Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

5-Inverse-trig-functions/5.5-Inverse-secant/157-5.5.2-Inverse-secant-functions

Nasser M. Abbasi

December 9, 2023 Compiled on December 9, 2023 at 4:41am

Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [50]. This is test number [157].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (50)	0.00 (0)
Mathematica	98.00 (49)	2.00 (1)
Maple	74.00 (37)	26.00 (13)
Fricas	56.00 (28)	44.00 (22)
Giac	54.00 (27)	46.00 (23)
Maxima	36.00 (18)	64.00 (32)
Sympy	26.00 (13)	74.00 (37)
Mupad	20.00 (10)	80.00 (40)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

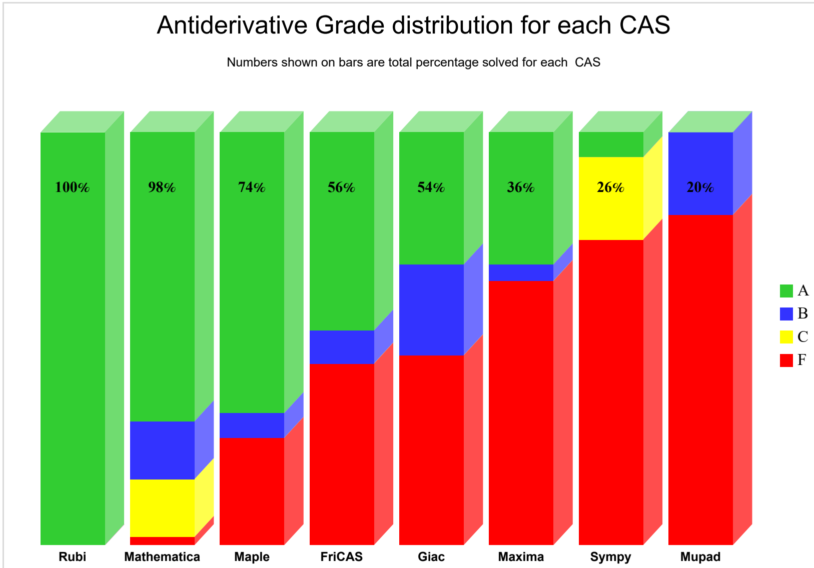
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

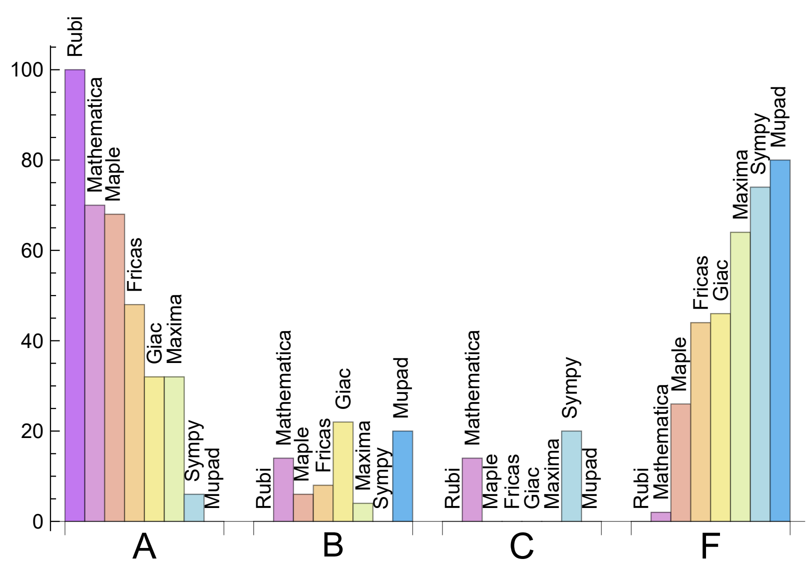
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	70.000	14.000	14.000	2.000
Maple	68.000	6.000	0.000	26.000
Fricas	48.000	8.000	0.000	44.000
Giac	32.000	22.000	0.000	46.000
Maxima	32.000	4.000	0.000	64.000
Sympy	6.000	0.000	20.000	74.000
Mupad	0.000	20.000	0.000	80.000

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	1	0.00	100.00	0.00
Maple	13	100.00	0.00	0.00
Fricas	22	90.91	0.00	9.09
Giac	23	91.30	0.00	8.70
Maxima	32	100.00	0.00	0.00
Sympy	37	91.89	8.11	0.00
Mupad	40	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.21
Fricas	0.29
Giac	0.31
Rubi	0.47
Mathematica	0.60
Maple	0.79
Mupad	0.92
Sympy	21.08

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	37.50	0.91	36.50	0.90
Maxima	56.67	1.21	54.50	1.20
Sympy	76.46	1.63	61.00	1.67
Fricas	102.61	1.37	56.50	0.92
Giac	120.67	1.65	82.00	1.72
Rubi	125.98	1.04	74.50	1.00
Maple	174.38	1.36	76.00	1.31
Mathematica	195.20	1.84	107.00	1.00

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

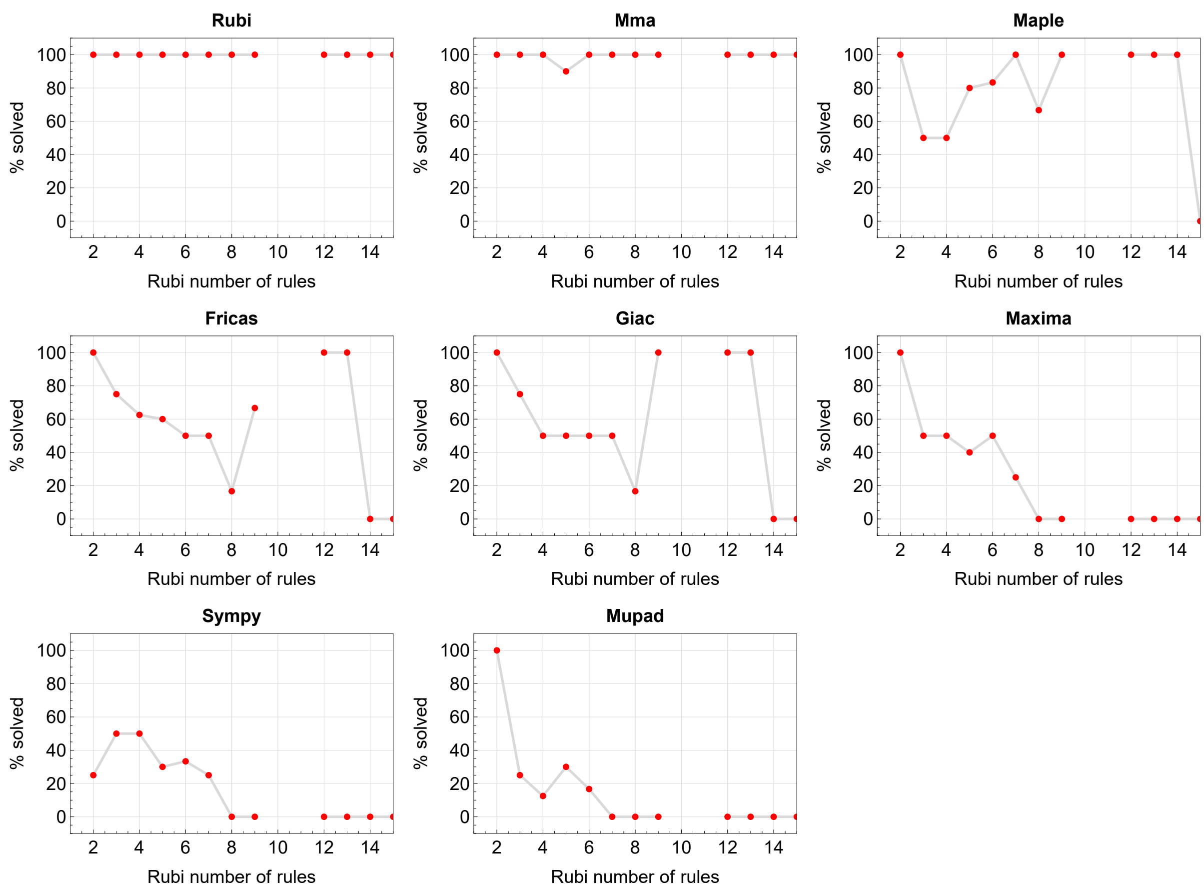


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

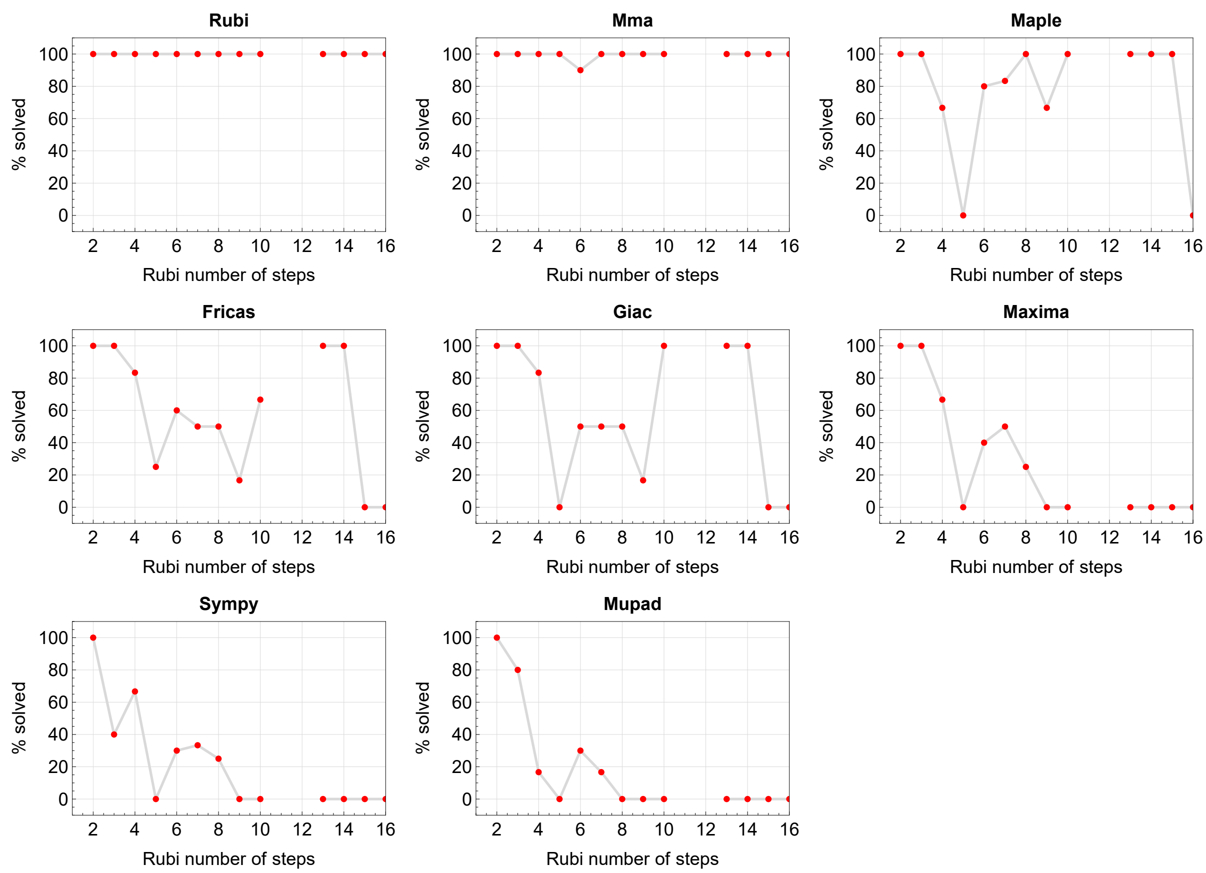


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

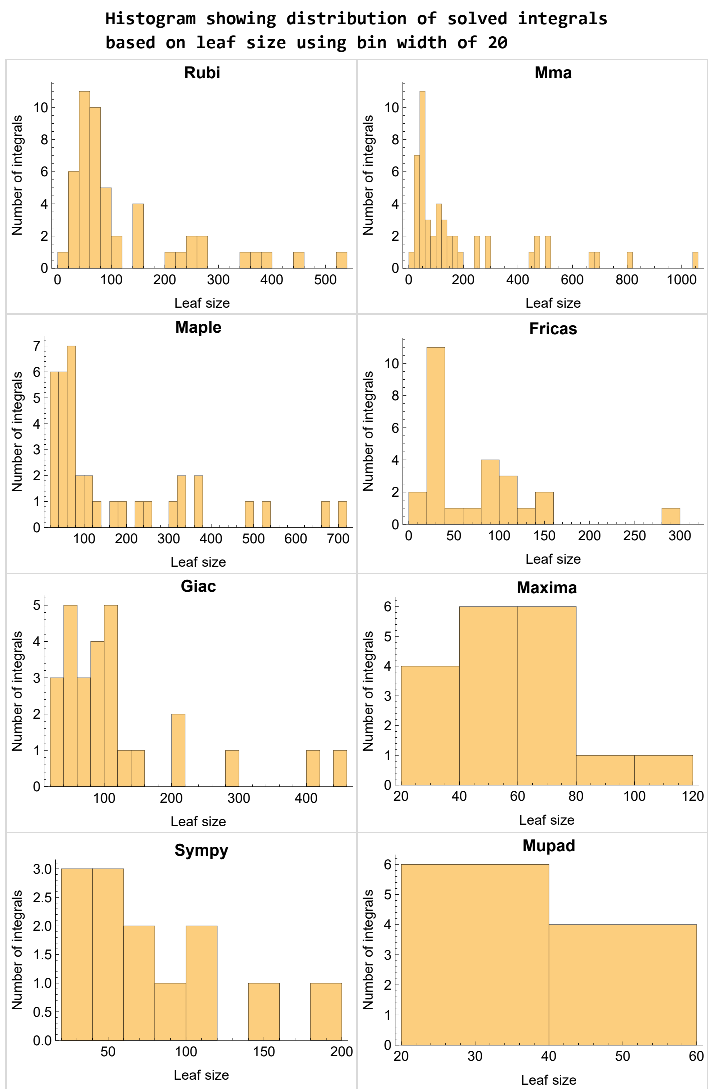


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

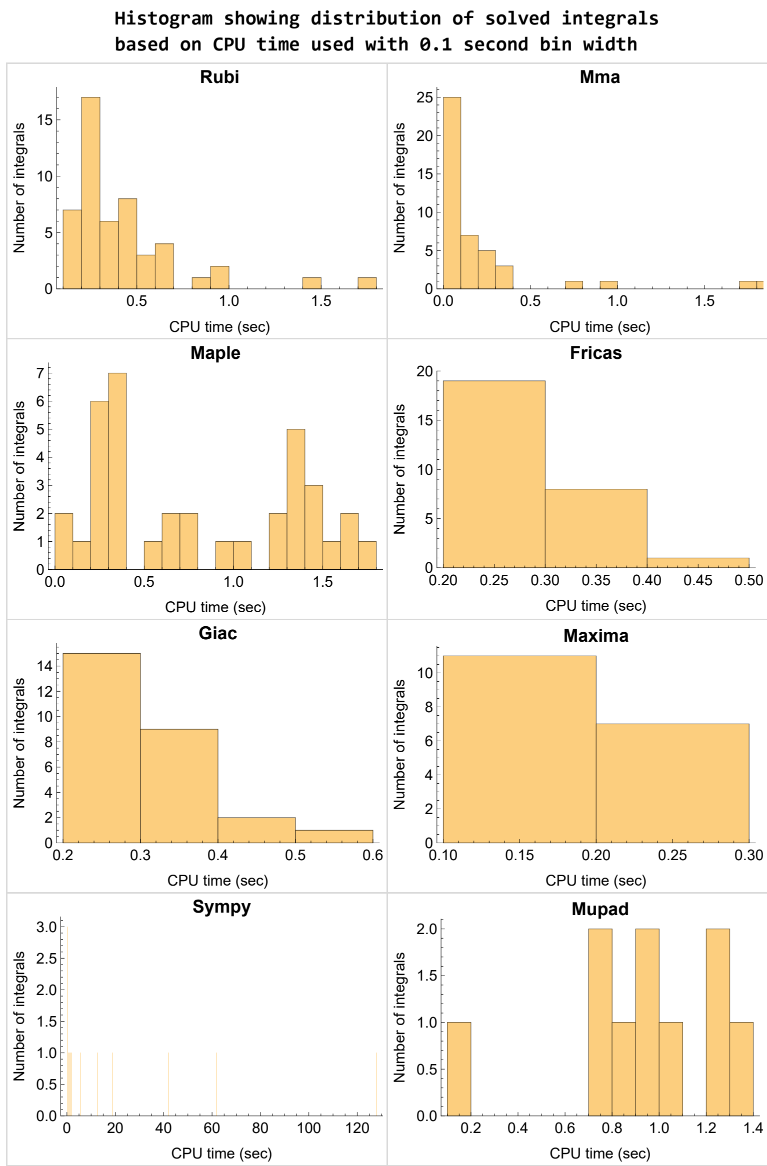


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

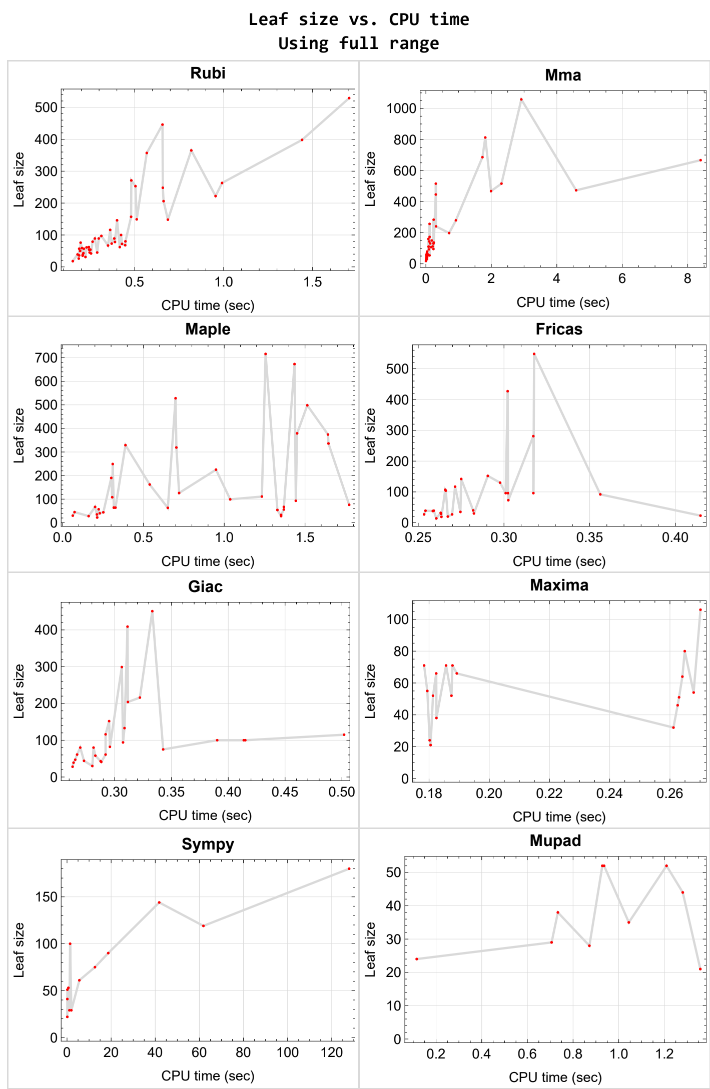


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{1, 6, 17, 22, 41, 42, 50\}$

Mathematica $\{27, 28, 31, 36\}$

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

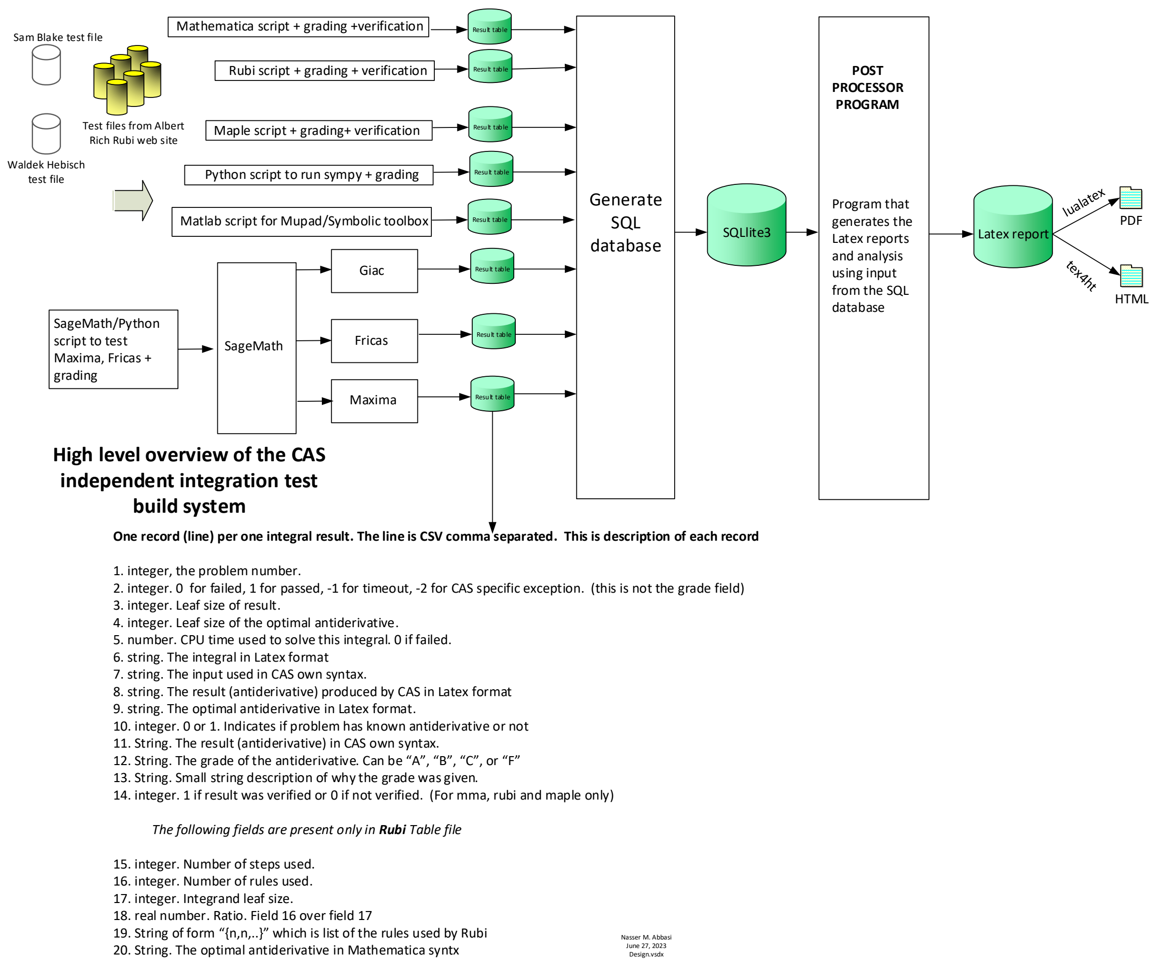
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 18, 19, 20, 21, 23, 27, 28, 29, 30, 33, 34, 35, 43, 44, 45, 46, 47, 48, 49, 50 }

B grade { 14, 31, 32, 36, 40, 41, 42 }

C grade { 17, 22, 24, 25, 26, 38, 39 }

F normal fail { }

F(-1) timedout fail { 37 }

F(-2) exception fail { }

2.1.3 Maple

A grade $\{ 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 27, 28, 29, 30, 32, 33, 34, 38, 39, 40, 42, 50 \}$

B grade $\{ 24, 25, 26 \}$

C grade $\{ \}$

F normal fail $\{ 1, 31, 35, 36, 37, 41, 43, 44, 45, 46, 47, 48, 49 \}$

F(-1) timedout fail $\{ \}$

F(-2) exception fail $\{ \}$

2.1.4 Fricas

A grade $\{ 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 15, 18, 19, 20, 21, 25, 26, 38, 39, 40, 41, 47, 48, 49 \}$

B grade $\{ 14, 16, 22, 24 \}$

C grade $\{ \}$

F normal fail $\{ 1, 6, 13, 23, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 43, 44, 45, 46, 50 \}$

F(-1) timedout fail $\{ \}$

F(-2) exception fail $\{ 17, 42 \}$

2.1.5 Maxima

A grade $\{ 2, 3, 4, 5, 7, 10, 11, 12, 14, 15, 16, 22, 38, 39, 40, 41 \}$

B grade $\{ 8, 9 \}$

C grade $\{ \}$

F normal fail $\{ 1, 6, 13, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 42, 43, 44, 45, 46, 47, 48, 49, 50 \}$

F(-1) timedout fail $\{ \}$

F(-2) exception fail $\{ \}$

2.1.6 Giac

A grade { 7, 8, 9, 10, 11, 12, 15, 16, 21, 24, 38, 39, 40, 41, 47, 50 }

B grade { 2, 3, 4, 5, 14, 18, 19, 20, 22, 25, 26 }

C grade { }

F normal fail { 1, 13, 17, 23, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 42, 43, 44, 45, 46, 48, 49 }

F(-1) timedout fail { }

F(-2) exception fail { 6, 27 }

2.1.7 Mupad

A grade { }

B grade { 5, 7, 11, 12, 14, 22, 38, 39, 40, 41 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 6, 8, 9, 10, 13, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 42, 43, 44, 45, 46, 47, 48, 49, 50 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 10, 11, 12 }

B grade { }

C grade { 2, 3, 4, 5, 7, 8, 9, 14, 15, 16 }

F normal fail { 1, 6, 13, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 42, 43, 44, 45, 46, 47, 48, 49, 50 }

F(-1) timedout fail { 39, 40, 41 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	72	56	0	0	0	0	0	0
N.S.	1	1.16	0.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.470	0.029	0.000	0.000	0.000	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	59	40	40	66	32	119	152	0
N.S.	1	1.02	0.69	0.69	1.14	0.55	2.05	2.62	0.00
time (sec)	N/A	0.225	0.021	0.230	0.182	0.263	61.845	0.295	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	50	35	35	52	27	90	116	0
N.S.	1	1.06	0.74	0.74	1.11	0.57	1.91	2.47	0.00
time (sec)	N/A	0.211	0.017	0.212	0.187	0.253	18.696	0.292	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	39	28	28	38	20	61	80	0
N.S.	1	1.08	0.78	0.78	1.06	0.56	1.69	2.22	0.00
time (sec)	N/A	0.195	0.014	0.161	0.182	0.267	5.586	0.281	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	22	21	14	29	41	21
N.S.	1	1.00	1.00	1.22	1.17	0.78	1.61	2.28	1.17
time (sec)	N/A	0.168	0.003	0.214	0.181	0.261	1.954	0.288	1.357

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	62	54	63	0	0	0	0	0
N.S.	1	1.11	0.96	1.12	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.463	0.018	0.651	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	36	32	44	51	19	75	30	28
N.S.	1	0.95	0.84	1.16	1.34	0.50	1.97	0.79	0.74
time (sec)	N/A	0.217	0.017	0.252	0.263	0.264	12.657	0.280	0.872

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	57	55	57	80	29	144	44	0
N.S.	1	1.06	1.02	1.06	1.48	0.54	2.67	0.81	0.00
time (sec)	N/A	0.214	0.029	0.223	0.265	0.263	41.805	0.273	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	76	45	67	106	35	180	58	0
N.S.	1	1.12	0.66	0.99	1.56	0.51	2.65	0.85	0.00
time (sec)	N/A	0.225	0.041	0.201	0.270	0.275	127.916	0.283	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	62	42	56	54	39	51	47	0
N.S.	1	1.11	0.75	1.00	0.96	0.70	0.91	0.84	0.00
time (sec)	N/A	0.272	0.025	1.369	0.268	0.259	0.169	0.265	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	57	44	67	46	38	41	39	38
N.S.	1	1.21	0.94	1.43	0.98	0.81	0.87	0.83	0.81
time (sec)	N/A	0.241	0.017	1.369	0.263	0.258	0.142	0.264	0.734

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	28	24	27	22	28	24
N.S.	1	1.00	1.00	1.08	0.92	1.04	0.85	1.08	0.92
time (sec)	N/A	0.216	0.008	1.352	0.180	0.270	0.128	0.263	0.115

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	67	59	76	0	0	0	0	0
N.S.	1	1.14	1.00	1.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.387	0.016	1.773	0.000	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	93	30	52	107	29	61	29
N.S.	1	1.00	3.00	0.97	1.68	3.45	0.94	1.97	0.94
time (sec)	N/A	0.256	0.093	0.063	0.181	0.266	1.072	0.267	0.706

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	36	33	32	39	53	61	0
N.S.	1	1.00	0.95	0.87	0.84	1.03	1.39	1.61	0.00
time (sec)	N/A	0.229	0.016	1.352	0.261	0.254	0.619	0.292	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	61	69	54	64	142	100	80	0
N.S.	1	1.02	1.15	0.90	1.07	2.37	1.67	1.33	0.00
time (sec)	N/A	0.257	0.031	1.329	0.264	0.275	1.399	0.270	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	80	60	93	0	0	0	0	0
N.S.	1	1.16	0.87	1.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.510	0.065	1.443	0.000	0.000	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	206	173	329	0	152	0	409	0
N.S.	1	1.05	0.88	1.67	0.00	0.77	0.00	2.08	0.00
time (sec)	N/A	0.737	0.115	0.389	0.000	0.291	0.000	0.311	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	157	150	249	0	130	0	299	0
N.S.	1	1.01	0.97	1.61	0.00	0.84	0.00	1.93	0.00
time (sec)	N/A	0.532	0.181	0.310	0.000	0.298	0.000	0.306	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	131	190	0	117	0	204	0
N.S.	1	1.00	1.13	1.64	0.00	1.01	0.00	1.76	0.00
time (sec)	N/A	0.374	0.132	0.301	0.000	0.271	0.000	0.312	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	110	108	0	104	0	133	0
N.S.	1	1.00	1.41	1.38	0.00	1.33	0.00	1.71	0.00
time (sec)	N/A	0.427	0.084	0.307	0.000	0.266	0.000	0.309	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	35	468	45	55	73	0	82	35
N.S.	1	0.95	12.65	1.22	1.49	1.97	0.00	2.22	0.95
time (sec)	N/A	0.223	1.992	0.074	0.179	0.303	0.000	0.296	1.044

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	263	284	374	0	0	0	0	0
N.S.	1	1.32	1.42	1.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.096	0.238	1.642	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	73	112	126	0	281	0	94	0
N.S.	1	1.04	1.60	1.80	0.00	4.01	0.00	1.34	0.00
time (sec)	N/A	0.408	0.209	0.721	0.000	0.317	0.000	0.307	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	148	198	319	0	427	0	216	0
N.S.	1	1.18	1.58	2.55	0.00	3.42	0.00	1.73	0.00
time (sec)	N/A	0.751	0.711	0.704	0.000	0.302	0.000	0.322	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	222	241	528	0	548	0	451	0
N.S.	1	1.23	1.33	2.92	0.00	3.03	0.00	2.49	0.00
time (sec)	N/A	1.065	0.311	0.699	0.000	0.318	0.000	0.333	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	381	357	667	673	0	0	0	0	0
N.S.	1	0.94	1.75	1.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.619	8.394	1.435	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	288	271	473	498	0	0	0	0	0
N.S.	1	0.94	1.64	1.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.541	4.591	1.514	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	146	144	225	0	0	0	0	0
N.S.	1	0.95	0.94	1.46	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.423	0.102	0.949	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	89	111	162	0	0	0	0	0
N.S.	1	0.95	1.18	1.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.411	0.086	0.539	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	310	398	813	0	0	0	0	0	0
N.S.	1	1.28	2.62	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.595	1.815	0.000	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	248	686	336	0	0	0	0	0
N.S.	1	1.02	2.81	1.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.742	1.728	1.645	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	494	446	446	716	0	0	0	0	0
N.S.	1	0.90	0.90	1.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.743	0.304	1.256	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	253	257	379	0	0	0	0	0
N.S.	1	0.91	0.92	1.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.550	0.113	1.451	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	154	149	160	0	0	0	0	0	0
N.S.	1	0.97	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.585	0.077	0.000	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	430	529	1058	0	0	0	0	0	0
N.S.	1	1.23	2.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.851	2.916	0.000	0.000	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	362	365	0	0	0	0	0	0	0
N.S.	1	1.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.922	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	54	516	64	71	96	0	100	52
N.S.	1	0.93	8.90	1.10	1.22	1.66	0.00	1.72	0.90
time (sec)	N/A	0.269	2.310	0.327	0.178	0.301	0.000	0.390	1.209

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	54	516	64	71	96	0	100	52
N.S.	1	0.93	8.90	1.10	1.22	1.66	0.00	1.72	0.90
time (sec)	N/A	0.280	0.304	0.317	0.186	0.317	0.000	0.414	0.928

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	54	137	64	71	96	0	100	52
N.S.	1	0.93	2.36	1.10	1.22	1.66	0.00	1.72	0.90
time (sec)	N/A	0.277	0.241	0.317	0.188	0.302	0.000	0.415	0.936

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	A	A	F(-1)	A	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	49	45	130	0	66	92	0	75	44
N.S.	1	0.92	2.65	0.00	1.35	1.88	0.00	1.53	0.90
time (sec)	N/A	0.326	0.232	0.000	0.189	0.356	0.000	0.343	1.280

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	100	280	111	0	0	0	0	0
N.S.	1	1.18	3.29	1.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.466	0.918	1.233	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	99	97	95	0	0	0	0	0	0
N.S.	1	0.98	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.350	0.231	0.000	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	89	107	0	0	0	0	0	0
N.S.	1	0.98	1.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.335	0.161	0.000	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	89	54	0	0	0	0	0	0
N.S.	1	0.98	0.59	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.303	0.046	0.000	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	45	45	79	0	0	0	0	0	0
N.S.	1	1.00	1.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.274	0.040	0.000	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	39	43	34	0	0	23	0	43	0
N.S.	1	1.10	0.87	0.00	0.00	0.59	0.00	1.10	0.00
time (sec)	N/A	0.238	0.030	0.000	0.000	0.415	0.000	0.288	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	41	42	30	0	0	30	0	0	0
N.S.	1	1.02	0.73	0.00	0.00	0.73	0.00	0.00	0.00
time (sec)	N/A	0.284	0.035	0.000	0.000	0.282	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	79	54	0	0	40	0	0	0
N.S.	1	0.94	0.64	0.00	0.00	0.48	0.00	0.00	0.00
time (sec)	N/A	0.293	0.113	0.000	0.000	0.282	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	68	59	99	0	0	0	115	0
N.S.	1	0.99	0.86	1.43	0.00	0.00	0.00	1.67	0.00
time (sec)	N/A	0.490	0.053	1.037	0.000	0.000	0.000	0.502	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [23] had the largest ratio of [1.39999999999999991]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	9	8	1.16	10	0.800
2	A	4	4	1.02	10	0.400
3	A	4	4	1.06	10	0.400
4	A	4	4	1.08	8	0.500
5	A	2	2	1.00	6	0.333
6	A	9	8	1.11	10	0.800
7	A	6	5	0.95	10	0.500
8	A	7	6	1.06	10	0.600
9	A	8	7	1.12	10	0.700
10	A	6	5	1.11	10	0.500
11	A	4	4	1.21	8	0.500
12	A	3	3	1.00	6	0.500
13	A	8	7	1.14	10	0.700
14	A	6	5	1.00	10	0.500
15	A	3	3	1.00	10	0.300
16	A	7	6	1.02	10	0.600
17	A	9	8	1.16	10	0.800
18	A	10	9	1.05	10	0.900
19	A	9	8	1.01	10	0.800
20	A	6	5	1.00	10	0.500
21	A	10	9	1.00	8	1.125
22	A	6	5	0.95	6	0.833

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	15	14	1.32	10	1.400
24	A	8	7	1.04	10	0.700
25	A	13	12	1.18	10	1.200
26	A	14	13	1.23	10	1.300
27	A	7	6	0.94	12	0.500
28	A	6	5	0.94	12	0.417
29	A	7	6	0.95	10	0.600
30	A	8	7	0.95	8	0.875
31	A	16	15	1.28	12	1.250
32	A	6	5	1.02	12	0.417
33	A	6	5	0.90	12	0.417
34	A	7	6	0.91	10	0.600
35	A	9	8	0.97	8	1.000
36	A	16	15	1.23	12	1.250
37	A	6	5	1.01	12	0.417
38	A	3	2	0.93	14	0.143
39	A	3	2	0.93	16	0.125
40	A	3	2	0.93	16	0.125
41	A	7	6	0.92	14	0.429
42	A	9	8	1.18	10	0.800
43	A	5	4	0.98	10	0.400
44	A	5	4	0.98	8	0.500
45	A	4	3	0.98	6	0.500
46	A	5	4	1.00	10	0.400
47	A	4	3	1.10	10	0.300
48	A	6	5	1.02	10	0.500
49	A	5	4	0.94	10	0.400
50	A	10	9	0.99	19	0.474

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{\sec^{-1}(ax^5)}{x} dx$

3.1.1 Optimal result

Integrand size = 10, antiderivative size = 62

$$\int \frac{\sec^{-1}(ax^5)}{x} dx = \frac{1}{10} i \sec^{-1}(ax^5)^2 - \frac{1}{5} \sec^{-1}(ax^5) \log\left(1 + e^{2i\sec^{-1}(ax^5)}\right) + \frac{1}{10} i \operatorname{PolyLog}\left(2, -e^{2i\sec^{-1}(ax^5)}\right)$$

output
```
1/10*I*arcsec(a*x^5)^2-1/5*arcsec(a*x^5)*ln(1+(1/a/x^5+I*(1-1/a^2/x^10)^(1
/2))^2)+1/10*I*polylog(2,-(1/a/x^5+I*(1-1/a^2/x^10)^(1/2))^2)
```

3.1.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int \frac{\sec^{-1}(ax^5)}{x} dx = \frac{1}{10} i\left(\sec^{-1}(ax^5)\left(\sec^{-1}(ax^5) + 2i \log\left(1 + e^{2i\sec^{-1}(ax^5)}\right)\right) + \operatorname{PolyLog}\left(2, -e^{2i\sec^{-1}(ax^5)}\right)\right)$$

input
```
Integrate[ArcSec[a*x^5]/x,x]
```

output
```
(I/10)*(ArcSec[a*x^5]*(ArcSec[a*x^5] + (2*I)*Log[1 + E^((2*I)*ArcSec[a*x^5
])]) + PolyLog[2, -E^((2*I)*ArcSec[a*x^5])])
```

3.1.3 Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {7282, 5741, 5137, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{-1}(ax^5)}{x}dx$$

$$\Big\downarrow \text{ 7282}$$

$$\frac{1}{5}\int \frac{\sec^{-1}(ax^5)}{x^5}dx^5$$

$$\Big\downarrow \text{ 5741}$$

$$-\frac{1}{5}\int \frac{\arccos\left(\frac{1}{ax^5}\right)}{x^5}d\frac{1}{x^5}$$

$$\Big\downarrow \text{ 5137}$$

$$\frac{1}{5}\int a\sqrt{1-\frac{1}{a^2x^{10}}}x^5\arccos\left(\frac{1}{ax^5}\right)d\arccos\left(\frac{1}{ax^5}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\int \arccos\left(\frac{1}{ax^5}\right)\tan\left(\arccos\left(\frac{1}{ax^5}\right)\right)d\arccos\left(\frac{1}{ax^5}\right)$$

$$\Big\downarrow \text{ 4202}$$

$$\frac{1}{5}\left(\frac{ix^{10}}{2}-2i\int \frac{e^{2i\arccos\left(\frac{1}{ax^5}\right)}\arccos\left(\frac{1}{ax^5}\right)}{1+e^{2i\arccos\left(\frac{1}{ax^5}\right)}}d\arccos\left(\frac{1}{ax^5}\right)\right)$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{1}{5}\left(\frac{ix^{10}}{2}-2i\left(\frac{1}{2}i\int \log\left(1+e^{2i\arccos\left(\frac{1}{ax^5}\right)}\right)d\arccos\left(\frac{1}{ax^5}\right)-\frac{1}{2}i\arccos\left(\frac{1}{ax^5}\right)\log\left(1+e^{2i\arccos\left(\frac{1}{ax^5}\right)}\right)\right)\right)$$

$$\Big\downarrow \text{ 2715}$$

$$\frac{1}{5}\left(\frac{ix^{10}}{2}-2i\left(\frac{1}{4}\int e^{2i\arccos\left(\frac{1}{ax^5}\right)}\log\left(1+e^{2i\arccos\left(\frac{1}{ax^5}\right)}\right)de^{2i\arccos\left(\frac{1}{ax^5}\right)}-\frac{1}{2}i\arccos\left(\frac{1}{ax^5}\right)\log\left(1+e^{2i\arccos\left(\frac{1}{ax^5}\right)}\right.\right.\right.$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{1}{5}\left(\frac{ix^{10}}{2}-2i\left(-\frac{1}{4}\operatorname{PolyLog}\left(2,-e^{2i\arccos\left(\frac{1}{ax^5}\right)}\right)-\frac{1}{2}i\arccos\left(\frac{1}{ax^5}\right)\log\left(1+e^{2i\arccos\left(\frac{1}{ax^5}\right)}\right)\right)\right)$$

input
```
Int[ArcSec[a*x^5]/x,x]
```

output
```
((I/2)*x^10 - (2*I)*((-1/2*I)*ArcCos[1/(a*x^5)]*Log[1 + E^((2*I)*ArcCos[1/
(a*x^5)])] - PolyLog[2, -E^((2*I)*ArcCos[1/(a*x^5)])]/4))/5
```

3.1.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 5741
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a + b
*ArcCos[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

3.1.4 Maple [F]

$$\int \frac{\operatorname{arcsec}\left(a\,x^{5}\right)}{x}dx$$

input
```
int(arcsec(a*x^5)/x,x)
```

output
```
int(arcsec(a*x^5)/x,x)
```

3.1.5 Fricas [F]

$$\int \frac{\sec^{-1}\left(ax^{5}\right)}{x}\,dx=\int \frac{\operatorname{arcsec}\left(ax^{5}\right)}{x}\,dx$$

input
```
integrate(arcsec(a*x^5)/x,x, algorithm="fricas")
```

output
```
integral(arcsec(a*x^5)/x, x)
```

3.1.6 Sympy [F]

$$\int \frac{\sec^{-1}\left(ax^{5}\right)}{x}\,dx=\int \frac{\operatorname{asec}\left(ax^{5}\right)}{x}\,dx$$

input
```
integrate(asec(a*x**5)/x,x)
```

output
```
Integral(asec(a*x**5)/x, x)
```

3.1.7 Maxima [F]

$$\int \frac{\sec^{-1}(ax^5)}{x}\,dx = \int \frac{\operatorname{arcsec}(ax^5)}{x}\,dx$$

input
```
integrate(arcsec(a*x^5)/x,x, algorithm="maxima")
```

output
```
-5*a^2*integrate(sqrt(a*x^5 + 1)*sqrt(a*x^5 - 1)*log(x)/(a^4*x^11 - a^2*x)
, x) - 5*I*a^2*integrate(log(x)/(a^4*x^11 - a^2*x), x) + arctan(sqrt(a*x^5
 + 1)*sqrt(a*x^5 - 1))*log(x) - 1/2*I*log(a^2*x^10)*log(x) + 1/2*I*log(a*x
^5 + 1)*log(x) + 1/2*I*log(-a*x^5 + 1)*log(x) + I*log(a)*log(x) + 5/2*I*lo
g(x)^2 + 1/10*I*dilog(a*x^5) + 1/10*I*dilog(-a*x^5)
```

3.1.8 Giac [F]

$$\int \frac{\sec^{-1}(ax^5)}{x}\,dx = \int \frac{\operatorname{arcsec}(ax^5)}{x}\,dx$$

input
```
integrate(arcsec(a*x^5)/x,x, algorithm="giac")
```

output
```
integrate(arcsec(a*x^5)/x, x)
```

3.1.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{-1}(ax^5)}{x}\,dx = \int \frac{\operatorname{acos}\left(\frac{1}{a\,x^5}\right)}{x}\,dx$$

input
```
int(acos(1/(a*x^5))/x,x)
```

output
```
int(acos(1/(a*x^5))/x, x)
```

3.2 $\int x^3 \sec^{-1}\left(\sqrt{x}\right) dx$

3.2.1 Optimal result

Integrand size = 10, antiderivative size = 58

$$\int x^3 \sec^{-1}\left(\sqrt{x}\right) dx = -\frac{1}{4}\sqrt{-1+x} - \frac{1}{4}(-1+x)^{3/2} - \frac{3}{20}(-1+x)^{5/2} - \frac{1}{28}(-1+x)^{7/2} + \frac{1}{4}x^4 \sec^{-1}\left(\sqrt{x}\right)$$

output
```
-1/4*(-1+x)^(3/2)-3/20*(-1+x)^(5/2)-1/28*(-1+x)^(7/2)+1/4*x^4*arcsec(x^(1/
2))-1/4*(-1+x)^(1/2)
```

3.2.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.69

$$\int x^3 \sec^{-1}\left(\sqrt{x}\right) dx = -\frac{1}{140}\sqrt{-1+x}\left(16+8x+6x^2+5x^3\right) + \frac{1}{4}x^4 \sec^{-1}\left(\sqrt{x}\right)$$

input
```
Integrate[x^3*ArcSec[Sqrt[x]],x]
```

output
```
-1/140*(Sqrt[-1 + x]*(16 + 8*x + 6*x^2 + 5*x^3)) + (x^4*ArcSec[Sqrt[x]])/4
```

3.2.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5793, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sec^{-1}\left(\sqrt{x}\right) dx$$

$$\Big\downarrow \text{5793}$$

$$\frac{1}{4}x^4 \sec^{-1}\left(\sqrt{x}\right) - \frac{1}{4}\int \frac{x^3}{2\sqrt{x-1}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}x^4 \sec^{-1}\left(\sqrt{x}\right) - \frac{1}{8}\int \frac{x^3}{\sqrt{x-1}}dx$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}x^4 \sec^{-1}\left(\sqrt{x}\right) - \frac{1}{8}\int \left((x-1)^{5/2} + 3(x-1)^{3/2} + 3\sqrt{x-1} + \frac{1}{\sqrt{x-1}}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}x^4 \sec^{-1}\left(\sqrt{x}\right) + \frac{1}{8}\left(-\frac{2}{7}(x-1)^{7/2} - \frac{6}{5}(x-1)^{5/2} - 2(x-1)^{3/2} - 2\sqrt{x-1}\right)$$

input
```
Int[x^3*ArcSec[Sqrt[x]],x]
```

output
```
(-2*Sqrt[-1 + x] - 2*(-1 + x)^(3/2) - (6*(-1 + x)^(5/2))/5 - (2*(-1 + x)^(
7/2))/7)/8 + (x^4*ArcSec[Sqrt[x]])/4
```

3.2.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5793
```
Int[((a_.) + ArcSec[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcSec[u])/(d*(m + 1))), x] - Simp[b*(u/(d*(m +
 1)*Sqrt[u^2]))   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(u*Sqrt[
u^2 - 1])), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && Invers
eFunctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !Functio
nOfExponentialQ[u, x]
```

3.2.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.69

method	result	size
parts	$\frac{x^4 \operatorname{arcsec}(\sqrt{x})}{4}-\frac{\sqrt{\frac{x-1}{x}}\,\sqrt{x}\,(5x^3+6x^2+8x+16)}{140}$	40
derivativedivides	$\frac{x^4 \operatorname{arcsec}(\sqrt{x})}{4}-\frac{(x-1)(5x^3+6x^2+8x+16)}{140\sqrt{\frac{x-1}{x}}\,\sqrt{x}}$	43
default	$\frac{x^4 \operatorname{arcsec}(\sqrt{x})}{4}-\frac{(x-1)(5x^3+6x^2+8x+16)}{140\sqrt{\frac{x-1}{x}}\,\sqrt{x}}$	43

input
```
int(x^3*arcsec(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*arcsec(x^(1/2))-1/140*((x-1)/x)^(1/2)*x^(1/2)*(5*x^3+6*x^2+8*x+16)
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.55

$$\int x^3 \sec^{-1}\left(\sqrt{x}\right)\,dx = \frac{1}{4}\,x^4 \operatorname{arcsec}\left(\sqrt{x}\right) - \frac{1}{140}\left(5\,x^3 + 6\,x^2 + 8\,x + 16\right)\sqrt{x-1}$$

input
```
integrate(x^3*arcsec(x^(1/2)),x, algorithm="fricas")
```

output
```
1/4*x^4*arcsec(sqrt(x)) - 1/140*(5*x^3 + 6*x^2 + 8*x + 16)*sqrt(x - 1)
```

3.2.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 61.84 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.05

$$\int x^3 \sec^{-1}\left(\sqrt{x}\right)\,dx = \frac{x^4 \operatorname{asec}\left(\sqrt{x}\right)}{4} - \frac{\begin{cases} \frac{2x^3\sqrt{x-1}}{7} + \frac{12x^2\sqrt{x-1}}{35} + \frac{16x\sqrt{x-1}}{35} + \frac{32\sqrt{x-1}}{35} & \text{for } |x| > 1 \\ \frac{2ix^3\sqrt{1-x}}{7} + \frac{12ix^2\sqrt{1-x}}{35} + \frac{16ix\sqrt{1-x}}{35} + \frac{32i\sqrt{1-x}}{35} & \text{otherwise} \end{cases}}{8}$$

input
```
integrate(x**3*asec(x**(1/2)),x)
```

output
```
x**4*asec(sqrt(x))/4 - Piecewise((2*x**3*sqrt(x - 1)/7 + 12*x**2*sqrt(x -
1)/35 + 16*x*sqrt(x - 1)/35 + 32*sqrt(x - 1)/35, Abs(x) > 1), (2*I*x**3*sq
rt(1 - x)/7 + 12*I*x**2*sqrt(1 - x)/35 + 16*I*x*sqrt(1 - x)/35 + 32*I*sqrt
(1 - x)/35, True))/8
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.14

$$\int x^3 \sec^{-1}\left(\sqrt{x}\right)\,dx = -\frac{1}{28}\,x^{\frac{7}{2}}\left(-\frac{1}{x}+1\right)^{\frac{7}{2}} - \frac{3}{20}\,x^{\frac{5}{2}}\left(-\frac{1}{x}+1\right)^{\frac{5}{2}} + \frac{1}{4}\,x^4 \operatorname{arcsec}\left(\sqrt{x}\right) - \frac{1}{4}\,x^{\frac{3}{2}}\left(-\frac{1}{x}+1\right)^{\frac{3}{2}} - \frac{1}{4}\,\sqrt{x}\sqrt{-\frac{1}{x}+1}$$

input
```
integrate(x^3*arcsec(x^(1/2)),x, algorithm="maxima")
```

output
```
-1/28*x^(7/2)*(-1/x + 1)^(7/2) - 3/20*x^(5/2)*(-1/x + 1)^(5/2) + 1/4*x^4*a
rcsec(sqrt(x)) - 1/4*x^(3/2)*(-1/x + 1)^(3/2) - 1/4*sqrt(x)*sqrt(-1/x + 1)
```

3.2.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(38) = 76$.

Time = 0.30 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.62

$$\begin{aligned}
&\int x^3 \sec^{-1}\left(\sqrt{x}\right) dx \\
&= -\frac{1}{3584} x^{\frac{7}{2}} \left(\sqrt{-\frac{1}{x}+1}-1\right)^7 - \frac{7}{2560} x^{\frac{5}{2}} \left(\sqrt{-\frac{1}{x}+1}-1\right)^5 \\
&+ \frac{1}{4} x^4 \arccos\left(\frac{1}{\sqrt{x}}\right) - \frac{7}{512} x^{\frac{3}{2}} \left(\sqrt{-\frac{1}{x}+1}-1\right)^3 - \frac{35}{512} \sqrt{x} \left(\sqrt{-\frac{1}{x}+1}-1\right) \\
&+ \frac{1225\, x^3 \left(\sqrt{-\frac{1}{x}+1}-1\right)^6 + 245\, x^2 \left(\sqrt{-\frac{1}{x}+1}-1\right)^4 + 49\, x \left(\sqrt{-\frac{1}{x}+1}-1\right)^2 + 5}{17920\, x^{\frac{7}{2}} \left(\sqrt{-\frac{1}{x}+1}-1\right)^7}
\end{aligned}$$

input
```
integrate(x^3*arcsec(x^(1/2)),x, algorithm="giac")
```

output
```
-1/3584*x^(7/2)*(sqrt(-1/x + 1) - 1)^7 - 7/2560*x^(5/2)*(sqrt(-1/x + 1) -
1)^5 + 1/4*x^4*arccos(1/sqrt(x)) - 7/512*x^(3/2)*(sqrt(-1/x + 1) - 1)^3 -
35/512*sqrt(x)*(sqrt(-1/x + 1) - 1) + 1/17920*(1225*x^3*(sqrt(-1/x + 1) -
1)^6 + 245*x^2*(sqrt(-1/x + 1) - 1)^4 + 49*x*(sqrt(-1/x + 1) - 1)^2 + 5)/(
x^(7/2)*(sqrt(-1/x + 1) - 1)^7)
```

3.2.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \sec^{-1}\left(\sqrt{x}\right) \, dx = \int x^3 \operatorname{acos}\left(\frac{1}{\sqrt{x}}\right) \, dx$$

input
```
int(x^3*acos(1/x^(1/2)),x)
```

output
```
int(x^3*acos(1/x^(1/2)), x)
```

3.3 $\int x^2 \sec^{-1}(\sqrt{x})\,dx$

3.3.1 Optimal result

Integrand size = 10, antiderivative size = 47

$$\int x^2 \sec^{-1}(\sqrt{x})\,dx = -\frac{1}{3}\sqrt{-1+x} - \frac{2}{9}(-1+x)^{3/2} - \frac{1}{15}(-1+x)^{5/2} + \frac{1}{3}x^3 \sec^{-1}(\sqrt{x})$$

output
```
-2/9*(-1+x)^(3/2)-1/15*(-1+x)^(5/2)+1/3*x^3*arcsec(x^(1/2))-1/3*(-1+x)^(1/
2)
```

3.3.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int x^2 \sec^{-1}(\sqrt{x})\,dx = -\frac{1}{45}\sqrt{-1+x}(8+4x+3x^2) + \frac{1}{3}x^3 \sec^{-1}(\sqrt{x})$$

input
```
Integrate[x^2*ArcSec[Sqrt[x]],x]
```

output
```
-1/45*(Sqrt[-1 + x]*(8 + 4*x + 3*x^2)) + (x^3*ArcSec[Sqrt[x]])/3
```

3.3.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5793, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sec^{-1}\left(\sqrt{x}\right) dx$$

$$\Big\downarrow \text{5793}$$

$$\frac{1}{3}x^3 \sec^{-1}\left(\sqrt{x}\right) - \frac{1}{3}\int \frac{x^2}{2\sqrt{x-1}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}x^3 \sec^{-1}\left(\sqrt{x}\right) - \frac{1}{6}\int \frac{x^2}{\sqrt{x-1}}dx$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3}x^3 \sec^{-1}\left(\sqrt{x}\right) - \frac{1}{6}\int \left((x-1)^{3/2} + 2\sqrt{x-1} + \frac{1}{\sqrt{x-1}}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3 \sec^{-1}\left(\sqrt{x}\right) + \frac{1}{6}\left(-\frac{2}{5}(x-1)^{5/2} - \frac{4}{3}(x-1)^{3/2} - 2\sqrt{x-1}\right)$$

input
```
Int[x^2*ArcSec[Sqrt[x]],x]
```

output
```
(-2*Sqrt[-1 + x] - (4*(-1 + x)^(3/2))/3 - (2*(-1 + x)^(5/2))/5)/6 + (x^3*A
rcSec[Sqrt[x]])/3
```

3.3.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5793
```
Int[((a_.) + ArcSec[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcSec[u])/(d*(m + 1))), x] - Simp[b*(u/(d*(m +
 1)*Sqrt[u^2]))   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(u*Sqrt[
u^2 - 1])), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && Invers
eFunctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !Functio
nOfExponentialQ[u, x]
```

3.3.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

method	result	size
parts	$\frac{x^3 \operatorname{arcsec}\left(\sqrt{x}\right)}{3}-\frac{\sqrt{\frac{x-1}{x}}\,\sqrt{x}\,\left(3x^2+4x+8\right)}{45}$	35
derivativedivides	$\frac{x^3 \operatorname{arcsec}\left(\sqrt{x}\right)}{3}-\frac{(x-1)\left(3x^2+4x+8\right)}{45\sqrt{\frac{x-1}{x}}\,\sqrt{x}}$	38
default	$\frac{x^3 \operatorname{arcsec}\left(\sqrt{x}\right)}{3}-\frac{(x-1)\left(3x^2+4x+8\right)}{45\sqrt{\frac{x-1}{x}}\,\sqrt{x}}$	38

input
```
int(x^2*arcsec(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*arcsec(x^(1/2))-1/45*((x-1)/x)^(1/2)*x^(1/2)*(3*x^2+4*x+8)
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.57

$$\int x^2 \sec^{-1}\left(\sqrt{x}\right)\, dx = \frac{1}{3}\, x^3 \operatorname{arcsec}\left(\sqrt{x}\right) - \frac{1}{45}\left(3\, x^2 + 4\, x + 8\right)\sqrt{x - 1}$$

input
```
integrate(x^2*arcsec(x^(1/2)),x, algorithm="fricas")
```

output
```
1/3*x^3*arcsec(sqrt(x)) - 1/45*(3*x^2 + 4*x + 8)*sqrt(x - 1)
```

3.3.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 18.70 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.91

$$\int x^2 \sec^{-1}\left(\sqrt{x}\right)\, dx = \frac{x^3 \operatorname{asec}\left(\sqrt{x}\right)}{3} - \frac{\begin{cases} \frac{2x^2\sqrt{x-1}}{5} + \frac{8x\sqrt{x-1}}{15} + \frac{16\sqrt{x-1}}{15} & \text{for } |x| > 1 \\ \frac{2ix^2\sqrt{1-x}}{5} + \frac{8ix\sqrt{1-x}}{15} + \frac{16i\sqrt{1-x}}{15} & \text{otherwise} \end{cases}}{6}$$

input
```
integrate(x**2*asec(x**(1/2)),x)
```

output
```
x**3*asec(sqrt(x))/3 - Piecewise((2*x**2*sqrt(x - 1)/5 + 8*x*sqrt(x - 1)/1
5 + 16*sqrt(x - 1)/15, Abs(x) > 1), (2*I*x**2*sqrt(1 - x)/5 + 8*I*x*sqrt(1
 - x)/15 + 16*I*sqrt(1 - x)/15, True))/6
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11

$$\int x^2 \sec^{-1}\left(\sqrt{x}\right)\, dx = -\frac{1}{15}\, x^{\frac{5}{2}}\left(-\frac{1}{x} + 1\right)^{\frac{5}{2}} + \frac{1}{3}\, x^3 \operatorname{arcsec}\left(\sqrt{x}\right) - \frac{2}{9}\, x^{\frac{3}{2}}\left(-\frac{1}{x} + 1\right)^{\frac{3}{2}} - \frac{1}{3}\,\sqrt{x}\sqrt{-\frac{1}{x} + 1}$$

input
```
integrate(x^2*arcsec(x^(1/2)),x, algorithm="maxima")
```

output
```
-1/15*x^(5/2)*(-1/x + 1)^(5/2) + 1/3*x^3*arcsec(sqrt(x)) - 2/9*x^(3/2)*(-1
/x + 1)^(3/2) - 1/3*sqrt(x)*sqrt(-1/x + 1)
```

3.3.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(31) = 62$.

