Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

6-Hyperbolic-functions/6.2-Hyperbolic-cosine/169-6.2.5-Hyperbolic-cosine-functions

Nasser M. Abbasi

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [336]. This is test number [169].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (336)	0.00 (0)
Mathematica	100.00 (336)	0.00 (0)
Fricas	96.73 (325)	3.27 (11)
Maple	91.67 (308)	8.33 (28)
Giac	77.08 (259)	22.92 (77)
Maxima	61.90 (208)	38.10 (128)
Mupad	56.55 (190)	43.45 (146)
Sympy	32.44 (109)	67.56 (227)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

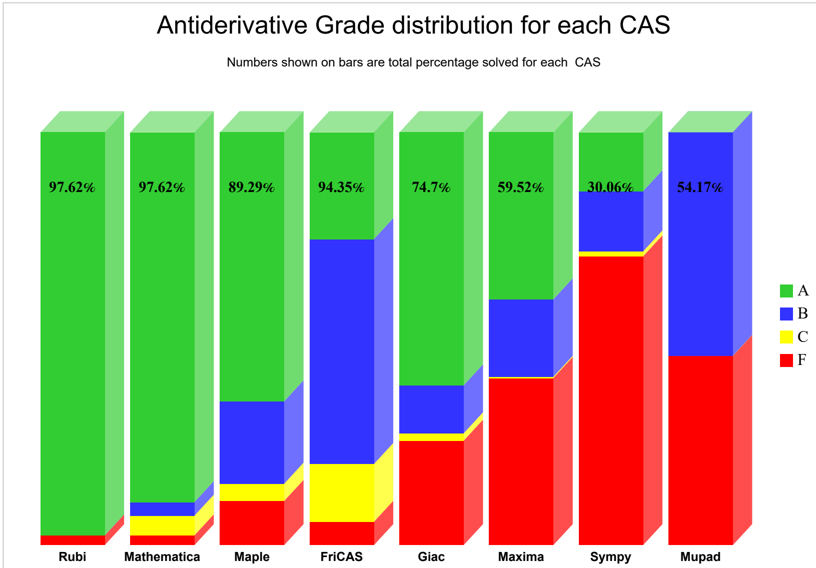
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

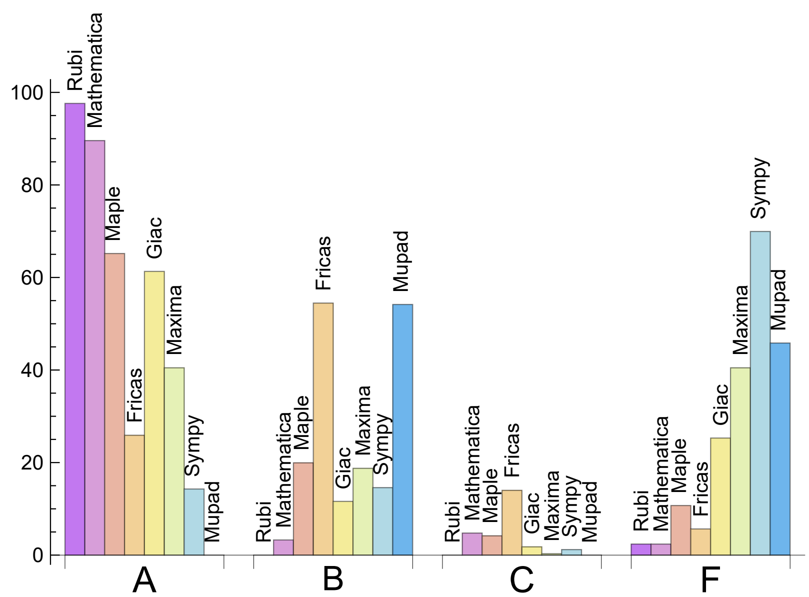
System	% A grade	% B grade	% C grade	% F grade
Rubi	90.476	0.000	7.143	2.381
Mathematica	89.583	3.274	4.762	2.381
Maple	65.179	19.940	4.167	10.714
Giac	61.310	11.607	1.786	25.298
Maxima	40.476	18.750	0.298	40.476
Fricas	25.893	54.464	13.988	5.655
Sympy	14.286	14.583	1.190	69.940
Mupad	0.000	54.167	0.000	45.833

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	11	72.73	0.00	27.27
Maple	28	100.00	0.00	0.00
Giac	77	93.51	3.90	2.60
Maxima	128	63.28	0.00	36.72
Mupad	146	0.00	100.00	0.00
Sympy	227	76.65	23.35	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.23
Fricas	0.25
Rubi	0.42
Mathematica	0.65
Giac	0.82
Mupad	1.67
Maple	3.94
Sympy	11.49

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	90.85	1.03	64.50	1.00
Mathematica	96.74	1.03	55.00	0.95
Maxima	108.00	1.72	81.50	1.29
Maple	124.00	1.46	69.50	1.00
Mupad	138.87	2.10	67.50	1.29
Giac	164.66	1.58	65.00	1.14
Sympy	349.16	5.29	58.00	1.62
Fricas	500.30	4.80	211.00	2.73

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

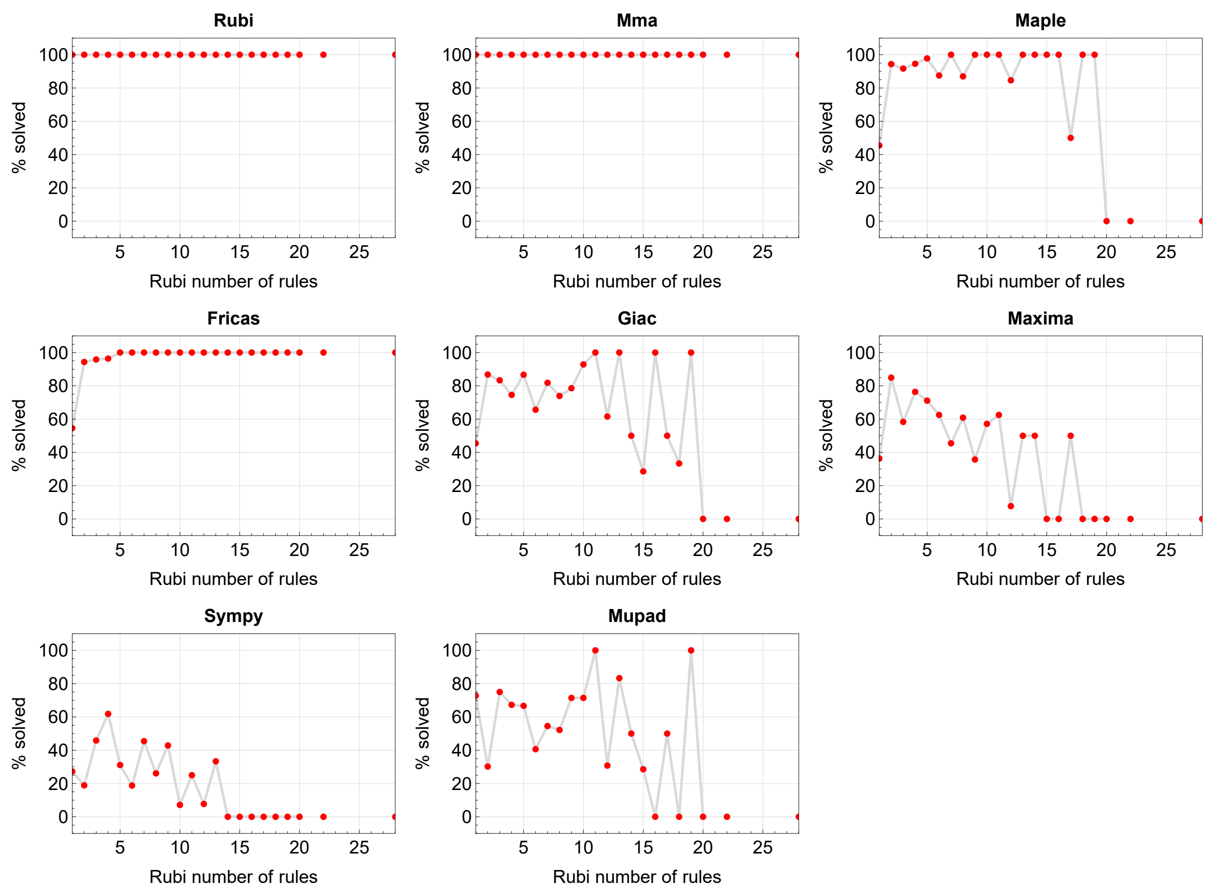


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

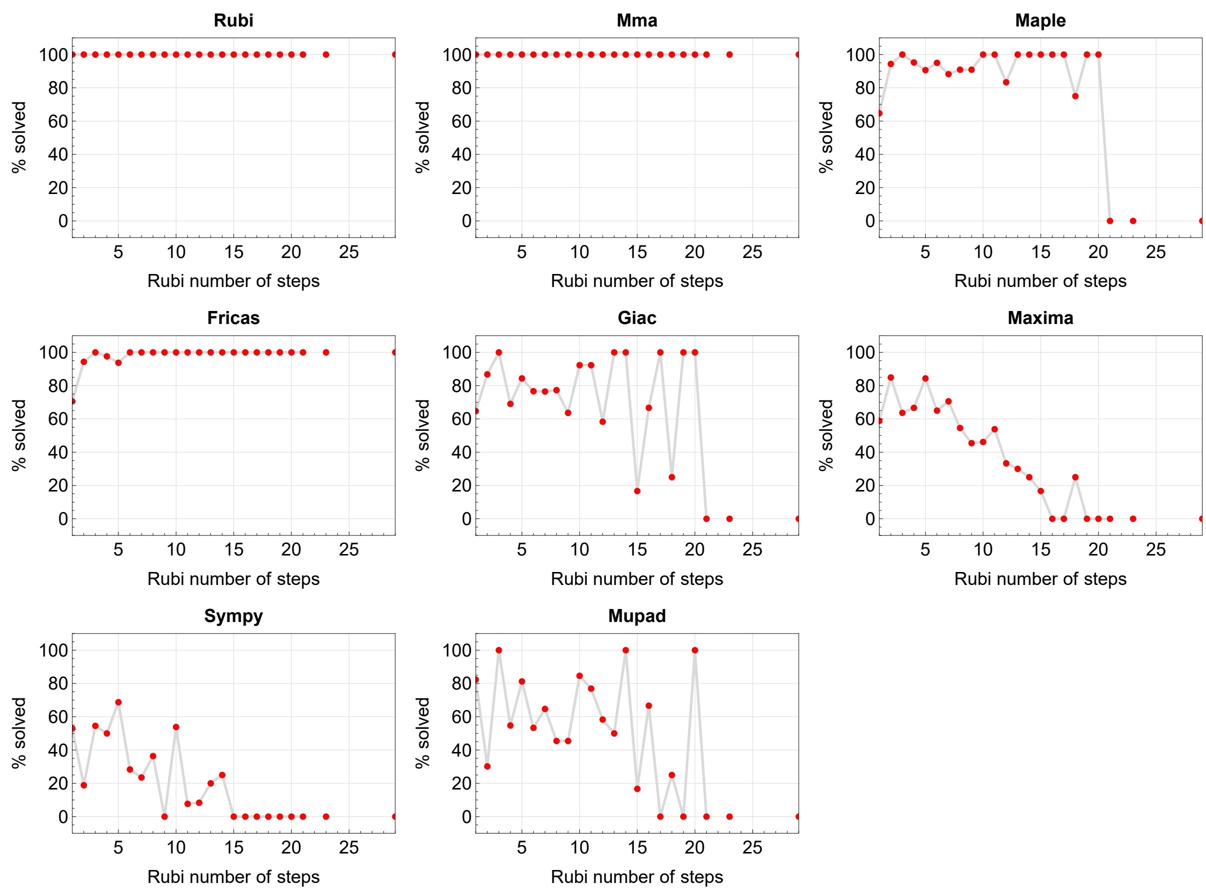


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

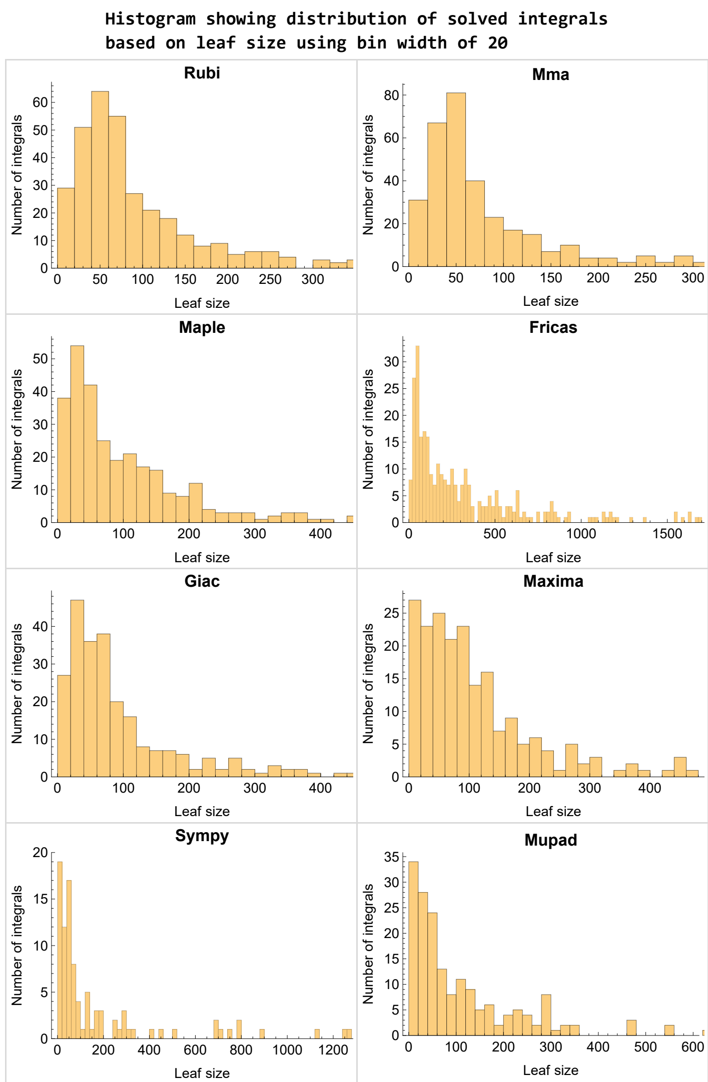


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

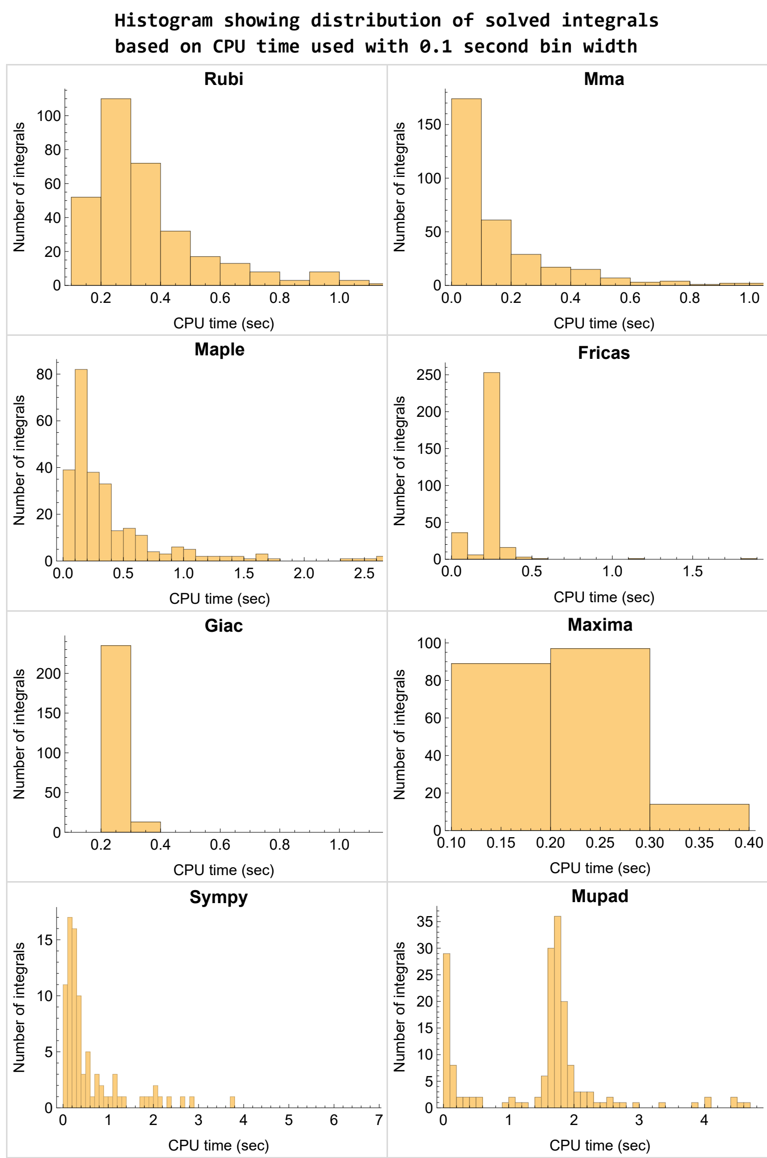


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

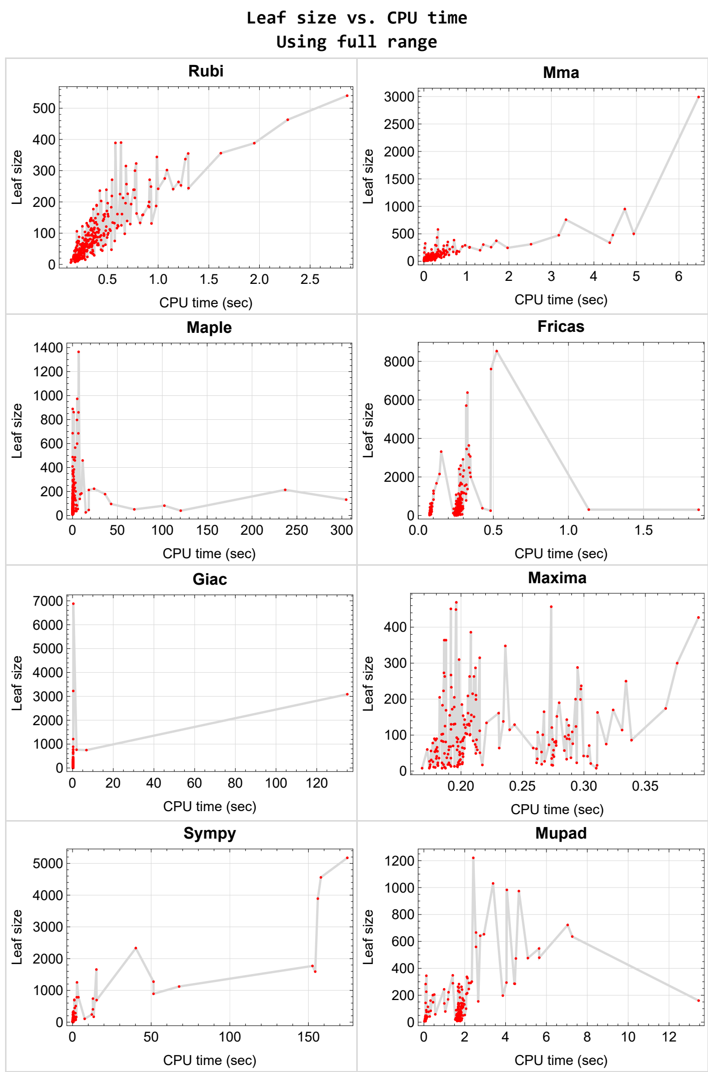


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{216, 217, 221, 226, 227, 232, 233, 238\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{258, 259, 265, 279, 292, 293\}$

Mathematica $\{326, 327, 329, 331\}$

Maple $\{84, 85, 119, 120, 292, 293, 294, 295, 296, 297, 298\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

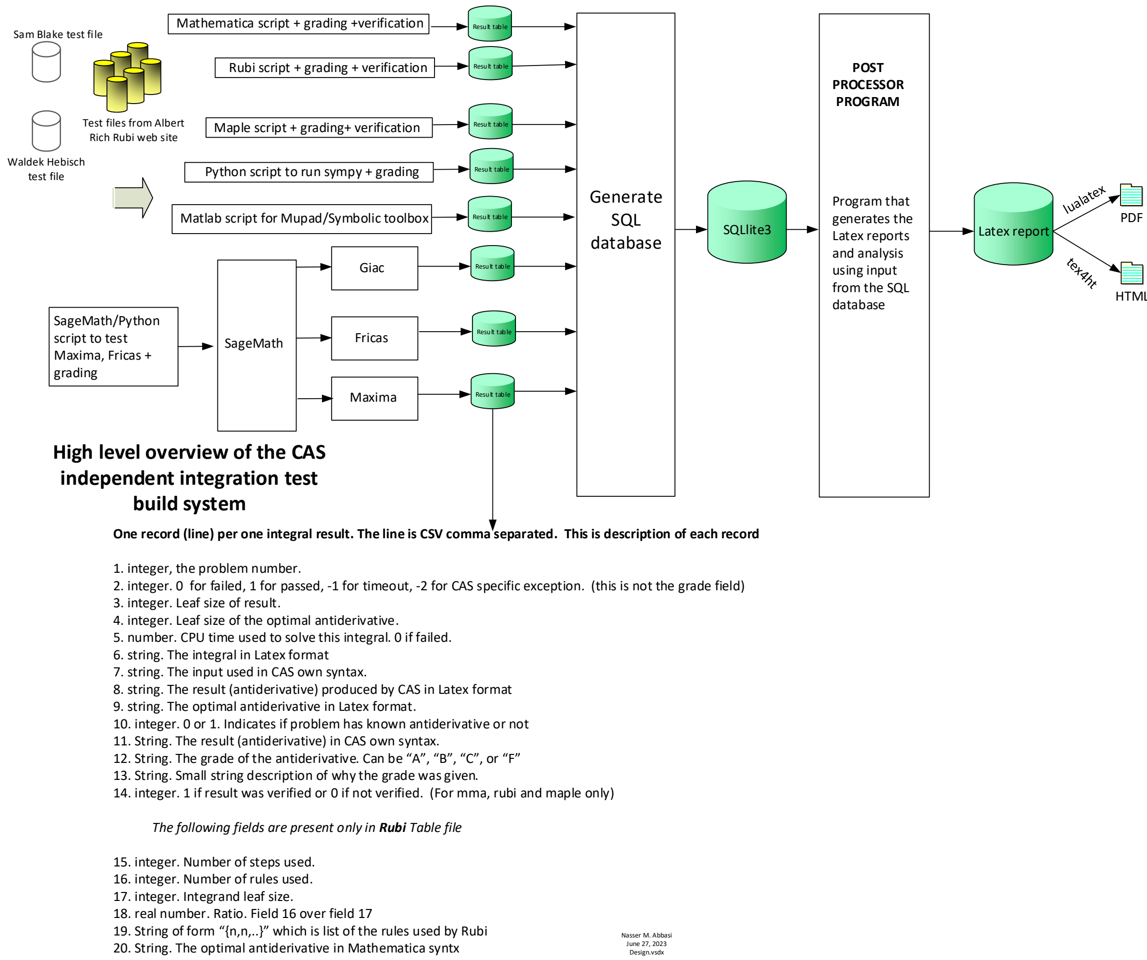
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 189, 191, 192, 197, 198, 199, 200, 201, 202, 203, 204, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 218, 219, 220, 222, 223, 224, 225, 230, 231, 237, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 250, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336 }

B grade { }

C grade { 3, 5, 24, 31, 138, 139, 163, 186, 188, 190, 193, 194, 195, 196, 205, 228, 229, 234, 235, 236, 249, 251, 262, 263 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14, 15, 16, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 218, 219, 220, 222, 223, 224, 225, 228, 229, 230, 231, 234, 235, 236, 237, 239, 240, 241, 242, 243, 244, 245, 246, 248, 249, 250, 251, 252, 254, 255, 256, 259, 260, 261, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 276, 277, 278, 281, 282, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 330, 331, 333, 334, 336 }

B grade { 1, 75, 76, 77, 78, 97, 247, 262, 263, 328, 329 }

C grade { 9, 13, 17, 21, 130, 143, 253, 257, 258, 275, 279, 280, 283, 284, 332, 335 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 12, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 104, 105, 106, 110, 111, 112, 113, 114, 115, 116, 117, 118, 121, 122, 123, 124, 134, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 153, 155, 157, 158, 159, 160, 161, 162, 163, 164, 165, 167, 169, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 200, 201, 202, 203, 204, 205, 207, 208, 209, 220, 225, 231, 237, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 256, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 277, 278, 281, 282, 285, 286, 287, 299, 300, 301, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 333, 334, 335, 336 }

B grade { 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 40, 45, 46, 47, 54, 55, 79, 80, 81, 82, 83, 84, 85, 101, 102, 103, 107, 108, 109, 119, 120, 125, 126, 127, 135, 136, 137, 152, 154, 156, 166, 168, 170, 197, 198, 199, 206, 210, 211, 212, 218, 219, 224, 230, 236, 252, 253, 254, 255, 257, 262, 263 }

C grade { 275, 276, 279, 280, 283, 284, 292, 293, 294, 295, 296, 297, 298, 302 }

F normal fail { 23, 128, 129, 130, 131, 132, 133, 213, 214, 215, 222, 223, 228, 229, 234, 235, 258, 259, 260, 261, 288, 289, 290, 291, 329, 330, 331, 332 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 25, 26, 27, 28, 32, 36, 40, 44, 45, 50, 51, 57, 58, 62, 63, 64, 65, 66, 67, 71, 75, 93, 94, 97, 98, 101, 110, 114, 115, 117, 142, 143, 154, 155, 156, 157, 158, 159, 172, 182, 183, 192, 203, 204, 205, 206, 220, 239, 240, 241, 242, 243, 244, 247, 248, 249, 250, 251, 258, 259, 260, 261, 262, 264, 266, 268, 274, 279, 292, 293, 294, 295, 299, 300, 301, 302, 303, 304, 312, 333, 334 }

B grade { 24, 29, 30, 31, 33, 34, 35, 37, 38, 39, 41, 42, 43, 46, 47, 48, 49, 52, 53, 54, 55, 56, 59, 60, 61, 68, 69, 70, 72, 73, 74, 76, 77, 78, 87, 88, 89, 90, 91, 92, 95, 96, 99, 100, 102, 103, 104, 105, 106, 111, 112, 113, 116, 121, 122, 123, 124, 125, 126, 127, 134, 135, 136, 137, 138, 139, 140, 141, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 184, 185, 186, 187, 188, 189, 190, 191, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 207, 208, 209, 210, 211, 212, 218, 219, 222, 223, 224, 225, 228, 229, 230, 231, 234, 235, 236, 237, 245, 246, 263, 265, 267, 269, 270, 271, 272, 273, 277, 278, 281, 282, 285, 286, 287, 296, 297, 298, 305, 306, 307, 308, 309, 310, 311, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 330, 335, 336 }

C grade { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 79, 80, 81, 82, 83, 84, 85, 86, 107, 108, 109, 118, 119, 120, 128, 129, 130, 131, 132, 133, 252, 253, 254, 255, 256, 257, 275, 276, 280, 283, 284 }

F normal fail { 23, 213, 214, 215, 288, 289, 290, 291 }

F(-1) timedout fail { }

F(-2) exception fail { 329, 331, 332 }

2.1.5 Maxima

A grade { 1, 2, 4, 6, 24, 25, 26, 27, 28, 29, 30, 32, 36, 42, 43, 44, 62, 63, 64, 65, 66, 71, 72, 73, 74, 75, 76, 77, 78, 93, 97, 117, 121, 122, 123, 124, 125, 126, 127, 134, 135, 136, 137, 140, 141, 142, 143, 145, 146, 147, 151, 153, 155, 158, 159, 171, 172, 174, 180, 182, 183, 185, 191, 192, 193, 200, 201, 220, 225, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 250, 264, 265, 266, 267, 268, 269, 271, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 285, 286, 287, 292, 293, 294, 295, 296, 297, 300, 301, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 333, 334 }

B grade { 3, 5, 31, 33, 34, 35, 37, 38, 39, 40, 45, 46, 47, 48, 49, 50, 87, 88, 89, 90, 91, 92, 94, 95, 96, 98, 99, 100, 101, 102, 103, 138, 139, 144, 148, 149, 150, 152, 154, 156, 157, 160, 161, 162, 163, 164, 165, 167, 169, 176, 187, 188, 189, 190, 194, 195, 196, 237, 249, 251, 270, 272, 298 }

C grade { 302 }

F normal fail { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 41, 51, 52, 53, 79, 80, 81, 82, 83, 84, 85, 86, 104, 105, 106, 107, 108, 109, 118, 119, 120, 128, 129, 130, 131, 132, 133, 197, 198, 210, 211, 212, 213, 214, 215, 222, 223, 224, 234, 235, 236, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 283, 284, 288, 289, 290, 291, 329, 330, 331, 332, 335 }

F(-1) timedout fail { }

F(-2) exception fail { 54, 55, 56, 57, 58, 59, 60, 61, 67, 68, 69, 70, 110, 111, 112, 113, 114, 115, 116, 166, 168, 170, 173, 175, 177, 178, 179, 181, 184, 186, 199, 202, 203, 204, 205, 206, 207, 208, 209, 218, 219, 228, 229, 230, 231, 299, 336 }

2.1.6 Giac

A grade { 2, 4, 6, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 87, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 110, 111, 114, 115, 117, 121, 122, 123, 124, 126, 127, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 196, 199, 200, 201, 202, 203, 204, 205, 206, 207, 220, 225, 231, 237, 239, 240, 250, 251, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 303, 304, 305, 307, 308, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 333, 334 }

B grade { 1, 3, 5, 48, 49, 50, 66, 88, 89, 90, 91, 92, 105, 106, 112, 113, 116, 144, 160, 162, 174, 176, 177, 180, 195, 208, 209, 241, 242, 243, 244, 245, 246, 247, 248, 249, 258, 262, 263 }

C grade { 285, 286, 287, 302, 306, 309 }

F normal fail { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 79, 80, 81, 82, 83, 84, 85, 86, 107, 108, 109, 118, 119, 120, 128, 129, 130, 131, 132, 133, 197, 198, 210, 211, 212, 213, 214, 215, 218, 219, 222, 223, 224, 228, 229, 230, 234, 235, 236, 252, 253, 254, 255, 256, 257, 288, 289, 290, 291, 329, 330, 331, 332, 335, 336 }

F(-1) timedout fail { 259, 260, 261 }

F(-2) exception fail { 47, 125 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 44, 50, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 71, 72, 73, 74, 75, 76, 77, 78, 93, 94, 95, 96, 97, 98, 99, 100, 110, 111, 114, 115, 116, 117, 124, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 199, 200, 201, 202, 203, 204, 205, 206, 207, 218, 225, 231, 237, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 294, 296, 297, 298, 299, 329, 330, 331, 333, 334 }

C grade { }

F normal fail { }

F(-1) timedout fail { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 40, 41, 42, 43, 45, 46, 47, 48, 49, 51, 52, 53, 69, 70, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 101, 102, 103, 104, 105, 106, 107, 108, 109, 112, 113, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 197, 198, 208, 209, 210, 211, 212, 213, 214, 215, 219, 220, 222, 223, 224, 228, 229, 230, 234, 235, 236, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 288, 289, 290, 291, 292, 293, 295, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 332, 335, 336 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 3, 5, 27, 32, 33, 34, 35, 36, 37, 38, 39, 62, 63, 64, 65, 66, 71, 75, 93, 94, 95, 96, 97, 98, 99, 100, 115, 117, 124, 140, 141, 142, 143, 146, 147, 148, 149, 159, 171, 200, 201, 251, 274, 278, 282, 333, 334 }

B grade { 2, 4, 6, 24, 25, 26, 55, 56, 57, 67, 68, 76, 77, 78, 110, 111, 114, 144, 145, 150, 151, 152, 153, 154, 155, 156, 157, 158, 169, 170, 199, 206, 207, 218, 225, 231, 247, 249, 264, 265, 266, 267, 273, 277, 281, 285, 286, 287, 299 }

C grade { 72, 73, 74, 294 }

F normal fail { 9, 10, 11, 12, 13, 17, 18, 19, 20, 21, 23, 28, 29, 30, 31, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 58, 59, 60, 61, 79, 80, 81, 82, 83, 84, 86, 88, 89, 91, 92, 101, 102, 103, 104, 105, 108, 109, 118, 125, 126, 130, 131, 160, 161, 162, 163, 164, 172, 173, 174, 175, 176, 177, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 202, 203, 204, 205, 210, 211, 212, 213, 214, 215, 220, 222, 223, 224, 228, 229, 230, 234, 235, 236, 239, 240, 241, 242, 243, 244, 245, 246, 248, 250, 253, 254, 255, 256, 259, 260, 262, 263, 268, 269, 270, 271, 272, 275, 276, 279, 280, 283, 284, 288, 289, 290, 291, 295, 296, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 329, 330, 332, 335, 336 }

F(-1) timedout fail { 7, 8, 14, 15, 16, 22, 54, 69, 70, 85, 87, 90, 106, 107, 112, 113, 116, 119, 120, 121, 122, 123, 127, 128, 129, 132, 133, 134, 135, 136, 137, 138, 139, 165, 166, 167, 168, 178, 208, 209, 219, 237, 252, 257, 258, 261, 292, 293, 297, 298, 327, 328, 331 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	21	11	10	10	12	26	10
N.S.	1	1.00	2.10	1.10	1.00	1.00	1.20	2.60	1.00
time (sec)	N/A	0.163	0.006	0.038	0.179	0.243	0.062	0.262	0.052

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	20	32	22	46	32	18
N.S.	1	1.00	0.92	0.80	1.28	0.88	1.84	1.28	0.72
time (sec)	N/A	0.175	0.011	0.069	0.200	0.244	0.080	0.260	1.542

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	32	26	23	54	32	36	54	22
N.S.	1	1.23	1.00	0.88	2.08	1.23	1.38	2.08	0.85
time (sec)	N/A	0.188	0.002	0.079	0.179	0.242	0.130	0.258	1.541

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	51	33	31	60	49	95	60	31
N.S.	1	1.11	0.72	0.67	1.30	1.07	2.07	1.30	0.67
time (sec)	N/A	0.242	0.032	0.141	0.180	0.241	0.161	0.263	0.090

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	46	41	33	82	66	58	82	31
N.S.	1	1.12	1.00	0.80	2.00	1.61	1.41	2.00	0.76
time (sec)	N/A	0.192	0.012	0.387	0.186	0.242	0.221	0.262	1.569

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	77	43	42	86	90	139	88	42
N.S.	1	1.15	0.64	0.63	1.28	1.34	2.07	1.31	0.63
time (sec)	N/A	0.331	0.038	0.533	0.179	0.240	0.328	0.271	1.739

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	74	55	201	0	326	0	0	0
N.S.	1	1.07	0.80	2.91	0.00	4.72	0.00	0.00	0.00
time (sec)	N/A	0.310	0.085	3.507	0.000	0.082	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	44	188	0	203	0	0	0
N.S.	1	1.00	0.96	4.09	0.00	4.41	0.00	0.00	0.00
time (sec)	N/A	0.240	0.044	0.772	0.000	0.090	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	81	174	0	102	0	0	0
N.S.	1	1.00	1.76	3.78	0.00	2.22	0.00	0.00	0.00
time (sec)	N/A	0.235	0.074	0.462	0.000	0.079	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	135	0	37	0	0	0
N.S.	1	1.00	1.00	6.75	0.00	1.85	0.00	0.00	0.00
time (sec)	N/A	0.169	0.024	0.384	0.000	0.076	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	135	0	24	0	0	0
N.S.	1	1.00	1.00	6.75	0.00	1.20	0.00	0.00	0.00
time (sec)	N/A	0.168	0.026	0.106	0.000	0.075	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	103	0	148	0	0	0
N.S.	1	1.00	1.00	2.45	0.00	3.52	0.00	0.00	0.00
time (sec)	N/A	0.234	0.049	0.304	0.000	0.076	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	84	217	0	310	0	0	0
N.S.	1	1.00	1.83	4.72	0.00	6.74	0.00	0.00	0.00
time (sec)	N/A	0.230	0.051	0.513	0.000	0.081	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	70	63	363	0	613	0	0	0
N.S.	1	1.01	0.91	5.26	0.00	8.88	0.00	0.00	0.00
time (sec)	N/A	0.303	0.105	0.958	0.000	0.080	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	71	53	145	0	256	0	0	0
N.S.	1	1.09	0.82	2.23	0.00	3.94	0.00	0.00	0.00
time (sec)	N/A	0.359	0.053	2.736	0.000	0.078	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	41	184	0	161	0	0	0
N.S.	1	1.00	0.85	3.83	0.00	3.35	0.00	0.00	0.00
time (sec)	N/A	0.289	0.044	0.881	0.000	0.085	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	57	130	0	68	0	0	0
N.S.	1	1.00	1.19	2.71	0.00	1.42	0.00	0.00	0.00
time (sec)	N/A	0.284	0.052	0.586	0.000	0.074	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	118	0	28	0	0	0
N.S.	1	1.00	1.00	4.37	0.00	1.04	0.00	0.00	0.00
time (sec)	N/A	0.213	0.020	0.516	0.000	0.075	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	100	0	16	0	0	0
N.S.	1	1.00	1.00	3.70	0.00	0.59	0.00	0.00	0.00
time (sec)	N/A	0.216	0.022	0.127	0.000	0.075	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	34	159	0	101	0	0	0
N.S.	1	1.00	0.74	3.46	0.00	2.20	0.00	0.00	0.00
time (sec)	N/A	0.278	0.030	0.344	0.000	0.076	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	56	177	0	213	0	0	0
N.S.	1	1.00	1.12	3.54	0.00	4.26	0.00	0.00	0.00
time (sec)	N/A	0.295	0.039	0.418	0.000	0.078	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	71	43	254	0	422	0	0	0
N.S.	1	1.06	0.64	3.79	0.00	6.30	0.00	0.00	0.00
time (sec)	N/A	0.364	0.053	0.679	0.000	0.077	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	65	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.206	0.049	0.000	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	62	53	37	66	100	337	70	70
N.S.	1	1.15	0.98	0.69	1.22	1.85	6.24	1.30	1.30
time (sec)	N/A	0.426	0.280	0.133	0.181	0.254	0.575	0.260	1.759

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	45	30	56	70	189	51	52
N.S.	1	1.00	1.05	0.70	1.30	1.63	4.40	1.19	1.21
time (sec)	N/A	0.280	0.221	0.102	0.176	0.259	0.373	0.255	1.680

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	24	32	20	41	47	63	35	34
N.S.	1	0.96	1.28	0.80	1.64	1.88	2.52	1.40	1.36
time (sec)	N/A	0.322	0.179	0.061	0.180	0.261	0.216	0.254	1.695

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	29	13	18	24	8	17	17
N.S.	1	1.00	1.61	0.72	1.00	1.33	0.44	0.94	0.94
time (sec)	N/A	0.241	0.129	0.039	0.186	0.245	0.146	0.255	1.669

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	22	19	23	29	0	20	31
N.S.	1	1.00	1.10	0.95	1.15	1.45	0.00	1.00	1.55
time (sec)	N/A	0.282	0.105	0.128	0.261	0.249	0.000	0.257	1.684

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	29	43	33	45	127	0	36	58
N.S.	1	1.04	1.54	1.18	1.61	4.54	0.00	1.29	2.07
time (sec)	N/A	0.395	0.161	0.236	0.275	0.234	0.000	0.258	1.725

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	44	49	46	73	325	0	48	73
N.S.	1	1.02	1.14	1.07	1.70	7.56	0.00	1.12	1.70
time (sec)	N/A	0.464	0.168	0.224	0.272	0.263	0.000	0.248	1.701

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	63	60	52	101	600	0	57	107
N.S.	1	1.12	1.07	0.93	1.80	10.71	0.00	1.02	1.91
time (sec)	N/A	0.501	0.239	0.299	0.267	0.251	0.000	0.254	1.676

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	14	14	18	22	17	15	15
N.S.	1	1.00	0.70	0.70	0.90	1.10	0.85	0.75	0.75
time (sec)	N/A	0.178	0.013	0.043	0.175	0.242	0.270	0.255	1.680

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	34	26	90	113	36	25	25
N.S.	1	1.00	0.72	0.55	1.91	2.40	0.77	0.53	0.53
time (sec)	N/A	0.253	0.023	0.060	0.178	0.243	0.419	0.250	0.068

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	75	44	37	205	174	51	36	36
N.S.	1	1.07	0.63	0.53	2.93	2.49	0.73	0.51	0.51
time (sec)	N/A	0.329	0.040	0.067	0.183	0.245	0.843	0.254	1.690

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	103	54	48	364	347	68	47	283
N.S.	1	1.11	0.58	0.52	3.91	3.73	0.73	0.51	3.04
time (sec)	N/A	0.419	0.066	0.083	0.188	0.245	1.753	0.260	1.720

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	14	16	18	24	19	15	15
N.S.	1	1.00	0.61	0.70	0.78	1.04	0.83	0.65	0.65
time (sec)	N/A	0.183	0.087	0.046	0.181	0.243	0.369	0.253	0.060

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	31	26	90	117	39	25	25
N.S.	1	1.00	0.61	0.51	1.76	2.29	0.76	0.49	0.49
time (sec)	N/A	0.254	0.063	0.067	0.180	0.233	0.560	0.259	0.062

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	81	41	37	205	174	56	36	36
N.S.	1	1.07	0.54	0.49	2.70	2.29	0.74	0.47	0.47
time (sec)	N/A	0.336	0.130	0.079	0.195	0.246	1.050	0.282	1.724

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	111	51	48	364	347	73	47	283
N.S.	1	1.10	0.50	0.48	3.60	3.44	0.72	0.47	2.80
time (sec)	N/A	0.438	0.091	0.094	0.186	0.236	2.073	0.249	0.088

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	34	92	114	62	0	37	0
N.S.	1	1.00	0.67	1.80	2.24	1.22	0.00	0.73	0.00
time (sec)	N/A	0.275	0.017	0.139	0.331	0.248	0.000	0.262	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	44	40	0	92	0	74	0
N.S.	1	1.00	0.83	0.75	0.00	1.74	0.00	1.40	0.00
time (sec)	N/A	0.284	0.122	0.147	0.000	0.266	0.000	0.261	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	93	71	73	121	327	0	105	0
N.S.	1	1.04	0.80	0.82	1.36	3.67	0.00	1.18	0.00
time (sec)	N/A	0.363	0.095	0.139	0.274	0.259	0.000	0.281	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	55	58	81	140	0	75	0
N.S.	1	1.00	0.93	0.98	1.37	2.37	0.00	1.27	0.00
time (sec)	N/A	0.268	0.051	0.114	0.276	0.249	0.000	0.256	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	29	43	40	41	0	35	26
N.S.	1	1.00	1.12	1.65	1.54	1.58	0.00	1.35	1.00
time (sec)	N/A	0.179	0.063	0.104	0.276	0.243	0.000	0.251	0.127

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	40	103	86	149	0	21	0
N.S.	1	1.00	0.87	2.24	1.87	3.24	0.00	0.46	0.00
time (sec)	N/A	0.201	0.014	0.107	0.339	0.251	0.000	0.259	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	63	144	170	222	0	70	0
N.S.	1	1.00	0.82	1.87	2.21	2.88	0.00	0.91	0.00
time (sec)	N/A	0.289	0.064	0.131	0.324	0.253	0.000	0.269	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	112	91	178	250	522	0	0	0
N.S.	1	1.05	0.85	1.66	2.34	4.88	0.00	0.00	0.00
time (sec)	N/A	0.379	0.192	0.143	0.334	0.266	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	96	72	71	190	328	0	194	0
N.S.	1	1.04	0.78	0.77	2.07	3.57	0.00	2.11	0.00
time (sec)	N/A	0.363	0.199	0.173	0.280	0.247	0.000	0.265	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	56	56	124	139	0	127	0
N.S.	1	1.00	0.92	0.92	2.03	2.28	0.00	2.08	0.00
time (sec)	N/A	0.276	0.149	0.104	0.294	0.248	0.000	0.266	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	30	41	58	42	0	63	27
N.S.	1	1.00	1.11	1.52	2.15	1.56	0.00	2.33	1.00
time (sec)	N/A	0.192	0.089	0.120	0.275	0.245	0.000	0.266	1.867

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	55	41	0	154	0	40	0
N.S.	1	1.00	1.15	0.85	0.00	3.21	0.00	0.83	0.00
time (sec)	N/A	0.206	0.115	0.091	0.000	0.253	0.000	0.263	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	98	87	0	274	0	115	0
N.S.	1	1.00	1.24	1.10	0.00	3.47	0.00	1.46	0.00
time (sec)	N/A	0.290	0.252	0.114	0.000	0.256	0.000	0.276	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	115	129	137	0	580	0	177	0
N.S.	1	1.05	1.17	1.25	0.00	5.27	0.00	1.61	0.00
time (sec)	N/A	0.391	0.283	0.131	0.000	0.253	0.000	0.284	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	129	99	203	0	1625	0	133	209
N.S.	1	1.15	0.88	1.81	0.00	14.51	0.00	1.19	1.87
time (sec)	N/A	0.767	0.171	0.306	0.000	0.290	0.000	0.250	2.080

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	95	78	153	0	903	4559	92	167
N.S.	1	1.12	0.92	1.80	0.00	10.62	53.64	1.08	1.96
time (sec)	N/A	0.529	0.114	0.179	0.000	0.277	157.924	0.258	1.919

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	65	57	94	0	449	1275	62	139
N.S.	1	1.05	0.92	1.52	0.00	7.24	20.56	1.00	2.24
time (sec)	N/A	0.369	0.097	0.109	0.000	0.263	51.494	0.250	1.866

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	48	64	0	218	241	42	109
N.S.	1	1.00	0.92	1.23	0.00	4.19	4.63	0.81	2.10
time (sec)	N/A	0.266	0.048	0.074	0.000	0.273	12.257	0.264	1.859

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	51	0	227	0	45	286
N.S.	1	1.00	1.00	0.94	0.00	4.20	0.00	0.83	5.30
time (sec)	N/A	0.324	0.068	0.221	0.000	0.273	0.000	0.254	4.430

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	67	63	73	0	515	0	61	294
N.S.	1	1.05	0.98	1.14	0.00	8.05	0.00	0.95	4.59
time (sec)	N/A	0.447	0.122	0.321	0.000	0.282	0.000	0.263	4.043

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	98	82	109	0	1370	0	89	476
N.S.	1	1.13	0.94	1.25	0.00	15.75	0.00	1.02	5.47
time (sec)	N/A	0.722	0.201	0.505	0.000	0.328	0.000	0.261	5.092

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	131	101	145	0	2483	0	123	547
N.S.	1	1.15	0.89	1.27	0.00	21.78	0.00	1.08	4.80
time (sec)	N/A	0.995	0.324	0.738	0.000	0.338	0.000	0.271	5.639

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	196	133	132	273	190	314	263	160
N.S.	1	1.07	0.73	0.72	1.49	1.04	1.72	1.44	0.87
time (sec)	N/A	0.733	0.765	0.588	0.186	0.266	0.277	0.292	0.326

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	145	104	101	183	123	240	196	114
N.S.	1	1.06	0.76	0.74	1.34	0.90	1.75	1.43	0.83
time (sec)	N/A	0.494	0.400	0.281	0.185	0.251	0.183	0.256	0.198

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	93	80	67	116	78	128	131	73
N.S.	1	1.03	0.89	0.74	1.29	0.87	1.42	1.46	0.81
time (sec)	N/A	0.320	0.213	0.166	0.192	0.251	0.146	0.263	1.765

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	46	43	55	40	78	75	41
N.S.	1	1.00	0.92	0.86	1.10	0.80	1.56	1.50	0.82
time (sec)	N/A	0.211	0.129	0.096	0.181	0.245	0.103	0.261	1.687

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	26	16	15	17	17	32	15
N.S.	1	1.00	1.73	1.07	1.00	1.13	1.13	2.13	1.00
time (sec)	N/A	0.147	0.003	0.034	0.181	0.254	0.066	0.254	0.059

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	48	44	0	237	163	39	53
N.S.	1	1.00	0.98	0.90	0.00	4.84	3.33	0.80	1.08
time (sec)	N/A	0.212	0.049	0.072	0.000	0.275	2.396	0.273	2.004

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	97	84	118	0	743	2332	99	215
N.S.	1	1.13	0.98	1.37	0.00	8.64	27.12	1.15	2.50
time (sec)	N/A	0.324	0.198	0.141	0.000	0.260	40.227	0.264	2.123

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	159	113	186	0	2591	0	195	0
N.S.	1	1.20	0.85	1.40	0.00	19.48	0.00	1.47	0.00
time (sec)	N/A	0.490	0.328	0.256	0.000	0.282	0.000	0.261	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	226	160	284	0	5705	0	329	0
N.S.	1	1.23	0.87	1.54	0.00	31.01	0.00	1.79	0.00
time (sec)	N/A	0.735	0.766	0.513	0.000	0.319	0.000	0.274	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	20	18	19	24	24	16	34
N.S.	1	1.00	0.91	0.82	0.86	1.09	1.09	0.73	1.55
time (sec)	N/A	0.188	0.035	0.120	0.266	0.262	0.412	0.266	1.708

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	43	46	64	147	291	54	74
N.S.	1	1.00	0.90	0.96	1.33	3.06	6.06	1.12	1.54
time (sec)	N/A	0.266	0.089	0.149	0.259	0.246	1.101	0.261	1.707

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	78	53	62	108	408	507	76	137
N.S.	1	1.07	0.73	0.85	1.48	5.59	6.95	1.04	1.88
time (sec)	N/A	0.392	0.136	0.195	0.262	0.252	2.040	0.257	1.732

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	108	63	75	152	793	784	98	223
N.S.	1	1.10	0.64	0.77	1.55	8.09	8.00	1.00	2.28
time (sec)	N/A	0.519	0.181	0.271	0.278	0.274	3.747	0.257	1.740

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	65	30	37	42	41	28	40
N.S.	1	1.00	2.10	0.97	1.19	1.35	1.32	0.90	1.29
time (sec)	N/A	0.186	0.034	0.091	0.188	0.257	0.312	0.274	1.735

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	61	144	64	81	212	199	65	77
N.S.	1	1.09	2.57	1.14	1.45	3.79	3.55	1.16	1.38
time (sec)	N/A	0.263	0.107	0.110	0.188	0.269	0.729	0.284	1.810

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	91	217	90	125	563	445	87	141
N.S.	1	1.12	2.68	1.11	1.54	6.95	5.49	1.07	1.74
time (sec)	N/A	0.379	0.180	0.159	0.187	0.259	1.384	0.273	1.824

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	121	296	112	169	1078	784	109	226
N.S.	1	1.14	2.79	1.06	1.59	10.17	7.40	1.03	2.13
time (sec)	N/A	0.513	0.270	0.209	0.188	0.252	2.694	0.282	0.119

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	159	150	685	0	464	0	0	0
N.S.	1	1.04	0.98	4.48	0.00	3.03	0.00	0.00	0.00
time (sec)	N/A	0.904	0.407	6.487	0.000	0.093	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	125	111	466	0	265	0	0	0
N.S.	1	1.01	0.90	3.76	0.00	2.14	0.00	0.00	0.00
time (sec)	N/A	0.691	0.181	1.430	0.000	0.088	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	61	276	0	196	0	0	0
N.S.	1	1.00	1.00	4.52	0.00	3.21	0.00	0.00	0.00
time (sec)	N/A	0.302	0.107	1.463	0.000	0.084	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	146	0	61	0	0	0
N.S.	1	1.00	1.00	3.17	0.00	1.33	0.00	0.00	0.00
time (sec)	N/A	0.274	0.033	0.198	0.000	0.076	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	68	298	0	413	0	0	0
N.S.	1	1.00	0.81	3.55	0.00	4.92	0.00	0.00	0.00
time (sec)	N/A	0.396	0.097	1.184	0.000	0.083	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	177	187	135	459	0	1281	0	0	0
N.S.	1	1.06	0.76	2.59	0.00	7.24	0.00	0.00	0.00
time (sec)	N/A	0.980	0.413	2.631	0.000	0.101	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	227	253	165	566	0	3315	0	0	0
N.S.	1	1.11	0.73	2.49	0.00	14.60	0.00	0.00	0.00
time (sec)	N/A	1.320	0.546	2.960	0.000	0.152	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	73	181	0	174	0	0	0
N.S.	1	1.00	0.73	1.81	0.00	1.74	0.00	0.00	0.00
time (sec)	N/A	0.545	0.284	0.988	0.000	0.081	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	89	60	71	237	563	0	153	0
N.S.	1	0.95	0.64	0.76	2.52	5.99	0.00	1.63	0.00
time (sec)	N/A	0.451	0.096	0.682	0.298	0.258	0.000	0.268	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	66	46	57	163	279	0	113	0
N.S.	1	0.97	0.68	0.84	2.40	4.10	0.00	1.66	0.00
time (sec)	N/A	0.358	0.071	0.311	0.311	0.262	0.000	0.272	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	31	39	90	100	0	71	0
N.S.	1	1.00	0.78	0.98	2.25	2.50	0.00	1.78	0.00
time (sec)	N/A	0.276	0.032	0.307	0.286	0.277	0.000	0.263	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	93	61	69	288	564	0	295	0
N.S.	1	0.95	0.62	0.70	2.94	5.76	0.00	3.01	0.00
time (sec)	N/A	0.467	0.249	0.663	0.295	0.264	0.000	0.286	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	69	47	55	199	279	0	212	0
N.S.	1	0.97	0.66	0.77	2.80	3.93	0.00	2.99	0.00
time (sec)	N/A	0.366	0.217	0.373	0.297	0.257	0.000	0.274	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	32	39	109	107	0	131	0
N.S.	1	1.00	0.73	0.89	2.48	2.43	0.00	2.98	0.00
time (sec)	N/A	0.300	0.154	0.321	0.290	0.258	0.000	0.264	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	32	15	26	29	15	17	19
N.S.	1	1.00	1.78	0.83	1.44	1.61	0.83	0.94	1.06
time (sec)	N/A	0.244	0.187	0.058	0.177	0.259	0.136	0.257	0.056

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	25	31	129	50	36	30	30
N.S.	1	1.00	0.71	0.89	3.69	1.43	1.03	0.86	0.86
time (sec)	N/A	0.277	0.055	0.086	0.203	0.251	0.227	0.248	0.085

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	54	41	35	263	127	46	46	141
N.S.	1	0.96	0.73	0.62	4.70	2.27	0.82	0.82	2.52
time (sec)	N/A	0.334	0.066	0.104	0.187	0.266	0.389	0.259	1.690

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	71	55	55	449	175	78	60	231
N.S.	1	0.95	0.73	0.73	5.99	2.33	1.04	0.80	3.08
time (sec)	N/A	0.417	0.069	0.147	0.196	0.243	0.780	0.253	1.685

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	42	19	27	31	15	16	19
N.S.	1	1.00	2.10	0.95	1.35	1.55	0.75	0.80	0.95
time (sec)	N/A	0.254	0.192	0.058	0.174	0.258	0.207	0.254	0.053

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	25	26	131	48	36	32	32
N.S.	1	1.00	0.68	0.70	3.54	1.30	0.97	0.86	0.86
time (sec)	N/A	0.269	0.049	0.077	0.195	0.257	0.324	0.249	1.699

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	58	41	39	267	127	46	46	143
N.S.	1	0.97	0.68	0.65	4.45	2.12	0.77	0.77	2.38
time (sec)	N/A	0.342	0.064	0.115	0.192	0.248	0.551	0.255	0.084

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	77	55	56	451	175	78	60	233
N.S.	1	0.95	0.68	0.69	5.57	2.16	0.96	0.74	2.88
time (sec)	N/A	0.426	0.060	0.125	0.192	0.246	0.996	0.250	1.685

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	41	128	174	72	0	44	0
N.S.	1	1.00	0.73	2.29	3.11	1.29	0.00	0.79	0.00
time (sec)	N/A	0.295	0.032	0.408	0.367	0.262	0.000	0.252	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	44	159	300	189	0	78	0
N.S.	1	1.00	0.68	2.45	4.62	2.91	0.00	1.20	0.00
time (sec)	N/A	0.317	0.062	0.447	0.376	0.269	0.000	0.266	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	91	57	209	427	509	0	118	0
N.S.	1	0.98	0.61	2.25	4.59	5.47	0.00	1.27	0.00
time (sec)	N/A	0.394	0.110	0.509	0.393	0.279	0.000	0.266	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	51	63	0	99	0	84	0
N.S.	1	1.00	0.89	1.11	0.00	1.74	0.00	1.47	0.00
time (sec)	N/A	0.301	0.186	0.335	0.000	0.261	0.000	0.260	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	81	83	0	217	0	111	0
N.S.	1	1.00	1.25	1.28	0.00	3.34	0.00	1.71	0.00
time (sec)	N/A	0.317	0.305	0.440	0.000	0.258	0.000	0.266	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	92	118	118	0	548	0	189	0
N.S.	1	0.98	1.26	1.26	0.00	5.83	0.00	2.01	0.00
time (sec)	N/A	0.414	0.495	0.431	0.000	0.266	0.000	0.270	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	244	203	1365	0	1141	0	0	0
N.S.	1	1.05	0.87	5.86	0.00	4.90	0.00	0.00	0.00
time (sec)	N/A	1.384	0.464	6.786	0.000	0.102	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	187	124	973	0	635	0	0	0
N.S.	1	1.03	0.69	5.38	0.00	3.51	0.00	0.00	0.00
time (sec)	N/A	1.047	0.519	5.224	0.000	0.092	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	139	123	599	0	325	0	0	0
N.S.	1	1.01	0.89	4.34	0.00	2.36	0.00	0.00	0.00
time (sec)	N/A	0.741	0.279	5.138	0.000	0.088	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	59	73	0	240	403	50	242
N.S.	1	1.00	0.98	1.22	0.00	4.00	6.72	0.83	4.03
time (sec)	N/A	0.292	0.092	0.093	0.000	0.283	12.972	0.248	1.871

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	93	81	108	0	828	3890	107	246
N.S.	1	1.13	0.99	1.32	0.00	10.10	47.44	1.30	3.00
time (sec)	N/A	0.335	0.149	0.145	0.000	0.288	155.986	0.268	2.200

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	161	134	207	0	3166	0	249	0
N.S.	1	1.19	0.99	1.53	0.00	23.45	0.00	1.84	0.00
time (sec)	N/A	0.542	0.304	0.274	0.000	0.337	0.000	0.258	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	239	196	342	0	7603	0	453	0
N.S.	1	1.21	0.99	1.74	0.00	38.59	0.00	2.30	0.00
time (sec)	N/A	0.832	0.520	0.499	0.000	0.484	0.000	0.274	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	63	56	81	0	190	168	57	205
N.S.	1	1.12	1.00	1.45	0.00	3.39	3.00	1.02	3.66
time (sec)	N/A	0.311	0.085	0.103	0.000	0.271	13.464	0.252	0.510

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	0	6	3	6	6
N.S.	1	1.00	1.00	1.17	0.00	1.00	0.50	1.00	1.00
time (sec)	N/A	0.150	0.000	0.030	0.000	0.252	0.082	0.269	0.026

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	0	54	0	26	51
N.S.	1	1.00	1.00	1.09	0.00	4.91	0.00	2.36	4.64
time (sec)	N/A	0.201	0.067	0.098	0.000	0.245	0.000	0.258	1.783

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	38	24	32	34	45	44	37	48
N.S.	1	1.06	0.67	0.89	0.94	1.25	1.22	1.03	1.33
time (sec)	N/A	0.262	0.055	0.060	0.262	0.261	0.316	0.253	0.121

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	80	218	0	183	0	0	0
N.S.	1	1.00	0.74	2.02	0.00	1.69	0.00	0.00	0.00
time (sec)	N/A	0.591	0.369	1.689	0.000	0.087	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	152	163	133	485	0	639	0	0	0
N.S.	1	1.07	0.88	3.19	0.00	4.20	0.00	0.00	0.00
time (sec)	N/A	0.829	0.293	2.762	0.000	0.089	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	231	241	172	797	0	2153	0	0	0
N.S.	1	1.04	0.74	3.45	0.00	9.32	0.00	0.00	0.00
time (sec)	N/A	1.216	0.692	4.979	0.000	0.142	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	78	39	38	71	817	0	79	0
N.S.	1	1.08	0.54	0.53	0.99	11.35	0.00	1.10	0.00
time (sec)	N/A	0.465	0.021	0.431	0.277	0.281	0.000	0.259	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	56	33	32	53	501	0	61	0
N.S.	1	1.06	0.62	0.60	1.00	9.45	0.00	1.15	0.00
time (sec)	N/A	0.370	0.017	0.360	0.266	0.261	0.000	0.302	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	23	24	35	222	0	29	0
N.S.	1	1.00	0.68	0.71	1.03	6.53	0.00	0.85	0.00
time (sec)	N/A	0.296	0.014	0.126	0.274	0.258	0.000	0.256	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	15	17	69	15	14	17
N.S.	1	1.00	1.00	1.15	1.31	5.31	1.15	1.08	1.31
time (sec)	N/A	0.215	0.018	0.102	0.274	0.247	0.182	0.265	0.056

