])^3,x]
```

output
```
$Aborted
```

3.148.3.1 Defintions of rubi rules used

rule 5561
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCot[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

rule 5571
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.148.4 Maple [N/A] (verified)

Not integrable

Time = 0.55 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (fx+e)^m (a+b \operatorname{arccot}(dx+c))^3 \, dx$$

input
```
int((f*x+e)^m*(a+b*arccot(d*x+c))^3,x)
```

output
```
int((f*x+e)^m*(a+b*arccot(d*x+c))^3,x)
```

3.148.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.60

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx = \int (b \operatorname{arccot}(dx+c)+a)^3 (fx+e)^m \, dx$$

input
```
integrate((f*x+e)^m*(a+b*arccot(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral((b^3*arccot(d*x + c)^3 + 3*a*b^2*arccot(d*x + c)^2 + 3*a^2*b*arcc
ot(d*x + c) + a^3)*(f*x + e)^m, x)
```

3.148.6 Sympy [F(-1)]

Timed out.

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx = \text{Timed out}$$

input
```
integrate((f*x+e)**m*(a+b*acot(d*x+c))**3,x)
```

output
```
Timed out
```

3.148.7 Maxima [N/A]

Not integrable

Time = 7.83 (sec) , antiderivative size = 880, normalized size of antiderivative = 44.00

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx = \int (b \operatorname{arccot}(dx+c)+a)^3 (fx+e)^m \, dx$$

input
```
integrate((f*x+e)^m*(a+b*arccot(d*x+c))^3,x, algorithm="maxima")
```

output
```
(f*x + e)^(m + 1)*a^3/(f*(m + 1)) - 1/32*(3*(b^3*f*x*arctan2(1, d*x + c) +
 b^3*e*arctan2(1, d*x + c))*(f*x + e)^m*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2
 - 4*(b^3*f*x*arctan2(1, d*x + c)^3 + b^3*e*arctan2(1, d*x + c)^3)*(f*x +
e)^m - 32*(f*m + f)*integrate(-1/32*(3*(b^3*d*e - (b^3*c^2*arctan2(1, d*x
+ c) + b^3*arctan2(1, d*x + c))*f*m - (b^3*d^2*f*m*arctan2(1, d*x + c) + b
^3*d^2*f*arctan2(1, d*x + c))*x^2 - (b^3*c^2*arctan2(1, d*x + c) + b^3*arc
tan2(1, d*x + c))*f - (2*b^3*c*d*f*m*arctan2(1, d*x + c) + (2*b^3*c*arctan
2(1, d*x + c) - b^3)*d*f)*x)*(f*x + e)^m*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^
2 - 12*(b^3*d^2*f*x^2*arctan2(1, d*x + c) + b^3*c*d*e*arctan2(1, d*x + c)
+ (b^3*d^2*e*arctan2(1, d*x + c) + b^3*c*d*f*arctan2(1, d*x + c))*x)*(f*x
+ e)^m*log(d^2*x^2 + 2*c*d*x + c^2 + 1) - 4*(3*b^3*d*e*arctan2(1, d*x + c)
^2 + (7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2 + 24*a^
2*b*arctan2(1, d*x + c) + (7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(
1, d*x + c)^2 + 24*a^2*b*arctan2(1, d*x + c))*c^2)*f*m + ((7*b^3*arctan2(1
, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2 + 24*a^2*b*arctan2(1, d*x +
c))*d^2*f*m + (7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^
2 + 24*a^2*b*arctan2(1, d*x + c))*d^2*f)*x^2 + (7*b^3*arctan2(1, d*x + c)^
3 + 24*a*b^2*arctan2(1, d*x + c)^2 + 24*a^2*b*arctan2(1, d*x + c) + (7*b^3
*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2 + 24*a^2*b*arctan2
(1, d*x + c))*c^2)*f + (2*(7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arcta...
```

3.148.8 Giac [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx = \int (b \operatorname{arccot}(dx+c)+a)^3 (fx+e)^m \, dx$$

input
```
integrate((f*x+e)^m*(a+b*arccot(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((b*arccot(d*x + c) + a)^3*(f*x + e)^m, x)
```

3.148.9 Mupad [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (e + fx)^m \left(a + b\cot^{-1}(c + dx)\right)^3 \, dx = \int (e + f\,x)^m \left(a + b\,\mathrm{acot}(c + d\,x)\right)^3 dx$$

input
```
int((e + f*x)^m*(a + b*acot(c + d*x))^3,x)
```

output
```
int((e + f*x)^m*(a + b*acot(c + d*x))^3, x)
```

3.149 $\int x^3 \cot^{-1}(a+bx^4)\,dx$

3.149.1 Optimal result

Integrand size = 12, antiderivative size = 42

$$\int x^3 \cot^{-1}(a+bx^4)\,dx = \frac{(a+bx^4)\cot^{-1}(a+bx^4)}{4b} + \frac{\log\left(1+(a+bx^4)^2\right)}{8b}$$

output
```
1/4*(b*x^4+a)*arccot(b*x^4+a)/b+1/8*ln(1+(b*x^4+a)^2)/b
```

3.149.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int x^3 \cot^{-1}(a+bx^4)\,dx = \frac{2(a+bx^4)\cot^{-1}(a+bx^4)+\log\left(1+(a+bx^4)^2\right)}{8b}$$

input
```
Integrate[x^3*ArcCot[a + b*x^4],x]
```

output
```
(2*(a + b*x^4)*ArcCot[a + b*x^4] + Log[1 + (a + b*x^4)^2])/(8*b)
```

3.149.3 Rubi [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {7266, 5563, 5346, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \cot^{-1}\left(a+bx^4\right) dx$$

$$\Big\downarrow \text{ 7266}$$

$$\frac{1}{4}\int \cot^{-1}\left(bx^4+a\right) dx^4$$

$$\Big\downarrow \text{ 5563}$$

$$\frac{\int \cot^{-1}\left(bx^4+a\right) d\left(bx^4+a\right)}{4b}$$

$$\Big\downarrow \text{ 5346}$$

$$\frac{\int \frac{bx^4+a}{x^8+1} d\left(bx^4+a\right)+\left(a+bx^4\right)\cot^{-1}\left(a+bx^4\right)}{4b}$$

$$\Big\downarrow \text{ 240}$$

$$\frac{\left(a+bx^4\right)\cot^{-1}\left(a+bx^4\right)+\frac{1}{2}\log\left(x^8+1\right)}{4b}$$

input
```
Int[x^3*ArcCot[a + b*x^4],x]
```

output
```
((a + b*x^4)*ArcCot[a + b*x^4] + Log[1 + x^8]/2)/(4*b)
```

3.149.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5563
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[1/d
  Subst[Int[(a + b*ArcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
 x] && IGtQ[p, 0]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.149.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{\operatorname{arccot}\left(b\,x^{4}+a\right)\left(b\,x^{4}+a\right)+\frac{\ln\left(1+\left(b\,x^{4}+a\right)^{2}\right)}{2}}{4b}$
default	$\frac{\operatorname{arccot}\left(b\,x^{4}+a\right)\left(b\,x^{4}+a\right)+\frac{\ln\left(1+\left(b\,x^{4}+a\right)^{2}\right)}{2}}{4b}$
parallelrisch	$\frac{2\,\operatorname{arccot}\left(b\,x^{4}+a\right)x^{4}b^{2}+2a\,\operatorname{arccot}\left(b\,x^{4}+a\right)b+\ln\left(b^{2}x^{8}+2ab\,x^{4}+a^{2}+1\right)b}{8b^{2}}$
parts	$\frac{x^{4}\operatorname{arccot}\left(b\,x^{4}+a\right)}{4}+b\left(\frac{\ln\left(b^{2}x^{8}+2ab\,x^{4}+a^{2}+1\right)}{8b^{2}}-\frac{a\arctan\left(\frac{2b^{2}x^{4}+2ab}{2b}\right)}{4b^{2}}\right)$
risch	$\frac{ix^{4}\ln\left(1+i\left(b\,x^{4}+a\right)\right)}{8}-\frac{ix^{4}\ln\left(1-i\left(b\,x^{4}+a\right)\right)}{8}+\frac{\pi\,x^{4}}{8}-\frac{a\arctan\left(\frac{b\,x^{4}}{a^{2}+1}+\frac{a^{2}b\,x^{4}}{a^{2}+1}+\frac{a^{3}}{a^{2}+1}+\frac{a}{a^{2}+1}\right)}{4b}+\frac{a\arctan\left(a\right)}{4b}+$

input
```
int(x^3*arccot(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/b*(arccot(b*x^4+a)*(b*x^4+a)+1/2*ln(1+(b*x^4+a)^2))
```

3.149.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int x^{3}\cot^{-1}\left(a+bx^{4}\right)\,dx = \frac{2\,bx^{4}\operatorname{arccot}\left(bx^{4}+a\right)-2\,a\arctan\left(bx^{4}+a\right)+\log\left(b^{2}x^{8}+2\,abx^{4}+a^{2}+1\right)}{8\,b}$$

input
```
integrate(x^3*arccot(b*x^4+a),x, algorithm="fricas")
```

output
```
1/8*(2*b*x^4*arccot(b*x^4 + a) - 2*a*arctan(b*x^4 + a) + log(b^2*x^8 + 2*a
*b*x^4 + a^2 + 1))/b
```

3.149.6 Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.43

$$\int x^3 \cot^{-1}\left(a+bx^4\right)\,dx = \begin{cases} \frac{a\,\mathrm{acot}\,(a+bx^4)}{4b} + \frac{x^4\,\mathrm{acot}\,(a+bx^4)}{4} + \frac{\log\,(a^2+2abx^4+b^2x^8+1)}{8b} & \text{for } b \neq 0 \\ \frac{x^4\,\mathrm{acot}\,(a)}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*acot(b*x**4+a),x)
```

output
```
Piecewise((a*acot(a + b*x**4)/(4*b) + x**4*acot(a + b*x**4)/4 + log(a**2 +
 2*a*b*x**4 + b**2*x**8 + 1)/(8*b), Ne(b, 0)), (x**4*acot(a)/4, True))
```

3.149.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int x^3 \cot^{-1}\left(a+bx^4\right)\,dx = \frac{2\,(bx^4+a)\,\mathrm{arccot}\,(bx^4+a) + \log\left((bx^4+a)^2+1\right)}{8\,b}$$

input
```
integrate(x^3*arccot(b*x^4+a),x, algorithm="maxima")
```

output
```
1/8*(2*(b*x^4 + a)*arccot(b*x^4 + a) + log((b*x^4 + a)^2 + 1))/b
```

3.149.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(38) = 76.

Time = 0.40 (sec) , antiderivative size = 127, normalized size of antiderivative = 3.02

$$\int x^3 \cot^{-1}\left(a+bx^4\right)\,dx =$$

$$-\frac{\arctan\left(\frac{1}{bx^4+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx^4+a}\right)\right)^2 + \log\left(\frac{16\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx^4+a}\right)\right)^2}{\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx^4+a}\right)\right)^4+2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx^4+a}\right)\right)^2+1}\right)\tan\left(\frac{1}{2}\,\mathrm{arct}\right.}{8\,b\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx^4+a}\right)\right)}$$

input
```
integrate(x^3*arccot(b*x^4+a),x, algorithm="giac")
```

output
```
-1/8*(arctan(1/(b*x^4 + a))*tan(1/2*arctan(1/(b*x^4 + a)))^2 + log(16*tan(
1/2*arctan(1/(b*x^4 + a)))^2/(tan(1/2*arctan(1/(b*x^4 + a)))^4 + 2*tan(1/2
*arctan(1/(b*x^4 + a)))^2 + 1))*tan(1/2*arctan(1/(b*x^4 + a))) - arctan(1/
(b*x^4 + a)))/(b*tan(1/2*arctan(1/(b*x^4 + a))))
```

3.149.9 Mupad [B] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 230, normalized size of antiderivative = 5.48

$$\int x^3 \cot^{-1}\left(a+bx^4\right) dx = \frac{\ln\left(a^2+2\,a\,b\,x^4+b^2\,x^8+1\right)}{8\,b}+\frac{x^4\,\mathrm{acot}(b\,x^4+a)}{4}$$
$$-\frac{a\,\mathrm{atan}\left(\frac{a}{a^6+3\,a^4+3\,a^2+1}+\frac{3\,a^3}{a^6+3\,a^4+3\,a^2+1}+\frac{3\,a^5}{a^6+3\,a^4+3\,a^2+1}+\frac{a^7}{a^6+3\,a^4+3\,a^2+1}+\frac{b\,x^4}{a^6+3\,a^4+3\,a^2+1}+\frac{3\,a^2\,b\,x^4}{a^6+3\,a^4+3\,a^2+1}+\right.}{4\,b}$$

input
```
int(x^3*acot(a + b*x^4),x)
```

output
```
log(a^2 + b^2*x^8 + 2*a*b*x^4 + 1)/(8*b) + (x^4*acot(a + b*x^4))/4 - (a*at
an(a/(3*a^2 + 3*a^4 + a^6 + 1) + (3*a^3)/(3*a^2 + 3*a^4 + a^6 + 1) + (3*a^
5)/(3*a^2 + 3*a^4 + a^6 + 1) + a^7/(3*a^2 + 3*a^4 + a^6 + 1) + (b*x^4)/(3*
a^2 + 3*a^4 + a^6 + 1) + (3*a^2*b*x^4)/(3*a^2 + 3*a^4 + a^6 + 1) + (3*a^4*
b*x^4)/(3*a^2 + 3*a^4 + a^6 + 1) + (a^6*b*x^4)/(3*a^2 + 3*a^4 + a^6 + 1)))
/(4*b)
```

3.150 $\int x^{-1+n} \cot^{-1}(a+bx^n)\, dx$

3.150.1 Optimal result

Integrand size = 14, antiderivative size = 45

$$\int x^{-1+n} \cot^{-1}(a+bx^n)\, dx = \frac{(a+bx^n)\cot^{-1}(a+bx^n)}{bn} + \frac{\log\left(1+(a+bx^n)^2\right)}{2bn}$$

output
```
(a+b*x^n)*arccot(a+b*x^n)/b/n+1/2*ln(1+(a+b*x^n)^2)/b/n
```

3.150.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

$$\int x^{-1+n} \cot^{-1}(a+bx^n)\, dx = \frac{2(a+bx^n)\cot^{-1}(a+bx^n) + \log\left(1+(a+bx^n)^2\right)}{2bn}$$

input
```
Integrate[x^(-1 + n)*ArcCot[a + b*x^n],x]
```

output
```
(2*(a + b*x^n)*ArcCot[a + b*x^n] + Log[1 + (a + b*x^n)^2])/(2*b*n)
```

3.150.3 Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {7266, 5563, 5346, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}\cot^{-1}(a+bx^n)\,dx$$

$$\Big\downarrow \text{ 7266}$$

$$\frac{\int \cot^{-1}(bx^n+a)\,dx^n}{n}$$

$$\Big\downarrow \text{ 5563}$$

$$\frac{\int \cot^{-1}(bx^n+a)\,d(bx^n+a)}{bn}$$

$$\Big\downarrow \text{ 5346}$$

$$\frac{\int \frac{bx^n+a}{x^{2n}+1}d(bx^n+a)+(a+bx^n)\cot^{-1}(a+bx^n)}{bn}$$

$$\Big\downarrow \text{ 240}$$

$$\frac{(a+bx^n)\cot^{-1}(a+bx^n)+\frac{1}{2}\log\left(x^{2n}+1\right)}{bn}$$

input
```
Int[x^(-1 + n)*ArcCot[a + b*x^n],x]
```

output
```
((a + b*x^n)*ArcCot[a + b*x^n] + Log[1 + x^(2*n)]/2)/(b*n)
```

3.150.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5563
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[1/d
  Subst[Int[(a + b*ArcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
 x] && IGtQ[p, 0]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.150.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.30 (sec) , antiderivative size = 149, normalized size of antiderivative = 3.31

method	result
risch	$\frac{i x^{n} \ln\left(1+i\left(a+b\,x^{n}\right)\right)}{2 n}-\frac{i x^{n} \ln\left(1-i\left(a+b\,x^{n}\right)\right)}{2 n}+\frac{\pi\,x^{n}}{2 n}+\frac{i \ln\left(x^{n}-\frac{i-a}{b}\right) a}{2 b n}-\frac{i \ln\left(\frac{i+a}{b}+x^{n}\right) a}{2 b n}+\frac{\ln\left(x^{n}-\frac{i-a}{b}\right)}{2 b n}+\frac{\ln\left(\frac{i+a}{b}+x^{n}\right)}{2 b n}$

input
```
int(x^(-1+n)*arccot(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/2*I/n*x^n*ln(1+I*(a+b*x^n))-1/2*I/n*x^n*ln(1-I*(a+b*x^n))+1/2/n*Pi*x^n+1
/2*I/b/n*ln(x^n-(I-a)/b)*a-1/2*I/b/n*ln((I+a)/b+x^n)*a+1/2/b/n*ln(x^n-(I-a
)/b)+1/2/b/n*ln((I+a)/b+x^n)
```

3.150.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.24

$$\int x^{-1+n}\cot^{-1}\left(a+bx^{n}\right)\,dx = \frac{2\,bx^{n}\operatorname{arccot}\left(bx^{n}+a\right)-2\,a\arctan\left(bx^{n}+a\right)+\log\left(b^{2}x^{2\,n}+2\,abx^{n}+a^{2}+1\right)}{2\,bn}$$

input
```
integrate(x^(-1+n)*arccot(a+b*x^n),x, algorithm="fricas")
```

output
```
1/2*(2*b*x^n*arccot(b*x^n + a) - 2*a*arctan(b*x^n + a) + log(b^2*x^(2*n) +
 2*a*b*x^n + a^2 + 1))/(b*n)
```

3.150.6 Sympy [F(-1)]

Timed out.

$$\int x^{-1+n}\cot^{-1}(a+bx^n)\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+n)*acot(a+b*x**n),x)
```

output
```
Timed out
```

3.150.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int x^{-1+n}\cot^{-1}(a+bx^n)\,dx = \frac{2\,(bx^n+a)\,\text{arccot}\,(bx^n+a)+\log\left((bx^n+a)^2+1\right)}{2\,bn}$$

input
```
integrate(x^(-1+n)*arccot(a+b*x^n),x, algorithm="maxima")
```

output
```
1/2*(2*(b*x^n + a)*arccot(b*x^n + a) + log((b*x^n + a)^2 + 1))/(b*n)
```

3.150.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.33

$$\int x^{-1+n}\cot^{-1}(a+bx^n)\,dx = \frac{b\left(\frac{2\,(bx^n+a)\arctan\left(\frac{1}{bx^n+a}\right)}{b^2}+\frac{\log\left(\frac{1}{(bx^n+a)^2}+1\right)}{b^2}-\frac{\log\left(\frac{1}{(bx^n+a)^2}\right)}{b^2}\right)}{2\,n}$$

input
```
integrate(x^(-1+n)*arccot(a+b*x^n),x, algorithm="giac")
```

output
```
1/2*b*(2*(b*x^n + a)*arctan(1/(b*x^n + a))/b^2 + log(1/(b*x^n + a)^2 + 1)/
b^2 - log((b*x^n + a)^(-2))/b^2)/n
```

3.150.9 Mupad [B] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.29

$$\int x^{-1+n} \cot^{-1}(a+bx^n)\, dx = \frac{\frac{\ln(a^2+b^2 x^{2n}+2 a b x^n+1)}{2} + a \operatorname{acot}(a+b x^n)}{b n} + \frac{x^n \operatorname{acot}(a+b x^n)}{n}$$

input
```
int(x^(n - 1)*acot(a + b*x^n),x)
```

output
```
(log(a^2 + b^2*x^(2*n) + 2*a*b*x^n + 1)/2 + a*acot(a + b*x^n))/(b*n) + (x^
n*acot(a + b*x^n))/n
```

3.151 $\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$

3.151.1 Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\text{Int}\left(\frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2},x\right)$$

output
```
Unintegrable((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

3.151.2 Mathematica [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

input
```
Integrate[(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

output
```
Integrate[(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

3.151.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7234}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^n}{1-c^2x^2} \, dx$$
$$\Big\downarrow \text{ 7234}$$
$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^n}{1-c^2x^2}dx$$

input
```
Int[(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

output
```
$Aborted
```

3.151.3.1 Defintions of rubi rules used

rule 7234
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_)/((A_) + (C_.)*(x_)^2), x_Symbol] :> Unintegrable[(a + b*F[(c*
Sqrt[d + e*x])/Sqrt[f + g*x]])^n/(A + C*x^2), x] /; FreeQ[{a, b, c, d, e, f
, g, A, C, F, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0] &&  !IGt
Q[n, 0]
```

3.151.4 Maple [N/A] (verified)

Not integrable

Time = 1.41 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+b\,\operatorname{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^n}{-c^2x^2+1}dx$$

input
```
int((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

output
```
int((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

3.151.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int -\frac{\left(b\,\mathrm{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="fricas")
```

output
```
integral(-(b*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

3.151.6 Sympy [N/A]

Not integrable

Time = 3.35 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=-\int \frac{\left(a+b\,\mathrm{acot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^n}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*acot((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**n/(-c**2*x**2+1),x)
```

output
```
-Integral((a + b*acot(sqrt(-c*x + 1)/sqrt(c*x + 1)))**n/(c**2*x**2 - 1), x
)
```

3.151.7 Maxima [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int -\frac{\left(b\operatorname{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="maxima")
```

output
```
-integrate((b*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x
)
```

3.151.8 Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int -\frac{\left(b\operatorname{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="giac")
```

output
```
integrate(-(b*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x
)
```

3.151.9 Mupad [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=-\int \frac{\left(a+b\,\mathrm{acot}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^n}{c^2\,x^2-1}\,dx$$

input
```
int(-(a + b*acot((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1),x)
```

output
```
-int((a + b*acot((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1), x)
```

3.151. $\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$

3.152 $\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

3.152.1 Optimal result

Integrand size = 40, antiderivative size = 488

$$
\begin{aligned}
\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx
&= -\frac{2\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\coth^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} \\
&\quad+\frac{3ib\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\operatorname{PolyLog}\left(2,1-\frac{2i}{i+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} \\
&\quad-\frac{3ib\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{1-cx}}{\sqrt{1+cx}\left(i+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c} \\
&\quad+\frac{3b^2\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\operatorname{PolyLog}\left(3,1-\frac{2i}{i+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} \\
&\quad-\frac{3b^2\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\operatorname{PolyLog}\left(3,1-\frac{2\sqrt{1-cx}}{\sqrt{1+cx}\left(i+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c} \\
&\quad-\frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2i}{i+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{4c}+\frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2\sqrt{1-cx}}{\sqrt{1+cx}\left(i+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{4c}
\end{aligned}
$$

output
```
-2*(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3*arccoth(1-2/(1+I*(-c*x+1)^
(1/2)/(c*x+1)^(1/2)))/c+3/2*I*b*(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))
^2*polylog(2,1-2*I/(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-3/2*I*b*(a+b*arccot
((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2*polylog(2,1-2*(-c*x+1)^(1/2)/(I+(-c*x+1)
^(1/2)/(c*x+1)^(1/2))/(c*x+1)^(1/2))/c+3/2*b^2*(a+b*arccot((-c*x+1)^(1/2)/
(c*x+1)^(1/2)))*polylog(3,1-2*I/(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-3/2*b^
2*(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polylog(3,1-2*(-c*x+1)^(1/2)/
(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/(c*x+1)^(1/2))/c-3/4*I*b^3*polylog(4,1-2*
I/(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c+3/4*I*b^3*polylog(4,1-2*(-c*x+1)^(1/
2)/(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/(c*x+1)^(1/2))/c
```

3.152.2 Mathematica [F]

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx=\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$$

input
```
Integrate[(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

output
```
Integrate[(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2), x]
```

3.152.3 Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {7232, 5358, 5524, 5528, 5532, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{1-c^2x^2}\,dx$$

$$\Big\downarrow \text{7232}$$

$$-\frac{\int \frac{\sqrt{cx+1}\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{\sqrt{1-cx}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}{c}$$

$$\Big\downarrow \text{5358}$$

$$-\frac{6b\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2\coth^{-1}\left(1-\frac{2}{\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)}{\frac{1-cx}{cx+1}+1}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+2\coth^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{c}$$

$\downarrow$ 5524

$$-\frac{6b\left(\frac{1}{2}\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2\log\left(\frac{2\sqrt{1-cx}}{\sqrt{cx+1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+i\right)}\right)}{\frac{1-cx}{cx+1}+1}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}-\frac{1}{2}\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2\log\left(\frac{2i}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+i}\right)}{\frac{1-cx}{cx+1}+1}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)+2\cot}{c}$$

$\downarrow$ 5528

$$-\frac{6b\left(\frac{1}{2}\left(-ib\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\text{PolyLog}\left(2,1-\frac{2i}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+i}\right)}{\frac{1-cx}{cx+1}+1}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}-\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2i}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+i}\right)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-}}{\sqrt{cx}}\right.\right.\right.\right.}{}$$

$\downarrow$ 5532

$$-\frac{6b\left(\frac{1}{2}\left(-ib\left(-\frac{1}{2}ib\int \frac{\text{PolyLog}\left(3,1-\frac{2i}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+i}\right)}{\frac{1-cx}{cx+1}+1}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}-\frac{1}{2}i\,\text{PolyLog}\left(3,1-\frac{2i}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+i}\right)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\right)-\frac{1}{2}\right.\right.}{}$$

$\downarrow$ 7164

$$-\frac{6b\left(\frac{1}{2}\left(-\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2i}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+i}\right)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2-ib\left(-\frac{1}{2}i\,\text{PolyLog}\left(3,1-\frac{2i}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+i}\right)\left(a+b\cot\right.\right.\right.\right.}{}$$

input
```
Int[(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

3.152. $\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

output
```
-((2*(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3*ArcCoth[1 - 2/(1 + (I*S
qrt[1 - c*x])/Sqrt[1 + c*x])] + 6*b*(((-1/2*I)*(a + b*ArcCot[Sqrt[1 - c*x]
/Sqrt[1 + c*x]])^2*PolyLog[2, 1 - (2*I)/(I + Sqrt[1 - c*x]/Sqrt[1 + c*x])]
 - I*b*((-1/2*I)*(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, 1
- (2*I)/(I + Sqrt[1 - c*x]/Sqrt[1 + c*x])] - (b*PolyLog[4, 1 - (2*I)/(I +
Sqrt[1 - c*x]/Sqrt[1 + c*x])])/4))/2 + ((I/2)*(a + b*ArcCot[Sqrt[1 - c*x]/
Sqrt[1 + c*x]])^2*PolyLog[2, 1 - (2*Sqrt[1 - c*x])/(Sqrt[1 + c*x]*(I + Sqr
t[1 - c*x]/Sqrt[1 + c*x]))] + I*b*((-1/2*I)*(a + b*ArcCot[Sqrt[1 - c*x]/Sq
rt[1 + c*x]])*PolyLog[3, 1 - (2*Sqrt[1 - c*x])/(Sqrt[1 + c*x]*(I + Sqrt[1
- c*x]/Sqrt[1 + c*x]))] - (b*PolyLog[4, 1 - (2*Sqrt[1 - c*x])/(Sqrt[1 + c*
x]*(I + Sqrt[1 - c*x]/Sqrt[1 + c*x]))])/4))/2))/c)
```

3.152.3.1 Defintions of rubi rules used

rule 5358
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcCot[c*x])^p*ArcCoth[1 - 2/(1 + I*c*x)], x] + Simp[2*b*c*p   Int[(a + b
*ArcCot[c*x])^(p - 1)*(ArcCoth[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
 FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 5524
```
Int[(ArcCoth[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[1/2   Int[Log[SimplifyIntegrand[1 + 1/u, x]]*((a +
 b*ArcCot[c*x])^p/(d + e*x^2)), x], x] - Simp[1/2   Int[Log[SimplifyIntegra
nd[1 - 1/u, x]]*((a + b*ArcCot[c*x])^p/(d + e*x^2)), x], x] /; FreeQ[{a, b,
 c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*
x)))^2, 0]
```

rule 5528
```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] + Simp[b*p*(I/2)   Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 5532
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(-I)*(a + b*ArcCot[c*x])^p*(PolyLog[k + 1, u]/
(2*c*d)), x] - Simp[b*p*(I/2)   Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[k
+ 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0]
&& EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

3.152. $\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

3.152.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1639 vs. 2(402) = 804.

Time = 1.32 (sec) , antiderivative size = 1640, normalized size of antiderivative = 3.36

method	result	size
default	Expression too large to display	1640
parts	Expression too large to display	1640

input
```
int((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x,method=_RE
TURNVERBOSE)
```

output
```
-1/2*a^3/c*ln(c*x-1)+1/2*a^3/c*ln(c*x+1)-b^3*(-1/c*arccot((-c*x+1)^(1/2)/(
c*x+1)^(1/2))^3*ln(1-(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)
^(1/2))+3*I/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,(I+(-c*x+1)
^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-6/c*arccot((-c*x+1)^(1/2
)/(c*x+1)^(1/2))*polylog(3,(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x
+1)+1)^(1/2))-6*I/c*polylog(4,(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(
c*x+1)+1)^(1/2))+1/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln(1+(I+(-c*x+
1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))-3/2*I/c*arccot((-c*x+1)^(1
/2)/(c*x+1)^(1/2))^2*polylog(2,-(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+
1)/(c*x+1)+1))+3/2/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(3,-(I+(-
c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))+3/4*I/c*polylog(4,-(I+
(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))-1/c*arccot((-c*x+1)^
(1/2)/(c*x+1)^(1/2))^3*ln(1+(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*
x+1)+1)^(1/2))+3*I/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,-(I+
(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-6/c*arccot((-c*x
+1)^(1/2)/(c*x+1)^(1/2))*polylog(3,-(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*
x+1)/(c*x+1)+1)^(1/2))-6*I/c*polylog(4,-(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/(
(-c*x+1)/(c*x+1)+1)^(1/2)))-3*a*b^2*(-1/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1
/2))^2*ln(1-(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+2
*I/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,(I+(-c*x+1)^(1/2)/(...
```

3.152.5 Fricas [F]

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx=\int -\frac{\left(b\,\mathrm{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^3}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="fricas")
```

output
```
integral(-(b^3*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1))^3 + 3*a*b^2*arccot(sqr
t(-c*x + 1)/sqrt(c*x + 1))^2 + 3*a^2*b*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1)
) + a^3)/(c^2*x^2 - 1), x)
```

3.152.6 Sympy [F]

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = -\int \frac{a^3}{c^2x^2-1}\,dx - \int \frac{b^3\operatorname{acot}^3\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx - \int \frac{3ab^2\operatorname{acot}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx - \int \frac{3a^2b\operatorname{acot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*acot((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3/(-c**2*x**2+1),x)
```

output
```
-Integral(a**3/(c**2*x**2 - 1), x) - Integral(b**3*acot(sqrt(-c*x + 1)/sqr
t(c*x + 1))**3/(c**2*x**2 - 1), x) - Integral(3*a*b**2*acot(sqrt(-c*x + 1)
/sqrt(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(3*a**2*b*acot(sqrt(-c*x
+ 1)/sqrt(c*x + 1))/(c**2*x**2 - 1), x)
```

3.152.7 Maxima [F]

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = \int -\frac{\left(b\operatorname{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^3}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="maxima")
```

output
```
1/2*a^3*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/64*(4*(b^3*log(c*x + 1) - b^
3*log(-c*x + 1))*arctan2(sqrt(c*x + 1), sqrt(-c*x + 1))^3 - 3*(b^3*log(2)^
2*log(c*x + 1) - b^3*log(2)^2*log(-c*x + 1))*arctan2(sqrt(c*x + 1), sqrt(-
c*x + 1)) + 64*c*integrate(-1/128*(112*b^3*arctan2(sqrt(c*x + 1), sqrt(-c*
x + 1))^3 + 384*a*b^2*arctan2(sqrt(c*x + 1), sqrt(-c*x + 1))^2 + 3*(b^3*lo
g(2)^2*log(c*x + 1) - b^3*log(2)^2*log(-c*x + 1) - 4*(b^3*log(c*x + 1) - b
^3*log(-c*x + 1))*arctan2(sqrt(c*x + 1), sqrt(-c*x + 1))^2)*sqrt(c*x + 1)*
sqrt(-c*x + 1) + 12*(b^3*log(2)^2 + 32*a^2*b)*arctan2(sqrt(c*x + 1), sqrt(
-c*x + 1)))/(c^2*x^2 - 1), x))/c
```

3.152.8 Giac [F]

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = \int -\frac{\left(b\,\mathrm{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^3}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="giac")
```

output
```
integrate(-(b*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^3/(c^2*x^2 - 1), x
)
```

3.152.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = \int -\frac{\left(a+b\,\mathrm{acot}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^3}{c^2\,x^2-1}\,dx$$

input
```
int(-(a + b*acot((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1),x)
```

output
```
int(-(a + b*acot((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1), x)
```

3.153 $\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

3.153.1 Optimal result

Integrand size = 40, antiderivative size = 321

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = -\frac{2\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\coth^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{ib\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\operatorname{PolyLog}\left(2,1-\frac{2i}{i+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{ib\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{1-cx}}{\sqrt{1+cx}\left(i+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2i}{i+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} - \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2\sqrt{1-cx}}{\sqrt{1+cx}\left(i+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c}$$

output
```
-2*(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2*arccoth(1-2/(1+I*(-c*x+1)^
(1/2)/(c*x+1)^(1/2)))/c+I*b*(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*pol
ylog(2,1-2*I/(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-I*b*(a+b*arccot((-c*x+1)^
(1/2)/(c*x+1)^(1/2)))*polylog(2,1-2*(-c*x+1)^(1/2)/(I+(-c*x+1)^(1/2)/(c*x+
1)^(1/2))/(c*x+1)^(1/2))/c+1/2*b^2*polylog(3,1-2*I/(I+(-c*x+1)^(1/2)/(c*x+
1)^(1/2)))/c-1/2*b^2*polylog(3,1-2*(-c*x+1)^(1/2)/(I+(-c*x+1)^(1/2)/(c*x+1
)^(1/2))/(c*x+1)^(1/2))/c
```

3.153.2 Mathematica [F]

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$$

input
```
Integrate[(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

output
```
Integrate[(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2), x]
```

3.153.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {7232, 5358, 5524, 5528, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{1-c^2x^2}\,dx$$

$$\Big\downarrow \ 7232$$

$$-\frac{\int \frac{\sqrt{cx+1}\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{\sqrt{1-cx}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}{c}$$

$$\Big\downarrow \ 5358$$

$$-\frac{4b\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\coth^{-1}\left(1-\frac{2}{\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)}{\frac{1-cx}{cx+1}+1}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+2\coth^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{c}$$

$$\Big\downarrow \ 5524$$

$$-\frac{4b\left(\frac{1}{2}\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\log\left(\frac{2\sqrt{1-cx}}{\sqrt{cx+1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+i\right)}\right)}{\frac{1-cx}{cx+1}+1}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}-\frac{1}{2}\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\log\left(\frac{2i}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+i}\right)}{\frac{1-cx}{cx+1}+1}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)+2\coth}{c}$$

$$\downarrow \text{5528}$$

$$-\frac{4b\left(\frac{1}{2}\left(-\frac{1}{2}ib\int \frac{\text{PolyLog}\left(2,1-\frac{2i}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+i}\right)}{\frac{1-cx}{cx+1}+1}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}-\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2i}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+i}\right)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\right)+\frac{1}{2}\left(\frac{1}{2}ib\right.\right.}{}$$

$$\downarrow \text{7164}$$

$$-\frac{4b\left(\frac{1}{2}\left(-\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2i}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+i}\right)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)-\frac{1}{4}b\,\text{PolyLog}\left(3,1-\frac{2i}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+i}\right)\right)+\frac{1}{2}\left(\frac{1}{2}i\,\text{PolyLo}\right.\right.}{}$$

input
```
Int[(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

output
```
-((2*(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*ArcCoth[1 - 2/(1 + (I*S
qrt[1 - c*x])/Sqrt[1 + c*x])] + 4*b*(((-1/2*I)*(a + b*ArcCot[Sqrt[1 - c*x]
/Sqrt[1 + c*x]])*PolyLog[2, 1 - (2*I)/(I + Sqrt[1 - c*x]/Sqrt[1 + c*x])] -
 (b*PolyLog[3, 1 - (2*I)/(I + Sqrt[1 - c*x]/Sqrt[1 + c*x])])/4)/2 + ((I/2)
*(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[2, 1 - (2*Sqrt[1 - c*
x])/(Sqrt[1 + c*x]*(I + Sqrt[1 - c*x]/Sqrt[1 + c*x]))] + (b*PolyLog[3, 1 -
 (2*Sqrt[1 - c*x])/(Sqrt[1 + c*x]*(I + Sqrt[1 - c*x]/Sqrt[1 + c*x]))])/4)/
2))/c)
```

3.153.3.1 Defintions of rubi rules used

rule 5358
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcCot[c*x])^p*ArcCoth[1 - 2/(1 + I*c*x)], x] + Simp[2*b*c*p   Int[(a + b
*ArcCot[c*x])^(p - 1)*(ArcCoth[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
 FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 5524
```
Int[(ArcCoth[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[1/2   Int[Log[SimplifyIntegrand[1 + 1/u, x]]*((a +
 b*ArcCot[c*x])^p/(d + e*x^2)), x], x] - Simp[1/2   Int[Log[SimplifyIntegra
nd[1 - 1/u, x]]*((a + b*ArcCot[c*x])^p/(d + e*x^2)), x], x] /; FreeQ[{a, b,
 c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*
x)))^2, 0]
```

3.153. $\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

rule 5528
```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] + Simp[b*p*(I/2)   Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

3.153.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 902 vs. $2(268) = 536$.

Time = 0.70 (sec) , antiderivative size = 903, normalized size of antiderivative = 2.81

method	result
default	$-\frac{a^2\ln(cx-1)}{2c}+\frac{a^2\ln(cx+1)}{2c}-b^2\left(-\frac{\operatorname{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2\ln\left(1-\frac{i+\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}}{\sqrt{\frac{-cx+1}{cx+1}+1}}\right)}{c}+\frac{2i\operatorname{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\operatorname{polylog}\left(2,\frac{i+\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}}{\sqrt{\frac{-cx+1}{cx+1}+1}}\right.}{c}\right.$
parts	$-\frac{a^2\ln(cx-1)}{2c}+\frac{a^2\ln(cx+1)}{2c}-b^2\left(-\frac{\operatorname{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2\ln\left(1-\frac{i+\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}}{\sqrt{\frac{-cx+1}{cx+1}+1}}\right)}{c}+\frac{2i\operatorname{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\operatorname{polylog}\left(2,\frac{i+\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}}{\sqrt{\frac{-cx+1}{cx+1}+1}}\right.}{c}\right.$

input
```
int((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x,method=_RE
TURNVERBOSE)
```

output
```
-1/2*a^2/c*ln(c*x-1)+1/2*a^2/c*ln(c*x+1)-b^2*(-1/c*arccot((-c*x+1)^(1/2)/(
c*x+1)^(1/2))^2*ln(1-(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)
^(1/2))+2*I/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,(I+(-c*x+1)^(
1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-2/c*polylog(3,(I+(-c*x+1)^
(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+1/c*arccot((-c*x+1)^(1/2)
/(c*x+1)^(1/2))^2*ln(1+(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1
)+1))-I/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-(I+(-c*x+1)^(1/2
)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))+1/2/c*polylog(3,-(I+(-c*x+1)^(1/2
)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))-1/c*arccot((-c*x+1)^(1/2)/(c*x+1)
^(1/2))^2*ln(1+(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2)
)+2*I/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-(I+(-c*x+1)^(1/2)/
(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-2/c*polylog(3,-(I+(-c*x+1)^(1/2
)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2)))-2*a*b*(-1/c*arccot((-c*x+1)^
(1/2)/(c*x+1)^(1/2))*ln(1-(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+
1)+1)^(1/2))+I/c*polylog(2,(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x
+1)+1)^(1/2))+1/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1+(I+(-c*x+1)^(1
/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))-1/2*I/c*polylog(2,-(I+(-c*x+1)^
(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))-1/c*arccot((-c*x+1)^(1/2)/(c*
x+1)^(1/2))*ln(1+(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/
2))+I/c*polylog(2,-(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1...
```

3.153.5 Fricas [F]

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=\int -\frac{\left(b\,\mathrm{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="fricas")
```

output
```
integral(-(b^2*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*b*arccot(sqrt(
-c*x + 1)/sqrt(c*x + 1)) + a^2)/(c^2*x^2 - 1), x)
```

3.153.6 Sympy [F]

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = -\int \frac{a^2}{c^2x^2-1}\,dx - \int \frac{b^2 \operatorname{acot}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx - \int \frac{2ab \operatorname{acot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*acot((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2/(-c**2*x**2+1),x)
```

output
```
-Integral(a**2/(c**2*x**2 - 1), x) - Integral(b**2*acot(sqrt(-c*x + 1)/sqr
t(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(2*a*b*acot(sqrt(-c*x + 1)/sq
rt(c*x + 1))/(c**2*x**2 - 1), x)
```

3.153.7 Maxima [F]

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = \int -\frac{\left(b\operatorname{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="maxima")
```

output
```
1/2*a^2*(log(c*x + 1)/c - log(c*x - 1)/c) - 1/32*(b^2*log(2)^2*log(c*x + 1
) - b^2*log(2)^2*log(-c*x + 1) - 4*(b^2*log(c*x + 1) - b^2*log(-c*x + 1))*
arctan2(sqrt(c*x + 1), sqrt(-c*x + 1))^2 - (b^2*(log(c*x + 1)/c - log(c*x
- 1)/c)*log(2)^2 + 64*b^2*integrate(1/16*sqrt(c*x + 1)*sqrt(-c*x + 1)*arct
an(sqrt(c*x + 1)/sqrt(-c*x + 1))*log(c*x + 1)/(c^2*x^2 - 1), x) - 64*b^2*i
ntegrate(1/16*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan(sqrt(c*x + 1)/sqrt(-c*x
+ 1))*log(-c*x + 1)/(c^2*x^2 - 1), x) - 384*b^2*integrate(1/16*arctan(sqrt
(c*x + 1)/sqrt(-c*x + 1))^2/(c^2*x^2 - 1), x) - 1024*a*b*integrate(1/16*ar
ctan(sqrt(c*x + 1)/sqrt(-c*x + 1))/(c^2*x^2 - 1), x))*c)/c
```

3.153.8 Giac [F]

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=\int -\frac{\left(b\operatorname{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="giac")
```

output
```
integrate(-(b*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2/(c^2*x^2 - 1), x
)
```

3.153.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=\int -\frac{\left(a+b\,\mathrm{acot}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2}{c^2\,x^2-1}\,dx$$

input
```
int(-(a + b*acot((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1),x)
```

output
```
int(-(a + b*acot((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1), x)
```

3.154 $\int \frac{a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$

3.154.1 Optimal result

Integrand size = 38, antiderivative size = 98

$$\int \frac{a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = -\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}+\frac{ib\,\text{PolyLog}\left(2,-\frac{i\sqrt{1+cx}}{\sqrt{1-cx}}\right)}{2c}-\frac{ib\,\text{PolyLog}\left(2,\frac{i\sqrt{1+cx}}{\sqrt{1-cx}}\right)}{2c}$$

output
```
-a*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2))/c+1/2*I*b*polylog(2,-I*(c*x+1)^(1/2)/(
-c*x+1)^(1/2))/c-1/2*I*b*polylog(2,I*(c*x+1)^(1/2)/(-c*x+1)^(1/2))/c
```

3.154.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.95

$$\int \frac{a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = -\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)-\frac{1}{2}ib\,\text{PolyLog}\left(2,-\frac{i\sqrt{1+cx}}{\sqrt{1-cx}}\right)+\frac{1}{2}ib\,\text{PolyLog}\left(2,\frac{i\sqrt{1+cx}}{\sqrt{1-cx}}\right)}{c}$$

input
```
Integrate[(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

output
```
-((a*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]] - (I/2)*b*PolyLog[2, ((-I)*Sqrt[1 +
c*x])/Sqrt[1 - c*x]] + (I/2)*b*PolyLog[2, (I*Sqrt[1 + c*x])/Sqrt[1 - c*x]]
)/c)
```

3.154.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {7232, 5356, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{1-c^2x^2}dx$$

$$\Big\downarrow \text{7232}$$

$$-\frac{\int \frac{\sqrt{cx+1}\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{\sqrt{1-cx}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}{c}$$

$$\Big\downarrow \text{5356}$$

$$-\frac{\frac{1}{2}ib\int \frac{\sqrt{cx+1}\log\left(1-\frac{i\sqrt{cx+1}}{\sqrt{1-cx}}\right)}{\sqrt{1-cx}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}-\frac{1}{2}ib\int \frac{\sqrt{cx+1}\log\left(\frac{i\sqrt{cx+1}}{\sqrt{1-cx}}+1\right)}{\sqrt{1-cx}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+a\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{c}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)-\frac{1}{2}ib\,\text{PolyLog}\left(2,-\frac{i\sqrt{cx+1}}{\sqrt{1-cx}}\right)+\frac{1}{2}ib\,\text{PolyLog}\left(2,\frac{i\sqrt{cx+1}}{\sqrt{1-cx}}\right)}{c}$$

input
```
Int[(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

output
```
-((a*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]] - (I/2)*b*PolyLog[2, ((-I)*Sqrt[1 +
c*x])/Sqrt[1 - c*x]] + (I/2)*b*PolyLog[2, (I*Sqrt[1 + c*x])/Sqrt[1 - c*x]]
)/c)
```

3.154.3.1 Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5356
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Simp[I*(b/2)   Int[Log[1 + I/(c*x)]/x, x], x] + Simp[I*(b/2)   Int[Log
[1 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

3.154.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(78) = 156$.

Time = 0.43 (sec) , antiderivative size = 363, normalized size of antiderivative = 3.70

method	result
default	$-\frac{a\ln(cx-1)}{2c}+\frac{a\ln(cx+1)}{2c}-b\left(-\frac{\operatorname{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\ln\left(1-\frac{i+\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}}{\sqrt{\frac{-cx+1}{cx+1}+1}}\right)}{c}+\frac{i\,\mathrm{polylog}\left(2,\frac{i+\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}}{\sqrt{\frac{-cx+1}{cx+1}+1}}\right)}{c}+\frac{\operatorname{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right.}{}\right.$
parts	$-\frac{a\ln(cx-1)}{2c}+\frac{a\ln(cx+1)}{2c}-b\left(-\frac{\operatorname{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\ln\left(1-\frac{i+\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}}{\sqrt{\frac{-cx+1}{cx+1}+1}}\right)}{c}+\frac{i\,\mathrm{polylog}\left(2,\frac{i+\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}}{\sqrt{\frac{-cx+1}{cx+1}+1}}\right)}{c}+\frac{\operatorname{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right.}{}\right.$

input
```
int((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x,method=_RETU
RNVERBOSE)
```

output
```
-1/2*a/c*ln(c*x-1)+1/2*a/c*ln(c*x+1)-b*(-1/c*arccot((-c*x+1)^(1/2)/(c*x+1)
^(1/2))*ln(1-(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+
I/c*polylog(2,(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))
+1/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1+(I+(-c*x+1)^(1/2)/(c*x+1)^(
1/2))^2/((-c*x+1)/(c*x+1)+1))-1/2*I/c*polylog(2,-(I+(-c*x+1)^(1/2)/(c*x+1)
^(1/2))^2/((-c*x+1)/(c*x+1)+1))-1/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))*l
n(1+(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+I/c*polyl
og(2,-(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2)))
```

3.154.5 Fricas [F]

$$\int \frac{a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=\int -\frac{b\,\mathrm{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="fricas")
```

output
```
integral(-(b*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

3.154.6 Sympy [F]

$$\int \frac{a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=-\int \frac{a}{c^2x^2-1}\,dx-\int \frac{b\,\mathrm{acot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*acot((-c*x+1)**(1/2)/(c*x+1)**(1/2)))/(-c**2*x**2+1),x)
```

output
```
-Integral(a/(c**2*x**2 - 1), x) - Integral(b*acot(sqrt(-c*x + 1)/sqrt(c*x
+ 1))/(c**2*x**2 - 1), x)
```

3.154.7 Maxima [F]

$$\int \frac{a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=\int -\frac{b\,\mathrm{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="maxima")
```

output
```
1/2*a*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/2*((log(c*x + 1) - log(-c*x +
1))*arctan2(sqrt(c*x + 1), sqrt(-c*x + 1)) + 2*c*integrate(1/2*(e^(1/2*log
(c*x + 1) + 1/2*log(-c*x + 1))*log(c*x + 1) - e^(1/2*log(c*x + 1) + 1/2*lo
g(-c*x + 1))*log(-c*x + 1))/((c^2*x^2 - 1)*(c*x + 1) - (c^2*x^2 - 1)*(c*x
- 1)), x))*b/c
```

3.154.8 Giac [F]

$$\int \frac{a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=\int -\frac{b\,\mathrm{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="giac")
```

output
```
integrate(-(b*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

3.154.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=\int -\frac{a+b\,\mathrm{acot}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)}{c^2\,x^2-1}\,dx$$

input
```
int(-(a + b*acot((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1),x)
```

output
```
int(-(a + b*acot((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1), x)
```

3.155 $\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$

3.155.1 Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx=\operatorname{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)},x\right)$$

output
```
Unintegrable(1/(-c^2*x^2+1)/(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

3.155.2 Mathematica [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx=\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

input
```
Integrate[1/((1 - c^2*x^2)*(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

output
```
Integrate[1/((1 - c^2*x^2)*(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])), x
]
```

3.155.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7234}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}dx$$

$$\Big\downarrow \text{7234}$$

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}dx$$

input
```
Int[1/((1 - c^2*x^2)*(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

output
```
$Aborted
```

3.155.3.1 Defintions of rubi rules used

rule 7234
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_)/((A_) + (C_.)*(x_)^2), x_Symbol] :> Unintegrable[(a + b*F[(c*
Sqrt[d + e*x])/Sqrt[f + g*x]])^n/(A + C*x^2), x] /; FreeQ[{a, b, c, d, e, f
, g, A, C, F, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0] &&  !IGt
Q[n, 0]
```

3.155.4 Maple [N/A] (verified)

Not integrable

Time = 0.77 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{1}{(-c^2x^2+1)\left(a+b\,\mathrm{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)}dx$$

input
```
int(1/(-c^2*x^2+1)/(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

output
```
int(1/(-c^2*x^2+1)/(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

3.155.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx=\int -\frac{1}{(c^2x^2-1)\left(b\,\mathrm{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="fricas")
```

output
```
integral(-1/(a*c^2*x^2 + (b*c^2*x^2 - b)*arccot(sqrt(-c*x + 1)/sqrt(c*x +
1)) - a), x)
```

3.155.6 Sympy [N/A]

Not integrable

Time = 2.83 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.52

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$
$$=-\int \frac{1}{ac^2x^2-a+bc^2x^2\,\mathrm{acot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-b\,\mathrm{acot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}\,dx$$

input
```
integrate(1/(-c**2*x**2+1)/(a+b*acot((-c*x+1)**(1/2)/(c*x+1)**(1/2))),x)
```

output
```
-Integral(1/(a*c**2*x**2 - a + b*c**2*x**2*acot(sqrt(-c*x + 1)/sqrt(c*x +
1)) - b*acot(sqrt(-c*x + 1)/sqrt(c*x + 1))), x)
```

3.155.7 Maxima [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\,\mathrm{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="maxima")
```

output
```
-integrate(1/((c^2*x^2 - 1)*(b*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
 x)
```

3.155.8 Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\,\mathrm{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="giac")
```

output
```
integrate(-1/((c^2*x^2 - 1)*(b*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
 x)
```

3.155.9 Mupad [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = -\int \frac{1}{\left(a+b\,\mathrm{acot}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)\,(c^2\,x^2-1)}\,dx$$

input
```
int(-1/((a + b*acot((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)),x)
```

output
```
-int(1/((a + b*acot((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)), x)
```

3.155. $\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$

3.156 $\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$

3.156.1 Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \text{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2},x\right)$$

output
```
Unintegrable(1/(-c^2*x^2+1)/(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x
)
```

3.156.2 Mathematica [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

input
```
Integrate[1/((1 - c^2*x^2)*(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
x]
```

output
```
Integrate[1/((1 - c^2*x^2)*(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
 x]
```

3.156.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7234}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}dx$$

$$\Big\downarrow \text{7234}$$

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}dx$$

input
```
Int[1/((1 - c^2*x^2)*(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x]
```

output
```
$Aborted
```

3.156.3.1 Defintions of rubi rules used

rule 7234
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_)/((A_) + (C_.)*(x_)^2), x_Symbol] :> Unintegrable[(a + b*F[(c*
Sqrt[d + e*x])/Sqrt[f + g*x]])^n/(A + C*x^2), x] /; FreeQ[{a, b, c, d, e, f
, g, A, C, F, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0] &&  !IGt
Q[n, 0]
```

3.156.4 Maple [N/A] (verified)

Not integrable

Time = 0.91 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{1}{(-c^2x^2+1)\left(a+b\,\mathrm{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^2}dx$$

input
```
int(1/(-c^2*x^2+1)/(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

output
```
int(1/(-c^2*x^2+1)/(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

3.156.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.28

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\,\mathrm{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="fricas")
```

output
```
integral(-1/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccot(sqrt(-c*x + 1)/sqrt(
c*x + 1))^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccot(sqrt(-c*x + 1)/sqrt(c*x +
 1))), x)
```

3.156.6 Sympy [N/A]

Not integrable

Time = 6.40 (sec) , antiderivative size = 126, normalized size of antiderivative = 3.15

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx =$$
$$-\int \frac{1}{a^2c^2x^2-a^2+2abc^2x^2\,\mathrm{acot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-2ab\,\mathrm{acot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+b^2c^2x^2\,\mathrm{acot}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-b^2\,\mathrm{acot}^2\left(\frac{\sqrt{-}}{\sqrt{c}}\right.}$$

input
```
integrate(1/(-c**2*x**2+1)/(a+b*acot((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2,x
)
```

output
```
-Integral(1/(a**2*c**2*x**2 - a**2 + 2*a*b*c**2*x**2*acot(sqrt(-c*x + 1)/s
qrt(c*x + 1)) - 2*a*b*acot(sqrt(-c*x + 1)/sqrt(c*x + 1)) + b**2*c**2*x**2*
acot(sqrt(-c*x + 1)/sqrt(c*x + 1))**2 - b**2*acot(sqrt(-c*x + 1)/sqrt(c*x
+ 1))**2), x)
```

3.156.7 Maxima [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 163, normalized size of antiderivative = 4.08

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\,\text{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="maxima")
```

output
```
-2*(2*(b^2*c^2*arctan2(sqrt(c*x + 1), sqrt(-c*x + 1)) + a*b*c^2)*sqrt(c*x
+ 1)*sqrt(-c*x + 1)*integrate(1/2*x/((a*b*c^2*x^2 - a*b + (b^2*c^2*x^2 - b
^2)*arctan2(sqrt(c*x + 1), sqrt(-c*x + 1)))*sqrt(c*x + 1)*sqrt(-c*x + 1)),
 x) + 1)/((b^2*c*arctan2(sqrt(c*x + 1), sqrt(-c*x + 1)) + a*b*c)*sqrt(c*x
+ 1)*sqrt(-c*x + 1))
```

3.156.8 Giac [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\,\text{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="giac")
```

output
```
integrate(-1/((c^2*x^2 - 1)*(b*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2
), x)
```

3.156.9 Mupad [N/A]

Not integrable

Time = 1.69 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = -\int \frac{1}{\left(a+b\,\mathrm{acot}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2\,(c^2\,x^2-1)}\,dx$$

input
```
int(-1/((a + b*acot((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)),x)
```

output
```
-int(1/((a + b*acot((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)), x)
```

3.157 $\int \cot^{-1}(\tan(a+bx))\,dx$

3.157.1 Optimal result

Integrand size = 7, antiderivative size = 16

$$\int \cot^{-1}(\tan(a+bx))\,dx = -\frac{\cot^{-1}(\tan(a+bx))^2}{2b}$$

output
```
-1/2*(1/2*Pi-arctan(tan(b*x+a)))^2/b
```

3.157.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \cot^{-1}(\tan(a+bx))\,dx = \frac{bx^2}{2} + x\cot^{-1}(\tan(a+bx))$$

input
```
Integrate[ArcCot[Tan[a + b*x]],x]
```

output
```
(b*x^2)/2 + x*ArcCot[Tan[a + b*x]]
```

3.157.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}(\tan(a+bx))\,dx$$

$$\Big\downarrow \ 2588$$

$$-\frac{\int \cot^{-1}(\tan(a+bx))d\cot^{-1}(\tan(a+bx))}{b}$$

$$\Big\downarrow \ 15$$

$$-\frac{\cot^{-1}(\tan(a+bx))^2}{2b}$$

input
```
Int[ArcCot[Tan[a + b*x]],x]
```

output
```
-1/2*ArcCot[Tan[a + b*x]]^2/b
```

3.157.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

3.157.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

method	result
default	$\frac{\pi x}{2}-\frac{\arctan\left(\tan\left(bx+a\right)\right)^{2}}{2b}$
parts	$\frac{\pi x}{2}-\frac{\arctan\left(\tan\left(bx+a\right)\right)^{2}}{2b}$
parallelrisch	$\frac{x^{2}b}{2}-x\arctan\left(\tan\left(bx+a\right)\right)+\frac{\pi x}{2}$
derivativedivides	$\frac{\pi\arctan\left(\tan\left(bx+a\right)\right)-\arctan\left(\tan\left(bx+a\right)\right)^{2}}{2b}$
risch	$\frac{\pi x}{2}+ix\ln\left(\mathrm{e}^{i(bx+a)}\right)+\frac{\pi x\,\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2i(bx+a)}+1}\right)\mathrm{csgn}\left(i\mathrm{e}^{2i(bx+a)}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2i(bx+a)}}{\mathrm{e}^{2i(bx+a)}+1}\right)}{4}-\frac{\pi x\,\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2i(bx+a)}+1}\right.}{}$

input
```
int(1/2*Pi-arctan(tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2*Pi*x-1/2*arctan(tan(b*x+a))^2/b
```

3.157.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \cot^{-1}(\tan(a+bx))\,dx = -\frac{1}{2}\,bx^2+\frac{1}{2}\,(\pi-2\,a)x$$

input
```
integrate(1/2*pi-arctan(tan(b*x+a)),x, algorithm="fricas")
```

output
```
-1/2*b*x^2 + 1/2*(pi - 2*a)*x
```

3.157.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(17) = 34$.

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 3.00

$$\int \cot^{-1}(\tan(a+bx))\,dx = \frac{\pi x}{2}-\begin{cases}\frac{\left(\operatorname{atan}\left(\tan\left(a+bx\right)\right)+\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)^2}{2b} & \text{for } b\neq 0\\ x\left(\operatorname{atan}\left(\tan\left(a\right)\right)+\pi\left\lfloor\frac{a-\frac{\pi}{2}}{\pi}\right\rfloor\right) & \text{otherwise}\end{cases}$$

input
```
integrate(1/2*pi-atan(tan(b*x+a)),x)
```

output
```
pi*x/2 - Piecewise(((atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))**
2/(2*b), Ne(b, 0)), (x*(atan(tan(a)) + pi*floor((a - pi/2)/pi)), True))
```

3.157.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \cot^{-1}(\tan(a+bx))\,dx = \frac{1}{2}\,\pi x - \frac{(bx+a)^2}{2\,b}$$

input
```
integrate(1/2*pi-arctan(tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/2*pi*x - 1/2*(b*x + a)^2/b
```

3.157.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

$$\int \cot^{-1}(\tan(a+bx))\,dx = -\frac{1}{2}\,bx^2 + \pi x\left\lfloor \frac{bx+a}{\pi} + \frac{1}{2}\right\rfloor + \frac{1}{2}\,\pi x - ax$$

input
```
integrate(1/2*pi-arctan(tan(b*x+a)),x, algorithm="giac")
```

output
```
-1/2*b*x^2 + pi*x*floor((b*x + a)/pi + 1/2) + 1/2*pi*x - a*x
```

3.157.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int \cot^{-1}(\tan(a+bx))\,dx = \frac{\Pi\,x}{2} + \frac{b\,x^2}{2} - x\,\mathrm{atan}(\tan(a+b\,x))$$

input
```
int(Pi/2 - atan(tan(a + b*x)),x)
```

output
```
(Pi*x)/2 + (b*x^2)/2 - x*atan(tan(a + b*x))
```

3.158 $\int x^2 \cot^{-1}(c + d\tan(a + bx))\,dx$

3.158.1 Optimal result

Integrand size = 15, antiderivative size = 403

$$\begin{aligned}\int x^2\cot^{-1}(c+d\tan(a+bx))\,dx &= \frac{1}{3}x^3\cot^{-1}(c+d\tan(a+bx)) \\
&\quad -\frac{1}{6}ix^3\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) \\
&\quad +\frac{1}{6}ix^3\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right) \\
&\quad -\frac{x^2\operatorname{PolyLog}\left(2,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{4b} \\
&\quad +\frac{x^2\operatorname{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{4b} \\
&\quad -\frac{ix\operatorname{PolyLog}\left(3,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{4b^2} \\
&\quad +\frac{ix\operatorname{PolyLog}\left(3,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{4b^2} \\
&\quad +\frac{\operatorname{PolyLog}\left(4,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{8b^3} \\
&\quad -\frac{\operatorname{PolyLog}\left(4,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{8b^3}\end{aligned}$$

output
```
1/3*x^3*arccot(c+d*tan(b*x+a))-1/6*I*x^3*ln(1+(1+I*c+d)*exp(2*I*a+2*I*b*x)
/(1+I*c-d))+1/6*I*x^3*ln(1+(c+I*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))-1/4
*x^2*polylog(2,-(1+I*c+d)*exp(2*I*a+2*I*b*x)/(1+I*c-d))/b+1/4*x^2*polylog(
2,-(c+I*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))/b-1/4*I*x*polylog(3,-(1+I*c
+d)*exp(2*I*a+2*I*b*x)/(1+I*c-d))/b^2+1/4*I*x*polylog(3,-(c+I*(1-d))*exp(2
*I*a+2*I*b*x)/(c+I*(1+d)))/b^2+1/8*polylog(4,-(1+I*c+d)*exp(2*I*a+2*I*b*x)
/(1+I*c-d))/b^3-1/8*polylog(4,-(c+I*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))
/b^3
```

3.158.2 Mathematica [A] (verified)

Time = 2.46 (sec) , antiderivative size = 371, normalized size of antiderivative = 0.92

$$\int x^2 \cot^{-1}(c + d\tan(a + bx))\,dx$$
$$= \frac{8b^3x^3\cot^{-1}(c + d\tan(a + bx)) - 4ib^3x^3\log\left(1 + \frac{(c+i(-1+d))e^{-2i(a+bx)}}{c-i(1+d)}\right) + 4ib^3x^3\log\left(1 + \frac{(c+i(1+d))e^{-2i(a+bx)}}{i+c-id}\right)}{}$$

input
```
Integrate[x^2*ArcCot[c + d*Tan[a + b*x]],x]
```

output
```
(8*b^3*x^3*ArcCot[c + d*Tan[a + b*x]] - (4*I)*b^3*x^3*Log[1 + (c + I*(-1 +
 d))/((c - I*(1 + d))*E^((2*I)*(a + b*x)))] + (4*I)*b^3*x^3*Log[1 + (c + I
*(1 + d))/((I + c - I*d)*E^((2*I)*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, (-c
- I*(1 + d))/((c - I*(-1 + d))*E^((2*I)*(a + b*x)))] + 6*b^2*x^2*PolyLog[2
, (I - c - I*d)/((c - I*(1 + d))*E^((2*I)*(a + b*x)))] + (6*I)*b*x*PolyLog
[3, (-c - I*(1 + d))/((I + c - I*d)*E^((2*I)*(a + b*x)))] - (6*I)*b*x*Poly
Log[3, (I - c - I*d)/((c - I*(1 + d))*E^((2*I)*(a + b*x)))] + 3*PolyLog[4,
 (-c - I*(1 + d))/((I + c - I*d)*E^((2*I)*(a + b*x)))] - 3*PolyLog[4, (I -
 c - I*d)/((c - I*(1 + d))*E^((2*I)*(a + b*x)))])/(24*b^3)
```

3.158.3 Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5699, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cot^{-1}(d\tan(a+bx)+c)\,dx$$

$$\Big\downarrow \text{5699}$$

$$-\frac{1}{3}b(-ic-d+1)\int \frac{e^{2ia+2ibx}x^3}{-ic+(-ic-d+1)e^{2ia+2ibx}+d+1}dx+\frac{1}{3}b(ic+d+1)\int \frac{e^{2ia+2ibx}x^3}{ic+(ic+d+1)e^{2ia+2ibx}-d+1}dx+\frac{1}{3}x^3\cot^{-1}(d\tan(a+bx)+c)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{3}b(ic+d+1)\left(\frac{3\int x^2\log\left(\frac{e^{2ia+2ibx}(ic+d+1)}{ic-d+1}+1\right)dx}{2b(c-i(d+1))}-\frac{x^3\log\left(1+\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{2b(c-i(d+1))}\right)-\frac{1}{3}b(-ic-d+1)\left(\frac{x^3\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{2b(c+i(1-d))}-\frac{3\int x^2\log\left(\frac{e^{2ia+2ibx}(c+i(1-d))}{c+i(d+1)}+1\right)dx}{2b(c+i(1-d))}\right)+\frac{1}{3}x^3\cot^{-1}(d\tan(a+bx)+c)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{3}b(ic+d+1)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{2b}-\frac{i\int x\operatorname{PolyLog}\left(2,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)dx}{b}\right)}{2b(c-i(d+1))}-\frac{x^3\log\left(1+\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{2b(c-i(d+1))}\right)-\frac{1}{3}b(-ic-d+1)\left(\frac{x^3\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{2b(c+i(1-d))}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{2b}-\frac{i\int x\operatorname{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)dx}{b}\right)}{2b(c+i(1-d))}\right)+\frac{1}{3}x^3\cot^{-1}(d\tan(a+bx)+c)$$

$$\Big\downarrow \text{7163}$$

3.158. $\int x^2\cot^{-1}(c+d\tan(a+bx))\,dx$

$$\frac{1}{3}b(ic+d+$$

$$1)\left(\frac{3\left(\frac{ix^2\,\mathrm{PolyLog}\left(2,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{2b}-\frac{i\left(\frac{i\int \mathrm{PolyLog}\left(3,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)dx}{2b}-\frac{ix\,\mathrm{PolyLog}\left(3,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{2b}\right)}{b}\right)}{2b(c-i(d+1))}-\frac{x^3\log\right.$$

$$\frac{1}{3}b(-ic-d+$$

$$1)\left(\frac{x^3\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{2b(c+i(1-d))}-\frac{3\left(\frac{ix^2\,\mathrm{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{2b}-\frac{i\left(\frac{i\int \mathrm{PolyLog}\left(3,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)dx}{2b}-\frac{ix\,\mathrm{Pol}}{}\right.}{b}\right.}{2b(c+i(1-d))}\right.$$

$$\frac{1}{3}x^3\cot^{-1}(d\tan(a+bx)+c)$$

$$\Big\downarrow \ 2720$$

3.158. $\int x^2\cot^{-1}(c+d\tan(a+bx))\,dx$

$$\frac{1}{3}b(ic+d+$$

$$1)\left(\frac{3\left(\frac{ix^2\,\text{PolyLog}\left(2,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\,\text{PolyLog}\left(3,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)de^{2ia+2ibx}}{4b^2}-\frac{ix\,\text{PolyLog}\left(3,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right.}{2b}\right.}{b}\right.}{2b(c-i(d+1))}\right.$$

$$\frac{1}{3}b(-ic-d+$$

$$1)\left(\frac{x^3\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{2b(c+i(1-d))}-\frac{3\left(\frac{ix^2\,\text{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\,\text{PolyLog}\left(3,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{4b^2}\right.}{}\right.}{2b(c+i(1-d))}\right.$$

$$\frac{1}{3}x^3\cot^{-1}(d\tan(a+bx)+c)$$

$$\Big\downarrow \; 7143$$

3.158. $\int x^2\cot^{-1}(c+d\tan(a+bx))\,dx$

$$\frac{1}{3}b(ic+d+1)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{2b}-\frac{i\left(\frac{\operatorname{PolyLog}\left(4,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{4b^2}-\frac{ix\operatorname{PolyLog}\left(3,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{2b}\right)}{b}\right)}{2b(c-i(d+1))}-\frac{x^3\log\left(1\right.}{2b(}\right.$$

$$\frac{1}{3}b(-ic-d+1)\left(\frac{x^3\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{2b(c+i(1-d))}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{2b}-\frac{i\left(\frac{\operatorname{PolyLog}\left(4,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{4b^2}-\frac{ix\operatorname{PolyLog}}{}\right.}{b}\right.}{2b(c+i(1-d))}\right.$$

$$\frac{1}{3}x^3\cot^{-1}(d\tan(a+bx)+c)$$

input
```
Int[x^2*ArcCot[c + d*Tan[a + b*x]],x]
```

output
```
(x^3*ArcCot[c + d*Tan[a + b*x]])/3 + (b*(1 + I*c + d)*(-1/2*(x^3*Log[1 + (
(1 + I*c + d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c - d)])/(b*(c - I*(1 + d)))
 + (3*(((I/2)*x^2*PolyLog[2, -(((1 + I*c + d)*E^((2*I)*a + (2*I)*b*x))/(1
+ I*c - d))])/b - (I*(((-1/2*I)*x*PolyLog[3, -(((1 + I*c + d)*E^((2*I)*a +
 (2*I)*b*x))/(1 + I*c - d))])/b + PolyLog[4, -(((1 + I*c + d)*E^((2*I)*a +
 (2*I)*b*x))/(1 + I*c - d))]/(4*b^2)))/b))/(2*b*(c - I*(1 + d)))))/3 - (b*
(1 - I*c - d)*((x^3*Log[1 + ((c + I*(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c +
 I*(1 + d))])/(2*b*(c + I*(1 - d))) - (3*(((I/2)*x^2*PolyLog[2, -(((c + I*
(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 + d)))])/b - (I*(((-1/2*I)*x*P
olyLog[3, -(((c + I*(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 + d)))])/b
 + PolyLog[4, -(((c + I*(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 + d)))
]/(4*b^2)))/b))/(2*b*(c + I*(1 - d)))))/3
```

3.158. $\int x^2\cot^{-1}(c+d\tan(a+bx))\,dx$

3.158.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5699
```
Int[ArcCot[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Tan[a + b*x]]/(f*(m +
1))), x] + (-Simp[b*((1 - I*c - d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^
(2*I*a + 2*I*b*x)/(1 - I*c + d + (1 - I*c - d)*E^(2*I*a + 2*I*b*x))), x], x
] + Simp[b*((1 + I*c + d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*I*a +
2*I*b*x)/(1 + I*c - d + (1 + I*c + d)*E^(2*I*a + 2*I*b*x))), x], x]) /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c + I*d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.158. $\int x^2 \cot^{-1}(c + d\tan(a + bx))\, dx$

3.158.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 43.73 (sec) , antiderivative size = 8038, normalized size of antiderivative = 19.95

method	result	size
risch	Expression too large to display	8038

input
```
int(x^2*arccot(c+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.158.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1965 vs. 2(290) = 580.

Time = 0.32 (sec) , antiderivative size = 1965, normalized size of antiderivative = 4.88

$$\int x^2 \cot^{-1}(c + d\tan(a + bx))\, dx = \text{Too large to display}$$

input
```
integrate(x^2*arccot(c+d*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/48*(16*b^3*x^3*arccot(d*tan(b*x + a) + c) - 6*b^2*x^2*dilog(2*((I*c*d -
d^2 + d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*
x + a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d +
1) + 1) + 6*b^2*x^2*dilog(2*((I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 - I*c*
d + (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d +
1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1) + 1) - 6*b^2*x^2*dilog(2*((-I*c*d
 - d^2 + d)*tan(b*x + a)^2 - c^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*ta
n(b*x + a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*
d + 1) + 1) + 6*b^2*x^2*dilog(2*((-I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 +
 I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*tan(b*x + a) - d - 1)/((c^2 + d^2 +
2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1) + 1) - 4*I*a^3*log(((I*c*d
+ d^2 + d)*tan(b*x + a)^2 - c^2 + I*c*d + (I*c^2 + I*d^2 + 2*I*d + I)*tan(
b*x + a) - d - 1)/(tan(b*x + a)^2 + 1)) + 4*I*a^3*log(((I*c*d + d^2 - d)*t
an(b*x + a)^2 - c^2 + I*c*d + (I*c^2 + I*d^2 - 2*I*d + I)*tan(b*x + a) + d
 - 1)/(tan(b*x + a)^2 + 1)) - 4*I*a^3*log(((I*c*d - d^2 + d)*tan(b*x + a)^
2 + c^2 + I*c*d + (I*c^2 + I*d^2 - 2*I*d + I)*tan(b*x + a) - d + 1)/(tan(b
*x + a)^2 + 1)) + 4*I*a^3*log(((I*c*d - d^2 - d)*tan(b*x + a)^2 + c^2 + I*
c*d + (I*c^2 + I*d^2 + 2*I*d + I)*tan(b*x + a) + d + 1)/(tan(b*x + a)^2 +
1)) - 6*I*b*x*polylog(3, ((c^2 + 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2 -
 2*I*c*d + d^2 - 2*(-I*c^2 + 2*c*d + I*d^2 - I)*tan(b*x + a) - 1)/((c^2...
```

3.158.6 Sympy [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(c + d\tan(a + bx))\, dx = \text{Timed out}$$

input
```
integrate(x**2*acot(c+d*tan(b*x+a)),x)
```

output
```
Timed out
```

3.158.7 Maxima [F]

$$\int x^2 \cot^{-1}(c + d\tan(a + bx))\,dx = \int x^2 \operatorname{arccot}(d\tan(bx + a) + c)\,dx$$

input
```
integrate(x^2*arccot(c+d*tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/6*x^3*arctan2(-(d + 1)*cos(2*b*x + 2*a) + c*sin(2*b*x + 2*a) + d - 1, -c
*cos(2*b*x + 2*a) - (d + 1)*sin(2*b*x + 2*a) - c) - 1/6*x^3*arctan2(-(d -
1)*cos(2*b*x + 2*a) + c*sin(2*b*x + 2*a) + d + 1, -c*cos(2*b*x + 2*a) - (d
 - 1)*sin(2*b*x + 2*a) - c) - 4*b*d*integrate(-1/3*(2*(c^2 + d^2 + 1)*x^3*
cos(2*b*x + 2*a)^2 + 2*c*d*x^3*sin(2*b*x + 2*a) + 2*(c^2 + d^2 + 1)*x^3*si
n(2*b*x + 2*a)^2 + (c^2 - d^2 + 1)*x^3*cos(2*b*x + 2*a) - (2*c*d*x^3*sin(2
*b*x + 2*a) - (c^2 - d^2 + 1)*x^3*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*
c*d*x^3*cos(2*b*x + 2*a) + (c^2 - d^2 + 1)*x^3*sin(2*b*x + 2*a))*sin(4*b*x
 + 4*a))/(c^4 + d^4 + 2*(c^2 - 1)*d^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c
^2 + 1)*cos(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*c
os(2*b*x + 2*a)^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*sin(4*b*x +
4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*sin(2*b*x + 2*a)^2 +
2*c^2 + 2*(c^4 + d^4 - 2*(3*c^2 + 1)*d^2 + 2*c^2 + 2*(c^4 - d^4 + 2*c^2 +
1)*cos(2*b*x + 2*a) - 4*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*
b*x + 4*a) + 4*(c^4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(2*c*d^3 - 2*(
c^3 + c)*d - 2*(c*d^3 + (c^3 + c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 + 2*c^2
 + 1)*sin(2*b*x + 2*a))*sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 + c)*d)*sin(2*b
*x + 2*a) + 1), x)
```

3.158.8 Giac [F]

$$\int x^2 \cot^{-1}(c + d\tan(a + bx))\,dx = \int x^2 \operatorname{arccot}(d\tan(bx + a) + c)\,dx$$

input
```
integrate(x^2*arccot(c+d*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arccot(d*tan(b*x + a) + c), x)
```

3.158.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(c + d\tan(a + bx))\,dx = \int x^2 \operatorname{acot}(c + d\tan(a + b\,x))\,dx$$

input
```
int(x^2*acot(c + d*tan(a + b*x)),x)
```

output
```
int(x^2*acot(c + d*tan(a + b*x)), x)
```

3.159 $\int x \cot^{-1}(c + d \tan(a + bx)) \, dx$

3.159.1 Optimal result

Integrand size = 13, antiderivative size = 305

$$\int x \cot^{-1}(c + d \tan(a + bx)) \, dx = \frac{1}{2} x^2 \cot^{-1}(c + d \tan(a + bx)) - \frac{1}{4} i x^2 \log \left(1 + \frac{(1 + ic + d) e^{2ia+2ibx}}{1 + ic - d}\right) + \frac{1}{4} i x^2 \log \left(1 + \frac{(c + i(1 - d)) e^{2ia+2ibx}}{c + i(1 + d)}\right) - \frac{x \operatorname{PolyLog}\left(2, -\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{4b} + \frac{x \operatorname{PolyLog}\left(2, -\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{4b} - \frac{i \operatorname{PolyLog}\left(3, -\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{8b^2} + \frac{i \operatorname{PolyLog}\left(3, -\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{8b^2}$$

output
```
1/2*x^2*arccot(c+d*tan(b*x+a))-1/4*I*x^2*ln(1+(1+I*c+d)*exp(2*I*a+2*I*b*x)
/(1+I*c-d))+1/4*I*x^2*ln(1+(c+I*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))-1/4
*x*polylog(2,-(1+I*c+d)*exp(2*I*a+2*I*b*x)/(1+I*c-d))/b+1/4*x*polylog(2,-(
c+I*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))/b-1/8*I*polylog(3,-(1+I*c+d)*ex
p(2*I*a+2*I*b*x)/(1+I*c-d))/b^2+1/8*I*polylog(3,-(c+I*(1-d))*exp(2*I*a+2*I
*b*x)/(c+I*(1+d)))/b^2
```

3.159.2 Mathematica [A] (verified)

Time = 2.24 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.92

$$\int x\cot^{-1}(c+d\tan(a+bx))\,dx$$
$$=\frac{4b^2x^2\cot^{-1}(c+d\tan(a+bx))-2ib^2x^2\log\left(1+\frac{(c+i(-1+d))e^{-2i(a+bx)}}{c-i(1+d)}\right)+2ib^2x^2\log\left(1+\frac{(c+i(1+d))e^{-2i(a+bx)}}{i+c-id}\right)}{}$$

input
```
Integrate[x*ArcCot[c + d*Tan[a + b*x]],x]
```

output
```
(4*b^2*x^2*ArcCot[c + d*Tan[a + b*x]] - (2*I)*b^2*x^2*Log[1 + (c + I*(-1 +
 d))/((c - I*(1 + d))*E^((2*I)*(a + b*x)))] + (2*I)*b^2*x^2*Log[1 + (c + I
*(1 + d))/((I + c - I*d)*E^((2*I)*(a + b*x)))] - 2*b*x*PolyLog[2, (-c - I*
(1 + d))/((c - I*(-1 + d))*E^((2*I)*(a + b*x)))] + 2*b*x*PolyLog[2, (I - c
 - I*d)/((c - I*(1 + d))*E^((2*I)*(a + b*x)))] + I*PolyLog[3, (-c - I*(1 +
 d))/((I + c - I*d)*E^((2*I)*(a + b*x)))] - I*PolyLog[3, (I - c - I*d)/((c
 - I*(1 + d))*E^((2*I)*(a + b*x)))])/(8*b^2)
```

3.159.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.30, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5699, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cot^{-1}(d\tan(a+bx)+c)\,dx$$

$$\Big\downarrow \text{5699}$$

$$-\frac{1}{2}b(-ic-d+1)\int\frac{e^{2ia+2ibx}x^2}{-ic+(-ic-d+1)e^{2ia+2ibx}+d+1}dx+\frac{1}{2}b(ic+d+1)\int\frac{e^{2ia+2ibx}x^2}{ic+(ic+d+1)e^{2ia+2ibx}-d+1}dx+\frac{1}{2}x^2\cot^{-1}(d\tan(a+bx)+c)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{2}b(ic+d+1)\left(\frac{\int x\log\left(\frac{e^{2ia+2ibx}(ic+d+1)}{ic-d+1}+1\right)dx}{b(c-i(d+1))}-\frac{x^2\log\left(1+\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{2b(c-i(d+1))}\right)-\frac{1}{2}b(-ic-d+1)\left(\frac{x^2\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{2b(c+i(1-d))}-\frac{\int x\log\left(\frac{e^{2ia+2ibx}(c+i(1-d))}{c+i(d+1)}+1\right)dx}{b(c+i(1-d))}\right)+\frac{1}{2}x^2\cot^{-1}(d\tan(a+bx)+c)$$

$\downarrow$ 3011

$$\frac{1}{2}b(ic+d+1)\left(\frac{\frac{ix\,\text{PolyLog}\left(2,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{2b}-\frac{i\int \text{PolyLog}\left(2,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)dx}{2b}}{b(c-i(d+1))}-\frac{x^2\log\left(1+\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{2b(c-i(d+1))}\right)-\frac{1}{2}b(-ic-d+1)\left(\frac{x^2\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{2b(c+i(1-d))}-\frac{\frac{ix\,\text{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{2b}-\frac{i\int \text{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)dx}{2b}}{b(c+i(1-d))}\right)+\frac{1}{2}x^2\cot^{-1}(d\tan(a+bx)+c)$$

$\downarrow$ 2720

$$\frac{1}{2}b(ic+d+1)\left(\frac{\frac{ix\,\text{PolyLog}\left(2,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{2b}-\frac{\int e^{-2ia-2ibx}\,\text{PolyLog}\left(2,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)de^{2ia+2ibx}}{4b^2}}{b(c-i(d+1))}-\frac{x^2\log\left(1+\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{2b(c-i(d+1))}\right)-\frac{1}{2}b(-ic-d+1)\left(\frac{x^2\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{2b(c+i(1-d))}-\frac{\frac{ix\,\text{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{2b}-\frac{\int e^{-2ia-2ibx}\,\text{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)de^{2ia+2ibx}}{4b^2}}{b(c+i(1-d))}\right)+\frac{1}{2}x^2\cot^{-1}(d\tan(a+bx)+c)$$

$\downarrow$ 7143

3.159. $\int x\cot^{-1}(c+d\tan(a+bx))\,dx$

$$\frac{1}{2}b(ic+d+1)\left(\frac{\frac{ix\operatorname{PolyLog}\left(2,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{2b}-\frac{\operatorname{PolyLog}\left(3,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{4b^2}}{b(c-i(d+1))}-\frac{x^2\log\left(1+\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{2b(c-i(d+1))}\right)-\frac{1}{2}b(-ic-d+1)\left(\frac{x^2\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{2b(c+i(1-d))}-\frac{\frac{ix\operatorname{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{2b}-\frac{\operatorname{PolyLog}\left(3,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{4b^2}}{b(c+i(1-d))}\right)+\frac{1}{2}x^2\cot^{-1}(d\tan(a+bx)+c)$$

input
```
Int[x*ArcCot[c + d*Tan[a + b*x]],x]
```

output
```
(x^2*ArcCot[c + d*Tan[a + b*x]])/2 + (b*(1 + I*c + d)*(-1/2*(x^2*Log[1 + (
(1 + I*c + d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c - d)])/(b*(c - I*(1 + d))
 + (((I/2)*x*PolyLog[2, -(((1 + I*c + d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c
 - d))])/b - PolyLog[3, -(((1 + I*c + d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c
 - d))]/(4*b^2))/(b*(c - I*(1 + d)))))/2 - (b*(1 - I*c - d)*((x^2*Log[1 +
((c + I*(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 + d))])/(2*b*(c + I*(1
 - d))) - (((I/2)*x*PolyLog[2, -(((c + I*(1 - d))*E^((2*I)*a + (2*I)*b*x))
/(c + I*(1 + d)))])/b - PolyLog[3, -(((c + I*(1 - d))*E^((2*I)*a + (2*I)*b
*x))/(c + I*(1 + d)))]/(4*b^2))/(b*(c + I*(1 - d)))))/2
```

3.159.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5699
```
Int[ArcCot[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Tan[a + b*x]]/(f*(m +
1))), x] + (-Simp[b*((1 - I*c - d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^
(2*I*a + 2*I*b*x)/(1 - I*c + d + (1 - I*c - d)*E^(2*I*a + 2*I*b*x))), x], x
] + Simp[b*((1 + I*c + d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*I*a +
2*I*b*x)/(1 + I*c - d + (1 + I*c + d)*E^(2*I*a + 2*I*b*x))), x], x]) /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c + I*d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.159.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.09 (sec) , antiderivative size = 7646, normalized size of antiderivative = 25.07

method	result	size
risch	Expression too large to display	7646

input
```
int(x*arccot(c+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.159.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1545 vs. 2(217) = 434.

Time = 0.33 (sec) , antiderivative size = 1545, normalized size of antiderivative = 5.07

$$\int x\cot^{-1}(c+d\tan(a+bx))\,dx = \text{Too large to display}$$

input
```
integrate(x*arccot(c+d*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/16*(8*b^2*x^2*arccot(d*tan(b*x + a) + c) - 2*b*x*dilog(2*((I*c*d - d^2 +
 d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x + a
) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1) +
1) + 2*b*x*dilog(2*((I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^
2 - 2*c*d - I*d^2 + I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*
x + a)^2 + c^2 + d^2 + 2*d + 1) + 1) - 2*b*x*dilog(2*((-I*c*d - d^2 + d)*t
an(b*x + a)^2 - c^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*tan(b*x + a) +
d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1) + 1) +
 2*b*x*dilog(2*((-I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 + I*c*d + (-I*c^2
- 2*c*d + I*d^2 - I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x
+ a)^2 + c^2 + d^2 + 2*d + 1) + 1) + 2*I*a^2*log(((I*c*d + d^2 + d)*tan(b*
x + a)^2 - c^2 + I*c*d + (I*c^2 + I*d^2 + 2*I*d + I)*tan(b*x + a) - d - 1)
/(tan(b*x + a)^2 + 1)) - 2*I*a^2*log(((I*c*d + d^2 - d)*tan(b*x + a)^2 - c
^2 + I*c*d + (I*c^2 + I*d^2 - 2*I*d + I)*tan(b*x + a) + d - 1)/(tan(b*x +
a)^2 + 1)) + 2*I*a^2*log(((I*c*d - d^2 + d)*tan(b*x + a)^2 + c^2 + I*c*d +
 (I*c^2 + I*d^2 - 2*I*d + I)*tan(b*x + a) - d + 1)/(tan(b*x + a)^2 + 1)) -
 2*I*a^2*log(((I*c*d - d^2 - d)*tan(b*x + a)^2 + c^2 + I*c*d + (I*c^2 + I*
d^2 + 2*I*d + I)*tan(b*x + a) + d + 1)/(tan(b*x + a)^2 + 1)) - 2*(-I*b^2*x
^2 + I*a^2)*log(-2*((I*c*d - d^2 + d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^
2 - 2*c*d - I*d^2 + I)*tan(b*x + a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan...
```

3.159.6 Sympy [F]

$$\int x\cot^{-1}(c+d\tan(a+bx))\,dx = \int x\,\mathrm{acot}\,(c+d\tan(a+bx))\,dx$$

input
```
integrate(x*acot(c+d*tan(b*x+a)),x)
```

output
```
Integral(x*acot(c + d*tan(a + b*x)), x)
```

3.159.7 Maxima [F]

$$\int x\cot^{-1}(c+d\tan(a+bx))\,dx=\int x\,\text{arccot}\left(d\tan\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x*arccot(c+d*tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*x^2*arctan2(-(d + 1)*cos(2*b*x + 2*a) + c*sin(2*b*x + 2*a) + d - 1, -c
*cos(2*b*x + 2*a) - (d + 1)*sin(2*b*x + 2*a) - c) - 1/4*x^2*arctan2(-(d -
1)*cos(2*b*x + 2*a) + c*sin(2*b*x + 2*a) + d + 1, -c*cos(2*b*x + 2*a) - (d
 - 1)*sin(2*b*x + 2*a) - c) - 2*b*d*integrate(-(2*(c^2 + d^2 + 1)*x^2*cos(
2*b*x + 2*a)^2 + 2*c*d*x^2*sin(2*b*x + 2*a) + 2*(c^2 + d^2 + 1)*x^2*sin(2*
b*x + 2*a)^2 + (c^2 - d^2 + 1)*x^2*cos(2*b*x + 2*a) - (2*c*d*x^2*sin(2*b*x
 + 2*a) - (c^2 - d^2 + 1)*x^2*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*
x^2*cos(2*b*x + 2*a) + (c^2 - d^2 + 1)*x^2*sin(2*b*x + 2*a))*sin(4*b*x + 4
*a))/(c^4 + d^4 + 2*(c^2 - 1)*d^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 +
 1)*cos(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*cos(2
*b*x + 2*a)^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*sin(4*b*x + 4*a)
^2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*sin(2*b*x + 2*a)^2 + 2*c^
2 + 2*(c^4 + d^4 - 2*(3*c^2 + 1)*d^2 + 2*c^2 + 2*(c^4 - d^4 + 2*c^2 + 1)*c
os(2*b*x + 2*a) - 4*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x
+ 4*a) + 4*(c^4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(2*c*d^3 - 2*(c^3
+ c)*d - 2*(c*d^3 + (c^3 + c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 + 2*c^2 + 1
)*sin(2*b*x + 2*a))*sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 + c)*d)*sin(2*b*x +
 2*a) + 1), x)
```

3.159.8 Giac [F]

$$\int x\cot^{-1}(c+d\tan(a+bx))\,dx=\int x\,\text{arccot}\left(d\tan\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x*arccot(c+d*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arccot(d*tan(b*x + a) + c), x)
```

3.159.9 Mupad [F(-1)]

Timed out.

$$\int x\cot^{-1}(c+d\tan(a+bx))\,dx=\int x\,\mathrm{acot}(c+d\tan(a+b\,x))\,\,dx$$

input
```
int(x*acot(c + d*tan(a + b*x)),x)
```

output
```
int(x*acot(c + d*tan(a + b*x)), x)
```

3.160 $\int \cot^{-1}(c + d\tan(a + bx))\,dx$

3.160.1 Optimal result

Integrand size = 11, antiderivative size = 198

$$\begin{aligned}\int \cot^{-1}(c + d\tan(a + bx))\,dx = {} & x\cot^{-1}(c + d\tan(a + bx)) \\ & - \frac{1}{2}ix\log\left(1 + \frac{(1 + ic + d)e^{2ia+2ibx}}{1 + ic - d}\right) \\ & + \frac{1}{2}ix\log\left(1 + \frac{(c + i(1 - d))e^{2ia+2ibx}}{c + i(1 + d)}\right) \\ & - \frac{\text{PolyLog}\left(2, -\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{4b} \\ & + \frac{\text{PolyLog}\left(2, -\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{4b}\end{aligned}$$

output
```
x*arccot(c+d*tan(b*x+a))-1/2*I*x*ln(1+(1+I*c+d)*exp(2*I*a+2*I*b*x)/(1+I*c-
d))+1/2*I*x*ln(1+(c+I*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))-1/4*polylog(2
,-(1+I*c+d)*exp(2*I*a+2*I*b*x)/(1+I*c-d))/b+1/4*polylog(2,-(c+I*(1-d))*exp
(2*I*a+2*I*b*x)/(c+I*(1+d)))/b
```

3.160.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 549 vs. 2(198) = 396.

Time = 0.54 (sec) , antiderivative size = 549, normalized size of antiderivative = 2.77

$$\int \cot^{-1}(c+d\tan(a+bx))\,dx = x\cot^{-1}(c+d\tan(a+bx)) + \frac{x\left(4a\sqrt{-d^2}\arctan(c+d\tan(a+bx)) - id\log(1-i\tan(a+bx))\log\left(\frac{-cd+\sqrt{-d^2}-d^2\tan(a+bx)}{-cd+id^2+\sqrt{-d^2}}\right) + id\log(1\right.}{}$$

input
```
Integrate[ArcCot[c + d*Tan[a + b*x]],x]
```

output
```
x*ArcCot[c + d*Tan[a + b*x]] + (x*(4*a*Sqrt[-d^2]*ArcTan[c + d*Tan[a + b*x
]] - I*d*Log[1 - I*Tan[a + b*x]]*Log[(-(c*d) + Sqrt[-d^2] - d^2*Tan[a + b*
x])/(-(c*d) + I*d^2 + Sqrt[-d^2])] + I*d*Log[1 + I*Tan[a + b*x]]*Log[(c*d
- Sqrt[-d^2] + d^2*Tan[a + b*x])/(c*d + I*d^2 - Sqrt[-d^2])] + I*d*Log[1 -
 I*Tan[a + b*x]]*Log[(c*d + Sqrt[-d^2] + d^2*Tan[a + b*x])/(c*d - I*d^2 +
Sqrt[-d^2])] - I*d*Log[1 + I*Tan[a + b*x]]*Log[(c*d + Sqrt[-d^2] + d^2*Tan
[a + b*x])/(c*d + I*d^2 + Sqrt[-d^2])] - I*d*PolyLog[2, (d^2*(1 - I*Tan[a
+ b*x]))/(I*c*d + d^2 - I*Sqrt[-d^2])] + I*d*PolyLog[2, (d^2*(1 - I*Tan[a
+ b*x]))/(I*c*d + d^2 + I*Sqrt[-d^2])] + I*d*PolyLog[2, (d^2*(1 + I*Tan[a
+ b*x]))/((-I)*c*d + d^2 + I*Sqrt[-d^2])] - I*d*PolyLog[2, (d^2*(1 + I*Tan
[a + b*x]))/(d^2 - I*(c*d + Sqrt[-d^2]))]))/(2*Sqrt[-d^2]*(2*a - I*Log[1 -
 I*Tan[a + b*x]] + I*Log[1 + I*Tan[a + b*x]]))
```

3.160.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.40, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5691, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}(d\tan(a+bx)+c)\,dx$$

$$\Big\downarrow \text{5691}$$

$$-b(-ic-d+1)\int \frac{e^{2ia+2ibx}x}{-ic+(-ic-d+1)e^{2ia+2ibx}+d+1}dx+b(ic+d+1)\int \frac{e^{2ia+2ibx}x}{ic+(ic+d+1)e^{2ia+2ibx}-d+1}dx+x\cot^{-1}(d\tan(a+bx)+c)$$

$$\Big\downarrow \text{2620}$$

$$b(ic+d+1)\left(\frac{\int \log\left(\frac{e^{2ia+2ibx}(ic+d+1)}{ic-d+1}+1\right)dx}{2b(c-i(d+1))}-\frac{x\log\left(1+\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{2b(c-i(d+1))}\right)-b(-ic-d+1)\left(\frac{x\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{2b(c+i(1-d))}-\frac{\int \log\left(\frac{e^{2ia+2ibx}(c+i(1-d))}{c+i(d+1)}+1\right)dx}{2b(c+i(1-d))}\right)+x\cot^{-1}(d\tan(a+bx)+c)$$

$$\Big\downarrow \text{2715}$$

$$b(ic+d+1)\left(-\frac{i\int e^{-2ia-2ibx}\log\left(\frac{e^{2ia+2ibx}(ic+d+1)}{ic-d+1}+1\right)de^{2ia+2ibx}}{4b^2(c-i(d+1))}-\frac{x\log\left(1+\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{2b(c-i(d+1))}\right)-b(-ic-d+1)\left(\frac{i\int e^{-2ia-2ibx}\log\left(\frac{e^{2ia+2ibx}(c+i(1-d))}{c+i(d+1)}+1\right)de^{2ia+2ibx}}{4b^2(c+i(1-d))}+\frac{x\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{2b(c+i(1-d))}\right)+x\cot^{-1}(d\tan(a+bx)+c)$$

$$\Big\downarrow \text{2838}$$

$$b(ic+d+1)\left(\frac{i\,\text{PolyLog}\left(2,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{4b^2(c-i(d+1))}-\frac{x\log\left(1+\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{2b(c-i(d+1))}\right)-b(-ic-d+1)\left(\frac{x\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{2b(c+i(1-d))}-\frac{i\,\text{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{4b^2(c+i(1-d))}\right)+x\cot^{-1}(d\tan(a+bx)+c)$$

input
```
Int[ArcCot[c + d*Tan[a + b*x]],x]
```

output
```
x*ArcCot[c + d*Tan[a + b*x]] + b*(1 + I*c + d)*(-1/2*(x*Log[1 + ((1 + I*c
+ d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c - d)])/(b*(c - I*(1 + d))) + ((I/4)
*PolyLog[2, -(((1 + I*c + d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c - d))])/(b^
2*(c - I*(1 + d)))) - b*(1 - I*c - d)*((x*Log[1 + ((c + I*(1 - d))*E^((2*I
)*a + (2*I)*b*x))/(c + I*(1 + d))])/(2*b*(c + I*(1 - d))) - ((I/4)*PolyLog
[2, -(((c + I*(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 + d)))])/(b^2*(c
 + I*(1 - d))))
```

3.160. $\int \cot^{-1}(c+d\tan(a+bx))\,dx$

3.160.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5691
```
Int[ArcCot[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcC
ot[c + d*Tan[a + b*x]], x] + (-Simp[b*(1 - I*c - d)   Int[x*(E^(2*I*a + 2*I
*b*x)/(1 - I*c + d + (1 - I*c - d)*E^(2*I*a + 2*I*b*x))), x], x] + Simp[b*(
1 + I*c + d)   Int[x*(E^(2*I*a + 2*I*b*x)/(1 + I*c - d + (1 + I*c + d)*E^(2
*I*a + 2*I*b*x))), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c + I*d)^2, -1
]
```

3.160.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1137 vs. 2(168) = 336.

Time = 4.35 (sec) , antiderivative size = 1138, normalized size of antiderivative = 5.75

method	result	size
derivativedivides	Expression too large to display	1138
default	Expression too large to display	1138
risch	Expression too large to display	4969

input
```
int(arccot(c+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/d*(d*arctan(tan(b*x+a))*arccot(c+d*tan(b*x+a))+d^2*(-1/d*arctan(d*((c+
d*tan(b*x+a))/d-c/d)+c)*arctan(-(c+d*tan(b*x+a))/d+c/d)-1/d^2*(1/2*I*d^2*l
n(1-(c-I*d+I)*(1+I*(d*((c+d*tan(b*x+a))/d-c/d)+c))^2/((d*((c+d*tan(b*x+a))
/d-c/d)+c)^2+1)/(I*d+I-c))*arctan(d*((c+d*tan(b*x+a))/d-c/d)+c)/(1+I*c+d)+
1/2*I*d*ln(1-(c-I*d+I)*(1+I*(d*((c+d*tan(b*x+a))/d-c/d)+c))^2/((d*((c+d*ta
n(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*arctan(d*((c+d*tan(b*x+a))/d-c/d)+c)/(
1+I*c+d)+1/2*I*d/(c-I*d-I)*ln(1-(c-I*d+I)*(1+I*(d*((c+d*tan(b*x+a))/d-c/d)
+c))^2/((d*((c+d*tan(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*c*arctan(d*((c+d*ta
n(b*x+a))/d-c/d)+c)+1/2*d^2*arctan(d*((c+d*tan(b*x+a))/d-c/d)+c)^2/(1+I*c+
d)+1/4*d^2*polylog(2,(c-I*d+I)*(1+I*(d*((c+d*tan(b*x+a))/d-c/d)+c))^2/((d*
((c+d*tan(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))/(1+I*c+d)+1/2*d*arctan(d*((c+d
*tan(b*x+a))/d-c/d)+c)^2/(1+I*c+d)+1/2*d/(c-I*d-I)*c*arctan(d*((c+d*tan(b*
x+a))/d-c/d)+c)^2+1/4*d*polylog(2,(c-I*d+I)*(1+I*(d*((c+d*tan(b*x+a))/d-c/
d)+c))^2/((d*((c+d*tan(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))/(1+I*c+d)+1/4*d/(
c-I*d-I)*polylog(2,(c-I*d+I)*(1+I*(d*((c+d*tan(b*x+a))/d-c/d)+c))^2/((d*((
c+d*tan(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*c-1/2*I*d*arctan(d*((c+d*tan(b*x
+a))/d-c/d)+c)*ln(1-(I+c+I*d)*(1+I*(d*((c+d*tan(b*x+a))/d-c/d)+c))^2/((d*(
(c+d*tan(b*x+a))/d-c/d)+c)^2+1)/(-I*d+I-c))-1/2*d*arctan(d*((c+d*tan(b*x+a
))/d-c/d)+c)^2-1/4*d*polylog(2,(I+c+I*d)*(1+I*(d*((c+d*tan(b*x+a))/d-c/d)+
c))^2/((d*((c+d*tan(b*x+a))/d-c/d)+c)^2+1)/(-I*d+I-c))))
```

3.160.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1101 vs. $2(141) = 282$.

Time = 0.34 (sec) , antiderivative size = 1101, normalized size of antiderivative = 5.56

$$\int \cot^{-1}(c + d\tan(a + bx))\, dx = \text{Too large to display}$$

input
```
integrate(arccot(c+d*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/8*(8*b*x*arccot(d*tan(b*x + a) + c) - 2*(-I*b*x - I*a)*log(-2*((I*c*d -
d^2 + d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*
x + a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d +
1)) - 2*(I*b*x + I*a)*log(-2*((I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 - I*c
*d + (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d +
 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) - 2*(I*b*x + I*a)*log(-2*((-I*c
*d - d^2 + d)*tan(b*x + a)^2 - c^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*
tan(b*x + a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 -
2*d + 1)) - 2*(-I*b*x - I*a)*log(-2*((-I*c*d - d^2 - d)*tan(b*x + a)^2 - c
^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*tan(b*x + a) - d - 1)/((c^2 + d^
2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) - 2*I*a*log(((I*c*d +
d^2 + d)*tan(b*x + a)^2 - c^2 + I*c*d + (I*c^2 + I*d^2 + 2*I*d + I)*tan(b*
x + a) - d - 1)/(tan(b*x + a)^2 + 1)) + 2*I*a*log(((I*c*d + d^2 - d)*tan(b
*x + a)^2 - c^2 + I*c*d + (I*c^2 + I*d^2 - 2*I*d + I)*tan(b*x + a) + d - 1
)/(tan(b*x + a)^2 + 1)) - 2*I*a*log(((I*c*d - d^2 + d)*tan(b*x + a)^2 + c^
2 + I*c*d + (I*c^2 + I*d^2 - 2*I*d + I)*tan(b*x + a) - d + 1)/(tan(b*x + a
)^2 + 1)) + 2*I*a*log(((I*c*d - d^2 - d)*tan(b*x + a)^2 + c^2 + I*c*d + (I
*c^2 + I*d^2 + 2*I*d + I)*tan(b*x + a) + d + 1)/(tan(b*x + a)^2 + 1)) - di
log(2*((I*c*d - d^2 + d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 - 2*c*d - I
*d^2 + I)*tan(b*x + a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 +...
```

3.160.6 Sympy [F]

$$\int \cot^{-1}(c + d\tan(a + bx))\,dx = \int \operatorname{acot}\left(c + d\tan\left(a + bx\right)\right)dx$$

input
```
integrate(acot(c+d*tan(b*x+a)),x)
```

output
```
Integral(acot(c + d*tan(a + b*x)), x)
```

3.160.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(141) = 282$.