Time = 0.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.47

$$\int x^2 \sec^{-1}\left(\sqrt{x}\right) dx = -\frac{1}{480} x^{\frac{5}{2}} \left(\sqrt{-\frac{1}{x}+1}-1\right)^5 - \frac{5}{288} x^{\frac{3}{2}} \left(\sqrt{-\frac{1}{x}+1}-1\right)^3 + \frac{1}{3} x^3 \arccos\left(\frac{1}{\sqrt{x}}\right) - \frac{5}{48} \sqrt{x} \left(\sqrt{-\frac{1}{x}+1}-1\right) + \frac{150 x^2 \left(\sqrt{-\frac{1}{x}+1}-1\right)^4 + 25 x \left(\sqrt{-\frac{1}{x}+1}-1\right)^2 + 3}{1440 x^{\frac{5}{2}} \left(\sqrt{-\frac{1}{x}+1}-1\right)^5}$$

input
```
integrate(x^2*arcsec(x^(1/2)),x, algorithm="giac")
```

output
```
-1/480*x^(5/2)*(sqrt(-1/x + 1) - 1)^5 - 5/288*x^(3/2)*(sqrt(-1/x + 1) - 1)
^3 + 1/3*x^3*arccos(1/sqrt(x)) - 5/48*sqrt(x)*(sqrt(-1/x + 1) - 1) + 1/144
0*(150*x^2*(sqrt(-1/x + 1) - 1)^4 + 25*x*(sqrt(-1/x + 1) - 1)^2 + 3)/(x^(5
/2)*(sqrt(-1/x + 1) - 1)^5)
```

3.3.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \sec^{-1}\left(\sqrt{x}\right) dx = \int x^2 \operatorname{acos}\left(\frac{1}{\sqrt{x}}\right) dx$$

input
```
int(x^2*acos(1/x^(1/2)),x)
```

output
```
int(x^2*acos(1/x^(1/2)), x)
```

3.4 $\int x \sec^{-1}(\sqrt{x}) \, dx$

3.4.1 Optimal result

Integrand size = 8, antiderivative size = 36

$$\int x \sec^{-1}(\sqrt{x}) \, dx = -\frac{1}{2}\sqrt{-1+x} - \frac{1}{6}(-1+x)^{3/2} + \frac{1}{2}x^2 \sec^{-1}(\sqrt{x})$$

output
```
-1/6*(-1+x)^(3/2)+1/2*x^2*arcsec(x^(1/2))-1/2*(-1+x)^(1/2)
```

3.4.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int x \sec^{-1}(\sqrt{x}) \, dx = -\frac{1}{6}\sqrt{-1+x}(2+x) + \frac{1}{2}x^2 \sec^{-1}(\sqrt{x})$$

input
```
Integrate[x*ArcSec[Sqrt[x]],x]
```

output
```
-1/6*(Sqrt[-1 + x]*(2 + x)) + (x^2*ArcSec[Sqrt[x]])/2
```

3.4.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5793, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sec^{-1}\left(\sqrt{x}\right) dx$$

$$\Big\downarrow \text{5793}$$

$$\frac{1}{2}x^2 \sec^{-1}\left(\sqrt{x}\right) - \frac{1}{2}\int \frac{x}{2\sqrt{x-1}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}x^2 \sec^{-1}\left(\sqrt{x}\right) - \frac{1}{4}\int \frac{x}{\sqrt{x-1}}dx$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}x^2 \sec^{-1}\left(\sqrt{x}\right) - \frac{1}{4}\int \left(\sqrt{x-1} + \frac{1}{\sqrt{x-1}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}x^2 \sec^{-1}\left(\sqrt{x}\right) + \frac{1}{4}\left(-\frac{2}{3}(x-1)^{3/2} - 2\sqrt{x-1}\right)$$

input
```
Int[x*ArcSec[Sqrt[x]],x]
```

output
```
(-2*Sqrt[-1 + x] - (2*(-1 + x)^(3/2))/3)/4 + (x^2*ArcSec[Sqrt[x]])/2
```

3.4.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5793
```
Int[((a_.) + ArcSec[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcSec[u])/(d*(m + 1))), x] - Simp[b*(u/(d*(m +
 1)*Sqrt[u^2]))   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(u*Sqrt[
u^2 - 1])), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && Invers
eFunctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !Functio
nOfExponentialQ[u, x]
```

3.4.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

method	result	size
parts	$\frac{x^2 \operatorname{arcsec}(\sqrt{x})}{2} - \frac{\sqrt{\frac{x-1}{x}}\,\sqrt{x}\,(2+x)}{6}$	28
derivativedivides	$\frac{x^2 \operatorname{arcsec}(\sqrt{x})}{2} - \frac{(x-1)(2+x)}{6\sqrt{\frac{x-1}{x}}\,\sqrt{x}}$	31
default	$\frac{x^2 \operatorname{arcsec}(\sqrt{x})}{2} - \frac{(x-1)(2+x)}{6\sqrt{\frac{x-1}{x}}\,\sqrt{x}}$	31

input
```
int(x*arcsec(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*arcsec(x^(1/2))-1/6*((x-1)/x)^(1/2)*x^(1/2)*(2+x)
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.56

$$\int x\sec^{-1}\left(\sqrt{x}\right)\,dx = \frac{1}{2}\,x^2\operatorname{arcsec}\left(\sqrt{x}\right) - \frac{1}{6}\,(x+2)\sqrt{x-1}$$

input
```
integrate(x*arcsec(x^(1/2)),x, algorithm="fricas")
```

output
```
1/2*x^2*arcsec(sqrt(x)) - 1/6*(x + 2)*sqrt(x - 1)
```

3.4.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.59 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.69

$$\int x\sec^{-1}\left(\sqrt{x}\right)\,dx = \frac{x^2\operatorname{asec}\left(\sqrt{x}\right)}{2} - \frac{\begin{cases} \frac{2x\sqrt{x-1}}{3} + \frac{4\sqrt{x-1}}{3} & \text{for } |x| > 1 \\ \frac{2ix\sqrt{1-x}}{3} + \frac{4i\sqrt{1-x}}{3} & \text{otherwise} \end{cases}}{4}$$

input
```
integrate(x*asec(x**(1/2)),x)
```

output
```
x**2*asec(sqrt(x))/2 - Piecewise((2*x*sqrt(x - 1)/3 + 4*sqrt(x - 1)/3, Abs
(x) > 1), (2*I*x*sqrt(1 - x)/3 + 4*I*sqrt(1 - x)/3, True))/4
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int x\sec^{-1}\left(\sqrt{x}\right)\,dx = -\frac{1}{6}\,x^{\frac{3}{2}}\left(-\frac{1}{x}+1\right)^{\frac{3}{2}} + \frac{1}{2}\,x^2\operatorname{arcsec}\left(\sqrt{x}\right) - \frac{1}{2}\,\sqrt{x}\sqrt{-\frac{1}{x}+1}$$

input
```
integrate(x*arcsec(x^(1/2)),x, algorithm="maxima")
```

output
```
-1/6*x^(3/2)*(-1/x + 1)^(3/2) + 1/2*x^2*arcsec(sqrt(x)) - 1/2*sqrt(x)*sqrt
(-1/x + 1)
```

3.4.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(24) = 48$.

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.22

$$\int x \sec^{-1}\left(\sqrt{x}\right) dx = -\frac{1}{48}x^{\frac{3}{2}}\left(\sqrt{-\frac{1}{x}+1}-1\right)^3 + \frac{1}{2}x^2\arccos\left(\frac{1}{\sqrt{x}}\right) - \frac{3}{16}\sqrt{x}\left(\sqrt{-\frac{1}{x}+1}-1\right) + \frac{9x\left(\sqrt{-\frac{1}{x}+1}-1\right)^2+1}{48x^{\frac{3}{2}}\left(\sqrt{-\frac{1}{x}+1}-1\right)^3}$$

input
```
integrate(x*arcsec(x^(1/2)),x, algorithm="giac")
```

output
```
-1/48*x^(3/2)*(sqrt(-1/x + 1) - 1)^3 + 1/2*x^2*arccos(1/sqrt(x)) - 3/16*sq
rt(x)*(sqrt(-1/x + 1) - 1) + 1/48*(9*x*(sqrt(-1/x + 1) - 1)^2 + 1)/(x^(3/2
)*(sqrt(-1/x + 1) - 1)^3)
```

3.4.9 Mupad [F(-1)]

Timed out.

$$\int x \sec^{-1}\left(\sqrt{x}\right) dx = \int x \operatorname{acos}\left(\frac{1}{\sqrt{x}}\right) dx$$

input
```
int(x*acos(1/x^(1/2)),x)
```

output
```
int(x*acos(1/x^(1/2)), x)
```

3.5 $\int \sec^{-1}\left(\sqrt{x}\right)\, dx$

3.5.1 Optimal result

Integrand size = 6, antiderivative size = 18

$$\int \sec^{-1}\left(\sqrt{x}\right)\, dx = -\sqrt{-1+x} + x\sec^{-1}\left(\sqrt{x}\right)$$

output
```
x*arcsec(x^(1/2))-(-1+x)^(1/2)
```

3.5.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \sec^{-1}\left(\sqrt{x}\right)\, dx = -\sqrt{-1+x} + x\sec^{-1}\left(\sqrt{x}\right)$$

input
```
Integrate[ArcSec[Sqrt[x]],x]
```

output
```
-Sqrt[-1 + x] + x*ArcSec[Sqrt[x]]
```

3.5.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5791, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{-1}\left(\sqrt{x}\right) dx$$

$$\Big\downarrow \text{ 5791}$$

$$x\sec^{-1}\left(\sqrt{x}\right)-\int \frac{1}{2\sqrt{x-1}}dx$$

$$\Big\downarrow \text{ 17}$$

$$x\sec^{-1}\left(\sqrt{x}\right)-\sqrt{x-1}$$

input
```
Int[ArcSec[Sqrt[x]],x]
```

output
```
-Sqrt[-1 + x] + x*ArcSec[Sqrt[x]]
```

3.5.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 5791
```
Int[ArcSec[u_], x_Symbol] :> Simp[x*ArcSec[u], x] - Simp[u/Sqrt[u^2]   Int[
SimplifyIntegrand[x*(D[u, x]/(u*Sqrt[u^2 - 1])), x], x], x] /; InverseFunct
ionFreeQ[u, x] &&  !FunctionOfExponentialQ[u, x]
```

3.5.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

method	result	size
parts	$x \operatorname{arcsec}\left(\sqrt{x}\right)-\sqrt{\frac{x-1}{x}}\, \sqrt{x}$	22
derivativedivides	$x \operatorname{arcsec}\left(\sqrt{x}\right)-\frac{x-1}{\sqrt{\frac{x-1}{x}}\, \sqrt{x}}$	25
default	$x \operatorname{arcsec}\left(\sqrt{x}\right)-\frac{x-1}{\sqrt{\frac{x-1}{x}}\, \sqrt{x}}$	25

input
```
int(arcsec(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
x*arcsec(x^(1/2))-((x-1)/x)^(1/2)*x^(1/2)
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \sec^{-1}\left(\sqrt{x}\right)\, dx = x \operatorname{arcsec}\left(\sqrt{x}\right) - \sqrt{x-1}$$

input
```
integrate(arcsec(x^(1/2)),x, algorithm="fricas")
```

output
```
x*arcsec(sqrt(x)) - sqrt(x - 1)
```

3.5.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.95 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.61

$$\int \sec^{-1}\left(\sqrt{x}\right)\, dx = x \operatorname{asec}\left(\sqrt{x}\right) - \frac{\begin{cases} 2\sqrt{x-1} & \text{for } |x| > 1 \\ 2i\sqrt{1-x} & \text{otherwise} \end{cases}}{2}$$

input
```
integrate(asec(x**(1/2)),x)
```

output
```
x*asec(sqrt(x)) - Piecewise((2*sqrt(x - 1), Abs(x) > 1), (2*I*sqrt(1 - x),
 True))/2
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \sec^{-1}\left(\sqrt{x}\right)\,dx = x\,\mathrm{arcsec}\left(\sqrt{x}\right) - \sqrt{x}\sqrt{-\frac{1}{x}+1}$$

input
```
integrate(arcsec(x^(1/2)),x, algorithm="maxima")
```

output
```
x*arcsec(sqrt(x)) - sqrt(x)*sqrt(-1/x + 1)
```

3.5.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(14) = 28$.

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.28

$$\int \sec^{-1}\left(\sqrt{x}\right)\,dx = x\arccos\left(\frac{1}{\sqrt{x}}\right) - \frac{1}{2}\sqrt{x}\left(\sqrt{-\frac{1}{x}+1}-1\right) + \frac{1}{2\sqrt{x}\left(\sqrt{-\frac{1}{x}+1}-1\right)}$$

input
```
integrate(arcsec(x^(1/2)),x, algorithm="giac")
```

output
```
x*arccos(1/sqrt(x)) - 1/2*sqrt(x)*(sqrt(-1/x + 1) - 1) + 1/2/(sqrt(x)*(sqr
t(-1/x + 1) - 1))
```

3.5.9 Mupad [B] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \sec^{-1}\left(\sqrt{x}\right)\,dx = x\,\mathrm{acos}\left(\frac{1}{\sqrt{x}}\right) - \sqrt{x}\sqrt{1-\frac{1}{x}}$$

input
```
int(acos(1/x^(1/2)),x)
```

output
```
x*acos(1/x^(1/2)) - x^(1/2)*(1 - 1/x)^(1/2)
```

3.6 $\int \frac{\sec^{-1}(\sqrt{x})}{x} dx$

3.6.1 Optimal result

Integrand size = 10, antiderivative size = 56

$$\int \frac{\sec^{-1}(\sqrt{x})}{x} dx = i\sec^{-1}(\sqrt{x})^2 - 2\sec^{-1}(\sqrt{x})\log\left(1+e^{2i\sec^{-1}(\sqrt{x})}\right) + i\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(\sqrt{x})}\right)$$

output
```
I*arcsec(x^(1/2))^2-2*arcsec(x^(1/2))*ln(1+(1/x^(1/2)+I*(1-1/x)^(1/2))^2)+
I*polylog(2,-(1/x^(1/2)+I*(1-1/x)^(1/2))^2)
```

3.6.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{\sec^{-1}(\sqrt{x})}{x} dx = i\left(\sec^{-1}(\sqrt{x})\left(\sec^{-1}(\sqrt{x}) + 2i\log\left(1+e^{2i\sec^{-1}(\sqrt{x})}\right)\right) + \text{PolyLog}\left(2,-e^{2i\sec^{-1}(\sqrt{x})}\right)\right)$$

input
```
Integrate[ArcSec[Sqrt[x]]/x,x]
```

output
```
I*(ArcSec[Sqrt[x]]*(ArcSec[Sqrt[x]] + (2*I)*Log[1 + E^((2*I)*ArcSec[Sqrt[x
]])]) + PolyLog[2, -E^((2*I)*ArcSec[Sqrt[x]])])
```

3.6.3 Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {7267, 5741, 5137, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x}\,dx$$

$$\Big\downarrow \; 7267$$

$$2\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \; 5741$$

$$-2\int \frac{\arccos\left(\frac{1}{\sqrt{x}}\right)}{\sqrt{x}}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \; 5137$$

$$2\int \sqrt{1-\frac{1}{x}}\sqrt{x}\arccos\left(\frac{1}{\sqrt{x}}\right)d\arccos\left(\frac{1}{\sqrt{x}}\right)$$

$$\Big\downarrow \; 3042$$

$$2\int \arccos\left(\frac{1}{\sqrt{x}}\right)\tan\left(\arccos\left(\frac{1}{\sqrt{x}}\right)\right)d\arccos\left(\frac{1}{\sqrt{x}}\right)$$

$$\Big\downarrow \; 4202$$

$$2\left(\frac{ix}{2}-2i\int \frac{e^{2i\arccos\left(\frac{1}{\sqrt{x}}\right)}\arccos\left(\frac{1}{\sqrt{x}}\right)}{1+e^{2i\arccos\left(\frac{1}{\sqrt{x}}\right)}}d\arccos\left(\frac{1}{\sqrt{x}}\right)\right)$$

$$\Big\downarrow \; 2620$$

$$2\left(\frac{ix}{2}-2i\left(\frac{1}{2}i\int \log\left(1+e^{2i\arccos\left(\frac{1}{\sqrt{x}}\right)}\right)d\arccos\left(\frac{1}{\sqrt{x}}\right)-\frac{1}{2}i\arccos\left(\frac{1}{\sqrt{x}}\right)\log\left(1+e^{2i\arccos\left(\frac{1}{\sqrt{x}}\right)}\right)\right)\right)$$

$$\Big\downarrow \; 2715$$

$$2\left(\frac{ix}{2}-2i\left(\frac{1}{4}\int e^{2i\arccos\left(\frac{1}{\sqrt{x}}\right)}\log\left(1+e^{2i\arccos\left(\frac{1}{\sqrt{x}}\right)}\right)de^{2i\arccos\left(\frac{1}{\sqrt{x}}\right)}-\frac{1}{2}i\arccos\left(\frac{1}{\sqrt{x}}\right)\log\left(1+e^{2i\arccos\left(\frac{1}{\sqrt{x}}\right)}\right)\right)\right)$$

$$\Big\downarrow \; 2838$$

$$2\left(\frac{ix}{2}-2i\left(-\frac{1}{4}\operatorname{PolyLog}\left(2,-e^{2i\arccos\left(\frac{1}{\sqrt{x}}\right)}\right)-\frac{1}{2}i\arccos\left(\frac{1}{\sqrt{x}}\right)\log\left(1+e^{2i\arccos\left(\frac{1}{\sqrt{x}}\right)}\right)\right)\right)$$

input
```
Int[ArcSec[Sqrt[x]]/x,x]
```

output
```
2*((I/2)*x - (2*I)*((-1/2*I)*ArcCos[1/Sqrt[x]]*Log[1 + E^((2*I)*ArcCos[1/S
qrt[x]])] - PolyLog[2, -E^((2*I)*ArcCos[1/Sqrt[x]])]/4))
```

3.6.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 5741
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a + b
*ArcCos[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.6.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.12

method	result
derivativedivides	$i \operatorname{arcsec}\left(\sqrt{x}\right)^{2}-2 \operatorname{arcsec}\left(\sqrt{x}\right) \ln\left(1+\left(\frac{1}{\sqrt{x}}+i \sqrt{1-\frac{1}{x}}\right)^{2}\right)+i \operatorname{polylog}\left(2,-\left(\frac{1}{\sqrt{x}}+i\right.\right.$
default	$i \operatorname{arcsec}\left(\sqrt{x}\right)^{2}-2 \operatorname{arcsec}\left(\sqrt{x}\right) \ln\left(1+\left(\frac{1}{\sqrt{x}}+i \sqrt{1-\frac{1}{x}}\right)^{2}\right)+i \operatorname{polylog}\left(2,-\left(\frac{1}{\sqrt{x}}+i\right.\right.$

input
```
int(arcsec(x^(1/2))/x,x,method=_RETURNVERBOSE)
```

output
```
I*arcsec(x^(1/2))^2-2*arcsec(x^(1/2))*ln(1+(1/x^(1/2)+I*(1-1/x)^(1/2))^2)+
I*polylog(2,-(1/x^(1/2)+I*(1-1/x)^(1/2))^2)
```

3.6.5 Fricas [F]

$$\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x}\,dx=\int \frac{\operatorname{arcsec}\left(\sqrt{x}\right)}{x}\,dx$$

input
```
integrate(arcsec(x^(1/2))/x,x, algorithm="fricas")
```

output
```
integral(arcsec(sqrt(x))/x, x)
```

3.6.6 Sympy [F]

$$\int \frac{\sec^{-1}(\sqrt{x})}{x}\,dx = \int \frac{\operatorname{asec}(\sqrt{x})}{x}\,dx$$

input
```
integrate(asec(x**(1/2))/x,x)
```

output
```
Integral(asec(sqrt(x))/x, x)
```

3.6.7 Maxima [F]

$$\int \frac{\sec^{-1}(\sqrt{x})}{x}\,dx = \int \frac{\operatorname{arcsec}(\sqrt{x})}{x}\,dx$$

input
```
integrate(arcsec(x^(1/2))/x,x, algorithm="maxima")
```

output
```
integrate(arcsec(sqrt(x))/x, x)
```

3.6.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^{-1}(\sqrt{x})}{x}\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate(arcsec(x^(1/2))/x,x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: Inva
lid series expansion: non tractable function acos at +infinity
```

3.6.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x}\,dx = \int \frac{\operatorname{acos}\left(\frac{1}{\sqrt{x}}\right)}{x}\,dx$$

input
```
int(acos(1/x^(1/2))/x,x)
```

output
```
int(acos(1/x^(1/2))/x, x)
```

3.7 $\int \frac{\sec^{-1}(\sqrt{x})}{x^2} dx$

3.7.1 Optimal result

Integrand size = 10, antiderivative size = 38

$$\int \frac{\sec^{-1}(\sqrt{x})}{x^2} dx = \frac{\sqrt{-1+x}}{2x} - \frac{\sec^{-1}(\sqrt{x})}{x} + \frac{1}{2}\arctan\left(\sqrt{-1+x}\right)$$

output
```
-arcsec(x^(1/2))/x+1/2*arctan((-1+x)^(1/2))+1/2*(-1+x)^(1/2)/x
```

3.7.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \frac{\sec^{-1}(\sqrt{x})}{x^2} dx = \frac{\sqrt{-1+x} - 2\sec^{-1}(\sqrt{x}) - x\arcsin\left(\frac{1}{\sqrt{x}}\right)}{2x}$$

input
```
Integrate[ArcSec[Sqrt[x]]/x^2,x]
```

output
```
(Sqrt[-1 + x] - 2*ArcSec[Sqrt[x]] - x*ArcSin[1/Sqrt[x]])/(2*x)
```

3.7.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5793, 27, 52, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x^2}dx$$

$$\Big\downarrow \text{5793}$$

$$\int \frac{1}{2\sqrt{x-1}x^2}dx-\frac{\sec^{-1}\left(\sqrt{x}\right)}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{x-1}x^2}dx-\frac{\sec^{-1}\left(\sqrt{x}\right)}{x}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{x-1}x}dx+\frac{\sqrt{x-1}}{x}\right)-\frac{\sec^{-1}\left(\sqrt{x}\right)}{x}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\int \frac{1}{x}d\sqrt{x-1}+\frac{\sqrt{x-1}}{x}\right)-\frac{\sec^{-1}\left(\sqrt{x}\right)}{x}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\arctan\left(\sqrt{x-1}\right)+\frac{\sqrt{x-1}}{x}\right)-\frac{\sec^{-1}\left(\sqrt{x}\right)}{x}$$

input
```
Int[ArcSec[Sqrt[x]]/x^2,x]
```

output
```
-(ArcSec[Sqrt[x]]/x) + (Sqrt[-1 + x]/x + ArcTan[Sqrt[-1 + x]])/2
```

3.7.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5793
```
Int[((a_.) + ArcSec[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcSec[u])/(d*(m + 1))), x] - Simp[b*(u/(d*(m +
 1)*Sqrt[u^2]))   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(u*Sqrt[
u^2 - 1])), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && Invers
eFunctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !Functio
nOfExponentialQ[u, x]
```

3.7.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

method	result	size
parts	$-\frac{\operatorname{arcsec}(\sqrt{x})}{x}+\frac{\sqrt{\frac{x-1}{x}}\,(\arctan(\sqrt{x-1})x+\sqrt{x-1})}{2\sqrt{x}\,\sqrt{x-1}}$	44
derivativedivides	$-\frac{\operatorname{arcsec}(\sqrt{x})}{x}-\frac{\sqrt{x-1}\left(\arctan\left(\frac{1}{\sqrt{x-1}}\right)x-\sqrt{x-1}\right)}{2\sqrt{\frac{x-1}{x}}\,x^{\frac{3}{2}}}$	46
default	$-\frac{\operatorname{arcsec}(\sqrt{x})}{x}-\frac{\sqrt{x-1}\left(\arctan\left(\frac{1}{\sqrt{x-1}}\right)x-\sqrt{x-1}\right)}{2\sqrt{\frac{x-1}{x}}\,x^{\frac{3}{2}}}$	46

input
```
int(arcsec(x^(1/2))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-arcsec(x^(1/2))/x+1/2*((x-1)/x)^(1/2)/x^(1/2)*(arctan((x-1)^(1/2))*x+(x-1
)^(1/2))/(x-1)^(1/2)
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.50

$$\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x^2}\,dx=\frac{(x-2)\operatorname{arcsec}\left(\sqrt{x}\right)+\sqrt{x-1}}{2\,x}$$

input
```
integrate(arcsec(x^(1/2))/x^2,x, algorithm="fricas")
```

output
```
1/2*((x - 2)*arcsec(sqrt(x)) + sqrt(x - 1))/x
```

3.7.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.66 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.97

$$\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x^2}\,dx=\frac{\begin{cases} i\operatorname{acosh}\left(\frac{1}{\sqrt{x}}\right)-\frac{i}{\sqrt{x}\sqrt{-1+\frac{1}{x}}}+\frac{i}{x^{\frac{3}{2}}\sqrt{-1+\frac{1}{x}}} & \text{for } \frac{1}{|x|}>1 \\ -\operatorname{asin}\left(\frac{1}{\sqrt{x}}\right)+\frac{\sqrt{1-\frac{1}{x}}}{\sqrt{x}} & \text{otherwise} \end{cases}}{2}-\frac{\operatorname{asec}\left(\sqrt{x}\right)}{x}$$

input
```
integrate(asec(x**(1/2))/x**2,x)
```

output
```
Piecewise((I*acosh(1/sqrt(x)) - I/(sqrt(x)*sqrt(-1 + 1/x)) + I/(x**(3/2)*s
qrt(-1 + 1/x)), 1/Abs(x) > 1), (-asin(1/sqrt(x)) + sqrt(1 - 1/x)/sqrt(x),
True))/2 - asec(sqrt(x))/x
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

$$\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x^2}\,dx = -\frac{\sqrt{x}\sqrt{-\frac{1}{x}+1}}{2\left(x\left(\frac{1}{x}-1\right)-1\right)} - \frac{\operatorname{arcsec}\left(\sqrt{x}\right)}{x} + \frac{1}{2}\arctan\left(\sqrt{x}\sqrt{-\frac{1}{x}+1}\right)$$

input
```
integrate(arcsec(x^(1/2))/x^2,x, algorithm="maxima")
```

output
```
-1/2*sqrt(x)*sqrt(-1/x + 1)/(x*(1/x - 1) - 1) - arcsec(sqrt(x))/x + 1/2*ar
ctan(sqrt(x)*sqrt(-1/x + 1))
```

3.7.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x^2}\,dx = \frac{\sqrt{-\frac{1}{x}+1}}{2\sqrt{x}} - \frac{\arccos\left(\frac{1}{\sqrt{x}}\right)}{x} + \frac{1}{2}\arccos\left(\frac{1}{\sqrt{x}}\right)$$

input
```
integrate(arcsec(x^(1/2))/x^2,x, algorithm="giac")
```

output
```
1/2*sqrt(-1/x + 1)/sqrt(x) - arccos(1/sqrt(x))/x + 1/2*arccos(1/sqrt(x))
```

3.7.9 Mupad [B] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x^2}\,dx = \frac{\sqrt{1-\frac{1}{x}}}{2\sqrt{x}} - \frac{\mathrm{acos}\left(\frac{1}{\sqrt{x}}\right)\left(\frac{2}{x}-1\right)}{2}$$

input
```
int(acos(1/x^(1/2))/x^2,x)
```

output
```
(1 - 1/x)^(1/2)/(2*x^(1/2)) - (acos(1/x^(1/2))*(2/x - 1))/2
```

3.8 $\int \frac{\sec^{-1}(\sqrt{x})}{x^3} dx$

3.8.1 Optimal result

Integrand size = 10, antiderivative size = 54

$$\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x^3} dx = \frac{\sqrt{-1+x}}{8x^2} + \frac{3\sqrt{-1+x}}{16x} - \frac{\sec^{-1}\left(\sqrt{x}\right)}{2x^2} + \frac{3}{16}\arctan\left(\sqrt{-1+x}\right)$$

output
```
-1/2*arcsec(x^(1/2))/x^2+3/16*arctan((-1+x)^(1/2))+1/8*(-1+x)^(1/2)/x^2+3/
16*(-1+x)^(1/2)/x
```

3.8.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x^3} dx = \left(\frac{1}{8x^{3/2}} + \frac{3}{16\sqrt{x}}\right)\sqrt{\frac{-1+x}{x}} - \frac{\sec^{-1}\left(\sqrt{x}\right)}{2x^2} - \frac{3}{16}\arcsin\left(\frac{1}{\sqrt{x}}\right)$$

input
```
Integrate[ArcSec[Sqrt[x]]/x^3,x]
```

output
```
(1/(8*x^(3/2)) + 3/(16*Sqrt[x]))*Sqrt[(-1 + x)/x] - ArcSec[Sqrt[x]]/(2*x^2
) - (3*ArcSin[1/Sqrt[x]])/16
```

3.8.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5793, 27, 52, 52, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x^3}dx$$

$$\Big\downarrow \text{5793}$$

$$\frac{1}{2}\int \frac{1}{2\sqrt{x-1}x^3}dx-\frac{\sec^{-1}\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int \frac{1}{\sqrt{x-1}x^3}dx-\frac{\sec^{-1}\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{4}\left(\frac{3}{4}\int \frac{1}{\sqrt{x-1}x^2}dx+\frac{\sqrt{x-1}}{2x^2}\right)-\frac{\sec^{-1}\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{4}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{x-1}x}dx+\frac{\sqrt{x-1}}{x}\right)+\frac{\sqrt{x-1}}{2x^2}\right)-\frac{\sec^{-1}\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(\frac{3}{4}\left(\int \frac{1}{x}d\sqrt{x-1}+\frac{\sqrt{x-1}}{x}\right)+\frac{\sqrt{x-1}}{2x^2}\right)-\frac{\sec^{-1}\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(\frac{3}{4}\left(\arctan\left(\sqrt{x-1}\right)+\frac{\sqrt{x-1}}{x}\right)+\frac{\sqrt{x-1}}{2x^2}\right)-\frac{\sec^{-1}\left(\sqrt{x}\right)}{2x^2}$$

input
```
Int[ArcSec[Sqrt[x]]/x^3,x]
```

output
```
-1/2*ArcSec[Sqrt[x]]/x^2 + (Sqrt[-1 + x]/(2*x^2) + (3*(Sqrt[-1 + x]/x + Ar
cTan[Sqrt[-1 + x]]))/4)/4
```

3.8.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5793
```
Int[((a_.) + ArcSec[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcSec[u])/(d*(m + 1))), x] - Simp[b*(u/(d*(m +
 1)*Sqrt[u^2]))   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(u*Sqrt[
u^2 - 1])), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && Invers
eFunctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !Functio
nOfExponentialQ[u, x]
```

3.8.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$-\frac{\operatorname{arcsec}\left(\sqrt{x}\right)}{2x^{2}}-\frac{\sqrt{x-1}\,\left(3\arctan\left(\frac{1}{\sqrt{x-1}}\right)x^{2}-3\sqrt{x-1}\,x-2\sqrt{x-1}\right)}{16\sqrt{\frac{x-1}{x}}\,x^{\frac{5}{2}}}$	57
default	$-\frac{\operatorname{arcsec}\left(\sqrt{x}\right)}{2x^{2}}-\frac{\sqrt{x-1}\,\left(3\arctan\left(\frac{1}{\sqrt{x-1}}\right)x^{2}-3\sqrt{x-1}\,x-2\sqrt{x-1}\right)}{16\sqrt{\frac{x-1}{x}}\,x^{\frac{5}{2}}}$	57
parts	$-\frac{\operatorname{arcsec}\left(\sqrt{x}\right)}{2x^{2}}+\frac{\sqrt{\frac{x-1}{x}}\,\left(3\arctan\left(\sqrt{x-1}\right)x^{2}+3\sqrt{x-1}\,x+2\sqrt{x-1}\right)}{16x^{\frac{3}{2}}\sqrt{x-1}}$	57

input
```
int(arcsec(x^(1/2))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*arcsec(x^(1/2))/x^2-1/16*(x-1)^(1/2)*(3*arctan(1/(x-1)^(1/2))*x^2-3*(
x-1)^(1/2)*x-2*(x-1)^(1/2))/((x-1)/x)^(1/2)/x^(5/2)
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.54

$$\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x^{3}}\,dx=\frac{\left(3\,x^{2}-8\right)\operatorname{arcsec}\left(\sqrt{x}\right)+\left(3\,x+2\right)\sqrt{x-1}}{16\,x^{2}}$$

input
```
integrate(arcsec(x^(1/2))/x^3,x, algorithm="fricas")
```

output
```
1/16*((3*x^2 - 8)*arcsec(sqrt(x)) + (3*x + 2)*sqrt(x - 1))/x^2
```

3.8.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 41.80 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.67

$$\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = \frac{\begin{cases} \frac{3i\operatorname{acosh}\left(\frac{1}{\sqrt{x}}\right)}{4} - \frac{3i}{4\sqrt{x}\sqrt{-1+\frac{1}{x}}} + \frac{i}{4x^{\frac{3}{2}}\sqrt{-1+\frac{1}{x}}} + \frac{i}{2x^{\frac{5}{2}}\sqrt{-1+\frac{1}{x}}} & \text{for } \frac{1}{|x|} > 1 \\ -\frac{3\operatorname{asin}\left(\frac{1}{\sqrt{x}}\right)}{4} + \frac{3}{4\sqrt{x}\sqrt{1-\frac{1}{x}}} - \frac{1}{4x^{\frac{3}{2}}\sqrt{1-\frac{1}{x}}} - \frac{1}{2x^{\frac{5}{2}}\sqrt{1-\frac{1}{x}}} & \text{otherwise} \end{cases}}{4} - \frac{\operatorname{asec}\left(\sqrt{x}\right)}{2x^2}$$

input
```
integrate(asec(x**(1/2))/x**3,x)
```

output
```
Piecewise((3*I*acosh(1/sqrt(x))/4 - 3*I/(4*sqrt(x)*sqrt(-1 + 1/x)) + I/(4*
x**(3/2)*sqrt(-1 + 1/x)) + I/(2*x**(5/2)*sqrt(-1 + 1/x)), 1/Abs(x) > 1), (
-3*asin(1/sqrt(x))/4 + 3/(4*sqrt(x)*sqrt(1 - 1/x)) - 1/(4*x**(3/2)*sqrt(1
- 1/x)) - 1/(2*x**(5/2)*sqrt(1 - 1/x)), True))/4 - asec(sqrt(x))/(2*x**2)
```

3.8.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(38) = 76.

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.48

$$\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = \frac{3\,x^{\frac{3}{2}}\left(-\frac{1}{x}+1\right)^{\frac{3}{2}} + 5\,\sqrt{x}\sqrt{-\frac{1}{x}+1}}{16\left(x^2\left(\frac{1}{x}-1\right)^2 - 2\,x\left(\frac{1}{x}-1\right)+1\right)} - \frac{\operatorname{arcsec}\left(\sqrt{x}\right)}{2\,x^2} + \frac{3}{16}\arctan\left(\sqrt{x}\sqrt{-\frac{1}{x}+1}\right)$$

input
```
integrate(arcsec(x^(1/2))/x^3,x, algorithm="maxima")
```

output
```
1/16*(3*x^(3/2)*(-1/x + 1)^(3/2) + 5*sqrt(x)*sqrt(-1/x + 1))/(x^2*(1/x - 1
)^2 - 2*x*(1/x - 1) + 1) - 1/2*arcsec(sqrt(x))/x^2 + 3/16*arctan(sqrt(x)*s
qrt(-1/x + 1))
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = \frac{3\sqrt{-\frac{1}{x}+1}}{16\sqrt{x}} + \frac{\sqrt{-\frac{1}{x}+1}}{8\,x^{\frac{3}{2}}} - \frac{\arccos\left(\frac{1}{\sqrt{x}}\right)}{2\,x^2} + \frac{3}{16}\arccos\left(\frac{1}{\sqrt{x}}\right)$$

input
```
integrate(arcsec(x^(1/2))/x^3,x, algorithm="giac")
```

output
```
3/16*sqrt(-1/x + 1)/sqrt(x) + 1/8*sqrt(-1/x + 1)/x^(3/2) - 1/2*arccos(1/sq
rt(x))/x^2 + 3/16*arccos(1/sqrt(x))
```

3.8.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = \int \frac{\mathrm{acos}\left(\frac{1}{\sqrt{x}}\right)}{x^3}\,dx$$

input
```
int(acos(1/x^(1/2))/x^3,x)
```

output
```
int(acos(1/x^(1/2))/x^3, x)
```

3.9 $\int \frac{\sec^{-1}(\sqrt{x})}{x^4} dx$

3.9.1 Optimal result

Integrand size = 10, antiderivative size = 68

$$\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x^4}\,dx=\frac{\sqrt{-1+x}}{18x^3}+\frac{5\sqrt{-1+x}}{72x^2}+\frac{5\sqrt{-1+x}}{48x}-\frac{\sec^{-1}\left(\sqrt{x}\right)}{3x^3}+\frac{5}{48}\arctan\left(\sqrt{-1+x}\right)$$

output
```
-1/3*arcsec(x^(1/2))/x^3+5/48*arctan((-1+x)^(1/2))+1/18*(-1+x)^(1/2)/x^3+5
/72*(-1+x)^(1/2)/x^2+5/48*(-1+x)^(1/2)/x
```

3.9.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.66

$$\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x^4}\,dx=\frac{\sqrt{-1+x}(8+10x+15x^2)-48\sec^{-1}\left(\sqrt{x}\right)-15x^3\arcsin\left(\frac{1}{\sqrt{x}}\right)}{144x^3}$$

input
```
Integrate[ArcSec[Sqrt[x]]/x^4,x]
```

output
```
(Sqrt[-1 + x]*(8 + 10*x + 15*x^2) - 48*ArcSec[Sqrt[x]] - 15*x^3*ArcSin[1/S
qrt[x]])/(144*x^3)
```

3.9.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5793, 27, 52, 52, 52, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x^4}dx$$

$$\Big\downarrow \text{5793}$$

$$\frac{1}{3}\int \frac{1}{2\sqrt{x-1}x^4}dx-\frac{\sec^{-1}\left(\sqrt{x}\right)}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\int \frac{1}{\sqrt{x-1}x^4}dx-\frac{\sec^{-1}\left(\sqrt{x}\right)}{3x^3}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{6}\left(\frac{5}{6}\int \frac{1}{\sqrt{x-1}x^3}dx+\frac{\sqrt{x-1}}{3x^3}\right)-\frac{\sec^{-1}\left(\sqrt{x}\right)}{3x^3}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{6}\left(\frac{5}{6}\left(\frac{3}{4}\int \frac{1}{\sqrt{x-1}x^2}dx+\frac{\sqrt{x-1}}{2x^2}\right)+\frac{\sqrt{x-1}}{3x^3}\right)-\frac{\sec^{-1}\left(\sqrt{x}\right)}{3x^3}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{6}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{x-1}x}dx+\frac{\sqrt{x-1}}{x}\right)+\frac{\sqrt{x-1}}{2x^2}\right)+\frac{\sqrt{x-1}}{3x^3}\right)-\frac{\sec^{-1}\left(\sqrt{x}\right)}{3x^3}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{6}\left(\frac{5}{6}\left(\frac{3}{4}\left(\int \frac{1}{x}d\sqrt{x-1}+\frac{\sqrt{x-1}}{x}\right)+\frac{\sqrt{x-1}}{2x^2}\right)+\frac{\sqrt{x-1}}{3x^3}\right)-\frac{\sec^{-1}\left(\sqrt{x}\right)}{3x^3}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{6}\left(\frac{5}{6}\left(\frac{3}{4}\left(\arctan\left(\sqrt{x-1}\right)+\frac{\sqrt{x-1}}{x}\right)+\frac{\sqrt{x-1}}{2x^2}\right)+\frac{\sqrt{x-1}}{3x^3}\right)-\frac{\sec^{-1}\left(\sqrt{x}\right)}{3x^3}$$

input
```
Int[ArcSec[Sqrt[x]]/x^4,x]
```

output
```
-1/3*ArcSec[Sqrt[x]]/x^3 + (Sqrt[-1 + x]/(3*x^3) + (5*(Sqrt[-1 + x]/(2*x^2
) + (3*(Sqrt[-1 + x]/x + ArcTan[Sqrt[-1 + x]]))/4))/6)/6
```

3.9.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5793
```
Int[((a_.) + ArcSec[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcSec[u])/(d*(m + 1))), x] - Simp[b*(u/(d*(m +
 1)*Sqrt[u^2]))   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(u*Sqrt[
u^2 - 1])), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && Invers
eFunctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !Functio
nOfExponentialQ[u, x]
```

3.9.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.99

method	result	size
derivativedivides	$-\frac{\operatorname{arcsec}\left(\sqrt{x}\right)}{3x^{3}}-\frac{\sqrt{x-1}\,\left(15\arctan\left(\frac{1}{\sqrt{x-1}}\right)x^{3}-15\sqrt{x-1}\,x^{2}-10\sqrt{x-1}\,x-8\sqrt{x-1}\right)}{144\sqrt{\frac{x-1}{x}}\,x^{\frac{7}{2}}}$	67
default	$-\frac{\operatorname{arcsec}\left(\sqrt{x}\right)}{3x^{3}}-\frac{\sqrt{x-1}\,\left(15\arctan\left(\frac{1}{\sqrt{x-1}}\right)x^{3}-15\sqrt{x-1}\,x^{2}-10\sqrt{x-1}\,x-8\sqrt{x-1}\right)}{144\sqrt{\frac{x-1}{x}}\,x^{\frac{7}{2}}}$	67
parts	$-\frac{\operatorname{arcsec}\left(\sqrt{x}\right)}{3x^{3}}+\frac{\sqrt{\frac{x-1}{x}}\,\left(15\arctan\left(\sqrt{x-1}\right)x^{3}+15\sqrt{x-1}\,x^{2}+10\sqrt{x-1}\,x+8\sqrt{x-1}\right)}{144x^{\frac{5}{2}}\sqrt{x-1}}$	67

input
```
int(arcsec(x^(1/2))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*arcsec(x^(1/2))/x^3-1/144*(x-1)^(1/2)*(15*arctan(1/(x-1)^(1/2))*x^3-1
5*(x-1)^(1/2)*x^2-10*(x-1)^(1/2)*x-8*(x-1)^(1/2))/((x-1)/x)^(1/2)/x^(7/2)
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.51

$$\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x^4}\,dx=\frac{3\,(5\,x^3-16)\operatorname{arcsec}\left(\sqrt{x}\right)+(15\,x^2+10\,x+8)\sqrt{x-1}}{144\,x^3}$$

input
```
integrate(arcsec(x^(1/2))/x^4,x, algorithm="fricas")
```

output
```
1/144*(3*(5*x^3 - 16)*arcsec(sqrt(x)) + (15*x^2 + 10*x + 8)*sqrt(x - 1))/x
^3
```

3.9.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 127.92 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.65

$$\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x^4}\,dx = \frac{\begin{cases} \frac{5i\operatorname{acosh}\left(\frac{1}{\sqrt{x}}\right)}{8} - \frac{5i}{8\sqrt{x}\sqrt{-1+\frac{1}{x}}} + \frac{5i}{24x^{\frac{3}{2}}\sqrt{-1+\frac{1}{x}}} + \frac{i}{12x^{\frac{5}{2}}\sqrt{-1+\frac{1}{x}}} + \frac{i}{3x^{\frac{7}{2}}\sqrt{-1+\frac{1}{x}}} & \text{for } \frac{1}{|x|} > 1 \\ -\frac{5\operatorname{asin}\left(\frac{1}{\sqrt{x}}\right)}{8} + \frac{5}{8\sqrt{x}\sqrt{1-\frac{1}{x}}} - \frac{5}{24x^{\frac{3}{2}}\sqrt{1-\frac{1}{x}}} - \frac{1}{12x^{\frac{5}{2}}\sqrt{1-\frac{1}{x}}} - \frac{1}{3x^{\frac{7}{2}}\sqrt{1-\frac{1}{x}}} & \text{otherwise} \end{cases}}{6} - \frac{\operatorname{asec}\left(\sqrt{x}\right)}{3x^3}$$

input
```
integrate(asec(x**(1/2))/x**4,x)
```

output
```
Piecewise((5*I*acosh(1/sqrt(x))/8 - 5*I/(8*sqrt(x)*sqrt(-1 + 1/x)) + 5*I/(
24*x**(3/2)*sqrt(-1 + 1/x)) + I/(12*x**(5/2)*sqrt(-1 + 1/x)) + I/(3*x**(7/
2)*sqrt(-1 + 1/x)), 1/Abs(x) > 1), (-5*asin(1/sqrt(x))/8 + 5/(8*sqrt(x)*sq
rt(1 - 1/x)) - 5/(24*x**(3/2)*sqrt(1 - 1/x)) - 1/(12*x**(5/2)*sqrt(1 - 1/x
)) - 1/(3*x**(7/2)*sqrt(1 - 1/x)), True))/6 - asec(sqrt(x))/(3*x**3)
```

3.9.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(48) = 96$.

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.56

$$\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x^4}\,dx = -\frac{15\,x^{\frac{5}{2}}\left(-\frac{1}{x}+1\right)^{\frac{5}{2}} + 40\,x^{\frac{3}{2}}\left(-\frac{1}{x}+1\right)^{\frac{3}{2}} + 33\,\sqrt{x}\sqrt{-\frac{1}{x}+1}}{144\left(x^3\left(\frac{1}{x}-1\right)^3 - 3\,x^2\left(\frac{1}{x}-1\right)^2 + 3\,x\left(\frac{1}{x}-1\right) - 1\right)} - \frac{\operatorname{arcsec}\left(\sqrt{x}\right)}{3\,x^3} + \frac{5}{48}\arctan\left(\sqrt{x}\sqrt{-\frac{1}{x}+1}\right)$$

input
```
integrate(arcsec(x^(1/2))/x^4,x, algorithm="maxima")
```

output
```
-1/144*(15*x^(5/2)*(-1/x + 1)^(5/2) + 40*x^(3/2)*(-1/x + 1)^(3/2) + 33*sqr
t(x)*sqrt(-1/x + 1))/(x^3*(1/x - 1)^3 - 3*x^2*(1/x - 1)^2 + 3*x*(1/x - 1)
- 1) - 1/3*arcsec(sqrt(x))/x^3 + 5/48*arctan(sqrt(x)*sqrt(-1/x + 1))
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.85

$$\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x^4}\,dx = \frac{5\sqrt{-\frac{1}{x}+1}}{48\sqrt{x}} + \frac{5\sqrt{-\frac{1}{x}+1}}{72\,x^{\frac{3}{2}}} + \frac{\sqrt{-\frac{1}{x}+1}}{18\,x^{\frac{5}{2}}} - \frac{\arccos\left(\frac{1}{\sqrt{x}}\right)}{3\,x^3} + \frac{5}{48}\arccos\left(\frac{1}{\sqrt{x}}\right)$$

input
```
integrate(arcsec(x^(1/2))/x^4,x, algorithm="giac")
```

output
```
5/48*sqrt(-1/x + 1)/sqrt(x) + 5/72*sqrt(-1/x + 1)/x^(3/2) + 1/18*sqrt(-1/x
 + 1)/x^(5/2) - 1/3*arccos(1/sqrt(x))/x^3 + 5/48*arccos(1/sqrt(x))
```

3.9.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x^4}\,dx = \int \frac{\mathrm{acos}\left(\frac{1}{\sqrt{x}}\right)}{x^4}\,dx$$

input
```
int(acos(1/x^(1/2))/x^4,x)
```

output
```
int(acos(1/x^(1/2))/x^4, x)
```

3.10 $\int x^2 \sec^{-1}\left(\frac{a}{x}\right) dx$

3.10.1 Optimal result

Integrand size = 10, antiderivative size = 56

$$\int x^2 \sec^{-1}\left(\frac{a}{x}\right) dx = -\frac{1}{3}a^3\sqrt{1-\frac{x^2}{a^2}}+\frac{1}{9}a^3\left(1-\frac{x^2}{a^2}\right)^{3/2}+\frac{1}{3}x^3\arccos\left(\frac{x}{a}\right)$$

output
```
1/9*a^3*(1-x^2/a^2)^(3/2)+1/3*x^3*arccos(x/a)-1/3*a^3*(1-x^2/a^2)^(1/2)
```

3.10.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.75

$$\int x^2 \sec^{-1}\left(\frac{a}{x}\right) dx = -\frac{1}{9}a(2a^2+x^2)\sqrt{1-\frac{x^2}{a^2}}+\frac{1}{3}x^3\sec^{-1}\left(\frac{a}{x}\right)$$

input
```
Integrate[x^2*ArcSec[a/x],x]
```

output
```
-1/9*(a*(2*a^2 + x^2)*Sqrt[1 - x^2/a^2]) + (x^3*ArcSec[a/x])/3
```

3.10.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5787, 5139, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sec^{-1}\left(\frac{a}{x}\right) dx$$

$$\Big\downarrow \text{ 5787}$$

$$\int x^2 \arccos\left(\frac{x}{a}\right) dx$$

$$\Big\downarrow \text{ 5139}$$

$$\frac{\int \frac{x^3}{\sqrt{1-\frac{x^2}{a^2}}}dx}{3a}+\frac{1}{3}x^3 \arccos\left(\frac{x}{a}\right)$$

$$\Big\downarrow \text{ 243}$$

$$\frac{\int \frac{x^2}{\sqrt{1-\frac{x^2}{a^2}}}dx^2}{6a}+\frac{1}{3}x^3 \arccos\left(\frac{x}{a}\right)$$

$$\Big\downarrow \text{ 53}$$

$$\frac{\int \left(\frac{a^2}{\sqrt{1-\frac{x^2}{a^2}}}-a^2\sqrt{1-\frac{x^2}{a^2}}\right)dx^2}{6a}+\frac{1}{3}x^3 \arccos\left(\frac{x}{a}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{2}{3}a^4\left(1-\frac{x^2}{a^2}\right)^{3/2}-2a^4\sqrt{1-\frac{x^2}{a^2}}}{6a}+\frac{1}{3}x^3 \arccos\left(\frac{x}{a}\right)$$

input
```
Int[x^2*ArcSec[a/x],x]
```

output
```
(-2*a^4*Sqrt[1 - x^2/a^2] + (2*a^4*(1 - x^2/a^2)^(3/2))/3)/(6*a) + (x^3*Ar
cCos[x/a])/3
```

3.10.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5787
```
Int[ArcSec[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcCos[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

3.10.4 Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

method	result	size
parts	$\frac{x^3 \operatorname{arcsec}\left(\frac{a}{x}\right)}{3}+\frac{-\frac{x^2 a^2 \sqrt{1-\frac{x^2}{a^2}}}{3}-\frac{2 a^4 \sqrt{1-\frac{x^2}{a^2}}}{3}}{3 a}$	56
derivativedivides	$-a^3\left(-\frac{x^3 \operatorname{arcsec}\left(\frac{a}{x}\right)}{3 a^3}+\frac{\left(\frac{a^2}{x^2}-1\right)\left(\frac{2 a^2}{x^2}+1\right) x^4}{9 \sqrt{\frac{\left(\frac{a^2}{x^2}-1\right) x^2}{a^2}}\, a^4}\right)$	66
default	$-a^3\left(-\frac{x^3 \operatorname{arcsec}\left(\frac{a}{x}\right)}{3 a^3}+\frac{\left(\frac{a^2}{x^2}-1\right)\left(\frac{2 a^2}{x^2}+1\right) x^4}{9 \sqrt{\frac{\left(\frac{a^2}{x^2}-1\right) x^2}{a^2}}\, a^4}\right)$	66

input
```
int(x^2*arcsec(a/x),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*arcsec(a/x)+1/3/a*(-1/3*x^2*a^2*(1-x^2/a^2)^(1/2)-2/3*a^4*(1-x^2/a
^2)^(1/2))
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.70

$$\int x^2 \sec^{-1}\left(\frac{a}{x}\right) dx = \frac{1}{3} x^3 \operatorname{arcsec}\left(\frac{a}{x}\right) - \frac{1}{9} \left(2 a^2 x + x^3\right) \sqrt{\frac{a^2 - x^2}{x^2}}$$

input
```
integrate(x^2*arcsec(a/x),x, algorithm="fricas")
```

output
```
1/3*x^3*arcsec(a/x) - 1/9*(2*a^2*x + x^3)*sqrt((a^2 - x^2)/x^2)
```

3.10.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int x^2 \sec^{-1}\left(\frac{a}{x}\right) dx = \begin{cases} -\frac{2a^3\sqrt{1-\frac{x^2}{a^2}}}{9} - \frac{ax^2\sqrt{1-\frac{x^2}{a^2}}}{9} + \frac{x^3 \operatorname{asec}\left(\frac{a}{x}\right)}{3} & \text{for } a \neq 0 \\ \tilde{\infty} x^3 & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*asec(a/x),x)
```

output
```
Piecewise((-2*a**3*sqrt(1 - x**2/a**2)/9 - a*x**2*sqrt(1 - x**2/a**2)/9 +
x**3*asec(a/x)/3, Ne(a, 0)), (zoo*x**3, True))
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int x^2 \sec^{-1}\left(\frac{a}{x}\right) dx = \frac{1}{3} x^3 \operatorname{arcsec}\left(\frac{a}{x}\right) - \frac{2 a^4 \sqrt{-\frac{x^2}{a^2}+1} + a^2 x^2 \sqrt{-\frac{x^2}{a^2}+1}}{9 a}$$

input
```
integrate(x^2*arcsec(a/x),x, algorithm="maxima")
```

output
```
1/3*x^3*arcsec(a/x) - 1/9*(2*a^4*sqrt(-x^2/a^2 + 1) + a^2*x^2*sqrt(-x^2/a^
2 + 1))/a
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

$$\int x^2 \sec^{-1}\left(\frac{a}{x}\right) dx = \frac{1}{3} x^3 \arccos\left(\frac{x}{a}\right) - \frac{2}{9} a^3 \sqrt{-\frac{x^2}{a^2}+1} - \frac{1}{9} a x^2 \sqrt{-\frac{x^2}{a^2}+1}$$

input
```
integrate(x^2*arcsec(a/x),x, algorithm="giac")
```

output
```
1/3*x^3*arccos(x/a) - 2/9*a^3*sqrt(-x^2/a^2 + 1) - 1/9*a*x^2*sqrt(-x^2/a^2
 + 1)
```

3.10.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \sec^{-1}\left(\frac{a}{x}\right) dx = \begin{cases} \frac{x^3 \operatorname{acos}\left(\frac{x}{a}\right)}{3} - \frac{\sqrt{a^2-x^2}\,(2a^2+x^2)}{9} & \text{if } 0 < a \\ \int x^2 \operatorname{acos}\left(\frac{x}{a}\right) dx & \text{if } \neg 0 < a \end{cases}$$

input
```
int(x^2*acos(x/a),x)
```

output
```
piecewise(0 < a, (x^3*acos(x/a))/3 - ((a^2 - x^2)^(1/2)*(2*a^2 + x^2))/9,
~0 < a, int(x^2*acos(x/a), x))
```

3.11 $\int x \sec^{-1}\left(\frac{a}{x}\right) dx$

3.11.1 Optimal result

Integrand size = 8, antiderivative size = 47

$$\int x \sec^{-1}\left(\frac{a}{x}\right) dx = -\frac{1}{4}ax\sqrt{1-\frac{x^2}{a^2}} + \frac{1}{2}x^2 \arccos\left(\frac{x}{a}\right) + \frac{1}{4}a^2 \arcsin\left(\frac{x}{a}\right)$$

output
```
1/2*x^2*arccos(x/a)+1/4*a^2*arcsin(x/a)-1/4*a*x*(1-x^2/a^2)^(1/2)
```

3.11.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int x \sec^{-1}\left(\frac{a}{x}\right) dx = \frac{1}{4}\left(-ax\sqrt{1-\frac{x^2}{a^2}} + 2x^2 \sec^{-1}\left(\frac{a}{x}\right) + a^2 \arcsin\left(\frac{x}{a}\right)\right)$$

input
```
Integrate[x*ArcSec[a/x],x]
```

output
```
(-(a*x*Sqrt[1 - x^2/a^2]) + 2*x^2*ArcSec[a/x] + a^2*ArcSin[x/a])/4
```

3.11.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5787, 5139, 262, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sec^{-1}\left(\frac{a}{x}\right) dx$$

$$\Big\downarrow \text{5787}$$

$$\int x \arccos\left(\frac{x}{a}\right) dx$$

$$\Big\downarrow \text{5139}$$

$$\frac{\int \frac{x^2}{\sqrt{1-\frac{x^2}{a^2}}}dx}{2a} + \frac{1}{2}x^2 \arccos\left(\frac{x}{a}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{1}{2}a^2 \int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}dx - \frac{1}{2}a^2 x\sqrt{1-\frac{x^2}{a^2}}}{2a} + \frac{1}{2}x^2 \arccos\left(\frac{x}{a}\right)$$

$$\Big\downarrow \text{223}$$

$$\frac{\frac{1}{2}a^3 \arcsin\left(\frac{x}{a}\right) - \frac{1}{2}a^2 x\sqrt{1-\frac{x^2}{a^2}}}{2a} + \frac{1}{2}x^2 \arccos\left(\frac{x}{a}\right)$$

input
```
Int[x*ArcSec[a/x],x]
```

output
```
(x^2*ArcCos[x/a])/2 + (-1/2*(a^2*x*Sqrt[1 - x^2/a^2]) + (a^3*ArcSin[x/a])/
2)/(2*a)
```

3.11.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5787
```
Int[ArcSec[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcCos[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

3.11.4 Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.43

method	result	size
parts	$\frac{x^2 \operatorname{arcsec}\left(\frac{a}{x}\right)}{2}+\frac{-\frac{x a^2 \sqrt{1-\frac{x^2}{a^2}}}{2}+\frac{a^2 \arctan\left(\frac{\sqrt{\frac{1}{a^2}}\, x}{\sqrt{1-\frac{x^2}{a^2}}}\right)}{2 \sqrt{\frac{1}{a^2}}}}{2a}$	67
derivativedivides	$-a^2 \left(-\frac{x^2 \operatorname{arcsec}\left(\frac{a}{x}\right)}{2a^2}-\frac{\sqrt{\frac{a^2}{x^2}-1}\left(\frac{\arctan\left(\frac{1}{\sqrt{\frac{a^2}{x^2}-1}}\right) a^2}{x^2}-\sqrt{\frac{a^2}{x^2}-1}\right) x^3}{4 \sqrt{\frac{\left(\frac{a^2}{x^2}-1\right) x^2}{a^2}}\, a^3}\right)$	91
default	$-a^2 \left(-\frac{x^2 \operatorname{arcsec}\left(\frac{a}{x}\right)}{2a^2}-\frac{\sqrt{\frac{a^2}{x^2}-1}\left(\frac{\arctan\left(\frac{1}{\sqrt{\frac{a^2}{x^2}-1}}\right) a^2}{x^2}-\sqrt{\frac{a^2}{x^2}-1}\right) x^3}{4 \sqrt{\frac{\left(\frac{a^2}{x^2}-1\right) x^2}{a^2}}\, a^3}\right)$	91

input
```
int(x*arcsec(a/x),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*arcsec(a/x)+1/2/a*(-1/2*x*a^2*(1-x^2/a^2)^(1/2)+1/2*a^2/(1/a^2)^(1
/2)*arctan((1/a^2)^(1/2)*x/(1-x^2/a^2)^(1/2)))
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int x \sec^{-1}\left(\frac{a}{x}\right) dx = -\frac{1}{4} x^2 \sqrt{\frac{a^2-x^2}{x^2}} - \frac{1}{4}\left(a^2-2\,x^2\right) \operatorname{arcsec}\left(\frac{a}{x}\right)$$

input
```
integrate(x*arcsec(a/x),x, algorithm="fricas")
```

output
```
-1/4*x^2*sqrt((a^2 - x^2)/x^2) - 1/4*(a^2 - 2*x^2)*arcsec(a/x)
```

3.11.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int x\sec^{-1}\left(\frac{a}{x}\right)\,dx = \begin{cases} -\frac{a^2\operatorname{asec}\left(\frac{a}{x}\right)}{4} - \frac{ax\sqrt{1-\frac{x^2}{a^2}}}{4} + \frac{x^2\operatorname{asec}\left(\frac{a}{x}\right)}{2} & \text{for } a \neq 0 \\ \tilde{\infty}x^2 & \text{otherwise} \end{cases}$$

input
```
integrate(x*asec(a/x),x)
```

output
```
Piecewise((-a**2*asec(a/x)/4 - a*x*sqrt(1 - x**2/a**2)/4 + x**2*asec(a/x)/
2, Ne(a, 0)), (zoo*x**2, True))
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int x\sec^{-1}\left(\frac{a}{x}\right)\,dx = \frac{1}{2}\,x^2\operatorname{arcsec}\left(\frac{a}{x}\right) + \frac{a^3\arcsin\left(\frac{x}{a}\right) - a^2x\sqrt{-\frac{x^2}{a^2}+1}}{4\,a}$$

input
```
integrate(x*arcsec(a/x),x, algorithm="maxima")
```

output
```
1/2*x^2*arcsec(a/x) + 1/4*(a^3*arcsin(x/a) - a^2*x*sqrt(-x^2/a^2 + 1))/a
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int x\sec^{-1}\left(\frac{a}{x}\right)\,dx = -\frac{1}{4}\,a^2\arccos\left(\frac{x}{a}\right) + \frac{1}{2}\,x^2\arccos\left(\frac{x}{a}\right) - \frac{1}{4}\,ax\sqrt{-\frac{x^2}{a^2}+1}$$

input
```
integrate(x*arcsec(a/x),x, algorithm="giac")
```

output
```
-1/4*a^2*arccos(x/a) + 1/2*x^2*arccos(x/a) - 1/4*a*x*sqrt(-x^2/a^2 + 1)
```

3.11.9 Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int x \sec^{-1}\left(\frac{a}{x}\right) dx = \frac{a^2 \operatorname{acos}\left(\frac{x}{a}\right) \left(\frac{2x^2}{a^2} - 1\right)}{4} - \frac{a x \sqrt{1 - \frac{x^2}{a^2}}}{4}$$

input

```
int(x*acos(x/a),x)
```

output

```
(a^2*acos(x/a)*((2*x^2)/a^2 - 1))/4 - (a*x*(1 - x^2/a^2)^(1/2))/4
```

3.12 $\int \sec^{-1}\left(\frac{a}{x}\right) dx$

3.12.1 Optimal result

Integrand size = 6, antiderivative size = 26

$$\int \sec^{-1}\left(\frac{a}{x}\right) dx = -a\sqrt{1-\frac{x^2}{a^2}} + x \arccos\left(\frac{x}{a}\right)$$

output
```
x*arccos(x/a)-a*(1-x^2/a^2)^(1/2)
```

3.12.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \sec^{-1}\left(\frac{a}{x}\right) dx = -a\sqrt{1-\frac{x^2}{a^2}} + x \sec^{-1}\left(\frac{a}{x}\right)$$

input
```
Integrate[ArcSec[a/x],x]
```

output
```
-(a*Sqrt[1 - x^2/a^2]) + x*ArcSec[a/x]
```

3.12.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5787, 5131, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{-1}\left(\frac{a}{x}\right) dx$$

$$\Big\downarrow \ 5787$$

$$\int \arccos\left(\frac{x}{a}\right) dx$$

$$\Big\downarrow \ 5131$$

$$\frac{\int \frac{x}{\sqrt{1-\frac{x^2}{a^2}}}dx}{a} + x \arccos\left(\frac{x}{a}\right)$$

$$\Big\downarrow \ 241$$

$$x \arccos\left(\frac{x}{a}\right) - a\sqrt{1-\frac{x^2}{a^2}}$$

input
```
Int[ArcSec[a/x],x]
```

output
```
-(a*Sqrt[1 - x^2/a^2]) + x*ArcCos[x/a]
```

3.12.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5787
```
Int[ArcSec[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcCos[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

3.12.4 Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

method	result	size
parts	$x \operatorname{arcsec}\left(\frac{a}{x}\right)-a\sqrt{\frac{a^{2}-x^{2}}{a^{2}}}$	28
derivativedivides	$-a\left(-\frac{x \operatorname{arcsec}\left(\frac{a}{x}\right)}{a}+\frac{x^{2}\left(\frac{a^{2}}{x^{2}}-1\right)}{\sqrt{\frac{\left(\frac{a^{2}}{x^{2}}-1\right)x^{2}}{a^{2}}}\, a^{2}}\right)$	51
default	$-a\left(-\frac{x \operatorname{arcsec}\left(\frac{a}{x}\right)}{a}+\frac{x^{2}\left(\frac{a^{2}}{x^{2}}-1\right)}{\sqrt{\frac{\left(\frac{a^{2}}{x^{2}}-1\right)x^{2}}{a^{2}}}\, a^{2}}\right)$	51

input
```
int(arcsec(a/x),x,method=_RETURNVERBOSE)
```

output
```
x*arcsec(a/x)-a*((a^2-x^2)/a^2)^(1/2)
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \sec^{-1}\left(\frac{a}{x}\right)\, dx = x \operatorname{arcsec}\left(\frac{a}{x}\right)-x\sqrt{\frac{a^{2}-x^{2}}{x^{2}}}$$

input
```
integrate(arcsec(a/x),x, algorithm="fricas")
```

output
```
x*arcsec(a/x) - x*sqrt((a^2 - x^2)/x^2)
```

3.12.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \sec^{-1}\left(\frac{a}{x}\right)\,dx = \begin{cases} -a\sqrt{1-\frac{x^2}{a^2}} + x\operatorname{asec}\left(\frac{a}{x}\right) & \text{for } a \neq 0 \\ \tilde{\infty} x & \text{otherwise} \end{cases}$$

input
```
integrate(asec(a/x),x)
```

output
```
Piecewise((-a*sqrt(1 - x**2/a**2) + x*asec(a/x), Ne(a, 0)), (zoo*x, True))
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \sec^{-1}\left(\frac{a}{x}\right)\,dx = x\operatorname{arcsec}\left(\frac{a}{x}\right) - a\sqrt{-\frac{x^2}{a^2}+1}$$

input
```
integrate(arcsec(a/x),x, algorithm="maxima")
```

output
```
x*arcsec(a/x) - a*sqrt(-x^2/a^2 + 1)
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \sec^{-1}\left(\frac{a}{x}\right)\,dx = a\left(\frac{x\arccos\left(\frac{x}{a}\right)}{a} - \sqrt{-\frac{x^2}{a^2}+1}\right)$$

input
```
integrate(arcsec(a/x),x, algorithm="giac")
```

output
```
a*(x*arccos(x/a)/a - sqrt(-x^2/a^2 + 1))
```

3.12.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \sec^{-1}\left(\frac{a}{x}\right)\,dx = x\,\mathrm{acos}\left(\frac{x}{a}\right) - a\sqrt{1-\frac{x^2}{a^2}}$$

input
```
int(acos(x/a),x)
```

output
```
x*acos(x/a) - a*(1 - x^2/a^2)^(1/2)
```

3.13 $\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x} \, dx$

3.13.1 Optimal result

Integrand size = 10, antiderivative size = 59

$$\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x} \, dx = -\frac{1}{2}i \arccos\left(\frac{x}{a}\right)^2 + \arccos\left(\frac{x}{a}\right) \log\left(1 + e^{2i \arccos\left(\frac{x}{a}\right)}\right) - \frac{1}{2}i \operatorname{PolyLog}\left(2, -e^{2i \arccos\left(\frac{x}{a}\right)}\right)$$

output
```
-1/2*I*arccos(x/a)^2+arccos(x/a)*ln(1+(x/a+I*(1-x^2/a^2)^(1/2))^2)-1/2*I*p
olylog(2,-(x/a+I*(1-x^2/a^2)^(1/2))^2)
```