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	55	8	186	0	0	0
N.S.	1	1.00	1.00	3.44	0.50	11.62	0.00	0.00	0.00
time (sec)	N/A	0.225	0.005	0.146	0.310	0.258	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	26	82	41	299	0	56	0
N.S.	1	1.00	0.62	1.95	0.98	7.12	0.00	1.33	0.00
time (sec)	N/A	0.302	0.015	0.191	0.303	0.257	0.000	0.264	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	69	36	102	75	837	0	67	0
N.S.	1	1.13	0.59	1.67	1.23	13.72	0.00	1.10	0.00
time (sec)	N/A	0.393	0.029	0.189	0.318	0.272	0.000	0.275	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	99	65	0	0	823	0	0	0
N.S.	1	0.82	0.54	0.00	0.00	6.80	0.00	0.00	0.00
time (sec)	N/A	0.500	0.112	0.000	0.000	0.101	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	63	54	0	0	317	0	0	0
N.S.	1	0.89	0.76	0.00	0.00	4.46	0.00	0.00	0.00
time (sec)	N/A	0.343	0.087	0.000	0.000	0.089	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	45	59	0	0	59	0	0	0
N.S.	1	0.94	1.23	0.00	0.00	1.23	0.00	0.00	0.00
time (sec)	N/A	0.285	0.062	0.000	0.000	0.082	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	46	41	36	0	0	93	0	0	0
N.S.	1	0.89	0.78	0.00	0.00	2.02	0.00	0.00	0.00
time (sec)	N/A	0.284	0.037	0.000	0.000	0.079	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	75	65	48	0	0	629	0	0	0
N.S.	1	0.87	0.64	0.00	0.00	8.39	0.00	0.00	0.00
time (sec)	N/A	0.362	0.061	0.000	0.000	0.091	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	95	61	0	0	1668	0	0	0
N.S.	1	0.79	0.50	0.00	0.00	13.79	0.00	0.00	0.00
time (sec)	N/A	0.495	0.098	0.000	0.000	0.121	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	92	53	177	100	1597	0	114	0
N.S.	1	0.70	0.40	1.34	0.76	12.10	0.00	0.86	0.00
time (sec)	N/A	0.497	0.105	8.666	0.287	0.320	0.000	0.293	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	60	38	131	62	659	0	52	0
N.S.	1	0.77	0.49	1.68	0.79	8.45	0.00	0.67	0.00
time (sec)	N/A	0.350	0.071	0.261	0.261	0.287	0.000	0.267	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	29	25	89	27	180	0	28	0
N.S.	1	0.81	0.69	2.47	0.75	5.00	0.00	0.78	0.00
time (sec)	N/A	0.228	0.025	0.219	0.268	0.251	0.000	0.270	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	29	16	116	0	13	39
N.S.	1	1.00	1.00	1.93	1.07	7.73	0.00	0.87	2.60
time (sec)	N/A	0.229	0.007	0.172	0.275	0.253	0.000	0.255	0.067

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	48	30	48	165	1137	0	27	48
N.S.	1	0.72	0.45	0.72	2.46	16.97	0.00	0.40	0.72
time (sec)	N/A	0.265	0.027	0.189	0.268	0.268	0.000	0.263	1.713

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	68	47	60	457	3065	0	39	256
N.S.	1	0.58	0.40	0.51	3.91	26.20	0.00	0.33	2.19
time (sec)	N/A	0.271	0.043	0.239	0.273	0.346	0.000	0.269	1.712

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	12	9	8	31	7	10	8
N.S.	1	1.00	1.50	1.12	1.00	3.88	0.88	1.25	1.00
time (sec)	N/A	0.190	0.010	0.094	0.174	0.245	0.160	0.256	0.085

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	12	9	8	31	7	10	8
N.S.	1	1.00	1.50	1.12	1.00	3.88	0.88	1.25	1.00
time (sec)	N/A	0.188	0.010	0.116	0.185	0.252	0.158	0.257	1.627

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	18	11	12	20	7	10	10
N.S.	1	1.00	1.50	0.92	1.00	1.67	0.58	0.83	0.83
time (sec)	N/A	0.201	0.006	0.143	0.175	0.255	0.207	0.255	1.650

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	24	11	12	22	7	10	10
N.S.	1	1.00	1.71	0.79	0.86	1.57	0.50	0.71	0.71
time (sec)	N/A	0.197	0.009	0.165	0.197	0.246	0.347	0.250	0.049

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	13	11	23	48	58	21	10
N.S.	1	1.00	1.30	1.10	2.30	4.80	5.80	2.10	1.00
time (sec)	N/A	0.214	0.016	0.552	0.181	0.250	0.192	0.248	1.703

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	13	11	23	54	58	22	10
N.S.	1	1.00	1.08	0.92	1.92	4.50	4.83	1.83	0.83
time (sec)	N/A	0.216	0.014	0.548	0.190	0.263	0.197	0.250	1.719

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	12	9	8	55	15	12	8
N.S.	1	1.00	1.20	0.90	0.80	5.50	1.50	1.20	0.80
time (sec)	N/A	0.185	0.009	0.188	0.187	0.249	0.265	0.253	1.783

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	8	55	14	12	8
N.S.	1	1.00	1.00	0.92	0.67	4.58	1.17	1.00	0.67
time (sec)	N/A	0.186	0.010	0.235	0.168	0.253	0.253	0.257	0.084

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	12	9	49	33	7	16	16
N.S.	1	1.00	0.86	0.64	3.50	2.36	0.50	1.14	1.14
time (sec)	N/A	0.197	0.096	0.307	0.187	0.254	0.376	0.245	0.088

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	12	9	49	33	8	16	16
N.S.	1	1.00	0.75	0.56	3.06	2.06	0.50	1.00	1.00
time (sec)	N/A	0.194	0.073	0.355	0.201	0.248	0.581	0.252	1.809

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	20	15	31	89	126	21	14
N.S.	1	1.00	1.43	1.07	2.21	6.36	9.00	1.50	1.00
time (sec)	N/A	0.221	0.011	0.439	0.176	0.257	0.260	0.249	1.841

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	27	17	35	90	126	20	16
N.S.	1	1.00	1.35	0.85	1.75	4.50	6.30	1.00	0.80
time (sec)	N/A	0.221	0.012	0.548	0.180	0.252	0.259	0.258	1.673

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	60	51	132	102	101	1253	90	131
N.S.	1	1.05	0.89	2.32	1.79	1.77	21.98	1.58	2.30
time (sec)	N/A	0.370	0.157	304.862	0.184	0.256	2.831	0.252	1.985

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	47	27	40	84	94	284	75	107
N.S.	1	1.02	0.59	0.87	1.83	2.04	6.17	1.63	2.33
time (sec)	N/A	0.251	0.022	120.533	0.184	0.254	1.835	0.258	1.832

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	46	39	96	78	57	692	66	95
N.S.	1	1.05	0.89	2.18	1.77	1.30	15.73	1.50	2.16
time (sec)	N/A	0.312	0.184	42.988	0.177	0.265	1.149	0.258	1.724

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	32	21	26	60	52	150	51	71
N.S.	1	0.97	0.64	0.79	1.82	1.58	4.55	1.55	2.15
time (sec)	N/A	0.236	0.018	14.749	0.173	0.257	0.759	0.255	1.683

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	30	25	60	54	27	294	40	59
N.S.	1	0.97	0.81	1.94	1.74	0.87	9.48	1.29	1.90
time (sec)	N/A	0.254	0.114	3.725	0.179	0.259	0.425	0.256	1.609

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	16	13	16	36	18	49	27	35
N.S.	1	0.84	0.68	0.84	1.89	0.95	2.58	1.42	1.84
time (sec)	N/A	0.213	0.012	1.083	0.182	0.260	0.267	0.252	1.638

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	17	24	23	11	46	17	23
N.S.	1	1.00	1.31	1.85	1.77	0.85	3.54	1.31	1.77
time (sec)	N/A	0.212	0.040	0.263	0.185	0.255	0.179	0.248	1.575

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	11	12	12	11	16	7	17	9
N.S.	1	1.22	1.33	1.33	1.22	1.78	0.78	1.89	1.00
time (sec)	N/A	0.194	0.006	0.059	0.178	0.264	0.058	0.254	0.046

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	29	42	20	47	103	0	52	51
N.S.	1	1.26	1.83	0.87	2.04	4.48	0.00	2.26	2.22
time (sec)	N/A	0.240	0.024	0.267	0.199	0.261	0.000	0.250	1.594

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	30	24	59	94	0	35	89
N.S.	1	1.00	1.25	1.00	2.46	3.92	0.00	1.46	3.71
time (sec)	N/A	0.274	0.173	0.468	0.190	0.242	0.000	0.252	1.579

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	61	60	38	103	631	0	94	114
N.S.	1	1.24	1.22	0.78	2.10	12.88	0.00	1.92	2.33
time (sec)	N/A	0.278	0.106	1.104	0.185	0.262	0.000	0.248	1.675

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	43	38	36	233	250	0	59	263
N.S.	1	1.16	1.03	0.97	6.30	6.76	0.00	1.59	7.11
time (sec)	N/A	0.296	0.144	2.640	0.192	0.250	0.000	0.251	1.668

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	93	89	54	155	1551	0	116	244
N.S.	1	1.19	1.14	0.69	1.99	19.88	0.00	1.49	3.13
time (sec)	N/A	0.310	0.207	5.886	0.191	0.275	0.000	0.269	0.992

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	144	148	310	2134	0	229	289
N.S.	1	1.00	1.03	1.06	2.21	15.24	0.00	1.64	2.06
time (sec)	N/A	0.370	0.180	0.045	0.198	0.279	0.000	0.259	1.420

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	184	154	409	0	2913	0	266	348
N.S.	1	1.19	1.00	2.66	0.00	18.92	0.00	1.73	2.26
time (sec)	N/A	0.958	0.213	0.053	0.000	0.297	0.000	0.255	1.416

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	82	84	82	178	866	0	124	169
N.S.	1	0.99	1.01	0.99	2.14	10.43	0.00	1.49	2.04
time (sec)	N/A	0.292	0.117	102.337	0.195	0.265	0.000	0.257	1.185

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	125	95	223	0	1099	0	146	222
N.S.	1	1.20	0.91	2.14	0.00	10.57	0.00	1.40	2.13
time (sec)	N/A	0.641	0.157	24.016	0.000	0.283	0.000	0.260	1.208

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	39	84	234	1591	56	79
N.S.	1	1.00	1.00	0.98	2.10	5.85	39.78	1.40	1.98
time (sec)	N/A	0.253	0.072	3.598	0.201	0.284	154.332	0.250	1.050

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	73	54	100	0	279	892	68	139
N.S.	1	1.24	0.92	1.69	0.00	4.73	15.12	1.15	2.36
time (sec)	N/A	0.413	0.089	0.673	0.000	0.283	51.501	0.262	1.002

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	27	14	19	11
N.S.	1	1.00	1.00	1.09	1.00	2.45	1.27	1.73	1.00
time (sec)	N/A	0.202	0.033	0.114	0.187	0.257	0.079	0.256	0.050

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	66	49	52	59	58	0	67	160
N.S.	1	1.25	0.92	0.98	1.11	1.09	0.00	1.26	3.02
time (sec)	N/A	0.298	0.075	0.340	0.213	0.293	0.000	0.256	1.960

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	78	77	78	0	470	0	76	327
N.S.	1	1.16	1.15	1.16	0.00	7.01	0.00	1.13	4.88
time (sec)	N/A	0.328	0.186	0.618	0.000	0.277	0.000	0.264	2.134

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	117	100	90	154	818	0	179	291
N.S.	1	1.29	1.10	0.99	1.69	8.99	0.00	1.97	3.20
time (sec)	N/A	0.377	0.351	1.543	0.202	0.292	0.000	0.254	2.206

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	134	141	127	0	2339	0	156	642
N.S.	1	1.22	1.28	1.15	0.00	21.26	0.00	1.42	5.84
time (sec)	N/A	0.596	0.433	3.519	0.000	0.303	0.000	0.259	2.758

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	196	165	138	348	3450	0	338	559
N.S.	1	1.30	1.09	0.91	2.30	22.85	0.00	2.24	3.70
time (sec)	N/A	0.493	0.553	8.071	0.236	0.321	0.000	0.267	2.550

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	200	201	213	0	6381	0	303	1031
N.S.	1	1.26	1.26	1.34	0.00	40.13	0.00	1.91	6.48
time (sec)	N/A	0.963	1.322	18.165	0.000	0.327	0.000	0.269	3.388

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	72	61	99	0	700	0	68	139
N.S.	1	1.07	0.91	1.48	0.00	10.45	0.00	1.01	2.07
time (sec)	N/A	0.384	0.105	0.651	0.000	0.279	0.000	0.256	1.903

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	133	100	154	0	2003	0	144	722
N.S.	1	1.18	0.88	1.36	0.00	17.73	0.00	1.27	6.39
time (sec)	N/A	0.883	0.332	0.662	0.000	0.349	0.000	0.254	7.039

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	59	46	95	96	450	0	115	1221
N.S.	1	1.04	0.81	1.67	1.68	7.89	0.00	2.02	21.42
time (sec)	N/A	0.289	0.103	0.337	0.285	0.272	0.000	0.267	2.421

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	75	61	78	0	326	0	67	285
N.S.	1	1.23	1.00	1.28	0.00	5.34	0.00	1.10	4.67
time (sec)	N/A	0.599	0.130	0.268	0.000	0.288	0.000	0.258	4.452

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	22	20	33	33	40	0	33	201
N.S.	1	1.10	1.00	1.65	1.65	2.00	0.00	1.65	10.05
time (sec)	N/A	0.215	0.010	0.110	0.286	0.281	0.000	0.252	0.439

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	59	50	53	59	60	0	67	148
N.S.	1	1.09	0.93	0.98	1.09	1.11	0.00	1.24	2.74
time (sec)	N/A	0.257	0.084	0.223	0.197	0.273	0.000	0.251	0.450

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	88	77	78	0	470	0	76	337
N.S.	1	1.14	1.00	1.01	0.00	6.10	0.00	0.99	4.38
time (sec)	N/A	0.459	0.196	0.377	0.000	0.283	0.000	0.254	2.105

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	135	101	91	156	839	0	178	291
N.S.	1	1.44	1.07	0.97	1.66	8.93	0.00	1.89	3.10
time (sec)	N/A	0.399	0.320	0.643	0.208	0.279	0.000	0.263	2.292

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	158	131	127	0	2417	0	172	666
N.S.	1	1.15	0.96	0.93	0.00	17.64	0.00	1.26	4.86
time (sec)	N/A	0.890	0.418	1.032	0.000	0.272	0.000	0.262	2.554

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	47	58	64	89	750	0	58	183
N.S.	1	1.02	1.26	1.39	1.93	16.30	0.00	1.26	3.98
time (sec)	N/A	0.458	0.238	0.605	0.288	0.253	0.000	0.251	1.768

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	38	25	43	223	174	0	48	117
N.S.	1	1.27	0.83	1.43	7.43	5.80	0.00	1.60	3.90
time (sec)	N/A	0.353	0.022	0.377	0.186	0.259	0.000	0.249	1.728

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	46	48	57	315	0	39	95
N.S.	1	1.00	1.39	1.45	1.73	9.55	0.00	1.18	2.88
time (sec)	N/A	0.391	0.147	0.281	0.284	0.293	0.000	0.252	1.676

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	27	17	26	70	66	0	22	25
N.S.	1	1.42	0.89	1.37	3.68	3.47	0.00	1.16	1.32
time (sec)	N/A	0.290	0.018	0.181	0.194	0.241	0.000	0.284	1.664

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	18	30	23	50	0	22	33
N.S.	1	1.00	1.20	2.00	1.53	3.33	0.00	1.47	2.20
time (sec)	N/A	0.306	0.175	0.130	0.294	0.254	0.000	0.259	1.647

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	22	12	14	24	28	0	22	26
N.S.	1	1.22	0.67	0.78	1.33	1.56	0.00	1.22	1.44
time (sec)	N/A	0.215	0.015	0.079	0.198	0.261	0.000	0.264	0.078

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	42	42	20	48	103	0	52	51
N.S.	1	1.27	1.27	0.61	1.45	3.12	0.00	1.58	1.55
time (sec)	N/A	0.375	0.031	0.083	0.202	0.271	0.000	0.256	1.650

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	36	25	29	121	91	0	35	92
N.S.	1	1.20	0.83	0.97	4.03	3.03	0.00	1.17	3.07
time (sec)	N/A	0.371	0.194	0.096	0.204	0.258	0.000	0.255	1.638

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	59	60	38	103	631	0	94	132
N.S.	1	1.28	1.30	0.83	2.24	13.72	0.00	2.04	2.87
time (sec)	N/A	0.476	0.082	0.142	0.205	0.257	0.000	0.261	1.683

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	46	41	45	469	224	0	59	263
N.S.	1	1.12	1.00	1.10	11.44	5.46	0.00	1.44	6.41
time (sec)	N/A	0.373	0.154	0.196	0.196	0.249	0.000	0.255	1.843

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	125	0	376	0	0	0
N.S.	1	1.00	1.00	3.38	0.00	10.16	0.00	0.00	0.00
time (sec)	N/A	0.238	0.017	0.261	0.000	0.427	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	104	0	356	0	0	0
N.S.	1	1.00	1.00	4.33	0.00	14.83	0.00	0.00	0.00
time (sec)	N/A	0.227	0.010	0.213	0.000	0.289	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	55	112	0	291	741	60	197
N.S.	1	1.00	0.98	2.00	0.00	5.20	13.23	1.07	3.52
time (sec)	N/A	0.326	0.077	0.207	0.000	0.274	12.981	0.272	3.865

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	19	23	19	46	20	22	22
N.S.	1	1.00	1.06	1.28	1.06	2.56	1.11	1.22	1.22
time (sec)	N/A	0.263	0.025	0.101	0.199	0.262	0.139	0.256	0.059

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	19	22	20	48	31	22	21
N.S.	1	1.00	0.79	0.92	0.83	2.00	1.29	0.92	0.88
time (sec)	N/A	0.276	0.261	0.108	0.201	0.259	0.217	0.254	1.710

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	61	100	0	315	0	66	160
N.S.	1	1.00	0.94	1.54	0.00	4.85	0.00	1.02	2.46
time (sec)	N/A	0.355	0.221	0.230	0.000	0.285	0.000	0.267	13.450

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	134	101	0	303	0	90	974
N.S.	1	1.00	1.34	1.01	0.00	3.03	0.00	0.90	9.74
time (sec)	N/A	0.381	0.340	0.229	0.000	1.135	0.000	0.265	4.651

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	63	59	0	249	0	53	636
N.S.	1	1.00	1.02	0.95	0.00	4.02	0.00	0.85	10.26
time (sec)	N/A	0.438	0.200	0.265	0.000	0.481	0.000	0.258	7.264

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	112	116	100	0	298	0	90	983
N.S.	1	1.13	1.17	1.01	0.00	3.01	0.00	0.91	9.93
time (sec)	N/A	0.559	0.270	0.670	0.000	1.867	0.000	0.259	4.062

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	81	156	0	405	695	94	653
N.S.	1	1.00	0.94	1.81	0.00	4.71	8.08	1.09	7.59
time (sec)	N/A	0.518	0.471	0.412	0.000	0.283	15.332	0.266	2.942

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	132	115	144	0	1044	5175	155	301
N.S.	1	1.09	0.95	1.19	0.00	8.63	42.77	1.28	2.49
time (sec)	N/A	0.582	0.590	0.785	0.000	0.277	174.733	0.284	2.348

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	213	175	273	0	3636	0	370	0
N.S.	1	1.14	0.94	1.46	0.00	19.44	0.00	1.98	0.00
time (sec)	N/A	0.841	0.829	3.280	0.000	0.337	0.000	0.303	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	302	245	459	0	8531	0	657	0
N.S.	1	1.16	0.94	1.77	0.00	32.81	0.00	2.53	0.00
time (sec)	N/A	1.142	1.969	11.281	0.000	0.522	0.000	0.304	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	193	284	487	0	780	0	0	0
N.S.	1	1.01	1.49	2.55	0.00	4.08	0.00	0.00	0.00
time (sec)	N/A	0.750	0.469	0.158	0.000	0.319	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	275	434	686	0	1162	0	0	0
N.S.	1	0.95	1.49	2.36	0.00	3.99	0.00	0.00	0.00
time (sec)	N/A	1.121	0.311	0.162	0.000	0.305	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	355	580	889	0	1542	0	0	0
N.S.	1	0.91	1.48	2.27	0.00	3.94	0.00	0.00	0.00
time (sec)	N/A	1.369	0.331	0.186	0.000	0.310	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	57	55	0	0	0	0	0	0
N.S.	1	0.98	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.356	0.068	0.000	0.000	0.000	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	57	57	0	0	0	0	0	0
N.S.	1	0.98	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.333	0.029	0.000	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.284	0.028	0.000	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	32	35	34	48	34	35
N.S.	1	1.00	1.06	0.94	1.03	1.00	1.41	1.00	1.03
time (sec)	N/A	0.287	8.826	0.064	0.410	0.276	7.980	0.429	1.759

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	36	38	32	120	34	51	34	35
N.S.	1	1.00	1.06	0.89	3.33	0.94	1.42	0.94	0.97
time (sec)	N/A	0.326	32.418	0.079	0.386	0.259	22.326	0.590	1.867

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	59	138	0	480	1770	0	110
N.S.	1	1.00	0.98	2.30	0.00	8.00	29.50	0.00	1.83
time (sec)	N/A	0.278	0.098	0.390	0.000	0.275	152.586	0.000	1.818

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	103	87	231	0	1692	0	0	0
N.S.	1	1.18	1.00	2.66	0.00	19.45	0.00	0.00	0.00
time (sec)	N/A	0.384	0.184	2.599	0.000	0.283	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	41	47	42	67	0	42	0
N.S.	1	1.00	0.87	1.00	0.89	1.43	0.00	0.89	0.00
time (sec)	N/A	0.600	0.095	0.929	0.300	0.260	0.000	0.260	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	202	24	20	24	24
N.S.	1	1.00	1.09	1.00	9.18	1.09	0.91	1.09	1.09
time (sec)	N/A	0.224	2.936	0.041	1.034	0.249	0.779	0.286	1.686

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	327	337	326	0	0	624	0	0	0
N.S.	1	1.03	1.00	0.00	0.00	1.91	0.00	0.00	0.00
time (sec)	N/A	1.312	0.032	0.000	0.000	0.270	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	245	249	244	0	0	497	0	0	0
N.S.	1	1.02	1.00	0.00	0.00	2.03	0.00	0.00	0.00
time (sec)	N/A	0.965	0.020	0.000	0.000	0.270	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	160	368	0	354	0	0	0
N.S.	1	1.00	0.99	2.29	0.00	2.20	0.00	0.00	0.00
time (sec)	N/A	0.593	0.011	0.263	0.000	0.271	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	44	41	31	18
N.S.	1	1.00	1.00	1.06	1.00	2.44	2.28	1.72	1.00
time (sec)	N/A	0.236	0.160	0.115	0.198	0.256	0.553	0.273	0.084

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	56	24	19	24	24
N.S.	1	1.00	1.09	1.00	2.55	1.09	0.86	1.09	1.09
time (sec)	N/A	0.217	8.903	0.054	0.297	0.259	4.672	0.289	1.842

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	22	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.92	1.08	1.08
time (sec)	N/A	0.239	6.924	0.080	0.367	0.252	1.280	0.284	1.781

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	495	463	386	0	0	1174	0	0	0
N.S.	1	0.94	0.78	0.00	0.00	2.37	0.00	0.00	0.00
time (sec)	N/A	2.361	0.709	0.000	0.000	0.295	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	370	356	293	0	0	937	0	0	0
N.S.	1	0.96	0.79	0.00	0.00	2.53	0.00	0.00	0.00
time (sec)	N/A	1.719	0.546	0.000	0.000	0.282	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	242	187	862	0	669	0	0	0
N.S.	1	0.99	0.77	3.53	0.00	2.74	0.00	0.00	0.00
time (sec)	N/A	1.081	0.782	1.268	0.000	0.286	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	87	69	129	0	415	1122	89	176
N.S.	1	1.19	0.95	1.77	0.00	5.68	15.37	1.22	2.41
time (sec)	N/A	0.457	0.297	0.733	0.000	0.269	67.780	0.273	1.897

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	92	26	20	26	26
N.S.	1	1.00	1.08	1.00	3.83	1.08	0.83	1.08	1.08
time (sec)	N/A	0.233	19.466	0.102	0.327	0.258	43.414	0.346	1.770

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	22	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.92	1.08	1.08
time (sec)	N/A	0.239	10.684	0.179	0.399	0.250	2.506	0.293	1.763

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	586	540	951	0	0	2025	0	0	0
N.S.	1	0.92	1.62	0.00	0.00	3.46	0.00	0.00	0.00
time (sec)	N/A	3.121	4.727	0.000	0.000	0.298	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	432	388	758	0	0	1622	0	0	0
N.S.	1	0.90	1.75	0.00	0.00	3.75	0.00	0.00	0.00
time (sec)	N/A	2.126	3.343	0.000	0.000	0.280	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	264	374	860	0	1196	0	0	0
N.S.	1	0.92	1.30	2.99	0.00	4.15	0.00	0.00	0.00
time (sec)	N/A	1.315	1.705	6.628	0.000	0.296	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	55	55	55	130	340	0	88	122
N.S.	1	0.90	0.90	0.90	2.13	5.57	0.00	1.44	2.00
time (sec)	N/A	0.277	0.268	4.124	0.194	0.265	0.000	0.269	1.852

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	143	26	20	26	26
N.S.	1	1.00	1.08	1.00	5.96	1.08	0.83	1.08	1.08
time (sec)	N/A	0.228	36.156	0.146	0.436	0.256	22.084	0.339	1.792

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	41	42	51	44	0	47	44
N.S.	1	1.00	0.76	0.78	0.94	0.81	0.00	0.87	0.81
time (sec)	N/A	0.200	0.050	0.198	0.215	0.262	0.000	0.262	1.763

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	56	59	67	90	0	169	53
N.S.	1	1.00	0.64	0.67	0.76	1.02	0.00	1.92	0.60
time (sec)	N/A	0.240	0.073	0.650	0.212	0.259	0.000	0.272	1.758

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	143	117	166	115	199	0	665	94
N.S.	1	0.96	0.79	1.11	0.77	1.34	0.00	4.46	0.63
time (sec)	N/A	0.350	0.358	2.405	0.240	0.260	0.000	0.293	1.752

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	177	167	184	129	293	0	777	102
N.S.	1	0.93	0.87	0.96	0.68	1.53	0.00	4.07	0.53
time (sec)	N/A	0.374	0.315	10.031	0.244	0.276	0.000	0.307	1.856

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	74	54	68	64	99	0	235	55
N.S.	1	1.01	0.74	0.93	0.88	1.36	0.00	3.22	0.75
time (sec)	N/A	0.225	0.087	0.511	0.231	0.270	0.000	0.271	1.830

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	122	87	84	87	250	0	759	73
N.S.	1	1.02	0.72	0.70	0.72	2.08	0.00	6.32	0.61
time (sec)	N/A	0.266	0.197	4.942	0.213	0.264	0.000	0.298	1.837

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	188	292	178	138	584	0	3225	117
N.S.	1	0.93	1.44	0.88	0.68	2.88	0.00	15.89	0.58
time (sec)	N/A	0.421	0.971	36.275	0.234	0.280	0.000	0.343	1.915

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	236	311	214	161	1123	0	6880	134
N.S.	1	0.89	1.17	0.80	0.61	4.22	0.00	25.86	0.50
time (sec)	N/A	0.460	2.521	236.928	0.231	0.318	0.000	0.399	1.971

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	37	19	18	19	34	42	18
N.S.	1	1.00	2.06	1.06	1.00	1.06	1.89	2.33	1.00
time (sec)	N/A	0.210	0.008	0.192	0.189	0.256	0.228	0.263	1.792

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	44	36	30	49	39	0	80	32
N.S.	1	1.13	0.92	0.77	1.26	1.00	0.00	2.05	0.82
time (sec)	N/A	0.222	0.022	0.903	0.212	0.260	0.000	0.257	1.801

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	45	42	36	86	53	71	81	35
N.S.	1	1.07	1.00	0.86	2.05	1.26	1.69	1.93	0.83
time (sec)	N/A	0.237	0.004	4.360	0.201	0.260	1.221	0.270	1.770

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	80	51	46	93	84	0	114	50
N.S.	1	1.10	0.70	0.63	1.27	1.15	0.00	1.56	0.68
time (sec)	N/A	0.301	0.030	18.102	0.197	0.261	0.000	0.267	1.859

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	64	65	51	130	105	105	116	49
N.S.	1	0.98	1.00	0.78	2.00	1.62	1.62	1.78	0.75
time (sec)	N/A	0.248	0.008	68.842	0.192	0.265	7.708	0.265	1.916

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	65	62	256	0	332	0	0	0
N.S.	1	0.97	0.93	3.82	0.00	4.96	0.00	0.00	0.00
time (sec)	N/A	0.303	0.034	1.682	0.000	0.088	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	65	114	237	0	170	0	0	0
N.S.	1	0.97	1.70	3.54	0.00	2.54	0.00	0.00	0.00
time (sec)	N/A	0.297	0.098	0.224	0.000	0.085	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	183	0	58	0	0	0
N.S.	1	1.00	1.00	6.54	0.00	2.07	0.00	0.00	0.00
time (sec)	N/A	0.215	0.011	0.208	0.000	0.084	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	183	0	39	0	0	0
N.S.	1	1.00	1.00	6.54	0.00	1.39	0.00	0.00	0.00
time (sec)	N/A	0.220	0.010	0.204	0.000	0.077	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	61	58	141	0	243	0	0	0
N.S.	1	0.97	0.92	2.24	0.00	3.86	0.00	0.00	0.00
time (sec)	N/A	0.297	0.037	0.240	0.000	0.086	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	65	122	295	0	501	0	0	0
N.S.	1	0.97	1.82	4.40	0.00	7.48	0.00	0.00	0.00
time (sec)	N/A	0.294	0.058	0.242	0.000	0.088	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	206	203	85	0	0	187	0	3088	0
N.S.	1	0.99	0.41	0.00	0.00	0.91	0.00	14.99	0.00
time (sec)	N/A	0.464	0.253	0.000	0.000	0.269	0.000	135.028	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F(-1)	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	102	109	74	0	0	141	0	0	0
N.S.	1	1.07	0.73	0.00	0.00	1.38	0.00	0.00	0.00
time (sec)	N/A	0.374	0.218	0.000	0.000	0.256	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	42	42	61	0	0	68	0	0	0
N.S.	1	1.00	1.45	0.00	0.00	1.62	0.00	0.00	0.00
time (sec)	N/A	0.305	0.087	0.000	0.000	0.260	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	141	121	0	0	128	0	0	0
N.S.	1	1.40	1.20	0.00	0.00	1.27	0.00	0.00	0.00
time (sec)	N/A	0.364	0.163	0.000	0.000	0.258	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	116	305	347	0	171	0	764	0
N.S.	1	1.15	3.02	3.44	0.00	1.69	0.00	7.56	0.00
time (sec)	N/A	0.632	1.398	0.213	0.000	0.273	0.000	1.963	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	122	475	358	0	366	0	749	0
N.S.	1	1.14	4.44	3.35	0.00	3.42	0.00	7.00	0.00
time (sec)	N/A	0.652	3.174	0.849	0.000	0.298	0.000	6.838	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	71	62	45	68	113	139	60	58
N.S.	1	0.86	0.75	0.54	0.82	1.36	1.67	0.72	0.70
time (sec)	N/A	0.210	0.035	1.071	0.190	0.257	2.145	0.279	0.559

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	47	53	47	53	95	182	57	42
N.S.	1	0.82	0.93	0.82	0.93	1.67	3.19	1.00	0.74
time (sec)	N/A	0.208	0.025	0.413	0.199	0.252	0.865	0.264	0.287

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	43	39	35	40	54	78	34	34
N.S.	1	0.88	0.80	0.71	0.82	1.10	1.59	0.69	0.69
time (sec)	N/A	0.200	0.018	0.172	0.202	0.240	0.399	0.278	1.730

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	30	23	19	24	50	80	22	18
N.S.	1	1.30	1.00	0.83	1.04	2.17	3.48	0.96	0.78
time (sec)	N/A	0.181	0.010	0.086	0.201	0.251	0.205	0.277	1.703

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	17	16	30	0	16	16
N.S.	1	1.00	1.00	1.00	0.94	1.76	0.00	0.94	0.94
time (sec)	N/A	0.170	0.013	0.094	0.311	0.264	0.000	0.261	0.072

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	43	38	25	37	105	0	35	48
N.S.	1	1.08	0.95	0.62	0.92	2.62	0.00	0.88	1.20
time (sec)	N/A	0.184	0.050	0.331	0.289	0.256	0.000	0.274	0.084

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	22	68	86	0	31	31
N.S.	1	1.00	1.00	0.76	2.34	2.97	0.00	1.07	1.07
time (sec)	N/A	0.180	0.015	0.308	0.192	0.254	0.000	0.267	1.693

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	106	64	37	83	513	0	60	130
N.S.	1	1.12	0.67	0.39	0.87	5.40	0.00	0.63	1.37
time (sec)	N/A	0.205	0.058	2.337	0.277	0.263	0.000	0.264	1.704

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	44	35	172	233	0	42	42
N.S.	1	1.00	0.73	0.58	2.87	3.88	0.00	0.70	0.70
time (sec)	N/A	0.212	0.027	1.737	0.193	0.247	0.000	0.267	1.713

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	28	26	17	17	47	42	17	17
N.S.	1	1.08	1.00	0.65	0.65	1.81	1.62	0.65	0.65
time (sec)	N/A	0.186	0.013	0.184	0.217	0.268	0.159	0.270	0.089

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	21	16	14	13	26	20	13	12
N.S.	1	1.11	0.84	0.74	0.68	1.37	1.05	0.68	0.63
time (sec)	N/A	0.185	0.008	0.055	0.195	0.249	0.098	0.258	1.639

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	104	24	25	76	81	0	76	77
N.S.	1	1.13	0.26	0.27	0.83	0.88	0.00	0.83	0.84
time (sec)	N/A	0.303	0.009	0.153	0.291	0.259	0.000	0.262	0.233

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	125	106	36	88	349	0	88	85
N.S.	1	1.13	0.95	0.32	0.79	3.14	0.00	0.79	0.77
time (sec)	N/A	0.339	0.071	0.346	0.275	0.272	0.000	0.276	1.800

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	28	26	17	17	67	42	17	17
N.S.	1	1.08	1.00	0.65	0.65	2.58	1.62	0.65	0.65
time (sec)	N/A	0.181	0.013	0.143	0.191	0.255	0.160	0.270	1.691

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	23	16	14	13	38	20	13	12
N.S.	1	1.21	0.84	0.74	0.68	2.00	1.05	0.68	0.63
time (sec)	N/A	0.176	0.010	0.096	0.192	0.250	0.097	0.267	1.656

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	35	24	79	71	83	0	44	65
N.S.	1	0.64	0.44	1.44	1.29	1.51	0.00	0.80	1.18
time (sec)	N/A	0.253	0.010	0.168	0.304	0.254	0.000	0.269	1.817

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	121	34	59	79	479	0	79	84
N.S.	1	1.10	0.31	0.54	0.72	4.35	0.00	0.72	0.76
time (sec)	N/A	0.346	0.017	0.359	0.277	0.297	0.000	0.275	0.335

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	28	26	17	17	87	42	17	17
N.S.	1	1.08	1.00	0.65	0.65	3.35	1.62	0.65	0.65
time (sec)	N/A	0.186	0.018	0.139	0.197	0.261	0.158	0.269	1.721

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	23	19	14	13	46	20	13	14
N.S.	1	1.21	1.00	0.74	0.68	2.42	1.05	0.68	0.74
time (sec)	N/A	0.182	0.013	0.064	0.197	0.249	0.098	0.270	0.054

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	389	24	25	0	167	0	249	479
N.S.	1	1.05	0.06	0.07	0.00	0.45	0.00	0.67	1.29
time (sec)	N/A	0.618	0.011	0.156	0.000	0.259	0.000	0.370	5.649

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	379	390	34	36	0	1215	0	261	473
N.S.	1	1.03	0.09	0.09	0.00	3.21	0.00	0.69	1.25
time (sec)	N/A	0.672	0.017	0.354	0.000	0.288	0.000	0.271	4.504

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	190	159	148	134	2218	1658	1211	154
N.S.	1	0.94	0.79	0.73	0.66	10.98	8.21	6.00	0.76
time (sec)	N/A	0.406	0.454	0.872	0.221	0.345	15.179	0.322	2.663

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	85	90	94	699	706	889	100
N.S.	1	1.00	0.64	0.68	0.71	5.30	5.35	6.73	0.76
time (sec)	N/A	0.328	0.146	0.368	0.203	0.269	1.135	0.301	2.067

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	50	52	63	246	265	597	74
N.S.	1	1.00	0.67	0.69	0.84	3.28	3.53	7.96	0.99
time (sec)	N/A	0.218	0.068	0.101	0.203	0.276	0.621	0.301	1.781

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	70	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.225	0.016	0.000	0.000	0.000	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	70	70	70	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.223	0.017	0.000	0.000	0.000	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	124	139	96	0	0	0	0	0	0
N.S.	1	1.12	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.344	0.164	0.000	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	133	148	101	0	0	0	0	0	0
N.S.	1	1.11	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.363	0.136	0.000	0.000	0.000	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	250	102	109	88	112	218	0	101	0
N.S.	1	0.41	0.44	0.35	0.45	0.87	0.00	0.40	0.00
time (sec)	N/A	0.438	0.073	4.182	0.215	0.258	0.000	0.283	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	162	74	81	75	74	126	0	73	0
N.S.	1	0.46	0.50	0.46	0.46	0.78	0.00	0.45	0.00
time (sec)	N/A	0.380	0.048	0.265	0.212	0.256	0.000	0.284	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	74	59	48	60	29	66	163	23	76
N.S.	1	0.80	0.65	0.81	0.39	0.89	2.20	0.31	1.03
time (sec)	N/A	0.363	0.025	0.206	0.199	0.268	1.942	0.262	0.129

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	44	44	42	29	21	42	0	20	0
N.S.	1	1.00	0.95	0.66	0.48	0.95	0.00	0.45	0.00
time (sec)	N/A	0.324	0.039	0.157	0.306	0.260	0.000	0.273	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	56	56	46	38	84	120	0	38	76
N.S.	1	1.00	0.82	0.68	1.50	2.14	0.00	0.68	1.36
time (sec)	N/A	0.346	0.053	0.187	0.210	0.258	0.000	0.284	1.710

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	141	87	72	65	209	315	0	51	89
N.S.	1	0.62	0.51	0.46	1.48	2.23	0.00	0.36	0.63
time (sec)	N/A	0.423	0.049	0.222	0.206	0.253	0.000	0.296	0.117

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	191	105	84	86	386	589	0	64	345
N.S.	1	0.55	0.44	0.45	2.02	3.08	0.00	0.34	1.81
time (sec)	N/A	0.453	0.060	0.235	0.208	0.261	0.000	0.298	0.122

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	28	28	0	45	99	32	45
N.S.	1	1.00	0.68	0.68	0.00	1.10	2.41	0.78	1.10
time (sec)	N/A	0.198	0.041	0.130	0.000	0.259	0.229	0.265	1.718

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	79	72	65	104	0	73	0
N.S.	1	1.00	0.93	0.85	0.76	1.22	0.00	0.86	0.00
time (sec)	N/A	0.292	0.062	0.200	0.201	0.270	0.000	0.267	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	91	97	81	130	0	91	0
N.S.	1	1.00	0.90	0.96	0.80	1.29	0.00	0.90	0.00
time (sec)	N/A	0.341	0.114	0.289	0.197	0.268	0.000	0.261	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	51	52	45	72	0	45	0
N.S.	1	1.00	0.78	0.80	0.69	1.11	0.00	0.69	0.00
time (sec)	N/A	0.263	0.052	0.190	0.199	0.256	0.000	0.274	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	71	48	47	76	0	49	0
N.S.	1	1.00	1.09	0.74	0.72	1.17	0.00	0.75	0.00
time (sec)	N/A	0.284	0.070	0.369	0.189	0.267	0.000	0.268	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	122	105	89	165	0	101	0
N.S.	1	1.00	1.06	0.91	0.77	1.43	0.00	0.88	0.00
time (sec)	N/A	0.378	0.250	0.386	0.198	0.263	0.000	0.276	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	102	100	90	211	0	106	0
N.S.	1	1.00	0.93	0.91	0.82	1.92	0.00	0.96	0.00
time (sec)	N/A	0.355	0.092	0.126	0.202	0.276	0.000	0.281	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	149	126	127	278	0	355	0
N.S.	1	1.00	1.01	0.85	0.86	1.88	0.00	2.40	0.00
time (sec)	N/A	0.424	0.474	0.394	0.288	0.265	0.000	0.306	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	286	207	200	443	0	223	0
N.S.	1	1.00	1.20	0.87	0.84	1.85	0.00	0.93	0.00
time (sec)	N/A	0.551	0.293	1.074	0.293	0.294	0.000	0.281	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	123	126	102	251	0	132	0
N.S.	1	1.00	1.07	1.10	0.89	2.18	0.00	1.15	0.00
time (sec)	N/A	0.434	0.206	0.146	0.207	0.275	0.000	0.268	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	220	158	143	334	0	387	0
N.S.	1	1.00	1.37	0.98	0.89	2.07	0.00	2.40	0.00
time (sec)	N/A	0.519	0.454	0.502	0.286	0.263	0.000	0.293	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	353	265	228	539	0	281	0
N.S.	1	1.00	1.37	1.03	0.89	2.10	0.00	1.09	0.00
time (sec)	N/A	0.764	0.564	1.339	0.297	0.264	0.000	0.288	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	104	117	105	216	0	132	0
N.S.	1	1.00	0.78	0.88	0.79	1.62	0.00	0.99	0.00
time (sec)	N/A	0.400	0.110	0.114	0.204	0.277	0.000	0.256	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	131	139	131	242	0	150	0
N.S.	1	1.00	0.81	0.86	0.81	1.50	0.00	0.93	0.00
time (sec)	N/A	0.434	0.171	0.331	0.206	0.260	0.000	0.272	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	214	234	211	426	0	264	0
N.S.	1	1.00	0.79	0.86	0.78	1.57	0.00	0.97	0.00
time (sec)	N/A	0.599	0.335	0.973	0.212	0.276	0.000	0.279	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	75	70	69	145	0	75	0
N.S.	1	1.00	0.93	0.86	0.85	1.79	0.00	0.93	0.00
time (sec)	N/A	0.366	0.253	0.106	0.198	0.268	0.000	0.268	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	179	101	100	254	0	107	0
N.S.	1	1.00	1.40	0.79	0.78	1.98	0.00	0.84	0.00
time (sec)	N/A	0.424	0.389	0.319	0.207	0.272	0.000	0.278	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	270	144	143	491	0	155	0
N.S.	1	1.00	1.58	0.84	0.84	2.87	0.00	0.91	0.00
time (sec)	N/A	0.532	0.907	0.954	0.203	0.280	0.000	0.283	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	165	147	127	321	0	172	0
N.S.	1	1.00	1.18	1.05	0.91	2.29	0.00	1.23	0.00
time (sec)	N/A	0.518	0.475	0.145	0.207	0.274	0.000	0.279	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	258	177	161	420	0	198	0
N.S.	1	1.00	1.41	0.97	0.88	2.30	0.00	1.08	0.00
time (sec)	N/A	0.604	1.078	0.438	0.206	0.292	0.000	0.275	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	478	302	263	847	0	352	0
N.S.	1	1.00	1.59	1.01	0.88	2.82	0.00	1.17	0.00
time (sec)	N/A	0.843	4.445	1.354	0.211	0.292	0.000	0.290	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	134	162	129	262	0	167	0
N.S.	1	1.00	0.88	1.06	0.84	1.71	0.00	1.09	0.00
time (sec)	N/A	0.554	0.201	0.122	0.205	0.276	0.000	0.285	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	183	210	185	341	0	223	0
N.S.	1	1.00	0.84	0.96	0.84	1.56	0.00	1.02	0.00
time (sec)	N/A	0.576	0.360	0.358	0.201	0.271	0.000	0.287	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	262	326	263	526	0	339	0
N.S.	1	1.00	0.83	1.03	0.83	1.67	0.00	1.08	0.00
time (sec)	N/A	0.744	0.675	1.081	0.207	0.265	0.000	0.282	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	185	160	139	324	0	181	0
N.S.	1	1.00	1.20	1.04	0.90	2.10	0.00	1.18	0.00
time (sec)	N/A	0.545	0.482	0.137	0.208	0.273	0.000	0.276	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	257	217	199	466	0	239	0
N.S.	1	1.00	1.14	0.96	0.88	2.07	0.00	1.06	0.00
time (sec)	N/A	0.642	1.582	0.398	0.212	0.273	0.000	0.295	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	501	326	287	851	0	369	0
N.S.	1	1.00	1.55	1.01	0.89	2.63	0.00	1.14	0.00
time (sec)	N/A	0.842	4.937	1.274	0.212	0.276	0.000	0.297	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	251	186	151	362	0	207	0
N.S.	1	1.00	1.56	1.16	0.94	2.25	0.00	1.29	0.00
time (sec)	N/A	0.635	1.073	0.164	0.211	0.270	0.000	0.293	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	339	249	215	516	0	271	0
N.S.	1	1.00	1.42	1.04	0.90	2.16	0.00	1.13	0.00
time (sec)	N/A	0.805	4.372	0.522	0.209	0.266	0.000	0.308	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	344	2991	384	315	939	0	427	0
N.S.	1	1.00	8.69	1.12	0.92	2.73	0.00	1.24	0.00
time (sec)	N/A	1.060	6.463	1.617	0.215	0.278	0.000	0.321	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	B
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	46	0	0	0	0	0	39
N.S.	1	1.00	2.30	0.00	0.00	0.00	0.00	0.00	1.95
time (sec)	N/A	0.198	0.246	0.000	0.000	0.000	0.000	0.000	0.172

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	16	0	0	109	0	0	42
N.S.	1	1.00	0.67	0.00	0.00	4.54	0.00	0.00	1.75
time (sec)	N/A	0.199	0.033	0.000	0.000	0.258	0.000	0.000	1.736

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	B
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	47	47	64	0	0	0	0	0	110
N.S.	1	1.00	1.36	0.00	0.00	0.00	0.00	0.00	2.34
time (sec)	N/A	0.219	0.411	0.000	0.000	0.000	0.000	0.000	1.914

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	36	36	76	0	0	0	0	0	0
N.S.	1	1.00	2.11	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.236	0.112	0.000	0.000	0.000	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	26	25	36	23	41	36	24
N.S.	1	1.00	0.87	0.83	1.20	0.77	1.37	1.20	0.80
time (sec)	N/A	0.220	0.049	0.107	0.192	0.242	0.063	0.264	0.053

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	51	48	81	54	85	75	48
N.S.	1	1.00	0.91	0.86	1.45	0.96	1.52	1.34	0.86
time (sec)	N/A	0.269	0.063	0.172	0.200	0.265	0.093	0.262	0.079

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	166	212	0	316	0	0	0
N.S.	1	1.00	0.78	1.00	0.00	1.48	0.00	0.00	0.00
time (sec)	N/A	0.724	0.397	0.254	0.000	0.262	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	220	376	0	671	0	0	0
N.S.	1	1.00	0.81	1.39	0.00	2.48	0.00	0.00	0.00
time (sec)	N/A	0.987	0.606	0.382	0.000	0.267	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [85] had the largest ratio of [1.8000000000000004]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	6	0.333
2	A	3	3	1.00	8	0.375
3	C	4	3	1.23	8	0.375
4	A	5	5	1.11	8	0.625
5	C	4	3	1.12	8	0.375
6	A	7	7	1.15	8	0.875
7	A	6	6	1.07	10	0.600
8	A	4	4	1.00	10	0.400
9	A	4	4	1.00	10	0.400
10	A	2	2	1.00	10	0.200
11	A	2	2	1.00	10	0.200
12	A	4	4	1.00	10	0.400
13	A	4	4	1.00	10	0.400
14	A	6	6	1.01	10	0.600
15	A	8	8	1.09	8	1.000
16	A	6	6	1.00	8	0.750
17	A	6	6	1.00	8	0.750
18	A	4	4	1.00	8	0.500
19	A	4	4	1.00	8	0.500
20	A	6	6	1.00	8	0.750
21	A	6	6	1.00	8	0.750
22	A	8	8	1.06	8	1.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	2	2	1.00	10	0.200
24	C	10	9	1.15	13	0.692
25	A	4	4	1.00	13	0.308
26	A	7	7	0.96	13	0.538
27	A	4	4	1.00	11	0.364
28	A	5	5	1.00	11	0.455
29	A	10	9	1.04	13	0.692
30	A	12	11	1.02	13	0.846
31	C	12	11	1.12	13	0.846
32	A	2	2	1.00	10	0.200
33	A	4	4	1.00	10	0.400
34	A	6	6	1.07	10	0.600
35	A	8	8	1.11	10	0.800
36	A	2	2	1.00	12	0.167
37	A	4	4	1.00	12	0.333
38	A	6	6	1.07	12	0.500
39	A	8	8	1.10	12	0.667
40	A	6	5	1.00	13	0.385
41	A	6	5	1.00	14	0.357
42	A	6	6	1.04	14	0.429
43	A	4	4	1.00	14	0.286
44	A	2	2	1.00	14	0.143
45	A	4	3	1.00	14	0.214
46	A	6	5	1.00	14	0.357
47	A	8	7	1.05	14	0.500
48	A	6	6	1.04	15	0.400
49	A	4	4	1.00	15	0.267
50	A	2	2	1.00	15	0.133
51	A	4	3	1.00	15	0.200
52	A	6	5	1.00	15	0.333
53	A	8	7	1.05	15	0.467
54	A	14	13	1.15	13	1.000
55	A	10	9	1.12	13	0.692
56	A	10	9	1.05	13	0.692

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	6	5	1.00	11	0.455
58	A	7	6	1.00	11	0.545
59	A	11	10	1.05	13	0.769
60	A	13	12	1.13	13	0.923
61	A	16	15	1.15	13	1.154
62	A	8	8	1.07	12	0.667
63	A	6	6	1.06	12	0.500
64	A	4	4	1.03	12	0.333
65	A	2	2	1.00	12	0.167
66	A	1	1	1.00	10	0.100
67	A	4	3	1.00	12	0.250
68	A	8	7	1.13	12	0.583
69	A	11	10	1.20	12	0.833
70	A	13	12	1.23	12	1.000
71	A	4	3	1.00	12	0.250
72	A	7	6	1.00	12	0.500
73	A	10	9	1.07	12	0.750
74	A	13	12	1.10	12	1.000
75	A	2	2	1.00	12	0.167
76	A	5	5	1.09	12	0.417
77	A	8	8	1.12	12	0.667
78	A	11	11	1.14	12	0.917
79	A	15	15	1.04	10	1.500
80	A	12	12	1.01	10	1.200
81	A	4	4	1.00	14	0.286
82	A	4	4	1.00	10	0.400
83	A	7	7	1.00	10	0.700
84	A	15	15	1.06	10	1.500
85	A	18	18	1.11	10	1.800
86	A	9	9	1.00	13	0.692
87	A	8	8	0.95	17	0.471
88	A	6	6	0.97	17	0.353
89	A	4	4	1.00	17	0.235
90	A	8	8	0.95	18	0.444

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	6	6	0.97	18	0.333
92	A	4	4	1.00	18	0.222
93	A	4	4	1.00	13	0.308
94	A	4	4	1.00	13	0.308
95	A	6	6	0.96	13	0.462
96	A	8	8	0.95	13	0.615
97	A	4	4	1.00	15	0.267
98	A	4	4	1.00	15	0.267
99	A	6	6	0.97	15	0.400
100	A	8	8	0.95	15	0.533
101	A	6	5	1.00	17	0.294
102	A	6	5	1.00	17	0.294
103	A	8	7	0.98	17	0.412
104	A	6	5	1.00	18	0.278
105	A	6	5	1.00	18	0.278
106	A	8	7	0.98	18	0.389
107	A	18	18	1.05	17	1.059
108	A	15	15	1.03	17	0.882
109	A	12	12	1.01	17	0.706
110	A	6	5	1.00	15	0.333
111	A	8	7	1.13	15	0.467
112	A	11	10	1.19	15	0.667
113	A	13	12	1.21	15	0.800
114	A	6	5	1.12	20	0.250
115	A	2	2	1.00	20	0.100
116	A	3	3	1.00	15	0.200
117	A	4	4	1.06	13	0.308
118	A	9	9	1.00	17	0.529
119	A	12	12	1.07	17	0.706
120	A	15	15	1.04	17	0.882
121	A	10	10	1.08	10	1.000
122	A	8	8	1.06	10	0.800
123	A	6	6	1.00	10	0.600
124	A	4	4	1.00	10	0.400

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
125	A	4	4	1.00	10	0.400
126	A	6	6	1.00	10	0.600
127	A	8	8	1.13	10	0.800
128	A	12	12	0.82	10	1.200
129	A	8	8	0.89	10	0.800
130	A	6	6	0.94	10	0.600
131	A	6	6	0.89	10	0.600
132	A	8	8	0.87	10	0.800
133	A	12	12	0.79	10	1.200
134	A	13	13	0.70	10	1.300
135	A	9	9	0.77	10	0.900
136	A	5	5	0.81	10	0.500
137	A	6	5	1.00	10	0.500
138	C	6	5	0.72	10	0.500
139	C	6	5	0.58	10	0.500
140	A	5	4	1.00	9	0.444
141	A	5	4	1.00	11	0.364
142	A	4	4	1.00	11	0.364
143	A	4	4	1.00	13	0.308
144	A	6	5	1.00	11	0.455
145	A	6	5	1.00	13	0.385
146	A	5	4	1.00	9	0.444
147	A	5	4	1.00	11	0.364
148	A	3	3	1.00	11	0.273
149	A	3	3	1.00	13	0.231
150	A	6	5	1.00	11	0.455
151	A	6	5	1.00	13	0.385
152	A	13	13	1.05	13	1.000
153	A	6	5	1.02	13	0.385
154	A	10	10	1.05	13	0.769
155	A	6	5	0.97	13	0.385
156	A	7	7	0.97	13	0.538
157	A	5	4	0.84	13	0.308
158	A	4	4	1.00	13	0.308

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
159	A	5	4	1.22	11	0.364
160	A	6	5	1.26	11	0.455
161	A	9	8	1.00	13	0.615
162	A	6	5	1.24	13	0.385
163	C	6	5	1.16	13	0.385
164	A	6	5	1.19	13	0.385
165	A	6	5	1.00	13	0.385
166	A	16	15	1.19	13	1.154
167	A	6	5	0.99	13	0.385
168	A	12	11	1.20	13	0.846
169	A	6	5	1.00	13	0.385
170	A	10	9	1.24	13	0.692
171	A	5	4	1.00	11	0.364
172	A	6	5	1.25	11	0.455
173	A	8	7	1.16	13	0.538
174	A	6	5	1.29	13	0.385
175	A	10	9	1.22	13	0.692
176	A	6	5	1.30	13	0.385
177	A	14	13	1.26	13	1.000
178	A	10	9	1.07	13	0.692
179	A	12	11	1.18	13	0.846
180	A	6	5	1.04	13	0.385
181	A	13	12	1.23	13	0.923
182	A	7	6	1.10	11	0.545
183	A	9	8	1.09	11	0.727
184	A	13	12	1.14	13	0.923
185	A	9	8	1.44	13	0.615
186	C	20	19	1.15	13	1.462
187	A	15	14	1.02	13	1.077
188	C	11	10	1.27	13	0.769
189	A	11	10	1.00	13	0.769
190	C	10	9	1.42	13	0.692
191	A	9	8	1.00	13	0.615
192	A	7	6	1.22	11	0.545

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
193	C	14	13	1.27	11	1.182
194	C	11	10	1.20	13	0.769
195	C	18	17	1.28	13	1.308
196	C	11	10	1.12	13	0.769
197	A	7	6	1.00	13	0.462
198	A	6	5	1.00	13	0.385
199	A	3	3	1.00	15	0.200
200	A	3	3	1.00	13	0.231
201	A	3	3	1.00	15	0.200
202	A	3	3	1.00	15	0.200
203	A	3	3	1.00	15	0.200
204	A	9	8	1.00	15	0.533
205	C	8	8	1.13	15	0.533
206	A	12	11	1.00	31	0.355
207	A	14	13	1.09	31	0.419
208	A	17	16	1.14	31	0.516
209	A	19	18	1.16	31	0.581
210	A	9	8	1.01	12	0.667
211	A	10	9	0.95	14	0.643
212	A	11	10	0.91	14	0.714
213	A	5	4	0.98	36	0.111
214	A	5	4	0.98	36	0.111
215	A	4	3	1.00	34	0.088
216	N/A	4	0	1.00	34	0.000
217	N/A	4	0	1.00	36	0.000
218	A	5	4	1.00	12	0.333
219	A	9	8	1.18	12	0.667
220	A	2	2	1.00	20	0.100
221	N/A	1	0	1.00	22	0.000
222	A	7	6	1.03	22	0.273
223	A	6	5	1.02	22	0.227
224	A	5	4	1.00	20	0.200
225	A	5	4	1.00	19	0.211

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
226	N/A	1	0	1.00	22	0.000
227	N/A	1	0	1.00	24	0.000
228	C	23	22	0.94	24	0.917
229	C	18	17	0.96	24	0.708
230	A	15	14	0.99	22	0.636
231	A	10	9	1.19	21	0.429
232	N/A	1	0	1.00	24	0.000
233	N/A	1	0	1.00	24	0.000
234	C	29	28	0.92	24	1.167
235	C	21	20	0.90	24	0.833
236	C	16	15	0.92	22	0.682
237	A	6	5	0.90	21	0.238
238	N/A	1	0	1.00	24	0.000
239	A	1	1	1.00	11	0.091
240	A	2	2	1.00	13	0.154
241	A	2	2	0.96	13	0.154
242	A	3	3	0.93	13	0.231
243	A	1	1	1.01	15	0.067
244	A	2	2	1.02	17	0.118
245	A	2	2	0.93	17	0.118
246	A	3	3	0.89	17	0.176
247	A	4	3	1.00	15	0.200
248	A	5	4	1.13	17	0.235
249	C	5	4	1.07	17	0.235
250	A	7	6	1.10	17	0.353
251	C	5	4	0.98	17	0.235
252	A	6	5	0.97	19	0.263
253	A	6	5	0.97	19	0.263
254	A	4	3	1.00	19	0.158
255	A	4	3	1.00	19	0.158
256	A	6	5	0.97	19	0.263
257	A	6	5	0.97	19	0.263
258	A	9	8	0.99	18	0.444

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
259	A	7	6	1.07	18	0.333
260	A	4	3	1.00	18	0.167
261	A	5	4	1.40	18	0.222
262	C	13	12	1.15	14	0.857
263	C	13	12	1.14	16	0.750
264	A	5	4	0.86	16	0.250
265	A	6	5	0.82	16	0.312
266	A	5	4	0.88	16	0.250
267	A	5	4	1.30	14	0.286
268	A	4	3	1.00	14	0.214
269	A	5	4	1.08	16	0.250
270	A	4	3	1.00	16	0.188
271	A	7	6	1.12	16	0.375
272	A	6	5	1.00	16	0.312
273	A	5	4	1.08	10	0.400
274	A	5	4	1.11	8	0.500
275	A	11	10	1.13	8	1.250
276	A	12	11	1.13	10	1.100
277	A	5	4	1.08	10	0.400
278	A	5	4	1.21	8	0.500
279	A	11	10	0.64	8	1.250
280	A	12	11	1.10	10	1.100
281	A	5	4	1.08	10	0.400
282	A	5	4	1.21	8	0.500
283	A	11	10	1.05	8	1.250
284	A	11	10	1.03	10	1.000
285	A	2	2	0.94	18	0.111
286	A	2	2	1.00	18	0.111
287	A	1	1	1.00	16	0.062
288	A	1	1	1.00	16	0.062
289	A	1	1	1.00	18	0.056
290	A	2	2	1.12	18	0.111
291	A	2	2	1.11	18	0.111
292	A	7	6	0.41	25	0.240

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
293	A	7	6	0.46	25	0.240
294	A	6	5	0.80	25	0.200
295	A	5	4	1.00	25	0.160
296	A	5	4	1.00	25	0.160
297	A	7	6	0.62	25	0.240
298	A	7	6	0.55	25	0.240
299	A	1	1	1.00	10	0.100
300	A	2	2	1.00	12	0.167
301	A	2	2	1.00	15	0.133
302	A	2	2	1.00	12	0.167
303	A	2	2	1.00	14	0.143
304	A	2	2	1.00	17	0.118
305	A	2	2	1.00	16	0.125
306	A	2	2	1.00	18	0.111
307	A	2	2	1.00	18	0.111
308	A	2	2	1.00	19	0.105
309	A	2	2	1.00	21	0.095
310	A	2	2	1.00	21	0.095
311	A	2	2	1.00	16	0.125
312	A	2	2	1.00	18	0.111
313	A	2	2	1.00	18	0.111
314	A	2	2	1.00	18	0.111
315	A	2	2	1.00	20	0.100
316	A	2	2	1.00	20	0.100
317	A	2	2	1.00	21	0.095
318	A	2	2	1.00	23	0.087
319	A	2	2	1.00	23	0.087
320	A	2	2	1.00	19	0.105
321	A	2	2	1.00	21	0.095
322	A	2	2	1.00	21	0.095
323	A	2	2	1.00	21	0.095
324	A	2	2	1.00	23	0.087
325	A	2	2	1.00	23	0.087
326	A	2	2	1.00	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
327	A	2	2	1.00	26	0.077
328	A	2	2	1.00	26	0.077
329	A	1	1	1.00	17	0.059
330	A	1	1	1.00	20	0.050
331	A	1	1	1.00	20	0.050
332	A	1	1	1.00	21	0.048
333	A	2	2	1.00	6	0.333
334	A	2	2	1.00	6	0.333
335	A	2	2	1.00	16	0.125
336	A	2	2	1.00	19	0.105

CHAPTER 3

Listing of integrals

3.1 $\int \cosh(a+bx)\,dx$

3.1.1 Optimal result

Integrand size = 6, antiderivative size = 10

$$\int \cosh(a+bx)\,dx=\frac{\sinh(a+bx)}{b}$$

output
```
sinh(b*x+a)/b
```

3.1.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.10

$$\int \cosh(a+bx)\,dx=\frac{\cosh(bx)\sinh(a)}{b}+\frac{\cosh(a)\sinh(bx)}{b}$$

input
```
Integrate[Cosh[a + b*x],x]
```

output
```
(Cosh[b*x]*Sinh[a])/b + (Cosh[a]*Sinh[b*x])/b
```

3.1.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \; 3117$$

$$\frac{\sinh(a+bx)}{b}$$

input
```
Int[Cosh[a + b*x],x]
```

output
```
Sinh[a + b*x]/b
```

3.1.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

3.1.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{\sinh(bx+a)}{b}$	11
default	$\frac{\sinh(bx+a)}{b}$	11
parallelrisch	$\frac{\sinh(bx+a)}{b}$	11
risch	$\frac{e^{bx+a}}{2b}-\frac{e^{-bx-a}}{2b}$	27
meijerg	$\frac{\cosh(a)\sinh(bx)}{b}-\frac{\sinh(a)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh(bx)}{\sqrt{\pi}}\right)}{b}$	35

input
```
int(cosh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
sinh(b*x+a)/b
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \cosh(a+bx)\,dx=\frac{\sinh(bx+a)}{b}$$

input
```
integrate(cosh(b*x+a),x, algorithm="fricas")
```

output
```
sinh(b*x + a)/b
```

3.1.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \cosh(a+bx)\,dx=\begin{cases}\frac{\sinh(a+bx)}{b} & \text{for } b\neq 0\\ x\cosh(a) & \text{otherwise}\end{cases}$$

input
```
integrate(cosh(b*x+a),x)
```

output
```
Piecewise((sinh(a + b*x)/b, Ne(b, 0)), (x*cosh(a), True))
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \cosh(a+bx)\,dx = \frac{\sinh\left(bx+a\right)}{b}$$

input
```
integrate(cosh(b*x+a),x, algorithm="maxima")
```

output
```
sinh(b*x + a)/b
```

3.1.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(10) = 20.