Time = 0.32 (sec) , antiderivative size = 433, normalized size of antiderivative = 2.19

$$\int \cot^{-1}(c+d\tan(a+bx))\,dx = -\frac{d\left(\frac{8(bx+a)\arctan\left(\frac{d^2\tan(bx+a)+cd}{d}\right)}{d} - \frac{4(bx+a)\arctan\left(\frac{cd+\left(d^2+d\right)\tan(bx+a)}{c^2+d^2+2d+1},\frac{cd\tan(bx+a)+c^2+d+1}{c^2+d^2+2d+1}\right)-4(bx+a)\arctan\left(\frac{cd+\left(d^2-d\right)\tan\cdots}{c^2+d^2-2d\cdots}\right.\cdots}{\cdots}\right.\cdots}{\cdots}$$

input
```
integrate(arccot(c+d*tan(b*x+a)),x, algorithm="maxima")
```

output
```
-1/8*(d*(8*(b*x + a)*arctan((d^2*tan(b*x + a) + c*d)/d)/d - (4*(b*x + a)*a
rctan2((c*d + (d^2 + d)*tan(b*x + a))/(c^2 + d^2 + 2*d + 1), (c*d*tan(b*x
+ a) + c^2 + d + 1)/(c^2 + d^2 + 2*d + 1)) - 4*(b*x + a)*arctan2((c*d + (d
^2 - d)*tan(b*x + a))/(c^2 + d^2 - 2*d + 1), (c*d*tan(b*x + a) + c^2 - d +
 1)/(c^2 + d^2 - 2*d + 1)) + log(tan(b*x + a)^2 + 1)*log((d^2*tan(b*x + a)
^2 + 2*c*d*tan(b*x + a) + c^2 + 1)/(c^2 + d^2 + 2*d + 1)) - log(tan(b*x +
a)^2 + 1)*log((d^2*tan(b*x + a)^2 + 2*c*d*tan(b*x + a) + c^2 + 1)/(c^2 + d
^2 - 2*d + 1)) + 2*dilog(-(I*d*tan(b*x + a) - d)/(I*c + d + 1)) - 2*dilog(
-(I*d*tan(b*x + a) - d)/(I*c + d - 1)) + 2*dilog((I*d*tan(b*x + a) + d)/(-
I*c + d + 1)) - 2*dilog((I*d*tan(b*x + a) + d)/(-I*c + d - 1)))/d) - 8*(b*
x + a)*arccot(d*tan(b*x + a) + c) - 8*(b*x + a)*arctan((d^2*tan(b*x + a) +
 c*d)/d))/b
```

3.160.8 Giac [F]

$$\int \cot^{-1}(c+d\tan(a+bx))\,dx = \int \operatorname{arccot}\left(d\tan\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(arccot(c+d*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccot(d*tan(b*x + a) + c), x)
```

3.160.9 Mupad [F(-1)]

Timed out.

$$\int \cot^{-1}(c+d\tan(a+bx))\,dx=\int \mathrm{acot}(c+d\tan(a+b\,x))\;dx$$

input
```
int(acot(c + d*tan(a + b*x)),x)
```

output
```
int(acot(c + d*tan(a + b*x)), x)
```

3.161 $\int \frac{\cot^{-1}(c+d\tan(a+bx))}{x}\,dx$

3.161.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\cot^{-1}(c+d\tan(a+bx))}{x}\,dx = \text{Int}\left(\frac{\cot^{-1}(c+d\tan(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(arccot(c+d*tan(b*x+a))/x,x)
```

3.161.2 Mathematica [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(c+d\tan(a+bx))}{x}\,dx = \int \frac{\cot^{-1}(c+d\tan(a+bx))}{x}\,dx$$

input
```
Integrate[ArcCot[c + d*Tan[a + b*x]]/x,x]
```

output
```
Integrate[ArcCot[c + d*Tan[a + b*x]]/x, x]
```

3.161.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(d\tan(a+bx)+c)}{x} \, dx$$

$$\Big\downarrow \ 7299$$

$$\int \frac{\cot^{-1}(d\tan(a+bx)+c)}{x}dx$$

input
```
Int[ArcCot[c + d*Tan[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.161.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.161.4 Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arccot}\left(c+d\tan\left(bx+a\right)\right)}{x}dx$$

input
```
int(arccot(c+d*tan(b*x+a))/x,x)
```

output
```
int(arccot(c+d*tan(b*x+a))/x,x)
```

3.161.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(c + d\tan(a + bx))}{x}\,dx = \int \frac{\operatorname{arccot}\left(d\tan\left(bx + a\right) + c\right)}{x}\,dx$$

input
```
integrate(arccot(c+d*tan(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(arccot(d*tan(b*x + a) + c)/x, x)
```

3.161.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(c + d\tan(a + bx))}{x}\,dx = \text{Timed out}$$

input
```
integrate(acot(c+d*tan(b*x+a))/x,x)
```

output
```
Timed out
```

3.161.7 Maxima [N/A]

Not integrable

Time = 227.57 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(c + d\tan(a + bx))}{x}\,dx = \int \frac{\operatorname{arccot}\left(d\tan\left(bx + a\right) + c\right)}{x}\,dx$$

input
```
integrate(arccot(c+d*tan(b*x+a))/x,x, algorithm="maxima")
```

output
```
integrate(arccot(d*tan(b*x + a) + c)/x, x)
```

3.161.8 Giac [N/A]

Not integrable

Time = 1.51 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(c+d\tan(a+bx))}{x}\,dx=\int \frac{\operatorname{arccot}\left(d\tan\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccot(c+d*tan(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arccot(d*tan(b*x + a) + c)/x, x)
```

3.161.9 Mupad [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(c+d\tan(a+bx))}{x}\,dx=\int \frac{\mathrm{acot}(c+d\tan(a+b\,x))}{x}\,dx$$

input
```
int(acot(c + d*tan(a + b*x))/x,x)
```

output
```
int(acot(c + d*tan(a + b*x))/x, x)
```

3.162 $\int x^2 \cot^{-1}(c+(1+ic)\tan(a+bx))\,dx$

3.162.1 Optimal result

Integrand size = 21, antiderivative size = 154

$$\int x^2 \cot^{-1}(c+(1+ic)\tan(a+bx))\,dx = \frac{bx^4}{12}+\frac{1}{3}x^3\cot^{-1}(c+(1+ic)\tan(a+bx)) + \frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right) + \frac{x^2\operatorname{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b} + \frac{ix\operatorname{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{4b^2} - \frac{\operatorname{PolyLog}\left(4,ice^{2ia+2ibx}\right)}{8b^3}$$

output
```
1/12*b*x^4+1/3*x^3*arccot(c+(1+I*c)*tan(b*x+a))+1/6*I*x^3*ln(1-I*c*exp(2*I
*a+2*I*b*x))+1/4*x^2*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b+1/4*I*x*polylog(3
,I*c*exp(2*I*a+2*I*b*x))/b^2-1/8*polylog(4,I*c*exp(2*I*a+2*I*b*x))/b^3
```

3.162.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.88

$$\int x^2 \cot^{-1}(c+(1+ic)\tan(a+bx))\,dx = \frac{1}{24}\left(8x^3\cot^{-1}(c+(1+ic)\tan(a+bx)) + 4ix^3\log\left(1+\frac{ie^{-2i(a+bx)}}{c}\right) - \frac{6x^2\,\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{c}\right)}{b} + \frac{6ix\,\text{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{c}\right)}{b^2} + \frac{3\,\text{PolyLog}\left(4,-\frac{ie^{-2i(a+bx)}}{c}\right)}{b^3}\right)$$

input
```
Integrate[x^2*ArcCot[c + (1 + I*c)*Tan[a + b*x]],x]
```

output
```
(8*x^3*ArcCot[c + (1 + I*c)*Tan[a + b*x]] + (4*I)*x^3*Log[1 + I/(c*E^((2*I
)*(a + b*x)))] - (6*x^2*PolyLog[2, (-I)/(c*E^((2*I)*(a + b*x)))])/b + ((6*
I)*x*PolyLog[3, (-I)/(c*E^((2*I)*(a + b*x)))])/b^2 + (3*PolyLog[4, (-I)/(c
*E^((2*I)*(a + b*x)))])/b^3)/24
```

3.162.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.29, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5695, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cot^{-1}(c+(1+ic)\tan(a+bx))\,dx$$

$$\Big\downarrow \; 5695$$

$$\frac{1}{3}ib\int \frac{x^3}{e^{2ia+2ibx}c+i}dx + \frac{1}{3}x^3\cot^{-1}(c+(1+ic)\tan(a+bx))$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{3} i b \left(i c \int \frac{e^{2 i a+2 i b x} x^3}{e^{2 i a+2 i b x} c+i} dx-\frac{i x^4}{4}\right)+\frac{1}{3} x^3 \cot^{-1}(c+(1+i c) \tan(a+b x))$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{3} i b \left(i c \left(\frac{3 i \int x^2 \log\left(1-i c e^{2 i a+2 i b x}\right) dx}{2 b c}-\frac{i x^3 \log\left(1-i c e^{2 i a+2 i b x}\right)}{2 b c}\right)-\frac{i x^4}{4}\right)+\frac{1}{3} x^3 \cot^{-1}(c+(1+ic) \tan(a+b x))$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{3} i b \left(i c \left(\frac{3 i \left(\frac{i x^2 \text{PolyLog}\left(2,i c e^{2 i a+2 i b x}\right)}{2 b}-\frac{i \int x \text{PolyLog}\left(2,i c e^{2 i a+2 i b x}\right) dx}{b}\right)}{2 b c}-\frac{i x^3 \log\left(1-i c e^{2 i a+2 i b x}\right)}{2 b c}\right)-\frac{i x^4}{4}\right)+\frac{1}{3} x^3 \cot^{-1}(c+(1+i c) \tan(a+b x))$$

$$\Big\downarrow \text{7163}$$

$$\frac{1}{3} i b \Bigg(i c \Bigg(\frac{3 i \left(\frac{i x^2 \text{PolyLog}\left(2,i c e^{2 i a+2 i b x}\right)}{2 b}-\frac{i \left(\frac{i \int \text{PolyLog}\left(3,i c e^{2 i a+2 i b x}\right) dx}{2 b}-\frac{i x \text{PolyLog}\left(3,i c e^{2 i a+2 i b x}\right)}{2 b}\right)}{b}\right)}{2 b c}-\frac{i x^3 \log\left(1-i c e^{2 i a+2 i b x}\right)}{2 b c}$$

$$\frac{1}{3} x^3 \cot^{-1}(c+(1+i c) \tan(a+b x))$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{3} i b \Bigg(i c \Bigg(\frac{3 i \left(\frac{i x^2 \text{PolyLog}\left(2,i c e^{2 i a+2 i b x}\right)}{2 b}-\frac{i \left(\frac{\int e^{-2 i a-2 i b x} \text{PolyLog}\left(3,i c e^{2 i a+2 i b x}\right) de^{2 i a+2 i b x}}{4 b^2}-\frac{i x \text{PolyLog}\left(3,i c e^{2 i a+2 i b x}\right)}{2 b}\right)}{b}\right)}{2 b c}-\frac{i x^3 \log (1}{}$$

$$\frac{1}{3} x^3 \cot^{-1}(c+(1+i c) \tan(a+b x))$$

$$\Big\downarrow \text{7143}$$

3.162. $\int x^2 \cot^{-1}(c+(1+i c) \tan(a+b x)) \, dx$

$$\frac{1}{3}ib\left(ic\left(\frac{3i\left(\frac{ix^2\,\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{2b}-\frac{i\left(\frac{\text{PolyLog}\left(4,ice^{2ia+2ibx}\right)}{4b^2}-\frac{ix\,\text{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{2b}\right)}{b}\right)}{2bc}-\frac{ix^3\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right)\right.$$

$$\frac{1}{3}x^3\cot^{-1}(c+(1+ic)\tan(a+bx))$$

input
```
Int[x^2*ArcCot[c + (1 + I*c)*Tan[a + b*x]],x]
```

output
```
(x^3*ArcCot[c + (1 + I*c)*Tan[a + b*x]])/3 + (I/3)*b*((-1/4*I)*x^4 + I*c*(
((-1/2*I)*x^3*Log[1 - I*c*E^((2*I)*a + (2*I)*b*x)])/(b*c) + (((3*I)/2)*(((
I/2)*x^2*PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)])/b - (I*(((-1/2*I)*x*Poly
Log[3, I*c*E^((2*I)*a + (2*I)*b*x)])/b + PolyLog[4, I*c*E^((2*I)*a + (2*I)
*b*x)]/(4*b^2)))/b))/(b*c)))
```

3.162.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5695
```
Int[ArcCot[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Tan[a + b*x]]/(f*(m +
1))), x] + Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(2
*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && E
qQ[(c + I*d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.162.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.88 (sec) , antiderivative size = 1448, normalized size of antiderivative = 9.40

method	result	size
risch	Expression too large to display	1448

input
```
int(x^2*arccot(c+(I*c+1)*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/4*I*x*polylog(3,I*exp(2*I*(b*x+a))*c)/b^2+1/12*(Pi*csgn(I/(exp(2*I*(b*x+
a))+1))*csgn(I*(c-I))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))-Pi*csgn(I/(exp(2*
I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*c+I))*csgn(I*(exp(2*I*(b*x+a))*c+I
)/(exp(2*I*(b*x+a))+1))-Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(c-I)/(exp(
2*I*(b*x+a))+1))^2+Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a)
)*c+I)/(exp(2*I*(b*x+a))+1))^2+Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*
(b*x+a)))-2*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2+Pi*csgn(I
*exp(2*I*(b*x+a)))^3+Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*(c-I)/(exp(2*I*(b*
x+a))+1))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))-Pi*csgn(I*ex
p(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2-Pi*c
sgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*
c+I)/(exp(2*I*(b*x+a))+1))+Pi*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a)
)+1))^2-Pi*csgn(I*(c-I))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))^2+Pi*csgn(I*(e
xp(2*I*(b*x+a))*c+I))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^
2+Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))*csgn(exp(2*I*(b*x
+a))*(c-I)/(exp(2*I*(b*x+a))+1))+Pi*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(
b*x+a))+1))^2+Pi*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))^3-Pi*csgn(I*(c-I)/(exp
(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2-Pi
*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^3+Pi*csgn(I*(exp(2*I*
(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*...
```

3.162.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(107) = 214.

Time = 0.27 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.08

$$\int x^2 \cot^{-1}(c+(1+ic)\tan(a+bx))\,dx = \frac{b^4x^4 - 2i\,b^3x^3\log\left(\frac{(ce^{(2i\,bx+2i\,a)}+i)e^{(-2i\,bx-2i\,a)}}{c-i}\right) + 6\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\sqrt{4i\,c}e^{(i\,bx+i\,a)}\right) + 6\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{4i\,c}e^{(i\,bx+i\,a)}\right)}{}$$

input
```
integrate(x^2*arccot(c+(1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(b^4*x^4 - 2*I*b^3*x^3*log((c*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x -
2*I*a)/(c - I)) + 6*b^2*x^2*dilog(1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + 6*b^2
*x^2*dilog(-1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) - a^4 - 2*I*a^3*log(1/2*(2*c*
e^(I*b*x + I*a) + I*sqrt(4*I*c))/c) - 2*I*a^3*log(1/2*(2*c*e^(I*b*x + I*a)
 - I*sqrt(4*I*c))/c) + 12*I*b*x*polylog(3, 1/2*sqrt(4*I*c)*e^(I*b*x + I*a)
) + 12*I*b*x*polylog(3, -1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) - 2*(-I*b^3*x^3
- I*a^3)*log(1/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1) - 2*(-I*b^3*x^3 - I*a^3)
*log(-1/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1) - 12*polylog(4, 1/2*sqrt(4*I*c)
*e^(I*b*x + I*a)) - 12*polylog(4, -1/2*sqrt(4*I*c)*e^(I*b*x + I*a)))/b^3
```

3.162.6 Sympy [F(-2)]

Exception generated.

$$\int x^2 \cot^{-1}(c + (1 + ic)\tan(a + bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(x**2*acot(c+(1+I*c)*tan(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert _t0**2*I + 2*c*exp(2*I*
a) - I*exp(2*I*a) of type <class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,e
xp(I*a)]
```

3.162.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(107) = 214.

Time = 0.21 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.01

$$\int x^2 \cot^{-1}(c + (1 + ic)\tan(a + bx))\,dx$$
$$= \frac{\frac{4\left((bx+a)^3-3(bx+a)^2a+3(bx+a)a^2\right)\operatorname{arccot}((ic+1)\tan(bx+a)+c)}{b^2} - \frac{\left(-3i(bx+a)^4+12i(bx+a)^3a-18i(bx+a)^2a^2-2\left(4i(bx+a)^3-9i(bx+a\right.\right.}{}}{}$$

input
```
integrate(x^2*arccot(c+(1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/12*(4*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arccot((I*c + 1)
*tan(b*x + a) + c)/b^2 - (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*
x + a)^2*a^2 - 2*(4*I*(b*x + a)^3 - 9*I*(b*x + a)^2*a + 9*I*(b*x + a)*a^2)
*arctan2(c*cos(2*b*x + 2*a), c*sin(2*b*x + 2*a) + 1) - 3*(4*I*(b*x + a)^2
- 6*I*(b*x + a)*a + 3*I*a^2)*dilog(I*c*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)
^3 - 9*(b*x + a)^2*a + 9*(b*x + a)*a^2)*log(c^2*cos(2*b*x + 2*a)^2 + c^2*s
in(2*b*x + 2*a)^2 + 2*c*sin(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, I
*c*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, I*c*e^(2*I*b*x + 2*I*a)))*(-I*c -
 1)/(b^2*(c - I)))/b
```

3.162.8 Giac [F]

$$\int x^2 \cot^{-1}(c+(1+ic)\tan(a+bx))\,dx=\int x^2\operatorname{arccot}\left((i\,c+1)\tan\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x^2*arccot(c+(1+I*c)*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arccot((I*c + 1)*tan(b*x + a) + c), x)
```

3.162.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(c+(1+ic)\tan(a+bx))\,dx=\int x^2\operatorname{acot}(c+\tan(a+b\,x)\,\left(1+c\,1\mathrm{i}\right))\,dx$$

input
```
int(x^2*acot(c + tan(a + b*x)*(c*1i + 1)),x)
```

output
```
int(x^2*acot(c + tan(a + b*x)*(c*1i + 1)), x)
```

3.163 $\int x \cot^{-1}(c+(1+ic)\tan(a+bx))\,dx$

3.163.1 Optimal result

Integrand size = 19, antiderivative size = 123

$$\int x \cot^{-1}(c+(1+ic)\tan(a+bx))\,dx = \frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c+(1+ic)\tan(a+bx)) + \frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right) + \frac{x\,\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b} + \frac{i\,\text{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{8b^2}$$

output
```
1/6*b*x^3+1/2*x^2*arccot(c+(1+I*c)*tan(b*x+a))+1/4*I*x^2*ln(1-I*c*exp(2*I*
a+2*I*b*x))+1/4*x*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b+1/8*I*polylog(3,I*c*
exp(2*I*a+2*I*b*x))/b^2
```

3.163.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89

$$\int x \cot^{-1}(c+(1+ic)\tan(a+bx))\,dx = \frac{1}{2}x^2\cot^{-1}(c+(1+ic)\tan(a+bx)) + \frac{i\left(2b^2x^2\log\left(1+\frac{ie^{-2i(a+bx)}}{c}\right)+2ibx\,\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{c}\right)+\text{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{c}\right)\right)}{8b^2}$$

input
```
Integrate[x*ArcCot[c + (1 + I*c)*Tan[a + b*x]],x]
```

output
```
(x^2*ArcCot[c + (1 + I*c)*Tan[a + b*x]])/2 + ((I/8)*(2*b^2*x^2*Log[1 + I/(
c*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, (-I)/(c*E^((2*I)*(a + b*x))
)] + PolyLog[3, (-I)/(c*E^((2*I)*(a + b*x)))]))/b^2
```

3.163.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.26, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5695, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \cot^{-1}(c+(1+ic)\tan(a+bx))\,dx$$

$$\Big\downarrow \text{5695}$$

$$\frac{1}{2}ib\int \frac{x^2}{e^{2ia+2ibx}c+i}dx+\frac{1}{2}x^2\cot^{-1}(c+(1+ic)\tan(a+bx))$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{2}ib\left(ic\int \frac{e^{2ia+2ibx}x^2}{e^{2ia+2ibx}c+i}dx-\frac{ix^3}{3}\right)+\frac{1}{2}x^2\cot^{-1}(c+(1+ic)\tan(a+bx))$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{2}ib\left(ic\left(\frac{i\int x\log\left(1-ice^{2ia+2ibx}\right)dx}{bc}-\frac{ix^2\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right)-\frac{ix^3}{3}\right)+\frac{1}{2}x^2\cot^{-1}(c+(1+ic)\tan(a+bx))$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}ib\left(ic\left(\frac{i\left(\frac{ix\,\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{2b}-\frac{i\int \text{PolyLog}\left(2,ice^{2ia+2ibx}\right)dx}{2b}\right)}{bc}-\frac{ix^2\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right)-\frac{ix^3}{3}\right)+\frac{1}{2}x^2\cot^{-1}(c+(1+ic)\tan(a+bx))$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}ib\left(ic\left(\frac{i\left(\frac{ix\operatorname{PolyLog}(2,ice^{2ia+2ibx})}{2b}-\frac{\int e^{-2ia-2ibx}\operatorname{PolyLog}(2,ice^{2ia+2ibx})de^{2ia+2ibx}}{4b^2}\right)}{bc}-\frac{ix^2\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right)-\frac{ix^3}{3}\right)+\frac{1}{2}x^2\cot^{-1}(c+(1+ic)\tan(a+bx))$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}ib\left(ic\left(\frac{i\left(\frac{ix\operatorname{PolyLog}(2,ice^{2ia+2ibx})}{2b}-\frac{\operatorname{PolyLog}(3,ice^{2ia+2ibx})}{4b^2}\right)}{bc}-\frac{ix^2\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right)-\frac{ix^3}{3}\right)+\frac{1}{2}x^2\cot^{-1}(c+(1+ic)\tan(a+bx))$$

input
```
Int[x*ArcCot[c + (1 + I*c)*Tan[a + b*x]],x]
```

output
```
(x^2*ArcCot[c + (1 + I*c)*Tan[a + b*x]])/2 + (I/2)*b*((-1/3*I)*x^3 + I*c*(
((-1/2*I)*x^2*Log[1 - I*c*E^((2*I)*a + (2*I)*b*x)])/(b*c) + (I*(((I/2)*x*P
olyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)])/b - PolyLog[3, I*c*E^((2*I)*a + (2
*I)*b*x)]/(4*b^2)))/(b*c)))
```

3.163.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5695
```
Int[ArcCot[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Tan[a + b*x]]/(f*(m +
1))), x] + Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(2
*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && E
qQ[(c + I*d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.163.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.55 (sec) , antiderivative size = 1413, normalized size of antiderivative = 11.49

method	result	size
risch	Expression too large to display	1413

input
```
int(x*arccot(c+(I*c+1)*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/8*I/b^2*polylog(3,I*exp(2*I*(b*x+a))*c)+1/8*(Pi*csgn(I/(exp(2*I*(b*x+a))
+1))*csgn(I*(c-I))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))-Pi*csgn(I/(exp(2*I*(
b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*c+I))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(
exp(2*I*(b*x+a))+1))-Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(c-I)/(exp(2*I
*(b*x+a))+1))^2+Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*c
+I)/(exp(2*I*(b*x+a))+1))^2+Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*
x+a)))-2*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2+Pi*csgn(I*ex
p(2*I*(b*x+a)))^3+Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*(c-I)/(exp(2*I*(b*x+a
))+1))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))-Pi*csgn(I*exp(2
*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2-Pi*csgn
(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*c+I
)/(exp(2*I*(b*x+a))+1))+Pi*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1
))^2-Pi*csgn(I*(c-I))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))^2+Pi*csgn(I*(exp(
2*I*(b*x+a))*c+I))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^2+P
i*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))*csgn(exp(2*I*(b*x+a)
)*(c-I)/(exp(2*I*(b*x+a))+1))+Pi*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x
+a))+1))^2+Pi*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))^3-Pi*csgn(I*(c-I)/(exp(2*
I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2-Pi*cs
gn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^3+Pi*csgn(I*(exp(2*I*(b*
x+a))*c+I)/(exp(2*I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(...
```

3.163.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(85) = 170$.

Time = 0.27 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.20

$$\int x \cot^{-1}(c + (1 + ic)\tan(a + bx))\,dx = \frac{2\,b^3x^3 - 3i\,b^2x^2\log\left(\frac{(ce^{(2i\,bx+2i\,a)}+i)e^{(-2i\,bx-2i\,a)}}{c-i}\right) + 2\,a^3 + 6\,bx\mathrm{Li}_2\left(\frac{1}{2}\sqrt{4i\,c}e^{(i\,bx+i\,a)}\right) + 6\,bx\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{4i\,c}e^{(i\,bx}\right.}{}$$

input
```
integrate(x*arccot(c+(1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(2*b^3*x^3 - 3*I*b^2*x^2*log((c*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x
- 2*I*a)/(c - I)) + 2*a^3 + 6*b*x*dilog(1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) +
 6*b*x*dilog(-1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + 3*I*a^2*log(1/2*(2*c*e^(I
*b*x + I*a) + I*sqrt(4*I*c))/c) + 3*I*a^2*log(1/2*(2*c*e^(I*b*x + I*a) - I
*sqrt(4*I*c))/c) - 3*(-I*b^2*x^2 + I*a^2)*log(1/2*sqrt(4*I*c)*e^(I*b*x + I
*a) + 1) - 3*(-I*b^2*x^2 + I*a^2)*log(-1/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1
) + 6*I*polylog(3, 1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + 6*I*polylog(3, -1/2*
sqrt(4*I*c)*e^(I*b*x + I*a)))/b^2
```

3.163.6 Sympy [F(-2)]

Exception generated.

$$\int x\cot^{-1}(c+(1+ic)\tan(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(x*acot(c+(1+I*c)*tan(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert _t0**2*I + 2*c*exp(2*I*
a) - I*exp(2*I*a) of type <class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,e
xp(I*a)]
```

3.163.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(85) = 170.

Time = 0.20 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.77

$$\int x\cot^{-1}(c+(1+ic)\tan(a+bx))\,dx = \frac{\frac{6\left((bx+a)^2-2\,(bx+a)a\right)\operatorname{arccot}((i\,c+1)\tan(bx+a)+c)}{b} - \frac{\left(-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\mathrm{Li}_2\left(i\,ce^{(2i\,bx+2i\,a)}\right)-6\left(i\,(bx+a)^2-2i\,(bx+a)a\right)\,\mathrm{ar}}{}}{}$$

input
```
integrate(x*arccot(c+(1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/12*(6*((b*x + a)^2 - 2*(b*x + a)*a)*arccot((I*c + 1)*tan(b*x + a) + c)/b
 - (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog(I*c*e^(2*I*b*x +
 2*I*a)) - 6*(I*(b*x + a)^2 - 2*I*(b*x + a)*a)*arctan2(c*cos(2*b*x + 2*a),
 c*sin(2*b*x + 2*a) + 1) + 3*((b*x + a)^2 - 2*(b*x + a)*a)*log(c^2*cos(2*b
*x + 2*a)^2 + c^2*sin(2*b*x + 2*a)^2 + 2*c*sin(2*b*x + 2*a) + 1) + 3*polyl
og(3, I*c*e^(2*I*b*x + 2*I*a)))*(-I*c - 1)/(b*(c - I)))/b
```

3.163.8 Giac [F]

$$\int x\cot^{-1}(c+(1+ic)\tan(a+bx))\,dx=\int x\,\mathrm{arccot}\left((i\,c+1)\tan\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x*arccot(c+(1+I*c)*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arccot((I*c + 1)*tan(b*x + a) + c), x)
```

3.163.9 Mupad [F(-1)]

Timed out.

$$\int x\cot^{-1}(c+(1+ic)\tan(a+bx))\,dx=\int x\,\mathrm{acot}(c+\tan(a+b\,x)\,\left(1+c\,1\mathrm{i}\right))\,dx$$

input
```
int(x*acot(c + tan(a + b*x)*(c*1i + 1)),x)
```

output
```
int(x*acot(c + tan(a + b*x)*(c*1i + 1)), x)
```

3.164 $\int \cot^{-1}(c+(1+ic)\tan(a+bx))\,dx$

3.164.1 Optimal result

Integrand size = 17, antiderivative size = 85

$$\int \cot^{-1}(c+(1+ic)\tan(a+bx))\,dx=\frac{bx^2}{2}+x\cot^{-1}(c+(1+ic)\tan(a+bx))+\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)+\frac{\operatorname{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}$$

output
```
1/2*b*x^2+x*arccot(c+(1+I*c)*tan(b*x+a))+1/2*I*x*ln(1-I*c*exp(2*I*a+2*I*b*
x))+1/4*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b
```

3.164.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 967 vs. 2(85) = 170.

Time = 4.19 (sec) , antiderivative size = 967, normalized size of antiderivative = 11.38

$$\int \cot^{-1}(c+(1+ic)\tan(a+bx))\,dx=x\cot^{-1}(c+(1+ic)\tan(a+bx))$$

$$-\frac{}{((i+c)\cos(a+bx)+(1+ic)\sin(a+bx))\left(2bx-i\log\left(1-\frac{\sec(bx)((-i+c)\cos(a)+i(i+c)\sin(a))(\cos(a+bx)-i\sin(a}{2c}\right.\right.}$$

input
```
Integrate[ArcCot[c + (1 + I*c)*Tan[a + b*x]],x]
```

output
```
x*ArcCot[c + (1 + I*c)*Tan[a + b*x]] - (I*x*((2*I)*b*x*Log[2*Cos[b*x]*(Cos
[b*x] - I*Sin[b*x])] - Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((I + c)*Cos[a +
b*x] + (1 + I*c)*Sin[a + b*x]))/(2*c)]*Log[1 - I*Tan[b*x]] + Log[(Sec[b*x]
*((1 - I*c)*Cos[a + b*x] + (-I + c)*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a
])]*Log[1 + I*Tan[b*x]] - PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] - PolyLog
[2, (Sec[b*x]*((-I + c)*Cos[a] + I*(I + c)*Sin[a])*(Cos[a + b*x] - I*Sin[a
 + b*x]))/(2*c)] + PolyLog[2, (Sec[b*x]*((1 + I*c)*Cos[a] - (I + c)*Sin[a]
)*(Cos[a + b*x] + I*Sin[a + b*x]))/2])*Sec[a + b*x]^2*(Cos[b*x] + I*Sin[b*
x])*(I*Cos[b*x] + Sin[b*x])*((1 - I*c)*Cos[a + b*x] + (-I + c)*Sin[a + b*x
]))/(((I + c)*Cos[a + b*x] + (1 + I*c)*Sin[a + b*x])*(2*b*x - I*Log[1 - (S
ec[b*x]*((-I + c)*Cos[a] + I*(I + c)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x
]))/(2*c)] - I*Log[1 + (Sec[b*x]*((-1 - I*c)*Cos[a] + (I + c)*Sin[a])*(Cos
[a + b*x] + I*Sin[a + b*x]))/2] - (I*(-I + c)*Cos[a + b*x]*(Log[1 - I*Tan[
b*x]] - Log[1 + I*Tan[b*x]]))/((I + c)*Cos[a + b*x] + (1 + I*c)*Sin[a + b*
x]) + ((I + c)*(Log[1 - I*Tan[b*x]] - Log[1 + I*Tan[b*x]])*Sin[a + b*x])/(
(I + c)*Cos[a + b*x] + (1 + I*c)*Sin[a + b*x]) - (2*I)*b*x*Tan[b*x] + Log[
1 - (Sec[b*x]*((-I + c)*Cos[a] + I*(I + c)*Sin[a])*(Cos[a + b*x] - I*Sin[a
 + b*x]))/(2*c)]*Tan[b*x] - Log[1 + (Sec[b*x]*((-1 - I*c)*Cos[a] + (I + c)
*Sin[a])*(Cos[a + b*x] + I*Sin[a + b*x]))/2]*Tan[b*x] - Log[1 - I*Tan[b*x]
]*Tan[b*x] + Cos[a]^2*Log[1 + I*Tan[b*x]]*Tan[b*x] + Log[1 + I*Tan[b*x]...
```

3.164.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {5687, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}(c+(1+ic)\tan(a+bx))\,dx$$

$$\Big\downarrow \text{5687}$$

$$ib\int \frac{x}{e^{2ia+2ibx}c+i}dx+x\cot^{-1}(c+(1+ic)\tan(a+bx))$$

$$\Big\downarrow \text{2615}$$

$$ib\left(ic\int \frac{e^{2ia+2ibx}x}{e^{2ia+2ibx}c+i}dx-\frac{ix^2}{2}\right)+x\cot^{-1}(c+(1+ic)\tan(a+bx))$$

$$\Big\downarrow \text{2620}$$

$$ib\left(ic\left(\frac{i\int\log\left(1-ice^{2ia+2ibx}\right)dx}{2bc}-\frac{ix\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right)-\frac{ix^2}{2}\right)+x\cot^{-1}(c+(1+ic)\tan(a+bx))$$

$\big\downarrow$ 2715

$$ib\left(ic\left(\frac{\int e^{-2ia-2ibx}\log\left(1-ice^{2ia+2ibx}\right)de^{2ia+2ibx}}{4b^2c}-\frac{ix\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right)-\frac{ix^2}{2}\right)+x\cot^{-1}(c+(1+ic)\tan(a+bx))$$

$\big\downarrow$ 2838

$$ib\left(ic\left(-\frac{\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b^2c}-\frac{ix\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right)-\frac{ix^2}{2}\right)+x\cot^{-1}(c+(1+ic)\tan(a+bx))$$

input
```
Int[ArcCot[c + (1 + I*c)*Tan[a + b*x]],x]
```

output
```
x*ArcCot[c + (1 + I*c)*Tan[a + b*x]] + I*b*((-1/2*I)*x^2 + I*c*(((-1/2*I)*
x*Log[1 - I*c*E^((2*I)*a + (2*I)*b*x)])/(b*c) - PolyLog[2, I*c*E^((2*I)*a
+ (2*I)*b*x)]/(4*b^2*c)))
```

3.164.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5687
```
Int[ArcCot[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcC
ot[c + d*Tan[a + b*x]], x] + Simp[I*b   Int[x/(c + I*d + c*E^(2*I*a + 2*I*b
*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c + I*d)^2, -1]
```

3.164.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 561 vs. 2(69) = 138.

Time = 2.06 (sec) , antiderivative size = 562, normalized size of antiderivative = 6.61

method	result
derivativedivides	$\frac{\frac{\operatorname{arccot}(c+(ic+1)\tan(bx+a))\ln(-c+(ic+1)\tan(bx+a)+i)c^2}{2i-2c}-\frac{2i\operatorname{arccot}(c+(ic+1)\tan(bx+a))\ln(-c+(ic+1)\tan(bx+a)+i)c}{2i-2c}-\operatorname{arccot}(c\cdots}{\cdots}$
default	$\frac{\frac{\operatorname{arccot}(c+(ic+1)\tan(bx+a))\ln(-c+(ic+1)\tan(bx+a)+i)c^2}{2i-2c}-\frac{2i\operatorname{arccot}(c+(ic+1)\tan(bx+a))\ln(-c+(ic+1)\tan(bx+a)+i)c}{2i-2c}-\operatorname{arccot}(c\cdots}{\cdots}$
risch	Expression too large to display

input
```
int(arccot(c+(I*c+1)*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/(I*c+1)*(arccot(c+(I*c+1)*tan(b*x+a))/(2*I-2*c)*ln(-c+(I*c+1)*tan(b*x+
a)+I)*c^2-2*I*arccot(c+(I*c+1)*tan(b*x+a))/(2*I-2*c)*ln(-c+(I*c+1)*tan(b*x
+a)+I)*c-arccot(c+(I*c+1)*tan(b*x+a))/(2*I-2*c)*ln(-c+(I*c+1)*tan(b*x+a)+I
)-arccot(c+(I*c+1)*tan(b*x+a))/(2*I-2*c)*ln(-I+c+(I*c+1)*tan(b*x+a))*c^2+2
*I*arccot(c+(I*c+1)*tan(b*x+a))/(2*I-2*c)*ln(-I+c+(I*c+1)*tan(b*x+a))*c+ar
ccot(c+(I*c+1)*tan(b*x+a))/(2*I-2*c)*ln(-I+c+(I*c+1)*tan(b*x+a))+(I*c+1)^2
*(1/2/(I-c)*(-1/4*I*ln(-I+c+(I*c+1)*tan(b*x+a))^2+1/2*I*(dilog(-1/2*I*(c+(
I*c+1)*tan(b*x+a)+I))+ln(-I+c+(I*c+1)*tan(b*x+a))*ln(-1/2*I*(c+(I*c+1)*tan
(b*x+a)+I))))-1/2/(I-c)*(1/2*I*(dilog(1/2*(c+(I*c+1)*tan(b*x+a)+I)/c)+ln(-
c+(I*c+1)*tan(b*x+a)+I)*ln(1/2*(c+(I*c+1)*tan(b*x+a)+I)/c))-1/2*I*(dilog((
-I+c+(I*c+1)*tan(b*x+a))/(-2*I+2*c))+ln(-c+(I*c+1)*tan(b*x+a)+I)*ln((-I+c+
(I*c+1)*tan(b*x+a))/(-2*I+2*c)))))
```

3.164.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(60) = 120$.

Time = 0.28 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.34

$$\int \cot^{-1}(c + (1 + ic)\tan(a + bx))\,dx$$
$$= \frac{b^2x^2 - i\,bx\log\left(\frac{(ce^{(2i\,bx+2i\,a)}+i)e^{(-2i\,bx-2i\,a)}}{c-i}\right) - a^2 + (i\,bx + i\,a)\log\left(\frac{1}{2}\sqrt{4i\,c}e^{(i\,bx+i\,a)} + 1\right) + (i\,bx + i\,a)\log\left(-\right.}{}$$

input
```
integrate(arccot(c+(1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(b^2*x^2 - I*b*x*log((c*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*I*a)/
(c - I)) - a^2 + (I*b*x + I*a)*log(1/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1) +
(I*b*x + I*a)*log(-1/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1) - I*a*log(1/2*(2*c
*e^(I*b*x + I*a) + I*sqrt(4*I*c))/c) - I*a*log(1/2*(2*c*e^(I*b*x + I*a) -
I*sqrt(4*I*c))/c) + dilog(1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + dilog(-1/2*sq
rt(4*I*c)*e^(I*b*x + I*a)))/b
```

3.164.6 Sympy [F(-2)]

Exception generated.

$$\int \cot^{-1}(c + (1 + ic)\tan(a + bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(acot(c+(1+I*c)*tan(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert _t0**2*I + 2*c*exp(2*I*
a) - I*exp(2*I*a) of type <class 'sympy.core.add.Add'> to QQ_I[b,c,_t0,exp
(I*a)]
```

3.164.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 455 vs. $2(60) = 120$.

Time = 0.29 (sec) , antiderivative size = 455, normalized size of antiderivative = 5.35

$$\int \cot^{-1}(c+(1+ic)\tan(a+bx))\,dx =$$

$$-\frac{(-i\,c-1)\left(\frac{4i\,(bx+a)\log\left(-\frac{2\left(-i\,c^2+\left(c^2-2i\,c-1\right)\tan(bx+a)-2\,c+i\right)}{2i\,c^2-2\left(c^2-2i\,c-1\right)\tan(bx+a)+2i}\right)}{i\,c+1}-\frac{i\left(4\,(bx+a)(\log(-i\,c^2+(c^2-2i\,c-1)\tan(bx+a)-2\,c+i)-\log(-\right.}{}\right.}{}$$

input
```
integrate(arccot(c+(1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

output
```
-1/8*((-I*c - 1)*(4*I*(b*x + a)*log(-2*(-I*c^2 + (c^2 - 2*I*c - 1)*tan(b*x
 + a) - 2*c + I)/(2*I*c^2 - 2*(c^2 - 2*I*c - 1)*tan(b*x + a) + 2*I))/(I*c
+ 1) - I*(4*(b*x + a)*(log(-I*c^2 + (c^2 - 2*I*c - 1)*tan(b*x + a) - 2*c +
 I) - log(-I*c^2 + (c^2 - 2*I*c - 1)*tan(b*x + a) - I)) + I*log(-I*c^2 + (
c^2 - 2*I*c - 1)*tan(b*x + a) - 2*c + I)^2 - 2*I*log(-I*c^2 + (c^2 - 2*I*c
 - 1)*tan(b*x + a) - I)*log(-1/2*(c - I)*tan(b*x + a) + 1/2*I*c + 1/2) + 2
*I*log(-I*c^2 + (c^2 - 2*I*c - 1)*tan(b*x + a) - I)*log(-1/2*((I*c + 1)*ta
n(b*x + a) + c + I)/c + 1) - 2*I*log(-I*c^2 + (c^2 - 2*I*c - 1)*tan(b*x +
a) - 2*c + I)*log(-1/2*I*tan(b*x + a) + 1/2) - 2*I*dilog(1/2*(c - I)*tan(b
*x + a) - 1/2*I*c + 1/2) + 2*I*dilog(1/2*((I*c + 1)*tan(b*x + a) + c + I)/
c) - 2*I*dilog(1/2*I*tan(b*x + a) + 1/2))/(I*c + 1)) - 8*(b*x + a)*arccot(
(I*c + 1)*tan(b*x + a) + c) - 4*(b*x + a)*(c - I)*log(-2*(-I*c^2 + (c^2 -
2*I*c - 1)*tan(b*x + a) - 2*c + I)/(2*I*c^2 - 2*(c^2 - 2*I*c - 1)*tan(b*x
+ a) + 2*I))/(I*c + 1))/b
```

3.164.8 Giac [F]

$$\int \cot^{-1}(c+(1+ic)\tan(a+bx))\,dx = \int \operatorname{arccot}\left((i\,c+1)\tan\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(arccot(c+(1+I*c)*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccot((I*c + 1)*tan(b*x + a) + c), x)
```

3.164.9 Mupad [F(-1)]

Timed out.

$$\int \cot^{-1}(c+(1+ic)\tan(a+bx))\,dx=\int \operatorname{acot}(c+\tan(a+b\,x)\,\,(1+c\,1\mathrm{i}))\,\,dx$$

input
```
int(acot(c + tan(a + b*x)*(c*1i + 1)),x)
```

output
```
int(acot(c + tan(a + b*x)*(c*1i + 1)), x)
```

3.165 $\int \frac{\cot^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx$

3.165.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\cot^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx = \text{Int}\left(\frac{\cot^{-1}(c+(1+ic)\tan(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(arccot(c+(1+I*c)*tan(b*x+a))/x,x)
```

3.165.2 Mathematica [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\cot^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx = \int \frac{\cot^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx$$

input
```
Integrate[ArcCot[c + (1 + I*c)*Tan[a + b*x]]/x,x]
```

output
```
Integrate[ArcCot[c + (1 + I*c)*Tan[a + b*x]]/x, x]
```

3.165.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx$$
$$\Big\downarrow \ 7299$$
$$\int \frac{\cot^{-1}(c+(1+ic)\tan(a+bx))}{x}dx$$

input
```
Int[ArcCot[c + (1 + I*c)*Tan[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.165.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.165.4 Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\operatorname{arccot}\left(c+(ic+1)\tan\left(bx+a\right)\right)}{x}dx$$

input
```
int(arccot(c+(I*c+1)*tan(b*x+a))/x,x)
```

output
```
int(arccot(c+(I*c+1)*tan(b*x+a))/x,x)
```

3.165.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.71

$$\int \frac{\cot^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx=\int \frac{\operatorname{arccot}\left((i\,c+1)\tan\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccot(c+(1+I*c)*tan(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(-1/2*I*log((c*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*I*a)/(c -
I))/x, x)
```

3.165.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx=\text{Timed out}$$

input
```
integrate(acot(c+(1+I*c)*tan(b*x+a))/x,x)
```

output
```
Timed out
```

3.165.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(arccot(c+(1+I*c)*tan(b*x+a))/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c-1>0)', see `assume?` for more
details)Is
```

3.165.8 Giac [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx = \int \frac{\operatorname{arccot}\left((i\,c+1)\tan\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccot(c+(1+I*c)*tan(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arccot((I*c + 1)*tan(b*x + a) + c)/x, x)
```

3.165.9 Mupad [N/A]

Not integrable

Time = 1.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{\cot^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx = \int \frac{\operatorname{acot}(c+\tan(a+b\,x)\ (1+c\,1\mathrm{i}))}{x}\,dx$$

input
```
int(acot(c + tan(a + b*x)*(c*1i + 1))/x,x)
```

output
```
int(acot(c + tan(a + b*x)*(c*1i + 1))/x, x)
```

3.166 $\int x^2 \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx$

3.166.1 Optimal result

Integrand size = 22, antiderivative size = 155

$$\int x^2 \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx = -\frac{bx^4}{12}+\frac{1}{3}x^3\cot^{-1}(c-(1-ic)\tan(a+bx)) -\frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right) -\frac{x^2\,\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b} -\frac{ix\,\text{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{4b^2} +\frac{\text{PolyLog}\left(4,-ice^{2ia+2ibx}\right)}{8b^3}$$

output
```
-1/12*b*x^4+1/3*x^3*arccot(c-(1-I*c)*tan(b*x+a))-1/6*I*x^3*ln(1+I*c*exp(2*
I*a+2*I*b*x))-1/4*x^2*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b-1/4*I*x*polylog
(3,-I*c*exp(2*I*a+2*I*b*x))/b^2+1/8*polylog(4,-I*c*exp(2*I*a+2*I*b*x))/b^3
```

3.166.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.91

$$\int x^2 \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx = \frac{1}{3}x^3\cot^{-1}(c+i(i+c)\tan(a+bx)) - \frac{4ib^3x^3\log\left(1-\frac{ie^{-2i(a+bx)}}{c}\right)-6b^2x^2\,\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{c}\right)+6ibx\,\text{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{c}\right)+3\,\text{PolyLog}\left(\right.}{24b^3}$$

input
```
Integrate[x^2*ArcCot[c - (1 - I*c)*Tan[a + b*x]],x]
```

output
```
(x^3*ArcCot[c + I*(I + c)*Tan[a + b*x]])/3 - ((4*I)*b^3*x^3*Log[1 - I/(c*E
^((2*I)*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, I/(c*E^((2*I)*(a + b*x)))] + (
6*I)*b*x*PolyLog[3, I/(c*E^((2*I)*(a + b*x)))] + 3*PolyLog[4, I/(c*E^((2*I
)*(a + b*x)))])/(24*b^3)
```

3.166.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.28, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5695, 25, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\cot^{-1}(c-(1-ic)\tan(a+bx))\,dx$$

$$\Big\downarrow \text{ 5695}$$

$$\frac{1}{3}ib\int -\frac{x^3}{i-ce^{2ia+2ibx}}dx+\frac{1}{3}x^3\cot^{-1}(c-(1-ic)\tan(a+bx))$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{3}x^3\cot^{-1}(c-(1-ic)\tan(a+bx))-\frac{1}{3}ib\int\frac{x^3}{i-ce^{2ia+2ibx}}dx$$

$$\Big\downarrow \text{ 2615}$$

$$\frac{1}{3}x^3\cot^{-1}(c-(1-ic)\tan(a+bx))-\frac{1}{3}ib\left(-ic\int\frac{e^{2ia+2ibx}x^3}{i-ce^{2ia+2ibx}}dx-\frac{ix^4}{4}\right)$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{1}{3}x^3\cot^{-1}(c-(1-ic)\tan(a+bx))-\frac{1}{3}ib\left(-ic\left(\frac{ix^3\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{3i\int x^2\log\left(ie^{2ia+2ibx}c+1\right)dx}{2bc}\right)-\frac{ix^4}{4}\right)$$

$\downarrow$ 3011

$$\frac{1}{3}x^3\cot^{-1}(c-(1-ic)\tan(a+bx))-\frac{1}{3}ib\left(-ic\left(\frac{ix^3\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{3i\left(\frac{ix^2\operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{2b}-\frac{i\int x\operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)dx}{b}\right)}{2bc}\right)-\frac{ix^4}{4}\right)$$

$\downarrow$ 7163

$$\frac{1}{3}x^3\cot^{-1}(c-(1-ic)\tan(a+bx))-\frac{1}{3}ib\left(-ic\left(\frac{ix^3\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{3i\left(\frac{ix^2\operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{2b}-\frac{i\left(\frac{i\int\operatorname{PolyLog}\left(3,-ice^{2ia+2ibx}\right)dx}{2b}-\frac{ix\operatorname{PolyLog}\left(3,-ice^{2ia+2ib\ldots}\right.}{2b}\right.}{b}\right.}{2bc}\right.\right.$$

$\downarrow$ 2720

$$\frac{1}{3}x^3\cot^{-1}(c-(1-ic)\tan(a+bx))-\frac{1}{3}ib\left(-ic\left(\frac{ix^3\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{3i\left(\frac{ix^2\operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\operatorname{PolyLog}\left(3,-ice^{2ia+2ibx}\right)de^{2ia+2ibx}}{4b^2}-\frac{ix\operatorname{Pol}\ldots}{}\right.}{b}\right.}{2bc}\right.\right.$$

$\downarrow$ 7143

$$\frac{1}{3}x^3\cot^{-1}(c-(1-ic)\tan(a+bx))-\frac{1}{3}ib\left(-ic\left(\frac{ix^3\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{3i\left(\frac{ix^2\operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{2b}-\frac{i\left(\frac{\operatorname{PolyLog}\left(4,-ice^{2ia+2ibx}\right)}{4b^2}-\frac{ix\operatorname{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{2b}\right)}{b}\right.}{2bc}\right.\right.$$

3.166. $\int x^2\cot^{-1}(c-(1-ic)\tan(a+bx))\,dx$

input
```
Int[x^2*ArcCot[c - (1 - I*c)*Tan[a + b*x]],x]
```

output
```
(x^3*ArcCot[c - (1 - I*c)*Tan[a + b*x]])/3 - (I/3)*b*((-1/4*I)*x^4 - I*c*(
((I/2)*x^3*Log[1 + I*c*E^((2*I)*a + (2*I)*b*x)])/(b*c) - (((3*I)/2)*(((I/2
)*x^2*PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b - (I*(((-1/2*I)*x*Poly
Log[3, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b + PolyLog[4, (-I)*c*E^((2*I)*a +
 (2*I)*b*x)]/(4*b^2)))/b))/(b*c)))
```

3.166.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*(F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5695
```
Int[ArcCot[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Tan[a + b*x]]/(f*(m +
1))), x] + Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(2
*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && E
qQ[(c + I*d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.166.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.06 (sec) , antiderivative size = 1449, normalized size of antiderivative = 9.35

method	result	size
risch	Expression too large to display	1449

input
```
int(x^2*arccot(c-(1-I*c)*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/b^3*a^3*ln(1-I*exp(I*(b*x+a))*(I*c)^(1/2))-1/12*(Pi*csgn(I/(exp(2*I
*(b*x+a))+1))*csgn(I*(I+c))*csgn(I/(exp(2*I*(b*x+a))+1)*(I+c))-Pi*csgn(I/(
exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*c-I))*csgn(I*(exp(2*I*(b*x+a
))*c-I)/(exp(2*I*(b*x+a))+1))-Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I/(exp(
2*I*(b*x+a))+1)*(I+c))^2+Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(
b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^2+Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*ex
p(2*I*(b*x+a)))-2*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2+Pi*
csgn(I*exp(2*I*(b*x+a)))^3+Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I/(exp(2*I*(b*
x+a))+1)*(I+c))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))-Pi*csg
n(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^
2+Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))*csgn(exp(2*I*(b*x
+a))*(I+c)/(exp(2*I*(b*x+a))+1))-Pi*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(
b*x+a))+1))^2-Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))*csgn(
(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))-Pi*csgn((exp(2*I*(b*x+a))*c-I
)/(exp(2*I*(b*x+a))+1))^2-Pi*csgn(I*(I+c))*csgn(I/(exp(2*I*(b*x+a))+1)*(I+
c))^2+Pi*csgn(I*(exp(2*I*(b*x+a))*c-I))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp
(2*I*(b*x+a))+1))^2+Pi*csgn(I/(exp(2*I*(b*x+a))+1)*(I+c))^3-Pi*csgn(I/(exp
(2*I*(b*x+a))+1)*(I+c))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1)
)^2-Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^3+Pi*csgn(I*(ex
p(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*c-I)/(...
```

3.166.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(108) = 216.

Time = 0.27 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.07

$$\int x^2 \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx =$$

$$-\frac{b^4x^4 + 2i\,b^3x^3\log\left(\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}-i}\right) + 6\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4i\,c}e^{(i\,bx+i\,a)}\right) + 6\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,c}e^{(i\,bx+i\,a)}\right) - a^4}{}$$

input
```
integrate(x^2*arccot(c-(1-I*c)*tan(b*x+a)),x, algorithm="fricas")
```

output
```
-1/12*(b^4*x^4 + 2*I*b^3*x^3*log((c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*x
 + 2*I*a) - I)) + 6*b^2*x^2*dilog(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) + 6*b^
2*x^2*dilog(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) - a^4 - 2*I*a^3*log(1/2*(2*
c*e^(I*b*x + I*a) + I*sqrt(-4*I*c))/c) - 2*I*a^3*log(1/2*(2*c*e^(I*b*x + I
*a) - I*sqrt(-4*I*c))/c) + 12*I*b*x*polylog(3, 1/2*sqrt(-4*I*c)*e^(I*b*x +
 I*a)) + 12*I*b*x*polylog(3, -1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) + 2*(I*b^3
*x^3 + I*a^3)*log(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a) + 1) + 2*(I*b^3*x^3 + I
*a^3)*log(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a) + 1) - 12*polylog(4, 1/2*sqrt(
-4*I*c)*e^(I*b*x + I*a)) - 12*polylog(4, -1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)
))/b^3
```

3.166.6 Sympy [F(-2)]

Exception generated.

$$\int x^2 \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(x**2*acot(c-(1-I*c)*tan(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert -_t0**4 - 3*_t0**2*I*c*
exp(2*I*a) + _t0**2*exp(2*I*a) + 2*c**2*exp(4*I*a) + I*c*exp(4*I*a) of typ
e <class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,exp(I*a)]
```

3.166.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 312 vs. 2(108) = 216.

Time = 0.20 (sec) , antiderivative size = 312, normalized size of antiderivative = 2.01

$$\int x^2 \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx =$$

$$-\frac{\frac{4\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\operatorname{arccot}((-i\,c+1)\tan(bx+a)-c)}{b^2}+\frac{\left(-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2-2\left(-4i\,(bx+a)^3+9i\right.}{}}{}$$

input
```
integrate(x^2*arccot(c-(1-I*c)*tan(b*x+a)),x, algorithm="maxima")
```

output
```
-1/12*(4*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arccot((-I*c +
1)*tan(b*x + a) - c)/b^2 + (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(
b*x + a)^2*a^2 - 2*(-4*I*(b*x + a)^3 + 9*I*(b*x + a)^2*a - 9*I*(b*x + a)*a
^2)*arctan2(c*cos(2*b*x + 2*a), -c*sin(2*b*x + 2*a) + 1) - 3*(4*I*(b*x + a
)^2 - 6*I*(b*x + a)*a + 3*I*a^2)*dilog(-I*c*e^(2*I*b*x + 2*I*a)) + (4*(b*x
 + a)^3 - 9*(b*x + a)^2*a + 9*(b*x + a)*a^2)*log(c^2*cos(2*b*x + 2*a)^2 +
c^2*sin(2*b*x + 2*a)^2 - 2*c*sin(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog
(3, -I*c*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, -I*c*e^(2*I*b*x + 2*I*a)))*
(I*c - 1)/(b^2*(c + I)))/b
```

3.166.8 Giac [F]

$$\int x^2 \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx = \int x^2 \operatorname{arccot}\left(-(-i\,c+1)\tan\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x^2*arccot(c-(1-I*c)*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arccot(-(-I*c + 1)*tan(b*x + a) + c), x)
```

3.166.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx = \int x^2\,\mathrm{acot}(c+\tan(a+b\,x)\,\left(-1+c\,1\mathrm{i}\right))\,dx$$

input
```
int(x^2*acot(c + tan(a + b*x)*(c*1i - 1)),x)
```

output
```
int(x^2*acot(c + tan(a + b*x)*(c*1i - 1)), x)
```

3.167 $\int x\cot^{-1}(c-(1-ic)\tan(a+bx))\,dx$

3.167.1 Optimal result

Integrand size = 20, antiderivative size = 124

$$\int x\cot^{-1}(c-(1-ic)\tan(a+bx))\,dx = -\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c-(1-ic)\tan(a+bx)) -\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right) -\frac{x\,\mathrm{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b} -\frac{i\,\mathrm{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{8b^2}$$

output
```
-1/6*b*x^3+1/2*x^2*arccot(c-(1-I*c)*tan(b*x+a))-1/4*I*x^2*ln(1+I*c*exp(2*I
*a+2*I*b*x))-1/4*x*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b-1/8*I*polylog(3,-I
*c*exp(2*I*a+2*I*b*x))/b^2
```

3.167.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90

$$\int x\cot^{-1}(c-(1-ic)\tan(a+bx))\,dx = \frac{1}{2}x^2\cot^{-1}(c+i(i+c)\tan(a+bx)) -\frac{i\left(2b^2x^2\log\left(1-\frac{ie^{-2i(a+bx)}}{c}\right)+2ibx\,\mathrm{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{c}\right)+\mathrm{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{c}\right)\right)}{8b^2}$$

input
```
Integrate[x*ArcCot[c - (1 - I*c)*Tan[a + b*x]],x]
```

output
```
(x^2*ArcCot[c + I*(I + c)*Tan[a + b*x]])/2 - ((I/8)*(2*b^2*x^2*Log[1 - I/(
c*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, I/(c*E^((2*I)*(a + b*x)))]
+ PolyLog[3, I/(c*E^((2*I)*(a + b*x)))]))/b^2
```

3.167.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.26, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {5695, 25, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cot^{-1}(c-(1-ic)\tan(a+bx))\,dx$$

$$\Big\downarrow \text{5695}$$

$$\frac{1}{2}ib\int -\frac{x^2}{i-ce^{2ia+2ibx}}dx+\frac{1}{2}x^2\cot^{-1}(c-(1-ic)\tan(a+bx))$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}x^2\cot^{-1}(c-(1-ic)\tan(a+bx))-\frac{1}{2}ib\int \frac{x^2}{i-ce^{2ia+2ibx}}dx$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{2}x^2\cot^{-1}(c-(1-ic)\tan(a+bx))-\frac{1}{2}ib\left(-ic\int \frac{e^{2ia+2ibx}x^2}{i-ce^{2ia+2ibx}}dx-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{2}x^2\cot^{-1}(c-(1-ic)\tan(a+bx))-\frac{1}{2}ib\left(-ic\left(\frac{ix^2\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{i\int x\log\left(ie^{2ia+2ibx}c+1\right)dx}{bc}\right)-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}x^2\cot^{-1}(c-(1-ic)\tan(a+bx))-\frac{1}{2}ib\left(-ic\left(\frac{ix^2\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{i\left(\frac{ix\,\text{PolyLog}(2,-ice^{2ia+2ibx})}{2b}-\frac{i\int \text{PolyLog}(2,-ice^{2ia+2ibx})dx}{2b}\right)}{bc}\right)-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}x^2\cot^{-1}(c-(1-ic)\tan(a+bx))-$$
$$\frac{1}{2}ib\left(-ic\left(\frac{ix^2\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{i\left(\frac{ix\,\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{2b}-\frac{\int e^{-2ia-2ibx}\,\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)de^{2ia+2ibx}}{4b^2}\right)}{bc}\right)-\frac{ix}{3}\right.$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}x^2\cot^{-1}(c-(1-ic)\tan(a+bx))-$$
$$\frac{1}{2}ib\left(-ic\left(\frac{ix^2\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{i\left(\frac{ix\,\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{2b}-\frac{\text{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{4b^2}\right)}{bc}\right)-\frac{ix^3}{3}\right)$$

input
```
Int[x*ArcCot[c - (1 - I*c)*Tan[a + b*x]],x]
```

output
```
(x^2*ArcCot[c - (1 - I*c)*Tan[a + b*x]])/2 - (I/2)*b*((-1/3*I)*x^3 - I*c*(
((I/2)*x^2*Log[1 + I*c*E^((2*I)*a + (2*I)*b*x)])/(b*c) - (I*(((I/2)*x*Poly
Log[2, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b - PolyLog[3, (-I)*c*E^((2*I)*a +
 (2*I)*b*x)]/(4*b^2)))/(b*c)))
```

3.167.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5695
```
Int[ArcCot[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Tan[a + b*x]]/(f*(m +
1))), x] + Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(2
*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && E
qQ[(c + I*d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.167.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.64 (sec) , antiderivative size = 1414, normalized size of antiderivative = 11.40

method	result	size
risch	Expression too large to display	1414

input
```
int(x*arccot(c-(1-I*c)*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2*I/b^2*a^2*ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))-1/8*(Pi*csgn(I/(exp(2*I*(
b*x+a))+1))*csgn(I*(I+c))*csgn(I/(exp(2*I*(b*x+a))+1)*(I+c))-Pi*csgn(I/(ex
p(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*c-I))*csgn(I*(exp(2*I*(b*x+a))
*c-I)/(exp(2*I*(b*x+a))+1))-Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I/(exp(2*
I*(b*x+a))+1)*(I+c))^2+Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*
x+a))*c-I)/(exp(2*I*(b*x+a))+1))^2+Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(
2*I*(b*x+a)))-2*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2+Pi*cs
gn(I*exp(2*I*(b*x+a)))^3+Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I/(exp(2*I*(b*x+
a))+1)*(I+c))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))-Pi*csgn(
I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^2+
Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))*csgn(exp(2*I*(b*x+a
))*(I+c)/(exp(2*I*(b*x+a))+1))-Pi*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*
x+a))+1))^2-Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))*csgn((e
xp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))-Pi*csgn((exp(2*I*(b*x+a))*c-I)/
(exp(2*I*(b*x+a))+1))^2-Pi*csgn(I*(I+c))*csgn(I/(exp(2*I*(b*x+a))+1)*(I+c)
)^2+Pi*csgn(I*(exp(2*I*(b*x+a))*c-I))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2
*I*(b*x+a))+1))^2+Pi*csgn(I/(exp(2*I*(b*x+a))+1)*(I+c))^3-Pi*csgn(I/(exp(2
*I*(b*x+a))+1)*(I+c))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^
2-Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^3+Pi*csgn(I*(exp(
2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*c-I)/(ex...
```

3.167.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(86) = 172$.

Time = 0.27 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.18

$$\int x \cot^{-1}(c - (1 - ic)\tan(a + bx))\,dx =$$

$$-\frac{2\,b^3x^3 + 3i\,b^2x^2\log\left(\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}-i}\right) + 2\,a^3 + 6\,bx\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4i\,c}e^{(i\,bx+i\,a)}\right) + 6\,bx\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,c}e^{(i\,bx+i\,a)}\right)}{}$$

input
```
integrate(x*arccot(c-(1-I*c)*tan(b*x+a)),x, algorithm="fricas")
```

output
```
-1/12*(2*b^3*x^3 + 3*I*b^2*x^2*log((c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b
*x + 2*I*a) - I)) + 2*a^3 + 6*b*x*dilog(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a))
+ 6*b*x*dilog(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) + 3*I*a^2*log(1/2*(2*c*e^
(I*b*x + I*a) + I*sqrt(-4*I*c))/c) + 3*I*a^2*log(1/2*(2*c*e^(I*b*x + I*a)
- I*sqrt(-4*I*c))/c) + 3*(I*b^2*x^2 - I*a^2)*log(1/2*sqrt(-4*I*c)*e^(I*b*x
 + I*a) + 1) + 3*(I*b^2*x^2 - I*a^2)*log(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)
 + 1) + 6*I*polylog(3, 1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) + 6*I*polylog(3,
-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)))/b^2
```

3.167.6 Sympy [F(-2)]

Exception generated.

$$\int x \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(x*acot(c-(1-I*c)*tan(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert -_t0**4 - 3*_t0**2*I*c*
exp(2*I*a) + _t0**2*exp(2*I*a) + 2*c**2*exp(4*I*a) + I*c*exp(4*I*a) of typ
e <class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,exp(I*a)]
```

3.167.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(86) = 172$.

Time = 0.20 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.77

$$\int x \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx =$$

$$-\frac{\frac{6\left((bx+a)^2-2\,(bx+a)a\right)\operatorname{arccot}((-i\,c+1)\tan(bx+a)-c)}{b}+\frac{\left(-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\operatorname{Li}_2\left(-i\,ce^{(2i\,bx+2i\,a)}\right)-6\left(-i\,(bx+a)^2+2i\,(bx\right.}{}}{}$$

input
```
integrate(x*arccot(c-(1-I*c)*tan(b*x+a)),x, algorithm="maxima")
```

output
```
-1/12*(6*((b*x + a)^2 - 2*(b*x + a)*a)*arccot((-I*c + 1)*tan(b*x + a) - c)
/b + (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog(-I*c*e^(2*I*b*
x + 2*I*a)) - 6*(-I*(b*x + a)^2 + 2*I*(b*x + a)*a)*arctan2(c*cos(2*b*x + 2
*a), -c*sin(2*b*x + 2*a) + 1) + 3*((b*x + a)^2 - 2*(b*x + a)*a)*log(c^2*co
s(2*b*x + 2*a)^2 + c^2*sin(2*b*x + 2*a)^2 - 2*c*sin(2*b*x + 2*a) + 1) + 3*
polylog(3, -I*c*e^(2*I*b*x + 2*I*a)))*(I*c - 1)/(b*(c + I)))/b
```

3.167.8 Giac [F]

$$\int x\cot^{-1}(c-(1-ic)\tan(a+bx))\,dx=\int x\,\mathrm{arccot}\left(-(-i\,c+1)\tan\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x*arccot(c-(1-I*c)*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arccot(-(-I*c + 1)*tan(b*x + a) + c), x)
```

3.167.9 Mupad [F(-1)]

Timed out.

$$\int x\cot^{-1}(c-(1-ic)\tan(a+bx))\,dx=\int x\,\mathrm{acot}(c+\tan(a+b\,x)\,\left(-1+c\,1\mathrm{i}\right))\,dx$$

input
```
int(x*acot(c + tan(a + b*x)*(c*1i - 1)),x)
```

output
```
int(x*acot(c + tan(a + b*x)*(c*1i - 1)), x)
```

3.168 $\int \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx$

3.168.1 Optimal result

Integrand size = 18, antiderivative size = 86

$$\int \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx = -\frac{bx^2}{2}+x\cot^{-1}(c-(1-ic)\tan(a+bx)) -\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)-\frac{\operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}$$

output
```
-1/2*b*x^2+x*arccot(c-(1-I*c)*tan(b*x+a))-1/2*I*x*ln(1+I*c*exp(2*I*a+2*I*b
*x))-1/4*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b
```

3.168.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 847 vs. 2(86) = 172.

Time = 2.35 (sec) , antiderivative size = 847, normalized size of antiderivative = 9.85

$$\int \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx = x\cot^{-1}(c+i(i+c)\tan(a+bx)) - \frac{ix\left(-2ibx\log(2\cos(bx)(\cos(bx)-i\sin(bx)))+\log\left(\frac{\sec(bx)(\cos(a)\cdots}{\cdots}\right.\right.}{((-i+c)\cos(a+bx)+i(i+c)\sin(a+bx))\left(-2bx+i\log\left(1-\frac{\sec(bx)((i+c)\cos(a)+(1+ic)\sin(a))(\cos(a+bx)-i\,\mathrm{si}\cdots}{2c}\right.\right.}$$

input
```
Integrate[ArcCot[c - (1 - I*c)*Tan[a + b*x]],x]
```

output
```
x*ArcCot[c + I*(I + c)*Tan[a + b*x]] - (I*x*((-2*I)*b*x*Log[2*Cos[b*x]*(Co
s[b*x] - I*Sin[b*x])] + Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((-I + c)*Cos[a
+ b*x] + I*(I + c)*Sin[a + b*x]))/(2*c)]*Log[1 - I*Tan[b*x]] - Log[(Sec[b*
x]*(Cos[a] + I*Sin[a])*((1 + I*c)*Cos[a + b*x] - (I + c)*Sin[a + b*x]))/2]
*Log[1 + I*Tan[b*x]] + PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] + PolyLog[2,
 (Sec[b*x]*((I + c)*Cos[a] + (1 + I*c)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b
*x]))/(2*c)] - PolyLog[2, ((Cos[a] + I*Sin[a])*((I + c)*Cos[a] + (1 + I*c)
*Sin[a])*(-I + Tan[b*x]))/2])*Sec[a + b*x]*(Cos[b*x] + I*Sin[b*x])*(I*Cos[
b*x] + Sin[b*x]))/(((-I + c)*Cos[a + b*x] + I*(I + c)*Sin[a + b*x])*(-2*b*
x + I*Log[1 - (Sec[b*x]*((I + c)*Cos[a] + (1 + I*c)*Sin[a])*(Cos[a + b*x]
- I*Sin[a + b*x]))/(2*c)] + (I*(I + c)*Cos[a + b*x]*(Log[1 - I*Tan[b*x]] -
 Log[1 + I*Tan[b*x]]))/((-I + c)*Cos[a + b*x] + I*(I + c)*Sin[a + b*x]) +
((1 + I*c)*(Log[1 - I*Tan[b*x]] - Log[1 + I*Tan[b*x]])*Sin[a + b*x])/((-1
- I*c)*Cos[a + b*x] + (I + c)*Sin[a + b*x]) + (2*I)*b*x*Tan[b*x] - Log[1 -
 (Sec[b*x]*((I + c)*Cos[a] + (1 + I*c)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b
*x]))/(2*c)]*Tan[b*x] + Log[1 - I*Tan[b*x]]*Tan[b*x] - Log[1 + I*Tan[b*x]]
*Tan[b*x] - (Log[(Sec[b*x]*(Cos[a] + I*Sin[a])*((1 + I*c)*Cos[a + b*x] - (
I + c)*Sin[a + b*x]))/2]*Sec[b*x]^2)/(-I + Tan[b*x]) + (Log[1 - ((Cos[a] +
 I*Sin[a])*((I + c)*Cos[a] + (1 + I*c)*Sin[a])*(-I + Tan[b*x]))/2]*Sec[b*x
]^2)/(-I + Tan[b*x]) + (Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((-I + c)*Cos...
```

3.168.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.26, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5687, 25, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx$$

$$\Big\downarrow \text{5687}$$

$$ib\int -\frac{x}{i-ce^{2ia+2ibx}}dx+x\cot^{-1}(c-(1-ic)\tan(a+bx))$$

$$\Big\downarrow \text{25}$$

$$x\cot^{-1}(c-(1-ic)\tan(a+bx))-ib\int \frac{x}{i-ce^{2ia+2ibx}}dx$$

$$\Big\downarrow \text{2615}$$

$$x\cot^{-1}(c-(1-ic)\tan(a+bx))-ib\left(-ic\int\frac{e^{2ia+2ibx}x}{i-ce^{2ia+2ibx}}dx-\frac{ix^2}{2}\right)$$

$\Big\downarrow$ 2620

$$x\cot^{-1}(c-(1-ic)\tan(a+bx))-ib\left(-ic\left(\frac{ix\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{i\int\log\left(ie^{2ia+2ibx}c+1\right)dx}{2bc}\right)-\frac{ix^2}{2}\right)$$

$\Big\downarrow$ 2715

$$x\cot^{-1}(c-(1-ic)\tan(a+bx))-ib\left(-ic\left(\frac{ix\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{\int e^{-2ia-2ibx}\log\left(ie^{2ia+2ibx}c+1\right)de^{2ia+2ibx}}{4b^2c}\right)-\frac{ix^2}{2}\right)$$

$\Big\downarrow$ 2838

$$x\cot^{-1}(c-(1-ic)\tan(a+bx))-ib\left(-ic\left(\frac{\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b^2c}+\frac{ix\log\left(1+ice^{2ia+2ibx}\right)}{2bc}\right)-\frac{ix^2}{2}\right)$$

input
```
Int[ArcCot[c - (1 - I*c)*Tan[a + b*x]],x]
```

output
```
x*ArcCot[c - (1 - I*c)*Tan[a + b*x]] - I*b*((-1/2*I)*x^2 - I*c*(((I/2)*x*L
og[1 + I*c*E^((2*I)*a + (2*I)*b*x)])/(b*c) + PolyLog[2, (-I)*c*E^((2*I)*a
+ (2*I)*b*x)]/(4*b^2*c)))
```

3.168.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5687
```
Int[ArcCot[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcC
ot[c + d*Tan[a + b*x]], x] + Simp[I*b   Int[x/(c + I*d + c*E^(2*I*a + 2*I*b
*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c + I*d)^2, -1]
```

3.168.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 594 vs. $2(70) = 140$.

Time = 2.14 (sec) , antiderivative size = 595, normalized size of antiderivative = 6.92

method	result
derivativedivides	$\frac{\frac{\operatorname{arccot}(c+(ic-1)\tan(bx+a))\ln(i+c+(ic-1)\tan(bx+a))c^2}{2i+2c}+\frac{2i\operatorname{arccot}(c+(ic-1)\tan(bx+a))\ln(i+c+(ic-1)\tan(bx+a))c}{2i+2c}-\frac{\operatorname{arccot}(c+(i}{}}{}$
default	$\frac{\frac{\operatorname{arccot}(c+(ic-1)\tan(bx+a))\ln(i+c+(ic-1)\tan(bx+a))c^2}{2i+2c}+\frac{2i\operatorname{arccot}(c+(ic-1)\tan(bx+a))\ln(i+c+(ic-1)\tan(bx+a))c}{2i+2c}-\frac{\operatorname{arccot}(c+(i}{}}{}$
risch	Expression too large to display

input
```
int(arccot(c-(1-I*c)*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/(-1+I*c)*(arccot(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(I+c+(-1+I*c)*tan(
b*x+a))*c^2+2*I*arccot(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(I+c+(-1+I*c)*ta
n(b*x+a))*c-arccot(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(I+c+(-1+I*c)*tan(b*
x+a))-arccot(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(c-(-1+I*c)*tan(b*x+a)+I)*
c^2-2*I*arccot(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(c-(-1+I*c)*tan(b*x+a)+I
)*c+arccot(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(c-(-1+I*c)*tan(b*x+a)+I)-(-
1+I*c)^2*(1/2/(I+c)*(1/4*I*ln(I+c+(-1+I*c)*tan(b*x+a))^2-1/2*I*((ln(I+c+(-
1+I*c)*tan(b*x+a))-ln(-1/2*I*(I+c+(-1+I*c)*tan(b*x+a))))*ln(-1/2*I*(I-c-(-
1+I*c)*tan(b*x+a)))-dilog(-1/2*I*(I+c+(-1+I*c)*tan(b*x+a)))))-1/2/(I+c)*(1
/2*I*(dilog((-I-c-(-1+I*c)*tan(b*x+a))/(-2*I-2*c))+ln(c-(-1+I*c)*tan(b*x+a
)+I)*ln((-I-c-(-1+I*c)*tan(b*x+a))/(-2*I-2*c)))-1/2*I*(dilog(-1/2*(I-c-(-1
+I*c)*tan(b*x+a))/c)+ln(c-(-1+I*c)*tan(b*x+a)+I)*ln(-1/2*(I-c-(-1+I*c)*tan
(b*x+a))/c)))))
```

3.168.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(61) = 122$.

Time = 0.26 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.34

$$\int \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx =$$

$$-\frac{b^2x^2 + i\,bx\log\left(\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}-i}\right) - a^2 - (-i\,bx - i\,a)\log\left(\frac{1}{2}\sqrt{-4i\,c}e^{(i\,bx+i\,a)} + 1\right) - (-i\,bx - i\,a)\log\left(-\frac{1}{2}\right.}{}$$

input
```
integrate(arccot(c-(1-I*c)*tan(b*x+a)),x, algorithm="fricas")
```

output
```
-1/2*(b^2*x^2 + I*b*x*log((c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*x + 2*I*
a) - I)) - a^2 - (-I*b*x - I*a)*log(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a) + 1)
- (-I*b*x - I*a)*log(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a) + 1) - I*a*log(1/2*
(2*c*e^(I*b*x + I*a) + I*sqrt(-4*I*c))/c) - I*a*log(1/2*(2*c*e^(I*b*x + I*
a) - I*sqrt(-4*I*c))/c) + dilog(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) + dilog(
-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)))/b
```

3.168.6 Sympy [F(-2)]

Exception generated.

$$\int \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(acot(c-(1-I*c)*tan(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert -_t0**4 - 3*_t0**2*I*c*
exp(2*I*a) + _t0**2*exp(2*I*a) + 2*c**2*exp(4*I*a) + I*c*exp(4*I*a) of typ
e <class 'sympy.core.add.Add'> to QQ_I[b,c,_t0,exp(I*a)]
```

3.168.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(61) = 122$.

Time = 0.29 (sec) , antiderivative size = 450, normalized size of antiderivative = 5.23

$$\int \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx$$
$$= \frac{(i\,c-1)\left(\frac{4i\,(bx+a)\log\left(-\frac{2\left(-i\,c^2+\left(c^2+2i\,c-1\right)\tan(bx+a)-i\right)}{2i\,c^2-2\left(c^2+2i\,c-1\right)\tan(bx+a)-4\,c-2i}\right)}{i\,c-1} + \frac{i\left(4\,(bx+a)(\log(-i\,c^2+(c^2+2i\,c-1)\tan(bx+a)+2\,c+i)-\log(-i\,c^2+(c\right.}{}\right.}{}$$

input
```
integrate(arccot(c-(1-I*c)*tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/8*((I*c - 1)*(4*I*(b*x + a)*log(-2*(-I*c^2 + (c^2 + 2*I*c - 1)*tan(b*x +
 a) - I)/(2*I*c^2 - 2*(c^2 + 2*I*c - 1)*tan(b*x + a) - 4*c - 2*I))/(I*c -
1) + I*(4*(b*x + a)*(log(-I*c^2 + (c^2 + 2*I*c - 1)*tan(b*x + a) + 2*c + I
) - log(-I*c^2 + (c^2 + 2*I*c - 1)*tan(b*x + a) - I)) + I*log(-I*c^2 + (c^
2 + 2*I*c - 1)*tan(b*x + a) + 2*c + I)^2 - 2*I*log(-I*c^2 + (c^2 + 2*I*c -
 1)*tan(b*x + a) - I)*log(1/2*(c + I)*tan(b*x + a) - 1/2*I*c + 1/2) + 2*I*
log(-I*c^2 + (c^2 + 2*I*c - 1)*tan(b*x + a) - I)*log(-1/2*((I*c - 1)*tan(b
*x + a) + c - I)/c + 1) - 2*I*log(-I*c^2 + (c^2 + 2*I*c - 1)*tan(b*x + a)
+ 2*c + I)*log(-1/2*I*tan(b*x + a) + 1/2) - 2*I*dilog(-1/2*(c + I)*tan(b*x
 + a) + 1/2*I*c + 1/2) + 2*I*dilog(1/2*((I*c - 1)*tan(b*x + a) + c - I)/c)
 - 2*I*dilog(1/2*I*tan(b*x + a) + 1/2))/(I*c - 1)) - 8*(b*x + a)*arccot((-
I*c + 1)*tan(b*x + a) - c) + 4*(-I*b*x - I*a)*log(-2*(-I*c^2 + (c^2 + 2*I*
c - 1)*tan(b*x + a) - I)/(2*I*c^2 - 2*(c^2 + 2*I*c - 1)*tan(b*x + a) - 4*c
 - 2*I)))/b
```

3.168.8 Giac [F]

$$\int \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx = \int \operatorname{arccot}\left(-(-i\,c+1)\tan\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(arccot(c-(1-I*c)*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccot(-(-I*c + 1)*tan(b*x + a) + c), x)
```

3.168.9 Mupad [F(-1)]

Timed out.

$$\int \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx = \int \mathrm{acot}(c+\tan(a+b\,x)\,\left(-1+c\,1\mathrm{i}\right))\,dx$$

input
```
int(acot(c + tan(a + b*x)*(c*1i - 1)),x)
```

output
```
int(acot(c + tan(a + b*x)*(c*1i - 1)), x)
```

3.169 $\int \frac{\cot^{-1}(c-(1-ic)\tan(a+bx))}{x}\,dx$

3.169.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\cot^{-1}(c-(1-ic)\tan(a+bx))}{x}\,dx = \text{Int}\left(\frac{\cot^{-1}(c-(1-ic)\tan(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(arccot(c-(1-I*c)*tan(b*x+a))/x,x)
```

3.169.2 Mathematica [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cot^{-1}(c-(1-ic)\tan(a+bx))}{x}\,dx = \int \frac{\cot^{-1}(c-(1-ic)\tan(a+bx))}{x}\,dx$$

input
```
Integrate[ArcCot[c - (1 - I*c)*Tan[a + b*x]]/x,x]
```

output
```
Integrate[ArcCot[c - (1 - I*c)*Tan[a + b*x]]/x, x]
```

3.169.3 Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(c-(1-ic)\tan(a+bx))}{x} \, dx$$
$$\Big\downarrow \text{7299}$$
$$\int \frac{\cot^{-1}(c-(1-ic)\tan(a+bx))}{x}dx$$

input
```
Int[ArcCot[c - (1 - I*c)*Tan[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.169.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.169.4 Maple [N/A] (verified)

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arccot}\left(c-(-ic+1)\tan\left(bx+a\right)\right)}{x}dx$$

input
```
int(arccot(c-(1-I*c)*tan(b*x+a))/x,x)
```

output
```
int(arccot(c-(1-I*c)*tan(b*x+a))/x,x)
```

3.169.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int \frac{\cot^{-1}(c-(1-ic)\tan(a+bx))}{x}\,dx = \int \frac{\operatorname{arccot}\left(-(-i\,c+1)\tan\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccot(c-(1-I*c)*tan(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(-1/2*I*log((c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*x + 2*I*a) - I
))/x, x)
```

3.169.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(c-(1-ic)\tan(a+bx))}{x}\,dx = \text{Timed out}$$

input
```
integrate(acot(c-(1-I*c)*tan(b*x+a))/x,x)
```

output
```
Timed out
```

3.169.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^{-1}(c-(1-ic)\tan(a+bx))}{x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(arccot(c-(1-I*c)*tan(b*x+a))/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c-1>0)', see `assume?` for more
details)Is
```

3.169.8 Giac [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(c-(1-ic)\tan(a+bx))}{x}\,dx = \int \frac{\operatorname{arccot}\left(-(-i\,c+1)\tan\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccot(c-(1-I*c)*tan(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arccot(-(-I*c + 1)*tan(b*x + a) + c)/x, x)
```

3.169.9 Mupad [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(c-(1-ic)\tan(a+bx))}{x}\,dx = \int \frac{\mathrm{acot}(c+\tan(a+b\,x)\,\left(-1+c\,1\mathrm{i}\right))}{x}\,dx$$

input
```
int(acot(c + tan(a + b*x)*(c*1i - 1))/x,x)
```

output
```
int(acot(c + tan(a + b*x)*(c*1i - 1))/x, x)
```

3.170 $\int \cot^{-1}(\cot(a+bx))\,dx$

3.170.1 Optimal result

Integrand size = 7, antiderivative size = 16

$$\int \cot^{-1}(\cot(a+bx))\,dx=\frac{\cot^{-1}(\cot(a+bx))^2}{2b}$$

output
```
1/2*arccot(cot(b*x+a))^2/b
```

3.170.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \cot^{-1}(\cot(a+bx))\,dx=-\frac{bx^2}{2}+x\cot^{-1}(\cot(a+bx))$$

input
```
Integrate[ArcCot[Cot[a + b*x]],x]
```

output
```
-1/2*(b*x^2) + x*ArcCot[Cot[a + b*x]]
```

3.170.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}(\cot(a+bx))\,dx$$

$$\Big\downarrow \text{2588}$$

$$\frac{\int \cot^{-1}(\cot(a+bx))d\cot^{-1}(\cot(a+bx))}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\cot^{-1}(\cot(a+bx))^2}{2b}$$

input
```
Int[ArcCot[Cot[a + b*x]],x]
```

output
```
ArcCot[Cot[a + b*x]]^2/(2*b)
```

3.170.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

3.170.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result
parallelrisch	$-\frac{x^2 b}{2}+x\,\mathrm{arccot}\left(\cot\left(bx+a\right)\right)$
parts	$x\,\mathrm{arccot}\left(\cot\left(bx+a\right)\right)+\frac{-\frac{\left(bx+a\right)^2}{2}+\left(bx+a\right)a}{b}$
derivativedivides	$\frac{-\left(\frac{\pi}{2}-\mathrm{arccot}(\cot(bx+a))\right)\mathrm{arccot}(\cot(bx+a))-\frac{\left(\frac{\pi}{2}-\mathrm{arccot}(\cot(bx+a))\right)^2}{2}}{b}$
default	$\frac{-\left(\frac{\pi}{2}-\mathrm{arccot}(\cot(bx+a))\right)\mathrm{arccot}(\cot(bx+a))-\frac{\left(\frac{\pi}{2}-\mathrm{arccot}(\cot(bx+a))\right)^2}{2}}{b}$
risch	$-ix\ln\left(\mathrm{e}^{i(bx+a)}\right)-\frac{\pi x\,\mathrm{csgn}\left(i\mathrm{e}^{i(bx+a)}\right)^2\mathrm{csgn}\left(i\mathrm{e}^{2i(bx+a)}\right)}{4}+\frac{\pi x\,\mathrm{csgn}\left(i\mathrm{e}^{i(bx+a)}\right)\mathrm{csgn}\left(i\mathrm{e}^{2i(bx+a)}\right)^2}{2}-\frac{\pi x\,\mathrm{csgn}(}{}$

input
```
int(arccot(cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^2*b+x*arccot(cot(b*x+a))
```

3.170.5 Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \cot^{-1}(\cot(a+bx))\,dx=\frac{1}{2}x^2b+xa$$

input
```
integrate(arccot(cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*x^2*b + x*a
```

3.170.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \cot^{-1}(\cot(a+bx))\,dx = \begin{cases} \frac{\operatorname{acot}^2(\cot(a+bx))}{2b} & \text{for } b\neq 0 \\ x\operatorname{acot}(\cot(a)) & \text{otherwise} \end{cases}$$

input
```
integrate(acot(cot(b*x+a)),x)
```

output
```
Piecewise((acot(cot(a + b*x))**2/(2*b), Ne(b, 0)), (x*acot(cot(a)), True))
```

3.170.7 Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \cot^{-1}(\cot(a+bx))\,dx = \frac{1}{2}bx^2 + ax$$

input
```
integrate(arccot(cot(b*x+a)),x, algorithm="maxima")
```

output
```
1/2*b*x^2 + a*x
```

3.170.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \cot^{-1}(\cot(a+bx))\,dx = \frac{1}{2}bx^2 + ax$$

input
```
integrate(arccot(cot(b*x+a)),x, algorithm="giac")
```

output
```
1/2*b*x^2 + a*x
```

3.170.9 Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \cot^{-1}(\cot(a+bx))\,dx = x\,\mathrm{acot}(\cot(a+b\,x)) - \frac{b\,x^2}{2}$$

input
```
int(acot(cot(a + b*x)),x)
```

output
```
x*acot(cot(a + b*x)) - (b*x^2)/2
```

3.171 $\int x^2 \cot^{-1}(c + d\cot(a + bx))\,dx$

3.171.1 Optimal result

Integrand size = 15, antiderivative size = 399

$$\begin{aligned}\int x^2 \cot^{-1}(c+d\cot(a+bx))\,dx =& \frac{1}{3}x^3\cot^{-1}(c+d\cot(a+bx)) \\ &- \frac{1}{6}ix^3\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) \\ &+ \frac{1}{6}ix^3\log\left(1-\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right) \\ &- \frac{x^2\,\text{PolyLog}\left(2,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{4b} \\ &+ \frac{x^2\,\text{PolyLog}\left(2,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b} \\ &- \frac{ix\,\text{PolyLog}\left(3,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{4b^2} \\ &+ \frac{ix\,\text{PolyLog}\left(3,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b^2} \\ &+ \frac{\text{PolyLog}\left(4,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{8b^3} \\ &- \frac{\text{PolyLog}\left(4,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{8b^3}\end{aligned}$$

output
```
1/3*x^3*arccot(c+d*cot(b*x+a))-1/6*I*x^3*ln(1-(1+I*c-d)*exp(2*I*a+2*I*b*x)
/(1+I*c+d))+1/6*I*x^3*ln(1-(c+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))-1/4
*x^2*polylog(2,(1+I*c-d)*exp(2*I*a+2*I*b*x)/(1+I*c+d))/b+1/4*x^2*polylog(2
,(c+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))/b-1/4*I*x*polylog(3,(1+I*c-d)
*exp(2*I*a+2*I*b*x)/(1+I*c+d))/b^2+1/4*I*x*polylog(3,(c+I*(1+d))*exp(2*I*a
+2*I*b*x)/(c+I*(1-d)))/b^2+1/8*polylog(4,(1+I*c-d)*exp(2*I*a+2*I*b*x)/(1+I
*c+d))/b^3-1/8*polylog(4,(c+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))/b^3
```

3.171.2 Mathematica [A] (verified)

Time = 2.33 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.90

$$\int x^2 \cot^{-1}(c + d\cot(a + bx))\,dx = \frac{8b^3x^3\cot^{-1}(c + d\cot(a + bx)) - 4ib^3x^3\log\left(1 + \frac{(-c+i(1+d))e^{-2i(a+bx)}}{c+i(-1+d)}\right) + 4ib^3x^3\log\left(1 + \frac{(-c+i(-1+d))e^{-2i(a+bx)}}{c+i(1+d)}\right.}{}$$

input
```
Integrate[x^2*ArcCot[c + d*Cot[a + b*x]],x]
```

output
```
(8*b^3*x^3*ArcCot[c + d*Cot[a + b*x]] - (4*I)*b^3*x^3*Log[1 + (-c + I*(1 +
 d))/((c + I*(-1 + d))*E^((2*I)*(a + b*x)))] + (4*I)*b^3*x^3*Log[1 + (-c +
 I*(-1 + d))/((c + I*(1 + d))*E^((2*I)*(a + b*x)))] + 6*b^2*x^2*PolyLog[2,
 (c - I*(1 + d))/((c + I*(-1 + d))*E^((2*I)*(a + b*x)))] - 6*b^2*x^2*PolyL
og[2, (I + c - I*d)/((c + I*(1 + d))*E^((2*I)*(a + b*x)))] - (6*I)*b*x*Pol
yLog[3, (c - I*(1 + d))/((c + I*(-1 + d))*E^((2*I)*(a + b*x)))] + (6*I)*b*
x*PolyLog[3, (I + c - I*d)/((c + I*(1 + d))*E^((2*I)*(a + b*x)))] - 3*Poly
Log[4, (c - I*(1 + d))/((c + I*(-1 + d))*E^((2*I)*(a + b*x)))] + 3*PolyLog
[4, (I + c - I*d)/((c + I*(1 + d))*E^((2*I)*(a + b*x)))])/(24*b^3)
```

3.171.3 Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5701, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cot^{-1}(d\cot(a+bx)+c)\,dx$$

$$\Big\downarrow \text{5701}$$

$$-\frac{1}{3}b(ic-d+1)\int \frac{e^{2ia+2ibx}x^3}{ic-(ic-d+1)e^{2ia+2ibx}+d+1}dx+\frac{1}{3}b(-ic+d+1)\int \frac{e^{2ia+2ibx}x^3}{-ic-(-ic+d+1)e^{2ia+2ibx}-d+1}dx+\frac{1}{3}x^3\cot^{-1}(d\cot(a+bx)+c)$$

$$\Big\downarrow \text{2620}$$

$$-\frac{1}{3}b(ic-d+1)\left(\frac{x^3\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{2b(c-i(1-d))}-\frac{3\int x^2\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)dx}{2b(c-i(1-d))}\right)+\frac{1}{3}b(-ic+d+1)\left(\frac{3\int x^2\log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)dx}{2b(c+i(d+1))}-\frac{x^3\log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{2b(c+i(d+1))}\right)+\frac{1}{3}x^3\cot^{-1}(d\cot(a+bx)+c)$$

$$\Big\downarrow \text{3011}$$

$$-\frac{1}{3}b(ic-d+1)\left(\frac{x^3\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{2b(c-i(1-d))}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{2b}-\frac{i\int x\operatorname{PolyLog}\left(2,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)dx}{b}\right)}{2b(c-i(1-d))}\right)+\frac{1}{3}b(-ic+d+1)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{2b}-\frac{i\int x\operatorname{PolyLog}\left(2,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)dx}{b}\right)}{2b(c+i(d+1))}-\frac{x^3\log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{2b(c+i(d+1))}\right)+\frac{1}{3}x^3\cot^{-1}(d\cot(a+bx)+c)$$

$$\Big\downarrow \text{7163}$$

3.171. $\int x^2 \cot^{-1}(c+d\cot(a+bx))\,dx$

$$-\frac{1}{3}b(ic-d+$$

$$1)\left(\frac{x^3\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{2b(c-i(1-d))}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{2b}-\frac{i\left(\frac{i\int\operatorname{PolyLog}\left(3,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)dx}{2b}-\frac{ix\operatorname{PolyLog}\left(3,\right.}{}\right.}{b}\right.}{2b(c-i(1-d))}\right.$$

$$\frac{1}{3}b(-ic+d+$$

$$1)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{2b}-\frac{i\left(\frac{i\int\operatorname{PolyLog}\left(3,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)dx}{2b}-\frac{ix\operatorname{PolyLog}\left(3,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{2b}\right)}{b}\right)}{2b(c+i(d+1))}-\frac{x^3\log}{}\right.$$

$$\frac{1}{3}x^3\cot^{-1}(d\cot(a+bx)+c)$$

$$\Big\downarrow \text{2720}$$

3.171. $\int x^2\cot^{-1}(c+d\cot(a+bx))\,dx$

$$-\frac{1}{3}b(ic-d+$$

$$1)\left(\frac{x^3\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{2b(c-i(1-d))}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\operatorname{PolyLog}\left(3,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)de^{2ia+2i}}{4b^2}\right.}{b}\right.}{2b(c-i(1-d))}\right.$$

$$\frac{1}{3}b(-ic+d+$$

$$1)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\operatorname{PolyLog}\left(3,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)de^{2ia+2ibx}}{4b^2}-\frac{ix\operatorname{PolyLog}\left(3,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right.}{2b}\right.}{b}\right.}{2b(c+i(d+1))}\right.$$

$$\frac{1}{3}x^3\cot^{-1}(d\cot(a+bx)+c)$$

$$\Big\downarrow \; 7143$$

3.171. $\int x^2\cot^{-1}(c+d\cot(a+bx))\,dx$

$$-\frac{1}{3}b(ic-d+1)\left(\frac{x^3\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{2b(c-i(1-d))}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{2b}-\frac{i\left(\frac{\operatorname{PolyLog}\left(4,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{4b^2}-\frac{ix\operatorname{PolyLog}\left(3,\frac{(ic-d\cdots}{\cdots}\right)}{2b}\cdots\right.}{b}\cdots\right.}{2b(c-i(1-d))}\cdots\right.$$

$$\frac{1}{3}b(-ic+d+1)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{2b}-\frac{i\left(\frac{\operatorname{PolyLog}\left(4,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b^2}-\frac{ix\operatorname{PolyLog}\left(3,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{2b}\right)}{b}\right)}{2b(c+i(d+1))}-\frac{x^3\log\left(1-\cdots\right.}{2b(\cdots}\cdots\right.$$

$$\frac{1}{3}x^3\cot^{-1}(d\cot(a+bx)+c)$$

input
```
Int[x^2*ArcCot[c + d*Cot[a + b*x]],x]
```

output
```
(x^3*ArcCot[c + d*Cot[a + b*x]])/3 - (b*(1 + I*c - d)*((x^3*Log[1 - ((1 +
I*c - d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c + d)])/(2*b*(c - I*(1 - d))) -
(3*(((I/2)*x^2*PolyLog[2, ((1 + I*c - d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c
 + d)])/b - (I*(((-1/2*I)*x*PolyLog[3, ((1 + I*c - d)*E^((2*I)*a + (2*I)*b
*x))/(1 + I*c + d)])/b + PolyLog[4, ((1 + I*c - d)*E^((2*I)*a + (2*I)*b*x)
)/(1 + I*c + d)]/(4*b^2)))/b))/(2*b*(c - I*(1 - d)))))/3 + (b*(1 - I*c + d
)*(-1/2*(x^3*Log[1 - ((c + I*(1 + d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 -
 d))]])/(b*(c + I*(1 + d))) + (3*(((I/2)*x^2*PolyLog[2, ((c + I*(1 + d))*E^
((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))]])/b - (I*(((-1/2*I)*x*PolyLog[3, ((
c + I*(1 + d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))]])/b + PolyLog[4, (
(c + I*(1 + d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))]/(4*b^2)))/b))/(2
*b*(c + I*(1 + d)))))/3
```

3.171. $\int x^2\cot^{-1}(c+d\cot(a+bx))\,dx$

3.171.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5701
```
Int[ArcCot[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Cot[a + b*x]]/(f*(m +
1))), x] + (-Simp[b*((1 + I*c - d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^
(2*I*a + 2*I*b*x)/(1 + I*c + d - (1 + I*c - d)*E^(2*I*a + 2*I*b*x))), x], x
] + Simp[b*((1 - I*c + d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*I*a +
2*I*b*x)/(1 - I*c - d - (1 - I*c + d)*E^(2*I*a + 2*I*b*x))), x], x]) /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c - I*d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.171.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 53.61 (sec) , antiderivative size = 7868, normalized size of antiderivative = 19.72

method	result	size
risch	Expression too large to display	7868

input
```
int(x^2*arccot(c+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.171.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1589 vs. 2(283) = 566.

Time = 0.46 (sec) , antiderivative size = 1589, normalized size of antiderivative = 3.98

$$\int x^2 \cot^{-1}(c + d\cot(a + bx))\, dx = \text{Too large to display}$$

input
```
integrate(x^2*arccot(c+d*cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/48*(16*b^3*x^3*arccot(d*cot(b*x + a) + c) - 6*b^2*x^2*dilog(-(c^2 + d^2
- (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I
)*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) - 6*b^2*x^2*dilog
(-(c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d
 - I*d^2 + I)*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) + 6*b
^2*x^2*dilog(-(c^2 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-
I*c^2 + 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d +
1) + 1) + 6*b^2*x^2*dilog(-(c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*
x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 +
d^2 - 2*d + 1) + 1) + 4*I*a^3*log(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2*(c^2 + d
^2 + 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d + I)*sin(2*b*x
 + 2*a) + 1/2) - 4*I*a^3*log(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2*(c^2 + d^2 -
2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(2*b*x + 2*
a) + 1/2) - 4*I*a^3*log(-1/2*c^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*d
+ 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d + I)*sin(2*b*x + 2*a) -
 1/2) + 4*I*a^3*log(-1/2*c^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*d + 1)
*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(2*b*x + 2*a) - 1/2
) - 6*I*b*x*polylog(3, ((c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^
2 - 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*d + 1)) + 6*I*b*x*
polylog(3, ((c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*c*d...
```

3.171.6 Sympy [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(c + d\cot(a + bx))\, dx = \text{Timed out}$$

input
```
integrate(x**2*acot(c+d*cot(b*x+a)),x)
```

output
```
Timed out
```

3.171.7 Maxima [F]

$$\int x^2 \cot^{-1}(c + d\cot(a + bx))\,dx = \int x^2 \operatorname{arccot}\left(d\cot\left(bx + a\right) + c\right)\,dx$$

input
```
integrate(x^2*arccot(c+d*cot(b*x+a)),x, algorithm="maxima")
```

output
```
1/6*x^3*arctan2((d + 1)*cos(2*b*x + 2*a) + c*sin(2*b*x + 2*a) + d - 1, c*c
os(2*b*x + 2*a) - (d + 1)*sin(2*b*x + 2*a) - c) - 1/6*x^3*arctan2((d - 1)*
cos(2*b*x + 2*a) + c*sin(2*b*x + 2*a) + d + 1, c*cos(2*b*x + 2*a) - (d - 1
)*sin(2*b*x + 2*a) - c) - 4*b*d*integrate(1/3*(2*(c^2 + d^2 + 1)*x^3*cos(2
*b*x + 2*a)^2 + 2*c*d*x^3*sin(2*b*x + 2*a) + 2*(c^2 + d^2 + 1)*x^3*sin(2*b
*x + 2*a)^2 - (c^2 - d^2 + 1)*x^3*cos(2*b*x + 2*a) - (2*c*d*x^3*sin(2*b*x
+ 2*a) + (c^2 - d^2 + 1)*x^3*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x
^3*cos(2*b*x + 2*a) - (c^2 - d^2 + 1)*x^3*sin(2*b*x + 2*a))*sin(4*b*x + 4*
a))/(c^4 + d^4 + 2*(c^2 - 1)*d^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 +
1)*cos(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*cos(2*
b*x + 2*a)^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*sin(4*b*x + 4*a)^
2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*sin(2*b*x + 2*a)^2 + 2*c^2
 + 2*(c^4 + d^4 - 2*(3*c^2 + 1)*d^2 + 2*c^2 - 2*(c^4 - d^4 + 2*c^2 + 1)*co
s(2*b*x + 2*a) - 4*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x +
 4*a) - 4*(c^4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a) + 4*(2*c*d^3 - 2*(c^3 +
 c)*d + 2*(c*d^3 + (c^3 + c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 + 2*c^2 + 1)
*sin(2*b*x + 2*a))*sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 + c)*d)*sin(2*b*x +
2*a) + 1), x)
```

3.171.8 Giac [F]

$$\int x^2 \cot^{-1}(c + d\cot(a + bx))\,dx = \int x^2 \operatorname{arccot}\left(d\cot\left(bx + a\right) + c\right)\,dx$$

input
```
integrate(x^2*arccot(c+d*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arccot(d*cot(b*x + a) + c), x)
```

3.171.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(c+d\cot(a+bx))\,dx = \int x^2\,\mathrm{acot}(c+d\cot(a+b\,x))\,dx$$

input
```
int(x^2*acot(c + d*cot(a + b*x)),x)
```

output
```
int(x^2*acot(c + d*cot(a + b*x)), x)
```

3.172 $\int x \cot^{-1}(c + d \cot(a + bx))\,dx$

3.172.1 Optimal result

Integrand size = 13, antiderivative size = 303

$$\begin{aligned}\int x \cot^{-1}(c + d \cot(a + bx))\,dx =& \frac{1}{2}x^2 \cot^{-1}(c + d \cot(a + bx)) \\ & - \frac{1}{4}ix^2 \log\left(1 - \frac{(1 + ic - d)e^{2ia+2ibx}}{1 + ic + d}\right) \\ & + \frac{1}{4}ix^2 \log\left(1 - \frac{(c + i(1 + d))e^{2ia+2ibx}}{c + i(1 - d)}\right) \\ & - \frac{x \operatorname{PolyLog}\left(2, \frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{4b} \\ & + \frac{x \operatorname{PolyLog}\left(2, \frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b} \\ & - \frac{i \operatorname{PolyLog}\left(3, \frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{8b^2} \\ & + \frac{i \operatorname{PolyLog}\left(3, \frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{8b^2}\end{aligned}$$

output
```
1/2*x^2*arccot(c+d*cot(b*x+a))-1/4*I*x^2*ln(1-(1+I*c-d)*exp(2*I*a+2*I*b*x)
/(1+I*c+d))+1/4*I*x^2*ln(1-(c+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))-1/4
*x*polylog(2,(1+I*c-d)*exp(2*I*a+2*I*b*x)/(1+I*c+d))/b+1/4*x*polylog(2,(c+
I*(1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))/b-1/8*I*polylog(3,(1+I*c-d)*exp(2
*I*a+2*I*b*x)/(1+I*c+d))/b^2+1/8*I*polylog(3,(c+I*(1+d))*exp(2*I*a+2*I*b*x
)/(c+I*(1-d)))/b^2
```

3.172.2 Mathematica [A] (verified)

Time = 1.89 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.91

$$\int x\cot^{-1}(c+d\cot(a+bx))\,dx$$
$$=\frac{4b^2x^2\cot^{-1}(c+d\cot(a+bx))-2ib^2x^2\log\left(1+\frac{(-c+i(1+d))e^{-2i(a+bx)}}{c+i(-1+d)}\right)+2ib^2x^2\log\left(1+\frac{(-c+i(-1+d))e^{-2i(a+bx)}}{c+i(1+d)}\right.}{}$$

input
```
Integrate[x*ArcCot[c + d*Cot[a + b*x]],x]
```

output
```
(4*b^2*x^2*ArcCot[c + d*Cot[a + b*x]] - (2*I)*b^2*x^2*Log[1 + (-c + I*(1 +
 d))/((c + I*(-1 + d))*E^((2*I)*(a + b*x)))] + (2*I)*b^2*x^2*Log[1 + (-c +
 I*(-1 + d))/((c + I*(1 + d))*E^((2*I)*(a + b*x)))] + 2*b*x*PolyLog[2, (c
- I*(1 + d))/((c + I*(-1 + d))*E^((2*I)*(a + b*x)))] - 2*b*x*PolyLog[2, (I
 + c - I*d)/((c + I*(1 + d))*E^((2*I)*(a + b*x)))] - I*PolyLog[3, (c - I*(
1 + d))/((c + I*(-1 + d))*E^((2*I)*(a + b*x)))] + I*PolyLog[3, (I + c - I*
d)/((c + I*(1 + d))*E^((2*I)*(a + b*x)))])/(8*b^2)
```

3.172.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.30, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5701, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cot^{-1}(d\cot(a+bx)+c)\,dx$$

$$\Big\downarrow \text{5701}$$

$$-\frac{1}{2}b(ic-d+1)\int\frac{e^{2ia+2ibx}x^2}{ic-(ic-d+1)e^{2ia+2ibx}+d+1}dx+\frac{1}{2}b(-ic+d+1)\int\frac{e^{2ia+2ibx}x^2}{-ic-(-ic+d+1)e^{2ia+2ibx}-d+1}dx+\frac{1}{2}x^2\cot^{-1}(d\cot(a+bx)+c)$$

$$\Big\downarrow \text{2620}$$

$$-\frac{1}{2}b(ic-d+1)\left(\frac{x^2\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{2b(c-i(1-d))}-\frac{\int x\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)dx}{b(c-i(1-d))}\right)+\frac{1}{2}b(-ic+d+1)\left(\frac{\int x\log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)dx}{b(c+i(d+1))}-\frac{x^2\log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{2b(c+i(d+1))}\right)+\frac{1}{2}x^2\cot^{-1}(d\cot(a+bx)+c)$$

$$\Big\downarrow \text{3011}$$

$$-\frac{1}{2}b(ic-d+1)\left(\frac{x^2\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{2b(c-i(1-d))}-\frac{\frac{ix\,\text{PolyLog}\left(2,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{2b}-\frac{i\int \text{PolyLog}\left(2,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)dx}{2b}}{b(c-i(1-d))}\right)+\frac{1}{2}b(-ic+d+1)\left(\frac{\frac{ix\,\text{PolyLog}\left(2,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{2b}-\frac{i\int \text{PolyLog}\left(2,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)dx}{2b}}{b(c+i(d+1))}-\frac{x^2\log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{2b(c+i(d+1))}\right)+\frac{1}{2}x^2\cot^{-1}(d\cot(a+bx)+c)$$

$$\Big\downarrow \text{2720}$$

$$-\frac{1}{2}b(ic-d+1)\left(\frac{x^2\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{2b(c-i(1-d))}-\frac{\frac{ix\,\text{PolyLog}\left(2,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{2b}-\frac{\int e^{-2ia-2ibx}\,\text{PolyLog}\left(2,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)de^{2ia+2ibx}}{4b^2}}{b(c-i(1-d))}\right)$$

$$\frac{1}{2}b(-ic+d+1)\left(\frac{\frac{ix\,\text{PolyLog}\left(2,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{2b}-\frac{\int e^{-2ia-2ibx}\,\text{PolyLog}\left(2,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)de^{2ia+2ibx}}{4b^2}}{b(c+i(d+1))}-\frac{x^2\log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right.}{2b(c+i(d+1))}\right.$$

$$\frac{1}{2}x^2\cot^{-1}(d\cot(a+bx)+c)$$

$$\Big\downarrow \text{7143}$$

3.172. $\int x\cot^{-1}(c+d\cot(a+bx))\,dx$

$$-\frac{1}{2}b(ic-d+1)\left(\frac{x^2\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{2b(c-i(1-d))}-\frac{\frac{ix\,\mathrm{PolyLog}\left(2,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{2b}-\frac{\mathrm{PolyLog}\left(3,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{4b^2}}{b(c-i(1-d))}\right)+\frac{1}{2}b(-ic+d+1)\left(\frac{\frac{ix\,\mathrm{PolyLog}\left(2,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{2b}-\frac{\mathrm{PolyLog}\left(3,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b^2}}{b(c+i(d+1))}-\frac{x^2\log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{2b(c+i(d+1))}\right)+\frac{1}{2}x^2\cot^{-1}(d\cot(a+bx)+c)$$

input
```
Int[x*ArcCot[c + d*Cot[a + b*x]],x]
```

output
```
(x^2*ArcCot[c + d*Cot[a + b*x]])/2 - (b*(1 + I*c - d)*((x^2*Log[1 - ((1 +
I*c - d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c + d)])/(2*b*(c - I*(1 - d))) -
(((I/2)*x*PolyLog[2, ((1 + I*c - d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c + d)
])/b - PolyLog[3, ((1 + I*c - d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c + d)]/(
4*b^2))/(b*(c - I*(1 - d)))))/2 + (b*(1 - I*c + d)*(-1/2*(x^2*Log[1 - ((c
+ I*(1 + d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))])/(b*(c + I*(1 + d))
) + (((I/2)*x*PolyLog[2, ((c + I*(1 + d))*E^((2*I)*a + (2*I)*b*x))/(c + I*
(1 - d))])/b - PolyLog[3, ((c + I*(1 + d))*E^((2*I)*a + (2*I)*b*x))/(c + I
*(1 - d))]/(4*b^2))/(b*(c + I*(1 + d)))))/2
```

3.172.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5701
```
Int[ArcCot[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Cot[a + b*x]]/(f*(m +
1))), x] + (-Simp[b*((1 + I*c - d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^
(2*I*a + 2*I*b*x)/(1 + I*c + d - (1 + I*c - d)*E^(2*I*a + 2*I*b*x))), x], x
] + Simp[b*((1 - I*c + d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*I*a +
2*I*b*x)/(1 - I*c - d - (1 - I*c + d)*E^(2*I*a + 2*I*b*x))), x], x]) /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c - I*d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.172.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.44 (sec) , antiderivative size = 7488, normalized size of antiderivative = 24.71

method	result	size
risch	Expression too large to display	7488

input
```
int(x*arccot(c+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.172.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1289 vs. $2(213) = 426$.

Time = 0.40 (sec) , antiderivative size = 1289, normalized size of antiderivative = 4.25

$$\int x \cot^{-1}(c + d \cot(a + bx))\, dx = \text{Too large to display}$$

input
```
integrate(x*arccot(c+d*cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/16*(8*b^2*x^2*arccot(d*cot(b*x + a) + c) - 2*b*x*dilog(-(c^2 + d^2 - (c^
2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin
(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) - 2*b*x*dilog(-(c^2 +
d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2
+ I)*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) + 2*b*x*dilog(
-(c^2 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d
 + I*d^2 - I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1) + 1) + 2*b
*x*dilog(-(c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2
 + 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1) +
1) - 2*I*a^2*log(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2*(c^2 + d^2 + 2*d + 1)*cos
(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d + I)*sin(2*b*x + 2*a) + 1/2) +
2*I*a^2*log(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2*(c^2 + d^2 - 2*d + 1)*cos(2*b*
x + 2*a) + 1/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(2*b*x + 2*a) + 1/2) + 2*I*a
^2*log(-1/2*c^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*d + 1)*cos(2*b*x +
2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d + I)*sin(2*b*x + 2*a) - 1/2) - 2*I*a^2*l
og(-1/2*c^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*d + 1)*cos(2*b*x + 2*a)
 + 1/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(2*b*x + 2*a) - 1/2) - 2*(I*b^2*x^2
- I*a^2)*log((c^2 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I
*c^2 + 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1
)) - 2*(-I*b^2*x^2 + I*a^2)*log((c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*...
```

3.172.6 Sympy [F(-1)]

Timed out.

$$\int x \cot^{-1}(c + d \cot(a + bx))\, dx = \text{Timed out}$$

input
```
integrate(x*acot(c+d*cot(b*x+a)),x)
```

output
```
Timed out
```

3.172.7 Maxima [F]

$$\int x\cot^{-1}(c+d\cot(a+bx))\,dx=\int x\,\text{arccot}\left(d\cot\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x*arccot(c+d*cot(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*x^2*arctan2((d + 1)*cos(2*b*x + 2*a) + c*sin(2*b*x + 2*a) + d - 1, c*c
os(2*b*x + 2*a) - (d + 1)*sin(2*b*x + 2*a) - c) - 1/4*x^2*arctan2((d - 1)*
cos(2*b*x + 2*a) + c*sin(2*b*x + 2*a) + d + 1, c*cos(2*b*x + 2*a) - (d - 1
)*sin(2*b*x + 2*a) - c) - 2*b*d*integrate((2*(c^2 + d^2 + 1)*x^2*cos(2*b*x
 + 2*a)^2 + 2*c*d*x^2*sin(2*b*x + 2*a) + 2*(c^2 + d^2 + 1)*x^2*sin(2*b*x +
 2*a)^2 - (c^2 - d^2 + 1)*x^2*cos(2*b*x + 2*a) - (2*c*d*x^2*sin(2*b*x + 2*
a) + (c^2 - d^2 + 1)*x^2*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^2*c
os(2*b*x + 2*a) - (c^2 - d^2 + 1)*x^2*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/
(c^4 + d^4 + 2*(c^2 - 1)*d^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*c
os(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*cos(2*b*x
+ 2*a)^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*sin(4*b*x + 4*a)^2 +
4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*sin(2*b*x + 2*a)^2 + 2*c^2 + 2
*(c^4 + d^4 - 2*(3*c^2 + 1)*d^2 + 2*c^2 - 2*(c^4 - d^4 + 2*c^2 + 1)*cos(2*
b*x + 2*a) - 4*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a
) - 4*(c^4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a) + 4*(2*c*d^3 - 2*(c^3 + c)*
d + 2*(c*d^3 + (c^3 + c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 + 2*c^2 + 1)*sin
(2*b*x + 2*a))*sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a)
 + 1), x)
```