3.13.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x} \, dx = -\frac{1}{2}i \sec^{-1}\left(\frac{a}{x}\right)^2 + \sec^{-1}\left(\frac{a}{x}\right) \log\left(1 + e^{2i \sec^{-1}\left(\frac{a}{x}\right)}\right) - \frac{1}{2}i \operatorname{PolyLog}\left(2, -e^{2i \sec^{-1}\left(\frac{a}{x}\right)}\right)$$

input
```
Integrate[ArcSec[a/x]/x,x]
```

output
```
(-1/2*I)*ArcSec[a/x]^2 + ArcSec[a/x]*Log[1 + E^((2*I)*ArcSec[a/x])] - (I/2
)*PolyLog[2, -E^((2*I)*ArcSec[a/x])]
```

3.13.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5787, 5137, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x}dx$$

$$\Big\downarrow \text{5787}$$

$$\int \frac{\arccos\left(\frac{x}{a}\right)}{x}dx$$

$$\Big\downarrow \text{5137}$$

$$-\int \frac{a\sqrt{1-\frac{x^2}{a^2}}\arccos\left(\frac{x}{a}\right)}{x}d\arccos\left(\frac{x}{a}\right)$$

$$\Big\downarrow \text{3042}$$

$$-\int \arccos\left(\frac{x}{a}\right)\tan\left(\arccos\left(\frac{x}{a}\right)\right)d\arccos\left(\frac{x}{a}\right)$$

$$\Big\downarrow \text{4202}$$

$$2i\int \frac{e^{2i\arccos\left(\frac{x}{a}\right)}\arccos\left(\frac{x}{a}\right)}{1+e^{2i\arccos\left(\frac{x}{a}\right)}}d\arccos\left(\frac{x}{a}\right)-\frac{1}{2}i\arccos\left(\frac{x}{a}\right)^2$$

$$\Big\downarrow \text{2620}$$

$$2i\left(\frac{1}{2}i\int \log\left(1+e^{2i\arccos\left(\frac{x}{a}\right)}\right)d\arccos\left(\frac{x}{a}\right)-\frac{1}{2}i\arccos\left(\frac{x}{a}\right)\log\left(1+e^{2i\arccos\left(\frac{x}{a}\right)}\right)\right)-\frac{1}{2}i\arccos\left(\frac{x}{a}\right)^2$$

$$\Big\downarrow \text{2715}$$

$$2i\left(\frac{1}{4}\int e^{-2i\arccos\left(\frac{x}{a}\right)}\log\left(1+e^{2i\arccos\left(\frac{x}{a}\right)}\right)de^{2i\arccos\left(\frac{x}{a}\right)}-\frac{1}{2}i\arccos\left(\frac{x}{a}\right)\log\left(1+e^{2i\arccos\left(\frac{x}{a}\right)}\right)\right)-\frac{1}{2}i\arccos\left(\frac{x}{a}\right)^2$$

$$\Big\downarrow \text{2838}$$

$$2i\left(-\frac{1}{4}\text{PolyLog}\left(2,-e^{2i\arccos\left(\frac{x}{a}\right)}\right)-\frac{1}{2}i\arccos\left(\frac{x}{a}\right)\log\left(1+e^{2i\arccos\left(\frac{x}{a}\right)}\right)\right)-\frac{1}{2}i\arccos\left(\frac{x}{a}\right)^2$$

input
```
Int[ArcSec[a/x]/x,x]
```

output
```
(-1/2*I)*ArcCos[x/a]^2 + (2*I)*((-1/2*I)*ArcCos[x/a]*Log[1 + E^((2*I)*ArcC
os[x/a])] - PolyLog[2, -E^((2*I)*ArcCos[x/a])]/4)
```

3.13.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 5787
```
Int[ArcSec[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcCos[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

3.13.4 Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

method	result	siz
derivativedivides	$-\frac{i \operatorname{arcsec}\left(\frac{a}{x}\right)^{2}}{2}+\operatorname{arcsec}\left(\frac{a}{x}\right) \ln \left(1+\left(\frac{x}{a}+i \sqrt{1-\frac{x^{2}}{a^{2}}}\right)^{2}\right)-\frac{i \operatorname{polylog}\left(2,-\left(\frac{x}{a}+i \sqrt{1-\frac{x^{2}}{a^{2}}}\right)^{2}\right)}{2}$	76
default	$-\frac{i \operatorname{arcsec}\left(\frac{a}{x}\right)^{2}}{2}+\operatorname{arcsec}\left(\frac{a}{x}\right) \ln \left(1+\left(\frac{x}{a}+i \sqrt{1-\frac{x^{2}}{a^{2}}}\right)^{2}\right)-\frac{i \operatorname{polylog}\left(2,-\left(\frac{x}{a}+i \sqrt{1-\frac{x^{2}}{a^{2}}}\right)^{2}\right)}{2}$	76

input
```
int(arcsec(a/x)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*arcsec(a/x)^2+arcsec(a/x)*ln(1+(x/a+I*(1-x^2/a^2)^(1/2))^2)-1/2*I*p
olylog(2,-(x/a+I*(1-x^2/a^2)^(1/2))^2)
```

3.13.5 Fricas [F]

$$\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x}\, dx = \int \frac{\operatorname{arcsec}\left(\frac{a}{x}\right)}{x}\, dx$$

input
```
integrate(arcsec(a/x)/x,x, algorithm="fricas")
```

output
```
integral(arcsec(a/x)/x, x)
```

3.13.6 Sympy [F]

$$\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x}\, dx = \int \frac{\operatorname{asec}\left(\frac{a}{x}\right)}{x}\, dx$$

input
```
integrate(asec(a/x)/x,x)
```

output
```
Integral(asec(a/x)/x, x)
```

3.13.7 Maxima [F]

$$\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x}\,dx = \int \frac{\operatorname{arcsec}\left(\frac{a}{x}\right)}{x}\,dx$$

input
```
integrate(arcsec(a/x)/x,x, algorithm="maxima")
```

output
```
integrate(arcsec(a/x)/x, x)
```

3.13.8 Giac [F]

$$\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x}\,dx = \int \frac{\operatorname{arcsec}\left(\frac{a}{x}\right)}{x}\,dx$$

input
```
integrate(arcsec(a/x)/x,x, algorithm="giac")
```

output
```
integrate(arcsec(a/x)/x, x)
```

3.13.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x}\,dx = \int \frac{\operatorname{acos}\left(\frac{x}{a}\right)}{x}\,dx$$

input
```
int(acos(x/a)/x,x)
```

output
```
int(acos(x/a)/x, x)
```

3.14 $\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^2} dx$

3.14.1 Optimal result

Integrand size = 10, antiderivative size = 31

$$\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^2} dx = -\frac{\arccos\left(\frac{x}{a}\right)}{x} + \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{x^2}{a^2}}\right)}{a}$$

output
```
-arccos(x/a)/x+arctanh((1-x^2/a^2)^(1/2))/a
```

3.14.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 93 vs. 2(31) = 62.

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.00

$$\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^2} dx = -\frac{\sec^{-1}\left(\frac{a}{x}\right)}{x} + \frac{\sqrt{-1+\frac{a^2}{x^2}}x\left(-\log\left(1-\frac{a}{\sqrt{-1+\frac{a^2}{x^2}}x}\right)+\log\left(1+\frac{a}{\sqrt{-1+\frac{a^2}{x^2}}x}\right)\right)}{2a^2\sqrt{1-\frac{x^2}{a^2}}}$$

input
```
Integrate[ArcSec[a/x]/x^2,x]
```

output
```
-(ArcSec[a/x]/x) + (Sqrt[-1 + a^2/x^2]*x*(-Log[1 - a/(Sqrt[-1 + a^2/x^2]*x
)] + Log[1 + a/(Sqrt[-1 + a^2/x^2]*x)]))/(2*a^2*Sqrt[1 - x^2/a^2])
```

3.14.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5787, 5139, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^2} \, dx$$

$$\Big\downarrow \; 5787$$

$$\int \frac{\arccos\left(\frac{x}{a}\right)}{x^2}dx$$

$$\Big\downarrow \; 5139$$

$$-\frac{\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}dx}{a}-\frac{\arccos\left(\frac{x}{a}\right)}{x}$$

$$\Big\downarrow \; 243$$

$$-\frac{\int \frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}}dx^2}{2a}-\frac{\arccos\left(\frac{x}{a}\right)}{x}$$

$$\Big\downarrow \; 73$$

$$a\int \frac{1}{a^2-a^2x^4}d\sqrt{1-\frac{x^2}{a^2}}-\frac{\arccos\left(\frac{x}{a}\right)}{x}$$

$$\Big\downarrow \; 221$$

$$\frac{\text{arctanh}\left(\sqrt{1-\frac{x^2}{a^2}}\right)}{a}-\frac{\arccos\left(\frac{x}{a}\right)}{x}$$

input
```
Int[ArcSec[a/x]/x^2,x]
```

output
```
-(ArcCos[x/a]/x) + ArcTanh[Sqrt[1 - x^2/a^2]]/a
```

3.14.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5787
```
Int[ArcSec[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcCos[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

3.14.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

method	result	size
parts	$-\frac{\operatorname{arcsec}\left(\frac{a}{x}\right)}{x}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{1-\frac{x^{2}}{a^{2}}}}\right)}{a}$	30
derivativedivides	$-\frac{\frac{a\,\operatorname{arcsec}\left(\frac{a}{x}\right)}{x}-\ln\left(\frac{a}{x}+\frac{a\sqrt{1-\frac{x^{2}}{a^{2}}}}{x}\right)}{a}$	44
default	$-\frac{\frac{a\,\operatorname{arcsec}\left(\frac{a}{x}\right)}{x}-\ln\left(\frac{a}{x}+\frac{a\sqrt{1-\frac{x^{2}}{a^{2}}}}{x}\right)}{a}$	44

input
```
int(arcsec(a/x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-arcsec(a/x)/x+1/a*arctanh(1/(1-x^2/a^2)^(1/2))
```

3.14.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(29) = 58$.

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.45

$$\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^2}\,dx = -\frac{2\,ax\arctan\left(-\frac{x^2\sqrt{\frac{a^2-x^2}{x^2}}}{a^2-x^2}\right) - 2\,(ax-a)\,\mathrm{arcsec}\left(\frac{a}{x}\right) - x\log\left(x\sqrt{\frac{a^2-x^2}{x^2}}+a\right) + x\log\left(x\sqrt{\frac{a^2-x^2}{x^2}}-a\right)}{2\,ax}$$

input
```
integrate(arcsec(a/x)/x^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a*x*arctan(-x^2*sqrt((a^2 - x^2)/x^2)/(a^2 - x^2)) - 2*(a*x - a)*a
rcsec(a/x) - x*log(x*sqrt((a^2 - x^2)/x^2) + a) + x*log(x*sqrt((a^2 - x^2)
/x^2) - a))/(a*x)
```

3.14.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^2}\,dx = -\frac{\mathrm{asec}\left(\frac{a}{x}\right)}{x} - \frac{\begin{cases} -\operatorname{acosh}\left(\frac{a}{x}\right) & \text{for } \left|\frac{a^2}{x^2}\right| > 1 \\ i\,\mathrm{asin}\left(\frac{a}{x}\right) & \text{otherwise} \end{cases}}{a}$$

input
```
integrate(asec(a/x)/x**2,x)
```

output
```
-asec(a/x)/x - Piecewise((-acosh(a/x), Abs(a**2/x**2) > 1), (I*asin(a/x),
True))/a
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.68

$$\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^2}\,dx = -\frac{\frac{2\,a\,\mathrm{arcsec}\left(\frac{a}{x}\right)}{x} - \log\left(\sqrt{-\frac{x^2}{a^2}+1}+1\right) + \log\left(-\sqrt{-\frac{x^2}{a^2}+1}+1\right)}{2\,a}$$

input
```
integrate(arcsec(a/x)/x^2,x, algorithm="maxima")
```

output
```
-1/2*(2*a*arcsec(a/x)/x - log(sqrt(-x^2/a^2 + 1) + 1) + log(-sqrt(-x^2/a^2
 + 1) + 1))/a
```

3.14.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(29) = 58.

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.97

$$\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^2}\,dx = \frac{a\left(\frac{\log\left(\left|a+\sqrt{a^2-x^2}\right|\right)}{a} - \frac{\log\left(\left|-a+\sqrt{a^2-x^2}\right|\right)}{a}\right)}{2\,|a|} - \frac{\arccos\left(\frac{x}{a}\right)}{x}$$

input
```
integrate(arcsec(a/x)/x^2,x, algorithm="giac")
```

output
```
1/2*a*(log(abs(a + sqrt(a^2 - x^2)))/a - log(abs(-a + sqrt(a^2 - x^2)))/a)
/abs(a) - arccos(x/a)/x
```

3.14.9 Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^2}\,dx = \frac{\mathrm{atanh}\left(\frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\right)}{a} - \frac{\mathrm{acos}\left(\frac{x}{a}\right)}{x}$$

input
```
int(acos(x/a)/x^2,x)
```

output
```
atanh(1/(1 - x^2/a^2)^(1/2))/a - acos(x/a)/x
```

3.15 $\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^3} dx$

3.15.1 Optimal result

Integrand size = 10, antiderivative size = 38

$$\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^3} dx = \frac{\sqrt{1-\frac{x^2}{a^2}}}{2ax} - \frac{\arccos\left(\frac{x}{a}\right)}{2x^2}$$

output
```
-1/2*arccos(x/a)/x^2+1/2*(1-x^2/a^2)^(1/2)/a/x
```

3.15.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^3} dx = \frac{x\sqrt{1-\frac{x^2}{a^2}} - a\sec^{-1}\left(\frac{a}{x}\right)}{2ax^2}$$

input
```
Integrate[ArcSec[a/x]/x^3,x]
```

output
```
(x*Sqrt[1 - x^2/a^2] - a*ArcSec[a/x])/(2*a*x^2)
```

3.15.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5787, 5139, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^3} \, dx$$

$$\Big\downarrow \; 5787$$

$$\int \frac{\arccos\left(\frac{x}{a}\right)}{x^3}dx$$

$$\Big\downarrow \; 5139$$

$$-\frac{\int \frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}}dx}{2a}-\frac{\arccos\left(\frac{x}{a}\right)}{2x^2}$$

$$\Big\downarrow \; 242$$

$$\frac{\sqrt{1-\frac{x^2}{a^2}}}{2ax}-\frac{\arccos\left(\frac{x}{a}\right)}{2x^2}$$

input
```
Int[ArcSec[a/x]/x^3,x]
```

output
```
Sqrt[1 - x^2/a^2]/(2*a*x) - ArcCos[x/a]/(2*x^2)
```

3.15.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5787
```
Int[ArcSec[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcCos[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

3.15.4 Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

method	result	size
parts	$-\frac{\operatorname{arcsec}\left(\frac{a}{x}\right)}{2x^2}+\frac{\sqrt{1-\frac{x^2}{a^2}}}{2ax}$	33
derivativedivides	$-\frac{\frac{a^2\operatorname{arcsec}\left(\frac{a}{x}\right)}{2x^2}-\frac{x\left(\frac{a^2}{x^2}-1\right)}{2\sqrt{\frac{\left(\frac{a^2}{x^2}-1\right)x^2}{a^2}}\,a}}{a^2}$	54
default	$-\frac{\frac{a^2\operatorname{arcsec}\left(\frac{a}{x}\right)}{2x^2}-\frac{x\left(\frac{a^2}{x^2}-1\right)}{2\sqrt{\frac{\left(\frac{a^2}{x^2}-1\right)x^2}{a^2}}\,a}}{a^2}$	54

input
```
int(arcsec(a/x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*arcsec(a/x)/x^2+1/2*(1-x^2/a^2)^(1/2)/a/x
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^3}\,dx=-\frac{a^2\operatorname{arcsec}\left(\frac{a}{x}\right)-x^2\sqrt{\frac{a^2-x^2}{x^2}}}{2\,a^2x^2}$$

input
```
integrate(arcsec(a/x)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(a^2*arcsec(a/x) - x^2*sqrt((a^2 - x^2)/x^2))/(a^2*x^2)
```

3.15.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.39

$$\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^3}\,dx = -\frac{\operatorname{asec}\left(\frac{a}{x}\right)}{2x^2} - \frac{\begin{cases} -\frac{\sqrt{\frac{a^2}{x^2}-1}}{a} & \text{for } \left|\frac{a^2}{x^2}\right| > 1 \\ -\frac{i\sqrt{-\frac{a^2}{x^2}+1}}{a} & \text{otherwise} \end{cases}}{2a}$$

input
```
integrate(asec(a/x)/x**3,x)
```

output
```
-asec(a/x)/(2*x**2) - Piecewise((-sqrt(a**2/x**2 - 1)/a, Abs(a**2/x**2) >
1), (-I*sqrt(-a**2/x**2 + 1)/a, True))/(2*a)
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^3}\,dx = -\frac{\operatorname{arcsec}\left(\frac{a}{x}\right)}{2\,x^2} + \frac{\sqrt{-\frac{x^2}{a^2}+1}}{2\,ax}$$

input
```
integrate(arcsec(a/x)/x^3,x, algorithm="maxima")
```

output
```
-1/2*arcsec(a/x)/x^2 + 1/2*sqrt(-x^2/a^2 + 1)/(a*x)
```

3.15.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.61

$$\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^3}\,dx = \frac{a\left(\frac{a+\sqrt{a^2-x^2}}{a^2x} - \frac{x}{\left(a+\sqrt{a^2-x^2}\right)a^2}\right)}{4\,|a|} - \frac{\arccos\left(\frac{x}{a}\right)}{2\,x^2}$$

input
```
integrate(arcsec(a/x)/x^3,x, algorithm="giac")
```

output
```
1/4*a*((a + sqrt(a^2 - x^2))/(a^2*x) - x/((a + sqrt(a^2 - x^2))*a^2))/abs(
a) - 1/2*arccos(x/a)/x^2
```

3.15.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^3}\,dx = \int \frac{\operatorname{acos}\left(\frac{x}{a}\right)}{x^3}\,dx$$

input
```
int(acos(x/a)/x^3,x)
```

output
```
int(acos(x/a)/x^3, x)
```

3.16 $\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^4}\,dx$

3.16.1 Optimal result

Integrand size = 10, antiderivative size = 60

$$\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^4}\,dx = \frac{\sqrt{1-\frac{x^2}{a^2}}}{6ax^2} - \frac{\arccos\left(\frac{x}{a}\right)}{3x^3} + \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{x^2}{a^2}}\right)}{6a^3}$$

output
```
-1/3*arccos(x/a)/x^3+1/6*arctanh((1-x^2/a^2)^(1/2))/a^3+1/6*(1-x^2/a^2)^(1
/2)/a/x^2
```

3.16.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.15

$$\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^4}\,dx = \frac{a^2x\sqrt{1-\frac{x^2}{a^2}} - 2a^3\sec^{-1}\left(\frac{a}{x}\right) - x^3\log(x) + x^3\log\left(1+\sqrt{1-\frac{x^2}{a^2}}\right)}{6a^3x^3}$$

input
```
Integrate[ArcSec[a/x]/x^4,x]
```

output
```
(a^2*x*Sqrt[1 - x^2/a^2] - 2*a^3*ArcSec[a/x] - x^3*Log[x] + x^3*Log[1 + Sq
rt[1 - x^2/a^2]])/(6*a^3*x^3)
```

3.16.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5787, 5139, 243, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^4}dx$$

$$\Big\downarrow \text{ 5787}$$

$$\int \frac{\arccos\left(\frac{x}{a}\right)}{x^4}dx$$

$$\Big\downarrow \text{ 5139}$$

$$-\frac{\int \frac{1}{x^3\sqrt{1-\frac{x^2}{a^2}}}dx}{3a}-\frac{\arccos\left(\frac{x}{a}\right)}{3x^3}$$

$$\Big\downarrow \text{ 243}$$

$$-\frac{\int \frac{1}{x^4\sqrt{1-\frac{x^2}{a^2}}}dx^2}{6a}-\frac{\arccos\left(\frac{x}{a}\right)}{3x^3}$$

$$\Big\downarrow \text{ 52}$$

$$-\frac{\frac{\int \frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}}dx^2}{2a^2}-\frac{\sqrt{1-\frac{x^2}{a^2}}}{x^2}}{6a}-\frac{\arccos\left(\frac{x}{a}\right)}{3x^3}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{-\int \frac{1}{a^2-a^2x^4}d\sqrt{1-\frac{x^2}{a^2}}-\frac{\sqrt{1-\frac{x^2}{a^2}}}{x^2}}{6a}-\frac{\arccos\left(\frac{x}{a}\right)}{3x^3}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{-\frac{\text{arctanh}\left(\sqrt{1-\frac{x^2}{a^2}}\right)}{a^2}-\frac{\sqrt{1-\frac{x^2}{a^2}}}{x^2}}{6a}-\frac{\arccos\left(\frac{x}{a}\right)}{3x^3}$$

input
```
Int[ArcSec[a/x]/x^4,x]
```

output
```
-1/3*ArcCos[x/a]/x^3 - (-(Sqrt[1 - x^2/a^2]/x^2) - ArcTanh[Sqrt[1 - x^2/a^
2]]/a^2)/(6*a)
```

3.16.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5787
```
Int[ArcSec[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcCos[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

3.16.4 Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

method	result	size
parts	$-\frac{\operatorname{arcsec}\left(\frac{a}{x}\right)}{3x^3}-\frac{-\frac{\sqrt{1-\frac{x^2}{a^2}}}{2x^2}-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\right)}{2a^2}}{3a}$	54
derivativedivides	$-\frac{\frac{a^3 \operatorname{arcsec}\left(\frac{a}{x}\right)}{3x^3}-\frac{\sqrt{\frac{a^2}{x^2}-1}\left(\frac{a\sqrt{\frac{a^2}{x^2}-1}}{x}+\ln\left(\frac{a}{x}+\sqrt{\frac{a^2}{x^2}-1}\right)\right)x}{6\sqrt{\frac{\left(\frac{a^2}{x^2}-1\right)x^2}{a^2}}\,a}}{a^3}$	91
default	$-\frac{\frac{a^3 \operatorname{arcsec}\left(\frac{a}{x}\right)}{3x^3}-\frac{\sqrt{\frac{a^2}{x^2}-1}\left(\frac{a\sqrt{\frac{a^2}{x^2}-1}}{x}+\ln\left(\frac{a}{x}+\sqrt{\frac{a^2}{x^2}-1}\right)\right)x}{6\sqrt{\frac{\left(\frac{a^2}{x^2}-1\right)x^2}{a^2}}\,a}}{a^3}$	91

input
```
int(arcsec(a/x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*arcsec(a/x)/x^3-1/3/a*(-1/2/x^2*(1-x^2/a^2)^(1/2)-1/2/a^2*arctanh(1/(
1-x^2/a^2)^(1/2)))
```

3.16.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(50) = 100$.

Time = 0.28 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.37

$$\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^4}\,dx =$$

$$-\frac{4\,a^3x^3\arctan\left(-\frac{x^2\sqrt{\frac{a^2-x^2}{x^2}}}{a^2-x^2}\right)-x^3\log\left(x\sqrt{\frac{a^2-x^2}{x^2}}+a\right)+x^3\log\left(x\sqrt{\frac{a^2-x^2}{x^2}}-a\right)-2\,ax^2\sqrt{\frac{a^2-x^2}{x^2}}-4\,(a^3}{12\,a^3x^3}$$

input
```
integrate(arcsec(a/x)/x^4,x, algorithm="fricas")
```

output
```
-1/12*(4*a^3*x^3*arctan(-x^2*sqrt((a^2 - x^2)/x^2)/(a^2 - x^2)) - x^3*log(
x*sqrt((a^2 - x^2)/x^2) + a) + x^3*log(x*sqrt((a^2 - x^2)/x^2) - a) - 2*a*
x^2*sqrt((a^2 - x^2)/x^2) - 4*(a^3*x^3 - a^3)*arcsec(a/x))/(a^3*x^3)
```

3.16.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.40 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.67

$$\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^4}\,dx = -\frac{\operatorname{asec}\left(\frac{a}{x}\right)}{3x^3} - \frac{\begin{cases} -\frac{\sqrt{\frac{a^2}{x^2}-1}}{2ax} - \frac{\operatorname{acosh}\left(\frac{a}{x}\right)}{2a^2} & \text{for } \left|\frac{a^2}{x^2}\right| > 1 \\ \frac{ia}{2x^3\sqrt{-\frac{a^2}{x^2}+1}} - \frac{i}{2ax\sqrt{-\frac{a^2}{x^2}+1}} + \frac{i\operatorname{asin}\left(\frac{a}{x}\right)}{2a^2} & \text{otherwise} \end{cases}}{3a}$$

input
```
integrate(asec(a/x)/x**4,x)
```

output
```
-asec(a/x)/(3*x**3) - Piecewise((-sqrt(a**2/x**2 - 1)/(2*a*x) - acosh(a/x)
/(2*a**2), Abs(a**2/x**2) > 1), (I*a/(2*x**3*sqrt(-a**2/x**2 + 1)) - I/(2*
a*x*sqrt(-a**2/x**2 + 1)) + I*asin(a/x)/(2*a**2), True))/(3*a)
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.07

$$\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^4}\,dx = \frac{\frac{\log\left(\frac{2\sqrt{-\frac{x^2}{a^2}+1}}{|x|}+\frac{2}{|x|}\right)}{a^2} + \frac{\sqrt{-\frac{x^2}{a^2}+1}}{x^2}}{6\,a} - \frac{\operatorname{arcsec}\left(\frac{a}{x}\right)}{3\,x^3}$$

input
```
integrate(arcsec(a/x)/x^4,x, algorithm="maxima")
```

output
```
1/6*(log(2*sqrt(-x^2/a^2 + 1)/abs(x) + 2/abs(x))/a^2 + sqrt(-x^2/a^2 + 1)/
x^2)/a - 1/3*arcsec(a/x)/x^3
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.33

$$\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^4}\,dx = \frac{a\left(\frac{\log\left(\left|a+\sqrt{a^2-x^2}\right|\right)}{a^3}-\frac{\log\left(\left|-a+\sqrt{a^2-x^2}\right|\right)}{a^3}+\frac{2\sqrt{a^2-x^2}}{a^2x^2}\right)}{12\,|a|}-\frac{\arccos\left(\frac{x}{a}\right)}{3\,x^3}$$

input
```
integrate(arcsec(a/x)/x^4,x, algorithm="giac")
```

output
```
1/12*a*(log(abs(a + sqrt(a^2 - x^2)))/a^3 - log(abs(-a + sqrt(a^2 - x^2)))
/a^3 + 2*sqrt(a^2 - x^2)/(a^2*x^2))/abs(a) - 1/3*arccos(x/a)/x^3
```

3.16.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^4}\,dx = \int \frac{\mathrm{acos}\left(\frac{x}{a}\right)}{x^4}\,dx$$

input
```
int(acos(x/a)/x^4,x)
```

output
```
int(acos(x/a)/x^4, x)
```

3.17 $\int \frac{\sec^{-1}(ax^n)}{x} dx$

3.17.1 Optimal result

Integrand size = 10, antiderivative size = 69

$$\int \frac{\sec^{-1}(ax^n)}{x} dx = \frac{i\sec^{-1}(ax^n)^2}{2n} - \frac{\sec^{-1}(ax^n)\log\left(1+e^{2i\sec^{-1}(ax^n)}\right)}{n} + \frac{i\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(ax^n)}\right)}{2n}$$

output
```
1/2*I*arcsec(a*x^n)^2/n-arcsec(a*x^n)*ln(1+(1/a/(x^n)+I*(1-1/a^2/(x^n)^2)^
(1/2))^2)/n+1/2*I*polylog(2,-(1/a/(x^n)+I*(1-1/a^2/(x^n)^2)^(1/2))^2)/n
```

3.17.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

$$\int \frac{\sec^{-1}(ax^n)}{x} dx = \frac{x^{-n}\,{}_3F_2\left(\frac{1}{2},\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{x^{-2n}}{a^2}\right)}{an} + \left(\sec^{-1}(ax^n)+\arcsin\left(\frac{x^{-n}}{a}\right)\right)\log(x)$$

input
```
Integrate[ArcSec[a*x^n]/x,x]
```

output
```
HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2, 3/2}, 1/(a^2*x^(2*n))]/(a*n*x^n)
+ (ArcSec[a*x^n] + ArcSin[1/(a*x^n)])*Log[x]
```

3.17.3 Rubi [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {7282, 5741, 5137, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{-1}(ax^n)}{x} \, dx$$

$$\Big\downarrow \; 7282$$

$$\frac{\int x^{-n} \sec^{-1}(ax^n)\, dx^n}{n}$$

$$\Big\downarrow \; 5741$$

$$-\frac{\int x^{-n} \arccos\left(\frac{x^{-n}}{a}\right) dx^{-n}}{n}$$

$$\Big\downarrow \; 5137$$

$$\frac{\int ax^n \sqrt{1-\frac{x^{-2n}}{a^2}} \arccos\left(\frac{x^{-n}}{a}\right) d\arccos\left(\frac{x^{-n}}{a}\right)}{n}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \arccos\left(\frac{x^{-n}}{a}\right) \tan\left(\arccos\left(\frac{x^{-n}}{a}\right)\right) d\arccos\left(\frac{x^{-n}}{a}\right)}{n}$$

$$\Big\downarrow \; 4202$$

$$\frac{\frac{1}{2} i x^{2n} - 2i \int \frac{e^{2i\arccos\left(\frac{x^{-n}}{a}\right)} \arccos\left(\frac{x^{-n}}{a}\right)}{1+e^{2i\arccos\left(\frac{x^{-n}}{a}\right)}} d\arccos\left(\frac{x^{-n}}{a}\right)}{n}$$

$$\Big\downarrow \; 2620$$

$$\frac{\frac{1}{2} i x^{2n} - 2i\left(\frac{1}{2} i \int \log\left(1+e^{2i\arccos\left(\frac{x^{-n}}{a}\right)}\right) d\arccos\left(\frac{x^{-n}}{a}\right) - \frac{1}{2} i \arccos\left(\frac{x^{-n}}{a}\right) \log\left(1+e^{2i\arccos\left(\frac{x^{-n}}{a}\right)}\right)\right)}{n}$$

$$\Big\downarrow \; 2715$$

$$\frac{\frac{1}{2} i x^{2n} - 2i\left(\frac{1}{4} \int e^{2i\arccos\left(\frac{x^{-n}}{a}\right)} \log\left(1+e^{2i\arccos\left(\frac{x^{-n}}{a}\right)}\right) de^{2i\arccos\left(\frac{x^{-n}}{a}\right)} - \frac{1}{2} i \arccos\left(\frac{x^{-n}}{a}\right) \log\left(1+e^{2i\arccos\left(\frac{x^{-n}}{a}\right)}\right)\right)}{n}$$

$$\Big\downarrow 2838$$

$$\frac{\frac{1}{2}ix^{2n}-2i\left(-\frac{1}{4}\operatorname{PolyLog}\left(2,-e^{2i\arccos\left(\frac{x^{-n}}{a}\right)}\right)-\frac{1}{2}i\arccos\left(\frac{x^{-n}}{a}\right)\log\left(1+e^{2i\arccos\left(\frac{x^{-n}}{a}\right)}\right)\right)}{n}$$

input
```
Int[ArcSec[a*x^n]/x,x]
```

output
```
((I/2)*x^(2*n) - (2*I)*((-1/2*I)*ArcCos[1/(a*x^n)]*Log[1 + E^((2*I)*ArcCos
[1/(a*x^n)])] - PolyLog[2, -E^((2*I)*ArcCos[1/(a*x^n)])]/4))/n
```

3.17.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 5741
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a + b
*ArcCos[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

3.17.4 Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.35

method	result	size
derivativedivides	$\frac{\frac{i\,\mathrm{arcsec}(a\,x^n)^2}{2}-\mathrm{arcsec}(a\,x^n)\ln\left(1+\left(\frac{x^{-n}}{a}+i\sqrt{1-\frac{x^{-2n}}{a^2}}\right)^2\right)+\frac{i\,\mathrm{polylog}\left(2,-\left(\frac{x^{-n}}{a}+i\sqrt{1-\frac{x^{-2n}}{a^2}}\right)^2\right)}{2}}{n}$	93
default	$\frac{\frac{i\,\mathrm{arcsec}(a\,x^n)^2}{2}-\mathrm{arcsec}(a\,x^n)\ln\left(1+\left(\frac{x^{-n}}{a}+i\sqrt{1-\frac{x^{-2n}}{a^2}}\right)^2\right)+\frac{i\,\mathrm{polylog}\left(2,-\left(\frac{x^{-n}}{a}+i\sqrt{1-\frac{x^{-2n}}{a^2}}\right)^2\right)}{2}}{n}$	93

input
```
int(arcsec(a*x^n)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(1/2*I*arcsec(a*x^n)^2-arcsec(a*x^n)*ln(1+(1/a/(x^n)+I*(1-1/a^2/(x^n)^
2)^(1/2))^2)+1/2*I*polylog(2,-(1/a/(x^n)+I*(1-1/a^2/(x^n)^2)^(1/2))^2))
```

3.17.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sec^{-1}(ax^n)}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsec(a*x^n)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.17.6 Sympy [F]

$$\int \frac{\sec^{-1}(ax^n)}{x}\,dx = \int \frac{\operatorname{asec}(ax^n)}{x}\,dx$$

input
```
integrate(asec(a*x**n)/x,x)
```

output
```
Integral(asec(a*x**n)/x, x)
```

3.17.7 Maxima [F]

$$\int \frac{\sec^{-1}(ax^n)}{x}\,dx = \int \frac{\operatorname{arcsec}(ax^n)}{x}\,dx$$

input
```
integrate(arcsec(a*x^n)/x,x, algorithm="maxima")
```

output
```
-a^2*n*integrate(sqrt(a*x^n + 1)*sqrt(a*x^n - 1)*log(x)/(a^4*x*x^(2*n) - a
^2*x), x) + arctan(sqrt(a*x^n + 1)*sqrt(a*x^n - 1))*log(x)
```

3.17.8 Giac [F]

$$\int \frac{\sec^{-1}(ax^n)}{x}\,dx = \int \frac{\operatorname{arcsec}(ax^n)}{x}\,dx$$

input
```
integrate(arcsec(a*x^n)/x,x, algorithm="giac")
```

output
```
integrate(arcsec(a*x^n)/x, x)
```

3.17.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{-1}(ax^n)}{x}\,dx = \int \frac{\operatorname{acos}\left(\frac{1}{a\,x^n}\right)}{x}\,dx$$

input
```
int(acos(1/(a*x^n))/x,x)
```

output
```
int(acos(1/(a*x^n))/x, x)
```

3.18 $\int x^4 \sec^{-1}(a+bx)\,dx$

3.18.1 Optimal result

Integrand size = 10, antiderivative size = 197

$$\int x^4 \sec^{-1}(a+bx)\,dx = \frac{a(20+53a^2)(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{30b^5} + \frac{11ax^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{60b^3} - \frac{x^3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{20b^2} - \frac{(9+58a^2)(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}}{120b^5} + \frac{a^5\sec^{-1}(a+bx)}{5b^5} + \frac{1}{5}x^5\sec^{-1}(a+bx) - \frac{(3+40a^2+40a^4)\operatorname{arctanh}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{40b^5}$$

output
```
1/5*a^5*arcsec(b*x+a)/b^5+1/5*x^5*arcsec(b*x+a)-1/40*(40*a^4+40*a^2+3)*arc
tanh((1-1/(b*x+a)^2)^(1/2))/b^5+1/30*a*(53*a^2+20)*(b*x+a)*(1-1/(b*x+a)^2)
^(1/2)/b^5+11/60*a*x^2*(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/b^3-1/20*x^3*(b*x+a)*
(1-1/(b*x+a)^2)^(1/2)/b^2-1/120*(58*a^2+9)*(b*x+a)^2*(1-1/(b*x+a)^2)^(1/2)
/b^5
```

3.18.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.88

$$\int x^4 \sec^{-1}(a+bx)\,dx$$
$$= \frac{\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}\left(a^2(71+154a^2)+2a(31+48a^2)\,bx-9(1+4a^2)\,b^2x^2+16ab^3x^3-6b^4x^4\right)+24b^5x^5\sec}{120b^5}$$

input
```
Integrate[x^4*ArcSec[a + b*x],x]
```

output
```
(Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2]*(a^2*(71 + 154*a^2) + 2*
a*(31 + 48*a^2)*b*x - 9*(1 + 4*a^2)*b^2*x^2 + 16*a*b^3*x^3 - 6*b^4*x^4) +
24*b^5*x^5*ArcSec[a + b*x] - 24*a^5*ArcSin[(a + b*x)^(-1)] - 3*(3 + 40*a^2
 + 40*a^4)*Log[(a + b*x)*(1 + Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x
)^2])])/(120*b^5)
```

3.18.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {5781, 4926, 3042, 4269, 3042, 4544, 3042, 4536, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \sec^{-1}(a+bx)\,dx$$
$$\Big\downarrow \text{5781}$$
$$\frac{\int b^4x^4(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)d\sec^{-1}(a+bx)}{b^5}$$
$$\Big\downarrow \text{4926}$$
$$\frac{\frac{1}{5}\int -b^5x^5d\sec^{-1}(a+bx)+\frac{1}{5}b^5x^5\sec^{-1}(a+bx)}{b^5}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\frac{1}{5}\int \left(a-\csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)\right)^5d\sec^{-1}(a+bx)+\frac{1}{5}b^5x^5\sec^{-1}(a+bx)}{b^5}$$

$$\Big\downarrow \text{4269}$$

$$\frac{\frac{1}{5}\left(\frac{1}{4}\int b^2x^2\left(4a^3+11(a+bx)^2a-3\left(4a^2+1\right)(a+bx)\right)d\sec^{-1}(a+bx)-\frac{1}{4}b^3x^3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\right)+\frac{1}{5}b^5x^5\text{se}}{b^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{5}\left(\frac{1}{4}\int\left(a-\csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)\right)^2\left(4a^3+11\csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)^2a-3\left(4a^2+1\right)\csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)\right.}{b^5}$$

$$\Big\downarrow \text{4544}$$

$$\frac{\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\int-bx\left(12a^4-\left(48a^2+31\right)(a+bx)a+\left(58a^2+9\right)(a+bx)^2\right)d\sec^{-1}(a+bx)+\frac{11}{3}ab^2x^2(a+bx)\sqrt{1-\frac{1}{(a+}}\right.\right.}{b^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\int\left(a-\csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)\right)\left(12a^4-\left(48a^2+31\right)\csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)a+\left(58a^2+9\right)\csc\left(\sec^{-1}(a\right.\right.\right.\right.$$

$$\Big\downarrow \text{4536}$$

$$\frac{\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int\left(24a^5+4\left(53a^2+20\right)(a+bx)^2a-3\left(40a^4+40a^2+3\right)(a+bx)\right)d\sec^{-1}(a+bx)-\frac{1}{2}\left(58a^2+9\right)(a+\right.\right.\right.}{b^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(24a^5\sec^{-1}(a+bx)+4\left(53a^2+20\right)a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}-3\left(40a^4+40a^2+3\right)\text{arctanh}\left(\sqrt{1-\frac{1}{(a+bx}}\right.\right.\right.\right.\right.$$

input
```
Int[x^4*ArcSec[a + b*x],x]
```

output
```
((b^5*x^5*ArcSec[a + b*x])/5 + (-1/4*(b^3*x^3*(a + b*x)*Sqrt[1 - (a + b*x)
^(-2)]) + ((11*a*b^2*x^2*(a + b*x)*Sqrt[1 - (a + b*x)^(-2)])/3 + (-1/2*((9
 + 58*a^2)*(a + b*x)^2*Sqrt[1 - (a + b*x)^(-2)]) + (4*a*(20 + 53*a^2)*(a +
 b*x)*Sqrt[1 - (a + b*x)^(-2)] + 24*a^5*ArcSec[a + b*x] - 3*(3 + 40*a^2 +
40*a^4)*ArcTanh[Sqrt[1 - (a + b*x)^(-2)]])/2)/3)/4)/5)/b^5
```

3.18. $\int x^4\sec^{-1}(a+bx)\,dx$

3.18.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4269
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Simp[1/(n - 1)
   Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*
a^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

rule 4536
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(-b)*C*Csc[e +
 f*x]*(Cot[e + f*x]/(2*f)), x] + Simp[1/2   Int[Simp[2*A*a + (2*B*a + b*(2*
A + C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a
, b, e, f, A, B, C}, x]
```

rule 4544
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> Simp[(-C)*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[(
a + b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m
)*Csc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

rule 4926
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(e + f*
x)^m*((a + b*Sec[c + d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n +
 1)))   Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5781
```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d^(m + 1)   Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d
*e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.18.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.67

method	result
derivativedivides	$\frac{-\frac{\operatorname{arcsec}(bx+a)a^5}{5}+\operatorname{arcsec}(bx+a)a^4(bx+a)-2\operatorname{arcsec}(bx+a)a^3(bx+a)^2+2\operatorname{arcsec}(bx+a)a^2(bx+a)^3-\operatorname{arcsec}(bx+a)a(bx+}{}$
default	$\frac{-\frac{\operatorname{arcsec}(bx+a)a^5}{5}+\operatorname{arcsec}(bx+a)a^4(bx+a)-2\operatorname{arcsec}(bx+a)a^3(bx+a)^2+2\operatorname{arcsec}(bx+a)a^2(bx+a)^3-\operatorname{arcsec}(bx+a)a(bx+}{}$
parts	$\frac{x^5\operatorname{arcsec}(bx+a)}{5}+\frac{\sqrt{b^2x^2+2abx+a^2-1}\left(-6x^3\sqrt{b^2x^2+2abx+a^2-1}\,b^3\sqrt{b^2}+22\sqrt{b^2x^2+2abx+a^2-1}\,\sqrt{b^2}\,ab^2x^2-24a^5\right.}{}$

input
```
int(x^4*arcsec(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^5*(-1/5*arcsec(b*x+a)*a^5+arcsec(b*x+a)*a^4*(b*x+a)-2*arcsec(b*x+a)*a^
3*(b*x+a)^2+2*arcsec(b*x+a)*a^2*(b*x+a)^3-arcsec(b*x+a)*a*(b*x+a)^4+1/5*ar
csec(b*x+a)*(b*x+a)^5+1/120*((b*x+a)^2-1)^(1/2)*(-24*a^5*arctan(1/((b*x+a)
^2-1)^(1/2))-120*a^4*ln(b*x+a+((b*x+a)^2-1)^(1/2))+240*a^3*((b*x+a)^2-1)^(
1/2)-120*a^2*(b*x+a)*((b*x+a)^2-1)^(1/2)+40*a*(b*x+a)^2*((b*x+a)^2-1)^(1/2
)-6*(b*x+a)^3*((b*x+a)^2-1)^(1/2)-120*a^2*ln(b*x+a+((b*x+a)^2-1)^(1/2))+80
*a*((b*x+a)^2-1)^(1/2)-9*(b*x+a)*((b*x+a)^2-1)^(1/2)-9*ln(b*x+a+((b*x+a)^2
-1)^(1/2)))/(((b*x+a)^2-1)/(b*x+a)^2)^(1/2)/(b*x+a))
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.77

$$\int x^4\sec^{-1}(a+bx)\,dx$$
$$=\frac{24\,b^5x^5\operatorname{arcsec}(bx+a)+48\,a^5\arctan\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2-1}\right)+3\,(40\,a^4+40\,a^2+3)\log\left(-\right.}{}$$

input
```
integrate(x^4*arcsec(b*x+a),x, algorithm="fricas")
```

output
```
1/120*(24*b^5*x^5*arcsec(b*x + a) + 48*a^5*arctan(-b*x - a + sqrt(b^2*x^2
+ 2*a*b*x + a^2 - 1)) + 3*(40*a^4 + 40*a^2 + 3)*log(-b*x - a + sqrt(b^2*x^
2 + 2*a*b*x + a^2 - 1)) - (6*b^3*x^3 - 22*a*b^2*x^2 - 154*a^3 + (58*a^2 +
9)*b*x - 71*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))/b^5
```

3.18.6 Sympy [F]

$$\int x^4 \sec^{-1}(a+bx)\,dx = \int x^4 \operatorname{asec}(a+bx)\,dx$$

input
```
integrate(x**4*asec(b*x+a),x)
```

output
```
Integral(x**4*asec(a + b*x), x)
```

3.18.7 Maxima [F]

$$\int x^4 \sec^{-1}(a+bx)\,dx = \int x^4 \operatorname{arcsec}(bx+a)\,dx$$

input
```
integrate(x^4*arcsec(b*x+a),x, algorithm="maxima")
```

output
```
1/5*x^5*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) - integrate(1/5*(b^2*x
^6 + a*b*x^5)*e^(1/2*log(b*x + a + 1) + 1/2*log(b*x + a - 1))/(b^2*x^2 + 2
*a*b*x + a^2 + (b^2*x^2 + 2*a*b*x + a^2 - 1)*e^(log(b*x + a + 1) + log(b*x
 + a - 1)) - 1), x)
```

3.18.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 409 vs. $2(173) = 346$.

Time = 0.31 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.08

$$\int x^4 \sec^{-1}(a+bx)\, dx =$$

$$-\frac{1}{960}b\left(\frac{192\,(bx+a)^5\left(\frac{5\,a}{bx+a}-\frac{10\,a^2}{(bx+a)^2}+\frac{10\,a^3}{(bx+a)^3}-\frac{5\,a^4}{(bx+a)^4}-1\right)\arccos\left(-\frac{1}{(bx+a)\left(\frac{a}{bx+a}-1\right)-a}\right)}{b^6}-\frac{3\,(bx+a)}{}\right.$$

input
```
integrate(x^4*arcsec(b*x+a),x, algorithm="giac")
```

output
```
-1/960*b*(192*(b*x + a)^5*(5*a/(b*x + a) - 10*a^2/(b*x + a)^2 + 10*a^3/(b*
x + a)^3 - 5*a^4/(b*x + a)^4 - 1)*arccos(-1/((b*x + a)*(a/(b*x + a) - 1) -
 a))/b^6 - (3*(b*x + a)^4*(sqrt(-1/(b*x + a)^2 + 1) - 1)^4 + 40*(b*x + a)^
3*a*(sqrt(-1/(b*x + a)^2 + 1) - 1)^3 + 240*(b*x + a)^2*a^2*(sqrt(-1/(b*x +
 a)^2 + 1) - 1)^2 + 960*(b*x + a)*a^3*(sqrt(-1/(b*x + a)^2 + 1) - 1) + 24*
(b*x + a)^2*(sqrt(-1/(b*x + a)^2 + 1) - 1)^2 + 360*(b*x + a)*a*(sqrt(-1/(b
*x + a)^2 + 1) - 1) + 24*(40*a^4 + 40*a^2 + 3)*log(-(sqrt(-1/(b*x + a)^2 +
 1) - 1)*abs(b*x + a)) - (120*(8*a^3 + 3*a)*(b*x + a)^3*(sqrt(-1/(b*x + a)
^2 + 1) - 1)^3 + 24*(10*a^2 + 1)*(b*x + a)^2*(sqrt(-1/(b*x + a)^2 + 1) - 1
)^2 + 40*(b*x + a)*a*(sqrt(-1/(b*x + a)^2 + 1) - 1) + 3)/((b*x + a)^4*(sqr
t(-1/(b*x + a)^2 + 1) - 1)^4))/b^6)
```

3.18.9 Mupad [F(-1)]

Timed out.

$$\int x^4 \sec^{-1}(a+bx)\, dx = \int x^4\, \mathrm{acos}\left(\frac{1}{a+b\,x}\right)\, dx$$

input
```
int(x^4*acos(1/(a + b*x)),x)
```

output
```
int(x^4*acos(1/(a + b*x)), x)
```

3.19 $\int x^3 \sec^{-1}(a+bx)\,dx$

3.19.1 Optimal result

Integrand size = 10, antiderivative size = 155

$$\int x^3 \sec^{-1}(a+bx)\,dx = -\frac{(2+17a^2)(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12b^4} - \frac{x^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12b^2} + \frac{a(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}}{3b^4} - \frac{a^4\sec^{-1}(a+bx)}{4b^4} + \frac{1}{4}x^4\sec^{-1}(a+bx) + \frac{a(1+2a^2)\operatorname{arctanh}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{2b^4}$$

output
```
-1/4*a^4*arcsec(b*x+a)/b^4+1/4*x^4*arcsec(b*x+a)+1/2*a*(2*a^2+1)*arctanh((
1-1/(b*x+a)^2)^(1/2))/b^4-1/12*(17*a^2+2)*(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/b^
4-1/12*x^2*(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/b^2+1/3*a*(b*x+a)^2*(1-1/(b*x+a)^
2)^(1/2)/b^4
```

3.19.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.97

$$\int x^3 \sec^{-1}(a+bx)\,dx = \frac{-\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}(2a+13a^3+2bx+9a^2bx-3ab^2x^2+b^3x^3)+3b^4x^4\sec^{-1}(a+bx)+3a^4\arcsin\left(\frac{1}{a+bx}\right)}{12b^4}$$

input
```
Integrate[x^3*ArcSec[a + b*x],x]
```

output
```
(-(Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2]*(2*a + 13*a^3 + 2*b*x
+ 9*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3)) + 3*b^4*x^4*ArcSec[a + b*x] + 3*a^4*
ArcSin[(a + b*x)^(-1)] + 6*a*(1 + 2*a^2)*Log[(a + b*x)*(1 + Sqrt[(-1 + a^2
 + 2*a*b*x + b^2*x^2)/(a + b*x)^2])])/(12*b^4)
```

3.19.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {5781, 25, 4926, 3042, 4269, 3042, 4536, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sec^{-1}(a+bx)\,dx$$

$$\Big\downarrow \ 5781$$

$$\frac{\int b^3x^3(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)d\sec^{-1}(a+bx)}{b^4}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int -b^3x^3(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)d\sec^{-1}(a+bx)}{b^4}$$

$$\Big\downarrow \ 4926$$

$$\frac{\frac{1}{4}b^4x^4\sec^{-1}(a+bx)-\frac{1}{4}\int b^4x^4d\sec^{-1}(a+bx)}{b^4}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{1}{4}b^4x^4\sec^{-1}(a+bx)-\frac{1}{4}\int \left(a-\csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)\right)^4d\sec^{-1}(a+bx)}{b^4}$$

$$\Big\downarrow \ 4269$$

$$\frac{\frac{1}{4}\left(-\frac{1}{3}\int -bx\left(3a^3+8(a+bx)^2a-\left(9a^2+2\right)(a+bx)\right)d\sec^{-1}(a+bx)-\frac{1}{3}b^2x^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\right)+\frac{1}{4}b^4x^4\sec}{b^4}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{1}{4}\left(-\frac{1}{3}\int\left(a-\csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)\right)\left(3a^3+8\csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)^2a+\left(-9a^2-2\right)\csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)\right.}{b^4}$$

$\downarrow$ 4536

$$\frac{\frac{1}{4}\left(\frac{1}{3}\left(4a(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}-\frac{1}{2}\int\left(6a^4-12\left(2a^2+1\right)(a+bx)a+2\left(17a^2+2\right)(a+bx)^2\right)d\sec^{-1}(a+bx)\right)-\frac{1}{3}\right.}{b^4}$$

$\downarrow$ 2009

$$\frac{\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(-6a^4\sec^{-1}(a+bx)+12\left(2a^2+1\right)a\text{arctanh}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)-2\left(17a^2+2\right)(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\right)+4a\sqrt{\right.\right.}{b^4}$$

input
```
Int[x^3*ArcSec[a + b*x],x]
```

output
```
((b^4*x^4*ArcSec[a + b*x])/4 + (-1/3*(b^2*x^2*(a + b*x)*Sqrt[1 - (a + b*x)
^(-2)]) + (4*a*(a + b*x)^2*Sqrt[1 - (a + b*x)^(-2)] + (-2*(2 + 17*a^2)*(a
+ b*x)*Sqrt[1 - (a + b*x)^(-2)] - 6*a^4*ArcSec[a + b*x] + 12*a*(1 + 2*a^2)
*ArcTanh[Sqrt[1 - (a + b*x)^(-2)]])/2)/3)/4)/b^4
```