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.60

$$\int \cosh(a+bx)\,dx = \frac{e^{(bx+a)}}{2\,b} - \frac{e^{(-bx-a)}}{2\,b}$$

input
```
integrate(cosh(b*x+a),x, algorithm="giac")
```

output
```
1/2*e^(b*x + a)/b - 1/2*e^(-b*x - a)/b
```

3.1.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \cosh(a+bx)\,dx = \frac{\sinh(a+b\,x)}{b}$$

input
```
int(cosh(a + b*x),x)
```

output
```
sinh(a + b*x)/b
```

3.2 $\int \cosh^2(a+bx)\,dx$

3.2.1 Optimal result

Integrand size = 8, antiderivative size = 25

$$\int \cosh^2(a+bx)\,dx = \frac{x}{2}+\frac{\cosh(a+bx)\sinh(a+bx)}{2b}$$

output
```
1/2*x+1/2*cosh(b*x+a)*sinh(b*x+a)/b
```

3.2.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \cosh^2(a+bx)\,dx = \frac{2(a+bx)+\sinh(2(a+bx))}{4b}$$

input
```
Integrate[Cosh[a + b*x]^2,x]
```

output
```
(2*(a + b*x) + Sinh[2*(a + b*x)])/(4*b)
```

3.2.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \; 3115$$

$$\frac{\int 1dx}{2}+\frac{\sinh(a+bx)\cosh(a+bx)}{2b}$$

$$\Big\downarrow \; 24$$

$$\frac{\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}$$

input
```
Int[Cosh[a + b*x]^2,x]
```

output
```
x/2 + (Cosh[a + b*x]*Sinh[a + b*x])/(2*b)
```

3.2.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.2.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
parallelrisch	$\frac{2bx+\sinh(2bx+2a)}{4b}$	20
derivativedivides	$\frac{\frac{\cosh(bx+a)\sinh(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}}{b}$	27
default	$\frac{\frac{\cosh(bx+a)\sinh(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}}{b}$	27
risch	$\frac{x}{2}+\frac{e^{2bx+2a}}{8b}-\frac{e^{-2bx-2a}}{8b}$	33

input
```
int(cosh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*b*x+sinh(2*b*x+2*a))/b
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \cosh^2(a+bx)\,dx = \frac{bx+\cosh(bx+a)\sinh(bx+a)}{2\,b}$$

input
```
integrate(cosh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(b*x + cosh(b*x + a)*sinh(b*x + a))/b
```

3.2.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(19) = 38.

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \cosh^2(a+bx)\,dx = \begin{cases} -\frac{x\sinh^2(a+bx)}{2}+\frac{x\cosh^2(a+bx)}{2}+\frac{\sinh(a+bx)\cosh(a+bx)}{2b} & \text{for } b\neq 0 \\ x\cosh^2(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(b*x+a)**2,x)
```

output
```
Piecewise((-x*sinh(a + b*x)**2/2 + x*cosh(a + b*x)**2/2 + sinh(a + b*x)*co
sh(a + b*x)/(2*b), Ne(b, 0)), (x*cosh(a)**2, True))
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \cosh^2(a+bx)\,dx = \frac{1}{2}\,x + \frac{e^{(2\,bx+2\,a)}}{8\,b} - \frac{e^{(-2\,bx-2\,a)}}{8\,b}$$

input
```
integrate(cosh(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*x + 1/8*e^(2*b*x + 2*a)/b - 1/8*e^(-2*b*x - 2*a)/b
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \cosh^2(a+bx)\,dx = \frac{1}{2}\,x + \frac{e^{(2\,bx+2\,a)}}{8\,b} - \frac{e^{(-2\,bx-2\,a)}}{8\,b}$$

input
```
integrate(cosh(b*x+a)^2,x, algorithm="giac")
```

output
```
1/2*x + 1/8*e^(2*b*x + 2*a)/b - 1/8*e^(-2*b*x - 2*a)/b
```

3.2.9 Mupad [B] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \cosh^2(a+bx)\,dx = \frac{x}{2} + \frac{\sinh(2\,a+2\,b\,x)}{4\,b}$$

input
```
int(cosh(a + b*x)^2,x)
```

output
```
x/2 + sinh(2*a + 2*b*x)/(4*b)
```

3.3 $\int \cosh^3(a + bx)\, dx$

3.3.1 Optimal result

Integrand size = 8, antiderivative size = 26

$$\int \cosh^3(a + bx)\, dx = \frac{\sinh(a + bx)}{b} + \frac{\sinh^3(a + bx)}{3b}$$

output
```
sinh(b*x+a)/b+1/3*sinh(b*x+a)^3/b
```

3.3.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \cosh^3(a + bx)\, dx = \frac{\sinh(a + bx)}{b} + \frac{\sinh^3(a + bx)}{3b}$$

input
```
Integrate[Cosh[a + b*x]^3,x]
```

output
```
Sinh[a + b*x]/b + Sinh[a + b*x]^3/(3*b)
```

3.3.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^3 dx$$

$$\Big\downarrow \text{3113}$$

$$\frac{i\int \left(\sinh^2(a+bx)+1\right)d(-i\sinh(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\left(-\frac{1}{3}i\sinh^3(a+bx)-i\sinh(a+bx)\right)}{b}$$

input
```
Int[Cosh[a + b*x]^3,x]
```

output
```
(I*((-I)*Sinh[a + b*x] - (I/3)*Sinh[a + b*x]^3))/b
```

3.3.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

3.3.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\left(\frac{2}{3}+\frac{\cosh(bx+a)^2}{3}\right)\sinh(bx+a)}{b}$	23
default	$\frac{\left(\frac{2}{3}+\frac{\cosh(bx+a)^2}{3}\right)\sinh(bx+a)}{b}$	23
parallelrisch	$\frac{\sinh(3bx+3a)+9\sinh(bx+a)}{12b}$	24
risch	$\frac{e^{3bx+3a}}{24b}+\frac{3\,e^{bx+a}}{8b}-\frac{3\,e^{-bx-a}}{8b}-\frac{e^{-3bx-3a}}{24b}$	55

input
```
int(cosh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(2/3+1/3*cosh(b*x+a)^2)*sinh(b*x+a)
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int \cosh^3(a+bx)\,dx=\frac{\sinh\left(bx+a\right)^3+3\left(\cosh\left(bx+a\right)^2+3\right)\sinh\left(bx+a\right)}{12\,b}$$

input
```
integrate(cosh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/12*(sinh(b*x + a)^3 + 3*(cosh(b*x + a)^2 + 3)*sinh(b*x + a))/b
```

3.3.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.38

$$\int \cosh^3(a+bx)\,dx=\begin{cases}-\frac{2\sinh^3\left(a+bx\right)}{3b}+\frac{\sinh\left(a+bx\right)\cosh^2\left(a+bx\right)}{b} & \text{for } b\neq 0\\ x\cosh^3\left(a\right) & \text{otherwise}\end{cases}$$

input
```
integrate(cosh(b*x+a)**3,x)
```

output
```
Piecewise((-2*sinh(a + b*x)**3/(3*b) + sinh(a + b*x)*cosh(a + b*x)**2/b, N
e(b, 0)), (x*cosh(a)**3, True))
```

3.3.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(24) = 48$.

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.08

$$\int \cosh^3(a+bx)\,dx = \frac{e^{(3\,bx+3\,a)}}{24\,b} + \frac{3\,e^{(bx+a)}}{8\,b} - \frac{3\,e^{(-bx-a)}}{8\,b} - \frac{e^{(-3\,bx-3\,a)}}{24\,b}$$

input
```
integrate(cosh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/24*e^(3*b*x + 3*a)/b + 3/8*e^(b*x + a)/b - 3/8*e^(-b*x - a)/b - 1/24*e^(
-3*b*x - 3*a)/b
```

3.3.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(24) = 48$.

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.08

$$\int \cosh^3(a+bx)\,dx = \frac{e^{(3\,bx+3\,a)}}{24\,b} + \frac{3\,e^{(bx+a)}}{8\,b} - \frac{3\,e^{(-bx-a)}}{8\,b} - \frac{e^{(-3\,bx-3\,a)}}{24\,b}$$

input
```
integrate(cosh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/24*e^(3*b*x + 3*a)/b + 3/8*e^(b*x + a)/b - 3/8*e^(-b*x - a)/b - 1/24*e^(
-3*b*x - 3*a)/b
```

3.3.9 Mupad [B] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \cosh^3(a+bx)\,dx = \frac{\sinh(a+b\,x)^3+3\sinh(a+b\,x)}{3\,b}$$

input
```
int(cosh(a + b*x)^3,x)
```

output
```
(3*sinh(a + b*x) + sinh(a + b*x)^3)/(3*b)
```

3.4 $\int \cosh^4(a+bx)\,dx$

3.4.1 Optimal result

Integrand size = 8, antiderivative size = 46

$$\int \cosh^4(a+bx)\,dx=\frac{3x}{8}+\frac{3\cosh(a+bx)\sinh(a+bx)}{8b}+\frac{\cosh^3(a+bx)\sinh(a+bx)}{4b}$$

output
```
3/8*x+3/8*cosh(b*x+a)*sinh(b*x+a)/b+1/4*cosh(b*x+a)^3*sinh(b*x+a)/b
```

3.4.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \cosh^4(a+bx)\,dx=\frac{12(a+bx)+8\sinh(2(a+bx))+\sinh(4(a+bx))}{32b}$$

input
```
Integrate[Cosh[a + b*x]^4,x]
```

output
```
(12*(a + b*x) + 8*Sinh[2*(a + b*x)] + Sinh[4*(a + b*x)])/(32*b)
```

3.4.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^4 dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{3}{4}\int \cosh^2(a+bx)dx+\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)+\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}\right)$$

input
```
Int[Cosh[a + b*x]^4,x]
```

output
```
(Cosh[a + b*x]^3*Sinh[a + b*x])/(4*b) + (3*(x/2 + (Cosh[a + b*x]*Sinh[a +
b*x])/(2*b)))/4
```

3.4.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.4.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$\frac{12bx+8\sinh(2bx+2a)+\sinh(4bx+4a)}{32b}$	31
derivativedivides	$\frac{\left(\frac{\cosh(bx+a)^3}{4}+\frac{3\cosh(bx+a)}{8}\right)\sinh(bx+a)+\frac{3bx}{8}+\frac{3a}{8}}{b}$	39
default	$\frac{\left(\frac{\cosh(bx+a)^3}{4}+\frac{3\cosh(bx+a)}{8}\right)\sinh(bx+a)+\frac{3bx}{8}+\frac{3a}{8}}{b}$	39
risch	$\frac{3x}{8}+\frac{e^{4bx+4a}}{64b}+\frac{e^{2bx+2a}}{8b}-\frac{e^{-2bx-2a}}{8b}-\frac{e^{-4bx-4a}}{64b}$	61

input
```
int(cosh(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/32*(12*b*x+8*sinh(2*b*x+2*a)+sinh(4*b*x+4*a))/b
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \cosh^4(a+bx)\,dx = \frac{\cosh(bx+a)\sinh(bx+a)^3 + 3\,bx + \left(\cosh(bx+a)^3 + 4\,\cosh(bx+a)\right)\sinh(bx+a)}{8\,b}$$

input
```
integrate(cosh(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/8*(cosh(b*x + a)*sinh(b*x + a)^3 + 3*b*x + (cosh(b*x + a)^3 + 4*cosh(b*x
 + a))*sinh(b*x + a))/b
```

3.4.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(41) = 82.

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.07

$$\int \cosh^4(a+bx)\,dx = \begin{cases} \frac{3x\sinh^4(a+bx)}{8} - \frac{3x\sinh^2(a+bx)\cosh^2(a+bx)}{4} + \frac{3x\cosh^4(a+bx)}{8} - \frac{3\sinh^3(a+bx)\cosh(a+bx)}{8b} + \frac{5\sinh(a+bx)\cosh^3(a+bx)}{8b} & \text{for} \\ x\cosh^4(a) & \text{oth} \end{cases}$$

input
```
integrate(cosh(b*x+a)**4,x)
```

output
```
Piecewise((3*x*sinh(a + b*x)**4/8 - 3*x*sinh(a + b*x)**2*cosh(a + b*x)**2/
4 + 3*x*cosh(a + b*x)**4/8 - 3*sinh(a + b*x)**3*cosh(a + b*x)/(8*b) + 5*si
nh(a + b*x)*cosh(a + b*x)**3/(8*b), Ne(b, 0)), (x*cosh(a)**4, True))
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int \cosh^4(a+bx)\,dx = \frac{3}{8}x + \frac{e^{(4\,bx+4\,a)}}{64\,b} + \frac{e^{(2\,bx+2\,a)}}{8\,b} - \frac{e^{(-2\,bx-2\,a)}}{8\,b} - \frac{e^{(-4\,bx-4\,a)}}{64\,b}$$

input
```
integrate(cosh(b*x+a)^4,x, algorithm="maxima")
```

output
```
3/8*x + 1/64*e^(4*b*x + 4*a)/b + 1/8*e^(2*b*x + 2*a)/b - 1/8*e^(-2*b*x - 2
*a)/b - 1/64*e^(-4*b*x - 4*a)/b
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int \cosh^4(a+bx)\,dx = \frac{3}{8}x + \frac{e^{(4\,bx+4\,a)}}{64\,b} + \frac{e^{(2\,bx+2\,a)}}{8\,b} - \frac{e^{(-2\,bx-2\,a)}}{8\,b} - \frac{e^{(-4\,bx-4\,a)}}{64\,b}$$

input
```
integrate(cosh(b*x+a)^4,x, algorithm="giac")
```

output
```
3/8*x + 1/64*e^(4*b*x + 4*a)/b + 1/8*e^(2*b*x + 2*a)/b - 1/8*e^(-2*b*x - 2
*a)/b - 1/64*e^(-4*b*x - 4*a)/b
```

3.4.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

$$\int \cosh^4(a+bx)\,dx = \frac{3\,x}{8} + \frac{\frac{\sinh(2\,a+2\,b\,x)}{4} + \frac{\sinh(4\,a+4\,b\,x)}{32}}{b}$$

input
```
int(cosh(a + b*x)^4,x)
```

output
```
(3*x)/8 + (sinh(2*a + 2*b*x)/4 + sinh(4*a + 4*b*x)/32)/b
```

3.5 $\int \cosh^5(a+bx)\,dx$

3.5.1 Optimal result

Integrand size = 8, antiderivative size = 41

$$\int \cosh^5(a+bx)\,dx=\frac{\sinh(a+bx)}{b}+\frac{2\sinh^3(a+bx)}{3b}+\frac{\sinh^5(a+bx)}{5b}$$

output
```
sinh(b*x+a)/b+2/3*sinh(b*x+a)^3/b+1/5*sinh(b*x+a)^5/b
```

3.5.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \cosh^5(a+bx)\,dx=\frac{\sinh(a+bx)}{b}+\frac{2\sinh^3(a+bx)}{3b}+\frac{\sinh^5(a+bx)}{5b}$$

input
```
Integrate[Cosh[a + b*x]^5,x]
```

output
```
Sinh[a + b*x]/b + (2*Sinh[a + b*x]^3)/(3*b) + Sinh[a + b*x]^5/(5*b)
```

3.5.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^5(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^5 dx$$

$$\Big\downarrow \text{3113}$$

$$\frac{i\int \left(\sinh^4(a+bx)+2\sinh^2(a+bx)+1\right)d(-i\sinh(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\left(-\frac{1}{5}i\sinh^5(a+bx)-\frac{2}{3}i\sinh^3(a+bx)-i\sinh(a+bx)\right)}{b}$$

input
```
Int[Cosh[a + b*x]^5,x]
```

output
```
(I*((-I)*Sinh[a + b*x] - ((2*I)/3)*Sinh[a + b*x]^3 - (I/5)*Sinh[a + b*x]^5
))/b
```

3.5.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

3.5.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{\left(\frac{8}{15}+\frac{\cosh(bx+a)^4}{5}+\frac{4\cosh(bx+a)^2}{15}\right)\sinh(bx+a)}{b}$	33
default	$\frac{\left(\frac{8}{15}+\frac{\cosh(bx+a)^4}{5}+\frac{4\cosh(bx+a)^2}{15}\right)\sinh(bx+a)}{b}$	33
parallelrisch	$\frac{25\sinh(3bx+3a)+150\sinh(bx+a)+3\sinh(5bx+5a)}{240b}$	37
risch	$\frac{e^{5bx+5a}}{160b}+\frac{5\,e^{3bx+3a}}{96b}+\frac{5\,e^{bx+a}}{16b}-\frac{5\,e^{-bx-a}}{16b}-\frac{5\,e^{-3bx-3a}}{96b}-\frac{e^{-5bx-5a}}{160b}$	83

input
```
int(cosh(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*(8/15+1/5*cosh(b*x+a)^4+4/15*cosh(b*x+a)^2)*sinh(b*x+a)
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.61

$$\int \cosh^5(a+bx)\,dx$$
$$=\frac{3\,\sinh\left(bx+a\right)^5+5\,\left(6\,\cosh\left(bx+a\right)^2+5\right)\sinh\left(bx+a\right)^3+15\,\left(\cosh\left(bx+a\right)^4+5\,\cosh\left(bx+a\right)^2+10\right)\mathrm{s}}{240\,b}$$

input
```
integrate(cosh(b*x+a)^5,x, algorithm="fricas")
```

output
```
1/240*(3*sinh(b*x + a)^5 + 5*(6*cosh(b*x + a)^2 + 5)*sinh(b*x + a)^3 + 15*
(cosh(b*x + a)^4 + 5*cosh(b*x + a)^2 + 10)*sinh(b*x + a))/b
```

3.5.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.41

$$\int \cosh^5(a+bx)\,dx = \begin{cases} \frac{8\sinh^5(a+bx)}{15b} - \frac{4\sinh^3(a+bx)\cosh^2(a+bx)}{3b} + \frac{\sinh(a+bx)\cosh^4(a+bx)}{b} & \text{for } b \neq 0 \\ x\cosh^5(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(b*x+a)**5,x)
```

output
```
Piecewise((8*sinh(a + b*x)**5/(15*b) - 4*sinh(a + b*x)**3*cosh(a + b*x)**2
/(3*b) + sinh(a + b*x)*cosh(a + b*x)**4/b, Ne(b, 0)), (x*cosh(a)**5, True)
)
```

3.5.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(37) = 74.

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.00

$$\int \cosh^5(a+bx)\,dx = \frac{e^{(5\,bx+5\,a)}}{160\,b} + \frac{5\,e^{(3\,bx+3\,a)}}{96\,b} + \frac{5\,e^{(bx+a)}}{16\,b} - \frac{5\,e^{(-bx-a)}}{16\,b} - \frac{5\,e^{(-3\,bx-3\,a)}}{96\,b} - \frac{e^{(-5\,bx-5\,a)}}{160\,b}$$

input
```
integrate(cosh(b*x+a)^5,x, algorithm="maxima")
```

output
```
1/160*e^(5*b*x + 5*a)/b + 5/96*e^(3*b*x + 3*a)/b + 5/16*e^(b*x + a)/b - 5/
16*e^(-b*x - a)/b - 5/96*e^(-3*b*x - 3*a)/b - 1/160*e^(-5*b*x - 5*a)/b
```

3.5.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(37) = 74.

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.00

$$\int \cosh^5(a+bx)\,dx = \frac{e^{(5\,bx+5\,a)}}{160\,b} + \frac{5\,e^{(3\,bx+3\,a)}}{96\,b} + \frac{5\,e^{(bx+a)}}{16\,b} - \frac{5\,e^{(-bx-a)}}{16\,b} - \frac{5\,e^{(-3\,bx-3\,a)}}{96\,b} - \frac{e^{(-5\,bx-5\,a)}}{160\,b}$$

input
```
integrate(cosh(b*x+a)^5,x, algorithm="giac")
```

output
```
1/160*e^(5*b*x + 5*a)/b + 5/96*e^(3*b*x + 3*a)/b + 5/16*e^(b*x + a)/b - 5/
16*e^(-b*x - a)/b - 5/96*e^(-3*b*x - 3*a)/b - 1/160*e^(-5*b*x - 5*a)/b
```

3.5.9 Mupad [B] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \cosh^5(a+bx)\,dx = \frac{\frac{\sinh(a+b\,x)^5}{5}+\frac{2\sinh(a+b\,x)^3}{3}+\sinh(a+b\,x)}{b}$$

input
```
int(cosh(a + b*x)^5,x)
```

output
```
(sinh(a + b*x) + (2*sinh(a + b*x)^3)/3 + sinh(a + b*x)^5/5)/b
```

3.6 $\int \cosh^6(a+bx)\,dx$

3.6.1 Optimal result

Integrand size = 8, antiderivative size = 67

$$\int \cosh^6(a+bx)\,dx = \frac{5x}{16} + \frac{5\cosh(a+bx)\sinh(a+bx)}{16b} + \frac{5\cosh^3(a+bx)\sinh(a+bx)}{24b} + \frac{\cosh^5(a+bx)\sinh(a+bx)}{6b}$$

output
```
5/16*x+5/16*cosh(b*x+a)*sinh(b*x+a)/b+5/24*cosh(b*x+a)^3*sinh(b*x+a)/b+1/6
*cosh(b*x+a)^5*sinh(b*x+a)/b
```

3.6.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.64

$$\int \cosh^6(a+bx)\,dx = \frac{60a + 60bx + 45\sinh(2(a+bx)) + 9\sinh(4(a+bx)) + \sinh(6(a+bx))}{192b}$$

input
```
Integrate[Cosh[a + b*x]^6,x]
```

output
```
(60*a + 60*b*x + 45*Sinh[2*(a + b*x)] + 9*Sinh[4*(a + b*x)] + Sinh[6*(a +
b*x)])/(192*b)
```

3.6.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^6(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^6dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{6}\int \cosh^4(a+bx)dx+\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}+\frac{5}{6}\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{6}\left(\frac{3}{4}\int \cosh^2(a+bx)dx+\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)+\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}+\frac{5}{6}\left(\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx\right)$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)+\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)+\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}+\frac{5}{6}\left(\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}\right)\right)$$

input
```
Int[Cosh[a + b*x]^6,x]
```

output
```
(Cosh[a + b*x]^5*Sinh[a + b*x])/(6*b) + (5*((Cosh[a + b*x]^3*Sinh[a + b*x]
)/(4*b) + (3*(x/2 + (Cosh[a + b*x]*Sinh[a + b*x])/(2*b)))/4))/6
```

3.6.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.6.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.63

method	result	size
parallelrisch	$\frac{60bx+45\sinh(2bx+2a)+9\sinh(4bx+4a)+\sinh(6bx+6a)}{192b}$	42
derivativedivides	$\frac{\left(\frac{\cosh(bx+a)^5}{6}+\frac{5\cosh(bx+a)^3}{24}+\frac{5\cosh(bx+a)}{16}\right)\sinh(bx+a)+\frac{5bx}{16}+\frac{5a}{16}}{b}$	49
default	$\frac{\left(\frac{\cosh(bx+a)^5}{6}+\frac{5\cosh(bx+a)^3}{24}+\frac{5\cosh(bx+a)}{16}\right)\sinh(bx+a)+\frac{5bx}{16}+\frac{5a}{16}}{b}$	49
risch	$\frac{5x}{16}+\frac{e^{6bx+6a}}{384b}+\frac{3e^{4bx+4a}}{128b}+\frac{15e^{2bx+2a}}{128b}-\frac{15e^{-2bx-2a}}{128b}-\frac{3e^{-4bx-4a}}{128b}-\frac{e^{-6bx-6a}}{384b}$	89

input
```
int(cosh(b*x+a)^6,x,method=_RETURNVERBOSE)
```

output
```
1/192*(60*b*x+45*sinh(2*b*x+2*a)+9*sinh(4*b*x+4*a)+sinh(6*b*x+6*a))/b
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.34

$$\int \cosh^6(a+bx)\,dx$$
$$= \frac{3\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^5+2\left(5\,\cosh\left(bx+a\right)^3+9\,\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)^3+30\,bx+3\left(\cosh\left(b\right.\right.}{96\,b}$$

input
```
integrate(cosh(b*x+a)^6,x, algorithm="fricas")
```

output
```
1/96*(3*cosh(b*x + a)*sinh(b*x + a)^5 + 2*(5*cosh(b*x + a)^3 + 9*cosh(b*x
+ a))*sinh(b*x + a)^3 + 30*b*x + 3*(cosh(b*x + a)^5 + 6*cosh(b*x + a)^3 +
15*cosh(b*x + a))*sinh(b*x + a))/b
```

3.6.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(61) = 122.

Time = 0.33 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.07

$$\int \cosh^6(a+bx)\,dx$$
$$= \begin{cases} -\frac{5x\sinh^6(a+bx)}{16}+\frac{15x\sinh^4(a+bx)\cosh^2(a+bx)}{16}-\frac{15x\sinh^2(a+bx)\cosh^4(a+bx)}{16}+\frac{5x\cosh^6(a+bx)}{16}+\frac{5\sinh^5(a+bx)\cosh(a+bx)}{16b} \\ x\cosh^6(a) \end{cases}$$

input
```
integrate(cosh(b*x+a)**6,x)
```

output
```
Piecewise((-5*x*sinh(a + b*x)**6/16 + 15*x*sinh(a + b*x)**4*cosh(a + b*x)*
*2/16 - 15*x*sinh(a + b*x)**2*cosh(a + b*x)**4/16 + 5*x*cosh(a + b*x)**6/1
6 + 5*sinh(a + b*x)**5*cosh(a + b*x)/(16*b) - 5*sinh(a + b*x)**3*cosh(a +
b*x)**3/(6*b) + 11*sinh(a + b*x)*cosh(a + b*x)**5/(16*b), Ne(b, 0)), (x*co
sh(a)**6, True))
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.28

$$\int \cosh^6(a+bx)\,dx = \frac{\left(9\,e^{(-2\,bx-2\,a)}+45\,e^{(-4\,bx-4\,a)}+1\right)e^{(6\,bx+6\,a)}}{384\,b}+\frac{5\,(bx+a)}{16\,b}-\frac{45\,e^{(-2\,bx-2\,a)}+9\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}}{384\,b}$$

input
```
integrate(cosh(b*x+a)^6,x, algorithm="maxima")
```

output
```
1/384*(9*e^(-2*b*x - 2*a) + 45*e^(-4*b*x - 4*a) + 1)*e^(6*b*x + 6*a)/b + 5
/16*(b*x + a)/b - 1/384*(45*e^(-2*b*x - 2*a) + 9*e^(-4*b*x - 4*a) + e^(-6*
b*x - 6*a))/b
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int \cosh^6(a+bx)\,dx = \frac{5}{16}\,x+\frac{e^{(6\,bx+6\,a)}}{384\,b}+\frac{3\,e^{(4\,bx+4\,a)}}{128\,b}+\frac{15\,e^{(2\,bx+2\,a)}}{128\,b}-\frac{15\,e^{(-2\,bx-2\,a)}}{128\,b}-\frac{3\,e^{(-4\,bx-4\,a)}}{128\,b}-\frac{e^{(-6\,bx-6\,a)}}{384\,b}$$

input
```
integrate(cosh(b*x+a)^6,x, algorithm="giac")
```

output
```
5/16*x + 1/384*e^(6*b*x + 6*a)/b + 3/128*e^(4*b*x + 4*a)/b + 15/128*e^(2*b
*x + 2*a)/b - 15/128*e^(-2*b*x - 2*a)/b - 3/128*e^(-4*b*x - 4*a)/b - 1/384
*e^(-6*b*x - 6*a)/b
```

3.6.9 Mupad [B] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.63

$$\int \cosh^6(a+bx)\,dx = \frac{5\,x}{16}+\frac{\frac{15\sinh(2\,a+2\,b\,x)}{64}+\frac{3\sinh(4\,a+4\,b\,x)}{64}+\frac{\sinh(6\,a+6\,b\,x)}{192}}{b}$$

input
```
int(cosh(a + b*x)^6,x)
```

output
```
(5*x)/16 + ((15*sinh(2*a + 2*b*x))/64 + (3*sinh(4*a + 4*b*x))/64 + sinh(6*
a + 6*b*x)/192)/b
```

3.7 $\int \cosh^{\frac{7}{2}}(a+bx)\,dx$

3.7.1 Optimal result

Integrand size = 10, antiderivative size = 69

$$\int \cosh^{\frac{7}{2}}(a+bx)\,dx = -\frac{10i\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{21b}+\frac{10\sqrt{\cosh(a+bx)}\sinh(a+bx)}{21b}+\frac{2\cosh^{\frac{5}{2}}(a+bx)\sinh(a+bx)}{7b}$$

output
```
-10/21*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticF(I*sin
h(1/2*a+1/2*b*x),2^(1/2))/b+2/7*cosh(b*x+a)^(5/2)*sinh(b*x+a)/b+10/21*sinh
(b*x+a)*cosh(b*x+a)^(1/2)/b
```

3.7.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.80

$$\int \cosh^{\frac{7}{2}}(a+bx)\,dx = \frac{-20i\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)+\sqrt{\cosh(a+bx)}(23\sinh(a+bx)+3\sinh(3(a+bx)))}{42b}$$

input
```
Integrate[Cosh[a + b*x]^(7/2),x]
```

output
```
((-20*I)*EllipticF[(I/2)*(a + b*x), 2] + Sqrt[Cosh[a + b*x]]*(23*Sinh[a +
b*x] + 3*Sinh[3*(a + b*x)]))/(42*b)
```

3.7.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3115, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^{\frac{7}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^{7/2}dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{7}\int \cosh^{\frac{3}{2}}(a+bx)dx+\frac{2\sinh(a+bx)\cosh^{\frac{5}{2}}(a+bx)}{7b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sinh(a+bx)\cosh^{\frac{5}{2}}(a+bx)}{7b}+\frac{5}{7}\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^{3/2}dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\cosh(a+bx)}}dx+\frac{2\sinh(a+bx)\sqrt{\cosh(a+bx)}}{3b}\right)+\frac{2\sinh(a+bx)\cosh^{\frac{5}{2}}(a+bx)}{7b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sinh(a+bx)\cosh^{\frac{5}{2}}(a+bx)}{7b}+\frac{5}{7}\left(\frac{2\sinh(a+bx)\sqrt{\cosh(a+bx)}}{3b}+\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(ia+ibx+\frac{\pi}{2}\right)}}dx\right)$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sinh(a+bx)\cosh^{\frac{5}{2}}(a+bx)}{7b}+\frac{5}{7}\left(\frac{2\sinh(a+bx)\sqrt{\cosh(a+bx)}}{3b}-\frac{2i\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{3b}\right)$$

input
```
Int[Cosh[a + b*x]^(7/2),x]
```

output
```
(2*Cosh[a + b*x]^(5/2)*Sinh[a + b*x])/(7*b) + (5*((((-2*I)/3)*EllipticF[(I
/2)*(a + b*x), 2])/b + (2*Sqrt[Cosh[a + b*x]]*Sinh[a + b*x])/(3*b)))/7
```

3.7.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.7.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(85) = 170$.

Time = 3.51 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.91

method	result
default	$\frac{2\sqrt{\left(-1+2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\left(48\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{9}-120\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{7}+128\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{5}-72\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{3}+5\sqrt{-s\cdots}\right.}{21\sqrt{2\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}+\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{-1+2\cdots}}$

input
```
int(cosh(b*x+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/21*((-1+2*cosh(1/2*b*x+1/2*a)^2)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(48*cosh(1
/2*b*x+1/2*a)^9-120*cosh(1/2*b*x+1/2*a)^7+128*cosh(1/2*b*x+1/2*a)^5-72*cos
h(1/2*b*x+1/2*a)^3+5*(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-2*cosh(1/2*b*x+1/2*a
)^2+1)^(1/2)*EllipticF(cosh(1/2*b*x+1/2*a),2^(1/2))+16*cosh(1/2*b*x+1/2*a)
)/(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2*b*x+1/2*a)^2)^(1/2)/sinh(1/2*b*x+1/2*a
)/(-1+2*cosh(1/2*b*x+1/2*a)^2)^(1/2)/b
```

3.7.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 326, normalized size of antiderivative = 4.72

$$\int \cosh^{\frac{7}{2}}(a+bx)\,dx$$
$$= \frac{40\left(\sqrt{2}\cosh(bx+a)^3 + 3\sqrt{2}\cosh(bx+a)^2\sinh(bx+a) + 3\sqrt{2}\cosh(bx+a)\sinh(bx+a)^2 + \sqrt{2}\sinh(\right.}{}$$

input
```
integrate(cosh(b*x+a)^(7/2),x, algorithm="fricas")
```

output
```
1/84*(40*(sqrt(2)*cosh(b*x + a)^3 + 3*sqrt(2)*cosh(b*x + a)^2*sinh(b*x + a
) + 3*sqrt(2)*cosh(b*x + a)*sinh(b*x + a)^2 + sqrt(2)*sinh(b*x + a)^3)*wei
erstrassPInverse(-4, 0, cosh(b*x + a) + sinh(b*x + a)) + (3*cosh(b*x + a)^
6 + 18*cosh(b*x + a)*sinh(b*x + a)^5 + 3*sinh(b*x + a)^6 + (45*cosh(b*x +
a)^2 + 23)*sinh(b*x + a)^4 + 23*cosh(b*x + a)^4 + 4*(15*cosh(b*x + a)^3 +
23*cosh(b*x + a))*sinh(b*x + a)^3 + (45*cosh(b*x + a)^4 + 138*cosh(b*x + a
)^2 - 23)*sinh(b*x + a)^2 - 23*cosh(b*x + a)^2 + 2*(9*cosh(b*x + a)^5 + 46
*cosh(b*x + a)^3 - 23*cosh(b*x + a))*sinh(b*x + a) - 3)*sqrt(cosh(b*x + a)
))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)^2*sinh(b*x + a) + 3*b*cosh(b*x +
 a)*sinh(b*x + a)^2 + b*sinh(b*x + a)^3)
```

3.7.6 Sympy [F(-1)]

Timed out.

$$\int \cosh^{\frac{7}{2}}(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(cosh(b*x+a)**(7/2),x)
```

output
```
Timed out
```

3.7.7 Maxima [F]

$$\int \cosh^{\frac{7}{2}}(a+bx)\,dx = \int \cosh\left(bx+a\right)^{\frac{7}{2}}\,dx$$

input
```
integrate(cosh(b*x+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(cosh(b*x + a)^(7/2), x)
```

3.7.8 Giac [F]

$$\int \cosh^{\frac{7}{2}}(a+bx)\,dx = \int \cosh\left(bx+a\right)^{\frac{7}{2}}\,dx$$

input
```
integrate(cosh(b*x+a)^(7/2),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^(7/2), x)
```

3.7.9 Mupad [F(-1)]

Timed out.

$$\int \cosh^{\frac{7}{2}}(a+bx)\,dx = \int \cosh(a+b\,x)^{7/2}\,dx$$

input
```
int(cosh(a + b*x)^(7/2),x)
```

output
```
int(cosh(a + b*x)^(7/2), x)
```

3.8 $\int \cosh^{\frac{5}{2}}(a+bx)\,dx$

3.8.1 Optimal result

Integrand size = 10, antiderivative size = 46

$$\int \cosh^{\frac{5}{2}}(a+bx)\,dx=-\frac{6iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{5b}+\frac{2\cosh^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{5b}$$

output
```
-6/5*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticE(I*sinh(
1/2*a+1/2*b*x),2^(1/2))/b+2/5*cosh(b*x+a)^(3/2)*sinh(b*x+a)/b
```

3.8.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \cosh^{\frac{5}{2}}(a+bx)\,dx=\frac{-6iE\left(\frac{1}{2}i(a+bx)\middle|2\right)+\sqrt{\cosh(a+bx)}\sinh(2(a+bx))}{5b}$$

input
```
Integrate[Cosh[a + b*x]^(5/2),x]
```

output
```
((-6*I)*EllipticE[(I/2)*(a + b*x), 2] + Sqrt[Cosh[a + b*x]]*Sinh[2*(a + b*
x)])/(5*b)
```

3.8.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3115, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^{\frac{5}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^{5/2}dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{3}{5}\int \sqrt{\cosh(a+bx)}dx+\frac{2\sinh(a+bx)\cosh^{\frac{3}{2}}(a+bx)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sinh(a+bx)\cosh^{\frac{3}{2}}(a+bx)}{5b}+\frac{3}{5}\int \sqrt{\sin\left(ia+ibx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\sinh(a+bx)\cosh^{\frac{3}{2}}(a+bx)}{5b}-\frac{6iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{5b}$$

input
```
Int[Cosh[a + b*x]^(5/2),x]
```

output
```
(((-6*I)/5)*EllipticE[(I/2)*(a + b*x), 2])/b + (2*Cosh[a + b*x]^(3/2)*Sinh
[a + b*x])/(5*b)
```

3.8.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.8.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(66) = 132$.

Time = 0.77 (sec) , antiderivative size = 188, normalized size of antiderivative = 4.09

method	result
default	$\frac{2\sqrt{\left(-1+2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\left(8\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{7}-16\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{5}+10\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{3}-3\sqrt{-\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\sqrt{-2\cos}\right.}{5\sqrt{2\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}+\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{-1+2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right.}}$

input
```
int(cosh(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*((-1+2*cosh(1/2*b*x+1/2*a)^2)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(8*cosh(1/2
*b*x+1/2*a)^7-16*cosh(1/2*b*x+1/2*a)^5+10*cosh(1/2*b*x+1/2*a)^3-3*(-sinh(1
/2*b*x+1/2*a)^2)^(1/2)*(-2*cosh(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cosh(1
/2*b*x+1/2*a),2^(1/2))-2*cosh(1/2*b*x+1/2*a))/(2*sinh(1/2*b*x+1/2*a)^4+sin
h(1/2*b*x+1/2*a)^2)^(1/2)/sinh(1/2*b*x+1/2*a)/(-1+2*cosh(1/2*b*x+1/2*a)^2)
^(1/2)/b
```

3.8.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 203, normalized size of antiderivative = 4.41

$$\int \cosh^{\frac{5}{2}}(a+bx)\,dx = -\frac{12\left(\sqrt{2}\cosh\left(bx+a\right)^{2}+2\sqrt{2}\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sqrt{2}\sinh\left(bx+a\right)^{2}\right)\text{weierstrassZeta}(-4,0,\text{w}}{}$$

input
```
integrate(cosh(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/10*(12*(sqrt(2)*cosh(b*x + a)^2 + 2*sqrt(2)*cosh(b*x + a)*sinh(b*x + a)
 + sqrt(2)*sinh(b*x + a)^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cosh(b*x + a) + sinh(b*x + a))) - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*s
inh(b*x + a)^3 + sinh(b*x + a)^4 + 6*(cosh(b*x + a)^2 - 2)*sinh(b*x + a)^2
 - 12*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - 6*cosh(b*x + a))*sinh(b*x + a
) - 1)*sqrt(cosh(b*x + a)))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*
x + a) + b*sinh(b*x + a)^2)
```

3.8.6 Sympy [F(-1)]

Timed out.

$$\int \cosh^{\frac{5}{2}}(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(cosh(b*x+a)**(5/2),x)
```

output
```
Timed out
```

3.8.7 Maxima [F]

$$\int \cosh^{\frac{5}{2}}(a+bx)\,dx = \int \cosh\left(bx+a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(cosh(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cosh(b*x + a)^(5/2), x)
```

3.8.8 Giac [F]

$$\int \cosh^{\frac{5}{2}}(a+bx)\,dx=\int \cosh\left(bx+a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(cosh(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^(5/2), x)
```

3.8.9 Mupad [F(-1)]

Timed out.

$$\int \cosh^{\frac{5}{2}}(a+bx)\,dx=\int \cosh(a+b\,x)^{5/2}\,dx$$

input
```
int(cosh(a + b*x)^(5/2),x)
```

output
```
int(cosh(a + b*x)^(5/2), x)
```

3.9 $\int \cosh^{\frac{3}{2}}(a+bx)\,dx$

3.9.1 Optimal result

Integrand size = 10, antiderivative size = 46

$$\int \cosh^{\frac{3}{2}}(a+bx)\,dx = -\frac{2i\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{3b}+\frac{2\sqrt{\cosh(a+bx)}\sinh(a+bx)}{3b}$$

output
```
-2/3*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticF(I*sinh(
1/2*a+1/2*b*x),2^(1/2))/b+2/3*sinh(b*x+a)*cosh(b*x+a)^(1/2)/b
```

3.9.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.76

$$\int \cosh^{\frac{3}{2}}(a+bx)\,dx = \frac{\sinh(2(a+bx)) + 2\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\cosh(2(a+bx))-\sinh(2(a+bx))\right)\sqrt{1+\cosh(2(a+bx))}}{3b\sqrt{\cosh(a+bx)}}$$

input
```
Integrate[Cosh[a + b*x]^(3/2),x]
```

output
```
(Sinh[2*(a + b*x)] + 2*Hypergeometric2F1[1/4, 1/2, 5/4, -Cosh[2*(a + b*x)]
 - Sinh[2*(a + b*x)]]*Sqrt[1 + Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)]])/(3*
b*Sqrt[Cosh[a + b*x]])
```

3.9.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^{\frac{3}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^{3/2}dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{3}\int \frac{1}{\sqrt{\cosh(a+bx)}}dx+\frac{2\sinh(a+bx)\sqrt{\cosh(a+bx)}}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sinh(a+bx)\sqrt{\cosh(a+bx)}}{3b}+\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(ia+ibx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sinh(a+bx)\sqrt{\cosh(a+bx)}}{3b}-\frac{2i\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{3b}$$

input
```
Int[Cosh[a + b*x]^(3/2),x]
```

output
```
(((-2*I)/3)*EllipticF[(I/2)*(a + b*x), 2])/b + (2*Sqrt[Cosh[a + b*x]]*Sinh
[a + b*x])/(3*b)
```

3.9.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.9.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(66) = 132$.

Time = 0.46 (sec) , antiderivative size = 174, normalized size of antiderivative = 3.78

method	result
default	$\frac{2\sqrt{\left(-1+2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,\left(4\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{5}-6\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{3}+\sqrt{-\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,\sqrt{-2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+1}\ \mathrm{Ellipti}}{3\sqrt{2\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}+\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{-1+2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,b}$

input
```
int(cosh(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*((-1+2*cosh(1/2*b*x+1/2*a)^2)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(4*cosh(1/2
*b*x+1/2*a)^5-6*cosh(1/2*b*x+1/2*a)^3+(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-2*c
osh(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticF(cosh(1/2*b*x+1/2*a),2^(1/2))+2*cos
h(1/2*b*x+1/2*a))/(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2*b*x+1/2*a)^2)^(1/2)/si
nh(1/2*b*x+1/2*a)/(-1+2*cosh(1/2*b*x+1/2*a)^2)^(1/2)/b
```

3.9.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.22

$$\int \cosh^{\frac{3}{2}}(a+bx)\,dx$$
$$= \frac{2\left(\sqrt{2}\cosh(bx+a)+\sqrt{2}\sinh(bx+a)\right)\text{weierstrassPInverse}(-4,0,\cosh(bx+a)+\sinh(bx+a))+(\cosh}{3\left(b\cosh(bx+a)+b\sinh(bx\right.}$$

input
```
integrate(cosh(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(2*(sqrt(2)*cosh(b*x + a) + sqrt(2)*sinh(b*x + a))*weierstrassPInverse
(-4, 0, cosh(b*x + a) + sinh(b*x + a)) + (cosh(b*x + a)^2 + 2*cosh(b*x + a
)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*sqrt(cosh(b*x + a)))/(b*cosh(b*x +
a) + b*sinh(b*x + a))
```

3.9.6 Sympy [F]

$$\int \cosh^{\frac{3}{2}}(a+bx)\,dx = \int \cosh^{\frac{3}{2}}(a+bx)\,dx$$

input
```
integrate(cosh(b*x+a)**(3/2),x)
```

output
```
Integral(cosh(a + b*x)**(3/2), x)
```

3.9.7 Maxima [F]

$$\int \cosh^{\frac{3}{2}}(a+bx)\,dx = \int \cosh(bx+a)^{\frac{3}{2}}\,dx$$

input
```
integrate(cosh(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cosh(b*x + a)^(3/2), x)
```

3.9.8 Giac [F]

$$\int \cosh^{\frac{3}{2}}(a+bx)\,dx = \int \cosh\left(bx+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(cosh(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^(3/2), x)
```

3.9.9 Mupad [F(-1)]

Timed out.

$$\int \cosh^{\frac{3}{2}}(a+bx)\,dx = \int \cosh(a+b\,x)^{3/2}\,dx$$

input
```
int(cosh(a + b*x)^(3/2),x)
```

output
```
int(cosh(a + b*x)^(3/2), x)
```

3.10 $\int \sqrt{\cosh(a+bx)}\,dx$

3.10.1 Optimal result

Integrand size = 10, antiderivative size = 20

$$\int \sqrt{\cosh(a+bx)}\,dx = -\frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

output
```
-2*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticE(I*sinh(1/
2*a+1/2*b*x),2^(1/2))/b
```

3.10.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{\cosh(a+bx)}\,dx = -\frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

input
```
Integrate[Sqrt[Cosh[a + b*x]],x]
```

output
```
((-2*I)*EllipticE[(I/2)*(a + b*x), 2])/b
```

3.10.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cosh(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(ia+ibx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3119}$$

$$-\frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

input
```
Int[Sqrt[Cosh[a + b*x]],x]
```

output
```
((-2*I)*EllipticE[(I/2)*(a + b*x), 2])/b
```

3.10.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.10.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(46) = 92.

Time = 0.38 (sec) , antiderivative size = 135, normalized size of antiderivative = 6.75

method	result
default	$-\frac{2\sqrt{\left(-1+2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,\sqrt{-\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,\sqrt{-2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+1}\;\operatorname{EllipticE}\left(\cosh\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)}{\sqrt{2\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}+\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{-1+2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,b}$
risch	$\frac{\sqrt{2}\,\sqrt{\left(1+e^{2bx+2a}\right)e^{-bx-a}}}{b}+\frac{\left(-\frac{2\left(1+e^{2bx+2a}\right)}{\sqrt{\left(1+e^{2bx+2a}\right)e^{bx+a}}}+\frac{i\sqrt{-i\left(e^{bx+a}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{bx+a}-i\right)}\,\sqrt{i e^{bx+a}}\,\left(-2i\,\operatorname{EllipticE}\left(\sqrt{-i\left(e^{bx+a}+i\right)},\frac{\sqrt{2}}{2}\right.\right.}{\sqrt{e^{3bx+3a}+e^{bx+a}}}\right.}{b\left(1+e^{2bx+2a}\right)}$

input
```
int(cosh(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*((-1+2*cosh(1/2*b*x+1/2*a)^2)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-sinh(1/2*b
*x+1/2*a)^2)^(1/2)*(-2*cosh(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cosh(1/2*b
*x+1/2*a),2^(1/2))/(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2*b*x+1/2*a)^2)^(1/2)/s
inh(1/2*b*x+1/2*a)/(-1+2*cosh(1/2*b*x+1/2*a)^2)^(1/2)/b
```

3.10.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \sqrt{\cosh(a+bx)}\,dx =$$

$$-\frac{2\left(\sqrt{2}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cosh\left(bx+a\right)+\sinh\left(bx+a\right)))+\sqrt{\cosh\left(bx+\right.}\right.}{b}$$

input
```
integrate(cosh(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-2*(sqrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(b*x + a
) + sinh(b*x + a))) + sqrt(cosh(b*x + a)))/b
```

3.10.6 Sympy [F]

$$\int \sqrt{\cosh(a+bx)}\,dx = \int \sqrt{\cosh{(a+bx)}}\,dx$$

input
```
integrate(cosh(b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(cosh(a + b*x)), x)
```

3.10.7 Maxima [F]

$$\int \sqrt{\cosh(a+bx)}\,dx = \int \sqrt{\cosh{(bx+a)}}\,dx$$

input
```
integrate(cosh(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(cosh(b*x + a)), x)
```

3.10.8 Giac [F]

$$\int \sqrt{\cosh(a+bx)}\,dx = \int \sqrt{\cosh{(bx+a)}}\,dx$$

input
```
integrate(cosh(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cosh(b*x + a)), x)
```

3.10.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cosh(a+bx)}\,dx = \int \sqrt{\cosh\left(a+b\,x\right)}\,dx$$

input
```
int(cosh(a + b*x)^(1/2),x)
```

output
```
int(cosh(a + b*x)^(1/2), x)
```

3.11 $\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx$

3.11.1 Optimal result

Integrand size = 10, antiderivative size = 20

$$\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx = -\frac{2i\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{b}$$

output
```
-2*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticF(I*sinh(1/
2*a+1/2*b*x),2^(1/2))/b
```

3.11.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx = -\frac{2i\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{b}$$

input
```
Integrate[1/Sqrt[Cosh[a + b*x]],x]
```

output
```
((-2*I)*EllipticF[(I/2)*(a + b*x), 2])/b
```

3.11.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cosh(a+bx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sqrt{\sin\left(ia+ibx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{3120}$$
$$-\frac{2i\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{b}$$

input
```
Int[1/Sqrt[Cosh[a + b*x]],x]
```

output
```
((-2*I)*EllipticF[(I/2)*(a + b*x), 2])/b
```

3.11.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.11.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(46) = 92.

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 6.75

method	result	size
default	$\frac{2\sqrt{\left(-1+2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,\sqrt{-\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,\sqrt{-2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+1}\,\mathrm{EllipticF}\left(\cosh\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)}{\sqrt{2\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}+\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{-1+2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,b}$	135

input
```
int(1/cosh(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*((-1+2*cosh(1/2*b*x+1/2*a)^2)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-sinh(1/2*b*
x+1/2*a)^2)^(1/2)*(-2*cosh(1/2*b*x+1/2*a)^2+1)^(1/2)/(2*sinh(1/2*b*x+1/2*a
)^4+sinh(1/2*b*x+1/2*a)^2)^(1/2)*EllipticF(cosh(1/2*b*x+1/2*a),2^(1/2))/si
nh(1/2*b*x+1/2*a)/(-1+2*cosh(1/2*b*x+1/2*a)^2)^(1/2)/b
```

3.11.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx = \frac{2\sqrt{2}\mathrm{weierstrassPInverse}(-4,0,\cosh(bx+a)+\sinh(bx+a))}{b}$$

input
```
integrate(1/cosh(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(2)*weierstrassPInverse(-4, 0, cosh(b*x + a) + sinh(b*x + a))/b
```

3.11.6 Sympy [F]

$$\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx = \int \frac{1}{\sqrt{\cosh{(a+bx)}}}\,dx$$

input
```
integrate(1/cosh(b*x+a)**(1/2),x)
```

output
```
Integral(1/sqrt(cosh(a + b*x)), x)
```

3.11.7 Maxima [F]

$$\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx = \int \frac{1}{\sqrt{\cosh{(bx+a)}}}\,dx$$

input
```
integrate(1/cosh(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(cosh(b*x + a)), x)
```

3.11.8 Giac [F]

$$\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx = \int \frac{1}{\sqrt{\cosh{(bx+a)}}}\,dx$$

input
```
integrate(1/cosh(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(cosh(b*x + a)), x)
```

3.11.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx = \int \frac{1}{\sqrt{\cosh\left(a+b\,x\right)}}\,dx$$

input
```
int(1/cosh(a + b*x)^(1/2),x)
```

output
```
int(1/cosh(a + b*x)^(1/2), x)
```

3.12 $\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}\,dx$

3.12.1 Optimal result

Integrand size = 10, antiderivative size = 42

$$\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = \frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b} + \frac{2\sinh(a+bx)}{b\sqrt{\cosh(a+bx)}}$$

output
```
2*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticE(I*sinh(1/2
*a+1/2*b*x),2^(1/2))/b+2*sinh(b*x+a)/b/cosh(b*x+a)^(1/2)
```

3.12.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = \frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b} + \frac{2\sinh(a+bx)}{b\sqrt{\cosh(a+bx)}}$$

input
```
Integrate[Cosh[a + b*x]^(-3/2),x]
```

output
```
((2*I)*EllipticE[(I/2)*(a + b*x), 2])/b + (2*Sinh[a + b*x])/(b*Sqrt[Cosh[a
 + b*x]])
```

3.12.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin\left(ia+ibx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3116}$$

$$\frac{2\sinh(a+bx)}{b\sqrt{\cosh(a+bx)}}-\int \sqrt{\cosh(a+bx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\sinh(a+bx)}{b\sqrt{\cosh(a+bx)}}-\int \sqrt{\sin\left(ia+ibx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{2\sinh(a+bx)}{b\sqrt{\cosh(a+bx)}}+\frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

input
```
Int[Cosh[a + b*x]^(-3/2),x]
```

output
```
((2*I)*EllipticE[(I/2)*(a + b*x), 2])/b + (2*Sinh[a + b*x])/(b*Sqrt[Cosh[a
 + b*x]])
```

3.12.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.12.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.45

method	result	size
default	$\frac{4\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^2+2\,\mathrm{EllipticE}\left(\cosh\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\sqrt{-\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^2}\,\sqrt{-2\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1}}{\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{-1+2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^2}\,b}$	103

input
```
int(1/cosh(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(2*cosh(1/2*b*x+1/2*a)*sinh(1/2*b*x+1/2*a)^2+EllipticE(cosh(1/2*b*x+1/2*
a),2^(1/2))*(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-2*sinh(1/2*b*x+1/2*a)^2-1)^(1
/2))/sinh(1/2*b*x+1/2*a)/(-1+2*cosh(1/2*b*x+1/2*a)^2)^(1/2)/b
```

3.12.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 3.52

$$\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = \frac{2\left(\left(\sqrt{2}\cosh\left(bx+a\right)^2+2\sqrt{2}\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sqrt{2}\sinh\left(bx+a\right)^2+\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,\right.}{b\cosh\left(bx+a\right)}$$

input
```
integrate(1/cosh(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
2*((sqrt(2)*cosh(b*x + a)^2 + 2*sqrt(2)*cosh(b*x + a)*sinh(b*x + a) + sqrt
(2)*sinh(b*x + a)^2 + sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cosh(b*x + a) + sinh(b*x + a))) + 2*(cosh(b*x + a)^2 + 2*cosh(b*x +
 a)*sinh(b*x + a) + sinh(b*x + a)^2)*sqrt(cosh(b*x + a)))/(b*cosh(b*x + a)
^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 + b)
```

3.12.6 Sympy [F]

$$\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}\,dx=\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}\,dx$$

input
```
integrate(1/cosh(b*x+a)**(3/2),x)
```

output
```
Integral(cosh(a + b*x)**(-3/2), x)
```

3.12.7 Maxima [F]

$$\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}\,dx=\int \frac{1}{\cosh(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cosh(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cosh(b*x + a)^(-3/2), x)
```

3.12.8 Giac [F]

$$\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\cosh(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cosh(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^(-3/2), x)
```

3.12.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\cosh(a+b\,x)^{3/2}}\,dx$$

input
```
int(1/cosh(a + b*x)^(3/2),x)
```

output
```
int(1/cosh(a + b*x)^(3/2), x)
```

3.13 $\int \frac{1}{\cosh^{\frac{5}{2}}(a+bx)}\,dx$

3.13.1 Optimal result

Integrand size = 10, antiderivative size = 46

$$\int \frac{1}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = -\frac{2i\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{3b} + \frac{2\sinh(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

output
```
-2/3*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticF(I*sinh(
1/2*a+1/2*b*x),2^(1/2))/b+2/3*sinh(b*x+a)/b/cosh(b*x+a)^(3/2)
```

3.13.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.83

$$\int \frac{1}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \frac{2\left(\sinh(a+bx) + \cosh(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\cosh(2(a+bx))-\sinh(2(a+bx))\right)\sqrt{1+\cdots}\right)}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

input
```
Integrate[Cosh[a + b*x]^(-5/2),x]
```

output
```
(2*(Sinh[a + b*x] + Cosh[a + b*x]*Hypergeometric2F1[1/4, 1/2, 5/4, -Cosh[2
*(a + b*x)] - Sinh[2*(a + b*x)]]*Sqrt[1 + Cosh[2*(a + b*x)] + Sinh[2*(a +
b*x)]]))/(3*b*Cosh[a + b*x]^(3/2))
```

3.13.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3116, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cosh^{\frac{5}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(ia+ibx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$\frac{1}{3}\int \frac{1}{\sqrt{\cosh(a+bx)}}dx+\frac{2\sinh(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sinh(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}+\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(ia+ibx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sinh(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}-\frac{2i\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{3b}$$

input
```
Int[Cosh[a + b*x]^(-5/2),x]
```

output
```
(((-2*I)/3)*EllipticF[(I/2)*(a + b*x), 2])/b + (2*Sinh[a + b*x])/(3*b*Cosh
[a + b*x]^(3/2))
```

3.13.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.13.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(66) = 132$.