3.172.8 Giac [F]

$$\int x\cot^{-1}(c+d\cot(a+bx))\,dx=\int x\,\text{arccot}\left(d\cot\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x*arccot(c+d*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arccot(d*cot(b*x + a) + c), x)
```

3.172.9 Mupad [F(-1)]

Timed out.

$$\int x \cot^{-1}(c + d\cot(a + bx))\,dx = \int x\,\mathrm{acot}(c + d\cot(a + b\,x))\,dx$$

input
```
int(x*acot(c + d*cot(a + b*x)),x)
```

output
```
int(x*acot(c + d*cot(a + b*x)), x)
```

3.173 $\int \cot^{-1}(c+d\cot(a+bx))\,dx$

3.173.1 Optimal result

Integrand size = 11, antiderivative size = 198

$$\int \cot^{-1}(c+d\cot(a+bx))\,dx = x\cot^{-1}(c+d\cot(a+bx)) - \frac{1}{2}ix\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) + \frac{1}{2}ix\log\left(1-\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right) - \frac{\operatorname{PolyLog}\left(2,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{4b} + \frac{\operatorname{PolyLog}\left(2,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b}$$

output
```
x*arccot(c+d*cot(b*x+a))-1/2*I*x*ln(1-(1+I*c-d)*exp(2*I*a+2*I*b*x)/(1+I*c+
d))+1/2*I*x*ln(1-(c+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))-1/4*polylog(2
,(1+I*c-d)*exp(2*I*a+2*I*b*x)/(1+I*c+d))/b+1/4*polylog(2,(c+I*(1+d))*exp(2
*I*a+2*I*b*x)/(c+I*(1-d)))/b
```

3.173.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1649 vs. 2(198) = 396.

Time = 12.13 (sec) , antiderivative size = 1649, normalized size of antiderivative = 8.33

$$\int \cot^{-1}(c+d\cot(a+bx))\,dx = \text{Too large to display}$$

input
```
Integrate[ArcCot[c + d*Cot[a + b*x]],x]
```

output
```
x*ArcCot[c + d*Cot[a + b*x]] - (d*(4*a*Sqrt[-d^2]*ArcTan[(c*d + Tan[a + b*
x] + c^2*Tan[a + b*x])/d] + I*d*Log[1 + I*Tan[a + b*x]]*Log[(c*d - Sqrt[-d
^2] + Tan[a + b*x] + c^2*Tan[a + b*x])/(I + I*c^2 + c*d - Sqrt[-d^2])] + I
*d*Log[1 - I*Tan[a + b*x]]*Log[(c*d + Sqrt[-d^2] + Tan[a + b*x] + c^2*Tan[
a + b*x])/(-I - I*c^2 + c*d + Sqrt[-d^2])] - I*d*Log[1 + I*Tan[a + b*x]]*L
og[(c*d + Sqrt[-d^2] + Tan[a + b*x] + c^2*Tan[a + b*x])/(I + I*c^2 + c*d +
 Sqrt[-d^2])] - I*d*Log[1 - I*Tan[a + b*x]]*Log[(-(c*d) + Sqrt[-d^2] - (1
+ c^2)*Tan[a + b*x])/(I + I*c^2 - c*d + Sqrt[-d^2])] - I*d*PolyLog[2, ((1
+ c^2)*(1 - I*Tan[a + b*x]))/(1 + c^2 + I*c*d - I*Sqrt[-d^2])] + I*d*PolyL
og[2, ((1 + c^2)*(1 - I*Tan[a + b*x]))/(1 + c^2 + I*c*d + I*Sqrt[-d^2])] -
 I*d*PolyLog[2, ((1 + c^2)*(1 + I*Tan[a + b*x]))/(1 + c^2 - I*c*d - I*Sqrt
[-d^2])] + I*d*PolyLog[2, ((1 + c^2)*(1 + I*Tan[a + b*x]))/(1 + c^2 - I*c*
d + I*Sqrt[-d^2])])*((2*a)/(b*(-1 - c^2 - d^2 + Cos[2*(a + b*x)] + c^2*Cos
[2*(a + b*x)] - d^2*Cos[2*(a + b*x)] - 2*c*d*Sin[2*(a + b*x)])) - (2*(a +
b*x))/(b*(-1 - c^2 - d^2 + Cos[2*(a + b*x)] + c^2*Cos[2*(a + b*x)] - d^2*C
os[2*(a + b*x)] - 2*c*d*Sin[2*(a + b*x)]))))/((d*Log[1 - ((1 + c^2)*(1 - I
*Tan[a + b*x]))/(1 + c^2 + I*c*d - I*Sqrt[-d^2])]*Sec[a + b*x]^2)/(1 - I*T
an[a + b*x]) - (d*Log[1 - ((1 + c^2)*(1 - I*Tan[a + b*x]))/(1 + c^2 + I*c*
d + I*Sqrt[-d^2])]*Sec[a + b*x]^2)/(1 - I*Tan[a + b*x]) + (d*Log[(c*d + Sq
rt[-d^2] + Tan[a + b*x] + c^2*Tan[a + b*x])/(-I - I*c^2 + c*d + Sqrt[-d...
```

3.173.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.40, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5693, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}(d\cot(a+bx)+c)\,dx$$

$$\Big\downarrow \text{5693}$$

$$-b(ic-d+1)\int \frac{e^{2ia+2ibx}x}{ic-(ic-d+1)e^{2ia+2ibx}+d+1}dx+b(-ic+d+1)\int \frac{e^{2ia+2ibx}x}{-ic-(-ic+d+1)e^{2ia+2ibx}-d+1}dx+x\cot^{-1}(d\cot(a+bx)+c)$$

$$\Big\downarrow \text{2620}$$

$$-b(ic-d+1)\left(\frac{x\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{2b(c-i(1-d))}-\frac{\int \log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)dx}{2b(c-i(1-d))}\right)+b(-ic+d+1)\left(\frac{\int \log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)dx}{2b(c+i(d+1))}-\frac{x\log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{2b(c+i(d+1))}\right)+x\cot^{-1}(d\cot(a+bx)+c)$$

$$\Big\downarrow \text{2715}$$

$$-b(ic-d+1)\left(\frac{i\int e^{-2ia-2ibx}\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)de^{2ia+2ibx}}{4b^2(c-i(1-d))}+\frac{x\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{2b(c-i(1-d))}\right)+b(-ic+d+1)\left(-\frac{i\int e^{-2ia-2ibx}\log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)de^{2ia+2ibx}}{4b^2(c+i(d+1))}-\frac{x\log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{2b(c+i(d+1))}\right)+x\cot^{-1}(d\cot(a+bx)+c)$$

$$\Big\downarrow \text{2838}$$

$$-b(ic-d+1)\left(\frac{x\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{2b(c-i(1-d))}-\frac{i\,\text{PolyLog}\left(2,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{4b^2(c-i(1-d))}\right)+b(-ic+d+1)\left(\frac{i\,\text{PolyLog}\left(2,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b^2(c+i(d+1))}-\frac{x\log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{2b(c+i(d+1))}\right)+x\cot^{-1}(d\cot(a+bx)+c)$$

input
```
Int[ArcCot[c + d*Cot[a + b*x]],x]
```

3.173. $\int \cot^{-1}(c+d\cot(a+bx))\,dx$

output
```
x*ArcCot[c + d*Cot[a + b*x]] - b*(1 + I*c - d)*((x*Log[1 - ((1 + I*c - d)*
E^((2*I)*a + (2*I)*b*x))/(1 + I*c + d)])/(2*b*(c - I*(1 - d))) - ((I/4)*Po
lyLog[2, ((1 + I*c - d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c + d)])/(b^2*(c -
 I*(1 - d)))) + b*(1 - I*c + d)*(-1/2*(x*Log[1 - ((c + I*(1 + d))*E^((2*I)
*a + (2*I)*b*x))/(c + I*(1 - d))])/(b*(c + I*(1 + d))) + ((I/4)*PolyLog[2,
 ((c + I*(1 + d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))])/(b^2*(c + I*(
1 + d))))
```

3.173.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5693
```
Int[ArcCot[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*ArcC
ot[c + d*Cot[a + b*x]], x] + (-Simp[b*(1 + I*c - d)   Int[x*(E^(2*I*a + 2*I
*b*x)/(1 + I*c + d - (1 + I*c - d)*E^(2*I*a + 2*I*b*x))), x], x] + Simp[b*(
1 - I*c + d)   Int[x*(E^(2*I*a + 2*I*b*x)/(1 - I*c - d - (1 - I*c + d)*E^(2
*I*a + 2*I*b*x))), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - I*d)^2, -1
]
```

3.173.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1145 vs. 2(168) = 336.

Time = 4.58 (sec) , antiderivative size = 1146, normalized size of antiderivative = 5.79

method	result	size
derivativedivides	Expression too large to display	1146
default	Expression too large to display	1146
risch	Expression too large to display	4982

input
```
int(arccot(c+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/d*(-d*(1/2*Pi-arccot(cot(b*x+a)))*arccot(c+d*cot(b*x+a))-d^2*(-1/d*arc
tan(d*((c+d*cot(b*x+a))/d-c/d)+c)*arctan(-(c+d*cot(b*x+a))/d+c/d)-1/d^2*(1
/2*I*d^2*ln(1-(c-I*d+I)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+c))^2/((d*((c+d*c
ot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*arctan(d*((c+d*cot(b*x+a))/d-c/d)+c)/
(1+I*c+d)+1/2*I*d*ln(1-(c-I*d+I)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+c))^2/((
d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*arctan(d*((c+d*cot(b*x+a))/d
-c/d)+c)/(1+I*c+d)+1/2*I*d/(c-I*d-I)*ln(1-(c-I*d+I)*(1+I*(d*((c+d*cot(b*x+
a))/d-c/d)+c))^2/((d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*c*arctan(
d*((c+d*cot(b*x+a))/d-c/d)+c)+1/2*d^2*arctan(d*((c+d*cot(b*x+a))/d-c/d)+c)
^2/(1+I*c+d)+1/4*d^2*polylog(2,(c-I*d+I)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+
c))^2/((d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))/(1+I*c+d)+1/2*d*arct
an(d*((c+d*cot(b*x+a))/d-c/d)+c)^2/(1+I*c+d)+1/2*d/(c-I*d-I)*c*arctan(d*((
c+d*cot(b*x+a))/d-c/d)+c)^2+1/4*d*polylog(2,(c-I*d+I)*(1+I*(d*((c+d*cot(b*
x+a))/d-c/d)+c))^2/((d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))/(1+I*c+
d)+1/4*d/(c-I*d-I)*polylog(2,(c-I*d+I)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+c)
)^2/((d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*c-1/2*I*d*arctan(d*((c
+d*cot(b*x+a))/d-c/d)+c)*ln(1-(I+c+I*d)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+c
))^2/((d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(-I*d+I-c))-1/2*d*arctan(d*((c+d
*cot(b*x+a))/d-c/d)+c)^2-1/4*d*polylog(2,(I+c+I*d)*(1+I*(d*((c+d*cot(b*x+a
))/d-c/d)+c))^2/((d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(-I*d+I-c))))
```

3.173.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 965 vs. 2(140) = 280.

Time = 0.40 (sec) , antiderivative size = 965, normalized size of antiderivative = 4.87

$$\int \cot^{-1}(c + d\cot(a + bx))\,dx = \text{Too large to display}$$

input
```
integrate(arccot(c+d*cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/8*(8*b*x*arccot(d*cot(b*x + a) + c) + 2*I*a*log(1/2*c^2 + I*c*d - 1/2*d^
2 - 1/2*(c^2 + d^2 + 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*
d + I)*sin(2*b*x + 2*a) + 1/2) - 2*I*a*log(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2
*(c^2 + d^2 - 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 - 2*I*d + I)*
sin(2*b*x + 2*a) + 1/2) - 2*I*a*log(-1/2*c^2 + I*c*d + 1/2*d^2 + 1/2*(c^2
+ d^2 + 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d + I)*sin(2*
b*x + 2*a) - 1/2) + 2*I*a*log(-1/2*c^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2
- 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(2*b*x +
2*a) - 1/2) - 2*(I*b*x + I*a)*log((c^2 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*c
os(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a) + 2*d + 1)
/(c^2 + d^2 + 2*d + 1)) - 2*(-I*b*x - I*a)*log((c^2 + d^2 - (c^2 - 2*I*c*d
 - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a
) + 2*d + 1)/(c^2 + d^2 + 2*d + 1)) - 2*(-I*b*x - I*a)*log((c^2 + d^2 - (c
^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*si
n(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1)) - 2*(I*b*x + I*a)*log((c^
2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*
d^2 + I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1)) - dilog(-(c^2
+ d^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d
^2 - I)*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) - dilog(-(c
^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d ...
```

3.173.6 Sympy [F]

$$\int \cot^{-1}(c + d\cot(a + bx))\,dx = \int \operatorname{acot}(c + d\cot(a + bx))\,dx$$

input
```
integrate(acot(c+d*cot(b*x+a)),x)
```

output
```
Integral(acot(c + d*cot(a + b*x)), x)
```

3.173.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 532 vs. 2(140) = 280.

Time = 0.35 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.69

$$\int \cot^{-1}(c+d\cot(a+bx))\,dx$$
$$= \frac{d\left(\frac{8\,(bx+a)\arctan\left(\frac{cd+\left(c^2+1\right)\tan(bx+a)}{d}\right)}{d} - \frac{8\,(bx+a)\arctan\left(\frac{cd+\left(c^2+1\right)\tan(bx+a)}{d}\right)-4\arctan\left(\frac{cd+\left(c^2+1\right)\tan(bx+a)}{d}\right)\arctan\left(\frac{cd+\left(c^2+d+\ldots\right)}{c^2+d^2\ldots}\right)}{\ldots}\right.}{\ldots}$$

input
```
integrate(arccot(c+d*cot(b*x+a)),x, algorithm="maxima")
```

output
```
1/8*(d*(8*(b*x + a)*arctan((c*d + (c^2 + 1)*tan(b*x + a))/d)/d - (8*(b*x +
 a)*arctan((c*d + (c^2 + 1)*tan(b*x + a))/d) - 4*arctan((c*d + (c^2 + 1)*t
an(b*x + a))/d)*arctan2((c*d + (c^2 + d + 1)*tan(b*x + a))/(c^2 + d^2 + 2*
d + 1), -(c*d*tan(b*x + a) - c^2 - d - 1)/(c^2 + d^2 + 2*d + 1)) + 4*arcta
n((c*d + (c^2 + 1)*tan(b*x + a))/d)*arctan2(-(c*d + (c^2 - d + 1)*tan(b*x
+ a))/(c^2 + d^2 - 2*d + 1), -(c*d*tan(b*x + a) - c^2 + d - 1)/(c^2 + d^2
- 2*d + 1)) - (log(((c^2 + 1)*tan(b*x + a)^2 + c^2 + 1)/(c^2 + d^2 + 2*d +
 1)) - log(((c^2 + 1)*tan(b*x + a)^2 + c^2 + 1)/(c^2 + d^2 - 2*d + 1)))*lo
g((c^2 + 1)*d^2 + 2*(c^3 + c)*d*tan(b*x + a) + (c^4 + 2*c^2 + 1)*tan(b*x +
 a)^2) - 2*dilog(((I*c - 1)*tan(b*x + a) + I*d)/(c + I*d + I)) + 2*dilog((
(I*c + 1)*tan(b*x + a) + I*d)/(c + I*d - I)) + 2*dilog(-((I*c - 1)*tan(b*x
 + a) + I*d)/(c - I*d + I)) - 2*dilog(-((I*c + 1)*tan(b*x + a) + I*d)/(c -
 I*d - I)))/d) + 8*(b*x + a)*arccot(c + d/tan(b*x + a)) - 8*(b*x + a)*arct
an((c*d + (c^2 + 1)*tan(b*x + a))/d))/b
```

3.173.8 Giac [F]

$$\int \cot^{-1}(c+d\cot(a+bx))\,dx = \int \operatorname{arccot}\left(d\cot\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(arccot(c+d*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccot(d*cot(b*x + a) + c), x)
```

3.173.9 Mupad [F(-1)]

Timed out.

$$\int \cot^{-1}(c+d\cot(a+bx))\,dx=\int \operatorname{acot}(c+d\cot(a+b\,x))\ dx$$

input
```
int(acot(c + d*cot(a + b*x)),x)
```

output
```
int(acot(c + d*cot(a + b*x)), x)
```

3.174 $\int \frac{\cot^{-1}(c+d\cot(a+bx))}{x}\,dx$

3.174.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\cot^{-1}(c+d\cot(a+bx))}{x}\,dx = \text{Int}\left(\frac{\cot^{-1}(c+d\cot(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(arccot(c+d*cot(b*x+a))/x,x)
```

3.174.2 Mathematica [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(c+d\cot(a+bx))}{x}\,dx = \int \frac{\cot^{-1}(c+d\cot(a+bx))}{x}\,dx$$

input
```
Integrate[ArcCot[c + d*Cot[a + b*x]]/x,x]
```

output
```
Integrate[ArcCot[c + d*Cot[a + b*x]]/x, x]
```

3.174.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(d\cot(a+bx)+c)}{x}\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{\cot^{-1}(d\cot(a+bx)+c)}{x}dx$$

input
```
Int[ArcCot[c + d*Cot[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.174.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.174.4 Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arccot}\left(c+d\cot\left(bx+a\right)\right)}{x}dx$$

input
```
int(arccot(c+d*cot(b*x+a))/x,x)
```

output
```
int(arccot(c+d*cot(b*x+a))/x,x)
```

3.174.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(c+d\cot(a+bx))}{x}\,dx = \int \frac{\operatorname{arccot}\left(d\cot\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccot(c+d*cot(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(arccot(d*cot(b*x + a) + c)/x, x)
```

3.174.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(c+d\cot(a+bx))}{x}\,dx = \text{Timed out}$$

input
```
integrate(acot(c+d*cot(b*x+a))/x,x)
```

output
```
Timed out
```

3.174.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(c+d\cot(a+bx))}{x}\,dx = \text{Timed out}$$

input
```
integrate(arccot(c+d*cot(b*x+a))/x,x, algorithm="maxima")
```

output
```
Timed out
```

3.174.8 Giac [N/A]

Not integrable

Time = 3.50 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(c + d\cot(a + bx))}{x}\, dx = \int \frac{\operatorname{arccot}\left(d\cot\left(bx + a\right) + c\right)}{x}\, dx$$

input
```
integrate(arccot(c+d*cot(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arccot(d*cot(b*x + a) + c)/x, x)
```

3.174.9 Mupad [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(c + d\cot(a + bx))}{x}\, dx = \int \frac{\operatorname{acot}(c + d\cot(a + b\,x))}{x}\, dx$$

input
```
int(acot(c + d*cot(a + b*x))/x,x)
```

output
```
int(acot(c + d*cot(a + b*x))/x, x)
```

3.175 $\int x^2 \cot^{-1}(c+(1-ic)\cot(a+bx))\,dx$

3.175.1 Optimal result

Integrand size = 21, antiderivative size = 154

$$\int x^2 \cot^{-1}(c+(1-ic)\cot(a+bx))\,dx = -\frac{bx^4}{12}+\frac{1}{3}x^3\cot^{-1}(c+(1-ic)\cot(a+bx)) - \frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right) - \frac{x^2\operatorname{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b} - \frac{ix\operatorname{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{4b^2} + \frac{\operatorname{PolyLog}\left(4,ice^{2ia+2ibx}\right)}{8b^3}$$

output
```
-1/12*b*x^4+1/3*x^3*(Pi-arccot(-c-(1-I*c)*cot(b*x+a)))-1/6*I*x^3*ln(1-I*c*
exp(2*I*a+2*I*b*x))-1/4*x^2*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b-1/4*I*x*po
lylog(3,I*c*exp(2*I*a+2*I*b*x))/b^2+1/8*polylog(4,I*c*exp(2*I*a+2*I*b*x))/
b^3
```

3.175.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.91

$$\int x^2 \cot^{-1}(c+(1-ic)\cot(a+bx))\,dx = \frac{1}{3}x^3\cot^{-1}(c+(1-ic)\cot(a+bx)) - \frac{4ib^3x^3\log\left(1+\frac{ie^{-2i(a+bx)}}{c}\right) - 6b^2x^2\operatorname{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{c}\right) + 6ibx\operatorname{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{c}\right) + 3\operatorname{PolyL}}{24b^3}$$

input
```
Integrate[x^2*ArcCot[c + (1 - I*c)*Cot[a + b*x]],x]
```

output
```
(x^3*ArcCot[c + (1 - I*c)*Cot[a + b*x]])/3 - ((4*I)*b^3*x^3*Log[1 + I/(c*E
^((2*I)*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, (-I)/(c*E^((2*I)*(a + b*x)))]
+ (6*I)*b*x*PolyLog[3, (-I)/(c*E^((2*I)*(a + b*x)))] + 3*PolyLog[4, (-I)/(
c*E^((2*I)*(a + b*x)))])/(24*b^3)
```

3.175.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.29, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {5697, 25, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cot^{-1}(c+(1-ic)\cot(a+bx))\,dx$$

$$\Big\downarrow \text{5697}$$

$$\frac{1}{3}ib\int -\frac{x^3}{e^{2ia+2ibx}c+i}dx + \frac{1}{3}x^3\cot^{-1}(c+(1-ic)\cot(a+bx))$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}x^3\cot^{-1}(c+(1-ic)\cot(a+bx)) - \frac{1}{3}ib\int \frac{x^3}{e^{2ia+2ibx}c+i}dx$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{3}x^3\cot^{-1}(c+(1-ic)\cot(a+bx)) - \frac{1}{3}ib\left(ic\int \frac{e^{2ia+2ibx}x^3}{e^{2ia+2ibx}c+i}dx - \frac{ix^4}{4}\right)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{3}x^3\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{1}{3}ib\left(ic\left(\frac{3i\int x^2\log\left(1-ice^{2ia+2ibx}\right)dx}{2bc}-\frac{ix^3\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right)-\frac{ix^4}{4}\right)$$

$\big\downarrow$ 3011

$$\frac{1}{3}x^3\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{1}{3}ib\left(ic\left(\frac{3i\left(\frac{ix^2\operatorname{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{2b}-\frac{i\int x\operatorname{PolyLog}\left(2,ice^{2ia+2ibx}\right)dx}{b}\right)}{2bc}-\frac{ix^3\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right)-\frac{ix^4}{4}\right)$$

$\big\downarrow$ 7163

$$\frac{1}{3}x^3\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{1}{3}ib\left(ic\left(\frac{3i\left(\frac{ix^2\operatorname{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{2b}-\frac{i\left(\frac{i\int\operatorname{PolyLog}\left(3,ice^{2ia+2ibx}\right)dx}{2b}-\frac{ix\operatorname{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{2b}\right)}{b}\right)}{2bc}-\frac{ix^3\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right.\right.$$

$\big\downarrow$ 2720

$$\frac{1}{3}x^3\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{1}{3}ib\left(ic\left(\frac{3i\left(\frac{ix^2\operatorname{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\operatorname{PolyLog}\left(3,ice^{2ia+2ibx}\right)de^{2ia+2ibx}}{4b^2}-\frac{ix\operatorname{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{2b}\right)}{b}\right)}{2bc}-\frac{ix^3\log(1}{}\right.\right.$$

$\big\downarrow$ 7143

$$\frac{1}{3}x^3\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{1}{3}ib\left(ic\left(\frac{3i\left(\frac{ix^2\operatorname{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{2b}-\frac{i\left(\frac{\operatorname{PolyLog}\left(4,ice^{2ia+2ibx}\right)}{4b^2}-\frac{ix\operatorname{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{2b}\right)}{b}\right)}{2bc}-\frac{ix^3\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right)-\right.$$

3.175. $\int x^2\cot^{-1}(c+(1-ic)\cot(a+bx))\,dx$

input
```
Int[x^2*ArcCot[c + (1 - I*c)*Cot[a + b*x]],x]
```

output
```
(x^3*ArcCot[c + (1 - I*c)*Cot[a + b*x]])/3 - (I/3)*b*((-1/4*I)*x^4 + I*c*(
((-1/2*I)*x^3*Log[1 - I*c*E^((2*I)*a + (2*I)*b*x)])/(b*c) + (((3*I)/2)*(((
I/2)*x^2*PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)])/b - (I*(((-1/2*I)*x*Poly
Log[3, I*c*E^((2*I)*a + (2*I)*b*x)])/b + PolyLog[4, I*c*E^((2*I)*a + (2*I)
*b*x)]/(4*b^2)))/b))/(b*c)))
```

3.175.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*(F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5697
```
Int[ArcCot[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Cot[a + b*x]]/(f*(m +
1))), x] + Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(2
*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && E
qQ[(c - I*d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.175.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.12 (sec) , antiderivative size = 1448, normalized size of antiderivative = 9.40

method	result	size
risch	Expression too large to display	1448

input
```
int(x^2*(Pi-arccot(-c-(1-I*c)*cot(b*x+a))),x,method=_RETURNVERBOSE)
```

output
```
-1/6*I*x^3*ln(1-I*exp(2*I*(b*x+a))*c)-1/12*(Pi*csgn(I*exp(I*(b*x+a)))^2*cs
gn(I*exp(2*I*(b*x+a)))-2*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))
)^2+Pi*csgn(I*exp(2*I*(b*x+a)))^3+Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*(I+c)
/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))
-Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a
))-1))^2-Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*c+I))*cs
gn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))+Pi*csgn(I/(exp(2*I*(b*x+
a))-1))*csgn(I*(I+c))*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))-Pi*csgn(I/(exp(2*
I*(b*x+a))-1))*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))^2+Pi*csgn(I/(exp(2*I*(b*
x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^2+Pi*csgn(I*
exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))*csgn(exp(2*I*(b*x+a))*(I+c)/(
exp(2*I*(b*x+a))-1))-Pi*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^
2+Pi*csgn(I*(exp(2*I*(b*x+a))*c+I))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I
*(b*x+a))-1))^2-Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))*csg
n((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))-Pi*csgn((exp(2*I*(b*x+a))*c
+I)/(exp(2*I*(b*x+a))-1))^2-Pi*csgn(I*(I+c))*csgn(I*(I+c)/(exp(2*I*(b*x+a)
)-1))^2+Pi*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))^3-Pi*csgn(I*(I+c)/(exp(2*I*(
b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^2-Pi*csgn(
I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^3+Pi*csgn(I*(exp(2*I*(b*x+a
))*c+I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x...
```

3.175.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.13

$$\int x^2 \cot^{-1}(c + (1 - ic)\cot(a + bx))\, dx =$$

$$-\frac{2\,b^4x^4 - 8\,\pi b^3x^3 - 4i\,b^3x^3\log\left(\frac{\left(ce^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{c+i}\right) + 6\,b^2x^2\mathrm{Li}_2\left(i\,ce^{(2i\,bx+2i\,a)}\right) - 2\,a^4 - 4i\,a^3\log\left(\frac{ce}{}\right.}{}$$

input
```
integrate(x^2*(pi-arccot(-c-(1-I*c)*cot(b*x+a))),x, algorithm="fricas")
```

output
```
-1/24*(2*b^4*x^4 - 8*pi*b^3*x^3 - 4*I*b^3*x^3*log((c*e^(2*I*b*x + 2*I*a) +
 I)*e^(-2*I*b*x - 2*I*a)/(c + I)) + 6*b^2*x^2*dilog(I*c*e^(2*I*b*x + 2*I*a
)) - 2*a^4 - 4*I*a^3*log((c*e^(2*I*b*x + 2*I*a) + I)/c) + 6*I*b*x*polylog(
3, I*c*e^(2*I*b*x + 2*I*a)) + 4*(I*b^3*x^3 + I*a^3)*log(-I*c*e^(2*I*b*x +
2*I*a) + 1) - 3*polylog(4, I*c*e^(2*I*b*x + 2*I*a)))/b^3
```

3.175.6 Sympy [F(-2)]

Exception generated.

$$\int x^2 \cot^{-1}(c + (1 - ic)\cot(a + bx))\, dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(x**2*(pi-acot(-c-(1-I*c)*cot(b*x+a))),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert -_t0**4 + 3*_t0**2*I*c*
exp(2*I*a) - _t0**2*exp(2*I*a) + 2*c**2*exp(4*I*a) + I*c*exp(4*I*a) of typ
e <class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,exp(I*a)]
```

3.175.7 Maxima [F(-2)]

Exception generated.

$$\int x^2 \cot^{-1}(c + (1 - ic)\cot(a + bx))\, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(pi-arccot(-c-(1-I*c)*cot(b*x+a))),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c-1>0)', see `assume?` for more
details)Is
```

3.175.8 Giac [F]

$$\int x^2 \cot^{-1}(c + (1 - ic)\cot(a + bx))\, dx = \int (\pi - \operatorname{arccot}(-(-i\,c + 1)\cot(bx + a) - c))x^2\, dx$$

input
```
integrate(x^2*(pi-arccot(-c-(1-I*c)*cot(b*x+a))),x, algorithm="giac")
```

output
```
integrate((pi - arccot(-(-I*c + 1)*cot(b*x + a) - c))*x^2, x)
```

3.175.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(c+(1-ic)\cot(a+bx))\,dx = \int x^2\,(\Pi+\mathrm{acot}(c-\cot(a+b\,x)\,(-1+c\,1\mathrm{i})))\,dx$$

input
```
int(x^2*(Pi + acot(c - cot(a + b*x)*(c*1i - 1))),x)
```

output
```
int(x^2*(Pi + acot(c - cot(a + b*x)*(c*1i - 1))), x)
```

3.176 $\int x\cot^{-1}(c+(1-ic)\cot(a+bx))\,dx$

3.176.1 Optimal result

Integrand size = 19, antiderivative size = 123

$$\int x\cot^{-1}(c+(1-ic)\cot(a+bx))\,dx=-\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right)-\frac{x\,\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}-\frac{i\,\text{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{8b^2}$$

output
```
-1/6*b*x^3+1/2*x^2*(Pi-arccot(-c-(1-I*c)*cot(b*x+a)))-1/4*I*x^2*ln(1-I*c*e
xp(2*I*a+2*I*b*x))-1/4*x*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b-1/8*I*polylog
(3,I*c*exp(2*I*a+2*I*b*x))/b^2
```

3.176.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89

$$\int x\cot^{-1}(c+(1-ic)\cot(a+bx))\,dx=\frac{1}{2}x^2\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{i\left(2b^2x^2\log\left(1+\frac{ie^{-2i(a+bx)}}{c}\right)+2ibx\,\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{c}\right)+\text{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{c}\right)\right)}{8b^2}$$

input
```
Integrate[x*ArcCot[c + (1 - I*c)*Cot[a + b*x]],x]
```

output
```
(x^2*ArcCot[c + (1 - I*c)*Cot[a + b*x]])/2 - ((I/8)*(2*b^2*x^2*Log[1 + I/(
c*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, (-I)/(c*E^((2*I)*(a + b*x))
)] + PolyLog[3, (-I)/(c*E^((2*I)*(a + b*x)))]))/b^2
```

3.176.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.26, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5697, 25, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \cot^{-1}(c+(1-ic)\cot(a+bx))\,dx$$

$$\Big\downarrow \text{5697}$$

$$\frac{1}{2}ib\int -\frac{x^2}{e^{2ia+2ibx}c+i}dx+\frac{1}{2}x^2\cot^{-1}(c+(1-ic)\cot(a+bx))$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}x^2\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{1}{2}ib\int \frac{x^2}{e^{2ia+2ibx}c+i}dx$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{2}x^2\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{1}{2}ib\left(ic\int \frac{e^{2ia+2ibx}x^2}{e^{2ia+2ibx}c+i}dx-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{2}x^2\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{1}{2}ib\left(ic\left(\frac{i\int x\log\left(1-ice^{2ia+2ibx}\right)dx}{bc}-\frac{ix^2\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right)-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}x^2\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{1}{2}ib\left(ic\left(\frac{i\left(\frac{ix\,\text{PolyLog}(2,ice^{2ia+2ibx})}{2b}-\frac{i\int \text{PolyLog}(2,ice^{2ia+2ibx})dx}{2b}\right)}{bc}-\frac{ix^2\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right)-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}x^2\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{1}{2}ib\left(ic\left(\frac{i\left(\frac{ix\,\text{PolyLog}(2,ice^{2ia+2ibx})}{2b}-\frac{\int e^{-2ia-2ibx}\,\text{PolyLog}(2,ice^{2ia+2ibx})de^{2ia+2ibx}}{4b^2}\right)}{bc}-\frac{ix^2\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right)-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}x^2\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{1}{2}ib\left(ic\left(\frac{i\left(\frac{ix\,\text{PolyLog}(2,ice^{2ia+2ibx})}{2b}-\frac{\text{PolyLog}(3,ice^{2ia+2ibx})}{4b^2}\right)}{bc}-\frac{ix^2\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right)-\frac{ix^3}{3}\right)$$

input
```
Int[x*ArcCot[c + (1 - I*c)*Cot[a + b*x]],x]
```

output
```
(x^2*ArcCot[c + (1 - I*c)*Cot[a + b*x]])/2 - (I/2)*b*((-1/3*I)*x^3 + I*c*(
((-1/2*I)*x^2*Log[1 - I*c*E^((2*I)*a + (2*I)*b*x)])/(b*c) + (I*(((I/2)*x*P
olyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)])/b - PolyLog[3, I*c*E^((2*I)*a + (2
*I)*b*x)]/(4*b^2)))/(b*c)))
```

3.176.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5697
```
Int[ArcCot[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Cot[a + b*x]]/(f*(m +
1))), x] + Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(2
*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && E
qQ[(c - I*d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.176.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.62 (sec) , antiderivative size = 1413, normalized size of antiderivative = 11.49

method	result	size
risch	Expression too large to display	1413

input
```
int(x*(Pi-arccot(-c-(1-I*c)*cot(b*x+a))),x,method=_RETURNVERBOSE)
```

output
```
1/2*I/b*a*ln(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))*x-1/8*(Pi*csgn(I*exp(I*(b*x+
a)))^2*csgn(I*exp(2*I*(b*x+a)))-2*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I
*(b*x+a)))^2+Pi*csgn(I*exp(2*I*(b*x+a)))^3+Pi*csgn(I*exp(2*I*(b*x+a)))*csg
n(I*(I+c)/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*
x+a))-1))-Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2
*I*(b*x+a))-1))^2-Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))
*c+I))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))+Pi*csgn(I/(exp(
2*I*(b*x+a))-1))*csgn(I*(I+c))*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))-Pi*csgn(
I/(exp(2*I*(b*x+a))-1))*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))^2+Pi*csgn(I/(ex
p(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^2+P
i*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))*csgn(exp(2*I*(b*x+a)
)*(I+c)/(exp(2*I*(b*x+a))-1))-Pi*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x
+a))-1))^2+Pi*csgn(I*(exp(2*I*(b*x+a))*c+I))*csgn(I*(exp(2*I*(b*x+a))*c+I)
/(exp(2*I*(b*x+a))-1))^2-Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a)
)-1))*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))-Pi*csgn((exp(2*I*(
b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^2-Pi*csgn(I*(I+c))*csgn(I*(I+c)/(exp(2*
I*(b*x+a))-1))^2+Pi*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))^3-Pi*csgn(I*(I+c)/(
exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^2
-Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^3+Pi*csgn(I*(exp(2
*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*c+I)/(exp...
```

3.176.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.24

$$\int x\cot^{-1}(c+(1-ic)\cot(a+bx))\,dx =$$

$$-\frac{4\,b^3x^3-12\,\pi b^2x^2-6i\,b^2x^2\log\left(\frac{(ce^{(2i\,bx+2i\,a)}+i)e^{(-2i\,bx-2i\,a)}}{c+i}\right)+4\,a^3+6\,bx\mathrm{Li}_2\left(i\,ce^{(2i\,bx+2i\,a)}\right)+6i\,a^2\log\left(\cdots\right.}{24\,b^2}$$

input
```
integrate(x*(pi-arccot(-c-(1-I*c)*cot(b*x+a))),x, algorithm="fricas")
```

output
```
-1/24*(4*b^3*x^3 - 12*pi*b^2*x^2 - 6*I*b^2*x^2*log((c*e^(2*I*b*x + 2*I*a)
+ I)*e^(-2*I*b*x - 2*I*a)/(c + I)) + 4*a^3 + 6*b*x*dilog(I*c*e^(2*I*b*x +
2*I*a)) + 6*I*a^2*log((c*e^(2*I*b*x + 2*I*a) + I)/c) + 6*(I*b^2*x^2 - I*a^
2)*log(-I*c*e^(2*I*b*x + 2*I*a) + 1) + 3*I*polylog(3, I*c*e^(2*I*b*x + 2*I
*a)))/b^2
```

3.176.6 Sympy [F(-2)]

Exception generated.

$$\int x \cot^{-1}(c+(1-ic)\cot(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(x*(pi-acot(-c-(1-I*c)*cot(b*x+a))),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert -_t0**4 + 3*_t0**2*I*c*
exp(2*I*a) - _t0**2*exp(2*I*a) + 2*c**2*exp(4*I*a) + I*c*exp(4*I*a) of typ
e <class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,exp(I*a)]
```

3.176.7 Maxima [F(-2)]

Exception generated.

$$\int x \cot^{-1}(c+(1-ic)\cot(a+bx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(pi-arccot(-c-(1-I*c)*cot(b*x+a))),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c-1>0)', see `assume?` for more
details)Is
```

3.176.8 Giac [F]

$$\int x \cot^{-1}(c+(1-ic)\cot(a+bx))\,dx = \int (\pi - \operatorname{arccot}(-(-i\,c+1)\cot(bx+a)-c))x\,dx$$

input
```
integrate(x*(pi-arccot(-c-(1-I*c)*cot(b*x+a))),x, algorithm="giac")
```

output
```
integrate((pi - arccot(-(-I*c + 1)*cot(b*x + a) - c))*x, x)
```

3.176.9 Mupad [F(-1)]

Timed out.

$$\int x\cot^{-1}(c+(1-ic)\cot(a+bx))\,dx=\int x\,(\Pi+\mathrm{acot}(c-\cot(a+b\,x)\,\,(-1+c\,1\mathrm{i})))\,\,dx$$

input
```
int(x*(Pi + acot(c - cot(a + b*x)*(c*1i - 1))),x)
```

output
```
int(x*(Pi + acot(c - cot(a + b*x)*(c*1i - 1))), x)
```

3.177 $\int \cot^{-1}(c+(1-ic)\cot(a+bx))\,dx$

3.177.1 Optimal result

Integrand size = 17, antiderivative size = 85

$$\int \cot^{-1}(c+(1-ic)\cot(a+bx))\,dx = -\frac{bx^2}{2}+x\cot^{-1}(c+(1-ic)\cot(a+bx)) -\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)-\frac{\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}$$

output
```
-1/2*b*x^2+x*(Pi-arccot(-c-(1-I*c)*cot(b*x+a)))-1/2*I*x*ln(1-I*c*exp(2*I*a
+2*I*b*x))-1/4*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b
```

3.177.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 929 vs. 2(85) = 170.

Time = 4.89 (sec) , antiderivative size = 929, normalized size of antiderivative = 10.93

$$\int \cot^{-1}(c+(1-ic)\cot(a+bx))\,dx = x\cot^{-1}(c+(1-ic)\cot(a+bx)) + \frac{}{(i+\cot(a+bx))(1+ic+(i+c)\cot(a+bx))\left(2ibx+\log\left(1-\frac{\sec(bx)((-i+c)\cos(a)+i(i+c)\sin(a))(\cos(a+bx)-i\text{s}}{2c}\right.\right.}$$

input
```
Integrate[ArcCot[c + (1 - I*c)*Cot[a + b*x]],x]
```

output
```
x*ArcCot[c + (1 - I*c)*Cot[a + b*x]] + (I*x*Csc[a + b*x]^2*(2*b*x*Log[2*Co
s[b*x]*(Cos[b*x] - I*Sin[b*x])] + I*Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((I
+ c)*Cos[a + b*x] + (1 + I*c)*Sin[a + b*x]))/(2*c)]*Log[1 - I*Tan[b*x]] -
I*Log[(Sec[b*x]*((1 - I*c)*Cos[a + b*x] + (-I + c)*Sin[a + b*x]))/(2*Cos[a
] - (2*I)*Sin[a])]*Log[1 + I*Tan[b*x]] + I*PolyLog[2, -Cos[2*b*x] + I*Sin[
2*b*x]] + I*PolyLog[2, (Sec[b*x]*((-I + c)*Cos[a] + I*(I + c)*Sin[a])*(Cos
[a + b*x] - I*Sin[a + b*x]))/(2*c)] - I*PolyLog[2, (Sec[b*x]*((1 + I*c)*Co
s[a] - (I + c)*Sin[a])*(Cos[a + b*x] + I*Sin[a + b*x]))/2])*(Cos[b*x] - I*
Sin[b*x])*(Cos[b*x] + I*Sin[b*x]))/((I + Cot[a + b*x])*(1 + I*c + (I + c)*
Cot[a + b*x])*((2*I)*b*x + Log[1 - (Sec[b*x]*((-I + c)*Cos[a] + I*(I + c)*
Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*c)] + Log[1 + (Sec[b*x]*((-1 -
 I*c)*Cos[a] + (I + c)*Sin[a])*(Cos[a + b*x] + I*Sin[a + b*x]))/2] + ((-I
+ c)*Cos[a + b*x]*(Log[1 - I*Tan[b*x]] - Log[1 + I*Tan[b*x]]))/((I + c)*Co
s[a + b*x] + (1 + I*c)*Sin[a + b*x]) + ((I + c)*(Log[1 - I*Tan[b*x]] - Log
[1 + I*Tan[b*x]])*Sin[a + b*x])/((1 - I*c)*Cos[a + b*x] + (-I + c)*Sin[a +
 b*x]) + 2*b*x*Tan[b*x] + I*Log[1 - (Sec[b*x]*((-I + c)*Cos[a] + I*(I + c)
*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*c)]*Tan[b*x] - I*Log[1 + (Sec
[b*x]*((-1 - I*c)*Cos[a] + (I + c)*Sin[a])*(Cos[a + b*x] + I*Sin[a + b*x])
)/2]*Tan[b*x] - I*Log[1 - I*Tan[b*x]]*Tan[b*x] + I*Cos[a]^2*Log[1 + I*Tan[
b*x]]*Tan[b*x] + I*Log[1 + I*Tan[b*x]]*Sin[a]^2*Tan[b*x] + (I*Log[(Sec[...
```

3.177.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.26, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {5689, 25, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}(c+(1-ic)\cot(a+bx))\,dx$$

$$\Big\downarrow \text{5689}$$

$$ib\int -\frac{x}{e^{2ia+2ibx}c+i}dx+x\cot^{-1}(c+(1-ic)\cot(a+bx))$$

$$\Big\downarrow \text{25}$$

$$x\cot^{-1}(c+(1-ic)\cot(a+bx))-ib\int \frac{x}{e^{2ia+2ibx}c+i}dx$$

$$\Big\downarrow \text{2615}$$

$$x\cot^{-1}(c+(1-ic)\cot(a+bx))-ib\left(ic\int\frac{e^{2ia+2ibx}x}{e^{2ia+2ibx}c+i}dx-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{ 2620}$$

$$x\cot^{-1}(c+(1-ic)\cot(a+bx))-ib\left(ic\left(\frac{i\int\log\left(1-ice^{2ia+2ibx}\right)dx}{2bc}-\frac{ix\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right)-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{ 2715}$$

$$x\cot^{-1}(c+(1-ic)\cot(a+bx))-ib\left(ic\left(\frac{\int e^{-2ia-2ibx}\log\left(1-ice^{2ia+2ibx}\right)de^{2ia+2ibx}}{4b^2c}-\frac{ix\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right)-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{ 2838}$$

$$x\cot^{-1}(c+(1-ic)\cot(a+bx))-ib\left(ic\left(-\frac{\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b^2c}-\frac{ix\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right)-\frac{ix^2}{2}\right)$$

input
```
Int[ArcCot[c + (1 - I*c)*Cot[a + b*x]],x]
```

output
```
x*ArcCot[c + (1 - I*c)*Cot[a + b*x]] - I*b*((-1/2*I)*x^2 + I*c*(((-1/2*I)*
x*Log[1 - I*c*E^((2*I)*a + (2*I)*b*x)])/(b*c) - PolyLog[2, I*c*E^((2*I)*a
+ (2*I)*b*x)]/(4*b^2*c)))
```

3.177.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5689
```
Int[ArcCot[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*ArcC
ot[c + d*Cot[a + b*x]], x] + Simp[I*b   Int[x/(c - I*d - c*E^(2*I*a + 2*I*b
*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - I*d)^2, -1]
```

3.177.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(76) = 152$.

Time = 1.57 (sec) , antiderivative size = 587, normalized size of antiderivative = 6.91

method	result
default	$\pi x-\frac{-\frac{\operatorname{arccot}(-c+\cot(bx+a)(ic-1))\ln(\cot(bx+a)(ic-1)+c+i)c^{2}}{2i+2c}-\frac{2i\operatorname{arccot}(-c+\cot(bx+a)(ic-1))\ln(\cot(bx+a)(ic-1)+c+i)c}{2i+2c}+}{}$
parts	$\pi x-\frac{-\frac{\operatorname{arccot}(-c+\cot(bx+a)(ic-1))\ln(\cot(bx+a)(ic-1)+c+i)c^{2}}{2i+2c}-\frac{2i\operatorname{arccot}(-c+\cot(bx+a)(ic-1))\ln(\cot(bx+a)(ic-1)+c+i)c}{2i+2c}+}{}$
derivativedivides	$\frac{-\frac{\pi\ln\left(4c^{2}+4(-c+\cot(bx+a)(ic-1))c+(-c+\cot(bx+a)(ic-1))^{2}+1\right)c^{2}}{2(2i+2c)}-\frac{i\pi\ln\left(4c^{2}+4(-c+\cot(bx+a)(ic-1))c+(-c+\cot(bx+a)(ic-1\right.}{2i+2c}}{}$
risch	Expression too large to display

input
```
int(Pi-arccot(-c-(1-I*c)*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
Pi*x-1/b/(-1+I*c)*(-arccot(-c+cot(b*x+a)*(-1+I*c))/(2*I+2*c)*ln(cot(b*x+a)
*(-1+I*c)+c+I)*c^2-2*I*arccot(-c+cot(b*x+a)*(-1+I*c))/(2*I+2*c)*ln(cot(b*x
+a)*(-1+I*c)+c+I)*c+arccot(-c+cot(b*x+a)*(-1+I*c))/(2*I+2*c)*ln(cot(b*x+a)
*(-1+I*c)+c+I)+arccot(-c+cot(b*x+a)*(-1+I*c))/(2*I+2*c)*ln(-I+cot(b*x+a)*(
-1+I*c)-c)*c^2+2*I*arccot(-c+cot(b*x+a)*(-1+I*c))/(2*I+2*c)*ln(-I+cot(b*x+
a)*(-1+I*c)-c)*c-arccot(-c+cot(b*x+a)*(-1+I*c))/(2*I+2*c)*ln(-I+cot(b*x+a)
*(-1+I*c)-c)+(-1+I*c)^2*(-1/2/(I+c)*(-1/4*I*ln(-I+cot(b*x+a)*(-1+I*c)-c)^2
+1/2*I*(dilog(-1/2*I*(cot(b*x+a)*(-1+I*c)-c+I))+ln(-I+cot(b*x+a)*(-1+I*c)-
c)*ln(-1/2*I*(cot(b*x+a)*(-1+I*c)-c+I))))+1/2/(I+c)*(1/2*I*(dilog(-1/2*(co
t(b*x+a)*(-1+I*c)-c+I)/c)+ln(cot(b*x+a)*(-1+I*c)+c+I)*ln(-1/2*(cot(b*x+a)*
(-1+I*c)-c+I)/c))-1/2*I*(dilog((-I+cot(b*x+a)*(-1+I*c)-c)/(-2*I-2*c))+ln(c
ot(b*x+a)*(-1+I*c)+c+I)*ln((-I+cot(b*x+a)*(-1+I*c)-c)/(-2*I-2*c)))))
```

3.177.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.36

$$\int \cot^{-1}(c+(1-ic)\cot(a+bx))\,dx = -\frac{2\,b^2x^2 - 4\,\pi bx - 2i\,bx\log\left(\frac{(ce^{(2i\,bx+2i\,a)}+i)e^{(-2i\,bx-2i\,a)}}{c+i}\right) - 2\,a^2 + 2\,(i\,bx + i\,a)\log\left(-i\,ce^{(2i\,bx+2i\,a)}+1\right) - 2i}{4\,b}$$

input
```
integrate(pi-arccot(-c-(1-I*c)*cot(b*x+a)),x, algorithm="fricas")
```

output
```
-1/4*(2*b^2*x^2 - 4*pi*b*x - 2*I*b*x*log((c*e^(2*I*b*x + 2*I*a) + I)*e^(-2
*I*b*x - 2*I*a)/(c + I)) - 2*a^2 + 2*(I*b*x + I*a)*log(-I*c*e^(2*I*b*x + 2
*I*a) + 1) - 2*I*a*log((c*e^(2*I*b*x + 2*I*a) + I)/c) + dilog(I*c*e^(2*I*b
*x + 2*I*a)))/b
```

3.177.6 Sympy [F(-2)]

Exception generated.

$$\int \cot^{-1}(c+(1-ic)\cot(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(pi-acot(-c-(1-I*c)*cot(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert -_t0**4 + 3*_t0**2*I*c*
exp(2*I*a) - _t0**2*exp(2*I*a) + 2*c**2*exp(4*I*a) + I*c*exp(4*I*a) of typ
e <class 'sympy.core.add.Add'> to QQ_I[b,c,_t0,exp(I*a)]
```

3.177.7 Maxima [F(-2)]

Exception generated.

$$\int \cot^{-1}(c+(1-ic)\cot(a+bx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(pi-arccot(-c-(1-I*c)*cot(b*x+a)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c-1>0)', see `assume?` for more
details)Is
```

3.177.8 Giac [F]

$$\int \cot^{-1}(c+(1-ic)\cot(a+bx))\,dx = \int \pi - \operatorname{arccot}\left(-(-i\,c+1)\cot\left(bx+a\right)-c\right)\,dx$$

input
```
integrate(pi-arccot(-c-(1-I*c)*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(pi - arccot(-(-I*c + 1)*cot(b*x + a) - c), x)
```

3.177.9 Mupad [F(-1)]

Timed out.

$$\int \cot^{-1}(c+(1-ic)\cot(a+bx))\,dx=\int \Pi+\mathrm{acot}(c-\cot(a+b\,x)\,(-1+c\,1\mathrm{i}))\,dx$$

input
```
int(Pi + acot(c - cot(a + b*x)*(c*1i - 1)),x)
```

output
```
int(Pi + acot(c - cot(a + b*x)*(c*1i - 1)), x)
```

3.178 $\int \frac{\cot^{-1}(c+(1-ic)\cot(a+bx))}{x} dx$

3.178.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\cot^{-1}(c+(1-ic)\cot(a+bx))}{x} dx = \text{Int}\left(\frac{\cot^{-1}(c+(1-ic)\cot(a+bx))}{x}, x\right)$$

output
```
CannotIntegrate((Pi-arccot(-c-(1-I*c)*cot(b*x+a)))/x,x)
```

3.178.2 Mathematica [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\cot^{-1}(c+(1-ic)\cot(a+bx))}{x} dx = \int \frac{\cot^{-1}(c+(1-ic)\cot(a+bx))}{x} dx$$

input
```
Integrate[ArcCot[c + (1 - I*c)*Cot[a + b*x]]/x,x]
```

output
```
Integrate[ArcCot[c + (1 - I*c)*Cot[a + b*x]]/x, x]
```

3.178.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(c+(1-ic)\cot(a+bx))}{x} \, dx$$
$$\Big\downarrow \text{7299}$$
$$\int \frac{\cot^{-1}(c+(1-ic)\cot(a+bx))}{x}dx$$

input
```
Int[ArcCot[c + (1 - I*c)*Cot[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.178.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.178.4 Maple [N/A] (verified)

Not integrable

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int \frac{\pi - \operatorname{arccot}\left(-c-(-ic+1)\cot\left(bx+a\right)\right)}{x}dx$$

input
```
int((Pi-arccot(-c-(1-I*c)*cot(b*x+a)))/x,x)
```

output
```
int((Pi-arccot(-c-(1-I*c)*cot(b*x+a)))/x,x)
```

3.178.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.00

$$\int \frac{\cot^{-1}(c+(1-ic)\cot(a+bx))}{x}\,dx = \int \frac{\pi - \operatorname{arccot}\left(-(-i\,c+1)\cot\left(bx+a\right)-c\right)}{x}\,dx$$

input
```
integrate((pi-arccot(-c-(1-I*c)*cot(b*x+a)))/x,x, algorithm="fricas")
```

output
```
integral(1/2*(2*pi + I*log((c*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*I*a
)/(c + I)))/x, x)
```

3.178.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(c+(1-ic)\cot(a+bx))}{x}\,dx = \text{Timed out}$$

input
```
integrate((pi-acot(-c-(1-I*c)*cot(b*x+a)))/x,x)
```

output
```
Timed out
```

3.178.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^{-1}(c+(1-ic)\cot(a+bx))}{x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((pi-arccot(-c-(1-I*c)*cot(b*x+a)))/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c-1>0)', see `assume?` for more
details)Is
```

3.178.8 Giac [N/A]

Not integrable

Time = 2.38 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.33

$$\int \frac{\cot^{-1}(c+(1-ic)\cot(a+bx))}{x}\,dx=\int \frac{\pi-\operatorname{arccot}\left(-(-i\,c+1)\cot\left(bx+a\right)-c\right)}{x}\,dx$$

input
```
integrate((pi-arccot(-c-(1-I*c)*cot(b*x+a)))/x,x, algorithm="giac")
```

output
```
integrate((pi - arccot(-(-I*c + 1)*cot(b*x + a) - c))/x, x)
```

3.178.9 Mupad [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{\cot^{-1}(c+(1-ic)\cot(a+bx))}{x}\,dx=\int \frac{\Pi+\operatorname{acot}(c-\cot(a+b\,x)\ (-1+c\,1\mathrm{i}))}{x}\,dx$$

input
```
int((Pi + acot(c - cot(a + b*x)*(c*1i - 1)))/x,x)
```

output
```
int((Pi + acot(c - cot(a + b*x)*(c*1i - 1)))/x, x)
```

3.179 $\int x^2 \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx$

3.179.1 Optimal result

Integrand size = 22, antiderivative size = 155

$$\int x^2 \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx = \frac{bx^4}{12}+\frac{1}{3}x^3\cot^{-1}(c-(1+ic)\cot(a+bx)) +\frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right) +\frac{x^2\operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b} +\frac{ix\operatorname{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{4b^2} -\frac{\operatorname{PolyLog}\left(4,-ice^{2ia+2ibx}\right)}{8b^3}$$

output
```
1/12*b*x^4+1/3*x^3*(Pi-arccot(-c+(1+I*c)*cot(b*x+a)))+1/6*I*x^3*ln(1+I*c*e
xp(2*I*a+2*I*b*x))+1/4*x^2*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b+1/4*I*x*po
lylog(3,-I*c*exp(2*I*a+2*I*b*x))/b^2-1/8*polylog(4,-I*c*exp(2*I*a+2*I*b*x)
)/b^3
```

3.179.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.88

$$\int x^2 \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx = \frac{1}{24}\left(8x^3\cot^{-1}(c+(-1-ic)\cot(a+bx)) + 4ix^3\log\left(1-\frac{ie^{-2i(a+bx)}}{c}\right) - \frac{6x^2\,\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{c}\right)}{b} + \frac{6ix\,\text{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{c}\right)}{b^2} + \frac{3\,\text{PolyLog}\left(4,\frac{ie^{-2i(a+bx)}}{c}\right)}{b^3}\right)$$

input
```
Integrate[x^2*ArcCot[c - (1 + I*c)*Cot[a + b*x]],x]
```

output
```
(8*x^3*ArcCot[c + (-1 - I*c)*Cot[a + b*x]] + (4*I)*x^3*Log[1 - I/(c*E^((2*
I)*(a + b*x)))] - (6*x^2*PolyLog[2, I/(c*E^((2*I)*(a + b*x)))])/b + ((6*I)
*x*PolyLog[3, I/(c*E^((2*I)*(a + b*x)))])/b^2 + (3*PolyLog[4, I/(c*E^((2*I
)*(a + b*x)))])/b^3)/24
```

3.179.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.28, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {5697, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx$$

$$\Big\downarrow \text{5697}$$

$$\frac{1}{3}ib\int \frac{x^3}{i-ce^{2ia+2ibx}}dx+\frac{1}{3}x^3\cot^{-1}(c-(1+ic)\cot(a+bx))$$

$$\Big\downarrow \; 2615$$

$$\frac{1}{3}ib\left(-ic\int \frac{e^{2ia+2ibx}x^3}{i-ce^{2ia+2ibx}}dx-\frac{ix^4}{4}\right)+\frac{1}{3}x^3\cot^{-1}(c-(1+ic)\cot(a+bx))$$

$$\Big\downarrow \; 2620$$

$$\frac{1}{3}ib\left(-ic\left(\frac{ix^3\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{3i\int x^2\log\left(ie^{2ia+2ibx}c+1\right)dx}{2bc}\right)-\frac{ix^4}{4}\right)+\frac{1}{3}x^3\cot^{-1}(c-(1+ic)\cot(a+bx))$$

$$\Big\downarrow \; 3011$$

$$\frac{1}{3}ib\left(-ic\left(\frac{ix^3\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{3i\left(\frac{ix^2\operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{2b}-\frac{i\int x\operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)dx}{b}\right)}{2bc}\right)-\frac{ix^4}{4}\right)+\frac{1}{3}x^3\cot^{-1}(c-(1+ic)\cot(a+bx))$$

$$\Big\downarrow \; 7163$$

$$\frac{1}{3}ib\Bigg(-ic\Bigg(\frac{ix^3\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{3i\left(\frac{ix^2\operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{2b}-\frac{i\left(\frac{i\int \operatorname{PolyLog}\left(3,-ice^{2ia+2ibx}\right)dx}{2b}-\frac{ix\operatorname{PolyLog}\left(3,-ice^{2ia+2ib\cdots\right.}}{2b}\cdots\right.}{b}\cdots\right.}{2bc}\cdots$$

$$\frac{1}{3}x^3\cot^{-1}(c-(1+ic)\cot(a+bx))$$

$$\Big\downarrow \; 2720$$

$$\frac{1}{3}ib\Bigg(-ic\Bigg(\frac{ix^3\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{3i\left(\frac{ix^2\operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\operatorname{PolyLog}\left(3,-ice^{2ia+2ibx}\right)de^{2ia+2ibx}}{4b^2}-\frac{ix\operatorname{Pol}\cdots}{\cdots}\right.}{b}\cdots\right.}{2bc}\cdots$$

$$\frac{1}{3}x^3\cot^{-1}(c-(1+ic)\cot(a+bx))$$

$$\Big\downarrow \; 7143$$

3.179. $\int x^2\cot^{-1}(c-(1+ic)\cot(a+bx))\,dx$

$$\frac{1}{3}ib\left(-ic\left(\frac{ix^3\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{3i\left(\frac{ix^2\operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{2b}-\frac{i\left(\frac{\operatorname{PolyLog}\left(4,-ice^{2ia+2ibx}\right)}{4b^2}-\frac{ix\operatorname{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{2b}\right)}{b}\right.}{2bc}\right.\right.$$

$$\frac{1}{3}x^3\cot^{-1}(c-(1+ic)\cot(a+bx))$$

input
```
Int[x^2*ArcCot[c - (1 + I*c)*Cot[a + b*x]],x]
```

output
```
(x^3*ArcCot[c - (1 + I*c)*Cot[a + b*x]])/3 + (I/3)*b*((-1/4*I)*x^4 - I*c*(
((I/2)*x^3*Log[1 + I*c*E^((2*I)*a + (2*I)*b*x)])/(b*c) - (((3*I)/2)*(((I/2
)*x^2*PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b - (I*(((-1/2*I)*x*Poly
Log[3, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b + PolyLog[4, (-I)*c*E^((2*I)*a +
 (2*I)*b*x)]/(4*b^2)))/b))/(b*c))
```

3.179.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5697
```
Int[ArcCot[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Cot[a + b*x]]/(f*(m +
1))), x] + Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(2
*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && E
qQ[(c - I*d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.179.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.42 (sec) , antiderivative size = 1449, normalized size of antiderivative = 9.35

method	result	size
risch	Expression too large to display	1449

input
```
int(x^2*(Pi-arccot(-c+(I*c+1)*cot(b*x+a))),x,method=_RETURNVERBOSE)
```

output
```
-1/6*I*x^3*ln(exp(2*I*(b*x+a))*c-I)+1/12*(Pi*csgn(I*exp(I*(b*x+a)))^2*csgn
(I*exp(2*I*(b*x+a)))-2*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^
2+Pi*csgn(I*exp(2*I*(b*x+a)))^3+Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I/(exp(2*
I*(b*x+a))-1)*(c-I))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))-P
i*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))
-1))^2+Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(c-I))*csgn(I/(exp(2*I*(b*x+
a))-1)*(c-I))-Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*c-I
))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))-Pi*csgn(I/(exp(2*I*
(b*x+a))-1))*csgn(I/(exp(2*I*(b*x+a))-1)*(c-I))^2+Pi*csgn(I/(exp(2*I*(b*x+
a))-1))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^2-Pi*csgn(I*(e
xp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*c-I)/(ex
p(2*I*(b*x+a))-1))+Pi*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^2-
Pi*csgn(I*(c-I))*csgn(I/(exp(2*I*(b*x+a))-1)*(c-I))^2+Pi*csgn(I*exp(2*I*(b
*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b
*x+a))-1))+Pi*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^2+Pi*csgn(
I*(exp(2*I*(b*x+a))*c-I))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-
1))^2+Pi*csgn(I/(exp(2*I*(b*x+a))-1)*(c-I))^3-Pi*csgn(I/(exp(2*I*(b*x+a))-
1)*(c-I))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^2-Pi*csgn(I*
(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^3+Pi*csgn(I*(exp(2*I*(b*x+a))
*c-I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a...
```

3.179.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.12

$$\int x^2 \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx = \frac{2\,b^4x^4 + 8\,\pi b^3x^3 + 4i\,b^3x^3\log\left(\frac{(c-i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}-i}\right) + 6\,b^2x^2\mathrm{Li}_2\left(-i\,ce^{(2i\,bx+2i\,a)}\right) - 2\,a^4 - 4i\,a^3\log\left(\frac{ce^{(2i\,bx+2i\,a)}-i}{c}\right.}{2}$$

input
```
integrate(x^2*(pi-arccot(-c+(1+I*c)*cot(b*x+a))),x, algorithm="fricas")
```

output
```
1/24*(2*b^4*x^4 + 8*pi*b^3*x^3 + 4*I*b^3*x^3*log((c - I)*e^(2*I*b*x + 2*I*
a)/(c*e^(2*I*b*x + 2*I*a) - I)) + 6*b^2*x^2*dilog(-I*c*e^(2*I*b*x + 2*I*a)
) - 2*a^4 - 4*I*a^3*log((c*e^(2*I*b*x + 2*I*a) - I)/c) + 6*I*b*x*polylog(3
, -I*c*e^(2*I*b*x + 2*I*a)) - 4*(-I*b^3*x^3 - I*a^3)*log(I*c*e^(2*I*b*x +
2*I*a) + 1) - 3*polylog(4, -I*c*e^(2*I*b*x + 2*I*a)))/b^3
```

3.179.6 Sympy [F(-2)]

Exception generated.

$$\int x^2 \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(x**2*(pi-acot(-c+(1+I*c)*cot(b*x+a))),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert -_t0**2*I + 2*c*exp(2*I
*a) - I*exp(2*I*a) of type <class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,
exp(I*a)]
```

3.179.7 Maxima [F(-2)]

Exception generated.

$$\int x^2 \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(pi-arccot(-c+(1+I*c)*cot(b*x+a))),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c-1>0)', see `assume?` for more
details)Is
```

3.179.8 Giac [F]

$$\int x^2 \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx = \int (\pi - \operatorname{arccot}((i\,c+1)\cot(bx+a)-c))x^2\,dx$$

input
```
integrate(x^2*(pi-arccot(-c+(1+I*c)*cot(b*x+a))),x, algorithm="giac")
```

output
```
integrate((pi - arccot((I*c + 1)*cot(b*x + a) - c))*x^2, x)
```

3.179.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx = \int x^2\,(\Pi+\mathrm{acot}(c-\cot(a+b\,x)\,\left(1+c\,1\mathrm{i}\right)))\,dx$$

input
```
int(x^2*(Pi + acot(c - cot(a + b*x)*(c*1i + 1))),x)
```

output
```
int(x^2*(Pi + acot(c - cot(a + b*x)*(c*1i + 1))), x)
```

3.180 $\int x \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx$

3.180.1 Optimal result

Integrand size = 20, antiderivative size = 124

$$\int x \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx=\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c-(1+ic)\cot(a+bx))+\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right)+\frac{x\,\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}+\frac{i\,\text{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{8b^2}$$

output
```
1/6*b*x^3+1/2*x^2*(Pi-arccot(-c+(1+I*c)*cot(b*x+a)))+1/4*I*x^2*ln(1+I*c*ex
p(2*I*a+2*I*b*x))+1/4*x*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b+1/8*I*polylog
(3,-I*c*exp(2*I*a+2*I*b*x))/b^2
```

3.180.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89

$$\int x\cot^{-1}(c-(1+ic)\cot(a+bx))\,dx=\frac{1}{2}x^2\cot^{-1}(c+(-1-ic)\cot(a+bx))+\frac{i\left(2b^2x^2\log\left(1-\frac{ie^{-2i(a+bx)}}{c}\right)+2ibx\,\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{c}\right)+\text{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{c}\right)\right)}{8b^2}$$

input
```
Integrate[x*ArcCot[c - (1 + I*c)*Cot[a + b*x]],x]
```

output
```
(x^2*ArcCot[c + (-1 - I*c)*Cot[a + b*x]])/2 + ((I/8)*(2*b^2*x^2*Log[1 - I/
(c*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, I/(c*E^((2*I)*(a + b*x)))]
 + PolyLog[3, I/(c*E^((2*I)*(a + b*x)))]))/b^2
```

3.180.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.26, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5697, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx$$

$$\Big\downarrow \text{5697}$$

$$\frac{1}{2}ib\int \frac{x^2}{i-ce^{2ia+2ibx}}dx+\frac{1}{2}x^2\cot^{-1}(c-(1+ic)\cot(a+bx))$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{2}ib\left(-ic\int \frac{e^{2ia+2ibx}x^2}{i-ce^{2ia+2ibx}}dx-\frac{ix^3}{3}\right)+\frac{1}{2}x^2\cot^{-1}(c-(1+ic)\cot(a+bx))$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{2}ib\left(-ic\left(\frac{ix^2\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{i\int x\log\left(ie^{2ia+2ibx}c+1\right)dx}{bc}\right)-\frac{ix^3}{3}\right)+\frac{1}{2}x^2\cot^{-1}(c-(1+ic)\cot(a+bx))$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}ib\left(-ic\left(\frac{ix^2\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{i\left(\frac{ix\operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{2b}-\frac{i\int \operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)dx}{2b}\right)}{bc}\right)-\frac{ix^3}{3}\right)+\frac{1}{2}x^2\cot^{-1}(c-(1+ic)\cot(a+bx))$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}ib\left(-ic\left(\frac{ix^2\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{i\left(\frac{ix\,\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{2b}-\frac{\int e^{-2ia-2ibx}\,\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)de^{2ia+2ibx}}{4b^2}\right)}{bc}\right)-\frac{ix^3}{3}\right)+$$
$$\frac{1}{2}x^2\cot^{-1}(c-(1+ic)\cot(a+bx))$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}ib\left(-ic\left(\frac{ix^2\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{i\left(\frac{ix\,\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{2b}-\frac{\text{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{4b^2}\right)}{bc}\right)-\frac{ix^3}{3}\right)+$$
$$\frac{1}{2}x^2\cot^{-1}(c-(1+ic)\cot(a+bx))$$

input
```
Int[x*ArcCot[c - (1 + I*c)*Cot[a + b*x]],x]
```

output
```
(x^2*ArcCot[c - (1 + I*c)*Cot[a + b*x]])/2 + (I/2)*b*((-1/3*I)*x^3 - I*c*(
((I/2)*x^2*Log[1 + I*c*E^((2*I)*a + (2*I)*b*x)])/(b*c) - (I*(((I/2)*x*Poly
Log[2, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b - PolyLog[3, (-I)*c*E^((2*I)*a +
 (2*I)*b*x)]/(4*b^2)))/(b*c)))
```

3.180.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5697
```
Int[ArcCot[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Cot[a + b*x]]/(f*(m +
1))), x] + Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(2
*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && E
qQ[(c - I*d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.180.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.86 (sec) , antiderivative size = 1414, normalized size of antiderivative = 11.40

method	result	size
risch	Expression too large to display	1414

input
```
int(x*(Pi-arccot(-c+(I*c+1)*cot(b*x+a))),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/b*a*ln(1-I*exp(I*(b*x+a))*(I*c)^(1/2))*x+1/8*(Pi*csgn(I*exp(I*(b*x+
a)))^2*csgn(I*exp(2*I*(b*x+a)))-2*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I
*(b*x+a)))^2+Pi*csgn(I*exp(2*I*(b*x+a)))^3+Pi*csgn(I*exp(2*I*(b*x+a)))*csg
n(I/(exp(2*I*(b*x+a))-1)*(c-I))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*
x+a))-1))-Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2
*I*(b*x+a))-1))^2+Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(c-I))*csgn(I/(ex
p(2*I*(b*x+a))-1)*(c-I))-Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(
b*x+a))*c-I))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))-Pi*csgn(
I/(exp(2*I*(b*x+a))-1))*csgn(I/(exp(2*I*(b*x+a))-1)*(c-I))^2+Pi*csgn(I/(ex
p(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^2-P
i*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a
))*c-I)/(exp(2*I*(b*x+a))-1))+Pi*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x
+a))-1))^2-Pi*csgn(I*(c-I))*csgn(I/(exp(2*I*(b*x+a))-1)*(c-I))^2+Pi*csgn(I
*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))*csgn(exp(2*I*(b*x+a))*(c-I)/
(exp(2*I*(b*x+a))-1))+Pi*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))
^2+Pi*csgn(I*(exp(2*I*(b*x+a))*c-I))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*
I*(b*x+a))-1))^2+Pi*csgn(I/(exp(2*I*(b*x+a))-1)*(c-I))^3-Pi*csgn(I/(exp(2*
I*(b*x+a))-1)*(c-I))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^2
-Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^3+Pi*csgn(I*(exp(2
*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*c-I)/(exp...
```

3.180.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.23

$$\int x\cot^{-1}(c-(1+ic)\cot(a+bx))\,dx$$
$$=\frac{4\,b^3x^3+12\,\pi b^2x^2+6i\,b^2x^2\log\left(\frac{(c-i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}-i}\right)+4\,a^3+6\,bx\mathrm{Li}_2\left(-i\,ce^{(2i\,bx+2i\,a)}\right)+6i\,a^2\log\left(\frac{ce^{(2i\,bx+2i\,a)}-i}{c}\right)}{24\,b^2}$$

input
```
integrate(x*(pi-arccot(-c+(1+I*c)*cot(b*x+a))),x, algorithm="fricas")
```

output
```
1/24*(4*b^3*x^3 + 12*pi*b^2*x^2 + 6*I*b^2*x^2*log((c - I)*e^(2*I*b*x + 2*I
*a)/(c*e^(2*I*b*x + 2*I*a) - I)) + 4*a^3 + 6*b*x*dilog(-I*c*e^(2*I*b*x + 2
*I*a)) + 6*I*a^2*log((c*e^(2*I*b*x + 2*I*a) - I)/c) - 6*(-I*b^2*x^2 + I*a^
2)*log(I*c*e^(2*I*b*x + 2*I*a) + 1) + 3*I*polylog(3, -I*c*e^(2*I*b*x + 2*I
*a)))/b^2
```

3.180.6 Sympy [F(-2)]

Exception generated.

$$\int x\cot^{-1}(c-(1+ic)\cot(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(x*(pi-acot(-c+(1+I*c)*cot(b*x+a))),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert -_t0**2*I + 2*c*exp(2*I
*a) - I*exp(2*I*a) of type <class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,
exp(I*a)]
```

3.180.7 Maxima [F(-2)]

Exception generated.

$$\int x\cot^{-1}(c-(1+ic)\cot(a+bx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(pi-arccot(-c+(1+I*c)*cot(b*x+a))),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c-1>0)', see `assume?` for more
details)Is
```

3.180.8 Giac [F]

$$\int x\cot^{-1}(c-(1+ic)\cot(a+bx))\,dx = \int (\pi - \operatorname{arccot}\left((i\,c+1)\cot\left(bx+a\right)-c\right))x\,dx$$

input
```
integrate(x*(pi-arccot(-c+(1+I*c)*cot(b*x+a))),x, algorithm="giac")
```

output
```
integrate((pi - arccot((I*c + 1)*cot(b*x + a) - c))*x, x)
```

3.180.9 Mupad [F(-1)]

Timed out.

$$\int x\cot^{-1}(c-(1+ic)\cot(a+bx))\,dx=\int x\,(\Pi+\mathrm{acot}(c-\cot(a+b\,x)\,\,(1+c\,1\mathrm{i})))\,\,dx$$

input
```
int(x*(Pi + acot(c - cot(a + b*x)*(c*1i + 1))),x)
```

output
```
int(x*(Pi + acot(c - cot(a + b*x)*(c*1i + 1))), x)
```

3.181 $\int \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx$

3.181.1 Optimal result

Integrand size = 18, antiderivative size = 86

$$\int \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx=\frac{bx^2}{2}+x\cot^{-1}(c-(1+ic)\cot(a+bx))+\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)+\frac{\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}$$

output
```
1/2*b*x^2+x*(Pi-arccot(-c+(1+I*c)*cot(b*x+a)))+1/2*I*x*ln(1+I*c*exp(2*I*a+
2*I*b*x))+1/4*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b
```

3.181.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 872 vs. 2(86) = 172.

Time = 2.40 (sec) , antiderivative size = 872, normalized size of antiderivative = 10.14

$$\int \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx=x\cot^{-1}(c+(-1-ic)\cot(a+bx))-\frac{ix\csc(a+bx)\left(2bx\log(2\cos(bx)(\cos(bx)-i\sin(bx)))+i\log\left(\frac{\text{se}}{}\right.\right.}{(i+\cot(a+bx))((-i+c)\cos(a+bx)+i(i+c)\sin(a+bx))\left(-2ibx-\log\left(1-\frac{\sec(bx)((i+c)\cos(a)+(1+ic)}{}\right.\right.}$$

input
```
Integrate[ArcCot[c - (1 + I*c)*Cot[a + b*x]],x]
```

output
```
x*ArcCot[c + (-1 - I*c)*Cot[a + b*x]] - (I*x*Csc[a + b*x]*(2*b*x*Log[2*Cos
[b*x]*(Cos[b*x] - I*Sin[b*x])] + I*Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((-I
+ c)*Cos[a + b*x] + I*(I + c)*Sin[a + b*x]))/(2*c)]*Log[1 - I*Tan[b*x]] -
I*Log[(Sec[b*x]*(Cos[a] + I*Sin[a])*((1 + I*c)*Cos[a + b*x] - (I + c)*Sin[
a + b*x]))/2]*Log[1 + I*Tan[b*x]] + I*PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x
]] + I*PolyLog[2, (Sec[b*x]*((I + c)*Cos[a] + (1 + I*c)*Sin[a])*(Cos[a + b
*x] - I*Sin[a + b*x]))/(2*c)] - I*PolyLog[2, ((Cos[a] + I*Sin[a])*((I + c)
*Cos[a] + (1 + I*c)*Sin[a])*(-I + Tan[b*x]))/2])*(Cos[b*x] - I*Sin[b*x])*(
Cos[b*x] + I*Sin[b*x]))/((I + Cot[a + b*x])*((-I + c)*Cos[a + b*x] + I*(I
+ c)*Sin[a + b*x])*((-2*I)*b*x - Log[1 - (Sec[b*x]*((I + c)*Cos[a] + (1 +
I*c)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*c)] - (Log[1 - I*Tan[b*x]
]*((I + c)*Cos[a + b*x] + (1 + I*c)*Sin[a + b*x]))/((-I + c)*Cos[a + b*x]
+ I*(I + c)*Sin[a + b*x]) + (Log[1 + I*Tan[b*x]]*((I + c)*Cos[a + b*x] + (
1 + I*c)*Sin[a + b*x]))/((-I + c)*Cos[a + b*x] + I*(I + c)*Sin[a + b*x]) +
 (Log[(Sec[b*x]*(Cos[a] + I*Sin[a])*((1 + I*c)*Cos[a + b*x] - (I + c)*Sin[
a + b*x]))/2]*Sec[b*x]^2)/(1 + I*Tan[b*x]) - 2*b*x*Tan[b*x] - I*Log[1 - (S
ec[b*x]*((I + c)*Cos[a] + (1 + I*c)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]
))/(2*c)]*Tan[b*x] + I*Log[1 - I*Tan[b*x]]*Tan[b*x] - I*Log[1 + I*Tan[b*x]
]*Tan[b*x] + (I*Log[1 - ((Cos[a] + I*Sin[a])*((I + c)*Cos[a] + (1 + I*c)*S
in[a])*(-I + Tan[b*x]))/2]*Sec[b*x]^2)/(-I + Tan[b*x]) + (I*Log[(Sec[b*...
```

3.181.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5689, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx$$

$$\Big\downarrow \text{5689}$$

$$ib\int \frac{x}{i-ce^{2ia+2ibx}}dx+x\cot^{-1}(c-(1+ic)\cot(a+bx))$$

$$\Big\downarrow \text{2615}$$

$$ib\left(-ic\int \frac{e^{2ia+2ibx}x}{i-ce^{2ia+2ibx}}dx-\frac{ix^2}{2}\right)+x\cot^{-1}(c-(1+ic)\cot(a+bx))$$

$$\Big\downarrow \text{2620}$$

$$ib\left(-ic\left(\frac{ix\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{i\int\log\left(ie^{2ia+2ibx}c+1\right)dx}{2bc}\right)-\frac{ix^2}{2}\right)+x\cot^{-1}(c-(1+ic)\cot(a+bx))$$

$$\Big\downarrow \text{2715}$$

$$ib\left(-ic\left(\frac{ix\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{\int e^{-2ia-2ibx}\log\left(ie^{2ia+2ibx}c+1\right)de^{2ia+2ibx}}{4b^2c}\right)-\frac{ix^2}{2}\right)+x\cot^{-1}(c-(1+ic)\cot(a+bx))$$

$$\Big\downarrow \text{2838}$$

$$ib\left(-ic\left(\frac{\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b^2c}+\frac{ix\log\left(1+ice^{2ia+2ibx}\right)}{2bc}\right)-\frac{ix^2}{2}\right)+x\cot^{-1}(c-(1+ic)\cot(a+bx))$$

input
```
Int[ArcCot[c - (1 + I*c)*Cot[a + b*x]],x]
```

output
```
x*ArcCot[c - (1 + I*c)*Cot[a + b*x]] + I*b*((-1/2*I)*x^2 - I*c*(((I/2)*x*L
og[1 + I*c*E^((2*I)*a + (2*I)*b*x)])/(b*c) + PolyLog[2, (-I)*c*E^((2*I)*a
+ (2*I)*b*x)]/(4*b^2*c)))
```

3.181.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5689
```
Int[ArcCot[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*ArcC
ot[c + d*Cot[a + b*x]], x] + Simp[I*b   Int[x/(c - I*d - c*E^(2*I*a + 2*I*b
*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - I*d)^2, -1]
```

3.181.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 629 vs. $2(75) = 150$.

Time = 1.37 (sec) , antiderivative size = 630, normalized size of antiderivative = 7.33

method	result
default	$\pi x - \frac{\frac{\operatorname{arccot}(-c+(ic+1)\cot(bx+a))\ln(-(ic+1)\cot(bx+a)-c+i)c^2}{2i-2c} - \frac{2i\operatorname{arccot}(-c+(ic+1)\cot(bx+a))\ln(-(ic+1)\cot(bx+a)-c+i)c}{2i-2c}}{}$
parts	$\pi x - \frac{\frac{\operatorname{arccot}(-c+(ic+1)\cot(bx+a))\ln(-(ic+1)\cot(bx+a)-c+i)c^2}{2i-2c} - \frac{2i\operatorname{arccot}(-c+(ic+1)\cot(bx+a))\ln(-(ic+1)\cot(bx+a)-c+i)c}{2i-2c}}{}$
derivativedivides	$\frac{(ic+1)^2\left(-\frac{\operatorname{arccot}(-c+(ic+1)\cot(bx+a))\ln(i+(ic+1)\cot(bx+a)-c)}{2i-2c}+\frac{\operatorname{arccot}(-c+(ic+1)\cot(bx+a))\ln(-(ic+1)\cot(bx+a)-c+i)}{2i-2c}-\right.}{}$
risch	Expression too large to display

input
```
int(Pi-arccot(-c+(I*c+1)*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
Pi*x-1/b/(I*c+1)*(arccot(-c+(I*c+1)*cot(b*x+a))/(2*I-2*c)*ln(-(I*c+1)*cot(
b*x+a)-c+I)*c^2-2*I*arccot(-c+(I*c+1)*cot(b*x+a))/(2*I-2*c)*ln(-(I*c+1)*co
t(b*x+a)-c+I)*c-arccot(-c+(I*c+1)*cot(b*x+a))/(2*I-2*c)*ln(-(I*c+1)*cot(b*
x+a)-c+I)-arccot(-c+(I*c+1)*cot(b*x+a))/(2*I-2*c)*ln(I+(I*c+1)*cot(b*x+a)-
c)*c^2+2*I*arccot(-c+(I*c+1)*cot(b*x+a))/(2*I-2*c)*ln(I+(I*c+1)*cot(b*x+a)
-c)*c+arccot(-c+(I*c+1)*cot(b*x+a))/(2*I-2*c)*ln(I+(I*c+1)*cot(b*x+a)-c)-(
I*c+1)^2*(-1/2/(I-c)*(1/4*I*ln(I+(I*c+1)*cot(b*x+a)-c)^2-1/2*I*((ln(I+(I*c
+1)*cot(b*x+a)-c)-ln(-1/2*I*(I+(I*c+1)*cot(b*x+a)-c)))*ln(-1/2*I*(I-(I*c+1
)*cot(b*x+a)+c))-dilog(-1/2*I*(I+(I*c+1)*cot(b*x+a)-c))))+1/2/(I-c)*(1/2*I
*(dilog((-I-(I*c+1)*cot(b*x+a)+c)/(-2*I+2*c))+ln(-(I*c+1)*cot(b*x+a)-c+I)*
ln((-I-(I*c+1)*cot(b*x+a)+c)/(-2*I+2*c)))-1/2*I*(dilog(1/2*(I-(I*c+1)*cot(
b*x+a)+c)/c)+ln(-(I*c+1)*cot(b*x+a)-c+I)*ln(1/2*(I-(I*c+1)*cot(b*x+a)+c)/c
)))))
```

3.181.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.35

$$\int \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx = \frac{2\,b^2x^2+4\,\pi bx+2i\,bx\log\left(\frac{(c-i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}-i}\right)-2\,a^2-2\left(-i\,bx-i\,a\right)\log\left(i\,ce^{(2i\,bx+2i\,a)}+1\right)-2i\,a\log\left(\frac{ce^{(2i\,bx}}{}\right.}{4\,b}$$

input
```
integrate(pi-arccot(-c+(1+I*c)*cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/4*(2*b^2*x^2 + 4*pi*b*x + 2*I*b*x*log((c - I)*e^(2*I*b*x + 2*I*a)/(c*e^(
2*I*b*x + 2*I*a) - I)) - 2*a^2 - 2*(-I*b*x - I*a)*log(I*c*e^(2*I*b*x + 2*I
*a) + 1) - 2*I*a*log((c*e^(2*I*b*x + 2*I*a) - I)/c) + dilog(-I*c*e^(2*I*b*
x + 2*I*a)))/b
```

3.181.6 Sympy [F(-2)]

Exception generated.

$$\int \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(pi-acot(-c+(1+I*c)*cot(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert -_t0**2*I + 2*c*exp(2*I
*a) - I*exp(2*I*a) of type <class 'sympy.core.add.Add'> to QQ_I[b,c,_t0,ex
p(I*a)]
```

3.181.7 Maxima [F(-2)]

Exception generated.

$$\int \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(pi-arccot(-c+(1+I*c)*cot(b*x+a)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c-1>0)', see `assume?` for more
details)Is
```

3.181.8 Giac [F]

$$\int \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx = \int \pi - \operatorname{arccot}\left((i\,c+1)\cot\left(bx+a\right)-c\right)\,dx$$

input
```
integrate(pi-arccot(-c+(1+I*c)*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(pi - arccot((I*c + 1)*cot(b*x + a) - c), x)
```

3.181.9 Mupad [F(-1)]

Timed out.

$$\int \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx = \int \Pi + \operatorname{acot}(c-\cot(a+b\,x)\,\left(1+c\,1\mathrm{i}\right))\,dx$$

input
```
int(Pi + acot(c - cot(a + b*x)*(c*1i + 1)),x)
```

output
```
int(Pi + acot(c - cot(a + b*x)*(c*1i + 1)), x)
```

3.182 $\int \frac{\cot^{-1}(c-(1+ic)\cot(a+bx))}{x} dx$

3.182.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\cot^{-1}(c-(1+ic)\cot(a+bx))}{x} dx = \text{Int}\left(\frac{\cot^{-1}(c-(1+ic)\cot(a+bx))}{x}, x\right)$$

output
```
CannotIntegrate((Pi-arccot(-c+(1+I*c)*cot(b*x+a)))/x,x)
```

3.182.2 Mathematica [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cot^{-1}(c-(1+ic)\cot(a+bx))}{x} dx = \int \frac{\cot^{-1}(c-(1+ic)\cot(a+bx))}{x} dx$$

input
```
Integrate[ArcCot[c - (1 + I*c)*Cot[a + b*x]]/x,x]
```

output
```
Integrate[ArcCot[c - (1 + I*c)*Cot[a + b*x]]/x, x]
```

3.182.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(c-(1+ic)\cot(a+bx))}{x} \, dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{\cot^{-1}(c-(1+ic)\cot(a+bx))}{x}dx$$

input
```
Int[ArcCot[c - (1 + I*c)*Cot[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.182.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.182.4 Maple [N/A] (verified)

Not integrable

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{\pi - \operatorname{arccot}\left(-c+(ic+1)\cot\left(bx+a\right)\right)}{x}dx$$

input
```
int((Pi-arccot(-c+(I*c+1)*cot(b*x+a)))/x,x)
```

output
```
int((Pi-arccot(-c+(I*c+1)*cot(b*x+a)))/x,x)
```

3.182.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int \frac{\cot^{-1}(c-(1+ic)\cot(a+bx))}{x}\,dx=\int \frac{\pi-\operatorname{arccot}\left((i\,c+1)\cot\left(bx+a\right)-c\right)}{x}\,dx$$

input
```
integrate((pi-arccot(-c+(1+I*c)*cot(b*x+a)))/x,x, algorithm="fricas")
```

output
```
integral(1/2*(2*pi + I*log((c - I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*x + 2*I
*a) - I)))/x, x)
```

3.182.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(c-(1+ic)\cot(a+bx))}{x}\,dx=\text{Timed out}$$

input
```
integrate((pi-acot(-c+(1+I*c)*cot(b*x+a)))/x,x)
```

output
```
Timed out
```

3.182.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^{-1}(c-(1+ic)\cot(a+bx))}{x}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((pi-arccot(-c+(1+I*c)*cot(b*x+a)))/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c-1>0)', see `assume?` for more
details)Is
```

3.182.8 Giac [N/A]

Not integrable

Time = 2.72 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{\cot^{-1}(c-(1+ic)\cot(a+bx))}{x}\,dx = \int \frac{\pi - \operatorname{arccot}\left((i\,c+1)\cot\left(bx+a\right)-c\right)}{x}\,dx$$

input
```
integrate((pi-arccot(-c+(1+I*c)*cot(b*x+a)))/x,x, algorithm="giac")
```

output
```
integrate((pi - arccot((I*c + 1)*cot(b*x + a) - c))/x, x)
```

3.182.9 Mupad [N/A]

Not integrable

Time = 1.54 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{\cot^{-1}(c-(1+ic)\cot(a+bx))}{x}\,dx = \int \frac{\Pi + \operatorname{acot}(c-\cot(a+b\,x)\ (1+c\,1\mathrm{i}))}{x}\,dx$$

input
```
int((Pi + acot(c - cot(a + b*x)*(c*1i + 1)))/x,x)
```

output
```
int((Pi + acot(c - cot(a + b*x)*(c*1i + 1)))/x, x)
```

3.183 $\int (e+fx)^3 \cot^{-1}(\tanh(a+bx))\,dx$

3.183.1 Optimal result

Integrand size = 15, antiderivative size = 299

$$\begin{aligned}\int (e+fx)^3 \cot^{-1}(\tanh(a+bx))\,dx =& \frac{(e+fx)^4 \cot^{-1}(\tanh(a+bx))}{4f} \\ &+ \frac{(e+fx)^4 \arctan\left(e^{2a+2bx}\right)}{4f} \\ &- \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} \\ &+ \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} \\ &+ \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} \\ &- \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2} \\ &- \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} \\ &+ \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3} \\ &+ \frac{3if^3 \operatorname{PolyLog}\left(5,-ie^{2a+2bx}\right)}{16b^4} \\ &- \frac{3if^3 \operatorname{PolyLog}\left(5,ie^{2a+2bx}\right)}{16b^4}\end{aligned}$$

output
```
1/4*(f*x+e)^4*arccot(tanh(b*x+a))/f+1/4*(f*x+e)^4*arctan(exp(2*b*x+2*a))/f
-1/4*I*(f*x+e)^3*polylog(2,-I*exp(2*b*x+2*a))/b+1/4*I*(f*x+e)^3*polylog(2,
I*exp(2*b*x+2*a))/b+3/8*I*f*(f*x+e)^2*polylog(3,-I*exp(2*b*x+2*a))/b^2-3/8
*I*f*(f*x+e)^2*polylog(3,I*exp(2*b*x+2*a))/b^2-3/8*I*f^2*(f*x+e)*polylog(4
,-I*exp(2*b*x+2*a))/b^3+3/8*I*f^2*(f*x+e)*polylog(4,I*exp(2*b*x+2*a))/b^3+
3/16*I*f^3*polylog(5,-I*exp(2*b*x+2*a))/b^4-3/16*I*f^3*polylog(5,I*exp(2*b
*x+2*a))/b^4
```

3.183.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 600 vs. 2(299) = 598.

Time = 0.23 (sec) , antiderivative size = 600, normalized size of antiderivative = 2.01

$$\int (e+fx)^3 \cot^{-1}(\tanh(a+bx))\,dx = \frac{1}{4}x\left(4e^3+6e^2fx+4ef^2x^2+f^3x^3\right)\cot^{-1}(\tanh(a+bx)) + \frac{i\left(8b^4e^3x\log\left(1-ie^{2(a+bx)}\right)+12b^4e^2fx^2\log\left(1-ie^{2(a+bx)}\right)+8b^4ef^2x^3\log\left(1-ie^{2(a+bx)}\right)+2b^4f^3x^4\log\right.}{}$$

input
```
Integrate[(e + f*x)^3*ArcCot[Tanh[a + b*x]],x]
```

output
```
(x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*ArcCot[Tanh[a + b*x]])/4 +
((I/16)*(8*b^4*e^3*x*Log[1 - I*E^(2*(a + b*x))] + 12*b^4*e^2*f*x^2*Log[1 -
 I*E^(2*(a + b*x))] + 8*b^4*e*f^2*x^3*Log[1 - I*E^(2*(a + b*x))] + 2*b^4*f
^3*x^4*Log[1 - I*E^(2*(a + b*x))] - 8*b^4*e^3*x*Log[1 + I*E^(2*(a + b*x))]
 - 12*b^4*e^2*f*x^2*Log[1 + I*E^(2*(a + b*x))] - 8*b^4*e*f^2*x^3*Log[1 + I
*E^(2*(a + b*x))] - 2*b^4*f^3*x^4*Log[1 + I*E^(2*(a + b*x))] - 4*b^3*(e +
f*x)^3*PolyLog[2, (-I)*E^(2*(a + b*x))] + 4*b^3*(e + f*x)^3*PolyLog[2, I*E
^(2*(a + b*x))] + 6*b^2*e^2*f*PolyLog[3, (-I)*E^(2*(a + b*x))] + 12*b^2*e*
f^2*x*PolyLog[3, (-I)*E^(2*(a + b*x))] + 6*b^2*f^3*x^2*PolyLog[3, (-I)*E^(
2*(a + b*x))] - 6*b^2*e^2*f*PolyLog[3, I*E^(2*(a + b*x))] - 12*b^2*e*f^2*x
*PolyLog[3, I*E^(2*(a + b*x))] - 6*b^2*f^3*x^2*PolyLog[3, I*E^(2*(a + b*x)
)] - 6*b*e*f^2*PolyLog[4, (-I)*E^(2*(a + b*x))] - 6*b*f^3*x*PolyLog[4, (-I
)*E^(2*(a + b*x))] + 6*b*e*f^2*PolyLog[4, I*E^(2*(a + b*x))] + 6*b*f^3*x*P
olyLog[4, I*E^(2*(a + b*x))] + 3*f^3*PolyLog[5, (-I)*E^(2*(a + b*x))] - 3*
f^3*PolyLog[5, I*E^(2*(a + b*x))]))/b^4
```

3.183.3 Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {5707, 3042, 4668, 3011, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^3 \cot^{-1}(\tanh(a+bx))\,dx$$

$$\Big\downarrow \text{5707}$$

$$\frac{b\int (e+fx)^4\text{sech}(2a+2bx)dx}{4f}+\frac{(e+fx)^4\cot^{-1}(\tanh(a+bx))}{4f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^4\cot^{-1}(\tanh(a+bx))}{4f}+\frac{b\int (e+fx)^4\csc\left(2ia+2ibx+\frac{\pi}{2}\right)dx}{4f}$$

$$\Big\downarrow \text{4668}$$

$$\frac{(e+fx)^4\cot^{-1}(\tanh(a+bx))}{4f}+\frac{b\left(-\frac{2if\int (e+fx)^3\log\left(1-ie^{2a+2bx}\right)dx}{b}+\frac{2if\int (e+fx)^3\log\left(1+ie^{2a+2bx}\right)dx}{b}+\frac{(e+fx)^4\arctan\left(e^{2a+2bx}\right)}{b}\right)}{4f}$$

$$\Big\downarrow \text{3011}$$

$$\frac{(e+fx)^4\cot^{-1}(\tanh(a+bx))}{4f}+\frac{b\left(\frac{2if\left(\frac{3f\int (e+fx)^2\text{PolyLog}\left(2,-ie^{2a+2bx}\right)dx}{2b}-\frac{(e+fx)^3\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{2b}\right)}{b}-\frac{2if\left(\frac{3f\int (e+fx)^2\text{PolyLog}\left(2,ie^{2a+2bx}\right)dx}{2b}-\frac{(e+fx)^3\text{PolyLog}\left(2,ie^{2\cdots}\right)}{2b}\cdots\right)}{b}\cdots\right)}{4f}$$

$$\Big\downarrow \text{7163}$$

$$\frac{(e+fx)^4\cot^{-1}(\tanh(a+bx))}{4f}+\frac{b\left(\frac{2if\left(\frac{3f\left(\frac{(e+fx)^2\text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{2b}-\frac{f\int (e+fx)\text{PolyLog}\left(3,-ie^{2a+2bx}\right)dx}{b}\right)}{2b}-\frac{(e+fx)^3\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{2b}\right)}{b}-\frac{2if\left(\frac{3f\left(\frac{(e+fx)^2\text{PolyLog}(\cdots}{2b}\cdots\right)}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{4f}$$

$$\downarrow \text{7163}$$

$$\frac{(e+fx)^4\cot^{-1}(\tanh(a+bx))}{4f}+$$

$$b\left(\frac{2if\left(\frac{3f\left(\frac{(e+fx)^2\operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{2b}-\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{2b}-\frac{f\int\operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)dx}{2b}\right)}{b}\right)}{2b}-\frac{(e+fx)^3\operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{2b}\right)}{b}\right. \cdots$$

$$\downarrow \text{2720}$$

$$\frac{(e+fx)^4\cot^{-1}(\tanh(a+bx))}{4f}+$$

$$b\left(\frac{2if\left(\frac{3f\left(\frac{(e+fx)^2\operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{2b}-\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{2b}-\frac{f\int e^{-2a-2bx}\operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)de^{2a+2bx}}{4b^2}\right)}{b}\right)}{2b}-\frac{(e+fx)^3\operatorname{PolyLog}\left(\cdots\right.}{2b}\cdots\right.}{b}\right. \cdots$$

$$\downarrow \text{7143}$$

3.183. $\int(e+fx)^3\cot^{-1}(\tanh(a+bx))\,dx$

$$\frac{(e+fx)^4\cot^{-1}(\tanh(a+bx))}{4f}+$$

$$b\left(\frac{(e+fx)^4\arctan\left(e^{2a+2bx}\right)}{b}+\frac{2if\left(\frac{3f\left(\frac{(e+fx)^2\operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{2b}-\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{2b}-\frac{f\operatorname{PolyLog}\left(5,-ie^{2a+2bx}\right)}{4b^2}\right)}{b}\right)}{2b}-\frac{(e+fx)^3}{}\right.}{b}\right.$$

input
```
Int[(e + f*x)^3*ArcCot[Tanh[a + b*x]],x]
```

output
```
((e + f*x)^4*ArcCot[Tanh[a + b*x]])/(4*f) + (b*(((e + f*x)^4*ArcTan[E^(2*a
 + 2*b*x)])/b + ((2*I)*f*(-1/2*((e + f*x)^3*PolyLog[2, (-I)*E^(2*a + 2*b*x
)])/b + (3*f*(((e + f*x)^2*PolyLog[3, (-I)*E^(2*a + 2*b*x)])/(2*b) - (f*((
(e + f*x)*PolyLog[4, (-I)*E^(2*a + 2*b*x)])/(2*b) - (f*PolyLog[5, (-I)*E^(
2*a + 2*b*x)])/(4*b^2)))/b))/(2*b)))/b - ((2*I)*f*(-1/2*((e + f*x)^3*PolyL
og[2, I*E^(2*a + 2*b*x)])/b + (3*f*(((e + f*x)^2*PolyLog[3, I*E^(2*a + 2*b
*x)])/(2*b) - (f*(((e + f*x)*PolyLog[4, I*E^(2*a + 2*b*x)])/(2*b) - (f*Pol
yLog[5, I*E^(2*a + 2*b*x)])/(4*b^2)))/b))/(2*b)))/b))/(4*f)
```

3.183.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5707
```
Int[ArcCot[Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcCot[Tanh[a + b*x]]/(f*(m + 1))), x] + Simp[b/
(f*(m + 1))   Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a,
b, e, f}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.183. $\int (e+fx)^3 \cot^{-1}(\tanh(a+bx))\,dx$

3.183.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 33.57 (sec) , antiderivative size = 3570, normalized size of antiderivative = 11.94

method	result	size
risch	Expression too large to display	3570

input
```
int((f*x+e)^3*arccot(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/b*e^3*dilog(1+exp(b*x+a)*(-1)^(3/4))-1/2*I/b*e^3*dilog(1-exp(b*x+a)
*(-1)^(3/4))-1/2*I*e^3*ln(1+exp(b*x+a)*(-1)^(3/4))*x-1/2*I*e^3*ln(1-exp(b*
x+a)*(-1)^(3/4))*x-1/8*I*f^3*ln(1+I*exp(2*b*x+2*a))*x^4-1/8*I/f*e^4*ln(-ex
p(2*b*x+2*a)+I)+1/8*I*f^3*ln(exp(2*b*x+2*a)-I)*x^4+1/2*I*ln(exp(2*b*x+2*a)
-I)*x*e^3+1/8*I/f*ln(exp(2*b*x+2*a)-I)*e^4+1/2*I/b*e^3*ln(((-I)^(1/2)-exp(
b*x+a))/(-I)^(1/2))*a+1/2*I/b*e^3*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))*a
-1/2*I*f^3/b^4*a^3*dilog(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))+3/8*I*f^2/b^3
*e*polylog(4,I*exp(2*b*x+2*a))+1/4*I*f^3/b*polylog(2,I*exp(2*b*x+2*a))*x^3
+1/4*I*f^3/b^4*polylog(2,I*exp(2*b*x+2*a))*a^3+3/8*I*f^3/b^4*ln(1-I*exp(2*
b*x+2*a))*a^4-1/2*I*f^3/b^4*a^4*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))-1/2
*I*f^3/b^4*a^4*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))+1/8*I*f^3/b^4*a^4*ln
(exp(2*b*x+2*a)+I)+3/8*I*f^3/b^3*polylog(4,I*exp(2*b*x+2*a))*x-3/8*I*f/b^2
*e^2*polylog(3,I*exp(2*b*x+2*a))+1/2*I*f^2*e*ln(1-I*exp(2*b*x+2*a))*x^3+3/
4*I*f*e^2*ln(1-I*exp(2*b*x+2*a))*x^2-1/2*I*f^3/b^4*a^3*dilog(((-I)^(1/2)-e
xp(b*x+a))/(-I)^(1/2))-3/8*I*f^3/b^2*polylog(3,I*exp(2*b*x+2*a))*x^2-1/2*I
/b*a*e^3*ln(exp(2*b*x+2*a)+I)+3/16*I*f^3*polylog(5,-I*exp(2*b*x+2*a))/b^4+
3/2*I*f^2/b^2*e*ln(1+I*exp(2*b*x+2*a))*a^2*x-3/2*I*f/b*e^2*ln(1+I*exp(2*b*
x+2*a))*a*x-3/2*I*f^2/b^2*a^2*e*ln(1+exp(b*x+a)*(-1)^(3/4))*x-3/2*I*f^2/b^
2*a^2*e*ln(1-exp(b*x+a)*(-1)^(3/4))*x+3/2*I*f/b*a*e^2*ln(1+exp(b*x+a)*(-1)
^(3/4))*x+3/2*I*f/b*a*e^2*ln(1-exp(b*x+a)*(-1)^(3/4))*x+I*f^2/b^3*e*ln(...
```

3.183.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1460 vs. 2(236) = 472.