3.19.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4269
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Simp[1/(n - 1)
   Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*
a^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

rule 4536
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(-b)*C*Csc[e +
 f*x]*(Cot[e + f*x]/(2*f)), x] + Simp[1/2   Int[Simp[2*A*a + (2*B*a + b*(2*
A + C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a
, b, e, f, A, B, C}, x]
```

rule 4926
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(e + f*
x)^m*((a + b*Sec[c + d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n +
 1)))   Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5781
```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d^(m + 1)   Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d
*e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.19.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.61

method	result
derivativedivides	$\frac{\frac{\operatorname{arcsec}(bx+a)a^4}{4}-\operatorname{arcsec}(bx+a)a^3(bx+a)+\frac{3\operatorname{arcsec}(bx+a)a^2(bx+a)^2}{2}-\operatorname{arcsec}(bx+a)a(bx+a)^3+\frac{\operatorname{arcsec}(bx+a)(bx+a)^4}{4}+\frac{\sqrt{(bx\cdots}}{\cdots}}{\cdots}$
default	$\frac{\frac{\operatorname{arcsec}(bx+a)a^4}{4}-\operatorname{arcsec}(bx+a)a^3(bx+a)+\frac{3\operatorname{arcsec}(bx+a)a^2(bx+a)^2}{2}-\operatorname{arcsec}(bx+a)a(bx+a)^3+\frac{\operatorname{arcsec}(bx+a)(bx+a)^4}{4}+\frac{\sqrt{(bx\cdots}}{\cdots}}{\cdots}$
parts	$\frac{x^4\operatorname{arcsec}(bx+a)}{4}+\frac{\sqrt{b^2x^2+2abx+a^2-1}\left(3a^4\arctan\left(\frac{1}{\sqrt{b^2x^2+2abx+a^2-1}}\right)\sqrt{b^2}-x^2\sqrt{b^2x^2+2abx+a^2-1}\,b^2\sqrt{b^2}+12\ln\cdots\right.}{\cdots}$

input
```
int(x^3*arcsec(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^4*(1/4*arcsec(b*x+a)*a^4-arcsec(b*x+a)*a^3*(b*x+a)+3/2*arcsec(b*x+a)*a
^2*(b*x+a)^2-arcsec(b*x+a)*a*(b*x+a)^3+1/4*arcsec(b*x+a)*(b*x+a)^4+1/12*((
b*x+a)^2-1)^(1/2)*(3*a^4*arctan(1/((b*x+a)^2-1)^(1/2))+12*a^3*ln(b*x+a+((b
*x+a)^2-1)^(1/2))-18*a^2*((b*x+a)^2-1)^(1/2)+6*a*(b*x+a)*((b*x+a)^2-1)^(1/
2)-(b*x+a)^2*((b*x+a)^2-1)^(1/2)+6*a*ln(b*x+a+((b*x+a)^2-1)^(1/2))-2*((b*x
+a)^2-1)^(1/2))/(((b*x+a)^2-1)/(b*x+a)^2)^(1/2)/(b*x+a))
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.84

$$\int x^3 \sec^{-1}(a+bx)\,dx$$
$$= \frac{3\,b^4x^4\,\mathrm{arcsec}\,(bx+a) - 6\,a^4\arctan\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2-1}\right) - 6\,(2\,a^3+a)\log\left(-bx-a+\sqrt{}\right.}{12\,b^4}$$

input
```
integrate(x^3*arcsec(b*x+a),x, algorithm="fricas")
```

output
```
1/12*(3*b^4*x^4*arcsec(b*x + a) - 6*a^4*arctan(-b*x - a + sqrt(b^2*x^2 + 2
*a*b*x + a^2 - 1)) - 6*(2*a^3 + a)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x +
 a^2 - 1)) - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*(b^2*x^2 - 4*a*b*x + 13*a^2
 + 2))/b^4
```

3.19.6 Sympy [F]

$$\int x^3 \sec^{-1}(a+bx)\,dx = \int x^3\,\mathrm{asec}\,(a+bx)\,dx$$

input
```
integrate(x**3*asec(b*x+a),x)
```

output
```
Integral(x**3*asec(a + b*x), x)
```

3.19.7 Maxima [F]

$$\int x^3 \sec^{-1}(a+bx)\,dx = \int x^3 \operatorname{arcsec}(bx+a)\ dx$$

input
```
integrate(x^3*arcsec(b*x+a),x, algorithm="maxima")
```

output
```
1/4*x^4*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) - integrate(1/4*(b^2*x
^5 + a*b*x^4)*e^(1/2*log(b*x + a + 1) + 1/2*log(b*x + a - 1))/(b^2*x^2 + 2
*a*b*x + a^2 + (b^2*x^2 + 2*a*b*x + a^2 - 1)*e^(log(b*x + a + 1) + log(b*x
 + a - 1)) - 1), x)
```

3.19.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(135) = 270$.

Time = 0.31 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.93

$$\int x^3 \sec^{-1}(a+bx)\,dx =$$

$$-\frac{1}{96}b\left(\frac{24\,(bx+a)^4\left(\frac{4\,a}{bx+a}-\frac{6\,a^2}{(bx+a)^2}+\frac{4\,a^3}{(bx+a)^3}-1\right)\arccos\left(-\frac{1}{(bx+a)\left(\frac{a}{bx+a}-1\right)-a}\right)}{b^5}+\frac{(bx+a)^3\left(\sqrt{-\frac{1}{(bx+a)^2}\right.}{}\right.$$

input
```
integrate(x^3*arcsec(b*x+a),x, algorithm="giac")
```

output
```
-1/96*b*(24*(b*x + a)^4*(4*a/(b*x + a) - 6*a^2/(b*x + a)^2 + 4*a^3/(b*x +
a)^3 - 1)*arccos(-1/((b*x + a)*(a/(b*x + a) - 1) - a))/b^5 + ((b*x + a)^3*
(sqrt(-1/(b*x + a)^2 + 1) - 1)^3 + 12*(b*x + a)^2*a*(sqrt(-1/(b*x + a)^2 +
 1) - 1)^2 + 72*(b*x + a)*a^2*(sqrt(-1/(b*x + a)^2 + 1) - 1) + 9*(b*x + a)
*(sqrt(-1/(b*x + a)^2 + 1) - 1) + 48*(2*a^3 + a)*log(-(sqrt(-1/(b*x + a)^2
 + 1) - 1)*abs(b*x + a)) - (9*(8*a^2 + 1)*(b*x + a)^2*(sqrt(-1/(b*x + a)^2
 + 1) - 1)^2 + 12*(b*x + a)*a*(sqrt(-1/(b*x + a)^2 + 1) - 1) + 1)/((b*x +
a)^3*(sqrt(-1/(b*x + a)^2 + 1) - 1)^3))/b^5)
```

3.19.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \sec^{-1}(a+bx)\, dx = \int x^3 \operatorname{acos}\left(\frac{1}{a+b\,x}\right)\, dx$$

input
```
int(x^3*acos(1/(a + b*x)),x)
```

output
```
int(x^3*acos(1/(a + b*x)), x)
```

3.20 $\int x^2 \sec^{-1}(a+bx)\,dx$

3.20.1 Optimal result

Integrand size = 10, antiderivative size = 116

$$\int x^2 \sec^{-1}(a+bx)\,dx = \frac{5a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6b^3} - \frac{x(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6b^2} + \frac{a^3\sec^{-1}(a+bx)}{3b^3} + \frac{1}{3}x^3\sec^{-1}(a+bx) - \frac{(1+6a^2)\operatorname{arctanh}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{6b^3}$$

output
```
1/3*a^3*arcsec(b*x+a)/b^3+1/3*x^3*arcsec(b*x+a)-1/6*(6*a^2+1)*arctanh((1-1
/(b*x+a)^2)^(1/2))/b^3+5/6*a*(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/b^3-1/6*x*(b*x+
a)*(1-1/(b*x+a)^2)^(1/2)/b^2
```

3.20.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.13

$$\int x^2 \sec^{-1}(a+bx)\,dx = \frac{(5a^2+4abx-b^2x^2)\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}} + 2b^3x^3\sec^{-1}(a+bx) - 2a^3\arcsin\left(\frac{1}{a+bx}\right) - (1+6a^2)\log\left((a+bx\right.}{6b^3}$$

input
```
Integrate[x^2*ArcSec[a + b*x],x]
```

output
```
((5*a^2 + 4*a*b*x - b^2*x^2)*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)
^2] + 2*b^3*x^3*ArcSec[a + b*x] - 2*a^3*ArcSin[(a + b*x)^(-1)] - (1 + 6*a^
2)*Log[(a + b*x)*(1 + Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2])])/
(6*b^3)
```

3.20.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5781, 4926, 3042, 4269, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sec^{-1}(a+bx)\,dx$$

$$\Big\downarrow \text{ 5781}$$

$$\frac{\int b^2x^2(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)d\sec^{-1}(a+bx)}{b^3}$$

$$\Big\downarrow \text{ 4926}$$

$$\frac{\frac{1}{3}\int -b^3x^3d\sec^{-1}(a+bx)+\frac{1}{3}b^3x^3\sec^{-1}(a+bx)}{b^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{1}{3}\int \left(a-\csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)\right)^3d\sec^{-1}(a+bx)+\frac{1}{3}b^3x^3\sec^{-1}(a+bx)}{b^3}$$

$$\Big\downarrow \text{ 4269}$$

$$\frac{\frac{1}{3}\left(\frac{1}{2}\int \left(2a^3+5(a+bx)^2a-\left(6a^2+1\right)(a+bx)\right)d\sec^{-1}(a+bx)-\frac{1}{2}bx(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\right)+\frac{1}{3}b^3x^3\sec^{-1}(a+}{b^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{3}\left(\frac{1}{2}\left(2a^3\sec^{-1}(a+bx)-\left(6a^2+1\right)\text{arctanh}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)+5a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\right)-\frac{1}{2}bx(a+bx)\sqrt{1-\frac{1}{(a+}}\right.}{b^3}$$

input
```
Int[x^2*ArcSec[a + b*x],x]
```

output
```
((b^3*x^3*ArcSec[a + b*x])/3 + (-1/2*(b*x*(a + b*x)*Sqrt[1 - (a + b*x)^(-2
)]) + (5*a*(a + b*x)*Sqrt[1 - (a + b*x)^(-2)] + 2*a^3*ArcSec[a + b*x] - (1
 + 6*a^2)*ArcTanh[Sqrt[1 - (a + b*x)^(-2)]])/2)/3)/b^3
```

3.20.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4269
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Simp[1/(n - 1)
   Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*
a^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

rule 4926
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(e + f*
x)^m*((a + b*Sec[c + d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n +
 1)))   Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5781
```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d^(m + 1)   Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d
*e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.20.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.64

method	result
derivativedivides	$\frac{-\frac{\operatorname{arcsec}(bx+a)a^3}{3}+\operatorname{arcsec}(bx+a)a^2(bx+a)-\operatorname{arcsec}(bx+a)a(bx+a)^2+\frac{\operatorname{arcsec}(bx+a)(bx+a)^3}{3}-\frac{\sqrt{(bx+a)^2-1}\left(2a^3\arctan\left(\frac{}{\sqrt{(b}}\right.\right.}{}}{b^3}$
default	$\frac{-\frac{\operatorname{arcsec}(bx+a)a^3}{3}+\operatorname{arcsec}(bx+a)a^2(bx+a)-\operatorname{arcsec}(bx+a)a(bx+a)^2+\frac{\operatorname{arcsec}(bx+a)(bx+a)^3}{3}-\frac{\sqrt{(bx+a)^2-1}\left(2a^3\arctan\left(\frac{}{\sqrt{(b}}\right.\right.}{}}{b^3}$
parts	$\frac{x^3\operatorname{arcsec}(bx+a)}{3}-\frac{\sqrt{b^2x^2+2abx+a^2-1}\left(2a^3\arctan\left(\frac{1}{\sqrt{b^2x^2+2abx+a^2-1}}\right)\sqrt{b^2}+6\ln\left(\frac{b^2x+\sqrt{b^2x^2+2abx+a^2-1}\sqrt{b^2}+ab}{\sqrt{b^2}}\right.\right.}{6b^3\sqrt{\frac{b^2x^2+2}{(bx}}}$

input
```
int(x^2*arcsec(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(-1/3*arcsec(b*x+a)*a^3+arcsec(b*x+a)*a^2*(b*x+a)-arcsec(b*x+a)*a*(b
*x+a)^2+1/3*arcsec(b*x+a)*(b*x+a)^3-1/6*((b*x+a)^2-1)^(1/2)*(2*a^3*arctan(
1/((b*x+a)^2-1)^(1/2))+6*a^2*ln(b*x+a+((b*x+a)^2-1)^(1/2))-6*a*((b*x+a)^2-
1)^(1/2)+(b*x+a)*((b*x+a)^2-1)^(1/2)+ln(b*x+a+((b*x+a)^2-1)^(1/2)))/(((b*x
+a)^2-1)/(b*x+a)^2)^(1/2)/(b*x+a))
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.01

$$\int x^2\sec^{-1}(a+bx)\,dx$$
$$=\frac{2\,b^3x^3\operatorname{arcsec}\left(bx+a\right)+4\,a^3\arctan\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2-1}\right)+\left(6\,a^2+1\right)\log\left(-bx-a+\sqrt{b^2}\right.}{6\,b^3}$$

input
```
integrate(x^2*arcsec(b*x+a),x, algorithm="fricas")
```

output
```
1/6*(2*b^3*x^3*arcsec(b*x + a) + 4*a^3*arctan(-b*x - a + sqrt(b^2*x^2 + 2*
a*b*x + a^2 - 1)) + (6*a^2 + 1)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^
2 - 1)) - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*(b*x - 5*a))/b^3
```

3.20.6 Sympy [F]

$$\int x^2 \sec^{-1}(a+bx)\,dx = \int x^2 \operatorname{asec}(a+bx)\,dx$$

input
```
integrate(x**2*asec(b*x+a),x)
```

output
```
Integral(x**2*asec(a + b*x), x)
```

3.20.7 Maxima [F]

$$\int x^2 \sec^{-1}(a+bx)\,dx = \int x^2 \operatorname{arcsec}(bx+a)\ dx$$

input
```
integrate(x^2*arcsec(b*x+a),x, algorithm="maxima")
```

output
```
1/3*x^3*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) - integrate(1/3*(b^2*x
^4 + a*b*x^3)*e^(1/2*log(b*x + a + 1) + 1/2*log(b*x + a - 1))/(b^2*x^2 + 2
*a*b*x + a^2 + (b^2*x^2 + 2*a*b*x + a^2 - 1)*e^(log(b*x + a + 1) + log(b*x
 + a - 1)) - 1), x)
```

3.20.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(100) = 200.

Time = 0.31 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.76

$$\int x^2 \sec^{-1}(a+bx)\,dx =$$

$$-\frac{1}{24}b\left(\frac{8\,(bx+a)^3\left(\frac{3\,a}{bx+a}-\frac{3\,a^2}{(bx+a)^2}-1\right)\arccos\left(-\frac{1}{(bx+a)\left(\frac{a}{bx+a}-1\right)-a}\right)}{b^4}-\frac{(bx+a)^2\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)^2}{\cdots}\right.$$

input
```
integrate(x^2*arcsec(b*x+a),x, algorithm="giac")
```

output
```
-1/24*b*(8*(b*x + a)^3*(3*a/(b*x + a) - 3*a^2/(b*x + a)^2 - 1)*arccos(-1/(
(b*x + a)*(a/(b*x + a) - 1) - a))/b^4 - ((b*x + a)^2*(sqrt(-1/(b*x + a)^2
+ 1) - 1)^2 + 12*(b*x + a)*a*(sqrt(-1/(b*x + a)^2 + 1) - 1) + 4*(6*a^2 + 1
)*log(-(sqrt(-1/(b*x + a)^2 + 1) - 1)*abs(b*x + a)) - (12*(b*x + a)*a*(sqr
t(-1/(b*x + a)^2 + 1) - 1) + 1)/((b*x + a)^2*(sqrt(-1/(b*x + a)^2 + 1) - 1
)^2))/b^4)
```

3.20.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \sec^{-1}(a+bx)\,dx = \int x^2 \operatorname{acos}\left(\frac{1}{a+b\,x}\right)\,dx$$

input
```
int(x^2*acos(1/(a + b*x)),x)
```

output
```
int(x^2*acos(1/(a + b*x)), x)
```

3.21 $\int x \sec^{-1}(a+bx)\,dx$

3.21.1 Optimal result

Integrand size = 8, antiderivative size = 78

$$\int x \sec^{-1}(a+bx)\,dx = -\frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2b^2} - \frac{a^2\sec^{-1}(a+bx)}{2b^2} + \frac{1}{2}x^2\sec^{-1}(a+bx) + \frac{a\operatorname{arctanh}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{b^2}$$

output
```
-1/2*a^2*arcsec(b*x+a)/b^2+1/2*x^2*arcsec(b*x+a)+a*arctanh((1-1/(b*x+a)^2)
^(1/2))/b^2-1/2*(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/b^2
```

3.21.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.41

$$\int x \sec^{-1}(a+bx)\,dx = \frac{-\left((a+bx)\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}\right) + b^2x^2\sec^{-1}(a+bx) + a^2\arcsin\left(\frac{1}{a+bx}\right) + 2a\log\left((a+bx)\left(1+\sqrt{\frac{-1+a^2+2\cdots}{(a+\cdots}}\right.\right.}{2b^2}$$

input
```
Integrate[x*ArcSec[a + b*x],x]
```

output
```
(-((a + b*x)*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2]) + b^2*x^2*A
rcSec[a + b*x] + a^2*ArcSin[(a + b*x)^(-1)] + 2*a*Log[(a + b*x)*(1 + Sqrt[
(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2])])/(2*b^2)
```

3.21.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {5781, 25, 4926, 3042, 4260, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sec^{-1}(a+bx)\,dx$$

$$\Big\downarrow \text{5781}$$

$$\frac{\int bx(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)d\sec^{-1}(a+bx)}{b^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -bx(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)d\sec^{-1}(a+bx)}{b^2}$$

$$\Big\downarrow \text{4926}$$

$$\frac{\frac{1}{2}b^2x^2\sec^{-1}(a+bx)-\frac{1}{2}\int b^2x^2d\sec^{-1}(a+bx)}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{2}b^2x^2\sec^{-1}(a+bx)-\frac{1}{2}\int\left(a-\csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)\right)^2d\sec^{-1}(a+bx)}{b^2}$$

$$\Big\downarrow \text{4260}$$

$$\frac{\frac{1}{2}\left(2a\int(a+bx)d\sec^{-1}(a+bx)-\int(a+bx)^2d\sec^{-1}(a+bx)+a^2\left(-\sec^{-1}(a+bx)\right)\right)+\frac{1}{2}b^2x^2\sec^{-1}(a+bx)}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{2}\left(2a\int\csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)d\sec^{-1}(a+bx)-\int\csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)^2d\sec^{-1}(a+bx)+a^2\left(-\sec^{-1}(a+bx\right.\right.}{b^2}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\frac{1}{2}\left(\int 1d\left(-\left((a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\right)\right)+2a\int\csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)d\sec^{-1}(a+bx)+a^2\left(-\sec^{-1}(a+bx)\right)\right)+\frac{1}{2}b}{b^2}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{1}{2}\left(2a\int \csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)d\sec^{-1}(a+bx)+a^2\left(-\sec^{-1}(a+bx)\right)-(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\right)+\frac{1}{2}b^2x^2\sec^{-1}(a}{b^2}$$

$\downarrow$ 4257

$$\frac{\frac{1}{2}\left(a^2\left(-\sec^{-1}(a+bx)\right)+2a\text{arctanh}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)-(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\right)+\frac{1}{2}b^2x^2\sec^{-1}(a+bx)}{b^2}$$

input
```
Int[x*ArcSec[a + b*x],x]
```

output
```
((b^2*x^2*ArcSec[a + b*x])/2 + (-((a + b*x)*Sqrt[1 - (a + b*x)^(-2)]) - a^
2*ArcSec[a + b*x] + 2*a*ArcTanh[Sqrt[1 - (a + b*x)^(-2)]])/2)/b^2
```

3.21.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4260
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Simp[2*a*b   Int[Csc[c + d*x], x], x] + Simp[b^2   Int[Csc[c + d*x]^2, x]
, x]) /; FreeQ[{a, b, c, d}, x]
```

rule 4926
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(e + f*
x)^m*((a + b*Sec[c + d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n +
 1)))   Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5781
```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d^(m + 1)   Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d
*e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.21.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.38

method	result
derivativedivides	$\frac{\frac{\operatorname{arcsec}(bx+a)(bx+a)^2}{2}-\operatorname{arcsec}(bx+a)a(bx+a)+\frac{\sqrt{(bx+a)^2-1}\left(2a\ln\left(bx+a+\sqrt{(bx+a)^2-1}\right)-\sqrt{(bx+a)^2-1}\right)}{2(bx+a)\sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}}}{b^2}$
default	$\frac{\frac{\operatorname{arcsec}(bx+a)(bx+a)^2}{2}-\operatorname{arcsec}(bx+a)a(bx+a)+\frac{\sqrt{(bx+a)^2-1}\left(2a\ln\left(bx+a+\sqrt{(bx+a)^2-1}\right)-\sqrt{(bx+a)^2-1}\right)}{2(bx+a)\sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}}}{b^2}$
parts	$\frac{x^2\operatorname{arcsec}(bx+a)}{2}+\frac{\sqrt{b^2x^2+2abx+a^2-1}\left(a^2\arctan\left(\frac{1}{\sqrt{b^2x^2+2abx+a^2-1}}\right)\sqrt{b^2}+2a\ln\left(\frac{b^2x+\sqrt{b^2x^2+2abx+a^2-1}\sqrt{b^2}+ab}{\sqrt{b^2}}\right.\right.}{2b^2\sqrt{\frac{b^2x^2+2abx+a^2-1}{(bx+a)^2}}(bx+a)\sqrt{b^2}}$

input
```
int(x*arcsec(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(1/2*arcsec(b*x+a)*(b*x+a)^2-arcsec(b*x+a)*a*(b*x+a)+1/2*((b*x+a)^2-
1)^(1/2)*(2*a*ln(b*x+a+((b*x+a)^2-1)^(1/2))-((b*x+a)^2-1)^(1/2))/(b*x+a)/(
((b*x+a)^2-1)/(b*x+a)^2)^(1/2))
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.33

$$\int x\sec^{-1}(a+bx)\,dx$$
$$=\frac{b^2x^2\operatorname{arcsec}(bx+a)-2a^2\arctan\left(-bx-a+\sqrt{b^2x^2+2abx+a^2-1}\right)-2a\log\left(-bx-a+\sqrt{b^2x^2+2a\dots\right.}{2b^2}$$

input
```
integrate(x*arcsec(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(b^2*x^2*arcsec(b*x + a) - 2*a^2*arctan(-b*x - a + sqrt(b^2*x^2 + 2*a*
b*x + a^2 - 1)) - 2*a*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)) -
sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))/b^2
```

3.21.6 Sympy [F]

$$\int x\sec^{-1}(a+bx)\,dx=\int x\operatorname{asec}(a+bx)\,dx$$

input
```
integrate(x*asec(b*x+a),x)
```

output
```
Integral(x*asec(a + b*x), x)
```

3.21.7 Maxima [F]

$$\int x\sec^{-1}(a+bx)\,dx=\int x\operatorname{arcsec}(bx+a)\,dx$$

input
```
integrate(x*arcsec(b*x+a),x, algorithm="maxima")
```

output
```
1/2*x^2*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) - integrate(1/2*(b^2*x
^3 + a*b*x^2)*e^(1/2*log(b*x + a + 1) + 1/2*log(b*x + a - 1))/(b^2*x^2 + 2
*a*b*x + a^2 + (b^2*x^2 + 2*a*b*x + a^2 - 1)*e^(log(b*x + a + 1) + log(b*x
 + a - 1)) - 1), x)
```

3.21.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.71

$$\int x\sec^{-1}(a+bx)\,dx =$$

$$-\frac{1}{4}b\left(\frac{2\,(bx+a)^2\left(\frac{2\,a}{bx+a}-1\right)\arccos\left(-\frac{1}{(bx+a)\left(\frac{a}{bx+a}-1\right)-a}\right)}{b^3}+\frac{(bx+a)\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)+4\,a\log\left(-\right.}{}\right.$$

input
```
integrate(x*arcsec(b*x+a),x, algorithm="giac")
```

output
```
-1/4*b*(2*(b*x + a)^2*(2*a/(b*x + a) - 1)*arccos(-1/((b*x + a)*(a/(b*x + a
) - 1) - a))/b^3 + ((b*x + a)*(sqrt(-1/(b*x + a)^2 + 1) - 1) + 4*a*log(-(s
qrt(-1/(b*x + a)^2 + 1) - 1)*abs(b*x + a)) - 1/((b*x + a)*(sqrt(-1/(b*x +
a)^2 + 1) - 1)))/b^3)
```

3.21.9 Mupad [F(-1)]

Timed out.

$$\int x\sec^{-1}(a+bx)\,dx = \int x\,\mathrm{acos}\left(\frac{1}{a+b\,x}\right)\,dx$$

input
```
int(x*acos(1/(a + b*x)),x)
```

output
```
int(x*acos(1/(a + b*x)), x)
```

3.22 $\int \sec^{-1}(a+bx)\,dx$

3.22.1 Optimal result

Integrand size = 6, antiderivative size = 37

$$\int \sec^{-1}(a+bx)\,dx=\frac{(a+bx)\sec^{-1}(a+bx)}{b}-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{b}$$

output
```
(b*x+a)*arcsec(b*x+a)/b-arctanh((1-1/(b*x+a)^2)^(1/2))/b
```

3.22.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.99 (sec) , antiderivative size = 468, normalized size of antiderivative = 12.65

$$\int \sec^{-1}(a+bx)\,dx = x\sec^{-1}(a+bx) + \frac{(a+bx)\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}\left(\sqrt[4]{-1}\left(-i+\sqrt{-1+a^2}\right)\sqrt{2i-ia^2+2\sqrt{-1+a^2}}\arctan\left(\frac{(-1)^{3/4}\sqrt{2i-ia^2+2\sqrt{\cdots}}}{a\sqrt{-1+a^2}-a\sqrt{-1+a^2+\cdots}}\right.\cdots}{\cdots}$$

input
```
Integrate[ArcSec[a + b*x],x]
```

output
```
x*ArcSec[a + b*x] + ((a + b*x)*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*
x)^2]*((-1)^(1/4)*(-I + Sqrt[-1 + a^2])*Sqrt[2*I - I*a^2 + 2*Sqrt[-1 + a^2
]]*ArcTan[((-1)^(3/4)*Sqrt[2*I - I*a^2 + 2*Sqrt[-1 + a^2]]*b*x)/(a*Sqrt[-1
 + a^2] - a*Sqrt[-1 + a^2 + 2*a*b*x + b^2*x^2])] + (-1)^(3/4)*(I + Sqrt[-1
 + a^2])*Sqrt[-2*I + I*a^2 + 2*Sqrt[-1 + a^2]]*ArcTan[((-1)^(1/4)*Sqrt[-2*
I + I*a^2 + 2*Sqrt[-1 + a^2]]*b*x)/(a*Sqrt[-1 + a^2] - a*Sqrt[-1 + a^2 + 2
*a*b*x + b^2*x^2])] + a*(a*ArcTan[(Sqrt[-1 + a^2]*b^2*x^2)/(a^4 + a^3*b*x
+ b^2*x^2 - a^2*(1 + Sqrt[-1 + a^2]*Sqrt[-1 + a^2 + 2*a*b*x + b^2*x^2]))]
- Log[Sqrt[-1 + a^2] - b*x - Sqrt[-1 + a^2 + 2*a*b*x + b^2*x^2]] + Log[b^2
*(Sqrt[-1 + a^2] + b*x - Sqrt[-1 + a^2 + 2*a*b*x + b^2*x^2])])))/(a*b*Sqrt
[-1 + a^2 + 2*a*b*x + b^2*x^2])
```

3.22.3 Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {5773, 895, 798, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{-1}(a+bx)\,dx$$

$$\Big\downarrow \; 5773$$

$$\frac{(a+bx)\sec^{-1}(a+bx)}{b}-\int \frac{1}{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}dx$$

$$\Big\downarrow \; 895$$

$$\frac{(a+bx)\sec^{-1}(a+bx)}{b}-\frac{\int \frac{1}{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}d(a+bx)}{b}$$

$$\Big\downarrow \; 798$$

$$\frac{\int \frac{(a+bx)^2}{\sqrt{-a-bx+1}}d\frac{1}{(a+bx)^2}}{2b}+\frac{(a+bx)\sec^{-1}(a+bx)}{b}$$

$$\Big\downarrow \; 73$$

$$\frac{(a+bx)\sec^{-1}(a+bx)}{b}-\frac{\int \frac{1}{1-\frac{1}{(a+bx)^4}}d\sqrt{-a-bx+1}}{b}$$

$$\Big\downarrow \; 219$$

$$\frac{(a+bx)\sec^{-1}(a+bx)}{b} - \frac{\operatorname{arctanh}\left(\sqrt{-a-bx+1}\right)}{b}$$

input
```
Int[ArcSec[a + b*x],x]
```

output
```
((a + b*x)*ArcSec[a + b*x])/b - ArcTanh[Sqrt[1 - a - b*x]]/b
```

3.22.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 5773
```
Int[ArcSec[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(c + d*x)*(ArcSec[c + d*x]
/d), x] - Int[1/((c + d*x)*Sqrt[1 - 1/(c + d*x)^2]), x] /; FreeQ[{c, d}, x]
```

3.22.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{(bx+a)\operatorname{arcsec}(bx+a)-\ln\left(bx+a+(bx+a)\sqrt{1-\frac{1}{(bx+a)^2}}\right)}{b}$
default	$\frac{(bx+a)\operatorname{arcsec}(bx+a)-\ln\left(bx+a+(bx+a)\sqrt{1-\frac{1}{(bx+a)^2}}\right)}{b}$
parts	$x\operatorname{arcsec}(bx+a)-\frac{\sqrt{b^2x^2+2abx+a^2-1}\left(a\arctan\left(\frac{1}{\sqrt{b^2x^2+2abx+a^2-1}}\right)\sqrt{b^2}+\ln\left(\frac{b^2x+\sqrt{b^2x^2+2abx+a^2-1}\sqrt{b^2}}{\sqrt{b^2}}\right.\right.}{b\sqrt{\frac{b^2x^2+2abx+a^2-1}{(bx+a)^2}}(bx+a)\sqrt{b^2}}$

input
```
int(arcsec(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*((b*x+a)*arcsec(b*x+a)-ln(b*x+a+(b*x+a)*(1-1/(b*x+a)^2)^(1/2)))
```

3.22.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(35) = 70.

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.97

$$\int \sec^{-1}(a+bx)\,dx$$
$$=\frac{bx\operatorname{arcsec}(bx+a)+2a\arctan\left(-bx-a+\sqrt{b^2x^2+2abx+a^2-1}\right)+\log\left(-bx-a+\sqrt{b^2x^2+2abx+a}\right.}{b}$$

input
```
integrate(arcsec(b*x+a),x, algorithm="fricas")
```

output
```
(b*x*arcsec(b*x + a) + 2*a*arctan(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2
- 1)) + log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)))/b
```

3.22.6 Sympy [F]

$$\int \sec^{-1}(a+bx)\,dx = \int \operatorname{asec}(a+bx)\,dx$$

input
```
integrate(asec(b*x+a),x)
```

output
```
Integral(asec(a + b*x), x)
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.49

$$\int \sec^{-1}(a+bx)\,dx = \frac{2\,(bx+a)\operatorname{arcsec}(bx+a) - \log\left(\sqrt{-\frac{1}{(bx+a)^2}+1}+1\right) + \log\left(-\sqrt{-\frac{1}{(bx+a)^2}+1}+1\right)}{2\,b}$$

input
```
integrate(arcsec(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(2*(b*x + a)*arcsec(b*x + a) - log(sqrt(-1/(b*x + a)^2 + 1) + 1) + log
(-sqrt(-1/(b*x + a)^2 + 1) + 1))/b
```

3.22.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(35) = 70.

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.22

$$\int \sec^{-1}(a+bx)\,dx = \frac{1}{2}\,b\left(\frac{2\,(bx+a)\arccos\left(-\frac{1}{(bx+a)\left(\frac{a}{bx+a}-1\right)-a}\right)}{b^2} - \frac{\log\left(\sqrt{-\frac{1}{(bx+a)^2}+1}+1\right) - \log\left(-\sqrt{-\frac{1}{(bx+a)^2}+1}+1\right)}{b^2}\right)$$

input
```
integrate(arcsec(b*x+a),x, algorithm="giac")
```

output
```
1/2*b*(2*(b*x + a)*arccos(-1/((b*x + a)*(a/(b*x + a) - 1) - a))/b^2 - (log
(sqrt(-1/(b*x + a)^2 + 1) + 1) - log(-sqrt(-1/(b*x + a)^2 + 1) + 1))/b^2)
```

3.22.9 Mupad [B] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \sec^{-1}(a+bx)\,dx = -\frac{\operatorname{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{(a+b\,x)^2}}}\right)-\operatorname{acos}\left(\frac{1}{a+b\,x}\right)\,(a+b\,x)}{b}$$

input
```
int(acos(1/(a + b*x)),x)
```

output
```
-(atanh(1/(1 - 1/(a + b*x)^2)^(1/2)) - acos(1/(a + b*x))*(a + b*x))/b
```

3.23 $\int \frac{\sec^{-1}(a+bx)}{x}\,dx$

3.23.1 Optimal result

Integrand size = 10, antiderivative size = 200

$$\begin{aligned}\int \frac{\sec^{-1}(a+bx)}{x}\,dx &= \sec^{-1}(a+bx)\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)\\&\quad+\sec^{-1}(a+bx)\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)\\&\quad-\sec^{-1}(a+bx)\log\left(1+e^{2i\sec^{-1}(a+bx)}\right)-i\,\mathrm{PolyLog}\left(2,\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)\\&\quad-i\,\mathrm{PolyLog}\left(2,\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)+\frac{1}{2}i\,\mathrm{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)\end{aligned}$$

output
```
-arcsec(b*x+a)*ln(1+(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)+arcsec(b*x+a)*l
n(1-a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2)))+arcsec(b*x+a
)*ln(1-a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1+(-a^2+1)^(1/2)))+1/2*I*pol
ylog(2,-(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)-I*polylog(2,a*(1/(b*x+a)+I*
(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2)))-I*polylog(2,a*(1/(b*x+a)+I*(1-1
/(b*x+a)^2)^(1/2))/(1+(-a^2+1)^(1/2)))
```

3.23.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.42

$$\int \frac{\sec^{-1}(a+bx)}{x}\,dx = -4i\arcsin\left(\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right)\arctan\left(\frac{(1+a)\tan\left(\frac{1}{2}\sec^{-1}(a+bx)\right)}{\sqrt{1-a^2}}\right) + \left(\sec^{-1}(a+bx) - 2\arcsin\left(\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right)\right)\log\left(1 + \frac{\left(-1+\sqrt{1-a^2}\right)e^{i\sec^{-1}(a+bx)}}{a}\right) + \left(\sec^{-1}(a+bx) + 2\arcsin\left(\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right)\right)\log\left(1 - \frac{\left(1+\sqrt{1-a^2}\right)e^{i\sec^{-1}(a+bx)}}{a}\right) - \sec^{-1}(a+bx)\log\left(1+e^{2i\sec^{-1}(a+bx)}\right) - i\left(\text{PolyLog}\left(2, -\frac{\left(-1+\sqrt{1-a^2}\right)e^{i\sec^{-1}(a+bx)}}{a}\right) + \text{PolyLog}\left(2, \frac{\left(1+\sqrt{1-a^2}\right)e^{i\sec^{-1}(a+bx)}}{a}\right)\right) + \frac{1}{2}i\,\text{PolyLog}\left(2, -e^{2i\sec^{-1}(a+bx)}\right)$$

input
```
Integrate[ArcSec[a + b*x]/x,x]
```

output
```
(-4*I)*ArcSin[Sqrt[(-1 + a)/a]/Sqrt[2]]*ArcTan[((1 + a)*Tan[ArcSec[a + b*x
]/2])/Sqrt[1 - a^2]] + (ArcSec[a + b*x] - 2*ArcSin[Sqrt[(-1 + a)/a]/Sqrt[2
]])*Log[1 + ((-1 + Sqrt[1 - a^2])*E^(I*ArcSec[a + b*x]))/a] + (ArcSec[a +
b*x] + 2*ArcSin[Sqrt[(-1 + a)/a]/Sqrt[2]])*Log[1 - ((1 + Sqrt[1 - a^2])*E^
(I*ArcSec[a + b*x]))/a] - ArcSec[a + b*x]*Log[1 + E^((2*I)*ArcSec[a + b*x]
)] - I*(PolyLog[2, -(((-1 + Sqrt[1 - a^2])*E^(I*ArcSec[a + b*x]))/a)] + Po
lyLog[2, ((1 + Sqrt[1 - a^2])*E^(I*ArcSec[a + b*x]))/a]) + (I/2)*PolyLog[2
, -E^((2*I)*ArcSec[a + b*x])]
```

3.23.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.32, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.400, Rules used = {5781, 25, 5062, 5041, 25, 3042, 4202, 2620, 2715, 2838, 5031, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{-1}(a+bx)}{x}\,dx$$

$$\Big\downarrow \text{5781}$$

$$\int \frac{(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{bx}d\sec^{-1}(a+bx)$$

$$\Big\downarrow \text{25}$$

$$-\int -\frac{(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{bx}d\sec^{-1}(a+bx)$$

$$\Big\downarrow \text{5062}$$

$$-\int \frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{\frac{a}{a+bx}-1}d\sec^{-1}(a+bx)$$

$$\Big\downarrow \text{5041}$$

$$\int (a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)d\sec^{-1}(a+bx)-a\int -\frac{\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{1-\frac{a}{a+bx}}d\sec^{-1}(a+bx)$$

$$\Big\downarrow \text{25}$$

$$\int (a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)d\sec^{-1}(a+bx)+a\int \frac{\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{1-\frac{a}{a+bx}}d\sec^{-1}(a+bx)$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{1-\frac{a}{a+bx}}d\sec^{-1}(a+bx)+\int \sec^{-1}(a+bx)\tan\left(\sec^{-1}(a+bx)\right)d\sec^{-1}(a+bx)$$

$$\Big\downarrow \text{4202}$$

$$-2i\int \frac{e^{2i\sec^{-1}(a+bx)}\sec^{-1}(a+bx)}{1+e^{2i\sec^{-1}(a+bx)}}d\sec^{-1}(a+bx)+a\int \frac{\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{1-\frac{a}{a+bx}}d\sec^{-1}(a+bx)+\frac{1}{2}i\sec^{-1}(a+bx)^2$$

$$\Big\downarrow \text{2620}$$

$$a\int \frac{\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{1-\frac{a}{a+bx}}d\sec^{-1}(a+bx)-2i\left(\frac{1}{2}i\int \log\left(1+e^{2i\sec^{-1}(a+bx)}\right)d\sec^{-1}(a+bx)-\frac{1}{2}i\sec^{-1}(a+bx)\log\left(1+e^{2i\sec^{-1}(a+bx)}\right)\right)+\frac{1}{2}i\sec^{-1}(a+bx)^2$$

$$\Big\downarrow \text{2715}$$

$$a\int \frac{\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{1-\frac{a}{a+bx}}d\sec^{-1}(a+bx)-2i\left(\frac{1}{4}\int e^{-2i\sec^{-1}(a+bx)}\log\left(1+e^{2i\sec^{-1}(a+bx)}\right)de^{2i\sec^{-1}(a+bx)}-\frac{1}{2}i\sec^{-1}(a+bx)\log\left(1+e^{2i\sec^{-1}(a+bx)}\right)\right)+\frac{1}{2}i\sec^{-1}(a+bx)^2$$

$$\Big\downarrow \text{2838}$$

$$a\int \frac{\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{1-\frac{a}{a+bx}}d\sec^{-1}(a+bx)-2i\left(-\frac{1}{4}\text{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)-\frac{1}{2}i\sec^{-1}(a+bx)\log\left(1+e^{2i\sec^{-1}(a+bx)}\right)\right)+\frac{1}{2}i\sec^{-1}(a+bx)^2$$

$$\Big\downarrow \text{5031}$$

$$a\left(-i\int \frac{e^{i\sec^{-1}(a+bx)}\sec^{-1}(a+bx)}{-e^{i\sec^{-1}(a+bx)}a-\sqrt{1-a^2}+1}d\sec^{-1}(a+bx)-i\int \frac{e^{i\sec^{-1}(a+bx)}\sec^{-1}(a+bx)}{-e^{i\sec^{-1}(a+bx)}a+\sqrt{1-a^2}+1}d\sec^{-1}(a+bx)-\frac{i\,\text{s}}{}\right.$$
$$2i\left(-\frac{1}{4}\text{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)-\frac{1}{2}i\sec^{-1}(a+bx)\log\left(1+e^{2i\sec^{-1}(a+bx)}\right)\right)+\frac{1}{2}i\sec^{-1}(a+bx)^2$$

$$\Big\downarrow \text{2620}$$

3.23. $\int \frac{\sec^{-1}(a+bx)}{x}\,dx$

$$a\left(-i\left(\frac{i\sec^{-1}(a+bx)\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a}-\frac{i\int\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)d\sec^{-1}(a+bx)}{a}\right)-i\left(\frac{i\sec^{-1}(a+bx}{}\right.\right.$$
$$2i\left(-\frac{1}{4}\text{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)-\frac{1}{2}i\sec^{-1}(a+bx)\log\left(1+e^{2i\sec^{-1}(a+bx)}\right)\right)+\frac{1}{2}i\sec^{-1}(a+$$
$$bx)^2$$

$\big\downarrow$ 2715

$$a\left(-i\left(\frac{i\sec^{-1}(a+bx)\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a}-\frac{\int e^{-i\sec^{-1}(a+bx)}\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)de^{i\sec^{-1}(a+bx)}}{a}\right)-i\left(\frac{i\,\text{se}}{}\right.\right.$$
$$2i\left(-\frac{1}{4}\text{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)-\frac{1}{2}i\sec^{-1}(a+bx)\log\left(1+e^{2i\sec^{-1}(a+bx)}\right)\right)+\frac{1}{2}i\sec^{-1}(a+$$
$$bx)^2$$

$\big\downarrow$ 2838

$$a\left(-i\left(\frac{\text{PolyLog}\left(2,\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a}+\frac{i\sec^{-1}(a+bx)\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a}\right)-i\left(\frac{\text{PolyLog}\left(2,\frac{ae^{i\sec^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a}+\right.\right.$$
$$2i\left(-\frac{1}{4}\text{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)-\frac{1}{2}i\sec^{-1}(a+bx)\log\left(1+e^{2i\sec^{-1}(a+bx)}\right)\right)+\frac{1}{2}i\sec^{-1}(a+$$
$$bx)^2$$

input
```
Int[ArcSec[a + b*x]/x,x]
```

output
```
(I/2)*ArcSec[a + b*x]^2 + a*(((-1/2*I)*ArcSec[a + b*x]^2)/a - I*((I*ArcSec
[a + b*x]*Log[1 - (a*E^(I*ArcSec[a + b*x]))/(1 - Sqrt[1 - a^2])])/a + Poly
Log[2, (a*E^(I*ArcSec[a + b*x]))/(1 - Sqrt[1 - a^2])]/a) - I*((I*ArcSec[a
+ b*x]*Log[1 - (a*E^(I*ArcSec[a + b*x]))/(1 + Sqrt[1 - a^2])])/a + PolyLog
[2, (a*E^(I*ArcSec[a + b*x]))/(1 + Sqrt[1 - a^2])]/a)) - (2*I)*((-1/2*I)*A
rcSec[a + b*x]*Log[1 + E^((2*I)*ArcSec[a + b*x])] - PolyLog[2, -E^((2*I)*A
rcSec[a + b*x])]/4)
```

3.23. $\int \frac{\sec^{-1}(a+bx)}{x}\,dx$

3.23.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5031
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[I*((e + f*x)^(m + 1)/(b*f*(m + 1)))
, x] + (-Simp[I   Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] +
b*E^(I*(c + d*x)))), x], x] - Simp[I   Int[(e + f*x)^m*(E^(I*(c + d*x))/(a
+ Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

3.23. $\int \frac{\sec^{-1}(a+bx)}{x}\,dx$

rule 5041
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tan[(c_.) + (d_.)*(x_)]^(n_.))/(Cos[(c_.) +
 (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Tan[c +
 d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*Sin[c + d*x]*(Tan[c + d*x]^(n
- 1)/(a + b*Cos[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[
m, 0] && IGtQ[n, 0]
```

rule 5062
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sec[(c_.) + (d_.)*(x_)]), x_Symbol] :> In
t[(e + f*x)^m*Cos[c + d*x]*F[c + d*x]^n*(G[c + d*x]^p/(b + a*Cos[c + d*x]))
, x] /; FreeQ[{a, b, c, d, e, f}, x] && TrigQ[F] && TrigQ[G] && IntegersQ[m
, n, p]
```

rule 5781
```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d^(m + 1)   Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d
*e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.23.4 Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.87

method	result
derivativedivides	$\operatorname{arcsec}(bx+a)\ln\left(\frac{-a\left(\frac{1}{bx+a}+i\sqrt{1-\frac{1}{(bx+a)^2}}\right)+\sqrt{-a^2+1}+1}{1+\sqrt{-a^2+1}}\right)+\operatorname{arcsec}(bx+a)\ln\left(\frac{a\left(\frac{1}{bx+a}+i\sqrt{1-\right.}{-\right.}\right.$
default	$\operatorname{arcsec}(bx+a)\ln\left(\frac{-a\left(\frac{1}{bx+a}+i\sqrt{1-\frac{1}{(bx+a)^2}}\right)+\sqrt{-a^2+1}+1}{1+\sqrt{-a^2+1}}\right)+\operatorname{arcsec}(bx+a)\ln\left(\frac{a\left(\frac{1}{bx+a}+i\sqrt{1-\right.}{-\right.}\right.$

input
```
int(arcsec(b*x+a)/x,x,method=_RETURNVERBOSE)
```

output
```
arcsec(b*x+a)*ln((-a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))+(-a^2+1)^(1/2)+1)
/(1+(-a^2+1)^(1/2)))+arcsec(b*x+a)*ln((a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2
))+(-a^2+1)^(1/2)-1)/(-1+(-a^2+1)^(1/2)))-arcsec(b*x+a)*ln(1+I*(1/(b*x+a)+
I*(1-1/(b*x+a)^2)^(1/2)))-arcsec(b*x+a)*ln(1-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2
)^(1/2)))+I*dilog(1+I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))+I*dilog(1-I*(1/
(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))-I*dilog((-a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)
^(1/2))+(-a^2+1)^(1/2)+1)/(1+(-a^2+1)^(1/2)))-I*dilog((a*(1/(b*x+a)+I*(1-1
/(b*x+a)^2)^(1/2))+(-a^2+1)^(1/2)-1)/(-1+(-a^2+1)^(1/2)))
```

3.23.5 Fricas [F]

$$\int \frac{\sec^{-1}(a+bx)}{x}\, dx = \int \frac{\operatorname{arcsec}(bx+a)}{x}\, dx$$

input
```
integrate(arcsec(b*x+a)/x,x, algorithm="fricas")
```

output
```
integral(arcsec(b*x + a)/x, x)
```

3.23.6 Sympy [F]

$$\int \frac{\sec^{-1}(a+bx)}{x}\, dx = \int \frac{\operatorname{asec}(a+bx)}{x}\, dx$$

input
```
integrate(asec(b*x+a)/x,x)
```

output
```
Integral(asec(a + b*x)/x, x)
```

3.23.7 Maxima [F]

$$\int \frac{\sec^{-1}(a+bx)}{x}\,dx = \int \frac{\operatorname{arcsec}\left(bx+a\right)}{x}\,dx$$

input
```
integrate(arcsec(b*x+a)/x,x, algorithm="maxima")
```

output
```
integrate(arcsec(b*x + a)/x, x)
```

3.23.8 Giac [F]

$$\int \frac{\sec^{-1}(a+bx)}{x}\,dx = \int \frac{\operatorname{arcsec}\left(bx+a\right)}{x}\,dx$$

input
```
integrate(arcsec(b*x+a)/x,x, algorithm="giac")
```

output
```
integrate(arcsec(b*x + a)/x, x)
```

3.23.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{-1}(a+bx)}{x}\,dx = \int \frac{\operatorname{acos}\left(\frac{1}{a+b\,x}\right)}{x}\,dx$$

input
```
int(acos(1/(a + b*x))/x,x)
```

output
```
int(acos(1/(a + b*x))/x, x)
```

3.24 $\int \frac{\sec^{-1}(a+bx)}{x^2} dx$

3.24.1 Optimal result

Integrand size = 10, antiderivative size = 70

$$\int \frac{\sec^{-1}(a+bx)}{x^2} dx = -\frac{b\sec^{-1}(a+bx)}{a} - \frac{\sec^{-1}(a+bx)}{x} + \frac{2b\arctan\left(\frac{\sqrt{1+a}\tan\left(\frac{1}{2}\sec^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{a\sqrt{1-a^2}}$$

output
```
-b*arcsec(b*x+a)/a-arcsec(b*x+a)/x+2*b*arctan((1+a)^(1/2)*tan(1/2*arcsec(b
*x+a))/(1-a)^(1/2))/a/(-a^2+1)^(1/2)
```

3.24.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.60

$$\int \frac{\sec^{-1}(a+bx)}{x^2} dx = -\frac{\sec^{-1}(a+bx)}{x} + \frac{b\left(\arcsin\left(\frac{1}{a+bx}\right) - \frac{i\log\left(\frac{2\left(\frac{ia(-1+a^2+abx)}{\sqrt{1-a^2}} + a(a+bx)\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}\right)}{bx}\right)}{\sqrt{1-a^2}}\right)}{a}$$

input
```
Integrate[ArcSec[a + b*x]/x^2,x]
```

output
```
-(ArcSec[a + b*x]/x) + (b*(ArcSin[(a + b*x)^(-1)] - (I*Log[(2*((I*a*(-1 +
a^2 + a*b*x))/Sqrt[1 - a^2] + a*(a + b*x)*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x
^2)/(a + b*x)^2]))/(b*x)])/Sqrt[1 - a^2]))/a
```

3.24.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5781, 4926, 3042, 4270, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{-1}(a+bx)}{x^2} dx$$

$$\Big\downarrow \text{ 5781}$$

$$b\int \frac{(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{b^2x^2}d\sec^{-1}(a+bx)$$

$$\Big\downarrow \text{ 4926}$$

$$b\left(-\int -\frac{1}{bx}d\sec^{-1}(a+bx)-\frac{\sec^{-1}(a+bx)}{bx}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b\left(-\int \frac{1}{a-\csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)}d\sec^{-1}(a+bx)-\frac{\sec^{-1}(a+bx)}{bx}\right)$$

$$\Big\downarrow \text{ 4270}$$

$$b\left(\frac{\int \frac{1}{1-\frac{a}{a+bx}}d\sec^{-1}(a+bx)}{a}-\frac{\sec^{-1}(a+bx)}{a}-\frac{\sec^{-1}(a+bx)}{bx}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b\left(\frac{\int \frac{1}{1-a\sin\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)}d\sec^{-1}(a+bx)}{a}-\frac{\sec^{-1}(a+bx)}{a}-\frac{\sec^{-1}(a+bx)}{bx}\right)$$

$$\Big\downarrow \text{ 3138}$$

$$b\left(\frac{2\int \frac{1}{(a+1)\tan^2\left(\frac{1}{2}\sec^{-1}(a+bx)\right)-a+1}d\tan\left(\frac{1}{2}\sec^{-1}(a+bx)\right)}{a}-\frac{\sec^{-1}(a+bx)}{a}-\frac{\sec^{-1}(a+bx)}{bx}\right)$$

$$\Big\downarrow 218$$

$$b\left(\frac{2\arctan\left(\frac{\sqrt{a+1}\tan\left(\frac{1}{2}\sec^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{a\sqrt{1-a^2}}-\frac{\sec^{-1}(a+bx)}{a}-\frac{\sec^{-1}(a+bx)}{bx}\right)$$

input
```
Int[ArcSec[a + b*x]/x^2,x]
```

output
```
b*(-(ArcSec[a + b*x]/a) - ArcSec[a + b*x]/(b*x) + (2*ArcTan[(Sqrt[1 + a]*T
an[ArcSec[a + b*x]/2])/Sqrt[1 - a]])/(a*Sqrt[1 - a^2]))
```

3.24.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4270
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Simp[1/a   Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[a^2 - b^2, 0]
```

rule 4926
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(e + f*
x)^m*((a + b*Sec[c + d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n +
 1)))   Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5781
```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d^(m + 1)   Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d
*e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.24.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(62) = 124$.

Time = 0.72 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.80

method	result
derivativedivides	$b\left(-\frac{\operatorname{arcsec}(bx+a)}{bx}+\frac{\sqrt{(bx+a)^2-1}\left(\arctan\left(\frac{1}{\sqrt{(bx+a)^2-1}}\right)\sqrt{a^2-1}-\ln\left(\frac{2\sqrt{a^2-1}\sqrt{(bx+a)^2-1}+2(bx+a)a-2}{bx}\right)\right)}{\sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}(bx+a)a\sqrt{a^2-1}}\right)$
default	$b\left(-\frac{\operatorname{arcsec}(bx+a)}{bx}+\frac{\sqrt{(bx+a)^2-1}\left(\arctan\left(\frac{1}{\sqrt{(bx+a)^2-1}}\right)\sqrt{a^2-1}-\ln\left(\frac{2\sqrt{a^2-1}\sqrt{(bx+a)^2-1}+2(bx+a)a-2}{bx}\right)\right)}{\sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}(bx+a)a\sqrt{a^2-1}}\right)$
parts	$-\frac{\operatorname{arcsec}(bx+a)}{x}+\frac{b\sqrt{b^2x^2+2abx+a^2-1}\left(\arctan\left(\frac{1}{\sqrt{b^2x^2+2abx+a^2-1}}\right)\sqrt{a^2-1}-\ln\left(\frac{2a^2-2+2abx+2\sqrt{a^2-1}\sqrt{b^2x^2+2ab\cdots}}{x}\right.\right.}{\sqrt{\frac{b^2x^2+2abx+a^2-1}{(bx+a)^2}}(bx+a)a\sqrt{a^2-1}}$

input
```
int(arcsec(b*x+a)/x^2,x,method=_RETURNVERBOSE)
```

output
```
b*(-1/b/x*arcsec(b*x+a)+((b*x+a)^2-1)^(1/2)*(arctan(1/((b*x+a)^2-1)^(1/2))
*(a^2-1)^(1/2)-ln(2*((a^2-1)^(1/2)*((b*x+a)^2-1)^(1/2)+(b*x+a)*a-1)/b/x))/
(((b*x+a)^2-1)/(b*x+a)^2)^(1/2)/(b*x+a)/a/(a^2-1)^(1/2))
```

3.24.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(62) = 124.

Time = 0.32 (sec) , antiderivative size = 281, normalized size of antiderivative = 4.01

$$\int \frac{\sec^{-1}(a+bx)}{x^2}\,dx$$
$$= \left[-\frac{2\,(a^2-1)bx\arctan\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2-1}\right)-\sqrt{a^2-1}bx\log\left(\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2-1}\left(a^2\right.}{}\right.}{(a^3-a)x}\right.$$
$$-\frac{2\,(a^2-1)bx\arctan\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2-1}\right)-2\,\sqrt{-a^2+1}bx\arctan\left(-\frac{\sqrt{-a^2+1}bx-\sqrt{b^2x^2+2\,a}}{a^2-1}\right.}{(a^3-a)x}$$

input
```
integrate(arcsec(b*x+a)/x^2,x, algorithm="fricas")
```

output
```
[-(2*(a^2 - 1)*b*x*arctan(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)) -
sqrt(a^2 - 1)*b*x*log((a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*(
a^2 - sqrt(a^2 - 1)*a - 1) - (a*b*x + a^2 - 1)*sqrt(a^2 - 1) - a)/x) + (a^
3 - a)*arcsec(b*x + a))/((a^3 - a)*x), -(2*(a^2 - 1)*b*x*arctan(-b*x - a +
 sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)) - 2*sqrt(-a^2 + 1)*b*x*arctan(-(sqrt(-
a^2 + 1)*b*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*sqrt(-a^2 + 1))/(a^2 - 1)
) + (a^3 - a)*arcsec(b*x + a))/((a^3 - a)*x)]
```

3.24.6 Sympy [F]

$$\int \frac{\sec^{-1}(a+bx)}{x^2}\,dx = \int \frac{\operatorname{asec}(a+bx)}{x^2}\,dx$$

input
```
integrate(asec(b*x+a)/x**2,x)
```

output
```
Integral(asec(a + b*x)/x**2, x)
```

3.24.7 Maxima [F]

$$\int \frac{\sec^{-1}(a+bx)}{x^2}\,dx = \int \frac{\operatorname{arcsec}(bx+a)}{x^2}\,dx$$

input
```
integrate(arcsec(b*x+a)/x^2,x, algorithm="maxima")
```

output
```
(x*integrate((b^2*x + a*b)*e^(1/2*log(b*x + a + 1) + 1/2*log(b*x + a - 1))
/(b^2*x^3 + 2*a*b*x^2 + (a^2 - 1)*x + (b^2*x^3 + 2*a*b*x^2 + (a^2 - 1)*x)*
e^(log(b*x + a + 1) + log(b*x + a - 1))), x) - arctan(sqrt(b*x + a + 1)*sq
rt(b*x + a - 1)))/x
```

3.24.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.34

$$\int \frac{\sec^{-1}(a+bx)}{x^2}\,dx = b\left(\frac{2\arctan\left(\frac{(bx+a)\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)+a}{\sqrt{-a^2+1}}\right)}{\sqrt{-a^2+1}a} + \frac{\arccos\left(-\frac{1}{(bx+a)\left(\frac{a}{bx+a}-1\right)-a}\right)}{a\left(\frac{a}{bx+a}-1\right)}\right)$$

input
```
integrate(arcsec(b*x+a)/x^2,x, algorithm="giac")
```

output
```
b*(2*arctan(((b*x + a)*(sqrt(-1/(b*x + a)^2 + 1) - 1) + a)/sqrt(-a^2 + 1))
/(sqrt(-a^2 + 1)*a) + arccos(-1/((b*x + a)*(a/(b*x + a) - 1) - a))/(a*(a/(
b*x + a) - 1)))
```

3.24.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{-1}(a+bx)}{x^2}\,dx = \int \frac{\mathrm{acos}\left(\frac{1}{a+b\,x}\right)}{x^2}\,dx$$

input
```
int(acos(1/(a + b*x))/x^2,x)
```

output
```
int(acos(1/(a + b*x))/x^2, x)
```

3.25 $\int \frac{\sec^{-1}(a+bx)}{x^3}\,dx$

3.25.1 Optimal result

Integrand size = 10, antiderivative size = 125

$$\int \frac{\sec^{-1}(a+bx)}{x^3}\,dx = \frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2a(1-a^2)\,x} + \frac{b^2\sec^{-1}(a+bx)}{2a^2} - \frac{\sec^{-1}(a+bx)}{2x^2} - \frac{(1-2a^2)\,b^2\arctan\left(\frac{\sqrt{1+a}\tan\left(\frac{1}{2}\sec^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{a^2(1-a^2)^{3/2}}$$

output
```
1/2*b^2*arcsec(b*x+a)/a^2-1/2*arcsec(b*x+a)/x^2-(-2*a^2+1)*b^2*arctan((1+a
)^(1/2)*tan(1/2*arcsec(b*x+a))/(1-a)^(1/2))/a^2/(-a^2+1)^(3/2)+1/2*b*(b*x+
a)*(1-1/(b*x+a)^2)^(1/2)/a/(-a^2+1)/x
```

3.25.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.58

$$\int \frac{\sec^{-1}(a+bx)}{x^3}\,dx = - \frac{\frac{bx(a+bx)\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}}{a(-1+a^2)} + \sec^{-1}(a+bx) + \frac{b^2x^2\arcsin\left(\frac{1}{a+bx}\right)}{a^2} + \frac{i(-1+2a^2)b^2x^2\log\left(\frac{4(-1+a)a^2(1+a)\left(-\frac{i(-1+a^2+abx)}{\sqrt{1-a^2}}-(a \cdots\right.}{(-1+2a^2)b^2 \cdots}\right.}{a^2(1-a^2)^{3/2}} \cdots}{2x^2}$$

input
```
Integrate[ArcSec[a + b*x]/x^3,x]
```

output
```
-1/2*((b*x*(a + b*x)*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2])/(a*
(-1 + a^2)) + ArcSec[a + b*x] + (b^2*x^2*ArcSin[(a + b*x)^(-1)])/a^2 + (I*
(-1 + 2*a^2)*b^2*x^2*Log[(4*(-1 + a)*a^2*(1 + a)*(((-I)*(-1 + a^2 + a*b*x)
)/Sqrt[1 - a^2] - (a + b*x)*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^
2]))/((-1 + 2*a^2)*b^2*x)])/(a^2*(1 - a^2)^(3/2)))/x^2
```

3.25.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.18, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.200$, Rules used = {5781, 25, 4926, 3042, 4272, 3042, 4407, 3042, 4318, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{-1}(a+bx)}{x^3}dx$$

$$\Big\downarrow \text{5781}$$

$$b^2\int \frac{(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{b^3x^3}d\sec^{-1}(a+bx)$$

$$\Big\downarrow \text{25}$$

$$-b^2\int -\frac{(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{b^3x^3}d\sec^{-1}(a+bx)$$

$$\Big\downarrow \text{4926}$$

$$b^2\left(\frac{1}{2}\int \frac{1}{b^2x^2}d\sec^{-1}(a+bx)-\frac{\sec^{-1}(a+bx)}{2b^2x^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{1}{2}\int \frac{1}{\left(a-\csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)\right)^2}d\sec^{-1}(a+bx)-\frac{\sec^{-1}(a+bx)}{2b^2x^2}\right)$$

$$\Big\downarrow \text{4272}$$

$$b^2\left(\frac{1}{2}\left(\frac{\int -\frac{-a^2-(a+bx)a+1}{bx}d\sec^{-1}(a+bx)}{a(1-a^2)}+\frac{\sqrt{1-\frac{1}{(a+bx)^2}}(a+bx)}{a(1-a^2)bx}\right)-\frac{\sec^{-1}(a+bx)}{2b^2x^2}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b^2\left(\frac{1}{2}\left(\frac{\int \frac{-a^2-\csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)a+1}{a-\csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)}d\sec^{-1}(a+bx)}{a\left(1-a^2\right)}+\frac{\sqrt{1-\frac{1}{(a+bx)^2}}(a+bx)}{a\left(1-a^2\right)bx}\right)-\frac{\sec^{-1}(a+bx)}{2b^2x^2}\right)$$

$$\Big\downarrow \text{ 4407}$$

$$b^2\left(\frac{1}{2}\left(\frac{\frac{\left(1-2a^2\right)\int -\frac{a+bx}{bx}d\sec^{-1}(a+bx)}{a}+\frac{\left(1-a^2\right)\sec^{-1}(a+bx)}{a}}{a\left(1-a^2\right)}+\frac{\sqrt{1-\frac{1}{(a+bx)^2}}(a+bx)}{a\left(1-a^2\right)bx}\right)-\frac{\sec^{-1}(a+bx)}{2b^2x^2}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b^2\left(\frac{1}{2}\left(\frac{\frac{\left(1-2a^2\right)\int \frac{\csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)}{a-\csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)}d\sec^{-1}(a+bx)}{a}+\frac{\left(1-a^2\right)\sec^{-1}(a+bx)}{a}}{a\left(1-a^2\right)}+\frac{\sqrt{1-\frac{1}{(a+bx)^2}}(a+bx)}{a\left(1-a^2\right)bx}\right)-\frac{\sec^{-1}(a+bx)}{2b^2x^2}\right)$$

$$\Big\downarrow \text{ 4318}$$

$$b^2\left(\frac{1}{2}\left(\frac{\frac{\left(1-a^2\right)\sec^{-1}(a+bx)}{a}-\frac{\left(1-2a^2\right)\int \frac{1}{1-\frac{a}{a+bx}}d\sec^{-1}(a+bx)}{a}}{a\left(1-a^2\right)}+\frac{\sqrt{1-\frac{1}{(a+bx)^2}}(a+bx)}{a\left(1-a^2\right)bx}\right)-\frac{\sec^{-1}(a+bx)}{2b^2x^2}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b^2\left(\frac{1}{2}\left(\frac{\frac{\left(1-a^2\right)\sec^{-1}(a+bx)}{a}-\frac{\left(1-2a^2\right)\int \frac{1}{1-a\sin\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)}d\sec^{-1}(a+bx)}{a}}{a\left(1-a^2\right)}+\frac{\sqrt{1-\frac{1}{(a+bx)^2}}(a+bx)}{a\left(1-a^2\right)bx}\right)-\frac{\sec^{-1}(a+bx)}{2b^2x^2}\right)$$

$$\Big\downarrow \text{ 3138}$$

$$b^2\left(\frac{1}{2}\left(\frac{\frac{\left(1-a^2\right)\sec^{-1}(a+bx)}{a}-\frac{2\left(1-2a^2\right)\int \frac{1}{(a+1)\tan^2\left(\frac{1}{2}\sec^{-1}(a+bx)\right)-a+1}d\tan\left(\frac{1}{2}\sec^{-1}(a+bx)\right)}{a}}{a\left(1-a^2\right)}+\frac{\sqrt{1-\frac{1}{(a+bx)^2}}(a+bx)}{a\left(1-a^2\right)bx}\right)-\sec\right.\right.$$

$$\Big\downarrow \text{ 218}$$

3.25. $\int \frac{\sec^{-1}(a+bx)}{x^3}dx$

$$b^2\left(\frac{1}{2}\left(\frac{\frac{(1-a^2)\sec^{-1}(a+bx)}{a}-\frac{2(1-2a^2)\arctan\left(\frac{\sqrt{a+1}\tan\left(\frac{1}{2}\sec^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{a\sqrt{1-a^2}}}{a(1-a^2)}+\frac{\sqrt{1-\frac{1}{(a+bx)^2}}(a+bx)}{a(1-a^2)bx}\right)-\frac{\sec^{-1}(a+bx)}{2b^2x^2}\right)$$

input
```
Int[ArcSec[a + b*x]/x^3,x]
```

output
```
b^2*(-1/2*ArcSec[a + b*x]/(b^2*x^2) + (((a + b*x)*Sqrt[1 - (a + b*x)^(-2)]
)/(a*(1 - a^2)*b*x) + (((1 - a^2)*ArcSec[a + b*x])/a - (2*(1 - 2*a^2)*ArcT
an[(Sqrt[1 + a]*Tan[ArcSec[a + b*x]/2])/Sqrt[1 - a]])/(a*Sqrt[1 - a^2]))/(
a*(1 - a^2)))/2)
```

3.25.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4926
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(e + f*
x)^m*((a + b*Sec[c + d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n +
 1)))   Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5781
```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d^(m + 1)   Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d
*e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.25.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(109) = 218$.

Time = 0.70 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.55

method	result
parts	$-\frac{\operatorname{arcsec}(bx+a)}{2x^2} - \frac{b\sqrt{b^2x^2+2abx+a^2-1}\,\big((a^2-1)^{\frac{3}{2}} \arctan\big(\frac{1}{\sqrt{b^2x^2+2abx+a^2-1}}\big)a^2bx-2\ln\big(\frac{2a^2-2+2abx+2\sqrt{a^2-1}\,\sqrt{\cdots}}{x}\cdots}{\cdots}$
derivativedivides	$b^2\Big(-\frac{\operatorname{arcsec}(bx+a)}{2b^2x^2}+\frac{\sqrt{(bx+a)^2-1}\,\big((a^2-1)^{\frac{3}{2}} \arctan\big(\frac{1}{\sqrt{(bx+a)^2-1}}\big)a^3-(a^2-1)^{\frac{3}{2}} \arctan\big(\frac{1}{\sqrt{(bx+a)^2-1}}\big)a^2(bx+\cdots}{\cdots}$
default	$b^2\Big(-\frac{\operatorname{arcsec}(bx+a)}{2b^2x^2}+\frac{\sqrt{(bx+a)^2-1}\,\big((a^2-1)^{\frac{3}{2}} \arctan\big(\frac{1}{\sqrt{(bx+a)^2-1}}\big)a^3-(a^2-1)^{\frac{3}{2}} \arctan\big(\frac{1}{\sqrt{(bx+a)^2-1}}\big)a^2(bx+\cdots}{\cdots}$

input
```
int(arcsec(b*x+a)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*arcsec(b*x+a)/x^2-1/2*b*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*((a^2-1)^(3/2)*
arctan(1/(b^2*x^2+2*a*b*x+a^2-1)^(1/2))*a^2*b*x-2*ln(2*(a*b*x+(a^2-1)^(1/2
)*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+a^2-1)/x)*a^4*b*x-b*arctan(1/(b^2*x^2+2*a*
b*x+a^2-1)^(1/2))*x*(a^2-1)^(3/2)+(a^2-1)^(3/2)*(b^2*x^2+2*a*b*x+a^2-1)^(1
/2)*a+3*ln(2*(a*b*x+(a^2-1)^(1/2)*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+a^2-1)/x)*
a^2*b*x-b*ln(2*(a*b*x+(a^2-1)^(1/2)*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+a^2-1)/x
)*x)/((b^2*x^2+2*a*b*x+a^2-1)/(b*x+a)^2)^(1/2)/(b*x+a)/a^2/(a^2-1)^(5/2)/x
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 427, normalized size of antiderivative = 3.42

$$\int \frac{\sec^{-1}(a+bx)}{x^3}\,dx$$

$$= \left[\frac{(2\,a^2-1)\sqrt{a^2-1}b^2x^2\log\left(\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2-1}\left(a^2+\sqrt{a^2-1}a-1\right)+(abx+a^2-1)\sqrt{a^2-1}-a}{x}\right)+2\,(a^4-2\,a^2+1}{}\right.$$

$$-\frac{2\,(2\,a^2-1)\sqrt{-a^2+1}b^2x^2\arctan\left(-\frac{\sqrt{-a^2+1}bx-\sqrt{b^2x^2+2\,abx+a^2-1}\sqrt{-a^2+1}}{a^2-1}\right)-2\,(a^4-2\,a^2+1)b^2x^2\arctan\left(\right.}{}$$

input
```
integrate(arcsec(b*x+a)/x^3,x, algorithm="fricas")
```

output
```
[1/2*((2*a^2 - 1)*sqrt(a^2 - 1)*b^2*x^2*log((a^2*b*x + a^3 + sqrt(b^2*x^2
+ 2*a*b*x + a^2 - 1)*(a^2 + sqrt(a^2 - 1)*a - 1) + (a*b*x + a^2 - 1)*sqrt(
a^2 - 1) - a)/x) + 2*(a^4 - 2*a^2 + 1)*b^2*x^2*arctan(-b*x - a + sqrt(b^2*
x^2 + 2*a*b*x + a^2 - 1)) - (a^3 - a)*b^2*x^2 - sqrt(b^2*x^2 + 2*a*b*x + a
^2 - 1)*(a^3 - a)*b*x - (a^6 - 2*a^4 + a^2)*arcsec(b*x + a))/((a^6 - 2*a^4
 + a^2)*x^2), -1/2*(2*(2*a^2 - 1)*sqrt(-a^2 + 1)*b^2*x^2*arctan(-(sqrt(-a^
2 + 1)*b*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*sqrt(-a^2 + 1))/(a^2 - 1))
- 2*(a^4 - 2*a^2 + 1)*b^2*x^2*arctan(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a
^2 - 1)) + (a^3 - a)*b^2*x^2 + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*(a^3 - a)
*b*x + (a^6 - 2*a^4 + a^2)*arcsec(b*x + a))/((a^6 - 2*a^4 + a^2)*x^2)]
```

3.25.6 Sympy [F]

$$\int \frac{\sec^{-1}(a+bx)}{x^3}\,dx = \int \frac{\operatorname{asec}(a+bx)}{x^3}\,dx$$

input
```
integrate(asec(b*x+a)/x**3,x)
```

output
```
Integral(asec(a + b*x)/x**3, x)
```

3.25.7 Maxima [F]

$$\int \frac{\sec^{-1}(a+bx)}{x^3}\,dx = \int \frac{\operatorname{arcsec}(bx+a)}{x^3}\,dx$$

input
```
integrate(arcsec(b*x+a)/x^3,x, algorithm="maxima")
```

output
```
1/2*(2*x^2*integrate(1/2*(b^2*x + a*b)*e^(1/2*log(b*x + a + 1) + 1/2*log(b
*x + a - 1))/(b^2*x^4 + 2*a*b*x^3 + (a^2 - 1)*x^2 + (b^2*x^4 + 2*a*b*x^3 +
 (a^2 - 1)*x^2)*e^(log(b*x + a + 1) + log(b*x + a - 1))), x) - arctan(sqrt
(b*x + a + 1)*sqrt(b*x + a - 1)))/x^2
```

3.25.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(106) = 212$.