Time = 0.51 (sec) , antiderivative size = 217, normalized size of antiderivative = 4.72

method	result
default	$\frac{2\left(2\sqrt{-\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,\sqrt{-2\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1}\,\operatorname{EllipticF}\left(\cosh\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+\sqrt{-\mathrm{si}}}{3\sqrt{2\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}+\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,\left(-1+2\cosh\right.}$

input
```
int(1/cosh(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(2*(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-2*sinh(1/2*b*x+1/2*a)^2-1)^(1/2)*E
llipticF(cosh(1/2*b*x+1/2*a),2^(1/2))*sinh(1/2*b*x+1/2*a)^2+2*cosh(1/2*b*x
+1/2*a)*sinh(1/2*b*x+1/2*a)^2+(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-2*sinh(1/2*
b*x+1/2*a)^2-1)^(1/2)*EllipticF(cosh(1/2*b*x+1/2*a),2^(1/2)))*((-1+2*cosh(
1/2*b*x+1/2*a)^2)*sinh(1/2*b*x+1/2*a)^2)^(1/2)/(2*sinh(1/2*b*x+1/2*a)^4+si
nh(1/2*b*x+1/2*a)^2)^(1/2)/(-1+2*cosh(1/2*b*x+1/2*a)^2)^(3/2)/sinh(1/2*b*x
+1/2*a)/b
```

3.13.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 310, normalized size of antiderivative = 6.74

$$\int \frac{1}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \frac{2\left(\left(\sqrt{2}\cosh\left(bx+a\right)^{4}+4\sqrt{2}\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+\sqrt{2}\sinh\left(bx+a\right)^{4}+2\left(3\sqrt{2}\cosh\left(bx+a\right)^{2}+\right.\right.\right.}{}$$

input
```
integrate(1/cosh(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
2/3*((sqrt(2)*cosh(b*x + a)^4 + 4*sqrt(2)*cosh(b*x + a)*sinh(b*x + a)^3 +
sqrt(2)*sinh(b*x + a)^4 + 2*(3*sqrt(2)*cosh(b*x + a)^2 + sqrt(2))*sinh(b*x
 + a)^2 + 2*sqrt(2)*cosh(b*x + a)^2 + 4*(sqrt(2)*cosh(b*x + a)^3 + sqrt(2)
*cosh(b*x + a))*sinh(b*x + a) + sqrt(2))*weierstrassPInverse(-4, 0, cosh(b
*x + a) + sinh(b*x + a)) + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x +
 a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x
 + a))*sqrt(cosh(b*x + a)))/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*
x + a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^
2 + b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x
+ a) + b)
```

3.13.6 Sympy [F]

$$\int \frac{1}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\cosh^{\frac{5}{2}}(a+bx)}\,dx$$

input
```
integrate(1/cosh(b*x+a)**(5/2),x)
```

output
```
Integral(cosh(a + b*x)**(-5/2), x)
```

3.13.7 Maxima [F]

$$\int \frac{1}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\cosh(bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cosh(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cosh(b*x + a)^(-5/2), x)
```

3.13.8 Giac [F]

$$\int \frac{1}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\cosh(bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cosh(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^(-5/2), x)
```

3.13.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\cosh(a+b\,x)^{5/2}}\,dx$$

input
```
int(1/cosh(a + b*x)^(5/2),x)
```

output
```
int(1/cosh(a + b*x)^(5/2), x)
```

3.14 $\int \frac{1}{\cosh^{\frac{7}{2}}(a+bx)}\,dx$

3.14.1 Optimal result

Integrand size = 10, antiderivative size = 69

$$\int \frac{1}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \frac{6iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{5b} + \frac{2\sinh(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)} + \frac{6\sinh(a+bx)}{5b\sqrt{\cosh(a+bx)}}$$

output
```
6/5*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticE(I*sinh(1
/2*a+1/2*b*x),2^(1/2))/b+2/5*sinh(b*x+a)/b/cosh(b*x+a)^(5/2)+6/5*sinh(b*x+
a)/b/cosh(b*x+a)^(1/2)
```

3.14.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \frac{1}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \frac{6i\cosh^{\frac{3}{2}}(a+bx)E\left(\frac{1}{2}i(a+bx)\middle|2\right) + 3\sinh(2(a+bx)) + 2\tanh(a+bx)}{5b\cosh^{\frac{3}{2}}(a+bx)}$$

input
```
Integrate[Cosh[a + b*x]^(-7/2),x]
```

output
```
((6*I)*Cosh[a + b*x]^(3/2)*EllipticE[(I/2)*(a + b*x), 2] + 3*Sinh[2*(a + b
*x)] + 2*Tanh[a + b*x])/(5*b*Cosh[a + b*x]^(3/2))
```

3.14.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3116, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cosh^{\frac{7}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(ia+ibx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$\frac{3}{5}\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}dx+\frac{2\sinh(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sinh(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}+\frac{3}{5}\int \frac{1}{\sin\left(ia+ibx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$\frac{3}{5}\left(\frac{2\sinh(a+bx)}{b\sqrt{\cosh(a+bx)}}-\int \sqrt{\cosh(a+bx)}dx\right)+\frac{2\sinh(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sinh(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}+\frac{3}{5}\left(\frac{2\sinh(a+bx)}{b\sqrt{\cosh(a+bx)}}-\int \sqrt{\sin\left(ia+ibx+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\sinh(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}+\frac{3}{5}\left(\frac{2\sinh(a+bx)}{b\sqrt{\cosh(a+bx)}}+\frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}\right)$$

input
```
Int[Cosh[a + b*x]^(-7/2),x]
```

output
```
(2*Sinh[a + b*x])/(5*b*Cosh[a + b*x]^(5/2)) + (3*(((2*I)*EllipticE[(I/2)*(
a + b*x), 2])/b + (2*Sinh[a + b*x])/(b*Sqrt[Cosh[a + b*x]])))/5
```

3.14.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.14.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(85) = 170$.

Time = 0.96 (sec) , antiderivative size = 363, normalized size of antiderivative = 5.26

method	result
default	$\frac{2\sqrt{\left(-1+2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^2}\left(24\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^6\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)+12\sqrt{-2\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1}\ \mathrm{EllipticE}\left(\cosh\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{}\right.\right.}{}$

input
```
int(1/cosh(b*x+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*((-1+2*cosh(1/2*b*x+1/2*a)^2)*sinh(1/2*b*x+1/2*a)^2)^(1/2)/(8*sinh(1/2
*b*x+1/2*a)^6+12*sinh(1/2*b*x+1/2*a)^4+6*sinh(1/2*b*x+1/2*a)^2+1)/sinh(1/2
*b*x+1/2*a)^3*(24*sinh(1/2*b*x+1/2*a)^6*cosh(1/2*b*x+1/2*a)+12*(-2*sinh(1/
2*b*x+1/2*a)^2-1)^(1/2)*EllipticE(cosh(1/2*b*x+1/2*a),2^(1/2))*(-sinh(1/2*
b*x+1/2*a)^2)^(1/2)*sinh(1/2*b*x+1/2*a)^4+24*sinh(1/2*b*x+1/2*a)^4*cosh(1/
2*b*x+1/2*a)+12*(-2*sinh(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticE(cosh(1/2*b*x+
1/2*a),2^(1/2))*(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*sinh(1/2*b*x+1/2*a)^2+8*cos
h(1/2*b*x+1/2*a)*sinh(1/2*b*x+1/2*a)^2+3*EllipticE(cosh(1/2*b*x+1/2*a),2^(
1/2))*(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-2*sinh(1/2*b*x+1/2*a)^2-1)^(1/2))*(
2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2*b*x+1/2*a)^2)^(1/2)/(-1+2*cosh(1/2*b*x+1/
2*a)^2)^(1/2)/b
```

3.14.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 613, normalized size of antiderivative = 8.88

$$\int \frac{1}{\cosh^{\frac{7}{2}}(a+bx)}\,dx$$
$$= \frac{2\left(3\left(\sqrt{2}\cosh\left(bx+a\right)^{6}+6\sqrt{2}\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{5}+\sqrt{2}\sinh\left(bx+a\right)^{6}+3\left(5\sqrt{2}\cosh\left(bx+a\right)^{2}\right.\right.\right.}{}$$

input
```
integrate(1/cosh(b*x+a)^(7/2),x, algorithm="fricas")
```

output
```
2/5*(3*(sqrt(2)*cosh(b*x + a)^6 + 6*sqrt(2)*cosh(b*x + a)*sinh(b*x + a)^5
+ sqrt(2)*sinh(b*x + a)^6 + 3*(5*sqrt(2)*cosh(b*x + a)^2 + sqrt(2))*sinh(b
*x + a)^4 + 3*sqrt(2)*cosh(b*x + a)^4 + 4*(5*sqrt(2)*cosh(b*x + a)^3 + 3*s
qrt(2)*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*sqrt(2)*cosh(b*x + a)^4 + 6*s
qrt(2)*cosh(b*x + a)^2 + sqrt(2))*sinh(b*x + a)^2 + 3*sqrt(2)*cosh(b*x + a
)^2 + 6*(sqrt(2)*cosh(b*x + a)^5 + 2*sqrt(2)*cosh(b*x + a)^3 + sqrt(2)*cos
h(b*x + a))*sinh(b*x + a) + sqrt(2))*weierstrassZeta(-4, 0, weierstrassPIn
verse(-4, 0, cosh(b*x + a) + sinh(b*x + a))) + 2*(3*cosh(b*x + a)^6 + 18*c
osh(b*x + a)*sinh(b*x + a)^5 + 3*sinh(b*x + a)^6 + (45*cosh(b*x + a)^2 + 8
)*sinh(b*x + a)^4 + 8*cosh(b*x + a)^4 + 4*(15*cosh(b*x + a)^3 + 8*cosh(b*x
 + a))*sinh(b*x + a)^3 + (45*cosh(b*x + a)^4 + 48*cosh(b*x + a)^2 + 1)*sin
h(b*x + a)^2 + cosh(b*x + a)^2 + 2*(9*cosh(b*x + a)^5 + 16*cosh(b*x + a)^3
 + cosh(b*x + a))*sinh(b*x + a))*sqrt(cosh(b*x + a)))/(b*cosh(b*x + a)^6 +
 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*x + a)^6 + 3*b*cosh(b*x + a)
^4 + 3*(5*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*x + a)^3
+ 3*b*cosh(b*x + a))*sinh(b*x + a)^3 + 3*b*cosh(b*x + a)^2 + 3*(5*b*cosh(b
*x + a)^4 + 6*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 6*(b*cosh(b*x + a)^
5 + 2*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) + b)
```

3.14.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(1/cosh(b*x+a)**(7/2),x)
```

output
```
Timed out
```

3.14.7 Maxima [F]

$$\int \frac{1}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{1}{\cosh\left(bx+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cosh(b*x+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(cosh(b*x + a)^(-7/2), x)
```

3.14.8 Giac [F]

$$\int \frac{1}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{1}{\cosh\left(bx+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cosh(b*x+a)^(7/2),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^(-7/2), x)
```

3.14.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{1}{\cosh\left(a+b\,x\right)^{7/2}}\,dx$$

input
```
int(1/cosh(a + b*x)^(7/2),x)
```

output
```
int(1/cosh(a + b*x)^(7/2), x)
```

3.15 $\int (a\cosh(x))^{7/2}\,dx$

3.15.1 Optimal result

Integrand size = 8, antiderivative size = 65

$$\int (a\cosh(x))^{7/2}\,dx = -\frac{10ia^4\sqrt{\cosh(x)}\,\text{EllipticF}\left(\frac{ix}{2},2\right)}{21\sqrt{a\cosh(x)}} + \frac{10}{21}a^3\sqrt{a\cosh(x)}\sinh(x) + \frac{2}{7}a(a\cosh(x))^{5/2}\sinh(x)$$

output
```
2/7*a*(a*cosh(x))^(5/2)*sinh(x)-10/21*I*a^4*(cosh(1/2*x)^2)^(1/2)/cosh(1/2
*x)*EllipticF(I*sinh(1/2*x),2^(1/2))*cosh(x)^(1/2)/(a*cosh(x))^(1/2)+10/21
*a^3*sinh(x)*(a*cosh(x))^(1/2)
```

3.15.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int (a\cosh(x))^{7/2}\,dx = \frac{a^3\sqrt{a\cosh(x)}\left(-20i\,\text{EllipticF}\left(\frac{ix}{2},2\right) + \sqrt{\cosh(x)}(23\sinh(x) + 3\sinh(3x))\right)}{42\sqrt{\cosh(x)}}$$

input
```
Integrate[(a*Cosh[x])^(7/2),x]
```

output
```
(a^3*Sqrt[a*Cosh[x]]*((-20*I)*EllipticF[(I/2)*x, 2] + Sqrt[Cosh[x]]*(23*Si
nh[x] + 3*Sinh[3*x])))/(42*Sqrt[Cosh[x]])
```

3.15.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3115, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cosh(x))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(\frac{\pi}{2}+ix\right)\right)^{7/2}dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{7}a^2\int (a\cosh(x))^{3/2}dx+\frac{2}{7}a\sinh(x)(a\cosh(x))^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}a\sinh(x)(a\cosh(x))^{5/2}+\frac{5}{7}a^2\int \left(a\sin\left(ix+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{7}a^2\left(\frac{1}{3}a^2\int \frac{1}{\sqrt{a\cosh(x)}}dx+\frac{2}{3}a\sinh(x)\sqrt{a\cosh(x)}\right)+\frac{2}{7}a\sinh(x)(a\cosh(x))^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}a\sinh(x)(a\cosh(x))^{5/2}+\frac{5}{7}a^2\left(\frac{2}{3}a\sinh(x)\sqrt{a\cosh(x)}+\frac{1}{3}a^2\int \frac{1}{\sqrt{a\sin\left(ix+\frac{\pi}{2}\right)}}dx\right)$$

$$\Big\downarrow \text{3121}$$

$$\frac{5}{7}a^2\left(\frac{a^2\sqrt{\cosh(x)}\int \frac{1}{\sqrt{\cosh(x)}}dx}{3\sqrt{a\cosh(x)}}+\frac{2}{3}a\sinh(x)\sqrt{a\cosh(x)}\right)+\frac{2}{7}a\sinh(x)(a\cosh(x))^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}a\sinh(x)(a\cosh(x))^{5/2}+\frac{5}{7}a^2\left(\frac{2}{3}a\sinh(x)\sqrt{a\cosh(x)}+\frac{a^2\sqrt{\cosh(x)}\int \frac{1}{\sqrt{\sin\left(ix+\frac{\pi}{2}\right)}}dx}{3\sqrt{a\cosh(x)}}\right)$$

$$\Big\downarrow \text{3120}$$

$$\frac{2}{7}a\sinh(x)(a\cosh(x))^{5/2}+\frac{5}{7}a^2\left(\frac{2}{3}a\sinh(x)\sqrt{a\cosh(x)}-\frac{2ia^2\sqrt{\cosh(x)}\,\text{EllipticF}\left(\frac{ix}{2},2\right)}{3\sqrt{a\cosh(x)}}\right)$$

input
```
Int[(a*Cosh[x])^(7/2),x]
```

output
```
(2*a*(a*Cosh[x])^(5/2)*Sinh[x])/7 + (5*a^2*((((-2*I)/3)*a^2*Sqrt[Cosh[x]]*
EllipticF[(I/2)*x, 2])/Sqrt[a*Cosh[x]] + (2*a*Sqrt[a*Cosh[x]]*Sinh[x])/3))
/7
```

3.15.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.15.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(66) = 132$.

Time = 2.74 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.23

method	result
default	$\frac{\sqrt{a\left(2\cosh\left(\frac{x}{2}\right)^2-1\right)\sinh\left(\frac{x}{2}\right)^2}\,a^4\left(96\cosh\left(\frac{x}{2}\right)^9-240\cosh\left(\frac{x}{2}\right)^7+256\cosh\left(\frac{x}{2}\right)^5+5\sqrt{2}\,\sqrt{-2\cosh\left(\frac{x}{2}\right)^2+1}\,\sqrt{-\sinh\left(\frac{x}{2}\right)^2}\,\text{EllipticF}\left(\sqrt{2}\right.\right.}{21\sqrt{a\left(2\sinh\left(\frac{x}{2}\right)^4+\sinh\left(\frac{x}{2}\right)^2\right)}\,\sinh\left(\frac{x}{2}\right)\sqrt{a\left(2\cosh\left(\frac{x}{2}\right)^2-1\right)}}$

input
```
int((a*cosh(x))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/21*(a*(2*cosh(1/2*x)^2-1)*sinh(1/2*x)^2)^(1/2)*a^4*(96*cosh(1/2*x)^9-240
*cosh(1/2*x)^7+256*cosh(1/2*x)^5+5*2^(1/2)*(-2*cosh(1/2*x)^2+1)^(1/2)*(-si
nh(1/2*x)^2)^(1/2)*EllipticF(2^(1/2)*cosh(1/2*x),1/2*2^(1/2))-144*cosh(1/2
*x)^3+32*cosh(1/2*x))/(a*(2*sinh(1/2*x)^4+sinh(1/2*x)^2))^(1/2)/sinh(1/2*x
)/(a*(2*cosh(1/2*x)^2-1))^(1/2)
```

3.15.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 256, normalized size of antiderivative = 3.94

$$\int (a\cosh(x))^{7/2}\,dx = \frac{40\left(\sqrt{2}a^3\cosh\left(x\right)^3 + 3\sqrt{2}a^3\cosh\left(x\right)^2\sinh\left(x\right) + 3\sqrt{2}a^3\cosh\left(x\right)\sinh\left(x\right)^2 + \sqrt{2}a^3\sin\right.}{}$$

input
```
integrate((a*cosh(x))^(7/2),x, algorithm="fricas")
```

output
```
1/84*(40*(sqrt(2)*a^3*cosh(x)^3 + 3*sqrt(2)*a^3*cosh(x)^2*sinh(x) + 3*sqrt
(2)*a^3*cosh(x)*sinh(x)^2 + sqrt(2)*a^3*sinh(x)^3)*sqrt(a)*weierstrassPInv
erse(-4, 0, cosh(x) + sinh(x)) + (3*a^3*cosh(x)^6 + 18*a^3*cosh(x)*sinh(x)
^5 + 3*a^3*sinh(x)^6 + 23*a^3*cosh(x)^4 - 23*a^3*cosh(x)^2 + (45*a^3*cosh(
x)^2 + 23*a^3)*sinh(x)^4 + 4*(15*a^3*cosh(x)^3 + 23*a^3*cosh(x))*sinh(x)^3
 - 3*a^3 + (45*a^3*cosh(x)^4 + 138*a^3*cosh(x)^2 - 23*a^3)*sinh(x)^2 + 2*(
9*a^3*cosh(x)^5 + 46*a^3*cosh(x)^3 - 23*a^3*cosh(x))*sinh(x))*sqrt(a*cosh(
x)))/(cosh(x)^3 + 3*cosh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)
```

3.15.6 Sympy [F(-1)]

Timed out.

$$\int (a\cosh(x))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((a*cosh(x))**(7/2),x)
```

output
```
Timed out
```

3.15.7 Maxima [F]

$$\int (a\cosh(x))^{7/2}\,dx = \int (a\cosh(x))^{\frac{7}{2}}\,dx$$

input
```
integrate((a*cosh(x))^(7/2),x, algorithm="maxima")
```

output
```
integrate((a*cosh(x))^(7/2), x)
```

3.15.8 Giac [F]

$$\int (a\cosh(x))^{7/2}\,dx = \int (a\cosh(x))^{\frac{7}{2}}\,dx$$

input
```
integrate((a*cosh(x))^(7/2),x, algorithm="giac")
```

output
```
integrate((a*cosh(x))^(7/2), x)
```

3.15.9 Mupad [F(-1)]

Timed out.

$$\int (a\cosh(x))^{7/2}\,dx = \int (a\cosh(x))^{7/2}\,dx$$

input
```
int((a*cosh(x))^(7/2),x)
```

output
```
int((a*cosh(x))^(7/2), x)
```

3.16 $\int (a\cosh(x))^{5/2}\,dx$

3.16.1 Optimal result

Integrand size = 8, antiderivative size = 48

$$\int (a\cosh(x))^{5/2}\,dx = -\frac{6ia^2\sqrt{a\cosh(x)}E\left(\frac{ix}{2}\middle|2\right)}{5\sqrt{\cosh(x)}} + \frac{2}{5}a(a\cosh(x))^{3/2}\sinh(x)$$

output
```
2/5*a*(a*cosh(x))^(3/2)*sinh(x)-6/5*I*a^2*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x
)*EllipticE(I*sinh(1/2*x),2^(1/2))*(a*cosh(x))^(1/2)/cosh(x)^(1/2)
```

3.16.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int (a\cosh(x))^{5/2}\,dx = \frac{2(a\cosh(x))^{5/2}\left(-3iE\left(\frac{ix}{2}\middle|2\right)+\cosh^{\frac{3}{2}}(x)\sinh(x)\right)}{5\cosh^{\frac{5}{2}}(x)}$$

input
```
Integrate[(a*Cosh[x])^(5/2),x]
```

output
```
(2*(a*Cosh[x])^(5/2)*((-3*I)*EllipticE[(I/2)*x, 2] + Cosh[x]^(3/2)*Sinh[x]
))/(5*Cosh[x]^(5/2))
```

3.16.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cosh(x))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(\frac{\pi}{2}+ix\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{3}{5}a^2\int \sqrt{a\cosh(x)}dx+\frac{2}{5}a\sinh(x)(a\cosh(x))^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}a\sinh(x)(a\cosh(x))^{3/2}+\frac{3}{5}a^2\int \sqrt{a\sin\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3121}$$

$$\frac{3a^2\sqrt{a\cosh(x)}\int \sqrt{\cosh(x)}dx}{5\sqrt{\cosh(x)}}+\frac{2}{5}a\sinh(x)(a\cosh(x))^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}a\sinh(x)(a\cosh(x))^{3/2}+\frac{3a^2\sqrt{a\cosh(x)}\int \sqrt{\sin\left(ix+\frac{\pi}{2}\right)}dx}{5\sqrt{\cosh(x)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2}{5}a\sinh(x)(a\cosh(x))^{3/2}-\frac{6ia^2E\left(\frac{ix}{2}|2\right)\sqrt{a\cosh(x)}}{5\sqrt{\cosh(x)}}$$

input
```
Int[(a*Cosh[x])^(5/2),x]
```

output
```
(((-6*I)/5)*a^2*Sqrt[a*Cosh[x]]*EllipticE[(I/2)*x, 2])/Sqrt[Cosh[x]] + (2*
a*(a*Cosh[x])^(3/2)*Sinh[x])/5
```

3.16.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.16.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(53) = 106$.

Time = 0.88 (sec) , antiderivative size = 184, normalized size of antiderivative = 3.83

method	result
default	$\frac{\sqrt{a\left(2\cosh\left(\frac{x}{2}\right)^{2}-1\right)\sinh\left(\frac{x}{2}\right)^{2}}\, a^{3}\left(16\cosh\left(\frac{x}{2}\right)\sinh\left(\frac{x}{2}\right)^{6}+16\sinh\left(\frac{x}{2}\right)^{4}\cosh\left(\frac{x}{2}\right)+3\sqrt{2}\,\sqrt{-2\sinh\left(\frac{x}{2}\right)^{2}-1}\,\sqrt{-\sinh\left(\frac{x}{2}\right)^{2}}\ \text{EllipticF}\left(\sqrt{2}\right.}{5\sqrt{a\left(2\sinh\left(\frac{x}{2}\right)^{4}+\sinh\left(\frac{x}{2}\right)^{2}\right)}\ \sinh\left(\frac{x}{2}\right)\sqrt{a}}$

input
```
int((a*cosh(x))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*(a*(2*cosh(1/2*x)^2-1)*sinh(1/2*x)^2)^(1/2)*a^3*(16*cosh(1/2*x)*sinh(1
/2*x)^6+16*sinh(1/2*x)^4*cosh(1/2*x)+3*2^(1/2)*(-2*sinh(1/2*x)^2-1)^(1/2)*
(-sinh(1/2*x)^2)^(1/2)*EllipticF(2^(1/2)*cosh(1/2*x),1/2*2^(1/2))-6*2^(1/2
)*(-2*sinh(1/2*x)^2-1)^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(2^(1/2)*cosh
(1/2*x),1/2*2^(1/2))+4*sinh(1/2*x)^2*cosh(1/2*x))/(a*(2*sinh(1/2*x)^4+sinh
(1/2*x)^2))^(1/2)/sinh(1/2*x)/(a*(2*cosh(1/2*x)^2-1))^(1/2)
```

3.16.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 161, normalized size of antiderivative = 3.35

$$\int (a\cosh(x))^{5/2}\,dx =$$
$$-\frac{12\left(\sqrt{2}a^2\cosh(x)^2+2\sqrt{2}a^2\cosh(x)\sinh(x)+\sqrt{2}a^2\sinh(x)^2\right)\sqrt{a}\text{weierstrassZeta}(-4,0,\text{weierstrassPInv}$$

input
```
integrate((a*cosh(x))^(5/2),x, algorithm="fricas")
```

output
```
-1/10*(12*(sqrt(2)*a^2*cosh(x)^2 + 2*sqrt(2)*a^2*cosh(x)*sinh(x) + sqrt(2)
*a^2*sinh(x)^2)*sqrt(a)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
cosh(x) + sinh(x))) - (a^2*cosh(x)^4 + 4*a^2*cosh(x)*sinh(x)^3 + a^2*sinh(
x)^4 - 12*a^2*cosh(x)^2 + 6*(a^2*cosh(x)^2 - 2*a^2)*sinh(x)^2 - a^2 + 4*(a
^2*cosh(x)^3 - 6*a^2*cosh(x))*sinh(x))*sqrt(a*cosh(x)))/(cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2)
```

3.16.6 Sympy [F(-1)]

Timed out.

$$\int (a\cosh(x))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a*cosh(x))**(5/2),x)
```

output
```
Timed out
```

3.16.7 Maxima [F]

$$\int (a\cosh(x))^{5/2}\,dx = \int (a\cosh(x))^{\frac{5}{2}}\,dx$$

input
```
integrate((a*cosh(x))^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*cosh(x))^(5/2), x)
```

3.16.8 Giac [F]

$$\int (a\cosh(x))^{5/2}\,dx = \int (a\cosh(x))^{\frac{5}{2}}\,dx$$

input
```
integrate((a*cosh(x))^(5/2),x, algorithm="giac")
```

output
```
integrate((a*cosh(x))^(5/2), x)
```

3.16.9 Mupad [F(-1)]

Timed out.

$$\int (a\cosh(x))^{5/2}\,dx = \int (a\cosh(x))^{5/2}\,dx$$

input
```
int((a*cosh(x))^(5/2),x)
```

output
```
int((a*cosh(x))^(5/2), x)
```

3.17 $\int (a\cosh(x))^{3/2}\,dx$

3.17.1 Optimal result

Integrand size = 8, antiderivative size = 48

$$\int (a\cosh(x))^{3/2}\,dx = -\frac{2ia^2\sqrt{\cosh(x)}\,\text{EllipticF}\left(\frac{ix}{2},2\right)}{3\sqrt{a\cosh(x)}} + \frac{2}{3}a\sqrt{a\cosh(x)}\sinh(x)$$

output
```
-2/3*I*a^2*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/
2))*cosh(x)^(1/2)/(a*cosh(x))^(1/2)+2/3*a*sinh(x)*(a*cosh(x))^(1/2)
```

3.17.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int (a\cosh(x))^{3/2}\,dx = \frac{2}{3}(a\cosh(x))^{3/2}\left(\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\cosh(2x)-\sinh(2x)\right)\text{sech}^2(x)\sqrt{1+\right.$$

input
```
Integrate[(a*Cosh[x])^(3/2),x]
```

output
```
(2*(a*Cosh[x])^(3/2)*(Hypergeometric2F1[1/4, 1/2, 5/4, -Cosh[2*x] - Sinh[2
*x]]*Sech[x]^2*Sqrt[1 + Cosh[2*x] + Sinh[2*x]] + Tanh[x]))/3
```

3.17.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cosh(x))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(\frac{\pi}{2}+ix\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{3}a^2\int \frac{1}{\sqrt{a\cosh(x)}}dx+\frac{2}{3}a\sinh(x)\sqrt{a\cosh(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}a\sinh(x)\sqrt{a\cosh(x)}+\frac{1}{3}a^2\int \frac{1}{\sqrt{a\sin\left(ix+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3121}$$

$$\frac{a^2\sqrt{\cosh(x)}\int \frac{1}{\sqrt{\cosh(x)}}dx}{3\sqrt{a\cosh(x)}}+\frac{2}{3}a\sinh(x)\sqrt{a\cosh(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}a\sinh(x)\sqrt{a\cosh(x)}+\frac{a^2\sqrt{\cosh(x)}\int \frac{1}{\sqrt{\sin\left(ix+\frac{\pi}{2}\right)}}dx}{3\sqrt{a\cosh(x)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2}{3}a\sinh(x)\sqrt{a\cosh(x)}-\frac{2ia^2\sqrt{\cosh(x)}\,\text{EllipticF}\left(\frac{ix}{2},2\right)}{3\sqrt{a\cosh(x)}}$$

input
```
Int[(a*Cosh[x])^(3/2),x]
```

output
```
(((-2*I)/3)*a^2*Sqrt[Cosh[x]]*EllipticF[(I/2)*x, 2])/Sqrt[a*Cosh[x]] + (2*
a*Sqrt[a*Cosh[x]]*Sinh[x])/3
```

3.17.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.17.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(53) = 106$.

Time = 0.59 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.71

method	result
default	$\frac{\sqrt{a\left(2\cosh\left(\frac{x}{2}\right)^{2}-1\right)\sinh\left(\frac{x}{2}\right)^{2}}\, a^{2}\left(8\sinh\left(\frac{x}{2}\right)^{4}\cosh\left(\frac{x}{2}\right)+\sqrt{2}\,\sqrt{-2\sinh\left(\frac{x}{2}\right)^{2}-1}\,\sqrt{-\sinh\left(\frac{x}{2}\right)^{2}}\ \mathrm{EllipticF}\left(\sqrt{2}\cosh\left(\frac{x}{2}\right),\frac{\sqrt{2}}{2}\right)+4\sinh\left(\frac{x}{2}\right.\right.}{3\sqrt{a\left(2\sinh\left(\frac{x}{2}\right)^{4}+\sinh\left(\frac{x}{2}\right)^{2}\right)}\,\sinh\left(\frac{x}{2}\right)\sqrt{a\left(2\cosh\left(\frac{x}{2}\right)^{2}-1\right)}}$

input
```
int((a*cosh(x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(a*(2*cosh(1/2*x)^2-1)*sinh(1/2*x)^2)^(1/2)*a^2*(8*sinh(1/2*x)^4*cosh(
1/2*x)+2^(1/2)*(-2*sinh(1/2*x)^2-1)^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF
(2^(1/2)*cosh(1/2*x),1/2*2^(1/2))+4*sinh(1/2*x)^2*cosh(1/2*x))/(a*(2*sinh(
1/2*x)^4+sinh(1/2*x)^2))^(1/2)/sinh(1/2*x)/(a*(2*cosh(1/2*x)^2-1))^(1/2)
```

3.17.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.42

$$\int (a\cosh(x))^{3/2}\,dx = \frac{2\left(\sqrt{2}a\cosh(x) + \sqrt{2}a\sinh(x)\right)\sqrt{a}\text{weierstrassPInverse}(-4,0,\cosh(x)+\sinh(x)) +}{3(\cosh(x)+\sinh(x)}$$

input
```
integrate((a*cosh(x))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(2*(sqrt(2)*a*cosh(x) + sqrt(2)*a*sinh(x))*sqrt(a)*weierstrassPInverse
(-4, 0, cosh(x) + sinh(x)) + (a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x
)^2 - a)*sqrt(a*cosh(x)))/(cosh(x) + sinh(x))
```

3.17.6 Sympy [F]

$$\int (a\cosh(x))^{3/2}\,dx = \int (a\cosh(x))^{\frac{3}{2}}\,dx$$

input
```
integrate((a*cosh(x))**(3/2),x)
```

output
```
Integral((a*cosh(x))**(3/2), x)
```

3.17.7 Maxima [F]

$$\int (a\cosh(x))^{3/2}\,dx = \int (a\cosh(x))^{\frac{3}{2}}\,dx$$

input
```
integrate((a*cosh(x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*cosh(x))^(3/2), x)
```

3.17.8 Giac [F]

$$\int (a\cosh(x))^{3/2}\,dx = \int (a\cosh(x))^{\frac{3}{2}}\,dx$$

input
```
integrate((a*cosh(x))^(3/2),x, algorithm="giac")
```

output
```
integrate((a*cosh(x))^(3/2), x)
```

3.17.9 Mupad [F(-1)]

Timed out.

$$\int (a\cosh(x))^{3/2}\,dx = \int (a\cosh(x))^{3/2}\,dx$$

input
```
int((a*cosh(x))^(3/2),x)
```

output
```
int((a*cosh(x))^(3/2), x)
```

3.18 $\int \sqrt{a \cosh(x)}\, dx$

3.18.1 Optimal result

Integrand size = 8, antiderivative size = 27

$$\int \sqrt{a \cosh(x)}\, dx = -\frac{2i\sqrt{a \cosh(x)}\, E\left(\frac{ix}{2}\middle|2\right)}{\sqrt{\cosh(x)}}$$

output
```
-2*I*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(1/2*x),2^(1/2))*(a
*cosh(x))^(1/2)/cosh(x)^(1/2)
```

3.18.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \sqrt{a \cosh(x)}\, dx = -\frac{2i\sqrt{a \cosh(x)}\, E\left(\frac{ix}{2}\middle|2\right)}{\sqrt{\cosh(x)}}$$

input
```
Integrate[Sqrt[a*Cosh[x]],x]
```

output
```
((-2*I)*Sqrt[a*Cosh[x]]*EllipticE[(I/2)*x, 2])/Sqrt[Cosh[x]]
```

3.18.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a \cosh(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a \sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3121}$$

$$\frac{\sqrt{a \cosh(x)}\int \sqrt{\cosh(x)}dx}{\sqrt{\cosh(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a \cosh(x)}\int \sqrt{\sin\left(ix+\frac{\pi}{2}\right)}dx}{\sqrt{\cosh(x)}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{2iE\left(\frac{ix}{2}\middle|2\right)\sqrt{a \cosh(x)}}{\sqrt{\cosh(x)}}$$

input
```
Int[Sqrt[a*Cosh[x]],x]
```

output
```
((-2*I)*Sqrt[a*Cosh[x]]*EllipticE[(I/2)*x, 2])/Sqrt[Cosh[x]]
```

3.18.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.18.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(38) = 76$.

Time = 0.52 (sec) , antiderivative size = 118, normalized size of antiderivative = 4.37

method	result
default	$\frac{\sqrt{a\left(2\cosh\left(\frac{x}{2}\right)^{2}-1\right)\sinh\left(\frac{x}{2}\right)^{2}}\, a\sqrt{2}\,\sqrt{-2\cosh\left(\frac{x}{2}\right)^{2}+1}\,\sqrt{-\sinh\left(\frac{x}{2}\right)^{2}}\left(\operatorname{EllipticF}\left(\sqrt{2}\,\cosh\left(\frac{x}{2}\right),\frac{\sqrt{2}}{2}\right)-2\operatorname{EllipticE}\left(\sqrt{2}\,\cosh\left(\frac{x}{2}\right),\frac{\sqrt{2}}{2}\right)\right)}{\sqrt{a\left(2\sinh\left(\frac{x}{2}\right)^{4}+\sinh\left(\frac{x}{2}\right)^{2}\right)}\,\sinh\left(\frac{x}{2}\right)\sqrt{a\left(2\cosh\left(\frac{x}{2}\right)^{2}-1\right)}}$
risch	$\sqrt{2}\,\sqrt{a\left(1+\mathrm{e}^{2x}\right)\mathrm{e}^{-x}}+\frac{\left(-\frac{4\left(a\,\mathrm{e}^{2x}+a\right)}{a\sqrt{\mathrm{e}^{x}\left(a\,\mathrm{e}^{2x}+a\right)}}+\frac{2i\sqrt{-i(\mathrm{e}^{x}+i)}\,\sqrt{2}\,\sqrt{i(\mathrm{e}^{x}-i)}\,\sqrt{i\mathrm{e}^{x}}\,\left(-2i\,\operatorname{EllipticE}\left(\sqrt{-i(\mathrm{e}^{x}+i)},\frac{\sqrt{2}}{2}\right)+i\,\operatorname{EllipticF}\left(\sqrt{-i(\mathrm{e}^{x}}\right.\right.}{\sqrt{a\,\mathrm{e}^{3x}+a\,\mathrm{e}^{x}}}\right.}{2+2\,\mathrm{e}^{2x}}$

input
```
int((a*cosh(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(a*(2*cosh(1/2*x)^2-1)*sinh(1/2*x)^2)^(1/2)*a*2^(1/2)*(-2*cosh(1/2*x)^2+1)
^(1/2)*(-sinh(1/2*x)^2)^(1/2)*(EllipticF(2^(1/2)*cosh(1/2*x),1/2*2^(1/2))-
2*EllipticE(2^(1/2)*cosh(1/2*x),1/2*2^(1/2)))/(a*(2*sinh(1/2*x)^4+sinh(1/2
*x)^2))^(1/2)/sinh(1/2*x)/(a*(2*cosh(1/2*x)^2-1))^(1/2)
```

3.18.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int \sqrt{a\cosh(x)}\,dx = -2\sqrt{2}\sqrt{a}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cosh(x) + \sinh(x))) - 2\sqrt{a\cosh(x)}$$

input
```
integrate((a*cosh(x))^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(2)*sqrt(a)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(
x) + sinh(x))) - 2*sqrt(a*cosh(x))
```

3.18.6 Sympy [F]

$$\int \sqrt{a\cosh(x)}\,dx = \int \sqrt{a\cosh(x)}\,dx$$

input
```
integrate((a*cosh(x))**(1/2),x)
```

output
```
Integral(sqrt(a*cosh(x)), x)
```

3.18.7 Maxima [F]

$$\int \sqrt{a\cosh(x)}\,dx = \int \sqrt{a\cosh(x)}\,dx$$

input
```
integrate((a*cosh(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*cosh(x)), x)
```

3.18.8 Giac [F]

$$\int \sqrt{a\cosh(x)}\,dx = \int \sqrt{a\cosh(x)}\,dx$$

input
```
integrate((a*cosh(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*cosh(x)), x)
```

3.18.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a\cosh(x)}\,dx = \int \sqrt{a\cosh(x)}\,dx$$

input
```
int((a*cosh(x))^(1/2),x)
```

output
```
int((a*cosh(x))^(1/2), x)
```

3.19 $\int \frac{1}{\sqrt{a \cosh(x)}} \, dx$

3.19.1 Optimal result

Integrand size = 8, antiderivative size = 27

$$\int \frac{1}{\sqrt{a \cosh(x)}} \, dx = -\frac{2i\sqrt{\cosh(x)} \operatorname{EllipticF}\left(\frac{ix}{2}, 2\right)}{\sqrt{a \cosh(x)}}$$

output
```
-2*I*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/2))*co
sh(x)^(1/2)/(a*cosh(x))^(1/2)
```

3.19.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a \cosh(x)}} \, dx = -\frac{2i\sqrt{\cosh(x)} \operatorname{EllipticF}\left(\frac{ix}{2}, 2\right)}{\sqrt{a \cosh(x)}}$$

input
```
Integrate[1/Sqrt[a*Cosh[x]],x]
```

output
```
((-2*I)*Sqrt[Cosh[x]]*EllipticF[(I/2)*x, 2])/Sqrt[a*Cosh[x]]
```

3.19.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\cosh(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\sin\left(\frac{\pi}{2}+ix\right)}}dx$$

$$\Big\downarrow \text{3121}$$

$$\frac{\sqrt{\cosh(x)}\int \frac{1}{\sqrt{\cosh(x)}}dx}{\sqrt{a\cosh(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cosh(x)}\int \frac{1}{\sqrt{\sin\left(ix+\frac{\pi}{2}\right)}}dx}{\sqrt{a\cosh(x)}}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{2i\sqrt{\cosh(x)}\,\text{EllipticF}\left(\frac{ix}{2},2\right)}{\sqrt{a\cosh(x)}}$$

input
```
Int[1/Sqrt[a*Cosh[x]],x]
```

output
```
((-2*I)*Sqrt[Cosh[x]]*EllipticF[(I/2)*x, 2])/Sqrt[a*Cosh[x]]
```

3.19.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.19.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(38) = 76.

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 3.70

method	result	size
default	$\frac{\sqrt{a\left(2\cosh\left(\frac{x}{2}\right)^{2}-1\right)\sinh\left(\frac{x}{2}\right)^{2}}\,\sqrt{2}\,\sqrt{-2\cosh\left(\frac{x}{2}\right)^{2}+1}\,\sqrt{-\sinh\left(\frac{x}{2}\right)^{2}}\,\operatorname{EllipticF}\left(\sqrt{2}\cosh\left(\frac{x}{2}\right),\frac{\sqrt{2}}{2}\right)}{\sqrt{a\left(2\sinh\left(\frac{x}{2}\right)^{4}+\sinh\left(\frac{x}{2}\right)^{2}\right)}\,\sinh\left(\frac{x}{2}\right)\sqrt{a\left(2\cosh\left(\frac{x}{2}\right)^{2}-1\right)}}$	100

input
```
int(1/(a*cosh(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(a*(2*cosh(1/2*x)^2-1)*sinh(1/2*x)^2)^(1/2)*2^(1/2)*(-2*cosh(1/2*x)^2+1)^(
1/2)*(-sinh(1/2*x)^2)^(1/2)/(a*(2*sinh(1/2*x)^4+sinh(1/2*x)^2))^(1/2)*Elli
pticF(2^(1/2)*cosh(1/2*x),1/2*2^(1/2))/sinh(1/2*x)/(a*(2*cosh(1/2*x)^2-1))
^(1/2)
```

3.19.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.59

$$\int \frac{1}{\sqrt{a\cosh(x)}}\,dx = \frac{2\sqrt{2}\text{weierstrassPInverse}(-4,0,\cosh(x)+\sinh(x))}{\sqrt{a}}$$

input
```
integrate(1/(a*cosh(x))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(2)*weierstrassPInverse(-4, 0, cosh(x) + sinh(x))/sqrt(a)
```

3.19.6 Sympy [F]

$$\int \frac{1}{\sqrt{a\cosh(x)}}\,dx = \int \frac{1}{\sqrt{a\cosh(x)}}\,dx$$

input
```
integrate(1/(a*cosh(x))**(1/2),x)
```

output
```
Integral(1/sqrt(a*cosh(x)), x)
```

3.19.7 Maxima [F]

$$\int \frac{1}{\sqrt{a\cosh(x)}}\,dx = \int \frac{1}{\sqrt{a\cosh(x)}}\,dx$$

input
```
integrate(1/(a*cosh(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(a*cosh(x)), x)
```

3.19.8 Giac [F]

$$\int \frac{1}{\sqrt{a\cosh(x)}}\,dx = \int \frac{1}{\sqrt{a\cosh(x)}}\,dx$$

input
```
integrate(1/(a*cosh(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(a*cosh(x)), x)
```

3.19.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\cosh(x)}}\,dx = \int \frac{1}{\sqrt{a\cosh(x)}}\,dx$$

input
```
int(1/(a*cosh(x))^(1/2),x)
```

output
```
int(1/(a*cosh(x))^(1/2), x)
```

3.20 $\int \frac{1}{(a\cosh(x))^{3/2}}\,dx$

3.20.1 Optimal result

Integrand size = 8, antiderivative size = 46

$$\int \frac{1}{(a\cosh(x))^{3/2}}\,dx=\frac{2i\sqrt{a\cosh(x)}E\left(\frac{ix}{2}\middle|2\right)}{a^2\sqrt{\cosh(x)}}+\frac{2\sinh(x)}{a\sqrt{a\cosh(x)}}$$

output
```
2*sinh(x)/a/(a*cosh(x))^(1/2)+2*I*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*Ellipt
icE(I*sinh(1/2*x),2^(1/2))*(a*cosh(x))^(1/2)/a^2/cosh(x)^(1/2)
```

3.20.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{1}{(a\cosh(x))^{3/2}}\,dx=\frac{2\cosh(x)\left(i\sqrt{\cosh(x)}E\left(\frac{ix}{2}\middle|2\right)+\sinh(x)\right)}{(a\cosh(x))^{3/2}}$$

input
```
Integrate[(a*Cosh[x])^(-3/2),x]
```

output
```
(2*Cosh[x]*(I*Sqrt[Cosh[x]]*EllipticE[(I/2)*x, 2] + Sinh[x]))/(a*Cosh[x])^
(3/2)
```

3.20.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cosh(x))^{3/2}}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\left(a\sin\left(\frac{\pi}{2}+ix\right)\right)^{3/2}}dx$$
$$\Big\downarrow \text{ 3116}$$
$$\frac{2\sinh(x)}{a\sqrt{a\cosh(x)}}-\frac{\int \sqrt{a\cosh(x)}dx}{a^2}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{2\sinh(x)}{a\sqrt{a\cosh(x)}}-\frac{\int \sqrt{a\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2}$$
$$\Big\downarrow \text{ 3121}$$
$$\frac{2\sinh(x)}{a\sqrt{a\cosh(x)}}-\frac{\sqrt{a\cosh(x)}\int \sqrt{\cosh(x)}dx}{a^2\sqrt{\cosh(x)}}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{2\sinh(x)}{a\sqrt{a\cosh(x)}}-\frac{\sqrt{a\cosh(x)}\int \sqrt{\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2\sqrt{\cosh(x)}}$$
$$\Big\downarrow \text{ 3119}$$
$$\frac{2\sinh(x)}{a\sqrt{a\cosh(x)}}+\frac{2iE\left(\frac{ix}{2}\middle|2\right)\sqrt{a\cosh(x)}}{a^2\sqrt{\cosh(x)}}$$

input
```
Int[(a*Cosh[x])^(-3/2),x]
```

output
```
((2*I)*Sqrt[a*Cosh[x]]*EllipticE[(I/2)*x, 2])/(a^2*Sqrt[Cosh[x]]) + (2*Sin
h[x])/(a*Sqrt[a*Cosh[x]])
```

3.20.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.20.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(55) = 110.

Time = 0.34 (sec) , antiderivative size = 159, normalized size of antiderivative = 3.46

method	result
default	$\frac{\sqrt{2\sinh\left(\frac{x}{2}\right)^{4}a+\sinh\left(\frac{x}{2}\right)^{2}a}\,\left(-\sqrt{2}\,\sqrt{-2\sinh\left(\frac{x}{2}\right)^{2}-1}\,\sqrt{-\sinh\left(\frac{x}{2}\right)^{2}}\,\operatorname{EllipticF}\left(\sqrt{2}\,\cosh\left(\frac{x}{2}\right),\frac{\sqrt{2}}{2}\right)+2\sqrt{2}\,\sqrt{-2\sinh\left(\frac{x}{2}\right)^{2}-1}\,\sqrt{-\sinh(\frac{x}{2}}\cdots}{a\sqrt{a\left(2\sinh\left(\frac{x}{2}\right)^{4}+\sinh\left(\frac{x}{2}\right)^{2}\right)}\,\sinh\left(\frac{x}{2}\right)\sqrt{a\left(2\cosh\left(\frac{x}{2}\right)^{2}-1\right)}}$

input
```
int(1/(a*cosh(x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(2*sinh(1/2*x)^4*a+sinh(1/2*x)^2*a)^(1/2)*(-2^(1/2)*(-2*sinh(1/2*x)^2-
1)^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(2^(1/2)*cosh(1/2*x),1/2*2^(1/2))
+2*2^(1/2)*(-2*sinh(1/2*x)^2-1)^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(2^(
1/2)*cosh(1/2*x),1/2*2^(1/2))+4*sinh(1/2*x)^2*cosh(1/2*x))/(a*(2*sinh(1/2*
x)^4+sinh(1/2*x)^2))^(1/2)/sinh(1/2*x)/(a*(2*cosh(1/2*x)^2-1))^(1/2)
```

3.20.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.20

$$\int \frac{1}{(a\cosh(x))^{3/2}}\,dx = \frac{2\left(\left(\sqrt{2}\cosh(x)^2 + 2\sqrt{2}\cosh(x)\sinh(x) + \sqrt{2}\sinh(x)^2 + \sqrt{2}\right)\sqrt{a}\text{weierstrassZeta}(-\right.}{a^2\cosh(x)^2}$$

input
```
integrate(1/(a*cosh(x))^(3/2),x, algorithm="fricas")
```

output
```
2*((sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 + sq
rt(2))*sqrt(a)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(x) +
 sinh(x))) + 2*sqrt(a*cosh(x))*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
)/(a^2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 + a^2)
```

3.20.6 Sympy [F]

$$\int \frac{1}{(a\cosh(x))^{3/2}}\,dx = \int \frac{1}{(a\cosh(x))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*cosh(x))**(3/2),x)
```

output
```
Integral((a*cosh(x))**(-3/2), x)
```

3.20.7 Maxima [F]

$$\int \frac{1}{(a\cosh(x))^{3/2}}\,dx = \int \frac{1}{(a\cosh(x))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*cosh(x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*cosh(x))^(-3/2), x)
```

3.20.8 Giac [F]

$$\int \frac{1}{(a\cosh(x))^{3/2}}\,dx = \int \frac{1}{(a\cosh(x))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*cosh(x))^(3/2),x, algorithm="giac")
```

output
```
integrate((a*cosh(x))^(-3/2), x)
```

3.20.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\cosh(x))^{3/2}}\,dx = \int \frac{1}{(a\cosh(x))^{3/2}}\,dx$$

input
```
int(1/(a*cosh(x))^(3/2),x)
```

output
```
int(1/(a*cosh(x))^(3/2), x)
```

3.21 $\int \frac{1}{(a\cosh(x))^{5/2}}\,dx$

3.21.1 Optimal result

Integrand size = 8, antiderivative size = 50

$$\int \frac{1}{(a\cosh(x))^{5/2}}\,dx = -\frac{2i\sqrt{\cosh(x)}\,\text{EllipticF}\left(\frac{ix}{2},2\right)}{3a^2\sqrt{a\cosh(x)}}+\frac{2\sinh(x)}{3a(a\cosh(x))^{3/2}}$$

output
```
2/3*sinh(x)/a/(a*cosh(x))^(3/2)-2/3*I*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*El
lipticF(I*sinh(1/2*x),2^(1/2))*cosh(x)^(1/2)/a^2/(a*cosh(x))^(1/2)
```

3.21.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12

$$\int \frac{1}{(a\cosh(x))^{5/2}}\,dx = \frac{2\left(\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\cosh(2x)-\sinh(2x)\right)\sqrt{1+\cosh(2x)+\sinh(2x)}+\right.}{3a^2\sqrt{a\cosh(x)}}$$

input
```
Integrate[(a*Cosh[x])^(-5/2),x]
```

output
```
(2*(Hypergeometric2F1[1/4, 1/2, 5/4, -Cosh[2*x] - Sinh[2*x]]*Sqrt[1 + Cosh
[2*x] + Sinh[2*x]] + Tanh[x]))/(3*a^2*Sqrt[a*Cosh[x]])
```

3.21.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3116, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cosh(x))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sin\left(\frac{\pi}{2}+ix\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$\frac{\int \frac{1}{\sqrt{a\cosh(x)}}dx}{3a^2}+\frac{2\sinh(x)}{3a(a\cosh(x))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sinh(x)}{3a(a\cosh(x))^{3/2}}+\frac{\int \frac{1}{\sqrt{a\sin\left(ix+\frac{\pi}{2}\right)}}dx}{3a^2}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\sqrt{\cosh(x)}\int \frac{1}{\sqrt{\cosh(x)}}dx}{3a^2\sqrt{a\cosh(x)}}+\frac{2\sinh(x)}{3a(a\cosh(x))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sinh(x)}{3a(a\cosh(x))^{3/2}}+\frac{\sqrt{\cosh(x)}\int \frac{1}{\sqrt{\sin\left(ix+\frac{\pi}{2}\right)}}dx}{3a^2\sqrt{a\cosh(x)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sinh(x)}{3a(a\cosh(x))^{3/2}}-\frac{2i\sqrt{\cosh(x)}\,\text{EllipticF}\left(\frac{ix}{2},2\right)}{3a^2\sqrt{a\cosh(x)}}$$

input
```
Int[(a*Cosh[x])^(-5/2),x]
```

output
```
(((-2*I)/3)*Sqrt[Cosh[x]]*EllipticF[(I/2)*x, 2])/(a^2*Sqrt[a*Cosh[x]]) + (
2*Sinh[x])/(3*a*(a*Cosh[x])^(3/2))
```

3.21.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.21.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(55) = 110$.

Time = 0.42 (sec) , antiderivative size = 177, normalized size of antiderivative = 3.54

method	result
default	$\frac{\left(2\sqrt{-2\sinh\left(\frac{x}{2}\right)^2-1}\,\sqrt{-\sinh\left(\frac{x}{2}\right)^2}\,\mathrm{EllipticF}\left(\sqrt{2}\,\cosh\left(\frac{x}{2}\right),\frac{\sqrt{2}}{2}\right)\sqrt{2}\,\sinh\left(\frac{x}{2}\right)^2+\sqrt{2}\,\sqrt{-2\sinh\left(\frac{x}{2}\right)^2-1}\,\sqrt{-\sinh\left(\frac{x}{2}\right)^2}\,\mathrm{EllipticF}\left(\sqrt{2}\,\mathrm{c}\cdots\right.\right.}{3a^2\sqrt{a\left(2\sinh\left(\frac{x}{2}\right)^4+\sinh\left(\frac{x}{2}\right)^2\right)}\,\left(2\cosh\left(\frac{x}{2}\right)^2-1\right)\sinh\left(\frac{x}{2}\right)\sqrt{a\left(2\cosh\left(\frac{x}{2}\right)\cdots\right.}}$

input
```
int(1/(a*cosh(x))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(2*(-2*sinh(1/2*x)^2-1)^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(2^(1/2)
*cosh(1/2*x),1/2*2^(1/2))*2^(1/2)*sinh(1/2*x)^2+2^(1/2)*(-2*sinh(1/2*x)^2-
1)^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(2^(1/2)*cosh(1/2*x),1/2*2^(1/2))
+4*sinh(1/2*x)^2*cosh(1/2*x))/a^2*(a*(2*cosh(1/2*x)^2-1)*sinh(1/2*x)^2)^(1
/2)/(a*(2*sinh(1/2*x)^4+sinh(1/2*x)^2))^(1/2)/(2*cosh(1/2*x)^2-1)/sinh(1/2
*x)/(a*(2*cosh(1/2*x)^2-1))^(1/2)
```

3.21.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 213, normalized size of antiderivative = 4.26

$$\int \frac{1}{(a\cosh(x))^{5/2}}\,dx = \frac{2\left(\left(\sqrt{2}\cosh\left(x\right)^{4}+4\sqrt{2}\cosh\left(x\right)\sinh\left(x\right)^{3}+\sqrt{2}\sinh\left(x\right)^{4}+2\left(3\sqrt{2}\cosh\left(x\right)^{2}+\sqrt{2}\right)\right.\right.}{3\,(}$$

input
```
integrate(1/(a*cosh(x))^(5/2),x, algorithm="fricas")
```

output
```
2/3*((sqrt(2)*cosh(x)^4 + 4*sqrt(2)*cosh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4
+ 2*(3*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^2 + 2*sqrt(2)*cosh(x)^2 + 4*(s
qrt(2)*cosh(x)^3 + sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*sqrt(a)*weierstrass
PInverse(-4, 0, cosh(x) + sinh(x)) + 2*(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 +
sinh(x)^3 + (3*cosh(x)^2 - 1)*sinh(x) - cosh(x))*sqrt(a*cosh(x)))/(a^3*cos
h(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4 + 2*a^3*cosh(x)^2 + a^3 +
 2*(3*a^3*cosh(x)^2 + a^3)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + a^3*cosh(x))*sin
h(x))
```

3.21.6 Sympy [F]

$$\int \frac{1}{(a\cosh(x))^{5/2}}\,dx = \int \frac{1}{\left(a\cosh\left(x\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*cosh(x))**(5/2),x)
```

output
```
Integral((a*cosh(x))**(-5/2), x)
```

3.21.7 Maxima [F]

$$\int \frac{1}{(a\cosh(x))^{5/2}}\,dx = \int \frac{1}{(a\cosh{(x)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*cosh(x))^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*cosh(x))^(-5/2), x)
```

3.21.8 Giac [F]

$$\int \frac{1}{(a\cosh(x))^{5/2}}\,dx = \int \frac{1}{(a\cosh{(x)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*cosh(x))^(5/2),x, algorithm="giac")
```

output
```
integrate((a*cosh(x))^(-5/2), x)
```

3.21.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\cosh(x))^{5/2}}\,dx = \int \frac{1}{(a\cosh{(x)})^{5/2}}\,dx$$

input
```
int(1/(a*cosh(x))^(5/2),x)
```

output
```
int(1/(a*cosh(x))^(5/2), x)
```

3.22 $\int \frac{1}{(a\cosh(x))^{7/2}}\,dx$

3.22.1 Optimal result

Integrand size = 8, antiderivative size = 67

$$\int \frac{1}{(a\cosh(x))^{7/2}}\,dx=\frac{6i\sqrt{a\cosh(x)}E\left(\frac{ix}{2}\middle|2\right)}{5a^4\sqrt{\cosh(x)}}+\frac{2\sinh(x)}{5a(a\cosh(x))^{5/2}}+\frac{6\sinh(x)}{5a^3\sqrt{a\cosh(x)}}$$

output
```
2/5*sinh(x)/a/(a*cosh(x))^(5/2)+6/5*sinh(x)/a^3/(a*cosh(x))^(1/2)+6/5*I*(c
osh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(1/2*x),2^(1/2))*(a*cosh(x
))^(1/2)/a^4/cosh(x)^(1/2)
```

3.22.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.64

$$\int \frac{1}{(a\cosh(x))^{7/2}}\,dx=\frac{2\left(3i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)+3\cosh(x)\sinh(x)+\tanh(x)\right)}{5a^2(a\cosh(x))^{3/2}}$$

input
```
Integrate[(a*Cosh[x])^(-7/2),x]
```

output
```
(2*((3*I)*Cosh[x]^(3/2)*EllipticE[(I/2)*x, 2] + 3*Cosh[x]*Sinh[x] + Tanh[x
]))/(5*a^2*(a*Cosh[x])^(3/2))
```

3.22.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3116, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cosh(x))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sin\left(\frac{\pi}{2}+ix\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$\frac{3\int \frac{1}{(a\cosh(x))^{3/2}}dx}{5a^2}+\frac{2\sinh(x)}{5a(a\cosh(x))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sinh(x)}{5a(a\cosh(x))^{5/2}}+\frac{3\int \frac{1}{\left(a\sin\left(ix+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5a^2}$$

$$\Big\downarrow \text{3116}$$

$$\frac{3\left(\frac{2\sinh(x)}{a\sqrt{a\cosh(x)}}-\frac{\int \sqrt{a\cosh(x)}dx}{a^2}\right)}{5a^2}+\frac{2\sinh(x)}{5a(a\cosh(x))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sinh(x)}{5a(a\cosh(x))^{5/2}}+\frac{3\left(\frac{2\sinh(x)}{a\sqrt{a\cosh(x)}}-\frac{\int \sqrt{a\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2}\right)}{5a^2}$$

$$\Big\downarrow \text{3121}$$

$$\frac{3\left(\frac{2\sinh(x)}{a\sqrt{a\cosh(x)}}-\frac{\sqrt{a\cosh(x)}\int \sqrt{\cosh(x)}dx}{a^2\sqrt{\cosh(x)}}\right)}{5a^2}+\frac{2\sinh(x)}{5a(a\cosh(x))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sinh(x)}{5a(a\cosh(x))^{5/2}}+\frac{3\left(\frac{2\sinh(x)}{a\sqrt{a\cosh(x)}}-\frac{\sqrt{a\cosh(x)}\int \sqrt{\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2\sqrt{\cosh(x)}}\right)}{5a^2}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\sinh(x)}{5a(a\cosh(x))^{5/2}}+\frac{3\left(\frac{2\sinh(x)}{a\sqrt{a\cosh(x)}}+\frac{2iE\left(\frac{ix}{2}|2\right)\sqrt{a\cosh(x)}}{a^2\sqrt{\cosh(x)}}\right)}{5a^2}$$

input
```
Int[(a*Cosh[x])^(-7/2),x]
```

output
```
(2*Sinh[x])/(5*a*(a*Cosh[x])^(5/2)) + (3*(((2*I)*Sqrt[a*Cosh[x]]*EllipticE
[(I/2)*x, 2])/(a^2*Sqrt[Cosh[x]]) + (2*Sinh[x])/(a*Sqrt[a*Cosh[x]])))/(5*a
^2)
```

3.22.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.22.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(68) = 136$.

Time = 0.68 (sec) , antiderivative size = 254, normalized size of antiderivative = 3.79

method	result
default	$\frac{2\sqrt{a\left(2\cosh\left(\frac{x}{2}\right)^{2}-1\right)\sinh\left(\frac{x}{2}\right)^{2}}\left(\frac{\cosh\left(\frac{x}{2}\right)\sqrt{a\left(2\sinh\left(\frac{x}{2}\right)^{4}+\sinh\left(\frac{x}{2}\right)^{2}\right)}}{20a\left(\cosh\left(\frac{x}{2}\right)^{2}-\frac{1}{2}\right)^{3}}+\frac{6\sinh\left(\frac{x}{2}\right)^{2}\cosh\left(\frac{x}{2}\right)}{5\sqrt{a\left(2\cosh\left(\frac{x}{2}\right)^{2}-1\right)\sinh\left(\frac{x}{2}\right)^{2}}}+\frac{3\sqrt{2}\sqrt{-2\cosh\left(\frac{x}{2}\right)^{2}+1}\sqrt{-\sinh\left(\cdots\right.}}{10\sqrt{a\left(2\sinh\left(\cdots\right.}}\right.}{a^{3}\sinh\left(\frac{x}{2}\right)\sqrt{a\left(2\,\mathrm{c}\cdots\right.}}$

input
```
int(1/(a*cosh(x))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2*(a*(2*cosh(1/2*x)^2-1)*sinh(1/2*x)^2)^(1/2)/a^3*(1/20*cosh(1/2*x)/a*(a*(
2*sinh(1/2*x)^4+sinh(1/2*x)^2))^(1/2)/(cosh(1/2*x)^2-1/2)^3+6/5*sinh(1/2*x
)^2*cosh(1/2*x)/(a*(2*cosh(1/2*x)^2-1)*sinh(1/2*x)^2)^(1/2)+3/10*2^(1/2)*(
-2*cosh(1/2*x)^2+1)^(1/2)*(-sinh(1/2*x)^2)^(1/2)/(a*(2*sinh(1/2*x)^4+sinh(
1/2*x)^2))^(1/2)*EllipticF(2^(1/2)*cosh(1/2*x),1/2*2^(1/2))-3/5*2^(1/2)*(-
2*cosh(1/2*x)^2+1)^(1/2)*(-sinh(1/2*x)^2)^(1/2)/(a*(2*sinh(1/2*x)^4+sinh(1
/2*x)^2))^(1/2)*(EllipticF(2^(1/2)*cosh(1/2*x),1/2*2^(1/2))-EllipticE(2^(1
/2)*cosh(1/2*x),1/2*2^(1/2))))/sinh(1/2*x)/(a*(2*cosh(1/2*x)^2-1))^(1/2)
```

3.22.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 422, normalized size of antiderivative = 6.30

$$\int \frac{1}{(a\cosh(x))^{7/2}}\,dx = \frac{2\left(3\left(\sqrt{2}\cosh\left(x\right)^{6}+6\sqrt{2}\cosh\left(x\right)\sinh\left(x\right)^{5}+\sqrt{2}\sinh\left(x\right)^{6}+3\left(5\sqrt{2}\cosh\left(x\right)^{2}+\sqrt{2}\cdots\right.\right.\right.}{\cdots}$$

input
```
integrate(1/(a*cosh(x))^(7/2),x, algorithm="fricas")
```

output
```
2/5*(3*(sqrt(2)*cosh(x)^6 + 6*sqrt(2)*cosh(x)*sinh(x)^5 + sqrt(2)*sinh(x)^
6 + 3*(5*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^4 + 3*sqrt(2)*cosh(x)^4 + 4*
(5*sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x))*sinh(x)^3 + 3*(5*sqrt(2)*cosh(x)
^4 + 6*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^2 + 3*sqrt(2)*cosh(x)^2 + 6*(s
qrt(2)*cosh(x)^5 + 2*sqrt(2)*cosh(x)^3 + sqrt(2)*cosh(x))*sinh(x) + sqrt(2
))*sqrt(a)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(x) + sin
h(x))) + 2*(3*cosh(x)^6 + 18*cosh(x)*sinh(x)^5 + 3*sinh(x)^6 + (45*cosh(x)
^2 + 8)*sinh(x)^4 + 8*cosh(x)^4 + 4*(15*cosh(x)^3 + 8*cosh(x))*sinh(x)^3 +
 (45*cosh(x)^4 + 48*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(9*cosh(x)^5
+ 16*cosh(x)^3 + cosh(x))*sinh(x))*sqrt(a*cosh(x)))/(a^4*cosh(x)^6 + 6*a^4
*cosh(x)*sinh(x)^5 + a^4*sinh(x)^6 + 3*a^4*cosh(x)^4 + 3*a^4*cosh(x)^2 + 3
*(5*a^4*cosh(x)^2 + a^4)*sinh(x)^4 + a^4 + 4*(5*a^4*cosh(x)^3 + 3*a^4*cosh
(x))*sinh(x)^3 + 3*(5*a^4*cosh(x)^4 + 6*a^4*cosh(x)^2 + a^4)*sinh(x)^2 + 6
*(a^4*cosh(x)^5 + 2*a^4*cosh(x)^3 + a^4*cosh(x))*sinh(x))
```

3.22.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a\cosh(x))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*cosh(x))**(7/2),x)
```

output
```
Timed out
```

3.22.7 Maxima [F]

$$\int \frac{1}{(a\cosh(x))^{7/2}}\,dx = \int \frac{1}{(a\cosh(x))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a*cosh(x))^(7/2),x, algorithm="maxima")
```

output
```
integrate((a*cosh(x))^(-7/2), x)
```

3.22.8 Giac [F]

$$\int \frac{1}{(a\cosh(x))^{7/2}}\,dx = \int \frac{1}{(a\cosh(x))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a*cosh(x))^(7/2),x, algorithm="giac")
```

output
```
integrate((a*cosh(x))^(-7/2), x)
```

3.22.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\cosh(x))^{7/2}}\,dx = \int \frac{1}{(a\cosh(x))^{7/2}}\,dx$$

input
```
int(1/(a*cosh(x))^(7/2),x)
```

output
```
int(1/(a*cosh(x))^(7/2), x)
```

3.23 $\int (b\cosh(c+dx))^n\,dx$

3.23.1 Optimal result

Integrand size = 10, antiderivative size = 71

$$\int (b\cosh(c+dx))^n\,dx = -\frac{(b\cosh(c+dx))^{1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cosh^2(c+dx)\right)\sinh(c+dx)}{bd(1+n)\sqrt{-\sinh^2(c+dx)}}$$

output
```
-(b*cosh(d*x+c))^(1+n)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cosh(d*x+c)^
2)*sinh(d*x+c)/b/d/(1+n)/(-sinh(d*x+c)^2)^(1/2)
```