Time = 0.38 (sec) , antiderivative size = 1460, normalized size of antiderivative = 4.88

$$\int (e+fx)^3 \cot^{-1}(\tanh(a+bx))\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*arccot(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/8*(-24*I*f^3*polylog(5, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) -
 24*I*f^3*polylog(5, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 24*
I*f^3*polylog(5, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 24*I*f^
3*polylog(5, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*(b^4*f^3
*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x)*arctan(cosh(b*x +
a)/sinh(b*x + a)) - 4*(-I*b^3*f^3*x^3 - 3*I*b^3*e*f^2*x^2 - 3*I*b^3*e^2*f*
x - I*b^3*e^3)*dilog(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 4*(-
I*b^3*f^3*x^3 - 3*I*b^3*e*f^2*x^2 - 3*I*b^3*e^2*f*x - I*b^3*e^3)*dilog(-1/
2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 4*(I*b^3*f^3*x^3 + 3*I*b^3*
e*f^2*x^2 + 3*I*b^3*e^2*f*x + I*b^3*e^3)*dilog(1/2*sqrt(-4*I)*(cosh(b*x +
a) + sinh(b*x + a))) - 4*(I*b^3*f^3*x^3 + 3*I*b^3*e*f^2*x^2 + 3*I*b^3*e^2*
f*x + I*b^3*e^3)*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) +
(I*b^4*f^3*x^4 + 4*I*b^4*e*f^2*x^3 + 6*I*b^4*e^2*f*x^2 + 4*I*b^4*e^3*x + 4
*I*a*b^3*e^3 - 6*I*a^2*b^2*e^2*f + 4*I*a^3*b*e*f^2 - I*a^4*f^3)*log(1/2*sq
rt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^4*f^3*x^4 + 4*I*b^4*e*
f^2*x^3 + 6*I*b^4*e^2*f*x^2 + 4*I*b^4*e^3*x + 4*I*a*b^3*e^3 - 6*I*a^2*b^2*
e^2*f + 4*I*a^3*b*e*f^2 - I*a^4*f^3)*log(-1/2*sqrt(4*I)*(cosh(b*x + a) + s
inh(b*x + a)) + 1) + (-I*b^4*f^3*x^4 - 4*I*b^4*e*f^2*x^3 - 6*I*b^4*e^2*f*x
^2 - 4*I*b^4*e^3*x - 4*I*a*b^3*e^3 + 6*I*a^2*b^2*e^2*f - 4*I*a^3*b*e*f^2 +
 I*a^4*f^3)*log(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (...
```

3.183.6 Sympy [F]

$$\int (e+fx)^3 \cot^{-1}(\tanh(a+bx))\,dx = \int (e+fx)^3 \operatorname{acot}(\tanh(a+bx))\,dx$$

input
```
integrate((f*x+e)**3*acot(tanh(b*x+a)),x)
```

output
```
Integral((e + f*x)**3*acot(tanh(a + b*x)), x)
```

3.183.7 Maxima [F]

$$\int (e+fx)^3 \cot^{-1}(\tanh(a+bx))\,dx = \int (fx+e)^3 \operatorname{arccot}(\tanh(bx+a))\,dx$$

input
```
integrate((f*x+e)^3*arccot(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*arctan2(e^(2*b*x + 2*a
) + 1, e^(2*b*x + 2*a) - 1) + integrate(1/2*(b*f^3*x^4*e^(2*a) + 4*b*e*f^2
*x^3*e^(2*a) + 6*b*e^2*f*x^2*e^(2*a) + 4*b*e^3*x*e^(2*a))*e^(2*b*x)/(e^(4*
b*x + 4*a) + 1), x)
```

3.183.8 Giac [F]

$$\int (e+fx)^3 \cot^{-1}(\tanh(a+bx))\,dx = \int (fx+e)^3 \operatorname{arccot}(\tanh(bx+a))\,dx$$

input
```
integrate((f*x+e)^3*arccot(tanh(b*x+a)),x, algorithm="giac")
```

output
```
sage0*x
```

3.183.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)^3 \cot^{-1}(\tanh(a+bx))\,dx = \int \operatorname{acot}(\tanh(a+b\,x))\,(e+f\,x)^3\,dx$$

input
```
int(acot(tanh(a + b*x))*(e + f*x)^3,x)
```

output
```
int(acot(tanh(a + b*x))*(e + f*x)^3, x)
```

3.184 $\int (e+fx)^2 \cot^{-1}(\tanh(a+bx))\,dx$

3.184.1 Optimal result

Integrand size = 15, antiderivative size = 229

$$\begin{aligned}\int (e+fx)^2 \cot^{-1}(\tanh(a+bx))\,dx =& \frac{(e+fx)^3 \cot^{-1}(\tanh(a+bx))}{3f} \\ &+ \frac{(e+fx)^3 \arctan\left(e^{2a+2bx}\right)}{3f} \\ &- \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} \\ &+ \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} \\ &+ \frac{if(e+fx) \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{4b^2} \\ &- \frac{if(e+fx) \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{4b^2} \\ &- \frac{if^2 \operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} \\ &+ \frac{if^2 \operatorname{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3}\end{aligned}$$

output
```
1/3*(f*x+e)^3*arccot(tanh(b*x+a))/f+1/3*(f*x+e)^3*arctan(exp(2*b*x+2*a))/f
-1/4*I*(f*x+e)^2*polylog(2,-I*exp(2*b*x+2*a))/b+1/4*I*(f*x+e)^2*polylog(2,
I*exp(2*b*x+2*a))/b+1/4*I*f*(f*x+e)*polylog(3,-I*exp(2*b*x+2*a))/b^2-1/4*I
*f*(f*x+e)*polylog(3,I*exp(2*b*x+2*a))/b^2-1/8*I*f^2*polylog(4,-I*exp(2*b*
x+2*a))/b^3+1/8*I*f^2*polylog(4,I*exp(2*b*x+2*a))/b^3
```

3.184.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.64

$$\int (e+fx)^2 \cot^{-1}(\tanh(a+bx))\,dx = \frac{1}{3}x\left(3e^2+3efx+f^2x^2\right)\cot^{-1}(\tanh(a+bx)) + \frac{i\left(12b^3e^2x\log\left(1-ie^{2(a+bx)}\right)+12b^3efx^2\log\left(1-ie^{2(a+bx)}\right)+4b^3f^2x^3\log\left(1-ie^{2(a+bx)}\right)-12b^3e^2x\log\left(\right.\right.}{}$$

input
```
Integrate[(e + f*x)^2*ArcCot[Tanh[a + b*x]],x]
```

output
```
(x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcCot[Tanh[a + b*x]])/3 + ((I/24)*(12*b^3*
e^2*x*Log[1 - I*E^(2*(a + b*x))] + 12*b^3*e*f*x^2*Log[1 - I*E^(2*(a + b*x)
)] + 4*b^3*f^2*x^3*Log[1 - I*E^(2*(a + b*x))] - 12*b^3*e^2*x*Log[1 + I*E^(
2*(a + b*x))] - 12*b^3*e*f*x^2*Log[1 + I*E^(2*(a + b*x))] - 4*b^3*f^2*x^3*
Log[1 + I*E^(2*(a + b*x))] - 6*b^2*(e + f*x)^2*PolyLog[2, (-I)*E^(2*(a + b
*x))] + 6*b^2*(e + f*x)^2*PolyLog[2, I*E^(2*(a + b*x))] + 6*b*e*f*PolyLog[
3, (-I)*E^(2*(a + b*x))] + 6*b*f^2*x*PolyLog[3, (-I)*E^(2*(a + b*x))] - 6*
b*e*f*PolyLog[3, I*E^(2*(a + b*x))] - 6*b*f^2*x*PolyLog[3, I*E^(2*(a + b*x
))] - 3*f^2*PolyLog[4, (-I)*E^(2*(a + b*x))] + 3*f^2*PolyLog[4, I*E^(2*(a
+ b*x))]))/b^3
```

3.184.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {5707, 3042, 4668, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2 \cot^{-1}(\tanh(a+bx))\,dx$$

$$\Big\downarrow \text{ 5707}$$

$$\frac{b\int (e+fx)^3\text{sech}(2a+2bx)dx}{3f}+\frac{(e+fx)^3\cot^{-1}(\tanh(a+bx))}{3f}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(e+fx)^3\cot^{-1}(\tanh(a+bx))}{3f}+\frac{b\int (e+fx)^3\csc\left(2ia+2ibx+\frac{\pi}{2}\right)dx}{3f}$$

$$\Big\downarrow \; 4668$$

$$\frac{(e+fx)^3\cot^{-1}(\tanh(a+bx))}{3f}+\frac{b\left(-\frac{3if\int(e+fx)^2\log\left(1-ie^{2a+2bx}\right)dx}{2b}+\frac{3if\int(e+fx)^2\log\left(1+ie^{2a+2bx}\right)dx}{2b}+\frac{(e+fx)^3\arctan\left(e^{2a+2bx}\right)}{b}\right)}{3f}$$

$$\Big\downarrow \; 3011$$

$$\frac{(e+fx)^3\cot^{-1}(\tanh(a+bx))}{3f}+\frac{b\left(\frac{3if\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)dx}{b}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{2b}\right)}{2b}-\frac{3if\left(\frac{f\int(e+fx)\operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)dx}{b}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,ie^{2a+2}\right.}{2b}\right.}{2b}\right.}{3f}$$

$$\Big\downarrow \; 7163$$

$$\frac{(e+fx)^3\cot^{-1}(\tanh(a+bx))}{3f}+\frac{b\left(\frac{3if\left(\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{2b}-\frac{f\int\operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)dx}{2b}\right)}{b}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{2b}\right)}{2b}-\frac{3if\left(\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{2b}\right.}{}\right.}{}\right.}{3f}$$

$$\Big\downarrow \; 2720$$

$$\frac{(e+fx)^3\cot^{-1}(\tanh(a+bx))}{3f}+\frac{b\left(\frac{3if\left(\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{2b}-\frac{f\int e^{-2a-2bx}\operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)de^{2a+2bx}}{4b^2}\right)}{b}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{2b}\right)}{2b}-\frac{3if\left(\frac{f\left(\frac{(e+fx)\operatorname{Po}}{}\right.}{}\right.}{}\right.}{3f}$$

$$\Big\downarrow \; 7143$$

3.184. $\int(e+fx)^2\cot^{-1}(\tanh(a+bx))\,dx$

$$\frac{(e+fx)^3\cot^{-1}(\tanh(a+bx))}{3f}+\frac{b\left(\frac{(e+fx)^3\arctan\left(e^{2a+2bx}\right)}{b}+\frac{3if\left(\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{2b}-\frac{f\operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{4b^2}\right)}{b}-\frac{(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{2b}\right)}{2b}-\frac{3if\left(\frac{f\left(\cdots\right.}{}\right.}{}\right.}{3f}$$

input
```
Int[(e + f*x)^2*ArcCot[Tanh[a + b*x]],x]
```

output
```
((e + f*x)^3*ArcCot[Tanh[a + b*x]])/(3*f) + (b*(((e + f*x)^3*ArcTan[E^(2*a
 + 2*b*x)])/b + (((3*I)/2)*f*(-1/2*((e + f*x)^2*PolyLog[2, (-I)*E^(2*a + 2
*b*x)])/b + (f*(((e + f*x)*PolyLog[3, (-I)*E^(2*a + 2*b*x)])/(2*b) - (f*Po
lyLog[4, (-I)*E^(2*a + 2*b*x)])/(4*b^2)))/b))/b - (((3*I)/2)*f*(-1/2*((e +
 f*x)^2*PolyLog[2, I*E^(2*a + 2*b*x)])/b + (f*(((e + f*x)*PolyLog[3, I*E^(
2*a + 2*b*x)])/(2*b) - (f*PolyLog[4, I*E^(2*a + 2*b*x)])/(4*b^2)))/b))/b))
/(3*f)
```

3.184.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5707
```
Int[ArcCot[Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcCot[Tanh[a + b*x]]/(f*(m + 1))), x] + Simp[b/
(f*(m + 1))   Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a,
b, e, f}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.184.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 24.43 (sec) , antiderivative size = 2668, normalized size of antiderivative = 11.65

method	result	size
risch	Expression too large to display	2668

input
```
int((f*x+e)^2*arccot(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*f*e*ln(1+I*exp(2*b*x+2*a))*x^2+1/6*I*f^2/b^3*a^3*ln(-exp(2*b*x+2*a)
+I)+I*f/b*e*ln(1-I*exp(2*b*x+2*a))*a*x-I*f/b*a*e*ln(((-I)^(1/2)-exp(b*x+a)
)/(-I)^(1/2))*x-I*f/b*a*e*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))*x+I*f/b*a
*e*ln(1+exp(b*x+a)*(-1)^(3/4))*x+I*f/b*a*e*ln(1-exp(b*x+a)*(-1)^(3/4))*x-I
*f/b*e*ln(1+I*exp(2*b*x+2*a))*a*x+1/8*I*f^2*polylog(4,I*exp(2*b*x+2*a))/b^
3-1/2*I*f^2/b^3*a^2*dilog(1-exp(b*x+a)*(-1)^(3/4))+1/4*I*f/b^2*e*polylog(3
,-I*exp(2*b*x+2*a))+1/3*I*f^2/b^3*ln(1+I*exp(2*b*x+2*a))*a^3-1/4*I*f^2/b*p
olylog(2,-I*exp(2*b*x+2*a))*x^2+1/4*I*f^2/b^3*polylog(2,-I*exp(2*b*x+2*a))
*a^2+1/4*I*f^2/b^2*polylog(3,-I*exp(2*b*x+2*a))*x-1/2*I*f^2/b^3*a^3*ln(1+e
xp(b*x+a)*(-1)^(3/4))-1/2*I*f^2/b^3*a^3*ln(1-exp(b*x+a)*(-1)^(3/4))-1/2*I*
f^2/b^3*a^2*dilog(1+exp(b*x+a)*(-1)^(3/4))+1/2*I/b*a*e^2*ln(-exp(2*b*x+2*a
)+I)-1/2*I/b*e^2*ln(1+exp(b*x+a)*(-1)^(3/4))*a-1/2*I/b*e^2*ln(1-exp(b*x+a)
*(-1)^(3/4))*a+1/2*I*f*ln(exp(2*b*x+2*a)-I)*x^2*e+1/2*I/b*e^2*ln(((-I)^(1/
2)-exp(b*x+a))/(-I)^(1/2))*a+1/2*I/b*e^2*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(
1/2))*a-1/2*I/b*a*e^2*ln(exp(2*b*x+2*a)+I)-1/6*I*f^2/b^3*a^3*ln(exp(2*b*x+
2*a)+I)-1/3*I*f^2/b^3*ln(1-I*exp(2*b*x+2*a))*a^3+1/4*I*f^2/b*polylog(2,I*e
xp(2*b*x+2*a))*x^2-1/4*I*f^2/b^3*polylog(2,I*exp(2*b*x+2*a))*a^2-1/4*I*f^2
/b^2*polylog(3,I*exp(2*b*x+2*a))*x+1/2*I*f^2/b^3*a^3*ln(((-I)^(1/2)-exp(b*
x+a))/(-I)^(1/2))+1/2*I*f^2/b^3*a^3*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))
+1/2*I*f^2/b^3*a^2*dilog(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))+1/2*I*f^2/...
```

3.184.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1002 vs. $2(180) = 360$.

Time = 0.34 (sec) , antiderivative size = 1002, normalized size of antiderivative = 4.38

$$\int (e + fx)^2 \cot^{-1}(\tanh(a + bx))\, dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*arccot(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/6*(6*I*f^2*polylog(4, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 6
*I*f^2*polylog(4, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 6*I*f^
2*polylog(4, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 6*I*f^2*pol
ylog(4, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*(b^3*f^2*x^3
+ 3*b^3*e*f*x^2 + 3*b^3*e^2*x)*arctan(cosh(b*x + a)/sinh(b*x + a)) - 3*(-I
*b^2*f^2*x^2 - 2*I*b^2*e*f*x - I*b^2*e^2)*dilog(1/2*sqrt(4*I)*(cosh(b*x +
a) + sinh(b*x + a))) - 3*(-I*b^2*f^2*x^2 - 2*I*b^2*e*f*x - I*b^2*e^2)*dilo
g(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 3*(I*b^2*f^2*x^2 + 2*I
*b^2*e*f*x + I*b^2*e^2)*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a
))) - 3*(I*b^2*f^2*x^2 + 2*I*b^2*e*f*x + I*b^2*e^2)*dilog(-1/2*sqrt(-4*I)*
(cosh(b*x + a) + sinh(b*x + a))) + (I*b^3*f^2*x^3 + 3*I*b^3*e*f*x^2 + 3*I*
b^3*e^2*x + 3*I*a*b^2*e^2 - 3*I*a^2*b*e*f + I*a^3*f^2)*log(1/2*sqrt(4*I)*(
cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^3*f^2*x^3 + 3*I*b^3*e*f*x^2 + 3
*I*b^3*e^2*x + 3*I*a*b^2*e^2 - 3*I*a^2*b*e*f + I*a^3*f^2)*log(-1/2*sqrt(4*
I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^3*f^2*x^3 - 3*I*b^3*e*f*x^
2 - 3*I*b^3*e^2*x - 3*I*a*b^2*e^2 + 3*I*a^2*b*e*f - I*a^3*f^2)*log(1/2*sqr
t(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^3*f^2*x^3 - 3*I*b^3*e
*f*x^2 - 3*I*b^3*e^2*x - 3*I*a*b^2*e^2 + 3*I*a^2*b*e*f - I*a^3*f^2)*log(-1
/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-3*I*a*b^2*e^2 + 3*I
*a^2*b*e*f - I*a^3*f^2)*log(I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x ...
```

3.184.6 Sympy [F]

$$\int (e+fx)^2 \cot^{-1}(\tanh(a+bx))\,dx = \int (e+fx)^2 \operatorname{acot}(\tanh(a+bx))\,dx$$

input
```
integrate((f*x+e)**2*acot(tanh(b*x+a)),x)
```

output
```
Integral((e + f*x)**2*acot(tanh(a + b*x)), x)
```

3.184.7 Maxima [F]

$$\int (e+fx)^2 \cot^{-1}(\tanh(a+bx))\,dx = \int (fx+e)^2 \operatorname{arccot}(\tanh(bx+a))\,dx$$

input
```
integrate((f*x+e)^2*arccot(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/3*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*arctan2(e^(2*b*x + 2*a) + 1, e^(2*b*x
+ 2*a) - 1) + integrate(2/3*(b*f^2*x^3*e^(2*a) + 3*b*e*f*x^2*e^(2*a) + 3*b
*e^2*x*e^(2*a))*e^(2*b*x)/(e^(4*b*x + 4*a) + 1), x)
```

3.184.8 Giac [F]

$$\int (e+fx)^2 \cot^{-1}(\tanh(a+bx))\,dx = \int (fx+e)^2 \operatorname{arccot}(\tanh(bx+a))\,dx$$

input
```
integrate((f*x+e)^2*arccot(tanh(b*x+a)),x, algorithm="giac")
```

output
```
sage0*x
```

3.184.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2 \cot^{-1}(\tanh(a+bx))\,dx = \int \operatorname{acot}(\tanh(a+b\,x))\,(e+f\,x)^2\,dx$$

input
```
int(acot(tanh(a + b*x))*(e + f*x)^2,x)
```

output
```
int(acot(tanh(a + b*x))*(e + f*x)^2, x)
```

3.185 $\int (e+fx)\cot^{-1}(\tanh(a+bx))\,dx$

3.185.1 Optimal result

Integrand size = 13, antiderivative size = 159

$$\int (e+fx)\cot^{-1}(\tanh(a+bx))\,dx = \frac{(e+fx)^2\cot^{-1}(\tanh(a+bx))}{2f} + \frac{(e+fx)^2\arctan\left(e^{2a+2bx}\right)}{2f} - \frac{i(e+fx)\operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i(e+fx)\operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} + \frac{if\operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} - \frac{if\operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2}$$

output
```
1/2*(f*x+e)^2*arccot(tanh(b*x+a))/f+1/2*(f*x+e)^2*arctan(exp(2*b*x+2*a))/f
-1/4*I*(f*x+e)*polylog(2,-I*exp(2*b*x+2*a))/b+1/4*I*(f*x+e)*polylog(2,I*ex
p(2*b*x+2*a))/b+1/8*I*f*polylog(3,-I*exp(2*b*x+2*a))/b^2-1/8*I*f*polylog(3
,I*exp(2*b*x+2*a))/b^2
```

3.185.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.49

$$\int (e+fx)\cot^{-1}(\tanh(a+bx))\,dx = ex\cot^{-1}(\tanh(a+bx)) + \frac{1}{2}fx^2\cot^{-1}(\tanh(a+bx)) + \frac{ie\left(2bx\left(\log\left(1-ie^{2(a+bx)}\right)-\log\left(1+ie^{2(a+bx)}\right)\right)-\text{PolyLog}\left(2,-ie^{2(a+bx)}\right)+\text{PolyLog}\left(2,ie^{2(a+bx)}\right)\right)}{4b} + \frac{if\left(2b^2x^2\log\left(1-ie^{2(a+bx)}\right)-2b^2x^2\log\left(1+ie^{2(a+bx)}\right)-2bx\,\text{PolyLog}\left(2,-ie^{2(a+bx)}\right)+2bx\,\text{PolyLog}\left(2,\right.\right.}{8b^2}$$

input
```
Integrate[(e + f*x)*ArcCot[Tanh[a + b*x]],x]
```

output
```
e*x*ArcCot[Tanh[a + b*x]] + (f*x^2*ArcCot[Tanh[a + b*x]])/2 + ((I/4)*e*(2*
b*x*(Log[1 - I*E^(2*(a + b*x))] - Log[1 + I*E^(2*(a + b*x))]) - PolyLog[2,
 (-I)*E^(2*(a + b*x))] + PolyLog[2, I*E^(2*(a + b*x))]))/b + ((I/8)*f*(2*b
^2*x^2*Log[1 - I*E^(2*(a + b*x))] - 2*b^2*x^2*Log[1 + I*E^(2*(a + b*x))] -
 2*b*x*PolyLog[2, (-I)*E^(2*(a + b*x))] + 2*b*x*PolyLog[2, I*E^(2*(a + b*x
))] + PolyLog[3, (-I)*E^(2*(a + b*x))] - PolyLog[3, I*E^(2*(a + b*x))]))/b
^2
```

3.185.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {5707, 3042, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\cot^{-1}(\tanh(a+bx))\,dx$$

$$\Big\downarrow \text{5707}$$

$$\frac{b\int (e+fx)^2\text{sech}(2a+2bx)dx}{2f}+\frac{(e+fx)^2\cot^{-1}(\tanh(a+bx))}{2f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^2\cot^{-1}(\tanh(a+bx))}{2f}+\frac{b\int (e+fx)^2\csc\left(2ia+2ibx+\frac{\pi}{2}\right)dx}{2f}$$

$$\Big\downarrow \text{4668}$$

$$\frac{(e+fx)^2\cot^{-1}(\tanh(a+bx))}{2f}+\frac{b\left(-\frac{if\int(e+fx)\log(1-ie^{2a+2bx})dx}{b}+\frac{if\int(e+fx)\log(1+ie^{2a+2bx})dx}{b}+\frac{(e+fx)^2\arctan(e^{2a+2bx})}{b}\right)}{2f}$$

$\big\downarrow$ 3011

$$\frac{(e+fx)^2\cot^{-1}(\tanh(a+bx))}{2f}+\frac{b\left(\frac{if\left(\frac{f\int \text{PolyLog}(2,-ie^{2a+2bx})dx}{2b}-\frac{(e+fx)\text{PolyLog}(2,-ie^{2a+2bx})}{2b}\right)}{b}-\frac{if\left(\frac{f\int \text{PolyLog}(2,ie^{2a+2bx})dx}{2b}-\frac{(e+fx)\text{PolyLog}(2,ie^{2a+2bx})}{2b}\right)}{b}+\frac{(e+fx)^2 \ldots}{}\right.}{2f}$$

$\big\downarrow$ 2720

$$\frac{(e+fx)^2\cot^{-1}(\tanh(a+bx))}{2f}+\frac{b\left(\frac{if\left(\frac{f\int e^{-2a-2bx}\text{PolyLog}(2,-ie^{2a+2bx})de^{2a+2bx}}{4b^2}-\frac{(e+fx)\text{PolyLog}(2,-ie^{2a+2bx})}{2b}\right)}{b}-\frac{if\left(\frac{f\int e^{-2a-2bx}\text{PolyLog}(2,ie^{2a+2bx})de^{2a+2bx}}{4b^2}-\frac{(e+fx)\text{P}\ldots}{}\right.}{b}\right.}{2f}$$

$\big\downarrow$ 7143

$$\frac{(e+fx)^2\cot^{-1}(\tanh(a+bx))}{2f}+\frac{b\left(\frac{(e+fx)^2\arctan(e^{2a+2bx})}{b}+\frac{if\left(\frac{f\text{PolyLog}(3,-ie^{2a+2bx})}{4b^2}-\frac{(e+fx)\text{PolyLog}(2,-ie^{2a+2bx})}{2b}\right)}{b}-\frac{if\left(\frac{f\text{PolyLog}(3,ie^{2a+2bx})}{4b^2}-\frac{(e+fx)\text{PolyLog}(2,ie\ldots}{2b}\right.}{b}\right.}{2f}$$

input
```
Int[(e + f*x)*ArcCot[Tanh[a + b*x]],x]
```

output
```
((e + f*x)^2*ArcCot[Tanh[a + b*x]])/(2*f) + (b*(((e + f*x)^2*ArcTan[E^(2*a
 + 2*b*x)])/b + (I*f*(-1/2*((e + f*x)*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b
+ (f*PolyLog[3, (-I)*E^(2*a + 2*b*x)])/(4*b^2)))/b - (I*f*(-1/2*((e + f*x)
*PolyLog[2, I*E^(2*a + 2*b*x)])/b + (f*PolyLog[3, I*E^(2*a + 2*b*x)])/(4*b
^2)))/b))/(2*f)
```

3.185. $\int (e+fx)\cot^{-1}(\tanh(a+bx))\,dx$

3.185.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5707
```
Int[ArcCot[Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcCot[Tanh[a + b*x]]/(f*(m + 1))), x] + Simp[b/
(f*(m + 1))   Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a,
b, e, f}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.185.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.57 (sec) , antiderivative size = 1776, normalized size of antiderivative = 11.17

method	result	size
risch	Expression too large to display	1776

input
```
int((f*x+e)*arccot(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/4*I*ln(exp(2*b*x+2*a)-I)*x^2*f+1/2*I*ln(exp(2*b*x+2*a)-I)*e*x-1/4*I*f*ln
(1+I*exp(2*b*x+2*a))*x^2-1/2*I*e*ln(1+exp(b*x+a)*(-1)^(3/4))*x-1/2*I*e*ln(
1-exp(b*x+a)*(-1)^(3/4))*x-1/2*I*e/b*dilog(1+exp(b*x+a)*(-1)^(3/4))-1/2*I*
e/b*dilog(1-exp(b*x+a)*(-1)^(3/4))+1/4*Pi*(csgn(I/(exp(2*b*x+2*a)+1))*csgn
(I*(exp(2*b*x+2*a)-I))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))-csgn(
I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)+I))*csgn(I*(exp(2*b*x+2*a)+I)
/(exp(2*b*x+2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)-I)/
(exp(2*b*x+2*a)+1))^2+csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)+I)
/(exp(2*b*x+2*a)+1))^2-csgn(I*(exp(2*b*x+2*a)-I))*csgn(I*(exp(2*b*x+2*a)-I
)/(exp(2*b*x+2*a)+1))^2+csgn(I*(exp(2*b*x+2*a)+I))*csgn(I*(exp(2*b*x+2*a)+
I)/(exp(2*b*x+2*a)+1))^2-csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))*csg
n((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))+csgn((1+I)*(exp(2*b*x+2*a)+
I)/(exp(2*b*x+2*a)+1))^2+csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))*csg
n((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))+csgn((1-I)*(exp(2*b*x+2*a)-
I)/(exp(2*b*x+2*a)+1))^2+csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^3-c
sgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))*csgn((1-I)*(exp(2*b*x+2*a)-I)
/(exp(2*b*x+2*a)+1))^2-csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^3+csg
n(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))*csgn((1+I)*(exp(2*b*x+2*a)+I)/(
exp(2*b*x+2*a)+1))^2-csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^3-c
sgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^3+1)*(1/2*f*x^2+e*x)-1...
```

3.185.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 600 vs. 2(130) = 260.

Time = 0.31 (sec) , antiderivative size = 600, normalized size of antiderivative = 3.77

$$\int (e + fx) \cot^{-1}(\tanh(a + bx))\, dx$$

$$= \frac{2\left(b^2fx^2 + 2\,b^2ex\right)\arctan\left(\frac{\cosh(bx+a)}{\sinh(bx+a)}\right) - 2\left(-i\,bfx - i\,be\right)\mathrm{Li}_2\left(\frac{1}{2}\sqrt{4i}(\cosh\left(bx+a\right) + \sinh\left(bx+a\right))\right) - 2\,(\cdots}{}$$

input
```
integrate((f*x+e)*arccot(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/4*(2*(b^2*f*x^2 + 2*b^2*e*x)*arctan(cosh(b*x + a)/sinh(b*x + a)) - 2*(-I
*b*f*x - I*b*e)*dilog(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*(
-I*b*f*x - I*b*e)*dilog(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) -
2*(I*b*f*x + I*b*e)*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)))
- 2*(I*b*f*x + I*b*e)*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)
)) + (I*b^2*f*x^2 + 2*I*b^2*e*x + 2*I*a*b*e - I*a^2*f)*log(1/2*sqrt(4*I)*(
cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^2*f*x^2 + 2*I*b^2*e*x + 2*I*a*b
*e - I*a^2*f)*log(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-
I*b^2*f*x^2 - 2*I*b^2*e*x - 2*I*a*b*e + I*a^2*f)*log(1/2*sqrt(-4*I)*(cosh(
b*x + a) + sinh(b*x + a)) + 1) + (-I*b^2*f*x^2 - 2*I*b^2*e*x - 2*I*a*b*e +
 I*a^2*f)*log(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-2*I
*a*b*e + I*a^2*f)*log(I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (
-2*I*a*b*e + I*a^2*f)*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)
) + (2*I*a*b*e - I*a^2*f)*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x
+ a)) + (2*I*a*b*e - I*a^2*f)*log(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh
(b*x + a)) - 2*I*f*polylog(3, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)
)) - 2*I*f*polylog(3, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*
I*f*polylog(3, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*I*f*pol
ylog(3, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

3.185.6 Sympy [F]

$$\int (e+fx)\cot^{-1}(\tanh(a+bx))\,dx=\int (e+fx)\operatorname{acot}(\tanh(a+bx))\,dx$$

input
```
integrate((f*x+e)*acot(tanh(b*x+a)),x)
```

output
```
Integral((e + f*x)*acot(tanh(a + b*x)), x)
```

3.185.7 Maxima [F]

$$\int (e+fx)\cot^{-1}(\tanh(a+bx))\,dx = \int (fx+e)\operatorname{arccot}(\tanh(bx+a))\,dx$$

input
```
integrate((f*x+e)*arccot(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/2*(f*x^2 + 2*e*x)*arctan2(e^(2*b*x + 2*a) + 1, e^(2*b*x + 2*a) - 1) + in
tegrate((b*f*x^2*e^(2*a) + 2*b*e*x*e^(2*a))*e^(2*b*x)/(e^(4*b*x + 4*a) + 1
), x)
```

3.185.8 Giac [F]

$$\int (e+fx)\cot^{-1}(\tanh(a+bx))\,dx = \int (fx+e)\operatorname{arccot}(\tanh(bx+a))\,dx$$

input
```
integrate((f*x+e)*arccot(tanh(b*x+a)),x, algorithm="giac")
```

output
```
sage0*x
```

3.185.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)\cot^{-1}(\tanh(a+bx))\,dx = \int \operatorname{acot}(\tanh(a+b\,x))\,(e+f\,x)\,dx$$

input
```
int(acot(tanh(a + b*x))*(e + f*x),x)
```

output
```
int(acot(tanh(a + b*x))*(e + f*x), x)
```

3.186 $\int \cot^{-1}(\tanh(a+bx))\,dx$

3.186.1 Optimal result

Integrand size = 7, antiderivative size = 73

$$\int \cot^{-1}(\tanh(a+bx))\,dx = x\cot^{-1}(\tanh(a+bx)) + x\arctan\left(e^{2a+2bx}\right) - \frac{i\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i\,\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}$$

output
```
x*arccot(tanh(b*x+a))+x*arctan(exp(2*b*x+2*a))-1/4*I*polylog(2,-I*exp(2*b*
x+2*a))/b+1/4*I*polylog(2,I*exp(2*b*x+2*a))/b
```

3.186.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int \cot^{-1}(\tanh(a+bx))\,dx = x\cot^{-1}(\tanh(a+bx)) + \frac{i\left(2bx\left(\log\left(1-ie^{2(a+bx)}\right)-\log\left(1+ie^{2(a+bx)}\right)\right)-\text{PolyLog}\left(2,-ie^{2(a+bx)}\right)+\text{PolyLog}\left(2,ie^{2(a+bx)}\right)\right)}{4b}$$

input
```
Integrate[ArcCot[Tanh[a + b*x]],x]
```

output
```
x*ArcCot[Tanh[a + b*x]] + ((I/4)*(2*b*x*(Log[1 - I*E^(2*(a + b*x))] - Log[
1 + I*E^(2*(a + b*x))]) - PolyLog[2, (-I)*E^(2*(a + b*x))] + PolyLog[2, I*
E^(2*(a + b*x))]))/b
```

3.186.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5703, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}(\tanh(a+bx))\,dx$$

$$\Big\downarrow \text{5703}$$

$$b\int x\text{sech}(2a+2bx)dx + x\cot^{-1}(\tanh(a+bx))$$

$$\Big\downarrow \text{3042}$$

$$x\cot^{-1}(\tanh(a+bx)) + b\int x\csc\left(2ia+2ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4668}$$

$$x\cot^{-1}(\tanh(a+bx)) + b\left(-\frac{i\int\log\left(1-ie^{2a+2bx}\right)dx}{2b}+\frac{i\int\log\left(1+ie^{2a+2bx}\right)dx}{2b}+\frac{x\arctan\left(e^{2a+2bx}\right)}{b}\right)$$

$$\Big\downarrow \text{2715}$$

$$x\cot^{-1}(\tanh(a+bx)) + b\left(-\frac{i\int e^{-2a-2bx}\log\left(1-ie^{2a+2bx}\right)de^{2a+2bx}}{4b^2}+\frac{i\int e^{-2a-2bx}\log\left(1+ie^{2a+2bx}\right)de^{2a+2bx}}{4b^2}+\frac{x\arctan\left(e^{2a+2bx}\right)}{b}\right)$$

$$\Big\downarrow \text{2838}$$

$$x\cot^{-1}(\tanh(a+bx)) + b\left(\frac{x\arctan\left(e^{2a+2bx}\right)}{b}-\frac{i\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b^2}+\frac{i\,\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b^2}\right)$$

input
```
Int[ArcCot[Tanh[a + b*x]],x]
```

output
```
x*ArcCot[Tanh[a + b*x]] + b*((x*ArcTan[E^(2*a + 2*b*x)])/b - ((I/4)*PolyLo
g[2, (-I)*E^(2*a + 2*b*x)])/b^2 + ((I/4)*PolyLog[2, I*E^(2*a + 2*b*x)])/b^
2)
```

3.186.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5703
```
Int[ArcCot[Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcCot[Tanh[a + b
*x]], x] + Simp[b   Int[x*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b}, x]
```

3.186.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(62) = 124$.

Time = 1.74 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.52

method	result
derivativedivides	$\operatorname{arctanh}(\tanh(bx+a))\operatorname{arccot}(\tanh(bx+a))+\arctan(\tanh(bx+a))\operatorname{arctanh}(\tanh(bx+a))+\frac{\arctan(\tanh(bx+a))\ln\left(1+\frac{i(1+i\,\mathrm{ta}}{\tanh(}\right.}{2}$
default	$\operatorname{arctanh}(\tanh(bx+a))\operatorname{arccot}(\tanh(bx+a))+\arctan(\tanh(bx+a))\operatorname{arctanh}(\tanh(bx+a))+\frac{\arctan(\tanh(bx+a))\ln\left(1+\frac{i(1+i\,\mathrm{ta}}{\tanh(}\right.}{2}$
risch	Expression too large to display

input
```
int(arccot(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b*(arctanh(tanh(b*x+a))*arccot(tanh(b*x+a))+arctan(tanh(b*x+a))*arctanh(
tanh(b*x+a))+1/2*arctan(tanh(b*x+a))*ln(1+I*(1+I*tanh(b*x+a))^2/(tanh(b*x+
a)^2+1))-1/2*arctan(tanh(b*x+a))*ln(1-I*(1+I*tanh(b*x+a))^2/(tanh(b*x+a)^2
+1))-1/4*I*dilog(1+I*(1+I*tanh(b*x+a))^2/(tanh(b*x+a)^2+1))+1/4*I*dilog(1-
I*(1+I*tanh(b*x+a))^2/(tanh(b*x+a)^2+1)))
```

3.186.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(56) = 112$.

Time = 0.30 (sec) , antiderivative size = 334, normalized size of antiderivative = 4.58

$$\int \cot^{-1}(\tanh(a+bx))\,dx$$
$$= \frac{2\,bx\arctan\left(\frac{\cosh(bx+a)}{\sinh(bx+a)}\right) + (i\,bx+i\,a)\log\left(\frac{1}{2}\sqrt{4i}(\cosh(bx+a)+\sinh(bx+a))+1\right) + (i\,bx+i\,a)\log\left(-\right.}{}$$

input
```
integrate(arccot(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(2*b*x*arctan(cosh(b*x + a)/sinh(b*x + a)) + (I*b*x + I*a)*log(1/2*sqr
t(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b*x + I*a)*log(-1/2*sqrt(
4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b*x - I*a)*log(1/2*sqrt(-4
*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b*x - I*a)*log(-1/2*sqrt(-4
*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - I*a*log(I*sqrt(4*I) + 2*cosh(b*
x + a) + 2*sinh(b*x + a)) - I*a*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sin
h(b*x + a)) + I*a*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) +
I*a*log(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + I*dilog(1/2*s
qrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + I*dilog(-1/2*sqrt(4*I)*(cosh(b
*x + a) + sinh(b*x + a))) - I*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b
*x + a))) - I*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

3.186.6 Sympy [F]

$$\int \cot^{-1}(\tanh(a+bx))\,dx = \int \operatorname{acot}\left(\tanh\left(a+bx\right)\right)dx$$

input
```
integrate(acot(tanh(b*x+a)),x)
```

output
```
Integral(acot(tanh(a + b*x)), x)
```

3.186.7 Maxima [F]

$$\int \cot^{-1}(\tanh(a+bx))\,dx = \int \operatorname{arccot}\left(\tanh\left(bx+a\right)\right)\,dx$$

input
```
integrate(arccot(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
x*arctan2(e^(2*b*x + 2*a) + 1, e^(2*b*x + 2*a) - 1) + 2*b*integrate(x*e^(2
*b*x + 2*a)/(e^(4*b*x + 4*a) + 1), x)
```

3.186.8 Giac [F]

$$\int \cot^{-1}(\tanh(a+bx))\,dx = \int \operatorname{arccot}\left(\tanh\left(bx+a\right)\right)\,dx$$

input
```
integrate(arccot(tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccot(tanh(b*x + a)), x)
```

3.186.9 Mupad [F(-1)]

Timed out.

$$\int \cot^{-1}(\tanh(a+bx))\,dx = \int \mathrm{acot}(\tanh(a+b\,x))\,dx$$

input
```
int(acot(tanh(a + b*x)),x)
```

output
```
int(acot(tanh(a + b*x)), x)
```

3.187 $\int \frac{\cot^{-1}(\tanh(a+bx))}{e+fx} dx$

3.187.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\cot^{-1}(\tanh(a+bx))}{e+fx} dx = \text{Int}\left(\frac{\cot^{-1}(\tanh(a+bx))}{e+fx}, x\right)$$

output
```
CannotIntegrate(arccot(tanh(b*x+a))/(f*x+e),x)
```

3.187.2 Mathematica [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(\tanh(a+bx))}{e+fx} dx = \int \frac{\cot^{-1}(\tanh(a+bx))}{e+fx} dx$$

input
```
Integrate[ArcCot[Tanh[a + b*x]]/(e + f*x),x]
```

output
```
Integrate[ArcCot[Tanh[a + b*x]]/(e + f*x), x]
```

3.187.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(\tanh(a+bx))}{e+fx} \, dx$$
$$\Big\downarrow \ 7299$$
$$\int \frac{\cot^{-1}(\tanh(a+bx))}{e+fx}dx$$

input
```
Int[ArcCot[Tanh[a + b*x]]/(e + f*x),x]
```

output
```
$Aborted
```

3.187.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.187.4 Maple [N/A] (verified)

Not integrable

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arccot}\left(\tanh\left(bx+a\right)\right)}{fx+e}dx$$

input
```
int(arccot(tanh(b*x+a))/(f*x+e),x)
```

output
```
int(arccot(tanh(b*x+a))/(f*x+e),x)
```

3.187.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(\tanh(a+bx))}{e+fx}\,dx = \int \frac{\operatorname{arccot}(\tanh(bx+a))}{fx+e}\,dx$$

input
```
integrate(arccot(tanh(b*x+a))/(f*x+e),x, algorithm="fricas")
```

output
```
integral(arccot(tanh(b*x + a))/(f*x + e), x)
```

3.187.6 Sympy [N/A]

Not integrable

Time = 3.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{\cot^{-1}(\tanh(a+bx))}{e+fx}\,dx = \int \frac{\operatorname{acot}(\tanh(a+bx))}{e+fx}\,dx$$

input
```
integrate(acot(tanh(b*x+a))/(f*x+e),x)
```

output
```
Integral(acot(tanh(a + b*x))/(e + f*x), x)
```

3.187.7 Maxima [N/A]

Not integrable

Time = 1.56 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(\tanh(a+bx))}{e+fx}\,dx = \int \frac{\operatorname{arccot}(\tanh(bx+a))}{fx+e}\,dx$$

input
```
integrate(arccot(tanh(b*x+a))/(f*x+e),x, algorithm="maxima")
```

output
```
integrate(arccot(tanh(b*x + a))/(f*x + e), x)
```

3.187.8 Giac [N/A]

Not integrable

Time = 90.59 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.20

$$\int \frac{\cot^{-1}(\tanh(a+bx))}{e+fx}\,dx=\int \frac{\operatorname{arccot}\left(\tanh\left(bx+a\right)\right)}{fx+e}\,dx$$

input
```
integrate(arccot(tanh(b*x+a))/(f*x+e),x, algorithm="giac")
```

output
```
sage0*x
```

3.187.9 Mupad [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(\tanh(a+bx))}{e+fx}\,dx=\int \frac{\operatorname{acot}(\tanh(a+b\,x))}{e+f\,x}\,dx$$

input
```
int(acot(tanh(a + b*x))/(e + f*x),x)
```

output
```
int(acot(tanh(a + b*x))/(e + f*x), x)
```

3.188 $\int x^2 \cot^{-1}(c+d\tanh(a+bx))\,dx$

3.188.1 Optimal result

Integrand size = 15, antiderivative size = 355

$$\begin{aligned}\int x^2\cot^{-1}(c+d\tanh(a+bx))\,dx &= \frac{1}{3}x^3\cot^{-1}(c+d\tanh(a+bx)) \\ &\quad -\frac{1}{6}ix^3\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) \\ &\quad +\frac{1}{6}ix^3\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) \\ &\quad -\frac{ix^2\operatorname{PolyLog}\left(2,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} \\ &\quad +\frac{ix^2\operatorname{PolyLog}\left(2,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b} \\ &\quad +\frac{ix\operatorname{PolyLog}\left(3,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b^2} \\ &\quad -\frac{ix\operatorname{PolyLog}\left(3,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b^2} \\ &\quad -\frac{i\operatorname{PolyLog}\left(4,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{8b^3} \\ &\quad +\frac{i\operatorname{PolyLog}\left(4,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{8b^3}\end{aligned}$$

output
```
1/3*x^3*arccot(c+d*tanh(b*x+a))-1/6*I*x^3*ln(1+(I-c-d)*exp(2*b*x+2*a)/(I-c
+d))+1/6*I*x^3*ln(1+(I+c+d)*exp(2*b*x+2*a)/(I+c-d))-1/4*I*x^2*polylog(2,-(
I-c-d)*exp(2*b*x+2*a)/(I-c+d))/b+1/4*I*x^2*polylog(2,-(I+c+d)*exp(2*b*x+2*
a)/(I+c-d))/b+1/4*I*x*polylog(3,-(I-c-d)*exp(2*b*x+2*a)/(I-c+d))/b^2-1/4*I
*x*polylog(3,-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^2-1/8*I*polylog(4,-(I-c-d)
*exp(2*b*x+2*a)/(I-c+d))/b^3+1/8*I*polylog(4,-(I+c+d)*exp(2*b*x+2*a)/(I+c-
d))/b^3
```

3.188.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.23

$$\int x^2 \cot^{-1}(c+d\tanh(a+bx))\,dx = \frac{1}{3}x^3\cot^{-1}(c+d\tanh(a+bx)) + \frac{d\left(4b^3x^3\log\left(1+\frac{2(1+(c+d)^2)e^{2(a+bx)}}{2+2c^2-2d^2-4\sqrt{-d^2}}\right)-4b^3x^3\log\left(1+\frac{(1+(c+d)^2)e^{2(a+bx)}}{1+c^2-d^2+2\sqrt{-d^2}}\right)+6b^2x^2\operatorname{PolyLog}\left(2,\frac{(1+c^2+2cd+d^2}{-1-c^2+d^2+}\right.\right.}{}$$

input
```
Integrate[x^2*ArcCot[c + d*Tanh[a + b*x]],x]
```

output
```
(x^3*ArcCot[c + d*Tanh[a + b*x]])/3 + (d*(4*b^3*x^3*Log[1 + (2*(1 + (c + d
)^2)*E^(2*(a + b*x)))/(2 + 2*c^2 - 2*d^2 - 4*Sqrt[-d^2])] - 4*b^3*x^3*Log[
1 + ((1 + (c + d)^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2])] + 6*
b^2*x^2*PolyLog[2, ((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(-1 - c^2 + d
^2 + 2*Sqrt[-d^2])] - 6*b^2*x^2*PolyLog[2, -(((1 + (c + d)^2)*E^(2*(a + b*
x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2]))] - 6*b*x*PolyLog[3, ((1 + c^2 + 2*c*d
 + d^2)*E^(2*(a + b*x)))/(-1 - c^2 + d^2 + 2*Sqrt[-d^2])] + 6*b*x*PolyLog[
3, -(((1 + (c + d)^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2]))] +
3*PolyLog[4, (-2*(1 + (c + d)^2)*E^(2*(a + b*x)))/(2 + 2*c^2 - 2*d^2 - 4*S
qrt[-d^2])] - 3*PolyLog[4, -(((1 + (c + d)^2)*E^(2*(a + b*x)))/(1 + c^2 -
d^2 + 2*Sqrt[-d^2]))]))/(24*b^3*Sqrt[-d^2])
```

3.188.3 Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.24, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5723, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cot^{-1}(d\tanh(a+bx)+c)\,dx$$

$$\Big\downarrow \text{5723}$$

$$\frac{1}{3}b(1+i(c+d))\int \frac{e^{2a+2bx}x^3}{-c+(-c-d+i)e^{2a+2bx}+d+i}dx-\frac{1}{3}b(1-i(c+d))\int \frac{e^{2a+2bx}x^3}{c+(c+d+i)e^{2a+2bx}-d+i}dx+\frac{1}{3}x^3\cot^{-1}(d\tanh(a+bx)+c)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{3}b(1+i(c+d))\left(\frac{x^3\log\left(1+\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}-\frac{3\int x^2\log\left(\frac{e^{2a+2bx}(-c-d+i)}{-c+d+i}+1\right)dx}{2b(-c-d+i)}\right)-\frac{1}{3}b(1-i(c+d))\left(\frac{x^3\log\left(1+\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}-\frac{3\int x^2\log\left(\frac{e^{2a+2bx}(c+d+i)}{c-d+i}+1\right)dx}{2b(c+d+i)}\right)+\frac{1}{3}x^3\cot^{-1}(d\tanh(a+bx)+c)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{3}b(1+i(c+d))\left(\frac{x^3\log\left(1+\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b}\right)}{2b(-c-d+i)}\right)-\frac{1}{3}b(1-i(c+d))\left(\frac{x^3\log\left(1+\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b}\right)}{2b(c+d+i)}\right)+\frac{1}{3}x^3\cot^{-1}(d\tanh(a+bx)+c)$$

$$\downarrow 7163$$

$$\frac{1}{3}b(1+i(c+d))\left(\frac{x^3\log\left(1+\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}-\frac{3\left(\frac{\frac{x\operatorname{PolyLog}\left(3,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b}-\frac{\int \operatorname{PolyLog}\left(3,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)dx}{2b}}{b}-\frac{x^2\operatorname{PolyLog}\left(2,-\frac{(-}{}\right.}{2b}\right.}{2b(-c-d+i)}\right.$$

$$\frac{1}{3}b(1-i(c+d))\left(\frac{x^3\log\left(1+\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}-\frac{3\left(\frac{\frac{x\operatorname{PolyLog}\left(3,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b}-\frac{\int \operatorname{PolyLog}\left(3,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)dx}{2b}}{b}-\frac{x^2\operatorname{PolyLog}\left(2,-\frac{(c+d+i)}{c-}\right.}{2b}\right.}{2b(c+d+i)}\right.$$

$$\frac{1}{3}x^3\cot^{-1}(d\tanh(a+bx)+c)$$

$$\downarrow 2720$$

$$\frac{1}{3}b(1+i(c+d))\left(\frac{x^3\log\left(1+\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}-\frac{3\left(\frac{\frac{x\operatorname{PolyLog}\left(3,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b}-\frac{\int e^{-2a-2bx}\operatorname{PolyLog}\left(3,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)de^{2a+2bx}}{4b^2}}{b}-\frac{x^2}{}\right.}{2b(-c-d+i)}\right.$$

$$\frac{1}{3}b(1-i(c+d))\left(\frac{x^3\log\left(1+\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}-\frac{3\left(\frac{\frac{x\operatorname{PolyLog}\left(3,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b}-\frac{\int e^{-2a-2bx}\operatorname{PolyLog}\left(3,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)de^{2a+2bx}}{4b^2}}{b}-\frac{x^2\operatorname{Poly}}{}\right.}{2b(c+d+i)}\right.$$

$$\frac{1}{3}x^3\cot^{-1}(d\tanh(a+bx)+c)$$

$$\downarrow 7143$$

3.188. $\int x^2\cot^{-1}(c+d\tanh(a+bx))\,dx$

$$\frac{1}{3}b(1+i(c+d))\left(\frac{x^3\log\left(1+\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}-\frac{3\left(\frac{\frac{x\,\mathrm{PolyLog}\left(3,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b}-\frac{\mathrm{PolyLog}\left(4,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b^2}}{b}-\frac{x^2\,\mathrm{PolyLog}\left(2,-\frac{(-c-d}{-}\right.}{2b}\right.}{2b(-c-d+i)}\right.$$

$$\frac{1}{3}b(1-i(c+d))\left(\frac{x^3\log\left(1+\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}-\frac{3\left(\frac{\frac{x\,\mathrm{PolyLog}\left(3,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b}-\frac{\mathrm{PolyLog}\left(4,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b^2}}{b}-\frac{x^2\,\mathrm{PolyLog}\left(2,-\frac{(c+d+i)e^{2a}}{c-d+i}\right.}{2b}\right.}{2b(c+d+i)}\right.$$

$$\frac{1}{3}x^3\cot^{-1}(d\tanh(a+bx)+c)$$

input
```
Int[x^2*ArcCot[c + d*Tanh[a + b*x]],x]
```

output
```
(x^3*ArcCot[c + d*Tanh[a + b*x]])/3 + (b*(1 + I*(c + d))*((x^3*Log[1 + ((I
 - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/(2*b*(I - c - d)) - (3*(-1/2*(x^2
*PolyLog[2, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c + d))])/b + ((x*PolyLog
[3, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c + d))])/(2*b) - PolyLog[4, -(((
I - c - d)*E^(2*a + 2*b*x))/(I - c + d))]/(4*b^2))/b))/(2*b*(I - c - d))))
/3 - (b*(1 - I*(c + d))*((x^3*Log[1 + ((I + c + d)*E^(2*a + 2*b*x))/(I + c
 - d)])/(2*b*(I + c + d)) - (3*(-1/2*(x^2*PolyLog[2, -(((I + c + d)*E^(2*a
 + 2*b*x))/(I + c - d))])/b + ((x*PolyLog[3, -(((I + c + d)*E^(2*a + 2*b*x
))/(I + c - d))])/(2*b) - PolyLog[4, -(((I + c + d)*E^(2*a + 2*b*x))/(I +
c - d))]/(4*b^2))/b))/(2*b*(I + c + d))))/3
```

3.188. $\int x^2\cot^{-1}(c+d\tanh(a+bx))\,dx$

3.188.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5723
```
Int[ArcCot[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Tanh[a + b*x]]/(f*(m
+ 1))), x] + (-Simp[I*b*((I - c - d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(
E^(2*a + 2*b*x)/(I - c + d + (I - c - d)*E^(2*a + 2*b*x))), x], x] + Simp[I
*b*((I + c + d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(I +
c - d + (I + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[m, 0] && NeQ[(c - d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.188. $\int x^2 \cot^{-1}(c + d \tanh(a + bx)) \, dx$

3.188.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 42.43 (sec) , antiderivative size = 6916, normalized size of antiderivative = 19.48

method	result	size
risch	Expression too large to display	6916

input
```
int(x^2*arccot(c+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.188.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1289 vs. 2(263) = 526.

Time = 0.34 (sec) , antiderivative size = 1289, normalized size of antiderivative = 3.63

$$\int x^2 \cot^{-1}(c + d \tanh(a + bx)) \, dx = \text{Too large to display}$$

input
```
integrate(x^2*arccot(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/6*(2*b^3*x^3*arctan(cosh(b*x + a)/(c*cosh(b*x + a) + d*sinh(b*x + a))) -
 3*I*b^2*x^2*dilog(sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*
(cosh(b*x + a) + sinh(b*x + a))) - 3*I*b^2*x^2*dilog(-sqrt(-(c^2 - d^2 + 2
*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*I*
b^2*x^2*dilog(sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh
(b*x + a) + sinh(b*x + a))) + 3*I*b^2*x^2*dilog(-sqrt(-(c^2 - d^2 - 2*I*d
+ 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + I*a^3*log
(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(
b*x + a) + 2*(c^2 - d^2 - 2*I*d + 1)*sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 -
2*c*d + d^2 + 1))) + I*a^3*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2
*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) - 2*(c^2 - d^2 - 2*I*d + 1)*sqrt(-(
c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) - I*a^3*log(2*(c^2 + 2*c*
d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(
c^2 - d^2 + 2*I*d + 1)*sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 +
1))) - I*a^3*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d
+ d^2 + 1)*sinh(b*x + a) - 2*(c^2 - d^2 + 2*I*d + 1)*sqrt(-(c^2 - d^2 - 2*
I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) + 6*I*b*x*polylog(3, sqrt(-(c^2 - d^2 +
 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 6*
I*b*x*polylog(3, -sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(
cosh(b*x + a) + sinh(b*x + a))) - 6*I*b*x*polylog(3, sqrt(-(c^2 - d^2 -...
```

3.188.6 Sympy [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(c + d \tanh(a + bx))\,dx = \text{Timed out}$$

input
```
integrate(x**2*acot(c+d*tanh(b*x+a)),x)
```

output
```
Timed out
```

3.188.7 Maxima [F]

$$\int x^2 \cot^{-1}(c + d\tanh(a + bx))\,dx = \int x^2 \operatorname{arccot}\left(d\tanh\left(bx + a\right) + c\right)\,dx$$

input
```
integrate(x^2*arccot(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/3*x^3*arctan2(e^(2*b*x + 2*a) + 1, (c*e^(2*a) + d*e^(2*a))*e^(2*b*x) + c
 - d) + 4*b*d*integrate(1/3*x^3*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*
e^(4*a) + 2*c*d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) + 2*(c^2*e^(2*a
) - d^2*e^(2*a) + e^(2*a))*e^(2*b*x) + 1), x)
```

3.188.8 Giac [F]

$$\int x^2 \cot^{-1}(c + d\tanh(a + bx))\,dx = \int x^2 \operatorname{arccot}\left(d\tanh\left(bx + a\right) + c\right)\,dx$$

input
```
integrate(x^2*arccot(c+d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arccot(d*tanh(b*x + a) + c), x)
```

3.188.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(c + d\tanh(a + bx))\,dx = \int x^2 \operatorname{acot}(c + d\tanh(a + b\,x))\,dx$$

input
```
int(x^2*acot(c + d*tanh(a + b*x)),x)
```

output
```
int(x^2*acot(c + d*tanh(a + b*x)), x)
```

3.189 $\int x \cot^{-1}(c+d\tanh(a+bx))\,dx$

3.189.1 Optimal result

Integrand size = 13, antiderivative size = 267

$$\begin{aligned}\int x\cot^{-1}(c+d\tanh(a+bx))\,dx =& \frac{1}{2}x^2\cot^{-1}(c+d\tanh(a+bx)) \\ &-\frac{1}{4}ix^2\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) \\ &+\frac{1}{4}ix^2\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) \\ &-\frac{ix\operatorname{PolyLog}\left(2,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} \\ &+\frac{ix\operatorname{PolyLog}\left(2,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b} \\ &+\frac{i\operatorname{PolyLog}\left(3,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{8b^2} \\ &-\frac{i\operatorname{PolyLog}\left(3,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{8b^2}\end{aligned}$$

output
```
1/2*x^2*arccot(c+d*tanh(b*x+a))-1/4*I*x^2*ln(1+(I-c-d)*exp(2*b*x+2*a)/(I-c
+d))+1/4*I*x^2*ln(1+(I+c+d)*exp(2*b*x+2*a)/(I+c-d))-1/4*I*x*polylog(2,-(I-
c-d)*exp(2*b*x+2*a)/(I-c+d))/b+1/4*I*x*polylog(2,-(I+c+d)*exp(2*b*x+2*a)/(
I+c-d))/b+1/8*I*polylog(3,-(I-c-d)*exp(2*b*x+2*a)/(I-c+d))/b^2-1/8*I*polyl
og(3,-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^2
```

3.189.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.24

$$\int x\cot^{-1}(c+d\tanh(a+bx))\,dx=\frac{1}{2}x^2\cot^{-1}(c+d\tanh(a+bx))$$
$$+\frac{d\left(2b^2x^2\log\left(1+\frac{2(1+(c+d)^2)e^{2(a+bx)}}{2+2c^2-2d^2-4\sqrt{-d^2}}\right)-2b^2x^2\log\left(1+\frac{(1+(c+d)^2)e^{2(a+bx)}}{1+c^2-d^2+2\sqrt{-d^2}}\right)+2bx\operatorname{PolyLog}\left(2,\frac{(1+c^2+2cd+d^2)e}{-1-c^2+d^2+2\sqrt{}}\right.\right.}{}$$

input
```
Integrate[x*ArcCot[c + d*Tanh[a + b*x]],x]
```

output
```
(x^2*ArcCot[c + d*Tanh[a + b*x]])/2 + (d*(2*b^2*x^2*Log[1 + (2*(1 + (c + d
)^2)*E^(2*(a + b*x)))/(2 + 2*c^2 - 2*d^2 - 4*Sqrt[-d^2])] - 2*b^2*x^2*Log[
1 + ((1 + (c + d)^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2])] + 2*
b*x*PolyLog[2, ((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(-1 - c^2 + d^2 +
 2*Sqrt[-d^2])] - 2*b*x*PolyLog[2, -(((1 + (c + d)^2)*E^(2*(a + b*x)))/(1
+ c^2 - d^2 + 2*Sqrt[-d^2]))] - PolyLog[3, (-2*(1 + (c + d)^2)*E^(2*(a + b
*x)))/(2 + 2*c^2 - 2*d^2 - 4*Sqrt[-d^2])] + PolyLog[3, -(((1 + (c + d)^2)*
E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2]))]))/(8*b^2*Sqrt[-d^2])
```

3.189.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.28, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5723, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cot^{-1}(d\tanh(a+bx)+c)\,dx$$

$$\Big\downarrow \text{5723}$$

$$\frac{1}{2}b(1+i(c+d))\int\frac{e^{2a+2bx}x^2}{-c+(-c-d+i)e^{2a+2bx}+d+i}dx-\frac{1}{2}b(1-i(c+d))\int\frac{e^{2a+2bx}x^2}{c+(c+d+i)e^{2a+2bx}-d+i}dx+\frac{1}{2}x^2\cot^{-1}(d\tanh(a+bx)+c)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{2}b(1+i(c+d))\left(\frac{x^2\log\left(1+\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}-\frac{\int x\log\left(\frac{e^{2a+2bx}(-c-d+i)}{-c+d+i}+1\right)dx}{b(-c-d+i)}\right)-\frac{1}{2}b(1-i(c+d))\left(\frac{x^2\log\left(1+\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}-\frac{\int x\log\left(\frac{e^{2a+2bx}(c+d+i)}{c-d+i}+1\right)dx}{b(c+d+i)}\right)+\frac{1}{2}x^2\cot^{-1}(d\tanh(a+bx)+c)$$

$$\Big\downarrow \ 3011$$

$$\frac{1}{2}b(1+i(c+d))\left(\frac{x^2\log\left(1+\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}-\frac{\frac{\int \text{PolyLog}\left(2,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)dx}{2b}-\frac{x\,\text{PolyLog}\left(2,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b}}{b(-c-d+i)}\right)-\frac{1}{2}b(1-i(c+d))\left(\frac{x^2\log\left(1+\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}-\frac{\frac{\int \text{PolyLog}\left(2,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)dx}{2b}-\frac{x\,\text{PolyLog}\left(2,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b}}{b(c+d+i)}\right)+\frac{1}{2}x^2\cot^{-1}(d\tanh(a+bx)+c)$$

$$\Big\downarrow \ 2720$$

$$\frac{1}{2}b(1+i(c+d))\left(\frac{x^2\log\left(1+\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}-\frac{\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(2,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)de^{2a+2bx}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b}}{b(-c-d+i)}\right)-\frac{1}{2}b(1-i(c+d))\left(\frac{x^2\log\left(1+\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}-\frac{\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(2,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)de^{2a+2bx}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b}}{b(c+d+i)}\right)+\frac{1}{2}x^2\cot^{-1}(d\tanh(a+bx)+c)$$

$$\Big\downarrow \ 7143$$

3.189. $\int x\cot^{-1}(c+d\tanh(a+bx))\,dx$

$$\frac{1}{2}b(1+i(c+d))\left(\frac{x^2\log\left(1+\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}-\frac{\frac{\operatorname{PolyLog}\left(3,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b^2}-\frac{x\operatorname{PolyLog}\left(2,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b}}{b(-c-d+i)}\right)-\frac{1}{2}b(1-i(c+d))\left(\frac{x^2\log\left(1+\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}-\frac{\frac{\operatorname{PolyLog}\left(3,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b^2}-\frac{x\operatorname{PolyLog}\left(2,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b}}{b(c+d+i)}\right)+\frac{1}{2}x^2\cot^{-1}(d\tanh(a+bx)+c)$$

input
```
Int[x*ArcCot[c + d*Tanh[a + b*x]],x]
```

output
```
(x^2*ArcCot[c + d*Tanh[a + b*x]])/2 + (b*(1 + I*(c + d))*((x^2*Log[1 + ((I
 - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/(2*b*(I - c - d)) - (-1/2*(x*Poly
Log[2, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c + d))])/b + PolyLog[3, -(((I
 - c - d)*E^(2*a + 2*b*x))/(I - c + d))]/(4*b^2))/(b*(I - c - d))))/2 - (b
*(1 - I*(c + d))*((x^2*Log[1 + ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])
/(2*b*(I + c + d)) - (-1/2*(x*PolyLog[2, -(((I + c + d)*E^(2*a + 2*b*x))/(
I + c - d))])/b + PolyLog[3, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))]
/(4*b^2))/(b*(I + c + d))))/2
```

3.189.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5723
```
Int[ArcCot[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Tanh[a + b*x]]/(f*(m
+ 1))), x] + (-Simp[I*b*((I - c - d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(
E^(2*a + 2*b*x)/(I - c + d + (I - c - d)*E^(2*a + 2*b*x))), x], x] + Simp[I
*b*((I + c + d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(I +
c - d + (I + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[m, 0] && NeQ[(c - d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.189.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.51 (sec) , antiderivative size = 6566, normalized size of antiderivative = 24.59

method	result	size
risch	Expression too large to display	6566

input
```
int(x*arccot(c+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.189.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1067 vs. $2(197) = 394$.

Time = 0.35 (sec) , antiderivative size = 1067, normalized size of antiderivative = 4.00

$$\int x \cot^{-1}(c + d \tanh(a + bx)) \, dx = \text{Too large to display}$$

input
```
integrate(x*arccot(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/4*(2*b^2*x^2*arctan(cosh(b*x + a)/(c*cosh(b*x + a) + d*sinh(b*x + a))) -
 2*I*b*x*dilog(sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cos
h(b*x + a) + sinh(b*x + a))) - 2*I*b*x*dilog(-sqrt(-(c^2 - d^2 + 2*I*d + 1
)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*I*b*x*dilo
g(sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) +
sinh(b*x + a))) + 2*I*b*x*dilog(-sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*
d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - I*a^2*log(2*(c^2 + 2*c*d
+ d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^
2 - d^2 - 2*I*d + 1)*sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1)
)) - I*a^2*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d +
d^2 + 1)*sinh(b*x + a) - 2*(c^2 - d^2 - 2*I*d + 1)*sqrt(-(c^2 - d^2 + 2*I*
d + 1)/(c^2 - 2*c*d + d^2 + 1))) + I*a^2*log(2*(c^2 + 2*c*d + d^2 + 1)*cos
h(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 - d^2 + 2*I*
d + 1)*sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) + I*a^2*log
(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(
b*x + a) - 2*(c^2 - d^2 + 2*I*d + 1)*sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 -
2*c*d + d^2 + 1))) + (-I*b^2*x^2 + I*a^2)*log(sqrt(-(c^2 - d^2 + 2*I*d + 1
)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^2*
x^2 + I*a^2)*log(-sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(
cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^2*x^2 - I*a^2)*log(sqrt(-(c^...
```

3.189.6 Sympy [F(-1)]

Timed out.

$$\int x \cot^{-1}(c + d \tanh(a + bx)) \, dx = \text{Timed out}$$

input
```
integrate(x*acot(c+d*tanh(b*x+a)),x)
```

output
```
Timed out
```

3.189.7 Maxima [F]

$$\int x\cot^{-1}(c+d\tanh(a+bx))\,dx=\int x\,\mathrm{arccot}\left(d\tanh\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x*arccot(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/2*x^2*arctan2(e^(2*b*x + 2*a) + 1, (c*e^(2*a) + d*e^(2*a))*e^(2*b*x) + c
 - d) + 2*b*d*integrate(x^2*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^(4
*a) + 2*c*d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) + 2*(c^2*e^(2*a) -
d^2*e^(2*a) + e^(2*a))*e^(2*b*x) + 1), x)
```

3.189.8 Giac [F]

$$\int x\cot^{-1}(c+d\tanh(a+bx))\,dx=\int x\,\mathrm{arccot}\left(d\tanh\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x*arccot(c+d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arccot(d*tanh(b*x + a) + c), x)
```

3.189.9 Mupad [F(-1)]

Timed out.

$$\int x\cot^{-1}(c+d\tanh(a+bx))\,dx=\int x\,\mathrm{acot}(c+d\tanh(a+b\,x))\,dx$$

input
```
int(x*acot(c + d*tanh(a + b*x)),x)
```

output
```
int(x*acot(c + d*tanh(a + b*x)), x)
```

3.190 $\int \cot^{-1}(c+d\tanh(a+bx))\,dx$

3.190.1 Optimal result

Integrand size = 11, antiderivative size = 174

$$\int \cot^{-1}(c+d\tanh(a+bx))\,dx = x\cot^{-1}(c+d\tanh(a+bx)) - \frac{1}{2}ix\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) + \frac{1}{2}ix\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) - \frac{i\,\mathrm{PolyLog}\left(2,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} + \frac{i\,\mathrm{PolyLog}\left(2,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b}$$

output
```
x*arccot(c+d*tanh(b*x+a))-1/2*I*x*ln(1+(I-c-d)*exp(2*b*x+2*a)/(I-c+d))+1/2
*I*x*ln(1+(I+c+d)*exp(2*b*x+2*a)/(I+c-d))-1/4*I*polylog(2,-(I-c-d)*exp(2*b
*x+2*a)/(I-c+d))/b+1/4*I*polylog(2,-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b
```

3.190.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.66

$$\int \cot^{-1}(c + d\tanh(a + bx))\, dx = x\cot^{-1}(c + d\tanh(a + bx))$$
$$- \frac{4a\sqrt{-d^2}\arctan\left(\frac{1+c^2-d^2+(1+c^2+2cd+d^2)e^{2(a+bx)}}{2d}\right) - 2d(a + bx)\log\left(1 + \frac{2(1+(c+d)^2)e^{2(a+bx)}}{2+2c^2-2d^2-4\sqrt{-d^2}}\right) + 2d(a + bx)\,\mathrm{lo}}{4b\sqrt{-}}$$

input
```
Integrate[ArcCot[c + d*Tanh[a + b*x]],x]
```

output
```
x*ArcCot[c + d*Tanh[a + b*x]] - (4*a*Sqrt[-d^2]*ArcTan[(1 + c^2 - d^2 + (1
 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(2*d)] - 2*d*(a + b*x)*Log[1 + (2*(
1 + (c + d)^2)*E^(2*(a + b*x)))/(2 + 2*c^2 - 2*d^2 - 4*Sqrt[-d^2])] + 2*d*
(a + b*x)*Log[1 + ((1 + (c + d)^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqr
t[-d^2])] + d*PolyLog[2, -(((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(1 +
c^2 - d^2 + 2*Sqrt[-d^2]))] - d*PolyLog[2, ((1 + c^2 + 2*c*d + d^2)*E^(2*(
a + b*x)))/(-1 - c^2 + d^2 + 2*Sqrt[-d^2])])/(4*b*Sqrt[-d^2])
```

3.190.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.36, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5715, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}(d\tanh(a + bx) + c)\, dx$$

$$\Big\downarrow \text{ 5715}$$

$$b(1 + i(c + d))\int \frac{e^{2a+2bx}x}{-c + (-c - d + i)e^{2a+2bx} + d + i}dx - b(1 - i(c + d))\int \frac{e^{2a+2bx}x}{c + (c + d + i)e^{2a+2bx} - d + i}dx + x\cot^{-1}(d\tanh(a + bx) + c)$$

$$\Big\downarrow \text{ 2620}$$

$$b(1+i(c+d))\left(\frac{x\log\left(1+\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}-\frac{\int\log\left(\frac{e^{2a+2bx}(-c-d+i)}{-c+d+i}+1\right)dx}{2b(-c-d+i)}\right)-b(1-i(c+d))\left(\frac{x\log\left(1+\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}-\frac{\int\log\left(\frac{e^{2a+2bx}(c+d+i)}{c-d+i}+1\right)dx}{2b(c+d+i)}\right)+x\cot^{-1}(d\tanh(a+bx)+c)$$

$\Big\downarrow$ 2715

$$b(1+i(c+d))\left(\frac{x\log\left(1+\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}-\frac{\int e^{-2a-2bx}\log\left(\frac{e^{2a+2bx}(-c-d+i)}{-c+d+i}+1\right)de^{2a+2bx}}{4b^2(-c-d+i)}\right)-b(1-i(c+d))\left(\frac{x\log\left(1+\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}-\frac{\int e^{-2a-2bx}\log\left(\frac{e^{2a+2bx}(c+d+i)}{c-d+i}+1\right)de^{2a+2bx}}{4b^2(c+d+i)}\right)+x\cot^{-1}(d\tanh(a+bx)+c)$$

$\Big\downarrow$ 2838

$$b(1+i(c+d))\left(\frac{\text{PolyLog}\left(2,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b^2(-c-d+i)}+\frac{x\log\left(1+\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}\right)-b(1-i(c+d))\left(\frac{\text{PolyLog}\left(2,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b^2(c+d+i)}+\frac{x\log\left(1+\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}\right)+x\cot^{-1}(d\tanh(a+bx)+c)$$

input
```
Int[ArcCot[c + d*Tanh[a + b*x]],x]
```

output
```
x*ArcCot[c + d*Tanh[a + b*x]] + b*(1 + I*(c + d))*((x*Log[1 + ((I - c - d)
*E^(2*a + 2*b*x))/(I - c + d)])/(2*b*(I - c - d)) + PolyLog[2, -(((I - c -
 d)*E^(2*a + 2*b*x))/(I - c + d))]/(4*b^2*(I - c - d))) - b*(1 - I*(c + d)
)*((x*Log[1 + ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/(2*b*(I + c + d)
) + PolyLog[2, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))]/(4*b^2*(I + c
 + d)))
```

3.190. $\int \cot^{-1}(c+d\tanh(a+bx))\,dx$

3.190.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5715
```
Int[ArcCot[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Cot[c + d*Tanh[a + b*x]], x] + (-Simp[I*b*(I - c - d)   Int[x*(E^(2*a + 2*b
*x)/(I - c + d + (I - c - d)*E^(2*a + 2*b*x))), x], x] + Simp[I*b*(I + c +
d)   Int[x*(E^(2*a + 2*b*x)/(I + c - d + (I + c + d)*E^(2*a + 2*b*x))), x],
 x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - d)^2, -1]
```

3.190.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(150) = 300$.

Time = 3.03 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.02

method	result
derivativedivides	$\frac{\operatorname{arccot}(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)-d)}{2}-\frac{\operatorname{arccot}(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}-\frac{d^2\left(\frac{i\ln(-d\tanh(bx+a)+d)\ln(\ldots}{2}\ldots\right.}{\ldots}$
default	$\frac{\operatorname{arccot}(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)-d)}{2}-\frac{\operatorname{arccot}(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}-\frac{d^2\left(\frac{i\ln(-d\tanh(bx+a)+d)\ln(\ldots}{2}\ldots\right.}{\ldots}$
risch	Expression too large to display

input
```
int(arccot(c+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/d*(1/2*arccot(c+d*tanh(b*x+a))*d*ln(-d*tanh(b*x+a)-d)-1/2*arccot(c+d*t
anh(b*x+a))*d*ln(-d*tanh(b*x+a)+d)-1/2*d^2*(1/d*(1/2*I*ln(-d*tanh(b*x+a)+d
)*ln((I+d*tanh(b*x+a)+c)/(I+c+d))-1/2*I*ln(-d*tanh(b*x+a)+d)*ln((I-d*tanh(
b*x+a)-c)/(I-c-d))+1/2*I*dilog((I+d*tanh(b*x+a)+c)/(I+c+d))-1/2*I*dilog((I
-d*tanh(b*x+a)-c)/(I-c-d)))-1/d*(1/2*I*ln(-d*tanh(b*x+a)-d)*ln((I+d*tanh(b
*x+a)+c)/(I+c-d))-1/2*I*ln(-d*tanh(b*x+a)-d)*ln((I-d*tanh(b*x+a)-c)/(I-c+d
))+1/2*I*dilog((I+d*tanh(b*x+a)+c)/(I+c-d))-1/2*I*dilog((I-d*tanh(b*x+a)-c
)/(I-c+d))))
```

3.190.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 825 vs. $2(128) = 256$.