Time = 0.32 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.73

$$\int \frac{\sec^{-1}(a+bx)}{x^3}\,dx =$$

$$-\frac{1}{2}b\left(\frac{2\,(2\,a^2b-b)\arctan\left(\frac{(bx+a)\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)+a}{\sqrt{-a^2+1}}\right)}{(a^4-a^2)\sqrt{-a^2+1}} + \frac{2\left((bx+a)ab\left(\sqrt{-\frac{1}{(bx+a)^2}\right.\right.}{\left((bx+a)^2\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)^2+2\,(bx+a)\right.}\right.$$

input
```
integrate(arcsec(b*x+a)/x^3,x, algorithm="giac")
```

output
```
-1/2*b*(2*(2*a^2*b - b)*arctan(((b*x + a)*(sqrt(-1/(b*x + a)^2 + 1) - 1) +
 a)/sqrt(-a^2 + 1))/((a^4 - a^2)*sqrt(-a^2 + 1)) + 2*((b*x + a)*a*b*(sqrt(
-1/(b*x + a)^2 + 1) - 1) + b)/(((b*x + a)^2*(sqrt(-1/(b*x + a)^2 + 1) - 1)
^2 + 2*(b*x + a)*a*(sqrt(-1/(b*x + a)^2 + 1) - 1) + 1)*(a^3 - a)) + (2*a*b
/(b*x + a) - b)*arccos(-1/((b*x + a)*(a/(b*x + a) - 1) - a))/(a^2*(a/(b*x
+ a) - 1)^2))
```

3.25.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{-1}(a+bx)}{x^3}\,dx = \int \frac{\operatorname{acos}\left(\frac{1}{a+b\,x}\right)}{x^3}\,dx$$

input
```
int(acos(1/(a + b*x))/x^3,x)
```

output
```
int(acos(1/(a + b*x))/x^3, x)
```

3.26 $\int \frac{\sec^{-1}(a+bx)}{x^4}\,dx$

3.26.1 Optimal result

Integrand size = 10, antiderivative size = 181

$$\int \frac{\sec^{-1}(a+bx)}{x^4}\,dx = \frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a(1-a^2)x^2} - \frac{(2-5a^2)\,b^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a^2(1-a^2)^2x} - \frac{b^3\sec^{-1}(a+bx)}{3a^3} - \frac{\sec^{-1}(a+bx)}{3x^3} + \frac{(2-5a^2+6a^4)\,b^3\arctan\left(\frac{\sqrt{1+a}\tan\left(\frac{1}{2}\sec^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{3a^3(1-a^2)^{5/2}}$$

output

```
-1/3*b^3*arcsec(b*x+a)/a^3-1/3*arcsec(b*x+a)/x^3+1/3*(6*a^4-5*a^2+2)*b^3*a
rctan((1+a)^(1/2)*tan(1/2*arcsec(b*x+a))/(1-a)^(1/2))/a^3/(-a^2+1)^(5/2)+1
/6*b*(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/a/(-a^2+1)/x^2-1/6*(-5*a^2+2)*b^2*(b*x+
a)*(1-1/(b*x+a)^2)^(1/2)/a^2/(-a^2+1)^2/x
```

3.26.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.33

$$\int \frac{\sec^{-1}(a+bx)}{x^4}\,dx = \frac{1}{6}\left(-\frac{b\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}\left(a^4+abx-4a^3bx+2b^2x^2-a^2(1+5b^2x^2)\right)}{a^2\left(-1+a^2\right)^2x^2}-\frac{2\sec^{-1}(a+bx)}{x^3}+\frac{2b^3\arcsin\left(\frac{1}{a+bx}\right)}{a^3}-\frac{i(2-5a^2+6a^4)\,b^3\log\left(\frac{12a^3(-1+a^2)^2\left(\frac{i(-1+a^2+abx)}{\sqrt{1-a^2}}+(a+bx)\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}\right)}{(2-5a^2+6a^4)b^3x}\right)}{a^3\left(1-a^2\right)^{5/2}}\right)$$

input
```
Integrate[ArcSec[a + b*x]/x^4,x]
```

output
```
(-((b*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2]*(a^4 + a*b*x - 4*a^
3*b*x + 2*b^2*x^2 - a^2*(1 + 5*b^2*x^2)))/(a^2*(-1 + a^2)^2*x^2)) - (2*Arc
Sec[a + b*x])/x^3 + (2*b^3*ArcSin[(a + b*x)^(-1)])/a^3 - (I*(2 - 5*a^2 + 6
*a^4)*b^3*Log[(12*a^3*(-1 + a^2)^2*((I*(-1 + a^2 + a*b*x))/Sqrt[1 - a^2] +
 (a + b*x)*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2]))/((2 - 5*a^2
+ 6*a^4)*b^3*x)])/(a^3*(1 - a^2)^(5/2)))/6
```

3.26.3 Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.23, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.300, Rules used = {5781, 4926, 3042, 4272, 3042, 4548, 3042, 4407, 3042, 4318, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{-1}(a+bx)}{x^4}\,dx$$

$$\Big\downarrow \text{ 5781}$$

$$b^3\int \frac{(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{b^4x^4}d\sec^{-1}(a+bx)$$

$$\Big\downarrow \text{ 4926}$$

$$b^3\left(-\frac{1}{3}\int -\frac{1}{b^3x^3}d\sec^{-1}(a+bx)-\frac{\sec^{-1}(a+bx)}{3b^3x^3}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b^3\left(-\frac{1}{3}\int \frac{1}{\left(a-\csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)\right)^3}d\sec^{-1}(a+bx)-\frac{\sec^{-1}(a+bx)}{3b^3x^3}\right)$$

$$\Big\downarrow \text{ 4272}$$

$$b^3\left(\frac{1}{3}\left(\frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2a(1-a^2)b^2x^2}-\frac{\int \frac{-(a+bx)^2-2a(a+bx)+2(1-a^2)}{b^2x^2}d\sec^{-1}(a+bx)}{2a(1-a^2)}\right)-\frac{\sec^{-1}(a+bx)}{3b^3x^3}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b^3\left(\frac{1}{3}\left(\frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2a(1-a^2)b^2x^2}-\frac{\int \frac{-\csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)^2-2a\csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)+2(1-a^2)}{\left(a-\csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)\right)^2}d\sec^{-1}(a+bx)}{2a(1-a^2)}\right)-\frac{\sec^{-1}(a+bx)}{3b^3x^3}\right)$$

$$\Big\downarrow \text{ 4548}$$

$$b^3\left(\frac{1}{3}\left(\frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2a(1-a^2)b^2x^2}-\frac{\frac{\int -\frac{2(1-a^2)^2-a(1-4a^2)(a+bx)}{bx}d\sec^{-1}(a+bx)}{a(1-a^2)}+\frac{(2-5a^2)\sqrt{1-\frac{1}{(a+bx)^2}}(a+bx)}{a(1-a^2)bx}}{2a(1-a^2)}\right)-\frac{\sec^{-1}(a+bx)}{3b^3x^3}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b^3\left(\frac{1}{3}\left(\frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2a(1-a^2)b^2x^2}-\frac{\frac{\int \frac{2(1-a^2)^2-a(1-4a^2)\csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)}{a-\csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)}d\sec^{-1}(a+bx)}{a(1-a^2)}+\frac{(2-5a^2)\sqrt{1-\frac{1}{(a+bx)^2}}(a+bx)}{a(1-a^2)bx}}{2a(1-a^2)}\right)-\frac{\sec^{-1}(a+bx)}{3b^3x^3}\right)$$

$$\Big\downarrow \text{ 4407}$$

3.26. $\int \frac{\sec^{-1}(a+bx)}{x^4}\,dx$

$$b^3\left(\frac{1}{3}\left(\frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2a(1-a^2)b^2x^2}-\frac{\frac{\frac{(6a^4-5a^2+2)\int -\frac{a+bx}{bx}d\sec^{-1}(a+bx)}{a}+\frac{2(1-a^2)^2\sec^{-1}(a+bx)}{a}}{a(1-a^2)}+\frac{(2-5a^2)\sqrt{1-\frac{1}{(a+bx)^2}}(a+bx)}{a(1-a^2)bx}}{2a(1-a^2)}\right)-$$

$\downarrow$ 3042

$$b^3\left(\frac{1}{3}\left(\frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2a(1-a^2)b^2x^2}-\frac{\frac{\frac{(6a^4-5a^2+2)\int \frac{\csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)}{a-\csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)}d\sec^{-1}(a+bx)}{a}+\frac{2(1-a^2)^2\sec^{-1}(a+bx)}{a}}{a(1-a^2)}+\frac{(2-5a^2)\sqrt{1-\frac{1}{(a+bx)^2}}}{a(1-a^2)bx}}{2a(1-a^2)}\right.\right.$$

$\downarrow$ 4318

$$b^3\left(\frac{1}{3}\left(\frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2a(1-a^2)b^2x^2}-\frac{\frac{\frac{2(1-a^2)^2\sec^{-1}(a+bx)}{a}-\frac{(6a^4-5a^2+2)\int \frac{1}{1-\frac{a}{a+bx}}d\sec^{-1}(a+bx)}{a}}{a(1-a^2)}+\frac{(2-5a^2)\sqrt{1-\frac{1}{(a+bx)^2}}(a+bx)}{a(1-a^2)bx}}{2a(1-a^2)}\right)-$$

$\downarrow$ 3042

$$b^3\left(\frac{1}{3}\left(\frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2a(1-a^2)b^2x^2}-\frac{\frac{\frac{2(1-a^2)^2\sec^{-1}(a+bx)}{a}-\frac{(6a^4-5a^2+2)\int \frac{1}{1-a\sin\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)}d\sec^{-1}(a+bx)}{a}}{a(1-a^2)}+\frac{(2-5a^2)\sqrt{1-\frac{1}{(a+bx)^2}}}{a(1-a^2)bx}}{2a(1-a^2)}\right.\right.$$

$\downarrow$ 3138

$$b^3\left(\frac{1}{3}\left(\frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2a(1-a^2)b^2x^2}-\frac{\frac{\frac{2(1-a^2)^2\sec^{-1}(a+bx)}{a}-\frac{2(6a^4-5a^2+2)\int \frac{1}{(a+1)\tan^2\left(\frac{1}{2}\sec^{-1}(a+bx)\right)-a+1}d\tan\left(\frac{1}{2}\sec^{-1}(a+bx)\right)}{a}}{a(1-a^2)}+\ldots}{2a(1-a^2)}\right.\right.$$

$\downarrow$ 218

$$b^3\left(\frac{1}{3}\left(\frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2a(1-a^2)b^2x^2}-\frac{\frac{(2-5a^2)\sqrt{1-\frac{1}{(a+bx)^2}}(a+bx)}{a(1-a^2)bx}+\frac{\frac{2(1-a^2)^2\sec^{-1}(a+bx)}{a}-\frac{2(6a^4-5a^2+2)\arctan\left(\frac{\sqrt{a+1}\tan\left(\frac{1}{2}\sec^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{a\sqrt{1-a^2}}}{a(1-a^2)}}{2a(1-a^2)}\right.\right.$$

3.26. $\int \frac{\sec^{-1}(a+bx)}{x^4}\,dx$

input
```
Int[ArcSec[a + b*x]/x^4,x]
```

output
```
b^3*(-1/3*ArcSec[a + b*x]/(b^3*x^3) + (((a + b*x)*Sqrt[1 - (a + b*x)^(-2)]
)/(2*a*(1 - a^2)*b^2*x^2) - (((2 - 5*a^2)*(a + b*x)*Sqrt[1 - (a + b*x)^(-2
)])/(a*(1 - a^2)*b*x) + ((2*(1 - a^2)^2*ArcSec[a + b*x])/a - (2*(2 - 5*a^2
 + 6*a^4)*ArcTan[(Sqrt[1 + a]*Tan[ArcSec[a + b*x]/2])/Sqrt[1 - a]])/(a*Sqr
t[1 - a^2]))/(a*(1 - a^2)))/(2*a*(1 - a^2)))/3)
```

3.26.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4548
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(
m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x
] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

rule 4926
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(e + f*
x)^m*((a + b*Sec[c + d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n +
 1)))   Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5781
```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d^(m + 1)   Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d
*e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.26.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(159) = 318$.

Time = 0.70 (sec) , antiderivative size = 528, normalized size of antiderivative = 2.92

method	result
parts	$-\frac{\operatorname{arcsec}(bx+a)}{3x^3} - \frac{b\sqrt{b^2x^2+2abx+a^2-1}\left(-2\left(a^2-1\right)^{\frac{3}{2}}\arctan\left(\frac{1}{\sqrt{b^2x^2+2abx+a^2-1}}\right)a^4b^2x^2+6\ln\left(\frac{2a^2-2+2abx+2\sqrt{a^2}}{}\right.\right.}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(arcsec(b*x+a)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*arcsec(b*x+a)/x^3-1/6*b*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*(-2*(a^2-1)^(3/
2)*arctan(1/(b^2*x^2+2*a*b*x+a^2-1)^(1/2))*a^4*b^2*x^2+6*ln(2*(a*b*x+(a^2-
1)^(1/2)*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+a^2-1)/x)*a^6*b^2*x^2+4*(a^2-1)^(3/
2)*arctan(1/(b^2*x^2+2*a*b*x+a^2-1)^(1/2))*a^2*b^2*x^2-5*(b^2*x^2+2*a*b*x+
a^2-1)^(1/2)*(a^2-1)^(3/2)*a^3*b*x-11*ln(2*(a*b*x+(a^2-1)^(1/2)*(b^2*x^2+2
*a*b*x+a^2-1)^(1/2)+a^2-1)/x)*a^4*b^2*x^2+(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*(a
^2-1)^(3/2)*a^4-2*b^2*arctan(1/(b^2*x^2+2*a*b*x+a^2-1)^(1/2))*x^2*(a^2-1)^
(3/2)+2*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*(a^2-1)^(3/2)*a*b*x+7*ln(2*(a*b*x+(a
^2-1)^(1/2)*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+a^2-1)/x)*a^2*b^2*x^2-(b^2*x^2+2
*a*b*x+a^2-1)^(1/2)*(a^2-1)^(3/2)*a^2-2*b^2*ln(2*(a*b*x+(a^2-1)^(1/2)*(b^2
*x^2+2*a*b*x+a^2-1)^(1/2)+a^2-1)/x)*x^2)/((b^2*x^2+2*a*b*x+a^2-1)/(b*x+a)^
2)^(1/2)/(b*x+a)/a^3/(a^2-1)^(7/2)/x^2
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 548, normalized size of antiderivative = 3.03

$$\int \frac{\sec^{-1}(a+bx)}{x^4}\,dx$$
$$= \left[\frac{(6\,a^4-5\,a^2+2)\sqrt{a^2-1}b^3x^3\log\left(\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2-1}\left(a^2-\sqrt{a^2-1}a-1\right)-(abx+a^2-1)\sqrt{a^2-1}-a}{x}\right)-4\,(a^6-3}{}\right.$$

input
```
integrate(arcsec(b*x+a)/x^4,x, algorithm="fricas")
```

output
```
[1/6*((6*a^4 - 5*a^2 + 2)*sqrt(a^2 - 1)*b^3*x^3*log((a^2*b*x + a^3 + sqrt(
b^2*x^2 + 2*a*b*x + a^2 - 1)*(a^2 - sqrt(a^2 - 1)*a - 1) - (a*b*x + a^2 -
1)*sqrt(a^2 - 1) - a)/x) - 4*(a^6 - 3*a^4 + 3*a^2 - 1)*b^3*x^3*arctan(-b*x
 - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)) + (5*a^5 - 7*a^3 + 2*a)*b^3*x^3
- 2*(a^9 - 3*a^7 + 3*a^5 - a^3)*arcsec(b*x + a) + ((5*a^5 - 7*a^3 + 2*a)*b
^2*x^2 - (a^6 - 2*a^4 + a^2)*b*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))/((a^9
 - 3*a^7 + 3*a^5 - a^3)*x^3), 1/6*(2*(6*a^4 - 5*a^2 + 2)*sqrt(-a^2 + 1)*b^
3*x^3*arctan(-(sqrt(-a^2 + 1)*b*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*sqrt
(-a^2 + 1))/(a^2 - 1)) - 4*(a^6 - 3*a^4 + 3*a^2 - 1)*b^3*x^3*arctan(-b*x -
 a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)) + (5*a^5 - 7*a^3 + 2*a)*b^3*x^3 -
2*(a^9 - 3*a^7 + 3*a^5 - a^3)*arcsec(b*x + a) + ((5*a^5 - 7*a^3 + 2*a)*b^2
*x^2 - (a^6 - 2*a^4 + a^2)*b*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))/((a^9 -
 3*a^7 + 3*a^5 - a^3)*x^3)]
```

3.26.6 Sympy [F]

$$\int \frac{\sec^{-1}(a+bx)}{x^4}\,dx = \int \frac{\operatorname{asec}(a+bx)}{x^4}\,dx$$

input
```
integrate(asec(b*x+a)/x**4,x)
```

output
```
Integral(asec(a + b*x)/x**4, x)
```

3.26.7 Maxima [F]

$$\int \frac{\sec^{-1}(a+bx)}{x^4}\,dx = \int \frac{\operatorname{arcsec}(bx+a)}{x^4}\,dx$$

input
```
integrate(arcsec(b*x+a)/x^4,x, algorithm="maxima")
```

output
```
1/3*(3*x^3*integrate(1/3*(b^2*x + a*b)*e^(1/2*log(b*x + a + 1) + 1/2*log(b
*x + a - 1))/(b^2*x^5 + 2*a*b*x^4 + (a^2 - 1)*x^3 + (b^2*x^5 + 2*a*b*x^4 +
 (a^2 - 1)*x^3)*e^(log(b*x + a + 1) + log(b*x + a - 1))), x) - arctan(sqrt
(b*x + a + 1)*sqrt(b*x + a - 1)))/x^3
```

3.26.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(155) = 310$.

Time = 0.33 (sec) , antiderivative size = 451, normalized size of antiderivative = 2.49

$$\int \frac{\sec^{-1}(a+bx)}{x^4}\,dx$$
$$= \frac{1}{3}b\left(\frac{(6a^4b^2 - 5a^2b^2 + 2b^2)\arctan\left(\frac{(bx+a)\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)+a}{\sqrt{-a^2+1}}\right)}{(a^7 - 2a^5 + a^3)\sqrt{-a^2+1}} + \frac{4(bx+a)^3a^3b^2\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)^3}{\ldots}\right.$$

input
```
integrate(arcsec(b*x+a)/x^4,x, algorithm="giac")
```

output
```
1/3*b*((6*a^4*b^2 - 5*a^2*b^2 + 2*b^2)*arctan(((b*x + a)*(sqrt(-1/(b*x + a
)^2 + 1) - 1) + a)/sqrt(-a^2 + 1))/((a^7 - 2*a^5 + a^3)*sqrt(-a^2 + 1)) +
(4*(b*x + a)^3*a^3*b^2*(sqrt(-1/(b*x + a)^2 + 1) - 1)^3 + 10*(b*x + a)^2*a
^4*b^2*(sqrt(-1/(b*x + a)^2 + 1) - 1)^2 - (b*x + a)^3*a*b^2*(sqrt(-1/(b*x
+ a)^2 + 1) - 1)^3 + (b*x + a)^2*a^2*b^2*(sqrt(-1/(b*x + a)^2 + 1) - 1)^2
+ 16*(b*x + a)*a^3*b^2*(sqrt(-1/(b*x + a)^2 + 1) - 1) - 2*(b*x + a)^2*b^2*
(sqrt(-1/(b*x + a)^2 + 1) - 1)^2 - 7*(b*x + a)*a*b^2*(sqrt(-1/(b*x + a)^2
+ 1) - 1) + 5*a^2*b^2 - 2*b^2)/((a^6 - 2*a^4 + a^2)*((b*x + a)^2*(sqrt(-1/
(b*x + a)^2 + 1) - 1)^2 + 2*(b*x + a)*a*(sqrt(-1/(b*x + a)^2 + 1) - 1) + 1
)^2) - (3*a*b^2/(b*x + a) - 3*a^2*b^2/(b*x + a)^2 - b^2)*arccos(-1/((b*x +
 a)*(a/(b*x + a) - 1) - a))/(a^3*(a/(b*x + a) - 1)^3))
```

3.26.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{-1}(a+bx)}{x^4}\,dx = \int \frac{\operatorname{acos}\left(\frac{1}{a+b\,x}\right)}{x^4}\,dx$$

input
```
int(acos(1/(a + b*x))/x^4,x)
```

output
```
int(acos(1/(a + b*x))/x^4, x)
```

3.27 $\int x^3 \sec^{-1}(a+bx)^2\,dx$

3.27.1 Optimal result

Integrand size = 12, antiderivative size = 381

$$\begin{aligned}\int x^3 \sec^{-1}(a+bx)^2\,dx = &-\frac{ax}{b^3}+\frac{(a+bx)^2}{12b^4}-\frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{3b^4}\\&-\frac{3a^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{b^4}\\&+\frac{a(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{b^4}\\&-\frac{(a+bx)^3\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{6b^4}-\frac{a^4\sec^{-1}(a+bx)^2}{4b^4}\\&+\frac{1}{4}x^4\sec^{-1}(a+bx)^2-\frac{2ia\sec^{-1}(a+bx)\arctan\left(e^{i\sec^{-1}(a+bx)}\right)}{b^4}\\&-\frac{4ia^3\sec^{-1}(a+bx)\arctan\left(e^{i\sec^{-1}(a+bx)}\right)}{b^4}+\frac{\log(a+bx)}{3b^4}\\&+\frac{3a^2\log(a+bx)}{b^4}+\frac{ia\,\mathrm{PolyLog}\left(2,-ie^{i\sec^{-1}(a+bx)}\right)}{b^4}\\&+\frac{2ia^3\,\mathrm{PolyLog}\left(2,-ie^{i\sec^{-1}(a+bx)}\right)}{b^4}\\&-\frac{ia\,\mathrm{PolyLog}\left(2,ie^{i\sec^{-1}(a+bx)}\right)}{b^4}-\frac{2ia^3\,\mathrm{PolyLog}\left(2,ie^{i\sec^{-1}(a+bx)}\right)}{b^4}\end{aligned}$$

output
```
-a*x/b^3+1/12*(b*x+a)^2/b^4-1/4*a^4*arcsec(b*x+a)^2/b^4+1/4*x^4*arcsec(b*x
+a)^2-2*I*a^3*polylog(2,I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^4+2*I*a^3
*polylog(2,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^4+1/3*ln(b*x+a)/b^4+3
*a^2*ln(b*x+a)/b^4+I*a*polylog(2,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b
^4-2*I*a*arcsec(b*x+a)*arctan(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/b^4-4*I*a
^3*arcsec(b*x+a)*arctan(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/b^4-I*a*polylog
(2,I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^4-1/3*(b*x+a)*arcsec(b*x+a)*(1
-1/(b*x+a)^2)^(1/2)/b^4-3*a^2*(b*x+a)*arcsec(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/
b^4+a*(b*x+a)^2*arcsec(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/b^4-1/6*(b*x+a)^3*arcs
ec(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/b^4
```

3.27.2 Mathematica [A] (warning: unable to verify)

Time = 8.39 (sec) , antiderivative size = 667, normalized size of antiderivative = 1.75

$$\int x^3 \sec^{-1}(a+bx)^2\,dx$$
$$= \frac{\left(1-\frac{a}{a+bx}\right)^3\left(24a(2+(1+2a^2)\sec^{-1}(a+bx)^2)+\frac{2+(-2+24a)\sec^{-1}(a+bx)+3(1-4a+12a^2)\sec^{-1}(a+bx)^2}{-1+\sqrt{1-\frac{1}{(a+bx)^2}}}+16(1+9\right.}{}$$

input
```
Integrate[x^3*ArcSec[a + b*x]^2,x]
```

output
```
((1 - a/(a + b*x))^3*(24*a*(2 + (1 + 2*a^2)*ArcSec[a + b*x]^2) + (2 + (-2
+ 24*a)*ArcSec[a + b*x] + 3*(1 - 4*a + 12*a^2)*ArcSec[a + b*x]^2)/(-1 + Sq
rt[1 - (a + b*x)^(-2)]) + 16*(1 + 9*a^2)*Log[(a + b*x)^(-1)] - 24*a*(1 + 2
*a^2)*((Pi - 2*ArcSec[a + b*x])*(Log[1 - I/E^(I*ArcSec[a + b*x])] - Log[1
+ I/E^(I*ArcSec[a + b*x])]) - Pi*Log[Cot[(Pi + 2*ArcSec[a + b*x])/4]] + (2
*I)*(PolyLog[2, (-I)/E^(I*ArcSec[a + b*x])] - PolyLog[2, I/E^(I*ArcSec[a +
 b*x])])) - (3*ArcSec[a + b*x]^2)/(Cos[ArcSec[a + b*x]/2] - Sin[ArcSec[a +
 b*x]/2])^4 + (4*ArcSec[a + b*x]*(1 + 6*a*ArcSec[a + b*x])*Sin[ArcSec[a +
b*x]/2])/(Cos[ArcSec[a + b*x]/2] - Sin[ArcSec[a + b*x]/2])^3 + (8*(2*ArcSe
c[a + b*x] + 18*a^2*ArcSec[a + b*x] + 6*a^3*ArcSec[a + b*x]^2 + 3*a*(2 + A
rcSec[a + b*x]^2))*Sin[ArcSec[a + b*x]/2])/(Cos[ArcSec[a + b*x]/2] - Sin[A
rcSec[a + b*x]/2]) - (3*ArcSec[a + b*x]^2)/(Cos[ArcSec[a + b*x]/2] + Sin[A
rcSec[a + b*x]/2])^4 + (4*ArcSec[a + b*x]*(1 - 6*a*ArcSec[a + b*x])*Sin[Ar
cSec[a + b*x]/2])/(Cos[ArcSec[a + b*x]/2] + Sin[ArcSec[a + b*x]/2])^3 - (2
 + (2 - 24*a)*ArcSec[a + b*x] + 3*(1 - 4*a + 12*a^2)*ArcSec[a + b*x]^2)/(C
os[ArcSec[a + b*x]/2] + Sin[ArcSec[a + b*x]/2])^2 - (8*(-2*ArcSec[a + b*x]
 - 18*a^2*ArcSec[a + b*x] + 6*a^3*ArcSec[a + b*x]^2 + 3*a*(2 + ArcSec[a +
b*x]^2))*Sin[ArcSec[a + b*x]/2])/(Cos[ArcSec[a + b*x]/2] + Sin[ArcSec[a +
b*x]/2])))/(48*b^4*(-1 + a/(a + b*x))^3)
```

3.27.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 357, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5781, 25, 4926, 3042, 4678, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sec^{-1}(a+bx)^2 \, dx$$

$$\Big\downarrow \text{ 5781}$$

$$\frac{\int b^3x^3(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^2d\sec^{-1}(a+bx)}{b^4}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int -b^3x^3(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^2d\sec^{-1}(a+bx)}{b^4}$$

$$\Big\downarrow \text{ 4926}$$

$$\frac{\frac{1}{4}b^4x^4\sec^{-1}(a+bx)^2-\frac{1}{2}\int b^4x^4\sec^{-1}(a+bx)d\sec^{-1}(a+bx)}{b^4}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{1}{4}b^4x^4\sec^{-1}(a+bx)^2-\frac{1}{2}\int \sec^{-1}(a+bx)\left(a-\csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)\right)^4d\sec^{-1}(a+bx)}{b^4}$$

$$\Big\downarrow 4678$$

$$\frac{\frac{1}{4}b^4x^4\sec^{-1}(a+bx)^2-\frac{1}{2}\int\left(\sec^{-1}(a+bx)a^4-4(a+bx)\sec^{-1}(a+bx)a^3+6(a+bx)^2\sec^{-1}(a+bx)a^2-4(a+b\right.}{b^4}$$

$$\Big\downarrow 2009$$

$$\frac{1}{4}b^4x^4\sec^{-1}(a+bx)^2+\frac{1}{2}\left(-\frac{1}{2}a^4\sec^{-1}(a+bx)^2-8ia^3\sec^{-1}(a+bx)\arctan\left(e^{i\sec^{-1}(a+bx)}\right)+4ia^3\operatorname{PolyLog}\left(2,-i\right.\right.$$

input
```
Int[x^3*ArcSec[a + b*x]^2,x]
```

output
```
((b^4*x^4*ArcSec[a + b*x]^2)/4 + (-2*a*(a + b*x) + (a + b*x)^2/6 - (2*(a +
 b*x)*Sqrt[1 - (a + b*x)^(-2)]*ArcSec[a + b*x])/3 - 6*a^2*(a + b*x)*Sqrt[1
 - (a + b*x)^(-2)]*ArcSec[a + b*x] + 2*a*(a + b*x)^2*Sqrt[1 - (a + b*x)^(-
2)]*ArcSec[a + b*x] - ((a + b*x)^3*Sqrt[1 - (a + b*x)^(-2)]*ArcSec[a + b*x
])/3 - (a^4*ArcSec[a + b*x]^2)/2 - (4*I)*a*ArcSec[a + b*x]*ArcTan[E^(I*Arc
Sec[a + b*x])] - (8*I)*a^3*ArcSec[a + b*x]*ArcTan[E^(I*ArcSec[a + b*x])] -
 (2*Log[(a + b*x)^(-1)])/3 - 6*a^2*Log[(a + b*x)^(-1)] + (2*I)*a*PolyLog[2
, (-I)*E^(I*ArcSec[a + b*x])] + (4*I)*a^3*PolyLog[2, (-I)*E^(I*ArcSec[a +
b*x])] - (2*I)*a*PolyLog[2, I*E^(I*ArcSec[a + b*x])] - (4*I)*a^3*PolyLog[2
, I*E^(I*ArcSec[a + b*x])])/2)/b^4
```

3.27.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4678
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 4926
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(e + f*
x)^m*((a + b*Sec[c + d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n +
 1)))   Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5781
```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d^(m + 1)   Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d
*e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.27.4 Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 673, normalized size of antiderivative = 1.77

method	result
derivativedivides	$-\operatorname{arcsec}(bx+a)^2 a^3(bx+a)+\frac{3\operatorname{arcsec}(bx+a)^2a^2(bx+a)^2}{2}-\operatorname{arcsec}(bx+a)^2a(bx+a)^3+\frac{\operatorname{arcsec}(bx+a)^2(bx+a)^4}{4}-3\operatorname{arcsec}(bx+a)$
default	$-\operatorname{arcsec}(bx+a)^2 a^3(bx+a)+\frac{3\operatorname{arcsec}(bx+a)^2a^2(bx+a)^2}{2}-\operatorname{arcsec}(bx+a)^2a(bx+a)^3+\frac{\operatorname{arcsec}(bx+a)^2(bx+a)^4}{4}-3\operatorname{arcsec}(bx+a)$

input
```
int(x^3*arcsec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^4*(-arcsec(b*x+a)^2*a^3*(b*x+a)+3/2*arcsec(b*x+a)^2*a^2*(b*x+a)^2-arcs
ec(b*x+a)^2*a*(b*x+a)^3+1/4*arcsec(b*x+a)^2*(b*x+a)^4-3*arcsec(b*x+a)*(((b
*x+a)^2-1)/(b*x+a)^2)^(1/2)*a^2*(b*x+a)+arcsec(b*x+a)*(((b*x+a)^2-1)/(b*x+
a)^2)^(1/2)*a*(b*x+a)^2-1/6*arcsec(b*x+a)*(((b*x+a)^2-1)/(b*x+a)^2)^(1/2)*
(b*x+a)^3-3*I*a^2*arcsec(b*x+a)-1/3*arcsec(b*x+a)*(((b*x+a)^2-1)/(b*x+a)^2
)^(1/2)*(b*x+a)+I*dilog(1+I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))*a-(b*x+a)
*a+1/12*(b*x+a)^2-1/3*ln(1+(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)+2/3*ln(1
/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))-3*ln(1+(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2
))^2)*a^2+6*ln(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))*a^2-2*ln(1+I*(1/(b*x+a)+
I*(1-1/(b*x+a)^2)^(1/2)))*a^3*arcsec(b*x+a)+2*ln(1-I*(1/(b*x+a)+I*(1-1/(b*
x+a)^2)^(1/2)))*a^3*arcsec(b*x+a)-1/3*I*arcsec(b*x+a)-2*I*dilog(1-I*(1/(b*
x+a)+I*(1-1/(b*x+a)^2)^(1/2)))*a^3-ln(1+I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/
2)))*a*arcsec(b*x+a)+ln(1-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))*a*arcsec(
b*x+a)-I*dilog(1-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))*a+2*I*dilog(1+I*(1
/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))*a^3)
```

3.27.5 Fricas [F]

$$\int x^3 \sec^{-1}(a+bx)^2\,dx = \int x^3 \operatorname{arcsec}\left(bx+a\right)^2\,dx$$

input
```
integrate(x^3*arcsec(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(x^3*arcsec(b*x + a)^2, x)
```

3.27.6 Sympy [F]

$$\int x^3 \sec^{-1}(a+bx)^2\,dx = \int x^3 \operatorname{asec}^2\left(a+bx\right)dx$$

input
```
integrate(x**3*asec(b*x+a)**2,x)
```

output
```
Integral(x**3*asec(a + b*x)**2, x)
```

3.27.7 Maxima [F]

$$\int x^3 \sec^{-1}(a+bx)^2 \, dx = \int x^3 \operatorname{arcsec}(bx+a)^2 \, dx$$

input
```
integrate(x^3*arcsec(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/4*x^4*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^2 - 1/16*x^4*log(b^2*x
^2 + 2*a*b*x + a^2)^2 - integrate(1/4*(2*sqrt(b*x + a + 1)*sqrt(b*x + a -
1)*b*x^4*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + 4*(b^3*x^6 + 3*a*b^
2*x^5 + (3*a^2 - 1)*b*x^4 + (a^3 - a)*x^3)*log(b*x + a)^2 - (b^3*x^6 + 2*a
*b^2*x^5 + (a^2 - 1)*b*x^4 + 4*(b^3*x^6 + 3*a*b^2*x^5 + (3*a^2 - 1)*b*x^4
+ (a^3 - a)*x^3)*log(b*x + a))*log(b^2*x^2 + 2*a*b*x + a^2))/(b^3*x^3 + 3*
a*b^2*x^2 + a^3 + (3*a^2 - 1)*b*x - a), x)
```

3.27.8 Giac [F(-2)]

Exception generated.

$$\int x^3 \sec^{-1}(a+bx)^2 \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arcsec(b*x+a)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.27.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \sec^{-1}(a+bx)^2 \, dx = \int x^3 \operatorname{acos}\left(\frac{1}{a+b\,x}\right)^2 \, dx$$

input
```
int(x^3*acos(1/(a + b*x))^2,x)
```

output
```
int(x^3*acos(1/(a + b*x))^2, x)
```

3.28 $\int x^2 \sec^{-1}(a+bx)^2\,dx$

3.28.1 Optimal result

Integrand size = 12, antiderivative size = 288

$$\begin{aligned}\int x^2 \sec^{-1}(a+bx)^2\,dx &= \frac{x}{3b^2} + \frac{2a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{b^3} \\ &\quad - \frac{(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{3b^3} + \frac{a^3\sec^{-1}(a+bx)^2}{3b^3} \\ &\quad + \frac{1}{3}x^3\sec^{-1}(a+bx)^2 + \frac{2i\sec^{-1}(a+bx)\arctan\left(e^{i\sec^{-1}(a+bx)}\right)}{3b^3} \\ &\quad + \frac{4ia^2\sec^{-1}(a+bx)\arctan\left(e^{i\sec^{-1}(a+bx)}\right)}{b^3} \\ &\quad - \frac{2a\log(a+bx)}{b^3} - \frac{i\,\mathrm{PolyLog}\left(2,-ie^{i\sec^{-1}(a+bx)}\right)}{3b^3} \\ &\quad - \frac{2ia^2\,\mathrm{PolyLog}\left(2,-ie^{i\sec^{-1}(a+bx)}\right)}{b^3} \\ &\quad + \frac{i\,\mathrm{PolyLog}\left(2,ie^{i\sec^{-1}(a+bx)}\right)}{3b^3} + \frac{2ia^2\,\mathrm{PolyLog}\left(2,ie^{i\sec^{-1}(a+bx)}\right)}{b^3}\end{aligned}$$

output
```
1/3*x/b^2+1/3*a^3*arcsec(b*x+a)^2/b^3+1/3*x^3*arcsec(b*x+a)^2+2/3*I*arcsec
(b*x+a)*arctan(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/b^3+4*I*a^2*arcsec(b*x+a
)*arctan(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/b^3-2*a*ln(b*x+a)/b^3-1/3*I*po
lylog(2,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^3-2*I*a^2*polylog(2,-I*(
1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^3+1/3*I*polylog(2,I*(1/(b*x+a)+I*(1-
1/(b*x+a)^2)^(1/2)))/b^3+2*I*a^2*polylog(2,I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^
(1/2)))/b^3+2*a*(b*x+a)*arcsec(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/b^3-1/3*(b*x+a
)^2*arcsec(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/b^3
```

3.28.2 Mathematica [A] (warning: unable to verify)

Time = 4.59 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.64

$$\int x^2 \sec^{-1}(a+bx)^2 \, dx$$
$$= \frac{4+2(1+6a^2)\sec^{-1}(a+bx)^2+\frac{\sec^{-1}(a+bx)(2+(-1+6a)\sec^{-1}(a+bx))}{-1+\sqrt{1-\frac{1}{(a+bx)^2}}}+24a\log\left(\frac{1}{a+bx}\right)+2(-1-6a^2)\left((\pi-2\,\mathrm{se}\right.}{}$$

input
```
Integrate[x^2*ArcSec[a + b*x]^2,x]
```

output
```
(4 + 2*(1 + 6*a^2)*ArcSec[a + b*x]^2 + (ArcSec[a + b*x]*(2 + (-1 + 6*a)*Ar
cSec[a + b*x]))/(-1 + Sqrt[1 - (a + b*x)^(-2)]) + 24*a*Log[(a + b*x)^(-1)]
 + 2*(-1 - 6*a^2)*((Pi - 2*ArcSec[a + b*x])*(Log[1 - I/E^(I*ArcSec[a + b*x
])] - Log[1 + I/E^(I*ArcSec[a + b*x])]) - Pi*Log[Cot[(Pi + 2*ArcSec[a + b*
x])/4]] + (2*I)*(PolyLog[2, (-I)/E^(I*ArcSec[a + b*x])] - PolyLog[2, I/E^(
I*ArcSec[a + b*x])])) + (2*ArcSec[a + b*x]^2*Sin[ArcSec[a + b*x]/2])/(Cos[
ArcSec[a + b*x]/2] - Sin[ArcSec[a + b*x]/2])^3 + (2*(2 + 12*a*ArcSec[a + b
*x] + (1 + 6*a^2)*ArcSec[a + b*x]^2)*Sin[ArcSec[a + b*x]/2])/(Cos[ArcSec[a
 + b*x]/2] - Sin[ArcSec[a + b*x]/2]) - (2*ArcSec[a + b*x]^2*Sin[ArcSec[a +
 b*x]/2])/(Cos[ArcSec[a + b*x]/2] + Sin[ArcSec[a + b*x]/2])^3 + (ArcSec[a
+ b*x]*(2 + (1 - 6*a)*ArcSec[a + b*x]))/(Cos[ArcSec[a + b*x]/2] + Sin[ArcS
ec[a + b*x]/2])^2 - (2*(2 - 12*a*ArcSec[a + b*x] + (1 + 6*a^2)*ArcSec[a +
b*x]^2)*Sin[ArcSec[a + b*x]/2])/(Cos[ArcSec[a + b*x]/2] + Sin[ArcSec[a + b
*x]/2]))/(12*b^3)
```

3.28.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5781, 4926, 3042, 4678, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sec^{-1}(a+bx)^2 \, dx$$

$$\Big\downarrow \text{5781}$$

$$\frac{\int b^2x^2(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^2d\sec^{-1}(a+bx)}{b^3}$$

$$\Big\downarrow \text{4926}$$

$$\frac{\frac{2}{3}\int -b^3x^3\sec^{-1}(a+bx)d\sec^{-1}(a+bx)+\frac{1}{3}b^3x^3\sec^{-1}(a+bx)^2}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2}{3}\int \sec^{-1}(a+bx)\left(a-\csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)\right)^3d\sec^{-1}(a+bx)+\frac{1}{3}b^3x^3\sec^{-1}(a+bx)^2}{b^3}$$

$$\Big\downarrow \text{4678}$$

$$\frac{\frac{2}{3}\int \left(\sec^{-1}(a+bx)a^3-3(a+bx)\sec^{-1}(a+bx)a^2+3(a+bx)^2\sec^{-1}(a+bx)a-(a+bx)^3\sec^{-1}(a+bx)\right)d\sec^{-1}}{b^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}b^3x^3\sec^{-1}(a+bx)^2+\frac{2}{3}\left(\frac{1}{2}a^3\sec^{-1}(a+bx)^2+6ia^2\sec^{-1}(a+bx)\arctan\left(e^{i\sec^{-1}(a+bx)}\right)-3ia^2\,\text{PolyLog}\left(2,-ie^{i}\right.\right.$$

input
```
Int[x^2*ArcSec[a + b*x]^2,x]
```

output
```
((b^3*x^3*ArcSec[a + b*x]^2)/3 + (2*((a + b*x)/2 + 3*a*(a + b*x)*Sqrt[1 -
(a + b*x)^(-2)]*ArcSec[a + b*x] - ((a + b*x)^2*Sqrt[1 - (a + b*x)^(-2)]*Ar
cSec[a + b*x])/2 + (a^3*ArcSec[a + b*x]^2)/2 + I*ArcSec[a + b*x]*ArcTan[E^
(I*ArcSec[a + b*x])] + (6*I)*a^2*ArcSec[a + b*x]*ArcTan[E^(I*ArcSec[a + b*
x])] + 3*a*Log[(a + b*x)^(-1)] - (I/2)*PolyLog[2, (-I)*E^(I*ArcSec[a + b*x
])] - (3*I)*a^2*PolyLog[2, (-I)*E^(I*ArcSec[a + b*x])] + (I/2)*PolyLog[2,
I*E^(I*ArcSec[a + b*x])] + (3*I)*a^2*PolyLog[2, I*E^(I*ArcSec[a + b*x])]))
/3)/b^3
```

3.28.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4678
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 4926
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(e + f*
x)^m*((a + b*Sec[c + d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n +
 1)))   Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5781
```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d^(m + 1)   Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d
*e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.28.4 Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.73

method	result
derivativedivides	$\frac{\operatorname{arcsec}(bx+a)^2 a^2 (bx+a) - \operatorname{arcsec}(bx+a)^2 a (bx+a)^2 + \frac{\operatorname{arcsec}(bx+a)^2 (bx+a)^3}{3} + 2 \operatorname{arcsec}(bx+a) \sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}\, a(bx+a) - \frac{\operatorname{arcse}}{}}{}$
default	$\frac{\operatorname{arcsec}(bx+a)^2 a^2 (bx+a) - \operatorname{arcsec}(bx+a)^2 a (bx+a)^2 + \frac{\operatorname{arcsec}(bx+a)^2 (bx+a)^3}{3} + 2 \operatorname{arcsec}(bx+a) \sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}\, a(bx+a) - \frac{\operatorname{arcse}}{}}{}$

input
```
int(x^2*arcsec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(arcsec(b*x+a)^2*a^2*(b*x+a)-arcsec(b*x+a)^2*a*(b*x+a)^2+1/3*arcsec(
b*x+a)^2*(b*x+a)^3+2*arcsec(b*x+a)*(((b*x+a)^2-1)/(b*x+a)^2)^(1/2)*a*(b*x+
a)-1/3*arcsec(b*x+a)*(((b*x+a)^2-1)/(b*x+a)^2)^(1/2)*(b*x+a)^2+2*I*dilog(1
-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))*a^2+1/3*b*x+1/3*a+1/3*arcsec(b*x+a
)*ln(1+I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))-1/3*arcsec(b*x+a)*ln(1-I*(1/
(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))+1/3*I*dilog(1-I*(1/(b*x+a)+I*(1-1/(b*x+a
)^2)^(1/2)))+2*I*arcsec(b*x+a)*a+2*ln(1+(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)
)^2)*a-4*ln(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))*a-2*I*dilog(1+I*(1/(b*x+a)+
I*(1-1/(b*x+a)^2)^(1/2)))*a^2-1/3*I*dilog(1+I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)
^(1/2)))+2*ln(1+I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))*a^2*arcsec(b*x+a)-2
*ln(1-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))*a^2*arcsec(b*x+a))
```

3.28.5 Fricas [F]

$$\int x^2 \sec^{-1}(a+bx)^2\,dx = \int x^2 \operatorname{arcsec}(bx+a)^2\,dx$$

input
```
integrate(x^2*arcsec(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(x^2*arcsec(b*x + a)^2, x)
```

3.28.6 Sympy [F]

$$\int x^2 \sec^{-1}(a+bx)^2\,dx = \int x^2 \operatorname{asec}^2(a+bx)\,dx$$

input
```
integrate(x**2*asec(b*x+a)**2,x)
```

output
```
Integral(x**2*asec(a + b*x)**2, x)
```

3.28.7 Maxima [F]

$$\int x^2 \sec^{-1}(a+bx)^2\,dx = \int x^2 \operatorname{arcsec}(bx+a)^2\,dx$$

input
```
integrate(x^2*arcsec(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/3*x^3*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^2 - 1/12*x^3*log(b^2*x
^2 + 2*a*b*x + a^2)^2 - integrate(1/3*(2*sqrt(b*x + a + 1)*sqrt(b*x + a -
1)*b*x^3*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + 3*(b^3*x^5 + 3*a*b^
2*x^4 + (3*a^2 - 1)*b*x^3 + (a^3 - a)*x^2)*log(b*x + a)^2 - (b^3*x^5 + 2*a
*b^2*x^4 + (a^2 - 1)*b*x^3 + 3*(b^3*x^5 + 3*a*b^2*x^4 + (3*a^2 - 1)*b*x^3
+ (a^3 - a)*x^2)*log(b*x + a))*log(b^2*x^2 + 2*a*b*x + a^2))/(b^3*x^3 + 3*
a*b^2*x^2 + a^3 + (3*a^2 - 1)*b*x - a), x)
```

3.28.8 Giac [F]

$$\int x^2 \sec^{-1}(a+bx)^2\,dx = \int x^2 \operatorname{arcsec}(bx+a)^2\,dx$$

input
```
integrate(x^2*arcsec(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^2*arcsec(b*x + a)^2, x)
```

3.28.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \sec^{-1}(a+bx)^2 \, dx = \int x^2 \operatorname{acos}\left(\frac{1}{a+b\,x}\right)^2 \, dx$$

input
```
int(x^2*acos(1/(a + b*x))^2,x)
```

output
```
int(x^2*acos(1/(a + b*x))^2, x)
```

3.29 $\int x \sec^{-1}(a+bx)^2 \, dx$

3.29.1 Optimal result

Integrand size = 10, antiderivative size = 154

$$\int x \sec^{-1}(a+bx)^2 \, dx = -\frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{b^2} - \frac{a^2\sec^{-1}(a+bx)^2}{2b^2} + \frac{1}{2}x^2\sec^{-1}(a+bx)^2 - \frac{4ia\sec^{-1}(a+bx)\arctan\left(e^{i\sec^{-1}(a+bx)}\right)}{b^2} + \frac{\log(a+bx)}{b^2} + \frac{2ia\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(a+bx)}\right)}{b^2} - \frac{2ia\,\text{PolyLog}\left(2,ie^{i\sec^{-1}(a+bx)}\right)}{b^2}$$

output
```
-1/2*a^2*arcsec(b*x+a)^2/b^2+1/2*x^2*arcsec(b*x+a)^2-4*I*a*arcsec(b*x+a)*a
rctan(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/b^2+ln(b*x+a)/b^2+2*I*a*polylog(2
,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^2-2*I*a*polylog(2,I*(1/(b*x+a)+
I*(1-1/(b*x+a)^2)^(1/2)))/b^2-(b*x+a)*arcsec(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/
b^2
```

3.29.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.94

$$\int x \sec^{-1}(a+bx)^2 \, dx$$
$$= \frac{-\left((a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)\right)-a(a+bx)\sec^{-1}(a+bx)^2+\frac{1}{2}(a+bx)^2\sec^{-1}(a+bx)^2-4ia\,\mathrm{se}}{}$$

input
```
Integrate[x*ArcSec[a + b*x]^2,x]
```

output
```
(-((a + b*x)*Sqrt[1 - (a + b*x)^(-2)]*ArcSec[a + b*x]) - a*(a + b*x)*ArcSe
c[a + b*x]^2 + ((a + b*x)^2*ArcSec[a + b*x]^2)/2 - (4*I)*a*ArcSec[a + b*x]
*ArcTan[E^(I*ArcSec[a + b*x])] + Log[a + b*x] + (2*I)*a*PolyLog[2, (-I)*E^
(I*ArcSec[a + b*x])] - (2*I)*a*PolyLog[2, I*E^(I*ArcSec[a + b*x])])/b^2
```

3.29.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5781, 25, 4926, 3042, 4678, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sec^{-1}(a+bx)^2 \, dx$$

$$\Big\downarrow \ 5781$$

$$\frac{\int bx(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^2 d\sec^{-1}(a+bx)}{b^2}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int -bx(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^2 d\sec^{-1}(a+bx)}{b^2}$$

$$\Big\downarrow \ 4926$$

$$\frac{\frac{1}{2}b^2x^2\sec^{-1}(a+bx)^2-\int b^2x^2\sec^{-1}(a+bx)d\sec^{-1}(a+bx)}{b^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{1}{2}b^2x^2\sec^{-1}(a+bx)^2-\int \sec^{-1}(a+bx)\left(a-\csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)\right)^2d\sec^{-1}(a+bx)}{b^2}$$

$$\Big\downarrow \text{4678}$$

$$\frac{\frac{1}{2}b^2x^2\sec^{-1}(a+bx)^2-\int \left(\sec^{-1}(a+bx)a^2-2(a+bx)\sec^{-1}(a+bx)a+(a+bx)^2\sec^{-1}(a+bx)\right)d\sec^{-1}(a+bx}{b^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{2}a^2\sec^{-1}(a+bx)^2-4ia\sec^{-1}(a+bx)\arctan\left(e^{i\sec^{-1}(a+bx)}\right)+\frac{1}{2}b^2x^2\sec^{-1}(a+bx)^2+2ia\,\text{PolyLog}\left(2,-ie^{i\sec}\right.}{b^2}$$

input
```
Int[x*ArcSec[a + b*x]^2,x]
```

output
```
(-((a + b*x)*Sqrt[1 - (a + b*x)^(-2)]*ArcSec[a + b*x]) - (a^2*ArcSec[a + b
*x]^2)/2 + (b^2*x^2*ArcSec[a + b*x]^2)/2 - (4*I)*a*ArcSec[a + b*x]*ArcTan[
E^(I*ArcSec[a + b*x])] - Log[(a + b*x)^(-1)] + (2*I)*a*PolyLog[2, (-I)*E^(
I*ArcSec[a + b*x])] - (2*I)*a*PolyLog[2, I*E^(I*ArcSec[a + b*x])])/b^2
```

3.29.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4678
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 4926
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(e + f*
x)^m*((a + b*Sec[c + d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n +
 1)))   Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5781
```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d^(m + 1)   Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d
*e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.29.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.46

method	result
derivativedivides	$\frac{-a\left(\operatorname{arcsec}(bx+a)^2(bx+a)+2\operatorname{arcsec}(bx+a)\ln\left(1+i\left(\frac{1}{bx+a}+i\sqrt{1-\frac{1}{(bx+a)^2}}\right)\right)-2\operatorname{arcsec}(bx+a)\ln\left(1-i\left(\frac{1}{bx+a}+i\sqrt{1-\right.\right.\right.}{}$
default	$\frac{-a\left(\operatorname{arcsec}(bx+a)^2(bx+a)+2\operatorname{arcsec}(bx+a)\ln\left(1+i\left(\frac{1}{bx+a}+i\sqrt{1-\frac{1}{(bx+a)^2}}\right)\right)-2\operatorname{arcsec}(bx+a)\ln\left(1-i\left(\frac{1}{bx+a}+i\sqrt{1-\right.\right.\right.}{}$

input
```
int(x*arcsec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(-a*(arcsec(b*x+a)^2*(b*x+a)+2*arcsec(b*x+a)*ln(1+I*(1/(b*x+a)+I*(1-
1/(b*x+a)^2)^(1/2)))-2*arcsec(b*x+a)*ln(1-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(
1/2)))-2*I*dilog(1+I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))+2*I*dilog(1-I*(1
/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))))+1/2*arcsec(b*x+a)^2*(b*x+a)^2-arcsec(b
*x+a)*(((b*x+a)^2-1)/(b*x+a)^2)^(1/2)*(b*x+a)-ln(1/(b*x+a)))
```

3.29.5 Fricas [F]

$$\int x\sec^{-1}(a+bx)^2\,dx=\int x\,\mathrm{arcsec}\left(bx+a\right)^2\,dx$$

input
```
integrate(x*arcsec(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(x*arcsec(b*x + a)^2, x)
```

3.29.6 Sympy [F]

$$\int x \sec^{-1}(a+bx)^2 \, dx = \int x \operatorname{asec}^2(a+bx) \, dx$$

input
```
integrate(x*asec(b*x+a)**2,x)
```

output
```
Integral(x*asec(a + b*x)**2, x)
```

3.29.7 Maxima [F]

$$\int x \sec^{-1}(a+bx)^2 \, dx = \int x \operatorname{arcsec}(bx+a)^2 \, dx$$

input
```
integrate(x*arcsec(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*x^2*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^2 - 1/8*x^2*log(b^2*x^
2 + 2*a*b*x + a^2)^2 - integrate(1/2*(2*sqrt(b*x + a + 1)*sqrt(b*x + a - 1
)*b*x^2*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + 2*(b^3*x^4 + 3*a*b^2
*x^3 + (3*a^2 - 1)*b*x^2 + (a^3 - a)*x)*log(b*x + a)^2 - (b^3*x^4 + 2*a*b^
2*x^3 + (a^2 - 1)*b*x^2 + 2*(b^3*x^4 + 3*a*b^2*x^3 + (3*a^2 - 1)*b*x^2 + (
a^3 - a)*x)*log(b*x + a))*log(b^2*x^2 + 2*a*b*x + a^2))/(b^3*x^3 + 3*a*b^2
*x^2 + a^3 + (3*a^2 - 1)*b*x - a), x)
```

3.29.8 Giac [F]

$$\int x \sec^{-1}(a+bx)^2 \, dx = \int x \operatorname{arcsec}(bx+a)^2 \, dx$$

input
```
integrate(x*arcsec(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x*arcsec(b*x + a)^2, x)
```

3.29.9 Mupad [F(-1)]

Timed out.

$$\int x\sec^{-1}(a+bx)^2\,dx = \int x\,\mathrm{acos}\left(\frac{1}{a+b\,x}\right)^2\,dx$$

input
```
int(x*acos(1/(a + b*x))^2,x)
```

output
```
int(x*acos(1/(a + b*x))^2, x)
```

3.30 $\int \sec^{-1}(a+bx)^2\,dx$

3.30.1 Optimal result

Integrand size = 8, antiderivative size = 94

$$\int \sec^{-1}(a+bx)^2\,dx = \frac{(a+bx)\sec^{-1}(a+bx)^2}{b} + \frac{4i\sec^{-1}(a+bx)\arctan\left(e^{i\sec^{-1}(a+bx)}\right)}{b} - \frac{2i\,\mathrm{PolyLog}\left(2,-ie^{i\sec^{-1}(a+bx)}\right)}{b} + \frac{2i\,\mathrm{PolyLog}\left(2,ie^{i\sec^{-1}(a+bx)}\right)}{b}$$

output
```
(b*x+a)*arcsec(b*x+a)^2/b+4*I*arcsec(b*x+a)*arctan(1/(b*x+a)+I*(1-1/(b*x+a
)^2)^(1/2))/b-2*I*polylog(2,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b+2*I*
polylog(2,I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b
```

3.30.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.18

$$\int \sec^{-1}(a+bx)^2\,dx$$
$$= \frac{\sec^{-1}(a+bx)\left((a+bx)\sec^{-1}(a+bx) - 2\log\left(1-ie^{i\sec^{-1}(a+bx)}\right) + 2\log\left(1+ie^{i\sec^{-1}(a+bx)}\right)\right) - 2i\,\mathrm{PolyL}}{b}$$

input
```
Integrate[ArcSec[a + b*x]^2,x]
```

output
```
(ArcSec[a + b*x]*((a + b*x)*ArcSec[a + b*x] - 2*Log[1 - I*E^(I*ArcSec[a +
b*x])] + 2*Log[1 + I*E^(I*ArcSec[a + b*x])]) - (2*I)*PolyLog[2, (-I)*E^(I*
ArcSec[a + b*x])] + (2*I)*PolyLog[2, I*E^(I*ArcSec[a + b*x])])/b
```

3.30.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5775, 5739, 4244, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{-1}(a+bx)^2\,dx$$

$$\Big\downarrow \text{5775}$$

$$\frac{\int \sec^{-1}(a+bx)^2 d(a+bx)}{b}$$

$$\Big\downarrow \text{5739}$$

$$\frac{\int (a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^2 d\sec^{-1}(a+bx)}{b}$$

$$\Big\downarrow \text{4244}$$

$$\frac{(a+bx)\sec^{-1}(a+bx)^2-2\int (a+bx)\sec^{-1}(a+bx)d\sec^{-1}(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a+bx)\sec^{-1}(a+bx)^2-2\int \sec^{-1}(a+bx)\csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)d\sec^{-1}(a+bx)}{b}$$

$$\Big\downarrow \text{4669}$$

$$\frac{(a+bx)\sec^{-1}(a+bx)^2-2\left(-\int \log\left(1-ie^{i\sec^{-1}(a+bx)}\right)d\sec^{-1}(a+bx)+\int \log\left(1+ie^{i\sec^{-1}(a+bx)}\right)d\sec^{-1}(a+b}{b}$$

$$\Big\downarrow \text{2715}$$

$$\frac{(a+bx)\sec^{-1}(a+bx)^2-2\left(i\int e^{-i\sec^{-1}(a+bx)}\log\left(1-ie^{i\sec^{-1}(a+bx)}\right)de^{i\sec^{-1}(a+bx)}-i\int e^{-i\sec^{-1}(a+bx)}\log\left(1+i\right.\right.}{b}$$

$$\Big\downarrow \text{2838}$$

$$\frac{(a+bx)\sec^{-1}(a+bx)^2 - 2\left(-2i\sec^{-1}(a+bx)\arctan\left(e^{i\sec^{-1}(a+bx)}\right) + i\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(a+bx)}\right) - i\,\text{PolyLo}\right.}{b}$$

input
```
Int[ArcSec[a + b*x]^2,x]
```

output
```
((a + b*x)*ArcSec[a + b*x]^2 - 2*((-2*I)*ArcSec[a + b*x]*ArcTan[E^(I*ArcSe
c[a + b*x])] + I*PolyLog[2, (-I)*E^(I*ArcSec[a + b*x])] - I*PolyLog[2, I*E
^(I*ArcSec[a + b*x])]))/b
```

3.30.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4244
```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^p/(b*n*p)), x]
 - Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Fre
eQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5739
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/c   Subst[
Int[(a + b*x)^n*Sec[x]*Tan[x], x], x, ArcSec[c*x]], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[n, 0]
```

rule 5775
```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[1/d
  Subst[Int[(a + b*ArcSec[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
 x] && IGtQ[p, 0]
```

3.30.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.72

method	result
derivativedivides	$\frac{\operatorname{arcsec}(bx+a)^2(bx+a)+2\operatorname{arcsec}(bx+a)\ln\left(1+i\left(\frac{1}{bx+a}+i\sqrt{1-\frac{1}{(bx+a)^2}}\right)\right)-2\operatorname{arcsec}(bx+a)\ln\left(1-i\left(\frac{1}{bx+a}+i\sqrt{1-\frac{1}{(bx+}}\right.\right.}{b}$
default	$\frac{\operatorname{arcsec}(bx+a)^2(bx+a)+2\operatorname{arcsec}(bx+a)\ln\left(1+i\left(\frac{1}{bx+a}+i\sqrt{1-\frac{1}{(bx+a)^2}}\right)\right)-2\operatorname{arcsec}(bx+a)\ln\left(1-i\left(\frac{1}{bx+a}+i\sqrt{1-\frac{1}{(bx+}}\right.\right.}{b}$

input
```
int(arcsec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(arcsec(b*x+a)^2*(b*x+a)+2*arcsec(b*x+a)*ln(1+I*(1/(b*x+a)+I*(1-1/(b*x
+a)^2)^(1/2)))-2*arcsec(b*x+a)*ln(1-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))
-2*I*dilog(1+I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))+2*I*dilog(1-I*(1/(b*x+
a)+I*(1-1/(b*x+a)^2)^(1/2))))
```

3.30.5 Fricas [F]

$$\int \sec^{-1}(a+bx)^2\,dx = \int \operatorname{arcsec}(bx+a)^2\,dx$$

input
```
integrate(arcsec(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(arcsec(b*x + a)^2, x)
```

3.30.6 Sympy [F]

$$\int \sec^{-1}(a+bx)^2\,dx = \int \operatorname{asec}^2(a+bx)\,dx$$

input
```
integrate(asec(b*x+a)**2,x)
```

output
```
Integral(asec(a + b*x)**2, x)
```

3.30.7 Maxima [F]

$$\int \sec^{-1}(a+bx)^2\,dx = \int \operatorname{arcsec}(bx+a)^2\,dx$$

input
```
integrate(arcsec(b*x+a)^2,x, algorithm="maxima")
```

output
```
x*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^2 - 1/4*x*log(b^2*x^2 + 2*a*
b*x + a^2)^2 - integrate((2*sqrt(b*x + a + 1)*sqrt(b*x + a - 1)*b*x*arctan
(sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*
a^2 - 1)*b*x - a)*log(b*x + a)^2 - (b^3*x^3 + 2*a*b^2*x^2 + (a^2 - 1)*b*x
+ (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2 - 1)*b*x - a)*log(b*x + a))*log(b^
2*x^2 + 2*a*b*x + a^2))/(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2 - 1)*b*x - a
), x)
```

3.30.8 Giac [F]

$$\int \sec^{-1}(a+bx)^2\,dx = \int \operatorname{arcsec}(bx+a)^2\,dx$$

input
```
integrate(arcsec(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(arcsec(b*x + a)^2, x)
```

3.30.9 Mupad [F(-1)]

Timed out.

$$\int \sec^{-1}(a+bx)^2\,dx = \int \mathrm{acos}\left(\frac{1}{a+b\,x}\right)^2\,dx$$

input
```
int(acos(1/(a + b*x))^2,x)
```

output
```
int(acos(1/(a + b*x))^2, x)
```

3.31 $\int \frac{\sec^{-1}(a+bx)^2}{x}\,dx$

3.31.1 Optimal result

Integrand size = 12, antiderivative size = 310

$$\begin{aligned}\int \frac{\sec^{-1}(a+bx)^2}{x}\,dx &= \sec^{-1}(a+bx)^2\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)\\
&+\sec^{-1}(a+bx)^2\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)\\
&-\sec^{-1}(a+bx)^2\log\left(1+e^{2i\sec^{-1}(a+bx)}\right)\\
&-2i\sec^{-1}(a+bx)\,\text{PolyLog}\left(2,\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)\\
&-2i\sec^{-1}(a+bx)\,\text{PolyLog}\left(2,\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)\\
&+i\sec^{-1}(a+bx)\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)\\
&+2\,\text{PolyLog}\left(3,\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)+2\,\text{PolyLog}\left(3,\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)\\
&-\frac{1}{2}\text{PolyLog}\left(3,-e^{2i\sec^{-1}(a+bx)}\right)\end{aligned}$$

output
```
-arcsec(b*x+a)^2*ln(1+(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)+arcsec(b*x+a)
^2*ln(1-a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2)))+arcsec(b
*x+a)^2*ln(1-a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1+(-a^2+1)^(1/2)))+I*a
rcsec(b*x+a)*polylog(2,-(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)-2*I*arcsec(
b*x+a)*polylog(2,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2)))
-2*I*arcsec(b*x+a)*polylog(2,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1+(-a^
2+1)^(1/2)))-1/2*polylog(3,-(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)+2*polyl
og(3,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2)))+2*polylog(3
,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1+(-a^2+1)^(1/2)))
```

3.31.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 813 vs. $2(310) = 620$.