3.23.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int (b\cosh(c+dx))^n\,dx = \frac{(b\cosh(c+dx))^n\coth(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cosh^2(c+dx)\right)\sqrt{-\sinh^2(c+dx)}}{d(1+n)}$$

input
```
Integrate[(b*Cosh[c + d*x])^n,x]
```

output
```
((b*Cosh[c + d*x])^n*Coth[c + d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 +
n)/2, Cosh[c + d*x]^2]*Sqrt[-Sinh[c + d*x]^2])/(d*(1 + n))
```

3.23.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\cosh(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{\sinh(c+dx)(b\cosh(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cosh^2(c+dx)\right)}{bd(n+1)\sqrt{-\sinh^2(c+dx)}}$$

input
```
Int[(b*Cosh[c + d*x])^n,x]
```

output
```
-(((b*Cosh[c + d*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2,
Cosh[c + d*x]^2]*Sinh[c + d*x])/(b*d*(1 + n)*Sqrt[-Sinh[c + d*x]^2]))
```

3.23.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.23.4 Maple [F]

$$\int (b\cosh (dx+c))^{n}\, dx$$

input
```
int((b*cosh(d*x+c))^n,x)
```

output
```
int((b*cosh(d*x+c))^n,x)
```

3.23.5 Fricas [F]

$$\int (b\cosh(c+dx))^n\, dx = \int (b\cosh (dx+c))^{n}\, dx$$

input
```
integrate((b*cosh(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*cosh(d*x + c))^n, x)
```

3.23.6 Sympy [F]

$$\int (b\cosh(c+dx))^n\, dx = \int (b\cosh (c+dx))^{n}\, dx$$

input
```
integrate((b*cosh(d*x+c))**n,x)
```

output
```
Integral((b*cosh(c + d*x))**n, x)
```

3.23.7 Maxima [F]

$$\int (b\cosh(c+dx))^n\,dx = \int (b\cosh(dx+c))^n\,dx$$

input
```
integrate((b*cosh(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*cosh(d*x + c))^n, x)
```

3.23.8 Giac [F]

$$\int (b\cosh(c+dx))^n\,dx = \int (b\cosh(dx+c))^n\,dx$$

input
```
integrate((b*cosh(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*cosh(d*x + c))^n, x)
```

3.23.9 Mupad [F(-1)]

Timed out.

$$\int (b\cosh(c+dx))^n\,dx = \int (b\cosh(c+d\,x))^n\,dx$$

input
```
int((b*cosh(c + d*x))^n,x)
```

output
```
int((b*cosh(c + d*x))^n, x)
```

3.24 $\int \frac{\cosh^4(x)}{a+a\cosh(x)}\,dx$

3.24.1 Optimal result

Integrand size = 13, antiderivative size = 54

$$\int \frac{\cosh^4(x)}{a+a\cosh(x)}\,dx = -\frac{3x}{2a}+\frac{4\sinh(x)}{a}-\frac{3\cosh(x)\sinh(x)}{2a}-\frac{\cosh^3(x)\sinh(x)}{a+a\cosh(x)}+\frac{4\sinh^3(x)}{3a}$$

output
```
-3/2*x/a+4*sinh(x)/a-3/2*cosh(x)*sinh(x)/a-cosh(x)^3*sinh(x)/(a+a*cosh(x))
+4/3*sinh(x)^3/a
```

3.24.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{\cosh^4(x)}{a+a\cosh(x)}\,dx = \frac{\operatorname{sech}\left(\frac{x}{2}\right)\left(-36x\cosh\left(\frac{x}{2}\right)+45\sinh\left(\frac{x}{2}\right)+18\sinh\left(\frac{3x}{2}\right)-2\sinh\left(\frac{5x}{2}\right)+\sinh\left(\frac{7x}{2}\right)\right)}{24a}$$

input
```
Integrate[Cosh[x]^4/(a + a*Cosh[x]),x]
```

output
```
(Sech[x/2]*(-36*x*Cosh[x/2] + 45*Sinh[x/2] + 18*Sinh[(3*x)/2] - 2*Sinh[(5*
x)/2] + Sinh[(7*x)/2]))/(24*a)
```

3.24.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {3042, 3246, 3042, 3227, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^4(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)^4}{a+a\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3246}$$

$$-\frac{\int \cosh^2(x)(3a-4a\cosh(x))dx}{a^2}-\frac{\sinh(x)\cosh^3(x)}{a\cosh(x)+a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(x)\cosh^3(x)}{a\cosh(x)+a}-\frac{\int \sin\left(ix+\frac{\pi}{2}\right)^2\left(3a-4a\sin\left(ix+\frac{\pi}{2}\right)\right)dx}{a^2}$$

$$\Big\downarrow \text{3227}$$

$$-\frac{3a\int \cosh^2(x)dx-4a\int \cosh^3(x)dx}{a^2}-\frac{\sinh(x)\cosh^3(x)}{a\cosh(x)+a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(x)\cosh^3(x)}{a\cosh(x)+a}-\frac{3a\int \sin\left(ix+\frac{\pi}{2}\right)^2dx-4a\int \sin\left(ix+\frac{\pi}{2}\right)^3dx}{a^2}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{\sinh(x)\cosh^3(x)}{a\cosh(x)+a}-\frac{3a\int \sin\left(ix+\frac{\pi}{2}\right)^2dx-4ia\int \left(\sinh^2(x)+1\right)d(-i\sinh(x))}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sinh(x)\cosh^3(x)}{a\cosh(x)+a}-\frac{3a\int \sin\left(ix+\frac{\pi}{2}\right)^2dx-4ia\left(-\frac{1}{3}i\sinh^3(x)-i\sinh(x)\right)}{a^2}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{\sinh(x)\cosh^3(x)}{a\cosh(x)+a}-\frac{3a\left(\frac{\int 1dx}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)-4ia\left(-\frac{1}{3}i\sinh^3(x)-i\sinh(x)\right)}{a^2}$$

$$\Big\downarrow \; 24$$

$$-\frac{\sinh(x)\cosh^3(x)}{a\cosh(x)+a}-\frac{3a\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)-4ia\left(-\frac{1}{3}i\sinh^3(x)-i\sinh(x)\right)}{a^2}$$

input
```
Int[Cosh[x]^4/(a + a*Cosh[x]),x]
```

output
```
-((Cosh[x]^3*Sinh[x])/(a + a*Cosh[x])) - (3*a*(x/2 + (Cosh[x]*Sinh[x])/2)
- (4*I)*a*((-I)*Sinh[x] - (I/3)*Sinh[x]^3))/a^2
```

3.24.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3246
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((c + d*Sin[e +
 f*x])^(n - 1)/(a*f*(a + b*Sin[e + f*x]))), x] - Simp[d/(a*b)   Int[(c + d*
Sin[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[
e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] ||
EqQ[c, 0])
```

3.24.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.69

method	result
parallelrisch	$-\frac{\operatorname{csch}(x)(36x\sinh(x)+3\cosh(3x)-27\cosh(x)-\cosh(4x)-20\cosh(2x)+45)}{24a}$
risch	$\frac{e^{4x}-2e^{3x}-69-18e^{-x}+2e^{-2x}+18e^{2x}-36x e^{x}-e^{-3x}+21e^{x}-36x}{24(e^{x}+1)a}$
default	$\frac{\tanh(\frac{x}{2})-\frac{1}{3(\tanh(\frac{x}{2})-1)^3}-\frac{1}{(\tanh(\frac{x}{2})-1)^2}-\frac{5}{2(\tanh(\frac{x}{2})-1)}+\frac{3\ln(\tanh(\frac{x}{2})-1)}{2}-\frac{1}{3(\tanh(\frac{x}{2})+1)^3}+\frac{1}{(\tanh(\frac{x}{2})+1)^2}-\frac{5}{2(\tanh(\frac{x}{2}}{a}$

input
```
int(cosh(x)^4/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/24*csch(x)*(36*x*sinh(x)+3*cosh(3*x)-27*cosh(x)-cosh(4*x)-20*cosh(2*x)+
45)/a
```

3.24.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(48) = 96$.

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.85

$$\int \frac{\cosh^4(x)}{a+a\cosh(x)}\,dx$$
$$= \frac{\cosh(x)^4 + (4\cosh(x)-1)\sinh(x)^3 + \sinh(x)^4 - 3\cosh(x)^3 + \left(6\cosh(x)^2 - 9\cosh(x) + 20\right)\sinh(x)}{24\,(a\cosh}$$

input
```
integrate(cosh(x)^4/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
1/24*(cosh(x)^4 + (4*cosh(x) - 1)*sinh(x)^3 + sinh(x)^4 - 3*cosh(x)^3 + (6
*cosh(x)^2 - 9*cosh(x) + 20)*sinh(x)^2 - 3*(12*x - 1)*cosh(x) + 20*cosh(x)
^2 + (4*cosh(x)^3 - 3*cosh(x)^2 - 36*x + 32*cosh(x) + 39)*sinh(x) - 36*x -
 69)/(a*cosh(x) + a*sinh(x) + a)
```

3.24.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(49) = 98$.

Time = 0.58 (sec) , antiderivative size = 337, normalized size of antiderivative = 6.24

$$\begin{aligned}\int \frac{\cosh^4(x)}{a + a\cosh(x)}\,dx = &-\frac{9x\tanh^6\left(\frac{x}{2}\right)}{6a\tanh^6\left(\frac{x}{2}\right) - 18a\tanh^4\left(\frac{x}{2}\right) + 18a\tanh^2\left(\frac{x}{2}\right) - 6a} \\ &+ \frac{27x\tanh^4\left(\frac{x}{2}\right)}{6a\tanh^6\left(\frac{x}{2}\right) - 18a\tanh^4\left(\frac{x}{2}\right) + 18a\tanh^2\left(\frac{x}{2}\right) - 6a} \\ &- \frac{27x\tanh^2\left(\frac{x}{2}\right)}{6a\tanh^6\left(\frac{x}{2}\right) - 18a\tanh^4\left(\frac{x}{2}\right) + 18a\tanh^2\left(\frac{x}{2}\right) - 6a} \\ &+ \frac{9x}{6a\tanh^6\left(\frac{x}{2}\right) - 18a\tanh^4\left(\frac{x}{2}\right) + 18a\tanh^2\left(\frac{x}{2}\right) - 6a} \\ &+ \frac{6\tanh^7\left(\frac{x}{2}\right)}{6a\tanh^6\left(\frac{x}{2}\right) - 18a\tanh^4\left(\frac{x}{2}\right) + 18a\tanh^2\left(\frac{x}{2}\right) - 6a} \\ &- \frac{48\tanh^5\left(\frac{x}{2}\right)}{6a\tanh^6\left(\frac{x}{2}\right) - 18a\tanh^4\left(\frac{x}{2}\right) + 18a\tanh^2\left(\frac{x}{2}\right) - 6a} \\ &+ \frac{50\tanh^3\left(\frac{x}{2}\right)}{6a\tanh^6\left(\frac{x}{2}\right) - 18a\tanh^4\left(\frac{x}{2}\right) + 18a\tanh^2\left(\frac{x}{2}\right) - 6a} \\ &- \frac{24\tanh\left(\frac{x}{2}\right)}{6a\tanh^6\left(\frac{x}{2}\right) - 18a\tanh^4\left(\frac{x}{2}\right) + 18a\tanh^2\left(\frac{x}{2}\right) - 6a}\end{aligned}$$

input
```
integrate(cosh(x)**4/(a+a*cosh(x)),x)
```

output
```
-9*x*tanh(x/2)**6/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4 + 18*a*tanh(x/2)**
2 - 6*a) + 27*x*tanh(x/2)**4/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4 + 18*a*
tanh(x/2)**2 - 6*a) - 27*x*tanh(x/2)**2/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)
**4 + 18*a*tanh(x/2)**2 - 6*a) + 9*x/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4
 + 18*a*tanh(x/2)**2 - 6*a) + 6*tanh(x/2)**7/(6*a*tanh(x/2)**6 - 18*a*tanh
(x/2)**4 + 18*a*tanh(x/2)**2 - 6*a) - 48*tanh(x/2)**5/(6*a*tanh(x/2)**6 -
18*a*tanh(x/2)**4 + 18*a*tanh(x/2)**2 - 6*a) + 50*tanh(x/2)**3/(6*a*tanh(x
/2)**6 - 18*a*tanh(x/2)**4 + 18*a*tanh(x/2)**2 - 6*a) - 24*tanh(x/2)/(6*a*
tanh(x/2)**6 - 18*a*tanh(x/2)**4 + 18*a*tanh(x/2)**2 - 6*a)
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.22

$$\int \frac{\cosh^4(x)}{a+a\cosh(x)}\,dx = -\frac{3\,x}{2\,a} - \frac{21\,e^{(-x)} - 3\,e^{(-2\,x)} + e^{(-3\,x)}}{24\,a} - \frac{2\,e^{(-x)} - 18\,e^{(-2\,x)} - 69\,e^{(-3\,x)} - 1}{24\,(ae^{(-3\,x)} + ae^{(-4\,x)})}$$

input
```
integrate(cosh(x)^4/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-3/2*x/a - 1/24*(21*e^(-x) - 3*e^(-2*x) + e^(-3*x))/a - 1/24*(2*e^(-x) - 1
8*e^(-2*x) - 69*e^(-3*x) - 1)/(a*e^(-3*x) + a*e^(-4*x))
```

3.24.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

$$\int \frac{\cosh^4(x)}{a+a\cosh(x)}\,dx = -\frac{3\,x}{2\,a} - \frac{\left(69\,e^{(3\,x)} + 18\,e^{(2\,x)} - 2\,e^{x} + 1\right)e^{(-3\,x)}}{24\,a(e^{x}+1)} + \frac{a^2e^{(3\,x)} - 3\,a^2e^{(2\,x)} + 21\,a^2e^{x}}{24\,a^3}$$

input
```
integrate(cosh(x)^4/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
-3/2*x/a - 1/24*(69*e^(3*x) + 18*e^(2*x) - 2*e^x + 1)*e^(-3*x)/(a*(e^x + 1
)) + 1/24*(a^2*e^(3*x) - 3*a^2*e^(2*x) + 21*a^2*e^x)/a^3
```

3.24.9 Mupad [B] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

$$\int \frac{\cosh^4(x)}{a+a\cosh(x)}\,dx = \frac{e^{-2x}}{8a} - \frac{7e^{-x}}{8a} - \frac{e^{2x}}{8a} - \frac{e^{-3x}}{24a} + \frac{e^{3x}}{24a} - \frac{3x}{2a} - \frac{2}{a(e^x+1)} + \frac{7e^x}{8a}$$

input
```
int(cosh(x)^4/(a + a*cosh(x)),x)
```

output
```
exp(-2*x)/(8*a) - (7*exp(-x))/(8*a) - exp(2*x)/(8*a) - exp(-3*x)/(24*a) +
exp(3*x)/(24*a) - (3*x)/(2*a) - 2/(a*(exp(x) + 1)) + (7*exp(x))/(8*a)
```

3.25 $\int \frac{\cosh^3(x)}{a+a\cosh(x)}\,dx$

3.25.1 Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \frac{\cosh^3(x)}{a+a\cosh(x)}\,dx = \frac{3x}{2a}-\frac{2\sinh(x)}{a}+\frac{3\cosh(x)\sinh(x)}{2a}-\frac{\cosh^2(x)\sinh(x)}{a+a\cosh(x)}$$

output
```
3/2*x/a-2*sinh(x)/a+3/2*cosh(x)*sinh(x)/a-cosh(x)^2*sinh(x)/(a+a*cosh(x))
```

3.25.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{\cosh^3(x)}{a+a\cosh(x)}\,dx = \frac{\operatorname{sech}\left(\frac{x}{2}\right)\left(12x\cosh\left(\frac{x}{2}\right)-12\sinh\left(\frac{x}{2}\right)-3\sinh\left(\frac{3x}{2}\right)+\sinh\left(\frac{5x}{2}\right)\right)}{8a}$$

input
```
Integrate[Cosh[x]^3/(a + a*Cosh[x]),x]
```

output
```
(Sech[x/2]*(12*x*Cosh[x/2] - 12*Sinh[x/2] - 3*Sinh[(3*x)/2] + Sinh[(5*x)/2
]))/(8*a)
```

3.25.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3246, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(x)}{a\cosh(x)+a}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)^3}{a+a\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{ 3246}$$

$$-\frac{\int \cosh(x)(2a-3a\cosh(x))dx}{a^2}-\frac{\sinh(x)\cosh^2(x)}{a\cosh(x)+a}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\sinh(x)\cosh^2(x)}{a\cosh(x)+a}-\frac{\int \sin\left(ix+\frac{\pi}{2}\right)\left(2a-3a\sin\left(ix+\frac{\pi}{2}\right)\right)dx}{a^2}$$

$$\Big\downarrow \text{ 3213}$$

$$-\frac{-\frac{3ax}{2}+2a\sinh(x)-\frac{3}{2}a\sinh(x)\cosh(x)}{a^2}-\frac{\sinh(x)\cosh^2(x)}{a\cosh(x)+a}$$

input
```
Int[Cosh[x]^3/(a + a*Cosh[x]),x]
```

output
```
-((Cosh[x]^2*Sinh[x])/(a + a*Cosh[x])) - ((-3*a*x)/2 + 2*a*Sinh[x] - (3*a*
Cosh[x]*Sinh[x])/2)/a^2
```

3.25.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3246
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((c + d*Sin[e +
 f*x])^(n - 1)/(a*f*(a + b*Sin[e + f*x]))), x] - Simp[d/(a*b)   Int[(c + d*
Sin[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[
e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] ||
EqQ[c, 0])
```

3.25.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

method	result	size
parallelrisch	$\frac{\coth(x)\cosh(2x)+(-4\cosh(x)-5)\coth(x)+6x+8\,\operatorname{csch}(x)}{4a}$	30
risch	$\frac{e^{3x}-3\,e^{2x}+20+3\,e^{-x}+12x\,e^{x}-4\,e^{x}-e^{-2x}+12x}{8(e^{x}+1)a}$	48
default	$\frac{-\tanh\left(\frac{x}{2}\right)+\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{3}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{3}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}}{a}$	70

input
```
int(cosh(x)^3/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*(coth(x)*cosh(2*x)+(-4*cosh(x)-5)*coth(x)+6*x+8*csch(x))/a
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.63

$$\int \frac{\cosh^3(x)}{a+a\cosh(x)}\,dx$$
$$= \frac{\cosh(x)^3 + (3\cosh(x) - 4)\sinh(x)^2 + \sinh(x)^3 + (12x - 1)\cosh(x) - 4\cosh(x)^2 + (3\cosh(x)^2 + 12}{8(a\cosh(x) + a\sinh(x) + a)}$$

input
```
integrate(cosh(x)^3/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
1/8*(cosh(x)^3 + (3*cosh(x) - 4)*sinh(x)^2 + sinh(x)^3 + (12*x - 1)*cosh(x
) - 4*cosh(x)^2 + (3*cosh(x)^2 + 12*x - 4*cosh(x) - 7)*sinh(x) + 12*x + 20
)/(a*cosh(x) + a*sinh(x) + a)
```

3.25.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(39) = 78.

Time = 0.37 (sec) , antiderivative size = 189, normalized size of antiderivative = 4.40

$$\int \frac{\cosh^3(x)}{a+a\cosh(x)}\,dx = \frac{3x\tanh^4\left(\frac{x}{2}\right)}{2a\tanh^4\left(\frac{x}{2}\right) - 4a\tanh^2\left(\frac{x}{2}\right) + 2a} - \frac{6x\tanh^2\left(\frac{x}{2}\right)}{2a\tanh^4\left(\frac{x}{2}\right) - 4a\tanh^2\left(\frac{x}{2}\right) + 2a}$$
$$+ \frac{3x}{2a\tanh^4\left(\frac{x}{2}\right) - 4a\tanh^2\left(\frac{x}{2}\right) + 2a}$$
$$- \frac{2\tanh^5\left(\frac{x}{2}\right)}{2a\tanh^4\left(\frac{x}{2}\right) - 4a\tanh^2\left(\frac{x}{2}\right) + 2a}$$
$$+ \frac{10\tanh^3\left(\frac{x}{2}\right)}{2a\tanh^4\left(\frac{x}{2}\right) - 4a\tanh^2\left(\frac{x}{2}\right) + 2a}$$
$$- \frac{4\tanh\left(\frac{x}{2}\right)}{2a\tanh^4\left(\frac{x}{2}\right) - 4a\tanh^2\left(\frac{x}{2}\right) + 2a}$$

input
```
integrate(cosh(x)**3/(a+a*cosh(x)),x)
```

output
```
3*x*tanh(x/2)**4/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a) - 6*x*tanh(x/
2)**2/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a) + 3*x/(2*a*tanh(x/2)**4
- 4*a*tanh(x/2)**2 + 2*a) - 2*tanh(x/2)**5/(2*a*tanh(x/2)**4 - 4*a*tanh(x/
2)**2 + 2*a) + 10*tanh(x/2)**3/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a)
 - 4*tanh(x/2)/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a)
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.30

$$\int \frac{\cosh^3(x)}{a + a\cosh(x)}\,dx = \frac{3\,x}{2\,a} + \frac{4\,e^{(-x)} - e^{(-2\,x)}}{8\,a} - \frac{3\,e^{(-x)} + 20\,e^{(-2\,x)} - 1}{8\,(ae^{(-2\,x)} + ae^{(-3\,x)})}$$

input
```
integrate(cosh(x)^3/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
3/2*x/a + 1/8*(4*e^(-x) - e^(-2*x))/a - 1/8*(3*e^(-x) + 20*e^(-2*x) - 1)/(
a*e^(-2*x) + a*e^(-3*x))
```

3.25.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int \frac{\cosh^3(x)}{a + a\cosh(x)}\,dx = \frac{3\,x}{2\,a} + \frac{(20\,e^{(2\,x)} + 3\,e^x - 1)e^{(-2\,x)}}{8\,a(e^x + 1)} + \frac{ae^{(2\,x)} - 4\,ae^x}{8\,a^2}$$

input
```
integrate(cosh(x)^3/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
3/2*x/a + 1/8*(20*e^(2*x) + 3*e^x - 1)*e^(-2*x)/(a*(e^x + 1)) + 1/8*(a*e^(
2*x) - 4*a*e^x)/a^2
```

3.25.9 Mupad [B] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.21

$$\int \frac{\cosh^3(x)}{a + a\cosh(x)}\,dx = \frac{e^{-x}}{2\,a} - \frac{e^{-2\,x}}{8\,a} + \frac{e^{2\,x}}{8\,a} + \frac{3\,x}{2\,a} + \frac{2}{a\,(e^x + 1)} - \frac{e^x}{2\,a}$$

input
```
int(cosh(x)^3/(a + a*cosh(x)),x)
```

output
```
exp(-x)/(2*a) - exp(-2*x)/(8*a) + exp(2*x)/(8*a) + (3*x)/(2*a) + 2/(a*(exp
(x) + 1)) - exp(x)/(2*a)
```

3.26 $\int \frac{\cosh^2(x)}{a+a\cosh(x)}\,dx$

3.26.1 Optimal result

Integrand size = 13, antiderivative size = 25

$$\int \frac{\cosh^2(x)}{a+a\cosh(x)}\,dx = -\frac{x}{a}+\frac{\sinh(x)}{a}+\frac{\sinh(x)}{a(1+\cosh(x))}$$

output
```
-x/a+sinh(x)/a+sinh(x)/a/(1+cosh(x))
```

3.26.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \frac{\cosh^2(x)}{a+a\cosh(x)}\,dx = \frac{-2x+\operatorname{sech}\left(\frac{x}{2}\right)\sinh\left(\frac{3x}{2}\right)+3\tanh\left(\frac{x}{2}\right)}{2a}$$

input
```
Integrate[Cosh[x]^2/(a + a*Cosh[x]),x]
```

output
```
(-2*x + Sech[x/2]*Sinh[(3*x)/2] + 3*Tanh[x/2])/(2*a)
```

3.26.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3042, 3225, 25, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(x)}{a\cosh(x)+a}dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)^2}{a+a\sin\left(\frac{\pi}{2}+ix\right)}dx$$
$$\Big\downarrow \ 3225$$
$$\frac{\int -\frac{\cosh(x)}{\cosh(x)+1}dx}{a}+\frac{\sinh(x)}{a}$$
$$\Big\downarrow \ 25$$
$$\frac{\sinh(x)}{a}-\frac{\int \frac{\cosh(x)}{\cosh(x)+1}dx}{a}$$
$$\Big\downarrow \ 3042$$
$$\frac{\sinh(x)}{a}-\frac{\int \frac{\sin\left(ix+\frac{\pi}{2}\right)}{\sin\left(ix+\frac{\pi}{2}\right)+1}dx}{a}$$
$$\Big\downarrow \ 3214$$
$$\frac{\sinh(x)}{a}-\frac{x-\int \frac{1}{\cosh(x)+1}dx}{a}$$
$$\Big\downarrow \ 3042$$
$$\frac{\sinh(x)}{a}-\frac{x-\int \frac{1}{\sin\left(ix+\frac{\pi}{2}\right)+1}dx}{a}$$
$$\Big\downarrow \ 3127$$
$$\frac{\sinh(x)}{a}-\frac{x-\frac{\sinh(x)}{\cosh(x)+1}}{a}$$

input
```
Int[Cosh[x]^2/(a + a*Cosh[x]),x]
```

output
```
Sinh[x]/a - (x - Sinh[x]/(1 + Cosh[x]))/a
```

3.26.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3225
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Simp[1/d
Int[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.26.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
parallelrisch	$\frac{\coth(x)\cosh(x)-x+\coth(x)-2\ \mathrm{csch}(x)}{a}$	20
risch	$-\frac{x}{a}+\frac{e^{x}}{2a}-\frac{e^{-x}}{2a}-\frac{2}{(e^{x}+1)a}$	35
default	$\frac{\tanh\left(\frac{x}{2}\right)-\frac{1}{\tanh\left(\frac{x}{2}\right)-1}+\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{1}{\tanh\left(\frac{x}{2}\right)+1}-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}$	46

input
```
int(cosh(x)^2/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
(coth(x)*cosh(x)-x+coth(x)-2*csch(x))/a
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.88

$$\int \frac{\cosh^2(x)}{a+a\cosh(x)}\,dx = -\frac{2\,x\cosh\left(x\right)-\cosh\left(x\right)^2+2\left(x-\cosh\left(x\right)-1\right)\sinh\left(x\right)-\sinh\left(x\right)^2+2\,x+5}{2\left(a\cosh\left(x\right)+a\sinh\left(x\right)+a\right)}$$

input
```
integrate(cosh(x)^2/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
-1/2*(2*x*cosh(x) - cosh(x)^2 + 2*(x - cosh(x) - 1)*sinh(x) - sinh(x)^2 +
2*x + 5)/(a*cosh(x) + a*sinh(x) + a)
```

3.26.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(17) = 34$.

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.52

$$\int \frac{\cosh^2(x)}{a+a\cosh(x)}\,dx = -\frac{x\tanh^2\left(\frac{x}{2}\right)}{a\tanh^2\left(\frac{x}{2}\right)-a}+\frac{x}{a\tanh^2\left(\frac{x}{2}\right)-a}+\frac{\tanh^3\left(\frac{x}{2}\right)}{a\tanh^2\left(\frac{x}{2}\right)-a}-\frac{3\tanh\left(\frac{x}{2}\right)}{a\tanh^2\left(\frac{x}{2}\right)-a}$$

input
```
integrate(cosh(x)**2/(a+a*cosh(x)),x)
```

output
```
-x*tanh(x/2)**2/(a*tanh(x/2)**2 - a) + x/(a*tanh(x/2)**2 - a) + tanh(x/2)*
*3/(a*tanh(x/2)**2 - a) - 3*tanh(x/2)/(a*tanh(x/2)**2 - a)
```

3.26.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.64

$$\int \frac{\cosh^2(x)}{a + a\cosh(x)}\, dx = -\frac{x}{a} + \frac{5\, e^{(-x)} + 1}{2\,(ae^{(-x)} + ae^{(-2\,x)})} - \frac{e^{(-x)}}{2\,a}$$

input
```
integrate(cosh(x)^2/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-x/a + 1/2*(5*e^(-x) + 1)/(a*e^(-x) + a*e^(-2*x)) - 1/2*e^(-x)/a
```

3.26.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.40

$$\int \frac{\cosh^2(x)}{a + a\cosh(x)}\, dx = -\frac{x}{a} - \frac{(5\, e^{x} + 1)e^{(-x)}}{2\,a(e^{x} + 1)} + \frac{e^{x}}{2\,a}$$

input
```
integrate(cosh(x)^2/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
-x/a - 1/2*(5*e^x + 1)*e^(-x)/(a*(e^x + 1)) + 1/2*e^x/a
```

3.26.9 Mupad [B] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int \frac{\cosh^2(x)}{a + a\cosh(x)}\, dx = \frac{e^{x}}{2\,a} - \frac{x}{a} - \frac{2}{a\,(e^{x} + 1)} - \frac{e^{-x}}{2\,a}$$

input
```
int(cosh(x)^2/(a + a*cosh(x)),x)
```

output
```
exp(x)/(2*a) - x/a - 2/(a*(exp(x) + 1)) - exp(-x)/(2*a)
```

3.27 $\int \frac{\cosh(x)}{a+a\cosh(x)}\,dx$

3.27.1 Optimal result

Integrand size = 11, antiderivative size = 18

$$\int \frac{\cosh(x)}{a+a\cosh(x)}\,dx = \frac{x}{a} - \frac{\sinh(x)}{a+a\cosh(x)}$$

output
```
x/a-sinh(x)/(a+a*cosh(x))
```

3.27.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.61

$$\int \frac{\cosh(x)}{a+a\cosh(x)}\,dx = \frac{\arcsin(\cosh(x))\operatorname{csch}(x)\sqrt{-\sinh^2(x)} - \tanh\left(\frac{x}{2}\right)}{a}$$

input
```
Integrate[Cosh[x]/(a + a*Cosh[x]),x]
```

output
```
(ArcSin[Cosh[x]]*Csch[x]*Sqrt[-Sinh[x]^2] - Tanh[x/2])/a
```

3.27.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)}{a+a\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{ 3214}$$

$$\frac{x}{a}-\int \frac{1}{\cosh(x)a+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{x}{a}-\int \frac{1}{\sin\left(ix+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{ 3127}$$

$$\frac{x}{a}-\frac{\sinh(x)}{a\cosh(x)+a}$$

input
```
Int[Cosh[x]/(a + a*Cosh[x]),x]
```

output
```
x/a - Sinh[x]/(a + a*Cosh[x])
```

3.27.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.27.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

method	result	size
parallelrisch	$\frac{x-\tanh\left(\frac{x}{2}\right)}{a}$	13
risch	$\frac{x}{a}+\frac{2}{\left({\mathrm e}^{x}+1\right) a}$	18
default	$\frac{-\tanh\left(\frac{x}{2}\right)-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}$	28

input
```
int(cosh(x)/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
(x-tanh(1/2*x))/a
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int \frac{\cosh(x)}{a+a\cosh(x)}\,dx=\frac{x\cosh\left(x\right)+x\sinh\left(x\right)+x+2}{a\cosh\left(x\right)+a\sinh\left(x\right)+a}$$

input
```
integrate(cosh(x)/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
(x*cosh(x) + x*sinh(x) + x + 2)/(a*cosh(x) + a*sinh(x) + a)
```

3.27.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.44

$$\int \frac{\cosh(x)}{a + a\cosh(x)}\,dx = \frac{x}{a} - \frac{\tanh\left(\frac{x}{2}\right)}{a}$$

input
```
integrate(cosh(x)/(a+a*cosh(x)),x)
```

output
```
x/a - tanh(x/2)/a
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)}{a + a\cosh(x)}\,dx = \frac{x}{a} - \frac{2}{ae^{(-x)} + a}$$

input
```
integrate(cosh(x)/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
x/a - 2/(a*e^(-x) + a)
```

3.27.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\cosh(x)}{a + a\cosh(x)}\,dx = \frac{x}{a} + \frac{2}{a(e^x + 1)}$$

input
```
integrate(cosh(x)/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
x/a + 2/(a*(e^x + 1))
```

3.27.9 Mupad [B] (verification not implemented)

Time = 1.67 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\cosh(x)}{a + a\cosh(x)}\,dx = \frac{x}{a} + \frac{2}{a\,(e^x + 1)}$$

input
```
int(cosh(x)/(a + a*cosh(x)),x)
```

output
```
x/a + 2/(a*(exp(x) + 1))
```

3.28 $\int \frac{\operatorname{sech}(x)}{a+a\cosh(x)}\,dx$

3.28.1 Optimal result

Integrand size = 11, antiderivative size = 20

$$\int \frac{\operatorname{sech}(x)}{a+a\cosh(x)}\,dx=\frac{\arctan(\sinh(x))}{a}-\frac{\sinh(x)}{a+a\cosh(x)}$$

output
```
arctan(sinh(x))/a-sinh(x)/(a+a*cosh(x))
```

3.28.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{sech}(x)}{a+a\cosh(x)}\,dx=\frac{2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)-\tanh\left(\frac{x}{2}\right)}{a}$$

input
```
Integrate[Sech[x]/(a + a*Cosh[x]),x]
```

output
```
(2*ArcTan[Tanh[x/2]] - Tanh[x/2])/a
```

3.28.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 3226, 3042, 3127, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sin\left(\frac{\pi}{2}+ix\right)\left(a+a\sin\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \ 3226$$

$$\frac{\int \operatorname{sech}(x)dx}{a}-\int \frac{1}{\cosh(x)a+a}dx$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}-\int \frac{1}{\sin\left(ix+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \ 3127$$

$$-\frac{\sinh(x)}{a\cosh(x)+a}+\frac{\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \ 4257$$

$$\frac{\arctan(\sinh(x))}{a}-\frac{\sinh(x)}{a\cosh(x)+a}$$

input
```
Int[Sech[x]/(a + a*Cosh[x]),x]
```

output
```
ArcTan[Sinh[x]]/a - Sinh[x]/(a + a*Cosh[x])
```

3.28.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3226
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]),
 x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.28.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{-\tanh\left(\frac{x}{2}\right)+2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	19
parallelrisch	$\frac{-i\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)+i\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)-\tanh\left(\frac{x}{2}\right)}{a}$	34
risch	$\frac{2}{\left(e^{x}+1\right)a}+\frac{i\ln\left(e^{x}+i\right)}{a}-\frac{i\ln\left(e^{x}-i\right)}{a}$	37

input
```
int(sech(x)/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/a*(-tanh(1/2*x)+2*arctan(tanh(1/2*x)))
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int \frac{\text{sech}(x)}{a+a\cosh(x)}\,dx = \frac{2\left((\cosh(x)+\sinh(x)+1)\arctan(\cosh(x)+\sinh(x))+1\right)}{a\cosh(x)+a\sinh(x)+a}$$

input
```
integrate(sech(x)/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
2*((cosh(x) + sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + 1)/(a*cosh(x) + a*s
inh(x) + a)
```

3.28.6 Sympy [F]

$$\int \frac{\text{sech}(x)}{a+a\cosh(x)}\,dx = \frac{\int \frac{\text{sech}(x)}{\cosh(x)+1}\,dx}{a}$$

input
```
integrate(sech(x)/(a+a*cosh(x)),x)
```

output
```
Integral(sech(x)/(cosh(x) + 1), x)/a
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{\text{sech}(x)}{a+a\cosh(x)}\,dx = -\frac{2\arctan\left(e^{(-x)}\right)}{a} - \frac{2}{ae^{(-x)}+a}$$

input
```
integrate(sech(x)/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-2*arctan(e^(-x))/a - 2/(a*e^(-x) + a)
```

3.28.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(x)}{a + a\cosh(x)}\,dx = \frac{2\arctan\left(e^x\right)}{a} + \frac{2}{a(e^x + 1)}$$

input
```
integrate(sech(x)/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
2*arctan(e^x)/a + 2/(a*(e^x + 1))
```

3.28.9 Mupad [B] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{\operatorname{sech}(x)}{a + a\cosh(x)}\,dx = \frac{2}{a\,\left(\mathrm{e}^x + 1\right)} + \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^x\,\sqrt{a^2}}{a}\right)}{\sqrt{a^2}}$$

input
```
int(1/(cosh(x)*(a + a*cosh(x))),x)
```

output
```
2/(a*(exp(x) + 1)) + (2*atan((exp(x)*(a^2)^(1/2))/a))/(a^2)^(1/2)
```

3.29 $\int \frac{\text{sech}^2(x)}{a+a\cosh(x)}\,dx$

3.29.1 Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \frac{\text{sech}^2(x)}{a+a\cosh(x)}\,dx=-\frac{\arctan(\sinh(x))}{a}+\frac{2\tanh(x)}{a}-\frac{\tanh(x)}{a+a\cosh(x)}$$

output
```
-arctan(sinh(x))/a+2*tanh(x)/a-tanh(x)/(a+a*cosh(x))
```

3.29.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.54

$$\int \frac{\text{sech}^2(x)}{a+a\cosh(x)}\,dx=\frac{2\cosh\left(\frac{x}{2}\right)\left(\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\left(-2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)+\tanh(x)\right)\right)}{a(1+\cosh(x))}$$

input
```
Integrate[Sech[x]^2/(a + a*Cosh[x]),x]
```

output
```
(2*Cosh[x/2]*(Sinh[x/2] + Cosh[x/2]*(-2*ArcTan[Tanh[x/2]] + Tanh[x])))/(a*
(1 + Cosh[x]))
```

3.29.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {3042, 3247, 25, 3042, 3227, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^2(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin\left(\frac{\pi}{2}+ix\right)^2\left(a+a\sin\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{ 3247}$$

$$-\frac{\int -\left((2a-a\cosh(x))\text{sech}^2(x)\right)dx}{a^2}-\frac{\tanh(x)}{a\cosh(x)+a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int (2a-a\cosh(x))\text{sech}^2(x)dx}{a^2}-\frac{\tanh(x)}{a\cosh(x)+a}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\tanh(x)}{a\cosh(x)+a}+\frac{\int \frac{2a-a\sin\left(ix+\frac{\pi}{2}\right)}{\sin\left(ix+\frac{\pi}{2}\right)^2}dx}{a^2}$$

$$\Big\downarrow \text{ 3227}$$

$$\frac{2a\int \text{sech}^2(x)dx-a\int \text{sech}(x)dx}{a^2}-\frac{\tanh(x)}{a\cosh(x)+a}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\tanh(x)}{a\cosh(x)+a}+\frac{2a\int \csc\left(ix+\frac{\pi}{2}\right)^2dx-a\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a^2}$$

$$\Big\downarrow \text{ 4254}$$

$$-\frac{\tanh(x)}{a\cosh(x)+a}+\frac{2ia\int 1d(-i\tanh(x))-a\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a^2}$$

$$\Big\downarrow \text{ 24}$$

$$-\frac{\tanh(x)}{a\cosh(x)+a}+\frac{2a\tanh(x)-a\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a^2}$$

$$\Big\downarrow \text{4257}$$

$$\frac{2a\tanh(x)-a\arctan(\sinh(x))}{a^2}-\frac{\tanh(x)}{a\cosh(x)+a}$$

input
```
Int[Sech[x]^2/(a + a*Cosh[x]),x]
```

output
```
-(Tanh[x]/(a + a*Cosh[x])) + (-(a*ArcTan[Sinh[x]]) + 2*a*Tanh[x])/a^2
```

3.29.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3247
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Simp[d/(a*(b*c - a*d))
   Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[
c^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.29.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18

method	result	size
default	$\frac{\tanh\left(\frac{x}{2}\right)+\frac{2\tanh\left(\frac{x}{2}\right)}{1+\tanh\left(\frac{x}{2}\right)^{2}}-2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	33
parallelrisch	$\frac{i\ln(-i+\coth(x)-\operatorname{csch}(x))-i\ln(i+\coth(x)-\operatorname{csch}(x))+(-\operatorname{sech}(x)-1)\operatorname{csch}(x)+2\coth(x)}{a}$	45
risch	$-\frac{2(\mathrm{e}^{2x}+\mathrm{e}^{x}+2)}{a(1+\mathrm{e}^{2x})(\mathrm{e}^{x}+1)}+\frac{i\ln(\mathrm{e}^{x}-i)}{a}-\frac{i\ln(\mathrm{e}^{x}+i)}{a}$	53

input
```
int(sech(x)^2/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/a*(tanh(1/2*x)+2*tanh(1/2*x)/(1+tanh(1/2*x)^2)-2*arctan(tanh(1/2*x)))
```

3.29.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(28) = 56$.

Time = 0.23 (sec) , antiderivative size = 127, normalized size of antiderivative = 4.54

$$\int \frac{\operatorname{sech}^2(x)}{a+a\cosh(x)}\,dx = -\frac{2\left(\left(\cosh(x)^3+(3\cosh(x)+1)\sinh(x)^2+\sinh(x)^3+\cosh(x)^2+\left(3\cosh(x)^2+2\cosh(x)+1\right)\sinh\cdots\right.\right.}{a\cosh(x)^3+a\sinh(x)^3+a\cosh(x)^2+(3a\cosh(x)+a)\cdots}$$

input
```
integrate(sech(x)^2/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
-2*((cosh(x)^3 + (3*cosh(x) + 1)*sinh(x)^2 + sinh(x)^3 + cosh(x)^2 + (3*co
sh(x)^2 + 2*cosh(x) + 1)*sinh(x) + cosh(x) + 1)*arctan(cosh(x) + sinh(x))
+ cosh(x)^2 + (2*cosh(x) + 1)*sinh(x) + sinh(x)^2 + cosh(x) + 2)/(a*cosh(x
)^3 + a*sinh(x)^3 + a*cosh(x)^2 + (3*a*cosh(x) + a)*sinh(x)^2 + a*cosh(x)
+ (3*a*cosh(x)^2 + 2*a*cosh(x) + a)*sinh(x) + a)
```

3.29.6 Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{a+a\cosh(x)}\,dx = \frac{\int \frac{\operatorname{sech}^2(x)}{\cosh(x)+1}\,dx}{a}$$

input
```
integrate(sech(x)**2/(a+a*cosh(x)),x)
```

output
```
Integral(sech(x)**2/(cosh(x) + 1), x)/a
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.61

$$\int \frac{\operatorname{sech}^2(x)}{a+a\cosh(x)}\,dx = \frac{2\left(e^{(-x)}+e^{(-2x)}+2\right)}{ae^{(-x)}+ae^{(-2x)}+ae^{(-3x)}+a}+\frac{2\arctan\left(e^{(-x)}\right)}{a}$$

input
```
integrate(sech(x)^2/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
2*(e^(-x) + e^(-2*x) + 2)/(a*e^(-x) + a*e^(-2*x) + a*e^(-3*x) + a) + 2*arc
tan(e^(-x))/a
```

3.29.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{\operatorname{sech}^2(x)}{a+a\cosh(x)}\,dx = -\frac{2\arctan(e^x)}{a}-\frac{2\left(e^{(2x)}+e^x+2\right)}{a(e^{(3x)}+e^{(2x)}+e^x+1)}$$

input
```
integrate(sech(x)^2/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
-2*arctan(e^x)/a - 2*(e^(2*x) + e^x + 2)/(a*(e^(3*x) + e^(2*x) + e^x + 1))
```

3.29.9 Mupad [B] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.07

$$\int \frac{\operatorname{sech}^2(x)}{a+a\cosh(x)}\,dx = -\frac{\frac{2\,\mathrm{e}^{2\,x}}{a}+\frac{4}{a}+\frac{2\,\mathrm{e}^{x}}{a}}{\mathrm{e}^{2\,x}+\mathrm{e}^{3\,x}+\mathrm{e}^{x}+1}-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{x}\,\sqrt{a^2}}{a}\right)}{\sqrt{a^2}}$$

input
```
int(1/(cosh(x)^2*(a + a*cosh(x))),x)
```

output
```
- ((2*exp(2*x))/a + 4/a + (2*exp(x))/a)/(exp(2*x) + exp(3*x) + exp(x) + 1)
 - (2*atan((exp(x)*(a^2)^(1/2))/a))/(a^2)^(1/2)
```

3.30 $\int \frac{\operatorname{sech}^3(x)}{a+a\cosh(x)}\,dx$

3.30.1 Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \frac{\operatorname{sech}^3(x)}{a+a\cosh(x)}\,dx=\frac{3\arctan(\sinh(x))}{2a}-\frac{2\tanh(x)}{a}+\frac{3\operatorname{sech}(x)\tanh(x)}{2a}-\frac{\operatorname{sech}(x)\tanh(x)}{a+a\cosh(x)}$$

output
```
3/2*arctan(sinh(x))/a-2*tanh(x)/a+3/2*sech(x)*tanh(x)/a-sech(x)*tanh(x)/(a
+a*cosh(x))
```

3.30.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{sech}^3(x)}{a+a\cosh(x)}\,dx =\frac{\cosh\left(\frac{x}{2}\right)\left(-2\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\left(6\arctan\left(\tanh\left(\frac{x}{2}\right)\right)+(-2+\operatorname{sech}(x))\tanh(x)\right)\right)}{a(1+\cosh(x))}$$

input
```
Integrate[Sech[x]^3/(a + a*Cosh[x]),x]
```

output
```
(Cosh[x/2]*(-2*Sinh[x/2] + Cosh[x/2]*(6*ArcTan[Tanh[x/2]] + (-2 + Sech[x])
*Tanh[x])))/(a*(1 + Cosh[x]))
```

3.30.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.846, Rules used = {3042, 3247, 25, 3042, 3227, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^3(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(\frac{\pi}{2}+ix\right)^3\left(a+a\sin\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{3247}$$

$$-\frac{\int -\left((3a-2a\cosh(x))\text{sech}^3(x)\right)dx}{a^2}-\frac{\tanh(x)\text{sech}(x)}{a\cosh(x)+a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int (3a-2a\cosh(x))\text{sech}^3(x)dx}{a^2}-\frac{\tanh(x)\text{sech}(x)}{a\cosh(x)+a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tanh(x)\text{sech}(x)}{a\cosh(x)+a}+\frac{\int \frac{3a-2a\sin\left(ix+\frac{\pi}{2}\right)}{\sin\left(ix+\frac{\pi}{2}\right)^3}dx}{a^2}$$

$$\Big\downarrow \text{3227}$$

$$\frac{3a\int \text{sech}^3(x)dx-2a\int \text{sech}^2(x)dx}{a^2}-\frac{\tanh(x)\text{sech}(x)}{a\cosh(x)+a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tanh(x)\text{sech}(x)}{a\cosh(x)+a}+\frac{3a\int \csc\left(ix+\frac{\pi}{2}\right)^3dx-2a\int \csc\left(ix+\frac{\pi}{2}\right)^2dx}{a^2}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\tanh(x)\text{sech}(x)}{a\cosh(x)+a}+\frac{3a\int \csc\left(ix+\frac{\pi}{2}\right)^3dx-2ia\int 1d(-i\tanh(x))}{a^2}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\tanh(x)\text{sech}(x)}{a\cosh(x)+a}+\frac{-2a\tanh(x)+3a\int \csc\left(ix+\frac{\pi}{2}\right)^3dx}{a^2}$$

$$\Big\downarrow \; 4255$$

$$\frac{3a\left(\frac{\int \text{sech}(x)dx}{2}+\frac{1}{2}\tanh(x)\text{sech}(x)\right)-2a\tanh(x)}{a^2}-\frac{\tanh(x)\text{sech}(x)}{a\cosh(x)+a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\tanh(x)\text{sech}(x)}{a\cosh(x)+a}+\frac{-2a\tanh(x)+3a\left(\frac{1}{2}\tanh(x)\text{sech}(x)+\frac{1}{2}\int \csc\left(ix+\frac{\pi}{2}\right)dx\right)}{a^2}$$

$$\Big\downarrow \; 4257$$

$$\frac{3a\left(\frac{1}{2}\arctan(\sinh(x))+\frac{1}{2}\tanh(x)\text{sech}(x)\right)-2a\tanh(x)}{a^2}-\frac{\tanh(x)\text{sech}(x)}{a\cosh(x)+a}$$

input
```
Int[Sech[x]^3/(a + a*Cosh[x]),x]
```

output
```
-((Sech[x]*Tanh[x])/(a + a*Cosh[x])) + (-2*a*Tanh[x] + 3*a*(ArcTan[Sinh[x]
]/2 + (Sech[x]*Tanh[x])/2))/a^2
```

3.30.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3247
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Simp[d/(a*(b*c - a*d))
   Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[
c^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.30.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

method	result	size
default	$\frac{-\tanh\left(\frac{x}{2}\right)+\frac{-3\tanh\left(\frac{x}{2}\right)^{3}-\tanh\left(\frac{x}{2}\right)}{\left(1+\tanh\left(\frac{x}{2}\right)^{2}\right)^{2}}+3\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	46
parallelrisch	$\frac{-3i\ln(-i+\coth(x)-\operatorname{csch}(x))+3i\ln(i+\coth(x)-\operatorname{csch}(x))+\left(-\operatorname{sech}(x)^{2}+2\operatorname{sech}(x)+3\right)\operatorname{csch}(x)-4\coth(x)}{2a}$	52
risch	$\frac{3\,\mathrm{e}^{4x}+3\,\mathrm{e}^{3x}+5\,\mathrm{e}^{2x}+\mathrm{e}^{x}+4}{a(1+\mathrm{e}^{2x})^{2}(\mathrm{e}^{x}+1)}+\frac{3i\ln(\mathrm{e}^{x}+i)}{2a}-\frac{3i\ln(\mathrm{e}^{x}-i)}{2a}$	66

input
```
int(sech(x)^3/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/a*(-tanh(1/2*x)+2*(-3/2*tanh(1/2*x)^3-1/2*tanh(1/2*x))/(1+tanh(1/2*x)^2)
^2+3*arctan(tanh(1/2*x)))
```

3.30.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. 2(39) = 78.

Time = 0.26 (sec) , antiderivative size = 325, normalized size of antiderivative = 7.56

$$\int \frac{\operatorname{sech}^3(x)}{a+a\cosh(x)}\,dx$$
$$= \frac{3\cosh(x)^4 + 3(4\cosh(x)+1)\sinh(x)^3 + 3\sinh(x)^4 + 3\cosh(x)^3 + (18\cosh(x)^2 + 9\cosh(x)+5)\sin$$

input
```
integrate(sech(x)^3/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
(3*cosh(x)^4 + 3*(4*cosh(x) + 1)*sinh(x)^3 + 3*sinh(x)^4 + 3*cosh(x)^3 + (
18*cosh(x)^2 + 9*cosh(x) + 5)*sinh(x)^2 + 3*(cosh(x)^5 + (5*cosh(x) + 1)*s
inh(x)^4 + sinh(x)^5 + cosh(x)^4 + 2*(5*cosh(x)^2 + 2*cosh(x) + 1)*sinh(x)
^3 + 2*cosh(x)^3 + 2*(5*cosh(x)^3 + 3*cosh(x)^2 + 3*cosh(x) + 1)*sinh(x)^2
 + 2*cosh(x)^2 + (5*cosh(x)^4 + 4*cosh(x)^3 + 6*cosh(x)^2 + 4*cosh(x) + 1)
*sinh(x) + cosh(x) + 1)*arctan(cosh(x) + sinh(x)) + 5*cosh(x)^2 + (12*cosh
(x)^3 + 9*cosh(x)^2 + 10*cosh(x) + 1)*sinh(x) + cosh(x) + 4)/(a*cosh(x)^5
+ a*sinh(x)^5 + a*cosh(x)^4 + (5*a*cosh(x) + a)*sinh(x)^4 + 2*a*cosh(x)^3
+ 2*(5*a*cosh(x)^2 + 2*a*cosh(x) + a)*sinh(x)^3 + 2*a*cosh(x)^2 + 2*(5*a*c
osh(x)^3 + 3*a*cosh(x)^2 + 3*a*cosh(x) + a)*sinh(x)^2 + a*cosh(x) + (5*a*c
osh(x)^4 + 4*a*cosh(x)^3 + 6*a*cosh(x)^2 + 4*a*cosh(x) + a)*sinh(x) + a)
```

3.30.6 Sympy [F]

$$\int \frac{\operatorname{sech}^3(x)}{a+a\cosh(x)}\,dx = \frac{\int \frac{\operatorname{sech}^3(x)}{\cosh(x)+1}\,dx}{a}$$

input
```
integrate(sech(x)**3/(a+a*cosh(x)),x)
```

output
```
Integral(sech(x)**3/(cosh(x) + 1), x)/a
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.70

$$\int \frac{\operatorname{sech}^3(x)}{a+a\cosh(x)}\,dx = -\frac{e^{(-x)}+5\,e^{(-2\,x)}+3\,e^{(-3\,x)}+3\,e^{(-4\,x)}+4}{ae^{(-x)}+2\,ae^{(-2\,x)}+2\,ae^{(-3\,x)}+ae^{(-4\,x)}+ae^{(-5\,x)}+a} - \frac{3\,\arctan\left(e^{(-x)}\right)}{a}$$

input
```
integrate(sech(x)^3/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-(e^(-x) + 5*e^(-2*x) + 3*e^(-3*x) + 3*e^(-4*x) + 4)/(a*e^(-x) + 2*a*e^(-2
*x) + 2*a*e^(-3*x) + a*e^(-4*x) + a*e^(-5*x) + a) - 3*arctan(e^(-x))/a
```

3.30.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}^3(x)}{a+a\cosh(x)}\,dx = \frac{3\,\arctan\left(e^{x}\right)}{a} + \frac{e^{(3\,x)}+2\,e^{(2\,x)}-e^{x}+2}{a\left(e^{(2\,x)}+1\right)^2} + \frac{2}{a(e^{x}+1)}$$

input
```
integrate(sech(x)^3/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
3*arctan(e^x)/a + (e^(3*x) + 2*e^(2*x) - e^x + 2)/(a*(e^(2*x) + 1)^2) + 2/
(a*(e^x + 1))
```

3.30.9 Mupad [B] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.70

$$\int \frac{\operatorname{sech}^3(x)}{a+a\cosh(x)}\,dx = \frac{2}{a\,\left(e^{x}+1\right)} + \frac{\frac{2}{a}+\frac{e^{x}}{a}}{e^{2\,x}+1} + \frac{3\,\mathrm{atan}\left(\frac{e^{x}\,\sqrt{a^2}}{a}\right)}{\sqrt{a^2}} - \frac{2\,e^{x}}{a\,\left(2\,e^{2\,x}+e^{4\,x}+1\right)}$$

input
```
int(1/(cosh(x)^3*(a + a*cosh(x))),x)
```

output
```
2/(a*(exp(x) + 1)) + (2/a + exp(x)/a)/(exp(2*x) + 1) + (3*atan((exp(x)*(a^
2)^(1/2))/a))/(a^2)^(1/2) - (2*exp(x))/(a*(2*exp(2*x) + exp(4*x) + 1))
```

3.31 $\int \frac{\operatorname{sech}^4(x)}{a+a\cosh(x)}\,dx$

3.31.1 Optimal result

Integrand size = 13, antiderivative size = 56

$$\int \frac{\operatorname{sech}^4(x)}{a+a\cosh(x)}\,dx = -\frac{3\arctan(\sinh(x))}{2a}+\frac{4\tanh(x)}{a}-\frac{3\operatorname{sech}(x)\tanh(x)}{2a}-\frac{\operatorname{sech}^2(x)\tanh(x)}{a+a\cosh(x)}-\frac{4\tanh^3(x)}{3a}$$

output
```
-3/2*arctan(sinh(x))/a+4*tanh(x)/a-3/2*sech(x)*tanh(x)/a-sech(x)^2*tanh(x)
/(a+a*cosh(x))-4/3*tanh(x)^3/a
```

3.31.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.07

$$\int \frac{\operatorname{sech}^4(x)}{a+a\cosh(x)}\,dx = \frac{\cosh\left(\frac{x}{2}\right)\left(6\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\left(-18\arctan\left(\tanh\left(\frac{x}{2}\right)\right)+\left(10-3\operatorname{sech}(x)+2\operatorname{sech}^2(x)\right)\tanh(x)\right)\right)}{3a(1+\cosh(x))}$$

input
```
Integrate[Sech[x]^4/(a + a*Cosh[x]),x]
```

output
```
(Cosh[x/2]*(6*Sinh[x/2] + Cosh[x/2]*(-18*ArcTan[Tanh[x/2]] + (10 - 3*Sech[
x] + 2*Sech[x]^2)*Tanh[x])))/(3*a*(1 + Cosh[x]))
```

3.31.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {3042, 3247, 25, 3042, 3227, 3042, 4254, 2009, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^4(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sin\left(\frac{\pi}{2}+ix\right)^4\left(a+a\sin\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \ 3247$$

$$-\frac{\int -\left((4a-3a\cosh(x))\text{sech}^4(x)\right)dx}{a^2}-\frac{\tanh(x)\text{sech}^2(x)}{a\cosh(x)+a}$$

$$\Big\downarrow \ 25$$

$$\frac{\int (4a-3a\cosh(x))\text{sech}^4(x)dx}{a^2}-\frac{\tanh(x)\text{sech}^2(x)}{a\cosh(x)+a}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\tanh(x)\text{sech}^2(x)}{a\cosh(x)+a}+\frac{\int \frac{4a-3a\sin\left(ix+\frac{\pi}{2}\right)}{\sin\left(ix+\frac{\pi}{2}\right)^4}dx}{a^2}$$

$$\Big\downarrow \ 3227$$

$$\frac{4a\int \text{sech}^4(x)dx-3a\int \text{sech}^3(x)dx}{a^2}-\frac{\tanh(x)\text{sech}^2(x)}{a\cosh(x)+a}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\tanh(x)\text{sech}^2(x)}{a\cosh(x)+a}+\frac{4a\int \csc\left(ix+\frac{\pi}{2}\right)^4dx-3a\int \csc\left(ix+\frac{\pi}{2}\right)^3dx}{a^2}$$

$$\Big\downarrow \ 4254$$

$$-\frac{\tanh(x)\text{sech}^2(x)}{a\cosh(x)+a}+\frac{4ia\int \left(1-\tanh^2(x)\right)d(-i\tanh(x))-3a\int \csc\left(ix+\frac{\pi}{2}\right)^3dx}{a^2}$$

$$\Big\downarrow \ 2009$$

$$-\frac{\tanh(x)\text{sech}^2(x)}{a\cosh(x)+a}+\frac{4ia\left(\frac{1}{3}i\tanh^3(x)-i\tanh(x)\right)-3a\int\csc\left(ix+\frac{\pi}{2}\right)^3dx}{a^2}$$

$$\Big\downarrow \text{4255}$$

$$-\frac{\tanh(x)\text{sech}^2(x)}{a\cosh(x)+a}+\frac{-3a\left(\frac{\int\text{sech}(x)dx}{2}+\frac{1}{2}\tanh(x)\text{sech}(x)\right)+4ia\left(\frac{1}{3}i\tanh^3(x)-i\tanh(x)\right)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tanh(x)\text{sech}^2(x)}{a\cosh(x)+a}+\frac{4ia\left(\frac{1}{3}i\tanh^3(x)-i\tanh(x)\right)-3a\left(\frac{1}{2}\tanh(x)\text{sech}(x)+\frac{1}{2}\int\csc\left(ix+\frac{\pi}{2}\right)dx\right)}{a^2}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\tanh(x)\text{sech}^2(x)}{a\cosh(x)+a}+\frac{-3a\left(\frac{1}{2}\arctan(\sinh(x))+\frac{1}{2}\tanh(x)\text{sech}(x)\right)+4ia\left(\frac{1}{3}i\tanh^3(x)-i\tanh(x)\right)}{a^2}$$

input
```
Int[Sech[x]^4/(a + a*Cosh[x]),x]
```

output
```
-((Sech[x]^2*Tanh[x])/(a + a*Cosh[x])) + (-3*a*(ArcTan[Sinh[x]]/2 + (Sech[
x]*Tanh[x])/2) + (4*I)*a*((-I)*Tanh[x] + (I/3)*Tanh[x]^3))/a^2
```

3.31.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

3.31. $\int \frac{\text{sech}^4(x)}{a+a\cosh(x)}\,dx$

rule 3247
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Simp[d/(a*(b*c - a*d))
   Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[
c^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.31.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\tanh\left(\frac{x}{2}\right)-\frac{8\left(-\frac{5\tanh\left(\frac{x}{2}\right)^{5}}{8}-\frac{2\tanh\left(\frac{x}{2}\right)^{3}}{3}-\frac{3\tanh\left(\frac{x}{2}\right)}{8}\right)}{\left(1+\tanh\left(\frac{x}{2}\right)^{2}\right)^{3}}-3\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	52
parallelrisch	$\frac{9i\ln(-i+\coth(x)-\operatorname{csch}(x))-9i\ln(i+\coth(x)-\operatorname{csch}(x))+\left(-2\operatorname{sech}(x)^{3}+3\operatorname{sech}(x)^{2}-8\operatorname{sech}(x)-9\right)\operatorname{csch}(x)+16\coth(x)}{6a}$	58
risch	$-\frac{9\,e^{6x}+9\,e^{5x}+24\,e^{4x}+24\,e^{3x}+39\,e^{2x}+7\,e^{x}+16}{3(1+e^{2x})^{3}a(e^{x}+1)}+\frac{3i\ln(e^{x}-i)}{2a}-\frac{3i\ln(e^{x}+i)}{2a}$	81

input
```
int(sech(x)^4/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/a*(tanh(1/2*x)-8*(-5/8*tanh(1/2*x)^5-2/3*tanh(1/2*x)^3-3/8*tanh(1/2*x))/
(1+tanh(1/2*x)^2)^3-3*arctan(tanh(1/2*x)))
```

3.31.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(50) = 100$.