Time = 0.40 (sec) , antiderivative size = 825, normalized size of antiderivative = 4.74

$$\int \cot^{-1}(c + d\tanh(a + bx))\,dx$$
$$= \frac{2\,bx\arctan\left(\frac{\cosh(bx+a)}{c\cosh(bx+a)+d\sinh(bx+a)}\right) + i\,a\log\left(2\left(c^2 + 2\,cd + d^2 + 1\right)\cosh\left(bx + a\right) + 2\left(c^2 + 2\,cd + d^2 + 1\right)\right.}{}$$

input
```
integrate(arccot(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(2*b*x*arctan(cosh(b*x + a)/(c*cosh(b*x + a) + d*sinh(b*x + a))) + I*a
*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*s
inh(b*x + a) + 2*(c^2 - d^2 - 2*I*d + 1)*sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^
2 - 2*c*d + d^2 + 1))) + I*a*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) +
 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) - 2*(c^2 - d^2 - 2*I*d + 1)*sqrt(
-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) - I*a*log(2*(c^2 + 2*c*
d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(
c^2 - d^2 + 2*I*d + 1)*sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 +
1))) - I*a*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d +
d^2 + 1)*sinh(b*x + a) - 2*(c^2 - d^2 + 2*I*d + 1)*sqrt(-(c^2 - d^2 - 2*I*
d + 1)/(c^2 - 2*c*d + d^2 + 1))) + (-I*b*x - I*a)*log(sqrt(-(c^2 - d^2 + 2
*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) +
(-I*b*x - I*a)*log(-sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))
*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b*x + I*a)*log(sqrt(-(c^2 - d^2
 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1
) + (I*b*x + I*a)*log(-sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 +
1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - I*dilog(sqrt(-(c^2 - d^2 + 2*I*
d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - I*dilog
(-sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) +
sinh(b*x + a))) + I*dilog(sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + ...
```

3.190.6 Sympy [F]

$$\int \cot^{-1}(c+d\tanh(a+bx))\,dx = \int \operatorname{acot}\left(c+d\tanh\left(a+bx\right)\right)dx$$

input
```
integrate(acot(c+d*tanh(b*x+a)),x)
```

output
```
Integral(acot(c + d*tanh(a + b*x)), x)
```

3.190.7 Maxima [F]

$$\int \cot^{-1}(c + d\tanh(a + bx))\,dx = \int \operatorname{arccot}\left(d\tanh\left(bx + a\right) + c\right)\,dx$$

input
```
integrate(arccot(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
4*b*d*integrate(x*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^(4*a) + 2*c*
d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) + 2*(c^2*e^(2*a) - d^2*e^(2*a
) + e^(2*a))*e^(2*b*x) + 1), x) + x*arctan2(e^(2*b*x + 2*a) + 1, (c*e^(2*a
) + d*e^(2*a))*e^(2*b*x) + c - d)
```

3.190.8 Giac [F]

$$\int \cot^{-1}(c + d\tanh(a + bx))\,dx = \int \operatorname{arccot}\left(d\tanh\left(bx + a\right) + c\right)\,dx$$

input
```
integrate(arccot(c+d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccot(d*tanh(b*x + a) + c), x)
```

3.190.9 Mupad [F(-1)]

Timed out.

$$\int \cot^{-1}(c + d\tanh(a + bx))\,dx = \int \operatorname{acot}(c + d\tanh(a + b\,x))\,dx$$

input
```
int(acot(c + d*tanh(a + b*x)),x)
```

output
```
int(acot(c + d*tanh(a + b*x)), x)
```

3.191 $\int \frac{\cot^{-1}(c+d\tanh(a+bx))}{x}\,dx$

3.191.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\cot^{-1}(c+d\tanh(a+bx))}{x}\,dx = \text{Int}\left(\frac{\cot^{-1}(c+d\tanh(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(arccot(c+d*tanh(b*x+a))/x,x)
```

3.191.2 Mathematica [N/A]

Not integrable

Time = 3.87 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(c+d\tanh(a+bx))}{x}\,dx = \int \frac{\cot^{-1}(c+d\tanh(a+bx))}{x}\,dx$$

input
```
Integrate[ArcCot[c + d*Tanh[a + b*x]]/x,x]
```

output
```
Integrate[ArcCot[c + d*Tanh[a + b*x]]/x, x]
```

3.191.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(d\tanh(a+bx)+c)}{x} \, dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{\cot^{-1}(d\tanh(a+bx)+c)}{x}dx$$

input
```
Int[ArcCot[c + d*Tanh[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.191.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.191.4 Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arccot}\left(c+d\tanh\left(bx+a\right)\right)}{x}dx$$

input
```
int(arccot(c+d*tanh(b*x+a))/x,x)
```

output
```
int(arccot(c+d*tanh(b*x+a))/x,x)
```

3.191.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(c+d\tanh(a+bx))}{x}\,dx=\int \frac{\operatorname{arccot}\left(d\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccot(c+d*tanh(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(arccot(d*tanh(b*x + a) + c)/x, x)
```

3.191.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(c+d\tanh(a+bx))}{x}\,dx=\text{Timed out}$$

input
```
integrate(acot(c+d*tanh(b*x+a))/x,x)
```

output
```
Timed out
```

3.191.7 Maxima [N/A]

Not integrable

Time = 1.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(c+d\tanh(a+bx))}{x}\,dx=\int \frac{\operatorname{arccot}\left(d\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccot(c+d*tanh(b*x+a))/x,x, algorithm="maxima")
```

output
```
integrate(arccot(d*tanh(b*x + a) + c)/x, x)
```

3.191.8 Giac [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(c + d\tanh(a + bx))}{x}\, dx = \int \frac{\operatorname{arccot}\left(d\tanh\left(bx + a\right) + c\right)}{x}\, dx$$

input
```
integrate(arccot(c+d*tanh(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arccot(d*tanh(b*x + a) + c)/x, x)
```

3.191.9 Mupad [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(c + d\tanh(a + bx))}{x}\, dx = \int \frac{\operatorname{acot}(c + d\tanh(a + b\, x))}{x}\, dx$$

input
```
int(acot(c + d*tanh(a + b*x))/x,x)
```

output
```
int(acot(c + d*tanh(a + b*x))/x, x)
```

3.192 $\int x^2 \cot^{-1}(c+(i+c)\tanh(a+bx))\,dx$

3.192.1 Optimal result

Integrand size = 19, antiderivative size = 142

$$\int x^2 \cot^{-1}(c+(i+c)\tanh(a+bx))\,dx = \frac{1}{12}ibx^4 + \frac{1}{3}x^3\cot^{-1}(c+(i+c)\tanh(a+bx)) - \frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right) - \frac{ix^2\operatorname{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b} + \frac{ix\operatorname{PolyLog}\left(3,-ice^{2a+2bx}\right)}{4b^2} - \frac{i\operatorname{PolyLog}\left(4,-ice^{2a+2bx}\right)}{8b^3}$$

output
```
1/12*I*b*x^4+1/3*x^3*arccot(c+(I+c)*tanh(b*x+a))-1/6*I*x^3*ln(1+I*c*exp(2*
b*x+2*a))-1/4*I*x^2*polylog(2,-I*c*exp(2*b*x+2*a))/b+1/4*I*x*polylog(3,-I*
c*exp(2*b*x+2*a))/b^2-1/8*I*polylog(4,-I*c*exp(2*b*x+2*a))/b^3
```

3.192.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.94

$$\int x^2 \cot^{-1}(c+(i+c)\tanh(a+bx))\,dx$$
$$=\frac{8b^3x^3\cot^{-1}(c+(i+c)\tanh(a+bx))-4ib^3x^3\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right)+6ib^2x^2\operatorname{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right)+6ibx}{24b^3}$$

input
```
Integrate[x^2*ArcCot[c + (I + c)*Tanh[a + b*x]],x]
```

output
```
(8*b^3*x^3*ArcCot[c + (I + c)*Tanh[a + b*x]] - (4*I)*b^3*x^3*Log[1 - I/(c*
E^(2*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, I/(c*E^(2*(a + b*x)))] + (6*I
)*b*x*PolyLog[3, I/(c*E^(2*(a + b*x)))] + (3*I)*PolyLog[4, I/(c*E^(2*(a +
b*x)))])/(24*b^3)
```

3.192.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {5719, 25, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cot^{-1}(c+(c+i)\tanh(a+bx))\,dx$$
$$\Big\downarrow \; 5719$$
$$\frac{1}{3}b\int -\frac{x^3}{i-ce^{2a+2bx}}dx+\frac{1}{3}x^3\cot^{-1}(c+(c+i)\tanh(a+bx))$$
$$\Big\downarrow \; 25$$
$$\frac{1}{3}x^3\cot^{-1}(c+(c+i)\tanh(a+bx))-\frac{1}{3}b\int \frac{x^3}{i-ce^{2a+2bx}}dx$$
$$\Big\downarrow \; 2615$$
$$\frac{1}{3}x^3\cot^{-1}(c+(c+i)\tanh(a+bx))-\frac{1}{3}b\left(-ic\int \frac{e^{2a+2bx}x^3}{i-ce^{2a+2bx}}dx-\frac{ix^4}{4}\right)$$
$$\Big\downarrow \; 2620$$

$$\frac{1}{3}x^3\cot^{-1}(c+(c+i)\tanh(a+bx))-\frac{1}{3}b\left(-ic\left(\frac{3\int x^2\log\left(ie^{2a+2bx}c+1\right)dx}{2bc}-\frac{x^3\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^4}{4}\right)$$

$\downarrow$ 3011

$$\frac{1}{3}x^3\cot^{-1}(c+(c+i)\tanh(a+bx))-\frac{1}{3}b\left(-ic\left(\frac{3\left(\frac{\int x\,\text{PolyLog}(2,-ice^{2a+2bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,-ice^{2a+2bx})}{2b}\right)}{2bc}-\frac{x^3\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^4}{4}\right)$$

$\downarrow$ 7163

$$\frac{1}{3}x^3\cot^{-1}(c+(c+i)\tanh(a+bx))-\frac{1}{3}b\left(-ic\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-ice^{2a+2bx}\right)}{2b}-\frac{\int \text{PolyLog}\left(3,-ice^{2a+2bx}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}(2,-ice^{2a+2bx})}{2b}\right)}{2bc}-\frac{x^3\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^4}{4}\right)$$

$\downarrow$ 2720

$$\frac{1}{3}x^3\cot^{-1}(c+(c+i)\tanh(a+bx))-\frac{1}{3}b\left(-ic\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-ice^{2a+2bx}\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(3,-ice^{2a+2bx}\right)de^{2a+2bx}}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}(2,-ice^{2a+2bx})}{2b}\right)}{2bc}-\frac{x^3\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^4}{4}\right)$$

$\downarrow$ 7143

$$\frac{1}{3}x^3\cot^{-1}(c+(c+i)\tanh(a+bx))-\frac{1}{3}b\left(-ic\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-ice^{2a+2bx}\right)}{2b}-\frac{\text{PolyLog}\left(4,-ice^{2a+2bx}\right)}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}(2,-ice^{2a+2bx})}{2b}\right)}{2bc}-\frac{x^3\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^4}{4}\right)$$

input
```
Int[x^2*ArcCot[c + (I + c)*Tanh[a + b*x]],x]
```

3.192. $\int x^2\cot^{-1}(c+(i+c)\tanh(a+bx))\,dx$

output
```
(x^3*ArcCot[c + (I + c)*Tanh[a + b*x]])/3 - (b*((-1/4*I)*x^4 - I*c*(-1/2*(
x^3*Log[1 + I*c*E^(2*a + 2*b*x)])/(b*c) + (3*(-1/2*(x^2*PolyLog[2, (-I)*c*
E^(2*a + 2*b*x)])/b + ((x*PolyLog[3, (-I)*c*E^(2*a + 2*b*x)])/(2*b) - Poly
Log[4, (-I)*c*E^(2*a + 2*b*x)]/(4*b^2))/b))/(2*b*c))))/3
```

3.192.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5719
```
Int[ArcCot[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Tanh[a + b*x]]/(f*(m
+ 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a +
 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.192.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.53 (sec) , antiderivative size = 1405, normalized size of antiderivative = 9.89

method	result	size
risch	Expression too large to display	1405

input
```
int(x^2*arccot(c+(I+c)*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/12*I/b^3*c/(I+c)*a^4+1/12*Pi*(csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*exp(
2*b*x+2*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))-csg
n(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*cs
gn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))-csgn(I/(e
xp(2*b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2+cs
gn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(e
xp(2*b*x+2*a)+1))^2-csgn(I*(2*exp(2*b*x+2*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2
*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2+csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*
a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^
2+csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^3-csgn(I*(2*exp(2*b*
x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x
+2*a)+1))^2+csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))*csgn((2*ex
p(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))-csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2
*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^3+csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+
2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(
exp(2*b*x+2*a)+1))^2-csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2
*b*x+2*a)+1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)
+1))-csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^3+cs
gn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2-csgn((2*e
xp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^3+csgn((2*exp(2*b*x+2*a)*c-2*I...
```

3.192.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(105) = 210$.

Time = 0.27 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.06

$$\int x^2 \cot^{-1}(c + (i + c)\tanh(a + bx))\,dx$$
$$= \frac{i\,b^4x^4 + 2i\,b^3x^3 \log\left(\frac{(ce^{(2\,bx+2\,a)}-i)e^{(-2\,bx-2\,a)}}{c+i}\right) - 6i\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4i\,c}e^{(bx+a)}\right) - 6i\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,c}e^{(bx+a)}\right)}{}$$

input
```
integrate(x^2*arccot(c+(I+c)*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(I*b^4*x^4 + 2*I*b^3*x^3*log((c*e^(2*b*x + 2*a) - I)*e^(-2*b*x - 2*a)
/(c + I)) - 6*I*b^2*x^2*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) - 6*I*b^2*x^2*
dilog(-1/2*sqrt(-4*I*c)*e^(b*x + a)) - I*a^4 + 2*I*a^3*log(1/2*(2*c*e^(b*x
 + a) + I*sqrt(-4*I*c))/c) + 2*I*a^3*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*
I*c))/c) + 12*I*b*x*polylog(3, 1/2*sqrt(-4*I*c)*e^(b*x + a)) + 12*I*b*x*po
lylog(3, -1/2*sqrt(-4*I*c)*e^(b*x + a)) - 2*(I*b^3*x^3 + I*a^3)*log(1/2*sq
rt(-4*I*c)*e^(b*x + a) + 1) - 2*(I*b^3*x^3 + I*a^3)*log(-1/2*sqrt(-4*I*c)*
e^(b*x + a) + 1) - 12*I*polylog(4, 1/2*sqrt(-4*I*c)*e^(b*x + a)) - 12*I*po
lylog(4, -1/2*sqrt(-4*I*c)*e^(b*x + a)))/b^3
```

3.192.6 Sympy [F(-2)]

Exception generated.

$$\int x^2 \cot^{-1}(c+(i+c)\tanh(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(x**2*acot(c+(I+c)*tanh(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert 2*_t0**4*c**2*exp(4*a)
+ _t0**4*I*c*exp(4*a) - 3*_t0**2*I*c*exp(2*a) + _t0**2*exp(2*a) - 1 of typ
e <class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,exp(a)]
```

3.192.7 Maxima [A] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.91

$$\int x^2 \cot^{-1}(c+(i+c)\tanh(a+bx))\,dx = \frac{1}{3}x^3\,\mathrm{arccot}\left((c+i)\tanh(bx+a)+c\right) - \frac{4}{9}\left(\frac{3x^4}{4ic-4} - \frac{4b^3x^3\log\left(ice^{(2bx+2a)}+1\right)+6b^2x^2\mathrm{Li}_2\left(-ice^{(2bx+2a)}\right)-6bx\mathrm{Li}_3(-ice^{(2bx+2a)})+3\,\mathrm{Li}_4(-}{-2b^4(-ic+1)}\right.$$

input
```
integrate(x^2*arccot(c+(I+c)*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/3*x^3*arccot((c + I)*tanh(b*x + a) + c) - 4/9*(3*x^4/(4*I*c - 4) - (4*b^
3*x^3*log(I*c*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-I*c*e^(2*b*x + 2*a))
 - 6*b*x*polylog(3, -I*c*e^(2*b*x + 2*a)) + 3*polylog(4, -I*c*e^(2*b*x + 2
*a)))/(b^4*(2*I*c - 2)))*b*(c + I)
```

3.192.8 Giac [F]

$$\int x^2 \cot^{-1}(c + (i + c)\tanh(a + bx))\, dx = \int x^2 \operatorname{arccot}\left((c + i)\tanh\left(bx + a\right) + c\right)\, dx$$

input
```
integrate(x^2*arccot(c+(I+c)*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arccot((c + I)*tanh(b*x + a) + c), x)
```

3.192.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(c + (i + c)\tanh(a + bx))\, dx = \int x^2 \operatorname{acot}(c + \tanh(a + b\,x)\ (c + 1\mathrm{i}))\, dx$$

input
```
int(x^2*acot(c + tanh(a + b*x)*(c + 1i)),x)
```

output
```
int(x^2*acot(c + tanh(a + b*x)*(c + 1i)), x)
```

3.193 $\int x \cot^{-1}(c+(i+c)\tanh(a+bx))\,dx$

3.193.1 Optimal result

Integrand size = 17, antiderivative size = 113

$$\int x \cot^{-1}(c+(i+c)\tanh(a+bx))\,dx = \frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c+(i+c)\tanh(a+bx)) - \frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right) - \frac{ix\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b} + \frac{i\,\text{PolyLog}\left(3,-ice^{2a+2bx}\right)}{8b^2}$$

output
```
1/6*I*b*x^3+1/2*x^2*arccot(c+(I+c)*tanh(b*x+a))-1/4*I*x^2*ln(1+I*c*exp(2*b
*x+2*a))-1/4*I*x*polylog(2,-I*c*exp(2*b*x+2*a))/b+1/8*I*polylog(3,-I*c*exp
(2*b*x+2*a))/b^2
```

3.193.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.91

$$\int x \cot^{-1}(c+(i+c)\tanh(a+bx))\,dx = \frac{2b^2x^2\left(2\cot^{-1}(c+(i+c)\tanh(a+bx)) - i\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right)\right) + 2ibx\,\text{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right) + i\,\text{PolyLo}}{8b^2}$$

input
```
Integrate[x*ArcCot[c + (I + c)*Tanh[a + b*x]],x]
```

output
```
(2*b^2*x^2*(2*ArcCot[c + (I + c)*Tanh[a + b*x]] - I*Log[1 - I/(c*E^(2*(a +
 b*x)))]) + (2*I)*b*x*PolyLog[2, I/(c*E^(2*(a + b*x)))] + I*PolyLog[3, I/(
c*E^(2*(a + b*x)))])/(8*b^2)
```

3.193.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {5719, 25, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cot^{-1}(c+(c+i)\tanh(a+bx))\,dx$$

$$\Big\downarrow \text{5719}$$

$$\frac{1}{2}b\int -\frac{x^2}{i-ce^{2a+2bx}}dx+\frac{1}{2}x^2\cot^{-1}(c+(c+i)\tanh(a+bx))$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}x^2\cot^{-1}(c+(c+i)\tanh(a+bx))-\frac{1}{2}b\int \frac{x^2}{i-ce^{2a+2bx}}dx$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{2}x^2\cot^{-1}(c+(c+i)\tanh(a+bx))-\frac{1}{2}b\left(-ic\int \frac{e^{2a+2bx}x^2}{i-ce^{2a+2bx}}dx-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{2}x^2\cot^{-1}(c+(c+i)\tanh(a+bx))-\\ \frac{1}{2}b\left(-ic\left(\frac{\int x\log\left(ie^{2a+2bx}c+1\right)dx}{bc}-\frac{x^2\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}x^2\cot^{-1}(c+(c+i)\tanh(a+bx))-\\ \frac{1}{2}b\left(-ic\left(\frac{\frac{\int \text{PolyLog}(2,-ice^{2a+2bx})dx}{2b}-\frac{x\,\text{PolyLog}(2,-ice^{2a+2bx})}{2b}}{bc}-\frac{x^2\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \; 2720$$

$$\frac{1}{2}x^2\cot^{-1}(c+(c+i)\tanh(a+bx))-\frac{1}{2}b\left(-ic\left(\frac{\frac{\int e^{-2a-2bx}\operatorname{PolyLog}(2,-ice^{2a+2bx})de^{2a+2bx}}{4b^2}-\frac{x\operatorname{PolyLog}(2,-ice^{2a+2bx})}{2b}}{bc}-\frac{x^2\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \; 7143$$

$$\frac{1}{2}x^2\cot^{-1}(c+(c+i)\tanh(a+bx))-\frac{1}{2}b\left(-ic\left(\frac{\frac{\operatorname{PolyLog}(3,-ice^{2a+2bx})}{4b^2}-\frac{x\operatorname{PolyLog}(2,-ice^{2a+2bx})}{2b}}{bc}-\frac{x^2\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^3}{3}\right)$$

input
```
Int[x*ArcCot[c + (I + c)*Tanh[a + b*x]],x]
```

output
```
(x^2*ArcCot[c + (I + c)*Tanh[a + b*x]])/2 - (b*((-1/3*I)*x^3 - I*c*(-1/2*(
x^2*Log[1 + I*c*E^(2*a + 2*b*x)])/(b*c) + (-1/2*(x*PolyLog[2, (-I)*c*E^(2*
a + 2*b*x)])/b + PolyLog[3, (-I)*c*E^(2*a + 2*b*x)]/(4*b^2))/(b*c))))/2
```

3.193.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5719
```
Int[ArcCot[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Tanh[a + b*x]]/(f*(m
+ 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a +
 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.193.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.96 (sec) , antiderivative size = 1369, normalized size of antiderivative = 12.12

method	result	size
risch	Expression too large to display	1369

input
```
int(x*arccot(c+(I+c)*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2*I/b*a*ln(1+I*exp(b*x+a)*(I*c)^(1/2))*x+1/8*Pi*(csgn(I/(exp(2*b*x+2*a)+
1))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(
2*b*x+2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp
(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+
2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2
*b*x+2*a)+1))^2+csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*I*exp(2*b*x+2*a)+2*ex
p(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2-csgn(I*(2*exp(2*b*x+2*a)*c-2*I))*csg
n(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2+csgn(I*(2*I*exp(2*b*x+2
*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(e
xp(2*b*x+2*a)+1))^2+csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^3-
csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))*csgn((2*exp(2*b*x+2*a)
*c-2*I)/(exp(2*b*x+2*a)+1))^2+csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2
*a)+1))*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))-csgn(I*(2*I*exp(
2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^3+csgn(I*(2*I*exp(2*b*x
+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((2*I*exp(2*b*x+2*a)+2*e
xp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2-csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*
b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*
c)/(exp(2*b*x+2*a)+1))-csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2
*b*x+2*a)+1))^3+csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*
a)+1))^2-csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^3+csgn((2*ex...
```

3.193.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(83) = 166$.

Time = 0.29 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.18

$$\int x\cot^{-1}(c+(i+c)\tanh(a+bx))\,dx$$
$$= \frac{2i\,b^3x^3 + 3i\,b^2x^2\log\left(\frac{(ce^{(2\,bx+2\,a)}-i)e^{(-2\,bx-2\,a)}}{c+i}\right) + 2i\,a^3 - 6i\,bx\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4i\,c}e^{(bx+a)}\right) - 6i\,bx\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,c}e^{(b}\right.}{}$$

input
```
integrate(x*arccot(c+(I+c)*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(2*I*b^3*x^3 + 3*I*b^2*x^2*log((c*e^(2*b*x + 2*a) - I)*e^(-2*b*x - 2*
a)/(c + I)) + 2*I*a^3 - 6*I*b*x*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) - 6*I*
b*x*dilog(-1/2*sqrt(-4*I*c)*e^(b*x + a)) - 3*I*a^2*log(1/2*(2*c*e^(b*x + a
) + I*sqrt(-4*I*c))/c) - 3*I*a^2*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c)
)/c) - 3*(I*b^2*x^2 - I*a^2)*log(1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) - 3*(I*
b^2*x^2 - I*a^2)*log(-1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) + 6*I*polylog(3, 1
/2*sqrt(-4*I*c)*e^(b*x + a)) + 6*I*polylog(3, -1/2*sqrt(-4*I*c)*e^(b*x + a
)))/b^2
```

3.193.6 Sympy [F(-2)]

Exception generated.

$$\int x\cot^{-1}(c+(i+c)\tanh(a+bx))\,dx=\text{Exception raised: CoercionFailed}$$

input
```
integrate(x*acot(c+(I+c)*tanh(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert 2*_t0**4*c**2*exp(4*a)
+ _t0**4*I*c*exp(4*a) - 3*_t0**2*I*c*exp(2*a) + _t0**2*exp(2*a) - 1 of typ
e <class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,exp(a)]
```

3.193.7 Maxima [A] (verification not implemented)

Time = 1.38 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.95

$$\int x\cot^{-1}(c+(i+c)\tanh(a+bx))\,dx = -\left(\frac{2\,x^3}{3i\,c-3}-\frac{2\,b^2x^2\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3\left(-i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^3(-i\,c+1)}\right)b(c+i) + \frac{1}{2}\,x^2\,\mathrm{arccot}\left((c+i)\tanh\left(bx+a\right)+c\right)$$

input
```
integrate(x*arccot(c+(I+c)*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-(2*x^3/(3*I*c - 3) - (2*b^2*x^2*log(I*c*e^(2*b*x + 2*a) + 1) + 2*b*x*dilo
g(-I*c*e^(2*b*x + 2*a)) - polylog(3, -I*c*e^(2*b*x + 2*a)))/(b^3*(2*I*c -
2)))*b*(c + I) + 1/2*x^2*arccot((c + I)*tanh(b*x + a) + c)
```

3.193.8 Giac [F]

$$\int x\cot^{-1}(c+(i+c)\tanh(a+bx))\,dx=\int x\,\mathrm{arccot}\left((c+i)\tanh\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x*arccot(c+(I+c)*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arccot((c + I)*tanh(b*x + a) + c), x)
```

3.193.9 Mupad [F(-1)]

Timed out.

$$\int x\cot^{-1}(c+(i+c)\tanh(a+bx))\,dx=\int x\,\mathrm{acot}(c+\tanh(a+b\,x)\,\left(c+1\mathrm{i}\right))\,dx$$

input
```
int(x*acot(c + tanh(a + b*x)*(c + 1i)),x)
```

output
```
int(x*acot(c + tanh(a + b*x)*(c + 1i)), x)
```

3.194 $\int \cot^{-1}(c+(i+c)\tanh(a+bx))\,dx$

3.194.1 Optimal result

Integrand size = 15, antiderivative size = 79

$$\int \cot^{-1}(c+(i+c)\tanh(a+bx))\,dx = \frac{1}{2}ibx^2+x\cot^{-1}(c+(i+c)\tanh(a+bx)) - \frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right)-\frac{i\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}$$

output
```
1/2*I*b*x^2+x*arccot(c+(I+c)*tanh(b*x+a))-1/2*I*x*ln(1+I*c*exp(2*b*x+2*a))
-1/4*I*polylog(2,-I*c*exp(2*b*x+2*a))/b
```

3.194.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int \cot^{-1}(c+(i+c)\tanh(a+bx))\,dx = x\cot^{-1}(c+(i+c)\tanh(a+bx)) - \frac{i\left(2bx\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right)-\text{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right)\right)}{4b}$$

input
```
Integrate[ArcCot[c + (I + c)*Tanh[a + b*x]],x]
```

output
```
x*ArcCot[c + (I + c)*Tanh[a + b*x]] - ((I/4)*(2*b*x*Log[1 - I/(c*E^(2*(a +
 b*x)))] - PolyLog[2, I/(c*E^(2*(a + b*x)))]))/b
```

3.194.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5711, 25, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}(c+(c+i)\tanh(a+bx))\,dx$$

$$\Big\downarrow \text{5711}$$

$$b\int -\frac{x}{i-ce^{2a+2bx}}dx+x\cot^{-1}(c+(c+i)\tanh(a+bx))$$

$$\Big\downarrow \text{25}$$

$$x\cot^{-1}(c+(c+i)\tanh(a+bx))-b\int \frac{x}{i-ce^{2a+2bx}}dx$$

$$\Big\downarrow \text{2615}$$

$$x\cot^{-1}(c+(c+i)\tanh(a+bx))-b\left(-ic\int \frac{e^{2a+2bx}x}{i-ce^{2a+2bx}}dx-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2620}$$

$$x\cot^{-1}(c+(c+i)\tanh(a+bx))-b\left(-ic\left(\frac{\int \log\left(ie^{2a+2bx}c+1\right)dx}{2bc}-\frac{x\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2715}$$

$$x\cot^{-1}(c+(c+i)\tanh(a+bx))-b\left(-ic\left(\frac{\int e^{-2a-2bx}\log\left(ie^{2a+2bx}c+1\right)de^{2a+2bx}}{4b^2c}-\frac{x\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2838}$$

$$x\cot^{-1}(c+(c+i)\tanh(a+bx))-b\left(-ic\left(-\frac{\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b^2c}-\frac{x\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^2}{2}\right)$$

input
```
Int[ArcCot[c + (I + c)*Tanh[a + b*x]],x]
```

output
```
x*ArcCot[c + (I + c)*Tanh[a + b*x]] - b*((-1/2*I)*x^2 - I*c*(-1/2*(x*Log[1
 + I*c*E^(2*a + 2*b*x)])/(b*c) - PolyLog[2, (-I)*c*E^(2*a + 2*b*x)]/(4*b^2
*c)))
```

3.194.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5711
```
Int[ArcCot[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Cot[c + d*Tanh[a + b*x]], x] + Simp[b   Int[x/(c - d + c*E^(2*a + 2*b*x)),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, -1]
```

3.194.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 543 vs. 2(65) = 130.

Time = 1.83 (sec) , antiderivative size = 544, normalized size of antiderivative = 6.89

method	result
derivativedivides	$-\frac{\operatorname{arccot}(c+(i+c)\tanh(bx+a))\ln(i+c+(i+c)\tanh(bx+a))}{2i+2c}+\frac{2i\operatorname{arccot}(c+(i+c)\tanh(bx+a))\ln(i+c+(i+c)\tanh(bx+a))c}{2i+2c}+\frac{\operatorname{arccot}(c+(i$
default	$-\frac{\operatorname{arccot}(c+(i+c)\tanh(bx+a))\ln(i+c+(i+c)\tanh(bx+a))}{2i+2c}+\frac{2i\operatorname{arccot}(c+(i+c)\tanh(bx+a))\ln(i+c+(i+c)\tanh(bx+a))c}{2i+2c}+\frac{\operatorname{arccot}(c+(i$
risch	Expression too large to display

input
```
int(arccot(c+(I+c)*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/(I+c)*(-arccot(c+(I+c)*tanh(b*x+a))/(2*I+2*c)*ln(I+c+(I+c)*tanh(b*x+a)
)+2*I*arccot(c+(I+c)*tanh(b*x+a))/(2*I+2*c)*ln(I+c+(I+c)*tanh(b*x+a))*c+ar
ccot(c+(I+c)*tanh(b*x+a))/(2*I+2*c)*ln(I+c+(I+c)*tanh(b*x+a))*c^2+arccot(c
+(I+c)*tanh(b*x+a))/(2*I+2*c)*ln(c-(I+c)*tanh(b*x+a)+I)-2*I*arccot(c+(I+c)
*tanh(b*x+a))/(2*I+2*c)*ln(c-(I+c)*tanh(b*x+a)+I)*c-arccot(c+(I+c)*tanh(b*
x+a))/(2*I+2*c)*ln(c-(I+c)*tanh(b*x+a)+I)*c^2+(I+c)^2*(1/2/(I+c)*(1/4*I*ln
(I+c+(I+c)*tanh(b*x+a))^2-1/2*I*((ln(I+c+(I+c)*tanh(b*x+a))-ln(-1/2*I*(I+c
+(I+c)*tanh(b*x+a))))*ln(-1/2*I*(I-c-(I+c)*tanh(b*x+a)))-dilog(-1/2*I*(I+c
+(I+c)*tanh(b*x+a)))))-1/2/(I+c)*(1/2*I*(dilog((-I-c-(I+c)*tanh(b*x+a))/(-
2*I-2*c))+ln(c-(I+c)*tanh(b*x+a)+I)*ln((-I-c-(I+c)*tanh(b*x+a))/(-2*I-2*c)
))-1/2*I*(dilog(-1/2*(I-c-(I+c)*tanh(b*x+a))/c)+ln(c-(I+c)*tanh(b*x+a)+I)*
ln(-1/2*(I-c-(I+c)*tanh(b*x+a))/c)))))
```

3.194.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(58) = 116.

Time = 0.27 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.35

$$\int \cot^{-1}(c+(i+c)\tanh(a+bx))\,dx = \frac{i\,b^2x^2 + i\,bx\log\left(\frac{(ce^{(2\,bx+2\,a)}-i)e^{(-2\,bx-2\,a)}}{c+i}\right) - i\,a^2 + (-i\,bx - i\,a)\log\left(\frac{1}{2}\sqrt{-4i\,c}e^{(bx+a)}+1\right) + (-i\,bx - i\,a)\,\mathrm{lo}}{}$$

input
```
integrate(arccot(c+(I+c)*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(I*b^2*x^2 + I*b*x*log((c*e^(2*b*x + 2*a) - I)*e^(-2*b*x - 2*a)/(c + I
)) - I*a^2 + (-I*b*x - I*a)*log(1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) + (-I*b*
x - I*a)*log(-1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) + I*a*log(1/2*(2*c*e^(b*x
+ a) + I*sqrt(-4*I*c))/c) + I*a*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c))
/c) - I*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) - I*dilog(-1/2*sqrt(-4*I*c)*e^
(b*x + a)))/b
```

3.194.6 Sympy [F(-2)]

Exception generated.

$$\int \cot^{-1}(c+(i+c)\tanh(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(acot(c+(I+c)*tanh(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert 2*_t0**4*c**2*exp(4*a)
+ _t0**4*I*c*exp(4*a) - 3*_t0**2*I*c*exp(2*a) + _t0**2*exp(2*a) - 1 of typ
e <class 'sympy.core.add.Add'> to QQ_I[b,c,_t0,exp(a)]
```

3.194.7 Maxima [A] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.01

$$\int \cot^{-1}(c+(i+c)\tanh(a+bx))\,dx$$
$$= -2\,b(c+i)\left(\frac{2\,x^2}{2i\,c-2} - \frac{2\,bx\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^2(-i\,c+1)}\right)$$
$$+ x\,\mathrm{arccot}\left((c+i)\tanh\left(bx+a\right)+c\right)$$

input
```
integrate(arccot(c+(I+c)*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-2*b*(c + I)*(2*x^2/(2*I*c - 2) - (2*b*x*log(I*c*e^(2*b*x + 2*a) + 1) + di
log(-I*c*e^(2*b*x + 2*a)))/(b^2*(2*I*c - 2))) + x*arccot((c + I)*tanh(b*x
+ a) + c)
```

3.194.8 Giac [F]

$$\int \cot^{-1}(c+(i+c)\tanh(a+bx))\,dx = \int \mathrm{arccot}\left((c+i)\tanh\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(arccot(c+(I+c)*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccot((c + I)*tanh(b*x + a) + c), x)
```

3.194.9 Mupad [F(-1)]

Timed out.

$$\int \cot^{-1}(c+(i+c)\tanh(a+bx))\,dx = \int \mathrm{acot}(c+\tanh(a+b\,x)\,\left(c+1\mathrm{i}\right))\,dx$$

input
```
int(acot(c + tanh(a + b*x)*(c + 1i)),x)
```

output
```
int(acot(c + tanh(a + b*x)*(c + 1i)), x)
```

3.195 $\int \frac{\cot^{-1}(c+(i+c)\tanh(a+bx))}{x}\,dx$

3.195.1 Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{\cot^{-1}(c+(i+c)\tanh(a+bx))}{x}\,dx = \text{Int}\left(\frac{\cot^{-1}(c+(i+c)\tanh(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(arccot(c+(I+c)*tanh(b*x+a))/x,x)
```

3.195.2 Mathematica [N/A]

Not integrable

Time = 3.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\cot^{-1}(c+(i+c)\tanh(a+bx))}{x}\,dx = \int \frac{\cot^{-1}(c+(i+c)\tanh(a+bx))}{x}\,dx$$

input
```
Integrate[ArcCot[c + (I + c)*Tanh[a + b*x]]/x,x]
```

output
```
Integrate[ArcCot[c + (I + c)*Tanh[a + b*x]]/x, x]
```

3.195.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(c+(c+i)\tanh(a+bx))}{x}\,dx$$
$$\Big\downarrow \ 7299$$
$$\int \frac{\cot^{-1}(c+(c+i)\tanh(a+bx))}{x}dx$$

input
```
Int[ArcCot[c + (I + c)*Tanh[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.195.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.195.4 Maple [N/A] (verified)

Not integrable

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\operatorname{arccot}(c+(i+c)\tanh(bx+a))}{x}dx$$

input
```
int(arccot(c+(I+c)*tanh(b*x+a))/x,x)
```

output
```
int(arccot(c+(I+c)*tanh(b*x+a))/x,x)
```

3.195.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{\cot^{-1}(c+(i+c)\tanh(a+bx))}{x}\,dx = \int \frac{\operatorname{arccot}\left((c+i)\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccot(c+(I+c)*tanh(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(1/2*I*log((c*e^(2*b*x + 2*a) - I)*e^(-2*b*x - 2*a)/(c + I))/x, x)
```

3.195.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(c+(i+c)\tanh(a+bx))}{x}\,dx = \text{Timed out}$$

input
```
integrate(acot(c+(I+c)*tanh(b*x+a))/x,x)
```

output
```
Timed out
```

3.195.7 Maxima [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 81, normalized size of antiderivative = 4.26

$$\int \frac{\cot^{-1}(c+(i+c)\tanh(a+bx))}{x}\,dx = \int \frac{\operatorname{arccot}\left((c+i)\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccot(c+(I+c)*tanh(b*x+a))/x,x, algorithm="maxima")
```

output
```
-I*b*x - 1/2*pi*log(x) - 1/4*(2*pi + 4*I*a - 2*arctan(1/c) + I*log(c^2 + 1
))*log(x) + 1/2*integrate(arctan(e^(-2*b*x - 2*a)/c)/x, x) + 1/4*I*integra
te(log(c^2*e^(4*b*x + 4*a) + 1)/x, x)
```

3.195.8 Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(c+(i+c)\tanh(a+bx))}{x}\,dx = \int \frac{\operatorname{arccot}\left((c+i)\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccot(c+(I+c)*tanh(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arccot((c + I)*tanh(b*x + a) + c)/x, x)
```

3.195.9 Mupad [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{\cot^{-1}(c+(i+c)\tanh(a+bx))}{x}\,dx = \int \frac{\operatorname{acot}(c+\tanh(a+b\,x)\ (c+1\mathrm{i}))}{x}\,dx$$

input
```
int(acot(c + tanh(a + b*x)*(c + 1i))/x,x)
```

output
```
int(acot(c + tanh(a + b*x)*(c + 1i))/x, x)
```

3.196 $\int x^2 \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx$

3.196.1 Optimal result

Integrand size = 22, antiderivative size = 145

$$\int x^2 \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx = -\frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c-(i-c)\tanh(a+bx))+\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right) + \frac{ix^2\operatorname{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}-\frac{ix\operatorname{PolyLog}\left(3,ice^{2a+2bx}\right)}{4b^2}+\frac{i\operatorname{PolyLog}\left(4,ice^{2a+2bx}\right)}{8b^3}$$

output
```
-1/12*I*b*x^4+1/3*x^3*arccot(c-(I-c)*tanh(b*x+a))+1/6*I*x^3*ln(1-I*c*exp(2
*b*x+2*a))+1/4*I*x^2*polylog(2,I*c*exp(2*b*x+2*a))/b-1/4*I*x*polylog(3,I*c
*exp(2*b*x+2*a))/b^2+1/8*I*polylog(4,I*c*exp(2*b*x+2*a))/b^3
```

3.196.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.92

$$\int x^2 \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx = \frac{8b^3x^3\cot^{-1}(c+(-i+c)\tanh(a+bx))+4ib^3x^3\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right)-6ib^2x^2\operatorname{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right)-6}{24b^3}$$

input
```
Integrate[x^2*ArcCot[c - (I - c)*Tanh[a + b*x]],x]
```

output
```
(8*b^3*x^3*ArcCot[c + (-I + c)*Tanh[a + b*x]] + (4*I)*b^3*x^3*Log[1 + I/(c
*E^(2*(a + b*x)))] - (6*I)*b^2*x^2*PolyLog[2, (-I)/(c*E^(2*(a + b*x)))] -
(6*I)*b*x*PolyLog[3, (-I)/(c*E^(2*(a + b*x)))] - (3*I)*PolyLog[4, (-I)/(c*
E^(2*(a + b*x)))])/(24*b^3)
```

3.196.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {5719, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cot^{-1}(c-(-c+i)\tanh(a+bx))\,dx$$

$$\Big\downarrow \; 5719$$

$$\frac{1}{3}b\int \frac{x^3}{e^{2a+2bx}c+i}dx+\frac{1}{3}x^3\cot^{-1}(c-(-c+i)\tanh(a+bx))$$

$$\Big\downarrow \; 2615$$

$$\frac{1}{3}b\left(ic\int \frac{e^{2a+2bx}x^3}{e^{2a+2bx}c+i}dx-\frac{ix^4}{4}\right)+\frac{1}{3}x^3\cot^{-1}(c-(-c+i)\tanh(a+bx))$$

$$\Big\downarrow \; 2620$$

$$\frac{1}{3}b\left(ic\left(\frac{x^3\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{3\int x^2\log\left(1-ice^{2a+2bx}\right)dx}{2bc}\right)-\frac{ix^4}{4}\right)+\frac{1}{3}x^3\cot^{-1}(c-(-c+i)\tanh(a+bx))$$

$$\Big\downarrow \; 3011$$

$$\frac{1}{3}b\left(ic\left(\frac{x^3\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{3\left(\frac{\int x\,\text{PolyLog}(2,ice^{2a+2bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,ice^{2a+2bx})}{2b}\right)}{2bc}\right)-\frac{ix^4}{4}\right)+\frac{1}{3}x^3\cot^{-1}(c-(-c+i)\tanh(a+bx))$$

$$\Big\downarrow \; 7163$$

$$\frac{1}{3}b\left(ic\left(\frac{x^3\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,ice^{2a+2bx}\right)}{2b}-\frac{\int \text{PolyLog}\left(3,ice^{2a+2bx}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{2b}\right)}{2bc}\right)-\frac{ix^4}{4}\right)+\frac{1}{3}x^3\cot^{-1}(c-(-c+i)\tanh(a+bx))$$

$\big\downarrow$ 2720

$$\frac{1}{3}b\left(ic\left(\frac{x^3\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,ice^{2a+2bx}\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(3,ice^{2a+2bx}\right)de^{2a+2bx}}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{2b}\right)}{2bc}\right.\right.$$

$$\frac{1}{3}x^3\cot^{-1}(c-(-c+i)\tanh(a+bx))$$

$\big\downarrow$ 7143

$$\frac{1}{3}b\left(ic\left(\frac{x^3\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,ice^{2a+2bx}\right)}{2b}-\frac{\text{PolyLog}\left(4,ice^{2a+2bx}\right)}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{2b}\right)}{2bc}\right)-\frac{ix^4}{4}\right)+\frac{1}{3}x^3\cot^{-1}(c-(-c+i)\tanh(a+bx))$$

input
```
Int[x^2*ArcCot[c - (I - c)*Tanh[a + b*x]],x]
```

output
```
(x^3*ArcCot[c - (I - c)*Tanh[a + b*x]])/3 + (b*((-1/4*I)*x^4 + I*c*((x^3*L
og[1 - I*c*E^(2*a + 2*b*x)])/(2*b*c) - (3*(-1/2*(x^2*PolyLog[2, I*c*E^(2*a
 + 2*b*x)])/b + ((x*PolyLog[3, I*c*E^(2*a + 2*b*x)])/(2*b) - PolyLog[4, I*
c*E^(2*a + 2*b*x)]/(4*b^2))/b))/(2*b*c))))/3
```

3.196. $\int x^2\cot^{-1}(c-(i-c)\tanh(a+bx))\,dx$

3.196.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5719
```
Int[ArcCot[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Tanh[a + b*x]]/(f*(m
+ 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a +
 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.196.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.47 (sec) , antiderivative size = 1409, normalized size of antiderivative = 9.72

method	result	size
risch	Expression too large to display	1409

input
```
int(x^2*arccot(c-(I-c)*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/6*I*x^3*ln(2*I*exp(2*b*x+2*a)-2*exp(2*b*x+2*a)*c)-1/12*Pi*(csgn(I/(exp(2
*b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp(2*b*x+2*a)*c+
2*I)/(exp(2*b*x+2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(-2*I*exp(2*b*x
+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)
/(exp(2*b*x+2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c
+2*I)/(exp(2*b*x+2*a)+1))^2+csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(-2*I*exp(2*
b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2+csgn(I*(2*exp(2*b*x+2*a
)*c+2*I))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^2-csgn(I*(-2
*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2
*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2-csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*
b*x+2*a)+1))^3+csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))*csgn((2
*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^2+csgn(I*(2*exp(2*b*x+2*a)*c+2*
I)/(exp(2*b*x+2*a)+1))*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))+c
sgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^3-csgn(
I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((-2*I*
exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2-csgn(I*(-2*I*exp(
2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((-2*I*exp(2*b*x+2*
a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))+csgn((-2*I*exp(2*b*x+2*a)+2*exp
(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^3+csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x
+2*a)*c)/(exp(2*b*x+2*a)+1))^2+csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x...
```

3.196.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(105) = 210$.

Time = 0.28 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.01

$$\int x^2 \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx = \frac{-i\,b^4x^4 + 2i\,b^3x^3\log\left(\frac{(c-i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}+i}\right) + 6i\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\sqrt{4i\,c}e^{(bx+a)}\right) + 6i\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{4i\,c}e^{(bx+a)}\right) + i\,a^4 -}{}$$

input
```
integrate(x^2*arccot(c-(I-c)*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(-I*b^4*x^4 + 2*I*b^3*x^3*log((c - I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2
*a) + I)) + 6*I*b^2*x^2*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) + 6*I*b^2*x^2*d
ilog(-1/2*sqrt(4*I*c)*e^(b*x + a)) + I*a^4 - 2*I*a^3*log(1/2*(2*c*e^(b*x +
 a) + I*sqrt(4*I*c))/c) - 2*I*a^3*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c)
)/c) - 12*I*b*x*polylog(3, 1/2*sqrt(4*I*c)*e^(b*x + a)) - 12*I*b*x*polylog
(3, -1/2*sqrt(4*I*c)*e^(b*x + a)) - 2*(-I*b^3*x^3 - I*a^3)*log(1/2*sqrt(4*
I*c)*e^(b*x + a) + 1) - 2*(-I*b^3*x^3 - I*a^3)*log(-1/2*sqrt(4*I*c)*e^(b*x
 + a) + 1) + 12*I*polylog(4, 1/2*sqrt(4*I*c)*e^(b*x + a)) + 12*I*polylog(4
, -1/2*sqrt(4*I*c)*e^(b*x + a)))/b^3
```

3.196.6 Sympy [F(-2)]

Exception generated.

$$\int x^2 \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(x**2*acot(c-(I-c)*tanh(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert 2*_t0**2*c*exp(2*a) - _
t0**2*I*exp(2*a) + I of type <class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t
0,exp(a)]
```

3.196.7 Maxima [A] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.89

$$\int x^2 \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx = \frac{1}{3}\,x^3 \operatorname{arccot}\left((c-i)\tanh\left(bx+a\right)+c\right) + \frac{4}{9}\left(\frac{3\,x^4}{4i\,c+4} - \frac{4\,b^3x^3\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right)+6\,b^2x^2\mathrm{Li}_2\left(i\,ce^{(2\,bx+2\,a)}\right)-6\,bx\mathrm{Li}_3\left(i\,ce^{(2\,bx+2\,a)}\right)+3\,\mathrm{Li}_4\left(i\,ce\right.}{-2\,b^4(-i\,c-1)}\right.$$

input
```
integrate(x^2*arccot(c-(I-c)*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/3*x^3*arccot((c - I)*tanh(b*x + a) + c) + 4/9*(3*x^4/(4*I*c + 4) - (4*b^
3*x^3*log(-I*c*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(I*c*e^(2*b*x + 2*a))
 - 6*b*x*polylog(3, I*c*e^(2*b*x + 2*a)) + 3*polylog(4, I*c*e^(2*b*x + 2*a
)))/(b^4*(2*I*c + 2)))*b*(c - I)
```

3.196.8 Giac [F]

$$\int x^2 \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx = \int x^2 \operatorname{arccot}\left((c-i)\tanh\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x^2*arccot(c-(I-c)*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arccot((c - I)*tanh(b*x + a) + c), x)
```

3.196.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx = \int x^2\,\mathrm{acot}(c+\tanh(a+b\,x)\,(c-\mathrm{i}))\,dx$$

input
```
int(x^2*acot(c + tanh(a + b*x)*(c - 1i)),x)
```

output
```
int(x^2*acot(c + tanh(a + b*x)*(c - 1i)), x)
```

3.197 $\int x \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx$

3.197.1 Optimal result

Integrand size = 20, antiderivative size = 116

$$\int x\cot^{-1}(c-(i-c)\tanh(a+bx))\,dx = -\frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c-(i-c)\tanh(a+bx)) + \frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right) + \frac{ix\,\mathrm{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b} - \frac{i\,\mathrm{PolyLog}\left(3,ice^{2a+2bx}\right)}{8b^2}$$

output
```
-1/6*I*b*x^3+1/2*x^2*arccot(c-(I-c)*tanh(b*x+a))+1/4*I*x^2*ln(1-I*c*exp(2*
b*x+2*a))+1/4*I*x*polylog(2,I*c*exp(2*b*x+2*a))/b-1/8*I*polylog(3,I*c*exp(
2*b*x+2*a))/b^2
```

3.197.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.89

$$\int x\cot^{-1}(c-(i-c)\tanh(a+bx))\,dx = \frac{2b^2x^2\left(2\cot^{-1}(c+(-i+c)\tanh(a+bx))+i\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right)\right)-2ibx\,\mathrm{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right)-i\,\mathrm{Po}}{8b^2}$$

input
```
Integrate[x*ArcCot[c - (I - c)*Tanh[a + b*x]],x]
```

output
```
(2*b^2*x^2*(2*ArcCot[c + (-I + c)*Tanh[a + b*x]] + I*Log[1 + I/(c*E^(2*(a
+ b*x)))]) - (2*I)*b*x*PolyLog[2, (-I)/(c*E^(2*(a + b*x)))] - I*PolyLog[3,
 (-I)/(c*E^(2*(a + b*x)))])/(8*b^2)
```

3.197.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5719, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cot^{-1}(c-(-c+i)\tanh(a+bx))\,dx$$

$$\Big\downarrow \text{5719}$$

$$\frac{1}{2}b\int \frac{x^2}{e^{2a+2bx}c+i}dx+\frac{1}{2}x^2\cot^{-1}(c-(-c+i)\tanh(a+bx))$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{2}b\left(ic\int \frac{e^{2a+2bx}x^2}{e^{2a+2bx}c+i}dx-\frac{ix^3}{3}\right)+\frac{1}{2}x^2\cot^{-1}(c-(-c+i)\tanh(a+bx))$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{2}b\left(ic\left(\frac{x^2\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{\int x\log\left(1-ice^{2a+2bx}\right)dx}{bc}\right)-\frac{ix^3}{3}\right)+\frac{1}{2}x^2\cot^{-1}(c-(-c+i)\tanh(a+bx))$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}b\left(ic\left(\frac{x^2\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{\frac{\int \text{PolyLog}(2,ice^{2a+2bx})dx}{2b}-\frac{x\,\text{PolyLog}(2,ice^{2a+2bx})}{2b}}{bc}\right)-\frac{ix^3}{3}\right)+\frac{1}{2}x^2\cot^{-1}(c-(-c+i)\tanh(a+bx))$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}b\left(ic\left(\frac{x^2\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{\frac{\int e^{-2a-2bx}\operatorname{PolyLog}(2,ice^{2a+2bx})de^{2a+2bx}}{4b^2}-\frac{x\operatorname{PolyLog}(2,ice^{2a+2bx})}{2b}}{bc}\right)-\frac{ix^3}{3}\right)+$$
$$\frac{1}{2}x^2\cot^{-1}(c-(-c+i)\tanh(a+bx))$$
$$\Big\downarrow \text{7143}$$
$$\frac{1}{2}b\left(ic\left(\frac{x^2\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{\frac{\operatorname{PolyLog}(3,ice^{2a+2bx})}{4b^2}-\frac{x\operatorname{PolyLog}(2,ice^{2a+2bx})}{2b}}{bc}\right)-\frac{ix^3}{3}\right)+$$
$$\frac{1}{2}x^2\cot^{-1}(c-(-c+i)\tanh(a+bx))$$

input
```
Int[x*ArcCot[c - (I - c)*Tanh[a + b*x]],x]
```

output
```
(x^2*ArcCot[c - (I - c)*Tanh[a + b*x]])/2 + (b*((-1/3*I)*x^3 + I*c*((x^2*L
og[1 - I*c*E^(2*a + 2*b*x)])/(2*b*c) - (-1/2*(x*PolyLog[2, I*c*E^(2*a + 2*
b*x)])/b + PolyLog[3, I*c*E^(2*a + 2*b*x)]/(4*b^2))/(b*c))))/2
```

3.197.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5719
```
Int[ArcCot[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Tanh[a + b*x]]/(f*(m
+ 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a +
 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.197.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.03 (sec) , antiderivative size = 1373, normalized size of antiderivative = 11.84

method	result	size
risch	Expression too large to display	1373

input
```
int(x*arccot(c-(I-c)*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/4*I*x^2*ln(-2*exp(2*b*x+2*a)*c-2*I)-1/8*Pi*(csgn(I/(exp(2*b*x+2*a)+1))*
csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*
x+2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*
b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*
a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b
*x+2*a)+1))^2+csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp
(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2+csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn
(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^2-csgn(I*(-2*I*exp(2*b*x+2
*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(
exp(2*b*x+2*a)+1))^2-csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^3
+csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))*csgn((2*exp(2*b*x+2*a
)*c+2*I)/(exp(2*b*x+2*a)+1))^2+csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+
2*a)+1))*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))+csgn(I*(-2*I*ex
p(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^3-csgn(I*(-2*I*exp(2*
b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((-2*I*exp(2*b*x+2*a)
+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2-csgn(I*(-2*I*exp(2*b*x+2*a)+2*e
xp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x
+2*a)*c)/(exp(2*b*x+2*a)+1))+csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)
/(exp(2*b*x+2*a)+1))^3+csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(
2*b*x+2*a)+1))^2+csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^3+cs...
```

3.197.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(83) = 166$.

Time $= 0.27$ (sec) , antiderivative size $= 246$, normalized size of antiderivative $= 2.12$

$$\int x \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx$$
$$= \frac{-2i\,b^3x^3 + 3i\,b^2x^2\log\left(\frac{(c-i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}+i}\right) - 2i\,a^3 + 6i\,bx\mathrm{Li}_2\left(\frac{1}{2}\sqrt{4i\,c}e^{(bx+a)}\right) + 6i\,bx\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{4i\,c}e^{(bx+a)}\right) + 3i}{}$$

input
```
integrate(x*arccot(c-(I-c)*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(-2*I*b^3*x^3 + 3*I*b^2*x^2*log((c - I)*e^(2*b*x + 2*a)/(c*e^(2*b*x +
 2*a) + I)) - 2*I*a^3 + 6*I*b*x*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) + 6*I*b
*x*dilog(-1/2*sqrt(4*I*c)*e^(b*x + a)) + 3*I*a^2*log(1/2*(2*c*e^(b*x + a)
+ I*sqrt(4*I*c))/c) + 3*I*a^2*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c)
 - 3*(-I*b^2*x^2 + I*a^2)*log(1/2*sqrt(4*I*c)*e^(b*x + a) + 1) - 3*(-I*b^2
*x^2 + I*a^2)*log(-1/2*sqrt(4*I*c)*e^(b*x + a) + 1) - 6*I*polylog(3, 1/2*s
qrt(4*I*c)*e^(b*x + a)) - 6*I*polylog(3, -1/2*sqrt(4*I*c)*e^(b*x + a)))/b^
2
```

3.197.6 Sympy [F(-2)]

Exception generated.

$$\int x\cot^{-1}(c-(i-c)\tanh(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(x*acot(c-(I-c)*tanh(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert 2*_t0**2*c*exp(2*a) - _
t0**2*I*exp(2*a) + I of type <class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t
0,exp(a)]
```

3.197.7 Maxima [A] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

$$\int x\cot^{-1}(c-(i-c)\tanh(a+bx))\,dx = \left(\frac{2\,x^3}{3i\,c+3} - \frac{2\,b^2x^2\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(i\,ce^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3\left(i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^3(-i\,c-1)}\right)b(c-i) + \frac{1}{2}\,x^2\,\mathrm{arccot}\left((c-i)\tanh\left(bx+a\right)+c\right)$$

input
```
integrate(x*arccot(c-(I-c)*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
(2*x^3/(3*I*c + 3) - (2*b^2*x^2*log(-I*c*e^(2*b*x + 2*a) + 1) + 2*b*x*dilo
g(I*c*e^(2*b*x + 2*a)) - polylog(3, I*c*e^(2*b*x + 2*a)))/(b^3*(2*I*c + 2)
))*b*(c - I) + 1/2*x^2*arccot((c - I)*tanh(b*x + a) + c)
```

3.197.8 Giac [F]

$$\int x\cot^{-1}(c-(i-c)\tanh(a+bx))\,dx=\int x\,\mathrm{arccot}\left((c-i)\tanh\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x*arccot(c-(I-c)*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arccot((c - I)*tanh(b*x + a) + c), x)
```

3.197.9 Mupad [F(-1)]

Timed out.

$$\int x\cot^{-1}(c-(i-c)\tanh(a+bx))\,dx=\int x\,\mathrm{acot}(c+\tanh(a+b\,x)\,\left(c-\mathrm{i}\right))\,dx$$

input
```
int(x*acot(c + tanh(a + b*x)*(c - 1i)),x)
```

output
```
int(x*acot(c + tanh(a + b*x)*(c - 1i)), x)
```

3.198 $\int \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx$

3.198.1 Optimal result

Integrand size = 18, antiderivative size = 82

$$\int \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx = -\frac{1}{2}ibx^2+x\cot^{-1}(c-(i-c)\tanh(a+bx)) + \frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right)+\frac{i\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}$$

output
```
-1/2*I*b*x^2+x*arccot(c-(I-c)*tanh(b*x+a))+1/2*I*x*ln(1-I*c*exp(2*b*x+2*a)
)+1/4*I*polylog(2,I*c*exp(2*b*x+2*a))/b
```

3.198.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.87

$$\int \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx = x\cot^{-1}(c+(-i+c)\tanh(a+bx))+\frac{i\left(2bx\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right)-\text{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right)\right)}{4b}$$

input
```
Integrate[ArcCot[c - (I - c)*Tanh[a + b*x]],x]
```

output
```
x*ArcCot[c + (-I + c)*Tanh[a + b*x]] + ((I/4)*(2*b*x*Log[1 + I/(c*E^(2*(a
+ b*x)))] - PolyLog[2, (-I)/(c*E^(2*(a + b*x)))]))/b
```

3.198.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5711, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}(c-(-c+i)\tanh(a+bx))\,dx$$

$$\Big\downarrow \text{5711}$$

$$b\int \frac{x}{e^{2a+2bx}c+i}dx+x\cot^{-1}(c-(-c+i)\tanh(a+bx))$$

$$\Big\downarrow \text{2615}$$

$$b\left(ic\int \frac{e^{2a+2bx}x}{e^{2a+2bx}c+i}dx-\frac{ix^2}{2}\right)+x\cot^{-1}(c-(-c+i)\tanh(a+bx))$$

$$\Big\downarrow \text{2620}$$

$$b\left(ic\left(\frac{x\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{\int \log\left(1-ice^{2a+2bx}\right)dx}{2bc}\right)-\frac{ix^2}{2}\right)+x\cot^{-1}(c-(-c+i)\tanh(a+bx))$$

$$\Big\downarrow \text{2715}$$

$$b\left(ic\left(\frac{x\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{\int e^{-2a-2bx}\log\left(1-ice^{2a+2bx}\right)de^{2a+2bx}}{4b^2c}\right)-\frac{ix^2}{2}\right)+x\cot^{-1}(c-(-c+i)\tanh(a+bx))$$

$$\Big\downarrow \text{2838}$$

$$b\left(ic\left(\frac{\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b^2c}+\frac{x\log\left(1-ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^2}{2}\right)+x\cot^{-1}(c-(-c+i)\tanh(a+bx))$$

input
```
Int[ArcCot[c - (I - c)*Tanh[a + b*x]],x]
```

output
```
x*ArcCot[c - (I - c)*Tanh[a + b*x]] + b*((-1/2*I)*x^2 + I*c*((x*Log[1 - I*
c*E^(2*a + 2*b*x)])/(2*b*c) + PolyLog[2, I*c*E^(2*a + 2*b*x)]/(4*b^2*c)))
```

3.198.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5711
```
Int[ArcCot[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Cot[c + d*Tanh[a + b*x]], x] + Simp[b   Int[x/(c - d + c*E^(2*a + 2*b*x)),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, -1]
```

3.198.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 516 vs. 2(68) = 136.

Time = 1.86 (sec) , antiderivative size = 517, normalized size of antiderivative = 6.30

method	result
derivativedivides	$\frac{-\frac{\operatorname{arccot}(c+\tanh(bx+a)(c-i))\ln(\tanh(bx+a)(c-i)-c+i)}{2i-2c}-\frac{2i\operatorname{arccot}(c+\tanh(bx+a)(c-i))\ln(\tanh(bx+a)(c-i)-c+i)c}{2i-2c}+\frac{\operatorname{arccot}(c+\tan}{}}{}$
default	$\frac{-\frac{\operatorname{arccot}(c+\tanh(bx+a)(c-i))\ln(\tanh(bx+a)(c-i)-c+i)}{2i-2c}-\frac{2i\operatorname{arccot}(c+\tanh(bx+a)(c-i))\ln(\tanh(bx+a)(c-i)-c+i)c}{2i-2c}+\frac{\operatorname{arccot}(c+\tan}{}}{}$
risch	Expression too large to display

input
```
int(arccot(c-(I-c)*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/(c-I)*(-arccot(c+tanh(b*x+a)*(c-I))/(2*I-2*c)*ln(tanh(b*x+a)*(c-I)-c+I
)-2*I*arccot(c+tanh(b*x+a)*(c-I))/(2*I-2*c)*ln(tanh(b*x+a)*(c-I)-c+I)*c+ar
ccot(c+tanh(b*x+a)*(c-I))/(2*I-2*c)*ln(tanh(b*x+a)*(c-I)-c+I)*c^2+arccot(c
+tanh(b*x+a)*(c-I))/(2*I-2*c)*ln(-I+tanh(b*x+a)*(c-I)+c)+2*I*arccot(c+tanh
(b*x+a)*(c-I))/(2*I-2*c)*ln(-I+tanh(b*x+a)*(c-I)+c)*c-arccot(c+tanh(b*x+a)
*(c-I))/(2*I-2*c)*ln(-I+tanh(b*x+a)*(c-I)+c)*c^2-(I-c)^2*(1/2/(I-c)*(-1/4*
I*ln(-I+tanh(b*x+a)*(c-I)+c)^2+1/2*I*(dilog(-1/2*I*(tanh(b*x+a)*(c-I)+c+I)
)+ln(-I+tanh(b*x+a)*(c-I)+c)*ln(-1/2*I*(tanh(b*x+a)*(c-I)+c+I))))-1/2/(I-c
)*(1/2*I*(dilog(1/2*(tanh(b*x+a)*(c-I)+c+I)/c)+ln(tanh(b*x+a)*(c-I)-c+I)*l
n(1/2*(tanh(b*x+a)*(c-I)+c+I)/c))-1/2*I*(dilog((-I+tanh(b*x+a)*(c-I)+c)/(-
2*I+2*c))+ln(tanh(b*x+a)*(c-I)-c+I)*ln((-I+tanh(b*x+a)*(c-I)+c)/(-2*I+2*c)
)))))
```

3.198.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(58) = 116.

Time = 0.27 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.27

$$\int \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx$$
$$=\frac{-i\,b^2x^2+i\,bx\log\left(\frac{(c-i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}+i}\right)+i\,a^2+(i\,bx+i\,a)\log\left(\frac{1}{2}\sqrt{4i\,c}e^{(bx+a)}+1\right)+(i\,bx+i\,a)\log\left(-\frac{1}{2}\sqrt{4i\,c}\right.}{}$$

input
```
integrate(arccot(c-(I-c)*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(-I*b^2*x^2 + I*b*x*log((c - I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2*a) + I
)) + I*a^2 + (I*b*x + I*a)*log(1/2*sqrt(4*I*c)*e^(b*x + a) + 1) + (I*b*x +
 I*a)*log(-1/2*sqrt(4*I*c)*e^(b*x + a) + 1) - I*a*log(1/2*(2*c*e^(b*x + a)
 + I*sqrt(4*I*c))/c) - I*a*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c) +
I*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) + I*dilog(-1/2*sqrt(4*I*c)*e^(b*x + a
)))/b
```

3.198.6 Sympy [F(-2)]

Exception generated.

$$\int \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(acot(c-(I-c)*tanh(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert 2*_t0**2*c*exp(2*a) - _
t0**2*I*exp(2*a) + I of type <class 'sympy.core.add.Add'> to QQ_I[b,c,_t0,
exp(a)]
```

3.198.7 Maxima [A] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\int \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx \\ = 2\,b(c-i)\left(\frac{2\,x^2}{2i\,c+2} - \frac{2\,bx\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^2(-i\,c-1)}\right) \\ + x\,\mathrm{arccot}\left((c-i)\tanh\left(bx+a\right)+c\right)$$

input
```
integrate(arccot(c-(I-c)*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
2*b*(c - I)*(2*x^2/(2*I*c + 2) - (2*b*x*log(-I*c*e^(2*b*x + 2*a) + 1) + di
log(I*c*e^(2*b*x + 2*a)))/(b^2*(2*I*c + 2))) + x*arccot((c - I)*tanh(b*x +
 a) + c)
```

3.198.8 Giac [F]

$$\int \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx = \int \operatorname{arccot}\left((c-i)\tanh\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(arccot(c-(I-c)*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccot((c - I)*tanh(b*x + a) + c), x)
```

3.198.9 Mupad [F(-1)]

Timed out.

$$\int \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx = \int \operatorname{acot}(c+\tanh(a+b\,x)\,\,(c-\mathrm{i}))\,dx$$

input
```
int(acot(c + tanh(a + b*x)*(c - 1i)),x)
```

output
```
int(acot(c + tanh(a + b*x)*(c - 1i)), x)
```

3.199 $\int \frac{\cot^{-1}(c-(i-c)\tanh(a+bx))}{x} dx$

3.199.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\cot^{-1}(c-(i-c)\tanh(a+bx))}{x} dx = \text{Int}\left(\frac{\cot^{-1}(c-(i-c)\tanh(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(arccot(c-(I-c)*tanh(b*x+a))/x,x)
```

3.199.2 Mathematica [N/A]

Not integrable

Time = 3.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cot^{-1}(c-(i-c)\tanh(a+bx))}{x} dx = \int \frac{\cot^{-1}(c-(i-c)\tanh(a+bx))}{x} dx$$

input
```
Integrate[ArcCot[c - (I - c)*Tanh[a + b*x]]/x,x]
```

output
```
Integrate[ArcCot[c - (I - c)*Tanh[a + b*x]]/x, x]
```

3.199.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(c-(-c+i)\tanh(a+bx))}{x}\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \frac{\cot^{-1}(c-(-c+i)\tanh(a+bx))}{x}dx$$

input
```
Int[ArcCot[c - (I - c)*Tanh[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.199.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.199.4 Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arccot}\left(c-(i-c)\tanh\left(bx+a\right)\right)}{x}dx$$

input
```
int(arccot(c-(I-c)*tanh(b*x+a))/x,x)
```

output
```
int(arccot(c-(I-c)*tanh(b*x+a))/x,x)
```

3.199.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int \frac{\cot^{-1}(c-(i-c)\tanh(a+bx))}{x}\,dx = \int \frac{\operatorname{arccot}\left((c-i)\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccot(c-(I-c)*tanh(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(1/2*I*log((c - I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2*a) + I))/x, x)
```

3.199.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(c-(i-c)\tanh(a+bx))}{x}\,dx = \text{Timed out}$$

input
```
integrate(acot(c-(I-c)*tanh(b*x+a))/x,x)
```

output
```
Timed out
```

3.199.7 Maxima [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.32

$$\int \frac{\cot^{-1}(c-(i-c)\tanh(a+bx))}{x}\,dx = \int \frac{\operatorname{arccot}\left((c-i)\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccot(c-(I-c)*tanh(b*x+a))/x,x, algorithm="maxima")
```

output
```
I*b*x - 1/4*(-4*I*a - 2*arctan(1/c) - I*log(c^2 + 1))*log(x) + 1/2*integra
te(arctan(e^(-2*b*x - 2*a)/c)/x, x) - 1/4*I*integrate(log(c^2*e^(4*b*x + 4
*a) + 1)/x, x)
```

3.199.8 Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\cot^{-1}(c-(i-c)\tanh(a+bx))}{x}\,dx = \int \frac{\text{arccot}\left((c-i)\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccot(c-(I-c)*tanh(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arccot((c - I)*tanh(b*x + a) + c)/x, x)
```

3.199.9 Mupad [N/A]

Not integrable

Time = 1.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\cot^{-1}(c-(i-c)\tanh(a+bx))}{x}\,dx = \int \frac{\text{acot}(c+\tanh(a+b\,x)\,\left(c-\mathrm{i}\right))}{x}\,dx$$

input
```
int(acot(c + tanh(a + b*x)*(c - 1i))/x,x)
```

output
```
int(acot(c + tanh(a + b*x)*(c - 1i))/x, x)
```

3.200 $\int (e+fx)^3 \cot^{-1}(\coth(a+bx))\,dx$

3.200.1 Optimal result

Integrand size = 15, antiderivative size = 299

$$\begin{aligned}\int (e+fx)^3 \cot^{-1}(\coth(a+bx))\,dx &= \frac{(e+fx)^4 \cot^{-1}(\coth(a+bx))}{4f} \\ &- \frac{(e+fx)^4 \arctan\left(e^{2a+2bx}\right)}{4f} \\ &+ \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} \\ &- \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} \\ &- \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} \\ &+ \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2} \\ &+ \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} \\ &- \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3} \\ &- \frac{3if^3 \operatorname{PolyLog}\left(5,-ie^{2a+2bx}\right)}{16b^4} \\ &+ \frac{3if^3 \operatorname{PolyLog}\left(5,ie^{2a+2bx}\right)}{16b^4}\end{aligned}$$

output
```
1/4*(f*x+e)^4*arccot(coth(b*x+a))/f-1/4*(f*x+e)^4*arctan(exp(2*b*x+2*a))/f
+1/4*I*(f*x+e)^3*polylog(2,-I*exp(2*b*x+2*a))/b-1/4*I*(f*x+e)^3*polylog(2,
I*exp(2*b*x+2*a))/b-3/8*I*f*(f*x+e)^2*polylog(3,-I*exp(2*b*x+2*a))/b^2+3/8
*I*f*(f*x+e)^2*polylog(3,I*exp(2*b*x+2*a))/b^2+3/8*I*f^2*(f*x+e)*polylog(4
,-I*exp(2*b*x+2*a))/b^3-3/8*I*f^2*(f*x+e)*polylog(4,I*exp(2*b*x+2*a))/b^3-
3/16*I*f^3*polylog(5,-I*exp(2*b*x+2*a))/b^4+3/16*I*f^3*polylog(5,I*exp(2*b
*x+2*a))/b^4
```

3.200.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 600 vs. $2(299) = 598$.

Time = 0.22 (sec) , antiderivative size = 600, normalized size of antiderivative = 2.01

$$\int (e+fx)^3 \cot^{-1}(\coth(a+bx))\,dx = \frac{1}{4}x\left(4e^3+6e^2fx+4ef^2x^2+f^3x^3\right)\cot^{-1}(\coth(a+bx))$$
$$-\frac{i\left(8b^4e^3x\log\left(1-ie^{2(a+bx)}\right)+12b^4e^2fx^2\log\left(1-ie^{2(a+bx)}\right)+8b^4ef^2x^3\log\left(1-ie^{2(a+bx)}\right)+2b^4f^3x^4\log\right.}{}$$

input
```
Integrate[(e + f*x)^3*ArcCot[Coth[a + b*x]],x]
```

output
```
(x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*ArcCot[Coth[a + b*x]])/4 -
((I/16)*(8*b^4*e^3*x*Log[1 - I*E^(2*(a + b*x))] + 12*b^4*e^2*f*x^2*Log[1 -
 I*E^(2*(a + b*x))] + 8*b^4*e*f^2*x^3*Log[1 - I*E^(2*(a + b*x))] + 2*b^4*f
^3*x^4*Log[1 - I*E^(2*(a + b*x))] - 8*b^4*e^3*x*Log[1 + I*E^(2*(a + b*x))]
 - 12*b^4*e^2*f*x^2*Log[1 + I*E^(2*(a + b*x))] - 8*b^4*e*f^2*x^3*Log[1 + I
*E^(2*(a + b*x))] - 2*b^4*f^3*x^4*Log[1 + I*E^(2*(a + b*x))] - 4*b^3*(e +
f*x)^3*PolyLog[2, (-I)*E^(2*(a + b*x))] + 4*b^3*(e + f*x)^3*PolyLog[2, I*E
^(2*(a + b*x))] + 6*b^2*e^2*f*PolyLog[3, (-I)*E^(2*(a + b*x))] + 12*b^2*e*
f^2*x*PolyLog[3, (-I)*E^(2*(a + b*x))] + 6*b^2*f^3*x^2*PolyLog[3, (-I)*E^(
2*(a + b*x))] - 6*b^2*e^2*f*PolyLog[3, I*E^(2*(a + b*x))] - 12*b^2*e*f^2*x
*PolyLog[3, I*E^(2*(a + b*x))] - 6*b^2*f^3*x^2*PolyLog[3, I*E^(2*(a + b*x)
)] - 6*b*e*f^2*PolyLog[4, (-I)*E^(2*(a + b*x))] - 6*b*f^3*x*PolyLog[4, (-I
)*E^(2*(a + b*x))] + 6*b*e*f^2*PolyLog[4, I*E^(2*(a + b*x))] + 6*b*f^3*x*P
olyLog[4, I*E^(2*(a + b*x))] + 3*f^3*PolyLog[5, (-I)*E^(2*(a + b*x))] - 3*
f^3*PolyLog[5, I*E^(2*(a + b*x))]))/b^4
```

3.200.3 Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {5709, 3042, 4668, 3011, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^3 \cot^{-1}(\coth(a+bx))\,dx$$

$$\Big\downarrow \text{5709}$$

$$\frac{(e+fx)^4 \cot^{-1}(\coth(a+bx))}{4f}-\frac{b\int (e+fx)^4\text{sech}(2a+2bx)dx}{4f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^4 \cot^{-1}(\coth(a+bx))}{4f}-\frac{b\int (e+fx)^4 \csc\left(2ia+2ibx+\frac{\pi}{2}\right)dx}{4f}$$

$$\Big\downarrow \text{4668}$$

$$\frac{(e+fx)^4 \cot^{-1}(\coth(a+bx))}{4f}-\frac{b\left(-\frac{2if\int (e+fx)^3\log\left(1-ie^{2a+2bx}\right)dx}{b}+\frac{2if\int (e+fx)^3\log\left(1+ie^{2a+2bx}\right)dx}{b}+\frac{(e+fx)^4\arctan\left(e^{2a+2bx}\right)}{b}\right)}{4f}$$

$$\Big\downarrow \text{3011}$$

$$\frac{(e+fx)^4 \cot^{-1}(\coth(a+bx))}{4f}-\frac{b\left(\frac{2if\left(\frac{3f\int (e+fx)^2\text{PolyLog}\left(2,-ie^{2a+2bx}\right)dx}{2b}-\frac{(e+fx)^3\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{2b}\right)}{b}-\frac{2if\left(\frac{3f\int (e+fx)^2\text{PolyLog}\left(2,ie^{2a+2bx}\right)dx}{2b}-\frac{(e+fx)^3\text{PolyLog}\left(2,ie^{2\cdots}\right.}{2b}\right.}{b}\cdots\right.}{4f}$$

$$\Big\downarrow \text{7163}$$

$$\frac{(e+fx)^4 \cot^{-1}(\coth(a+bx))}{4f}-\frac{b\left(\frac{2if\left(\frac{3f\left(\frac{(e+fx)^2\text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{2b}-\frac{f\int (e+fx)\text{PolyLog}\left(3,-ie^{2a+2bx}\right)dx}{b}\right)}{2b}-\frac{(e+fx)^3\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{2b}\right)}{b}-\frac{2if\left(\frac{3f\left(\frac{(e+fx)^2\text{PolyLog}\left(\cdots\right.}{2b}\right.}{\cdots}\right.}{\cdots}\right.}{4f}$$

$\downarrow$ 7163

$$\frac{(e+fx)^4\cot^{-1}(\coth(a+bx))}{4f}-$$

$$b\left(\frac{2if\left(\frac{3f\left(\frac{(e+fx)^2\operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{2b}-\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{2b}-\frac{f\int \operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)dx}{2b}\right)}{b}\right)}{2b}-\frac{(e+fx)^3\operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{2b}\right)}{b}\right.$$

$\downarrow$ 2720

$$\frac{(e+fx)^4\cot^{-1}(\coth(a+bx))}{4f}-$$

$$b\left(\frac{2if\left(\frac{3f\left(\frac{(e+fx)^2\operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{2b}-\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{2b}-\frac{f\int e^{-2a-2bx}\operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)de^{2a+2bx}}{4b^2}\right)}{b}\right)}{2b}-\frac{(e+fx)^3\operatorname{PolyLog}(}{2b}\right.}{b}\right.$$

$\downarrow$ 7143

3.200. $\int (e+fx)^3\cot^{-1}(\coth(a+bx))\,dx$

$$\frac{(e+fx)^4\cot^{-1}(\coth(a+bx))}{4f}-$$

$$b\left(\frac{(e+fx)^4\arctan\left(e^{2a+2bx}\right)}{b}+\frac{2if\left(\frac{3f\left(\frac{(e+fx)^2\operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{2b}-\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{2b}-\frac{f\operatorname{PolyLog}\left(5,-ie^{2a+2bx}\right)}{4b^2}\right)}{b}\right)}{2b}-\frac{(e+fx)^3}{}\right.}{b}\right.$$

input
```
Int[(e + f*x)^3*ArcCot[Coth[a + b*x]],x]
```

output
```
((e + f*x)^4*ArcCot[Coth[a + b*x]])/(4*f) - (b*(((e + f*x)^4*ArcTan[E^(2*a
 + 2*b*x)])/b + ((2*I)*f*(-1/2*((e + f*x)^3*PolyLog[2, (-I)*E^(2*a + 2*b*x
)])/b + (3*f*(((e + f*x)^2*PolyLog[3, (-I)*E^(2*a + 2*b*x)])/(2*b) - (f*((
(e + f*x)*PolyLog[4, (-I)*E^(2*a + 2*b*x)])/(2*b) - (f*PolyLog[5, (-I)*E^(
2*a + 2*b*x)])/(4*b^2)))/b))/(2*b)))/b - ((2*I)*f*(-1/2*((e + f*x)^3*PolyL
og[2, I*E^(2*a + 2*b*x)])/b + (3*f*(((e + f*x)^2*PolyLog[3, I*E^(2*a + 2*b
*x)])/(2*b) - (f*(((e + f*x)*PolyLog[4, I*E^(2*a + 2*b*x)])/(2*b) - (f*Pol
yLog[5, I*E^(2*a + 2*b*x)])/(4*b^2)))/b))/(2*b)))/b))/(4*f)
```

3.200.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5709
```
Int[ArcCot[Coth[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcCot[Coth[a + b*x]]/(f*(m + 1))), x] - Simp[b/
(f*(m + 1))   Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a,
b, e, f}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.200. $\int (e+fx)^3 \cot^{-1}(\coth(a+bx))\,dx$

3.200.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 33.25 (sec) , antiderivative size = 3570, normalized size of antiderivative = 11.94

method	result	size
risch	Expression too large to display	3570

input
```
int((f*x+e)^3*arccot(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
3/16*I*f^3*polylog(5,I*exp(2*b*x+2*a))/b^4-3/16*I*f^3*polylog(5,-I*exp(2*b
*x+2*a))/b^4+3/2*I*f/b*e^2*ln(1+I*exp(2*b*x+2*a))*a*x+3/2*I*f^2/b^2*a^2*e*
ln(1+exp(b*x+a)*(-1)^(3/4))*x+3/2*I*f^2/b^2*a^2*e*ln(1-exp(b*x+a)*(-1)^(3/
4))*x-3/2*I*f/b*a*e^2*ln(1+exp(b*x+a)*(-1)^(3/4))*x-3/2*I*f/b*a*e^2*ln(1-e
xp(b*x+a)*(-1)^(3/4))*x-3/8*I*f^3/b^3*polylog(4,I*exp(2*b*x+2*a))*x+1/2*I*
f^3/b^4*a^4*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))+1/2*I*f^3/b^4*a^4*ln(((
-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))+1/2*I*f^3/b^4*a^3*dilog(((-I)^(1/2)-exp(
b*x+a))/(-I)^(1/2))+1/2*I*f^3/b^4*a^3*dilog(((-I)^(1/2)+exp(b*x+a))/(-I)^(
1/2))-1/8*I*f^3/b^4*a^4*ln(exp(2*b*x+2*a)+I)-1/2*I*f^2*e*ln(1-I*exp(2*b*x+
2*a))*x^3-3/4*I*f*e^2*ln(1-I*exp(2*b*x+2*a))*x^2-3/8*I*f^2/b^3*e*polylog(4
,I*exp(2*b*x+2*a))+3/8*I*f/b^2*e^2*polylog(3,I*exp(2*b*x+2*a))-3/8*I*f^3/b
^4*ln(1-I*exp(2*b*x+2*a))*a^4-1/4*I*f^3/b*polylog(2,I*exp(2*b*x+2*a))*x^3-
1/4*I*f^3/b^4*polylog(2,I*exp(2*b*x+2*a))*a^3+3/8*I*f^3/b^2*polylog(3,I*ex
p(2*b*x+2*a))*x^2+1/2*I/b*a*e^3*ln(exp(2*b*x+2*a)+I)-1/2*I/b*e^3*ln(((-I)^
(1/2)-exp(b*x+a))/(-I)^(1/2))*a-1/2*I/b*e^3*ln(((-I)^(1/2)+exp(b*x+a))/(-I
)^(1/2))*a-3/2*I*f/b^2*a^2*e^2*ln(1+exp(b*x+a)*(-1)^(3/4))-3/2*I*f/b^2*a^2
*e^2*ln(1-exp(b*x+a)*(-1)^(3/4))-3/2*I*f/b^2*a*e^2*dilog(1+exp(b*x+a)*(-1)
^(3/4))-3/2*I*f/b^2*a*e^2*dilog(1-exp(b*x+a)*(-1)^(3/4))-1/2*I*f^3/b^4*a^4
*ln(1-exp(b*x+a)*(-1)^(3/4))-1/2*I*f^3/b^4*a^3*dilog(1+exp(b*x+a)*(-1)^(3/
4))-1/2*I*f^3/b^4*a^3*dilog(1-exp(b*x+a)*(-1)^(3/4))+3/8*I*f^2/b^3*e*po...
```

3.200.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1460 vs. 2(236) = 472.

Time = 0.33 (sec) , antiderivative size = 1460, normalized size of antiderivative = 4.88

$$\int (e + fx)^3 \cot^{-1}(\coth(a + bx))\, dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*arccot(coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/8*(24*I*f^3*polylog(5, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) +
24*I*f^3*polylog(5, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 24*I
*f^3*polylog(5, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 24*I*f^3
*polylog(5, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*(b^4*f^3*
x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x)*arctan(sinh(b*x + a
)/cosh(b*x + a)) - 4*(I*b^3*f^3*x^3 + 3*I*b^3*e*f^2*x^2 + 3*I*b^3*e^2*f*x
+ I*b^3*e^3)*dilog(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 4*(I*b
^3*f^3*x^3 + 3*I*b^3*e*f^2*x^2 + 3*I*b^3*e^2*f*x + I*b^3*e^3)*dilog(-1/2*s
qrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 4*(-I*b^3*f^3*x^3 - 3*I*b^3*e*
f^2*x^2 - 3*I*b^3*e^2*f*x - I*b^3*e^3)*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a)
 + sinh(b*x + a))) - 4*(-I*b^3*f^3*x^3 - 3*I*b^3*e*f^2*x^2 - 3*I*b^3*e^2*f
*x - I*b^3*e^3)*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (
-I*b^4*f^3*x^4 - 4*I*b^4*e*f^2*x^3 - 6*I*b^4*e^2*f*x^2 - 4*I*b^4*e^3*x - 4
*I*a*b^3*e^3 + 6*I*a^2*b^2*e^2*f - 4*I*a^3*b*e*f^2 + I*a^4*f^3)*log(1/2*sq
rt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^4*f^3*x^4 - 4*I*b^4*e
*f^2*x^3 - 6*I*b^4*e^2*f*x^2 - 4*I*b^4*e^3*x - 4*I*a*b^3*e^3 + 6*I*a^2*b^2
*e^2*f - 4*I*a^3*b*e*f^2 + I*a^4*f^3)*log(-1/2*sqrt(4*I)*(cosh(b*x + a) +
sinh(b*x + a)) + 1) + (I*b^4*f^3*x^4 + 4*I*b^4*e*f^2*x^3 + 6*I*b^4*e^2*f*x
^2 + 4*I*b^4*e^3*x + 4*I*a*b^3*e^3 - 6*I*a^2*b^2*e^2*f + 4*I*a^3*b*e*f^2 -
 I*a^4*f^3)*log(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (...
```

3.200.6 Sympy [F]

$$\int (e+fx)^3 \cot^{-1}(\coth(a+bx))\,dx = \int (e+fx)^3 \operatorname{acot}(\coth(a+bx))\,dx$$

input
```
integrate((f*x+e)**3*acot(coth(b*x+a)),x)
```

output
```
Integral((e + f*x)**3*acot(coth(a + b*x)), x)
```

3.200.7 Maxima [F]

$$\int (e+fx)^3 \cot^{-1}(\coth(a+bx))\,dx = \int (fx+e)^3 \operatorname{arccot}(\coth(bx+a))\,dx$$

input
```
integrate((f*x+e)^3*arccot(coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*arctan((e^(2*b*x + 2*a
) - 1)/(e^(2*b*x + 2*a) + 1)) - integrate(1/2*(b*f^3*x^4*e^(2*a) + 4*b*e*f
^2*x^3*e^(2*a) + 6*b*e^2*f*x^2*e^(2*a) + 4*b*e^3*x*e^(2*a))*e^(2*b*x)/(e^(
4*b*x + 4*a) + 1), x)
```

3.200.8 Giac [F(-1)]

Timed out.

$$\int (e+fx)^3 \cot^{-1}(\coth(a+bx))\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*arccot(coth(b*x+a)),x, algorithm="giac")
```

output
```
Timed out
```

3.200.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)^3 \cot^{-1}(\coth(a+bx))\,dx = \int \operatorname{acot}(\coth(a+b\,x))\,(e+f\,x)^3\,dx$$

input
```
int(acot(coth(a + b*x))*(e + f*x)^3,x)
```

output
```
int(acot(coth(a + b*x))*(e + f*x)^3, x)
```

3.201 $\int (e+fx)^2 \cot^{-1}(\coth(a+bx))\,dx$

3.201.1 Optimal result

Integrand size = 15, antiderivative size = 229

$$\begin{aligned}\int (e+fx)^2 \cot^{-1}(\coth(a+bx))\,dx =\ & \frac{(e+fx)^3 \cot^{-1}(\coth(a+bx))}{3f} \\ & - \frac{(e+fx)^3 \arctan\left(e^{2a+2bx}\right)}{3f} \\ & + \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} \\ & - \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} \\ & - \frac{if(e+fx) \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{4b^2} \\ & + \frac{if(e+fx) \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{4b^2} \\ & + \frac{if^2 \operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} \\ & - \frac{if^2 \operatorname{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3}\end{aligned}$$

output
```
1/3*(f*x+e)^3*arccot(coth(b*x+a))/f-1/3*(f*x+e)^3*arctan(exp(2*b*x+2*a))/f
+1/4*I*(f*x+e)^2*polylog(2,-I*exp(2*b*x+2*a))/b-1/4*I*(f*x+e)^2*polylog(2,
I*exp(2*b*x+2*a))/b-1/4*I*f*(f*x+e)*polylog(3,-I*exp(2*b*x+2*a))/b^2+1/4*I
*f*(f*x+e)*polylog(3,I*exp(2*b*x+2*a))/b^2+1/8*I*f^2*polylog(4,-I*exp(2*b*
x+2*a))/b^3-1/8*I*f^2*polylog(4,I*exp(2*b*x+2*a))/b^3
```

3.201.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.64

$$\int (e+fx)^2\cot^{-1}(\coth(a+bx))\,dx=\frac{1}{3}x\big(3e^2+3efx+f^2x^2\big)\cot^{-1}(\coth(a+bx))$$
$$-\frac{i\big(12b^3e^2x\log\big(1-ie^{2(a+bx)}\big)+12b^3efx^2\log\big(1-ie^{2(a+bx)}\big)+4b^3f^2x^3\log\big(1-ie^{2(a+bx)}\big)-12b^3e^2x\log\big(\right.}{}$$

input
```
Integrate[(e + f*x)^2*ArcCot[Coth[a + b*x]],x]
```

output
```
(x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcCot[Coth[a + b*x]])/3 - ((I/24)*(12*b^3*
e^2*x*Log[1 - I*E^(2*(a + b*x))] + 12*b^3*e*f*x^2*Log[1 - I*E^(2*(a + b*x)
)] + 4*b^3*f^2*x^3*Log[1 - I*E^(2*(a + b*x))] - 12*b^3*e^2*x*Log[1 + I*E^(
2*(a + b*x))] - 12*b^3*e*f*x^2*Log[1 + I*E^(2*(a + b*x))] - 4*b^3*f^2*x^3*
Log[1 + I*E^(2*(a + b*x))] - 6*b^2*(e + f*x)^2*PolyLog[2, (-I)*E^(2*(a + b
*x))] + 6*b^2*(e + f*x)^2*PolyLog[2, I*E^(2*(a + b*x))] + 6*b*e*f*PolyLog[
3, (-I)*E^(2*(a + b*x))] + 6*b*f^2*x*PolyLog[3, (-I)*E^(2*(a + b*x))] - 6*
b*e*f*PolyLog[3, I*E^(2*(a + b*x))] - 6*b*f^2*x*PolyLog[3, I*E^(2*(a + b*x
))] - 3*f^2*PolyLog[4, (-I)*E^(2*(a + b*x))] + 3*f^2*PolyLog[4, I*E^(2*(a
+ b*x))]))/b^3
```

3.201.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {5709, 3042, 4668, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2\cot^{-1}(\coth(a+bx))\,dx$$

$$\Big\downarrow \text{5709}$$

$$\frac{(e+fx)^3\cot^{-1}(\coth(a+bx))}{3f}-\frac{b\int (e+fx)^3\text{sech}(2a+2bx)dx}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^3\cot^{-1}(\coth(a+bx))}{3f}-\frac{b\int (e+fx)^3\csc\left(2ia+2ibx+\frac{\pi}{2}\right)dx}{3f}$$

$$\Big\downarrow \text{4668}$$

$$\frac{(e+fx)^3\cot^{-1}(\coth(a+bx))}{3f}-\frac{b\left(-\frac{3if\int(e+fx)^2\log\left(1-ie^{2a+2bx}\right)dx}{2b}+\frac{3if\int(e+fx)^2\log\left(1+ie^{2a+2bx}\right)dx}{2b}+\frac{(e+fx)^3\arctan\left(e^{2a+2bx}\right)}{b}\right)}{3f}$$

$$\Big\downarrow \text{3011}$$

$$\frac{(e+fx)^3\cot^{-1}(\coth(a+bx))}{3f}-\frac{b\left(\frac{3if\left(\frac{f\int(e+fx)\text{PolyLog}\left(2,-ie^{2a+2bx}\right)dx}{b}-\frac{(e+fx)^2\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{2b}\right)}{2b}-\frac{3if\left(\frac{f\int(e+fx)\text{PolyLog}\left(2,ie^{2a+2bx}\right)dx}{b}-\frac{(e+fx)^2\text{PolyLog}\left(2,ie^{2a+2\cdots}\right)}{2b}\right.}{2b}\right.}{3f}$$

$$\Big\downarrow \text{7163}$$

$$\frac{(e+fx)^3\cot^{-1}(\coth(a+bx))}{3f}-\frac{b\left(\frac{3if\left(\frac{f\left(\frac{(e+fx)\text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{2b}-\frac{f\int\text{PolyLog}\left(3,-ie^{2a+2bx}\right)dx}{2b}\right)}{b}-\frac{(e+fx)^2\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{2b}\right)}{2b}-\frac{3if\left(\frac{f\left(\frac{(e+fx)\text{PolyLog}\left(3,ie^{2a+2bx}\right)}{2b}\cdots\right.}{\cdots}\right.}{\cdots}\right.}{3f}$$

$$\Big\downarrow \text{2720}$$

$$\frac{(e+fx)^3\cot^{-1}(\coth(a+bx))}{3f}-\frac{b\left(\frac{3if\left(\frac{f\left(\frac{(e+fx)\text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{2b}-\frac{f\int e^{-2a-2bx}\text{PolyLog}\left(3,-ie^{2a+2bx}\right)de^{2a+2bx}}{4b^2}\right)}{b}-\frac{(e+fx)^2\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{2b}\right)}{2b}-\frac{3if\left(\frac{f\left(\frac{(e+fx)\text{Po}\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{3f}$$

$$\Big\downarrow \text{7143}$$

3.201. $\int(e+fx)^2\cot^{-1}(\coth(a+bx))\,dx$

$$\frac{(e+fx)^3\cot^{-1}(\coth(a+bx))}{3f} - \frac{b\left(\frac{(e+fx)^3\arctan(e^{2a+2bx})}{b} + \frac{3if\left(\frac{f\left(\frac{(e+fx)\,\mathrm{PolyLog}(3,-ie^{2a+2bx})}{2b}-\frac{f\,\mathrm{PolyLog}(4,-ie^{2a+2bx})}{4b^2}\right)}{b}-\frac{(e+fx)^2\,\mathrm{PolyLog}(2,-ie^{2a+2bx})}{2b}\right)}{2b} - \frac{3if\left(\frac{f\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{3f}$$

input
```
Int[(e + f*x)^2*ArcCot[Coth[a + b*x]],x]
```

output
```
((e + f*x)^3*ArcCot[Coth[a + b*x]])/(3*f) - (b*(((e + f*x)^3*ArcTan[E^(2*a
 + 2*b*x)])/b + (((3*I)/2)*f*(-1/2*((e + f*x)^2*PolyLog[2, (-I)*E^(2*a + 2
*b*x)])/b + (f*(((e + f*x)*PolyLog[3, (-I)*E^(2*a + 2*b*x)])/(2*b) - (f*Po
lyLog[4, (-I)*E^(2*a + 2*b*x)])/(4*b^2)))/b))/b - (((3*I)/2)*f*(-1/2*((e +
 f*x)^2*PolyLog[2, I*E^(2*a + 2*b*x)])/b + (f*(((e + f*x)*PolyLog[3, I*E^(
2*a + 2*b*x)])/(2*b) - (f*PolyLog[4, I*E^(2*a + 2*b*x)])/(4*b^2)))/b))/b))
/(3*f)
```

3.201.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5709
```
Int[ArcCot[Coth[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcCot[Coth[a + b*x]]/(f*(m + 1))), x] - Simp[b/
(f*(m + 1))   Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a,
b, e, f}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.201.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 21.01 (sec) , antiderivative size = 2668, normalized size of antiderivative = 11.65

method	result	size
risch	Expression too large to display	2668

input
```
int((f*x+e)^2*arccot(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*f^2/b^3*a^2*dilog(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))-1/2*I*f^2/b^3
*a^2*dilog(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))+1/4*I*f/b^2*e*polylog(3,I*e
xp(2*b*x+2*a))+1/6*I*f^2/b^3*a^3*ln(exp(2*b*x+2*a)+I)+1/3*I*f^2/b^3*ln(1-I
*exp(2*b*x+2*a))*a^3-1/4*I*f^2/b*polylog(2,I*exp(2*b*x+2*a))*x^2+1/2*I/b*a
*e^2*ln(exp(2*b*x+2*a)+I)-1/2*I/b*e^2*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2
))*a-1/2*I/b*e^2*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))*a+1/8*I*f^2*polylo
g(4,-I*exp(2*b*x+2*a))/b^3-1/8*I*f^2*polylog(4,I*exp(2*b*x+2*a))/b^3-1/6*I
*f^2/b^3*a^3*ln(-exp(2*b*x+2*a)+I)+1/2*I*f*e*ln(1+I*exp(2*b*x+2*a))*x^2-1/
3*I*f^2/b^3*ln(1+I*exp(2*b*x+2*a))*a^3+1/4*I*f^2/b*polylog(2,-I*exp(2*b*x+
2*a))*x^2-1/4*I*f^2/b^3*polylog(2,-I*exp(2*b*x+2*a))*a^2-1/4*I*f^2/b^2*pol
ylog(3,-I*exp(2*b*x+2*a))*x+1/2*I*f^2/b^3*a^3*ln(1+exp(b*x+a)*(-1)^(3/4))+
1/2*I*f^2/b^3*a^3*ln(1-exp(b*x+a)*(-1)^(3/4))+1/2*I*f^2/b^3*a^2*dilog(1+ex
p(b*x+a)*(-1)^(3/4))+1/2*I*f^2/b^3*a^2*dilog(1-exp(b*x+a)*(-1)^(3/4))-1/4*
I*f/b^2*e*polylog(3,-I*exp(2*b*x+2*a))+1/2*I/b*e^2*ln(1+exp(b*x+a)*(-1)^(3
/4))*a+1/2*I/b*e^2*ln(1-exp(b*x+a)*(-1)^(3/4))*a-1/2*I/b*a*e^2*ln(-exp(2*b
*x+2*a)+I)-1/2*I*f*ln(exp(2*b*x+2*a)-I)*x^2*e-1/2*I*f*e*ln(1-I*exp(2*b*x+2
*a))*x^2+1/4*I*f^2/b^3*polylog(2,I*exp(2*b*x+2*a))*a^2+1/4*I*f^2/b^2*polyl
og(3,I*exp(2*b*x+2*a))*x-1/2*I*f^2/b^3*a^3*ln(((-I)^(1/2)-exp(b*x+a))/(-I)
^(1/2))-1/2*I*f^2/b^3*a^3*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))+1/2*I*f/b
^2*e*ln(1+I*exp(2*b*x+2*a))*a^2+1/2*I*f/b*e*polylog(2,-I*exp(2*b*x+2*a)...
```

3.201.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1002 vs. $2(180) = 360$.

Time = 0.33 (sec) , antiderivative size = 1002, normalized size of antiderivative = 4.38

$$\int (e+fx)^2 \cot^{-1}(\coth(a+bx))\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*arccot(coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/6*(-6*I*f^2*polylog(4, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) -
6*I*f^2*polylog(4, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 6*I*f
^2*polylog(4, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 6*I*f^2*po
lylog(4, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*(b^3*f^2*x^3
 + 3*b^3*e*f*x^2 + 3*b^3*e^2*x)*arctan(sinh(b*x + a)/cosh(b*x + a)) - 3*(I
*b^2*f^2*x^2 + 2*I*b^2*e*f*x + I*b^2*e^2)*dilog(1/2*sqrt(4*I)*(cosh(b*x +
a) + sinh(b*x + a))) - 3*(I*b^2*f^2*x^2 + 2*I*b^2*e*f*x + I*b^2*e^2)*dilog
(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 3*(-I*b^2*f^2*x^2 - 2*I
*b^2*e*f*x - I*b^2*e^2)*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a
))) - 3*(-I*b^2*f^2*x^2 - 2*I*b^2*e*f*x - I*b^2*e^2)*dilog(-1/2*sqrt(-4*I)
*(cosh(b*x + a) + sinh(b*x + a))) + (-I*b^3*f^2*x^3 - 3*I*b^3*e*f*x^2 - 3*
I*b^3*e^2*x - 3*I*a*b^2*e^2 + 3*I*a^2*b*e*f - I*a^3*f^2)*log(1/2*sqrt(4*I)
*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^3*f^2*x^3 - 3*I*b^3*e*f*x^2
- 3*I*b^3*e^2*x - 3*I*a*b^2*e^2 + 3*I*a^2*b*e*f - I*a^3*f^2)*log(-1/2*sqrt
(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^3*f^2*x^3 + 3*I*b^3*e*f*
x^2 + 3*I*b^3*e^2*x + 3*I*a*b^2*e^2 - 3*I*a^2*b*e*f + I*a^3*f^2)*log(1/2*s
qrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^3*f^2*x^3 + 3*I*b^3*
e*f*x^2 + 3*I*b^3*e^2*x + 3*I*a*b^2*e^2 - 3*I*a^2*b*e*f + I*a^3*f^2)*log(-
1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (3*I*a*b^2*e^2 - 3*I
*a^2*b*e*f + I*a^3*f^2)*log(I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x ...
```

3.201.6 Sympy [F]

$$\int (e+fx)^2 \cot^{-1}(\coth(a+bx))\,dx = \int (e+fx)^2 \operatorname{acot}(\coth(a+bx))\,dx$$

input
```
integrate((f*x+e)**2*acot(coth(b*x+a)),x)
```

output
```
Integral((e + f*x)**2*acot(coth(a + b*x)), x)
```

3.201.7 Maxima [F]

$$\int (e+fx)^2 \cot^{-1}(\coth(a+bx))\,dx = \int (fx+e)^2 \operatorname{arccot}(\coth(bx+a))\,dx$$

input
```
integrate((f*x+e)^2*arccot(coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/3*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*arctan((e^(2*b*x + 2*a) - 1)/(e^(2*b*x
 + 2*a) + 1)) - integrate(2/3*(b*f^2*x^3*e^(2*a) + 3*b*e*f*x^2*e^(2*a) + 3
*b*e^2*x*e^(2*a))*e^(2*b*x)/(e^(4*b*x + 4*a) + 1), x)
```

3.201.8 Giac [F]

$$\int (e+fx)^2 \cot^{-1}(\coth(a+bx))\,dx = \int (fx+e)^2 \operatorname{arccot}(\coth(bx+a))\,dx$$

input
```
integrate((f*x+e)^2*arccot(coth(b*x+a)),x, algorithm="giac")
```

output
```
sage0*x
```

3.201.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2 \cot^{-1}(\coth(a+bx))\,dx = \int \operatorname{acot}(\coth(a+b\,x))\,(e+f\,x)^2\,dx$$

input
```
int(acot(coth(a + b*x))*(e + f*x)^2,x)
```

output
```
int(acot(coth(a + b*x))*(e + f*x)^2, x)
```

3.202 $\int (e+fx)\cot^{-1}(\coth(a+bx))\,dx$

3.202.1 Optimal result

Integrand size = 13, antiderivative size = 159

$$\int (e+fx)\cot^{-1}(\coth(a+bx))\,dx = \frac{(e+fx)^2\cot^{-1}(\coth(a+bx))}{2f} - \frac{(e+fx)^2\arctan\left(e^{2a+2bx}\right)}{2f} + \frac{i(e+fx)\,\mathrm{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i(e+fx)\,\mathrm{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} - \frac{if\,\mathrm{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} + \frac{if\,\mathrm{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2}$$

output
```
1/2*(f*x+e)^2*arccot(coth(b*x+a))/f-1/2*(f*x+e)^2*arctan(exp(2*b*x+2*a))/f
+1/4*I*(f*x+e)*polylog(2,-I*exp(2*b*x+2*a))/b-1/4*I*(f*x+e)*polylog(2,I*ex
p(2*b*x+2*a))/b-1/8*I*f*polylog(3,-I*exp(2*b*x+2*a))/b^2+1/8*I*f*polylog(3
,I*exp(2*b*x+2*a))/b^2
```

3.202.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.49

$$\int (e+fx)\cot^{-1}(\coth(a+bx))\,dx = ex\cot^{-1}(\coth(a+bx))+\frac{1}{2}fx^2\cot^{-1}(\coth(a+bx)) - \frac{ie\left(2bx\left(\log\left(1-ie^{2(a+bx)}\right)-\log\left(1+ie^{2(a+bx)}\right)\right)-\text{PolyLog}\left(2,-ie^{2(a+bx)}\right)+\text{PolyLog}\left(2,ie^{2(a+bx)}\right)\right)}{4b} - \frac{if\left(2b^2x^2\log\left(1-ie^{2(a+bx)}\right)-2b^2x^2\log\left(1+ie^{2(a+bx)}\right)-2bx\,\text{PolyLog}\left(2,-ie^{2(a+bx)}\right)+2bx\,\text{PolyLog}\left(2,\right.\right.}{8b^2}$$

input
```
Integrate[(e + f*x)*ArcCot[Coth[a + b*x]],x]
```

output
```
e*x*ArcCot[Coth[a + b*x]] + (f*x^2*ArcCot[Coth[a + b*x]])/2 - ((I/4)*e*(2*
b*x*(Log[1 - I*E^(2*(a + b*x))] - Log[1 + I*E^(2*(a + b*x))]) - PolyLog[2,
 (-I)*E^(2*(a + b*x))] + PolyLog[2, I*E^(2*(a + b*x))]))/b - ((I/8)*f*(2*b
^2*x^2*Log[1 - I*E^(2*(a + b*x))] - 2*b^2*x^2*Log[1 + I*E^(2*(a + b*x))] -
 2*b*x*PolyLog[2, (-I)*E^(2*(a + b*x))] + 2*b*x*PolyLog[2, I*E^(2*(a + b*x
))] + PolyLog[3, (-I)*E^(2*(a + b*x))] - PolyLog[3, I*E^(2*(a + b*x))]))/b
^2
```

3.202.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {5709, 3042, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\cot^{-1}(\coth(a+bx))\,dx$$

$$\Big\downarrow \text{ 5709}$$

$$\frac{(e+fx)^2\cot^{-1}(\coth(a+bx))}{2f}-\frac{b\int (e+fx)^2\text{sech}(2a+2bx)dx}{2f}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(e+fx)^2\cot^{-1}(\coth(a+bx))}{2f}-\frac{b\int (e+fx)^2\csc\left(2ia+2ibx+\frac{\pi}{2}\right)dx}{2f}$$

$$\Big\downarrow \text{ 4668}$$

$$\frac{(e+fx)^2\cot^{-1}(\coth(a+bx))}{2f}-\frac{b\left(-\frac{if\int(e+fx)\log\left(1-ie^{2a+2bx}\right)dx}{b}+\frac{if\int(e+fx)\log\left(1+ie^{2a+2bx}\right)dx}{b}+\frac{(e+fx)^2\arctan\left(e^{2a+2bx}\right)}{b}\right)}{2f}$$

$\big\downarrow$ 3011

$$\frac{(e+fx)^2\cot^{-1}(\coth(a+bx))}{2f}-\frac{b\left(\frac{if\left(\frac{f\int\text{PolyLog}\left(2,-ie^{2a+2bx}\right)dx}{2b}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{2b}\right)}{b}-\frac{if\left(\frac{f\int\text{PolyLog}\left(2,ie^{2a+2bx}\right)dx}{2b}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{2b}\right)}{b}+\frac{(e+fx)^2}{\cdots}\right.}{2f}$$

$\big\downarrow$ 2720

$$\frac{(e+fx)^2\cot^{-1}(\coth(a+bx))}{2f}-\frac{b\left(\frac{if\left(\frac{f\int e^{-2a-2bx}\text{PolyLog}\left(2,-ie^{2a+2bx}\right)de^{2a+2bx}}{4b^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{2b}\right)}{b}-\frac{if\left(\frac{f\int e^{-2a-2bx}\text{PolyLog}\left(2,ie^{2a+2bx}\right)de^{2a+2bx}}{4b^2}-\frac{(e+fx)\text{P}\cdots}{\cdots}\right.}{b}\right.}{2f}$$

$\big\downarrow$ 7143

$$\frac{(e+fx)^2\cot^{-1}(\coth(a+bx))}{2f}-\frac{b\left(\frac{(e+fx)^2\arctan\left(e^{2a+2bx}\right)}{b}+\frac{if\left(\frac{f\text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{4b^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{2b}\right)}{b}-\frac{if\left(\frac{f\text{PolyLog}\left(3,ie^{2a+2bx}\right)}{4b^2}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{\cdots}\right.}{2b}\right.}{b}\right.}{2f}$$

input
```
Int[(e + f*x)*ArcCot[Coth[a + b*x]],x]
```

output
```
((e + f*x)^2*ArcCot[Coth[a + b*x]])/(2*f) - (b*(((e + f*x)^2*ArcTan[E^(2*a
 + 2*b*x)])/b + (I*f*(-1/2*((e + f*x)*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b
+ (f*PolyLog[3, (-I)*E^(2*a + 2*b*x)])/(4*b^2)))/b - (I*f*(-1/2*((e + f*x)
*PolyLog[2, I*E^(2*a + 2*b*x)])/b + (f*PolyLog[3, I*E^(2*a + 2*b*x)])/(4*b
^2)))/b))/(2*f)
```

3.202. $\int(e+fx)\cot^{-1}(\coth(a+bx))\,dx$

3.202.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5709
```
Int[ArcCot[Coth[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcCot[Coth[a + b*x]]/(f*(m + 1))), x] - Simp[b/
(f*(m + 1))   Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a,
b, e, f}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.202.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.32 (sec) , antiderivative size = 1776, normalized size of antiderivative = 11.17

method	result	size
risch	Expression too large to display	1776

input
```
int((f*x+e)*arccot(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/4*I*f/b^2*ln(1+I*exp(2*b*x+2*a))*a^2+1/4*I*f/b*polylog(2,-I*exp(2*b*x+2*
a))*x+1/4*I*f/b^2*polylog(2,-I*exp(2*b*x+2*a))*a+1/4*I*f/b^2*a^2*ln(-exp(2
*b*x+2*a)+I)+1/2*I*e/b*ln(1+exp(b*x+a)*(-1)^(3/4))*a+1/2*I*e/b*ln(1-exp(b*
x+a)*(-1)^(3/4))*a-1/2*I*e/b*a*ln(-exp(2*b*x+2*a)+I)+1/2*I*(1/2*f*x^2+e*x)
*ln(exp(2*b*x+2*a)+I)-1/4*Pi*(csgn(I*(exp(2*b*x+2*a)-I))*csgn(I/(exp(2*b*x
+2*a)-1))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))-csgn(I*(exp(2*b*x+
2*a)-I))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2-csgn(I*(exp(2*b*x
+2*a)+I))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+
2*a)-1))+csgn(I*(exp(2*b*x+2*a)+I))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2
*a)-1))^2+csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))*csgn((1+I)*(exp(2*
b*x+2*a)-I)/(exp(2*b*x+2*a)-1))-csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2
*a)-1))^2-csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))*csgn((1-I)*(exp(2*
b*x+2*a)+I)/(exp(2*b*x+2*a)-1))-csgn((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2
*a)-1))^2-csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+
2*a)-1))^2+csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x
+2*a)-1))^2+csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^3-csgn(I*(exp(2*
b*x+2*a)-I)/(exp(2*b*x+2*a)-1))*csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2
*a)-1))^2-csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^3+csgn(I*(exp(2*b*
x+2*a)+I)/(exp(2*b*x+2*a)-1))*csgn((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a
)-1))^2+csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^3+csgn((1-I)*...
```

3.202.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 600 vs. 2(130) = 260.