Time = 1.82 (sec) , antiderivative size = 813, normalized size of antiderivative = 2.62

$$\int \frac{\sec^{-1}(a+bx)^2}{x}\,dx = \sec^{-1}(a+bx)^2 \log\left(1+\frac{ae^{i\sec^{-1}(a+bx)}}{-1+\sqrt{1-a^2}}\right) + \sec^{-1}(a+bx)^2 \log\left(1+\frac{\left(-1+\sqrt{1-a^2}\right)e^{i\sec^{-1}(a+bx)}}{a}\right) - 4\sec^{-1}(a+bx)\arcsin\left(\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right)\log\left(1+\frac{\left(-1+\sqrt{1-a^2}\right)e^{i\sec^{-1}(a+bx)}}{a}\right) + \sec^{-1}(a+bx)^2 \log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) + \sec^{-1}(a+bx)^2 \log\left(1-\frac{\left(1+\sqrt{1-a^2}\right)e^{i\sec^{-1}(a+bx)}}{a}\right) + 4\sec^{-1}(a+bx)\arcsin\left(\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right)\log\left(1-\frac{\left(1+\sqrt{1-a^2}\right)e^{i\sec^{-1}(a+bx)}}{a}\right) - 2\sec^{-1}(a+bx)^2\log\left(1+e^{2i\sec^{-1}(a+bx)}\right) + \sec^{-1}(a+bx)^2\log\left(\frac{2\left(\frac{1}{a+bx}+i\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{a+bx}\right) - \sec^{-1}(a+bx)^2\log\left(1+\frac{\left(-1+\sqrt{1-a^2}\right)\left(\frac{1}{a+bx}+i\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{a}\right) + 4\sec^{-1}(a+bx)\arcsin\left(\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right)\log\left(1+\frac{\left(-1+\sqrt{1-a^2}\right)\left(\frac{1}{a+bx}+i\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{a}\right) - \sec^{-1}(a+bx)^2\log\left(1-\frac{\left(1+\sqrt{1-a^2}\right)\left(\frac{1}{a+bx}+i\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{a}\right) - 4\sec^{-1}(a+bx)\arcsin\left(\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right)\log\left(1-\frac{\left(1+\sqrt{1-a^2}\right)\left(\frac{1}{a+bx}+i\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{a}\right) - 2i\sec^{-1}(a+bx)\,\mathrm{PolyLog}\left(2,-\frac{ae^{i\sec^{-1}(a+bx)}}{-1+\sqrt{1-a^2}}\right) - 2i\sec^{-1}(a+bx)\,\mathrm{PolyLog}\left(2,\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)$$

input
```
Integrate[ArcSec[a + b*x]^2/x,x]
```

output
```
ArcSec[a + b*x]^2*Log[1 + (a*E^(I*ArcSec[a + b*x]))/(-1 + Sqrt[1 - a^2])]
+ ArcSec[a + b*x]^2*Log[1 + ((-1 + Sqrt[1 - a^2])*E^(I*ArcSec[a + b*x]))/a
] - 4*ArcSec[a + b*x]*ArcSin[Sqrt[(-1 + a)/a]/Sqrt[2]]*Log[1 + ((-1 + Sqrt
[1 - a^2])*E^(I*ArcSec[a + b*x]))/a] + ArcSec[a + b*x]^2*Log[1 - (a*E^(I*A
rcSec[a + b*x]))/(1 + Sqrt[1 - a^2])] + ArcSec[a + b*x]^2*Log[1 - ((1 + Sq
rt[1 - a^2])*E^(I*ArcSec[a + b*x]))/a] + 4*ArcSec[a + b*x]*ArcSin[Sqrt[(-1
 + a)/a]/Sqrt[2]]*Log[1 - ((1 + Sqrt[1 - a^2])*E^(I*ArcSec[a + b*x]))/a] -
 2*ArcSec[a + b*x]^2*Log[1 + E^((2*I)*ArcSec[a + b*x])] + ArcSec[a + b*x]^
2*Log[(2*((a + b*x)^(-1) + I*Sqrt[1 - (a + b*x)^(-2)]))/(a + b*x)] - ArcSe
c[a + b*x]^2*Log[1 + ((-1 + Sqrt[1 - a^2])*((a + b*x)^(-1) + I*Sqrt[1 - (a
 + b*x)^(-2)]))/a] + 4*ArcSec[a + b*x]*ArcSin[Sqrt[(-1 + a)/a]/Sqrt[2]]*Lo
g[1 + ((-1 + Sqrt[1 - a^2])*((a + b*x)^(-1) + I*Sqrt[1 - (a + b*x)^(-2)]))
/a] - ArcSec[a + b*x]^2*Log[1 - ((1 + Sqrt[1 - a^2])*((a + b*x)^(-1) + I*S
qrt[1 - (a + b*x)^(-2)]))/a] - 4*ArcSec[a + b*x]*ArcSin[Sqrt[(-1 + a)/a]/S
qrt[2]]*Log[1 - ((1 + Sqrt[1 - a^2])*((a + b*x)^(-1) + I*Sqrt[1 - (a + b*x
)^(-2)]))/a] - (2*I)*ArcSec[a + b*x]*PolyLog[2, -((a*E^(I*ArcSec[a + b*x])
)/(-1 + Sqrt[1 - a^2]))] - (2*I)*ArcSec[a + b*x]*PolyLog[2, (a*E^(I*ArcSec
[a + b*x]))/(1 + Sqrt[1 - a^2])] + I*ArcSec[a + b*x]*PolyLog[2, -E^((2*I)*
ArcSec[a + b*x])] + 2*PolyLog[3, -((a*E^(I*ArcSec[a + b*x]))/(-1 + Sqrt[1
- a^2]))] + 2*PolyLog[3, (a*E^(I*ArcSec[a + b*x]))/(1 + Sqrt[1 - a^2])]...
```

3.31.3 Rubi [A] (verified)

Time = 1.60 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.28, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {5781, 25, 5062, 5041, 25, 3042, 4202, 2620, 3011, 2720, 5031, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{-1}(a+bx)^2}{x}\,dx$$

$$\Big\downarrow \; 5781$$

$$\int \frac{(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^2}{bx}d\sec^{-1}(a+bx)$$

$$\Big\downarrow \; 25$$

$$-\int -\frac{(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^2}{bx}d\sec^{-1}(a+bx)$$

$\downarrow$ 5062

$$-\int \frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^2}{\frac{a}{a+bx}-1}d\sec^{-1}(a+bx)$$

$\downarrow$ 5041

$$\int (a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^2d\sec^{-1}(a+bx)-a\int -\frac{\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^2}{1-\frac{a}{a+bx}}d\sec^{-1}(a+bx)$$

$\downarrow$ 25

$$\int (a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^2d\sec^{-1}(a+bx)+a\int \frac{\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^2}{1-\frac{a}{a+bx}}d\sec^{-1}(a+bx)$$

$\downarrow$ 3042

$$a\int \frac{\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^2}{1-\frac{a}{a+bx}}d\sec^{-1}(a+bx)+\int \sec^{-1}(a+bx)^2\tan\left(\sec^{-1}(a+bx)\right)d\sec^{-1}(a+bx)$$

$\downarrow$ 4202

$$-2i\int \frac{e^{2i\sec^{-1}(a+bx)}\sec^{-1}(a+bx)^2}{1+e^{2i\sec^{-1}(a+bx)}}d\sec^{-1}(a+bx)+a\int \frac{\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^2}{1-\frac{a}{a+bx}}d\sec^{-1}(a+bx)+\frac{1}{3}i\sec^{-1}(a+bx)^3$$

$\downarrow$ 2620

$$a\int \frac{\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^2}{1-\frac{a}{a+bx}}d\sec^{-1}(a+bx)-2i\left(i\int \sec^{-1}(a+bx)\log\left(1+e^{2i\sec^{-1}(a+bx)}\right)d\sec^{-1}(a+bx)-\frac{1}{2}i\sec^{-1}(a+bx)^2\log\left(1+e^{2i\sec^{-1}(a+bx)}\right)\right)+\frac{1}{3}i\sec^{-1}(a+bx)^3$$

$\downarrow$ 3011

3.31. $\int \frac{\sec^{-1}(a+bx)^2}{x}\,dx$

$$-2i\left(i\left(\frac{1}{2}i\sec^{-1}(a+bx)\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)-\frac{1}{2}i\int \operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)d\sec^{-1}(a+bx)\right)-\frac{1}{2}\right.$$
$$a\int\frac{\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^2}{1-\frac{a}{a+bx}}d\sec^{-1}(a+bx)+\frac{1}{3}i\sec^{-1}(a+bx)^3$$

$\downarrow$ 2720

$$-2i\left(i\left(\frac{1}{2}i\sec^{-1}(a+bx)\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)-\frac{1}{4}\int e^{-2i\sec^{-1}(a+bx)}\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)de^{2i\sec^{-}}\right.\right.$$
$$a\int\frac{\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^2}{1-\frac{a}{a+bx}}d\sec^{-1}(a+bx)+\frac{1}{3}i\sec^{-1}(a+bx)^3$$

$\downarrow$ 5031

$$a\left(-i\int\frac{e^{i\sec^{-1}(a+bx)}\sec^{-1}(a+bx)^2}{-e^{i\sec^{-1}(a+bx)}a-\sqrt{1-a^2}+1}d\sec^{-1}(a+bx)-i\int\frac{e^{i\sec^{-1}(a+bx)}\sec^{-1}(a+bx)^2}{-e^{i\sec^{-1}(a+bx)}a+\sqrt{1-a^2}+1}d\sec^{-1}(a+bx)-\frac{i\,s}{}\right.$$
$$2i\left(i\left(\frac{1}{2}i\sec^{-1}(a+bx)\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)-\frac{1}{4}\int e^{-2i\sec^{-1}(a+bx)}\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)de^{2i\sec^{-1}(}\right.\right.$$
$$\frac{1}{3}i\sec^{-1}(a+bx)^3$$

$\downarrow$ 2620

$$a\left(-i\left(\frac{i\sec^{-1}(a+bx)^2\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a}-\frac{2i\int\sec^{-1}(a+bx)\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)d\sec^{-1}(a+bx)}{a}\right)-i\right.$$
$$2i\left(i\left(\frac{1}{2}i\sec^{-1}(a+bx)\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)-\frac{1}{4}\int e^{-2i\sec^{-1}(a+bx)}\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)de^{2i\sec^{-1}(}\right.\right.$$
$$\frac{1}{3}i\sec^{-1}(a+bx)^3$$

$\downarrow$ 3011

$$a\left(-i\left(\frac{i\sec^{-1}(a+bx)^2\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a}-\frac{2i\left(i\sec^{-1}(a+bx)\operatorname{PolyLog}\left(2,\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)-i\int\operatorname{PolyLog}\left(2,\right.\right.}{a}\right.\right.$$
$$2i\left(i\left(\frac{1}{2}i\sec^{-1}(a+bx)\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)-\frac{1}{4}\int e^{-2i\sec^{-1}(a+bx)}\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)de^{2i\sec^{-1}(}\right.\right.$$
$$\frac{1}{3}i\sec^{-1}(a+bx)^3$$

$\downarrow$ 2720

3.31. $\int\frac{\sec^{-1}(a+bx)^2}{x}\,dx$

$$a\left(-i\left(\frac{i\sec^{-1}(a+bx)^2\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a}-\frac{2i\left(i\sec^{-1}(a+bx)\operatorname{PolyLog}\left(2,\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)-\int e^{-i\sec^{-1}(a+bx)}\right.}{a}\right.\right.$$

$$2i\left(i\left(\frac{1}{2}i\sec^{-1}(a+bx)\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)-\frac{1}{4}\int e^{-2i\sec^{-1}(a+bx)}\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)de^{2i\sec^{-1}(}\right.\right.$$

$$\frac{1}{3}i\sec^{-1}(a+bx)^3$$

$$\Big\downarrow \text{7143}$$

$$a\left(-i\left(\frac{i\sec^{-1}(a+bx)^2\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a}-\frac{2i\left(i\sec^{-1}(a+bx)\operatorname{PolyLog}\left(2,\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)-\operatorname{PolyLog}\left(3,\frac{ae^{i}}{}\right.\right.}{a}\right.\right.$$

$$2i\left(i\left(\frac{1}{2}i\sec^{-1}(a+bx)\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)-\frac{1}{4}\operatorname{PolyLog}\left(3,-e^{2i\sec^{-1}(a+bx)}\right)\right)-\frac{1}{2}i\sec^{-1}(a+bx)^2\log\left(\right.\right.$$

$$\frac{1}{3}i\sec^{-1}(a+bx)^3$$

input
```
Int[ArcSec[a + b*x]^2/x,x]
```

output
```
(I/3)*ArcSec[a + b*x]^3 + a*(((-1/3*I)*ArcSec[a + b*x]^3)/a - I*((I*ArcSec
[a + b*x]^2*Log[1 - (a*E^(I*ArcSec[a + b*x]))/(1 - Sqrt[1 - a^2])])/a - ((
2*I)*(I*ArcSec[a + b*x]*PolyLog[2, (a*E^(I*ArcSec[a + b*x]))/(1 - Sqrt[1 -
 a^2])] - PolyLog[3, (a*E^(I*ArcSec[a + b*x]))/(1 - Sqrt[1 - a^2])]))/a) -
 I*((I*ArcSec[a + b*x]^2*Log[1 - (a*E^(I*ArcSec[a + b*x]))/(1 + Sqrt[1 - a
^2])])/a - ((2*I)*(I*ArcSec[a + b*x]*PolyLog[2, (a*E^(I*ArcSec[a + b*x]))/
(1 + Sqrt[1 - a^2])] - PolyLog[3, (a*E^(I*ArcSec[a + b*x]))/(1 + Sqrt[1 -
a^2])]))/a)) - (2*I)*((-1/2*I)*ArcSec[a + b*x]^2*Log[1 + E^((2*I)*ArcSec[a
 + b*x])] + I*((I/2)*ArcSec[a + b*x]*PolyLog[2, -E^((2*I)*ArcSec[a + b*x])
] - PolyLog[3, -E^((2*I)*ArcSec[a + b*x])]/4))
```

3.31. $\int \frac{\sec^{-1}(a+bx)^2}{x}\,dx$

3.31.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5031
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[I*((e + f*x)^(m + 1)/(b*f*(m + 1)))
, x] + (-Simp[I   Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] +
b*E^(I*(c + d*x)))), x], x] - Simp[I   Int[(e + f*x)^m*(E^(I*(c + d*x))/(a
+ Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

rule 5041
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tan[(c_.) + (d_.)*(x_)]^(n_.))/(Cos[(c_.) +
 (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Tan[c +
 d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*Sin[c + d*x]*(Tan[c + d*x]^(n
- 1)/(a + b*Cos[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[
m, 0] && IGtQ[n, 0]
```

rule 5062
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sec[(c_.) + (d_.)*(x_)]), x_Symbol] :> In
t[(e + f*x)^m*Cos[c + d*x]*F[c + d*x]^n*(G[c + d*x]^p/(b + a*Cos[c + d*x]))
, x] /; FreeQ[{a, b, c, d, e, f}, x] && TrigQ[F] && TrigQ[G] && IntegersQ[m
, n, p]
```

rule 5781
```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d^(m + 1)   Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d
*e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.31.4 Maple [F]

$$\int \frac{\operatorname{arcsec}(bx+a)^2}{x}dx$$

input
```
int(arcsec(b*x+a)^2/x,x)
```

output
```
int(arcsec(b*x+a)^2/x,x)
```

3.31.5 Fricas [F]

$$\int \frac{\sec^{-1}(a+bx)^2}{x}\,dx = \int \frac{\operatorname{arcsec}\left(bx+a\right)^2}{x}\,dx$$

input
```
integrate(arcsec(b*x+a)^2/x,x, algorithm="fricas")
```

output
```
integral(arcsec(b*x + a)^2/x, x)
```

3.31.6 Sympy [F]

$$\int \frac{\sec^{-1}(a+bx)^2}{x}\,dx = \int \frac{\operatorname{asec}^2\left(a+bx\right)}{x}\,dx$$

input
```
integrate(asec(b*x+a)**2/x,x)
```

output
```
Integral(asec(a + b*x)**2/x, x)
```

3.31.7 Maxima [F]

$$\int \frac{\sec^{-1}(a+bx)^2}{x}\,dx = \int \frac{\operatorname{arcsec}\left(bx+a\right)^2}{x}\,dx$$

input
```
integrate(arcsec(b*x+a)^2/x,x, algorithm="maxima")
```

output
```
integrate(arcsec(b*x + a)^2/x, x)
```

3.31.8 Giac [F]

$$\int \frac{\sec^{-1}(a+bx)^2}{x} \, dx = \int \frac{\operatorname{arcsec}(bx+a)^2}{x} \, dx$$

input
```
integrate(arcsec(b*x+a)^2/x,x, algorithm="giac")
```

output
```
integrate(arcsec(b*x + a)^2/x, x)
```

3.31.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{-1}(a+bx)^2}{x} \, dx = \int \frac{\operatorname{acos}\left(\frac{1}{a+b\,x}\right)^2}{x} \, dx$$

input
```
int(acos(1/(a + b*x))^2/x,x)
```

output
```
int(acos(1/(a + b*x))^2/x, x)
```

3.32 $\int \frac{\sec^{-1}(a+bx)^2}{x^2}\,dx$

3.32.1 Optimal result

Integrand size = 12, antiderivative size = 244

$$\begin{aligned}\int \frac{\sec^{-1}(a+bx)^2}{x^2}\,dx = &-\frac{b\sec^{-1}(a+bx)^2}{a}-\frac{\sec^{-1}(a+bx)^2}{x}\\ &-\frac{2ib\sec^{-1}(a+bx)\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}\\ &+\frac{2ib\sec^{-1}(a+bx)\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}\\ &-\frac{2b\,\text{PolyLog}\left(2,\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}+\frac{2b\,\text{PolyLog}\left(2,\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}\end{aligned}$$

output
```
-b*arcsec(b*x+a)^2/a-arcsec(b*x+a)^2/x-2*I*b*arcsec(b*x+a)*ln(1-a*(1/(b*x+
a)+I*(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2)))/a/(-a^2+1)^(1/2)+2*I*b*arc
sec(b*x+a)*ln(1-a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1+(-a^2+1)^(1/2)))/
a/(-a^2+1)^(1/2)-2*b*polylog(2,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1-(-
a^2+1)^(1/2)))/a/(-a^2+1)^(1/2)+2*b*polylog(2,a*(1/(b*x+a)+I*(1-1/(b*x+a)^
2)^(1/2))/(1+(-a^2+1)^(1/2)))/a/(-a^2+1)^(1/2)
```

3.32.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 686 vs. $2(244) = 488$.

Time = 1.73 (sec) , antiderivative size = 686, normalized size of antiderivative = 2.81

$$\int \frac{\sec^{-1}(a+bx)^2}{x^2}\,dx =$$

$$-\frac{\frac{(a+bx)\sec^{-1}(a+bx)^2}{x} + \frac{2b\left(2\sec^{-1}(a+bx)\operatorname{arctanh}\left(\frac{(-1+a)\cot\left(\frac{1}{2}\sec^{-1}(a+bx)\right)}{\sqrt{-1+a^2}}\right) - 2\arccos\left(\frac{1}{a}\right)\operatorname{arctanh}\left(\frac{(1+a)\tan\left(\frac{1}{2}\sec^{-1}(a+bx)\right)}{\sqrt{-1+a^2}}\right) +}{}}{}$$

input
```
Integrate[ArcSec[a + b*x]^2/x^2,x]
```

output
```
-((((a + b*x)*ArcSec[a + b*x]^2)/x + (2*b*(2*ArcSec[a + b*x]*ArcTanh[((-1
+ a)*Cot[ArcSec[a + b*x]/2])/Sqrt[-1 + a^2]] - 2*ArcCos[a^(-1)]*ArcTanh[((
1 + a)*Tan[ArcSec[a + b*x]/2])/Sqrt[-1 + a^2]] + (ArcCos[a^(-1)] - (2*I)*A
rcTanh[((-1 + a)*Cot[ArcSec[a + b*x]/2])/Sqrt[-1 + a^2]] + (2*I)*ArcTanh[(
(1 + a)*Tan[ArcSec[a + b*x]/2])/Sqrt[-1 + a^2]])*Log[Sqrt[-1 + a^2]/(Sqrt[
2]*Sqrt[a]*E^((I/2)*ArcSec[a + b*x])*Sqrt[-((b*x)/(a + b*x))])] + (ArcCos[
a^(-1)] + (2*I)*(ArcTanh[((-1 + a)*Cot[ArcSec[a + b*x]/2])/Sqrt[-1 + a^2]]
 - ArcTanh[((1 + a)*Tan[ArcSec[a + b*x]/2])/Sqrt[-1 + a^2]]))*Log[(Sqrt[-1
 + a^2]*E^((I/2)*ArcSec[a + b*x]))/(Sqrt[2]*Sqrt[a]*Sqrt[-((b*x)/(a + b*x)
)])] - (ArcCos[a^(-1)] - (2*I)*ArcTanh[((1 + a)*Tan[ArcSec[a + b*x]/2])/Sq
rt[-1 + a^2]])*Log[((-1 + a)*(I + I*a + Sqrt[-1 + a^2])*(-I + Tan[ArcSec[a
 + b*x]/2]))/(a*(-1 + a + Sqrt[-1 + a^2]*Tan[ArcSec[a + b*x]/2]))] - (ArcC
os[a^(-1)] + (2*I)*ArcTanh[((1 + a)*Tan[ArcSec[a + b*x]/2])/Sqrt[-1 + a^2]
])*Log[((-1 + a)*(-I - I*a + Sqrt[-1 + a^2])*(I + Tan[ArcSec[a + b*x]/2]))
/(a*(-1 + a + Sqrt[-1 + a^2]*Tan[ArcSec[a + b*x]/2]))] + I*(-PolyLog[2, ((
1 - I*Sqrt[-1 + a^2])*(1 - a + Sqrt[-1 + a^2]*Tan[ArcSec[a + b*x]/2]))/(a*
(-1 + a + Sqrt[-1 + a^2]*Tan[ArcSec[a + b*x]/2]))] + PolyLog[2, ((1 + I*Sq
rt[-1 + a^2])*(1 - a + Sqrt[-1 + a^2]*Tan[ArcSec[a + b*x]/2]))/(a*(-1 + a
+ Sqrt[-1 + a^2]*Tan[ArcSec[a + b*x]/2]))])))/Sqrt[-1 + a^2])/a)
```

3.32.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5781, 4926, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{-1}(a+bx)^2}{x^2} \, dx$$

$$\Big\downarrow \text{5781}$$

$$b \int \frac{(a+bx)^2 \sqrt{1-\frac{1}{(a+bx)^2}} \sec^{-1}(a+bx)^2}{b^2x^2} d\sec^{-1}(a+bx)$$

$$\Big\downarrow \text{4926}$$

$$b\left(-2\int -\frac{\sec^{-1}(a+bx)}{bx}d\sec^{-1}(a+bx)-\frac{\sec^{-1}(a+bx)^2}{bx}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(-2\int \frac{\sec^{-1}(a+bx)}{a-\csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)}d\sec^{-1}(a+bx)-\frac{\sec^{-1}(a+bx)^2}{bx}\right)$$

$$\Big\downarrow \text{4679}$$

$$b\left(-2\int \left(\frac{\sec^{-1}(a+bx)}{a}+\frac{\sec^{-1}(a+bx)}{a\left(\frac{a}{a+bx}-1\right)}\right)d\sec^{-1}(a+bx)-\frac{\sec^{-1}(a+bx)^2}{bx}\right)$$

$$\Big\downarrow \text{2009}$$

$$b\left(-\frac{\sec^{-1}(a+bx)^2}{bx}-2\left(\frac{\text{PolyLog}\left(2,\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}-\frac{\text{PolyLog}\left(2,\frac{ae^{i\sec^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a\sqrt{1-a^2}}+\frac{i\sec^{-1}(a+bx)\log\left(1-\frac{ae^{i s}}{1\cdots}\right.}{a\sqrt{1-a^2}}\right.\right.$$

input
```
Int[ArcSec[a + b*x]^2/x^2,x]
```

output
```
b*(-(ArcSec[a + b*x]^2/(b*x)) - 2*(ArcSec[a + b*x]^2/(2*a) + (I*ArcSec[a +
 b*x]*Log[1 - (a*E^(I*ArcSec[a + b*x]))/(1 - Sqrt[1 - a^2])])/(a*Sqrt[1 -
a^2]) - (I*ArcSec[a + b*x]*Log[1 - (a*E^(I*ArcSec[a + b*x]))/(1 + Sqrt[1 -
 a^2])])/(a*Sqrt[1 - a^2]) + PolyLog[2, (a*E^(I*ArcSec[a + b*x]))/(1 - Sqr
t[1 - a^2])]/(a*Sqrt[1 - a^2]) - PolyLog[2, (a*E^(I*ArcSec[a + b*x]))/(1 +
 Sqrt[1 - a^2])]/(a*Sqrt[1 - a^2])))
```

3.32.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4679
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

rule 4926
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(e + f*
x)^m*((a + b*Sec[c + d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n +
 1)))   Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5781
```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d^(m + 1)   Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d
*e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.32.4 Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.38

method	result
derivativedivides	$b\left(-\frac{(bx+a)\,\text{arcsec}(bx+a)^2}{abx}-\frac{2i\sqrt{-a^2+1}\,\text{arcsec}(bx+a)\ln\left(\frac{-a\left(\frac{1}{bx+a}+i\sqrt{1-\frac{1}{(bx+a)^2}}\right)+\sqrt{-a^2+1}+1}{1+\sqrt{-a^2+1}}\right)}{a(a^2-1)}+\frac{2i\sqrt{-a^2+}}{}\right.$
default	$b\left(-\frac{(bx+a)\,\text{arcsec}(bx+a)^2}{abx}-\frac{2i\sqrt{-a^2+1}\,\text{arcsec}(bx+a)\ln\left(\frac{-a\left(\frac{1}{bx+a}+i\sqrt{1-\frac{1}{(bx+a)^2}}\right)+\sqrt{-a^2+1}+1}{1+\sqrt{-a^2+1}}\right)}{a(a^2-1)}+\frac{2i\sqrt{-a^2+}}{}\right.$

input
```
int(arcsec(b*x+a)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
b*(-(b*x+a)*arcsec(b*x+a)^2/a/b/x-2*I*(-a^2+1)^(1/2)/a/(a^2-1)*arcsec(b*x+
a)*ln((-a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))+(-a^2+1)^(1/2)+1)/(1+(-a^2+1
)^(1/2)))+2*I*(-a^2+1)^(1/2)/a/(a^2-1)*arcsec(b*x+a)*ln((a*(1/(b*x+a)+I*(1
-1/(b*x+a)^2)^(1/2))+(-a^2+1)^(1/2)-1)/(-1+(-a^2+1)^(1/2)))-2*(-a^2+1)^(1/
2)/a/(a^2-1)*dilog((-a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))+(-a^2+1)^(1/2)+
1)/(1+(-a^2+1)^(1/2)))+2*(-a^2+1)^(1/2)/a/(a^2-1)*dilog((a*(1/(b*x+a)+I*(1
-1/(b*x+a)^2)^(1/2))+(-a^2+1)^(1/2)-1)/(-1+(-a^2+1)^(1/2))))
```

3.32.5 Fricas [F]

$$\int \frac{\sec^{-1}(a+bx)^2}{x^2}\,dx=\int \frac{\text{arcsec}\left(bx+a\right)^2}{x^2}\,dx$$

input
```
integrate(arcsec(b*x+a)^2/x^2,x, algorithm="fricas")
```

output
```
integral(arcsec(b*x + a)^2/x^2, x)
```

3.32.6 Sympy [F]

$$\int \frac{\sec^{-1}(a+bx)^2}{x^2} dx = \int \frac{\operatorname{asec}^2(a+bx)}{x^2} dx$$

input
```
integrate(asec(b*x+a)**2/x**2,x)
```

output
```
Integral(asec(a + b*x)**2/x**2, x)
```

3.32.7 Maxima [F]

$$\int \frac{\sec^{-1}(a+bx)^2}{x^2} dx = \int \frac{\operatorname{arcsec}(bx+a)^2}{x^2} dx$$

input
```
integrate(arcsec(b*x+a)^2/x^2,x, algorithm="maxima")
```

output
```
-1/4*(4*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^2 - 4*x*integrate((2*s
qrt(b*x + a + 1)*sqrt(b*x + a - 1)*b*x*arctan(sqrt(b*x + a + 1)*sqrt(b*x +
 a - 1)) - (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2 - 1)*b*x - a)*log(b*x + a
)^2 - (b^3*x^3 + 2*a*b^2*x^2 + (a^2 - 1)*b*x - (b^3*x^3 + 3*a*b^2*x^2 + a^
3 + (3*a^2 - 1)*b*x - a)*log(b*x + a))*log(b^2*x^2 + 2*a*b*x + a^2))/(b^3*
x^5 + 3*a*b^2*x^4 + (3*a^2 - 1)*b*x^3 + (a^3 - a)*x^2), x) - log(b^2*x^2 +
 2*a*b*x + a^2)^2)/x
```

3.32.8 Giac [F]

$$\int \frac{\sec^{-1}(a+bx)^2}{x^2} dx = \int \frac{\operatorname{arcsec}(bx+a)^2}{x^2} dx$$

input
```
integrate(arcsec(b*x+a)^2/x^2,x, algorithm="giac")
```

output
```
integrate(arcsec(b*x + a)^2/x^2, x)
```

3.32.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{-1}(a+bx)^2}{x^2}\,dx = \int \frac{\operatorname{acos}\left(\frac{1}{a+b\,x}\right)^2}{x^2}\,dx$$

input
```
int(acos(1/(a + b*x))^2/x^2,x)
```

output
```
int(acos(1/(a + b*x))^2/x^2, x)
```

3.33 $\int x^2 \sec^{-1}(a+bx)^3 \, dx$

3.33.1 Optimal result

Integrand size = 12, antiderivative size = 494

$$\int x^2 \sec^{-1}(a+bx)^3 \, dx = \frac{(a+bx)\sec^{-1}(a+bx)}{b^3} - \frac{3ia\sec^{-1}(a+bx)^2}{b^3} + \frac{3a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^2}{b^3} - \frac{(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^2}{2b^3} + \frac{a^3\sec^{-1}(a+bx)^3}{3b^3} + \frac{1}{3}x^3\sec^{-1}(a+bx)^3 + \frac{i\sec^{-1}(a+bx)^2\arctan\left(e^{i\sec^{-1}(a+bx)}\right)}{b^3} + \frac{6ia^2\sec^{-1}(a+bx)^2\arctan\left(e^{i\sec^{-1}(a+bx)}\right)}{b^3} - \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{b^3} + \frac{6a\sec^{-1}(a+bx)\log\left(1+e^{2i\sec^{-1}(a+bx)}\right)}{b^3} - \frac{i\sec^{-1}(a+bx)\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(a+bx)}\right)}{b^3} - \frac{6ia^2\sec^{-1}(a+bx)\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(a+bx)}\right)}{b^3} + \frac{i\sec^{-1}(a+bx)\,\text{PolyLog}\left(2,ie^{i\sec^{-1}(a+bx)}\right)}{b^3} + \frac{6ia^2\sec^{-1}(a+bx)\,\text{PolyLog}\left(2,ie^{i\sec^{-1}(a+bx)}\right)}{b^3} - \frac{3ia\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)}{b^3} + \frac{\text{PolyLog}\left(3,-ie^{i\sec^{-1}(a+bx)}\right)}{b^3} + \frac{6a^2\,\text{PolyLog}\left(3,-ie^{i\sec^{-1}(a+bx)}\right)}{b^3} - \frac{\text{PolyLog}\left(3,ie^{i\sec^{-1}(a+bx)}\right)}{b^3} - \frac{6a^2\,\text{PolyLog}\left(3,ie^{i\sec^{-1}(a+bx)}\right)}{b^3}$$

output
```
(b*x+a)*arcsec(b*x+a)/b^3-I*arcsec(b*x+a)*polylog(2,-I*(1/(b*x+a)+I*(1-1/(
b*x+a)^2)^(1/2)))/b^3+1/3*a^3*arcsec(b*x+a)^3/b^3+1/3*x^3*arcsec(b*x+a)^3-
3*I*a*polylog(2,-(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)/b^3+I*arcsec(b*x+a
)^2*arctan(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/b^3-arctanh((1-1/(b*x+a)^2)^
(1/2))/b^3+6*a*arcsec(b*x+a)*ln(1+(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)/b
^3+I*arcsec(b*x+a)*polylog(2,I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^3+6*
I*a^2*arcsec(b*x+a)^2*arctan(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/b^3-6*I*a^
2*arcsec(b*x+a)*polylog(2,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^3-3*I*
a*arcsec(b*x+a)^2/b^3+6*I*a^2*arcsec(b*x+a)*polylog(2,I*(1/(b*x+a)+I*(1-1/
(b*x+a)^2)^(1/2)))/b^3+polylog(3,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b
^3+6*a^2*polylog(3,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^3-polylog(3,I
*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^3-6*a^2*polylog(3,I*(1/(b*x+a)+I*(
1-1/(b*x+a)^2)^(1/2)))/b^3+3*a*(b*x+a)*arcsec(b*x+a)^2*(1-1/(b*x+a)^2)^(1/
2)/b^3-1/2*(b*x+a)^2*arcsec(b*x+a)^2*(1-1/(b*x+a)^2)^(1/2)/b^3
```

3.33.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 446, normalized size of antiderivative = 0.90

$$\int x^2 \sec^{-1}(a+bx)^3 \, dx$$
$$= \frac{(a+bx)\sec^{-1}(a+bx) - 3ia\sec^{-1}(a+bx)^2 + 3a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^2 - \frac{1}{2}(a+bx)^2\sqrt{1-}}{}$$

input
```
Integrate[x^2*ArcSec[a + b*x]^3,x]
```

output
```
((a + b*x)*ArcSec[a + b*x] - (3*I)*a*ArcSec[a + b*x]^2 + 3*a*(a + b*x)*Sqr
t[1 - (a + b*x)^(-2)]*ArcSec[a + b*x]^2 - ((a + b*x)^2*Sqrt[1 - (a + b*x)^
(-2)]*ArcSec[a + b*x]^2)/2 + (a^3*ArcSec[a + b*x]^3)/3 + (b^3*x^3*ArcSec[a
 + b*x]^3)/3 + I*ArcSec[a + b*x]^2*ArcTan[E^(I*ArcSec[a + b*x])] + (6*I)*a
^2*ArcSec[a + b*x]^2*ArcTan[E^(I*ArcSec[a + b*x])] - ArcTanh[Sqrt[1 - (a +
 b*x)^(-2)]] + 6*a*ArcSec[a + b*x]*Log[1 + E^((2*I)*ArcSec[a + b*x])] - I*
ArcSec[a + b*x]*PolyLog[2, (-I)*E^(I*ArcSec[a + b*x])] - (6*I)*a^2*ArcSec[
a + b*x]*PolyLog[2, (-I)*E^(I*ArcSec[a + b*x])] + I*ArcSec[a + b*x]*PolyLo
g[2, I*E^(I*ArcSec[a + b*x])] + (6*I)*a^2*ArcSec[a + b*x]*PolyLog[2, I*E^(
I*ArcSec[a + b*x])] - (3*I)*a*PolyLog[2, -E^((2*I)*ArcSec[a + b*x])] + Pol
yLog[3, (-I)*E^(I*ArcSec[a + b*x])] + 6*a^2*PolyLog[3, (-I)*E^(I*ArcSec[a
+ b*x])] - PolyLog[3, I*E^(I*ArcSec[a + b*x])] - 6*a^2*PolyLog[3, I*E^(I*A
rcSec[a + b*x])])/b^3
```

3.33.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 446, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5781, 4926, 3042, 4678, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sec^{-1}(a+bx)^3\,dx$$

$$\Big\downarrow \text{ 5781}$$

$$\frac{\int b^2x^2(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^3d\sec^{-1}(a+bx)}{b^3}$$

$$\Big\downarrow \text{ 4926}$$

$$\frac{\int -b^3x^3\sec^{-1}(a+bx)^2d\sec^{-1}(a+bx)+\frac{1}{3}b^3x^3\sec^{-1}(a+bx)^3}{b^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \sec^{-1}(a+bx)^2\left(a-\csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)\right)^3d\sec^{-1}(a+bx)+\frac{1}{3}b^3x^3\sec^{-1}(a+bx)^3}{b^3}$$

$$\Big\downarrow \text{ 4678}$$

$$\frac{\int \left(\sec^{-1}(a+bx)^2a^3-3(a+bx)\sec^{-1}(a+bx)^2a^2+3(a+bx)^2\sec^{-1}(a+bx)^2a-(a+bx)^3\sec^{-1}(a+bx)^2\right)d\sec}{b^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}a^3\sec^{-1}(a+bx)^3+6ia^2\sec^{-1}(a+bx)^2\arctan\left(e^{i\sec^{-1}(a+bx)}\right)-6ia^2\sec^{-1}(a+bx)\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(a+bx)}\right)$$

input
```
Int[x^2*ArcSec[a + b*x]^3,x]
```

output
```
((a + b*x)*ArcSec[a + b*x] - (3*I)*a*ArcSec[a + b*x]^2 + 3*a*(a + b*x)*Sqr
t[1 - (a + b*x)^(-2)]*ArcSec[a + b*x]^2 - ((a + b*x)^2*Sqrt[1 - (a + b*x)^
(-2)]*ArcSec[a + b*x]^2)/2 + (a^3*ArcSec[a + b*x]^3)/3 + (b^3*x^3*ArcSec[a
 + b*x]^3)/3 + I*ArcSec[a + b*x]^2*ArcTan[E^(I*ArcSec[a + b*x])] + (6*I)*a
^2*ArcSec[a + b*x]^2*ArcTan[E^(I*ArcSec[a + b*x])] - ArcTanh[Sqrt[1 - (a +
 b*x)^(-2)]] + 6*a*ArcSec[a + b*x]*Log[1 + E^((2*I)*ArcSec[a + b*x])] - I*
ArcSec[a + b*x]*PolyLog[2, (-I)*E^(I*ArcSec[a + b*x])] - (6*I)*a^2*ArcSec[
a + b*x]*PolyLog[2, (-I)*E^(I*ArcSec[a + b*x])] + I*ArcSec[a + b*x]*PolyLo
g[2, I*E^(I*ArcSec[a + b*x])] + (6*I)*a^2*ArcSec[a + b*x]*PolyLog[2, I*E^(
I*ArcSec[a + b*x])] - (3*I)*a*PolyLog[2, -E^((2*I)*ArcSec[a + b*x])] + Pol
yLog[3, (-I)*E^(I*ArcSec[a + b*x])] + 6*a^2*PolyLog[3, (-I)*E^(I*ArcSec[a
+ b*x])] - PolyLog[3, I*E^(I*ArcSec[a + b*x])] - 6*a^2*PolyLog[3, I*E^(I*A
rcSec[a + b*x])])/b^3
```

3.33.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4678
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 4926
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(e + f*
x)^m*((a + b*Sec[c + d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n +
 1)))   Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5781
```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d^(m + 1)   Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d
*e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.33.4 Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 716, normalized size of antiderivative = 1.45

method	result
derivativedivides	$\frac{\frac{\operatorname{arcsec}(bx+a)\left(6\operatorname{arcsec}(bx+a)^2a^2(bx+a)-6\operatorname{arcsec}(bx+a)^2a(bx+a)^2+2\operatorname{arcsec}(bx+a)^2(bx+a)^3+18\operatorname{arcsec}(bx+a)\sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}\,a(bx+\right.}{6}}{}$
default	$\frac{\frac{\operatorname{arcsec}(bx+a)\left(6\operatorname{arcsec}(bx+a)^2a^2(bx+a)-6\operatorname{arcsec}(bx+a)^2a(bx+a)^2+2\operatorname{arcsec}(bx+a)^2(bx+a)^3+18\operatorname{arcsec}(bx+a)\sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}\,a(bx+\right.}{6}}{}$

input
```
int(x^2*arcsec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/6*arcsec(b*x+a)*(6*arcsec(b*x+a)^2*a^2*(b*x+a)-6*arcsec(b*x+a)^2*
a*(b*x+a)^2+2*arcsec(b*x+a)^2*(b*x+a)^3+18*arcsec(b*x+a)*(((b*x+a)^2-1)/(b
*x+a)^2)^(1/2)*a*(b*x+a)-3*arcsec(b*x+a)*(((b*x+a)^2-1)/(b*x+a)^2)^(1/2)*(
b*x+a)^2+18*I*a*arcsec(b*x+a)+6*b*x+6*a)+2*I*arctan(1/(b*x+a)+I*(1-1/(b*x+
a)^2)^(1/2))+I*arcsec(b*x+a)*polylog(2,I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2
)))+3*ln(1+I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))*a^2*arcsec(b*x+a)^2+6*po
lylog(3,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))*a^2-I*arcsec(b*x+a)*polylo
g(2,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))-3*ln(1-I*(1/(b*x+a)+I*(1-1/(b*
x+a)^2)^(1/2)))*a^2*arcsec(b*x+a)^2+6*I*polylog(2,I*(1/(b*x+a)+I*(1-1/(b*x
+a)^2)^(1/2)))*a^2*arcsec(b*x+a)-6*polylog(3,I*(1/(b*x+a)+I*(1-1/(b*x+a)^2
)^(1/2)))*a^2+6*ln(1+(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)*a*arcsec(b*x+a
)-6*I*polylog(2,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))*a^2*arcsec(b*x+a)-
3*I*polylog(2,-(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)*a+1/2*arcsec(b*x+a)^
2*ln(1+I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))-6*I*arcsec(b*x+a)^2*a+polylo
g(3,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))-1/2*arcsec(b*x+a)^2*ln(1-I*(1/
(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))-polylog(3,I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)
^(1/2))))
```

3.33.5 Fricas [F]

$$\int x^2\sec^{-1}(a+bx)^3\,dx=\int x^2\operatorname{arcsec}(bx+a)^3\,dx$$

input
```
integrate(x^2*arcsec(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(x^2*arcsec(b*x + a)^3, x)
```

3.33.6 Sympy [F]

$$\int x^2 \sec^{-1}(a+bx)^3\,dx = \int x^2 \operatorname{asec}^3(a+bx)\,dx$$

input
```
integrate(x**2*asec(b*x+a)**3,x)
```

output
```
Integral(x**2*asec(a + b*x)**3, x)
```

3.33.7 Maxima [F]

$$\int x^2 \sec^{-1}(a+bx)^3\,dx = \int x^2 \operatorname{arcsec}(bx+a)^3\,dx$$

input
```
integrate(x^2*arcsec(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/3*x^3*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^3 - 1/4*x^3*arctan(sqr
t(b*x + a + 1)*sqrt(b*x + a - 1))*log(b^2*x^2 + 2*a*b*x + a^2)^2 - integra
te(1/4*((4*b*x^3*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^2 - b*x^3*log
(b^2*x^2 + 2*a*b*x + a^2)^2)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + 4*(3*(b
^3*x^5 + 3*a*b^2*x^4 + (3*a^2 - 1)*b*x^3 + (a^3 - a)*x^2)*log(b*x + a)^2 -
 (b^3*x^5 + 2*a*b^2*x^4 + (a^2 - 1)*b*x^3 + 3*(b^3*x^5 + 3*a*b^2*x^4 + (3*
a^2 - 1)*b*x^3 + (a^3 - a)*x^2)*log(b*x + a))*log(b^2*x^2 + 2*a*b*x + a^2)
)*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1)))/(b^3*x^3 + 3*a*b^2*x^2 + a^
3 + (3*a^2 - 1)*b*x - a), x)
```

3.33.8 Giac [F]

$$\int x^2 \sec^{-1}(a+bx)^3\,dx = \int x^2 \operatorname{arcsec}(bx+a)^3\,dx$$

input
```
integrate(x^2*arcsec(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x^2*arcsec(b*x + a)^3, x)
```

3.33.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \sec^{-1}(a+bx)^3\,dx = \int x^2 \operatorname{acos}\left(\frac{1}{a+b\,x}\right)^3\,dx$$

input
```
int(x^2*acos(1/(a + b*x))^3,x)
```

output
```
int(x^2*acos(1/(a + b*x))^3, x)
```

3.34 $\int x \sec^{-1}(a+bx)^3\,dx$

3.34.1 Optimal result

Integrand size = 10, antiderivative size = 278

$$\begin{aligned}\int x \sec^{-1}(a+bx)^3\,dx = {} & \frac{3i\sec^{-1}(a+bx)^2}{2b^2} - \frac{3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^2}{2b^2} \\ & - \frac{a^2\sec^{-1}(a+bx)^3}{2b^2} + \frac{1}{2}x^2\sec^{-1}(a+bx)^3 \\ & - \frac{6ia\sec^{-1}(a+bx)^2\arctan\left(e^{i\sec^{-1}(a+bx)}\right)}{b^2} \\ & - \frac{3\sec^{-1}(a+bx)\log\left(1+e^{2i\sec^{-1}(a+bx)}\right)}{b^2} \\ & + \frac{6ia\sec^{-1}(a+bx)\operatorname{PolyLog}\left(2,-ie^{i\sec^{-1}(a+bx)}\right)}{b^2} \\ & - \frac{6ia\sec^{-1}(a+bx)\operatorname{PolyLog}\left(2,ie^{i\sec^{-1}(a+bx)}\right)}{b^2} \\ & + \frac{3i\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)}{2b^2} \\ & - \frac{6a\operatorname{PolyLog}\left(3,-ie^{i\sec^{-1}(a+bx)}\right)}{b^2} + \frac{6a\operatorname{PolyLog}\left(3,ie^{i\sec^{-1}(a+bx)}\right)}{b^2}\end{aligned}$$

output
```
3/2*I*arcsec(b*x+a)^2/b^2-1/2*a^2*arcsec(b*x+a)^3/b^2+1/2*x^2*arcsec(b*x+a
)^3-6*I*a*arcsec(b*x+a)^2*arctan(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/b^2-3*
arcsec(b*x+a)*ln(1+(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)/b^2+6*I*a*arcsec
(b*x+a)*polylog(2,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^2-6*I*a*arcsec
(b*x+a)*polylog(2,I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^2+3/2*I*polylog
(2,-(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)/b^2-6*a*polylog(3,-I*(1/(b*x+a)
+I*(1-1/(b*x+a)^2)^(1/2)))/b^2+6*a*polylog(3,I*(1/(b*x+a)+I*(1-1/(b*x+a)^2
)^(1/2)))/b^2-3/2*(b*x+a)*arcsec(b*x+a)^2*(1-1/(b*x+a)^2)^(1/2)/b^2
```

3.34.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.92

$$\int x \sec^{-1}(a+bx)^3 \, dx$$
$$= \frac{\frac{3}{2} i \sec^{-1}(a+bx)^2 - \frac{3}{2}(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}} \sec^{-1}(a+bx)^2 - a(a+bx)\sec^{-1}(a+bx)^3 + \frac{1}{2}(a+bx)^2 \sec^{-1}(}{}$$

input
```
Integrate[x*ArcSec[a + b*x]^3,x]
```

output
```
(((3*I)/2)*ArcSec[a + b*x]^2 - (3*(a + b*x)*Sqrt[1 - (a + b*x)^(-2)]*ArcSe
c[a + b*x]^2)/2 - a*(a + b*x)*ArcSec[a + b*x]^3 + ((a + b*x)^2*ArcSec[a +
b*x]^3)/2 - (6*I)*a*ArcSec[a + b*x]^2*ArcTan[E^(I*ArcSec[a + b*x])] - 3*Ar
cSec[a + b*x]*Log[1 + E^((2*I)*ArcSec[a + b*x])] + (6*I)*a*ArcSec[a + b*x]
*PolyLog[2, (-I)*E^(I*ArcSec[a + b*x])] - (6*I)*a*ArcSec[a + b*x]*PolyLog[
2, I*E^(I*ArcSec[a + b*x])] + ((3*I)/2)*PolyLog[2, -E^((2*I)*ArcSec[a + b*
x])] - 6*a*PolyLog[3, (-I)*E^(I*ArcSec[a + b*x])] + 6*a*PolyLog[3, I*E^(I*
ArcSec[a + b*x])])/b^2
```

3.34.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5781, 25, 4926, 3042, 4678, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sec^{-1}(a+bx)^3 \, dx$$

$\Big\downarrow$ 5781

$$\frac{\int bx(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^3 d\sec^{-1}(a+bx)}{b^2}$$

$\Big\downarrow$ 25

$$-\frac{\int -bx(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^3 d\sec^{-1}(a+bx)}{b^2}$$

$\Big\downarrow$ 4926

$$\frac{\frac{1}{2}b^2x^2\sec^{-1}(a+bx)^3-\frac{3}{2}\int b^2x^2\sec^{-1}(a+bx)^2 d\sec^{-1}(a+bx)}{b^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{1}{2}b^2x^2\sec^{-1}(a+bx)^3-\frac{3}{2}\int \sec^{-1}(a+bx)^2\left(a-\csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)\right)^2 d\sec^{-1}(a+bx)}{b^2}$$

$\Big\downarrow$ 4678

$$\frac{\frac{1}{2}b^2x^2\sec^{-1}(a+bx)^3-\frac{3}{2}\int \left(a^2\sec^{-1}(a+bx)^2+(a+bx)^2\sec^{-1}(a+bx)^2-2a(a+bx)\sec^{-1}(a+bx)^2\right) d\sec^{-1}(a}{b^2}$$

$\Big\downarrow$ 2009

$$\frac{\frac{1}{2}b^2x^2\sec^{-1}(a+bx)^3-\frac{3}{2}\left(\frac{1}{3}a^2\sec^{-1}(a+bx)^3+4ia\sec^{-1}(a+bx)^2\arctan\left(e^{i\sec^{-1}(a+bx)}\right)-4ia\sec^{-1}(a+bx)\text{Pol}\right.}{}$$

input
```
Int[x*ArcSec[a + b*x]^3,x]
```

output
```
((b^2*x^2*ArcSec[a + b*x]^3)/2 - (3*((-I)*ArcSec[a + b*x]^2 + (a + b*x)*Sq
rt[1 - (a + b*x)^(-2)]*ArcSec[a + b*x]^2 + (a^2*ArcSec[a + b*x]^3)/3 + (4*
I)*a*ArcSec[a + b*x]^2*ArcTan[E^(I*ArcSec[a + b*x])] + 2*ArcSec[a + b*x]*L
og[1 + E^((2*I)*ArcSec[a + b*x])] - (4*I)*a*ArcSec[a + b*x]*PolyLog[2, (-I
)*E^(I*ArcSec[a + b*x])] + (4*I)*a*ArcSec[a + b*x]*PolyLog[2, I*E^(I*ArcSe
c[a + b*x])] - I*PolyLog[2, -E^((2*I)*ArcSec[a + b*x])] + 4*a*PolyLog[3, (
-I)*E^(I*ArcSec[a + b*x])] - 4*a*PolyLog[3, I*E^(I*ArcSec[a + b*x])]))/2)/
b^2
```

3.34. $\int x \sec^{-1}(a+bx)^3 \, dx$

3.34.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4678
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 4926
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(e + f*
x)^m*((a + b*Sec[c + d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n +
 1)))   Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5781
```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d^(m + 1)   Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d
*e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.34.4 Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.36

method	result
derivativedivides	$\frac{-\frac{\operatorname{arcsec}(bx+a)^2\left(2\operatorname{arcsec}(bx+a)a(bx+a)-\operatorname{arcsec}(bx+a)(bx+a)^2+3\sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}\,(bx+a)+3i\right)}{2}-3\ln\left(1+i\left(\frac{1}{bx+a}+i\sqrt{1-\frac{1}{(bx+a)^2}}\right.\right.}{}$
default	$\frac{-\frac{\operatorname{arcsec}(bx+a)^2\left(2\operatorname{arcsec}(bx+a)a(bx+a)-\operatorname{arcsec}(bx+a)(bx+a)^2+3\sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}\,(bx+a)+3i\right)}{2}-3\ln\left(1+i\left(\frac{1}{bx+a}+i\sqrt{1-\frac{1}{(bx+a)^2}}\right.\right.}{}$

input
```
int(x*arcsec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(-1/2*arcsec(b*x+a)^2*(2*arcsec(b*x+a)*a*(b*x+a)-arcsec(b*x+a)*(b*x+
a)^2+3*(((b*x+a)^2-1)/(b*x+a)^2)^(1/2)*(b*x+a)+3*I)-3*ln(1+I*(1/(b*x+a)+I*
(1-1/(b*x+a)^2)^(1/2)))*a*arcsec(b*x+a)^2+6*I*polylog(2,-I*(1/(b*x+a)+I*(1
-1/(b*x+a)^2)^(1/2)))*a*arcsec(b*x+a)-6*polylog(3,-I*(1/(b*x+a)+I*(1-1/(b*
x+a)^2)^(1/2)))*a+3*ln(1-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))*a*arcsec(b
*x+a)^2-6*I*polylog(2,I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))*a*arcsec(b*x+
a)+6*polylog(3,I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))*a+3*I*arcsec(b*x+a)^
2-3*arcsec(b*x+a)*ln(1+(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)+3/2*I*polylo
g(2,-(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2))
```

3.34.5 Fricas [F]

$$\int x \sec^{-1}(a+bx)^3\,dx=\int x\,\mathrm{arcsec}\left(bx+a\right)^3\,dx$$

input
```
integrate(x*arcsec(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(x*arcsec(b*x + a)^3, x)
```

3.34.6 Sympy [F]

$$\int x \sec^{-1}(a+bx)^3\,dx=\int x\,\mathrm{asec}^3\left(a+bx\right)dx$$

input
```
integrate(x*asec(b*x+a)**3,x)
```

output
```
Integral(x*asec(a + b*x)**3, x)
```

3.34.7 Maxima [F]

$$\int x \sec^{-1}(a+bx)^3 \, dx = \int x \operatorname{arcsec}(bx+a)^3 \, dx$$

input
```
integrate(x*arcsec(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/2*x^2*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^3 - 3/8*x^2*arctan(sqr
t(b*x + a + 1)*sqrt(b*x + a - 1))*log(b^2*x^2 + 2*a*b*x + a^2)^2 - integra
te(3/8*((4*b*x^2*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^2 - b*x^2*log
(b^2*x^2 + 2*a*b*x + a^2)^2)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + 4*(2*(b
^3*x^4 + 3*a*b^2*x^3 + (3*a^2 - 1)*b*x^2 + (a^3 - a)*x)*log(b*x + a)^2 - (
b^3*x^4 + 2*a*b^2*x^3 + (a^2 - 1)*b*x^2 + 2*(b^3*x^4 + 3*a*b^2*x^3 + (3*a^
2 - 1)*b*x^2 + (a^3 - a)*x)*log(b*x + a))*log(b^2*x^2 + 2*a*b*x + a^2))*ar
ctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1)))/(b^3*x^3 + 3*a*b^2*x^2 + a^3 +
(3*a^2 - 1)*b*x - a), x)
```

3.34.8 Giac [F]

$$\int x \sec^{-1}(a+bx)^3 \, dx = \int x \operatorname{arcsec}(bx+a)^3 \, dx$$

input
```
integrate(x*arcsec(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x*arcsec(b*x + a)^3, x)
```

3.34.9 Mupad [F(-1)]

Timed out.

$$\int x \sec^{-1}(a+bx)^3 \, dx = \int x \operatorname{acos}\left(\frac{1}{a+b\,x}\right)^3 \, dx$$

input
```
int(x*acos(1/(a + b*x))^3,x)
```

output
```
int(x*acos(1/(a + b*x))^3, x)
```

3.35 $\int \sec^{-1}(a+bx)^3\,dx$

3.35.1 Optimal result

Integrand size = 8, antiderivative size = 154

$$\int \sec^{-1}(a+bx)^3\,dx = \frac{(a+bx)\sec^{-1}(a+bx)^3}{b} + \frac{6i\sec^{-1}(a+bx)^2\arctan\left(e^{i\sec^{-1}(a+bx)}\right)}{b} - \frac{6i\sec^{-1}(a+bx)\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(a+bx)}\right)}{b} + \frac{6i\sec^{-1}(a+bx)\,\text{PolyLog}\left(2,ie^{i\sec^{-1}(a+bx)}\right)}{b} + \frac{6\,\text{PolyLog}\left(3,-ie^{i\sec^{-1}(a+bx)}\right)}{b} - \frac{6\,\text{PolyLog}\left(3,ie^{i\sec^{-1}(a+bx)}\right)}{b}$$

output

```
(b*x+a)*arcsec(b*x+a)^3/b+6*I*arcsec(b*x+a)^2*arctan(1/(b*x+a)+I*(1-1/(b*x
+a)^2)^(1/2))/b-6*I*arcsec(b*x+a)*polylog(2,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2
)^(1/2)))/b+6*I*arcsec(b*x+a)*polylog(2,I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/
2)))/b+6*polylog(3,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b-6*polylog(3,I
*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b
```

3.35.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.04

$$\int \sec^{-1}(a+bx)^3\,dx$$
$$= \frac{(a+bx)\sec^{-1}(a+bx)^3 - 3\sec^{-1}(a+bx)^2\left(\log\left(1-ie^{i\sec^{-1}(a+bx)}\right)-\log\left(1+ie^{i\sec^{-1}(a+bx)}\right)\right) - 6i\sec^{-1}}{}$$

input
```
Integrate[ArcSec[a + b*x]^3,x]
```

output
```
((a + b*x)*ArcSec[a + b*x]^3 - 3*ArcSec[a + b*x]^2*(Log[1 - I*E^(I*ArcSec[
a + b*x])] - Log[1 + I*E^(I*ArcSec[a + b*x])]) - (6*I)*ArcSec[a + b*x]*(Po
lyLog[2, (-I)*E^(I*ArcSec[a + b*x])] - PolyLog[2, I*E^(I*ArcSec[a + b*x])]
) + 6*(PolyLog[3, (-I)*E^(I*ArcSec[a + b*x])] - PolyLog[3, I*E^(I*ArcSec[a
 + b*x])]))/b
```

3.35.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5775, 5739, 4244, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{-1}(a+bx)^3\,dx$$
$$\Big\downarrow \text{ 5775}$$
$$\frac{\int \sec^{-1}(a+bx)^3d(a+bx)}{b}$$
$$\Big\downarrow \text{ 5739}$$
$$\frac{\int (a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^3d\sec^{-1}(a+bx)}{b}$$
$$\Big\downarrow \text{ 4244}$$
$$\frac{(a+bx)\sec^{-1}(a+bx)^3-3\int (a+bx)\sec^{-1}(a+bx)^2d\sec^{-1}(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a+bx)\sec^{-1}(a+bx)^3-3\int \sec^{-1}(a+bx)^2\csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)d\sec^{-1}(a+bx)}{b}$$

$$\Big\downarrow \text{4669}$$

$$\frac{(a+bx)\sec^{-1}(a+bx)^3-3\Big(-2\int \sec^{-1}(a+bx)\log\left(1-ie^{i\sec^{-1}(a+bx)}\right)d\sec^{-1}(a+bx)+2\int \sec^{-1}(a+bx)\log\Big(}{b}$$

$$\Big\downarrow \text{3011}$$

$$(a+bx)\sec^{-1}(a+bx)^3-3\Big(2\Big(i\sec^{-1}(a+bx)\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(a+bx)}\right)-i\int \text{PolyLog}\left(2,-ie^{i\sec^{-1}(a+bx)}\right)d\,\text{s}$$

$$\Big\downarrow \text{2720}$$

$$(a+bx)\sec^{-1}(a+bx)^3-3\Big(2\Big(i\sec^{-1}(a+bx)\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(a+bx)}\right)-\int e^{-i\sec^{-1}(a+bx)}\,\text{PolyLog}\Big(2,-ie^{i\,\text{se}}$$

$$\Big\downarrow \text{7143}$$

$$(a+bx)\sec^{-1}(a+bx)^3-3\Big(-2i\sec^{-1}(a+bx)^2\arctan\left(e^{i\sec^{-1}(a+bx)}\right)+2\Big(i\sec^{-1}(a+bx)\,\text{PolyLog}\Big(2,-ie^{i\sec^{-1}(}$$

input
```
Int[ArcSec[a + b*x]^3,x]
```

output
```
((a + b*x)*ArcSec[a + b*x]^3 - 3*((-2*I)*ArcSec[a + b*x]^2*ArcTan[E^(I*Arc
Sec[a + b*x])] + 2*(I*ArcSec[a + b*x]*PolyLog[2, (-I)*E^(I*ArcSec[a + b*x]
)] - PolyLog[3, (-I)*E^(I*ArcSec[a + b*x])]) - 2*(I*ArcSec[a + b*x]*PolyLo
g[2, I*E^(I*ArcSec[a + b*x])] - PolyLog[3, I*E^(I*ArcSec[a + b*x])])))/b
```