Time = 0.25 (sec) , antiderivative size = 600, normalized size of antiderivative = 10.71

$$\int \frac{\operatorname{sech}^4(x)}{a + a\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(x)^4/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
-1/3*(9*cosh(x)^6 + 9*(6*cosh(x) + 1)*sinh(x)^5 + 9*sinh(x)^6 + 9*cosh(x)^
5 + 3*(45*cosh(x)^2 + 15*cosh(x) + 8)*sinh(x)^4 + 24*cosh(x)^4 + 6*(30*cos
h(x)^3 + 15*cosh(x)^2 + 16*cosh(x) + 4)*sinh(x)^3 + 24*cosh(x)^3 + 3*(45*c
osh(x)^4 + 30*cosh(x)^3 + 48*cosh(x)^2 + 24*cosh(x) + 13)*sinh(x)^2 + 9*(c
osh(x)^7 + (7*cosh(x) + 1)*sinh(x)^6 + sinh(x)^7 + cosh(x)^6 + 3*(7*cosh(x
)^2 + 2*cosh(x) + 1)*sinh(x)^5 + 3*cosh(x)^5 + (35*cosh(x)^3 + 15*cosh(x)^
2 + 15*cosh(x) + 3)*sinh(x)^4 + 3*cosh(x)^4 + (35*cosh(x)^4 + 20*cosh(x)^3
 + 30*cosh(x)^2 + 12*cosh(x) + 3)*sinh(x)^3 + 3*cosh(x)^3 + 3*(7*cosh(x)^5
 + 5*cosh(x)^4 + 10*cosh(x)^3 + 6*cosh(x)^2 + 3*cosh(x) + 1)*sinh(x)^2 + 3
*cosh(x)^2 + (7*cosh(x)^6 + 6*cosh(x)^5 + 15*cosh(x)^4 + 12*cosh(x)^3 + 9*
cosh(x)^2 + 6*cosh(x) + 1)*sinh(x) + cosh(x) + 1)*arctan(cosh(x) + sinh(x)
) + 39*cosh(x)^2 + (54*cosh(x)^5 + 45*cosh(x)^4 + 96*cosh(x)^3 + 72*cosh(x
)^2 + 78*cosh(x) + 7)*sinh(x) + 7*cosh(x) + 16)/(a*cosh(x)^7 + a*sinh(x)^7
 + a*cosh(x)^6 + (7*a*cosh(x) + a)*sinh(x)^6 + 3*a*cosh(x)^5 + 3*(7*a*cosh
(x)^2 + 2*a*cosh(x) + a)*sinh(x)^5 + 3*a*cosh(x)^4 + (35*a*cosh(x)^3 + 15*
a*cosh(x)^2 + 15*a*cosh(x) + 3*a)*sinh(x)^4 + 3*a*cosh(x)^3 + (35*a*cosh(x
)^4 + 20*a*cosh(x)^3 + 30*a*cosh(x)^2 + 12*a*cosh(x) + 3*a)*sinh(x)^3 + 3*
a*cosh(x)^2 + 3*(7*a*cosh(x)^5 + 5*a*cosh(x)^4 + 10*a*cosh(x)^3 + 6*a*cosh
(x)^2 + 3*a*cosh(x) + a)*sinh(x)^2 + a*cosh(x) + (7*a*cosh(x)^6 + 6*a*cosh
(x)^5 + 15*a*cosh(x)^4 + 12*a*cosh(x)^3 + 9*a*cosh(x)^2 + 6*a*cosh(x) +...
```

3.31.6 Sympy [F]

$$\int \frac{\operatorname{sech}^4(x)}{a + a\cosh(x)}\,dx = \frac{\int \frac{\operatorname{sech}^4(x)}{\cosh(x)+1}\,dx}{a}$$

input
```
integrate(sech(x)**4/(a+a*cosh(x)),x)
```

output
```
Integral(sech(x)**4/(cosh(x) + 1), x)/a
```

3.31.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(50) = 100$.

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.80

$$\int \frac{\operatorname{sech}^4(x)}{a+a\cosh(x)}\,dx = \frac{7e^{(-x)}+39e^{(-2x)}+24e^{(-3x)}+24e^{(-4x)}+9e^{(-5x)}+9e^{(-6x)}+16}{3\left(ae^{(-x)}+3ae^{(-2x)}+3ae^{(-3x)}+3ae^{(-4x)}+3ae^{(-5x)}+ae^{(-6x)}+ae^{(-7x)}+a\right)} + \frac{3\arctan\left(e^{(-x)}\right)}{a}$$

input
```
integrate(sech(x)^4/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
1/3*(7*e^(-x) + 39*e^(-2*x) + 24*e^(-3*x) + 24*e^(-4*x) + 9*e^(-5*x) + 9*e
^(-6*x) + 16)/(a*e^(-x) + 3*a*e^(-2*x) + 3*a*e^(-3*x) + 3*a*e^(-4*x) + 3*a
*e^(-5*x) + a*e^(-6*x) + a*e^(-7*x) + a) + 3*arctan(e^(-x))/a
```

3.31.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int \frac{\operatorname{sech}^4(x)}{a+a\cosh(x)}\,dx = -\frac{3\arctan(e^x)}{a} - \frac{2}{a(e^x+1)} - \frac{3e^{(5x)}+6e^{(4x)}+24e^{(2x)}-3e^x+10}{3a(e^{(2x)}+1)^3}$$

input
```
integrate(sech(x)^4/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
-3*arctan(e^x)/a - 2/(a*(e^x + 1)) - 1/3*(3*e^(5*x) + 6*e^(4*x) + 24*e^(2*
x) - 3*e^x + 10)/(a*(e^(2*x) + 1)^3)
```

3.31.9 Mupad [B] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.91

$$\int \frac{\operatorname{sech}^4(x)}{a+a\cosh(x)}\,dx = \frac{8}{3\,a\,\left(3\,\mathrm{e}^{2\,x}+3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}+1\right)}-\frac{\frac{4}{a}-\frac{2\,\mathrm{e}^{x}}{a}}{2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1}-\frac{2}{a\,\left(\mathrm{e}^{x}+1\right)}-\frac{\frac{2}{a}+\frac{\mathrm{e}^{x}}{a}}{\mathrm{e}^{2\,x}+1}-\frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{x}\,\sqrt{a^2}}{a}\right)}{\sqrt{a^2}}$$

input
```
int(1/(cosh(x)^4*(a + a*cosh(x))),x)
```

output
```
8/(3*a*(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1)) - (4/a - (2*exp(x))/a)/(2
*exp(2*x) + exp(4*x) + 1) - 2/(a*(exp(x) + 1)) - (2/a + exp(x)/a)/(exp(2*x
) + 1) - (3*atan((exp(x)*(a^2)^(1/2))/a))/(a^2)^(1/2)
```

3.32 $\int \frac{1}{1+\cosh(c+dx)}\,dx$

3.32.1 Optimal result

Integrand size = 10, antiderivative size = 20

$$\int \frac{1}{1+\cosh(c+dx)}\,dx=\frac{\sinh(c+dx)}{d(1+\cosh(c+dx))}$$

output
```
sinh(d*x+c)/d/(1+cosh(d*x+c))
```

3.32.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{1}{1+\cosh(c+dx)}\,dx=\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{d}$$

input
```
Integrate[(1 + Cosh[c + d*x])^(-1),x]
```

output
```
Tanh[(c + d*x)/2]/d
```

3.32.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cosh(c+dx)+1}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{1+\sin\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 3127$$

$$\frac{\sinh(c+dx)}{d(\cosh(c+dx)+1)}$$

input
```
Int[(1 + Cosh[c + d*x])^(-1),x]
```

output
```
Sinh[c + d*x]/(d*(1 + Cosh[c + d*x]))
```

3.32.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

3.32.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}$	14
default	$\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}$	14
parallelrisch	$\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}$	14
risch	$-\frac{2}{d(e^{dx+c}+1)}$	16

input
```
int(1/(cosh(d*x+c)+1),x,method=_RETURNVERBOSE)
```

output
```
1/d*tanh(1/2*d*x+1/2*c)
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{1+\cosh(c+dx)}\,dx = -\frac{2}{d\cosh\left(dx+c\right)+d\sinh\left(dx+c\right)+d}$$

input
```
integrate(1/(1+cosh(d*x+c)),x, algorithm="fricas")
```

output
```
-2/(d*cosh(d*x + c) + d*sinh(d*x + c) + d)
```

3.32.6 Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{1+\cosh(c+dx)}\,dx = \begin{cases} \frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d} & \text{for } d\neq 0 \\ \frac{x}{\cosh\left(c\right)+1} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(1+cosh(d*x+c)),x)
```

output
```
Piecewise((tanh(c/2 + d*x/2)/d, Ne(d, 0)), (x/(cosh(c) + 1), True))
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{1+\cosh(c+dx)}\,dx = \frac{2}{d(e^{(-dx-c)}+1)}$$

input
```
integrate(1/(1+cosh(d*x+c)),x, algorithm="maxima")
```

output
```
2/(d*(e^(-d*x - c) + 1))
```

3.32.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{1}{1+\cosh(c+dx)}\,dx = -\frac{2}{d(e^{(dx+c)}+1)}$$

input
```
integrate(1/(1+cosh(d*x+c)),x, algorithm="giac")
```

output
```
-2/(d*(e^(d*x + c) + 1))
```

3.32.9 Mupad [B] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{1}{1+\cosh(c+dx)}\,dx = -\frac{2}{d\,\left(\mathrm{e}^{c+d\,x}+1\right)}$$

input
```
int(1/(cosh(c + d*x) + 1),x)
```

output
```
-2/(d*(exp(c + d*x) + 1))
```

3.33 $\int \frac{1}{(1+\cosh(c+dx))^2}\,dx$

3.33.1 Optimal result

Integrand size = 10, antiderivative size = 47

$$\int \frac{1}{(1+\cosh(c+dx))^2}\,dx = \frac{\sinh(c+dx)}{3d(1+\cosh(c+dx))^2} + \frac{\sinh(c+dx)}{3d(1+\cosh(c+dx))}$$

output
```
1/3*sinh(d*x+c)/d/(1+cosh(d*x+c))^2+1/3*sinh(d*x+c)/d/(1+cosh(d*x+c))
```

3.33.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.72

$$\int \frac{1}{(1+\cosh(c+dx))^2}\,dx = \frac{4\sinh(c+dx)+\sinh(2(c+dx))}{6d(1+\cosh(c+dx))^2}$$

input
```
Integrate[(1 + Cosh[c + d*x])^(-2),x]
```

output
```
(4*Sinh[c + d*x] + Sinh[2*(c + d*x)])/(6*d*(1 + Cosh[c + d*x])^2)
```

3.33.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\cosh(c+dx)+1)^2} \, dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\left(1+\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \ 3129$$

$$\frac{1}{3}\int \frac{1}{\cosh(c+dx)+1}dx+\frac{\sinh(c+dx)}{3d(\cosh(c+dx)+1)^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sinh(c+dx)}{3d(\cosh(c+dx)+1)^2}+\frac{1}{3}\int \frac{1}{\sin\left(ic+idx+\frac{\pi}{2}\right)+1}dx$$

$$\Big\downarrow \ 3127$$

$$\frac{\sinh(c+dx)}{3d(\cosh(c+dx)+1)}+\frac{\sinh(c+dx)}{3d(\cosh(c+dx)+1)^2}$$

input
```
Int[(1 + Cosh[c + d*x])^(-2),x]
```

output
```
Sinh[c + d*x]/(3*d*(1 + Cosh[c + d*x])^2) + Sinh[c + d*x]/(3*d*(1 + Cosh[c
 + d*x]))
```

3.33.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.33.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.55

method	result	size
risch	$-\frac{2\left(3\,\mathrm{e}^{dx+c}+1\right)}{3d\left(\mathrm{e}^{dx+c}+1\right)^{3}}$	26
parallelrisch	$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-3\right)}{6d}$	28
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{6}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}}{d}$	30
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{6}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}}{d}$	30

input
```
int(1/(cosh(d*x+c)+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(3*exp(d*x+c)+1)/d/(exp(d*x+c)+1)^3
```

3.33.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(43) = 86.

Time = 0.24 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.40

$$\int \frac{1}{(1+\cosh(c+dx))^2}\,dx =$$

$$-\frac{2\left(3\cosh\left(dx+c\right)+3\sinh\left(dx+\right.\right.}{3\left(d\cosh\left(dx+c\right)^{3}+d\sinh\left(dx+c\right)^{3}+3\,d\cosh\left(dx+c\right)^{2}+3\left(d\cosh\left(dx+c\right)+d\right)\sinh\left(dx+c\right)^{2}+3\,d\right.}$$

input
```
integrate(1/(1+cosh(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2/3*(3*cosh(d*x + c) + 3*sinh(d*x + c) + 1)/(d*cosh(d*x + c)^3 + d*sinh(d
*x + c)^3 + 3*d*cosh(d*x + c)^2 + 3*(d*cosh(d*x + c) + d)*sinh(d*x + c)^2
+ 3*d*cosh(d*x + c) + 3*(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c) + d)*sinh(d
*x + c) + d)
```

3.33.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{1}{(1+\cosh(c+dx))^2}\,dx = \begin{cases} -\frac{\tanh^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6d} + \frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2d} & \text{for } d \neq 0 \\ \frac{x}{(\cosh(c)+1)^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(1+cosh(d*x+c))**2,x)
```

output
```
Piecewise((-tanh(c/2 + d*x/2)**3/(6*d) + tanh(c/2 + d*x/2)/(2*d), Ne(d, 0)
), (x/(cosh(c) + 1)**2, True))
```

3.33.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(43) = 86.

Time = 0.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.91

$$\int \frac{1}{(1+\cosh(c+dx))^2}\,dx = \frac{2\,e^{(-dx-c)}}{d(3\,e^{(-dx-c)}+3\,e^{(-2\,dx-2\,c)}+e^{(-3\,dx-3\,c)}+1)} + \frac{2}{3\,d(3\,e^{(-dx-c)}+3\,e^{(-2\,dx-2\,c)}+e^{(-3\,dx-3\,c)}+1)}$$

input
```
integrate(1/(1+cosh(d*x+c))^2,x, algorithm="maxima")
```

output
```
2*e^(-d*x - c)/(d*(3*e^(-d*x - c) + 3*e^(-2*d*x - 2*c) + e^(-3*d*x - 3*c)
+ 1)) + 2/3/(d*(3*e^(-d*x - c) + 3*e^(-2*d*x - 2*c) + e^(-3*d*x - 3*c) + 1
))
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.53

$$\int \frac{1}{(1+\cosh(c+dx))^2}\,dx = -\frac{2\left(3\,e^{(dx+c)}+1\right)}{3\,d\big(e^{(dx+c)}+1\big)^3}$$

input
```
integrate(1/(1+cosh(d*x+c))^2,x, algorithm="giac")
```

output
```
-2/3*(3*e^(d*x + c) + 1)/(d*(e^(d*x + c) + 1)^3)
```

3.33.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.53

$$\int \frac{1}{(1+\cosh(c+dx))^2}\,dx = -\frac{2\left(3\,\mathrm{e}^{c+d\,x}+1\right)}{3\,d\,\left(\mathrm{e}^{c+d\,x}+1\right)^3}$$

input
```
int(1/(cosh(c + d*x) + 1)^2,x)
```

output
```
-(2*(3*exp(c + d*x) + 1))/(3*d*(exp(c + d*x) + 1)^3)
```

3.34 $\int \frac{1}{(1+\cosh(c+dx))^3} \, dx$

3.34.1 Optimal result

Integrand size = 10, antiderivative size = 70

$$\int \frac{1}{(1+\cosh(c+dx))^3} \, dx = \frac{\sinh(c+dx)}{5d(1+\cosh(c+dx))^3} + \frac{2\sinh(c+dx)}{15d(1+\cosh(c+dx))^2} + \frac{2\sinh(c+dx)}{15d(1+\cosh(c+dx))}$$

output
```
1/5*sinh(d*x+c)/d/(1+cosh(d*x+c))^3+2/15*sinh(d*x+c)/d/(1+cosh(d*x+c))^2+2
/15*sinh(d*x+c)/d/(1+cosh(d*x+c))
```

3.34.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.63

$$\int \frac{1}{(1+\cosh(c+dx))^3} \, dx = \frac{15\sinh(c+dx) + 6\sinh(2(c+dx)) + \sinh(3(c+dx))}{30d(1+\cosh(c+dx))^3}$$

input
```
Integrate[(1 + Cosh[c + d*x])^(-3),x]
```

output
```
(15*Sinh[c + d*x] + 6*Sinh[2*(c + d*x)] + Sinh[3*(c + d*x)])/(30*d*(1 + Co
sh[c + d*x])^3)
```

3.34.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3129, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\cosh(c+dx)+1)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(1+\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{2}{5}\int \frac{1}{(\cosh(c+dx)+1)^2}dx+\frac{\sinh(c+dx)}{5d(\cosh(c+dx)+1)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(c+dx)}{5d(\cosh(c+dx)+1)^3}+\frac{2}{5}\int \frac{1}{\left(\sin\left(ic+idx+\frac{\pi}{2}\right)+1\right)^2}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{2}{5}\left(\frac{1}{3}\int \frac{1}{\cosh(c+dx)+1}dx+\frac{\sinh(c+dx)}{3d(\cosh(c+dx)+1)^2}\right)+\frac{\sinh(c+dx)}{5d(\cosh(c+dx)+1)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(c+dx)}{5d(\cosh(c+dx)+1)^3}+\frac{2}{5}\left(\frac{\sinh(c+dx)}{3d(\cosh(c+dx)+1)^2}+\frac{1}{3}\int \frac{1}{\sin\left(ic+idx+\frac{\pi}{2}\right)+1}dx\right)$$

$$\Big\downarrow \text{3127}$$

$$\frac{\sinh(c+dx)}{5d(\cosh(c+dx)+1)^3}+\frac{2}{5}\left(\frac{\sinh(c+dx)}{3d(\cosh(c+dx)+1)}+\frac{\sinh(c+dx)}{3d(\cosh(c+dx)+1)^2}\right)$$

input
```
Int[(1 + Cosh[c + d*x])^(-3),x]
```

output
```
Sinh[c + d*x]/(5*d*(1 + Cosh[c + d*x])^3) + (2*(Sinh[c + d*x]/(3*d*(1 + Co
sh[c + d*x])^2) + Sinh[c + d*x]/(3*d*(1 + Cosh[c + d*x]))))/5
```

3.34.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.34.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.53

method	result	size
risch	$-\frac{4\left(10\,{\mathrm e}^{2 d x +2 c}+5\,{\mathrm e}^{d x +c}+1\right)}{15 d \left({\mathrm e}^{d x +c}+1\right)^{5}}$	37
derivativedivides	$\frac{\frac{\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{5}}{20}-\frac{\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{6}+\frac{\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)}{4}}{d}$	43
default	$\frac{\frac{\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{5}}{20}-\frac{\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{6}+\frac{\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)}{4}}{d}$	43
parallelrisch	$\frac{3 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{5}-10 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{3}+15 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)}{60 d}$	44

input
```
int(1/(cosh(d*x+c)+1)^3,x,method=_RETURNVERBOSE)
```

output
```
-4/15*(10*exp(2*d*x+2*c)+5*exp(d*x+c)+1)/d/(exp(d*x+c)+1)^5
```

3.34.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(64) = 128.

Time = 0.25 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.49

$$\int \frac{1}{(1+\cosh(c+dx))^3}\,dx =$$

$$-\frac{}{15\left(d\cosh\left(dx+c\right)^4+d\sinh\left(dx+c\right)^4+5\,d\cosh\left(dx+c\right)^3+\left(4\,d\cosh\left(dx+c\right)+5\,d\right)\sinh\left(dx+c\right)^3+\right.}$$

input
```
integrate(1/(1+cosh(d*x+c))^3,x, algorithm="fricas")
```

output
```
-4/15*(11*cosh(d*x + c) + 9*sinh(d*x + c) + 5)/(d*cosh(d*x + c)^4 + d*sinh
(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + (4*d*cosh(d*x + c) + 5*d)*sinh(d*x + c
)^3 + 10*d*cosh(d*x + c)^2 + (6*d*cosh(d*x + c)^2 + 15*d*cosh(d*x + c) + 1
0*d)*sinh(d*x + c)^2 + 11*d*cosh(d*x + c) + (4*d*cosh(d*x + c)^3 + 15*d*co
sh(d*x + c)^2 + 20*d*cosh(d*x + c) + 9*d)*sinh(d*x + c) + 5*d)
```

3.34.6 Sympy [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.73

$$\int \frac{1}{(1+\cosh(c+dx))^3}\,dx = \begin{cases} \frac{\tanh^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20d} - \frac{\tanh^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6d} + \frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{4d} & \text{for } d \neq 0 \\ \frac{x}{(\cosh(c)+1)^3} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(1+cosh(d*x+c))**3,x)
```

output
```
Piecewise((tanh(c/2 + d*x/2)**5/(20*d) - tanh(c/2 + d*x/2)**3/(6*d) + tanh
(c/2 + d*x/2)/(4*d), Ne(d, 0)), (x/(cosh(c) + 1)**3, True))
```

3.34.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(64) = 128$.

Time = 0.18 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.93

$$\int \frac{1}{(1+\cosh(c+dx))^3}\,dx = \frac{4\,e^{(-dx-c)}}{3\,d(5\,e^{(-dx-c)}+10\,e^{(-2\,dx-2\,c)}+10\,e^{(-3\,dx-3\,c)}+5\,e^{(-4\,dx-4\,c)}+e^{(-5\,dx-5\,c)}+1)} + \frac{8\,e^{(-2\,dx-2\,c)}}{3\,d(5\,e^{(-dx-c)}+10\,e^{(-2\,dx-2\,c)}+10\,e^{(-3\,dx-3\,c)}+5\,e^{(-4\,dx-4\,c)}+e^{(-5\,dx-5\,c)}+1)} + \frac{4}{15\,d(5\,e^{(-dx-c)}+10\,e^{(-2\,dx-2\,c)}+10\,e^{(-3\,dx-3\,c)}+5\,e^{(-4\,dx-4\,c)}+e^{(-5\,dx-5\,c)}+1)}$$

input
```
integrate(1/(1+cosh(d*x+c))^3,x, algorithm="maxima")
```

output
```
4/3*e^(-d*x - c)/(d*(5*e^(-d*x - c) + 10*e^(-2*d*x - 2*c) + 10*e^(-3*d*x -
 3*c) + 5*e^(-4*d*x - 4*c) + e^(-5*d*x - 5*c) + 1)) + 8/3*e^(-2*d*x - 2*c)
/(d*(5*e^(-d*x - c) + 10*e^(-2*d*x - 2*c) + 10*e^(-3*d*x - 3*c) + 5*e^(-4*
d*x - 4*c) + e^(-5*d*x - 5*c) + 1)) + 4/15/(d*(5*e^(-d*x - c) + 10*e^(-2*d
*x - 2*c) + 10*e^(-3*d*x - 3*c) + 5*e^(-4*d*x - 4*c) + e^(-5*d*x - 5*c) +
1))
```

3.34.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.51

$$\int \frac{1}{(1+\cosh(c+dx))^3}\,dx = -\frac{4\left(10\,e^{(2\,dx+2\,c)}+5\,e^{(dx+c)}+1\right)}{15\,d\left(e^{(dx+c)}+1\right)^5}$$

input
```
integrate(1/(1+cosh(d*x+c))^3,x, algorithm="giac")
```

output
```
-4/15*(10*e^(2*d*x + 2*c) + 5*e^(d*x + c) + 1)/(d*(e^(d*x + c) + 1)^5)
```

3.34.9 Mupad [B] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.51

$$\int \frac{1}{(1+\cosh(c+dx))^3}\,dx = -\frac{4\left(5\,\mathrm{e}^{c+d\,x}+10\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}{15\,d\,\left(\mathrm{e}^{c+d\,x}+1\right)^5}$$

input
```
int(1/(cosh(c + d*x) + 1)^3,x)
```

output
```
-(4*(5*exp(c + d*x) + 10*exp(2*c + 2*d*x) + 1))/(15*d*(exp(c + d*x) + 1)^5
)
```

3.35 $\int \frac{1}{(1+\cosh(c+dx))^4}\,dx$

3.35.1 Optimal result

Integrand size = 10, antiderivative size = 93

$$\int \frac{1}{(1+\cosh(c+dx))^4}\,dx = \frac{\sinh(c+dx)}{7d(1+\cosh(c+dx))^4} + \frac{3\sinh(c+dx)}{35d(1+\cosh(c+dx))^3} + \frac{2\sinh(c+dx)}{35d(1+\cosh(c+dx))^2} + \frac{2\sinh(c+dx)}{35d(1+\cosh(c+dx))}$$

output
```
1/7*sinh(d*x+c)/d/(1+cosh(d*x+c))^4+3/35*sinh(d*x+c)/d/(1+cosh(d*x+c))^3+2
/35*sinh(d*x+c)/d/(1+cosh(d*x+c))^2+2/35*sinh(d*x+c)/d/(1+cosh(d*x+c))
```

3.35.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.58

$$\int \frac{1}{(1+\cosh(c+dx))^4}\,dx = \frac{56\sinh(c+dx)+28\sinh(2(c+dx))+8\sinh(3(c+dx))+\sinh(4(c+dx))}{140d(1+\cosh(c+dx))^4}$$

input
```
Integrate[(1 + Cosh[c + d*x])^(-4),x]
```

output
```
(56*Sinh[c + d*x] + 28*Sinh[2*(c + d*x)] + 8*Sinh[3*(c + d*x)] + Sinh[4*(c
 + d*x)])/(140*d*(1 + Cosh[c + d*x])^4)
```

3.35.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 3129, 3042, 3129, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\cosh(c+dx)+1)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(1+\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{3}{7}\int \frac{1}{(\cosh(c+dx)+1)^3}dx+\frac{\sinh(c+dx)}{7d(\cosh(c+dx)+1)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(c+dx)}{7d(\cosh(c+dx)+1)^4}+\frac{3}{7}\int \frac{1}{\left(\sin\left(ic+idx+\frac{\pi}{2}\right)+1\right)^3}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{3}{7}\left(\frac{2}{5}\int \frac{1}{(\cosh(c+dx)+1)^2}dx+\frac{\sinh(c+dx)}{5d(\cosh(c+dx)+1)^3}\right)+\frac{\sinh(c+dx)}{7d(\cosh(c+dx)+1)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(c+dx)}{7d(\cosh(c+dx)+1)^4}+\frac{3}{7}\left(\frac{\sinh(c+dx)}{5d(\cosh(c+dx)+1)^3}+\frac{2}{5}\int \frac{1}{\left(\sin\left(ic+idx+\frac{\pi}{2}\right)+1\right)^2}dx\right)$$

$$\Big\downarrow \text{3129}$$

$$\frac{3}{7}\left(\frac{2}{5}\left(\frac{1}{3}\int \frac{1}{\cosh(c+dx)+1}dx+\frac{\sinh(c+dx)}{3d(\cosh(c+dx)+1)^2}\right)+\frac{\sinh(c+dx)}{5d(\cosh(c+dx)+1)^3}\right)+\frac{\sinh(c+dx)}{7d(\cosh(c+dx)+1)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(c+dx)}{7d(\cosh(c+dx)+1)^4}+\frac{3}{7}\left(\frac{\sinh(c+dx)}{5d(\cosh(c+dx)+1)^3}+\frac{2}{5}\left(\frac{\sinh(c+dx)}{3d(\cosh(c+dx)+1)^2}+\frac{1}{3}\int \frac{1}{\sin\left(ic+idx+\frac{\pi}{2}\right)+1}dx\right)\right)$$

$$\Big\downarrow \text{3127}$$

$$\frac{\sinh(c+dx)}{7d(\cosh(c+dx)+1)^4}+\frac{3}{7}\left(\frac{\sinh(c+dx)}{5d(\cosh(c+dx)+1)^3}+\frac{2}{5}\left(\frac{\sinh(c+dx)}{3d(\cosh(c+dx)+1)}+\frac{\sinh(c+dx)}{3d(\cosh(c+dx)+1)^2}\right)\right)$$

input
```
Int[(1 + Cosh[c + d*x])^(-4),x]
```

output
```
Sinh[c + d*x]/(7*d*(1 + Cosh[c + d*x])^4) + (3*(Sinh[c + d*x]/(5*d*(1 + Co
sh[c + d*x])^3) + (2*(Sinh[c + d*x]/(3*d*(1 + Cosh[c + d*x])^2) + Sinh[c +
 d*x]/(3*d*(1 + Cosh[c + d*x]))))/5))/7
```

3.35.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.35.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.52

method	result	size
risch	$-\frac{4\left(35\,\mathrm{e}^{3dx+3c}+21\,\mathrm{e}^{2dx+2c}+7\,\mathrm{e}^{dx+c}+1\right)}{35d\left(\mathrm{e}^{dx+c}+1\right)^{7}}$	48
parallelrisch	$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-\frac{21\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{5}+7\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-7\right)}{56d}$	54
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{56}+\frac{3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{40}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{8}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8}}{d}$	56
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{56}+\frac{3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{40}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{8}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8}}{d}$	56

input
```
int(1/(cosh(d*x+c)+1)^4,x,method=_RETURNVERBOSE)
```

output
```
-4/35*(35*exp(3*d*x+3*c)+21*exp(2*d*x+2*c)+7*exp(d*x+c)+1)/d/(exp(d*x+c)+1
)^7
```

3.35.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(85) = 170.

Time = 0.25 (sec) , antiderivative size = 347, normalized size of antiderivative = 3.73

$$\int \frac{1}{(1+\cosh(c+dx))^4}\,dx =$$

$$-\frac{}{35\left(d\cosh\left(dx+c\right)^{6}+d\sinh\left(dx+c\right)^{6}+7\,d\cosh\left(dx+c\right)^{5}+\left(6\,d\cosh\left(dx+c\right)+7\,d\right)\sinh\left(dx+c\right)^{5}+\right.}$$

input
```
integrate(1/(1+cosh(d*x+c))^4,x, algorithm="fricas")
```

output
```
-4/35*(35*cosh(d*x + c)^2 + 10*(7*cosh(d*x + c) + 2)*sinh(d*x + c) + 35*si
nh(d*x + c)^2 + 22*cosh(d*x + c) + 7)/(d*cosh(d*x + c)^6 + d*sinh(d*x + c)
^6 + 7*d*cosh(d*x + c)^5 + (6*d*cosh(d*x + c) + 7*d)*sinh(d*x + c)^5 + 21*
d*cosh(d*x + c)^4 + (15*d*cosh(d*x + c)^2 + 35*d*cosh(d*x + c) + 21*d)*sin
h(d*x + c)^4 + 35*d*cosh(d*x + c)^3 + (20*d*cosh(d*x + c)^3 + 70*d*cosh(d*
x + c)^2 + 84*d*cosh(d*x + c) + 35*d)*sinh(d*x + c)^3 + 35*d*cosh(d*x + c)
^2 + (15*d*cosh(d*x + c)^4 + 70*d*cosh(d*x + c)^3 + 126*d*cosh(d*x + c)^2
+ 105*d*cosh(d*x + c) + 35*d)*sinh(d*x + c)^2 + 22*d*cosh(d*x + c) + (6*d*
cosh(d*x + c)^5 + 35*d*cosh(d*x + c)^4 + 84*d*cosh(d*x + c)^3 + 105*d*cosh
(d*x + c)^2 + 70*d*cosh(d*x + c) + 20*d)*sinh(d*x + c) + 7*d)
```

3.35.6 Sympy [A] (verification not implemented)

Time = 1.75 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.73

$$\int \frac{1}{(1+\cosh(c+dx))^4}\,dx = \begin{cases} -\frac{\tanh^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56d} + \frac{3\tanh^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40d} - \frac{\tanh^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{8d} + \frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{8d} & \text{for } d \neq 0 \\ \frac{x}{(\cosh(c)+1)^4} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(1+cosh(d*x+c))**4,x)
```

output
```
Piecewise((-tanh(c/2 + d*x/2)**7/(56*d) + 3*tanh(c/2 + d*x/2)**5/(40*d) -
tanh(c/2 + d*x/2)**3/(8*d) + tanh(c/2 + d*x/2)/(8*d), Ne(d, 0)), (x/(cosh(
c) + 1)**4, True))
```

3.35.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(85) = 170$.

Time = 0.19 (sec) , antiderivative size = 364, normalized size of antiderivative = 3.91

$$\int \frac{1}{(1+\cosh(c+dx))^4}\,dx$$
$$= \frac{4\,e^{(-dx-c)}}{5\,d(7\,e^{(-dx-c)} + 21\,e^{(-2\,dx-2\,c)} + 35\,e^{(-3\,dx-3\,c)} + 35\,e^{(-4\,dx-4\,c)} + 21\,e^{(-5\,dx-5\,c)} + 7\,e^{(-6\,dx-6\,c)} + e^{(-7\,dx-7\,c)} +}$$
$$+ \frac{12\,e^{(-2\,dx-2\,c)}}{5\,d(7\,e^{(-dx-c)} + 21\,e^{(-2\,dx-2\,c)} + 35\,e^{(-3\,dx-3\,c)} + 35\,e^{(-4\,dx-4\,c)} + 21\,e^{(-5\,dx-5\,c)} + 7\,e^{(-6\,dx-6\,c)} + e^{(-7\,dx-7\,c}}$$
$$+ \frac{4\,e^{(-3\,dx-3\,c)}}{d(7\,e^{(-dx-c)} + 21\,e^{(-2\,dx-2\,c)} + 35\,e^{(-3\,dx-3\,c)} + 35\,e^{(-4\,dx-4\,c)} + 21\,e^{(-5\,dx-5\,c)} + 7\,e^{(-6\,dx-6\,c)} + e^{(-7\,dx-7\,c)}}$$
$$+ \frac{4}{35\,d(7\,e^{(-dx-c)} + 21\,e^{(-2\,dx-2\,c)} + 35\,e^{(-3\,dx-3\,c)} + 35\,e^{(-4\,dx-4\,c)} + 21\,e^{(-5\,dx-5\,c)} + 7\,e^{(-6\,dx-6\,c)} + e^{(-7\,dx-7}}$$

input
```
integrate(1/(1+cosh(d*x+c))^4,x, algorithm="maxima")
```

output
```
4/5*e^(-d*x - c)/(d*(7*e^(-d*x - c) + 21*e^(-2*d*x - 2*c) + 35*e^(-3*d*x -
 3*c) + 35*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) + 7*e^(-6*d*x - 6*c) + e
^(-7*d*x - 7*c) + 1)) + 12/5*e^(-2*d*x - 2*c)/(d*(7*e^(-d*x - c) + 21*e^(-
2*d*x - 2*c) + 35*e^(-3*d*x - 3*c) + 35*e^(-4*d*x - 4*c) + 21*e^(-5*d*x -
5*c) + 7*e^(-6*d*x - 6*c) + e^(-7*d*x - 7*c) + 1)) + 4*e^(-3*d*x - 3*c)/(d
*(7*e^(-d*x - c) + 21*e^(-2*d*x - 2*c) + 35*e^(-3*d*x - 3*c) + 35*e^(-4*d*
x - 4*c) + 21*e^(-5*d*x - 5*c) + 7*e^(-6*d*x - 6*c) + e^(-7*d*x - 7*c) + 1
)) + 4/35/(d*(7*e^(-d*x - c) + 21*e^(-2*d*x - 2*c) + 35*e^(-3*d*x - 3*c) +
 35*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) + 7*e^(-6*d*x - 6*c) + e^(-7*d*
x - 7*c) + 1))
```

3.35.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.51

$$\int \frac{1}{(1+\cosh(c+dx))^4}\,dx = -\frac{4\left(35\,e^{(3\,dx+3\,c)} + 21\,e^{(2\,dx+2\,c)} + 7\,e^{(dx+c)} + 1\right)}{35\,d\left(e^{(dx+c)} + 1\right)^7}$$

input
```
integrate(1/(1+cosh(d*x+c))^4,x, algorithm="giac")
```

output
```
-4/35*(35*e^(3*d*x + 3*c) + 21*e^(2*d*x + 2*c) + 7*e^(d*x + c) + 1)/(d*(e^
(d*x + c) + 1)^7)
```

3.35.9 Mupad [B] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 283, normalized size of antiderivative = 3.04

$$\int \frac{1}{(1+\cosh(c+dx))^4}\,dx = -\frac{4}{35\,d\,\left(4\,e^{c+d\,x}+6\,e^{2\,c+2\,d\,x}+4\,e^{3\,c+3\,d\,x}+e^{4\,c+4\,d\,x}+1\right)} - \frac{16\,e^{c+d\,x}}{35\,d\,\left(5\,e^{c+d\,x}+10\,e^{2\,c+2\,d\,x}+10\,e^{3\,c+3\,d\,x}+5\,e^{4\,c+4\,d\,x}+e^{5\,c+5\,d\,x}+1\right)} - \frac{8\,e^{2\,c+2\,d\,x}}{7\,d\,\left(6\,e^{c+d\,x}+15\,e^{2\,c+2\,d\,x}+20\,e^{3\,c+3\,d\,x}+15\,e^{4\,c+4\,d\,x}+6\,e^{5\,c+5\,d\,x}+e^{6\,c+6\,d\,x}+1\right)} - \frac{16\,e^{3\,c+3\,d\,x}}{7\,d\,\left(7\,e^{c+d\,x}+21\,e^{2\,c+2\,d\,x}+35\,e^{3\,c+3\,d\,x}+35\,e^{4\,c+4\,d\,x}+21\,e^{5\,c+5\,d\,x}+7\,e^{6\,c+6\,d\,x}+e^{7\,c+7\,d\,x}+1\right)}$$

input
```
int(1/(cosh(c + d*x) + 1)^4,x)
```

output
```
- 4/(35*d*(4*exp(c + d*x) + 6*exp(2*c + 2*d*x) + 4*exp(3*c + 3*d*x) + exp(
4*c + 4*d*x) + 1)) - (16*exp(c + d*x))/(35*d*(5*exp(c + d*x) + 10*exp(2*c
+ 2*d*x) + 10*exp(3*c + 3*d*x) + 5*exp(4*c + 4*d*x) + exp(5*c + 5*d*x) + 1
)) - (8*exp(2*c + 2*d*x))/(7*d*(6*exp(c + d*x) + 15*exp(2*c + 2*d*x) + 20*
exp(3*c + 3*d*x) + 15*exp(4*c + 4*d*x) + 6*exp(5*c + 5*d*x) + exp(6*c + 6*
d*x) + 1)) - (16*exp(3*c + 3*d*x))/(7*d*(7*exp(c + d*x) + 21*exp(2*c + 2*d
*x) + 35*exp(3*c + 3*d*x) + 35*exp(4*c + 4*d*x) + 21*exp(5*c + 5*d*x) + 7*
exp(6*c + 6*d*x) + exp(7*c + 7*d*x) + 1))
```

3.36 $\int \frac{1}{1-\cosh(c+dx)}\,dx$

3.36.1 Optimal result

Integrand size = 12, antiderivative size = 23

$$\int \frac{1}{1-\cosh(c+dx)}\,dx = -\frac{\sinh(c+dx)}{d(1-\cosh(c+dx))}$$

output
```
-sinh(d*x+c)/d/(1-cosh(d*x+c))
```

3.36.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

$$\int \frac{1}{1-\cosh(c+dx)}\,dx = \frac{\coth\left(\frac{1}{2}(c+dx)\right)}{d}$$

input
```
Integrate[(1 - Cosh[c + d*x])^(-1),x]
```

output
```
Coth[(c + d*x)/2]/d
```

3.36.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-\cosh(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{1-\sin\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3127}$$

$$-\frac{\sinh(c+dx)}{d(1-\cosh(c+dx))}$$

input
```
Int[(1 - Cosh[c + d*x])^(-1),x]
```

output
```
-(Sinh[c + d*x]/(d*(1 - Cosh[c + d*x])))
```

3.36.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

3.36.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{1}{d\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}$	16
default	$\frac{1}{d\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}$	16
risch	$\frac{2}{d\left(e^{dx+c}-1\right)}$	16
parallelrisch	$\frac{1}{d\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}$	16

input
```
int(1/(1-cosh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/tanh(1/2*d*x+1/2*c)
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1}{1-\cosh(c+dx)}\,dx = \frac{2}{d\cosh\left(dx+c\right)+d\sinh\left(dx+c\right)-d}$$

input
```
integrate(1/(1-cosh(d*x+c)),x, algorithm="fricas")
```

output
```
2/(d*cosh(d*x + c) + d*sinh(d*x + c) - d)
```

3.36.6 Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{1-\cosh(c+dx)}\,dx = \begin{cases} \frac{1}{d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)} & \text{for } d\neq 0 \\ \frac{x}{1-\cosh\left(c\right)} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(1-cosh(d*x+c)),x)
```

output
```
Piecewise((1/(d*tanh(c/2 + d*x/2)), Ne(d, 0)), (x/(1 - cosh(c)), True))
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{1}{1-\cosh(c+dx)}\,dx = -\frac{2}{d(e^{(-dx-c)}-1)}$$

input
```
integrate(1/(1-cosh(d*x+c)),x, algorithm="maxima")
```

output
```
-2/(d*(e^(-d*x - c) - 1))
```

3.36.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{1}{1-\cosh(c+dx)}\,dx = \frac{2}{d(e^{(dx+c)}-1)}$$

input
```
integrate(1/(1-cosh(d*x+c)),x, algorithm="giac")
```

output
```
2/(d*(e^(d*x + c) - 1))
```

3.36.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{1}{1-\cosh(c+dx)}\,dx = \frac{2}{d\,(\mathrm{e}^{c+d\,x}-1)}$$

input
```
int(-1/(cosh(c + d*x) - 1),x)
```

output
```
2/(d*(exp(c + d*x) - 1))
```

3.37 $\int \frac{1}{(1-\cosh(c+dx))^2} dx$

3.37.1 Optimal result

Integrand size = 12, antiderivative size = 51

$$\int \frac{1}{(1-\cosh(c+dx))^2} dx = -\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))^2} - \frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))}$$

output
```
-1/3*sinh(d*x+c)/d/(1-cosh(d*x+c))^2-1/3*sinh(d*x+c)/d/(1-cosh(d*x+c))
```

3.37.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.61

$$\int \frac{1}{(1-\cosh(c+dx))^2} dx = \frac{(-2+\cosh(c+dx))\sinh(c+dx)}{3d(-1+\cosh(c+dx))^2}$$

input
```
Integrate[(1 - Cosh[c + d*x])^(-2),x]
```

output
```
((-2 + Cosh[c + d*x])*Sinh[c + d*x])/(3*d*(-1 + Cosh[c + d*x])^2)
```

3.37.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-\cosh(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(1-\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{1}{3}\int \frac{1}{1-\cosh(c+dx)}dx-\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))^2}+\frac{1}{3}\int \frac{1}{1-\sin\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3127}$$

$$-\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))}-\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))^2}$$

input
```
Int[(1 - Cosh[c + d*x])^(-2),x]
```

output
```
-1/3*Sinh[c + d*x]/(d*(1 - Cosh[c + d*x])^2) - Sinh[c + d*x]/(3*d*(1 - Cos
h[c + d*x]))
```

3.37.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.37.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.51

method	result	size
risch	$-\frac{2\left(-1+3\,{\mathrm e}^{d x +c}\right)}{3 d \left({\mathrm e}^{d x +c}-1\right)^{3}}$	26
parallelrisch	$-\frac{\coth\left(\frac{d x}{2}+\frac{c}{2}\right)\left(\coth\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}-3\right)}{6 d}$	28
derivativedivides	$\frac{-\frac{1}{6 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}+\frac{1}{2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)}}{d}$	32
default	$\frac{-\frac{1}{6 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}+\frac{1}{2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)}}{d}$	32

input
```
int(1/(1-cosh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-1+3*exp(d*x+c))/d/(exp(d*x+c)-1)^3
```

3.37.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(43) = 86.

Time = 0.23 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.29

$$\int \frac{1}{(1-\cosh(c+dx))^2}\,dx =$$

$$-\frac{2\left(3\,\cosh\left(dx+c\right)+3\,\sinh\left(dx+\right.\right.}{3\left(d\cosh\left(dx+c\right)^{3}+d\sinh\left(dx+c\right)^{3}-3\,d\cosh\left(dx+c\right)^{2}+3\left(d\cosh\left(dx+c\right)-d\right)\sinh\left(dx+c\right)^{2}+3\,d\right.}$$

input
```
integrate(1/(1-cosh(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2/3*(3*cosh(d*x + c) + 3*sinh(d*x + c) - 1)/(d*cosh(d*x + c)^3 + d*sinh(d
*x + c)^3 - 3*d*cosh(d*x + c)^2 + 3*(d*cosh(d*x + c) - d)*sinh(d*x + c)^2
+ 3*d*cosh(d*x + c) + 3*(d*cosh(d*x + c)^2 - 2*d*cosh(d*x + c) + d)*sinh(d
*x + c) - d)
```

3.37.6 Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \frac{1}{(1-\cosh(c+dx))^2}\,dx = \begin{cases} \frac{1}{2d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)} - \frac{1}{6d\tanh^3\left(\frac{c}{2}+\frac{dx}{2}\right)} & \text{for } d\neq 0 \\ \frac{x}{(1-\cosh(c))^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(1-cosh(d*x+c))**2,x)
```

output
```
Piecewise((1/(2*d*tanh(c/2 + d*x/2)) - 1/(6*d*tanh(c/2 + d*x/2)**3), Ne(d,
 0)), (x/(1 - cosh(c))**2, True))
```

3.37.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(43) = 86$.

Time = 0.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.76

$$\int \frac{1}{(1-\cosh(c+dx))^2}\,dx = \frac{2\,e^{(-dx-c)}}{d(3\,e^{(-dx-c)} - 3\,e^{(-2\,dx-2\,c)} + e^{(-3\,dx-3\,c)} - 1)} - \frac{2}{3\,d(3\,e^{(-dx-c)} - 3\,e^{(-2\,dx-2\,c)} + e^{(-3\,dx-3\,c)} - 1)}$$

input
```
integrate(1/(1-cosh(d*x+c))^2,x, algorithm="maxima")
```

output
```
2*e^(-d*x - c)/(d*(3*e^(-d*x - c) - 3*e^(-2*d*x - 2*c) + e^(-3*d*x - 3*c)
- 1)) - 2/3/(d*(3*e^(-d*x - c) - 3*e^(-2*d*x - 2*c) + e^(-3*d*x - 3*c) - 1
))
```

3.37.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.49

$$\int \frac{1}{(1-\cosh(c+dx))^2}\,dx = -\frac{2\left(3\,e^{(dx+c)}-1\right)}{3\,d(e^{(dx+c)}-1)^3}$$

input
```
integrate(1/(1-cosh(d*x+c))^2,x, algorithm="giac")
```

output
```
-2/3*(3*e^(d*x + c) - 1)/(d*(e^(d*x + c) - 1)^3)
```

3.37.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.49

$$\int \frac{1}{(1-\cosh(c+dx))^2}\,dx = -\frac{2\left(3\,\mathrm{e}^{c+d\,x}-1\right)}{3\,d\left(\mathrm{e}^{c+d\,x}-1\right)^3}$$

input
```
int(1/(cosh(c + d*x) - 1)^2,x)
```

output
```
-(2*(3*exp(c + d*x) - 1))/(3*d*(exp(c + d*x) - 1)^3)
```

3.38 $\int \frac{1}{(1-\cosh(c+dx))^3}\,dx$

3.38.1 Optimal result

Integrand size = 12, antiderivative size = 76

$$\int \frac{1}{(1-\cosh(c+dx))^3}\,dx = -\frac{\sinh(c+dx)}{5d(1-\cosh(c+dx))^3} - \frac{2\sinh(c+dx)}{15d(1-\cosh(c+dx))^2} - \frac{2\sinh(c+dx)}{15d(1-\cosh(c+dx))}$$

output
```
-1/5*sinh(d*x+c)/d/(1-cosh(d*x+c))^3-2/15*sinh(d*x+c)/d/(1-cosh(d*x+c))^2-
2/15*sinh(d*x+c)/d/(1-cosh(d*x+c))
```

3.38.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.54

$$\int \frac{1}{(1-\cosh(c+dx))^3}\,dx = \frac{(8-6\cosh(c+dx)+\cosh(2(c+dx)))\sinh(c+dx)}{15d(-1+\cosh(c+dx))^3}$$

input
```
Integrate[(1 - Cosh[c + d*x])^(-3),x]
```

output
```
((8 - 6*Cosh[c + d*x] + Cosh[2*(c + d*x)])*Sinh[c + d*x])/(15*d*(-1 + Cosh
[c + d*x])^3)
```

3.38.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3129, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-\cosh(c+dx))^3}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\left(1-\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \; 3129$$

$$\frac{2}{5}\int \frac{1}{(1-\cosh(c+dx))^2}dx-\frac{\sinh(c+dx)}{5d(1-\cosh(c+dx))^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\sinh(c+dx)}{5d(1-\cosh(c+dx))^3}+\frac{2}{5}\int \frac{1}{\left(1-\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \; 3129$$

$$\frac{2}{5}\left(\frac{1}{3}\int \frac{1}{1-\cosh(c+dx)}dx-\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))^2}\right)-\frac{\sinh(c+dx)}{5d(1-\cosh(c+dx))^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\sinh(c+dx)}{5d(1-\cosh(c+dx))^3}+\frac{2}{5}\left(-\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))^2}+\frac{1}{3}\int \frac{1}{1-\sin\left(ic+idx+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \; 3127$$

$$\frac{2}{5}\left(-\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))}-\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))^2}\right)-\frac{\sinh(c+dx)}{5d(1-\cosh(c+dx))^3}$$

input
```
Int[(1 - Cosh[c + d*x])^(-3),x]
```

output
```
-1/5*Sinh[c + d*x]/(d*(1 - Cosh[c + d*x])^3) + (2*(-1/3*Sinh[c + d*x]/(d*(
1 - Cosh[c + d*x])^2) - Sinh[c + d*x]/(3*d*(1 - Cosh[c + d*x]))))/5
```

3.38.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.38.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.49

method	result	size
risch	$\frac{\frac{8\,\mathrm{e}^{2dx+2c}}{3}-\frac{4\,\mathrm{e}^{dx+c}}{3}+\frac{4}{15}}{d\left(\mathrm{e}^{dx+c}-1\right)^{5}}$	37
parallelrisch	$\frac{3\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-10\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+15\coth\left(\frac{dx}{2}+\frac{c}{2}\right)}{60d}$	44
derivativedivides	$\frac{-\frac{1}{6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}+\frac{1}{4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{1}{20\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}}{d}$	45
default	$\frac{-\frac{1}{6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}+\frac{1}{4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{1}{20\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}}{d}$	45

input
```
int(1/(1-cosh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
4/15*(10*exp(2*d*x+2*c)-5*exp(d*x+c)+1)/d/(exp(d*x+c)-1)^5
```

3.38.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(64) = 128$.