Time = 0.32 (sec) , antiderivative size = 600, normalized size of antiderivative = 3.77

$$\int (e+fx)\cot^{-1}(\coth(a+bx))\,dx$$
$$= \frac{2\left(b^2fx^2+2\,b^2ex\right)\arctan\left(\frac{\sinh(bx+a)}{\cosh(bx+a)}\right)-2\,(i\,bfx+i\,be)\mathrm{Li}_2\left(\frac{1}{2}\sqrt{4i}(\cosh(bx+a)+\sinh(bx+a))\right)-2\,(i\,b$$

input
```
integrate((f*x+e)*arccot(coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/4*(2*(b^2*f*x^2 + 2*b^2*e*x)*arctan(sinh(b*x + a)/cosh(b*x + a)) - 2*(I*
b*f*x + I*b*e)*dilog(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*(I
*b*f*x + I*b*e)*dilog(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*
(-I*b*f*x - I*b*e)*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) -
 2*(-I*b*f*x - I*b*e)*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)
)) + (-I*b^2*f*x^2 - 2*I*b^2*e*x - 2*I*a*b*e + I*a^2*f)*log(1/2*sqrt(4*I)*
(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^2*f*x^2 - 2*I*b^2*e*x - 2*I*a
*b*e + I*a^2*f)*log(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) +
(I*b^2*f*x^2 + 2*I*b^2*e*x + 2*I*a*b*e - I*a^2*f)*log(1/2*sqrt(-4*I)*(cosh
(b*x + a) + sinh(b*x + a)) + 1) + (I*b^2*f*x^2 + 2*I*b^2*e*x + 2*I*a*b*e -
 I*a^2*f)*log(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (2*I*
a*b*e - I*a^2*f)*log(I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (2
*I*a*b*e - I*a^2*f)*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a))
+ (-2*I*a*b*e + I*a^2*f)*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x +
 a)) + (-2*I*a*b*e + I*a^2*f)*log(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh
(b*x + a)) + 2*I*f*polylog(3, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)
)) + 2*I*f*polylog(3, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*
I*f*polylog(3, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*I*f*pol
ylog(3, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

3.202.6 Sympy [F]

$$\int (e+fx)\cot^{-1}(\coth(a+bx))\,dx = \int (e+fx)\operatorname{acot}(\coth(a+bx))\,dx$$

input
```
integrate((f*x+e)*acot(coth(b*x+a)),x)
```

output
```
Integral((e + f*x)*acot(coth(a + b*x)), x)
```

3.202.7 Maxima [F]

$$\int (e+fx)\cot^{-1}(\coth(a+bx))\,dx = \int (fx+e)\,\mathrm{arccot}\left(\coth\left(bx+a\right)\right)\,dx$$

input
```
integrate((f*x+e)*arccot(coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/2*(f*x^2 + 2*e*x)*arctan((e^(2*b*x + 2*a) - 1)/(e^(2*b*x + 2*a) + 1)) -
integrate((b*f*x^2*e^(2*a) + 2*b*e*x*e^(2*a))*e^(2*b*x)/(e^(4*b*x + 4*a) +
 1), x)
```

3.202.8 Giac [F]

$$\int (e+fx)\cot^{-1}(\coth(a+bx))\,dx = \int (fx+e)\,\mathrm{arccot}\left(\coth\left(bx+a\right)\right)\,dx$$

input
```
integrate((f*x+e)*arccot(coth(b*x+a)),x, algorithm="giac")
```

output
```
sage0*x
```

3.202.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)\cot^{-1}(\coth(a+bx))\,dx = \int \mathrm{acot}(\coth(a+b\,x))\,\left(e+f\,x\right)\,dx$$

input
```
int(acot(coth(a + b*x))*(e + f*x),x)
```

output
```
int(acot(coth(a + b*x))*(e + f*x), x)
```

3.203 $\int \cot^{-1}(\coth(a+bx))\,dx$

3.203.1 Optimal result

Integrand size = 7, antiderivative size = 74

$$\int \cot^{-1}(\coth(a+bx))\,dx = x\cot^{-1}(\coth(a+bx)) - x\arctan\left(e^{2a+2bx}\right) + \frac{i\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i\,\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}$$

output
```
x*arccot(coth(b*x+a))-x*arctan(exp(2*b*x+2*a))+1/4*I*polylog(2,-I*exp(2*b*
x+2*a))/b-1/4*I*polylog(2,I*exp(2*b*x+2*a))/b
```

3.203.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.23

$$\int \cot^{-1}(\coth(a+bx))\,dx = x\cot^{-1}(\coth(a+bx)) - \frac{i\left(2bx\left(\log\left(1-ie^{2(a+bx)}\right)-\log\left(1+ie^{2(a+bx)}\right)\right)-\text{PolyLog}\left(2,-ie^{2(a+bx)}\right)+\text{PolyLog}\left(2,ie^{2(a+bx)}\right)\right)}{4b}$$

input
```
Integrate[ArcCot[Coth[a + b*x]],x]
```

output
```
x*ArcCot[Coth[a + b*x]] - ((I/4)*(2*b*x*(Log[1 - I*E^(2*(a + b*x))] - Log[
1 + I*E^(2*(a + b*x))]) - PolyLog[2, (-I)*E^(2*(a + b*x))] + PolyLog[2, I*
E^(2*(a + b*x))]))/b
```

3.203.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5705, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}(\coth(a+bx))\,dx$$

$$\Big\downarrow \text{5705}$$

$$x\cot^{-1}(\coth(a+bx))-b\int x\text{sech}(2a+2bx)dx$$

$$\Big\downarrow \text{3042}$$

$$x\cot^{-1}(\coth(a+bx))-b\int x\csc\left(2ia+2ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4668}$$

$$x\cot^{-1}(\coth(a+bx))- b\left(-\frac{i\int\log\left(1-ie^{2a+2bx}\right)dx}{2b}+\frac{i\int\log\left(1+ie^{2a+2bx}\right)dx}{2b}+\frac{x\arctan\left(e^{2a+2bx}\right)}{b}\right)$$

$$\Big\downarrow \text{2715}$$

$$x\cot^{-1}(\coth(a+bx))- b\left(-\frac{i\int e^{-2a-2bx}\log\left(1-ie^{2a+2bx}\right)de^{2a+2bx}}{4b^2}+\frac{i\int e^{-2a-2bx}\log\left(1+ie^{2a+2bx}\right)de^{2a+2bx}}{4b^2}+\frac{x\arctan\left(e^{2a+2bx}\right)}{b}\right)$$

$$\Big\downarrow \text{2838}$$

$$x\cot^{-1}(\coth(a+bx))- b\left(\frac{x\arctan\left(e^{2a+2bx}\right)}{b}-\frac{i\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b^2}+\frac{i\,\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b^2}\right)$$

input
```
Int[ArcCot[Coth[a + b*x]],x]
```

output
```
x*ArcCot[Coth[a + b*x]] - b*((x*ArcTan[E^(2*a + 2*b*x)])/b - ((I/4)*PolyLo
g[2, (-I)*E^(2*a + 2*b*x)])/b^2 + ((I/4)*PolyLog[2, I*E^(2*a + 2*b*x)])/b^
2)
```

3.203.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5705
```
Int[ArcCot[Coth[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcCot[Coth[a + b
*x]], x] - Simp[b   Int[x*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b}, x]
```

3.203.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(63) = 126$.

Time = 1.51 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.49

method	result
derivativedivides	$\frac{\operatorname{arctanh}(\coth(bx+a))\operatorname{arccot}(\coth(bx+a))+\arctan(\coth(bx+a))\operatorname{arctanh}(\coth(bx+a))+\frac{\arctan(\coth(bx+a))\ln\left(1+\frac{i(1+i\cot}{\coth(b}\right.}{2}}{}$
default	$\frac{\operatorname{arctanh}(\coth(bx+a))\operatorname{arccot}(\coth(bx+a))+\arctan(\coth(bx+a))\operatorname{arctanh}(\coth(bx+a))+\frac{\arctan(\coth(bx+a))\ln\left(1+\frac{i(1+i\cot}{\coth(b}\right.}{2}}{}$
risch	Expression too large to display

input
```
int(arccot(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b*(arctanh(coth(b*x+a))*arccot(coth(b*x+a))+arctan(coth(b*x+a))*arctanh(
coth(b*x+a))+1/2*arctan(coth(b*x+a))*ln(1+I*(1+I*coth(b*x+a))^2/(coth(b*x+
a)^2+1))-1/2*arctan(coth(b*x+a))*ln(1-I*(1+I*coth(b*x+a))^2/(coth(b*x+a)^2
+1))-1/4*I*dilog(1+I*(1+I*coth(b*x+a))^2/(coth(b*x+a)^2+1))+1/4*I*dilog(1-
I*(1+I*coth(b*x+a))^2/(coth(b*x+a)^2+1)))
```

3.203.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(57) = 114.

Time = 0.28 (sec) , antiderivative size = 334, normalized size of antiderivative = 4.51

$$\int \cot^{-1}(\coth(a+bx))\,dx = \frac{2\,bx\arctan\left(\frac{\sinh(bx+a)}{\cosh(bx+a)}\right) + (-i\,bx - i\,a)\log\left(\frac{1}{2}\sqrt{4i}(\cosh(bx+a) + \sinh(bx+a)) + 1\right) + (-i\,bx - i\,a)\log$$

input
```
integrate(arccot(coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(2*b*x*arctan(sinh(b*x + a)/cosh(b*x + a)) + (-I*b*x - I*a)*log(1/2*sq
rt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b*x - I*a)*log(-1/2*sqr
t(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b*x + I*a)*log(1/2*sqrt(-
4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b*x + I*a)*log(-1/2*sqrt(-4
*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + I*a*log(I*sqrt(4*I) + 2*cosh(b*
x + a) + 2*sinh(b*x + a)) + I*a*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sin
h(b*x + a)) - I*a*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) -
I*a*log(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) - I*dilog(1/2*s
qrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - I*dilog(-1/2*sqrt(4*I)*(cosh(b
*x + a) + sinh(b*x + a))) + I*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b
*x + a))) + I*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

3.203.6 Sympy [F]

$$\int \cot^{-1}(\coth(a+bx))\,dx = \int \operatorname{acot}\left(\coth\left(a+bx\right)\right)dx$$

input
```
integrate(acot(coth(b*x+a)),x)
```

output
```
Integral(acot(coth(a + b*x)), x)
```

3.203.7 Maxima [F]

$$\int \cot^{-1}(\coth(a+bx))\,dx = \int \operatorname{arccot}\left(\coth\left(bx+a\right)\right)\,dx$$

input
```
integrate(arccot(coth(b*x+a)),x, algorithm="maxima")
```

output
```
x*arctan((e^(2*b*x + 2*a) - 1)/(e^(2*b*x + 2*a) + 1)) - 2*b*integrate(x*e^
(2*b*x + 2*a)/(e^(4*b*x + 4*a) + 1), x)
```

3.203.8 Giac [F]

$$\int \cot^{-1}(\coth(a+bx))\,dx = \int \operatorname{arccot}\left(\coth\left(bx+a\right)\right)\,dx$$

input
```
integrate(arccot(coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccot(coth(b*x + a)), x)
```

3.203.9 Mupad [F(-1)]

Timed out.

$$\int \cot^{-1}(\coth(a+bx))\,dx = \int \mathrm{acot}(\coth(a+b\,x))\,dx$$

input
```
int(acot(coth(a + b*x)),x)
```

output
```
int(acot(coth(a + b*x)), x)
```

3.204 $\int \frac{\cot^{-1}(\coth(a+bx))}{e+fx}\,dx$

3.204.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\cot^{-1}(\coth(a+bx))}{e+fx}\,dx = \text{Int}\left(\frac{\cot^{-1}(\coth(a+bx))}{e+fx},x\right)$$

output
```
CannotIntegrate(arccot(coth(b*x+a))/(f*x+e),x)
```

3.204.2 Mathematica [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(\coth(a+bx))}{e+fx}\,dx = \int \frac{\cot^{-1}(\coth(a+bx))}{e+fx}\,dx$$

input
```
Integrate[ArcCot[Coth[a + b*x]]/(e + f*x),x]
```

output
```
Integrate[ArcCot[Coth[a + b*x]]/(e + f*x), x]
```

3.204.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(\coth(a+bx))}{e+fx} \, dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{\cot^{-1}(\coth(a+bx))}{e+fx}dx$$

input
```
Int[ArcCot[Coth[a + b*x]]/(e + f*x),x]
```

output
```
$Aborted
```

3.204.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.204.4 Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arccot}\left(\coth \left(bx+a\right)\right)}{fx+e}dx$$

input
```
int(arccot(coth(b*x+a))/(f*x+e),x)
```

output
```
int(arccot(coth(b*x+a))/(f*x+e),x)
```

3.204.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(\coth(a+bx))}{e+fx}\,dx = \int \frac{\operatorname{arccot}\left(\coth\left(bx+a\right)\right)}{fx+e}\,dx$$

input
```
integrate(arccot(coth(b*x+a))/(f*x+e),x, algorithm="fricas")
```

output
```
integral(arccot(coth(b*x + a))/(f*x + e), x)
```

3.204.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(\coth(a+bx))}{e+fx}\,dx = \text{Timed out}$$

input
```
integrate(acot(coth(b*x+a))/(f*x+e),x)
```

output
```
Timed out
```

3.204.7 Maxima [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(\coth(a+bx))}{e+fx}\,dx = \int \frac{\operatorname{arccot}\left(\coth\left(bx+a\right)\right)}{fx+e}\,dx$$

input
```
integrate(arccot(coth(b*x+a))/(f*x+e),x, algorithm="maxima")
```

output
```
integrate(arccot(coth(b*x + a))/(f*x + e), x)
```

3.204.8 Giac [N/A]

Not integrable

Time = 105.03 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.20

$$\int \frac{\cot^{-1}(\coth(a+bx))}{e+fx}\,dx = \int \frac{\operatorname{arccot}\left(\coth\left(bx+a\right)\right)}{fx+e}\,dx$$

input
```
integrate(arccot(coth(b*x+a))/(f*x+e),x, algorithm="giac")
```

output
```
sage0*x
```

3.204.9 Mupad [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(\coth(a+bx))}{e+fx}\,dx = \int \frac{\mathrm{acot}(\coth(a+b\,x))}{e+f\,x}\,dx$$

input
```
int(acot(coth(a + b*x))/(e + f*x),x)
```

output
```
int(acot(coth(a + b*x))/(e + f*x), x)
```

3.205 $\int x^2 \cot^{-1}(c+d\coth(a+bx))\,dx$

3.205.1 Optimal result

Integrand size = 15, antiderivative size = 351

$$\begin{aligned}\int x^2\cot^{-1}(c+d\coth(a+bx))\,dx &= \frac{1}{3}x^3\cot^{-1}(c+d\coth(a+bx))\\
&-\frac{1}{6}ix^3\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)\\
&+\frac{1}{6}ix^3\log\left(1-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)\\
&-\frac{ix^2\operatorname{PolyLog}\left(2,\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b}\\
&+\frac{ix^2\operatorname{PolyLog}\left(2,\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b}\\
&+\frac{ix\operatorname{PolyLog}\left(3,\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b^2}\\
&-\frac{ix\operatorname{PolyLog}\left(3,\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b^2}\\
&-\frac{i\operatorname{PolyLog}\left(4,\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{8b^3}\\
&+\frac{i\operatorname{PolyLog}\left(4,\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{8b^3}\end{aligned}$$

output
```
1/3*x^3*arccot(c+d*coth(b*x+a))-1/6*I*x^3*ln(1-(I-c-d)*exp(2*b*x+2*a)/(I-c
+d))+1/6*I*x^3*ln(1-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))-1/4*I*x^2*polylog(2,(I
-c-d)*exp(2*b*x+2*a)/(I-c+d))/b+1/4*I*x^2*polylog(2,(I+c+d)*exp(2*b*x+2*a)
/(I+c-d))/b+1/4*I*x*polylog(3,(I-c-d)*exp(2*b*x+2*a)/(I-c+d))/b^2-1/4*I*x*
polylog(3,(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^2-1/8*I*polylog(4,(I-c-d)*exp(
2*b*x+2*a)/(I-c+d))/b^3+1/8*I*polylog(4,(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^
3
```

3.205.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.26

$$\int x^2\cot^{-1}(c+d\coth(a+bx))\,dx = \frac{1}{3}x^3\cot^{-1}(c+d\coth(a+bx)) - \frac{d\left(4b^3x^3\log\left(1-\frac{(1+(c+d)^2)e^{2(a+bx)}}{1+c^2-d^2+2\sqrt{-d^2}}\right)-4b^3x^3\log\left(1+\frac{(1+(c+d)^2)e^{2(a+bx)}}{-1-c^2+d^2+2\sqrt{-d^2}}\right)+6b^2x^2\operatorname{PolyLog}\left(2,\frac{(1+c^2+2cd+d^2)}{1+c^2-d^2+2}\right.\right.}{}$$

input
```
Integrate[x^2*ArcCot[c + d*Coth[a + b*x]],x]
```

output
```
(x^3*ArcCot[c + d*Coth[a + b*x]])/3 - (d*(4*b^3*x^3*Log[1 - ((1 + (c + d)^
2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2])] - 4*b^3*x^3*Log[1 + ((
1 + (c + d)^2)*E^(2*(a + b*x)))/(-1 - c^2 + d^2 + 2*Sqrt[-d^2])] + 6*b^2*x
^2*PolyLog[2, ((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2
*Sqrt[-d^2])] - 6*b^2*x^2*PolyLog[2, -(((1 + (c + d)^2)*E^(2*(a + b*x)))/(
-1 - c^2 + d^2 + 2*Sqrt[-d^2]))] - 6*b*x*PolyLog[3, ((1 + c^2 + 2*c*d + d^
2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2])] + 6*b*x*PolyLog[3, -((
(1 + (c + d)^2)*E^(2*(a + b*x)))/(-1 - c^2 + d^2 + 2*Sqrt[-d^2]))] - 3*Pol
yLog[4, ((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 - 2*Sqrt[
-d^2])] + 3*PolyLog[4, ((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(1 + c^2
- d^2 + 2*Sqrt[-d^2])]))/(24*b^3*Sqrt[-d^2])
```

3.205.3 Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5725, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cot^{-1}(d\coth(a+bx)+c)\,dx$$

$$\Big\downarrow \; 5725$$

$$-\frac{1}{3}b(1+i(c+d))\int \frac{e^{2a+2bx}x^3}{-c-(-c-d+i)e^{2a+2bx}+d+i}dx+\frac{1}{3}b(1-i(c+d))\int \frac{e^{2a+2bx}x^3}{c-(c+d+i)e^{2a+2bx}-d+i}dx+\frac{1}{3}x^3\cot^{-1}(d\coth(a+bx)+c)$$

$$\Big\downarrow \; 2620$$

$$-\frac{1}{3}b(1+i(c+d))\left(\frac{3\int x^2\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)dx}{2b(-c-d+i)}-\frac{x^3\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}\right)+\frac{1}{3}b(1-i(c+d))\left(\frac{3\int x^2\log\left(1-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)dx}{2b(c+d+i)}-\frac{x^3\log\left(1-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}\right)+\frac{1}{3}x^3\cot^{-1}(d\coth(a+bx)+c)$$

$$\Big\downarrow \; 3011$$

$$-\frac{1}{3}b(1+i(c+d))\left(\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b}\right)}{2b(-c-d+i)}-\frac{x^3\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}\right)+\frac{1}{3}b(1-i(c+d))\left(\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b}\right)}{2b(c+d+i)}-\frac{x^3\log\left(1-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}\right)+\frac{1}{3}x^3\cot^{-1}(d\coth(a+bx)+c)$$

$\Big\downarrow$ 7163

$$-\frac{1}{3}b(1+i(c+d))\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b}-\frac{\int \text{PolyLog}\left(3,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}\left(2,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b}\right)}{2b(-c-d+i)}-\frac{x^3\log\left(1-\frac{(-c-\ldots}{\ldots}\right)}{2b(-c-\ldots}\right.$$

$$\frac{1}{3}b(1-i(c+d))\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b}-\frac{\int \text{PolyLog}\left(3,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}\left(2,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b}\right)}{2b(c+d+i)}-\frac{x^3\log\left(1-\frac{(c+d+i)e^{2\ldots}}{c-d+\ldots}\right)}{2b(c+d+i)}\right.$$

$$\frac{1}{3}x^3\cot^{-1}(d\coth(a+bx)+c)$$

$\Big\downarrow$ 2720

$$-\frac{1}{3}b(1+i(c+d))\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(3,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)de^{2a+2bx}}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b}\right)}{2b(-c-d+i)}-\frac{x^3\,\text{l}\ldots}{\ldots}\right.$$

$$\frac{1}{3}b(1-i(c+d))\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(3,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)de^{2a+2bx}}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b}\right)}{2b(c+d+i)}-\frac{x^3\log\left(\ldots\right.}{2\ldots}\right.$$

$$\frac{1}{3}x^3\cot^{-1}(d\coth(a+bx)+c)$$

$\Big\downarrow$ 7143

3.205. $\int x^2\cot^{-1}(c+d\coth(a+bx))\,dx$

$$-\frac{1}{3}b(1+i(c+d))\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b}-\frac{\text{PolyLog}\left(4,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b}\right)}{2b(-c-d+i)}-\frac{x^3\log\left(1-\frac{(-c-d+i}{-c+}\right.}{2b(-c-d+}\right.$$

$$\frac{1}{3}b(1-i(c+d))\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b}-\frac{\text{PolyLog}\left(4,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b}\right)}{2b(c+d+i)}-\frac{x^3\log\left(1-\frac{(c+d+i)e^{2a+2}}{c-d+i}\right.}{2b(c+d+i)}\right.$$

$$\frac{1}{3}x^3\cot^{-1}(d\coth(a+bx)+c)$$

input
```
Int[x^2*ArcCot[c + d*Coth[a + b*x]],x]
```

output
```
(x^3*ArcCot[c + d*Coth[a + b*x]])/3 - (b*(1 + I*(c + d))*(-1/2*(x^3*Log[1
- ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/(b*(I - c - d)) + (3*(-1/2*(
x^2*PolyLog[2, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/b + ((x*PolyLog
[3, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/(2*b) - PolyLog[4, ((I - c
 - d)*E^(2*a + 2*b*x))/(I - c + d)]/(4*b^2))/b))/(2*b*(I - c - d))))/3 + (
b*(1 - I*(c + d))*(-1/2*(x^3*Log[1 - ((I + c + d)*E^(2*a + 2*b*x))/(I + c
- d)])/(b*(I + c + d)) + (3*(-1/2*(x^2*PolyLog[2, ((I + c + d)*E^(2*a + 2*
b*x))/(I + c - d)])/b + ((x*PolyLog[3, ((I + c + d)*E^(2*a + 2*b*x))/(I +
c - d)])/(2*b) - PolyLog[4, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)]/(4*
b^2))/b))/(2*b*(I + c + d))))/3
```

3.205. $\int x^2\cot^{-1}(c+d\coth(a+bx))\,dx$

3.205.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5725
```
Int[ArcCot[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Coth[a + b*x]]/(f*(m
+ 1))), x] + (Simp[I*b*((I - c - d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E
^(2*a + 2*b*x)/(I - c + d - (I - c - d)*E^(2*a + 2*b*x))), x], x] - Simp[I*
b*((I + c + d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(I + c
 - d - (I + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f},
x] && IGtQ[m, 0] && NeQ[(c - d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.205.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 39.01 (sec) , antiderivative size = 6844, normalized size of antiderivative = 19.50

method	result	size
risch	Expression too large to display	6844

input
```
int(x^2*arccot(c+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.205.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1269 vs. 2(259) = 518.

Time = 0.36 (sec) , antiderivative size = 1269, normalized size of antiderivative = 3.62

$$\int x^2 \cot^{-1}(c + d\coth(a + bx))\, dx = \text{Too large to display}$$

input
```
integrate(x^2*arccot(c+d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/6*(2*b^3*x^3*arctan(sinh(b*x + a)/(d*cosh(b*x + a) + c*sinh(b*x + a))) -
 3*I*b^2*x^2*dilog(sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(
cosh(b*x + a) + sinh(b*x + a))) - 3*I*b^2*x^2*dilog(-sqrt((c^2 - d^2 + 2*I
*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*I*b^
2*x^2*dilog(sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*
x + a) + sinh(b*x + a))) + 3*I*b^2*x^2*dilog(-sqrt((c^2 - d^2 - 2*I*d + 1)
/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + I*a^3*log(2*(
c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x
+ a) + 2*(c^2 - d^2 - 2*I*d + 1)*sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d
 + d^2 + 1))) + I*a^3*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2
 + 2*c*d + d^2 + 1)*sinh(b*x + a) - 2*(c^2 - d^2 - 2*I*d + 1)*sqrt((c^2 -
d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) - I*a^3*log(2*(c^2 + 2*c*d + d^
2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 -
d^2 + 2*I*d + 1)*sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) -
I*a^3*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 +
 1)*sinh(b*x + a) - 2*(c^2 - d^2 + 2*I*d + 1)*sqrt((c^2 - d^2 - 2*I*d + 1)
/(c^2 - 2*c*d + d^2 + 1))) + 6*I*b*x*polylog(3, sqrt((c^2 - d^2 + 2*I*d +
1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 6*I*b*x*pol
ylog(3, -sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x +
 a) + sinh(b*x + a))) - 6*I*b*x*polylog(3, sqrt((c^2 - d^2 - 2*I*d + 1)...
```

3.205.6 Sympy [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(c + d\coth(a + bx))\, dx = \text{Timed out}$$

input
```
integrate(x**2*acot(c+d*coth(b*x+a)),x)
```

output
```
Timed out
```

3.205.7 Maxima [F]

$$\int x^2 \cot^{-1}(c + d\coth(a + bx))\, dx = \int x^2 \operatorname{arccot}\left(d\coth\left(bx + a\right) + c\right)\, dx$$

input
```
integrate(x^2*arccot(c+d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/3*x^3*arctan2(e^(2*b*x + 2*a) - 1, (c*e^(2*a) + d*e^(2*a))*e^(2*b*x) - c
 + d) - 4*b*d*integrate(1/3*x^3*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*
e^(4*a) + 2*c*d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) - 2*(c^2*e^(2*a
) - d^2*e^(2*a) + e^(2*a))*e^(2*b*x) + 1), x)
```

3.205.8 Giac [F]

$$\int x^2 \cot^{-1}(c + d\coth(a + bx))\, dx = \int x^2 \operatorname{arccot}\left(d\coth\left(bx + a\right) + c\right)\, dx$$

input
```
integrate(x^2*arccot(c+d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arccot(d*coth(b*x + a) + c), x)
```

3.205.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(c + d\coth(a + bx))\, dx = \int x^2\, \mathrm{acot}(c + d\coth(a + b\,x))\, dx$$

input
```
int(x^2*acot(c + d*coth(a + b*x)),x)
```

output
```
int(x^2*acot(c + d*coth(a + b*x)), x)
```

3.206 $\int x \cot^{-1}(c + d \coth(a + bx)) \, dx$

3.206.1 Optimal result

Integrand size = 13, antiderivative size = 265

$$\begin{aligned}\int x \cot^{-1}(c + d \coth(a + bx)) \, dx = & \frac{1}{2} x^2 \cot^{-1}(c + d \coth(a + bx)) \\ & - \frac{1}{4} i x^2 \log\left(1 - \frac{(i - c - d) e^{2a+2bx}}{i - c + d}\right) \\ & + \frac{1}{4} i x^2 \log\left(1 - \frac{(i + c + d) e^{2a+2bx}}{i + c - d}\right) \\ & - \frac{i x \operatorname{PolyLog}\left(2, \frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} \\ & + \frac{i x \operatorname{PolyLog}\left(2, \frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b} \\ & + \frac{i \operatorname{PolyLog}\left(3, \frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{8b^2} \\ & - \frac{i \operatorname{PolyLog}\left(3, \frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{8b^2}\end{aligned}$$

output
```
1/2*x^2*arccot(c+d*coth(b*x+a))-1/4*I*x^2*ln(1-(I-c-d)*exp(2*b*x+2*a)/(I-c
+d))+1/4*I*x^2*ln(1-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))-1/4*I*x*polylog(2,(I-c
-d)*exp(2*b*x+2*a)/(I-c+d))/b+1/4*I*x*polylog(2,(I+c+d)*exp(2*b*x+2*a)/(I+
c-d))/b+1/8*I*polylog(3,(I-c-d)*exp(2*b*x+2*a)/(I-c+d))/b^2-1/8*I*polylog(
3,(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^2
```

3.206.2 Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.26

$$\int x\cot^{-1}(c+d\coth(a+bx))\,dx=\frac{1}{2}x^2\cot^{-1}(c+d\coth(a+bx))$$
$$-\frac{d\left(2b^2x^2\log\left(1-\frac{(1+(c+d)^2)e^{2(a+bx)}}{1+c^2-d^2+2\sqrt{-d^2}}\right)-2b^2x^2\log\left(1+\frac{(1+(c+d)^2)e^{2(a+bx)}}{-1-c^2+d^2+2\sqrt{-d^2}}\right)+2bx\,\text{PolyLog}\left(2,\frac{(1+c^2+2cd+d^2)e^{2}}{1+c^2-d^2+2\sqrt{-}}\right.\right.}{}$$

input
```
Integrate[x*ArcCot[c + d*Coth[a + b*x]],x]
```

output
```
(x^2*ArcCot[c + d*Coth[a + b*x]])/2 - (d*(2*b^2*x^2*Log[1 - ((1 + (c + d)^
2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2])] - 2*b^2*x^2*Log[1 + ((
1 + (c + d)^2)*E^(2*(a + b*x)))/(-1 - c^2 + d^2 + 2*Sqrt[-d^2])] + 2*b*x*P
olyLog[2, ((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqr
t[-d^2])] - 2*b*x*PolyLog[2, -(((1 + (c + d)^2)*E^(2*(a + b*x)))/(-1 - c^2
 + d^2 + 2*Sqrt[-d^2]))] + PolyLog[3, ((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b
*x)))/(1 + c^2 - d^2 - 2*Sqrt[-d^2])] - PolyLog[3, ((1 + c^2 + 2*c*d + d^2
)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2])]))/(8*b^2*Sqrt[-d^2])
```

3.206.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.28, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5725, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cot^{-1}(d\coth(a+bx)+c)\,dx$$

$$\Big\downarrow \text{5725}$$

$$-\frac{1}{2}b(1+i(c+d))\int\frac{e^{2a+2bx}x^2}{-c-(-c-d+i)e^{2a+2bx}+d+i}dx+\frac{1}{2}b(1-i(c+d))\int\frac{e^{2a+2bx}x^2}{c-(c+d+i)e^{2a+2bx}-d+i}dx+\frac{1}{2}x^2\cot^{-1}(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{2620}$$

$$-\frac{1}{2}b(1+i(c+d))\left(\frac{\int x\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)dx}{b(-c-d+i)}-\frac{x^2\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}\right)+\frac{1}{2}b(1-i(c+d))\left(\frac{\int x\log\left(1-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)dx}{b(c+d+i)}-\frac{x^2\log\left(1-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}\right)+\frac{1}{2}x^2\cot^{-1}(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{3011}$$

$$-\frac{1}{2}b(1+i(c+d))\left(\frac{\frac{\int \text{PolyLog}\left(2,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)dx}{2b}-\frac{x\,\text{PolyLog}\left(2,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b}}{b(-c-d+i)}-\frac{x^2\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}\right)+\frac{1}{2}b(1-i(c+d))\left(\frac{\frac{\int \text{PolyLog}\left(2,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)dx}{2b}-\frac{x\,\text{PolyLog}\left(2,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b}}{b(c+d+i)}-\frac{x^2\log\left(1-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}\right)+\frac{1}{2}x^2\cot^{-1}(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{2720}$$

$$-\frac{1}{2}b(1+i(c+d))\left(\frac{\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(2,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)de^{2a+2bx}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b}}{b(-c-d+i)}-\frac{x^2\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}\right)+\frac{1}{2}b(1-i(c+d))\left(\frac{\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(2,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)de^{2a+2bx}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b}}{b(c+d+i)}-\frac{x^2\log\left(1-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}\right)+\frac{1}{2}x^2\cot^{-1}(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{7143}$$

3.206. $\int x\cot^{-1}(c+d\coth(a+bx))\,dx$

$$-\frac{1}{2}b(1+i(c+d))\left(\frac{\frac{\text{PolyLog}\left(3,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b^2}-\frac{x\,\text{PolyLog}\left(2,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b}}{b(-c-d+i)}-\frac{x^2\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}\right)+\frac{1}{2}b(1-i(c+d))\left(\frac{\frac{\text{PolyLog}\left(3,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b^2}-\frac{x\,\text{PolyLog}\left(2,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b}}{b(c+d+i)}-\frac{x^2\log\left(1-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}\right)+\frac{1}{2}x^2\cot^{-1}(d\coth(a+bx)+c)$$

input
```
Int[x*ArcCot[c + d*Coth[a + b*x]],x]
```

output
```
(x^2*ArcCot[c + d*Coth[a + b*x]])/2 - (b*(1 + I*(c + d))*(-1/2*(x^2*Log[1
- ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/(b*(I - c - d)) + (-1/2*(x*P
olyLog[2, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/b + PolyLog[3, ((I -
 c - d)*E^(2*a + 2*b*x))/(I - c + d)]/(4*b^2))/(b*(I - c - d))))/2 + (b*(1
 - I*(c + d))*(-1/2*(x^2*Log[1 - ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)
])/(b*(I + c + d)) + (-1/2*(x*PolyLog[2, ((I + c + d)*E^(2*a + 2*b*x))/(I
+ c - d)])/b + PolyLog[3, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)]/(4*b^
2))/(b*(I + c + d))))/2
```

3.206.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5725
```
Int[ArcCot[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Coth[a + b*x]]/(f*(m
+ 1))), x] + (Simp[I*b*((I - c - d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E
^(2*a + 2*b*x)/(I - c + d - (I - c - d)*E^(2*a + 2*b*x))), x], x] - Simp[I*
b*((I + c + d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(I + c
 - d - (I + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f},
x] && IGtQ[m, 0] && NeQ[(c - d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.206.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.40 (sec) , antiderivative size = 6494, normalized size of antiderivative = 24.51

method	result	size
risch	Expression too large to display	6494

input
```
int(x*arccot(c+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.206.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1051 vs. $2(195) = 390$.

Time = 0.33 (sec) , antiderivative size = 1051, normalized size of antiderivative = 3.97

$$\int x \cot^{-1}(c + d\coth(a + bx))\, dx = \text{Too large to display}$$

input
```
integrate(x*arccot(c+d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/4*(2*b^2*x^2*arctan(sinh(b*x + a)/(d*cosh(b*x + a) + c*sinh(b*x + a))) -
 2*I*b*x*dilog(sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh
(b*x + a) + sinh(b*x + a))) - 2*I*b*x*dilog(-sqrt((c^2 - d^2 + 2*I*d + 1)/
(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*I*b*x*dilog(
sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sin
h(b*x + a))) + 2*I*b*x*dilog(-sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d +
d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - I*a^2*log(2*(c^2 + 2*c*d + d^
2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 -
d^2 - 2*I*d + 1)*sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) -
I*a^2*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 +
 1)*sinh(b*x + a) - 2*(c^2 - d^2 - 2*I*d + 1)*sqrt((c^2 - d^2 + 2*I*d + 1)
/(c^2 - 2*c*d + d^2 + 1))) + I*a^2*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x
+ a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 - d^2 + 2*I*d + 1)
*sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) + I*a^2*log(2*(c^2
 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a
) - 2*(c^2 - d^2 + 2*I*d + 1)*sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d +
d^2 + 1))) + (-I*b^2*x^2 + I*a^2)*log(sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 -
2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^2*x^2 + I*a
^2)*log(-sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x +
 a) + sinh(b*x + a)) + 1) + (I*b^2*x^2 - I*a^2)*log(sqrt((c^2 - d^2 - 2...
```

3.206.6 Sympy [F(-1)]

Timed out.

$$\int x \cot^{-1}(c + d\coth(a + bx))\, dx = \text{Timed out}$$

input
```
integrate(x*acot(c+d*coth(b*x+a)),x)
```

output
```
Timed out
```

3.206.7 Maxima [F]

$$\int x\cot^{-1}(c+d\coth(a+bx))\,dx=\int x\,\text{arccot}\left(d\coth\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x*arccot(c+d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/2*x^2*arctan2(e^(2*b*x + 2*a) - 1, (c*e^(2*a) + d*e^(2*a))*e^(2*b*x) - c
 + d) - 2*b*d*integrate(x^2*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^(4
*a) + 2*c*d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) - 2*(c^2*e^(2*a) -
d^2*e^(2*a) + e^(2*a))*e^(2*b*x) + 1), x)
```

3.206.8 Giac [F]

$$\int x\cot^{-1}(c+d\coth(a+bx))\,dx=\int x\,\text{arccot}\left(d\coth\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x*arccot(c+d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arccot(d*coth(b*x + a) + c), x)
```

3.206.9 Mupad [F(-1)]

Timed out.

$$\int x\cot^{-1}(c+d\coth(a+bx))\,dx=\int x\,\text{acot}(c+d\coth(a+b\,x))\,dx$$

input
```
int(x*acot(c + d*coth(a + b*x)),x)
```

output
```
int(x*acot(c + d*coth(a + b*x)), x)
```

3.207 $\int \cot^{-1}(c+d\coth(a+bx))\,dx$

3.207.1 Optimal result

Integrand size = 11, antiderivative size = 174

$$\int \cot^{-1}(c+d\coth(a+bx))\,dx = x\cot^{-1}(c+d\coth(a+bx)) - \frac{1}{2}ix\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) + \frac{1}{2}ix\log\left(1-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) - \frac{i\,\mathrm{PolyLog}\left(2,\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} + \frac{i\,\mathrm{PolyLog}\left(2,\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b}$$

output
```
x*arccot(c+d*coth(b*x+a))-1/2*I*x*ln(1-(I-c-d)*exp(2*b*x+2*a)/(I-c+d))+1/2
*I*x*ln(1-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))-1/4*I*polylog(2,(I-c-d)*exp(2*b*
x+2*a)/(I-c+d))/b+1/4*I*polylog(2,(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b
```

3.207.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.65

$$\int \cot^{-1}(c+d\coth(a+bx))\,dx = x\cot^{-1}(c+d\coth(a+bx))$$
$$-\frac{4a\sqrt{-d^2}\arctan\left(\frac{1+c^2-d^2-(1+c^2+2cd+d^2)e^{2(a+bx)}}{2d}\right)+2d(a+bx)\log\left(1-\frac{(1+(c+d)^2)e^{2(a+bx)}}{1+c^2-d^2+2\sqrt{-d^2}}\right)-2d(a+bx)\log}{4b\sqrt{-}}$$

input
```
Integrate[ArcCot[c + d*Coth[a + b*x]],x]
```

output
```
x*ArcCot[c + d*Coth[a + b*x]] - (4*a*Sqrt[-d^2]*ArcTan[(1 + c^2 - d^2 - (1
 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(2*d)] + 2*d*(a + b*x)*Log[1 - ((1
+ (c + d)^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2])] - 2*d*(a + b
*x)*Log[1 + ((1 + (c + d)^2)*E^(2*(a + b*x)))/(-1 - c^2 + d^2 + 2*Sqrt[-d^
2])] + d*PolyLog[2, ((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(1 + c^2 - d
^2 + 2*Sqrt[-d^2])] - d*PolyLog[2, -(((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*
x)))/(-1 - c^2 + d^2 + 2*Sqrt[-d^2]))])/(4*b*Sqrt[-d^2])
```

3.207.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.36, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5717, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}(d\coth(a+bx)+c)\,dx$$

$$\Big\downarrow \text{5717}$$

$$-b(1+i(c+d))\int \frac{e^{2a+2bx}x}{-c-(-c-d+i)e^{2a+2bx}+d+i}dx+b(1-i(c+d))\int \frac{e^{2a+2bx}x}{c-(c+d+i)e^{2a+2bx}-d+i}dx+x\cot^{-1}(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{2620}$$

$$-b(1+i(c+d))\left(\frac{\int \log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)dx}{2b(-c-d+i)}-\frac{x\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}\right)+b(1-i(c+d))\left(\frac{\int \log\left(1-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)dx}{2b(c+d+i)}-\frac{x\log\left(1-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}\right)+x\cot^{-1}(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{2715}$$

$$-b(1+i(c+d))\left(\frac{\int e^{-2a-2bx}\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)de^{2a+2bx}}{4b^2(-c-d+i)}-\frac{x\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}\right)+b(1-i(c+d))\left(\frac{\int e^{-2a-2bx}\log\left(1-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)de^{2a+2bx}}{4b^2(c+d+i)}-\frac{x\log\left(1-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}\right)+x\cot^{-1}(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{2838}$$

$$-b(1+i(c+d))\left(-\frac{\text{PolyLog}\left(2,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b^2(-c-d+i)}-\frac{x\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}\right)+b(1-i(c+d))\left(-\frac{\text{PolyLog}\left(2,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b^2(c+d+i)}-\frac{x\log\left(1-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}\right)+x\cot^{-1}(d\coth(a+bx)+c)$$

input
```
Int[ArcCot[c + d*Coth[a + b*x]],x]
```

output
```
x*ArcCot[c + d*Coth[a + b*x]] - b*(1 + I*(c + d))*(-1/2*(x*Log[1 - ((I - c
 - d)*E^(2*a + 2*b*x))/(I - c + d)])/(b*(I - c - d)) - PolyLog[2, ((I - c
- d)*E^(2*a + 2*b*x))/(I - c + d)]/(4*b^2*(I - c - d))) + b*(1 - I*(c + d)
)*(-1/2*(x*Log[1 - ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/(b*(I + c +
 d)) - PolyLog[2, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)]/(4*b^2*(I + c
 + d)))
```

3.207. $\int \cot^{-1}(c+d\coth(a+bx))\,dx$

3.207.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5717
```
Int[ArcCot[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Cot[c + d*Coth[a + b*x]], x] + (Simp[I*b*(I - c - d)   Int[x*(E^(2*a + 2*b*
x)/(I - c + d - (I - c - d)*E^(2*a + 2*b*x))), x], x] - Simp[I*b*(I + c + d
)   Int[x*(E^(2*a + 2*b*x)/(I + c - d - (I + c + d)*E^(2*a + 2*b*x))), x],
x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - d)^2, -1]
```

3.207.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(150) = 300$.

Time = 2.92 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.02

method	result
derivativedivides	$-\frac{\operatorname{arccot}(c+d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}+\frac{\operatorname{arccot}(c+d\coth(bx+a))d\ln(-d\coth(bx+a)-d)}{2}-\frac{d^2\left(\frac{i\ln(-d\coth(bx+a)+d)\ln(}{2}\right.}{}$
default	$-\frac{\operatorname{arccot}(c+d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}+\frac{\operatorname{arccot}(c+d\coth(bx+a))d\ln(-d\coth(bx+a)-d)}{2}-\frac{d^2\left(\frac{i\ln(-d\coth(bx+a)+d)\ln(}{2}\right.}{}$
risch	Expression too large to display

input
```
int(arccot(c+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/d*(-1/2*arccot(c+d*coth(b*x+a))*d*ln(-d*coth(b*x+a)+d)+1/2*arccot(c+d*
coth(b*x+a))*d*ln(-d*coth(b*x+a)-d)-1/2*d^2*(1/d*(1/2*I*ln(-d*coth(b*x+a)+
d)*ln((I+d*coth(b*x+a)+c)/(I+c+d))-1/2*I*ln(-d*coth(b*x+a)+d)*ln((I-d*coth
(b*x+a)-c)/(I-c-d))+1/2*I*dilog((I+d*coth(b*x+a)+c)/(I+c+d))-1/2*I*dilog((
I-d*coth(b*x+a)-c)/(I-c-d)))-1/d*(1/2*I*ln(-d*coth(b*x+a)-d)*ln((I+d*coth(
b*x+a)+c)/(I+c-d))-1/2*I*ln(-d*coth(b*x+a)-d)*ln((I-d*coth(b*x+a)-c)/(I-c+
d))+1/2*I*dilog((I+d*coth(b*x+a)+c)/(I+c-d))-1/2*I*dilog((I-d*coth(b*x+a)-
c)/(I-c+d))))
```

3.207.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 813 vs. 2(128) = 256.

Time = 0.39 (sec) , antiderivative size = 813, normalized size of antiderivative = 4.67

$$\int \cot^{-1}(c+d\coth(a+bx))\,dx$$
$$= \frac{2\,bx\arctan\left(\frac{\sinh(bx+a)}{d\cosh(bx+a)+c\sinh(bx+a)}\right)+i\,a\log\left(2\left(c^2+2\,cd+d^2+1\right)\cosh\left(bx+a\right)+2\left(c^2+2\,cd+d^2+1\right)\right.}{}$$

input
```
integrate(arccot(c+d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(2*b*x*arctan(sinh(b*x + a)/(d*cosh(b*x + a) + c*sinh(b*x + a))) + I*a
*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*s
inh(b*x + a) + 2*(c^2 - d^2 - 2*I*d + 1)*sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2
 - 2*c*d + d^2 + 1))) + I*a*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) +
2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) - 2*(c^2 - d^2 - 2*I*d + 1)*sqrt((
c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) - I*a*log(2*(c^2 + 2*c*d
+ d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^
2 - d^2 + 2*I*d + 1)*sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))
) - I*a*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2
 + 1)*sinh(b*x + a) - 2*(c^2 - d^2 + 2*I*d + 1)*sqrt((c^2 - d^2 - 2*I*d +
1)/(c^2 - 2*c*d + d^2 + 1))) + (-I*b*x - I*a)*log(sqrt((c^2 - d^2 + 2*I*d
+ 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b
*x - I*a)*log(-sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh
(b*x + a) + sinh(b*x + a)) + 1) + (I*b*x + I*a)*log(sqrt((c^2 - d^2 - 2*I*
d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*
b*x + I*a)*log(-sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cos
h(b*x + a) + sinh(b*x + a)) + 1) - I*dilog(sqrt((c^2 - d^2 + 2*I*d + 1)/(c
^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - I*dilog(-sqrt((c
^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x +
 a))) + I*dilog(sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(...
```

3.207.6 Sympy [F]

$$\int \cot^{-1}(c+d\coth(a+bx))\,dx=\int \operatorname{acot}\left(c+d\coth\left(a+bx\right)\right)dx$$

input
```
integrate(acot(c+d*coth(b*x+a)),x)
```

output
```
Integral(acot(c + d*coth(a + b*x)), x)
```

3.207.7 Maxima [F]

$$\int \cot^{-1}(c+d\coth(a+bx))\,dx=\int \operatorname{arccot}\left(d\coth\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(arccot(c+d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
-4*b*d*integrate(x*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^(4*a) + 2*c
*d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) - 2*(c^2*e^(2*a) - d^2*e^(2*
a) + e^(2*a))*e^(2*b*x) + 1), x) + x*arctan2(e^(2*b*x + 2*a) - 1, (c*e^(2*
a) + d*e^(2*a))*e^(2*b*x) - c + d)
```

3.207.8 Giac [F]

$$\int \cot^{-1}(c+d\coth(a+bx))\,dx=\int \operatorname{arccot}\left(d\coth\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(arccot(c+d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccot(d*coth(b*x + a) + c), x)
```

3.207.9 Mupad [F(-1)]

Timed out.

$$\int \cot^{-1}(c+d\coth(a+bx))\,dx=\int \operatorname{acot}(c+d\coth(a+b\,x))\,dx$$

input
```
int(acot(c + d*coth(a + b*x)),x)
```

output
```
int(acot(c + d*coth(a + b*x)), x)
```

3.208 $\int \frac{\cot^{-1}(c+d\coth(a+bx))}{x}\,dx$

3.208.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\cot^{-1}(c+d\coth(a+bx))}{x}\,dx = \text{Int}\left(\frac{\cot^{-1}(c+d\coth(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(arccot(c+d*coth(b*x+a))/x,x)
```

3.208.2 Mathematica [N/A]

Not integrable

Time = 3.84 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(c+d\coth(a+bx))}{x}\,dx = \int \frac{\cot^{-1}(c+d\coth(a+bx))}{x}\,dx$$

input
```
Integrate[ArcCot[c + d*Coth[a + b*x]]/x,x]
```

output
```
Integrate[ArcCot[c + d*Coth[a + b*x]]/x, x]
```

3.208.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(d\coth(a+bx)+c)}{x}\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{\cot^{-1}(d\coth(a+bx)+c)}{x}dx$$

input
```
Int[ArcCot[c + d*Coth[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.208.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.208.4 Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arccot}\left(c+d\coth\left(bx+a\right)\right)}{x}dx$$

input
```
int(arccot(c+d*coth(b*x+a))/x,x)
```

output
```
int(arccot(c+d*coth(b*x+a))/x,x)
```

3.208.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(c+d\coth(a+bx))}{x}\,dx=\int \frac{\operatorname{arccot}\left(d\coth\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccot(c+d*coth(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(arccot(d*coth(b*x + a) + c)/x, x)
```

3.208.6 Sympy [N/A]

Not integrable

Time = 170.89 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{\cot^{-1}(c+d\coth(a+bx))}{x}\,dx=\int \frac{\operatorname{acot}\left(c+d\coth\left(a+bx\right)\right)}{x}\,dx$$

input
```
integrate(acot(c+d*coth(b*x+a))/x,x)
```

output
```
Integral(acot(c + d*coth(a + b*x))/x, x)
```

3.208.7 Maxima [N/A]

Not integrable

Time = 1.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(c+d\coth(a+bx))}{x}\,dx=\int \frac{\operatorname{arccot}\left(d\coth\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccot(c+d*coth(b*x+a))/x,x, algorithm="maxima")
```

output
```
integrate(arccot(d*coth(b*x + a) + c)/x, x)
```

3.208.8 Giac [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(c+d\coth(a+bx))}{x}\,dx=\int \frac{\operatorname{arccot}\left(d\coth\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccot(c+d*coth(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arccot(d*coth(b*x + a) + c)/x, x)
```

3.208.9 Mupad [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}(c+d\coth(a+bx))}{x}\,dx=\int \frac{\mathrm{acot}(c+d\coth(a+b\,x))}{x}\,dx$$

input
```
int(acot(c + d*coth(a + b*x))/x,x)
```

output
```
int(acot(c + d*coth(a + b*x))/x, x)
```

3.209 $\int x^2 \cot^{-1}(c+(i+c)\coth(a+bx))\,dx$

3.209.1 Optimal result

Integrand size = 19, antiderivative size = 142

$$
\begin{aligned}
&\int x^2 \cot^{-1}(c+(i+c)\coth(a+bx))\,dx \\
&= \frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c+(i+c)\coth(a+bx))-\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right) \\
&\quad -\frac{ix^2\operatorname{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}+\frac{ix\operatorname{PolyLog}\left(3,ice^{2a+2bx}\right)}{4b^2}-\frac{i\operatorname{PolyLog}\left(4,ice^{2a+2bx}\right)}{8b^3}
\end{aligned}
$$

output
```
1/12*I*b*x^4+1/3*x^3*arccot(c+(I+c)*coth(b*x+a))-1/6*I*x^3*ln(1-I*c*exp(2*
b*x+2*a))-1/4*I*x^2*polylog(2,I*c*exp(2*b*x+2*a))/b+1/4*I*x*polylog(3,I*c*
exp(2*b*x+2*a))/b^2-1/8*I*polylog(4,I*c*exp(2*b*x+2*a))/b^3
```

3.209.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.94

$$
\begin{aligned}
&\int x^2 \cot^{-1}(c+(i+c)\coth(a+bx))\,dx \\
&= \frac{8b^3x^3\cot^{-1}(c+(i+c)\coth(a+bx))-4ib^3x^3\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right)+6ib^2x^2\operatorname{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right)+6ib}{24b^3}
\end{aligned}
$$

input
```
Integrate[x^2*ArcCot[c + (I + c)*Coth[a + b*x]],x]
```

output
```
(8*b^3*x^3*ArcCot[c + (I + c)*Coth[a + b*x]] - (4*I)*b^3*x^3*Log[1 + I/(c*
E^(2*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, (-I)/(c*E^(2*(a + b*x)))] + (
6*I)*b*x*PolyLog[3, (-I)/(c*E^(2*(a + b*x)))] + (3*I)*PolyLog[4, (-I)/(c*E
^(2*(a + b*x)))])/(24*b^3)
```

3.209.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {5721, 25, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cot^{-1}(c+(c+i)\coth(a+bx))\,dx$$

$$\Big\downarrow \text{5721}$$

$$\frac{1}{3}b\int -\frac{x^3}{e^{2a+2bx}c+i}dx+\frac{1}{3}x^3\cot^{-1}(c+(c+i)\coth(a+bx))$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}x^3\cot^{-1}(c+(c+i)\coth(a+bx))-\frac{1}{3}b\int \frac{x^3}{e^{2a+2bx}c+i}dx$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{3}x^3\cot^{-1}(c+(c+i)\coth(a+bx))-\frac{1}{3}b\left(ic\int \frac{e^{2a+2bx}x^3}{e^{2a+2bx}c+i}dx-\frac{ix^4}{4}\right)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{3}x^3\cot^{-1}(c+(c+i)\coth(a+bx))-\frac{1}{3}b\left(ic\left(\frac{x^3\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{3\int x^2\log\left(1-ice^{2a+2bx}\right)dx}{2bc}\right)-\frac{ix^4}{4}\right)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{3}x^3\cot^{-1}(c+(c+i)\coth(a+bx))-\frac{1}{3}b\left(ic\left(\frac{x^3\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{3\left(\frac{\int x\,\text{PolyLog}(2,ice^{2a+2bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,ice^{2a+2bx})}{2b}\right)}{2bc}\right)-\frac{ix^4}{4}\right)$$

$$\Big\downarrow \text{7163}$$

$$\frac{1}{3}x^3\cot^{-1}(c+(c+i)\coth(a+bx))-\frac{1}{3}b\left(ic\left(\frac{x^3\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{3\left(\frac{\frac{x\operatorname{PolyLog}\left(3,ice^{2a+2bx}\right)}{2b}-\frac{\int\operatorname{PolyLog}\left(3,ice^{2a+2bx}\right)dx}{2b}}{b}-\frac{x^2\operatorname{PolyLog}\left(2,ice^{2a+2bx}\right)}{2b}\right)}{2bc}\right)-\frac{ix^4}{4}\right)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{3}x^3\cot^{-1}(c+(c+i)\coth(a+bx))-\frac{1}{3}b\left(ic\left(\frac{x^3\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{3\left(\frac{\frac{x\operatorname{PolyLog}\left(3,ice^{2a+2bx}\right)}{2b}-\frac{\int e^{-2a-2bx}\operatorname{PolyLog}\left(3,ice^{2a+2bx}\right)de^{2a+2bx}}{4b^2}}{b}-\frac{x^2\operatorname{PolyLog}\left(2,ice^{2a+2bx}\right)}{2b}\right)}{2bc}\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{3}x^3\cot^{-1}(c+(c+i)\coth(a+bx))-\frac{1}{3}b\left(ic\left(\frac{x^3\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{3\left(\frac{\frac{x\operatorname{PolyLog}\left(3,ice^{2a+2bx}\right)}{2b}-\frac{\operatorname{PolyLog}\left(4,ice^{2a+2bx}\right)}{4b^2}}{b}-\frac{x^2\operatorname{PolyLog}\left(2,ice^{2a+2bx}\right)}{2b}\right)}{2bc}\right)-\frac{ix^4}{4}\right)$$

input
```
Int[x^2*ArcCot[c + (I + c)*Coth[a + b*x]],x]
```

output
```
(x^3*ArcCot[c + (I + c)*Coth[a + b*x]])/3 - (b*((-1/4*I)*x^4 + I*c*((x^3*L
og[1 - I*c*E^(2*a + 2*b*x)])/(2*b*c) - (3*(-1/2*(x^2*PolyLog[2, I*c*E^(2*a
 + 2*b*x)])/b + ((x*PolyLog[3, I*c*E^(2*a + 2*b*x)])/(2*b) - PolyLog[4, I*
c*E^(2*a + 2*b*x)]/(4*b^2))/b))/(2*b*c))))/3
```

3.209. $\int x^2\cot^{-1}(c+(i+c)\coth(a+bx))\,dx$

3.209.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5721
```
Int[ArcCot[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Coth[a + b*x]]/(f*(m
+ 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a +
 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.209.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.94 (sec) , antiderivative size = 1404, normalized size of antiderivative = 9.89

method	result	size
risch	Expression too large to display	1404

input
```
int(x^2*arccot(c+(I+c)*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/3*I/b^3*ln(1-I*c*exp(2*b*x+2*a))*a^3+1/12*Pi*(csgn(I/(exp(2*b*x+2*a)-1))
*csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b
*x+2*a)-1))-csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*
b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a
)-1))-csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*
x+2*a)-1))^2+csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2
*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2-csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I
*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^2+csgn(I*(2*I*exp(2*b*x+2*a)
+2*exp(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(
2*b*x+2*a)-1))^2+csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^3-csg
n(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))*csgn((2*exp(2*b*x+2*a)*c+
2*I)/(exp(2*b*x+2*a)-1))^2+csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)
-1))*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))-csgn(I*(2*I*exp(2*b
*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^3+csgn(I*(2*I*exp(2*b*x+2*
a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(
2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2-csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x
+2*a)*c)/(exp(2*b*x+2*a)-1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/
(exp(2*b*x+2*a)-1))-csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*
x+2*a)-1))^3+csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-
1))^2-csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^3+csgn((2*exp(2...
```

3.209.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 292 vs. 2(105) = 210.

Time = 0.26 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.06

$$\int x^2 \cot^{-1}(c + (i + c)\coth(a + bx))\, dx$$
$$= \frac{i\, b^4x^4 + 2i\, b^3x^3 \log\left(\frac{(ce^{(2\,bx+2\,a)}+i)e^{(-2\,bx-2\,a)}}{c+i}\right) - 6i\, b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\sqrt{4i\,c}e^{(bx+a)}\right) - 6i\, b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{4i\,c}e^{(bx+a)}\right) -}{}$$

input
```
integrate(x^2*arccot(c+(I+c)*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(I*b^4*x^4 + 2*I*b^3*x^3*log((c*e^(2*b*x + 2*a) + I)*e^(-2*b*x - 2*a)
/(c + I)) - 6*I*b^2*x^2*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) - 6*I*b^2*x^2*d
ilog(-1/2*sqrt(4*I*c)*e^(b*x + a)) - I*a^4 + 2*I*a^3*log(1/2*(2*c*e^(b*x +
 a) + I*sqrt(4*I*c))/c) + 2*I*a^3*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c)
)/c) + 12*I*b*x*polylog(3, 1/2*sqrt(4*I*c)*e^(b*x + a)) + 12*I*b*x*polylog
(3, -1/2*sqrt(4*I*c)*e^(b*x + a)) - 2*(I*b^3*x^3 + I*a^3)*log(1/2*sqrt(4*I
*c)*e^(b*x + a) + 1) - 2*(I*b^3*x^3 + I*a^3)*log(-1/2*sqrt(4*I*c)*e^(b*x +
 a) + 1) - 12*I*polylog(4, 1/2*sqrt(4*I*c)*e^(b*x + a)) - 12*I*polylog(4,
-1/2*sqrt(4*I*c)*e^(b*x + a)))/b^3
```

3.209.6 Sympy [F(-2)]

Exception generated.

$$\int x^2 \cot^{-1}(c + (i + c)\coth(a + bx))\, dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(x**2*acot(c+(I+c)*coth(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert 2*_t0**4*c**2*exp(4*a)
+ _t0**4*I*c*exp(4*a) + 3*_t0**2*I*c*exp(2*a) - _t0**2*exp(2*a) - 1 of typ
e <class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,exp(a)]
```

3.209.7 Maxima [A] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.91

$$\int x^2 \cot^{-1}(c+(i+c)\coth(a+bx))\,dx = \frac{1}{3}\,x^3 \operatorname{arccot}\left((c+i)\coth\left(bx+a\right)+c\right) - \frac{4}{9}\left(\frac{3\,x^4}{4i\,c-4} - \frac{4\,b^3x^3\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right)+6\,b^2x^2\mathrm{Li}_2\left(i\,ce^{(2\,bx+2\,a)}\right)-6\,bx\mathrm{Li}_3(i\,ce^{(2\,bx+2\,a)})+3\,\mathrm{Li}_4(i\,ce^{(2\,bx+2\,a)})}{-2\,b^4(-i\,c+1)}\right)$$

input
```
integrate(x^2*arccot(c+(I+c)*coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/3*x^3*arccot((c + I)*coth(b*x + a) + c) - 4/9*(3*x^4/(4*I*c - 4) - (4*b^
3*x^3*log(-I*c*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(I*c*e^(2*b*x + 2*a))
 - 6*b*x*polylog(3, I*c*e^(2*b*x + 2*a)) + 3*polylog(4, I*c*e^(2*b*x + 2*a
)))/(b^4*(2*I*c - 2)))*b*(c + I)
```

3.209.8 Giac [F]

$$\int x^2 \cot^{-1}(c+(i+c)\coth(a+bx))\,dx = \int x^2 \operatorname{arccot}\left((c+i)\coth\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x^2*arccot(c+(I+c)*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arccot((c + I)*coth(b*x + a) + c), x)
```

3.209.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(c+(i+c)\coth(a+bx))\,dx = \int x^2\,\mathrm{acot}(c+\coth(a+b\,x)\,\left(c+1\mathrm{i}\right))\,dx$$

input
```
int(x^2*acot(c + coth(a + b*x)*(c + 1i)),x)
```

output
```
int(x^2*acot(c + coth(a + b*x)*(c + 1i)), x)
```

3.210 $\int x \cot^{-1}(c+(i+c)\coth(a+bx))\,dx$

3.210.1 Optimal result

Integrand size = 17, antiderivative size = 113

$$\int x\cot^{-1}(c+(i+c)\coth(a+bx))\,dx=\frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c+(i+c)\coth(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)-\frac{ix\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}+\frac{i\,\text{PolyLog}\left(3,ice^{2a+2bx}\right)}{8b^2}$$

output
```
1/6*I*b*x^3+1/2*x^2*arccot(c+(I+c)*coth(b*x+a))-1/4*I*x^2*ln(1-I*c*exp(2*b
*x+2*a))-1/4*I*x*polylog(2,I*c*exp(2*b*x+2*a))/b+1/8*I*polylog(3,I*c*exp(2
*b*x+2*a))/b^2
```

3.210.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.91

$$\int x\cot^{-1}(c+(i+c)\coth(a+bx))\,dx$$
$$=\frac{2b^2x^2\left(2\cot^{-1}(c+(i+c)\coth(a+bx))-i\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right)\right)+2ibx\,\text{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right)+i\,\text{PolyI}}{8b^2}$$

input
```
Integrate[x*ArcCot[c + (I + c)*Coth[a + b*x]],x]
```

output
```
(2*b^2*x^2*(2*ArcCot[c + (I + c)*Coth[a + b*x]] - I*Log[1 + I/(c*E^(2*(a +
 b*x)))]) + (2*I)*b*x*PolyLog[2, (-I)/(c*E^(2*(a + b*x)))] + I*PolyLog[3,
(-I)/(c*E^(2*(a + b*x)))])/(8*b^2)
```

3.210.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {5721, 25, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \cot^{-1}(c+(c+i)\coth(a+bx))\,dx$$

$$\Big\downarrow \text{5721}$$

$$\frac{1}{2}b\int -\frac{x^2}{e^{2a+2bx}c+i}dx+\frac{1}{2}x^2\cot^{-1}(c+(c+i)\coth(a+bx))$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}x^2\cot^{-1}(c+(c+i)\coth(a+bx))-\frac{1}{2}b\int \frac{x^2}{e^{2a+2bx}c+i}dx$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{2}x^2\cot^{-1}(c+(c+i)\coth(a+bx))-\frac{1}{2}b\left(ic\int \frac{e^{2a+2bx}x^2}{e^{2a+2bx}c+i}dx-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{2}x^2\cot^{-1}(c+(c+i)\coth(a+bx))-\frac{1}{2}b\left(ic\left(\frac{x^2\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{\int x\log\left(1-ice^{2a+2bx}\right)dx}{bc}\right)-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}x^2\cot^{-1}(c+(c+i)\coth(a+bx))-\frac{1}{2}b\left(ic\left(\frac{x^2\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{\frac{\int \text{PolyLog}(2,ice^{2a+2bx})dx}{2b}-\frac{x\,\text{PolyLog}(2,ice^{2a+2bx})}{2b}}{bc}\right)-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \; 2720$$

$$\frac{1}{2}x^2\cot^{-1}(c+(c+i)\coth(a+bx))-\frac{1}{2}b\left(ic\left(\frac{x^2\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{\frac{\int e^{-2a-2bx}\text{PolyLog}(2,ice^{2a+2bx})de^{2a+2bx}}{4b^2}-\frac{x\,\text{PolyLog}(2,ice^{2a+2bx})}{2b}}{bc}\right)-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \; 7143$$

$$\frac{1}{2}x^2\cot^{-1}(c+(c+i)\coth(a+bx))-\frac{1}{2}b\left(ic\left(\frac{x^2\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{\frac{\text{PolyLog}(3,ice^{2a+2bx})}{4b^2}-\frac{x\,\text{PolyLog}(2,ice^{2a+2bx})}{2b}}{bc}\right)-\frac{ix^3}{3}\right)$$

input
```
Int[x*ArcCot[c + (I + c)*Coth[a + b*x]],x]
```

output
```
(x^2*ArcCot[c + (I + c)*Coth[a + b*x]])/2 - (b*((-1/3*I)*x^3 + I*c*((x^2*L
og[1 - I*c*E^(2*a + 2*b*x)])/(2*b*c) - (-1/2*(x*PolyLog[2, I*c*E^(2*a + 2*
b*x)])/b + PolyLog[3, I*c*E^(2*a + 2*b*x)]/(4*b^2))/(b*c))))/2
```

3.210.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5721
```
Int[ArcCot[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Coth[a + b*x]]/(f*(m
+ 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a +
 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.210.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.54 (sec) , antiderivative size = 1368, normalized size of antiderivative = 12.11

method	result	size
risch	Expression too large to display	1368

input
```
int(x*arccot(c+(I+c)*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/4*I*x^2*ln(2*exp(2*b*x+2*a)*c+2*I)+1/8*Pi*(csgn(I/(exp(2*b*x+2*a)-1))*cs
gn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+
2*a)-1))-csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x
+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1
))-csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2
*a)-1))^2+csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*
x+2*a)*c)/(exp(2*b*x+2*a)-1))^2-csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2
*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^2+csgn(I*(2*I*exp(2*b*x+2*a)+2*
exp(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b
*x+2*a)-1))^2+csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^3-csgn(I
*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))*csgn((2*exp(2*b*x+2*a)*c+2*I
)/(exp(2*b*x+2*a)-1))^2+csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1)
)*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))-csgn(I*(2*I*exp(2*b*x+
2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^3+csgn(I*(2*I*exp(2*b*x+2*a)+
2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b
*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2-csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*
a)*c)/(exp(2*b*x+2*a)-1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(ex
p(2*b*x+2*a)-1))-csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2
*a)-1))^3+csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))
^2-csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^3+csgn((2*exp(2*b*...
```

3.210.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(83) = 166$.

Time = 0.26 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.18

$$\int x\cot^{-1}(c+(i+c)\coth(a+bx))\,dx$$
$$= \frac{2i\,b^3x^3+3i\,b^2x^2\log\left(\frac{(ce^{(2\,bx+2\,a)}+i)e^{(-2\,bx-2\,a)}}{c+i}\right)+2i\,a^3-6i\,bx\mathrm{Li}_2\left(\frac{1}{2}\sqrt{4i\,c}e^{(bx+a)}\right)-6i\,bx\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{4i\,c}e^{(bx+c}\right.}{}$$

input
```
integrate(x*arccot(c+(I+c)*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(2*I*b^3*x^3 + 3*I*b^2*x^2*log((c*e^(2*b*x + 2*a) + I)*e^(-2*b*x - 2*
a)/(c + I)) + 2*I*a^3 - 6*I*b*x*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) - 6*I*b
*x*dilog(-1/2*sqrt(4*I*c)*e^(b*x + a)) - 3*I*a^2*log(1/2*(2*c*e^(b*x + a)
+ I*sqrt(4*I*c))/c) - 3*I*a^2*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c)
 - 3*(I*b^2*x^2 - I*a^2)*log(1/2*sqrt(4*I*c)*e^(b*x + a) + 1) - 3*(I*b^2*x
^2 - I*a^2)*log(-1/2*sqrt(4*I*c)*e^(b*x + a) + 1) + 6*I*polylog(3, 1/2*sqr
t(4*I*c)*e^(b*x + a)) + 6*I*polylog(3, -1/2*sqrt(4*I*c)*e^(b*x + a)))/b^2
```

3.210.6 Sympy [F(-2)]

Exception generated.

$$\int x\cot^{-1}(c+(i+c)\coth(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(x*acot(c+(I+c)*coth(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert 2*_t0**4*c**2*exp(4*a)
+ _t0**4*I*c*exp(4*a) + 3*_t0**2*I*c*exp(2*a) - _t0**2*exp(2*a) - 1 of typ
e <class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,exp(a)]
```

3.210.7 Maxima [A] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.95

$$\int x\cot^{-1}(c+(i+c)\coth(a+bx))\,dx = -\left(\frac{2\,x^3}{3i\,c-3}-\frac{2\,b^2x^2\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(i\,ce^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3\left(i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^3(-i\,c+1)}\right)b(c+i) + \frac{1}{2}\,x^2\,\mathrm{arccot}\left((c+i)\coth\left(bx+a\right)+c\right)$$

input
```
integrate(x*arccot(c+(I+c)*coth(b*x+a)),x, algorithm="maxima")
```

output
```
-(2*x^3/(3*I*c - 3) - (2*b^2*x^2*log(-I*c*e^(2*b*x + 2*a) + 1) + 2*b*x*dil
og(I*c*e^(2*b*x + 2*a)) - polylog(3, I*c*e^(2*b*x + 2*a)))/(b^3*(2*I*c - 2
)))*b*(c + I) + 1/2*x^2*arccot((c + I)*coth(b*x + a) + c)
```

3.210.8 Giac [F]

$$\int x \cot^{-1}(c+(i+c)\coth(a+bx))\,dx = \int x\,\mathrm{arccot}\left((c+i)\coth\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x*arccot(c+(I+c)*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arccot((c + I)*coth(b*x + a) + c), x)
```

3.210.9 Mupad [F(-1)]

Timed out.

$$\int x \cot^{-1}(c+(i+c)\coth(a+bx))\,dx = \int x\,\mathrm{acot}(c+\coth(a+b\,x)\,\,(c+1\mathrm{i}))\,dx$$

input
```
int(x*acot(c + coth(a + b*x)*(c + 1i)),x)
```

output
```
int(x*acot(c + coth(a + b*x)*(c + 1i)), x)
```

3.211 $\int \cot^{-1}(c+(i+c)\coth(a+bx))\,dx$

3.211.1 Optimal result

Integrand size = 15, antiderivative size = 79

$$\int \cot^{-1}(c+(i+c)\coth(a+bx))\,dx=\frac{1}{2}ibx^2+x\cot^{-1}(c+(i+c)\coth(a+bx))-\frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right)-\frac{i\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}$$

output
```
1/2*I*b*x^2+x*arccot(c+(I+c)*coth(b*x+a))-1/2*I*x*ln(1-I*c*exp(2*b*x+2*a))
-1/4*I*polylog(2,I*c*exp(2*b*x+2*a))/b
```

3.211.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int \cot^{-1}(c+(i+c)\coth(a+bx))\,dx=x\cot^{-1}(c+(i+c)\coth(a+bx))-\frac{i\left(2bx\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right)-\text{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right)\right)}{4b}$$

input
```
Integrate[ArcCot[c + (I + c)*Coth[a + b*x]],x]
```

output
```
x*ArcCot[c + (I + c)*Coth[a + b*x]] - ((I/4)*(2*b*x*Log[1 + I/(c*E^(2*(a +
 b*x))] - PolyLog[2, (-I)/(c*E^(2*(a + b*x)))]))/b
```

3.211.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5713, 25, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}(c+(c+i)\coth(a+bx))\,dx$$

$$\Big\downarrow \text{5713}$$

$$b\int -\frac{x}{e^{2a+2bx}c+i}dx+x\cot^{-1}(c+(c+i)\coth(a+bx))$$

$$\Big\downarrow \text{25}$$

$$x\cot^{-1}(c+(c+i)\coth(a+bx))-b\int \frac{x}{e^{2a+2bx}c+i}dx$$

$$\Big\downarrow \text{2615}$$

$$x\cot^{-1}(c+(c+i)\coth(a+bx))-b\left(ic\int \frac{e^{2a+2bx}x}{e^{2a+2bx}c+i}dx-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2620}$$

$$x\cot^{-1}(c+(c+i)\coth(a+bx))-b\left(ic\left(\frac{x\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{\int \log\left(1-ice^{2a+2bx}\right)dx}{2bc}\right)-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2715}$$

$$x\cot^{-1}(c+(c+i)\coth(a+bx))-b\left(ic\left(\frac{x\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{\int e^{-2a-2bx}\log\left(1-ice^{2a+2bx}\right)de^{2a+2bx}}{4b^2c}\right)-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2838}$$

$$x\cot^{-1}(c+(c+i)\coth(a+bx))-b\left(ic\left(\frac{\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b^2c}+\frac{x\log\left(1-ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^2}{2}\right)$$

input
```
Int[ArcCot[c + (I + c)*Coth[a + b*x]],x]
```

output
```
x*ArcCot[c + (I + c)*Coth[a + b*x]] - b*((-1/2*I)*x^2 + I*c*((x*Log[1 - I*
c*E^(2*a + 2*b*x)])/(2*b*c) + PolyLog[2, I*c*E^(2*a + 2*b*x)]/(4*b^2*c)))
```

3.211.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5713
```
Int[ArcCot[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Cot[c + d*Coth[a + b*x]], x] + Simp[b   Int[x/(c - d - c*E^(2*a + 2*b*x)),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, -1]
```

3.211.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 543 vs. 2(65) = 130.

Time = 1.67 (sec) , antiderivative size = 544, normalized size of antiderivative = 6.89

method	result
derivativedivides	$\frac{\frac{\operatorname{arccot}(c+(i+c)\coth(bx+a))\ln(c-(i+c)\coth(bx+a)+i)}{2i+2c}-\frac{2i\operatorname{arccot}(c+(i+c)\coth(bx+a))\ln(c-(i+c)\coth(bx+a)+i)c}{2i+2c}-\frac{\operatorname{arccot}(c+(i+c)}{}}{}$
default	$\frac{\frac{\operatorname{arccot}(c+(i+c)\coth(bx+a))\ln(c-(i+c)\coth(bx+a)+i)}{2i+2c}-\frac{2i\operatorname{arccot}(c+(i+c)\coth(bx+a))\ln(c-(i+c)\coth(bx+a)+i)c}{2i+2c}-\frac{\operatorname{arccot}(c+(i+c)}{}}{}$
risch	Expression too large to display

input
```
int(arccot(c+(I+c)*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/(I+c)*(arccot(c+(I+c)*coth(b*x+a))/(2*I+2*c)*ln(c-(I+c)*coth(b*x+a)+I)
-2*I*arccot(c+(I+c)*coth(b*x+a))/(2*I+2*c)*ln(c-(I+c)*coth(b*x+a)+I)*c-arc
cot(c+(I+c)*coth(b*x+a))/(2*I+2*c)*ln(c-(I+c)*coth(b*x+a)+I)*c^2-arccot(c+
(I+c)*coth(b*x+a))/(2*I+2*c)*ln(I+c+(I+c)*coth(b*x+a))+2*I*arccot(c+(I+c)*
coth(b*x+a))/(2*I+2*c)*ln(I+c+(I+c)*coth(b*x+a))*c+arccot(c+(I+c)*coth(b*x
+a))/(2*I+2*c)*ln(I+c+(I+c)*coth(b*x+a))*c^2+(I+c)^2*(1/2/(I+c)*(1/4*I*ln(
I+c+(I+c)*coth(b*x+a))^2-1/2*I*((ln(I+c+(I+c)*coth(b*x+a))-ln(-1/2*I*(I+c+
(I+c)*coth(b*x+a))))*ln(-1/2*I*(I-c-(I+c)*coth(b*x+a)))-dilog(-1/2*I*(I+c+
(I+c)*coth(b*x+a)))))-1/2/(I+c)*(1/2*I*(dilog((-I-c-(I+c)*coth(b*x+a))/(-2
*I-2*c))+ln(c-(I+c)*coth(b*x+a)+I)*ln((-I-c-(I+c)*coth(b*x+a))/(-2*I-2*c))
)-1/2*I*(dilog(-1/2*(I-c-(I+c)*coth(b*x+a))/c)+ln(c-(I+c)*coth(b*x+a)+I)*l
n(-1/2*(I-c-(I+c)*coth(b*x+a))/c))))
```

3.211.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(58) = 116.

Time = 0.26 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.35

$$\int \cot^{-1}(c+(i+c)\coth(a+bx))\,dx$$
$$=\frac{i\,b^2x^2+i\,bx\log\left(\frac{(ce^{(2\,bx+2\,a)}+i)e^{(-2\,bx-2\,a)}}{c+i}\right)-i\,a^2+(-i\,bx-i\,a)\log\left(\frac{1}{2}\sqrt{4i\,c}e^{(bx+a)}+1\right)+(-i\,bx-i\,a)\log}{}$$

input
```
integrate(arccot(c+(I+c)*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(I*b^2*x^2 + I*b*x*log((c*e^(2*b*x + 2*a) + I)*e^(-2*b*x - 2*a)/(c + I
)) - I*a^2 + (-I*b*x - I*a)*log(1/2*sqrt(4*I*c)*e^(b*x + a) + 1) + (-I*b*x
 - I*a)*log(-1/2*sqrt(4*I*c)*e^(b*x + a) + 1) + I*a*log(1/2*(2*c*e^(b*x +
a) + I*sqrt(4*I*c))/c) + I*a*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c)
- I*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) - I*dilog(-1/2*sqrt(4*I*c)*e^(b*x +
 a)))/b
```

3.211.6 Sympy [F(-2)]

Exception generated.

$$\int \cot^{-1}(c+(i+c)\coth(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(acot(c+(I+c)*coth(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert 2*_t0**4*c**2*exp(4*a)
+ _t0**4*I*c*exp(4*a) + 3*_t0**2*I*c*exp(2*a) - _t0**2*exp(2*a) - 1 of typ
e <class 'sympy.core.add.Add'> to QQ_I[b,c,_t0,exp(a)]
```

3.211.7 Maxima [A] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.01

$$\int \cot^{-1}(c+(i+c)\coth(a+bx))\,dx$$
$$= -2\,b(c+i)\left(\frac{2\,x^2}{2i\,c-2} - \frac{2\,bx\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^2(-i\,c+1)}\right)$$
$$+ x\,\mathrm{arccot}\left((c+i)\coth\left(bx+a\right)+c\right)$$

input
```
integrate(arccot(c+(I+c)*coth(b*x+a)),x, algorithm="maxima")
```

output
```
-2*b*(c + I)*(2*x^2/(2*I*c - 2) - (2*b*x*log(-I*c*e^(2*b*x + 2*a) + 1) + d
ilog(I*c*e^(2*b*x + 2*a)))/(b^2*(2*I*c - 2))) + x*arccot((c + I)*coth(b*x
+ a) + c)
```

3.211.8 Giac [F]

$$\int \cot^{-1}(c+(i+c)\coth(a+bx))\,dx = \int \operatorname{arccot}\left((c+i)\coth\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(arccot(c+(I+c)*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccot((c + I)*coth(b*x + a) + c), x)
```

3.211.9 Mupad [F(-1)]

Timed out.

$$\int \cot^{-1}(c+(i+c)\coth(a+bx))\,dx = \int \operatorname{acot}(c+\coth(a+b\,x)\,\left(c+1\mathrm{i}\right))\,dx$$

input
```
int(acot(c + coth(a + b*x)*(c + 1i)),x)
```

output
```
int(acot(c + coth(a + b*x)*(c + 1i)), x)
```

3.212 $\int \frac{\cot^{-1}(c+(i+c)\coth(a+bx))}{x}\,dx$

3.212.1 Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{\cot^{-1}(c+(i+c)\coth(a+bx))}{x}\,dx = \text{Int}\left(\frac{\cot^{-1}(c+(i+c)\coth(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(arccot(c+(I+c)*coth(b*x+a))/x,x)
```

3.212.2 Mathematica [N/A]

Not integrable

Time = 3.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\cot^{-1}(c+(i+c)\coth(a+bx))}{x}\,dx = \int \frac{\cot^{-1}(c+(i+c)\coth(a+bx))}{x}\,dx$$

input
```
Integrate[ArcCot[c + (I + c)*Coth[a + b*x]]/x,x]
```

output
```
Integrate[ArcCot[c + (I + c)*Coth[a + b*x]]/x, x]
```

3.212.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(c+(c+i)\coth(a+bx))}{x}\,dx$$
$$\Big\downarrow \text{7299}$$
$$\int \frac{\cot^{-1}(c+(c+i)\coth(a+bx))}{x}dx$$

input
```
Int[ArcCot[c + (I + c)*Coth[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.212.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.212.4 Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\operatorname{arccot}\left(c+(i+c)\coth\left(bx+a\right)\right)}{x}dx$$

input
```
int(arccot(c+(I+c)*coth(b*x+a))/x,x)
```

output
```
int(arccot(c+(I+c)*coth(b*x+a))/x,x)
```

3.212.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{\cot^{-1}(c+(i+c)\coth(a+bx))}{x}\,dx = \int \frac{\operatorname{arccot}\left((c+i)\coth\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccot(c+(I+c)*coth(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(1/2*I*log((c*e^(2*b*x + 2*a) + I)*e^(-2*b*x - 2*a)/(c + I))/x, x)
```

3.212.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(c+(i+c)\coth(a+bx))}{x}\,dx = \text{Timed out}$$

input
```
integrate(acot(c+(I+c)*coth(b*x+a))/x,x)
```

output
```
Timed out
```

3.212.7 Maxima [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.84

$$\int \frac{\cot^{-1}(c+(i+c)\coth(a+bx))}{x}\,dx = \int \frac{\operatorname{arccot}\left((c+i)\coth\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccot(c+(I+c)*coth(b*x+a))/x,x, algorithm="maxima")
```

output
```
-I*b*x + 1/4*(-4*I*a + 2*arctan(1/c) - I*log(c^2 + 1))*log(x) - 1/2*integr
ate(arctan(e^(-2*b*x - 2*a)/c)/x, x) + 1/4*I*integrate(log(c^2*e^(4*b*x +
4*a) + 1)/x, x)
```

3.212.8 Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(c+(i+c)\coth(a+bx))}{x}\,dx = \int \frac{\operatorname{arccot}\left((c+i)\coth\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccot(c+(I+c)*coth(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arccot((c + I)*coth(b*x + a) + c)/x, x)
```

3.212.9 Mupad [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{\cot^{-1}(c+(i+c)\coth(a+bx))}{x}\,dx = \int \frac{\operatorname{acot}(c+\coth(a+b\,x)\ (c+1\mathrm{i}))}{x}\,dx$$

input
```
int(acot(c + coth(a + b*x)*(c + 1i))/x,x)
```

output
```
int(acot(c + coth(a + b*x)*(c + 1i))/x, x)
```

3.213 $\int x^2 \cot^{-1}(c-(i-c)\coth(a+bx))\,dx$

3.213.1 Optimal result

Integrand size = 22, antiderivative size = 145

$$\int x^2 \cot^{-1}(c-(i-c)\coth(a+bx))\,dx = -\frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c-(i-c)\coth(a+bx))+\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right)+\frac{ix^2\operatorname{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}-\frac{ix\operatorname{PolyLog}\left(3,-ice^{2a+2bx}\right)}{4b^2}+\frac{i\operatorname{PolyLog}\left(4,-ice^{2a+2bx}\right)}{8b^3}$$

output
```
-1/12*I*b*x^4+1/3*x^3*arccot(c-(I-c)*coth(b*x+a))+1/6*I*x^3*ln(1+I*c*exp(2
*b*x+2*a))+1/4*I*x^2*polylog(2,-I*c*exp(2*b*x+2*a))/b-1/4*I*x*polylog(3,-I
*c*exp(2*b*x+2*a))/b^2+1/8*I*polylog(4,-I*c*exp(2*b*x+2*a))/b^3
```

3.213.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.92

$$\int x^2 \cot^{-1}(c-(i-c)\coth(a+bx))\,dx = \frac{8b^3x^3\cot^{-1}(c+(-i+c)\coth(a+bx))+4ib^3x^3\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right)-6ib^2x^2\operatorname{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right)-6ib\cdots}{24b^3}$$

input
```
Integrate[x^2*ArcCot[c - (I - c)*Coth[a + b*x]],x]
```

output
```
(8*b^3*x^3*ArcCot[c + (-I + c)*Coth[a + b*x]] + (4*I)*b^3*x^3*Log[1 - I/(c
*E^(2*(a + b*x)))] - (6*I)*b^2*x^2*PolyLog[2, I/(c*E^(2*(a + b*x)))] - (6*
I)*b*x*PolyLog[3, I/(c*E^(2*(a + b*x)))] - (3*I)*PolyLog[4, I/(c*E^(2*(a +
 b*x)))])/(24*b^3)
```

3.213.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {5721, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cot^{-1}(c-(-c+i)\coth(a+bx))\,dx$$

$$\Big\downarrow \text{5721}$$

$$\frac{1}{3}b\int \frac{x^3}{i-ce^{2a+2bx}}dx+\frac{1}{3}x^3\cot^{-1}(c-(-c+i)\coth(a+bx))$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{3}b\left(-ic\int \frac{e^{2a+2bx}x^3}{i-ce^{2a+2bx}}dx-\frac{ix^4}{4}\right)+\frac{1}{3}x^3\cot^{-1}(c-(-c+i)\coth(a+bx))$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{3}b\left(-ic\left(\frac{3\int x^2\log\left(ie^{2a+2bx}c+1\right)dx}{2bc}-\frac{x^3\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^4}{4}\right)+\frac{1}{3}x^3\cot^{-1}(c-(-c+i)\coth(a+bx))$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{3}b\left(-ic\left(\frac{3\left(\frac{\int x\,\text{PolyLog}(2,-ice^{2a+2bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,-ice^{2a+2bx})}{2b}\right)}{2bc}-\frac{x^3\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^4}{4}\right)+\frac{1}{3}x^3\cot^{-1}(c-(-c+i)\coth(a+bx))$$

$$\Big\downarrow \text{7163}$$

$$\frac{1}{3}b\left(-ic\left(\frac{3\left(\frac{\frac{x\operatorname{PolyLog}\left(3,-ice^{2a+2bx}\right)}{2b}-\frac{\int \operatorname{PolyLog}\left(3,-ice^{2a+2bx}\right)dx}{2b}}{b}-\frac{x^2\operatorname{PolyLog}\left(2,-ice^{2a+2bx}\right)}{2b}\right)}{2bc}-\frac{x^3\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix}{4}\right.$$

$$\frac{1}{3}x^3\cot^{-1}(c-(-c+i)\coth(a+bx))$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{3}b\left(-ic\left(\frac{3\left(\frac{\frac{x\operatorname{PolyLog}\left(3,-ice^{2a+2bx}\right)}{2b}-\frac{\int e^{-2a-2bx}\operatorname{PolyLog}\left(3,-ice^{2a+2bx}\right)de^{2a+2bx}}{4b^2}}{b}-\frac{x^2\operatorname{PolyLog}\left(2,-ice^{2a+2bx}\right)}{2b}\right)}{2bc}-\frac{x^3\log\left(1+ice^{2a}\right.}{2bc}\right.\right.$$

$$\frac{1}{3}x^3\cot^{-1}(c-(-c+i)\coth(a+bx))$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{3}b\left(-ic\left(\frac{3\left(\frac{\frac{x\operatorname{PolyLog}\left(3,-ice^{2a+2bx}\right)}{2b}-\frac{\operatorname{PolyLog}\left(4,-ice^{2a+2bx}\right)}{4b^2}}{b}-\frac{x^2\operatorname{PolyLog}\left(2,-ice^{2a+2bx}\right)}{2b}\right)}{2bc}-\frac{x^3\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^4}{4}\right)$$

$$\frac{1}{3}x^3\cot^{-1}(c-(-c+i)\coth(a+bx))$$

input
```
Int[x^2*ArcCot[c - (I - c)*Coth[a + b*x]],x]
```

output
```
(x^3*ArcCot[c - (I - c)*Coth[a + b*x]])/3 + (b*((-1/4*I)*x^4 - I*c*(-1/2*(
x^3*Log[1 + I*c*E^(2*a + 2*b*x)])/(b*c) + (3*(-1/2*(x^2*PolyLog[2, (-I)*c*
E^(2*a + 2*b*x)])/b + ((x*PolyLog[3, (-I)*c*E^(2*a + 2*b*x)])/(2*b) - Poly
Log[4, (-I)*c*E^(2*a + 2*b*x)]/(4*b^2))/b))/(2*b*c))))/3
```

3.213. $\int x^2\cot^{-1}(c-(i-c)\coth(a+bx))\,dx$

3.213.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5721
```
Int[ArcCot[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Coth[a + b*x]]/(f*(m
+ 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a +
 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.213. $\int x^2 \cot^{-1}(c-(i-c)\coth(a+bx))\,dx$

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.213.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.31 (sec) , antiderivative size = 1410, normalized size of antiderivative = 9.72

method	result	size
risch	Expression too large to display	1410

input
```
int(x^2*arccot(c-(I-c)*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2*I/b^3*a^2*dilog(1+I*exp(b*x+a)*(I*c)^(1/2))-1/12*Pi*(csgn(I/(exp(2*b*x
+2*a)-1))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)
/(exp(2*b*x+2*a)-1))-csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(-2*I*exp(2*b*x+2*a
)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(ex
p(2*b*x+2*a)-1))-csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I
)/(exp(2*b*x+2*a)-1))^2+csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(-2*I*exp(2*b*x+
2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2+csgn(I*(2*exp(2*b*x+2*a)*c-
2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^2-csgn(I*(-2*I*e
xp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x
+2*a)*c)/(exp(2*b*x+2*a)-1))^2-csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+
2*a)-1))^3+csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))*csgn((2*exp
(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^2+csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(
exp(2*b*x+2*a)-1))*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))+csgn(
I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^3-csgn(I*(-
2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))*csgn((-2*I*exp(
2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2-csgn(I*(-2*I*exp(2*b*
x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))*csgn((-2*I*exp(2*b*x+2*a)+2
*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))+csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b
*x+2*a)*c)/(exp(2*b*x+2*a)-1))^3+csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a
)*c)/(exp(2*b*x+2*a)-1))^2+csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a...
```

3.213.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 292 vs. 2(105) = 210.

Time = 0.27 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.01

$$\int x^2 \cot^{-1}(c-(i-c)\coth(a+bx))\,dx = \frac{-i\,b^4x^4 + 2i\,b^3x^3\log\left(\frac{(c-i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}-i}\right) + 6i\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4i\,c}e^{(bx+a)}\right) + 6i\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,c}e^{(bx+a)}\right) + i\,a^4}{}$$

input
```
integrate(x^2*arccot(c-(I-c)*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(-I*b^4*x^4 + 2*I*b^3*x^3*log((c - I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2
*a) - I)) + 6*I*b^2*x^2*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) + 6*I*b^2*x^2*
dilog(-1/2*sqrt(-4*I*c)*e^(b*x + a)) + I*a^4 - 2*I*a^3*log(1/2*(2*c*e^(b*x
 + a) + I*sqrt(-4*I*c))/c) - 2*I*a^3*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*
I*c))/c) - 12*I*b*x*polylog(3, 1/2*sqrt(-4*I*c)*e^(b*x + a)) - 12*I*b*x*po
lylog(3, -1/2*sqrt(-4*I*c)*e^(b*x + a)) - 2*(-I*b^3*x^3 - I*a^3)*log(1/2*s
qrt(-4*I*c)*e^(b*x + a) + 1) - 2*(-I*b^3*x^3 - I*a^3)*log(-1/2*sqrt(-4*I*c
)*e^(b*x + a) + 1) + 12*I*polylog(4, 1/2*sqrt(-4*I*c)*e^(b*x + a)) + 12*I*
polylog(4, -1/2*sqrt(-4*I*c)*e^(b*x + a)))/b^3
```

3.213.6 Sympy [F(-2)]

Exception generated.

$$\int x^2 \cot^{-1}(c-(i-c)\coth(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(x**2*acot(c-(I-c)*coth(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert 2*_t0**2*c*exp(2*a) - _
t0**2*I*exp(2*a) - I of type <class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t
0,exp(a)]
```

3.213.7 Maxima [A] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.89

$$\int x^2 \cot^{-1}(c-(i-c)\coth(a+bx))\,dx = \frac{1}{3}x^3\,\mathrm{arccot}\left((c-i)\coth(bx+a)+c\right)$$
$$+\frac{4}{9}\left(\frac{3\,x^4}{4i\,c+4}-\frac{4\,b^3x^3\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+6\,b^2x^2\mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right)-6\,bx\mathrm{Li}_3(-i\,ce^{(2\,bx+2\,a)})+3\,\mathrm{Li}_4(-}{-2\,b^4(-i\,c-1)}\right.$$

input
```
integrate(x^2*arccot(c-(I-c)*coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/3*x^3*arccot((c - I)*coth(b*x + a) + c) + 4/9*(3*x^4/(4*I*c + 4) - (4*b^
3*x^3*log(I*c*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-I*c*e^(2*b*x + 2*a))
 - 6*b*x*polylog(3, -I*c*e^(2*b*x + 2*a)) + 3*polylog(4, -I*c*e^(2*b*x + 2
*a)))/(b^4*(2*I*c + 2)))*b*(c - I)
```

3.213.8 Giac [F]

$$\int x^2 \cot^{-1}(c-(i-c)\coth(a+bx))\,dx = \int x^2\,\mathrm{arccot}\left((c-i)\coth(bx+a)+c\right)\,dx$$

input
```
integrate(x^2*arccot(c-(I-c)*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arccot((c - I)*coth(b*x + a) + c), x)
```

3.213.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(c-(i-c)\coth(a+bx))\,dx = \int x^2\,\mathrm{acot}(c+\coth(a+b\,x)\,(c-\mathrm{i}))\,dx$$

input
```
int(x^2*acot(c + coth(a + b*x)*(c - 1i)),x)
```

output
```
int(x^2*acot(c + coth(a + b*x)*(c - 1i)), x)
```

3.214 $\int x \cot^{-1}(c-(i-c)\coth(a+bx))\,dx$

3.214.1 Optimal result

Integrand size = 20, antiderivative size = 116

$$\int x\cot^{-1}(c-(i-c)\coth(a+bx))\,dx=-\frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c-(i-c)\coth(a+bx))+\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)+\frac{ix\,\mathrm{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}-\frac{i\,\mathrm{PolyLog}\left(3,-ice^{2a+2bx}\right)}{8b^2}$$

output
```
-1/6*I*b*x^3+1/2*x^2*arccot(c-(I-c)*coth(b*x+a))+1/4*I*x^2*ln(1+I*c*exp(2*
b*x+2*a))+1/4*I*x*polylog(2,-I*c*exp(2*b*x+2*a))/b-1/8*I*polylog(3,-I*c*ex
p(2*b*x+2*a))/b^2
```

3.214.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.89

$$\int x\cot^{-1}(c-(i-c)\coth(a+bx))\,dx$$
$$=\frac{2b^2x^2\left(2\cot^{-1}(c+(-i+c)\coth(a+bx))+i\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right)\right)-2ibx\,\mathrm{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right)-i\,\mathrm{Poly}}{8b^2}$$

input
```
Integrate[x*ArcCot[c - (I - c)*Coth[a + b*x]],x]
```

output
```
(2*b^2*x^2*(2*ArcCot[c + (-I + c)*Coth[a + b*x]] + I*Log[1 - I/(c*E^(2*(a
+ b*x)))]) - (2*I)*b*x*PolyLog[2, I/(c*E^(2*(a + b*x)))] - I*PolyLog[3, I/
(c*E^(2*(a + b*x)))])/(8*b^2)
```

3.214.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5721, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \cot^{-1}(c-(-c+i)\coth(a+bx))\,dx$$

$$\Big\downarrow \text{ 5721}$$

$$\frac{1}{2}b\int \frac{x^2}{i-ce^{2a+2bx}}dx+\frac{1}{2}x^2\cot^{-1}(c-(-c+i)\coth(a+bx))$$

$$\Big\downarrow \text{ 2615}$$

$$\frac{1}{2}b\left(-ic\int \frac{e^{2a+2bx}x^2}{i-ce^{2a+2bx}}dx-\frac{ix^3}{3}\right)+\frac{1}{2}x^2\cot^{-1}(c-(-c+i)\coth(a+bx))$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{1}{2}b\left(-ic\left(\frac{\int x\log\left(ie^{2a+2bx}c+1\right)dx}{bc}-\frac{x^2\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^3}{3}\right)+\frac{1}{2}x^2\cot^{-1}(c-(-c+i)\coth(a+bx))$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{1}{2}b\left(-ic\left(\frac{\frac{\int \text{PolyLog}(2,-ice^{2a+2bx})dx}{2b}-\frac{x\,\text{PolyLog}(2,-ice^{2a+2bx})}{2b}}{bc}-\frac{x^2\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^3}{3}\right)+\frac{1}{2}x^2\cot^{-1}(c-(-c+i)\coth(a+bx))$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{1}{2}b\left(-ic\left(\frac{\frac{\int e^{-2a-2bx}\operatorname{PolyLog}\left(2,-ice^{2a+2bx}\right)de^{2a+2bx}}{4b^2}-\frac{x\operatorname{PolyLog}\left(2,-ice^{2a+2bx}\right)}{2b}}{bc}-\frac{x^2\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^3}{3}\right)+\frac{1}{2}x^2\cot^{-1}(c-(-c+i)\coth(a+bx))$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}b\left(-ic\left(\frac{\frac{\operatorname{PolyLog}\left(3,-ice^{2a+2bx}\right)}{4b^2}-\frac{x\operatorname{PolyLog}\left(2,-ice^{2a+2bx}\right)}{2b}}{bc}-\frac{x^2\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^3}{3}\right)+\frac{1}{2}x^2\cot^{-1}(c-(-c+i)\coth(a+bx))$$

input
```
Int[x*ArcCot[c - (I - c)*Coth[a + b*x]],x]
```

output
```
(x^2*ArcCot[c - (I - c)*Coth[a + b*x]])/2 + (b*((-1/3*I)*x^3 - I*c*(-1/2*(
x^2*Log[1 + I*c*E^(2*a + 2*b*x)])/(b*c) + (-1/2*(x*PolyLog[2, (-I)*c*E^(2*
a + 2*b*x)])/b + PolyLog[3, (-I)*c*E^(2*a + 2*b*x)]/(4*b^2))/(b*c))))/2
```

3.214.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5721
```
Int[ArcCot[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Coth[a + b*x]]/(f*(m
+ 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a +
 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.214.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.91 (sec) , antiderivative size = 1374, normalized size of antiderivative = 11.84

method	result	size
risch	Expression too large to display	1374

input
```
int(x*arccot(c-(I-c)*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2*I/b*ln(1+I*c*exp(2*b*x+2*a))*a*x-1/8*Pi*(csgn(I/(exp(2*b*x+2*a)-1))*cs
gn(I*(2*exp(2*b*x+2*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+
2*a)-1))-csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*
x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)
-1))-csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x
+2*a)-1))^2+csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2
*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2+csgn(I*(2*exp(2*b*x+2*a)*c-2*I))*csgn(I
*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^2-csgn(I*(-2*I*exp(2*b*x+2*a
)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(ex
p(2*b*x+2*a)-1))^2-csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^3+c
sgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))*csgn((2*exp(2*b*x+2*a)*
c-2*I)/(exp(2*b*x+2*a)-1))^2+csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*
a)-1))*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))+csgn(I*(-2*I*exp(
2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^3-csgn(I*(-2*I*exp(2*b*
x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))*csgn((-2*I*exp(2*b*x+2*a)+2
*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2-csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp
(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2
*a)*c)/(exp(2*b*x+2*a)-1))+csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(
exp(2*b*x+2*a)-1))^3+csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*
b*x+2*a)-1))^2+csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^3+csgn...
```

3.214.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(83) = 166$.

Time = 0.27 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.12

$$\int x\cot^{-1}(c-(i-c)\coth(a+bx))\,dx$$
$$= \frac{-2i\,b^3x^3+3i\,b^2x^2\log\left(\frac{(c-i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}-i}\right)-2i\,a^3+6i\,bx\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4i\,c}e^{(bx+a)}\right)+6i\,bx\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,c}e^{(bx+a)}\right)+}{}$$

input
```
integrate(x*arccot(c-(I-c)*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(-2*I*b^3*x^3 + 3*I*b^2*x^2*log((c - I)*e^(2*b*x + 2*a)/(c*e^(2*b*x +
 2*a) - I)) - 2*I*a^3 + 6*I*b*x*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) + 6*I*
b*x*dilog(-1/2*sqrt(-4*I*c)*e^(b*x + a)) + 3*I*a^2*log(1/2*(2*c*e^(b*x + a
) + I*sqrt(-4*I*c))/c) + 3*I*a^2*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c)
)/c) - 3*(-I*b^2*x^2 + I*a^2)*log(1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) - 3*(-
I*b^2*x^2 + I*a^2)*log(-1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) - 6*I*polylog(3,
 1/2*sqrt(-4*I*c)*e^(b*x + a)) - 6*I*polylog(3, -1/2*sqrt(-4*I*c)*e^(b*x +
 a)))/b^2
```

3.214.6 Sympy [F(-2)]

Exception generated.

$$\int x\cot^{-1}(c-(i-c)\coth(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(x*acot(c-(I-c)*coth(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert 2*_t0**2*c*exp(2*a) - _
t0**2*I*exp(2*a) - I of type <class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t
0,exp(a)]
```

3.214.7 Maxima [A] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

$$\int x\cot^{-1}(c-(i-c)\coth(a+bx))\,dx$$
$$= \left(\frac{2\,x^3}{3i\,c+3} - \frac{2\,b^2x^2\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3\left(-i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^3(-i\,c-1)}\right)b(c-i) + \frac{1}{2}\,x^2\,\mathrm{arccot}\left((c-i)\coth\left(bx+a\right)+c\right)$$

input
```
integrate(x*arccot(c-(I-c)*coth(b*x+a)),x, algorithm="maxima")
```

output
```
(2*x^3/(3*I*c + 3) - (2*b^2*x^2*log(I*c*e^(2*b*x + 2*a) + 1) + 2*b*x*dilog
(-I*c*e^(2*b*x + 2*a)) - polylog(3, -I*c*e^(2*b*x + 2*a)))/(b^3*(2*I*c + 2
)))*b*(c - I) + 1/2*x^2*arccot((c - I)*coth(b*x + a) + c)
```