3.35.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4244
```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^p/(b*n*p)), x]
 - Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Fre
eQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5739
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/c   Subst[
Int[(a + b*x)^n*Sec[x]*Tan[x], x], x, ArcSec[c*x]], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[n, 0]
```

rule 5775
```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[1/d
  Subst[Int[(a + b*ArcSec[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
 x] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.35. $\int \sec^{-1}(a+bx)^3\,dx$

3.35.4 Maple [F]

$$\int \operatorname{arcsec}(bx+a)^3\,dx$$

input
```
int(arcsec(b*x+a)^3,x)
```

output
```
int(arcsec(b*x+a)^3,x)
```

3.35.5 Fricas [F]

$$\int \sec^{-1}(a+bx)^3\,dx = \int \operatorname{arcsec}(bx+a)^3\,dx$$

input
```
integrate(arcsec(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(arcsec(b*x + a)^3, x)
```

3.35.6 Sympy [F]

$$\int \sec^{-1}(a+bx)^3\,dx = \int \operatorname{asec}^3(a+bx)\,dx$$

input
```
integrate(asec(b*x+a)**3,x)
```

output
```
Integral(asec(a + b*x)**3, x)
```

3.35.7 Maxima [F]

$$\int \sec^{-1}(a+bx)^3\,dx = \int \operatorname{arcsec}\left(bx+a\right)^3\,dx$$

input
```
integrate(arcsec(b*x+a)^3,x, algorithm="maxima")
```

output
```
x*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^3 - 3/4*x*arctan(sqrt(b*x +
a + 1)*sqrt(b*x + a - 1))*log(b^2*x^2 + 2*a*b*x + a^2)^2 - integrate(3/4*(
(4*b*x*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^2 - b*x*log(b^2*x^2 + 2
*a*b*x + a^2)^2)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + 4*((b^3*x^3 + 3*a*b
^2*x^2 + a^3 + (3*a^2 - 1)*b*x - a)*log(b*x + a)^2 - (b^3*x^3 + 2*a*b^2*x^
2 + (a^2 - 1)*b*x + (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2 - 1)*b*x - a)*lo
g(b*x + a))*log(b^2*x^2 + 2*a*b*x + a^2))*arctan(sqrt(b*x + a + 1)*sqrt(b*
x + a - 1)))/(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2 - 1)*b*x - a), x)
```

3.35.8 Giac [F]

$$\int \sec^{-1}(a+bx)^3\,dx = \int \operatorname{arcsec}\left(bx+a\right)^3\,dx$$

input
```
integrate(arcsec(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(arcsec(b*x + a)^3, x)
```

3.35.9 Mupad [F(-1)]

Timed out.

$$\int \sec^{-1}(a+bx)^3\,dx = \int \operatorname{acos}\left(\frac{1}{a+b\,x}\right)^3\,dx$$

input
```
int(acos(1/(a + b*x))^3,x)
```

output
```
int(acos(1/(a + b*x))^3, x)
```

3.36 $\int \frac{\sec^{-1}(a+bx)^3}{x}\,dx$

3.36.1 Optimal result

Integrand size = 12, antiderivative size = 430

$$\begin{aligned}\int \frac{\sec^{-1}(a+bx)^3}{x}\,dx &= \sec^{-1}(a+bx)^3\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)\\
&+\sec^{-1}(a+bx)^3\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)\\
&-\sec^{-1}(a+bx)^3\log\left(1+e^{2i\sec^{-1}(a+bx)}\right)\\
&-3i\sec^{-1}(a+bx)^2\operatorname{PolyLog}\left(2,\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)\\
&-3i\sec^{-1}(a+bx)^2\operatorname{PolyLog}\left(2,\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)\\
&+\frac{3}{2}i\sec^{-1}(a+bx)^2\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)\\
&+6\sec^{-1}(a+bx)\operatorname{PolyLog}\left(3,\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)\\
&+6\sec^{-1}(a+bx)\operatorname{PolyLog}\left(3,\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)\\
&-\frac{3}{2}\sec^{-1}(a+bx)\operatorname{PolyLog}\left(3,-e^{2i\sec^{-1}(a+bx)}\right)\\
&+6i\operatorname{PolyLog}\left(4,\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)+6i\operatorname{PolyLog}\left(4,\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)\\
&-\frac{3}{4}i\operatorname{PolyLog}\left(4,-e^{2i\sec^{-1}(a+bx)}\right)\end{aligned}$$

output
```
-arcsec(b*x+a)^3*ln(1+(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)+arcsec(b*x+a)
^3*ln(1-a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2)))+arcsec(b
*x+a)^3*ln(1-a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1+(-a^2+1)^(1/2)))+3/2
*I*arcsec(b*x+a)^2*polylog(2,-(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)-3*I*a
rcsec(b*x+a)^2*polylog(2,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)
^(1/2)))-3*I*arcsec(b*x+a)^2*polylog(2,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2
))/(1+(-a^2+1)^(1/2)))-3/2*arcsec(b*x+a)*polylog(3,-(1/(b*x+a)+I*(1-1/(b*x
+a)^2)^(1/2))^2)+6*arcsec(b*x+a)*polylog(3,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^
(1/2))/(1-(-a^2+1)^(1/2)))+6*arcsec(b*x+a)*polylog(3,a*(1/(b*x+a)+I*(1-1/(
b*x+a)^2)^(1/2))/(1+(-a^2+1)^(1/2)))-3/4*I*polylog(4,-(1/(b*x+a)+I*(1-1/(b
*x+a)^2)^(1/2))^2)+6*I*polylog(4,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1-
(-a^2+1)^(1/2)))+6*I*polylog(4,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1+(-
a^2+1)^(1/2)))
```

3.36.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1058 vs. $2(430) = 860$.

Time = 2.92 (sec) , antiderivative size = 1058, normalized size of antiderivative = 2.46

$$
\begin{aligned}
\int \frac{\sec^{-1}(a+bx)^3}{x}\,dx = {} & 2\sec^{-1}(a+bx)^3\log\left(1+\frac{ae^{i\sec^{-1}(a+bx)}}{-1+\sqrt{1-a^2}}\right) \\
& + \sec^{-1}(a+bx)^3\log\left(1+\frac{\left(-1+\sqrt{1-a^2}\right)e^{i\sec^{-1}(a+bx)}}{a}\right) - 6\sec^{-1}(a \\
& + bx)^2\arcsin\left(\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right)\log\left(1+\frac{\left(-1+\sqrt{1-a^2}\right)e^{i\sec^{-1}(a+bx)}}{a}\right) \\
& + 2\sec^{-1}(a+bx)^3\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) \\
& + \sec^{-1}(a+bx)^3\log\left(1-\frac{\left(1+\sqrt{1-a^2}\right)e^{i\sec^{-1}(a+bx)}}{a}\right) + 6\sec^{-1}(a \\
& + bx)^2\arcsin\left(\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right)\log\left(1-\frac{\left(1+\sqrt{1-a^2}\right)e^{i\sec^{-1}(a+bx)}}{a}\right) \\
& - 3\sec^{-1}(a+bx)^3\log\left(1+e^{2i\sec^{-1}(a+bx)}\right) \\
& + 2\sec^{-1}(a+bx)^3\log\left(\frac{2\left(\frac{1}{a+bx}+i\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{a+bx}\right) \\
& - \sec^{-1}(a+bx)^3\log\left(1+\frac{a\left(\frac{1}{a+bx}+i\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{-1+\sqrt{1-a^2}}\right) \\
& - \sec^{-1}(a+bx)^3\log\left(1+\frac{\left(-1+\sqrt{1-a^2}\right)\left(\frac{1}{a+bx}+i\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{a}\right) \\
& + 6\sec^{-1}(a+bx)^2\arcsin\left(\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right)\log\left(1\right. \\
& \left. + \frac{\left(-1+\sqrt{1-a^2}\right)\left(\frac{1}{a+bx}+i\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{a}\right) \\
& - \sec^{-1}(a+bx)^3\log\left(1-\frac{a\left(\frac{1}{a+bx}+i\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{1+\sqrt{1-a^2}}\right) \\
& - \sec^{-1}(a+bx)^3\log\left(1-\frac{\left(1+\sqrt{1-a^2}\right)\left(\frac{1}{a+bx}+i\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{a}\right) \\
& - 6\sec^{-1}(a+bx)^2\arcsin\left(\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right)\log\left(1\right. \\
& \left. \frac{\left(1+\sqrt{1-a^2}\right)\left(\frac{1}{a+bx}+i\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{}\right)
\end{aligned}
$$

input
```
Integrate[ArcSec[a + b*x]^3/x,x]
```

output
```
2*ArcSec[a + b*x]^3*Log[1 + (a*E^(I*ArcSec[a + b*x]))/(-1 + Sqrt[1 - a^2])
] + ArcSec[a + b*x]^3*Log[1 + ((-1 + Sqrt[1 - a^2])*E^(I*ArcSec[a + b*x]))
/a] - 6*ArcSec[a + b*x]^2*ArcSin[Sqrt[(-1 + a)/a]/Sqrt[2]]*Log[1 + ((-1 +
Sqrt[1 - a^2])*E^(I*ArcSec[a + b*x]))/a] + 2*ArcSec[a + b*x]^3*Log[1 - (a*
E^(I*ArcSec[a + b*x]))/(1 + Sqrt[1 - a^2])] + ArcSec[a + b*x]^3*Log[1 - ((
1 + Sqrt[1 - a^2])*E^(I*ArcSec[a + b*x]))/a] + 6*ArcSec[a + b*x]^2*ArcSin[
Sqrt[(-1 + a)/a]/Sqrt[2]]*Log[1 - ((1 + Sqrt[1 - a^2])*E^(I*ArcSec[a + b*x
]))/a] - 3*ArcSec[a + b*x]^3*Log[1 + E^((2*I)*ArcSec[a + b*x])] + 2*ArcSec
[a + b*x]^3*Log[(2*((a + b*x)^(-1) + I*Sqrt[1 - (a + b*x)^(-2)]))/(a + b*x
)] - ArcSec[a + b*x]^3*Log[1 + (a*((a + b*x)^(-1) + I*Sqrt[1 - (a + b*x)^(
-2)]))/(-1 + Sqrt[1 - a^2])] - ArcSec[a + b*x]^3*Log[1 + ((-1 + Sqrt[1 - a
^2])*((a + b*x)^(-1) + I*Sqrt[1 - (a + b*x)^(-2)]))/a] + 6*ArcSec[a + b*x]
^2*ArcSin[Sqrt[(-1 + a)/a]/Sqrt[2]]*Log[1 + ((-1 + Sqrt[1 - a^2])*((a + b*
x)^(-1) + I*Sqrt[1 - (a + b*x)^(-2)]))/a] - ArcSec[a + b*x]^3*Log[1 - (a*(
(a + b*x)^(-1) + I*Sqrt[1 - (a + b*x)^(-2)]))/(1 + Sqrt[1 - a^2])] - ArcSe
c[a + b*x]^3*Log[1 - ((1 + Sqrt[1 - a^2])*((a + b*x)^(-1) + I*Sqrt[1 - (a
+ b*x)^(-2)]))/a] - 6*ArcSec[a + b*x]^2*ArcSin[Sqrt[(-1 + a)/a]/Sqrt[2]]*L
og[1 - ((1 + Sqrt[1 - a^2])*((a + b*x)^(-1) + I*Sqrt[1 - (a + b*x)^(-2)]))
/a] - (3*I)*ArcSec[a + b*x]^2*PolyLog[2, -((a*E^(I*ArcSec[a + b*x]))/(-1 +
 Sqrt[1 - a^2]))] - (3*I)*ArcSec[a + b*x]^2*PolyLog[2, (a*E^(I*ArcSec[a...
```

3.36.3 Rubi [A] (verified)

Time = 1.85 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.23, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {5781, 25, 5062, 5041, 25, 3042, 4202, 2620, 3011, 5031, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{-1}(a+bx)^3}{x} \, dx$$

$$\Big\downarrow \; 5781$$

$$\int \frac{(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^3}{bx}d\sec^{-1}(a+bx)$$

$$\Big\downarrow \; 25$$

$$-\int -\frac{(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^3}{bx}d\sec^{-1}(a+bx)$$

$$\Big\downarrow \text{5062}$$

$$-\int \frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^3}{\frac{a}{a+bx}-1}d\sec^{-1}(a+bx)$$

$$\Big\downarrow \text{5041}$$

$$\int (a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^3d\sec^{-1}(a+bx)-a\int -\frac{\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^3}{1-\frac{a}{a+bx}}d\sec^{-1}(a+bx)$$

$$\Big\downarrow \text{25}$$

$$\int (a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^3d\sec^{-1}(a+bx)+a\int \frac{\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^3}{1-\frac{a}{a+bx}}d\sec^{-1}(a+bx)$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^3}{1-\frac{a}{a+bx}}d\sec^{-1}(a+bx)+\int \sec^{-1}(a+bx)^3\tan\left(\sec^{-1}(a+bx)\right)d\sec^{-1}(a+bx)$$

$$\Big\downarrow \text{4202}$$

$$-2i\int \frac{e^{2i\sec^{-1}(a+bx)}\sec^{-1}(a+bx)^3}{1+e^{2i\sec^{-1}(a+bx)}}d\sec^{-1}(a+bx)+a\int \frac{\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^3}{1-\frac{a}{a+bx}}d\sec^{-1}(a+bx)+\frac{1}{4}i\sec^{-1}(a+bx)^4$$

$$\Big\downarrow \text{2620}$$

$$a\int \frac{\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^3}{1-\frac{a}{a+bx}}d\sec^{-1}(a+bx)-2i\left(\frac{3}{2}i\int \sec^{-1}(a+bx)^2\log\left(1+e^{2i\sec^{-1}(a+bx)}\right)d\sec^{-1}(a+bx)-\frac{1}{2}i\sec^{-1}(a+bx)^3\log\left(1+e^{2i\sec^{-1}(a+bx)}\right)\right)+\frac{1}{4}i\sec^{-1}(a+bx)^4$$

$$\Big\downarrow \text{3011}$$

3.36. $\int \frac{\sec^{-1}(a+bx)^3}{x}dx$

$$-2i\left(\frac{3}{2}i\left(\frac{1}{2}i\sec^{-1}(a+bx)^2\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)-i\int\sec^{-1}(a+bx)\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)d\sec^{-1}\right.\right.$$
$$a\int\frac{\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^3}{1-\frac{a}{a+bx}}d\sec^{-1}(a+bx)+\frac{1}{4}i\sec^{-1}(a+bx)^4$$

$\big\downarrow$ 5031

$$a\left(-i\int\frac{e^{i\sec^{-1}(a+bx)}\sec^{-1}(a+bx)^3}{-e^{i\sec^{-1}(a+bx)}a-\sqrt{1-a^2}+1}d\sec^{-1}(a+bx)-i\int\frac{e^{i\sec^{-1}(a+bx)}\sec^{-1}(a+bx)^3}{-e^{i\sec^{-1}(a+bx)}a+\sqrt{1-a^2}+1}d\sec^{-1}(a+bx)-\frac{i\,s}{}\right.$$
$$2i\left(\frac{3}{2}i\left(\frac{1}{2}i\sec^{-1}(a+bx)^2\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)-i\int\sec^{-1}(a+bx)\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)d\sec^{-1}(a\right.\right.$$
$$\frac{1}{4}i\sec^{-1}(a+bx)^4$$

$\big\downarrow$ 2620

$$a\left(-i\left(\frac{i\sec^{-1}(a+bx)^3\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a}-\frac{3i\int\sec^{-1}(a+bx)^2\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)d\sec^{-1}(a+bx)}{a}\right)-\right.$$
$$2i\left(\frac{3}{2}i\left(\frac{1}{2}i\sec^{-1}(a+bx)^2\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)-i\int\sec^{-1}(a+bx)\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)d\sec^{-1}(a\right.\right.$$
$$\frac{1}{4}i\sec^{-1}(a+bx)^4$$

$\big\downarrow$ 3011

$$a\left(-i\left(\frac{i\sec^{-1}(a+bx)^3\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a}-\frac{3i\left(i\sec^{-1}(a+bx)^2\operatorname{PolyLog}\left(2,\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)-2i\int\sec^{-1}(a+\right.}{a}\right.\right.$$
$$2i\left(\frac{3}{2}i\left(\frac{1}{2}i\sec^{-1}(a+bx)^2\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)-i\int\sec^{-1}(a+bx)\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)d\sec^{-1}(a\right.\right.$$
$$\frac{1}{4}i\sec^{-1}(a+bx)^4$$

$\big\downarrow$ 7163

$$a\left(-i\left(\frac{i\sec^{-1}(a+bx)^3\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a}-\frac{3i\left(i\sec^{-1}(a+bx)^2\operatorname{PolyLog}\left(2,\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)-2i\left(i\int\operatorname{PolyLo}\right.\right.}{}\right.\right.$$
$$2i\left(\frac{3}{2}i\left(\frac{1}{2}i\sec^{-1}(a+bx)^2\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)-i\left(\frac{1}{2}i\int\operatorname{PolyLog}\left(3,-e^{2i\sec^{-1}(a+bx)}\right)d\sec^{-1}(a+bx)-\right.\right.\right.$$
$$\frac{1}{4}i\sec^{-1}(a+bx)^4$$

3.36. $\int\frac{\sec^{-1}(a+bx)^3}{x}\,dx$

$$\Big\downarrow \text{2720}$$

$$a\left(-i\left(\frac{i\sec^{-1}(a+bx)^3\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a}-\frac{3i\left(i\sec^{-1}(a+bx)^2\text{PolyLog}\left(2,\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)-2i\left(\int e^{-i\sec^{-1}(\right.\right.}{}\right.\right.$$

$$2i\left(\frac{3}{2}i\left(\frac{1}{2}i\sec^{-1}(a+bx)^2\text{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)-i\left(\frac{1}{4}\int e^{-2i\sec^{-1}(a+bx)}\text{PolyLog}\left(3,-e^{2i\sec^{-1}(a+bx)}\right)de^{2i}\right.\right.\right.$$

$$\frac{1}{4}i\sec^{-1}(a+bx)^4$$

$$\Big\downarrow \text{7143}$$

$$a\left(-i\left(\frac{i\sec^{-1}(a+bx)^3\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a}-\frac{3i\left(i\sec^{-1}(a+bx)^2\text{PolyLog}\left(2,\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)-2i\left(\text{PolyLog}\left(\right.\right.\right.}{a}\right.\right.$$

$$2i\left(\frac{3}{2}i\left(\frac{1}{2}i\sec^{-1}(a+bx)^2\text{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)-i\left(\frac{1}{4}\text{PolyLog}\left(4,-e^{2i\sec^{-1}(a+bx)}\right)-\frac{1}{2}i\sec^{-1}(a+bx)\,\text{Po}\right.\right.\right.$$

$$\frac{1}{4}i\sec^{-1}(a+bx)^4$$

input

```
Int[ArcSec[a + b*x]^3/x,x]
```

output

```
(I/4)*ArcSec[a + b*x]^4 + a*(((-1/4*I)*ArcSec[a + b*x]^4)/a - I*((I*ArcSec
[a + b*x]^3*Log[1 - (a*E^(I*ArcSec[a + b*x]))/(1 - Sqrt[1 - a^2])])/a - ((
3*I)*(I*ArcSec[a + b*x]^2*PolyLog[2, (a*E^(I*ArcSec[a + b*x]))/(1 - Sqrt[1
 - a^2])] - (2*I)*((-I)*ArcSec[a + b*x]*PolyLog[3, (a*E^(I*ArcSec[a + b*x]
))/(1 - Sqrt[1 - a^2])] + PolyLog[4, (a*E^(I*ArcSec[a + b*x]))/(1 - Sqrt[1
 - a^2])])))/a) - I*((I*ArcSec[a + b*x]^3*Log[1 - (a*E^(I*ArcSec[a + b*x])
)/(1 + Sqrt[1 - a^2])])/a - ((3*I)*(I*ArcSec[a + b*x]^2*PolyLog[2, (a*E^(I
*ArcSec[a + b*x]))/(1 + Sqrt[1 - a^2])] - (2*I)*((-I)*ArcSec[a + b*x]*Poly
Log[3, (a*E^(I*ArcSec[a + b*x]))/(1 + Sqrt[1 - a^2])] + PolyLog[4, (a*E^(I
*ArcSec[a + b*x]))/(1 + Sqrt[1 - a^2])])))/a)) - (2*I)*((-1/2*I)*ArcSec[a
+ b*x]^3*Log[1 + E^((2*I)*ArcSec[a + b*x])] + ((3*I)/2)*((I/2)*ArcSec[a +
b*x]^2*PolyLog[2, -E^((2*I)*ArcSec[a + b*x])] - I*((-1/2*I)*ArcSec[a + b*x
]*PolyLog[3, -E^((2*I)*ArcSec[a + b*x])] + PolyLog[4, -E^((2*I)*ArcSec[a +
 b*x])]/4)))
```

3.36. $\int \frac{\sec^{-1}(a+bx)^3}{x}\,dx$

3.36.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5031
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[I*((e + f*x)^(m + 1)/(b*f*(m + 1)))
, x] + (-Simp[I   Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] +
b*E^(I*(c + d*x)))), x], x] - Simp[I   Int[(e + f*x)^m*(E^(I*(c + d*x))/(a
+ Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

rule 5041
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tan[(c_.) + (d_.)*(x_)]^(n_.))/(Cos[(c_.) +
 (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Tan[c +
 d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*Sin[c + d*x]*(Tan[c + d*x]^(n
- 1)/(a + b*Cos[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[
m, 0] && IGtQ[n, 0]
```

rule 5062
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sec[(c_.) + (d_.)*(x_)]), x_Symbol] :> In
t[(e + f*x)^m*Cos[c + d*x]*F[c + d*x]^n*(G[c + d*x]^p/(b + a*Cos[c + d*x]))
, x] /; FreeQ[{a, b, c, d, e, f}, x] && TrigQ[F] && TrigQ[G] && IntegersQ[m
, n, p]
```

rule 5781
```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d^(m + 1)    Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d
*e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.36.4 Maple [F]

$$\int \frac{\operatorname{arcsec}(bx+a)^3}{x}dx$$

input
```
int(arcsec(b*x+a)^3/x,x)
```

output
```
int(arcsec(b*x+a)^3/x,x)
```

3.36.5 Fricas [F]

$$\int \frac{\sec^{-1}(a+bx)^3}{x}\,dx = \int \frac{\operatorname{arcsec}\left(bx+a\right)^3}{x}\,dx$$

input
```
integrate(arcsec(b*x+a)^3/x,x, algorithm="fricas")
```

output
```
integral(arcsec(b*x + a)^3/x, x)
```

3.36.6 Sympy [F]

$$\int \frac{\sec^{-1}(a+bx)^3}{x}\,dx = \int \frac{\operatorname{asec}^3\left(a+bx\right)}{x}\,dx$$

input
```
integrate(asec(b*x+a)**3/x,x)
```

output
```
Integral(asec(a + b*x)**3/x, x)
```

3.36.7 Maxima [F]

$$\int \frac{\sec^{-1}(a+bx)^3}{x}\,dx = \int \frac{\operatorname{arcsec}\left(bx+a\right)^3}{x}\,dx$$

input
```
integrate(arcsec(b*x+a)^3/x,x, algorithm="maxima")
```

output
```
integrate(arcsec(b*x + a)^3/x, x)
```

3.36.8 Giac [F]

$$\int \frac{\sec^{-1}(a+bx)^3}{x}\,dx = \int \frac{\operatorname{arcsec}\left(bx+a\right)^3}{x}\,dx$$

input
```
integrate(arcsec(b*x+a)^3/x,x, algorithm="giac")
```

output
```
integrate(arcsec(b*x + a)^3/x, x)
```

3.36.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{-1}(a+bx)^3}{x}\,dx = \int \frac{\operatorname{acos}\left(\frac{1}{a+b\,x}\right)^3}{x}\,dx$$

input
```
int(acos(1/(a + b*x))^3/x,x)
```

output
```
int(acos(1/(a + b*x))^3/x, x)
```

3.37 $\int \frac{\sec^{-1}(a+bx)^3}{x^2} \, dx$

3.37.1 Optimal result

Integrand size = 12, antiderivative size = 362

$$\begin{aligned}\int \frac{\sec^{-1}(a+bx)^3}{x^2} \, dx = & -\frac{b\sec^{-1}(a+bx)^3}{a} - \frac{\sec^{-1}(a+bx)^3}{x} \\ & - \frac{3ib\sec^{-1}(a+bx)^2 \log\left(1 - \frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} \\ & + \frac{3ib\sec^{-1}(a+bx)^2 \log\left(1 - \frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} \\ & - \frac{6b\sec^{-1}(a+bx) \operatorname{PolyLog}\left(2, \frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} \\ & + \frac{6b\sec^{-1}(a+bx) \operatorname{PolyLog}\left(2, \frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} \\ & - \frac{6ib \operatorname{PolyLog}\left(3, \frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} + \frac{6ib \operatorname{PolyLog}\left(3, \frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}\end{aligned}$$

output
```
-b*arcsec(b*x+a)^3/a-arcsec(b*x+a)^3/x-3*I*b*arcsec(b*x+a)^2*ln(1-a*(1/(b*
x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2)))/a/(-a^2+1)^(1/2)+3*I*b*a
rcsec(b*x+a)^2*ln(1-a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1+(-a^2+1)^(1/2
)))/a/(-a^2+1)^(1/2)-6*b*arcsec(b*x+a)*polylog(2,a*(1/(b*x+a)+I*(1-1/(b*x+
a)^2)^(1/2))/(1-(-a^2+1)^(1/2)))/a/(-a^2+1)^(1/2)+6*b*arcsec(b*x+a)*polylo
g(2,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1+(-a^2+1)^(1/2)))/a/(-a^2+1)^(
1/2)-6*I*b*polylog(3,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/
2)))/a/(-a^2+1)^(1/2)+6*I*b*polylog(3,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)
)/(1+(-a^2+1)^(1/2)))/a/(-a^2+1)^(1/2)
```

3.37.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{\sec^{-1}(a+bx)^3}{x^2}\,dx = \$\text{Aborted}$$

input
```
Integrate[ArcSec[a + b*x]^3/x^2,x]
```

output
```
$Aborted
```

3.37.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5781, 4926, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{-1}(a+bx)^3}{x^2}\,dx$$

$$\Big\downarrow \text{5781}$$

$$b\int \frac{(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^3}{b^2x^2}d\sec^{-1}(a+bx)$$

$$\Big\downarrow \text{4926}$$

$$b\left(-3\int -\frac{\sec^{-1}(a+bx)^2}{bx}d\sec^{-1}(a+bx)-\frac{\sec^{-1}(a+bx)^3}{bx}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(-3\int \frac{\sec^{-1}(a+bx)^2}{a-\csc\left(\sec^{-1}(a+bx)+\frac{\pi}{2}\right)}d\sec^{-1}(a+bx)-\frac{\sec^{-1}(a+bx)^3}{bx}\right)$$

$$\Big\downarrow \text{4679}$$

$$b\left(-3\int \left(\frac{\sec^{-1}(a+bx)^2}{a}+\frac{\sec^{-1}(a+bx)^2}{a\left(\frac{a}{a+bx}-1\right)}\right)d\sec^{-1}(a+bx)-\frac{\sec^{-1}(a+bx)^3}{bx}\right)$$

$$\Big\downarrow \text{2009}$$

$$b\left(-\frac{\sec^{-1}(a+bx)^3}{bx}-3\left(\frac{2\sec^{-1}(a+bx)\operatorname{PolyLog}\left(2,\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}-\frac{2\sec^{-1}(a+bx)\operatorname{PolyLog}\left(2,\frac{ae^{i\sec^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a\sqrt{1-a^2}}\right.\right.$$

input
```
Int[ArcSec[a + b*x]^3/x^2,x]
```

output
```
b*(-(ArcSec[a + b*x]^3/(b*x)) - 3*(ArcSec[a + b*x]^3/(3*a) + (I*ArcSec[a +
 b*x]^2*Log[1 - (a*E^(I*ArcSec[a + b*x]))/(1 - Sqrt[1 - a^2])])/(a*Sqrt[1
- a^2]) - (I*ArcSec[a + b*x]^2*Log[1 - (a*E^(I*ArcSec[a + b*x]))/(1 + Sqrt
[1 - a^2])])/(a*Sqrt[1 - a^2]) + (2*ArcSec[a + b*x]*PolyLog[2, (a*E^(I*Arc
Sec[a + b*x]))/(1 - Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2]) - (2*ArcSec[a + b*x
]*PolyLog[2, (a*E^(I*ArcSec[a + b*x]))/(1 + Sqrt[1 - a^2])])/(a*Sqrt[1 - a
^2]) + ((2*I)*PolyLog[3, (a*E^(I*ArcSec[a + b*x]))/(1 - Sqrt[1 - a^2])])/(
a*Sqrt[1 - a^2]) - ((2*I)*PolyLog[3, (a*E^(I*ArcSec[a + b*x]))/(1 + Sqrt[1
 - a^2])])/(a*Sqrt[1 - a^2])))
```

3.37.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4679
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

rule 4926
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(e + f*
x)^m*((a + b*Sec[c + d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n +
 1)))   Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5781
```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d^(m + 1)   Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d
*e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.37.4 Maple [F]

$$\int \frac{\operatorname{arcsec}(bx+a)^3}{x^2}dx$$

input
```
int(arcsec(b*x+a)^3/x^2,x)
```

output
```
int(arcsec(b*x+a)^3/x^2,x)
```

3.37.5 Fricas [F]

$$\int \frac{\sec^{-1}(a+bx)^3}{x^2}\,dx=\int \frac{\operatorname{arcsec}(bx+a)^3}{x^2}\,dx$$

input
```
integrate(arcsec(b*x+a)^3/x^2,x, algorithm="fricas")
```

output
```
integral(arcsec(b*x + a)^3/x^2, x)
```

3.37.6 Sympy [F]

$$\int \frac{\sec^{-1}(a+bx)^3}{x^2}\,dx = \int \frac{\operatorname{asec}^3(a+bx)}{x^2}\,dx$$

input
```
integrate(asec(b*x+a)**3/x**2,x)
```

output
```
Integral(asec(a + b*x)**3/x**2, x)
```

3.37.7 Maxima [F]

$$\int \frac{\sec^{-1}(a+bx)^3}{x^2}\,dx = \int \frac{\operatorname{arcsec}(bx+a)^3}{x^2}\,dx$$

input
```
integrate(arcsec(b*x+a)^3/x^2,x, algorithm="maxima")
```

output
```
-1/4*(4*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^3 - 3*arctan(sqrt(b*x
+ a + 1)*sqrt(b*x + a - 1))*log(b^2*x^2 + 2*a*b*x + a^2)^2 - 4*x*integrate
(3/4*((4*b*x*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^2 - b*x*log(b^2*x
^2 + 2*a*b*x + a^2)^2)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) - 4*((b^3*x^3 +
 3*a*b^2*x^2 + a^3 + (3*a^2 - 1)*b*x - a)*log(b*x + a)^2 + (b^3*x^3 + 2*a*
b^2*x^2 + (a^2 - 1)*b*x - (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2 - 1)*b*x -
 a)*log(b*x + a))*log(b^2*x^2 + 2*a*b*x + a^2))*arctan(sqrt(b*x + a + 1)*s
qrt(b*x + a - 1)))/(b^3*x^5 + 3*a*b^2*x^4 + (3*a^2 - 1)*b*x^3 + (a^3 - a)*
x^2), x))/x
```

3.37.8 Giac [F]

$$\int \frac{\sec^{-1}(a+bx)^3}{x^2}\,dx = \int \frac{\operatorname{arcsec}(bx+a)^3}{x^2}\,dx$$

input
```
integrate(arcsec(b*x+a)^3/x^2,x, algorithm="giac")
```

output
```
integrate(arcsec(b*x + a)^3/x^2, x)
```

3.37.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{-1}(a+bx)^3}{x^2}\,dx = \int \frac{\operatorname{acos}\left(\frac{1}{a+b\,x}\right)^3}{x^2}\,dx$$

input
```
int(acos(1/(a + b*x))^3/x^2,x)
```

output
```
int(acos(1/(a + b*x))^3/x^2, x)
```

3.38 $\int x(a+b\sec^{-1}(c+dx^2))\,dx$

3.38.1 Optimal result

Integrand size = 14, antiderivative size = 58

$$\int x\left(a+b\sec^{-1}\left(c+dx^2\right)\right)\,dx=\frac{ax^2}{2}+\frac{b(c+dx^2)\sec^{-1}\left(c+dx^2\right)}{2d}-\frac{b\operatorname{arctanh}\left(\sqrt{1-\frac{1}{(c+dx^2)^2}}\right)}{2d}$$

output
```
1/2*a*x^2+1/2*b*(d*x^2+c)*arcsec(d*x^2+c)/d-1/2*b*arctanh((1-1/(d*x^2+c)^2
)^(1/2))/d
```

3.38.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.31 (sec) , antiderivative size = 516, normalized size of antiderivative = 8.90

$$\int x\left(a+b\sec^{-1}\left(c+dx^2\right)\right)\,dx=\frac{ax^2}{2}+\frac{1}{2}bx^2\sec^{-1}\left(c+dx^2\right)$$
$$+\frac{b(c+dx^2)\sqrt{\frac{-1+c^2+2cdx^2+d^2x^4}{(c+dx^2)^2}}\left(\sqrt[4]{-1}\left(-i+\sqrt{-1+c^2}\right)\sqrt{2i-ic^2+2\sqrt{-1+c^2}}\arctan\left(\frac{(-1)^{3/4}\sqrt{2i-ic^2+\cdots}}{c\sqrt{-1+c^2}-c\sqrt{-1+c^2}\cdots}\right.\right.}{\cdots}$$

input
```
Integrate[x*(a + b*ArcSec[c + d*x^2]),x]
```

output
```
(a*x^2)/2 + (b*x^2*ArcSec[c + d*x^2])/2 + (b*(c + d*x^2)*Sqrt[(-1 + c^2 +
2*c*d*x^2 + d^2*x^4)/(c + d*x^2)^2]*((-1)^(1/4)*(-I + Sqrt[-1 + c^2])*Sqrt
[2*I - I*c^2 + 2*Sqrt[-1 + c^2]]*ArcTan[((-1)^(3/4)*Sqrt[2*I - I*c^2 + 2*S
qrt[-1 + c^2]]*d*x^2)/(c*Sqrt[-1 + c^2] - c*Sqrt[-1 + c^2 + 2*c*d*x^2 + d^
2*x^4])] + (-1)^(3/4)*(I + Sqrt[-1 + c^2])*Sqrt[-2*I + I*c^2 + 2*Sqrt[-1 +
 c^2]]*ArcTan[((-1)^(1/4)*Sqrt[-2*I + I*c^2 + 2*Sqrt[-1 + c^2]]*d*x^2)/(c*
Sqrt[-1 + c^2] - c*Sqrt[-1 + c^2 + 2*c*d*x^2 + d^2*x^4])] + c*(c*ArcTan[(S
qrt[-1 + c^2]*d^2*x^4)/(c^4 + c^3*d*x^2 + d^2*x^4 - c^2*(1 + Sqrt[-1 + c^2
]*Sqrt[-1 + c^2 + 2*c*d*x^2 + d^2*x^4]))] - Log[Sqrt[-1 + c^2] - d*x^2 - S
qrt[-1 + c^2 + 2*c*d*x^2 + d^2*x^4]] + Log[d^2*(Sqrt[-1 + c^2] + d*x^2 - S
qrt[-1 + c^2 + 2*c*d*x^2 + d^2*x^4])])))/(2*c*d*Sqrt[-1 + c^2 + 2*c*d*x^2
+ d^2*x^4])
```

3.38.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {7266, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\sec^{-1}\left(c+dx^2\right)\right)\,dx$$

$$\Big\downarrow \text{7266}$$

$$\frac{1}{2}\int \left(a+b\sec^{-1}\left(dx^2+c\right)\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(ax^2-\frac{b\,\text{arctanh}\left(\sqrt{1-\frac{1}{(c+dx^2)^2}}\right)}{d}+\frac{b(c+dx^2)\sec^{-1}\left(c+dx^2\right)}{d}\right)$$

input
```
Int[x*(a + b*ArcSec[c + d*x^2]),x]
```

output
```
(a*x^2 + (b*(c + d*x^2)*ArcSec[c + d*x^2])/d - (b*ArcTanh[Sqrt[1 - (c + d*
x^2)^(-2)]])/d)/2
```

3.38.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.38.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.10

method	result	size
parts	$\frac{a\,x^{2}}{2}+\frac{b\left(\left(d\,x^{2}+c\right)\operatorname{arcsec}\left(d\,x^{2}+c\right)-\ln\left(d\,x^{2}+c+\left(d\,x^{2}+c\right)\sqrt{1-\frac{1}{\left(d\,x^{2}+c\right)^{2}}}\right)\right)}{2d}$	64
derivativedivides	$\frac{\left(d\,x^{2}+c\right)a+b\left(\left(d\,x^{2}+c\right)\operatorname{arcsec}\left(d\,x^{2}+c\right)-\ln\left(d\,x^{2}+c+\left(d\,x^{2}+c\right)\sqrt{1-\frac{1}{\left(d\,x^{2}+c\right)^{2}}}\right)\right)}{2d}$	68
default	$\frac{\left(d\,x^{2}+c\right)a+b\left(\left(d\,x^{2}+c\right)\operatorname{arcsec}\left(d\,x^{2}+c\right)-\ln\left(d\,x^{2}+c+\left(d\,x^{2}+c\right)\sqrt{1-\frac{1}{\left(d\,x^{2}+c\right)^{2}}}\right)\right)}{2d}$	68

input
```
int(x*(a+b*arcsec(d*x^2+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2+1/2*b/d*((d*x^2+c)*arcsec(d*x^2+c)-ln(d*x^2+c+(d*x^2+c)*(1-1/(d*
x^2+c)^2)^(1/2)))
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.66

$$\int x\left(a+b\sec^{-1}\left(c+dx^2\right)\right)\,dx$$
$$=\frac{bdx^2\operatorname{arcsec}\left(dx^2+c\right)+adx^2+2\,bc\arctan\left(-dx^2-c+\sqrt{d^2x^4+2\,cdx^2+c^2-1}\right)+b\log\left(-dx^2-c+\sqrt{c}\right.}{2\,d}$$

input
```
integrate(x*(a+b*arcsec(d*x^2+c)),x, algorithm="fricas")
```

output
```
1/2*(b*d*x^2*arcsec(d*x^2 + c) + a*d*x^2 + 2*b*c*arctan(-d*x^2 - c + sqrt(
d^2*x^4 + 2*c*d*x^2 + c^2 - 1)) + b*log(-d*x^2 - c + sqrt(d^2*x^4 + 2*c*d*
x^2 + c^2 - 1)))/d
```

3.38.6 Sympy [F]

$$\int x\left(a+b\sec^{-1}\left(c+dx^2\right)\right)\,dx=\int x\left(a+b\operatorname{asec}\left(c+dx^2\right)\right)\,dx$$

input
```
integrate(x*(a+b*asec(d*x**2+c)),x)
```

output
```
Integral(x*(a + b*asec(c + d*x**2)), x)
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.22

$$\int x\left(a+b\sec^{-1}\left(c+dx^2\right)\right)\,dx=\frac{1}{2}ax^2+\frac{\left(2\left(dx^2+c\right)\operatorname{arcsec}\left(dx^2+c\right)-\log\left(\sqrt{-\frac{1}{(dx^2+c)^2}+1}+1\right)+\log\left(-\sqrt{-\frac{1}{(dx^2+c)^2}+1}+1\right)\right)b}{4\,d}$$

input
```
integrate(x*(a+b*arcsec(d*x^2+c)),x, algorithm="maxima")
```

output
```
1/2*a*x^2 + 1/4*(2*(d*x^2 + c)*arcsec(d*x^2 + c) - log(sqrt(-1/(d*x^2 + c)
^2 + 1) + 1) + log(-sqrt(-1/(d*x^2 + c)^2 + 1) + 1))*b/d
```

3.38.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.72

$$\int x\left(a+b\sec^{-1}\left(c+dx^2\right)\right)\,dx=\frac{1}{2}ax^2+\frac{1}{4}bd\left(\frac{2\left(dx^2+c\right)\arccos\left(-\frac{1}{(dx^2+c)\left(\frac{c}{dx^2+c}-1\right)-c}\right)}{d^2}-\frac{\log\left(\sqrt{-\frac{1}{(dx^2+c)^2}+1}+1\right)-\log\left(-\sqrt{-\frac{1}{(dx^2+c)^2}+1}+1\right.}{d^2}\right.$$

input
```
integrate(x*(a+b*arcsec(d*x^2+c)),x, algorithm="giac")
```

output
```
1/2*a*x^2 + 1/4*b*d*(2*(d*x^2 + c)*arccos(-1/((d*x^2 + c)*(c/(d*x^2 + c) -
 1) - c))/d^2 - (log(sqrt(-1/(d*x^2 + c)^2 + 1) + 1) - log(-sqrt(-1/(d*x^2
 + c)^2 + 1) + 1))/d^2)
```

3.38.9 Mupad [B] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

$$\int x\left(a+b\sec^{-1}\left(c+dx^2\right)\right)\,dx=\frac{a\,x^2}{2}-\frac{b\,\mathrm{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{{\left(d\,x^2+c\right)}^2}}}\right)}{2\,d}+\frac{b\,\mathrm{acos}\left(\frac{1}{d\,x^2+c}\right)\,\left(d\,x^2+c\right)}{2\,d}$$

input
```
int(x*(a + b*acos(1/(c + d*x^2))),x)
```

output
```
(a*x^2)/2 - (b*atanh(1/(1 - 1/(c + d*x^2)^2)^(1/2)))/(2*d) + (b*acos(1/(c
+ d*x^2))*(c + d*x^2))/(2*d)
```

3.39 $\int x^2(a+b\sec^{-1}(c+dx^3))\,dx$

3.39.1 Optimal result

Integrand size = 16, antiderivative size = 58

$$\int x^2\left(a+b\sec^{-1}\left(c+dx^3\right)\right)\,dx=\frac{ax^3}{3}+\frac{b(c+dx^3)\sec^{-1}(c+dx^3)}{3d}-\frac{b\,\mathrm{arctanh}\left(\sqrt{1-\frac{1}{(c+dx^3)^2}}\right)}{3d}$$

output
```
1/3*a*x^3+1/3*b*(d*x^3+c)*arcsec(d*x^3+c)/d-1/3*b*arctanh((1-1/(d*x^3+c)^2
)^(1/2))/d
```

3.39.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 516, normalized size of antiderivative = 8.90

$$\int x^2\left(a+b\sec^{-1}\left(c+dx^3\right)\right)\,dx=\frac{ax^3}{3}+\frac{1}{3}bx^3\sec^{-1}\left(c+dx^3\right)$$
$$+\frac{b(c+dx^3)\sqrt{\frac{-1+c^2+2cdx^3+d^2x^6}{(c+dx^3)^2}}\left(\sqrt[4]{-1}\left(-i+\sqrt{-1+c^2}\right)\sqrt{2i-ic^2+2\sqrt{-1+c^2}}\arctan\left(\frac{(-1)^{3/4}\sqrt{2i-ic^2+2}}{c\sqrt{-1+c^2}-c\sqrt{-1+c^2}}\right.\right.}{}$$

input
```
Integrate[x^2*(a + b*ArcSec[c + d*x^3]),x]
```

output
```
(a*x^3)/3 + (b*x^3*ArcSec[c + d*x^3])/3 + (b*(c + d*x^3)*Sqrt[(-1 + c^2 +
2*c*d*x^3 + d^2*x^6)/(c + d*x^3)^2]*((-1)^(1/4)*(-I + Sqrt[-1 + c^2])*Sqrt
[2*I - I*c^2 + 2*Sqrt[-1 + c^2]]*ArcTan[((-1)^(3/4)*Sqrt[2*I - I*c^2 + 2*S
qrt[-1 + c^2]]*d*x^3)/(c*Sqrt[-1 + c^2] - c*Sqrt[-1 + c^2 + 2*c*d*x^3 + d^
2*x^6])] + (-1)^(3/4)*(I + Sqrt[-1 + c^2])*Sqrt[-2*I + I*c^2 + 2*Sqrt[-1 +
 c^2]]*ArcTan[((-1)^(1/4)*Sqrt[-2*I + I*c^2 + 2*Sqrt[-1 + c^2]]*d*x^3)/(c*
Sqrt[-1 + c^2] - c*Sqrt[-1 + c^2 + 2*c*d*x^3 + d^2*x^6])] + c*(c*ArcTan[(S
qrt[-1 + c^2]*d^2*x^6)/(c^4 + c^3*d*x^3 + d^2*x^6 - c^2*(1 + Sqrt[-1 + c^2
]*Sqrt[-1 + c^2 + 2*c*d*x^3 + d^2*x^6]))] - Log[Sqrt[-1 + c^2] - d*x^3 - S
qrt[-1 + c^2 + 2*c*d*x^3 + d^2*x^6]] + Log[d^2*(Sqrt[-1 + c^2] + d*x^3 - S
qrt[-1 + c^2 + 2*c*d*x^3 + d^2*x^6])])))/(3*c*d*Sqrt[-1 + c^2 + 2*c*d*x^3
+ d^2*x^6])
```

3.39.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {7266, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b \sec^{-1}\left(c+d x^3\right)\right) \, dx$$

$$\Big\downarrow \text{7266}$$

$$\frac{1}{3} \int \left(a+b \sec^{-1}\left(d x^3+c\right)\right) dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(a x^3-\frac{b \text{arctanh}\left(\sqrt{1-\frac{1}{\left(c+d x^3\right)^2}}\right)}{d}+\frac{b \left(c+d x^3\right) \sec^{-1}\left(c+d x^3\right)}{d}\right)$$

input
```
Int[x^2*(a + b*ArcSec[c + d*x^3]),x]
```

output
```
(a*x^3 + (b*(c + d*x^3)*ArcSec[c + d*x^3])/d - (b*ArcTanh[Sqrt[1 - (c + d*
x^3)^(-2)]])/d)/3
```

3.39.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.39.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.10

method	result	size
parts	$\frac{x^{3} a}{3}+\frac{b \left(\left(d \,x^{3}+c \right) \operatorname{arcsec}\left(d \,x^{3}+c \right)-\ln \left(d \,x^{3}+c +\left(d \,x^{3}+c \right) \sqrt{1-\frac{1}{\left(d \,x^{3}+c \right)^{2}}}\right)\right)}{3 d}$	64
derivativedivides	$\frac{\left(d \,x^{3}+c \right) a +b \left(\left(d \,x^{3}+c \right) \operatorname{arcsec}\left(d \,x^{3}+c \right)-\ln \left(d \,x^{3}+c +\left(d \,x^{3}+c \right) \sqrt{1-\frac{1}{\left(d \,x^{3}+c \right)^{2}}}\right)\right)}{3 d}$	68
default	$\frac{\left(d \,x^{3}+c \right) a +b \left(\left(d \,x^{3}+c \right) \operatorname{arcsec}\left(d \,x^{3}+c \right)-\ln \left(d \,x^{3}+c +\left(d \,x^{3}+c \right) \sqrt{1-\frac{1}{\left(d \,x^{3}+c \right)^{2}}}\right)\right)}{3 d}$	68

input
```
int(x^2*(a+b*arcsec(d*x^3+c)),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*a+1/3*b/d*((d*x^3+c)*arcsec(d*x^3+c)-ln(d*x^3+c+(d*x^3+c)*(1-1/(d*
x^3+c)^2)^(1/2)))
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.66

$$\int x^2 \left(a+b\sec^{-1}\left(c+dx^3\right)\right)\,dx$$
$$= \frac{bdx^3 \operatorname{arcsec}\left(dx^3+c\right)+adx^3+2\,bc\arctan\left(-dx^3-c+\sqrt{d^2x^6+2\,cdx^3+c^2-1}\right)+b\log\left(-dx^3-c+\sqrt{c}\right.}{3\,d}$$

input
```
integrate(x^2*(a+b*arcsec(d*x^3+c)),x, algorithm="fricas")
```

output
```
1/3*(b*d*x^3*arcsec(d*x^3 + c) + a*d*x^3 + 2*b*c*arctan(-d*x^3 - c + sqrt(
d^2*x^6 + 2*c*d*x^3 + c^2 - 1)) + b*log(-d*x^3 - c + sqrt(d^2*x^6 + 2*c*d*
x^3 + c^2 - 1)))/d
```

3.39.6 Sympy [F(-1)]

Timed out.

$$\int x^2 \left(a + b \sec^{-1}\left(c + dx^3\right)\right) dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*asec(d*x**3+c)),x)
```

output
```
Timed out
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.22

$$\int x^2 \left(a + b \sec^{-1}\left(c + dx^3\right)\right) dx = \frac{1}{3} a x^3 + \frac{\left(2 \left(dx^3 + c\right) \operatorname{arcsec}\left(dx^3 + c\right) - \log\left(\sqrt{-\frac{1}{(dx^3+c)^2} + 1} + 1\right) + \log\left(-\sqrt{-\frac{1}{(dx^3+c)^2} + 1} + 1\right)\right) b}{6\, d}$$

input
```
integrate(x^2*(a+b*arcsec(d*x^3+c)),x, algorithm="maxima")
```

output
```
1/3*a*x^3 + 1/6*(2*(d*x^3 + c)*arcsec(d*x^3 + c) - log(sqrt(-1/(d*x^3 + c)
^2 + 1) + 1) + log(-sqrt(-1/(d*x^3 + c)^2 + 1) + 1))*b/d
```

3.39.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.72

$$\int x^2 \left(a + b \sec^{-1}\left(c + dx^3\right)\right) \, dx = \frac{1}{3} a x^3 + \frac{1}{6} b d \left(\frac{2 \left(dx^3 + c\right) \arccos\left(-\frac{1}{\left(dx^3+c\right)\left(\frac{c}{dx^3+c} - 1\right) - c}\right)}{d^2} - \frac{\log\left(\sqrt{-\frac{1}{\left(dx^3+c\right)^2} + 1} + 1\right) - \log\left(-\sqrt{-\frac{1}{\left(dx^3+c\right)^2} + 1}\right.}{d^2}\right.$$

input
```
integrate(x^2*(a+b*arcsec(d*x^3+c)),x, algorithm="giac")
```

output
```
1/3*a*x^3 + 1/6*b*d*(2*(d*x^3 + c)*arccos(-1/((d*x^3 + c)*(c/(d*x^3 + c) -
 1) - c))/d^2 - (log(sqrt(-1/(d*x^3 + c)^2 + 1) + 1) - log(-sqrt(-1/(d*x^3
 + c)^2 + 1) + 1))/d^2)
```

3.39.9 Mupad [B] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

$$\int x^2 \left(a + b \sec^{-1}\left(c + dx^3\right)\right) \, dx = \frac{a\,x^3}{3} - \frac{b\,\mathrm{atanh}\left(\frac{1}{\sqrt{1 - \frac{1}{{\left(d\,x^3+c\right)}^2}}}\right)}{3\,d} + \frac{b\,\mathrm{acos}\left(\frac{1}{d\,x^3+c}\right)\,\left(d\,x^3 + c\right)}{3\,d}$$

input
```
int(x^2*(a + b*acos(1/(c + d*x^3))),x)
```

output
```
(a*x^3)/3 - (b*atanh(1/(1 - 1/(c + d*x^3)^2)^(1/2)))/(3*d) + (b*acos(1/(c
+ d*x^3))*(c + d*x^3))/(3*d)
```

3.40 $\int x^3(a+b\sec^{-1}(c+dx^4))\,dx$

3.40.1 Optimal result

Integrand size = 16, antiderivative size = 58

$$\int x^3\left(a+b\sec^{-1}\left(c+dx^4\right)\right)\,dx=\frac{ax^4}{4}+\frac{b(c+dx^4)\sec^{-1}(c+dx^4)}{4d}-\frac{b\,\text{arctanh}\left(\sqrt{1-\frac{1}{(c+dx^4)^2}}\right)}{4d}$$

output
```
1/4*a*x^4+1/4*b*(d*x^4+c)*arcsec(d*x^4+c)/d-1/4*b*arctanh((1-1/(d*x^4+c)^2
)^(1/2))/d
```

3.40.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 137 vs. 2(58) = 116.

Time = 0.24 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.36

$$\int x^3\left(a+b\sec^{-1}\left(c+dx^4\right)\right)\,dx=\frac{ax^4}{4}+\frac{b(c+dx^4)\sec^{-1}(c+dx^4)}{4d}-\frac{b\sqrt{-1+(c+dx^4)^2}\left(-\log\left(1-\frac{c+dx^4}{\sqrt{-1+(c+dx^4)^2}}\right)+\log\left(1+\frac{c+dx^4}{\sqrt{-1+(c+dx^4)^2}}\right)\right)}{8d\,(c+dx^4)\sqrt{1-\frac{1}{(c+dx^4)^2}}}$$

input
```
Integrate[x^3*(a + b*ArcSec[c + d*x^4]),x]
```

output
```
(a*x^4)/4 + (b*(c + d*x^4)*ArcSec[c + d*x^4])/(4*d) - (b*Sqrt[-1 + (c + d*
x^4)^2]*(-Log[1 - (c + d*x^4)/Sqrt[-1 + (c + d*x^4)^2]] + Log[1 + (c + d*x
^4)/Sqrt[-1 + (c + d*x^4)^2]]))/(8*d*(c + d*x^4)*Sqrt[1 - (c + d*x^4)^(-2)
])
```

3.40.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {7266, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b \sec^{-1}\left(c+d x^4\right)\right) \, dx$$

$$\Big\downarrow \text{ 7266}$$

$$\frac{1}{4} \int \left(a+b \sec^{-1}\left(d x^4+c\right)\right)dx^4$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4} \left(a x^4-\frac{b \text{arctanh}\left(\sqrt{1-\frac{1}{\left(c+d x^4\right)^2}}\right)}{d}+\frac{b \left(c+d x^4\right) \sec^{-1}\left(c+d x^4\right)}{d}\right)$$

input
```
Int[x^3*(a + b*ArcSec[c + d*x^4]),x]
```

output
```
(a*x^4 + (b*(c + d*x^4)*ArcSec[c + d*x^4])/d - (b*ArcTanh[Sqrt[1 - (c + d*
x^4)^(-2)]])/d)/4
```

3.40.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.40.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.10

method	result	size
parts	$\frac{x^{4} a}{4}+\frac{b \left(\left(d \,x^{4}+c \right) \operatorname{arcsec}\left(d \,x^{4}+c \right)-\ln \left(d \,x^{4}+c +\left(d \,x^{4}+c \right) \sqrt{1-\frac{1}{\left(d \,x^{4}+c \right)^{2}}}\right)\right)}{4 d}$	64
derivativedivides	$\frac{\left(d \,x^{4}+c \right) a +b \left(\left(d \,x^{4}+c \right) \operatorname{arcsec}\left(d \,x^{4}+c \right)-\ln \left(d \,x^{4}+c +\left(d \,x^{4}+c \right) \sqrt{1-\frac{1}{\left(d \,x^{4}+c \right)^{2}}}\right)\right)}{4 d}$	68
default	$\frac{\left(d \,x^{4}+c \right) a +b \left(\left(d \,x^{4}+c \right) \operatorname{arcsec}\left(d \,x^{4}+c \right)-\ln \left(d \,x^{4}+c +\left(d \,x^{4}+c \right) \sqrt{1-\frac{1}{\left(d \,x^{4}+c \right)^{2}}}\right)\right)}{4 d}$	68

input
```
int(x^3*(a+b*arcsec(d*x^4+c)),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*a+1/4*b/d*((d*x^4+c)*arcsec(d*x^4+c)-ln(d*x^4+c+(d*x^4+c)*(1-1/(d*
x^4+c)^2)^(1/2)))
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.66

$$\int x^3\left(a+b\sec^{-1}\left(c+dx^4\right)\right)\,dx$$
$$= \frac{bdx^4 \operatorname{arcsec}\left(dx^4+c\right)+adx^4+2\,bc\arctan\left(-dx^4-c+\sqrt{d^2x^8+2\,cdx^4+c^2-1}\right)+b\log\left(-dx^4-c+\sqrt{c}\right.}{4\,d}$$

input
```
integrate(x^3*(a+b*arcsec(d*x^4+c)),x, algorithm="fricas")
```

output
```
1/4*(b*d*x^4*arcsec(d*x^4 + c) + a*d*x^4 + 2*b*c*arctan(-d*x^4 - c + sqrt(
d^2*x^8 + 2*c*d*x^4 + c^2 - 1)) + b*log(-d*x^4 - c + sqrt(d^2*x^8 + 2*c*d*
x^4 + c^2 - 1)))/d
```

3.40.6 Sympy [F(-1)]

Timed out.

$$\int x^3 \left(a + b \sec^{-1}\left(c + dx^4\right)\right) dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*asec(d*x**4+c)),x)
```

output
```
Timed out
```

3.40.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.22

$$\int x^3 \left(a + b \sec^{-1}\left(c + dx^4\right)\right) dx = \frac{1}{4} a x^4 + \frac{\left(2 \left(dx^4 + c\right) \operatorname{arcsec}\left(dx^4 + c\right) - \log\left(\sqrt{-\frac{1}{(dx^4+c)^2} + 1} + 1\right) + \log\left(-\sqrt{-\frac{1}{(dx^4+c)^2} + 1} + 1\right)\right) b}{8 d}$$

input
```
integrate(x^3*(a+b*arcsec(d*x^4+c)),x, algorithm="maxima")
```

output
```
1/4*a*x^4 + 1/8*(2*(d*x^4 + c)*arcsec(d*x^4 + c) - log(sqrt(-1/(d*x^4 + c)
^2 + 1) + 1) + log(-sqrt(-1/(d*x^4 + c)^2 + 1) + 1))*b/d
```

3.40.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.72

$$\int x^3 \left(a + b\sec^{-1}\left(c + dx^4\right)\right)\,dx = \frac{1}{4}ax^4 + \frac{1}{8}bd\left(\frac{2\left(dx^4+c\right)\arccos\left(-\frac{1}{\left(dx^4+c\right)\left(\frac{c}{dx^4+c}-1\right)-c}\right)}{d^2} - \frac{\log\left(\sqrt{-\frac{1}{\left(dx^4+c\right)^2}+1}+1\right) - \log\left(-\sqrt{-\frac{1}{\left(dx^4+c\right)^2}+1}\right.}{d^2}\right.$$

input
```
integrate(x^3*(a+b*arcsec(d*x^4+c)),x, algorithm="giac")
```

output
```
1/4*a*x^4 + 1/8*b*d*(2*(d*x^4 + c)*arccos(-1/((d*x^4 + c)*(c/(d*x^4 + c) -
 1) - c))/d^2 - (log(sqrt(-1/(d*x^4 + c)^2 + 1) + 1) - log(-sqrt(-1/(d*x^4
 + c)^2 + 1) + 1))/d^2)
```

3.40.9 Mupad [B] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

$$\int x^3 \left(a + b\sec^{-1}\left(c + dx^4\right)\right)\,dx = \frac{a\,x^4}{4} - \frac{b\,\mathrm{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{\left(d\,x^4+c\right)^2}}}\right)}{4\,d} + \frac{b\,\mathrm{acos}\left(\frac{1}{d\,x^4+c}\right)\,\left(d\,x^4+c\right)}{4\,d}$$

input
```
int(x^3*(a + b*acos(1/(c + d*x^4))),x)
```

output
```
(a*x^4)/4 - (b*atanh(1/(1 - 1/(c + d*x^4)^2)^(1/2)))/(4*d) + (b*acos(1/(c
+ d*x^4))*(c + d*x^4))/(4*d)
```

3.41 $\int x^{-1+n} \sec^{-1}(a+bx^n) \, dx$

3.41.1 Optimal result

Integrand size = 14, antiderivative size = 49

$$\int x^{-1+n} \sec^{-1}(a+bx^n) \, dx = \frac{(a+bx^n)\sec^{-1}(a+bx^n)}{bn} - \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{(a+bx^n)^2}}\right)}{bn}$$

output
```
(a+b*x^n)*arcsec(a+b*x^n)/b/n-arctanh((1-1/(a+b*x^n)^2)^(1/2))/b/n
```

3.41.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 130 vs. 2(49) = 98.