Time = 0.25 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.29

$$\int \frac{1}{(1-\cosh(c+dx))^3}\,dx$$

$$= \frac{}{15\left(d\cosh\left(dx+c\right)^4 + d\sinh\left(dx+c\right)^4 - 5\,d\cosh\left(dx+c\right)^3 + \left(4\,d\cosh\left(dx+c\right) - 5\,d\right)\sinh\left(dx+c\right)^3 + 10\right.}$$

input
```
integrate(1/(1-cosh(d*x+c))^3,x, algorithm="fricas")
```

output
```
4/15*(11*cosh(d*x + c) + 9*sinh(d*x + c) - 5)/(d*cosh(d*x + c)^4 + d*sinh(
d*x + c)^4 - 5*d*cosh(d*x + c)^3 + (4*d*cosh(d*x + c) - 5*d)*sinh(d*x + c)
^3 + 10*d*cosh(d*x + c)^2 + (6*d*cosh(d*x + c)^2 - 15*d*cosh(d*x + c) + 10
*d)*sinh(d*x + c)^2 - 11*d*cosh(d*x + c) + (4*d*cosh(d*x + c)^3 - 15*d*cos
h(d*x + c)^2 + 20*d*cosh(d*x + c) - 9*d)*sinh(d*x + c) + 5*d)
```

3.38.6 Sympy [A] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.74

$$\int \frac{1}{(1-\cosh(c+dx))^3}\,dx = \begin{cases} \frac{1}{4d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)} - \frac{1}{6d\tanh^3\left(\frac{c}{2}+\frac{dx}{2}\right)} + \frac{1}{20d\tanh^5\left(\frac{c}{2}+\frac{dx}{2}\right)} & \text{for } d \neq 0 \\ \frac{x}{(1-\cosh(c))^3} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(1-cosh(d*x+c))**3,x)
```

output
```
Piecewise((1/(4*d*tanh(c/2 + d*x/2)) - 1/(6*d*tanh(c/2 + d*x/2)**3) + 1/(2
0*d*tanh(c/2 + d*x/2)**5), Ne(d, 0)), (x/(1 - cosh(c))**3, True))
```

3.38.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(64) = 128$.

Time = 0.19 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.70

$$\int \frac{1}{(1-\cosh(c+dx))^3}\,dx$$
$$= \frac{4\,e^{(-dx-c)}}{3\,d(5\,e^{(-dx-c)} - 10\,e^{(-2\,dx-2\,c)} + 10\,e^{(-3\,dx-3\,c)} - 5\,e^{(-4\,dx-4\,c)} + e^{(-5\,dx-5\,c)} - 1)}$$
$$- \frac{8\,e^{(-2\,dx-2\,c)}}{3\,d(5\,e^{(-dx-c)} - 10\,e^{(-2\,dx-2\,c)} + 10\,e^{(-3\,dx-3\,c)} - 5\,e^{(-4\,dx-4\,c)} + e^{(-5\,dx-5\,c)} - 1)}$$
$$- \frac{4}{15\,d(5\,e^{(-dx-c)} - 10\,e^{(-2\,dx-2\,c)} + 10\,e^{(-3\,dx-3\,c)} - 5\,e^{(-4\,dx-4\,c)} + e^{(-5\,dx-5\,c)} - 1)}$$

input
```
integrate(1/(1-cosh(d*x+c))^3,x, algorithm="maxima")
```

output
```
4/3*e^(-d*x - c)/(d*(5*e^(-d*x - c) - 10*e^(-2*d*x - 2*c) + 10*e^(-3*d*x -
 3*c) - 5*e^(-4*d*x - 4*c) + e^(-5*d*x - 5*c) - 1)) - 8/3*e^(-2*d*x - 2*c)
/(d*(5*e^(-d*x - c) - 10*e^(-2*d*x - 2*c) + 10*e^(-3*d*x - 3*c) - 5*e^(-4*
d*x - 4*c) + e^(-5*d*x - 5*c) - 1)) - 4/15/(d*(5*e^(-d*x - c) - 10*e^(-2*d
*x - 2*c) + 10*e^(-3*d*x - 3*c) - 5*e^(-4*d*x - 4*c) + e^(-5*d*x - 5*c) -
1))
```

3.38.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.47

$$\int \frac{1}{(1-\cosh(c+dx))^3}\,dx = \frac{4\left(10\,e^{(2\,dx+2\,c)} - 5\,e^{(dx+c)} + 1\right)}{15\,d\left(e^{(dx+c)} - 1\right)^5}$$

input
```
integrate(1/(1-cosh(d*x+c))^3,x, algorithm="giac")
```

output
```
4/15*(10*e^(2*d*x + 2*c) - 5*e^(d*x + c) + 1)/(d*(e^(d*x + c) - 1)^5)
```

3.38.9 Mupad [B] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.47

$$\int \frac{1}{(1-\cosh(c+dx))^3}\,dx = \frac{4\left(10\,e^{2\,c+2\,d\,x}-5\,e^{c+d\,x}+1\right)}{15\,d\,\left(e^{c+d\,x}-1\right)^5}$$

input
```
int(-1/(cosh(c + d*x) - 1)^3,x)
```

output
```
(4*(10*exp(2*c + 2*d*x) - 5*exp(c + d*x) + 1))/(15*d*(exp(c + d*x) - 1)^5)
```

3.39 $\int \frac{1}{(1-\cosh(c+dx))^4}\,dx$

3.39.1 Optimal result

Integrand size = 12, antiderivative size = 101

$$\int \frac{1}{(1-\cosh(c+dx))^4}\,dx = -\frac{\sinh(c+dx)}{7d(1-\cosh(c+dx))^4} - \frac{3\sinh(c+dx)}{35d(1-\cosh(c+dx))^3} - \frac{2\sinh(c+dx)}{35d(1-\cosh(c+dx))^2} - \frac{2\sinh(c+dx)}{35d(1-\cosh(c+dx))}$$

output
```
-1/7*sinh(d*x+c)/d/(1-cosh(d*x+c))^4-3/35*sinh(d*x+c)/d/(1-cosh(d*x+c))^3-
2/35*sinh(d*x+c)/d/(1-cosh(d*x+c))^2-2/35*sinh(d*x+c)/d/(1-cosh(d*x+c))
```

3.39.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.50

$$\int \frac{1}{(1-\cosh(c+dx))^4}\,dx = \frac{(-32+29\cosh(c+dx)-8\cosh(2(c+dx))+\cosh(3(c+dx)))\sinh(c+dx)}{70d(-1+\cosh(c+dx))^4}$$

input
```
Integrate[(1 - Cosh[c + d*x])^(-4),x]
```

output
```
((-32 + 29*Cosh[c + d*x] - 8*Cosh[2*(c + d*x)] + Cosh[3*(c + d*x)])*Sinh[c
 + d*x])/(70*d*(-1 + Cosh[c + d*x])^4)
```

3.39.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3129, 3042, 3129, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-\cosh(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(1-\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{3}{7}\int \frac{1}{(1-\cosh(c+dx))^3}dx-\frac{\sinh(c+dx)}{7d(1-\cosh(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(c+dx)}{7d(1-\cosh(c+dx))^4}+\frac{3}{7}\int \frac{1}{\left(1-\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{3}{7}\left(\frac{2}{5}\int \frac{1}{(1-\cosh(c+dx))^2}dx-\frac{\sinh(c+dx)}{5d(1-\cosh(c+dx))^3}\right)-\frac{\sinh(c+dx)}{7d(1-\cosh(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(c+dx)}{7d(1-\cosh(c+dx))^4}+\frac{3}{7}\left(-\frac{\sinh(c+dx)}{5d(1-\cosh(c+dx))^3}+\frac{2}{5}\int \frac{1}{\left(1-\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^2}dx\right)$$

$$\Big\downarrow \text{3129}$$

$$\frac{3}{7}\left(\frac{2}{5}\left(\frac{1}{3}\int \frac{1}{1-\cosh(c+dx)}dx-\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))^2}\right)-\frac{\sinh(c+dx)}{5d(1-\cosh(c+dx))^3}\right)-\frac{\sinh(c+dx)}{7d(1-\cosh(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(c+dx)}{7d(1-\cosh(c+dx))^4}+\frac{3}{7}\left(-\frac{\sinh(c+dx)}{5d(1-\cosh(c+dx))^3}+\frac{2}{5}\left(-\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))^2}+\frac{1}{3}\int \frac{1}{1-\sin\left(ic+idx+\frac{\pi}{2}\right)}dx\right)\right)$$

$$\Big\downarrow \text{3127}$$

$$\frac{3}{7}\left(\frac{2}{5}\left(-\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))}-\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))^2}\right)-\frac{\sinh(c+dx)}{5d(1-\cosh(c+dx))^3}\right)-\frac{\sinh(c+dx)}{7d(1-\cosh(c+dx))^4}$$

input
```
Int[(1 - Cosh[c + d*x])^(-4),x]
```

output
```
-1/7*Sinh[c + d*x]/(d*(1 - Cosh[c + d*x])^4) + (3*(-1/5*Sinh[c + d*x]/(d*(
1 - Cosh[c + d*x])^3) + (2*(-1/3*Sinh[c + d*x]/(d*(1 - Cosh[c + d*x])^2) -
 Sinh[c + d*x]/(3*d*(1 - Cosh[c + d*x]))))/5))/7
```

3.39.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.39.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.48

method	result	size
risch	$-\frac{4\left(35\,{\mathrm e}^{3 d x +3 c}-21\,{\mathrm e}^{2 d x +2 c}+7\,{\mathrm e}^{d x +c}-1\right)}{35 d \left({\mathrm e}^{d x +c}-1\right)^{7}}$	48
parallelrisch	$-\frac{\coth\left(\frac{d x}{2}+\frac{c}{2}\right) \left(\coth\left(\frac{d x}{2}+\frac{c}{2}\right)^{6}-\frac{21 \coth\left(\frac{d x}{2}+\frac{c}{2}\right)^{4}}{5}+7 \coth\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}-7\right)}{56 d}$	54
derivativedivides	$\frac{\frac{3}{40 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{5}}-\frac{1}{56 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{7}}-\frac{1}{8 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}+\frac{1}{8 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)}}{d}$	58
default	$\frac{\frac{3}{40 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{5}}-\frac{1}{56 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{7}}-\frac{1}{8 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}+\frac{1}{8 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)}}{d}$	58

input
```
int(1/(1-cosh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-4/35*(35*exp(3*d*x+3*c)-21*exp(2*d*x+2*c)+7*exp(d*x+c)-1)/d/(exp(d*x+c)-1
)^7
```

3.39.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(85) = 170.

Time = 0.24 (sec) , antiderivative size = 347, normalized size of antiderivative = 3.44

$$\int \frac{1}{(1-\cosh(c+dx))^4}\,dx =$$

$$-\frac{}{35\left(d\cosh\left(dx+c\right)^{6}+d\sinh\left(dx+c\right)^{6}-7\,d\cosh\left(dx+c\right)^{5}+\left(6\,d\cosh\left(dx+c\right)-7\,d\right)\sinh\left(dx+c\right)^{5}+\right.}$$

input
```
integrate(1/(1-cosh(d*x+c))^4,x, algorithm="fricas")
```

output
```
-4/35*(35*cosh(d*x + c)^2 + 10*(7*cosh(d*x + c) - 2)*sinh(d*x + c) + 35*si
nh(d*x + c)^2 - 22*cosh(d*x + c) + 7)/(d*cosh(d*x + c)^6 + d*sinh(d*x + c)
^6 - 7*d*cosh(d*x + c)^5 + (6*d*cosh(d*x + c) - 7*d)*sinh(d*x + c)^5 + 21*
d*cosh(d*x + c)^4 + (15*d*cosh(d*x + c)^2 - 35*d*cosh(d*x + c) + 21*d)*sin
h(d*x + c)^4 - 35*d*cosh(d*x + c)^3 + (20*d*cosh(d*x + c)^3 - 70*d*cosh(d*
x + c)^2 + 84*d*cosh(d*x + c) - 35*d)*sinh(d*x + c)^3 + 35*d*cosh(d*x + c)
^2 + (15*d*cosh(d*x + c)^4 - 70*d*cosh(d*x + c)^3 + 126*d*cosh(d*x + c)^2
- 105*d*cosh(d*x + c) + 35*d)*sinh(d*x + c)^2 - 22*d*cosh(d*x + c) + (6*d*
cosh(d*x + c)^5 - 35*d*cosh(d*x + c)^4 + 84*d*cosh(d*x + c)^3 - 105*d*cosh
(d*x + c)^2 + 70*d*cosh(d*x + c) - 20*d)*sinh(d*x + c) + 7*d)
```

3.39.6 Sympy [A] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.72

$$\int \frac{1}{(1-\cosh(c+dx))^4}\,dx = \begin{cases} \frac{1}{8d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)} - \frac{1}{8d\tanh^3\left(\frac{c}{2}+\frac{dx}{2}\right)} + \frac{3}{40d\tanh^5\left(\frac{c}{2}+\frac{dx}{2}\right)} - \frac{1}{56d\tanh^7\left(\frac{c}{2}+\frac{dx}{2}\right)} & \text{for } d \neq 0 \\ \frac{x}{(1-\cosh(c))^4} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(1-cosh(d*x+c))**4,x)
```

output
```
Piecewise((1/(8*d*tanh(c/2 + d*x/2)) - 1/(8*d*tanh(c/2 + d*x/2)**3) + 3/(4
0*d*tanh(c/2 + d*x/2)**5) - 1/(56*d*tanh(c/2 + d*x/2)**7), Ne(d, 0)), (x/(
1 - cosh(c))**4, True))
```

3.39.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(85) = 170$.

Time = 0.19 (sec) , antiderivative size = 364, normalized size of antiderivative = 3.60

$$\int \frac{1}{(1-\cosh(c+dx))^4}\,dx$$
$$= \frac{4\,e^{(-dx-c)}}{5\,d(7\,e^{(-dx-c)} - 21\,e^{(-2\,dx-2\,c)} + 35\,e^{(-3\,dx-3\,c)} - 35\,e^{(-4\,dx-4\,c)} + 21\,e^{(-5\,dx-5\,c)} - 7\,e^{(-6\,dx-6\,c)} + e^{(-7\,dx-7\,c)} - \cdots}$$
$$- \frac{12\,e^{(-2\,dx-2\,c)}}{5\,d(7\,e^{(-dx-c)} - 21\,e^{(-2\,dx-2\,c)} + 35\,e^{(-3\,dx-3\,c)} - 35\,e^{(-4\,dx-4\,c)} + 21\,e^{(-5\,dx-5\,c)} - 7\,e^{(-6\,dx-6\,c)} + e^{(-7\,dx-7\,c \cdots}}$$
$$+ \frac{4\,e^{(-3\,dx-3\,c)}}{d(7\,e^{(-dx-c)} - 21\,e^{(-2\,dx-2\,c)} + 35\,e^{(-3\,dx-3\,c)} - 35\,e^{(-4\,dx-4\,c)} + 21\,e^{(-5\,dx-5\,c)} - 7\,e^{(-6\,dx-6\,c)} + e^{(-7\,dx-7\,c)} \cdots}$$
$$- \frac{4}{35\,d(7\,e^{(-dx-c)} - 21\,e^{(-2\,dx-2\,c)} + 35\,e^{(-3\,dx-3\,c)} - 35\,e^{(-4\,dx-4\,c)} + 21\,e^{(-5\,dx-5\,c)} - 7\,e^{(-6\,dx-6\,c)} + e^{(-7\,dx-7 \cdots}}$$

input
```
integrate(1/(1-cosh(d*x+c))^4,x, algorithm="maxima")
```

output
```
4/5*e^(-d*x - c)/(d*(7*e^(-d*x - c) - 21*e^(-2*d*x - 2*c) + 35*e^(-3*d*x -
 3*c) - 35*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) - 7*e^(-6*d*x - 6*c) + e
^(-7*d*x - 7*c) - 1)) - 12/5*e^(-2*d*x - 2*c)/(d*(7*e^(-d*x - c) - 21*e^(-
2*d*x - 2*c) + 35*e^(-3*d*x - 3*c) - 35*e^(-4*d*x - 4*c) + 21*e^(-5*d*x -
5*c) - 7*e^(-6*d*x - 6*c) + e^(-7*d*x - 7*c) - 1)) + 4*e^(-3*d*x - 3*c)/(d
*(7*e^(-d*x - c) - 21*e^(-2*d*x - 2*c) + 35*e^(-3*d*x - 3*c) - 35*e^(-4*d*
x - 4*c) + 21*e^(-5*d*x - 5*c) - 7*e^(-6*d*x - 6*c) + e^(-7*d*x - 7*c) - 1
)) - 4/35/(d*(7*e^(-d*x - c) - 21*e^(-2*d*x - 2*c) + 35*e^(-3*d*x - 3*c) -
 35*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) - 7*e^(-6*d*x - 6*c) + e^(-7*d*
x - 7*c) - 1))
```

3.39.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.47

$$\int \frac{1}{(1-\cosh(c+dx))^4}\,dx = -\frac{4\left(35\,e^{(3\,dx+3\,c)} - 21\,e^{(2\,dx+2\,c)} + 7\,e^{(dx+c)} - 1\right)}{35\,d\left(e^{(dx+c)} - 1\right)^7}$$

input
```
integrate(1/(1-cosh(d*x+c))^4,x, algorithm="giac")
```

output
```
-4/35*(35*e^(3*d*x + 3*c) - 21*e^(2*d*x + 2*c) + 7*e^(d*x + c) - 1)/(d*(e^
(d*x + c) - 1)^7)
```

3.39.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.80

$$\int \frac{1}{(1-\cosh(c+dx))^4}\,dx = -\frac{4}{35\,d\,\left(6\,{\mathrm{e}}^{2\,c+2\,d\,x}-4\,{\mathrm{e}}^{c+d\,x}-4\,{\mathrm{e}}^{3\,c+3\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1\right)} - \frac{16\,{\mathrm{e}}^{c+d\,x}}{35\,d\,\left(5\,{\mathrm{e}}^{c+d\,x}-10\,{\mathrm{e}}^{2\,c+2\,d\,x}+10\,{\mathrm{e}}^{3\,c+3\,d\,x}-5\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{5\,c+5\,d\,x}-1\right)} - \frac{8\,{\mathrm{e}}^{2\,c+2\,d\,x}}{7\,d\,\left(15\,{\mathrm{e}}^{2\,c+2\,d\,x}-6\,{\mathrm{e}}^{c+d\,x}-20\,{\mathrm{e}}^{3\,c+3\,d\,x}+15\,{\mathrm{e}}^{4\,c+4\,d\,x}-6\,{\mathrm{e}}^{5\,c+5\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1\right)} - \frac{16\,{\mathrm{e}}^{3\,c+3\,d\,x}}{7\,d\,\left(7\,{\mathrm{e}}^{c+d\,x}-21\,{\mathrm{e}}^{2\,c+2\,d\,x}+35\,{\mathrm{e}}^{3\,c+3\,d\,x}-35\,{\mathrm{e}}^{4\,c+4\,d\,x}+21\,{\mathrm{e}}^{5\,c+5\,d\,x}-7\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{7\,c+7\,d\,x}-1\right)}$$

input
```
int(1/(cosh(c + d*x) - 1)^4,x)
```

output
```
- 4/(35*d*(6*exp(2*c + 2*d*x) - 4*exp(c + d*x) - 4*exp(3*c + 3*d*x) + exp(
4*c + 4*d*x) + 1)) - (16*exp(c + d*x))/(35*d*(5*exp(c + d*x) - 10*exp(2*c
+ 2*d*x) + 10*exp(3*c + 3*d*x) - 5*exp(4*c + 4*d*x) + exp(5*c + 5*d*x) - 1
)) - (8*exp(2*c + 2*d*x))/(7*d*(15*exp(2*c + 2*d*x) - 6*exp(c + d*x) - 20*
exp(3*c + 3*d*x) + 15*exp(4*c + 4*d*x) - 6*exp(5*c + 5*d*x) + exp(6*c + 6*
d*x) + 1)) - (16*exp(3*c + 3*d*x))/(7*d*(7*exp(c + d*x) - 21*exp(2*c + 2*d
*x) + 35*exp(3*c + 3*d*x) - 35*exp(4*c + 4*d*x) + 21*exp(5*c + 5*d*x) - 7*
exp(6*c + 6*d*x) + exp(7*c + 7*d*x) - 1))
```

3.40 $\int \frac{\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx$

3.40.1 Optimal result

Integrand size = 13, antiderivative size = 51

$$\int \frac{\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a+a\cosh(x)}}\right)}{\sqrt{a}}+\frac{2\sinh(x)}{\sqrt{a+a\cosh(x)}}$$

output
```
-arctan(1/2*sinh(x)*a^(1/2)*2^(1/2)/(a+a*cosh(x))^(1/2))*2^(1/2)/a^(1/2)+2
*sinh(x)/(a+a*cosh(x))^(1/2)
```

3.40.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int \frac{\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx = -\frac{2\cosh\left(\frac{x}{2}\right)\left(\arctan\left(\sinh\left(\frac{x}{2}\right)\right)-2\sinh\left(\frac{x}{2}\right)\right)}{\sqrt{a(1+\cosh(x))}}$$

input
```
Integrate[Cosh[x]/Sqrt[a + a*Cosh[x]],x]
```

output
```
(-2*Cosh[x/2]*(ArcTan[Sinh[x/2]] - 2*Sinh[x/2]))/Sqrt[a*(1 + Cosh[x])]
```

3.40.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(x)}{\sqrt{a\cosh(x)+a}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)}{\sqrt{a+a\sin\left(\frac{\pi}{2}+ix\right)}}dx$$
$$\Big\downarrow \text{3230}$$
$$\frac{2\sinh(x)}{\sqrt{a\cosh(x)+a}}-\int \frac{1}{\sqrt{\cosh(x)a+a}}dx$$
$$\Big\downarrow \text{3042}$$
$$\frac{2\sinh(x)}{\sqrt{a\cosh(x)+a}}-\int \frac{1}{\sqrt{\sin\left(ix+\frac{\pi}{2}\right)a+a}}dx$$
$$\Big\downarrow \text{3128}$$
$$\frac{2\sinh(x)}{\sqrt{a\cosh(x)+a}}-2i\int \frac{1}{\frac{a^2\sinh^2(x)}{\cosh(x)a+a}+2a}d\left(-\frac{ia\sinh(x)}{\sqrt{\cosh(x)a+a}}\right)$$
$$\Big\downarrow \text{219}$$
$$\frac{2\sinh(x)}{\sqrt{a\cosh(x)+a}}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a\cosh(x)+a}}\right)}{\sqrt{a}}$$

input
```
Int[Cosh[x]/Sqrt[a + a*Cosh[x]],x]
```

output
```
-((Sqrt[2]*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a + a*Cosh[x]])])/Sqrt[a
]) + (2*Sinh[x])/Sqrt[a + a*Cosh[x]]
```

3.40.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

3.40.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(40) = 80$.

Time = 0.14 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.80

method	result	size
default	$\frac{\cosh\left(\frac{x}{2}\right)\sqrt{\sinh\left(\frac{x}{2}\right)^{2}a}\,\left(2\sqrt{\sinh\left(\frac{x}{2}\right)^{2}a}\,\sqrt{-a}+\ln\left(\frac{2\sqrt{\sinh\left(\frac{x}{2}\right)^{2}a}\,\sqrt{-a}-2a}{\cosh\left(\frac{x}{2}\right)}\right)a\right)\sqrt{2}}{a\sqrt{-a}\,\sinh\left(\frac{x}{2}\right)\sqrt{a\cosh\left(\frac{x}{2}\right)^{2}}}$	92

input
```
int(cosh(x)/(a+a*cosh(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
cosh(1/2*x)*(sinh(1/2*x)^2*a)^(1/2)*(2*(sinh(1/2*x)^2*a)^(1/2)*(-a)^(1/2)+
ln(2/cosh(1/2*x)*((sinh(1/2*x)^2*a)^(1/2)*(-a)^(1/2)-a))*a)/a/(-a)^(1/2)/s
inh(1/2*x)*2^(1/2)/(a*cosh(1/2*x)^2)^(1/2)
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.22

$$\int \frac{\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx = \frac{2\left(\sqrt{\frac{1}{2}}\sqrt{\frac{a}{\cosh(x)+\sinh(x)}}(\cosh(x)+\sinh(x)-1)-\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{\cosh(x)+\sinh(x)}}(\cosh(x)+\sinh(x))}{\sqrt{a}}\right)\right)}{a}$$

input
```
integrate(cosh(x)/(a+a*cosh(x))^(1/2),x, algorithm="fricas")
```

output
```
2*(sqrt(1/2)*sqrt(a/(cosh(x) + sinh(x)))*(cosh(x) + sinh(x) - 1) - sqrt(2)
*sqrt(a)*arctan(sqrt(2)*sqrt(1/2)*sqrt(a/(cosh(x) + sinh(x)))*(cosh(x) + s
inh(x))/sqrt(a)))/a
```

3.40.6 Sympy [F]

$$\int \frac{\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx = \int \frac{\cosh(x)}{\sqrt{a(\cosh(x)+1)}}\,dx$$

input
```
integrate(cosh(x)/(a+a*cosh(x))**(1/2),x)
```

output
```
Integral(cosh(x)/sqrt(a*(cosh(x) + 1)), x)
```

3.40.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(40) = 80.

Time = 0.33 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.24

$$\begin{aligned}\int \frac{\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx &= -\sqrt{2}\left(\frac{\arctan\left(e^{\left(\frac{1}{2}x\right)}\right)}{\sqrt{a}}-\frac{e^{\left(\frac{1}{2}x\right)}}{\sqrt{a}e^{x}+\sqrt{a}}\right)\\ &\quad+\frac{1}{3}\sqrt{2}\left(\frac{3\arctan\left(e^{\left(-\frac{1}{2}x\right)}\right)}{\sqrt{a}}-\frac{2e^{\left(-\frac{1}{2}x\right)}}{\sqrt{a}}-\frac{e^{\left(-\frac{1}{2}x\right)}}{\sqrt{a}e^{(-x)}+\sqrt{a}}\right)\\ &\quad+\frac{3\sqrt{2}\sqrt{a}e^{\left(\frac{3}{2}x\right)}-\sqrt{2}\sqrt{a}e^{\left(-\frac{1}{2}x\right)}}{3(ae^{x}+a)}\end{aligned}$$

input
```
integrate(cosh(x)/(a+a*cosh(x))^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(2)*(arctan(e^(1/2*x))/sqrt(a) - e^(1/2*x)/(sqrt(a)*e^x + sqrt(a))) +
 1/3*sqrt(2)*(3*arctan(e^(-1/2*x))/sqrt(a) - 2*e^(-1/2*x)/sqrt(a) - e^(-1/
2*x)/(sqrt(a)*e^(-x) + sqrt(a))) + 1/3*(3*sqrt(2)*sqrt(a)*e^(3/2*x) - sqrt
(2)*sqrt(a)*e^(-1/2*x))/(a*e^x + a)
```

3.40.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int \frac{\cosh(x)}{\sqrt{a + a\cosh(x)}}\, dx = -\frac{2\sqrt{2}\arctan\left(e^{\left(\frac{1}{2}x\right)}\right)}{\sqrt{a}} + \frac{\sqrt{2}e^{\left(\frac{1}{2}x\right)}}{\sqrt{a}} - \frac{\sqrt{2}e^{\left(-\frac{1}{2}x\right)}}{\sqrt{a}}$$

input
```
integrate(cosh(x)/(a+a*cosh(x))^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(2)*arctan(e^(1/2*x))/sqrt(a) + sqrt(2)*e^(1/2*x)/sqrt(a) - sqrt(2)
*e^(-1/2*x)/sqrt(a)
```

3.40.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(x)}{\sqrt{a + a\cosh(x)}}\, dx = \int \frac{\cosh(x)}{\sqrt{a + a\cosh(x)}}\, dx$$

input
```
int(cosh(x)/(a + a*cosh(x))^(1/2),x)
```

output
```
int(cosh(x)/(a + a*cosh(x))^(1/2), x)
```

3.41 $\int \frac{\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx$

3.41.1 Optimal result

Integrand size = 14, antiderivative size = 53

$$\int \frac{\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a-a\cosh(x)}}\right)}{\sqrt{a}}+\frac{2\sinh(x)}{\sqrt{a-a\cosh(x)}}$$

output
```
-arctan(1/2*sinh(x)*a^(1/2)*2^(1/2)/(a-a*cosh(x))^(1/2))*2^(1/2)/a^(1/2)+2
*sinh(x)/(a-a*cosh(x))^(1/2)
```

3.41.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \frac{\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx = \frac{2\left(2\cosh\left(\frac{x}{2}\right)-\log\left(\cosh\left(\frac{x}{4}\right)\right)+\log\left(\sinh\left(\frac{x}{4}\right)\right)\right)\sinh\left(\frac{x}{2}\right)}{\sqrt{a-a\cosh(x)}}$$

input
```
Integrate[Cosh[x]/Sqrt[a - a*Cosh[x]],x]
```

output
```
(2*(2*Cosh[x/2] - Log[Cosh[x/4]] + Log[Sinh[x/4]])*Sinh[x/2])/Sqrt[a - a*C
osh[x]]
```

3.41.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(x)}{\sqrt{a - a\cosh(x)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin\left(\frac{\pi}{2} + ix\right)}{\sqrt{a - a\sin\left(\frac{\pi}{2} + ix\right)}}dx$$

$$\Big\downarrow \; 3230$$

$$\int \frac{1}{\sqrt{a - a\cosh(x)}}dx + \frac{2\sinh(x)}{\sqrt{a - a\cosh(x)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{2\sinh(x)}{\sqrt{a - a\cosh(x)}} + \int \frac{1}{\sqrt{a - a\sin\left(ix + \frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 3128$$

$$\frac{2\sinh(x)}{\sqrt{a - a\cosh(x)}} + 2i\int \frac{1}{\frac{a^2\sinh^2(x)}{a - a\cosh(x)} + 2a}d\frac{ia\sinh(x)}{\sqrt{a - a\cosh(x)}}$$

$$\Big\downarrow \; 219$$

$$\frac{2\sinh(x)}{\sqrt{a - a\cosh(x)}} - \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a - a\cosh(x)}}\right)}{\sqrt{a}}$$

input
```
Int[Cosh[x]/Sqrt[a - a*Cosh[x]],x]
```

output
```
-((Sqrt[2]*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a - a*Cosh[x]])])/Sqrt[a
]) + (2*Sinh[x])/Sqrt[a - a*Cosh[x]]
```

3.41.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

3.41.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{\sinh\left(\frac{x}{2}\right)\left(4\cosh\left(\frac{x}{2}\right)+\ln\left(\cosh\left(\frac{x}{2}\right)-1\right)-\ln\left(\cosh\left(\frac{x}{2}\right)+1\right)\right)}{\sqrt{-2\sinh\left(\frac{x}{2}\right)^{2}a}}$	40

input
```
int(cosh(x)/(a-a*cosh(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
sinh(1/2*x)*(4*cosh(1/2*x)+ln(cosh(1/2*x)-1)-ln(cosh(1/2*x)+1))/(-2*sinh(1
/2*x)^2*a)^(1/2)
```

3.41.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(42) = 84$.

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.74

$$\int \frac{\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx = \frac{\sqrt{2}a\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a}{\cosh(x)+\sinh(x)}}\sqrt{-\frac{1}{a}}(\cosh(x)+\sinh(x))-\cosh(x)-\sinh(x)-1}{\cosh(x)+\sinh(x)-1}\right)-2\sqrt{\frac{1}{2}}\sqrt{-\frac{a}{\cosh(x)+\sinh(x)}}(\cosh (x}{a}$$

input
```
integrate(cosh(x)/(a-a*cosh(x))^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(2)*a*sqrt(-1/a)*log((2*sqrt(2)*sqrt(1/2)*sqrt(-a/(cosh(x) + sinh(x))
)*sqrt(-1/a)*(cosh(x) + sinh(x)) - cosh(x) - sinh(x) - 1)/(cosh(x) + sinh(
x) - 1)) - 2*sqrt(1/2)*sqrt(-a/(cosh(x) + sinh(x)))*(cosh(x) + sinh(x) + 1
))/a
```

3.41.6 Sympy [F]

$$\int \frac{\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx = \int \frac{\cosh{(x)}}{\sqrt{-a\left(\cosh{(x)}-1\right)}}\,dx$$

input
```
integrate(cosh(x)/(a-a*cosh(x))**(1/2),x)
```

output
```
Integral(cosh(x)/sqrt(-a*(cosh(x) - 1)), x)
```

3.41.7 Maxima [F]

$$\int \frac{\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx = \int \frac{\cosh{(x)}}{\sqrt{-a\cosh{(x)}+a}}\,dx$$

input
```
integrate(cosh(x)/(a-a*cosh(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cosh(x)/sqrt(-a*cosh(x) + a), x)
```

3.41.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.40

$$\int \frac{\cosh(x)}{\sqrt{a - a\cosh(x)}}\,dx = -\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{-ae^x}}{\sqrt{a}}\right)}{\sqrt{a}\mathrm{sgn}\left(-e^x+1\right)} - \frac{\sqrt{2}}{\sqrt{-ae^x}\mathrm{sgn}\left(-e^x+1\right)} + \frac{\sqrt{2}\sqrt{-ae^x}}{a\mathrm{sgn}\left(-e^x+1\right)}$$

input
```
integrate(cosh(x)/(a-a*cosh(x))^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(2)*arctan(sqrt(-a*e^x)/sqrt(a))/(sqrt(a)*sgn(-e^x + 1)) - sqrt(2)/
(sqrt(-a*e^x)*sgn(-e^x + 1)) + sqrt(2)*sqrt(-a*e^x)/(a*sgn(-e^x + 1))
```

3.41.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(x)}{\sqrt{a - a\cosh(x)}}\,dx = \int \frac{\cosh(x)}{\sqrt{a - a\cosh(x)}}\,dx$$

input
```
int(cosh(x)/(a - a*cosh(x))^(1/2),x)
```

output
```
int(cosh(x)/(a - a*cosh(x))^(1/2), x)
```

3.42 $\int (a+a\cosh(c+dx))^{5/2}\,dx$

3.42.1 Optimal result

Integrand size = 14, antiderivative size = 89

$$\int (a+a\cosh(c+dx))^{5/2}\,dx = \frac{64a^3\sinh(c+dx)}{15d\sqrt{a+a\cosh(c+dx)}} + \frac{16a^2\sqrt{a+a\cosh(c+dx)}\sinh(c+dx)}{15d} + \frac{2a(a+a\cosh(c+dx))^{3/2}\sinh(c+dx)}{5d}$$

output
```
2/5*a*(a+a*cosh(d*x+c))^(3/2)*sinh(d*x+c)/d+64/15*a^3*sinh(d*x+c)/d/(a+a*c
osh(d*x+c))^(1/2)+16/15*a^2*sinh(d*x+c)*(a+a*cosh(d*x+c))^(1/2)/d
```

3.42.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.80

$$\int (a+a\cosh(c+dx))^{5/2}\,dx = \frac{a^2\sqrt{a(1+\cosh(c+dx))}\operatorname{sech}\left(\frac{1}{2}(c+dx)\right)\left(150\sinh\left(\frac{1}{2}(c+dx)\right)+25\sinh\left(\frac{3}{2}(c+dx)\right)+3\sinh\ldots\right.}{30d}$$

input
```
Integrate[(a + a*Cosh[c + d*x])^(5/2),x]
```

output
```
(a^2*Sqrt[a*(1 + Cosh[c + d*x])]*Sech[(c + d*x)/2]*(150*Sinh[(c + d*x)/2]
+ 25*Sinh[(3*(c + d*x))/2] + 3*Sinh[(5*(c + d*x))/2]))/(30*d)
```

3.42.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 3126, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cosh(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+a\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{3126}$$

$$\frac{8}{5}a\int (\cosh(c+dx)a+a)^{3/2}dx+\frac{2a\sinh(c+dx)(a\cosh(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a\sinh(c+dx)(a\cosh(c+dx)+a)^{3/2}}{5d}+\frac{8}{5}a\int \left(\sin\left(ic+idx+\frac{\pi}{2}\right)a+a\right)^{3/2}dx$$

$$\Big\downarrow \text{3126}$$

$$\frac{8}{5}a\left(\frac{4}{3}a\int \sqrt{\cosh(c+dx)a+a}dx+\frac{2a\sinh(c+dx)\sqrt{a\cosh(c+dx)+a}}{3d}\right)+\frac{2a\sinh(c+dx)(a\cosh(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a\sinh(c+dx)(a\cosh(c+dx)+a)^{3/2}}{5d}+\frac{8}{5}a\left(\frac{2a\sinh(c+dx)\sqrt{a\cosh(c+dx)+a}}{3d}+\frac{4}{3}a\int \sqrt{\sin\left(ic+idx+\frac{\pi}{2}\right)a+a}dx\right)$$

$$\Big\downarrow \text{3125}$$

$$\frac{8}{5}a\left(\frac{8a^2\sinh(c+dx)}{3d\sqrt{a\cosh(c+dx)+a}}+\frac{2a\sinh(c+dx)\sqrt{a\cosh(c+dx)+a}}{3d}\right)+\frac{2a\sinh(c+dx)(a\cosh(c+dx)+a)^{3/2}}{5d}$$

input
```
Int[(a + a*Cosh[c + d*x])^(5/2),x]
```

output
```
(2*a*(a + a*Cosh[c + d*x])^(3/2)*Sinh[c + d*x])/(5*d) + (8*a*((8*a^2*Sinh[
c + d*x])/(3*d*Sqrt[a + a*Cosh[c + d*x]]) + (2*a*Sqrt[a + a*Cosh[c + d*x]]
*Sinh[c + d*x])/(3*d)))/5
```

3.42.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

3.42.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{8a^3\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\left(3\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+4\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+8\right)\sqrt{2}}{15\sqrt{a\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,d}$	73

input
```
int((a+a*cosh(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
8/15*a^3*cosh(1/2*d*x+1/2*c)*sinh(1/2*d*x+1/2*c)*(3*cosh(1/2*d*x+1/2*c)^4+
4*cosh(1/2*d*x+1/2*c)^2+8)*2^(1/2)/(a*cosh(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.42.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. 2(77) = 154.

Time = 0.26 (sec) , antiderivative size = 327, normalized size of antiderivative = 3.67

$$\int (a + a\cosh(c + dx))^{5/2}\, dx = \frac{\sqrt{\frac{1}{2}}\left(3\,a^2\cosh\left(dx+c\right)^5 + 3\,a^2\sinh\left(dx+c\right)^5 + 25\,a^2\cosh\left(dx+c\right)^4 + 150\,a^2\cosh\left(dx+c\right)^3 + \right.}{}$$

input
```
integrate((a+a*cosh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/30*sqrt(1/2)*(3*a^2*cosh(d*x + c)^5 + 3*a^2*sinh(d*x + c)^5 + 25*a^2*cos
h(d*x + c)^4 + 150*a^2*cosh(d*x + c)^3 + 5*(3*a^2*cosh(d*x + c) + 5*a^2)*s
inh(d*x + c)^4 - 150*a^2*cosh(d*x + c)^2 + 10*(3*a^2*cosh(d*x + c)^2 + 10*
a^2*cosh(d*x + c) + 15*a^2)*sinh(d*x + c)^3 - 25*a^2*cosh(d*x + c) + 30*(a
^2*cosh(d*x + c)^3 + 5*a^2*cosh(d*x + c)^2 + 15*a^2*cosh(d*x + c) - 5*a^2)
*sinh(d*x + c)^2 - 3*a^2 + 5*(3*a^2*cosh(d*x + c)^4 + 20*a^2*cosh(d*x + c)
^3 + 90*a^2*cosh(d*x + c)^2 - 60*a^2*cosh(d*x + c) - 5*a^2)*sinh(d*x + c))
*sqrt(a/(cosh(d*x + c) + sinh(d*x + c)))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x
 + c)*sinh(d*x + c) + d*sinh(d*x + c)^2)
```

3.42.6 Sympy [F]

$$\int (a + a\cosh(c + dx))^{5/2}\, dx = \int \left(a\cosh\left(c + dx\right) + a\right)^{\frac{5}{2}}\, dx$$

input
```
integrate((a+a*cosh(d*x+c))**(5/2),x)
```

output
```
Integral((a*cosh(c + d*x) + a)**(5/2), x)
```

3.42.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.36

$$\int (a + a\cosh(c + dx))^{5/2}\,dx = \frac{\sqrt{2}a^{\frac{5}{2}}e^{\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)}}{20\,d} + \frac{5\,\sqrt{2}a^{\frac{5}{2}}e^{\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)}}{12\,d} + \frac{5\,\sqrt{2}a^{\frac{5}{2}}e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d} - \frac{5\,\sqrt{2}a^{\frac{5}{2}}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}}{2\,d} - \frac{5\,\sqrt{2}a^{\frac{5}{2}}e^{\left(-\frac{3}{2}\,dx-\frac{3}{2}\,c\right)}}{12\,d} - \frac{\sqrt{2}a^{\frac{5}{2}}e^{\left(-\frac{5}{2}\,dx-\frac{5}{2}\,c\right)}}{20\,d}$$

input
```
integrate((a+a*cosh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/20*sqrt(2)*a^(5/2)*e^(5/2*d*x + 5/2*c)/d + 5/12*sqrt(2)*a^(5/2)*e^(3/2*d
*x + 3/2*c)/d + 5/2*sqrt(2)*a^(5/2)*e^(1/2*d*x + 1/2*c)/d - 5/2*sqrt(2)*a^
(5/2)*e^(-1/2*d*x - 1/2*c)/d - 5/12*sqrt(2)*a^(5/2)*e^(-3/2*d*x - 3/2*c)/d
 - 1/20*sqrt(2)*a^(5/2)*e^(-5/2*d*x - 5/2*c)/d
```

3.42.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.18

$$\int (a + a\cosh(c + dx))^{5/2}\,dx = -\frac{\sqrt{2}\left(\left(150\,a^{\frac{5}{2}}e^{\left(2\,dx+\frac{5}{2}\,c\right)} + 25\,a^{\frac{5}{2}}e^{\left(dx+\frac{3}{2}\,c\right)} + 3\,a^{\frac{5}{2}}e^{\left(\frac{1}{2}\,c\right)}\right)e^{\left(-\frac{5}{2}\,dx-3\,c\right)} - \left(3\,a^{\frac{5}{2}}e^{\left(\frac{5}{2}\,dx+\frac{35}{2}\,c\right)} + 25\,a^{\frac{5}{2}}e^{\left(\frac{3}{2}\,dx+\frac{33}{2}\,c\right)} + 15\right.}{60\,d}$$

input
```
integrate((a+a*cosh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
-1/60*sqrt(2)*((150*a^(5/2)*e^(2*d*x + 5/2*c) + 25*a^(5/2)*e^(d*x + 3/2*c)
 + 3*a^(5/2)*e^(1/2*c))*e^(-5/2*d*x - 3*c) - (3*a^(5/2)*e^(5/2*d*x + 35/2*
c) + 25*a^(5/2)*e^(3/2*d*x + 33/2*c) + 150*a^(5/2)*e^(1/2*d*x + 31/2*c))*e
^(-15*c))/d
```

3.42.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\cosh(c + dx))^{5/2}\, dx = \int (a + a\cosh(c + d\,x))^{5/2}\, dx$$

input
```
int((a + a*cosh(c + d*x))^(5/2),x)
```

output
```
int((a + a*cosh(c + d*x))^(5/2), x)
```

3.43 $\int (a+a\cosh(c+dx))^{3/2}\,dx$

3.43.1 Optimal result

Integrand size = 14, antiderivative size = 59

$$\int (a+a\cosh(c+dx))^{3/2}\,dx=\frac{8a^2\sinh(c+dx)}{3d\sqrt{a+a\cosh(c+dx)}}+\frac{2a\sqrt{a+a\cosh(c+dx)}\sinh(c+dx)}{3d}$$

output
```
8/3*a^2*sinh(d*x+c)/d/(a+a*cosh(d*x+c))^(1/2)+2/3*a*sinh(d*x+c)*(a+a*cosh(
d*x+c))^(1/2)/d
```

3.43.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int (a+a\cosh(c+dx))^{3/2}\,dx=\frac{a\sqrt{a(1+\cosh(c+dx))}\operatorname{sech}\left(\frac{1}{2}(c+dx)\right)\left(9\sinh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{3}{2}(c+dx)\right)\right)}{3d}$$

input
```
Integrate[(a + a*Cosh[c + d*x])^(3/2),x]
```

output
```
(a*Sqrt[a*(1 + Cosh[c + d*x])]*Sech[(c + d*x)/2]*(9*Sinh[(c + d*x)/2] + Si
nh[(3*(c + d*x))/2]))/(3*d)
```

3.43.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cosh(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+a\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{3126}$$

$$\frac{4}{3}a\int \sqrt{\cosh(c+dx)a+a}dx+\frac{2a\sinh(c+dx)\sqrt{a\cosh(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a\sinh(c+dx)\sqrt{a\cosh(c+dx)+a}}{3d}+\frac{4}{3}a\int \sqrt{\sin\left(ic+idx+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{3125}$$

$$\frac{8a^2\sinh(c+dx)}{3d\sqrt{a\cosh(c+dx)+a}}+\frac{2a\sinh(c+dx)\sqrt{a\cosh(c+dx)+a}}{3d}$$

input
```
Int[(a + a*Cosh[c + d*x])^(3/2),x]
```

output
```
(8*a^2*Sinh[c + d*x])/(3*d*Sqrt[a + a*Cosh[c + d*x]]) + (2*a*Sqrt[a + a*Co
sh[c + d*x]]*Sinh[c + d*x])/(3*d)
```

3.43.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

3.43.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{4a^2\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\right)\sqrt{2}}{3\sqrt{a\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,d}$	58

input
```
int((a+a*cosh(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
4/3*a^2*cosh(1/2*d*x+1/2*c)*sinh(1/2*d*x+1/2*c)*(cosh(1/2*d*x+1/2*c)^2+2)*
2^(1/2)/(a*cosh(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.43.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(51) = 102.

Time = 0.25 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.37

$$\int (a+a\cosh(c+dx))^{3/2}\,dx = \frac{\sqrt{\frac{1}{2}}\left(a\cosh(dx+c)^3+a\sinh(dx+c)^3+9\,a\cosh(dx+c)^2+3\,(a\cosh(dx+c)+3\,a)\sinh(a\cdots\right.}{3\,(d\cos\cdots}$$

input
```
integrate((a+a*cosh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(1/2)*(a*cosh(d*x + c)^3 + a*sinh(d*x + c)^3 + 9*a*cosh(d*x + c)^2
 + 3*(a*cosh(d*x + c) + 3*a)*sinh(d*x + c)^2 - 9*a*cosh(d*x + c) + 3*(a*co
sh(d*x + c)^2 + 6*a*cosh(d*x + c) - 3*a)*sinh(d*x + c) - a)*sqrt(a/(cosh(d
*x + c) + sinh(d*x + c)))/(d*cosh(d*x + c) + d*sinh(d*x + c))
```

3.43.6 Sympy [F]

$$\int (a+a\cosh(c+dx))^{3/2}\,dx=\int (a\cosh{(c+dx)}+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*cosh(d*x+c))**(3/2),x)
```

output
```
Integral((a*cosh(c + d*x) + a)**(3/2), x)
```

3.43.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.37

$$\int (a+a\cosh(c+dx))^{3/2}\,dx=\frac{\sqrt{2}a^{\frac{3}{2}}e^{\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)}}{6\,d}+\frac{3\,\sqrt{2}a^{\frac{3}{2}}e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}-\frac{3\,\sqrt{2}a^{\frac{3}{2}}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}}{2\,d}-\frac{\sqrt{2}a^{\frac{3}{2}}e^{\left(-\frac{3}{2}\,dx-\frac{3}{2}\,c\right)}}{6\,d}$$

input
```
integrate((a+a*cosh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/6*sqrt(2)*a^(3/2)*e^(3/2*d*x + 3/2*c)/d + 3/2*sqrt(2)*a^(3/2)*e^(1/2*d*x
 + 1/2*c)/d - 3/2*sqrt(2)*a^(3/2)*e^(-1/2*d*x - 1/2*c)/d - 1/6*sqrt(2)*a^(
3/2)*e^(-3/2*d*x - 3/2*c)/d
```

3.43.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27

$$\int (a + a\cosh(c + dx))^{3/2}\,dx = -\frac{\sqrt{2}\left(\left(9\,a^{\frac{3}{2}}e^{\left(dx+\frac{3}{2}\,c\right)} + a^{\frac{3}{2}}e^{\left(\frac{1}{2}\,c\right)}\right)e^{\left(-\frac{3}{2}\,dx-2\,c\right)} - \left(a^{\frac{3}{2}}e^{\left(\frac{3}{2}\,dx+\frac{15}{2}\,c\right)} + 9\,a^{\frac{3}{2}}e^{\left(\frac{1}{2}\,dx+\frac{13}{2}\,c\right)}\right)e^{(-6\,c)}\right)}{6\,d}$$

input
```
integrate((a+a*cosh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
-1/6*sqrt(2)*((9*a^(3/2)*e^(d*x + 3/2*c) + a^(3/2)*e^(1/2*c))*e^(-3/2*d*x
- 2*c) - (a^(3/2)*e^(3/2*d*x + 15/2*c) + 9*a^(3/2)*e^(1/2*d*x + 13/2*c))*e
^(-6*c))/d
```

3.43.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\cosh(c + dx))^{3/2}\,dx = \int (a + a\cosh(c + d\,x))^{3/2}\,dx$$

input
```
int((a + a*cosh(c + d*x))^(3/2),x)
```

output
```
int((a + a*cosh(c + d*x))^(3/2), x)
```

3.44 $\int \sqrt{a+a\cosh(c+dx)}\,dx$

3.44.1 Optimal result

Integrand size = 14, antiderivative size = 26

$$\int \sqrt{a+a\cosh(c+dx)}\,dx=\frac{2a\sinh(c+dx)}{d\sqrt{a+a\cosh(c+dx)}}$$

output
```
2*a*sinh(d*x+c)/d/(a+a*cosh(d*x+c))^(1/2)
```

3.44.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int \sqrt{a+a\cosh(c+dx)}\,dx=\frac{2\sqrt{a(1+\cosh(c+dx))}\tanh\left(\frac{1}{2}(c+dx)\right)}{d}$$

input
```
Integrate[Sqrt[a + a*Cosh[c + d*x]],x]
```

output
```
(2*Sqrt[a*(1 + Cosh[c + d*x])]*Tanh[(c + d*x)/2])/d
```

3.44.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\cosh(c+dx)+a}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{a+a\sin\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 3125$$

$$\frac{2a\sinh(c+dx)}{d\sqrt{a\cosh(c+dx)+a}}$$

input
```
Int[Sqrt[a + a*Cosh[c + d*x]],x]
```

output
```
(2*a*Sinh[c + d*x])/(d*Sqrt[a + a*Cosh[c + d*x]])
```

3.44.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

3.44.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.65

method	result	size
default	$\frac{2a\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2}}{\sqrt{a\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, d}$	43
risch	$\frac{\sqrt{2}\,\sqrt{a\left(e^{dx+c}+1\right)^{2}e^{-dx-c}}\,\left(e^{dx+c}-1\right)}{\left(e^{dx+c}+1\right)d}$	49

input
```
int((a+a*cosh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*a*cosh(1/2*d*x+1/2*c)*sinh(1/2*d*x+1/2*c)*2^(1/2)/(a*cosh(1/2*d*x+1/2*c)
^2)^(1/2)/d
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.58

$$\int \sqrt{a+a\cosh(c+dx)}\,dx = \frac{2\sqrt{\frac{1}{2}}\sqrt{\frac{a}{\cosh(dx+c)+\sinh(dx+c)}}\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)-1\right)}{d}$$

input
```
integrate((a+a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(1/2)*sqrt(a/(cosh(d*x + c) + sinh(d*x + c)))*(cosh(d*x + c) + sinh(
d*x + c) - 1)/d
```

3.44.6 Sympy [F]

$$\int \sqrt{a+a\cosh(c+dx)}\,dx = \int \sqrt{a\cosh\left(c+dx\right)+a}\,dx$$

input
```
integrate((a+a*cosh(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*cosh(c + d*x) + a), x)
```

3.44.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.54

$$\int \sqrt{a+a\cosh(c+dx)}\,dx = \frac{\sqrt{2}\sqrt{a}e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{d} - \frac{\sqrt{2}\sqrt{a}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}}{d}$$

input
```
integrate((a+a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
sqrt(2)*sqrt(a)*e^(1/2*d*x + 1/2*c)/d - sqrt(2)*sqrt(a)*e^(-1/2*d*x - 1/2*
c)/d
```

3.44.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.35

$$\int \sqrt{a+a\cosh(c+dx)}\,dx = \frac{\sqrt{2}\left(\sqrt{a}e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)} - \sqrt{a}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}\right)}{d}$$

input
```
integrate((a+a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sqrt(2)*(sqrt(a)*e^(1/2*d*x + 1/2*c) - sqrt(a)*e^(-1/2*d*x - 1/2*c))/d
```

3.44.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \sqrt{a+a\cosh(c+dx)}\,dx = \frac{2\tanh\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a+a\cosh\left(c+d\,x\right)}}{d}$$

input
```
int((a + a*cosh(c + d*x))^(1/2),x)
```

output
```
(2*tanh(c/2 + (d*x)/2)*(a + a*cosh(c + d*x))^(1/2))/d
```

3.45 $\int \frac{1}{\sqrt{a+a\cosh(c+dx)}}\,dx$

3.45.1 Optimal result

Integrand size = 14, antiderivative size = 46

$$\int \frac{1}{\sqrt{a+a\cosh(c+dx)}}\,dx=\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a+a\cosh(c+dx)}}\right)}{\sqrt{a}d}$$

output
```
arctan(1/2*sinh(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cosh(d*x+c))^(1/2))*2^(1/2)/d/
a^(1/2)
```

3.45.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{a+a\cosh(c+dx)}}\,dx=\frac{2\arctan\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)\cosh\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{a(1+\cosh(c+dx))}}$$

input
```
Integrate[1/Sqrt[a + a*Cosh[c + d*x]],x]
```

output
```
(2*ArcTan[Sinh[(c + d*x)/2]]*Cosh[(c + d*x)/2])/(d*Sqrt[a*(1 + Cosh[c + d*
x])])
```

3.45.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\cosh(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+a\sin\left(ic+idx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3128}$$

$$\frac{2i\int \frac{1}{\frac{a^2\sinh^2(c+dx)}{\cosh(c+dx)a+a}+2a}d\left(-\frac{ia\sinh(c+dx)}{\sqrt{\cosh(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a\cosh(c+dx)+a}}\right)}{\sqrt{a}d}$$

input
```
Int[1/Sqrt[a + a*Cosh[c + d*x]],x]
```

output
```
(Sqrt[2]*ArcTan[(Sqrt[a]*Sinh[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cosh[c + d*x]]
)])/(Sqrt[a]*d)
```

3.45.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

3.45.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(37) = 74$.

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.24

method	result	size
default	$-\frac{\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2} a}\,\ln\left(\frac{2\sqrt{-a}\,\sqrt{\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2} a}-2a}{\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\sqrt{2}}{\sqrt{-a}\,\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,d}$	103

input
```
int(1/(a+a*cosh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-cosh(1/2*d*x+1/2*c)*(sinh(1/2*d*x+1/2*c)^2*a)^(1/2)/(-a)^(1/2)*ln(2*((-a)
^(1/2)*(sinh(1/2*d*x+1/2*c)^2*a)^(1/2)-a)/cosh(1/2*d*x+1/2*c))/sinh(1/2*d*
x+1/2*c)*2^(1/2)/(a*cosh(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 149, normalized size of antiderivative = 3.24

$$\int \frac{1}{\sqrt{a+a\cosh(c+dx)}}\,dx = \left[\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{\cosh(dx+c)+\sinh(dx+c)}}\sqrt{-\frac{1}{a}}(\cosh(dx+c)+\sinh(dx+c))+\cosh(dx+c)+\sinh(dx+c)-1}{\cosh(dx+c)+\sinh(dx+c)+1}\right)}{d}, \frac{2\sqrt{2}\arctan\Bigg(\right.$$

input
```
integrate(1/(a+a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[sqrt(2)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt(1/2)*sqrt(a/(cosh(d*x + c) + sinh
(d*x + c)))*sqrt(-1/a)*(cosh(d*x + c) + sinh(d*x + c)) + cosh(d*x + c) + s
inh(d*x + c) - 1)/(cosh(d*x + c) + sinh(d*x + c) + 1))/d, 2*sqrt(2)*arctan
(sqrt(2)*sqrt(1/2)*sqrt(a/(cosh(d*x + c) + sinh(d*x + c)))*(cosh(d*x + c)
+ sinh(d*x + c))/sqrt(a))/(sqrt(a)*d)]
```

3.45.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+a\cosh(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\cosh(c+dx)+a}}\,dx$$

input
```
integrate(1/(a+a*cosh(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(a*cosh(c + d*x) + a), x)
```

3.45.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(37) = 74.

Time = 0.34 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.87

$$\int \frac{1}{\sqrt{a+a\cosh(c+dx)}}\,dx = 2\sqrt{2}\left(\frac{\arctan\left(e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}\right)}{\sqrt{a}d} + \frac{e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{\left(\sqrt{a}e^{(dx+c)}+\sqrt{a}\right)d}\right) - \frac{2\sqrt{2}e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{\sqrt{a}de^{(dx+c)}+\sqrt{a}d}$$

input
```
integrate(1/(a+a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(2)*(arctan(e^(1/2*d*x + 1/2*c))/(sqrt(a)*d) + e^(1/2*d*x + 1/2*c)/(
(sqrt(a)*e^(d*x + c) + sqrt(a))*d)) - 2*sqrt(2)*e^(1/2*d*x + 1/2*c)/(sqrt(
a)*d*e^(d*x + c) + sqrt(a)*d)
```

3.45.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.46

$$\int \frac{1}{\sqrt{a+a\cosh(c+dx)}}\,dx = \frac{2\sqrt{2}\arctan\left(e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}\right)}{\sqrt{a}d}$$

input
```
integrate(1/(a+a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(2)*arctan(e^(1/2*d*x + 1/2*c))/(sqrt(a)*d)
```

3.45.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+a\cosh(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+a\cosh\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(a + a*cosh(c + d*x))^(1/2),x)
```

output
```
int(1/(a + a*cosh(c + d*x))^(1/2), x)
```

3.46 $\int \frac{1}{(a+a\cosh(c+dx))^{3/2}}\,dx$

3.46.1 Optimal result

Integrand size = 14, antiderivative size = 77

$$\int \frac{1}{(a+a\cosh(c+dx))^{3/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a+a\cosh(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{\sinh(c+dx)}{2d(a+a\cosh(c+dx))^{3/2}}$$

output
```
1/2*sinh(d*x+c)/d/(a+a*cosh(d*x+c))^(3/2)+1/4*arctan(1/2*sinh(d*x+c)*a^(1/
2)*2^(1/2)/(a+a*cosh(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)
```

3.46.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.82

$$\int \frac{1}{(a+a\cosh(c+dx))^{3/2}}\,dx = \frac{\cosh^2\left(\frac{1}{2}(c+dx)\right)\left(\arctan\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)\cosh\left(\frac{1}{2}(c+dx)\right)+\tanh\left(\frac{1}{2}(c\right.\right.}{d(a(1+\cosh(c+dx)))^{3/2}}$$

input
```
Integrate[(a + a*Cosh[c + d*x])^(-3/2),x]
```

output
```
(Cosh[(c + d*x)/2]^2*(ArcTan[Sinh[(c + d*x)/2]]*Cosh[(c + d*x)/2] + Tanh[(
c + d*x)/2]))/(d*(a*(1 + Cosh[c + d*x]))^(3/2))
```

3.46.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cosh(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+a\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{\int \frac{1}{\sqrt{\cosh(c+dx)a+a}}dx}{4a}+\frac{\sinh(c+dx)}{2d(a\cosh(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(c+dx)}{2d(a\cosh(c+dx)+a)^{3/2}}+\frac{\int \frac{1}{\sqrt{\sin\left(ic+idx+\frac{\pi}{2}\right)a+a}}dx}{4a}$$

$$\Big\downarrow \text{3128}$$

$$\frac{\sinh(c+dx)}{2d(a\cosh(c+dx)+a)^{3/2}}+\frac{i\int \frac{1}{\frac{a^2\sinh^2(c+dx)}{\cosh(c+dx)a+a}+2a}d\left(-\frac{ia\sinh(c+dx)}{\sqrt{\cosh(c+dx)a+a}}\right)}{2ad}$$

$$\Big\downarrow \text{219}$$

$$\frac{\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a\cosh(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sinh(c+dx)}{2d(a\cosh(c+dx)+a)^{3/2}}$$

input
```
Int[(a + a*Cosh[c + d*x])^(-3/2),x]
```

output
```
ArcTan[(Sqrt[a]*Sinh[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cosh[c + d*x]])]/(2*Sqr
t[2]*a^(3/2)*d) + Sinh[c + d*x]/(2*d*(a + a*Cosh[c + d*x])^(3/2))
```

3.46.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.46.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(62) = 124$.

Time = 0.13 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.87

method	result	size
default	$-\frac{\sqrt{\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2} a}\,\left(\ln\left(\frac{2\sqrt{-a}\,\sqrt{\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2} a}-2a}{\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)}\right) a \cosh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\sqrt{-a}\,\sqrt{\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2} a}\right)\sqrt{2}}{4a^{2}\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-a}\,\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,d}$	144

input
```
int(1/(a+a*cosh(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(sinh(1/2*d*x+1/2*c)^2*a)^(1/2)*(ln(2*((-a)^(1/2)*(sinh(1/2*d*x+1/2*c
)^2*a)^(1/2)-a)/cosh(1/2*d*x+1/2*c))*a*cosh(1/2*d*x+1/2*c)^2-(-a)^(1/2)*(s
inh(1/2*d*x+1/2*c)^2*a)^(1/2))/a^2/cosh(1/2*d*x+1/2*c)/(-a)^(1/2)/sinh(1/2
*d*x+1/2*c)*2^(1/2)/(a*cosh(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.46.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(62) = 124$.

Time = 0.25 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.88

$$\int \frac{1}{(a + a\cosh(c + dx))^{3/2}}\, dx =$$

$$-\frac{\sqrt{2}\left(\cosh\left(dx + c\right)^2 + 2\left(\cosh\left(dx + c\right) + 1\right)\sinh\left(dx + c\right) + \sinh\left(dx + c\right)^2 + 2\cosh\left(dx + c\right) + 1\right)\sqrt{a}\arctan}{2\left(a^2 d\cosh\left(dx + c\right)^2 + a^2 d\sinh\left(dx\right.\right.}$$

input
```
integrate(1/(a+a*cosh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(2)*(cosh(d*x + c)^2 + 2*(cosh(d*x + c) + 1)*sinh(d*x + c) + sin
h(d*x + c)^2 + 2*cosh(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(1/2)*sqrt(
a)*sqrt(a/(cosh(d*x + c) + sinh(d*x + c)))/a) - 2*sqrt(1/2)*(cosh(d*x + c)
^2 + (2*cosh(d*x + c) - 1)*sinh(d*x + c) + sinh(d*x + c)^2 - cosh(d*x + c)
)*sqrt(a/(cosh(d*x + c) + sinh(d*x + c))))/(a^2*d*cosh(d*x + c)^2 + a^2*d*
sinh(d*x + c)^2 + 2*a^2*d*cosh(d*x + c) + a^2*d + 2*(a^2*d*cosh(d*x + c) +
 a^2*d)*sinh(d*x + c))
```

3.46.6 Sympy [F]

$$\int \frac{1}{(a + a\cosh(c + dx))^{3/2}}\, dx = \int \frac{1}{\left(a\cosh\left(c + dx\right) + a\right)^{\frac{3}{2}}}\, dx$$

input
```
integrate(1/(a+a*cosh(d*x+c))**(3/2),x)
```

output
```
Integral((a*cosh(c + d*x) + a)**(-3/2), x)
```

3.46.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(62) = 124$.