3.214.8 Giac [F]

$$\int x\cot^{-1}(c-(i-c)\coth(a+bx))\,dx=\int x\,\mathrm{arccot}\left((c-i)\coth\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x*arccot(c-(I-c)*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arccot((c - I)*coth(b*x + a) + c), x)
```

3.214.9 Mupad [F(-1)]

Timed out.

$$\int x\cot^{-1}(c-(i-c)\coth(a+bx))\,dx=\int x\,\mathrm{acot}(c+\coth(a+b\,x)\,\left(c-\mathrm{i}\right))\,dx$$

input
```
int(x*acot(c + coth(a + b*x)*(c - 1i)),x)
```

output
```
int(x*acot(c + coth(a + b*x)*(c - 1i)), x)
```

3.215 $\int \cot^{-1}(c-(i-c)\coth(a+bx))\,dx$

3.215.1 Optimal result

Integrand size = 18, antiderivative size = 82

$$\int \cot^{-1}(c-(i-c)\coth(a+bx))\,dx = -\frac{1}{2}ibx^2 + x\cot^{-1}(c-(i-c)\coth(a+bx)) + \frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right) + \frac{i\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}$$

output
```
-1/2*I*b*x^2+x*arccot(c-(I-c)*coth(b*x+a))+1/2*I*x*ln(1+I*c*exp(2*b*x+2*a)
)+1/4*I*polylog(2,-I*c*exp(2*b*x+2*a))/b
```

3.215.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.87

$$\int \cot^{-1}(c-(i-c)\coth(a+bx))\,dx = x\cot^{-1}(c+(-i+c)\coth(a+bx)) + \frac{i\left(2bx\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right) - \text{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right)\right)}{4b}$$

input
```
Integrate[ArcCot[c - (I - c)*Coth[a + b*x]],x]
```

output
```
x*ArcCot[c + (-I + c)*Coth[a + b*x]] + ((I/4)*(2*b*x*Log[1 - I/(c*E^(2*(a
+ b*x)))] - PolyLog[2, I/(c*E^(2*(a + b*x)))]))/b
```

3.215.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5713, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}(c-(-c+i)\coth(a+bx))\,dx$$

$$\Big\downarrow \text{5713}$$

$$b\int \frac{x}{i-ce^{2a+2bx}}dx+x\cot^{-1}(c-(-c+i)\coth(a+bx))$$

$$\Big\downarrow \text{2615}$$

$$b\left(-ic\int \frac{e^{2a+2bx}x}{i-ce^{2a+2bx}}dx-\frac{ix^2}{2}\right)+x\cot^{-1}(c-(-c+i)\coth(a+bx))$$

$$\Big\downarrow \text{2620}$$

$$b\left(-ic\left(\frac{\int \log\left(ie^{2a+2bx}c+1\right)dx}{2bc}-\frac{x\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^2}{2}\right)+x\cot^{-1}(c-(-c+i)\coth(a+bx))$$

$$\Big\downarrow \text{2715}$$

$$b\left(-ic\left(\frac{\int e^{-2a-2bx}\log\left(ie^{2a+2bx}c+1\right)de^{2a+2bx}}{4b^2c}-\frac{x\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^2}{2}\right)+x\cot^{-1}(c-(-c+i)\coth(a+bx))$$

$$\Big\downarrow \text{2838}$$

$$b\left(-ic\left(-\frac{\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b^2c}-\frac{x\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^2}{2}\right)+x\cot^{-1}(c-(-c+i)\coth(a+bx))$$

input
```
Int[ArcCot[c - (I - c)*Coth[a + b*x]],x]
```

output
```
x*ArcCot[c - (I - c)*Coth[a + b*x]] + b*((-1/2*I)*x^2 - I*c*(-1/2*(x*Log[1
 + I*c*E^(2*a + 2*b*x)])/(b*c) - PolyLog[2, (-I)*c*E^(2*a + 2*b*x)]/(4*b^2
*c)))
```

3.215.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5713
```
Int[ArcCot[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Cot[c + d*Coth[a + b*x]], x] + Simp[b   Int[x/(c - d - c*E^(2*a + 2*b*x)),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, -1]
```

3.215.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 516 vs. 2(68) = 136.

Time = 1.87 (sec) , antiderivative size = 517, normalized size of antiderivative = 6.30

method	result
derivativedivides	$\frac{-\frac{\operatorname{arccot}(c+\coth(bx+a)(c-i))\ln(\coth(bx+a)(c-i)-c+i)}{2i-2c}-\frac{2i\operatorname{arccot}(c+\coth(bx+a)(c-i))\ln(\coth(bx+a)(c-i)-c+i)c}{2i-2c}+\frac{\operatorname{arccot}(c+\coth}{}}{}$
default	$\frac{-\frac{\operatorname{arccot}(c+\coth(bx+a)(c-i))\ln(\coth(bx+a)(c-i)-c+i)}{2i-2c}-\frac{2i\operatorname{arccot}(c+\coth(bx+a)(c-i))\ln(\coth(bx+a)(c-i)-c+i)c}{2i-2c}+\frac{\operatorname{arccot}(c+\coth}{}}{}$
risch	Expression too large to display

input
```
int(arccot(c-(I-c)*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/(c-I)*(-arccot(c+coth(b*x+a)*(c-I))/(2*I-2*c)*ln(coth(b*x+a)*(c-I)-c+I
)-2*I*arccot(c+coth(b*x+a)*(c-I))/(2*I-2*c)*ln(coth(b*x+a)*(c-I)-c+I)*c+ar
ccot(c+coth(b*x+a)*(c-I))/(2*I-2*c)*ln(coth(b*x+a)*(c-I)-c+I)*c^2+arccot(c
+coth(b*x+a)*(c-I))/(2*I-2*c)*ln(-I+coth(b*x+a)*(c-I)+c)+2*I*arccot(c+coth
(b*x+a)*(c-I))/(2*I-2*c)*ln(-I+coth(b*x+a)*(c-I)+c)*c-arccot(c+coth(b*x+a)
*(c-I))/(2*I-2*c)*ln(-I+coth(b*x+a)*(c-I)+c)*c^2-(I-c)^2*(1/2/(I-c)*(-1/4*
I*ln(-I+coth(b*x+a)*(c-I)+c)^2+1/2*I*(dilog(-1/2*I*(coth(b*x+a)*(c-I)+c+I)
)+ln(-I+coth(b*x+a)*(c-I)+c)*ln(-1/2*I*(coth(b*x+a)*(c-I)+c+I))))-1/2/(I-c
)*(1/2*I*(dilog(1/2*(coth(b*x+a)*(c-I)+c+I)/c)+ln(coth(b*x+a)*(c-I)-c+I)*l
n(1/2*(coth(b*x+a)*(c-I)+c+I)/c))-1/2*I*(dilog((-I+coth(b*x+a)*(c-I)+c)/(-
2*I+2*c))+ln(coth(b*x+a)*(c-I)-c+I)*ln((-I+coth(b*x+a)*(c-I)+c)/(-2*I+2*c)
)))))
```

3.215.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(58) = 116.

Time = 0.25 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.27

$$\int \cot^{-1}(c-(i-c)\coth(a+bx))\,dx$$
$$=\frac{-i\,b^2x^2+i\,bx\log\left(\frac{(c-i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}-i}\right)+i\,a^2+(i\,bx+i\,a)\log\left(\frac{1}{2}\sqrt{-4i\,c}e^{(bx+a)}+1\right)+(i\,bx+i\,a)\log\left(-\frac{1}{2}\sqrt{-4}\right.}{}$$

input
```
integrate(arccot(c-(I-c)*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(-I*b^2*x^2 + I*b*x*log((c - I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2*a) - I
)) + I*a^2 + (I*b*x + I*a)*log(1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) + (I*b*x
+ I*a)*log(-1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) - I*a*log(1/2*(2*c*e^(b*x +
a) + I*sqrt(-4*I*c))/c) - I*a*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c))/c
) + I*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) + I*dilog(-1/2*sqrt(-4*I*c)*e^(b
*x + a)))/b
```

3.215.6 Sympy [F(-2)]

Exception generated.

$$\int \cot^{-1}(c-(i-c)\coth(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(acot(c-(I-c)*coth(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert 2*_t0**2*c*exp(2*a) - _
t0**2*I*exp(2*a) - I of type <class 'sympy.core.add.Add'> to QQ_I[b,c,_t0,
exp(a)]
```

3.215.7 Maxima [A] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\begin{aligned}
&\int \cot^{-1}(c-(i-c)\coth(a+bx))\,dx \\
&= 2\,b(c-i)\left(\frac{2\,x^2}{2i\,c+2}-\frac{2\,bx\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^2(-i\,c-1)}\right) \\
&\quad + x\,\mathrm{arccot}\left((c-i)\coth\left(bx+a\right)+c\right)
\end{aligned}$$

input
```
integrate(arccot(c-(I-c)*coth(b*x+a)),x, algorithm="maxima")
```

output
```
2*b*(c - I)*(2*x^2/(2*I*c + 2) - (2*b*x*log(I*c*e^(2*b*x + 2*a) + 1) + dil
og(-I*c*e^(2*b*x + 2*a)))/(b^2*(2*I*c + 2))) + x*arccot((c - I)*coth(b*x +
 a) + c)
```

3.215.8 Giac [F]

$$\int \cot^{-1}(c-(i-c)\coth(a+bx))\,dx=\int \operatorname{arccot}\left((c-i)\coth\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(arccot(c-(I-c)*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccot((c - I)*coth(b*x + a) + c), x)
```

3.215.9 Mupad [F(-1)]

Timed out.

$$\int \cot^{-1}(c-(i-c)\coth(a+bx))\,dx=\int \operatorname{acot}(c+\coth(a+b\,x)\,\,(c-\mathrm{i}))\,dx$$

input
```
int(acot(c + coth(a + b*x)*(c - 1i)),x)
```

output
```
int(acot(c + coth(a + b*x)*(c - 1i)), x)
```

3.216 $\int \frac{\cot^{-1}(c-(i-c)\coth(a+bx))}{x}\,dx$

3.216.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\cot^{-1}(c-(i-c)\coth(a+bx))}{x}\,dx = \text{Int}\left(\frac{\cot^{-1}(c-(i-c)\coth(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(arccot(c-(I-c)*coth(b*x+a))/x,x)
```

3.216.2 Mathematica [N/A]

Not integrable

Time = 3.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cot^{-1}(c-(i-c)\coth(a+bx))}{x}\,dx = \int \frac{\cot^{-1}(c-(i-c)\coth(a+bx))}{x}\,dx$$

input
```
Integrate[ArcCot[c - (I - c)*Coth[a + b*x]]/x,x]
```

output
```
Integrate[ArcCot[c - (I - c)*Coth[a + b*x]]/x, x]
```

3.216.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(c-(-c+i)\coth(a+bx))}{x}\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{\cot^{-1}(c-(-c+i)\coth(a+bx))}{x}dx$$

input
```
Int[ArcCot[c - (I - c)*Coth[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.216.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.216.4 Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arccot}\left(c-(i-c)\coth\left(bx+a\right)\right)}{x}dx$$

input
```
int(arccot(c-(I-c)*coth(b*x+a))/x,x)
```

output
```
int(arccot(c-(I-c)*coth(b*x+a))/x,x)
```

3.216.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int \frac{\cot^{-1}(c-(i-c)\coth(a+bx))}{x}\,dx = \int \frac{\operatorname{arccot}\left((c-i)\coth\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccot(c-(I-c)*coth(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(1/2*I*log((c - I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2*a) - I))/x, x)
```

3.216.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(c-(i-c)\coth(a+bx))}{x}\,dx = \text{Timed out}$$

input
```
integrate(acot(c-(I-c)*coth(b*x+a))/x,x)
```

output
```
Timed out
```

3.216.7 Maxima [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 81, normalized size of antiderivative = 3.68

$$\int \frac{\cot^{-1}(c-(i-c)\coth(a+bx))}{x}\,dx = \int \frac{\operatorname{arccot}\left((c-i)\coth\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccot(c-(I-c)*coth(b*x+a))/x,x, algorithm="maxima")
```

output
```
I*b*x + 1/2*pi*log(x) - 1/4*(2*pi - 4*I*a - 2*arctan(1/c) - I*log(c^2 + 1)
)*log(x) - 1/2*integrate(arctan(e^(-2*b*x - 2*a)/c)/x, x) - 1/4*I*integrat
e(log(c^2*e^(4*b*x + 4*a) + 1)/x, x)
```

3.216.8 Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\cot^{-1}(c-(i-c)\coth(a+bx))}{x}\,dx=\int \frac{\operatorname{arccot}\left((c-i)\coth\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccot(c-(I-c)*coth(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arccot((c - I)*coth(b*x + a) + c)/x, x)
```

3.216.9 Mupad [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\cot^{-1}(c-(i-c)\coth(a+bx))}{x}\,dx=\int \frac{\operatorname{acot}(c+\coth(a+b\,x)\,\,(c-\mathrm{i}))}{x}\,dx$$

input
```
int(acot(c + coth(a + b*x)*(c - 1i))/x,x)
```

output
```
int(acot(c + coth(a + b*x)*(c - 1i))/x, x)
```

3.217 $\int \frac{(a+b\cot^{-1}(cx^n))(d+e\log(fx^m))}{x}\,dx$

3.217.1 Optimal result

Integrand size = 24, antiderivative size = 187

$$\int \frac{(a+b\cot^{-1}(cx^n))(d+e\log(fx^m))}{x}\,dx = ad\log(x)+\frac{ae\log^2(fx^m)}{2m}-\frac{ibd\operatorname{PolyLog}\left(2,-\frac{ix^{-n}}{c}\right)}{2n}-\frac{ibe\log(fx^m)\operatorname{PolyLog}\left(2,-\frac{ix^{-n}}{c}\right)}{2n}+\frac{ibd\operatorname{PolyLog}\left(2,\frac{ix^{-n}}{c}\right)}{2n}+\frac{ibe\log(fx^m)\operatorname{PolyLog}\left(2,\frac{ix^{-n}}{c}\right)}{2n}-\frac{ibem\operatorname{PolyLog}\left(3,-\frac{ix^{-n}}{c}\right)}{2n^2}+\frac{ibem\operatorname{PolyLog}\left(3,\frac{ix^{-n}}{c}\right)}{2n^2}$$

output
```
a*d*ln(x)+1/2*a*e*ln(f*x^m)^2/m-1/2*I*b*d*polylog(2,-I/c/(x^n))/n-1/2*I*b*
e*ln(f*x^m)*polylog(2,-I/c/(x^n))/n+1/2*I*b*d*polylog(2,I/c/(x^n))/n+1/2*I
*b*e*ln(f*x^m)*polylog(2,I/c/(x^n))/n-1/2*I*b*e*m*polylog(3,-I/c/(x^n))/n^
2+1/2*I*b*e*m*polylog(3,I/c/(x^n))/n^2
```

3.217.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.26 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.71

$$\int \frac{(a+b\cot^{-1}(cx^n))(d+e\log(fx^m))}{x}\,dx = \frac{bcemx^n\,{}_4F_3\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},1;\frac{3}{2},\frac{3}{2},\frac{3}{2};-c^2x^{2n}\right)}{n^2} - \frac{bcx^n\,{}_3F_2\left(\frac{1}{2},\frac{1}{2},1;\frac{3}{2},\frac{3}{2};-c^2x^{2n}\right)(d+e\log(fx^m))}{n} - \frac{1}{2}\left(a+b\cot^{-1}(cx^n)+b\arctan(cx^n)\right)\log(x)\left(em\log(x)-2(d+e\log(fx^m))\right)$$

input
```
Integrate[((a + b*ArcCot[c*x^n])*(d + e*Log[f*x^m]))/x,x]
```

output
```
(b*c*e*m*x^n*HypergeometricPFQ[{1/2, 1/2, 1/2, 1}, {3/2, 3/2, 3/2}, -(c^2*
x^(2*n))])/n^2 - (b*c*x^n*HypergeometricPFQ[{1/2, 1/2, 1}, {3/2, 3/2}, -(c
^2*x^(2*n))]*(d + e*Log[f*x^m]))/n - ((a + b*ArcCot[c*x^n] + b*ArcTan[c*x^
n])*Log[x]*(e*m*Log[x] - 2*(d + e*Log[f*x^m])))/2
```

3.217.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cot^{-1}(cx^n))(d+e\log(fx^m))}{x}\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{d(a+b\cot^{-1}(cx^n))}{x}+\frac{e\log(fx^m)(a+b\cot^{-1}(cx^n))}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$ad\log(x)+\frac{ae\log^2(fx^m)}{2m}-\frac{ibd\,\text{PolyLog}\left(2,-\frac{ix^{-n}}{c}\right)}{2n}+\frac{ibd\,\text{PolyLog}\left(2,\frac{ix^{-n}}{c}\right)}{2n}-\frac{ibe\,\text{PolyLog}\left(2,-\frac{ix^{-n}}{c}\right)\log(fx^m)}{2n}+\frac{ibe\,\text{PolyLog}\left(2,\frac{ix^{-n}}{c}\right)\log(fx^m)}{2n}-\frac{ibem\,\text{PolyLog}\left(3,-\frac{ix^{-n}}{c}\right)}{2n^2}+\frac{ibem\,\text{PolyLog}\left(3,\frac{ix^{-n}}{c}\right)}{2n^2}$$

input
```
Int[((a + b*ArcCot[c*x^n])*(d + e*Log[f*x^m]))/x,x]
```

output
```
a*d*Log[x] + (a*e*Log[f*x^m]^2)/(2*m) - ((I/2)*b*d*PolyLog[2, (-I)/(c*x^n)
])/n - ((I/2)*b*e*Log[f*x^m]*PolyLog[2, (-I)/(c*x^n)])/n + ((I/2)*b*d*Poly
Log[2, I/(c*x^n)])/n + ((I/2)*b*e*Log[f*x^m]*PolyLog[2, I/(c*x^n)])/n - ((
I/2)*b*e*m*PolyLog[3, (-I)/(c*x^n)])/n^2 + ((I/2)*b*e*m*PolyLog[3, I/(c*x^
n)])/n^2
```

3.217.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.217.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 224.52 (sec) , antiderivative size = 566, normalized size of antiderivative = 3.03

method	result
risch	$\frac{\left(-\frac{ie\pi\,\text{csgn}(if)\,\text{csgn}(ix^m)\,\text{csgn}(if\,x^m)}{4}+\frac{ie\pi\,\text{csgn}(if)\,\text{csgn}(if\,x^m)^2}{4}+\frac{ie\pi\,\text{csgn}(ix^m)\,\text{csgn}(if\,x^m)^2}{4}-\frac{ie\pi\,\text{csgn}(if\,x^m)^3}{4}+\frac{e\ln(f)}{2}+\frac{d}{2}\right)((b\pi+2a)}{n}$

input
```
int((a+b*arccot(c*x^n))*(d+e*ln(f*x^m))/x,x,method=_RETURNVERBOSE)
```

output
```
(-1/4*I*e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/4*I*e*Pi*csgn(I*f)*csgn
(I*f*x^m)^2+1/4*I*e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2-1/4*I*e*Pi*csgn(I*f*x^m
)^3+1/2*e*ln(f)+1/2*d)/n*((Pi*b+2*a)*ln(x^n)-I*b*dilog(1+I*c*x^n)+I*b*dilo
g(1-I*c*x^n))+1/4*e/m*ln(x^m)^2*b*Pi+1/2*e/m*ln(x^m)^2*a+1/2*I*e*b*ln(x)*l
n(-I*(c*x^n+I))*ln(x^m)-1/2*I*e*b/n*dilog(-I*c*x^n)*m*ln(x)-1/2*I*e*b*ln(x
)*ln(1-I*c*x^n)*ln(x^m)-1/2*I*e*b*m/n^2*polylog(3,I*c*x^n)+1/2*I*e*b*ln(x)
^2*ln(1-I*c*x^n)*m-1/2*I*e*b*ln(x)*ln(-I*(-c*x^n+I))*ln(x^m)-1/2*I*e*b*ln(
x)^2*ln(1+I*c*x^n)*m+1/2*I*e*b*m/n*ln(x)*polylog(2,I*c*x^n)+1/2*I*e*b*ln(x
)^2*ln(-I*(-c*x^n+I))*m+1/2*I*e*b*m/n^2*polylog(3,-I*c*x^n)-1/2*I*e*b/n*di
log(-I*(c*x^n+I))*m*ln(x)-1/2*I*e*b*m/n*ln(x)*polylog(2,-I*c*x^n)-1/2*I*e*
b/n*ln(-I*(-c*x^n+I))*ln(-I*c*x^n)*m*ln(x)+1/2*I*e*b/n*dilog(-I*(c*x^n+I))
*ln(x^m)+1/2*I*e*b/n*ln(-I*(-c*x^n+I))*ln(-I*c*x^n)*ln(x^m)+1/2*I*e*b*ln(x
)*ln(1+I*c*x^n)*ln(x^m)-1/2*I*e*b*ln(x)^2*ln(-I*(c*x^n+I))*m+1/2*I*e*b/n*d
ilog(-I*c*x^n)*ln(x^m)
```

3.217.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.34

$$\int \frac{(a + b\cot^{-1}(cx^n))\,(d + e\log(fx^m))}{x}\,dx$$
$$= \frac{2\,aemn^2\log(x)^2 - 2i\,bem\text{polylog}(3, i\,cx^n) + 2i\,bem\text{polylog}(3, -i\,cx^n) + 2\left(bemn^2\log(x)^2 + 2\,(ben^2\log(\ldots\right.}{}$$

input
```
integrate((a+b*arccot(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="fricas")
```

output
```
1/4*(2*a*e*m*n^2*log(x)^2 - 2*I*b*e*m*polylog(3, I*c*x^n) + 2*I*b*e*m*poly
log(3, -I*c*x^n) + 2*(b*e*m*n^2*log(x)^2 + 2*(b*e*n^2*log(f) + b*d*n^2)*lo
g(x))*arccot(c*x^n) - 2*(-I*b*e*m*n*log(x) - I*b*e*n*log(f) - I*b*d*n)*dil
og(I*c*x^n) - 2*(I*b*e*m*n*log(x) + I*b*e*n*log(f) + I*b*d*n)*dilog(-I*c*x
^n) + (-I*b*e*m*n^2*log(x)^2 - 2*(I*b*e*n^2*log(f) + I*b*d*n^2)*log(x))*lo
g(I*c*x^n + 1) + (I*b*e*m*n^2*log(x)^2 - 2*(-I*b*e*n^2*log(f) - I*b*d*n^2)
*log(x))*log(-I*c*x^n + 1) + 4*(a*e*n^2*log(f) + a*d*n^2)*log(x))/n^2
```

3.217.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cot^{-1}(cx^n))\,(d+e\log(fx^m))}{x}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acot(c*x**n))*(d+e*ln(f*x**m))/x,x)
```

output
```
Timed out
```

3.217.7 Maxima [F]

$$\int \frac{(a+b\cot^{-1}(cx^n))\,(d+e\log(fx^m))}{x}\,dx = \int \frac{(b\,\mathrm{arccot}\,(cx^n)+a)(e\log(fx^m)+d)}{x}\,dx$$

input
```
integrate((a+b*arccot(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="maxima")
```

output
```
1/2*a*e*log(f*x^m)^2/m + a*d*log(x) - 1/2*(b*e*m*log(x)^2 - 2*b*e*log(x)*l
og(x^m) - 2*(b*e*log(f) + b*d)*log(x))*arctan(1/(c*x^n)) + integrate(-1/2*
(b*c*e*m*n*x^n*log(x)^2 - 2*b*c*e*n*x^n*log(x)*log(x^m) - 2*(b*c*e*log(f)
+ b*c*d)*n*x^n*log(x))/(c^2*x*x^(2*n) + x), x)
```

3.217.8 Giac [F]

$$\int \frac{(a+b\cot^{-1}(cx^n))\,(d+e\log(fx^m))}{x}\,dx = \int \frac{(b\,\mathrm{arccot}\,(cx^n)+a)(e\log(fx^m)+d)}{x}\,dx$$

input
```
integrate((a+b*arccot(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="giac")
```

output
```
integrate((b*arccot(c*x^n) + a)*(e*log(f*x^m) + d)/x, x)
```

3.217.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cot^{-1}(cx^n))\,(d+e\log(fx^m))}{x}\,dx=\int \frac{(a+b\,\mathrm{acot}(c\,x^n))\,\left(d+e\,\ln\left(f\,x^m\right)\right)}{x}\,dx$$

input
```
int(((a + b*acot(c*x^n))*(d + e*log(f*x^m)))/x,x)
```

output
```
int(((a + b*acot(c*x^n))*(d + e*log(f*x^m)))/x, x)
```

3.218 $\int \cot^{-1}(e^x)\,dx$

3.218.1 Optimal result

Integrand size = 4, antiderivative size = 35

$$\int \cot^{-1}(e^x)\,dx=-\frac{1}{2}i\,\text{PolyLog}\left(2,-ie^{-x}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,ie^{-x}\right)$$

output
```
-1/2*I*polylog(2,-I/exp(x))+1/2*I*polylog(2,I/exp(x))
```

3.218.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \cot^{-1}(e^x)\,dx=x\cot^{-1}(e^x)+\frac{1}{2}i\left(x\left(\log\left(1-ie^x\right)-\log\left(1+ie^x\right)\right)-\text{PolyLog}\left(2,-ie^x\right)+\text{PolyLog}\left(2,ie^x\right)\right)$$

input
```
Integrate[ArcCot[E^x],x]
```

output
```
x*ArcCot[E^x] + (I/2)*(x*(Log[1 - I*E^x] - Log[1 + I*E^x]) - PolyLog[2, (-
I)*E^x] + PolyLog[2, I*E^x])
```

3.218.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {2720, 5356, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}(e^x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int e^{-x}\cot^{-1}(e^x)\,de^x$$

$$\Big\downarrow \text{5356}$$

$$\frac{1}{2}i\int e^{-x}\log\left(1-ie^{-x}\right)de^x-\frac{1}{2}i\int e^{-x}\log\left(1+ie^{-x}\right)de^x$$

$$\Big\downarrow \text{2838}$$

$$\frac{1}{2}i\,\text{PolyLog}\left(2,ie^{-x}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,-ie^{-x}\right)$$

input
```
Int[ArcCot[E^x],x]
```

output
```
(-1/2*I)*PolyLog[2, (-I)/E^x] + (I/2)*PolyLog[2, I/E^x]
```

3.218.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5356
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Simp[I*(b/2)   Int[Log[1 + I/(c*x)]/x, x], x] + Simp[I*(b/2)   Int[Log
[1 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

3.218.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(25) = 50.

Time = 0.82 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.51

method	result	size
parts	$x \operatorname{arccot}(e^{x})-\frac{i x \ln(1+i e^{x})}{2}+\frac{i x \ln(1-i e^{x})}{2}-\frac{i \operatorname{dilog}(1+i e^{x})}{2}+\frac{i \operatorname{dilog}(1-i e^{x})}{2}$	53
derivativedivides	$\ln(e^{x}) \operatorname{arccot}(e^{x})-\frac{i \ln(e^{x}) \ln(1+i e^{x})}{2}+\frac{i \ln(e^{x}) \ln(1-i e^{x})}{2}-\frac{i \operatorname{dilog}(1+i e^{x})}{2}+\frac{i \operatorname{dilog}(1-i e^{x})}{2}$	59
default	$\ln(e^{x}) \operatorname{arccot}(e^{x})-\frac{i \ln(e^{x}) \ln(1+i e^{x})}{2}+\frac{i \ln(e^{x}) \ln(1-i e^{x})}{2}-\frac{i \operatorname{dilog}(1+i e^{x})}{2}+\frac{i \operatorname{dilog}(1-i e^{x})}{2}$	59
risch	$\frac{i x \ln(1+i e^{x})}{2}+\frac{\pi x}{2}+\frac{i \operatorname{dilog}(1-i e^{x})}{2}+\frac{i \ln(-i e^{x}) \ln(-i(-e^{x}+i))}{2}-\frac{i \ln(-i(-e^{x}+i)) x}{2}+\frac{i \operatorname{dilog}(-i e^{x})}{2}$	73

input
```
int(arccot(exp(x)),x,method=_RETURNVERBOSE)
```

output
```
x*arccot(exp(x))-1/2*I*x*ln(1+I*exp(x))+1/2*I*x*ln(1-I*exp(x))-1/2*I*dilog
(1+I*exp(x))+1/2*I*dilog(1-I*exp(x))
```

3.218.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(19) = 38.

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

$$\int \cot^{-1}(e^{x})\,dx = x \operatorname{arccot}(e^{x}) - \frac{1}{2} i\, x \log(i\, e^{x}+1) + \frac{1}{2} i\, x \log(-i\, e^{x}+1) + \frac{1}{2} i \operatorname{Li}_2(i\, e^{x}) - \frac{1}{2} i \operatorname{Li}_2(-i\, e^{x})$$

input
```
integrate(arccot(exp(x)),x, algorithm="fricas")
```

output
```
x*arccot(e^x) - 1/2*I*x*log(I*e^x + 1) + 1/2*I*x*log(-I*e^x + 1) + 1/2*I*d
ilog(I*e^x) - 1/2*I*dilog(-I*e^x)
```

3.218.6 Sympy [F]

$$\int \cot^{-1}(e^x)\,dx = \int \operatorname{acot}(e^x)\,dx$$

input
```
integrate(acot(exp(x)),x)
```

output
```
Integral(acot(exp(x)), x)
```

3.218.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \cot^{-1}(e^x)\,dx = x\operatorname{arccot}(e^x) + \frac{1}{4}\pi\log\left(e^{(2x)} + 1\right) + \frac{1}{2}i\operatorname{Li}_2(ie^x + 1) - \frac{1}{2}i\operatorname{Li}_2(-ie^x + 1)$$

input
```
integrate(arccot(exp(x)),x, algorithm="maxima")
```

output
```
x*arccot(e^x) + 1/4*pi*log(e^(2*x) + 1) + 1/2*I*dilog(I*e^x + 1) - 1/2*I*d
ilog(-I*e^x + 1)
```

3.218.8 Giac [F]

$$\int \cot^{-1}(e^x)\,dx = \int \operatorname{arccot}(e^x)\,dx$$

input
```
integrate(arccot(exp(x)),x, algorithm="giac")
```

output
```
integrate(arccot(e^x), x)
```

3.218.9 Mupad [F(-1)]

Timed out.

$$\int \cot^{-1}(e^x)\,dx = \int \operatorname{acot}(\mathrm{e}^x)\,dx$$

input
```
int(acot(exp(x)),x)
```

output
```
int(acot(exp(x)), x)
```

3.219 $\int x \cot^{-1}(e^x)\,dx$

3.219.1 Optimal result

Integrand size = 6, antiderivative size = 71

$$\int x \cot^{-1}(e^x)\,dx = -\frac{1}{2}ix\,\text{PolyLog}\left(2,-ie^{-x}\right)+\frac{1}{2}ix\,\text{PolyLog}\left(2,ie^{-x}\right) -\frac{1}{2}i\,\text{PolyLog}\left(3,-ie^{-x}\right)+\frac{1}{2}i\,\text{PolyLog}\left(3,ie^{-x}\right)$$

output
```
-1/2*I*x*polylog(2,-I/exp(x))+1/2*I*x*polylog(2,I/exp(x))-1/2*I*polylog(3,
-I/exp(x))+1/2*I*polylog(3,I/exp(x))
```

3.219.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.82

$$\int x \cot^{-1}(e^x)\,dx = -\frac{1}{2}i\left(x\,\text{PolyLog}\left(2,-ie^{-x}\right)-x\,\text{PolyLog}\left(2,ie^{-x}\right)+\text{PolyLog}\left(3,-ie^{-x}\right) -\text{PolyLog}\left(3,ie^{-x}\right)\right)$$

input
```
Integrate[x*ArcCot[E^x],x]
```

output
```
(-1/2*I)*(x*PolyLog[2, (-I)/E^x] - x*PolyLog[2, I/E^x] + PolyLog[3, (-I)/E
^x] - PolyLog[3, I/E^x])
```

3.219.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5667, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cot^{-1}(e^x)\,dx$$

$$\Big\downarrow \text{5667}$$

$$\frac{1}{2}i\int x\log\left(1-ie^{-x}\right)dx-\frac{1}{2}i\int x\log\left(1+ie^{-x}\right)dx$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}i\left(x\,\text{PolyLog}\left(2,ie^{-x}\right)-\int \text{PolyLog}\left(2,ie^{-x}\right)dx\right)-\frac{1}{2}i\left(x\,\text{PolyLog}\left(2,-ie^{-x}\right)-\int \text{PolyLog}\left(2,-ie^{-x}\right)dx\right)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}i\left(\int e^x\,\text{PolyLog}\left(2,ie^{-x}\right)de^{-x}+x\,\text{PolyLog}\left(2,ie^{-x}\right)\right)-\frac{1}{2}i\left(\int e^x\,\text{PolyLog}\left(2,-ie^{-x}\right)de^{-x}+x\,\text{PolyLog}\left(2,-ie^{-x}\right)\right)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}i\left(x\,\text{PolyLog}\left(2,ie^{-x}\right)+\text{PolyLog}\left(3,ie^{-x}\right)\right)-\frac{1}{2}i\left(x\,\text{PolyLog}\left(2,-ie^{-x}\right)+\text{PolyLog}\left(3,-ie^{-x}\right)\right)$$

input
```
Int[x*ArcCot[E^x],x]
```

output
```
(-1/2*I)*(x*PolyLog[2, (-I)/E^x] + PolyLog[3, (-I)/E^x]) + (I/2)*(x*PolyLo
g[2, I/E^x] + PolyLog[3, I/E^x])
```

3.219.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5667
```
Int[ArcCot[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Simp[I/2   Int[x^m*Log[1 - I/(a + b*f^(c + d*x))], x], x] - Simp[I/2   In
t[x^m*Log[1 + I/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] &
& IntegerQ[m] && m > 0
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.219.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.70

method	result	size
risch	$\frac{\pi x^2}{4}+\frac{i \operatorname{polylog}(2, i e^x) x}{2}-\frac{i \operatorname{polylog}(3, i e^x)}{2}-\frac{i x \operatorname{polylog}(2,-i e^x)}{2}+\frac{i \operatorname{polylog}(3,-i e^x)}{2}$	50

input
```
int(x*arccot(exp(x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*Pi*x^2+1/2*I*polylog(2,I*exp(x))*x-1/2*I*polylog(3,I*exp(x))-1/2*I*x*p
olylog(2,-I*exp(x))+1/2*I*polylog(3,-I*exp(x))
```

3.219.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int x\cot^{-1}(e^x)\,dx = \frac{1}{2}x^2\operatorname{arccot}(e^x) - \frac{1}{4}i\,x^2\log(i\,e^x+1) + \frac{1}{4}i\,x^2\log(-i\,e^x+1) + \frac{1}{2}i\,x\mathrm{Li}_2(i\,e^x) - \frac{1}{2}i\,x\mathrm{Li}_2(-i\,e^x) - \frac{1}{2}i\operatorname{polylog}(3,i\,e^x) + \frac{1}{2}i\operatorname{polylog}(3,-i\,e^x)$$

input
```
integrate(x*arccot(exp(x)),x, algorithm="fricas")
```

output
```
1/2*x^2*arccot(e^x) - 1/4*I*x^2*log(I*e^x + 1) + 1/4*I*x^2*log(-I*e^x + 1)
 + 1/2*I*x*dilog(I*e^x) - 1/2*I*x*dilog(-I*e^x) - 1/2*I*polylog(3, I*e^x)
+ 1/2*I*polylog(3, -I*e^x)
```

3.219.6 Sympy [F]

$$\int x\cot^{-1}(e^x)\,dx = \int x\operatorname{acot}(e^x)\,dx$$

input
```
integrate(x*acot(exp(x)),x)
```

output
```
Integral(x*acot(exp(x)), x)
```

3.219.7 Maxima [F]

$$\int x\cot^{-1}(e^x)\,dx = \int x\operatorname{arccot}(e^x)\,dx$$

input
```
integrate(x*arccot(exp(x)),x, algorithm="maxima")
```

output
```
1/2*x^2*arctan(e^(-x)) + integrate(1/2*x^2*e^x/(e^(2*x) + 1), x)
```

3.219.8 Giac [F]

$$\int x \cot^{-1}(e^x)\,dx = \int x \operatorname{arccot}(e^x)\,dx$$

input
```
integrate(x*arccot(exp(x)),x, algorithm="giac")
```

output
```
integrate(x*arccot(e^x), x)
```

3.219.9 Mupad [F(-1)]

Timed out.

$$\int x \cot^{-1}(e^x)\,dx = \int x \operatorname{acot}(e^x)\,dx$$

input
```
int(x*acot(exp(x)),x)
```

output
```
int(x*acot(exp(x)), x)
```

3.220 $\int x^2 \cot^{-1}(e^x)\, dx$

3.220.1 Optimal result

Integrand size = 8, antiderivative size = 103

$$\int x^2 \cot^{-1}(e^x)\, dx = -\frac{1}{2} i x^2 \operatorname{PolyLog}\left(2,-ie^{-x}\right) + \frac{1}{2} i x^2 \operatorname{PolyLog}\left(2,ie^{-x}\right) - ix \operatorname{PolyLog}\left(3,-ie^{-x}\right) + ix \operatorname{PolyLog}\left(3,ie^{-x}\right) - i \operatorname{PolyLog}\left(4,-ie^{-x}\right) + i \operatorname{PolyLog}\left(4,ie^{-x}\right)$$

output
```
-1/2*I*x^2*polylog(2,-I/exp(x))+1/2*I*x^2*polylog(2,I/exp(x))-I*x*polylog(
3,-I/exp(x))+I*x*polylog(3,I/exp(x))-I*polylog(4,-I/exp(x))+I*polylog(4,I/
exp(x))
```

3.220.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.89

$$\int x^2 \cot^{-1}(e^x)\, dx = -\frac{1}{2} i \left(x^2 \operatorname{PolyLog}\left(2,-ie^{-x}\right) - x^2 \operatorname{PolyLog}\left(2,ie^{-x}\right) + 2\left(x \operatorname{PolyLog}\left(3,-ie^{-x}\right) - x \operatorname{PolyLog}\left(3,ie^{-x}\right) + \operatorname{PolyLog}\left(4,-ie^{-x}\right) - \operatorname{PolyLog}\left(4,ie^{-x}\right)\right)\right)$$

input
```
Integrate[x^2*ArcCot[E^x],x]
```

output
```
(-1/2*I)*(x^2*PolyLog[2, (-I)/E^x] - x^2*PolyLog[2, I/E^x] + 2*(x*PolyLog[
3, (-I)/E^x] - x*PolyLog[3, I/E^x] + PolyLog[4, (-I)/E^x] - PolyLog[4, I/E
^x]))
```

3.220.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5667, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cot^{-1}\left(e^x\right) dx$$

$$\Big\downarrow \text{ 5667}$$

$$\frac{1}{2} i \int x^2 \log \left(1-i e^{-x}\right) dx-\frac{1}{2} i \int x^2 \log \left(1+i e^{-x}\right) dx$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{1}{2} i \left(x^2 \text{PolyLog}\left(2,i e^{-x}\right)-2 \int x \text{PolyLog}\left(2,i e^{-x}\right) dx\right)-\frac{1}{2} i \left(x^2 \text{PolyLog}\left(2,-i e^{-x}\right)-2 \int x \text{PolyLog}\left(2,-i e^{-x}\right) dx\right)$$

$$\Big\downarrow \text{ 7163}$$

$$\frac{1}{2} i \left(x^2 \text{PolyLog}\left(2,i e^{-x}\right)-2 \left(\int \text{PolyLog}\left(3,i e^{-x}\right) dx-x \text{PolyLog}\left(3,i e^{-x}\right)\right)\right)-\frac{1}{2} i \left(x^2 \text{PolyLog}\left(2,-i e^{-x}\right)-2 \left(\int \text{PolyLog}\left(3,-i e^{-x}\right) dx-x \text{PolyLog}\left(3,-i e^{-x}\right)\right)\right)$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{1}{2} i \left(x^2 \text{PolyLog}\left(2,i e^{-x}\right)-2 \left(-\int e^x \text{PolyLog}\left(3,i e^{-x}\right) de^{-x}-x \text{PolyLog}\left(3,i e^{-x}\right)\right)\right)-\frac{1}{2} i \left(x^2 \text{PolyLog}\left(2,-i e^{-x}\right)-2 \left(-\int e^x \text{PolyLog}\left(3,-i e^{-x}\right) de^{-x}-x \text{PolyLog}\left(3,-i e^{-x}\right)\right)\right)$$

$$\Big\downarrow \text{ 7143}$$

$$\frac{1}{2} i (x^2 \text{PolyLog}\left(2,i e^{-x}\right)-2(-x \text{PolyLog}\left(3,i e^{-x}\right)-\text{PolyLog}\left(4,i e^{-x}\right)))-\frac{1}{2} i (x^2 \text{PolyLog}\left(2,-i e^{-x}\right)-2(-x \text{PolyLog}\left(3,-i e^{-x}\right)-\text{PolyLog}\left(4,-i e^{-x}\right)))$$

input
```
Int[x^2*ArcCot[E^x],x]
```

output
```
(-1/2*I)*(x^2*PolyLog[2, (-I)/E^x] - 2*(-(x*PolyLog[3, (-I)/E^x]) - PolyLo
g[4, (-I)/E^x])) + (I/2)*(x^2*PolyLog[2, I/E^x] - 2*(-(x*PolyLog[3, I/E^x]
) - PolyLog[4, I/E^x]))
```

3.220.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5667
```
Int[ArcCot[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Simp[I/2   Int[x^m*Log[1 - I/(a + b*f^(c + d*x))], x], x] - Simp[I/2   In
t[x^m*Log[1 + I/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] &
& IntegerQ[m] && m > 0
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.220. $\int x^2 \cot^{-1}(e^x) \, dx$

3.220.4 Maple [A] (verified)

Time = 1.69 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.74

method	result
risch	$\frac{\pi x^3}{6}+\frac{ix^2 \operatorname{polylog}(2,ie^x)}{2}-ix \operatorname{polylog}(3,ie^x)+i \operatorname{polylog}(4,ie^x)-\frac{ix^2 \operatorname{polylog}(2,-ie^x)}{2}+ix \operatorname{polylog}(3,-$

input
```
int(x^2*arccot(exp(x)),x,method=_RETURNVERBOSE)
```

output
```
1/6*Pi*x^3+1/2*I*polylog(2,I*exp(x))*x^2-I*x*polylog(3,I*exp(x))+I*polylog
(4,I*exp(x))-1/2*I*x^2*polylog(2,-I*exp(x))+I*polylog(3,-I*exp(x))*x-I*pol
ylog(4,-I*exp(x))
```

3.220.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\begin{aligned}\int x^2\cot^{-1}(e^x)\,dx &= \frac{1}{3}x^3\operatorname{arccot}(e^x)-\frac{1}{6}i\,x^3\log(i\,e^x+1)+\frac{1}{6}i\,x^3\log(-i\,e^x+1)\\&\quad+\frac{1}{2}i\,x^2\mathrm{Li}_2(i\,e^x)-\frac{1}{2}i\,x^2\mathrm{Li}_2(-i\,e^x)-i\,x\operatorname{polylog}(3,i\,e^x)\\&\quad+i\,x\operatorname{polylog}(3,-i\,e^x)+i\operatorname{polylog}(4,i\,e^x)-i\operatorname{polylog}(4,-i\,e^x)\end{aligned}$$

input
```
integrate(x^2*arccot(exp(x)),x, algorithm="fricas")
```

output
```
1/3*x^3*arccot(e^x) - 1/6*I*x^3*log(I*e^x + 1) + 1/6*I*x^3*log(-I*e^x + 1)
 + 1/2*I*x^2*dilog(I*e^x) - 1/2*I*x^2*dilog(-I*e^x) - I*x*polylog(3, I*e^x
) + I*x*polylog(3, -I*e^x) + I*polylog(4, I*e^x) - I*polylog(4, -I*e^x)
```

3.220.6 Sympy [F]

$$\int x^2\cot^{-1}(e^x)\,dx=\int x^2\operatorname{acot}(e^x)\,dx$$

input
```
integrate(x**2*acot(exp(x)),x)
```

output
```
Integral(x**2*acot(exp(x)), x)
```

3.220.7 Maxima [F]

$$\int x^2 \cot^{-1}(e^x)\, dx = \int x^2 \operatorname{arccot}(e^x)\, dx$$

input
```
integrate(x^2*arccot(exp(x)),x, algorithm="maxima")
```

output
```
1/3*x^3*arctan(e^(-x)) + integrate(1/3*x^3*e^x/(e^(2*x) + 1), x)
```

3.220.8 Giac [F]

$$\int x^2 \cot^{-1}(e^x)\, dx = \int x^2 \operatorname{arccot}(e^x)\, dx$$

input
```
integrate(x^2*arccot(exp(x)),x, algorithm="giac")
```

output
```
integrate(x^2*arccot(e^x), x)
```

3.220.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(e^x)\, dx = \int x^2 \operatorname{acot}(\mathrm{e}^x)\, dx$$

input
```
int(x^2*acot(exp(x)),x)
```

output
```
int(x^2*acot(exp(x)), x)
```

3.221 $\int \cot^{-1}\left(e^{a+bx}\right) dx$

3.221.1 Optimal result

Integrand size = 8, antiderivative size = 51

$$\int \cot^{-1}\left(e^{a+bx}\right) dx = -\frac{i \operatorname{PolyLog}\left(2,-ie^{-a-bx}\right)}{2b} + \frac{i \operatorname{PolyLog}\left(2,ie^{-a-bx}\right)}{2b}$$

output
```
-1/2*I*polylog(2,-I*exp(-b*x-a))/b+1/2*I*polylog(2,I*exp(-b*x-a))/b
```

3.221.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.63

$$\int \cot^{-1}\left(e^{a+bx}\right) dx = x\cot^{-1}\left(e^{a+bx}\right) + \frac{i\left(bx\left(\log\left(1-ie^{a+bx}\right)-\log\left(1+ie^{a+bx}\right)\right)-\operatorname{PolyLog}\left(2,-ie^{a+bx}\right)+\operatorname{PolyLog}\left(2,ie^{a+bx}\right)\right)}{2b}$$

input
```
Integrate[ArcCot[E^(a + b*x)],x]
```

output
```
x*ArcCot[E^(a + b*x)] + ((I/2)*(b*x*(Log[1 - I*E^(a + b*x)] - Log[1 + I*E^
(a + b*x)]) - PolyLog[2, (-I)*E^(a + b*x)] + PolyLog[2, I*E^(a + b*x)]))/b
```

3.221.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2720, 5356, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}\left(e^{a+bx}\right) dx$$

$$\Big\downarrow \ 2720$$

$$\frac{\int e^{-a-bx}\cot^{-1}\left(e^{a+bx}\right)de^{a+bx}}{b}$$

$$\Big\downarrow \ 5356$$

$$\frac{\frac{1}{2}i\int e^{-a-bx}\log\left(1-ie^{-a-bx}\right)de^{a+bx}-\frac{1}{2}i\int e^{-a-bx}\log\left(1+ie^{-a-bx}\right)de^{a+bx}}{b}$$

$$\Big\downarrow \ 2838$$

$$\frac{\frac{1}{2}i\,\text{PolyLog}\left(2,ie^{-a-bx}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,-ie^{-a-bx}\right)}{b}$$

input
```
Int[ArcCot[E^(a + b*x)],x]
```

output
```
((-1/2*I)*PolyLog[2, (-I)*E^(-a - b*x)] + (I/2)*PolyLog[2, I*E^(-a - b*x)]
)/b
```

3.221.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5356
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Simp[I*(b/2)   Int[Log[1 + I/(c*x)]/x, x], x] + Simp[I*(b/2)   Int[Log
[1 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

3.221.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(41) = 82.

Time = 0.35 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.86

method	result
derivativedivides	$\frac{\ln\left(\mathrm{e}^{bx+a}\right)\operatorname{arccot}\left(\mathrm{e}^{bx+a}\right)-\frac{i\ln\left(\mathrm{e}^{bx+a}\right)\ln\left(1+i\mathrm{e}^{bx+a}\right)}{2}+\frac{i\ln\left(\mathrm{e}^{bx+a}\right)\ln\left(1-i\mathrm{e}^{bx+a}\right)}{2}-\frac{i\operatorname{dilog}\left(1+i\mathrm{e}^{bx+a}\right)}{2}+\frac{i\operatorname{dilog}\left(1-i\mathrm{e}^{bx+a}\right)}{2}}{b}$
default	$\frac{\ln\left(\mathrm{e}^{bx+a}\right)\operatorname{arccot}\left(\mathrm{e}^{bx+a}\right)-\frac{i\ln\left(\mathrm{e}^{bx+a}\right)\ln\left(1+i\mathrm{e}^{bx+a}\right)}{2}+\frac{i\ln\left(\mathrm{e}^{bx+a}\right)\ln\left(1-i\mathrm{e}^{bx+a}\right)}{2}-\frac{i\operatorname{dilog}\left(1+i\mathrm{e}^{bx+a}\right)}{2}+\frac{i\operatorname{dilog}\left(1-i\mathrm{e}^{bx+a}\right)}{2}}{b}$
parts	$x\operatorname{arccot}\left(\mathrm{e}^{bx+a}\right)+\frac{-\frac{i(bx+a)\ln\left(1+i\mathrm{e}^{bx+a}\right)}{2}+\frac{i(bx+a)\ln\left(1-i\mathrm{e}^{bx+a}\right)}{2}-\frac{i\operatorname{dilog}\left(1+i\mathrm{e}^{bx+a}\right)}{2}+\frac{i\operatorname{dilog}\left(1-i\mathrm{e}^{bx+a}\right)}{2}-a\operatorname{arcta}}{b}$
risch	$\frac{ix\ln\left(1+i\mathrm{e}^{bx+a}\right)}{2}+\frac{\pi x}{2}+\frac{i\operatorname{dilog}\left(1-i\mathrm{e}^{bx+a}\right)}{2b}-\frac{i\ln\left(-i\left(-\mathrm{e}^{bx+a}+i\right)\right)x}{2}+\frac{ia\ln\left(1+i\mathrm{e}^{bx+a}\right)}{2b}+\frac{i\ln\left(-i\mathrm{e}^{bx+a}\right)\ln(-i}{2b}$

input
```
int(arccot(exp(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b*(ln(exp(b*x+a))*arccot(exp(b*x+a))-1/2*I*ln(exp(b*x+a))*ln(1+I*exp(b*x
+a))+1/2*I*ln(exp(b*x+a))*ln(1-I*exp(b*x+a))-1/2*I*dilog(1+I*exp(b*x+a))+1
/2*I*dilog(1-I*exp(b*x+a)))
```

3.221.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(35) = 70.

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.02

$$\int \cot^{-1}\left(e^{a+bx}\right)\,dx$$
$$=\frac{2\,bx\operatorname{arccot}\left(e^{(bx+a)}\right)-i\,a\log\left(e^{(bx+a)}+i\right)+i\,a\log\left(e^{(bx+a)}-i\right)+(-i\,bx-i\,a)\log\left(i\,e^{(bx+a)}+1\right)+(i\,bx-}{2\,b}$$

input
```
integrate(arccot(exp(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(2*b*x*arccot(e^(b*x + a)) - I*a*log(e^(b*x + a) + I) + I*a*log(e^(b*x
 + a) - I) + (-I*b*x - I*a)*log(I*e^(b*x + a) + 1) + (I*b*x + I*a)*log(-I*
e^(b*x + a) + 1) + I*dilog(I*e^(b*x + a)) - I*dilog(-I*e^(b*x + a)))/b
```

3.221.6 Sympy [F]

$$\int \cot^{-1}\left(e^{a+bx}\right)\,dx = \int \operatorname{acot}\left(e^{a+bx}\right)dx$$

input
```
integrate(acot(exp(b*x+a)),x)
```

output
```
Integral(acot(exp(a + b*x)), x)
```

3.221.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24

$$\int \cot^{-1}\left(e^{a+bx}\right)\,dx = \frac{(bx+a)\operatorname{arccot}\left(e^{(bx+a)}\right)}{b} + \frac{\pi\log\left(e^{(2\,bx+2\,a)}+1\right)+2i\operatorname{Li}_2\left(i\,e^{(bx+a)}+1\right)-2i\operatorname{Li}_2\left(-i\,e^{(bx+a)}+1\right)}{4\,b}$$

input
```
integrate(arccot(exp(b*x+a)),x, algorithm="maxima")
```

output
```
(b*x + a)*arccot(e^(b*x + a))/b + 1/4*(pi*log(e^(2*b*x + 2*a) + 1) + 2*I*d
ilog(I*e^(b*x + a) + 1) - 2*I*dilog(-I*e^(b*x + a) + 1))/b
```

3.221.8 Giac [F]

$$\int \cot^{-1}\left(e^{a+bx}\right)\,dx = \int \operatorname{arccot}\left(e^{(bx+a)}\right)dx$$

input
```
integrate(arccot(exp(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccot(e^(b*x + a)), x)
```

3.221.9 Mupad [F(-1)]

Timed out.

$$\int \cot^{-1}\left(e^{a+bx}\right) \, dx = \int \mathrm{acot}\left(\mathrm{e}^{a+b\,x}\right) \, dx$$

input
```
int(acot(exp(a + b*x)),x)
```

output
```
int(acot(exp(a + b*x)), x)
```

3.222 $\int x\cot^{-1}\left(e^{a+bx}\right)\,dx$

3.222.1 Optimal result

Integrand size = 10, antiderivative size = 103

$$\int x\cot^{-1}\left(e^{a+bx}\right)\,dx=-\frac{ix\,\text{PolyLog}\left(2,-ie^{-a-bx}\right)}{2b}+\frac{ix\,\text{PolyLog}\left(2,ie^{-a-bx}\right)}{2b}-\frac{i\,\text{PolyLog}\left(3,-ie^{-a-bx}\right)}{2b^2}+\frac{i\,\text{PolyLog}\left(3,ie^{-a-bx}\right)}{2b^2}$$

output
```
-1/2*I*x*polylog(2,-I*exp(-b*x-a))/b+1/2*I*x*polylog(2,I*exp(-b*x-a))/b-1/
2*I*polylog(3,-I*exp(-b*x-a))/b^2+1/2*I*polylog(3,I*exp(-b*x-a))/b^2
```

3.222.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.81

$$\int x\cot^{-1}\left(e^{a+bx}\right)\,dx=-\frac{i\left(bx\,\text{PolyLog}\left(2,-ie^{-a-bx}\right)-bx\,\text{PolyLog}\left(2,ie^{-a-bx}\right)+\text{PolyLog}\left(3,-ie^{-a-bx}\right)-\text{PolyLog}\left(3,ie^{-a-bx}\right)\right)}{2b^2}$$

input
```
Integrate[x*ArcCot[E^(a + b*x)],x]
```

output
```
((-1/2*I)*(b*x*PolyLog[2, (-I)*E^(-a - b*x)] - b*x*PolyLog[2, I*E^(-a - b*
x)] + PolyLog[3, (-I)*E^(-a - b*x)] - PolyLog[3, I*E^(-a - b*x)]))/b^2
```

3.222.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5667, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \cot^{-1}\left(e^{a+bx}\right) dx$$

$$\Big\downarrow \text{5667}$$

$$\frac{1}{2} i \int x \log \left(1-i e^{-a-bx}\right) dx-\frac{1}{2} i \int x \log \left(1+i e^{-a-bx}\right) dx$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2} i \left(\frac{x \text{PolyLog}\left(2,i e^{-a-bx}\right)}{b}-\frac{\int \text{PolyLog}\left(2,i e^{-a-bx}\right)dx}{b}\right)-\frac{1}{2} i \left(\frac{x \text{PolyLog}\left(2,-i e^{-a-bx}\right)}{b}-\frac{\int \text{PolyLog}\left(2,-i e^{-a-bx}\right)dx}{b}\right)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2} i \left(\frac{\int e^{a+bx} \text{PolyLog}\left(2,i e^{-a-bx}\right)de^{-a-bx}}{b^2}+\frac{x \text{PolyLog}\left(2,i e^{-a-bx}\right)}{b}\right)-\frac{1}{2} i \left(\frac{\int e^{a+bx} \text{PolyLog}\left(2,-i e^{-a-bx}\right)de^{-a-bx}}{b^2}+\frac{x \text{PolyLog}\left(2,-i e^{-a-bx}\right)}{b}\right)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2} i \left(\frac{\text{PolyLog}\left(3,i e^{-a-bx}\right)}{b^2}+\frac{x \text{PolyLog}\left(2,i e^{-a-bx}\right)}{b}\right)-\frac{1}{2} i \left(\frac{\text{PolyLog}\left(3,-i e^{-a-bx}\right)}{b^2}+\frac{x \text{PolyLog}\left(2,-i e^{-a-bx}\right)}{b}\right)$$

input
```
Int[x*ArcCot[E^(a + b*x)],x]
```

output
```
(-1/2*I)*((x*PolyLog[2, (-I)*E^(-a - b*x)])/b + PolyLog[3, (-I)*E^(-a - b*
x)]/b^2) + (I/2)*((x*PolyLog[2, I*E^(-a - b*x)])/b + PolyLog[3, I*E^(-a -
b*x)]/b^2)
```

3.222.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5667
```
Int[ArcCot[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Simp[I/2   Int[x^m*Log[1 - I/(a + b*f^(c + d*x))], x], x] - Simp[I/2   In
t[x^m*Log[1 + I/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] &
& IntegerQ[m] && m > 0
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.222.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(83) = 166$.

Time = 0.80 (sec) , antiderivative size = 355, normalized size of antiderivative = 3.45

method	result
risch	$-\frac{i\,\mathrm{polylog}\left(3, i\,\mathrm{e}^{bx+a}\right)}{2b^{2}}+\frac{\pi\,x^{2}}{4}+\frac{i\,\mathrm{polylog}\left(3, -i\,\mathrm{e}^{bx+a}\right)}{2b^{2}}-\frac{i\ln\left(-i\left(\mathrm{e}^{bx+a}+i\right)\right)a^{2}}{2b^{2}}+\frac{i a^{2}\ln\left(1-i\,\mathrm{e}^{bx+a}\right)}{2b^{2}}+\frac{i x\,\mathrm{polylog}\left(2, i\,\mathrm{e}^{bx+a}\right)}{2b}$

input
```
int(x*arccot(exp(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/b^2*polylog(3,I*exp(b*x+a))+1/4*Pi*x^2+1/2*I/b^2*polylog(3,-I*exp(b
*x+a))-1/2*I/b^2*ln(-I*(exp(b*x+a)+I))*a^2+1/2*I/b^2*a^2*ln(1-I*exp(b*x+a)
)+1/2*I/b*polylog(2,I*exp(b*x+a))*x+1/2*I/b*ln(1-I*exp(b*x+a))*a*x-1/2*I/b
^2*dilog(-I*(exp(b*x+a)+I))*a-1/2*I/b^2*polylog(2,-I*exp(b*x+a))*a+1/2*I/b
^2*polylog(2,I*exp(b*x+a))*a-1/2*I/b*ln(1+I*exp(b*x+a))*a*x-1/2*I/b^2*dilo
g(-I*exp(b*x+a))*a+1/2*I/b*ln(-I*(-exp(b*x+a)+I))*a*x-1/2*I/b*ln(-I*(exp(b
*x+a)+I))*a*x-1/2*I/b^2*a^2*ln(1+I*exp(b*x+a))-1/2*I/b^2*ln(-I*exp(b*x+a))
*ln(-I*(-exp(b*x+a)+I))*a-1/2*I/b*polylog(2,-I*exp(b*x+a))*x+1/2*I/b^2*ln(
-I*(-exp(b*x+a)+I))*a^2
```

3.222.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(73) = 146$.

Time = 0.36 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.47

$$\int x\cot^{-1}\left(e^{a+bx}\right)\,dx$$
$$= \frac{2\,b^2x^2\,\mathrm{arccot}\left(e^{(bx+a)}\right)+2i\,bx\mathrm{Li}_2\left(i\,e^{(bx+a)}\right)-2i\,bx\mathrm{Li}_2\left(-i\,e^{(bx+a)}\right)+i\,a^2\log\left(e^{(bx+a)}+i\right)-i\,a^2\log\left(e^{(bx+a)}\right.}{}$$

input
```
integrate(x*arccot(exp(b*x+a)),x, algorithm="fricas")
```

output
```
1/4*(2*b^2*x^2*arccot(e^(b*x + a)) + 2*I*b*x*dilog(I*e^(b*x + a)) - 2*I*b*
x*dilog(-I*e^(b*x + a)) + I*a^2*log(e^(b*x + a) + I) - I*a^2*log(e^(b*x +
a) - I) + (-I*b^2*x^2 + I*a^2)*log(I*e^(b*x + a) + 1) + (I*b^2*x^2 - I*a^2
)*log(-I*e^(b*x + a) + 1) - 2*I*polylog(3, I*e^(b*x + a)) + 2*I*polylog(3,
 -I*e^(b*x + a)))/b^2
```

3.222.6 Sympy [F]

$$\int x\cot^{-1}\left(e^{a+bx}\right)\,dx=\int x\,\mathrm{acot}\left(e^ae^{bx}\right)dx$$

input
```
integrate(x*acot(exp(b*x+a)),x)
```

output
```
Integral(x*acot(exp(a)*exp(b*x)), x)
```

3.222.7 Maxima [F]

$$\int x\cot^{-1}\left(e^{a+bx}\right)\,dx=\int x\,\mathrm{arccot}\left(e^{(bx+a)}\right)\,dx$$

input
```
integrate(x*arccot(exp(b*x+a)),x, algorithm="maxima")
```

output
```
1/2*x^2*arctan(e^(-b*x - a)) + b*integrate(1/2*x^2*e^(b*x + a)/(e^(2*b*x +
 2*a) + 1), x)
```

3.222.8 Giac [F]

$$\int x\cot^{-1}\left(e^{a+bx}\right)\,dx=\int x\,\mathrm{arccot}\left(e^{(bx+a)}\right)\,dx$$

input
```
integrate(x*arccot(exp(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arccot(e^(b*x + a)), x)
```

3.222.9 Mupad [F(-1)]

Timed out.

$$\int x\cot^{-1}\left(e^{a+bx}\right)\,dx=\int x\,\mathrm{acot}\left(\mathrm{e}^{a+b\,x}\right)\,dx$$

input
```
int(x*acot(exp(a + b*x)),x)
```

output
```
int(x*acot(exp(a + b*x)), x)
```

3.223 $\int x^2 \cot^{-1}\left(e^{a+bx}\right) dx$

3.223.1 Optimal result

Integrand size = 12, antiderivative size = 151

$$\int x^2 \cot^{-1}\left(e^{a+bx}\right) dx = -\frac{ix^2 \operatorname{PolyLog}\left(2,-ie^{-a-bx}\right)}{2b} + \frac{ix^2 \operatorname{PolyLog}\left(2,ie^{-a-bx}\right)}{2b} - \frac{ix \operatorname{PolyLog}\left(3,-ie^{-a-bx}\right)}{b^2} + \frac{ix \operatorname{PolyLog}\left(3,ie^{-a-bx}\right)}{b^2} - \frac{i \operatorname{PolyLog}\left(4,-ie^{-a-bx}\right)}{b^3} + \frac{i \operatorname{PolyLog}\left(4,ie^{-a-bx}\right)}{b^3}$$

output
```
-1/2*I*x^2*polylog(2,-I*exp(-b*x-a))/b+1/2*I*x^2*polylog(2,I*exp(-b*x-a))/
b-I*x*polylog(3,-I*exp(-b*x-a))/b^2+I*x*polylog(3,I*exp(-b*x-a))/b^2-I*pol
ylog(4,-I*exp(-b*x-a))/b^3+I*polylog(4,I*exp(-b*x-a))/b^3
```

3.223.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.88

$$\int x^2 \cot^{-1}\left(e^{a+bx}\right) dx = -\frac{i\left(b^2x^2 \operatorname{PolyLog}\left(2,-ie^{-a-bx}\right) - b^2x^2 \operatorname{PolyLog}\left(2,ie^{-a-bx}\right) + 2\left(bx \operatorname{PolyLog}\left(3,-ie^{-a-bx}\right) - bx \operatorname{PolyLog}\right.\right.}{2b^3}$$

input
```
Integrate[x^2*ArcCot[E^(a + b*x)],x]
```

output
```
((-1/2*I)*(b^2*x^2*PolyLog[2, (-I)*E^(-a - b*x)] - b^2*x^2*PolyLog[2, I*E^
(-a - b*x)] + 2*(b*x*PolyLog[3, (-I)*E^(-a - b*x)] - b*x*PolyLog[3, I*E^(-
a - b*x)] + PolyLog[4, (-I)*E^(-a - b*x)] - PolyLog[4, I*E^(-a - b*x)])))/
b^3
```

3.223.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5667, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cot^{-1}\left(e^{a+bx}\right) dx$$

$$\Big\downarrow \text{ 5667}$$

$$\frac{1}{2}i\int x^2\log\left(1-ie^{-a-bx}\right)dx-\frac{1}{2}i\int x^2\log\left(1+ie^{-a-bx}\right)dx$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{1}{2}i\left(\frac{x^2\operatorname{PolyLog}\left(2,ie^{-a-bx}\right)}{b}-\frac{2\int x\operatorname{PolyLog}\left(2,ie^{-a-bx}\right)dx}{b}\right)-\frac{1}{2}i\left(\frac{x^2\operatorname{PolyLog}\left(2,-ie^{-a-bx}\right)}{b}-\frac{2\int x\operatorname{PolyLog}\left(2,-ie^{-a-bx}\right)dx}{b}\right)$$

$$\Big\downarrow \text{ 7163}$$

$$\frac{1}{2}i\left(\frac{x^2\operatorname{PolyLog}\left(2,ie^{-a-bx}\right)}{b}-\frac{2\left(\frac{\int \operatorname{PolyLog}(3,ie^{-a-bx})dx}{b}-\frac{x\operatorname{PolyLog}(3,ie^{-a-bx})}{b}\right)}{b}\right)-\frac{1}{2}i\left(\frac{x^2\operatorname{PolyLog}\left(2,-ie^{-a-bx}\right)}{b}-\frac{2\left(\frac{\int \operatorname{PolyLog}(3,-ie^{-a-bx})dx}{b}-\frac{x\operatorname{PolyLog}(3,-ie^{-a-bx})}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{1}{2}i\left(\frac{x^2\operatorname{PolyLog}\left(2,ie^{-a-bx}\right)}{b}-\frac{2\left(-\frac{\int e^{a+bx}\operatorname{PolyLog}(3,ie^{-a-bx})de^{-a-bx}}{b^2}-\frac{x\operatorname{PolyLog}(3,ie^{-a-bx})}{b}\right)}{b}\right)-\frac{1}{2}i\left(\frac{x^2\operatorname{PolyLog}\left(2,-ie^{-a-bx}\right)}{b}-\frac{2\left(-\frac{\int e^{a+bx}\operatorname{PolyLog}(3,-ie^{-a-bx})de^{-a-bx}}{b^2}-\frac{x\operatorname{PolyLog}(3,-ie^{-a-bx})}{b}\right)}{b}\right)$$

$$\Big\downarrow 7143$$

$$\frac{1}{2}i\left(\frac{x^2\operatorname{PolyLog}\left(2,ie^{-a-bx}\right)}{b}-\frac{2\left(-\frac{\operatorname{PolyLog}\left(4,ie^{-a-bx}\right)}{b^2}-\frac{x\operatorname{PolyLog}\left(3,ie^{-a-bx}\right)}{b}\right)}{b}\right)-\frac{1}{2}i\left(\frac{x^2\operatorname{PolyLog}\left(2,-ie^{-a-bx}\right)}{b}-\frac{2\left(-\frac{\operatorname{PolyLog}\left(4,-ie^{-a-bx}\right)}{b^2}-\frac{x\operatorname{PolyLog}\left(3,-ie^{-a-bx}\right)}{b}\right)}{b}\right)$$

input
```
Int[x^2*ArcCot[E^(a + b*x)],x]
```

output
```
(-1/2*I)*((x^2*PolyLog[2, (-I)*E^(-a - b*x)])/b - (2*(-((x*PolyLog[3, (-I)
*E^(-a - b*x)])/b) - PolyLog[4, (-I)*E^(-a - b*x)]/b^2))/b) + (I/2)*((x^2*
PolyLog[2, I*E^(-a - b*x)])/b - (2*(-((x*PolyLog[3, I*E^(-a - b*x)])/b) -
PolyLog[4, I*E^(-a - b*x)]/b^2))/b)
```

3.223.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5667
```
Int[ArcCot[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Simp[I/2   Int[x^m*Log[1 - I/(a + b*f^(c + d*x))], x], x] - Simp[I/2   In
t[x^m*Log[1 + I/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] &
& IntegerQ[m] && m > 0
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.223.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 412 vs. 2(129) = 258.

Time = 0.80 (sec) , antiderivative size = 413, normalized size of antiderivative = 2.74

method	result
risch	$\frac{i \ln\left(-i\left(e^{bx+a}+i\right)\right) a^{3}}{2 b^{3}}+\frac{\pi x^{3}}{6}+\frac{i \ln\left(1+i e^{bx+a}\right) a^{2} x}{2 b^{2}}-\frac{i \operatorname{polylog}\left(3, i e^{bx+a}\right) x}{b^{2}}-\frac{i \ln\left(1-i e^{bx+a}\right) a^{3}}{2 b^{3}}+\frac{i \operatorname{polylog}\left(3,-i e^{bx+a}\right) x}{b^{2}}+$

input
```
int(x^2*arccot(exp(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2*I/b^3*ln(-I*(exp(b*x+a)+I))*a^3+1/6*Pi*x^3+1/2*I/b^2*ln(1+I*exp(b*x+a)
)*a^2*x-I/b^2*polylog(3,I*exp(b*x+a))*x-1/2*I/b^3*ln(1-I*exp(b*x+a))*a^3+I
/b^2*polylog(3,-I*exp(b*x+a))*x+I/b^3*polylog(4,I*exp(b*x+a))+1/2*I/b^2*ln
(-I*(exp(b*x+a)+I))*x*a^2-1/2*I/b^2*ln(1-I*exp(b*x+a))*x*a^2-1/2*I/b*polyl
og(2,-I*exp(b*x+a))*x^2+1/2*I/b^3*a^3*ln(1+I*exp(b*x+a))-1/2*I/b^2*ln(-I*(
-exp(b*x+a)+I))*a^2*x-1/2*I/b^3*ln(-I*(-exp(b*x+a)+I))*a^3+1/2*I/b^3*polyl
og(2,-I*exp(b*x+a))*a^2-1/2*I/b^3*polylog(2,I*exp(b*x+a))*a^2+1/2*I/b^3*di
log(-I*exp(b*x+a))*a^2-I*polylog(4,-I*exp(b*x+a))/b^3+1/2*I/b^3*dilog(-I*(
exp(b*x+a)+I))*a^2+1/2*I/b*polylog(2,I*exp(b*x+a))*x^2+1/2*I/b^3*ln(-I*exp
(b*x+a))*ln(-I*(-exp(b*x+a)+I))*a^2
```

3.223.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.24

$$\int x^2 \cot^{-1}\left(e^{a+bx}\right)\,dx$$
$$= \frac{2\,b^3x^3\,\mathrm{arccot}\left(e^{(bx+a)}\right) + 3i\,b^2x^2\mathrm{Li}_2\left(i\,e^{(bx+a)}\right) - 3i\,b^2x^2\mathrm{Li}_2\left(-i\,e^{(bx+a)}\right) - i\,a^3\log\left(e^{(bx+a)}+i\right) + i\,a^3\log\left(e^{(b}\right.}{}$$

input
```
integrate(x^2*arccot(exp(b*x+a)),x, algorithm="fricas")
```

output
```
1/6*(2*b^3*x^3*arccot(e^(b*x + a)) + 3*I*b^2*x^2*dilog(I*e^(b*x + a)) - 3*
I*b^2*x^2*dilog(-I*e^(b*x + a)) - I*a^3*log(e^(b*x + a) + I) + I*a^3*log(e
^(b*x + a) - I) - 6*I*b*x*polylog(3, I*e^(b*x + a)) + 6*I*b*x*polylog(3, -
I*e^(b*x + a)) + (-I*b^3*x^3 - I*a^3)*log(I*e^(b*x + a) + 1) + (I*b^3*x^3
+ I*a^3)*log(-I*e^(b*x + a) + 1) + 6*I*polylog(4, I*e^(b*x + a)) - 6*I*pol
ylog(4, -I*e^(b*x + a)))/b^3
```

3.223.6 Sympy [F]

$$\int x^2 \cot^{-1}\left(e^{a+bx}\right)\,dx = \int x^2\,\mathrm{acot}\left(e^a e^{bx}\right)dx$$

input
```
integrate(x**2*acot(exp(b*x+a)),x)
```

output
```
Integral(x**2*acot(exp(a)*exp(b*x)), x)
```

3.223.7 Maxima [F]

$$\int x^2 \cot^{-1}\left(e^{a+bx}\right)\,dx = \int x^2\,\mathrm{arccot}\left(e^{(bx+a)}\right)\,dx$$

input
```
integrate(x^2*arccot(exp(b*x+a)),x, algorithm="maxima")
```

output
```
1/3*x^3*arctan(e^(-b*x - a)) + b*integrate(1/3*x^3*e^(b*x + a)/(e^(2*b*x +
 2*a) + 1), x)
```

3.223.8 Giac [F]

$$\int x^2 \cot^{-1}\left(e^{a+bx}\right) \, dx = \int x^2 \operatorname{arccot}\left(e^{(bx+a)}\right) \, dx$$

input
```
integrate(x^2*arccot(exp(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arccot(e^(b*x + a)), x)
```

3.223.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}\left(e^{a+bx}\right) \, dx = \int x^2 \operatorname{acot}\left(e^{a+b\,x}\right) \, dx$$

input
```
int(x^2*acot(exp(a + b*x)),x)
```

output
```
int(x^2*acot(exp(a + b*x)), x)
```

3.224 $\int \cot^{-1}\left(a+bf^{c+dx}\right) dx$

3.224.1 Optimal result

Integrand size = 12, antiderivative size = 196

$$\int \cot^{-1}\left(a+bf^{c+dx}\right) dx = -\frac{\cot^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1-i(a+bf^{c+dx})}\right)}{d\log(f)} + \frac{\cot^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(i-a)(1-i(a+bf^{c+dx}))}\right)}{d\log(f)} - \frac{i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(a+bf^{c+dx})}\right)}{2d\log(f)} + \frac{i\,\text{PolyLog}\left(2,1-\frac{2bf^{c+dx}}{(i-a)(1-i(a+bf^{c+dx}))}\right)}{2d\log(f)}$$

output
```
-arccot(a+b*f^(d*x+c))*ln(2/(1-I*(a+b*f^(d*x+c))))/d/ln(f)+arccot(a+b*f^(d
*x+c))*ln(2*b*f^(d*x+c)/(I-a)/(1-I*(a+b*f^(d*x+c))))/d/ln(f)-1/2*I*polylog
(2,1-2/(1-I*(a+b*f^(d*x+c))))/d/ln(f)+1/2*I*polylog(2,1-2*b*f^(d*x+c)/(I-a
)/(1-I*(a+b*f^(d*x+c))))/d/ln(f)
```

3.224.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.85

$$\int \cot^{-1}\left(a+bf^{c+dx}\right)\,dx = x\cot^{-1}\left(a+bf^{c+dx}\right) + \frac{b\left(dx\log(f)\left(\log\left(1+\frac{b^2f^{c+dx}}{ab-\sqrt{-b^2}}\right)-\log\left(1+\frac{b^2f^{c+dx}}{ab+\sqrt{-b^2}}\right)\right)+\text{PolyLog}\left(2,-\frac{b^2f^{c+dx}}{ab-\sqrt{-b^2}}\right)-\text{PolyLog}\left(2,-\frac{b^2f^{c-}}{ab+\sqrt{}}\right.}{2\sqrt{-b^2}d\log(f)}$$

input
```
Integrate[ArcCot[a + b*f^(c + d*x)],x]
```

output
```
x*ArcCot[a + b*f^(c + d*x)] + (b*(d*x*Log[f]*(Log[1 + (b^2*f^(c + d*x))/(a
*b - Sqrt[-b^2])] - Log[1 + (b^2*f^(c + d*x))/(a*b + Sqrt[-b^2])]) + PolyL
og[2, -((b^2*f^(c + d*x))/(a*b - Sqrt[-b^2]))] - PolyLog[2, -((b^2*f^(c +
d*x))/(a*b + Sqrt[-b^2]))]))/(2*Sqrt[-b^2]*d*Log[f])
```

3.224.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.90, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2720, 5571, 25, 27, 5382, 2849, 2752, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}\left(a+bf^{c+dx}\right)dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int f^{-c-dx}\cot^{-1}\left(bf^{c+dx}+a\right)df^{c+dx}}{d\log(f)}$$

$$\Big\downarrow \text{ 5571}$$

$$\frac{\int f^{-c-dx}\cot^{-1}\left(bf^{c+dx}+a\right)d\left(bf^{c+dx}+a\right)}{bd\log(f)}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int -f^{-c-dx}\cot^{-1}\left(bf^{c+dx}+a\right)d\left(bf^{c+dx}+a\right)}{bd\log(f)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int -\frac{f^{-c-dx}\cot^{-1}(bf^{c+dx}+a)}{b}d(bf^{c+dx}+a)}{d\log(f)}$$

$\Big\downarrow$ 5382

$$-\frac{\int \frac{\log\left(\frac{2}{1-i\left(bf^{c+dx}+a\right)}\right)}{f^{2c+2dx}+1}d(bf^{c+dx}+a)-\int \frac{\log\left(\frac{2bf^{c+dx}}{(i-a)\left(1-i\left(bf^{c+dx}+a\right)\right)}\right)}{f^{2c+2dx}+1}d(bf^{c+dx}+a)+\log\left(\frac{2}{1-i(a+bf^{c+dx})}\right)\cot^{-1}(a+b}{d\log(f)}$$

$\Big\downarrow$ 2849

$$-\frac{i\int \frac{\log\left(\frac{2}{1-i\left(bf^{c+dx}+a\right)}\right)}{1-\frac{2}{1-i\left(bf^{c+dx}+a\right)}}d\frac{1}{1-i(bf^{c+dx}+a)}-\int \frac{\log\left(\frac{2bf^{c+dx}}{(i-a)\left(1-i\left(bf^{c+dx}+a\right)\right)}\right)}{f^{2c+2dx}+1}d(bf^{c+dx}+a)+\log\left(\frac{2}{1-i(a+bf^{c+dx})}\right)\cot^{-1}(a+}{d\log(f)}$$

$\Big\downarrow$ 2752

$$-\frac{-\int \frac{\log\left(\frac{2bf^{c+dx}}{(i-a)\left(1-i\left(bf^{c+dx}+a\right)\right)}\right)}{f^{2c+2dx}+1}d(bf^{c+dx}+a)+\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(bf^{c+dx}+a)}\right)+\log\left(\frac{2}{1-i(a+bf^{c+dx})}\right)\cot^{-1}(a+}{d\log(f)}$$

$\Big\downarrow$ 2897

$$-\frac{\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(bf^{c+dx}+a)}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2bf^{c+dx}}{(i-a)(1-i(bf^{c+dx}+a))}\right)+\log\left(\frac{2}{1-i(a+bf^{c+dx})}\right)\cot^{-1}(a+b}{d\log(f)}$$

input
```
Int[ArcCot[a + b*f^(c + d*x)],x]
```

output
```
-((ArcCot[a + b*f^(c + d*x)]*Log[2/(1 - I*(a + b*f^(c + d*x)))] - ArcCot[a
 + b*f^(c + d*x)]*Log[(2*b*f^(c + d*x))/((I - a)*(1 - I*(a + b*f^(c + d*x)
)))] + (I/2)*PolyLog[2, 1 - 2/(1 - I*(a + b*f^(c + d*x)))] - (I/2)*PolyLog
[2, 1 - (2*b*f^(c + d*x))/((I - a)*(1 - I*(a + b*f^(c + d*x))))])/(d*Log[f
]))
```

3.224. $\int \cot^{-1}\left(a+bf^{c+dx}\right)\,dx$

3.224.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5382
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcCot[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*ArcCot[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] - Simp[b*(c/e)
   Int[Log[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] + Simp[b*(c/e)   Int[Log[2*
c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x]) /; FreeQ[{a
, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

rule 5571
```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.224.4 Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{\ln\left(-b f^{dx+c}\right) \operatorname{arccot}\left(a+b f^{dx+c}\right)+\frac{i \ln\left(-b f^{dx+c}\right) \ln\left(\frac{i+b f^{dx+c}+a}{i+a}\right)}{2}-\frac{i \ln\left(-b f^{dx+c}\right) \ln\left(\frac{i-b f^{dx+c}-a}{i-a}\right)}{2}+\frac{i \operatorname{dilog}\left(\frac{i+b f^{dx \cdots}}{i+c \cdots}\right.}{2}}{d \ln(f)}$
default	$\frac{\ln\left(-b f^{dx+c}\right) \operatorname{arccot}\left(a+b f^{dx+c}\right)+\frac{i \ln\left(-b f^{dx+c}\right) \ln\left(\frac{i+b f^{dx+c}+a}{i+a}\right)}{2}-\frac{i \ln\left(-b f^{dx+c}\right) \ln\left(\frac{i-b f^{dx+c}-a}{i-a}\right)}{2}+\frac{i \operatorname{dilog}\left(\frac{i+b f^{dx \cdots}}{i+c \cdots}\right.}{2}}{d \ln(f)}$
risch	$\frac{i x \ln\left(1+i\left(a+b f^{dx+c}\right)\right)}{2}+\frac{\pi x}{2}-\frac{i \ln\left(-i b f^{dx+c}-i a+1\right) \ln\left(-\frac{i f^{dx+c} b}{i a-1}\right)}{2 d \ln(f)}-\frac{i \operatorname{dilog}\left(-\frac{i f^{dx+c} b}{i a-1}\right)}{2 d \ln(f)}-\frac{i \operatorname{dilog}\left(\frac{b f^{dx} f^{c \cdots}}{a- \cdots}\right.}{2 \ln(f) d \cdots}$

input
```
int(arccot(a+b*f^(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/ln(f)*(ln(-b*f^(d*x+c))*arccot(a+b*f^(d*x+c))+1/2*I*ln(-b*f^(d*x+c))*l
n((I+b*f^(d*x+c)+a)/(I+a))-1/2*I*ln(-b*f^(d*x+c))*ln((I-b*f^(d*x+c)-a)/(I-
a))+1/2*I*dilog((I+b*f^(d*x+c)+a)/(I+a))-1/2*I*dilog((I-b*f^(d*x+c)-a)/(I-
a)))
```

3.224.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.08

$$\int \cot^{-1}\left(a+bf^{c+dx}\right)\,dx$$
$$= \frac{2\,dx \operatorname{arccot}\left(bf^{dx+c}+a\right)\log\left(f\right)-i\,c\log\left(bf^{dx+c}+a+i\right)\log\left(f\right)+i\,c\log\left(bf^{dx+c}+a-i\right)\log\left(f\right)+\left(-i\,d \cdots\right.}{\cdots}$$

input
```
integrate(arccot(a+b*f^(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(2*d*x*arccot(b*f^(d*x + c) + a)*log(f) - I*c*log(b*f^(d*x + c) + a +
I)*log(f) + I*c*log(b*f^(d*x + c) + a - I)*log(f) + (-I*d*x - I*c)*log(f)*
log((a^2 + (a*b + I*b)*f^(d*x + c) + 1)/(a^2 + 1)) + (I*d*x + I*c)*log(f)*
log((a^2 + (a*b - I*b)*f^(d*x + c) + 1)/(a^2 + 1)) - I*dilog(-(a^2 + (a*b
+ I*b)*f^(d*x + c) + 1)/(a^2 + 1) + 1) + I*dilog(-(a^2 + (a*b - I*b)*f^(d*
x + c) + 1)/(a^2 + 1) + 1))/(d*log(f))
```

3.224.6 Sympy [F]

$$\int \cot^{-1}\left(a+bf^{c+dx}\right)\,dx=\int \operatorname{acot}\left(a+bf^{c+dx}\right)dx$$

input
```
integrate(acot(a+b*f**(d*x+c)),x)
```

output
```
Integral(acot(a + b*f**(c + d*x)), x)
```

3.224.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.96

$$\int \cot^{-1}\left(a+bf^{c+dx}\right)\,dx=\frac{(dx+c)\operatorname{arccot}\left(bf^{dx+c}+a\right)}{d}+\frac{2\,(dx+c)\arctan\left(\frac{b^2f^{dx+c}+ab}{b}\right)\log\left(f\right)+\left(\pi-\arctan\left(\frac{1}{a}\right)\right)\log\left(b^2f^{2\,dx+2\,c}+2\,abf^{dx+c}+a^2+1\right)-\mathrm{arctar}}{2\,d\log\left(f\right)}$$

input
```
integrate(arccot(a+b*f^(d*x+c)),x, algorithm="maxima")
```

output
```
(d*x + c)*arccot(b*f^(d*x + c) + a)/d + 1/2*(2*(d*x + c)*arctan((b^2*f^(d*
x + c) + a*b)/b)*log(f) + (pi - arctan(1/a))*log(b^2*f^(2*d*x + 2*c) + 2*a
*b*f^(d*x + c) + a^2 + 1) - arctan(b*f^(d*x + c) + a)*log(b^2*f^(2*d*x + 2
*c)/(a^2 + 1)) + I*dilog((I*b*f^(d*x + c) + I*a + 1)/(I*a + 1)) - I*dilog(
(I*b*f^(d*x + c) + I*a - 1)/(I*a - 1)))/(d*log(f))
```

3.224.8 Giac [F]

$$\int \cot^{-1}\left(a+bf^{c+dx}\right)\,dx=\int \operatorname{arccot}\left(bf^{dx+c}+a\right)\,dx$$

input
```
integrate(arccot(a+b*f^(d*x+c)),x, algorithm="giac")
```

output
```
integrate(arccot(b*f^(d*x + c) + a), x)
```

3.224.9 Mupad [F(-1)]

Timed out.

$$\int \cot^{-1}\left(a+bf^{c+dx}\right)\,dx=\int \mathrm{acot}\left(a+b\,f^{c+d\,x}\right)\,dx$$

input
```
int(acot(a + b*f^(c + d*x)),x)
```

output
```
int(acot(a + b*f^(c + d*x)), x)
```

3.225 $\int x \cot^{-1}\left(a+bf^{c+dx}\right) dx$

3.225.1 Optimal result

Integrand size = 14, antiderivative size = 250

$$\begin{aligned}\int x\cot^{-1}\left(a+bf^{c+dx}\right)\,dx =& -\frac{1}{4}ix^2\log\left(1-\frac{bf^{c+dx}}{i-a}\right)+\frac{1}{4}ix^2\log\left(1+\frac{bf^{c+dx}}{i+a}\right)\\&+\frac{1}{4}ix^2\log\left(1-\frac{i}{a+bf^{c+dx}}\right)-\frac{1}{4}ix^2\log\left(1+\frac{i}{a+bf^{c+dx}}\right)\\&-\frac{ix\,\text{PolyLog}\left(2,\frac{bf^{c+dx}}{i-a}\right)}{2d\log(f)}+\frac{ix\,\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{i+a}\right)}{2d\log(f)}\\&+\frac{i\,\text{PolyLog}\left(3,\frac{bf^{c+dx}}{i-a}\right)}{2d^2\log^2(f)}-\frac{i\,\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{i+a}\right)}{2d^2\log^2(f)}\end{aligned}$$

output
```
-1/4*I*x^2*ln(1-b*f^(d*x+c)/(I-a))+1/4*I*x^2*ln(1+b*f^(d*x+c)/(I+a))+1/4*I
*x^2*ln(1-I/(a+b*f^(d*x+c)))-1/4*I*x^2*ln(1+I/(a+b*f^(d*x+c)))-1/2*I*x*pol
ylog(2,b*f^(d*x+c)/(I-a))/d/ln(f)+1/2*I*x*polylog(2,-b*f^(d*x+c)/(I+a))/d/
ln(f)+1/2*I*polylog(3,b*f^(d*x+c)/(I-a))/d^2/ln(f)^2-1/2*I*polylog(3,-b*f^
(d*x+c)/(I+a))/d^2/ln(f)^2
```

3.225.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00

$$\int x\cot^{-1}\left(a+bf^{c+dx}\right)\,dx=-\frac{1}{4}ix^2\log\left(1-\frac{bf^{c+dx}}{i-a}\right)+\frac{1}{4}ix^2\log\left(1+\frac{bf^{c+dx}}{i+a}\right)+\frac{1}{4}ix^2\log\left(1-\frac{i}{a+bf^{c+dx}}\right)-\frac{1}{4}ix^2\log\left(1+\frac{i}{a+bf^{c+dx}}\right)-\frac{ix\,\text{PolyLog}\left(2,\frac{bf^{c+dx}}{i-a}\right)}{2d\log(f)}+\frac{ix\,\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{i+a}\right)}{2d\log(f)}+\frac{i\,\text{PolyLog}\left(3,\frac{bf^{c+dx}}{i-a}\right)}{2d^2\log^2(f)}-\frac{i\,\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{i+a}\right)}{2d^2\log^2(f)}$$

input
```
Integrate[x*ArcCot[a + b*f^(c + d*x)],x]
```

output
```
(-1/4*I)*x^2*Log[1 - (b*f^(c + d*x))/(I - a)] + (I/4)*x^2*Log[1 + (b*f^(c
+ d*x))/(I + a)] + (I/4)*x^2*Log[1 - I/(a + b*f^(c + d*x))] - (I/4)*x^2*Lo
g[1 + I/(a + b*f^(c + d*x))] - ((I/2)*x*PolyLog[2, (b*f^(c + d*x))/(I - a)
])/(d*Log[f]) + ((I/2)*x*PolyLog[2, -((b*f^(c + d*x))/(I + a))])/(d*Log[f]
) + ((I/2)*PolyLog[3, (b*f^(c + d*x))/(I - a)])/(d^2*Log[f]^2) - ((I/2)*Po
lyLog[3, -((b*f^(c + d*x))/(I + a))])/(d^2*Log[f]^2)
```

3.225.3 Rubi [A] (verified)

Time = 4.42 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.94, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5667, 3031, 26, 27, 7292, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cot^{-1}\left(a+bf^{c+dx}\right)\,dx$$

$$\Big\downarrow \text{5667}$$

$$\frac{1}{2}i\int x\log\left(1-\frac{i}{bf^{c+dx}+a}\right)dx-\frac{1}{2}i\int x\log\left(1+\frac{i}{bf^{c+dx}+a}\right)dx$$

$$\Big\downarrow \text{3031}$$

$$\frac{1}{2}i\left(\frac{1}{2}x^2\log\left(1-\frac{i}{a+bf^{c+dx}}\right)-\frac{1}{2}\int -\frac{ibdf^{c+dx}x^2\log(f)}{(-bf^{c+dx}-a+i)\left(bf^{c+dx}+a\right)}dx\right)-\frac{1}{2}i\left(\frac{1}{2}x^2\log\left(1+\frac{i}{a+bf^{c+dx}}\right)-\frac{1}{2}\int -\frac{ibdf^{c+dx}x^2\log(f)}{\left(bf^{c+dx}+a\right)\left(bf^{c+dx}+a+i\right)}dx\right)$$

$\Big\downarrow$ 26

$$\frac{1}{2}i\left(\frac{1}{2}i\int \frac{bdf^{c+dx}x^2\log(f)}{(-bf^{c+dx}-a+i)\left(bf^{c+dx}+a\right)}dx+\frac{1}{2}x^2\log\left(1-\frac{i}{a+bf^{c+dx}}\right)\right)-\frac{1}{2}i\left(\frac{1}{2}i\int \frac{bdf^{c+dx}x^2\log(f)}{\left(bf^{c+dx}+a\right)\left(bf^{c+dx}+a+i\right)}dx+\frac{1}{2}x^2\log\left(1+\frac{i}{a+bf^{c+dx}}\right)\right)$$

$\Big\downarrow$ 27

$$\frac{1}{2}i\left(\frac{1}{2}ibd\log(f)\int \frac{f^{c+dx}x^2}{(-bf^{c+dx}-a+i)\left(bf^{c+dx}+a\right)}dx+\frac{1}{2}x^2\log\left(1-\frac{i}{a+bf^{c+dx}}\right)\right)-\frac{1}{2}i\left(\frac{1}{2}ibd\log(f)\int \frac{f^{c+dx}x^2}{\left(bf^{c+dx}+a\right)\left(bf^{c+dx}+a+i\right)}dx+\frac{1}{2}x^2\log\left(1+\frac{i}{a+bf^{c+dx}}\right)\right)$$

$\Big\downarrow$ 7292

$$\frac{1}{2}i\left(\frac{1}{2}ibd\log(f)\int \frac{f^{c+dx}x^2}{(i(ia+1)-bf^{c+dx})\left(bf^{c+dx}+a\right)}dx+\frac{1}{2}x^2\log\left(1-\frac{i}{a+bf^{c+dx}}\right)\right)-\frac{1}{2}i\left(\frac{1}{2}ibd\log(f)\int \frac{f^{c+dx}x^2}{\left(bf^{c+dx}+i(1-ia)\right)\left(bf^{c+dx}+a\right)}dx+\frac{1}{2}x^2\log\left(1+\frac{i}{a+bf^{c+dx}}\right)\right)$$

$\Big\downarrow$ 7293

$$\frac{1}{2}i\left(\frac{1}{2}ibd\log(f)\int \left(\frac{if^{c+dx}x^2}{bf^{c+dx}+a-i}-\frac{if^{c+dx}x^2}{bf^{c+dx}+a}\right)dx+\frac{1}{2}x^2\log\left(1-\frac{i}{a+bf^{c+dx}}\right)\right)-\frac{1}{2}i\left(\frac{1}{2}ibd\log(f)\int \left(\frac{if^{c+dx}x^2}{bf^{c+dx}+a+i}-\frac{if^{c+dx}x^2}{bf^{c+dx}+a}\right)dx+\frac{1}{2}x^2\log\left(1+\frac{i}{a+bf^{c+dx}}\right)\right)$$

$\Big\downarrow$ 2009

$$\frac{1}{2}i\left(\frac{1}{2}ibd\log(f)\left(-\frac{2i\,\text{PolyLog}\left(3,\frac{bf^{c+dx}}{i-a}\right)}{bd^3\log^3(f)}+\frac{2i\,\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a}\right)}{bd^3\log^3(f)}+\frac{2ix\,\text{PolyLog}\left(2,\frac{bf^{c+dx}}{i-a}\right)}{bd^2\log^2(f)}-\frac{2ix\,\text{PolyLog}}{bd^2\log}\right.\right.$$
$$\frac{1}{2}i\left(\frac{1}{2}ibd\log(f)\left(\frac{2i\,\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a}\right)}{bd^3\log^3(f)}-\frac{2i\,\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a+i}\right)}{bd^3\log^3(f)}-\frac{2ix\,\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{a}\right)}{bd^2\log^2(f)}+\frac{2ix\,\text{PolyLo}}{bd^2}\right.\right.$$

input
```
Int[x*ArcCot[a + b*f^(c + d*x)],x]
```

3.225. $\int x\cot^{-1}\left(a+bf^{c+dx}\right)\,dx$

output
```
(I/2)*((x^2*Log[1 - I/(a + b*f^(c + d*x))])/2 + (I/2)*b*d*Log[f]*((I*x^2*L
og[1 - (b*f^(c + d*x))/(I - a)])/(b*d*Log[f]) - (I*x^2*Log[1 + (b*f^(c + d
*x))/a])/(b*d*Log[f]) + ((2*I)*x*PolyLog[2, (b*f^(c + d*x))/(I - a)])/(b*d
^2*Log[f]^2) - ((2*I)*x*PolyLog[2, -((b*f^(c + d*x))/a)])/(b*d^2*Log[f]^2)
 - ((2*I)*PolyLog[3, (b*f^(c + d*x))/(I - a)])/(b*d^3*Log[f]^3) + ((2*I)*P
olyLog[3, -((b*f^(c + d*x))/a)])/(b*d^3*Log[f]^3))) - (I/2)*((x^2*Log[1 +
I/(a + b*f^(c + d*x))])/2 + (I/2)*b*d*Log[f]*(((-I)*x^2*Log[1 + (b*f^(c +
d*x))/a])/(b*d*Log[f]) + (I*x^2*Log[1 + (b*f^(c + d*x))/(I + a)])/(b*d*Log
[f]) - ((2*I)*x*PolyLog[2, -((b*f^(c + d*x))/a)])/(b*d^2*Log[f]^2) + ((2*I
)*x*PolyLog[2, -((b*f^(c + d*x))/(I + a))])/(b*d^2*Log[f]^2) + ((2*I)*Poly
Log[3, -((b*f^(c + d*x))/a)])/(b*d^3*Log[f]^3) - ((2*I)*PolyLog[3, -((b*f^
(c + d*x))/(I + a))])/(b*d^3*Log[f]^3)))
```

3.225.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3031
```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)
*(Log[u]/(b*(m + 1))), x] - Simp[1/(b*(m + 1))   Int[SimplifyIntegrand[(a +
 b*x)^(m + 1)*(D[u, x]/u), x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunc
tionFreeQ[u, x] && NeQ[m, -1]
```

rule 5667
```
Int[ArcCot[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Simp[I/2   Int[x^m*Log[1 - I/(a + b*f^(c + d*x))], x], x] - Simp[I/2   In
t[x^m*Log[1 + I/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] &
& IntegerQ[m] && m > 0
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.225.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 677 vs. 2(218) = 436.

Time = 1.04 (sec) , antiderivative size = 678, normalized size of antiderivative = 2.71

method	result
risch	$-\frac{i x^2 \ln(1-i(a+b f^{dx+c}))}{4}+\frac{\pi x^2}{4}-\frac{i c \ln\left(\frac{b f^{dx} f^c+a+i}{i+a}\right) x}{2d}-\frac{i \operatorname{polylog}\left(2,\frac{i b f^{dx} f^c}{-ia-1}\right) c}{2\ln(f) d^2}+\frac{i c \ln\left(\frac{b f^{dx} f^c+a-i}{a-i}\right) x}{2d}+\frac{i c^2 \ln(1-i}{4}$

input
```
int(x*arccot(a+b*f^(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/4*I*x^2*ln(1-I*(a+b*f^(d*x+c)))+1/4*Pi*x^2-1/2*I/d*c*ln((b*f^(d*x)*f^c+
a+I)/(I+a))*x-1/2*I/ln(f)/d^2*polylog(2,I*b/(-I*a-1)*f^(d*x)*f^c)*c+1/2*I/
d*c*ln((b*f^(d*x)*f^c+a-I)/(a-I))*x+1/4*I/d^2*c^2*ln(1-I*a-I*f^(d*x)*f^c*b
)-1/2*I/ln(f)/d^2*c*dilog((b*f^(d*x)*f^c+a+I)/(I+a))+1/4*I*x^2*ln(1+I*(a+b
*f^(d*x+c)))+1/2*I/ln(f)/d^2*polylog(2,I*b/(1-I*a)*f^(d*x)*f^c)*c-1/4*I*ln
(1-I*b/(-I*a-1)*f^(d*x)*f^c)*x^2-1/4*I/d^2*ln(1-I*b/(-I*a-1)*f^(d*x)*f^c)*
c^2+1/2*I/d*ln(1-I*b/(1-I*a)*f^(d*x)*f^c)*c*x+1/2*I/ln(f)^2/d^2*polylog(3,
I*b/(-I*a-1)*f^(d*x)*f^c)+1/2*I/ln(f)/d*polylog(2,I*b/(1-I*a)*f^(d*x)*f^c)
*x-1/4*I/d^2*c^2*ln(I*f^(d*x)*f^c*b+I*a+1)-1/2*I/d^2*c^2*ln((b*f^(d*x)*f^c
+a+I)/(I+a))+1/2*I/d^2*c^2*ln((b*f^(d*x)*f^c+a-I)/(a-I))-1/2*I/ln(f)/d*pol
ylog(2,I*b/(-I*a-1)*f^(d*x)*f^c)*x-1/2*I/ln(f)^2/d^2*polylog(3,I*b/(1-I*a)
*f^(d*x)*f^c)-1/2*I/d*ln(1-I*b/(-I*a-1)*f^(d*x)*f^c)*c*x+1/4*I/d^2*ln(1-I*
b/(1-I*a)*f^(d*x)*f^c)*c^2+1/4*I*ln(1-I*b/(1-I*a)*f^(d*x)*f^c)*x^2+1/2*I/l
n(f)/d^2*c*dilog((b*f^(d*x)*f^c+a-I)/(a-I))
```

3.225.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.22

$$\int x\cot^{-1}\left(a+bf^{c+dx}\right)\,dx$$
$$=\frac{2\,d^2x^2\operatorname{arccot}\left(bf^{dx+c}+a\right)\log\left(f\right)^2+i\,c^2\log\left(bf^{dx+c}+a+i\right)\log\left(f\right)^2-i\,c^2\log\left(bf^{dx+c}+a-i\right)\log\left(f\right)^2-}{}$$

input
```
integrate(x*arccot(a+b*f^(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(2*d^2*x^2*arccot(b*f^(d*x + c) + a)*log(f)^2 + I*c^2*log(b*f^(d*x + c
) + a + I)*log(f)^2 - I*c^2*log(b*f^(d*x + c) + a - I)*log(f)^2 - 2*I*d*x*
dilog(-(a^2 + (a*b + I*b)*f^(d*x + c) + 1)/(a^2 + 1) + 1)*log(f) + 2*I*d*x
*dilog(-(a^2 + (a*b - I*b)*f^(d*x + c) + 1)/(a^2 + 1) + 1)*log(f) + (-I*d^
2*x^2 + I*c^2)*log(f)^2*log((a^2 + (a*b + I*b)*f^(d*x + c) + 1)/(a^2 + 1))
 + (I*d^2*x^2 - I*c^2)*log(f)^2*log((a^2 + (a*b - I*b)*f^(d*x + c) + 1)/(a
^2 + 1)) + 2*I*polylog(3, -(a*b + I*b)*f^(d*x + c)/(a^2 + 1)) - 2*I*polylo
g(3, -(a*b - I*b)*f^(d*x + c)/(a^2 + 1)))/(d^2*log(f)^2)
```

3.225.6 Sympy [F]

$$\int x\cot^{-1}\left(a+bf^{c+dx}\right)\,dx=\int x\operatorname{acot}\left(a+bf^{c+dx}\right)dx$$

input
```
integrate(x*acot(a+b*f**(d*x+c)),x)
```

output
```
Integral(x*acot(a + b*f**(c + d*x)), x)
```

3.225.7 Maxima [F]

$$\int x\cot^{-1}\left(a+bf^{c+dx}\right)\,dx=\int x\,\mathrm{arccot}\left(bf^{dx+c}+a\right)\,dx$$

input
```
integrate(x*arccot(a+b*f^(d*x+c)),x, algorithm="maxima")
```

output
```
b*d*f^c*integrate(1/2*f^(d*x)*x^2/(b^2*f^(2*d*x)*f^(2*c) + 2*a*b*f^(d*x)*f
^c + a^2 + 1), x)*log(f) + 1/2*x^2*arctan(1/(b*f^(d*x)*f^c + a))
```

3.225.8 Giac [F]

$$\int x\cot^{-1}\left(a+bf^{c+dx}\right)\,dx=\int x\,\mathrm{arccot}\left(bf^{dx+c}+a\right)\,dx$$

input
```
integrate(x*arccot(a+b*f^(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x*arccot(b*f^(d*x + c) + a), x)
```

3.225.9 Mupad [F(-1)]

Timed out.

$$\int x\cot^{-1}\left(a+bf^{c+dx}\right)\,dx=\int x\,\mathrm{acot}\left(a+b\,f^{c+d\,x}\right)\,dx$$

input
```
int(x*acot(a + b*f^(c + d*x)),x)
```

output
```
int(x*acot(a + b*f^(c + d*x)), x)
```

3.226 $\int x^2 \cot^{-1}\left(a+bf^{c+dx}\right)\,dx$

3.226.1 Optimal result

Integrand size = 16, antiderivative size = 313

$$\begin{aligned}\int x^2 \cot^{-1}\left(a+bf^{c+dx}\right)\,dx =& -\frac{1}{6}ix^3\log\left(1-\frac{bf^{c+dx}}{i-a}\right)+\frac{1}{6}ix^3\log\left(1+\frac{bf^{c+dx}}{i+a}\right)\\&+\frac{1}{6}ix^3\log\left(1-\frac{i}{a+bf^{c+dx}}\right)-\frac{1}{6}ix^3\log\left(1+\frac{i}{a+bf^{c+dx}}\right)\\&-\frac{ix^2\operatorname{PolyLog}\left(2,\frac{bf^{c+dx}}{i-a}\right)}{2d\log(f)}+\frac{ix^2\operatorname{PolyLog}\left(2,-\frac{bf^{c+dx}}{i+a}\right)}{2d\log(f)}\\&+\frac{ix\operatorname{PolyLog}\left(3,\frac{bf^{c+dx}}{i-a}\right)}{d^2\log^2(f)}-\frac{ix\operatorname{PolyLog}\left(3,-\frac{bf^{c+dx}}{i+a}\right)}{d^2\log^2(f)}\\&-\frac{i\operatorname{PolyLog}\left(4,\frac{bf^{c+dx}}{i-a}\right)}{d^3\log^3(f)}+\frac{i\operatorname{PolyLog}\left(4,-\frac{bf^{c+dx}}{i+a}\right)}{d^3\log^3(f)}\end{aligned}$$

output
```
-1/6*I*x^3*ln(1-b*f^(d*x+c)/(I-a))+1/6*I*x^3*ln(1+b*f^(d*x+c)/(I+a))+1/6*I
*x^3*ln(1-I/(a+b*f^(d*x+c)))-1/6*I*x^3*ln(1+I/(a+b*f^(d*x+c)))-1/2*I*x^2*p
olylog(2,b*f^(d*x+c)/(I-a))/d/ln(f)+1/2*I*x^2*polylog(2,-b*f^(d*x+c)/(I+a)
)/d/ln(f)+I*x*polylog(3,b*f^(d*x+c)/(I-a))/d^2/ln(f)^2-I*x*polylog(3,-b*f^
(d*x+c)/(I+a))/d^2/ln(f)^2-I*polylog(4,b*f^(d*x+c)/(I-a))/d^3/ln(f)^3+I*po
lylog(4,-b*f^(d*x+c)/(I+a))/d^3/ln(f)^3
```

3.226.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.00

$$\int x^2 \cot^{-1}\left(a+bf^{c+dx}\right)\,dx = -\frac{1}{6}ix^3\log\left(1-\frac{bf^{c+dx}}{i-a}\right)+\frac{1}{6}ix^3\log\left(1+\frac{bf^{c+dx}}{i+a}\right)+\frac{1}{6}ix^3\log\left(1-\frac{i}{a+bf^{c+dx}}\right)-\frac{1}{6}ix^3\log\left(1+\frac{i}{a+bf^{c+dx}}\right)-\frac{ix^2\operatorname{PolyLog}\left(2,\frac{bf^{c+dx}}{i-a}\right)}{2d\log(f)}+\frac{ix^2\operatorname{PolyLog}\left(2,-\frac{bf^{c+dx}}{i+a}\right)}{2d\log(f)}+\frac{ix\operatorname{PolyLog}\left(3,\frac{bf^{c+dx}}{i-a}\right)}{d^2\log^2(f)}-\frac{ix\operatorname{PolyLog}\left(3,-\frac{bf^{c+dx}}{i+a}\right)}{d^2\log^2(f)}-\frac{i\operatorname{PolyLog}\left(4,\frac{bf^{c+dx}}{i-a}\right)}{d^3\log^3(f)}+\frac{i\operatorname{PolyLog}\left(4,-\frac{bf^{c+dx}}{i+a}\right)}{d^3\log^3(f)}$$

input
```
Integrate[x^2*ArcCot[a + b*f^(c + d*x)],x]
```

output
```
(-1/6*I)*x^3*Log[1 - (b*f^(c + d*x))/(I - a)] + (I/6)*x^3*Log[1 + (b*f^(c
+ d*x))/(I + a)] + (I/6)*x^3*Log[1 - I/(a + b*f^(c + d*x))] - (I/6)*x^3*Lo
g[1 + I/(a + b*f^(c + d*x))] - ((I/2)*x^2*PolyLog[2, (b*f^(c + d*x))/(I -
a)])/(d*Log[f]) + ((I/2)*x^2*PolyLog[2, -((b*f^(c + d*x))/(I + a))])/(d*Lo
g[f]) + (I*x*PolyLog[3, (b*f^(c + d*x))/(I - a)])/(d^2*Log[f]^2) - (I*x*Po
lyLog[3, -((b*f^(c + d*x))/(I + a))])/(d^2*Log[f]^2) - (I*PolyLog[4, (b*f^
(c + d*x))/(I - a)])/(d^3*Log[f]^3) + (I*PolyLog[4, -((b*f^(c + d*x))/(I +
 a))])/(d^3*Log[f]^3)
```

3.226.3 Rubi [A] (verified)

Time = 4.23 (sec) , antiderivative size = 621, normalized size of antiderivative = 1.98, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5667, 3031, 26, 27, 7292, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cot^{-1}\left(a+bf^{c+dx}\right)\,dx$$

$$\Big\downarrow \text{ 5667}$$

$$\frac{1}{2}i\int x^2\log\left(1-\frac{i}{bf^{c+dx}+a}\right)dx-\frac{1}{2}i\int x^2\log\left(1+\frac{i}{bf^{c+dx}+a}\right)dx$$

$$\Big\downarrow \text{3031}$$

$$\frac{1}{2}i\left(\frac{1}{3}x^3\log\left(1-\frac{i}{a+bf^{c+dx}}\right)-\frac{1}{3}\int -\frac{ibdf^{c+dx}x^3\log(f)}{(-bf^{c+dx}-a+i)(bf^{c+dx}+a)}dx\right)-\frac{1}{2}i\left(\frac{1}{3}x^3\log\left(1+\frac{i}{a+bf^{c+dx}}\right)-\frac{1}{3}\int -\frac{ibdf^{c+dx}x^3\log(f)}{(bf^{c+dx}+a)(bf^{c+dx}+a+i)}dx\right)$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}i\left(\frac{1}{3}i\int \frac{bdf^{c+dx}x^3\log(f)}{(-bf^{c+dx}-a+i)(bf^{c+dx}+a)}dx+\frac{1}{3}x^3\log\left(1-\frac{i}{a+bf^{c+dx}}\right)\right)-\frac{1}{2}i\left(\frac{1}{3}i\int \frac{bdf^{c+dx}x^3\log(f)}{(bf^{c+dx}+a)(bf^{c+dx}+a+i)}dx+\frac{1}{3}x^3\log\left(1+\frac{i}{a+bf^{c+dx}}\right)\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}i\left(\frac{1}{3}ibd\log(f)\int \frac{f^{c+dx}x^3}{(-bf^{c+dx}-a+i)(bf^{c+dx}+a)}dx+\frac{1}{3}x^3\log\left(1-\frac{i}{a+bf^{c+dx}}\right)\right)-\frac{1}{2}i\left(\frac{1}{3}ibd\log(f)\int \frac{f^{c+dx}x^3}{(bf^{c+dx}+a)(bf^{c+dx}+a+i)}dx+\frac{1}{3}x^3\log\left(1+\frac{i}{a+bf^{c+dx}}\right)\right)$$

$$\Big\downarrow \text{7292}$$

$$\frac{1}{2}i\left(\frac{1}{3}ibd\log(f)\int \frac{f^{c+dx}x^3}{(i(ia+1)-bf^{c+dx})(bf^{c+dx}+a)}dx+\frac{1}{3}x^3\log\left(1-\frac{i}{a+bf^{c+dx}}\right)\right)-\frac{1}{2}i\left(\frac{1}{3}ibd\log(f)\int \frac{f^{c+dx}x^3}{(bf^{c+dx}+i(1-ia))(bf^{c+dx}+a)}dx+\frac{1}{3}x^3\log\left(1+\frac{i}{a+bf^{c+dx}}\right)\right)$$

$$\Big\downarrow \text{7293}$$

$$\frac{1}{2}i\left(\frac{1}{3}ibd\log(f)\int \left(\frac{if^{c+dx}x^3}{bf^{c+dx}+a-i}-\frac{if^{c+dx}x^3}{bf^{c+dx}+a}\right)dx+\frac{1}{3}x^3\log\left(1-\frac{i}{a+bf^{c+dx}}\right)\right)-\frac{1}{2}i\left(\frac{1}{3}ibd\log(f)\int \left(\frac{if^{c+dx}x^3}{bf^{c+dx}+a+i}-\frac{if^{c+dx}x^3}{bf^{c+dx}+a}\right)dx+\frac{1}{3}x^3\log\left(1+\frac{i}{a+bf^{c+dx}}\right)\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}i\left(\frac{1}{3}ibd\log(f)\left(\frac{6i\,\text{PolyLog}\left(4,\frac{bf^{c+dx}}{i-a}\right)}{bd^4\log^4(f)}-\frac{6i\,\text{PolyLog}\left(4,-\frac{bf^{c+dx}}{a}\right)}{bd^4\log^4(f)}-\frac{6ix\,\text{PolyLog}\left(3,\frac{bf^{c+dx}}{i-a}\right)}{bd^3\log^3(f)}+\frac{6ix\,\text{PolyLog}\left(3\right.}{bd^3\log^3}\right.\right.$$

$$\frac{1}{2}i\left(\frac{1}{3}ibd\log(f)\left(-\frac{6i\,\text{PolyLog}\left(4,-\frac{bf^{c+dx}}{a}\right)}{bd^4\log^4(f)}+\frac{6i\,\text{PolyLog}\left(4,-\frac{bf^{c+dx}}{a+i}\right)}{bd^4\log^4(f)}+\frac{6ix\,\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a}\right)}{bd^3\log^3(f)}-\frac{6ix\,\text{PolyL}}{bo}\right.\right.$$

input
```
Int[x^2*ArcCot[a + b*f^(c + d*x)],x]
```

3.226. $\int x^2\cot^{-1}\left(a+bf^{c+dx}\right)\,dx$

output
```
(I/2)*((x^3*Log[1 - I/(a + b*f^(c + d*x))])/3 + (I/3)*b*d*Log[f]*((I*x^3*L
og[1 - (b*f^(c + d*x))/(I - a)])/(b*d*Log[f]) - (I*x^3*Log[1 + (b*f^(c + d
*x))/a])/(b*d*Log[f]) + ((3*I)*x^2*PolyLog[2, (b*f^(c + d*x))/(I - a)])/(b
*d^2*Log[f]^2) - ((3*I)*x^2*PolyLog[2, -((b*f^(c + d*x))/a)])/(b*d^2*Log[f
]^2) - ((6*I)*x*PolyLog[3, (b*f^(c + d*x))/(I - a)])/(b*d^3*Log[f]^3) + ((
6*I)*x*PolyLog[3, -((b*f^(c + d*x))/a)])/(b*d^3*Log[f]^3) + ((6*I)*PolyLog
[4, (b*f^(c + d*x))/(I - a)])/(b*d^4*Log[f]^4) - ((6*I)*PolyLog[4, -((b*f^
(c + d*x))/a)])/(b*d^4*Log[f]^4))) - (I/2)*((x^3*Log[1 + I/(a + b*f^(c + d
*x))])/3 + (I/3)*b*d*Log[f]*(((-I)*x^3*Log[1 + (b*f^(c + d*x))/a])/(b*d*Lo
g[f]) + (I*x^3*Log[1 + (b*f^(c + d*x))/(I + a)])/(b*d*Log[f]) - ((3*I)*x^2
*PolyLog[2, -((b*f^(c + d*x))/a)])/(b*d^2*Log[f]^2) + ((3*I)*x^2*PolyLog[2
, -((b*f^(c + d*x))/(I + a))])/(b*d^2*Log[f]^2) + ((6*I)*x*PolyLog[3, -((b
*f^(c + d*x))/a)])/(b*d^3*Log[f]^3) - ((6*I)*x*PolyLog[3, -((b*f^(c + d*x)
)/(I + a))])/(b*d^3*Log[f]^3) - ((6*I)*PolyLog[4, -((b*f^(c + d*x))/a)])/(
b*d^4*Log[f]^4) + ((6*I)*PolyLog[4, -((b*f^(c + d*x))/(I + a))])/(b*d^4*Lo
g[f]^4)))
```

3.226.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3031
```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)
*(Log[u]/(b*(m + 1))), x] - Simp[1/(b*(m + 1))   Int[SimplifyIntegrand[(a +
 b*x)^(m + 1)*(D[u, x]/u), x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunc
tionFreeQ[u, x] && NeQ[m, -1]
```

rule 5667
```
Int[ArcCot[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Simp[I/2   Int[x^m*Log[1 - I/(a + b*f^(c + d*x))], x], x] - Simp[I/2   In
t[x^m*Log[1 + I/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] &
& IntegerQ[m] && m > 0
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.226.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 763 vs. $2(281) = 562$.