Time = 0.23 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.65

$$\int x^{-1+n} \sec^{-1}(a+bx^n) \, dx = \frac{(a+bx^n)\sec^{-1}(a+bx^n)}{bn} - \frac{\sqrt{-1+(a+bx^n)^2}\left(-\log\left(1-\frac{a+bx^n}{\sqrt{-1+(a+bx^n)^2}}\right)+\log\left(1+\frac{a+bx^n}{\sqrt{-1+(a+bx^n)^2}}\right)\right)}{2bn(a+bx^n)\sqrt{1-\frac{1}{(a+bx^n)^2}}}$$

input
```
Integrate[x^(-1 + n)*ArcSec[a + b*x^n],x]
```

output
```
((a + b*x^n)*ArcSec[a + b*x^n])/(b*n) - (Sqrt[-1 + (a + b*x^n)^2]*(-Log[1
- (a + b*x^n)/Sqrt[-1 + (a + b*x^n)^2]] + Log[1 + (a + b*x^n)/Sqrt[-1 + (a
 + b*x^n)^2]]))/(2*b*n*(a + b*x^n)*Sqrt[1 - (a + b*x^n)^(-2)])
```

3.41.3 Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {7266, 5773, 895, 798, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}\sec^{-1}\left(a+bx^n\right)\,dx$$

$$\Big\downarrow \text{7266}$$

$$\frac{\int \sec^{-1}\left(bx^n+a\right)dx^n}{n}$$

$$\Big\downarrow \text{5773}$$

$$\frac{\frac{(a+bx^n)\sec^{-1}(a+bx^n)}{b}-\int \frac{1}{(bx^n+a)\sqrt{1-\frac{1}{(bx^n+a)^2}}}dx^n}{n}$$

$$\Big\downarrow \text{895}$$

$$\frac{\frac{(a+bx^n)\sec^{-1}(a+bx^n)}{b}-\frac{\int \frac{x^{-n}}{\sqrt{1-x^{-2n}}}d(bx^n+a)}{b}}{n}$$

$$\Big\downarrow \text{798}$$

$$\frac{\frac{\int \frac{x^{-n}}{\sqrt{-bx^n-a+1}}dx^{-2n}}{2b}+\frac{(a+bx^n)\sec^{-1}(a+bx^n)}{b}}{n}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{(a+bx^n)\sec^{-1}(a+bx^n)}{b}-\frac{\int \frac{1}{1-x^{2n}}d\sqrt{-bx^n-a+1}}{b}}{n}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{(a+bx^n)\sec^{-1}(a+bx^n)}{b}-\frac{\text{arctanh}\left(\sqrt{-a-bx^n+1}\right)}{b}}{n}$$

input
```
Int[x^(-1 + n)*ArcSec[a + b*x^n],x]
```

output
```
(((a + b*x^n)*ArcSec[a + b*x^n])/b - ArcTanh[Sqrt[1 - a - b*x^n]]/b)/n
```

3.41.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 5773
```
Int[ArcSec[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(c + d*x)*(ArcSec[c + d*x]
/d), x] - Int[1/((c + d*x)*Sqrt[1 - 1/(c + d*x)^2]), x] /; FreeQ[{c, d}, x]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.41.4 Maple [F]

$$\int x^{-1+n} \operatorname{arcsec}(a+b\,x^{n})\,dx$$

input
```
int(x^(-1+n)*arcsec(a+b*x^n),x)
```

output
```
int(x^(-1+n)*arcsec(a+b*x^n),x)
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.88

$$\int x^{-1+n}\sec^{-1}(a+bx^n)\ dx = \frac{bx^n \operatorname{arcsec}(bx^n+a) + 2\,a\arctan\left(-bx^n - a + \sqrt{b^2x^{2\,n} + 2\,abx^n + a^2 - 1}\right) + \log\left(-bx^n - a + \sqrt{b^2x^{2\,n} + 2}\right.}{bn}$$

input
```
integrate(x^(-1+n)*arcsec(a+b*x^n),x, algorithm="fricas")
```

output
```
(b*x^n*arcsec(b*x^n + a) + 2*a*arctan(-b*x^n - a + sqrt(b^2*x^(2*n) + 2*a*
b*x^n + a^2 - 1)) + log(-b*x^n - a + sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2 -
1)))/(b*n)
```

3.41.6 Sympy [F(-1)]

Timed out.

$$\int x^{-1+n}\sec^{-1}(a+bx^n)\ dx = \text{Timed out}$$

input
```
integrate(x**(-1+n)*asec(a+b*x**n),x)
```

output
```
Timed out
```

3.41.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.35

$$\int x^{-1+n}\sec^{-1}(a+bx^n)\,dx = \frac{2\,(bx^n+a)\operatorname{arcsec}(bx^n+a)-\log\left(\sqrt{-\frac{1}{(bx^n+a)^2}+1}+1\right)+\log\left(-\sqrt{-\frac{1}{(bx^n+a)^2}+1}+1\right)}{2\,bn}$$

input
```
integrate(x^(-1+n)*arcsec(a+b*x^n),x, algorithm="maxima")
```

output
```
1/2*(2*(b*x^n + a)*arcsec(b*x^n + a) - log(sqrt(-1/(b*x^n + a)^2 + 1) + 1)
 + log(-sqrt(-1/(b*x^n + a)^2 + 1) + 1))/(b*n)
```

3.41.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.53

$$\int x^{-1+n}\sec^{-1}(a+bx^n)\,dx = \frac{b\left(\frac{2\,(bx^n+a)\arccos\left(\frac{1}{bx^n+a}\right)}{b^2}-\frac{\log\left(\sqrt{-\frac{1}{(bx^n+a)^2}+1}+1\right)-\log\left(-\sqrt{-\frac{1}{(bx^n+a)^2}+1}+1\right)}{b^2}\right)}{2\,n}$$

input
```
integrate(x^(-1+n)*arcsec(a+b*x^n),x, algorithm="giac")
```

output
```
1/2*b*(2*(b*x^n + a)*arccos(1/(b*x^n + a))/b^2 - (log(sqrt(-1/(b*x^n + a)^
2 + 1) + 1) - log(-sqrt(-1/(b*x^n + a)^2 + 1) + 1))/b^2)/n
```

3.41.9 Mupad [B] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int x^{-1+n}\sec^{-1}(a+bx^n)\,dx = -\frac{\operatorname{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{(a+b\,x^n)^2}}}\right)-\operatorname{acos}\left(\frac{1}{a+b\,x^n}\right)\,(a+b\,x^n)}{b\,n}$$

input
```
int(x^(n - 1)*acos(1/(a + b*x^n)),x)
```

output
```
-(atanh(1/(1 - 1/(a + b*x^n)^2)^(1/2)) - acos(1/(a + b*x^n))*(a + b*x^n))/
(b*n)
```

3.41. $\int x^{-1+n} \sec^{-1}(a+bx^n)\,dx$

3.42 $\int \sec^{-1}\left(ce^{a+bx}\right)\,dx$

3.42.1 Optimal result

Integrand size = 10, antiderivative size = 85

$$\int \sec^{-1}\left(ce^{a+bx}\right)\,dx = \frac{i\sec^{-1}\left(ce^{a+bx}\right)^2}{2b} - \frac{\sec^{-1}\left(ce^{a+bx}\right)\log\left(1+e^{2i\sec^{-1}\left(ce^{a+bx}\right)}\right)}{b} + \frac{i\,\mathrm{PolyLog}\left(2,-e^{2i\sec^{-1}\left(ce^{a+bx}\right)}\right)}{2b}$$

output
```
1/2*I*arcsec(c*exp(b*x+a))^2/b-arcsec(c*exp(b*x+a))*ln(1+(1/c/exp(b*x+a)+I
*(1-1/c^2/exp(b*x+a)^2)^(1/2))^2)/b+1/2*I*polylog(2,-(1/c/exp(b*x+a)+I*(1-
1/c^2/exp(b*x+a)^2)^(1/2))^2)/b
```

3.42.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 280 vs. 2(85) = 170.

Time = 0.92 (sec) , antiderivative size = 280, normalized size of antiderivative = 3.29

$$\int \sec^{-1}\left(ce^{a+bx}\right)\,dx = x\sec^{-1}\left(ce^{a+bx}\right) - \frac{e^{-a-bx}\left(4\sqrt{-1+c^2e^{2(a+bx)}}\arctan\left(\sqrt{-1+c^2e^{2(a+bx)}}\right)\left(2bx-\log\left(c^2e^{2(a+bx)}\right)\right)+\sqrt{1-c^2e^{2(a+bx)}}\left(\log^2\left(\right.\right.\right.}{}$$

input
```
Integrate[ArcSec[c*E^(a + b*x)],x]
```

output
```
x*ArcSec[c*E^(a + b*x)] - (E^(-a - b*x)*(4*Sqrt[-1 + c^2*E^(2*(a + b*x))]*
ArcTan[Sqrt[-1 + c^2*E^(2*(a + b*x))]]*(2*b*x - Log[c^2*E^(2*(a + b*x))])
+ Sqrt[1 - c^2*E^(2*(a + b*x))]*(Log[c^2*E^(2*(a + b*x))]^2 - 4*Log[c^2*E^
(2*(a + b*x))]*Log[(1 + Sqrt[1 - c^2*E^(2*(a + b*x))])/2] + 2*Log[(1 + Sqr
t[1 - c^2*E^(2*(a + b*x))])/2]^2) - 4*Sqrt[1 - c^2*E^(2*(a + b*x))]*PolyLo
g[2, (1 - Sqrt[1 - c^2*E^(2*(a + b*x))])/2]))/(8*b*c*Sqrt[1 - 1/(c^2*E^(2*
(a + b*x)))])
```

3.42.3 Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {2720, 5741, 5137, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{-1}\left(ce^{a+bx}\right)dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int e^{-a-bx}\sec^{-1}\left(ce^{a+bx}\right)de^{a+bx}}{b}$$

$$\Big\downarrow \text{5741}$$

$$-\frac{\int e^{-a-bx}\arccos\left(\frac{e^{-a-bx}}{c}\right)de^{-a-bx}}{b}$$

$$\Big\downarrow \text{5137}$$

$$\frac{\int ce^{a+bx}\sqrt{1-\frac{e^{-2a-2bx}}{c^2}}\arccos\left(\frac{e^{-a-bx}}{c}\right)d\arccos\left(\frac{e^{-a-bx}}{c}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \arccos\left(\frac{e^{-a-bx}}{c}\right)\tan\left(\arccos\left(\frac{e^{-a-bx}}{c}\right)\right)d\arccos\left(\frac{e^{-a-bx}}{c}\right)}{b}$$

$$\Big\downarrow \text{4202}$$

$$\frac{\frac{1}{2}ie^{2a+2bx}-2i\int \frac{e^{a+bx+2i\arccos\left(\frac{e^{-a-bx}}{c}\right)}}{1+e^{2i\arccos\left(\frac{e^{-a-bx}}{c}\right)}}d\arccos\left(\frac{e^{-a-bx}}{c}\right)}{b}$$

$\big\downarrow$ 2620

$$\frac{\frac{1}{2}ie^{2a+2bx}-2i\left(\frac{1}{2}i\int \log\left(1+e^{2i\arccos\left(\frac{e^{-a-bx}}{c}\right)}\right)d\arccos\left(\frac{e^{-a-bx}}{c}\right)-\frac{1}{2}i\arccos\left(\frac{e^{-a-bx}}{c}\right)\log\left(1+e^{2i\arccos\left(\frac{e^{-a-bx}}{c}\right)}\right.\right.}{b}$$

$\big\downarrow$ 2715

$$\frac{\frac{1}{2}ie^{2a+2bx}-2i\left(\frac{1}{4}\int e^{2i\arccos\left(\frac{e^{-a-bx}}{c}\right)}\log\left(1+e^{2i\arccos\left(\frac{e^{-a-bx}}{c}\right)}\right)de^{2i\arccos\left(\frac{e^{-a-bx}}{c}\right)}-\frac{1}{2}i\arccos\left(\frac{e^{-a-bx}}{c}\right)\log\left(1+\right.\right.}{b}$$

$\big\downarrow$ 2838

$$\frac{\frac{1}{2}ie^{2a+2bx}-2i\left(-\frac{1}{4}\text{PolyLog}\left(2,-e^{2i\arccos\left(\frac{e^{-a-bx}}{c}\right)}\right)-\frac{1}{2}i\arccos\left(\frac{e^{-a-bx}}{c}\right)\log\left(1+e^{2i\arccos\left(\frac{e^{-a-bx}}{c}\right)}\right)\right)}{b}$$

input
```
Int[ArcSec[c*E^(a + b*x)],x]
```

output
```
((I/2)*E^(2*a + 2*b*x) - (2*I)*((-1/2*I)*ArcCos[E^(-a - b*x)/c]*Log[1 + E^
((2*I)*ArcCos[E^(-a - b*x)/c])] - PolyLog[2, -E^((2*I)*ArcCos[E^(-a - b*x)
/c])]/4))/b
```

3.42.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 5741
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a + b
*ArcCos[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

3.42.4 Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.31

method	result
derivativedivides	$\frac{\frac{i \operatorname{arcsec}\left(e^{b x+a} c\right)^{2}}{2}-\operatorname{arcsec}\left(e^{b x+a} c\right) \ln\left(1+\left(\frac{e^{-b x-a}}{c}+i \sqrt{1-\frac{e^{-2 b x-2 a}}{c^{2}}}\right)^{2}\right)+\frac{i \operatorname{polylog}\left(2,-\left(\frac{e^{-b x-a}}{c}+i \sqrt{1-\frac{e^{-2 b x-2 a}}{c^{2}}}\right)^{2}\right)}{2}}{b}$
default	$\frac{\frac{i \operatorname{arcsec}\left(e^{b x+a} c\right)^{2}}{2}-\operatorname{arcsec}\left(e^{b x+a} c\right) \ln\left(1+\left(\frac{e^{-b x-a}}{c}+i \sqrt{1-\frac{e^{-2 b x-2 a}}{c^{2}}}\right)^{2}\right)+\frac{i \operatorname{polylog}\left(2,-\left(\frac{e^{-b x-a}}{c}+i \sqrt{1-\frac{e^{-2 b x-2 a}}{c^{2}}}\right)^{2}\right)}{2}}{b}$

input
```
int(arcsec(exp(b*x+a)*c),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*I*arcsec(exp(b*x+a)*c)^2-arcsec(exp(b*x+a)*c)*ln(1+(1/c/exp(b*x+a
)+I*(1-1/c^2/exp(b*x+a)^2)^(1/2))^2)+1/2*I*polylog(2,-(1/c/exp(b*x+a)+I*(1
-1/c^2/exp(b*x+a)^2)^(1/2))^2))
```

3.42.5 Fricas [F(-2)]

Exception generated.

$$\int \sec^{-1}\left(ce^{a+bx}\right) dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsec(c*exp(b*x+a)),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.42.6 Sympy [F]

$$\int \sec^{-1}\left(ce^{a+bx}\right) dx = \int \operatorname{asec}\left(ce^{a+bx}\right) dx$$

input
```
integrate(asec(c*exp(b*x+a)),x)
```

output
```
Integral(asec(c*exp(a + b*x)), x)
```

3.42.7 Maxima [F]

$$\int \sec^{-1}\left(ce^{a+bx}\right) dx = \int \operatorname{arcsec}\left(ce^{(bx+a)}\right) dx$$

input
```
integrate(arcsec(c*exp(b*x+a)),x, algorithm="maxima")
```

output
```
-1/2*(2*b^2*c^2*integrate(x*e^(2*b*x + 2*a + 1/2*log(c*e^(b*x + a) + 1) +
1/2*log(c*e^(b*x + a) - 1))/(c^2*e^(2*b*x + 2*a) + (c^2*e^(2*b*x + 2*a) -
1)*e^(log(c*e^(b*x + a) + 1) + log(c*e^(b*x + a) - 1)) - 1), x) + 2*I*b^2*
c^2*integrate(x*e^(2*b*x + 2*a)/(c^2*e^(2*b*x + 2*a) + (c^2*e^(2*b*x + 2*a
) - 1)*e^(log(c*e^(b*x + a) + 1) + log(c*e^(b*x + a) - 1)) - 1), x) - I*b^
2*x^2 - 2*b*x*arctan(sqrt(c*e^(b*x + a) + 1)*sqrt(c*e^(b*x + a) - 1)) + I*
b*x*log(c^2*e^(2*b*x + 2*a)) - I*b*x*log(c*e^(b*x + a) + 1) - I*b*x*log(-c
*e^(b*x + a) + 1) - 2*(I*a*b + I*b*log(c))*x - I*dilog(c*e^(b*x + a)) - I*
dilog(-c*e^(b*x + a)))/b
```

3.42.8 Giac [F]

$$\int \sec^{-1}\left(ce^{a+bx}\right)\,dx = \int \operatorname{arcsec}\left(ce^{(bx+a)}\right)\,dx$$

input
```
integrate(arcsec(c*exp(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arcsec(c*e^(b*x + a)), x)
```

3.42.9 Mupad [F(-1)]

Timed out.

$$\int \sec^{-1}\left(ce^{a+bx}\right)\,dx = \int \operatorname{acos}\left(\frac{e^{-a-b\,x}}{c}\right)\,dx$$

input
```
int(acos(exp(- a - b*x)/c),x)
```

output
```
int(acos(exp(- a - b*x)/c), x)
```

3.43 $\int e^{\sec^{-1}(ax)}x^2\,dx$

3.43.1 Optimal result

Integrand size = 10, antiderivative size = 99

$$\int e^{\sec^{-1}(ax)}x^2\,dx = -\frac{\left(\frac{12}{5}+\frac{4i}{5}\right)e^{(1+3i)\sec^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{3}{2}-\frac{i}{2},3,\frac{5}{2}-\frac{i}{2},-e^{2i\sec^{-1}(ax)}\right)}{a^3} + \frac{\left(\frac{24}{5}+\frac{8i}{5}\right)e^{(1+3i)\sec^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{3}{2}-\frac{i}{2},4,\frac{5}{2}-\frac{i}{2},-e^{2i\sec^{-1}(ax)}\right)}{a^3}$$

output
```
(-12/5-4/5*I)*exp((1+3*I)*arcsec(a*x))*hypergeom([3, 3/2-1/2*I],[5/2-1/2*I
],-(1/a/x+I*(1-1/a^2/x^2)^(1/2))^2)/a^3+(24/5+8/5*I)*exp((1+3*I)*arcsec(a*
x))*hypergeom([4, 3/2-1/2*I],[5/2-1/2*I],-(1/a/x+I*(1-1/a^2/x^2)^(1/2))^2)
/a^3
```

3.43.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.96

$$\int e^{\sec^{-1}(ax)}x^2\,dx = \frac{e^{\sec^{-1}(ax)}\left((-4-4i)\left(-i+a\sqrt{1-\frac{1}{a^2x^2}}x\right)\text{Hypergeometric2F1}\left(\frac{1}{2}-\frac{i}{2},1,\frac{3}{2}-\frac{i}{2},-e^{2i\sec^{-1}(ax)}\right)+a^4x^4(5+\right.}{12a^4x}$$

input
```
Integrate[E^ArcSec[a*x]*x^2,x]
```

output
```
(E^ArcSec[a*x]*((-4 - 4*I)*(-I + a*Sqrt[1 - 1/(a^2*x^2)]*x)*Hypergeometric
2F1[1/2 - I/2, 1, 3/2 - I/2, -E^((2*I)*ArcSec[a*x])] + a^4*x^4*(5 + Cos[2*
ArcSec[a*x]] - Sin[2*ArcSec[a*x]])))/(12*a^4*x)
```

3.43.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5789, 27, 4974, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{\sec^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{5789}$$

$$\frac{\int a^2 e^{\sec^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^4d\sec^{-1}(ax)}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int a^4 e^{\sec^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^4d\sec^{-1}(ax)}{a^3}$$

$$\Big\downarrow \text{4974}$$

$$\frac{\int \left(\frac{16ie^{(1+3i)\sec^{-1}(ax)}}{\left(1+e^{2i\sec^{-1}(ax)}\right)^4}-\frac{8ie^{(1+3i)\sec^{-1}(ax)}}{\left(1+e^{2i\sec^{-1}(ax)}\right)^3}\right)d\sec^{-1}(ax)}{a^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{24}{5}+\frac{8i}{5}\right)e^{(1+3i)\sec^{-1}(ax)}\text{Hypergeometric2F1}\left(\frac{3}{2}-\frac{i}{2},4,\frac{5}{2}-\frac{i}{2},-e^{2i\sec^{-1}(ax)}\right)-\left(\frac{12}{5}+\frac{4i}{5}\right)e^{(1+3i)\sec^{-1}(ax)}\text{Hyperge}}{a^3}$$

input
```
Int[E^ArcSec[a*x]*x^2,x]
```

output
```
((-12/5 - (4*I)/5)*E^((1 + 3*I)*ArcSec[a*x])*Hypergeometric2F1[3/2 - I/2,
3, 5/2 - I/2, -E^((2*I)*ArcSec[a*x])] + (24/5 + (8*I)/5)*E^((1 + 3*I)*ArcS
ec[a*x])*Hypergeometric2F1[3/2 - I/2, 4, 5/2 - I/2, -E^((2*I)*ArcSec[a*x])
])/a^3
```

3.43.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4974
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && TrigQ[G] && TrigQ[H]
```

rule 5789
```
Int[(u_.)*(f_)^(ArcSec[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Simp[
1/b   Subst[Int[(u /. x -> -a/b + Sec[x]/b)*f^(c*x^n)*Sec[x]*Tan[x], x], x,
 ArcSec[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

3.43.4 Maple [F]

$$\int e^{\operatorname{arcsec}(ax)} x^2 dx$$

input
```
int(exp(arcsec(a*x))*x^2,x)
```

output
```
int(exp(arcsec(a*x))*x^2,x)
```

3.43.5 Fricas [F]

$$\int e^{\sec^{-1}(ax)}x^2\,dx=\int x^2e^{(\operatorname{arcsec}(ax))}\,dx$$

input
```
integrate(exp(arcsec(a*x))*x^2,x, algorithm="fricas")
```

output
```
integral(x^2*e^(arcsec(a*x)), x)
```

3.43.6 Sympy [F]

$$\int e^{\sec^{-1}(ax)}x^2\,dx=\int x^2e^{\operatorname{asec}(ax)}\,dx$$

input
```
integrate(exp(asec(a*x))*x**2,x)
```

output
```
Integral(x**2*exp(asec(a*x)), x)
```

3.43.7 Maxima [F]

$$\int e^{\sec^{-1}(ax)}x^2\,dx=\int x^2e^{(\operatorname{arcsec}(ax))}\,dx$$

input
```
integrate(exp(arcsec(a*x))*x^2,x, algorithm="maxima")
```

output
```
integrate(x^2*e^(arcsec(a*x)), x)
```

3.43.8 Giac [F]

$$\int e^{\sec^{-1}(ax)}x^2\,dx = \int x^2 e^{(\operatorname{arcsec}(ax))}\,dx$$

input
```
integrate(exp(arcsec(a*x))*x^2,x, algorithm="giac")
```

output
```
integrate(x^2*e^(arcsec(a*x)), x)
```

3.43.9 Mupad [F(-1)]

Timed out.

$$\int e^{\sec^{-1}(ax)}x^2\,dx = \int x^2\,\mathrm{e}^{\operatorname{acos}\left(\frac{1}{a\,x}\right)}\,dx$$

input
```
int(x^2*exp(acos(1/(a*x))),x)
```

output
```
int(x^2*exp(acos(1/(a*x))), x)
```

3.44 $\int e^{\sec^{-1}(ax)}x\,dx$

3.44.1 Optimal result

Integrand size = 8, antiderivative size = 91

$$\int e^{\sec^{-1}(ax)}x\,dx = -\frac{\left(\frac{8}{5}+\frac{4i}{5}\right)e^{(1+2i)\sec^{-1}(ax)}\,\text{Hypergeometric2F1}\left(1-\frac{i}{2},2,2-\frac{i}{2},-e^{2i\sec^{-1}(ax)}\right)}{a^2} + \frac{\left(\frac{16}{5}+\frac{8i}{5}\right)e^{(1+2i)\sec^{-1}(ax)}\,\text{Hypergeometric2F1}\left(1-\frac{i}{2},3,2-\frac{i}{2},-e^{2i\sec^{-1}(ax)}\right)}{a^2}$$

output
```
(-8/5-4/5*I)*exp((1+2*I)*arcsec(a*x))*hypergeom([2, 1-1/2*I],[2-1/2*I],-(1
/a/x+I*(1-1/a^2/x^2)^(1/2))^2)/a^2+(16/5+8/5*I)*exp((1+2*I)*arcsec(a*x))*h
ypergeom([3, 1-1/2*I],[2-1/2*I],-(1/a/x+I*(1-1/a^2/x^2)^(1/2))^2)/a^2
```

3.44.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.18

$$\int e^{\sec^{-1}(ax)}x\,dx = \frac{\left(\frac{1}{5}+\frac{i}{10}\right)e^{\sec^{-1}(ax)}\left((-2+i)ax\left(\sqrt{1-\frac{1}{a^2x^2}}-ax\right)+(1+2i)\,\text{Hypergeometric2F1}\left(-\frac{i}{2},1,1-\frac{i}{2},-e^{2i\sec^{-1}(\right.\right.}{a^2}$$

input
```
Integrate[E^ArcSec[a*x]*x,x]
```

output
```
((1/5 + I/10)*E^ArcSec[a*x]*((-2 + I)*a*x*(Sqrt[1 - 1/(a^2*x^2)] - a*x) +
(1 + 2*I)*Hypergeometric2F1[-1/2*I, 1, 1 - I/2, -E^((2*I)*ArcSec[a*x])] -
E^((2*I)*ArcSec[a*x])*Hypergeometric2F1[1, 1 - I/2, 2 - I/2, -E^((2*I)*Arc
Sec[a*x])]))/a^2
```

3.44.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5789, 27, 4974, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{\sec^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{5789}$$

$$\frac{\int a^2e^{\sec^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^3d\sec^{-1}(ax)}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int a^3e^{\sec^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^3d\sec^{-1}(ax)}{a^2}$$

$$\Big\downarrow \text{4974}$$

$$\frac{\int \left(\frac{8ie^{(1+2i)\sec^{-1}(ax)}}{\left(1+e^{2i\sec^{-1}(ax)}\right)^3}-\frac{4ie^{(1+2i)\sec^{-1}(ax)}}{\left(1+e^{2i\sec^{-1}(ax)}\right)^2}\right)d\sec^{-1}(ax)}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{16}{5}+\frac{8i}{5}\right)e^{(1+2i)\sec^{-1}(ax)}\text{Hypergeometric2F1}\left(1-\frac{i}{2},3,2-\frac{i}{2},-e^{2i\sec^{-1}(ax)}\right)-\left(\frac{8}{5}+\frac{4i}{5}\right)e^{(1+2i)\sec^{-1}(ax)}\text{Hypergeor}}{a^2}$$

input
```
Int[E^ArcSec[a*x]*x,x]
```

output
```
((-8/5 - (4*I)/5)*E^((1 + 2*I)*ArcSec[a*x])*Hypergeometric2F1[1 - I/2, 2,
2 - I/2, -E^((2*I)*ArcSec[a*x])] + (16/5 + (8*I)/5)*E^((1 + 2*I)*ArcSec[a*
x])*Hypergeometric2F1[1 - I/2, 3, 2 - I/2, -E^((2*I)*ArcSec[a*x])])/a^2
```

3.44.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4974
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && TrigQ[G] && TrigQ[H]
```

rule 5789
```
Int[(u_.)*(f_)^(ArcSec[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Simp[
1/b   Subst[Int[(u /. x -> -a/b + Sec[x]/b)*f^(c*x^n)*Sec[x]*Tan[x], x], x,
 ArcSec[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

3.44.4 Maple [F]

$$\int \mathrm{e}^{\mathrm{arcsec}(ax)} x dx$$

input
```
int(exp(arcsec(a*x))*x,x)
```

output
```
int(exp(arcsec(a*x))*x,x)
```

3.44.5 Fricas [F]

$$\int e^{\sec^{-1}(ax)}x\,dx = \int xe^{(\operatorname{arcsec}(ax))}\,dx$$

input
```
integrate(exp(arcsec(a*x))*x,x, algorithm="fricas")
```

output
```
integral(x*e^(arcsec(a*x)), x)
```

3.44.6 Sympy [F]

$$\int e^{\sec^{-1}(ax)}x\,dx = \int xe^{\operatorname{asec}(ax)}\,dx$$

input
```
integrate(exp(asec(a*x))*x,x)
```

output
```
Integral(x*exp(asec(a*x)), x)
```

3.44.7 Maxima [F]

$$\int e^{\sec^{-1}(ax)}x\,dx = \int xe^{(\operatorname{arcsec}(ax))}\,dx$$

input
```
integrate(exp(arcsec(a*x))*x,x, algorithm="maxima")
```

output
```
integrate(x*e^(arcsec(a*x)), x)
```

3.44.8 Giac [F]

$$\int e^{\sec^{-1}(ax)}x\,dx = \int xe^{(\mathrm{arcsec}(ax))}\,dx$$

input
```
integrate(exp(arcsec(a*x))*x,x, algorithm="giac")
```

output
```
integrate(x*e^(arcsec(a*x)), x)
```

3.44.9 Mupad [F(-1)]

Timed out.

$$\int e^{\sec^{-1}(ax)}x\,dx = \int x\,\mathrm{e}^{\mathrm{acos}\left(\frac{1}{a\,x}\right)}\,dx$$

input
```
int(x*exp(acos(1/(a*x))),x)
```

output
```
int(x*exp(acos(1/(a*x))), x)
```

3.45 $\int e^{\sec^{-1}(ax)}\,dx$

3.45.1 Optimal result

Integrand size = 6, antiderivative size = 91

$$\int e^{\sec^{-1}(ax)}\,dx = -\frac{(1+i)e^{(1+i)\sec^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}-\frac{i}{2},1,\frac{3}{2}-\frac{i}{2},-e^{2i\sec^{-1}(ax)}\right)}{a} + \frac{(2+2i)e^{(1+i)\sec^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}-\frac{i}{2},2,\frac{3}{2}-\frac{i}{2},-e^{2i\sec^{-1}(ax)}\right)}{a}$$

output
```
(-1-I)*exp((1+I)*arcsec(a*x))*hypergeom([1, 1/2-1/2*I],[3/2-1/2*I],-(1/a/x
+I*(1-1/a^2/x^2)^(1/2))^2)/a+(2+2*I)*exp((1+I)*arcsec(a*x))*hypergeom([2,
1/2-1/2*I],[3/2-1/2*I],-(1/a/x+I*(1-1/a^2/x^2)^(1/2))^2)/a
```

3.45.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.59

$$\int e^{\sec^{-1}(ax)}\,dx = e^{\sec^{-1}(ax)}x - \frac{(1-i)e^{(1+i)\sec^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}-\frac{i}{2},1,\frac{3}{2}-\frac{i}{2},-e^{2i\sec^{-1}(ax)}\right)}{a}$$

input
```
Integrate[E^ArcSec[a*x],x]
```

output
```
E^ArcSec[a*x]*x - ((1 - I)*E^((1 + I)*ArcSec[a*x])*Hypergeometric2F1[1/2 -
 I/2, 1, 3/2 - I/2, -E^((2*I)*ArcSec[a*x])])/a
```

3.45.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5789, 4974, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\sec^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 5789}$$

$$\frac{\int a^2 e^{\sec^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^2 d\sec^{-1}(ax)}{a}$$

$$\Big\downarrow \text{ 4974}$$

$$\frac{\int \left(\frac{4ie^{(1+i)\sec^{-1}(ax)}}{\left(1+e^{2i\sec^{-1}(ax)}\right)^2}-\frac{2ie^{(1+i)\sec^{-1}(ax)}}{1+e^{2i\sec^{-1}(ax)}}\right)d\sec^{-1}(ax)}{a}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(2+2i)e^{(1+i)\sec^{-1}(ax)}\text{Hypergeometric2F1}\left(\frac{1}{2}-\frac{i}{2},2,\frac{3}{2}-\frac{i}{2},-e^{2i\sec^{-1}(ax)}\right)-(1+i)e^{(1+i)\sec^{-1}(ax)}\text{Hypergeometri}}{a}$$

input
```
Int[E^ArcSec[a*x],x]
```

output
```
((-1 - I)*E^((1 + I)*ArcSec[a*x])*Hypergeometric2F1[1/2 - I/2, 1, 3/2 - I/
2, -E^((2*I)*ArcSec[a*x])] + (2 + 2*I)*E^((1 + I)*ArcSec[a*x])*Hypergeomet
ric2F1[1/2 - I/2, 2, 3/2 - I/2, -E^((2*I)*ArcSec[a*x])])/a
```

3.45.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4974
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && TrigQ[G] && TrigQ[H]
```

rule 5789
```
Int[(u_.)*(f_)^(ArcSec[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Simp[
1/b   Subst[Int[(u /. x -> -a/b + Sec[x]/b)*f^(c*x^n)*Sec[x]*Tan[x], x], x,
 ArcSec[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

3.45.4 Maple [F]

$$\int \mathrm{e}^{\mathrm{arcsec}(ax)}dx$$

input
```
int(exp(arcsec(a*x)),x)
```

output
```
int(exp(arcsec(a*x)),x)
```

3.45.5 Fricas [F]

$$\int e^{\sec^{-1}(ax)}\, dx = \int e^{(\mathrm{arcsec}(ax))}\, dx$$

input
```
integrate(exp(arcsec(a*x)),x, algorithm="fricas")
```

output
```
integral(e^(arcsec(a*x)), x)
```

3.45.6 Sympy [F]

$$\int e^{\sec^{-1}(ax)}\, dx = \int e^{\mathrm{asec}\,(ax)}\, dx$$

input
```
integrate(exp(asec(a*x)),x)
```

output
```
Integral(exp(asec(a*x)), x)
```

3.45.7 Maxima [F]

$$\int e^{\sec^{-1}(ax)}\,dx = \int e^{(\operatorname{arcsec}(ax))}\,dx$$

input
```
integrate(exp(arcsec(a*x)),x, algorithm="maxima")
```

output
```
integrate(e^(arcsec(a*x)), x)
```

3.45.8 Giac [F]

$$\int e^{\sec^{-1}(ax)}\,dx = \int e^{(\operatorname{arcsec}(ax))}\,dx$$

input
```
integrate(exp(arcsec(a*x)),x, algorithm="giac")
```

output
```
integrate(e^(arcsec(a*x)), x)
```

3.45.9 Mupad [F(-1)]

Timed out.

$$\int e^{\sec^{-1}(ax)}\,dx = \int e^{\operatorname{acos}\left(\frac{1}{a\,x}\right)}\,dx$$

input
```
int(exp(acos(1/(a*x))),x)
```

output
```
int(exp(acos(1/(a*x))), x)
```

3.46 $\int \frac{e^{\sec^{-1}(ax)}}{x} dx$

3.46.1 Optimal result

Integrand size = 10, antiderivative size = 45

$$\int \frac{e^{\sec^{-1}(ax)}}{x} dx = -ie^{\sec^{-1}(ax)} + 2ie^{\sec^{-1}(ax)} \text{Hypergeometric2F1}\left(-\frac{i}{2},1,1-\frac{i}{2},-e^{2i\sec^{-1}(ax)}\right)$$

output
```
-I*exp(arcsec(a*x))+2*I*exp(arcsec(a*x))*hypergeom([1, -1/2*I],[1-1/2*I],-
(1/a/x+I*(1-1/a^2/x^2)^(1/2))^2)
```

3.46.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.76

$$\int \frac{e^{\sec^{-1}(ax)}}{x} dx = -i\left(-e^{\sec^{-1}(ax)} \text{Hypergeometric2F1}\left(-\frac{i}{2},1,1-\frac{i}{2},-e^{2i\sec^{-1}(ax)}\right) + \left(\frac{1}{5} - \frac{2i}{5}\right) e^{(1+2i)\sec^{-1}(ax)} \text{Hypergeometric2F1}\left(1,1-\frac{i}{2},2-\frac{i}{2},-e^{2i\sec^{-1}(ax)}\right)\right)$$

input
```
Integrate[E^ArcSec[a*x]/x,x]
```

output
```
(-I)*(-(E^ArcSec[a*x]*Hypergeometric2F1[-1/2*I, 1, 1 - I/2, -E^((2*I)*ArcS
ec[a*x])]) + (1/5 - (2*I)/5)*E^((1 + 2*I)*ArcSec[a*x])*Hypergeometric2F1[1
, 1 - I/2, 2 - I/2, -E^((2*I)*ArcSec[a*x])])
```

3.46.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5789, 27, 4942, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\sec^{-1}(ax)}}{x}\,dx$$

$$\Big\downarrow \text{ 5789}$$

$$\frac{\int a^2 e^{\sec^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x d\sec^{-1}(ax)}{a}$$

$$\Big\downarrow \text{ 27}$$

$$\int ax\sqrt{1-\frac{1}{a^2x^2}}e^{\sec^{-1}(ax)}d\sec^{-1}(ax)$$

$$\Big\downarrow \text{ 4942}$$

$$i\int\left(\frac{2e^{\sec^{-1}(ax)}}{1+e^{2i\sec^{-1}(ax)}}-e^{\sec^{-1}(ax)}\right)d\sec^{-1}(ax)$$

$$\Big\downarrow \text{ 2009}$$

$$i\left(-e^{\sec^{-1}(ax)}+2e^{\sec^{-1}(ax)}\text{Hypergeometric2F1}\left(-\frac{i}{2},1,1-\frac{i}{2},-e^{2i\sec^{-1}(ax)}\right)\right)$$

input
```
Int[E^ArcSec[a*x]/x,x]
```

output
```
I*(-E^ArcSec[a*x] + 2*E^ArcSec[a*x]*Hypergeometric2F1[-1/2*I, 1, 1 - I/2,
-E^((2*I)*ArcSec[a*x])])
```

3.46.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4942
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[I^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x
)))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x
] && IntegerQ[n]
```

rule 5789
```
Int[(u_.)*(f_)^(ArcSec[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Simp[
1/b   Subst[Int[(u /. x -> -a/b + Sec[x]/b)*f^(c*x^n)*Sec[x]*Tan[x], x], x,
 ArcSec[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

3.46.4 Maple [F]

$$\int \frac{\mathrm{e}^{\operatorname{arcsec}(ax)}}{x}dx$$

input
```
int(exp(arcsec(a*x))/x,x)
```

output
```
int(exp(arcsec(a*x))/x,x)
```

3.46.5 Fricas [F]

$$\int \frac{e^{\sec^{-1}(ax)}}{x}\,dx = \int \frac{e^{(\operatorname{arcsec}(ax))}}{x}\,dx$$

input
```
integrate(exp(arcsec(a*x))/x,x, algorithm="fricas")
```

output
```
integral(e^(arcsec(a*x))/x, x)
```

3.46.6 Sympy [F]

$$\int \frac{e^{\sec^{-1}(ax)}}{x}\,dx = \int \frac{e^{\operatorname{asec}(ax)}}{x}\,dx$$

input
```
integrate(exp(asec(a*x))/x,x)
```

output
```
Integral(exp(asec(a*x))/x, x)
```

3.46.7 Maxima [F]

$$\int \frac{e^{\sec^{-1}(ax)}}{x}\,dx = \int \frac{e^{(\operatorname{arcsec}(ax))}}{x}\,dx$$

input
```
integrate(exp(arcsec(a*x))/x,x, algorithm="maxima")
```

output
```
integrate(e^(arcsec(a*x))/x, x)
```

3.46.8 Giac [F]

$$\int \frac{e^{\sec^{-1}(ax)}}{x}\,dx = \int \frac{e^{(\operatorname{arcsec}(ax))}}{x}\,dx$$

input
```
integrate(exp(arcsec(a*x))/x,x, algorithm="giac")
```

output
```
integrate(e^(arcsec(a*x))/x, x)
```

3.46.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\sec^{-1}(ax)}}{x} \, dx = \int \frac{e^{\mathrm{acos}\left(\frac{1}{a\,x}\right)}}{x} \, dx$$

input
```
int(exp(acos(1/(a*x)))/x,x)
```

output
```
int(exp(acos(1/(a*x)))/x, x)
```

3.47 $\int \frac{e^{\sec^{-1}(ax)}}{x^2}\,dx$

3.47.1 Optimal result

Integrand size = 10, antiderivative size = 39

$$\int \frac{e^{\sec^{-1}(ax)}}{x^2}\,dx = \frac{1}{2}ae^{\sec^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}-\frac{e^{\sec^{-1}(ax)}}{2x}$$

output
```
-1/2*exp(arcsec(a*x))/x+1/2*a*exp(arcsec(a*x))*(1-1/a^2/x^2)^(1/2)
```

3.47.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{e^{\sec^{-1}(ax)}}{x^2}\,dx = \frac{1}{2}ae^{\sec^{-1}(ax)}\left(\sqrt{1-\frac{1}{a^2x^2}}-\frac{1}{ax}\right)$$

input
```
Integrate[E^ArcSec[a*x]/x^2,x]
```

output
```
(a*E^ArcSec[a*x]*(Sqrt[1 - 1/(a^2*x^2)] - 1/(a*x)))/2
```

3.47.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5789, 27, 4932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\sec^{-1}(ax)}}{x^2} dx$$

$$\Big\downarrow \text{5789}$$

$$\frac{\int a^2 e^{\sec^{-1}(ax)} \sqrt{1-\frac{1}{a^2x^2}} d\sec^{-1}(ax)}{a}$$

$$\Big\downarrow \text{27}$$

$$a \int e^{\sec^{-1}(ax)} \sqrt{1-\frac{1}{a^2x^2}} d\sec^{-1}(ax)$$

$$\Big\downarrow \text{4932}$$

$$a \left(\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}} e^{\sec^{-1}(ax)} - \frac{e^{\sec^{-1}(ax)}}{2ax}\right)$$

input
```
Int[E^ArcSec[a*x]/x^2,x]
```

output
```
a*((E^ArcSec[a*x]*Sqrt[1 - 1/(a^2*x^2)])/2 - E^ArcSec[a*x]/(2*a*x))
```

3.47.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 4932
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

rule 5789
```
Int[(u_.)*(f_)^(ArcSec[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Simp[
1/b   Subst[Int[(u /. x -> -a/b + Sec[x]/b)*f^(c*x^n)*Sec[x]*Tan[x], x], x,
 ArcSec[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

3.47.4 Maple [F]

$$\int \frac{\mathrm{e}^{\operatorname{arcsec}(ax)}}{x^2}dx$$

input
```
int(exp(arcsec(a*x))/x^2,x)
```

output
```
int(exp(arcsec(a*x))/x^2,x)
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.59

$$\int \frac{e^{\sec^{-1}(ax)}}{x^2}\,dx = \frac{\left(\sqrt{a^2x^2-1}-1\right)e^{(\operatorname{arcsec}(ax))}}{2\,x}$$

input
```
integrate(exp(arcsec(a*x))/x^2,x, algorithm="fricas")
```

output
```
1/2*(sqrt(a^2*x^2 - 1) - 1)*e^(arcsec(a*x))/x
```

3.47.6 Sympy [F]

$$\int \frac{e^{\sec^{-1}(ax)}}{x^2}\,dx = \int \frac{e^{\operatorname{asec}(ax)}}{x^2}\,dx$$

input
```
integrate(exp(asec(a*x))/x**2,x)
```

output
```
Integral(exp(asec(a*x))/x**2, x)
```

3.47.7 Maxima [F]

$$\int \frac{e^{\sec^{-1}(ax)}}{x^2}\,dx = \int \frac{e^{(\operatorname{arcsec}(ax))}}{x^2}\,dx$$

input
```
integrate(exp(arcsec(a*x))/x^2,x, algorithm="maxima")
```

output
```
integrate(e^(arcsec(a*x))/x^2, x)
```

3.47.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{e^{\sec^{-1}(ax)}}{x^2}\,dx = \frac{1}{2}\left(\sqrt{-\frac{1}{a^2x^2}+1}e^{\left(\arccos\left(\frac{1}{ax}\right)\right)} - \frac{e^{\left(\arccos\left(\frac{1}{ax}\right)\right)}}{ax}\right)a$$

input
```
integrate(exp(arcsec(a*x))/x^2,x, algorithm="giac")
```

output
```
1/2*(sqrt(-1/(a^2*x^2) + 1)*e^(arccos(1/(a*x))) - e^(arccos(1/(a*x)))/(a*x
))*a
```

3.47.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\sec^{-1}(ax)}}{x^2}\,dx = \int \frac{\mathrm{e}^{\operatorname{acos}\left(\frac{1}{a\,x}\right)}}{x^2}\,dx$$

input
```
int(exp(acos(1/(a*x)))/x^2,x)
```

output
```
int(exp(acos(1/(a*x)))/x^2, x)
```

3.48 $\int \frac{e^{\sec^{-1}(ax)}}{x^3} \, dx$

3.48.1 Optimal result

Integrand size = 10, antiderivative size = 41

$$\int \frac{e^{\sec^{-1}(ax)}}{x^3} \, dx = -\frac{1}{5}a^2 e^{\sec^{-1}(ax)} \cos \left(2\sec^{-1}(ax)\right) + \frac{1}{10}a^2 e^{\sec^{-1}(ax)} \sin \left(2\sec^{-1}(ax)\right)$$

output
```
-1/5*a^2*exp(arcsec(a*x))*cos(2*arcsec(a*x))+1/10*a^2*exp(arcsec(a*x))*sin
(2*arcsec(a*x))
```

3.48.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int \frac{e^{\sec^{-1}(ax)}}{x^3} \, dx = \frac{1}{10}a^2 e^{\sec^{-1}(ax)} \left(-2\cos \left(2\sec^{-1}(ax)\right) + \sin \left(2\sec^{-1}(ax)\right)\right)$$

input
```
Integrate[E^ArcSec[a*x]/x^3,x]
```

output
```
(a^2*E^ArcSec[a*x]*(-2*Cos[2*ArcSec[a*x]] + Sin[2*ArcSec[a*x]]))/10
```

3.48.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5789, 27, 4972, 27, 4932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\sec^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{ 5789}$$

$$\frac{\int \frac{a^2e^{\sec^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}d\sec^{-1}(ax)}{a}$$

$$\Big\downarrow \text{ 27}$$

$$a^2\int \frac{e^{\sec^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{ax}d\sec^{-1}(ax)$$

$$\Big\downarrow \text{ 4972}$$

$$a^2\int \frac{1}{2}e^{\sec^{-1}(ax)}\sin\left(2\sec^{-1}(ax)\right)d\sec^{-1}(ax)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}a^2\int e^{\sec^{-1}(ax)}\sin\left(2\sec^{-1}(ax)\right)d\sec^{-1}(ax)$$

$$\Big\downarrow \text{ 4932}$$

$$\frac{1}{2}a^2\left(\frac{1}{5}e^{\sec^{-1}(ax)}\sin\left(2\sec^{-1}(ax)\right)-\frac{2}{5}e^{\sec^{-1}(ax)}\cos\left(2\sec^{-1}(ax)\right)\right)$$

input
```
Int[E^ArcSec[a*x]/x^3,x]
```

output
```
(a^2*((-2*E^ArcSec[a*x]*Cos[2*ArcSec[a*x]])/5 + (E^ArcSec[a*x]*Sin[2*ArcSe
c[a*x]])/5))/2
```

3.48.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 4932
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

rule 4972
```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 5789
```
Int[(u_.)*(f_)^(ArcSec[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Simp[
1/b   Subst[Int[(u /. x -> -a/b + Sec[x]/b)*f^(c*x^n)*Sec[x]*Tan[x], x], x,
 ArcSec[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

3.48.4 Maple [F]

$$\int \frac{\mathrm{e}^{\operatorname{arcsec}(ax)}}{x^3}dx$$

input
```
int(exp(arcsec(a*x))/x^3,x)
```

output
```
int(exp(arcsec(a*x))/x^3,x)
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int \frac{e^{\sec^{-1}(ax)}}{x^3}\,dx = \frac{\left(a^2x^2 + \sqrt{a^2x^2-1} - 2\right)e^{(\operatorname{arcsec}(ax))}}{5\,x^2}$$

input
```
integrate(exp(arcsec(a*x))/x^3,x, algorithm="fricas")
```

output
```
1/5*(a^2*x^2 + sqrt(a^2*x^2 - 1) - 2)*e^(arcsec(a*x))/x^2
```

3.48.6 Sympy [F]

$$\int \frac{e^{\sec^{-1}(ax)}}{x^3}\,dx = \int \frac{e^{\operatorname{asec}(ax)}}{x^3}\,dx$$

input
```
integrate(exp(asec(a*x))/x**3,x)
```

output
```
Integral(exp(asec(a*x))/x**3, x)
```

3.48.7 Maxima [F]

$$\int \frac{e^{\sec^{-1}(ax)}}{x^3}\,dx = \int \frac{e^{(\operatorname{arcsec}(ax))}}{x^3}\,dx$$

input
```
integrate(exp(arcsec(a*x))/x^3,x, algorithm="maxima")
```

output
```
integrate(e^(arcsec(a*x))/x^3, x)
```

3.48.8 Giac [F]

$$\int \frac{e^{\sec^{-1}(ax)}}{x^3}\,dx = \int \frac{e^{(\operatorname{arcsec}(ax))}}{x^3}\,dx$$

input
```
integrate(exp(arcsec(a*x))/x^3,x, algorithm="giac")
```

output
```
integrate(e^(arcsec(a*x))/x^3, x)
```

3.48.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\sec^{-1}(ax)}}{x^3}\,dx = \int \frac{\mathrm{e}^{\mathrm{acos}\left(\frac{1}{a\,x}\right)}}{x^3}\,dx$$

input
```
int(exp(acos(1/(a*x)))/x^3,x)
```

output
```
int(exp(acos(1/(a*x)))/x^3, x)
```

3.49 $\int \frac{e^{\sec^{-1}(ax)}}{x^4} dx$

3.49.1 Optimal result

Integrand size = 10, antiderivative size = 84

$$\int \frac{e^{\sec^{-1}(ax)}}{x^4} dx = \frac{1}{8}a^3e^{\sec^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}} - \frac{a^2e^{\sec^{-1}(ax)}}{8x} - \frac{3}{40}a^3e^{\sec^{-1}(ax)}\cos\left(3\sec^{-1}(ax)\right) + \frac{1}{40}a^3e^{\sec^{-1}(ax)}\sin\left(3\sec^{-1}(ax)\right)$$

output
```
-1/8*a^2*exp(arcsec(a*x))/x-3/40*a^3*exp(arcsec(a*x))*cos(3*arcsec(a*x))+1
/40*a^3*exp(arcsec(a*x))*sin(3*arcsec(a*x))+1/8*a^3*exp(arcsec(a*x))*(1-1/
a^2/x^2)^(1/2)
```

3.49.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.64

$$\int \frac{e^{\sec^{-1}(ax)}}{x^4} dx = \frac{1}{40}a^3e^{\sec^{-1}(ax)}\left(5\sqrt{1-\frac{1}{a^2x^2}} - \frac{5}{ax} - 3\cos\left(3\sec^{-1}(ax)\right) + \sin\left(3\sec^{-1}(ax)\right)\right)$$

input
```
Integrate[E^ArcSec[a*x]/x^4,x]
```

output
```
(a^3*E^ArcSec[a*x]*(5*Sqrt[1 - 1/(a^2*x^2)] - 5/(a*x) - 3*Cos[3*ArcSec[a*x
]] + Sin[3*ArcSec[a*x]]))/40
```

3.49.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5789, 27, 4972, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\sec^{-1}(ax)}}{x^4} \, dx$$

$$\Big\downarrow \text{ 5789}$$

$$\frac{\int \frac{a^2 e^{\sec^{-1}(ax)} \sqrt{1-\frac{1}{a^2x^2}}}{x^2} d\sec^{-1}(ax)}{a}$$

$$\Big\downarrow \text{ 27}$$

$$a^3 \int \frac{e^{\sec^{-1}(ax)} \sqrt{1-\frac{1}{a^2x^2}}}{a^2x^2} d\sec^{-1}(ax)$$

$$\Big\downarrow \text{ 4972}$$

$$a^3 \int \left(\frac{1}{4} e^{\sec^{-1}(ax)} \sin\left(3\sec^{-1}(ax)\right) + \frac{1}{4} e^{\sec^{-1}(ax)} \sqrt{1-\frac{1}{a^2x^2}}\right) d\sec^{-1}(ax)$$

$$\Big\downarrow \text{ 2009}$$

$$a^3 \left(\frac{1}{8} \sqrt{1-\frac{1}{a^2x^2}} e^{\sec^{-1}(ax)} - \frac{e^{\sec^{-1}(ax)}}{8ax} - \frac{3}{40} e^{\sec^{-1}(ax)} \cos\left(3\sec^{-1}(ax)\right) + \frac{1}{40} e^{\sec^{-1}(ax)} \sin\left(3\sec^{-1}(ax)\right)\right)$$

input
```
Int[E^ArcSec[a*x]/x^4,x]
```

output
```
a^3*((E^ArcSec[a*x]*Sqrt[1 - 1/(a^2*x^2)])/8 - E^ArcSec[a*x]/(8*a*x) - (3*
E^ArcSec[a*x]*Cos[3*ArcSec[a*x]])/40 + (E^ArcSec[a*x]*Sin[3*ArcSec[a*x]])/
40)
```

3.49.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4972
```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 5789
```
Int[(u_.)*(f_)^(ArcSec[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Simp[
1/b   Subst[Int[(u /. x -> -a/b + Sec[x]/b)*f^(c*x^n)*Sec[x]*Tan[x], x], x,
 ArcSec[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

3.49.4 Maple [F]

$$\int \frac{\mathrm{e}^{\operatorname{arcsec}(ax)}}{x^4}dx$$

input
```
int(exp(arcsec(a*x))/x^4,x)
```

output
```
int(exp(arcsec(a*x))/x^4,x)
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.48

$$\int \frac{e^{\sec^{-1}(ax)}}{x^4}\,dx = \frac{\left(a^2x^2 + (a^2x^2+1)\sqrt{a^2x^2-1} - 3\right)e^{(\operatorname{arcsec}(ax))}}{10\,x^3}$$

input
```
integrate(exp(arcsec(a*x))/x^4,x, algorithm="fricas")
```

output
```
1/10*(a^2*x^2 + (a^2*x^2 + 1)*sqrt(a^2*x^2 - 1) - 3)*e^(arcsec(a*x))/x^3
```

3.49.6 Sympy [F]

$$\int \frac{e^{\sec^{-1}(ax)}}{x^4}\,dx = \int \frac{e^{\operatorname{asec}(ax)}}{x^4}\,dx$$

input
```
integrate(exp(asec(a*x))/x**4,x)
```

output
```
Integral(exp(asec(a*x))/x**4, x)
```

3.49.7 Maxima [F]

$$\int \frac{e^{\sec^{-1}(ax)}}{x^4}\,dx = \int \frac{e^{(\operatorname{arcsec}(ax))}}{x^4}\,dx$$

input
```
integrate(exp(arcsec(a*x))/x^4,x, algorithm="maxima")
```

output
```
integrate(e^(arcsec(a*x))/x^4, x)
```

3.49.8 Giac [F]

$$\int \frac{e^{\sec^{-1}(ax)}}{x^4}\,dx = \int \frac{e^{(\operatorname{arcsec}(ax))}}{x^4}\,dx$$

input
```
integrate(exp(arcsec(a*x))/x^4,x, algorithm="giac")
```

output
```
integrate(e^(arcsec(a*x))/x^4, x)
```

3.49.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\sec^{-1}(ax)}}{x^4}\,dx = \int \frac{\mathrm{e}^{\mathrm{acos}\left(\frac{1}{a\,x}\right)}}{x^4}\,dx$$

input
```
int(exp(acos(1/(a*x)))/x^4,x)
```

output
```
int(exp(acos(1/(a*x)))/x^4, x)
```

3.50 $\int \frac{\sec^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx$

3.50.1 Optimal result

Integrand size = 19, antiderivative size = 69

$$\int \frac{\sec^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx = \frac{i\sec^{-1}(a+bx)^2}{2d} - \frac{\sec^{-1}(a+bx)\log\left(1+e^{2i\sec^{-1}(a+bx)}\right)}{d} + \frac{i\,\mathrm{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)}{2d}$$

output
```
1/2*I*arcsec(b*x+a)^2/d-arcsec(b*x+a)*ln(1+(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1
/2))^2)/d+1/2*I*polylog(2,-(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)/d
```

3.50.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{\sec^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx = \frac{i\left(\sec^{-1}(a+bx)\left(\sec^{-1}(a+bx)+2i\log\left(1+e^{2i\sec^{-1}(a+bx)}\right)\right)+\mathrm{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)\right)}{2d}$$

input
```
Integrate[ArcSec[a + b*x]/((a*d)/b + d*x),x]
```

output
```
((I/2)*(ArcSec[a + b*x]*(ArcSec[a + b*x] + (2*I)*Log[1 + E^((2*I)*ArcSec[a
 + b*x])]) + PolyLog[2, -E^((2*I)*ArcSec[a + b*x])]))/d
```

3.50.3 Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {5779, 27, 5741, 5137, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{-1}(a+bx)}{\frac{ad}{b}+dx}dx$$

$$\Big\downarrow \text{5779}$$

$$\frac{\int \frac{b\sec^{-1}(a+bx)}{d(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sec^{-1}(a+bx)}{a+bx}d(a+bx)}{d}$$

$$\Big\downarrow \text{5741}$$

$$-\frac{\int (a+bx)\arccos\left(\frac{1}{a+bx}\right)d\frac{1}{a+bx}}{d}$$

$$\Big\downarrow \text{5137}$$

$$\frac{\int (a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\arccos\left(\frac{1}{a+bx}\right)d\arccos\left(\frac{1}{a+bx}\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \arccos\left(\frac{1}{a+bx}\right)\tan\left(\arccos\left(\frac{1}{a+bx}\right)\right)d\arccos\left(\frac{1}{a+bx}\right)}{d}$$

$$\Big\downarrow \text{4202}$$

$$\frac{\frac{i}{2(a+bx)^2}-2i\int \frac{e^{2i\arccos\left(\frac{1}{a+bx}\right)}\arccos\left(\frac{1}{a+bx}\right)}{1+e^{2i\arccos\left(\frac{1}{a+bx}\right)}}d\arccos\left(\frac{1}{a+bx}\right)}{d}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\frac{i}{2(a+bx)^2}-2i\left(\frac{1}{2}i\int \log\left(1+e^{2i\arccos\left(\frac{1}{a+bx}\right)}\right)d\arccos\left(\frac{1}{a+bx}\right)-\frac{1}{2}i\arccos\left(\frac{1}{a+bx}\right)\log\left(1+e^{2i\arccos\left(\frac{1}{a+bx}\right)}\right)\right)}{d}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\frac{i}{2(a+bx)^2}-2i\left(\frac{1}{4}\int (a+bx)\log\left(1+e^{2i\arccos\left(\frac{1}{a+bx}\right)}\right)de^{2i\arccos\left(\frac{1}{a+bx}\right)}-\frac{1}{2}i\arccos\left(\frac{1}{a+bx}\right)\log\left(1+e^{2i\arccos\left(\frac{1}{a+bx}\right)}\right)\right)}{d}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\frac{i}{2(a+bx)^2}-2i\left(-\frac{1}{4}\operatorname{PolyLog}(2,-a-bx)-\frac{1}{2}i\arccos\left(\frac{1}{a+bx}\right)\log\left(1+e^{2i\arccos\left(\frac{1}{a+bx}\right)}\right)\right)}{d}$$

input
```
Int[ArcSec[a + b*x]/((a*d)/b + d*x),x]
```

output
```
((I/2)/(a + b*x)^2 - (2*I)*((-1/2*I)*ArcCos[(a + b*x)^(-1)]*Log[1 + E^((2*
I)*ArcCos[(a + b*x)^(-1)])] - PolyLog[2, -a - b*x]/4))/d
```

3.50.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 5741
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a + b
*ArcCos[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

rule 5779
```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcSec[x])^p, x],
 x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
 IGtQ[p, 0]
```

3.50.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.43

method	result	size
derivativedivides	$\frac{\frac{ib\,\mathrm{arcsec}(bx+a)^2}{2d}-\frac{b\,\mathrm{arcsec}(bx+a)\ln\left(1+\left(\frac{1}{bx+a}+i\sqrt{1-\frac{1}{(bx+a)^2}}\right)^2\right)}{d}+\frac{ib\,\mathrm{polylog}\left(2,-\left(\frac{1}{bx+a}+i\sqrt{1-\frac{1}{(bx+a)^2}}\right)^2\right)}{2d}}{b}$	99
default	$\frac{\frac{ib\,\mathrm{arcsec}(bx+a)^2}{2d}-\frac{b\,\mathrm{arcsec}(bx+a)\ln\left(1+\left(\frac{1}{bx+a}+i\sqrt{1-\frac{1}{(bx+a)^2}}\right)^2\right)}{d}+\frac{ib\,\mathrm{polylog}\left(2,-\left(\frac{1}{bx+a}+i\sqrt{1-\frac{1}{(bx+a)^2}}\right)^2\right)}{2d}}{b}$	99

input
```
int(arcsec(b*x+a)/(a*d/b+d*x),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*I*b/d*arcsec(b*x+a)^2-b/d*arcsec(b*x+a)*ln(1+(1/(b*x+a)+I*(1-1/(b
*x+a)^2)^(1/2))^2)+1/2*I*b/d*polylog(2,-(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)
)^2))
```

3.50.5 Fricas [F]

$$\int \frac{\sec^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx=\int \frac{\operatorname{arcsec}(bx+a)}{dx+\frac{ad}{b}}\,dx$$

input
```
integrate(arcsec(b*x+a)/(a*d/b+d*x),x, algorithm="fricas")
```

output
```
integral(b*arcsec(b*x + a)/(b*d*x + a*d), x)
```

3.50.6 Sympy [F]

$$\int \frac{\sec^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx=\frac{b\int \frac{\operatorname{asec}(a+bx)}{a+bx}\,dx}{d}$$

input
```
integrate(asec(b*x+a)/(a*d/b+d*x),x)
```

output
```
b*Integral(asec(a + b*x)/(a + b*x), x)/d
```

3.50.7 Maxima [F]

$$\int \frac{\sec^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx=\int \frac{\operatorname{arcsec}(bx+a)}{dx+\frac{ad}{b}}\,dx$$

input
```
integrate(arcsec(b*x+a)/(a*d/b+d*x),x, algorithm="maxima")
```

output
```
-1/2*(2*b*d*integrate(sqrt(b*x + a + 1)*sqrt(b*x + a - 1)*log(b*x + a)/(b^
3*d*x^3 + 3*a*b^2*d*x^2 + (3*a^2 - 1)*b*d*x + (a^3 - a)*d), x) + 2*I*b*d*i
ntegrate(log(b*x + a)/(b^3*d*x^3 + 3*a*b^2*d*x^2 + (3*a^2 - 1)*b*d*x + (a^
3 - a)*d), x) - 2*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))*log(b*x + a)
 + I*log(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a) - I*log(b*x + a + 1)*log(b*
x + a) - I*log(b*x + a)^2 - I*log(b*x + a)*log(-b*x - a + 1) - I*dilog(b*x
 + a) - I*dilog(-b*x - a))/d
```

3.50.8 Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.67

$$\int \frac{\sec^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx = -\frac{1}{4}b^2\left(\frac{2\,(bx+a)^2\arccos\left(\frac{1}{\left((bx+a)\left(\frac{a}{bx+a}-1\right)-a\right)\left(\frac{a}{bx+a}-1\right)+a}\right)}{b^3d}-\frac{(bx+a)\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)-\frac{}{(bx+a)\left(\sqrt{-\right.}\right.}}{b^3d}\right.$$

input
```
integrate(arcsec(b*x+a)/(a*d/b+d*x),x, algorithm="giac")
```

output
```
-1/4*b^2*(2*(b*x + a)^2*arccos(1/(((b*x + a)*(a/(b*x + a) - 1) - a)*(a/(b*
x + a) - 1) + a))/(b^3*d) - ((b*x + a)*(sqrt(-1/(b*x + a)^2 + 1) - 1) - 1/
((b*x + a)*(sqrt(-1/(b*x + a)^2 + 1) - 1)))/(b^3*d))
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx = \int \frac{\mathrm{acos}\left(\frac{1}{a+b\,x}\right)}{d\,x+\frac{a\,d}{b}}\,dx$$

input
```
int(acos(1/(a + b*x))/(d*x + (a*d)/b),x)
```

output
```
int(acos(1/(a + b*x))/(d*x + (a*d)/b), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```