Time = 1.54 (sec) , antiderivative size = 764, normalized size of antiderivative = 2.44

method	result
risch	$\frac{i x^3 \ln(1+i(a+b f^{dx+c}))}{6} - \frac{i \ln\left(1-\frac{i b f^{dx} f^c}{-ia-1}\right) x^3}{6} - \frac{i x^3 \ln(1-i(a+b f^{dx+c}))}{6} + \frac{i \ln\left(1-\frac{i b f^{dx} f^c}{-ia+1}\right) x^3}{6} + \frac{\pi x^3}{6} - \frac{i \operatorname{polylog}\left(4, \frac{ib}{\cdots}\right.}{\ln(f)^3 d}$

input
```
int(x^2*arccot(a+b*f^(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*I/ln(f)/d^3*c^2*dilog((b*f^(d*x)*f^c+a+I)/(I+a))-I/ln(f)^2/d^2*polylog
(3,I*b/(1-I*a)*f^(d*x)*f^c)*x-1/2*I/d^2*ln(1-I*b/(1-I*a)*f^(d*x)*f^c)*c^2*
x-1/2*I/ln(f)/d^3*polylog(2,I*b/(1-I*a)*f^(d*x)*f^c)*c^2+1/2*I/d^2*c^2*ln(
(b*f^(d*x)*f^c+a+I)/(I+a))*x+1/2*I/ln(f)/d*polylog(2,I*b/(1-I*a)*f^(d*x)*f
^c)*x^2+I/ln(f)^2/d^2*polylog(3,I*b/(-I*a-1)*f^(d*x)*f^c)*x+I/ln(f)^3/d^3*
polylog(4,I*b/(1-I*a)*f^(d*x)*f^c)+1/2*I/d^3*c^3*ln((b*f^(d*x)*f^c+a+I)/(I
+a))-1/6*I/d^3*c^3*ln(1-I*a-I*f^(d*x)*f^c*b)-1/3*I/d^3*ln(1-I*b/(1-I*a)*f^
(d*x)*f^c)*c^3+1/6*I*x^3*ln(1+I*(a+b*f^(d*x+c)))-1/6*I*ln(1-I*b/(-I*a-1)*f
^(d*x)*f^c)*x^3-1/6*I*x^3*ln(1-I*(a+b*f^(d*x+c)))+1/6*I*ln(1-I*b/(1-I*a)*f
^(d*x)*f^c)*x^3-1/2*I/d^2*c^2*ln((b*f^(d*x)*f^c+a-I)/(a-I))*x-1/2*I/ln(f)/
d^3*c^2*dilog((b*f^(d*x)*f^c+a-I)/(a-I))+1/2*I/ln(f)/d^3*polylog(2,I*b/(-I
*a-1)*f^(d*x)*f^c)*c^2-1/2*I/ln(f)/d*polylog(2,I*b/(-I*a-1)*f^(d*x)*f^c)*x
^2+1/2*I/d^2*ln(1-I*b/(-I*a-1)*f^(d*x)*f^c)*c^2*x-1/2*I/d^3*c^3*ln((b*f^(d
*x)*f^c+a-I)/(a-I))-I/ln(f)^3/d^3*polylog(4,I*b/(-I*a-1)*f^(d*x)*f^c)+1/3*
I/d^3*ln(1-I*b/(-I*a-1)*f^(d*x)*f^c)*c^3+1/6*I/d^3*c^3*ln(I*f^(d*x)*f^c*b+
I*a+1)+1/6*Pi*x^3
```

3.226.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.21

$$\int x^2 \cot^{-1}\left(a+bf^{c+dx}\right)\,dx$$
$$=\frac{2\,d^3x^3\operatorname{arccot}\left(bf^{dx+c}+a\right)\log\left(f\right)^3-3i\,d^2x^2\mathrm{Li}_2\left(-\frac{a^2+(ab+i\,b)f^{dx+c}+1}{a^2+1}+1\right)\log\left(f\right)^2+3i\,d^2x^2\mathrm{Li}_2\left(-\frac{a^2+(ab-i\,b)}{a^2+}\right.}{}$$

input
```
integrate(x^2*arccot(a+b*f^(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(2*d^3*x^3*arccot(b*f^(d*x + c) + a)*log(f)^3 - 3*I*d^2*x^2*dilog(-(a^
2 + (a*b + I*b)*f^(d*x + c) + 1)/(a^2 + 1) + 1)*log(f)^2 + 3*I*d^2*x^2*dil
og(-(a^2 + (a*b - I*b)*f^(d*x + c) + 1)/(a^2 + 1) + 1)*log(f)^2 - I*c^3*lo
g(b*f^(d*x + c) + a + I)*log(f)^3 + I*c^3*log(b*f^(d*x + c) + a - I)*log(f
)^3 + (-I*d^3*x^3 - I*c^3)*log(f)^3*log((a^2 + (a*b + I*b)*f^(d*x + c) + 1
)/(a^2 + 1)) + (I*d^3*x^3 + I*c^3)*log(f)^3*log((a^2 + (a*b - I*b)*f^(d*x
+ c) + 1)/(a^2 + 1)) + 6*I*d*x*log(f)*polylog(3, -(a*b + I*b)*f^(d*x + c)/
(a^2 + 1)) - 6*I*d*x*log(f)*polylog(3, -(a*b - I*b)*f^(d*x + c)/(a^2 + 1))
 - 6*I*polylog(4, -(a*b + I*b)*f^(d*x + c)/(a^2 + 1)) + 6*I*polylog(4, -(a
*b - I*b)*f^(d*x + c)/(a^2 + 1)))/(d^3*log(f)^3)
```

3.226.6 Sympy [F]

$$\int x^2 \cot^{-1}\left(a+bf^{c+dx}\right)\,dx=\int x^2\operatorname{acot}\left(a+bf^{c+dx}\right)dx$$

input
```
integrate(x**2*acot(a+b*f**(d*x+c)),x)
```

output
```
Integral(x**2*acot(a + b*f**(c + d*x)), x)
```

3.226.7 Maxima [F]

$$\int x^2 \cot^{-1}\left(a + bf^{c+dx}\right)\,dx = \int x^2 \operatorname{arccot}\left(bf^{dx+c} + a\right)\,dx$$

input
```
integrate(x^2*arccot(a+b*f^(d*x+c)),x, algorithm="maxima")
```

output
```
b*d*f^c*integrate(1/3*f^(d*x)*x^3/(b^2*f^(2*d*x)*f^(2*c) + 2*a*b*f^(d*x)*f
^c + a^2 + 1), x)*log(f) + 1/3*x^3*arctan(1/(b*f^(d*x)*f^c + a))
```

3.226.8 Giac [F]

$$\int x^2 \cot^{-1}\left(a + bf^{c+dx}\right)\,dx = \int x^2 \operatorname{arccot}\left(bf^{dx+c} + a\right)\,dx$$

input
```
integrate(x^2*arccot(a+b*f^(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x^2*arccot(b*f^(d*x + c) + a), x)
```

3.226.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}\left(a + bf^{c+dx}\right)\,dx = \int x^2 \operatorname{acot}\left(a + b\,f^{c+d\,x}\right)\,dx$$

input
```
int(x^2*acot(a + b*f^(c + d*x)),x)
```

output
```
int(x^2*acot(a + b*f^(c + d*x)), x)
```

3.227 $\int e^{-x} \cot^{-1}(e^x)\,dx$

3.227.1 Optimal result

Integrand size = 10, antiderivative size = 27

$$\int e^{-x} \cot^{-1}(e^x)\,dx = -x - e^{-x} \cot^{-1}(e^x) + \frac{1}{2}\log\left(1 + e^{2x}\right)$$

output
```
-x-arccot(exp(x))/exp(x)+1/2*ln(1+exp(2*x))
```

3.227.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int e^{-x} \cot^{-1}(e^x)\,dx = -x - e^{-x} \cot^{-1}(e^x) + \frac{1}{2}\log\left(1 + e^{2x}\right)$$

input
```
Integrate[ArcCot[E^x]/E^x,x]
```

output
```
-x - ArcCot[E^x]/E^x + Log[1 + E^(2*x)]/2
```

3.227.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5731, 25, 2720, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-x}\cot^{-1}\left(e^x\right)dx$$

$$\Big\downarrow \text{5731}$$

$$\int -\frac{1}{1+e^{2x}}dx-e^{-x}\cot^{-1}\left(e^x\right)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{1+e^{2x}}dx-e^{-x}\cot^{-1}\left(e^x\right)$$

$$\Big\downarrow \text{2720}$$

$$-\frac{1}{2}\int \frac{e^{-2x}}{1+e^{2x}}de^{2x}-e^{-x}\cot^{-1}\left(e^x\right)$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2}\left(\int \frac{1}{1+e^{2x}}de^{2x}-\int e^{-2x}de^{2x}\right)-e^{-x}\cot^{-1}\left(e^x\right)$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}\left(\int \frac{1}{1+e^{2x}}de^{2x}-\log\left(e^{2x}\right)\right)-e^{-x}\cot^{-1}\left(e^x\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\log\left(e^{2x}+1\right)-\log\left(e^{2x}\right)\right)-e^{-x}\cot^{-1}\left(e^x\right)$$

input
```
Int[ArcCot[E^x]/E^x,x]
```

output
```
-(ArcCot[E^x]/E^x) + (-Log[E^(2*x)] + Log[1 + E^(2*x)])/2
```

3.227.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 5731
```
Int[((a_.) + ArcCot[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Simp[(a + b*ArcCot[u])*w, x] + Simp[b   Int[SimplifyIntegrand[w*(D[u, x]/(
1 + u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCot[u]), x]]
```

3.227.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\operatorname{arccot}\left({\mathrm e}^{x}\right) {\mathrm e}^{-x}-\ln \left({\mathrm e}^{x}\right)+\frac{\ln \left(1+{\mathrm e}^{2 x}\right)}{2}$	25
default	$-\operatorname{arccot}\left({\mathrm e}^{x}\right) {\mathrm e}^{-x}-\ln \left({\mathrm e}^{x}\right)+\frac{\ln \left(1+{\mathrm e}^{2 x}\right)}{2}$	25
parallelrisch	$\frac{\left(\ln \left(1+{\mathrm e}^{2 x}\right) {\mathrm e}^{x}-2 x \,{\mathrm e}^{x}-2 \,\operatorname{arccot}\left({\mathrm e}^{x}\right)\right) {\mathrm e}^{-x}}{2}$	28
risch	$-\frac{i {\mathrm e}^{-x} \ln \left(1+i {\mathrm e}^{x}\right)}{2}+\frac{\ln \left(1+{\mathrm e}^{2 x}\right)}{2}-x +\frac{i {\mathrm e}^{-x} \ln \left(1-i {\mathrm e}^{x}\right)}{2}-\frac{{\mathrm e}^{-x} \pi}{2}$	51

input
```
int(arccot(exp(x))/exp(x),x,method=_RETURNVERBOSE)
```

output
```
-arccot(exp(x))/exp(x)-ln(exp(x))+1/2*ln(exp(x)^2+1)
```

3.227.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int e^{-x}\cot^{-1}\left(e^{x}\right)\,dx = -\frac{1}{2}\left(2\,xe^{x} - e^{x}\log\left(e^{(2\,x)}+1\right) + 2\,\mathrm{arccot}\left(e^{x}\right)\right)e^{(-x)}$$

input
```
integrate(arccot(exp(x))/exp(x),x, algorithm="fricas")
```

output
```
-1/2*(2*x*e^x - e^x*log(e^(2*x) + 1) + 2*arccot(e^x))*e^(-x)
```

3.227.6 Sympy [A] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{-x}\cot^{-1}\left(e^{x}\right)\,dx = -x + \frac{\log\left(e^{2x}+1\right)}{2} - e^{-x}\,\mathrm{acot}\left(e^{x}\right)$$

input
```
integrate(acot(exp(x))/exp(x),x)
```

output
```
-x + log(exp(2*x) + 1)/2 - exp(-x)*acot(exp(x))
```

3.227.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{-x}\cot^{-1}\left(e^{x}\right)\,dx = -\,\mathrm{arccot}\left(e^{x}\right)e^{(-x)} + \frac{1}{2}\log\left(e^{(-2\,x)}+1\right)$$

input
```
integrate(arccot(exp(x))/exp(x),x, algorithm="maxima")
```

output
```
-arccot(e^x)*e^(-x) + 1/2*log(e^(-2*x) + 1)
```

3.227.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int e^{-x}\cot^{-1}\left(e^{x}\right)\,dx = -\arctan\left(e^{(-x)}\right)e^{(-x)} + \frac{1}{2}\log\left(e^{(-2\,x)} + 1\right)$$

input
```
integrate(arccot(exp(x))/exp(x),x, algorithm="giac")
```

output
```
-arctan(e^(-x))*e^(-x) + 1/2*log(e^(-2*x) + 1)
```

3.227.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int e^{-x}\cot^{-1}\left(e^{x}\right)\,dx = \frac{\ln\left(\mathrm{e}^{2\,x} + 1\right)}{2} - x - \mathrm{acot}(\mathrm{e}^{x})\,\mathrm{e}^{-x}$$

input
```
int(acot(exp(x))*exp(-x),x)
```

output
```
log(exp(2*x) + 1)/2 - x - acot(exp(x))*exp(-x)
```

3.228 $\int \frac{1}{(a+ax^2)(b-2b\cot^{-1}(x))} dx$

3.228.1 Optimal result

Integrand size = 19, antiderivative size = 17

$$\int \frac{1}{(a+ax^2)(b-2b\cot^{-1}(x))} dx = \frac{\log(1-2\cot^{-1}(x))}{2ab}$$

output
```
1/2*ln(1-2*arccot(x))/a/b
```

3.228.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+ax^2)(b-2b\cot^{-1}(x))} dx = \frac{\log(-1+2\cot^{-1}(x))}{2ab}$$

input
```
Integrate[1/((a + a*x^2)*(b - 2*b*ArcCot[x])),x]
```

output
```
Log[-1 + 2*ArcCot[x]]/(2*a*b)
```

3.228.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {5418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^2+a)\left(b-2b\cot^{-1}(x)\right)}dx$$
$$\Big\downarrow \text{ 5418}$$
$$\frac{\log\left(1-2\cot^{-1}(x)\right)}{2ab}$$

input
```
Int[1/((a + a*x^2)*(b - 2*b*ArcCot[x])),x]
```

output
```
Log[1 - 2*ArcCot[x]]/(2*a*b)
```

3.228.3.1 Defintions of rubi rules used

rule 5418
```
Int[1/(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)), x_Symbol]
 :> Simp[-Log[RemoveContent[a + b*ArcCot[c*x], x]]/(b*c*d), x] /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[e, c^2*d]
```

3.228.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
parallelrisch	$\frac{\ln\left(\operatorname{arccot}(x)-\frac{1}{2}\right)}{2ab}$	14
default	$\frac{\ln(2b\ \operatorname{arccot}(x)-b)}{2ab}$	19
risch	$\frac{\ln(\ln(ix+1)-i(-i\ln(-ix+1)+\pi-1))}{2ab}$	34

input
```
int(1/(a*x^2+a)/(b-2*b*arccot(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(arccot(x)-1/2)/a/b
```

3.228.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+ax^2)(b-2b\cot^{-1}(x))}\,dx = \frac{\log(2\,\mathrm{arccot}\,(x)-1)}{2\,ab}$$

input
```
integrate(1/(a*x^2+a)/(b-2*b*arccot(x)),x, algorithm="fricas")
```

output
```
1/2*log(2*arccot(x) - 1)/(a*b)
```

3.228.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{1}{(a+ax^2)(b-2b\cot^{-1}(x))}\,dx = \frac{\log\left(\mathrm{acot}\,(x)-\frac{1}{2}\right)}{2ab}$$

input
```
integrate(1/(a*x**2+a)/(b-2*b*acot(x)),x)
```

output
```
log(acot(x) - 1/2)/(2*a*b)
```

3.228.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+ax^2)(b-2b\cot^{-1}(x))}\,dx = \frac{\log(|2\,\arctan(1,x)-1|)}{2\,ab}$$

input
```
integrate(1/(a*x^2+a)/(b-2*b*arccot(x)),x, algorithm="maxima")
```

output
```
1/2*log(abs(2*arctan2(1, x) - 1))/(a*b)
```

3.228.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1}{(a+ax^2)\left(b-2b\cot^{-1}(x)\right)}\,dx = \frac{\log\left(\left|2\,\arctan\left(\frac{1}{x}\right)-1\right|\right)}{2\,ab}$$

input
```
integrate(1/(a*x^2+a)/(b-2*b*arccot(x)),x, algorithm="giac")
```

output
```
1/2*log(abs(2*arctan(1/x) - 1))/(a*b)
```

3.228.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+ax^2)\left(b-2b\cot^{-1}(x)\right)}\,dx = \frac{\ln\left(2\,\mathrm{acot}(x)-1\right)}{2\,a\,b}$$

input
```
int(1/((a + a*x^2)*(b - 2*b*acot(x))),x)
```

output
```
log(2*acot(x) - 1)/(2*a*b)
```

3.229 $\int e^{c(a+bx)} \cot^{-1}(\sinh(ac+bcx))\,dx$

3.229.1 Optimal result

Integrand size = 20, antiderivative size = 47

$$\int e^{c(a+bx)} \cot^{-1}(\sinh(ac+bcx))\,dx = \frac{e^{ac+bcx}\cot^{-1}(\sinh(c(a+bx)))}{bc} + \frac{\log\left(1+e^{2c(a+bx)}\right)}{bc}$$

output
```
exp(b*c*x+a*c)*arccot(sinh(c*(b*x+a)))/b/c+ln(1+exp(2*c*(b*x+a)))/b/c
```

3.229.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.30

$$\int e^{c(a+bx)} \cot^{-1}(\sinh(ac+bcx))\,dx = \frac{-e^{c(a+bx)}\cot^{-1}\left(\frac{1}{2}e^{-c(a+bx)} - \frac{1}{2}e^{c(a+bx)}\right) + \log\left(1+e^{2c(a+bx)}\right)}{bc}$$

input
```
Integrate[E^(c*(a + b*x))*ArcCot[Sinh[a*c + b*c*x]],x]
```

output
```
(-(E^(c*(a + b*x))*ArcCot[1/(2*E^(c*(a + b*x))) - E^(c*(a + b*x))/2]) + Lo
g[1 + E^(2*c*(a + b*x))])/(b*c)
```

3.229.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {7281, 5731, 2720, 27, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)}\cot^{-1}(\sinh(ac+bcx))\,dx$$

$$\Big\downarrow \text{7281}$$

$$\frac{\int e^{ac+bxc}\cot^{-1}(\sinh(ac+bxc))d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{5731}$$

$$\frac{\int e^{ac+bxc}\text{sech}(ac+bxc)d(ac+bxc)+e^{ac+bcx}\cot^{-1}(\sinh(ac+bcx))}{bc}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{2e^{ac+bxc}}{1+e^{2ac+2bxc}}de^{ac+bxc}+e^{ac+bcx}\cot^{-1}(\sinh(ac+bcx))}{bc}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{e^{ac+bxc}}{1+e^{2ac+2bxc}}de^{ac+bxc}+e^{ac+bcx}\cot^{-1}(\sinh(ac+bcx))}{bc}$$

$$\Big\downarrow \text{240}$$

$$\frac{\log\left(e^{2ac+2bcx}+1\right)+e^{ac+bcx}\cot^{-1}(\sinh(ac+bcx))}{bc}$$

input
```
Int[E^(c*(a + b*x))*ArcCot[Sinh[a*c + b*c*x]],x]
```

output
```
(E^(a*c + b*c*x)*ArcCot[Sinh[a*c + b*c*x]] + Log[1 + E^(2*a*c + 2*b*c*x)])
/(b*c)
```

3.229.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 5731
```
Int[((a_.) + ArcCot[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Simp[(a + b*ArcCot[u])*w, x] + Simp[b   Int[SimplifyIntegrand[w*(D[u, x]/(
1 + u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCot[u]), x]]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

3.229.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.25 (sec) , antiderivative size = 1281, normalized size of antiderivative = 27.26

method	result	size
risch	Expression too large to display	1281

input
```
int(exp(c*(b*x+a))*arccot(sinh(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

output
```
-2*a/b-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I)^2)*csgn(I*exp(-c*(b*x+a)))*csg
n(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I
*(exp(c*(b*x+a))-I)^2)*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(exp
(c*(b*x+a))-I)^2)*exp(c*(b*x+a))-I/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a))+I)
-1/4/b/c*Pi*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)^3*exp(c*(b*x+a))-1/
4/b/c*Pi*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)^3*exp(c*(b*x+a))-1/4/b
/c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)^3*exp(c*(b*x+a))+1/4/b/
c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)^3*exp(c*(b*x+a))+1/4/b/c
*Pi*csgn(I*(exp(c*(b*x+a))-I)^2)^3*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c
*(b*x+a))+I)^2)^3*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(exp(-c*(b*x+a))*(exp(c*(b
*x+a))+I)^2)^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+
a))-I)^2)^2*exp(c*(b*x+a))+I/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a))-I)+ln(1+
exp(2*c*(b*x+a)))/b/c+1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)
^2)*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)*exp(c*(b*x+a))+1/4/b/c*Pi*c
sgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)*csgn(exp(-c*(b*x+a))*(exp(c*(b
*x+a))+I)^2)^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*
x+a))-I)^2)*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)^2*exp(c*(b*x+a))+1/
4/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I)^2)*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a
))+I)^2)^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c
*(b*x+a))*(exp(c*(b*x+a))+I)^2)^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp...
```

3.229.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(45) = 90$.

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.79

$$\int e^{c(a+bx)}\cot^{-1}(\sinh(ac+bcx))\,dx$$
$$= \frac{(\cosh(bcx+ac)+\sinh(bcx+ac))\arctan\left(\frac{2\,(\cosh(bcx+ac)+\sinh(bcx+ac))}{\cosh(bcx+ac)^2+2\,\cosh(bcx+ac)\sinh(bcx+ac)+\sinh(bcx+ac)^2-1}\right)+\log\left(\frac{\ldots}{\cosh(\ldots}\right.}{bc}$$

input
```
integrate(exp(c*(b*x+a))*arccot(sinh(b*c*x+a*c)),x, algorithm="fricas")
```

output
```
((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*arctan(2*(cosh(b*c*x + a*c) + sin
h(b*c*x + a*c))/(cosh(b*c*x + a*c)^2 + 2*cosh(b*c*x + a*c)*sinh(b*c*x + a*
c) + sinh(b*c*x + a*c)^2 - 1)) + log(2*cosh(b*c*x + a*c)/(cosh(b*c*x + a*c
) - sinh(b*c*x + a*c))))/(b*c)
```

3.229.6 Sympy [F]

$$\int e^{c(a+bx)}\cot^{-1}(\sinh(ac+bcx))\,dx = e^{ac}\int e^{bcx}\operatorname{acot}\left(\sinh\left(ac+bcx\right)\right)dx$$

input
```
integrate(exp(c*(b*x+a))*acot(sinh(b*c*x+a*c)),x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)*acot(sinh(a*c + b*c*x)), x)
```

3.229.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int e^{c(a+bx)}\cot^{-1}(\sinh(ac+bcx))\,dx = \frac{\operatorname{arccot}\left(\sinh\left(bcx+ac\right)\right)e^{((bx+a)c)}}{bc} + \frac{\log\left(e^{(2\,bcx+2\,ac)}+1\right)}{bc}$$

input
```
integrate(exp(c*(b*x+a))*arccot(sinh(b*c*x+a*c)),x, algorithm="maxima")
```

output
```
arccot(sinh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + log(e^(2*b*c*x + 2*a*c)
+ 1)/(b*c)
```

3.229.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.40

$$\int e^{c(a+bx)}\cot^{-1}(\sinh(ac+bcx))\,dx = \frac{\left(\arctan\left(\frac{2}{e^{(bcx+ac)}-e^{(-bcx-ac)}}\right)e^{(bcx)} + e^{(-ac)}\log\left(e^{(2\,bcx+2\,ac)}+1\right)\right)e^{(ac)}}{bc}$$

input
```
integrate(exp(c*(b*x+a))*arccot(sinh(b*c*x+a*c)),x, algorithm="giac")
```

output
```
(arctan(2/(e^(b*c*x + a*c) - e^(-b*c*x - a*c)))*e^(b*c*x) + e^(-a*c)*log(e
^(2*b*c*x + 2*a*c) + 1))*e^(a*c)/(b*c)
```

3.229.9 Mupad [B] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.38

$$\int e^{c(a+bx)} \cot^{-1}(\sinh(ac+bcx)) \, dx = \frac{\ln\left(e^{2\,b\,c\,x}\,e^{2\,a\,c}+1\right)}{b\,c} + \frac{e^{b\,c\,x}\,e^{a\,c}\,\mathrm{acot}\left(\frac{e^{b\,c\,x}\,e^{a\,c}}{2}-\frac{e^{-b\,c\,x}\,e^{-a\,c}}{2}\right)}{b\,c}$$

input
```
int(exp(c*(a + b*x))*acot(sinh(a*c + b*c*x)),x)
```

output
```
log(exp(2*b*c*x)*exp(2*a*c) + 1)/(b*c) + (exp(b*c*x)*exp(a*c)*acot((exp(b*
c*x)*exp(a*c))/2 - (exp(-b*c*x)*exp(-a*c))/2))/(b*c)
```

3.230 $\int e^{c(a+bx)} \cot^{-1}(\cosh(ac+bcx))\,dx$

3.230.1 Optimal result

Integrand size = 20, antiderivative size = 103

$$\int e^{c(a+bx)} \cot^{-1}(\cosh(ac+bcx))\,dx = \frac{e^{ac+bcx}\cot^{-1}(\cosh(c(a+bx)))}{bc} + \frac{(1-\sqrt{2})\log\left(3-2\sqrt{2}+e^{2c(a+bx)}\right)}{2bc} + \frac{(1+\sqrt{2})\log\left(3+2\sqrt{2}+e^{2c(a+bx)}\right)}{2bc}$$

output
```
exp(b*c*x+a*c)*arccot(cosh(c*(b*x+a)))/b/c+1/2*ln(3+exp(2*c*(b*x+a))-2*2^(
1/2))*(1-2^(1/2))/b/c+1/2*ln(3+exp(2*c*(b*x+a))+2*2^(1/2))*(1+2^(1/2))/b/c
```

3.230.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.42

$$\int e^{c(a+bx)} \cot^{-1}(\cosh(ac+bcx))\,dx$$

$$= \frac{4c(a+bx) + 2e^{c(a+bx)}\cot^{-1}\left(\frac{1}{2}e^{-c(a+bx)}\left(1+e^{2c(a+bx)}\right)\right) + \text{RootSum}\left[1+6\#1^2+\#1^4\&, \frac{-ac-bcx+\log\left(e^{c(a+bx)}\right.}{\ldots}\right.}{2bc}$$

input
```
Integrate[E^(c*(a + b*x))*ArcCot[Cosh[a*c + b*c*x]],x]
```

output
```
(4*c*(a + b*x) + 2*E^(c*(a + b*x))*ArcCot[(1 + E^(2*c*(a + b*x)))/(2*E^(c*
(a + b*x)))] + RootSum[1 + 6*#1^2 + #1^4 & , (-(a*c) - b*c*x + Log[E^(c*(a
 + b*x)) - #1] - 7*a*c*#1^2 - 7*b*c*x*#1^2 + 7*Log[E^(c*(a + b*x)) - #1]*#
1^2)/(1 + 3*#1^2) & ])/(2*b*c)
```

3.230.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {7281, 5731, 2720, 27, 1576, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)}\cot^{-1}(\cosh(ac+bcx))\,dx$$

$$\Big\downarrow \text{7281}$$

$$\frac{\int e^{ac+bxc}\cot^{-1}(\cosh(ac+bxc))d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{5731}$$

$$\frac{\int \frac{e^{ac+bxc}\sinh(ac+bxc)}{\cosh^2(ac+bxc)+1}d(ac+bxc)+e^{ac+bcx}\cot^{-1}(\cosh(ac+bcx))}{bc}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{2e^{ac+bxc}\left(1-e^{2ac+2bxc}\right)}{1+6e^{2ac+2bxc}+e^{4ac+4bxc}}de^{ac+bxc}+e^{ac+bcx}\cot^{-1}(\cosh(ac+bcx))}{bc}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^{ac+bcx}\cot^{-1}(\cosh(ac+bcx))-2\int \frac{e^{ac+bxc}\left(1-e^{2ac+2bxc}\right)}{1+6e^{2ac+2bxc}+e^{4ac+4bxc}}de^{ac+bxc}}{bc}$$

$$\Big\downarrow \text{1576}$$

$$\frac{e^{ac+bcx}\cot^{-1}(\cosh(ac+bcx))-\int \frac{-ac-bxc+1}{1+7e^{2ac+2bxc}}de^{2ac+2bxc}}{bc}$$

$$\Big\downarrow \text{1141}$$

$$\frac{e^{ac+bcx}\cot^{-1}(\cosh(ac+bcx)) - \int \left(-\frac{2-\sqrt{2}}{2\left(4-3\sqrt{2}-\sqrt{2}e^{2ac+2bxc}\right)} - \frac{1+\sqrt{2}}{2\left(3+2\sqrt{2}+e^{2ac+2bxc}\right)}\right) de^{2ac+2bxc}}{bc}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{1}{2}(1+\sqrt{2})\log\left(e^{2ac+2bcx}+3+2\sqrt{2}\right)+\frac{1}{2}(1-\sqrt{2})\log\left(-\sqrt{2}e^{2ac+2bcx}+4-3\sqrt{2}\right)+e^{ac+bcx}\cot^{-1}(\cosh(ac+bcx))}{bc}$$

input
```
Int[E^(c*(a + b*x))*ArcCot[Cosh[a*c + b*c*x]],x]
```

output
```
(E^(a*c + b*c*x)*ArcCot[Cosh[a*c + b*c*x]] + ((1 + Sqrt[2])*Log[3 + 2*Sqrt
[2] + E^(2*a*c + 2*b*c*x)])/2 + ((1 - Sqrt[2])*Log[4 - 3*Sqrt[2] - Sqrt[2]
*E^(2*a*c + 2*b*c*x)])/2)/(b*c)
```

3.230.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 5731
```
Int[((a_.) + ArcCot[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Simp[(a + b*ArcCot[u])*w, x] + Simp[b   Int[SimplifyIntegrand[w*(D[u, x]/(
1 + u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCot[u]), x]]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

3.230.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 6.52 (sec) , antiderivative size = 1354, normalized size of antiderivative = 13.15

method	result	size
risch	Expression too large to display	1354

input
```
int(exp(c*(b*x+a))*arccot(cosh(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*I/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+1-2*I*exp(c*(b*x+a)))-1/4/b/c
*Pi*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))^3*exp(
c*(b*x+a))+1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(exp(
2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(-I*ex
p(2*c*(b*x+a))+2*exp(c*(b*x+a))-I)*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a)
)+1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a)
)*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))*csgn(exp(-c*(b*x+a))*(exp(2*c*(
b*x+a))+1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*exp(-c*(
b*x+a)))*csgn(-I*exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-I)*csgn(I*exp(-c*(b*x+a
))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))*exp(c*(b*x+a))-1/4/b/c*Pi*csgn
(exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))^3*exp(c*(b*x+a))
-1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))
))^3*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+
a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*
csgn(I*exp(2*c*(b*x+a))+I+2*exp(c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(-exp(2
*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*exp(
-c*(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))*csgn(exp(-c*(b*x+a))
*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csg
n(I*exp(-c*(b*x+a)))*csgn(I*exp(2*c*(b*x+a))+I+2*exp(c*(b*x+a)))*csgn(I*ex
p(-c*(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))*exp(c*(b*x+a))+...
```

3.230.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(86) = 172$.

Time = 0.30 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.68

$$\int e^{c(a+bx)}\cot^{-1}(\cosh(ac+bcx))\,dx$$

$$= \frac{2\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)\arctan\left(\frac{2\left(\cosh(bcx+ac)+\sinh(bcx+ac)\right)}{\cosh(bcx+ac)^2+2\,\cosh(bcx+ac)\sinh(bcx+ac)+\sinh(bcx+ac)^2+1}\right)+\sqrt{2}\log}{}$$

input
```
integrate(exp(c*(b*x+a))*arccot(cosh(b*c*x+a*c)),x, algorithm="fricas")
```

output
```
1/2*(2*(cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*arctan(2*(cosh(b*c*x + a*c)
 + sinh(b*c*x + a*c))/(cosh(b*c*x + a*c)^2 + 2*cosh(b*c*x + a*c)*sinh(b*c*
x + a*c) + sinh(b*c*x + a*c)^2 + 1)) + sqrt(2)*log((3*(2*sqrt(2) + 3)*cosh
(b*c*x + a*c)^2 - 4*(3*sqrt(2) + 4)*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) +
3*(2*sqrt(2) + 3)*sinh(b*c*x + a*c)^2 + 2*sqrt(2) + 3)/(cosh(b*c*x + a*c)^
2 + sinh(b*c*x + a*c)^2 + 3)) + log(2*(cosh(b*c*x + a*c)^2 + sinh(b*c*x +
a*c)^2 + 3)/(cosh(b*c*x + a*c)^2 - 2*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) +
 sinh(b*c*x + a*c)^2)))/(b*c)
```

3.230.6 Sympy [F]

$$\int e^{c(a+bx)} \cot^{-1}(\cosh(ac+bcx))\,dx = e^{ac} \int e^{bcx} \operatorname{acot}(\cosh(ac+bcx))\,dx$$

input
```
integrate(exp(c*(b*x+a))*acot(cosh(b*c*x+a*c)),x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)*acot(cosh(a*c + b*c*x)), x)
```

3.230.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.27

$$\int e^{c(a+bx)} \cot^{-1}(\cosh(ac+bcx))\,dx = \frac{\operatorname{arccot}(\cosh(bcx+ac))\,e^{((bx+a)c)}}{bc} + \frac{\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(-2\,bcx-2\,ac)}-3}{2\sqrt{2}+e^{(-2\,bcx-2\,ac)}+3}\right)}{2\,bc} + \frac{2\,(bcx+ac)}{bc} + \frac{\log\left(6\,e^{(-2\,bcx-2\,ac)}+e^{(-4\,bcx-4\,ac)}+1\right)}{2\,bc}$$

input
```
integrate(exp(c*(b*x+a))*arccot(cosh(b*c*x+a*c)),x, algorithm="maxima")
```

output
```
arccot(cosh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + 1/2*sqrt(2)*log(-(2*sqrt
(2) - e^(-2*b*c*x - 2*a*c) - 3)/(2*sqrt(2) + e^(-2*b*c*x - 2*a*c) + 3))/(b
*c) + 2*(b*c*x + a*c)/(b*c) + 1/2*log(6*e^(-2*b*c*x - 2*a*c) + e^(-4*b*c*x
 - 4*a*c) + 1)/(b*c)
```

3.230.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.50

$$\int e^{c(a+bx)}\cot^{-1}(\cosh(ac+bcx))\,dx = -\frac{\left(\sqrt{2}e^{(-ac)}\log\left(-\frac{2\sqrt{2}e^{(2ac)}-e^{(2bcx+4ac)}-3e^{(2ac)}}{2\sqrt{2}e^{(2ac)}+e^{(2bcx+4ac)}+3e^{(2ac)}}\right)-2\arctan\left(\frac{2}{e^{(bcx+ac)}+e^{(-bcx-ac)}}\right)e^{(bcx)}-e^{(-ac)}\log\left(e^{(4bcx+4ac)}\right.\right.}{2bc}$$

input
```
integrate(exp(c*(b*x+a))*arccot(cosh(b*c*x+a*c)),x, algorithm="giac")
```

output
```
-1/2*(sqrt(2)*e^(-a*c)*log(-(2*sqrt(2)*e^(2*a*c) - e^(2*b*c*x + 4*a*c) - 3
*e^(2*a*c))/(2*sqrt(2)*e^(2*a*c) + e^(2*b*c*x + 4*a*c) + 3*e^(2*a*c))) - 2
*arctan(2/(e^(b*c*x + a*c) + e^(-b*c*x - a*c)))*e^(b*c*x) - e^(-a*c)*log(e
^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1))*e^(a*c)/(b*c)
```

3.230.9 Mupad [B] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.29

$$\int e^{c(a+bx)}\cot^{-1}(\cosh(ac+bcx))\,dx = \frac{\ln\left(8\,e^{2\,c\,(a+b\,x)}-2\,\sqrt{2}-6\,\sqrt{2}\,e^{2\,c\,(a+b\,x)}\right)\,\left(\sqrt{2}+1\right)}{2\,b\,c} - \frac{\ln\left(8\,e^{2\,c\,(a+b\,x)}+2\,\sqrt{2}+6\,\sqrt{2}\,e^{2\,c\,(a+b\,x)}\right)\,\left(\sqrt{2}-1\right)}{2\,b\,c} + \frac{e^{a\,c+b\,c\,x}\,\mathrm{acot}\left(\frac{e^{b\,c\,x}\,e^{a\,c}}{2}+\frac{e^{-b\,c\,x}\,e^{-a\,c}}{2}\right)}{b\,c}$$

input
```
int(exp(c*(a + b*x))*acot(cosh(a*c + b*c*x)),x)
```

output
```
(log(8*exp(2*c*(a + b*x)) - 2*2^(1/2) - 6*2^(1/2)*exp(2*c*(a + b*x)))*(2^(
1/2) + 1))/(2*b*c) - (log(8*exp(2*c*(a + b*x)) + 2*2^(1/2) + 6*2^(1/2)*exp
(2*c*(a + b*x)))*(2^(1/2) - 1))/(2*b*c) + (exp(a*c + b*c*x)*acot((exp(b*c*
x)*exp(a*c))/2 + (exp(-b*c*x)*exp(-a*c))/2))/(b*c)
```

3.231 $\int e^{c(a+bx)} \cot^{-1}(\tanh(ac+bcx))\,dx$

3.231.1 Optimal result

Integrand size = 20, antiderivative size = 180

$$\int e^{c(a+bx)} \cot^{-1}(\tanh(ac+bcx))\,dx = \frac{e^{ac+bcx}\cot^{-1}(\tanh(c(a+bx)))}{bc} - \frac{\arctan\left(1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} + \frac{\arctan\left(1+\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} + \frac{\log\left(1+e^{2c(a+bx)}-\sqrt{2}e^{ac+bcx}\right)}{2\sqrt{2}bc} - \frac{\log\left(1+e^{2c(a+bx)}+\sqrt{2}e^{ac+bcx}\right)}{2\sqrt{2}bc}$$

output
```
exp(b*c*x+a*c)*arccot(tanh(c*(b*x+a)))/b/c+1/2*arctan(-1+exp(b*c*x+a*c)*2^
(1/2))/b/c*2^(1/2)+1/2*arctan(1+exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)+1/4*ln
(1+exp(2*c*(b*x+a))-exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)-1/4*ln(1+exp(2*c*(
b*x+a))+exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)
```

3.231.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.49

$$\int e^{c(a+bx)}\cot^{-1}(\tanh(ac+bcx))\,dx = \frac{2e^{c(a+bx)}\cot^{-1}\left(\frac{-1+e^{2c(a+bx)}}{1+e^{2c(a+bx)}}\right)+\text{RootSum}\left[1+\#1^4\&,\frac{-ac-bcx+\log\left(e^{c(a+bx)}-\#1\right)}{\#1}\&\right]}{2bc}$$

input
```
Integrate[E^(c*(a + b*x))*ArcCot[Tanh[a*c + b*c*x]],x]
```

output
```
(2*E^(c*(a + b*x))*ArcCot[(-1 + E^(2*c*(a + b*x)))/(1 + E^(2*c*(a + b*x)))
] + RootSum[1 + #1^4 & , (-(a*c) - b*c*x + Log[E^(c*(a + b*x)) - #1])/#1 &
 ])/(2*b*c)
```

3.231.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.97, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {7281, 5731, 27, 2679, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)}\cot^{-1}(\tanh(ac+bcx))\,dx$$

$$\Big\downarrow \text{7281}$$

$$\frac{\int e^{ac+bxc}\cot^{-1}(\tanh(ac+bxc))d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{5731}$$

$$\frac{\int \frac{2e^{3(ac+bxc)}}{1+e^{4(ac+bxc)}}d(ac+bxc)+e^{ac+bcx}\cot^{-1}(\tanh(ac+bcx))}{bc}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{e^{3(ac+bxc)}}{1+e^{4(ac+bxc)}}d(ac+bxc)+e^{ac+bcx}\cot^{-1}(\tanh(ac+bcx))}{bc}$$

$$\Big\downarrow \text{2679}$$

$$\frac{2\int \frac{e^{2ac+2bxc}}{1+e^{4ac+4bxc}}de^{ac+bxc}+e^{ac+bcx}\cot^{-1}(\tanh(ac+bcx))}{bc}$$

$$\Big\downarrow \text{826}$$

$$\frac{2\left(\frac{1}{2}\int \frac{1+e^{2ac+2bxc}}{1+e^{4ac+4bxc}}de^{ac+bxc}-\frac{1}{2}\int \frac{1-e^{2ac+2bxc}}{1+e^{4ac+4bxc}}de^{ac+bxc}\right)+e^{ac+bcx}\cot^{-1}(\tanh(ac+bcx))}{bc}$$

$$\Big\downarrow \text{1476}$$

$$\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{1-\sqrt{2}e^{ac+bxc}+e^{2ac+2bxc}}de^{ac+bxc}+\frac{1}{2}\int \frac{1}{1+\sqrt{2}e^{ac+bxc}+e^{2ac+2bxc}}de^{ac+bxc}\right)-\frac{1}{2}\int \frac{1-e^{2ac+2bxc}}{1+e^{4ac+4bxc}}de^{ac+bxc}\right)+e^{ac+bcx}\text{co}}{bc}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-1-e^{2ac+2bxc}}d\left(1-\sqrt{2}e^{ac+bxc}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-1-e^{2ac+2bxc}}d\left(1+\sqrt{2}e^{ac+bxc}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-e^{2ac+2bxc}}{1+e^{4ac+4bxc}}de^{ac+bxc}\right)+e^{ac+bcx}\cot^{-1}(\text{tan}}{bc}$$

$$\Big\downarrow \text{217}$$

$$\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^{ac+bcx}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-e^{2ac+2bxc}}{1+e^{4ac+4bxc}}de^{ac+bxc}\right)+e^{ac+bcx}\cot^{-1}(\tanh(ac+bcx))}{bc}$$

$$\Big\downarrow \text{1479}$$

$$\frac{2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2e^{ac+bxc}}{1-\sqrt{2}e^{ac+bxc}+e^{2ac+2bxc}}de^{ac+bxc}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(1+\sqrt{2}e^{ac+bxc}\right)}{1+\sqrt{2}e^{ac+bxc}+e^{2ac+2bxc}}de^{ac+bxc}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^{ac+bcx}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^{ac+}\right.}{\sqrt{2}}\right.\right.}{bc}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2e^{ac+bxc}}{1-\sqrt{2}e^{ac+bxc}+e^{2ac+2bxc}}de^{ac+bxc}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(1+\sqrt{2}e^{ac+bxc}\right)}{1+\sqrt{2}e^{ac+bxc}+e^{2ac+2bxc}}de^{ac+bxc}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^{ac+bcx}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^{ac+b}\right.}{\sqrt{2}}\right.\right.}{bc}$$

$$\Big\downarrow \text{27}$$

3.231. $\int e^{c(a+bx)}\cot^{-1}(\tanh(ac+bcx))\,dx$

$$\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2e^{ac+bxc}}{1-\sqrt{2}e^{ac+bxc}+e^{2ac+2bxc}}de^{ac+bxc}}{2\sqrt{2}}-\frac{1}{2}\int \frac{1+\sqrt{2}e^{ac+bxc}}{1+\sqrt{2}e^{ac+bxc}+e^{2ac+2bxc}}de^{ac+bxc}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^{ac+bcx}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e\right.}{\sqrt{2}}\right.}{bc}$$

$\Big\downarrow$ 1103

$$\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^{ac+bcx}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^{ac+bcx}+e^{2ac+2bcx}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{2}e^{ac+bcx}+e^{2ac+2bcx}+1\right)}{2\sqrt{2}}\right)\right)}{bc}$$

input
```
Int[E^(c*(a + b*x))*ArcCot[Tanh[a*c + b*c*x]],x]
```

output
```
(E^(a*c + b*c*x)*ArcCot[Tanh[a*c + b*c*x]] + 2*((-(ArcTan[1 - Sqrt[2]*E^(a
*c + b*c*x)]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*E^(a*c + b*c*x)]/Sqrt[2])/2 + (
Log[1 - Sqrt[2]*E^(a*c + b*c*x) + E^(2*a*c + 2*b*c*x)]/(2*Sqrt[2]) - Log[1
 + Sqrt[2]*E^(a*c + b*c*x) + E^(2*a*c + 2*b*c*x)]/(2*Sqrt[2]))/2))/(b*c)
```

3.231.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

rule 5731
```
Int[((a_.) + ArcCot[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Simp[(a + b*ArcCot[u])*w, x] + Simp[b   Int[SimplifyIntegrand[w*(D[u, x]/(
1 + u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCot[u]), x]]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

3.231. $\int e^{c(a+bx)} \cot^{-1}(\tanh(ac+bcx))\,dx$

3.231.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.48 (sec) , antiderivative size = 1323, normalized size of antiderivative = 7.35

method	result	size
risch	Expression too large to display	1323

input
```
int(exp(c*(b*x+a))*arccot(tanh(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b/c*Pi*csgn((1-I)*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))^3*exp(c*
(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))^3*ex
p(c*(b*x+a))-1/4/b/c*Pi*csgn((1+I)*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a)
)))^3*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*
x+a))))^3*exp(c*(b*x+a))+1/4/b/c*Pi*csgn((1-I)*(exp(2*c*(b*x+a))-I)/(1+exp
(2*c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn((1+I)*(exp(2*c*(b*x+a))+I
)/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a
))-I)/(1+exp(2*c*(b*x+a))))*csgn((1-I)*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*
x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I/(1+exp(2*c*(b*x+a))))*csgn(I*(e
xp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(
I*(exp(2*c*(b*x+a))+I))*csgn(I*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))^
2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I/(1+exp(2*c*(b*x+a))))*csgn(I*(exp(2*c*(
b*x+a))-I))*csgn(I*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))*exp(c*(b*x+a
))-1/4/b/c*Pi*csgn(I/(1+exp(2*c*(b*x+a))))*csgn(I*(exp(2*c*(b*x+a))+I))*cs
gn(I*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))*exp(c*(b*x+a))+1/4/b/c*Pi*
csgn(I*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))*csgn((1+I)*(exp(2*c*(b*x
+a))+I)/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I/(1+exp(2*
c*(b*x+a))))*csgn(I*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*
x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I))*csgn(I*(exp(2*c*(b*x+a))-I)/
(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+...
```

3.231.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.43

$$\int e^{c(a+bx)} \cot^{-1}(\tanh(ac+bcx))\,dx$$
$$= \frac{bc\left(-\frac{1}{b^4c^4}\right)^{\frac{1}{4}} \log\left(b^3c^3\left(-\frac{1}{b^4c^4}\right)^{\frac{3}{4}} + \cosh(bcx+ac) + \sinh(bcx+ac)\right) - i\,bc\left(-\frac{1}{b^4c^4}\right)^{\frac{1}{4}} \log\left(i\,b^3c^3\left(-\frac{1}{b^4c^4}\right)^{\frac{3}{4}} + c\right.}{}$$

input
```
integrate(exp(c*(b*x+a))*arccot(tanh(b*c*x+a*c)),x, algorithm="fricas")
```

output
```
1/2*(b*c*(-1/(b^4*c^4))^(1/4)*log(b^3*c^3*(-1/(b^4*c^4))^(3/4) + cosh(b*c*
x + a*c) + sinh(b*c*x + a*c)) - I*b*c*(-1/(b^4*c^4))^(1/4)*log(I*b^3*c^3*(
-1/(b^4*c^4))^(3/4) + cosh(b*c*x + a*c) + sinh(b*c*x + a*c)) + I*b*c*(-1/(
b^4*c^4))^(1/4)*log(-I*b^3*c^3*(-1/(b^4*c^4))^(3/4) + cosh(b*c*x + a*c) +
sinh(b*c*x + a*c)) - b*c*(-1/(b^4*c^4))^(1/4)*log(-b^3*c^3*(-1/(b^4*c^4))^
(3/4) + cosh(b*c*x + a*c) + sinh(b*c*x + a*c)) + 2*(cosh(b*c*x + a*c) + si
nh(b*c*x + a*c))*arctan(cosh(b*c*x + a*c)/sinh(b*c*x + a*c)))/(b*c)
```

3.231.6 Sympy [F]

$$\int e^{c(a+bx)}\cot^{-1}(\tanh(ac+bcx))\,dx=e^{ac}\int e^{bcx}\operatorname{acot}\left(\tanh\left(ac+bcx\right)\right)dx$$

input
```
integrate(exp(c*(b*x+a))*acot(tanh(b*c*x+a*c)),x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)*acot(tanh(a*c + b*c*x)), x)
```

3.231.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.93

$$\begin{aligned}\int e^{c(a+bx)}\cot^{-1}(\tanh(ac+bcx))\,dx=&\frac{\operatorname{arccot}\left(\tanh\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc}\\&+\frac{\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{(bcx+ac)}\right)\right)}{2bc}\\&+\frac{\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{(bcx+ac)}\right)\right)}{2bc}\\&-\frac{\sqrt{2}\log\left(\sqrt{2}e^{(bcx+ac)}+e^{(2bcx+2ac)}+1\right)}{4bc}\\&+\frac{\sqrt{2}\log\left(-\sqrt{2}e^{(bcx+ac)}+e^{(2bcx+2ac)}+1\right)}{4bc}\end{aligned}$$

input
```
integrate(exp(c*(b*x+a))*arccot(tanh(b*c*x+a*c)),x, algorithm="maxima")
```

output
```
arccot(tanh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + 1/2*sqrt(2)*arctan(1/2*s
qrt(2)*(sqrt(2) + 2*e^(b*c*x + a*c)))/(b*c) + 1/2*sqrt(2)*arctan(-1/2*sqrt
(2)*(sqrt(2) - 2*e^(b*c*x + a*c)))/(b*c) - 1/4*sqrt(2)*log(sqrt(2)*e^(b*c*
x + a*c) + e^(2*b*c*x + 2*a*c) + 1)/(b*c) + 1/4*sqrt(2)*log(-sqrt(2)*e^(b*
c*x + a*c) + e^(2*b*c*x + 2*a*c) + 1)/(b*c)
```

3.231.8 Giac [F]

$$\int e^{c(a+bx)}\cot^{-1}(\tanh(ac+bcx))\,dx=\int \operatorname{arccot}\left(\tanh\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}\,dx$$

input
```
integrate(exp(c*(b*x+a))*arccot(tanh(b*c*x+a*c)),x, algorithm="giac")
```

output
```
sage0*x
```

3.231.9 Mupad [B] (verification not implemented)

Time = 2.91 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.91

$$\int e^{c(a+bx)}\cot^{-1}(\tanh(ac+bcx))\,dx$$
$$=\frac{4\,\mathrm{e}^{a\,c+b\,c\,x}\,\mathrm{acot}\left(\frac{\mathrm{e}^{2\,b\,c\,x}\,\mathrm{e}^{2\,a\,c}-1}{\mathrm{e}^{2\,b\,c\,x}\,\mathrm{e}^{2\,a\,c}+1}\right)+\sqrt{2}\,\ln\left(\sqrt{2}\,(-4-4\mathrm{i})-\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}\,8\mathrm{i}\right)\,(-1-\mathrm{i})+\sqrt{2}\,\ln\left(\sqrt{2}\,(-4+4\mathrm{i})+\mathrm{e}^{b\,c\,x}\right.}{4\,b}$$

input
```
int(exp(c*(a + b*x))*acot(tanh(a*c + b*c*x)),x)
```

output
```
(2^(1/2)*log(2^(1/2)*(4 - 4i) + exp(b*c*x)*exp(a*c)*8i)*(1 - 1i) - 2^(1/2)
*log(exp(b*c*x)*exp(a*c)*8i - 2^(1/2)*(4 - 4i))*(1 - 1i) - 2^(1/2)*log(- 2
^(1/2)*(4 + 4i) - exp(b*c*x)*exp(a*c)*8i)*(1 + 1i) + 2^(1/2)*log(2^(1/2)*(
4 + 4i) - exp(b*c*x)*exp(a*c)*8i)*(1 + 1i) + 4*exp(a*c + b*c*x)*acot((exp(
2*b*c*x)*exp(2*a*c) - 1)/(exp(2*b*c*x)*exp(2*a*c) + 1)))/(4*b*c)
```

3.232 $\int e^{c(a+bx)} \cot^{-1}(\coth(ac+bcx))\,dx$

3.232.1 Optimal result

Integrand size = 20, antiderivative size = 180

$$\int e^{c(a+bx)} \cot^{-1}(\coth(ac+bcx))\,dx = \frac{e^{ac+bcx}\cot^{-1}(\coth(c(a+bx)))}{bc} + \frac{\arctan\left(1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} - \frac{\arctan\left(1+\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} - \frac{\log\left(1+e^{2c(a+bx)}-\sqrt{2}e^{ac+bcx}\right)}{2\sqrt{2}bc} + \frac{\log\left(1+e^{2c(a+bx)}+\sqrt{2}e^{ac+bcx}\right)}{2\sqrt{2}bc}$$

output
```
exp(b*c*x+a*c)*arccot(coth(c*(b*x+a)))/b/c-1/2*arctan(-1+exp(b*c*x+a*c)*2^
(1/2))/b/c*2^(1/2)-1/2*arctan(1+exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)-1/4*ln
(1+exp(2*c*(b*x+a))-exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)+1/4*ln(1+exp(2*c*(
b*x+a))+exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)
```

3.232.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.49

$$\int e^{c(a+bx)} \cot^{-1}(\coth(ac+bcx))\,dx = \frac{2e^{c(a+bx)}\cot^{-1}\left(\frac{1+e^{2c(a+bx)}}{-1+e^{2c(a+bx)}}\right)+\text{RootSum}\left[1+\#1^4\&,\frac{ac+bcx-\log\left(e^{c(a+bx)}-\#1\right)}{\#1}\&\right]}{2bc}$$

input
```
Integrate[E^(c*(a + b*x))*ArcCot[Coth[a*c + b*c*x]],x]
```

output
```
(2*E^(c*(a + b*x))*ArcCot[(1 + E^(2*c*(a + b*x)))/(-1 + E^(2*c*(a + b*x)))
] + RootSum[1 + #1^4 & , (a*c + b*c*x - Log[E^(c*(a + b*x)) - #1])/#1 & ])
/(2*b*c)
```

3.232.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.97, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {7281, 5731, 27, 2679, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)} \cot^{-1}(\coth(ac+bcx))\,dx$$

$$\Big\downarrow \; 7281$$

$$\frac{\int e^{ac+bxc}\cot^{-1}(\coth(ac+bxc))d(ac+bxc)}{bc}$$

$$\Big\downarrow \; 5731$$

$$\frac{\int -\frac{2e^{3(ac+bxc)}}{1+e^{4(ac+bxc)}}d(ac+bxc)+e^{ac+bcx}\cot^{-1}(\coth(ac+bcx))}{bc}$$

$$\Big\downarrow \; 27$$

$$\frac{e^{ac+bcx}\cot^{-1}(\coth(ac+bcx))-2\int \frac{e^{3(ac+bxc)}}{1+e^{4(ac+bxc)}}d(ac+bxc)}{bc}$$

$$\Big\downarrow \; 2679$$

$$\frac{e^{ac+bcx}\cot^{-1}(\coth(ac+bcx))-2\int \frac{e^{2ac+2bxc}}{1+e^{4ac+4bxc}}de^{ac+bxc}}{bc}$$

$$\Big\downarrow \; 826$$

$$\frac{e^{ac+bcx}\cot^{-1}(\coth(ac+bcx))-2\left(\frac{1}{2}\int \frac{1+e^{2ac+2bxc}}{1+e^{4ac+4bxc}}de^{ac+bxc}-\frac{1}{2}\int \frac{1-e^{2ac+2bxc}}{1+e^{4ac+4bxc}}de^{ac+bxc}\right)}{bc}$$

$$\Big\downarrow \; 1476$$

$$\frac{e^{ac+bcx}\cot^{-1}(\coth(ac+bcx))-2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{1-\sqrt{2}e^{ac+bxc}+e^{2ac+2bxc}}de^{ac+bxc}+\frac{1}{2}\int \frac{1}{1+\sqrt{2}e^{ac+bxc}+e^{2ac+2bxc}}de^{ac+bxc}\right)-\frac{1}{2}\int\right.}{bc}$$

$$\Big\downarrow \; 1082$$

$$\frac{e^{ac+bcx}\cot^{-1}(\coth(ac+bcx))-2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-1-e^{2ac+2bxc}}d\left(1-\sqrt{2}e^{ac+bxc}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-1-e^{2ac+2bxc}}d\left(1+\sqrt{2}e^{ac+bxc}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-e^{2ac+2b}}{1+e^{4ac+4b}}\right.}{bc}$$

$$\Big\downarrow \; 217$$

$$\frac{e^{ac+bcx}\cot^{-1}(\coth(ac+bcx))-2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^{ac+bcx}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-e^{2ac+2bxc}}{1+e^{4ac+4bxc}}de^{ac+bxc}\right)}{bc}$$

$$\Big\downarrow \; 1479$$

$$\frac{e^{ac+bcx}\cot^{-1}(\coth(ac+bcx))-2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2e^{ac+bxc}}{1-\sqrt{2}e^{ac+bxc}+e^{2ac+2bxc}}de^{ac+bxc}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(1+\sqrt{2}e^{ac+bxc}\right)}{1+\sqrt{2}e^{ac+bxc}+e^{2ac+2bxc}}de^{ac+bxc}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\text{arct}}{}\right.\right.}{bc}$$

$$\Big\downarrow \; 25$$

$$\frac{e^{ac+bcx}\cot^{-1}(\coth(ac+bcx))-2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2e^{ac+bxc}}{1-\sqrt{2}e^{ac+bxc}+e^{2ac+2bxc}}de^{ac+bxc}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(1+\sqrt{2}e^{ac+bxc}\right)}{1+\sqrt{2}e^{ac+bxc}+e^{2ac+2bxc}}de^{ac+bxc}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\text{arcta}}{}\right.\right.}{bc}$$

$$\Big\downarrow \; 27$$

3.232. $\int e^{c(a+bx)}\cot^{-1}(\coth(ac+bcx))\,dx$

$$\frac{e^{ac+bcx}\cot^{-1}(\coth(ac+bcx))-2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2e^{ac+bxc}}{1-\sqrt{2}e^{ac+bxc}+e^{2ac+2bxc}}de^{ac+bxc}}{2\sqrt{2}}-\frac{1}{2}\int\frac{1+\sqrt{2}e^{ac+bxc}}{1+\sqrt{2}e^{ac+bxc}+e^{2ac+2bxc}}de^{ac+bxc}\right)+\frac{1}{2}\left(\cdots\right.\right.}{bc}$$

$$\Big\downarrow \ 1103$$

$$\frac{e^{ac+bcx}\cot^{-1}(\coth(ac+bcx))-2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^{ac+bcx}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^{ac+bcx}+e^{2ac+2bcx}+1\right)}{2\sqrt{2}}\cdots\right.\right.}{bc}$$

input
```
Int[E^(c*(a + b*x))*ArcCot[Coth[a*c + b*c*x]],x]
```

output
```
(E^(a*c + b*c*x)*ArcCot[Coth[a*c + b*c*x]] - 2*((-(ArcTan[1 - Sqrt[2]*E^(a
*c + b*c*x)]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*E^(a*c + b*c*x)]/Sqrt[2])/2 + (
Log[1 - Sqrt[2]*E^(a*c + b*c*x) + E^(2*a*c + 2*b*c*x)]/(2*Sqrt[2]) - Log[1
 + Sqrt[2]*E^(a*c + b*c*x) + E^(2*a*c + 2*b*c*x)]/(2*Sqrt[2]))/2))/(b*c)
```

3.232.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

rule 5731
```
Int[((a_.) + ArcCot[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Simp[(a + b*ArcCot[u])*w, x] + Simp[b   Int[SimplifyIntegrand[w*(D[u, x]/(
1 + u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCot[u]), x]]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

3.232. $\int e^{c(a+bx)} \cot^{-1}(\coth(ac+bcx))\,dx$

3.232.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.32 (sec) , antiderivative size = 1323, normalized size of antiderivative = 7.35

method	result	size
risch	Expression too large to display	1323

input
```
int(exp(c*(b*x+a))*arccot(coth(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

output
```
1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I))*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(
I*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))*exp(c*(b*x+a))+1/4/b/c*Pi*csg
n(I*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))*csgn((1-I)*(exp(2*c*(b*x+a)
)+I)/(exp(2*c*(b*x+a))-1))*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+
a))-I)/(exp(2*c*(b*x+a))-1))*csgn((1+I)*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x
+a))-1))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I))*csgn(I*(
exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn
(I/(exp(2*c*(b*x+a))-1))*csgn(I*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))
^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I))*csgn(I/(exp(2*c*
(b*x+a))-1))*csgn(I*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))*exp(c*(b*x+
a))-1/4/b/c*Pi*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I*(exp(2*c*(b*x+a))+I)/(e
xp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I
)/(exp(2*c*(b*x+a))-1))*csgn((1-I)*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-
1))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I))*csgn(I*(exp(2
*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(e
xp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))*csgn((1+I)*(exp(2*c*(b*x+a))-I)/(
exp(2*c*(b*x+a))-1))*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I)
/(exp(2*c*(b*x+a))-1))^3*exp(c*(b*x+a))-1/4/b/c*Pi*csgn((1-I)*(exp(2*c*(b*
x+a))+I)/(exp(2*c*(b*x+a))-1))^3*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(2*c
*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))^3*exp(c*(b*x+a))+1/4/b/c*Pi*csgn((1+...
```

3.232.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.43

$$\int e^{c(a+bx)}\cot^{-1}(\coth(ac+bcx))\,dx =$$

$$-\frac{bc\left(-\frac{1}{b^4c^4}\right)^{\frac{1}{4}}\log\left(b^3c^3\left(-\frac{1}{b^4c^4}\right)^{\frac{3}{4}}+\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)-i\,bc\left(-\frac{1}{b^4c^4}\right)^{\frac{1}{4}}\log\left(i\,b^3c^3\left(-\frac{1}{b^4c^4}\right)^{\frac{3}{4}}\right.}{}$$

input
```
integrate(exp(c*(b*x+a))*arccot(coth(b*c*x+a*c)),x, algorithm="fricas")
```

output
```
-1/2*(b*c*(-1/(b^4*c^4))^(1/4)*log(b^3*c^3*(-1/(b^4*c^4))^(3/4) + cosh(b*c
*x + a*c) + sinh(b*c*x + a*c)) - I*b*c*(-1/(b^4*c^4))^(1/4)*log(I*b^3*c^3*
(-1/(b^4*c^4))^(3/4) + cosh(b*c*x + a*c) + sinh(b*c*x + a*c)) + I*b*c*(-1/
(b^4*c^4))^(1/4)*log(-I*b^3*c^3*(-1/(b^4*c^4))^(3/4) + cosh(b*c*x + a*c) +
 sinh(b*c*x + a*c)) - b*c*(-1/(b^4*c^4))^(1/4)*log(-b^3*c^3*(-1/(b^4*c^4))
^(3/4) + cosh(b*c*x + a*c) + sinh(b*c*x + a*c)) - 2*(cosh(b*c*x + a*c) + s
inh(b*c*x + a*c))*arctan(sinh(b*c*x + a*c)/cosh(b*c*x + a*c)))/(b*c)
```

3.232.6 Sympy [F]

$$\int e^{c(a+bx)}\cot^{-1}(\coth(ac+bcx))\,dx = e^{ac}\int e^{bcx}\operatorname{acot}\left(\coth\left(ac+bcx\right)\right)dx$$

input
```
integrate(exp(c*(b*x+a))*acot(coth(b*c*x+a*c)),x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)*acot(coth(a*c + b*c*x)), x)
```

3.232.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.93

$$\begin{aligned}\int e^{c(a+bx)}\cot^{-1}(\coth(ac+bcx))\,dx =& \frac{\operatorname{arccot}\left(\coth\left(bcx+ac\right)\right)e^{((bx+a)c)}}{bc}\\&-\frac{\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{(bcx+ac)}\right)\right)}{2bc}\\&-\frac{\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{(bcx+ac)}\right)\right)}{2bc}\\&+\frac{\sqrt{2}\log\left(\sqrt{2}e^{(bcx+ac)}+e^{(2bcx+2ac)}+1\right)}{4bc}\\&-\frac{\sqrt{2}\log\left(-\sqrt{2}e^{(bcx+ac)}+e^{(2bcx+2ac)}+1\right)}{4bc}\end{aligned}$$

input
```
integrate(exp(c*(b*x+a))*arccot(coth(b*c*x+a*c)),x, algorithm="maxima")
```

output
```
arccot(coth(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - 1/2*sqrt(2)*arctan(1/2*s
qrt(2)*(sqrt(2) + 2*e^(b*c*x + a*c)))/(b*c) - 1/2*sqrt(2)*arctan(-1/2*sqrt
(2)*(sqrt(2) - 2*e^(b*c*x + a*c)))/(b*c) + 1/4*sqrt(2)*log(sqrt(2)*e^(b*c*
x + a*c) + e^(2*b*c*x + 2*a*c) + 1)/(b*c) - 1/4*sqrt(2)*log(-sqrt(2)*e^(b*
c*x + a*c) + e^(2*b*c*x + 2*a*c) + 1)/(b*c)
```

3.232.8 Giac [F]

$$\int e^{c(a+bx)}\cot^{-1}(\coth(ac+bcx))\,dx=\int \operatorname{arccot}\left(\coth\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}\,dx$$

input
```
integrate(exp(c*(b*x+a))*arccot(coth(b*c*x+a*c)),x, algorithm="giac")
```

output
```
sage0*x
```

3.232.9 Mupad [B] (verification not implemented)

Time = 2.74 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.91

$$\int e^{c(a+bx)}\cot^{-1}(\coth(ac+bcx))\,dx$$
$$=\frac{4\,\mathrm{e}^{a\,c+b\,c\,x}\,\mathrm{acot}\left(\frac{\mathrm{e}^{2\,b\,c\,x}\,\mathrm{e}^{2\,a\,c}+1}{\mathrm{e}^{2\,b\,c\,x}\,\mathrm{e}^{2\,a\,c}-1}\right)+\sqrt{2}\,\ln\left(\sqrt{2}\,(-4-4\mathrm{i})+\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}\,8\mathrm{i}\right)\,(-1-\mathrm{i})+\sqrt{2}\,\ln\left(\sqrt{2}\,(-4+4\mathrm{i})-\mathrm{e}^{b\,c\,x}\cdots\right.}{4\,b\cdots}$$

input
```
int(exp(c*(a + b*x))*acot(coth(a*c + b*c*x)),x)
```

output
```
(2^(1/2)*log(2^(1/2)*(4 - 4i) - exp(b*c*x)*exp(a*c)*8i)*(1 - 1i) - 2^(1/2)
*log(- 2^(1/2)*(4 - 4i) - exp(b*c*x)*exp(a*c)*8i)*(1 - 1i) - 2^(1/2)*log(e
xp(b*c*x)*exp(a*c)*8i - 2^(1/2)*(4 + 4i))*(1 + 1i) + 2^(1/2)*log(2^(1/2)*(
4 + 4i) + exp(b*c*x)*exp(a*c)*8i)*(1 + 1i) + 4*exp(a*c + b*c*x)*acot((exp(
2*b*c*x)*exp(2*a*c) + 1)/(exp(2*b*c*x)*exp(2*a*c) - 1)))/(4*b*c)
```

3.233 $\int e^{c(a+bx)} \cot^{-1}(\operatorname{sech}(ac+bcx))\,dx$

3.233.1 Optimal result

Integrand size = 20, antiderivative size = 103

$$\int e^{c(a+bx)} \cot^{-1}(\operatorname{sech}(ac+bcx))\,dx = \frac{e^{ac+bcx}\cot^{-1}(\operatorname{sech}(c(a+bx)))}{bc} - \frac{\left(1-\sqrt{2}\right)\log\left(3-2\sqrt{2}+e^{2c(a+bx)}\right)}{2bc} - \frac{\left(1+\sqrt{2}\right)\log\left(3+2\sqrt{2}+e^{2c(a+bx)}\right)}{2bc}$$

output
```
exp(b*c*x+a*c)*arccot(sech(c*(b*x+a)))/b/c-1/2*ln(3+exp(2*c*(b*x+a))-2*2^(
1/2))*(1-2^(1/2))/b/c-1/2*ln(3+exp(2*c*(b*x+a))+2*2^(1/2))*(1+2^(1/2))/b/c
```

3.233.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.41

$$\int e^{c(a+bx)} \cot^{-1}(\operatorname{sech}(ac+bcx))\,dx = \frac{-4c(a+bx)+2e^{c(a+bx)}\cot^{-1}\left(\frac{2e^{c(a+bx)}}{1+e^{2c(a+bx)}}\right)+\text{RootSum}\left[1+6\#1^2+\#1^4\&,\frac{ac+bcx-\log\left(e^{c(a+bx)}-\#1\right)+7ac\#1^2}{1+3}\right.}{2bc}$$

input
```
Integrate[E^(c*(a + b*x))*ArcCot[Sech[a*c + b*c*x]],x]
```

output
```
(-4*c*(a + b*x) + 2*E^(c*(a + b*x))*ArcCot[(2*E^(c*(a + b*x)))/(1 + E^(2*c
*(a + b*x)))] + RootSum[1 + 6*#1^2 + #1^4 & , (a*c + b*c*x - Log[E^(c*(a +
 b*x)) - #1] + 7*a*c*#1^2 + 7*b*c*x*#1^2 - 7*Log[E^(c*(a + b*x)) - #1]*#1^
2)/(1 + 3*#1^2) & ])/(2*b*c)
```

3.233.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {7281, 5731, 25, 2720, 27, 1576, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)} \cot^{-1}(\text{sech}(ac+bcx))\,dx$$

$$\Big\downarrow \text{7281}$$

$$\frac{\int e^{ac+bxc} \cot^{-1}(\text{sech}(ac+bxc))d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{5731}$$

$$\frac{\int -\frac{e^{ac+bxc}\text{sech}(ac+bxc)\tanh(ac+bxc)}{\text{sech}^2(ac+bxc)+1}d(ac+bxc)+e^{ac+bcx} \cot^{-1}(\text{sech}(ac+bcx))}{bc}$$

$$\Big\downarrow \text{25}$$

$$\frac{e^{ac+bcx} \cot^{-1}(\text{sech}(ac+bcx))-\int \frac{e^{ac+bxc}\text{sech}(ac+bxc)\tanh(ac+bxc)}{\text{sech}^2(ac+bxc)+1}d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{2720}$$

$$\frac{e^{ac+bcx} \cot^{-1}(\text{sech}(ac+bcx))-\int -\frac{2e^{ac+bxc}\left(1-e^{2ac+2bxc}\right)}{1+6e^{2ac+2bxc}+e^{4ac+4bxc}}de^{ac+bxc}}{bc}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{e^{ac+bxc}\left(1-e^{2ac+2bxc}\right)}{1+6e^{2ac+2bxc}+e^{4ac+4bxc}}de^{ac+bxc}+e^{ac+bcx} \cot^{-1}(\text{sech}(ac+bcx))}{bc}$$

$$\Big\downarrow \text{1576}$$

$$\frac{\int \frac{-ac-bxc+1}{1+7e^{2ac+2bxc}}de^{2ac+2bxc}+e^{ac+bcx}\cot^{-1}(\operatorname{sech}(ac+bcx))}{bc}$$

$$\Big\downarrow \text{1141}$$

$$\frac{\int \left(-\frac{2-\sqrt{2}}{2\left(4-3\sqrt{2}-\sqrt{2}e^{2ac+2bxc}\right)}-\frac{1+\sqrt{2}}{2\left(3+2\sqrt{2}+e^{2ac+2bxc}\right)}\right)de^{2ac+2bxc}+e^{ac+bcx}\cot^{-1}(\operatorname{sech}(ac+bcx))}{bc}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{2}(1+\sqrt{2})\log\left(e^{2ac+2bcx}+3+2\sqrt{2}\right)-\frac{1}{2}(1-\sqrt{2})\log\left(-\sqrt{2}e^{2ac+2bcx}+4-3\sqrt{2}\right)+e^{ac+bcx}\cot^{-1}(\operatorname{sech}(ac+bcx}{bc}$$

input
```
Int[E^(c*(a + b*x))*ArcCot[Sech[a*c + b*c*x]],x]
```

output
```
(E^(a*c + b*c*x)*ArcCot[Sech[a*c + b*c*x]] - ((1 + Sqrt[2])*Log[3 + 2*Sqrt
[2] + E^(2*a*c + 2*b*c*x)])/2 - ((1 - Sqrt[2])*Log[4 - 3*Sqrt[2] - Sqrt[2]
*E^(2*a*c + 2*b*c*x)])/2)/(b*c)
```

3.233.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 5731
```
Int[((a_.) + ArcCot[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Simp[(a + b*ArcCot[u])*w, x] + Simp[b   Int[SimplifyIntegrand[w*(D[u, x]/(
1 + u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCot[u]), x]]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

3.233.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 6.55 (sec) , antiderivative size = 855, normalized size of antiderivative = 8.30

method	result	size
risch	Expression too large to display	855

input
```
int(exp(c*(b*x+a))*arccot(sech(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+1-2*I*exp(c*(b*x+a)))+1/4/b/
c*Pi*csgn(I*exp(2*c*(b*x+a))+I+2*exp(c*(b*x+a)))*csgn(I/(1+exp(2*c*(b*x+a)
)))*csgn(I*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))*
exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*exp(2*c*(b*x+a))+I+2*exp(c*(b*x+a)))*csgn
(I*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^2*exp(c*
(b*x+a))+1/4/b/c*Pi*csgn(I/(1+exp(2*c*(b*x+a))))*csgn(I*(-exp(2*c*(b*x+a))
-1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*c
sgn(-I*exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-I)*csgn(I/(1+exp(2*c*(b*x+a))))*c
sgn(I*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))*exp(c*
(b*x+a))-1/4/b/c*Pi*csgn(I/(1+exp(2*c*(b*x+a))))*csgn(I*(exp(2*c*(b*x+a))+
1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*cs
gn(I*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^3*exp(
c*(b*x+a))+1/4/b/c*Pi*csgn(-I*exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-I)*csgn(I*
(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x
+a))+1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*
(b*x+a))))^3*exp(c*(b*x+a))+1/2*I/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+1
+2*I*exp(c*(b*x+a)))+1/2*Pi/b/c*exp(c*(b*x+a))+1/2/b/c*ln(exp(2*c*(b*x+a))
+(2^(1/2)-1)^2)*2^(1/2)-1/2/b/c*ln(exp(2*c*(b*x+a))+(1+2^(1/2))^2)*2^(1/2)
+2*a/b-1/2/b/c*ln(exp(2*c*(b*x+a))+(2^(1/2)-1)^2)-1/2/b/c*ln(exp(2*c*(b*x+
a))+(1+2^(1/2))^2)
```

3.233.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(86) = 172$.

Time = 0.30 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.15

$$\int e^{c(a+bx)}\cot^{-1}(\operatorname{sech}(ac+bcx))\,dx$$

$$= \frac{2\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)\arctan\left(\cosh\left(bcx+ac\right)\right)+\sqrt{2}\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh(bcx+ac)^2-4\left(3\sqrt{2}-4\right)}{\cosh(}\right.}{2}$$

input
```
integrate(exp(c*(b*x+a))*arccot(sech(b*c*x+a*c)),x, algorithm="fricas")
```

output
```
1/2*(2*(cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*arctan(cosh(b*c*x + a*c)) +
 sqrt(2)*log(-(3*(2*sqrt(2) - 3)*cosh(b*c*x + a*c)^2 - 4*(3*sqrt(2) - 4)*c
osh(b*c*x + a*c)*sinh(b*c*x + a*c) + 3*(2*sqrt(2) - 3)*sinh(b*c*x + a*c)^2
 + 2*sqrt(2) - 3)/(cosh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2 + 3)) - log(2
*(cosh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2 + 3)/(cosh(b*c*x + a*c)^2 - 2*
cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + sinh(b*c*x + a*c)^2)))/(b*c)
```

3.233.6 Sympy [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\cot^{-1}(\operatorname{sech}(ac+bcx))\,dx = \text{Timed out}$$

input
```
integrate(exp(c*(b*x+a))*acot(sech(b*c*x+a*c)),x)
```

output
```
Timed out
```

3.233.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.64

$$\int e^{c(a+bx)}\cot^{-1}(\operatorname{sech}(ac+bcx))\,dx = \frac{\operatorname{arccot}(\operatorname{sech}(bcx+ac))\,e^{((bx+a)c)}}{bc} + \frac{3\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(2\,bcx+2\,ac)}-3}{2\sqrt{2}+e^{(2\,bcx+2\,ac)}+3}\right)}{8\,bc} - \frac{\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(-2\,bcx-2\,ac)}-3}{2\sqrt{2}+e^{(-2\,bcx-2\,ac)}+3}\right)}{8\,bc} - \frac{\log\left(e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1\right)}{2\,bc}$$

input
```
integrate(exp(c*(b*x+a))*arccot(sech(b*c*x+a*c)),x, algorithm="maxima")
```

output
```
arccot(sech(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + 3/8*sqrt(2)*log(-(2*sqrt
(2) - e^(2*b*c*x + 2*a*c) - 3)/(2*sqrt(2) + e^(2*b*c*x + 2*a*c) + 3))/(b*c
) - 1/8*sqrt(2)*log(-(2*sqrt(2) - e^(-2*b*c*x - 2*a*c) - 3)/(2*sqrt(2) + e
^(-2*b*c*x - 2*a*c) + 3))/(b*c) - 1/2*log(e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c
*x + 2*a*c) + 1)/(b*c)
```

3.233.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.50

$$\int e^{c(a+bx)} \cot^{-1}(\operatorname{sech}(ac+bcx))\,dx = \frac{\left(\sqrt{2}e^{(-ac)}\log\left(-\frac{2\sqrt{2}e^{(2\,ac)}-e^{(2\,bcx+4\,ac)}-3\,e^{(2\,ac)}}{2\sqrt{2}e^{(2\,ac)}+e^{(2\,bcx+4\,ac)}+3\,e^{(2\,ac)}}\right)+2\arctan\left(\frac{1}{2}e^{(bcx+ac)}+\frac{1}{2}e^{(-bcx-ac)}\right)e^{(bcx)}-e^{(-ac)}\log\left(e^{(4\,bc}\right.\right.}{2\,bc}$$

input
```
integrate(exp(c*(b*x+a))*arccot(sech(b*c*x+a*c)),x, algorithm="giac")
```

output
```
1/2*(sqrt(2)*e^(-a*c)*log(-(2*sqrt(2)*e^(2*a*c) - e^(2*b*c*x + 4*a*c) - 3*
e^(2*a*c))/(2*sqrt(2)*e^(2*a*c) + e^(2*b*c*x + 4*a*c) + 3*e^(2*a*c))) + 2*
arctan(1/2*e^(b*c*x + a*c) + 1/2*e^(-b*c*x - a*c))*e^(b*c*x) - e^(-a*c)*lo
g(e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1))*e^(a*c)/(b*c)
```

3.233.9 Mupad [B] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.31

$$\int e^{c(a+bx)} \cot^{-1}(\operatorname{sech}(ac+bcx))\,dx = \frac{e^{a\,c+b\,c\,x}\,\mathrm{acot}\left(\frac{1}{\frac{e^{b\,c\,x}\,e^{a\,c}}{2}+\frac{e^{-b\,c\,x}\,e^{-a\,c}}{2}}\right)}{b\,c} + \frac{\ln\left(-8\,e^{2\,c\,(a+b\,x)}-2\,\sqrt{2}-6\,\sqrt{2}\,e^{2\,c\,(a+b\,x)}\right)\,\left(\sqrt{2}-1\right)}{2\,b\,c} - \frac{\ln\left(2\,\sqrt{2}-8\,e^{2\,c\,(a+b\,x)}+6\,\sqrt{2}\,e^{2\,c\,(a+b\,x)}\right)\,\left(\sqrt{2}+1\right)}{2\,b\,c}$$

input
```
int(acot(1/cosh(a*c + b*c*x))*exp(c*(a + b*x)),x)
```

output
```
(exp(a*c + b*c*x)*acot(1/((exp(b*c*x)*exp(a*c))/2 + (exp(-b*c*x)*exp(-a*c)
)/2)))/(b*c) + (log(- 8*exp(2*c*(a + b*x)) - 2*2^(1/2) - 6*2^(1/2)*exp(2*c
*(a + b*x)))*(2^(1/2) - 1))/(2*b*c) - (log(2*2^(1/2) - 8*exp(2*c*(a + b*x)
) + 6*2^(1/2)*exp(2*c*(a + b*x)))*(2^(1/2) + 1))/(2*b*c)
```

3.234 $\int e^{c(a+bx)} \cot^{-1}(\mathbf{csch}(ac+bcx))\,dx$

3.234.1 Optimal result

Integrand size = 20, antiderivative size = 48

$$\int e^{c(a+bx)} \cot^{-1}(\operatorname{csch}(ac+bcx))\,dx = \frac{e^{ac+bcx}\cot^{-1}(\operatorname{csch}(c(a+bx)))}{bc} - \frac{\log\left(1+e^{2c(a+bx)}\right)}{bc}$$

output
```
exp(b*c*x+a*c)*arccot(csch(c*(b*x+a)))/b/c-ln(1+exp(2*c*(b*x+a)))/b/c
```

3.234.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.23

$$\int e^{c(a+bx)} \cot^{-1}(\operatorname{csch}(ac+bcx))\,dx = \frac{e^{c(a+bx)}\cot^{-1}\left(\frac{2e^{c(a+bx)}}{-1+e^{2c(a+bx)}}\right) - \log\left(1+e^{2c(a+bx)}\right)}{bc}$$

input
```
Integrate[E^(c*(a + b*x))*ArcCot[Csch[a*c + b*c*x]],x]
```

output
```
(E^(c*(a + b*x))*ArcCot[(2*E^(c*(a + b*x)))/(-1 + E^(2*c*(a + b*x)))] - Lo
g[1 + E^(2*c*(a + b*x))])/(b*c)
```

3.234.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {7281, 5731, 25, 2720, 27, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)}\cot^{-1}(\operatorname{csch}(ac+bcx))\,dx$$

$$\Big\downarrow \text{7281}$$

$$\frac{\int e^{ac+bxc}\cot^{-1}(\operatorname{csch}(ac+bxc))d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{5731}$$

$$\frac{\int -e^{ac+bxc}\text{sech}(ac+bxc)d(ac+bxc)+e^{ac+bcx}\cot^{-1}(\operatorname{csch}(ac+bcx))}{bc}$$

$$\Big\downarrow \text{25}$$

$$\frac{e^{ac+bcx}\cot^{-1}(\operatorname{csch}(ac+bcx))-\int e^{ac+bxc}\text{sech}(ac+bxc)d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{2720}$$

$$\frac{e^{ac+bcx}\cot^{-1}(\operatorname{csch}(ac+bcx))-\int \frac{2e^{ac+bxc}}{1+e^{2ac+2bxc}}de^{ac+bxc}}{bc}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^{ac+bcx}\cot^{-1}(\operatorname{csch}(ac+bcx))-2\int \frac{e^{ac+bxc}}{1+e^{2ac+2bxc}}de^{ac+bxc}}{bc}$$

$$\Big\downarrow \text{240}$$

$$\frac{e^{ac+bcx}\cot^{-1}(\operatorname{csch}(ac+bcx))-\log\left(e^{2ac+2bcx}+1\right)}{bc}$$

input
```
Int[E^(c*(a + b*x))*ArcCot[Csch[a*c + b*c*x]],x]
```

output
```
(E^(a*c + b*c*x)*ArcCot[Csch[a*c + b*c*x]] - Log[1 + E^(2*a*c + 2*b*c*x)])
/(b*c)
```

3.234.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 5731
```
Int[((a_.) + ArcCot[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Simp[(a + b*ArcCot[u])*w, x] + Simp[b   Int[SimplifyIntegrand[w*(D[u, x]/(
1 + u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCot[u]), x]]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

3.234.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.35 (sec) , antiderivative size = 903, normalized size of antiderivative = 18.81

method	result	size
risch	Expression too large to display	903

input
```
int(exp(c*(b*x+a))*arccot(csch(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

output
```
-I/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a))-I)-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a
))-I))^2*csgn(I*(exp(c*(b*x+a))-I)^2)*exp(c*(b*x+a))+1/2/b/c*Pi*csgn(I*(ex
p(c*(b*x+a))-I))*csgn(I*(exp(c*(b*x+a))-I)^2)^2*exp(c*(b*x+a))-1/4/b/c*Pi*
csgn(I*(exp(c*(b*x+a))-I)^2)^3*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*
x+a))-I)^2)*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I*(exp(c*(b*x+a))-I)^2/(exp(
2*c*(b*x+a))-1))*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I)^2)*cs
gn(I*(exp(c*(b*x+a))-I)^2/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))+1/4/b/c*P
i*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I*(exp(c*(b*x+a))-I)^2/(exp(2*c*(b*x+a
))-1))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I)^2)*csgn(I/(ex
p(2*c*(b*x+a))-1))*csgn(I*(exp(c*(b*x+a))+I)^2/(exp(2*c*(b*x+a))-1))*exp(c
*(b*x+a))-1/4/b/c*Pi*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I*(exp(c*(b*x+a))+I
)^2/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a
))-I)^2/(exp(2*c*(b*x+a))-1))^3*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(c*(b
*x+a))+I))^2*csgn(I*(exp(c*(b*x+a))+I)^2)*exp(c*(b*x+a))-1/2/b/c*Pi*csgn(I
*(exp(c*(b*x+a))+I))*csgn(I*(exp(c*(b*x+a))+I)^2)^2*exp(c*(b*x+a))+1/4/b/c
*Pi*csgn(I*(exp(c*(b*x+a))+I)^2)^3*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c
*(b*x+a))+I)^2)*csgn(I*(exp(c*(b*x+a))+I)^2/(exp(2*c*(b*x+a))-1))^2*exp(c*
(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I)^2/(exp(2*c*(b*x+a))-1))^3*ex
p(c*(b*x+a))+I/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a))+I)+1/2*Pi/b/c*exp(c*(b
*x+a))+2*a/b-ln(1+exp(2*c*(b*x+a)))/b/c
```

3.234.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.56

$$\int e^{c(a+bx)}\cot^{-1}(\operatorname{csch}(ac+bcx))\,dx = \frac{(\cosh(bcx+ac)+\sinh(bcx+ac))\arctan(\sinh(bcx+ac))-\log\left(\frac{2\cosh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{bc}$$

input
```
integrate(exp(c*(b*x+a))*arccot(csch(b*c*x+a*c)),x, algorithm="fricas")
```

output
```
((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*arctan(sinh(b*c*x + a*c)) - log(2
*cosh(b*c*x + a*c)/(cosh(b*c*x + a*c) - sinh(b*c*x + a*c))))/(b*c)
```

3.234.6 Sympy [F]

$$\int e^{c(a+bx)} \cot^{-1}(\operatorname{csch}(ac+bcx))\,dx = e^{ac} \int e^{bcx} \operatorname{acot}\left(\operatorname{csch}\left(ac+bcx\right)\right)dx$$

input
```
integrate(exp(c*(b*x+a))*acot(csch(b*c*x+a*c)),x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)*acot(csch(a*c + b*c*x)), x)
```

3.234.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int e^{c(a+bx)} \cot^{-1}(\operatorname{csch}(ac+bcx))\,dx = \frac{\operatorname{arccot}\left(\operatorname{csch}\left(bcx+ac\right)\right)e^{((bx+a)c)}}{bc} - \frac{\log\left(e^{(2\,bcx+2\,ac)}+1\right)}{bc}$$

input
```
integrate(exp(c*(b*x+a))*arccot(csch(b*c*x+a*c)),x, algorithm="maxima")
```

output
```
arccot(csch(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - log(e^(2*b*c*x + 2*a*c)
+ 1)/(b*c)
```

3.234.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.35

$$\int e^{c(a+bx)} \cot^{-1}(\operatorname{csch}(ac+bcx))\,dx = \frac{\left(\arctan\left(\frac{1}{2}\,e^{(bcx+ac)} - \frac{1}{2}\,e^{(-bcx-ac)}\right)e^{(bcx)} - e^{(-ac)}\log\left(e^{(2\,bcx+2\,ac)}+1\right)\right)e^{(ac)}}{bc}$$

input
```
integrate(exp(c*(b*x+a))*arccot(csch(b*c*x+a*c)),x, algorithm="giac")
```

output
```
(arctan(1/2*e^(b*c*x + a*c) - 1/2*e^(-b*c*x - a*c))*e^(b*c*x) - e^(-a*c)*l
og(e^(2*b*c*x + 2*a*c) + 1))*e^(a*c)/(b*c)
```

3.234.9 Mupad [B] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.42

$$\int e^{c(a+bx)}\cot^{-1}(\operatorname{csch}(ac+bcx))\,dx=\frac{e^{b\,c\,x}\,e^{a\,c}\,\mathrm{acot}\left(\frac{1}{\frac{e^{b\,c\,x}\,e^{a\,c}}{2}-\frac{e^{-b\,c\,x}\,e^{-a\,c}}{2}}\right)}{b\,c}-\frac{\ln\left(e^{2\,b\,c\,x}\,e^{2\,a\,c}+1\right)}{b\,c}$$

input
```
int(acot(1/sinh(a*c + b*c*x))*exp(c*(a + b*x)),x)
```

output
```
(exp(b*c*x)*exp(a*c)*acot(1/((exp(b*c*x)*exp(a*c))/2 - (exp(-b*c*x)*exp(-a
*c))/2)))/(b*c) - log(exp(2*b*c*x)*exp(2*a*c) + 1)/(b*c)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
          if is_contains_complex(optimal) then
              if debug then
                      print("both result and optimal complex");
              fi;
              if leaf_count_result<=2*leaf_count_optimal then
                  return "A"," ";
              else
                 return "B",cat("Both result and optimal contain complex but leaf count of
                                 convert(leaf_count_result,string)," vs. $2 (",
                                 convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
              end if
          else #result contains complex but optimal is not
              if debug then
                      print("result contains complex but optimal is not");
              fi;
              return "C","Result contains complex when optimal does not.";
          fi;
      else # result do not contain complex
           # this assumes optimal do not as well. No check is needed here.
          if debug then
                 print("result do not contain complex, this assumes optimal do not as well"
          fi;
          if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                  print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A"," ";
          else
              if debug then
                  print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                 convert(leaf_count_result,string),"$ vs. $2(",
                                 convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
          fi;
      fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =" "
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


    print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```