Time = 0.32 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.21

$$\int \frac{1}{(a+a\cosh(c+dx))^{3/2}}\,dx = \frac{1}{6}\sqrt{2}\left(\frac{3\,e^{\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)}+8\,e^{\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)}-3\,e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{\left(a^{\frac{3}{2}}e^{(3\,dx+3\,c)}+3\,a^{\frac{3}{2}}e^{(2\,dx+2\,c)}+3\,a^{\frac{3}{2}}e^{(dx+c)}+a^{\frac{3}{2}}\right)d}+\frac{3\arctan\left(e^{\left(\frac{1}{2}\,\cdots\right.}\right.}{a^{\frac{3}{2}}d}\right.$$
$$-\frac{4\sqrt{2}e^{\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)}}{3\left(a^{\frac{3}{2}}de^{(3\,dx+3\,c)}+3\,a^{\frac{3}{2}}de^{(2\,dx+2\,c)}+3\,a^{\frac{3}{2}}de^{(dx+c)}+a^{\frac{3}{2}}d\right)}$$

input
```
integrate(1/(a+a*cosh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/6*sqrt(2)*((3*e^(5/2*d*x + 5/2*c) + 8*e^(3/2*d*x + 3/2*c) - 3*e^(1/2*d*x
 + 1/2*c))/((a^(3/2)*e^(3*d*x + 3*c) + 3*a^(3/2)*e^(2*d*x + 2*c) + 3*a^(3/
2)*e^(d*x + c) + a^(3/2))*d) + 3*arctan(e^(1/2*d*x + 1/2*c))/(a^(3/2)*d))
- 4/3*sqrt(2)*e^(3/2*d*x + 3/2*c)/(a^(3/2)*d*e^(3*d*x + 3*c) + 3*a^(3/2)*d
*e^(2*d*x + 2*c) + 3*a^(3/2)*d*e^(d*x + c) + a^(3/2)*d)
```

3.46.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91

$$\int \frac{1}{(a+a\cosh(c+dx))^{3/2}}\,dx = \frac{\frac{\sqrt{2}\arctan\left(e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}\right)}{a^{\frac{3}{2}}}+\frac{\sqrt{2}\left(\sqrt{a}e^{\left(\frac{3}{2}\,dx+2\,c\right)}-\sqrt{a}e^{\left(\frac{1}{2}\,dx+c\right)}\right)e^{\left(-\frac{1}{2}\,c\right)}}{a^2\left(e^{(dx+c)}+1\right)^2}}{2\,d}$$

input
```
integrate(1/(a+a*cosh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/2*(sqrt(2)*arctan(e^(1/2*d*x + 1/2*c))/a^(3/2) + sqrt(2)*(sqrt(a)*e^(3/2
*d*x + 2*c) - sqrt(a)*e^(1/2*d*x + c))*e^(-1/2*c)/(a^2*(e^(d*x + c) + 1)^2
))/d
```

3.46.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cosh(c+dx))^{3/2}}\,dx=\int \frac{1}{{\left(a+a\,\mathrm{cosh}\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

input
```
int(1/(a + a*cosh(c + d*x))^(3/2),x)
```

output
```
int(1/(a + a*cosh(c + d*x))^(3/2), x)
```

3.47 $\int \frac{1}{(a+a\cosh(c+dx))^{5/2}}\,dx$

3.47.1 Optimal result

Integrand size = 14, antiderivative size = 107

$$\int \frac{1}{(a+a\cosh(c+dx))^{5/2}}\,dx = \frac{3\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a+a\cosh(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{\sinh(c+dx)}{4d(a+a\cosh(c+dx))^{5/2}} + \frac{3\sinh(c+dx)}{16ad(a+a\cosh(c+dx))^{3/2}}$$

output
```
1/4*sinh(d*x+c)/d/(a+a*cosh(d*x+c))^(5/2)+3/16*sinh(d*x+c)/a/d/(a+a*cosh(d
*x+c))^(3/2)+3/32*arctan(1/2*sinh(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cosh(d*x+c))
^(1/2))/a^(5/2)/d*2^(1/2)
```

3.47.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.85

$$\int \frac{1}{(a+a\cosh(c+dx))^{5/2}}\,dx = \frac{\cosh^5\left(\frac{1}{2}(c+dx)\right)\left(32\operatorname{csch}^4(c+dx)\sinh^5\left(\frac{1}{2}(c+dx)\right)+3\left(\arctan\left(\sinh\left(\frac{1}{2}(c\right.\right.\right.\right.}{4d(a(1+\cosh(c+dx)))^{5/2}}$$

input
```
Integrate[(a + a*Cosh[c + d*x])^(-5/2),x]
```

output
```
(Cosh[(c + d*x)/2]^5*(32*Csch[c + d*x]^4*Sinh[(c + d*x)/2]^5 + 3*(ArcTan[S
inh[(c + d*x)/2]] + Sech[(c + d*x)/2]*Tanh[(c + d*x)/2])))/(4*d*(a*(1 + Co
sh[c + d*x]))^(5/2))
```

3.47.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3129, 3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cosh(c+dx)+a)^{5/2}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\left(a+a\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$
$$\Big\downarrow \text{ 3129}$$
$$\frac{3\int \frac{1}{(\cosh(c+dx)a+a)^{3/2}}dx}{8a}+\frac{\sinh(c+dx)}{4d(a\cosh(c+dx)+a)^{5/2}}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\sinh(c+dx)}{4d(a\cosh(c+dx)+a)^{5/2}}+\frac{3\int \frac{1}{\left(\sin\left(ic+idx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a}$$
$$\Big\downarrow \text{ 3129}$$
$$\frac{3\left(\frac{\int \frac{1}{\sqrt{\cosh(c+dx)a+a}}dx}{4a}+\frac{\sinh(c+dx)}{2d(a\cosh(c+dx)+a)^{3/2}}\right)}{8a}+\frac{\sinh(c+dx)}{4d(a\cosh(c+dx)+a)^{5/2}}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\sinh(c+dx)}{4d(a\cosh(c+dx)+a)^{5/2}}+\frac{3\left(\frac{\sinh(c+dx)}{2d(a\cosh(c+dx)+a)^{3/2}}+\frac{\int \frac{1}{\sqrt{\sin\left(ic+idx+\frac{\pi}{2}\right)a+a}}dx}{4a}\right)}{8a}$$
$$\Big\downarrow \text{ 3128}$$
$$\frac{\sinh(c+dx)}{4d(a\cosh(c+dx)+a)^{5/2}}+\frac{3\left(\frac{\sinh(c+dx)}{2d(a\cosh(c+dx)+a)^{3/2}}+\frac{i\int \frac{1}{\frac{a^2\sinh^2(c+dx)}{\cosh(c+dx)a+a}+2a}d\left(-\frac{ia\sinh(c+dx)}{\sqrt{\cosh(c+dx)a+a}}\right)}{2ad}\right)}{8a}$$
$$\Big\downarrow \text{ 219}$$

$$\frac{3\left(\frac{\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a\cosh(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sinh(c+dx)}{2d(a\cosh(c+dx)+a)^{3/2}}\right)}{8a}+\frac{\sinh(c+dx)}{4d(a\cosh(c+dx)+a)^{5/2}}$$

input
```
Int[(a + a*Cosh[c + d*x])^(-5/2),x]
```

output
```
Sinh[c + d*x]/(4*d*(a + a*Cosh[c + d*x])^(5/2)) + (3*(ArcTan[(Sqrt[a]*Sinh
[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cosh[c + d*x]])]/(2*Sqrt[2]*a^(3/2)*d) + Si
nh[c + d*x]/(2*d*(a + a*Cosh[c + d*x])^(3/2))))/(8*a)
```

3.47.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.47.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(88) = 176$.

Time = 0.14 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.66

method	result
default	$-\frac{\sqrt{\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2} a}\,\left(3\ln\left(\frac{2\sqrt{-a}\,\sqrt{\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2} a}-2a}{\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)}\right) a\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-3\sqrt{-a}\,\sqrt{\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2} a}\,\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-2\sqrt{-a}\,\sqrt{\sinh(}}{32a^{3}\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}\sqrt{-a}\,\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,d}$

input
```
int(1/(a+a*cosh(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32*(sinh(1/2*d*x+1/2*c)^2*a)^(1/2)*(3*ln(2*((-a)^(1/2)*(sinh(1/2*d*x+1/
2*c)^2*a)^(1/2)-a)/cosh(1/2*d*x+1/2*c))*a*cosh(1/2*d*x+1/2*c)^4-3*(-a)^(1/
2)*(sinh(1/2*d*x+1/2*c)^2*a)^(1/2)*cosh(1/2*d*x+1/2*c)^2-2*(-a)^(1/2)*(sin
h(1/2*d*x+1/2*c)^2*a)^(1/2))/a^3/cosh(1/2*d*x+1/2*c)^3/(-a)^(1/2)/sinh(1/2
*d*x+1/2*c)*2^(1/2)/(a*cosh(1/2*d*x+1/2*c)^2)^(1/2)/d
```

3.47.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 522 vs. $2(88) = 176$.

Time = 0.27 (sec) , antiderivative size = 522, normalized size of antiderivative = 4.88

$$\int \frac{1}{(a+a\cosh(c+dx))^{5/2}}\,dx =$$

$$-\frac{3\sqrt{2}\left(\cosh\left(dx+c\right)^{4}+4\left(\cosh\left(dx+c\right)+1\right)\sinh\left(dx+c\right)^{3}+\sinh\left(dx+c\right)^{4}+4\cosh\left(dx+c\right)^{3}+6\left(\cosh\left(\right.\right.}{}$$

input
```
integrate(1/(a+a*cosh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1/16*(3*sqrt(2)*(cosh(d*x + c)^4 + 4*(cosh(d*x + c) + 1)*sinh(d*x + c)^3
+ sinh(d*x + c)^4 + 4*cosh(d*x + c)^3 + 6*(cosh(d*x + c)^2 + 2*cosh(d*x +
c) + 1)*sinh(d*x + c)^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + 3*cosh(
d*x + c)^2 + 3*cosh(d*x + c) + 1)*sinh(d*x + c) + 4*cosh(d*x + c) + 1)*sqr
t(a)*arctan(sqrt(2)*sqrt(1/2)*sqrt(a/(cosh(d*x + c) + sinh(d*x + c)))/sqrt
(a)) - 2*sqrt(1/2)*(3*cosh(d*x + c)^4 + (12*cosh(d*x + c) + 11)*sinh(d*x +
 c)^3 + 3*sinh(d*x + c)^4 + 11*cosh(d*x + c)^3 + (18*cosh(d*x + c)^2 + 33*
cosh(d*x + c) - 11)*sinh(d*x + c)^2 - 11*cosh(d*x + c)^2 + (12*cosh(d*x +
c)^3 + 33*cosh(d*x + c)^2 - 22*cosh(d*x + c) - 3)*sinh(d*x + c) - 3*cosh(d
*x + c))*sqrt(a/(cosh(d*x + c) + sinh(d*x + c))))/(a^3*d*cosh(d*x + c)^4 +
 a^3*d*sinh(d*x + c)^4 + 4*a^3*d*cosh(d*x + c)^3 + 6*a^3*d*cosh(d*x + c)^2
 + 4*a^3*d*cosh(d*x + c) + a^3*d + 4*(a^3*d*cosh(d*x + c) + a^3*d)*sinh(d*
x + c)^3 + 6*(a^3*d*cosh(d*x + c)^2 + 2*a^3*d*cosh(d*x + c) + a^3*d)*sinh(
d*x + c)^2 + 4*(a^3*d*cosh(d*x + c)^3 + 3*a^3*d*cosh(d*x + c)^2 + 3*a^3*d*
cosh(d*x + c) + a^3*d)*sinh(d*x + c))
```

3.47.6 Sympy [F]

$$\int \frac{1}{(a+a\cosh(c+dx))^{5/2}}\,dx = \int \frac{1}{(a\cosh(c+dx)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+a*cosh(d*x+c))**(5/2),x)
```

output
```
Integral((a*cosh(c + d*x) + a)**(-5/2), x)
```

3.47.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(88) = 176$.

Time = 0.33 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.34

$$\int \frac{1}{(a+a\cosh(c+dx))^{5/2}}\,dx = \frac{1}{80}\sqrt{2}\left(\frac{15\,e^{\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)}+70\,e^{\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)}+128\,e^{\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)}-70\,e^{\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)}-}{\left(a^{\frac{5}{2}}e^{(5\,dx+5\,c)}+5\,a^{\frac{5}{2}}e^{(4\,dx+4\,c)}+10\,a^{\frac{5}{2}}e^{(3\,dx+3\,c)}+10\,a^{\frac{5}{2}}e^{(2\,dx+2\,c)}+5\right.}\right.$$

$$-\frac{8\sqrt{2}e^{\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)}}{5\left(a^{\frac{5}{2}}de^{(5\,dx+5\,c)}+5\,a^{\frac{5}{2}}de^{(4\,dx+4\,c)}+10\,a^{\frac{5}{2}}de^{(3\,dx+3\,c)}+10\,a^{\frac{5}{2}}de^{(2\,dx+2\,c)}+5\,a^{\frac{5}{2}}de^{(dx+c)}+a^{\frac{5}{2}}d\right)}$$

input
```
integrate(1/(a+a*cosh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/80*sqrt(2)*((15*e^(9/2*d*x + 9/2*c) + 70*e^(7/2*d*x + 7/2*c) + 128*e^(5/
2*d*x + 5/2*c) - 70*e^(3/2*d*x + 3/2*c) - 15*e^(1/2*d*x + 1/2*c))/((a^(5/2
)*e^(5*d*x + 5*c) + 5*a^(5/2)*e^(4*d*x + 4*c) + 10*a^(5/2)*e^(3*d*x + 3*c)
 + 10*a^(5/2)*e^(2*d*x + 2*c) + 5*a^(5/2)*e^(d*x + c) + a^(5/2))*d) + 15*a
rctan(e^(1/2*d*x + 1/2*c))/(a^(5/2)*d)) - 8/5*sqrt(2)*e^(5/2*d*x + 5/2*c)/
(a^(5/2)*d*e^(5*d*x + 5*c) + 5*a^(5/2)*d*e^(4*d*x + 4*c) + 10*a^(5/2)*d*e^
(3*d*x + 3*c) + 10*a^(5/2)*d*e^(2*d*x + 2*c) + 5*a^(5/2)*d*e^(d*x + c) + a
^(5/2)*d)
```

3.47.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+a\cosh(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+a*cosh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{%%{[147456,0]:[1,0,-2]%%},[0]%%%},0]:[1,0,%%%{-1,[
1]%%%}]%%
```

3.47.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cosh(c+dx))^{5/2}}\,dx = \int \frac{1}{(a+a\cosh(c+d\,x))^{5/2}}\,dx$$

input
```
int(1/(a + a*cosh(c + d*x))^(5/2),x)
```

output
```
int(1/(a + a*cosh(c + d*x))^(5/2), x)
```

3.48 $\int (a-a\cosh(c+dx))^{5/2}\,dx$

3.48.1 Optimal result

Integrand size = 15, antiderivative size = 92

$$\int (a-a\cosh(c+dx))^{5/2}\,dx=-\frac{64a^3\sinh(c+dx)}{15d\sqrt{a-a\cosh(c+dx)}}-\frac{16a^2\sqrt{a-a\cosh(c+dx)}\sinh(c+dx)}{15d}-\frac{2a(a-a\cosh(c+dx))^{3/2}\sinh(c+dx)}{5d}$$

output
```
-2/5*a*(a-a*cosh(d*x+c))^(3/2)*sinh(d*x+c)/d-64/15*a^3*sinh(d*x+c)/d/(a-a*
cosh(d*x+c))^(1/2)-16/15*a^2*sinh(d*x+c)*(a-a*cosh(d*x+c))^(1/2)/d
```

3.48.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.78

$$\int (a-a\cosh(c+dx))^{5/2}\,dx=\frac{a^2\sqrt{a-a\cosh(c+dx)}\left(150\cosh\left(\frac{1}{2}(c+dx)\right)-25\cosh\left(\frac{3}{2}(c+dx)\right)+3\cosh\left(\frac{5}{2}(c+dx)\right)\right)\operatorname{csc}}{30d}$$

input
```
Integrate[(a - a*Cosh[c + d*x])^(5/2),x]
```

output
```
(a^2*Sqrt[a - a*Cosh[c + d*x]]*(150*Cosh[(c + d*x)/2] - 25*Cosh[(3*(c + d*
x))/2] + 3*Cosh[(5*(c + d*x))/2])*Csch[(c + d*x)/2])/(30*d)
```

3.48.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3126, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-a\cosh(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{3126}$$

$$\frac{8}{5}a\int (a-a\cosh(c+dx))^{3/2}dx-\frac{2a\sinh(c+dx)(a-a\cosh(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2a\sinh(c+dx)(a-a\cosh(c+dx))^{3/2}}{5d}+\frac{8}{5}a\int \left(a-a\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{3126}$$

$$\frac{8}{5}a\left(\frac{4}{3}a\int \sqrt{a-a\cosh(c+dx)}dx-\frac{2a\sinh(c+dx)\sqrt{a-a\cosh(c+dx)}}{3d}\right)-\frac{2a\sinh(c+dx)(a-a\cosh(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2a\sinh(c+dx)(a-a\cosh(c+dx))^{3/2}}{5d}+\frac{8}{5}a\left(-\frac{2a\sinh(c+dx)\sqrt{a-a\cosh(c+dx)}}{3d}+\frac{4}{3}a\int \sqrt{a-a\sin\left(ic+idx+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \text{3125}$$

$$\frac{8}{5}a\left(-\frac{8a^2\sinh(c+dx)}{3d\sqrt{a-a\cosh(c+dx)}}-\frac{2a\sinh(c+dx)\sqrt{a-a\cosh(c+dx)}}{3d}\right)-\frac{2a\sinh(c+dx)(a-a\cosh(c+dx))^{3/2}}{5d}$$

input
```
Int[(a - a*Cosh[c + d*x])^(5/2),x]
```

output
```
(-2*a*(a - a*Cosh[c + d*x])^(3/2)*Sinh[c + d*x])/(5*d) + (8*a*((-8*a^2*Sin
h[c + d*x])/(3*d*Sqrt[a - a*Cosh[c + d*x]]) - (2*a*Sqrt[a - a*Cosh[c + d*x
]]*Sinh[c + d*x])/(3*d)))/5
```

3.48.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

3.48.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{16 \sinh\left(\frac{dx}{2}+\frac{c}{2}\right) a^{3} \cosh\left(\frac{dx}{2}+\frac{c}{2}\right) \left(3 \sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-4 \sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+8\right)}{15 \sqrt{-2 \sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2} a}\, d}$	71

input
```
int((a-a*cosh(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-16/15*sinh(1/2*d*x+1/2*c)*a^3*cosh(1/2*d*x+1/2*c)*(3*sinh(1/2*d*x+1/2*c)^
4-4*sinh(1/2*d*x+1/2*c)^2+8)/(-2*sinh(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

3.48.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. 2(80) = 160.

Time = 0.25 (sec) , antiderivative size = 328, normalized size of antiderivative = 3.57

$$\int (a - a\cosh(c + dx))^{5/2}\,dx = \frac{\sqrt{\frac{1}{2}}\left(3\,a^2\cosh\left(dx+c\right)^5 + 3\,a^2\sinh\left(dx+c\right)^5 - 25\,a^2\cosh\left(dx+c\right)^4 + 150\,a^2\cosh\left(dx+c\right)^3 + \right.}{}$$

input
```
integrate((a-a*cosh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/30*sqrt(1/2)*(3*a^2*cosh(d*x + c)^5 + 3*a^2*sinh(d*x + c)^5 - 25*a^2*cos
h(d*x + c)^4 + 150*a^2*cosh(d*x + c)^3 + 5*(3*a^2*cosh(d*x + c) - 5*a^2)*s
inh(d*x + c)^4 + 150*a^2*cosh(d*x + c)^2 + 10*(3*a^2*cosh(d*x + c)^2 - 10*
a^2*cosh(d*x + c) + 15*a^2)*sinh(d*x + c)^3 - 25*a^2*cosh(d*x + c) + 30*(a
^2*cosh(d*x + c)^3 - 5*a^2*cosh(d*x + c)^2 + 15*a^2*cosh(d*x + c) + 5*a^2)
*sinh(d*x + c)^2 + 3*a^2 + 5*(3*a^2*cosh(d*x + c)^4 - 20*a^2*cosh(d*x + c)
^3 + 90*a^2*cosh(d*x + c)^2 + 60*a^2*cosh(d*x + c) - 5*a^2)*sinh(d*x + c))
*sqrt(-a/(cosh(d*x + c) + sinh(d*x + c)))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*
x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2)
```

3.48.6 Sympy [F]

$$\int (a - a\cosh(c + dx))^{5/2}\,dx = \int \left(-a\cosh\left(c+dx\right) + a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a-a*cosh(d*x+c))**(5/2),x)
```

output
```
Integral((-a*cosh(c + d*x) + a)**(5/2), x)
```

3.48.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(80) = 160$.

Time = 0.28 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.07

$$\int (a-a\cosh(c+dx))^{5/2}\,dx = \frac{5\sqrt{2}a^{\frac{5}{2}}e^{(-dx-c)}}{12\,d\left(-e^{(-dx-c)}\right)^{\frac{5}{2}}} - \frac{5\sqrt{2}a^{\frac{5}{2}}e^{(-2\,dx-2\,c)}}{2\,d\left(-e^{(-dx-c)}\right)^{\frac{5}{2}}} - \frac{5\sqrt{2}a^{\frac{5}{2}}e^{(-3\,dx-3\,c)}}{2\,d\left(-e^{(-dx-c)}\right)^{\frac{5}{2}}} + \frac{5\sqrt{2}a^{\frac{5}{2}}e^{(-4\,dx-4\,c)}}{12\,d\left(-e^{(-dx-c)}\right)^{\frac{5}{2}}} - \frac{\sqrt{2}a^{\frac{5}{2}}e^{(-5\,dx-5\,c)}}{20\,d\left(-e^{(-dx-c)}\right)^{\frac{5}{2}}} - \frac{\sqrt{2}a^{\frac{5}{2}}}{20\,d\left(-e^{(-dx-c)}\right)^{\frac{5}{2}}}$$

input
```
integrate((a-a*cosh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
5/12*sqrt(2)*a^(5/2)*e^(-d*x - c)/(d*(-e^(-d*x - c))^(5/2)) - 5/2*sqrt(2)*
a^(5/2)*e^(-2*d*x - 2*c)/(d*(-e^(-d*x - c))^(5/2)) - 5/2*sqrt(2)*a^(5/2)*e
^(-3*d*x - 3*c)/(d*(-e^(-d*x - c))^(5/2)) + 5/12*sqrt(2)*a^(5/2)*e^(-4*d*x
 - 4*c)/(d*(-e^(-d*x - c))^(5/2)) - 1/20*sqrt(2)*a^(5/2)*e^(-5*d*x - 5*c)/
(d*(-e^(-d*x - c))^(5/2)) - 1/20*sqrt(2)*a^(5/2)/(d*(-e^(-d*x - c))^(5/2))
```

3.48.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(80) = 160$.

Time = 0.26 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.11

$$\int (a-a\cosh(c+dx))^{5/2}\,dx = -\frac{\sqrt{2}\Big(3\sqrt{-ae^{(dx+c)}}a^2e^{(2\,dx+2\,c)}\mathrm{sgn}\left(-e^{(dx+c)}+1\right) - 25\sqrt{-ae^{(dx+c)}}a^2e^{(dx+c)}\mathrm{sgn}\left(-e^{(dx+c)}+1\right) + 150\sqrt{-ae^{(d}}}{60}$$

input
```
integrate((a-a*cosh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
-1/60*sqrt(2)*(3*sqrt(-a*e^(d*x + c))*a^2*e^(2*d*x + 2*c)*sgn(-e^(d*x + c)
 + 1) - 25*sqrt(-a*e^(d*x + c))*a^2*e^(d*x + c)*sgn(-e^(d*x + c) + 1) + 15
0*sqrt(-a*e^(d*x + c))*a^2*sgn(-e^(d*x + c) + 1) - (150*a^5*e^(2*d*x + 2*c
)*sgn(-e^(d*x + c) + 1) - 25*a^5*e^(d*x + c)*sgn(-e^(d*x + c) + 1) + 3*a^5
*sgn(-e^(d*x + c) + 1))*e^(-2*d*x - 2*c)/(sqrt(-a*e^(d*x + c))*a^2))/d
```

3.48.9 Mupad [F(-1)]

Timed out.

$$\int (a - a\cosh(c + dx))^{5/2}\, dx = \int (a - a\cosh(c + d\,x))^{5/2}\, dx$$

input
```
int((a - a*cosh(c + d*x))^(5/2),x)
```

output
```
int((a - a*cosh(c + d*x))^(5/2), x)
```

3.49 $\int (a - a\cosh(c + dx))^{3/2} dx$

3.49.1 Optimal result

Integrand size = 15, antiderivative size = 61

$$\int (a-a\cosh(c+dx))^{3/2} dx = -\frac{8a^2\sinh(c+dx)}{3d\sqrt{a-a\cosh(c+dx)}} - \frac{2a\sqrt{a-a\cosh(c+dx)}\sinh(c+dx)}{3d}$$

output
```
-8/3*a^2*sinh(d*x+c)/d/(a-a*cosh(d*x+c))^(1/2)-2/3*a*sinh(d*x+c)*(a-a*cosh
(d*x+c))^(1/2)/d
```

3.49.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int (a-a\cosh(c+dx))^{3/2} dx = -\frac{a\sqrt{a-a\cosh(c+dx)}\left(-9\cosh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{3}{2}(c+dx)\right)\right)\operatorname{csch}\left(\frac{1}{2}(c+dx)\right)}{3d}$$

input
```
Integrate[(a - a*Cosh[c + d*x])^(3/2),x]
```

output
```
-1/3*(a*Sqrt[a - a*Cosh[c + d*x]]*(-9*Cosh[(c + d*x)/2] + Cosh[(3*(c + d*x
))/2])*Csch[(c + d*x)/2])/d
```

3.49.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-a\cosh(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(a-a\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{ 3126}$$

$$\frac{4}{3}a\int \sqrt{a-a\cosh(c+dx)}dx-\frac{2a\sinh(c+dx)\sqrt{a-a\cosh(c+dx)}}{3d}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{2a\sinh(c+dx)\sqrt{a-a\cosh(c+dx)}}{3d}+\frac{4}{3}a\int \sqrt{a-a\sin\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3125}$$

$$-\frac{8a^2\sinh(c+dx)}{3d\sqrt{a-a\cosh(c+dx)}}-\frac{2a\sinh(c+dx)\sqrt{a-a\cosh(c+dx)}}{3d}$$

input
```
Int[(a - a*Cosh[c + d*x])^(3/2),x]
```

output
```
(-8*a^2*Sinh[c + d*x])/(3*d*Sqrt[a - a*Cosh[c + d*x]]) - (2*a*Sqrt[a - a*C
osh[c + d*x]]*Sinh[c + d*x])/(3*d)
```

3.49.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

3.49.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{8\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)a^{2}\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-3\right)}{3\sqrt{-2\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a}\,d}$	56

input
```
int((a-a*cosh(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
8/3*sinh(1/2*d*x+1/2*c)*a^2*cosh(1/2*d*x+1/2*c)*(cosh(1/2*d*x+1/2*c)^2-3)/
(-2*sinh(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

3.49.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(53) = 106.

Time = 0.25 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.28

$$\int (a-a\cosh(c+dx))^{3/2}\,dx =$$

$$-\frac{\sqrt{\frac{1}{2}}\left(a\cosh\left(dx+c\right)^{3}+a\sinh\left(dx+c\right)^{3}-9\,a\cosh\left(dx+c\right)^{2}+3\left(a\cosh\left(dx+c\right)-3\,a\right)\sinh\left(dx+c\right)^{2}-9}{3\left(d\cosh\left(dx+c\right)-\right.}$$

input
```
integrate((a-a*cosh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/3*sqrt(1/2)*(a*cosh(d*x + c)^3 + a*sinh(d*x + c)^3 - 9*a*cosh(d*x + c)^
2 + 3*(a*cosh(d*x + c) - 3*a)*sinh(d*x + c)^2 - 9*a*cosh(d*x + c) + 3*(a*c
osh(d*x + c)^2 - 6*a*cosh(d*x + c) - 3*a)*sinh(d*x + c) + a)*sqrt(-a/(cosh
(d*x + c) + sinh(d*x + c)))/(d*cosh(d*x + c) + d*sinh(d*x + c))
```

3.49.6 Sympy [F]

$$\int (a-a\cosh(c+dx))^{3/2}\,dx=\int(-a\cosh{(c+dx)}+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a-a*cosh(d*x+c))**(3/2),x)
```

output
```
Integral((-a*cosh(c + d*x) + a)**(3/2), x)
```

3.49.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(53) = 106$.

Time = 0.29 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.03

$$\int (a-a\cosh(c+dx))^{3/2}\,dx=\frac{3\sqrt{2}a^{\frac{3}{2}}e^{(-dx-c)}}{2\,d\,(-e^{(-dx-c)})^{\frac{3}{2}}}+\frac{3\sqrt{2}a^{\frac{3}{2}}e^{(-2\,dx-2\,c)}}{2\,d\,(-e^{(-dx-c)})^{\frac{3}{2}}}-\frac{\sqrt{2}a^{\frac{3}{2}}e^{(-3\,dx-3\,c)}}{6\,d\,(-e^{(-dx-c)})^{\frac{3}{2}}}-\frac{\sqrt{2}a^{\frac{3}{2}}}{6\,d\,(-e^{(-dx-c)})^{\frac{3}{2}}}$$

input
```
integrate((a-a*cosh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
3/2*sqrt(2)*a^(3/2)*e^(-d*x - c)/(d*(-e^(-d*x - c))^(3/2)) + 3/2*sqrt(2)*a
^(3/2)*e^(-2*d*x - 2*c)/(d*(-e^(-d*x - c))^(3/2)) - 1/6*sqrt(2)*a^(3/2)*e^
(-3*d*x - 3*c)/(d*(-e^(-d*x - c))^(3/2)) - 1/6*sqrt(2)*a^(3/2)/(d*(-e^(-d*
x - c))^(3/2))
```

3.49.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(53) = 106$.

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.08

$$\int (a - a\cosh(c + dx))^{3/2}\,dx = \frac{\sqrt{2}\left(\sqrt{-ae^{(dx+c)}}ae^{(dx+c)}\mathrm{sgn}\left(-e^{(dx+c)} + 1\right) - 9\sqrt{-ae^{(dx+c)}}a\mathrm{sgn}\left(-e^{(dx+c)} + 1\right) + \frac{(9\,a^3e^{(dx+c)}\mathrm{sgn}(\cdots}{\cdots}\right.}{6\,d}$$

input
```
integrate((a-a*cosh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/6*sqrt(2)*(sqrt(-a*e^(d*x + c))*a*e^(d*x + c)*sgn(-e^(d*x + c) + 1) - 9*
sqrt(-a*e^(d*x + c))*a*sgn(-e^(d*x + c) + 1) + (9*a^3*e^(d*x + c)*sgn(-e^(
d*x + c) + 1) - a^3*sgn(-e^(d*x + c) + 1))*e^(-d*x - c)/(sqrt(-a*e^(d*x +
c))*a))/d
```

3.49.9 Mupad [F(-1)]

Timed out.

$$\int (a - a\cosh(c + dx))^{3/2}\,dx = \int (a - a\cosh(c + d\,x))^{3/2}\,dx$$

input
```
int((a - a*cosh(c + d*x))^(3/2),x)
```

output
```
int((a - a*cosh(c + d*x))^(3/2), x)
```

3.50 $\int \sqrt{a-a\cosh(c+dx)}\,dx$

3.50.1 Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \sqrt{a-a\cosh(c+dx)}\,dx=-\frac{2a\sinh(c+dx)}{d\sqrt{a-a\cosh(c+dx)}}$$

output
```
-2*a*sinh(d*x+c)/d/(a-a*cosh(d*x+c))^(1/2)
```

3.50.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \sqrt{a-a\cosh(c+dx)}\,dx=\frac{2\sqrt{a-a\cosh(c+dx)}\coth\left(\frac{1}{2}(c+dx)\right)}{d}$$

input
```
Integrate[Sqrt[a - a*Cosh[c + d*x]],x]
```

output
```
(2*Sqrt[a - a*Cosh[c + d*x]]*Coth[(c + d*x)/2])/d
```

3.50.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-a\cosh(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a-a\sin\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3125}$$

$$-\frac{2a\sinh(c+dx)}{d\sqrt{a-a\cosh(c+dx)}}$$

input
```
Int[Sqrt[a - a*Cosh[c + d*x]],x]
```

output
```
(-2*a*Sinh[c + d*x])/(d*Sqrt[a - a*Cosh[c + d*x]])
```

3.50.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

3.50.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

method	result	size
default	$-\frac{4 \sinh\left(\frac{dx}{2}+\frac{c}{2}\right) a \cosh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{-2 \sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2} a}\, d}$	41
risch	$\frac{\sqrt{2}\,\sqrt{-a\left(\mathrm{e}^{dx+c}-1\right)^{2} \mathrm{e}^{-dx-c}}\,\left(\mathrm{e}^{dx+c}+1\right)}{\left(\mathrm{e}^{dx+c}-1\right) d}$	50

input
```
int((a-a*cosh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4*sinh(1/2*d*x+1/2*c)*a*cosh(1/2*d*x+1/2*c)/(-2*sinh(1/2*d*x+1/2*c)^2*a)^
(1/2)/d
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

$$\int \sqrt{a-a\cosh(c+dx)}\,dx = \frac{2\sqrt{\frac{1}{2}}\sqrt{-\frac{a}{\cosh(dx+c)+\sinh(dx+c)}}\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)+1\right)}{d}$$

input
```
integrate((a-a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(1/2)*sqrt(-a/(cosh(d*x + c) + sinh(d*x + c)))*(cosh(d*x + c) + sinh
(d*x + c) + 1)/d
```

3.50.6 Sympy [F]

$$\int \sqrt{a-a\cosh(c+dx)}\,dx = \int \sqrt{-a\cosh\left(c+dx\right)+a}\,dx$$

input
```
integrate((a-a*cosh(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(-a*cosh(c + d*x) + a), x)
```

3.50.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(25) = 50$.

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \sqrt{a - a\cosh(c + dx)}\,dx = -\frac{\sqrt{2}\sqrt{a}e^{(-dx-c)}}{d\sqrt{-e^{(-dx-c)}}} - \frac{\sqrt{2}\sqrt{a}}{d\sqrt{-e^{(-dx-c)}}}$$

input
```
integrate((a-a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(2)*sqrt(a)*e^(-d*x - c)/(d*sqrt(-e^(-d*x - c))) - sqrt(2)*sqrt(a)/(d
*sqrt(-e^(-d*x - c)))
```

3.50.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(25) = 50$.

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.33

$$\int \sqrt{a - a\cosh(c + dx)}\,dx = -\frac{\sqrt{2}\left(\sqrt{-ae^{(dx+c)}}a\mathrm{sgn}\left(-e^{(dx+c)} + 1\right) - \frac{a^2\mathrm{sgn}\left(-e^{(dx+c)}+1\right)}{\sqrt{-ae^{(dx+c)}}}\right)}{ad}$$

input
```
integrate((a-a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-sqrt(2)*(sqrt(-a*e^(d*x + c))*a*sgn(-e^(d*x + c) + 1) - a^2*sgn(-e^(d*x +
 c) + 1)/sqrt(-a*e^(d*x + c)))/(a*d)
```

3.50.9 Mupad [B] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \sqrt{a - a\cosh(c + dx)}\,dx = \frac{2\coth\left(\frac{c}{2} + \frac{d\,x}{2}\right)\,\sqrt{a - a\cosh\left(c + d\,x\right)}}{d}$$

input
```
int((a - a*cosh(c + d*x))^(1/2),x)
```

output
```
(2*coth(c/2 + (d*x)/2)*(a - a*cosh(c + d*x))^(1/2))/d
```

3.51 $\int \frac{1}{\sqrt{a-a\cosh(c+dx)}}\,dx$

3.51.1 Optimal result

Integrand size = 15, antiderivative size = 48

$$\int \frac{1}{\sqrt{a-a\cosh(c+dx)}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a-a\cosh(c+dx)}}\right)}{\sqrt{a}d}$$

output
```
-arctan(1/2*sinh(d*x+c)*a^(1/2)*2^(1/2)/(a-a*cosh(d*x+c))^(1/2))*2^(1/2)/d
/a^(1/2)
```

3.51.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.15

$$\int \frac{1}{\sqrt{a-a\cosh(c+dx)}}\,dx = \frac{2\left(-\log\left(\cosh\left(\frac{1}{4}(c+dx)\right)\right)+\log\left(\sinh\left(\frac{1}{4}(c+dx)\right)\right)\right)\sinh\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{a-a\cosh(c+dx)}}$$

input
```
Integrate[1/Sqrt[a - a*Cosh[c + d*x]],x]
```

output
```
(2*(-Log[Cosh[(c + d*x)/4]] + Log[Sinh[(c + d*x)/4]])*Sinh[(c + d*x)/2])/(
d*Sqrt[a - a*Cosh[c + d*x]])
```

3.51.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-a\cosh(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a-a\sin\left(ic+idx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3128}$$

$$\frac{2i\int \frac{1}{\frac{a^2\sinh^2(c+dx)}{a-a\cosh(c+dx)}+2a}d\frac{ia\sinh(c+dx)}{\sqrt{a-a\cosh(c+dx)}}}{d}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a-a\cosh(c+dx)}}\right)}{\sqrt{a}d}$$

input
```
Int[1/Sqrt[a - a*Cosh[c + d*x]],x]
```

output
```
-((Sqrt[2]*ArcTan[(Sqrt[a]*Sinh[c + d*x])/(Sqrt[2]*Sqrt[a - a*Cosh[c + d*x
]])])/(Sqrt[a]*d))
```

3.51.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

3.51.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{2\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\operatorname{arctanh}\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\sqrt{-2\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a}\,d}$	41

input
```
int(1/(a-a*cosh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*sinh(1/2*d*x+1/2*c)*arctanh(cosh(1/2*d*x+1/2*c))/(-2*sinh(1/2*d*x+1/2*c
)^2*a)^(1/2)/d
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 154, normalized size of antiderivative = 3.21

$$\int \frac{1}{\sqrt{a-a\cosh(c+dx)}}\,dx$$
$$=\left[\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a}{\cosh(dx+c)+\sinh(dx+c)}}\sqrt{-\frac{1}{a}}(\cosh(dx+c)+\sinh(dx+c))-\cosh(dx+c)-\sinh(dx+c)-1}{\cosh(dx+c)+\sinh(dx+c)-1}\right)}{d}, \frac{2\sqrt{2}\arctan\left(\right.}{}\right.$$

input
```
integrate(1/(a-a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[sqrt(2)*sqrt(-1/a)*log((2*sqrt(2)*sqrt(1/2)*sqrt(-a/(cosh(d*x + c) + sinh
(d*x + c)))*sqrt(-1/a)*(cosh(d*x + c) + sinh(d*x + c)) - cosh(d*x + c) - s
inh(d*x + c) - 1)/(cosh(d*x + c) + sinh(d*x + c) - 1))/d, 2*sqrt(2)*arctan
(sqrt(2)*sqrt(1/2)*sqrt(-a/(cosh(d*x + c) + sinh(d*x + c)))*(cosh(d*x + c)
 + sinh(d*x + c))/sqrt(a))/(sqrt(a)*d)]
```

3.51.6 Sympy [F]

$$\int \frac{1}{\sqrt{a-a\cosh(c+dx)}}\,dx = \int \frac{1}{\sqrt{-a\cosh{(c+dx)}+a}}\,dx$$

input
```
integrate(1/(a-a*cosh(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(-a*cosh(c + d*x) + a), x)
```

3.51.7 Maxima [F]

$$\int \frac{1}{\sqrt{a-a\cosh(c+dx)}}\,dx = \int \frac{1}{\sqrt{-a\cosh{(dx+c)}+a}}\,dx$$

input
```
integrate(1/(a-a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-a*cosh(d*x + c) + a), x)
```

3.51.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{a-a\cosh(c+dx)}}\,dx = -\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{-ae^{(dx+c)}}}{\sqrt{a}}\right)}{\sqrt{a}d\mathrm{sgn}\left(-e^{(dx+c)}+1\right)}$$

input
```
integrate(1/(a-a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(2)*arctan(sqrt(-a*e^(d*x + c))/sqrt(a))/(sqrt(a)*d*sgn(-e^(d*x + c
) + 1))
```

3.51.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a-a\cosh(c+dx)}}\,dx = \int \frac{1}{\sqrt{a-a\cosh\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(a - a*cosh(c + d*x))^(1/2),x)
```

output
```
int(1/(a - a*cosh(c + d*x))^(1/2), x)
```

3.52 $\int \frac{1}{(a-a\cosh(c+dx))^{3/2}}\,dx$

3.52.1 Optimal result

Integrand size = 15, antiderivative size = 79

$$\int \frac{1}{(a-a\cosh(c+dx))^{3/2}}\,dx=-\frac{\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a-a\cosh(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sinh(c+dx)}{2d(a-a\cosh(c+dx))^{3/2}}$$

output
```
-1/2*sinh(d*x+c)/d/(a-a*cosh(d*x+c))^(3/2)-1/4*arctan(1/2*sinh(d*x+c)*a^(1
/2)*2^(1/2)/(a-a*cosh(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)
```

3.52.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.24

$$\int \frac{1}{(a-a\cosh(c+dx))^{3/2}}\,dx=\frac{\left(\operatorname{csch}^2\left(\frac{1}{4}(c+dx)\right)-4\log\left(\cosh\left(\frac{1}{4}(c+dx)\right)\right)+4\log\left(\sinh\left(\frac{1}{4}(c+dx)\right)\right)+\mathrm{s}\right.}{4ad(-1+\cosh(c+dx))\sqrt{a-a\cosh(c+d}}$$

input
```
Integrate[(a - a*Cosh[c + d*x])^(-3/2),x]
```

output
```
((Csch[(c + d*x)/4]^2 - 4*Log[Cosh[(c + d*x)/4]] + 4*Log[Sinh[(c + d*x)/4]
] + Sech[(c + d*x)/4]^2)*Sinh[(c + d*x)/2]^3)/(4*a*d*(-1 + Cosh[c + d*x])*
Sqrt[a - a*Cosh[c + d*x]])
```

3.52.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-a\cosh(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a-a\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{\int \frac{1}{\sqrt{a-a\cosh(c+dx)}}dx}{4a}-\frac{\sinh(c+dx)}{2d(a-a\cosh(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(c+dx)}{2d(a-a\cosh(c+dx))^{3/2}}+\frac{\int \frac{1}{\sqrt{a-a\sin\left(ic+idx+\frac{\pi}{2}\right)}}dx}{4a}$$

$$\Big\downarrow \text{3128}$$

$$-\frac{\sinh(c+dx)}{2d(a-a\cosh(c+dx))^{3/2}}+\frac{i\int \frac{1}{\frac{a^2\sinh^2(c+dx)}{a-a\cosh(c+dx)}+2a}d\frac{ia\sinh(c+dx)}{\sqrt{a-a\cosh(c+dx)}}}{2ad}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a-a\cosh(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sinh(c+dx)}{2d(a-a\cosh(c+dx))^{3/2}}$$

input
```
Int[(a - a*Cosh[c + d*x])^(-3/2),x]
```

output
```
-1/2*ArcTan[(Sqrt[a]*Sinh[c + d*x])/(Sqrt[2]*Sqrt[a - a*Cosh[c + d*x]])]/(
Sqrt[2]*a^(3/2)*d) - Sinh[c + d*x]/(2*d*(a - a*Cosh[c + d*x])^(3/2))
```

3.52.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.52.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{2\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)+\left(\ln\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\ln\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\right)\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{4a\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a}\,d}$	87

input
```
int(1/(a-a*cosh(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/a*(2*cosh(1/2*d*x+1/2*c)+(ln(cosh(1/2*d*x+1/2*c)-1)-ln(cosh(1/2*d*x+1/
2*c)+1))*sinh(1/2*d*x+1/2*c)^2)/sinh(1/2*d*x+1/2*c)/(-2*sinh(1/2*d*x+1/2*c
)^2*a)^(1/2)/d
```

3.52.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(64) = 128$.

Time = 0.26 (sec) , antiderivative size = 274, normalized size of antiderivative = 3.47

$$\int \frac{1}{(a-a\cosh(c+dx))^{3/2}}\,dx =$$

$$-\frac{\sqrt{2}\left(\cosh\left(dx+c\right)^2+2\left(\cosh\left(dx+c\right)-1\right)\sinh\left(dx+c\right)+\sinh\left(dx+c\right)^2-2\,\cosh\left(dx+c\right)+1\right)\sqrt{-a}\log\left(\right.}{4\left(a^2d\cosh\right.}$$

input
```
integrate(1/(a-a*cosh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(2)*(cosh(d*x + c)^2 + 2*(cosh(d*x + c) - 1)*sinh(d*x + c) + sin
h(d*x + c)^2 - 2*cosh(d*x + c) + 1)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(1/2)*sqr
t(-a)*sqrt(-a/(cosh(d*x + c) + sinh(d*x + c)))*(cosh(d*x + c) + sinh(d*x +
 c)) + a*cosh(d*x + c) + a*sinh(d*x + c) + a)/(cosh(d*x + c) + sinh(d*x +
c) - 1)) + 4*sqrt(1/2)*(cosh(d*x + c)^2 + (2*cosh(d*x + c) + 1)*sinh(d*x +
 c) + sinh(d*x + c)^2 + cosh(d*x + c))*sqrt(-a/(cosh(d*x + c) + sinh(d*x +
 c))))/(a^2*d*cosh(d*x + c)^2 + a^2*d*sinh(d*x + c)^2 - 2*a^2*d*cosh(d*x +
 c) + a^2*d + 2*(a^2*d*cosh(d*x + c) - a^2*d)*sinh(d*x + c))
```

3.52.6 Sympy [F]

$$\int \frac{1}{(a-a\cosh(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(-a\cosh\left(c+dx\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a-a*cosh(d*x+c))**(3/2),x)
```

output
```
Integral((-a*cosh(c + d*x) + a)**(-3/2), x)
```

3.52.7 Maxima [F]

$$\int \frac{1}{(a-a\cosh(c+dx))^{3/2}}\,dx = \int \frac{1}{(-a\cosh(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a-a*cosh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((-a*cosh(d*x + c) + a)^(-3/2), x)
```

3.52.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.46

$$\int \frac{1}{(a-a\cosh(c+dx))^{3/2}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{-ae^{(dx+c)}}}{\sqrt{a}}\right)}{2\,a^{\frac{3}{2}}d\mathrm{sgn}\left(-e^{(dx+c)}+1\right)} + \frac{\sqrt{2}\sqrt{-ae^{(dx+c)}}ae^{(dx+c)}+\sqrt{2}\sqrt{-ae^{(dx+c)}}a}{2\left(ae^{(dx+c)}-a\right)^2 ad\mathrm{sgn}\left(-e^{(dx+c)}+1\right)}$$

input
```
integrate(1/(a-a*cosh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*arctan(sqrt(-a*e^(d*x + c))/sqrt(a))/(a^(3/2)*d*sgn(-e^(d*x +
 c) + 1)) + 1/2*(sqrt(2)*sqrt(-a*e^(d*x + c))*a*e^(d*x + c) + sqrt(2)*sqrt
(-a*e^(d*x + c))*a)/((a*e^(d*x + c) - a)^2*a*d*sgn(-e^(d*x + c) + 1))
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-a\cosh(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a-a\cosh\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/(a - a*cosh(c + d*x))^(3/2),x)
```

output
```
int(1/(a - a*cosh(c + d*x))^(3/2), x)
```

3.53 $\int \frac{1}{(a-a\cosh(c+dx))^{5/2}}\,dx$

3.53.1 Optimal result

Integrand size = 15, antiderivative size = 110

$$\int \frac{1}{(a-a\cosh(c+dx))^{5/2}}\,dx = -\frac{3\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a-a\cosh(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sinh(c+dx)}{4d(a-a\cosh(c+dx))^{5/2}} - \frac{3\sinh(c+dx)}{16ad(a-a\cosh(c+dx))^{3/2}}$$

output
```
-1/4*sinh(d*x+c)/d/(a-a*cosh(d*x+c))^(5/2)-3/16*sinh(d*x+c)/a/d/(a-a*cosh(
d*x+c))^(3/2)-3/32*arctan(1/2*sinh(d*x+c)*a^(1/2)*2^(1/2)/(a-a*cosh(d*x+c)
)^(1/2))/a^(5/2)/d*2^(1/2)
```

3.53.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.17

$$\int \frac{1}{(a-a\cosh(c+dx))^{5/2}}\,dx = \frac{\left(6\operatorname{csch}^2\left(\frac{1}{4}(c+dx)\right) - \operatorname{csch}^4\left(\frac{1}{4}(c+dx)\right) + 24\left(-\log\left(\cosh\left(\frac{1}{4}(c+dx)\right)\right) + \text{lo}\right.\right.}{32a^2d(-1+\cosh(c+}$$

input
```
Integrate[(a - a*Cosh[c + d*x])^(-5/2),x]
```

output
```
((6*Csch[(c + d*x)/4]^2 - Csch[(c + d*x)/4]^4 + 24*(-Log[Cosh[(c + d*x)/4]
] + Log[Sinh[(c + d*x)/4]]) + 6*Sech[(c + d*x)/4]^2 + Sech[(c + d*x)/4]^4)
*Sinh[(c + d*x)/2]^5)/(32*a^2*d*(-1 + Cosh[c + d*x])^2*Sqrt[a - a*Cosh[c +
 d*x]])
```

3.53.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3042, 3129, 3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-a\cosh(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a-a\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{3\int \frac{1}{(a-a\cosh(c+dx))^{3/2}}dx}{8a}-\frac{\sinh(c+dx)}{4d(a-a\cosh(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(c+dx)}{4d(a-a\cosh(c+dx))^{5/2}}+\frac{3\int \frac{1}{\left(a-a\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{8a}$$

$$\Big\downarrow \text{3129}$$

$$\frac{3\left(\frac{\int \frac{1}{\sqrt{a-a\cosh(c+dx)}}dx}{4a}-\frac{\sinh(c+dx)}{2d(a-a\cosh(c+dx))^{3/2}}\right)}{8a}-\frac{\sinh(c+dx)}{4d(a-a\cosh(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(c+dx)}{4d(a-a\cosh(c+dx))^{5/2}}+\frac{3\left(-\frac{\sinh(c+dx)}{2d(a-a\cosh(c+dx))^{3/2}}+\frac{\int \frac{1}{\sqrt{a-a\sin\left(ic+idx+\frac{\pi}{2}\right)}}dx}{4a}\right)}{8a}$$

$$\Big\downarrow \text{3128}$$

$$-\frac{\sinh(c+dx)}{4d(a-a\cosh(c+dx))^{5/2}}+\frac{3\left(-\frac{\sinh(c+dx)}{2d(a-a\cosh(c+dx))^{3/2}}+\frac{i\int \frac{1}{\frac{a^2\sinh^2(c+dx)}{a-a\cosh(c+dx)}+2a}d\frac{ia\sinh(c+dx)}{\sqrt{a-a\cosh(c+dx)}}}{2ad}\right)}{8a}$$

$$\Big\downarrow \text{219}$$

$$\frac{3\left(-\frac{\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a-a\cosh(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sinh(c+dx)}{2d(a-a\cosh(c+dx))^{3/2}}\right)}{8a}-\frac{\sinh(c+dx)}{4d(a-a\cosh(c+dx))^{5/2}}$$

input
```
Int[(a - a*Cosh[c + d*x])^(-5/2),x]
```

output
```
-1/4*Sinh[c + d*x]/(d*(a - a*Cosh[c + d*x])^(5/2)) + (3*(-1/2*ArcTan[(Sqrt
[a]*Sinh[c + d*x])/(Sqrt[2]*Sqrt[a - a*Cosh[c + d*x]])]/(Sqrt[2]*a^(3/2)*d
) - Sinh[c + d*x]/(2*d*(a - a*Cosh[c + d*x])^(3/2))))/(8*a)
```

3.53.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.53.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.25

method	result	size
default	$\frac{6\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-4\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)+\left(3\ln\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-3\ln\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\right)\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{32a^{2}\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a}\,d}$	137

input
```
int(1/(a-a*cosh(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/a^2*(6*cosh(1/2*d*x+1/2*c)*sinh(1/2*d*x+1/2*c)^2-4*cosh(1/2*d*x+1/2*c
)+(3*ln(cosh(1/2*d*x+1/2*c)-1)-3*ln(cosh(1/2*d*x+1/2*c)+1))*sinh(1/2*d*x+1
/2*c)^4)/(cosh(1/2*d*x+1/2*c)+1)/(cosh(1/2*d*x+1/2*c)-1)/sinh(1/2*d*x+1/2*
c)/(-2*sinh(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

3.53.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. 2(91) = 182.

Time = 0.25 (sec) , antiderivative size = 580, normalized size of antiderivative = 5.27

$$\int \frac{1}{(a-a\cosh(c+dx))^{5/2}}\,dx =$$

$$-\frac{3\sqrt{2}\left(\cosh\left(dx+c\right)^{4}+4\left(\cosh\left(dx+c\right)-1\right)\sinh\left(dx+c\right)^{3}+\sinh\left(dx+c\right)^{4}-4\cosh\left(dx+c\right)^{3}+6\left(\cosh\left(\right.\right.}{}$$

input
```
integrate(1/(a-a*cosh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1/32*(3*sqrt(2)*(cosh(d*x + c)^4 + 4*(cosh(d*x + c) - 1)*sinh(d*x + c)^3
+ sinh(d*x + c)^4 - 4*cosh(d*x + c)^3 + 6*(cosh(d*x + c)^2 - 2*cosh(d*x +
c) + 1)*sinh(d*x + c)^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - 3*cosh(
d*x + c)^2 + 3*cosh(d*x + c) - 1)*sinh(d*x + c) - 4*cosh(d*x + c) + 1)*sqr
t(-a)*log(-(2*sqrt(2)*sqrt(1/2)*sqrt(-a)*sqrt(-a/(cosh(d*x + c) + sinh(d*x
 + c)))*(cosh(d*x + c) + sinh(d*x + c)) + a*cosh(d*x + c) + a*sinh(d*x + c
) + a)/(cosh(d*x + c) + sinh(d*x + c) - 1)) + 4*sqrt(1/2)*(3*cosh(d*x + c)
^4 + (12*cosh(d*x + c) - 11)*sinh(d*x + c)^3 + 3*sinh(d*x + c)^4 - 11*cosh
(d*x + c)^3 + (18*cosh(d*x + c)^2 - 33*cosh(d*x + c) - 11)*sinh(d*x + c)^2
 - 11*cosh(d*x + c)^2 + (12*cosh(d*x + c)^3 - 33*cosh(d*x + c)^2 - 22*cosh
(d*x + c) + 3)*sinh(d*x + c) + 3*cosh(d*x + c))*sqrt(-a/(cosh(d*x + c) + s
inh(d*x + c))))/(a^3*d*cosh(d*x + c)^4 + a^3*d*sinh(d*x + c)^4 - 4*a^3*d*c
osh(d*x + c)^3 + 6*a^3*d*cosh(d*x + c)^2 - 4*a^3*d*cosh(d*x + c) + a^3*d +
 4*(a^3*d*cosh(d*x + c) - a^3*d)*sinh(d*x + c)^3 + 6*(a^3*d*cosh(d*x + c)^
2 - 2*a^3*d*cosh(d*x + c) + a^3*d)*sinh(d*x + c)^2 + 4*(a^3*d*cosh(d*x + c
)^3 - 3*a^3*d*cosh(d*x + c)^2 + 3*a^3*d*cosh(d*x + c) - a^3*d)*sinh(d*x +
c))
```

3.53.6 Sympy [F]

$$\int \frac{1}{(a-a\cosh(c+dx))^{5/2}}\,dx=\int \frac{1}{(-a\cosh{(c+dx)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a-a*cosh(d*x+c))**(5/2),x)
```

output
```
Integral((-a*cosh(c + d*x) + a)**(-5/2), x)
```

3.53.7 Maxima [F]

$$\int \frac{1}{(a-a\cosh(c+dx))^{5/2}}\,dx=\int \frac{1}{(-a\cosh{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a-a*cosh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((-a*cosh(d*x + c) + a)^(-5/2), x)
```

3.53.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.61

$$\int \frac{1}{(a-a\cosh(c+dx))^{5/2}}\,dx = -\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{-ae^{(dx+c)}}}{\sqrt{a}}\right)}{16\,a^{\frac{5}{2}}d\mathrm{sgn}\left(-e^{(dx+c)}+1\right)}$$
$$+\frac{3\sqrt{2}\sqrt{-ae^{(dx+c)}}a^3e^{(3\,dx+3\,c)}-11\sqrt{2}\sqrt{-ae^{(dx+c)}}a^3e^{(2\,dx+2\,c)}-11\sqrt{2}\sqrt{-ae^{(dx+c)}}a^3e^{(dx+c)}+3\sqrt{2}\sqrt{-ae^{(dx+c}}}{16\left(ae^{(dx+c)}-a\right)^4a^2d\mathrm{sgn}\left(-e^{(dx+c)}+1\right)}$$

input
```
integrate(1/(a-a*cosh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
-3/16*sqrt(2)*arctan(sqrt(-a*e^(d*x + c))/sqrt(a))/(a^(5/2)*d*sgn(-e^(d*x
+ c) + 1)) + 1/16*(3*sqrt(2)*sqrt(-a*e^(d*x + c))*a^3*e^(3*d*x + 3*c) - 11
*sqrt(2)*sqrt(-a*e^(d*x + c))*a^3*e^(2*d*x + 2*c) - 11*sqrt(2)*sqrt(-a*e^(
d*x + c))*a^3*e^(d*x + c) + 3*sqrt(2)*sqrt(-a*e^(d*x + c))*a^3)/((a*e^(d*x
 + c) - a)^4*a^2*d*sgn(-e^(d*x + c) + 1))
```

3.53.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-a\cosh(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a-a\cosh\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(1/(a - a*cosh(c + d*x))^(5/2),x)
```

output
```
int(1/(a - a*cosh(c + d*x))^(5/2), x)
```

3.54 $\int \frac{\cosh^4(x)}{a+b\cosh(x)}\,dx$

3.54.1 Optimal result

Integrand size = 13, antiderivative size = 112

$$\int \frac{\cosh^4(x)}{a+b\cosh(x)}\,dx = -\frac{a(2a^2+b^2)\,x}{2b^4}+\frac{2a^4\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^4\sqrt{a+b}}+\frac{(3a^2+2b^2)\sinh(x)}{3b^3}-\frac{a\cosh(x)\sinh(x)}{2b^2}+\frac{\cosh^2(x)\sinh(x)}{3b}$$

output
```
-1/2*a*(2*a^2+b^2)*x/b^4+1/3*(3*a^2+2*b^2)*sinh(x)/b^3-1/2*a*cosh(x)*sinh(
x)/b^2+1/3*cosh(x)^2*sinh(x)/b+2*a^4*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)
^(1/2))/b^4/(a-b)^(1/2)/(a+b)^(1/2)
```

3.54.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{\cosh^4(x)}{a+b\cosh(x)}\,dx = \frac{-6a(2a^2+b^2)\,x-\frac{24a^4\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+3b(4a^2+3b^2)\sinh(x)-3ab^2\sinh(2x)+b^3\sinh(3x)}{12b^4}$$

input
```
Integrate[Cosh[x]^4/(a + b*Cosh[x]),x]
```

output
```
(-6*a*(2*a^2 + b^2)*x - (24*a^4*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2
]])/Sqrt[-a^2 + b^2] + 3*b*(4*a^2 + 3*b^2)*Sinh[x] - 3*a*b^2*Sinh[2*x] + b
^3*Sinh[3*x])/(12*b^4)
```

3.54.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.15, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 3272, 3042, 3528, 25, 3042, 3502, 27, 3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^4(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)^4}{a+b\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3272}$$

$$\frac{\int \frac{\cosh(x)\left(-3a\cosh^2(x)+2b\cosh(x)+2a\right)}{a+b\cosh(x)}dx}{3b}+\frac{\sinh(x)\cosh^2(x)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(x)\cosh^2(x)}{3b}+\frac{\int \frac{\sin\left(ix+\frac{\pi}{2}\right)\left(-3a\sin\left(ix+\frac{\pi}{2}\right)^2+2b\sin\left(ix+\frac{\pi}{2}\right)+2a\right)}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{3b}$$

$$\Big\downarrow \text{3528}$$

$$\frac{\frac{\int -\frac{3a^2-b\cosh(x)a-2\left(3a^2+2b^2\right)\cosh^2(x)}{a+b\cosh(x)}dx}{2b}-\frac{3a\sinh(x)\cosh(x)}{2b}}{3b}+\frac{\sinh(x)\cosh^2(x)}{3b}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{\int \frac{3a^2-b\cosh(x)a-2\left(3a^2+2b^2\right)\cosh^2(x)}{a+b\cosh(x)}dx}{2b}-\frac{3a\sinh(x)\cosh(x)}{2b}}{3b}+\frac{\sinh(x)\cosh^2(x)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(x)\cosh^2(x)}{3b}+\frac{-\frac{3a\sinh(x)\cosh(x)}{2b}-\frac{\int\frac{3a^2-b\sin\left(ix+\frac{\pi}{2}\right)a-2\left(3a^2+2b^2\right)\sin\left(ix+\frac{\pi}{2}\right)^2}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{2b}}{3b}$$

$$\Big\downarrow \text{ 3502}$$

$$-\frac{\frac{\frac{\int\frac{3\left(ba^2+\left(2a^2+b^2\right)\cosh(x)a\right)}{a+b\cosh(x)}dx}{b}-\frac{2\left(3a^2+2b^2\right)\sinh(x)}{b}}{2b}-\frac{3a\sinh(x)\cosh(x)}{2b}}{3b}+\frac{\sinh(x)\cosh^2(x)}{3b}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\frac{\frac{3\int\frac{ba^2+\left(2a^2+b^2\right)\cosh(x)a}{a+b\cosh(x)}dx}{b}-\frac{2\left(3a^2+2b^2\right)\sinh(x)}{b}}{2b}-\frac{3a\sinh(x)\cosh(x)}{2b}}{3b}+\frac{\sinh(x)\cosh^2(x)}{3b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sinh(x)\cosh^2(x)}{3b}+\frac{-\frac{3a\sinh(x)\cosh(x)}{2b}-\frac{-\frac{2\left(3a^2+2b^2\right)\sinh(x)}{b}+\frac{3\int\frac{ba^2+\left(2a^2+b^2\right)\sin\left(ix+\frac{\pi}{2}\right)a}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}}{2b}}{3b}$$

$$\Big\downarrow \text{ 3214}$$

$$-\frac{\frac{\frac{3\left(\frac{ax\left(2a^2+b^2\right)}{b}-\frac{2a^4\int\frac{1}{a+b\cosh(x)}dx}{b}\right)}{b}-\frac{2\left(3a^2+2b^2\right)\sinh(x)}{b}}{2b}-\frac{3a\sinh(x)\cosh(x)}{2b}}{3b}+\frac{\sinh(x)\cosh^2(x)}{3b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sinh(x)\cosh^2(x)}{3b}+\frac{-\frac{3a\sinh(x)\cosh(x)}{2b}-\frac{-\frac{2\left(3a^2+2b^2\right)\sinh(x)}{b}+\frac{3\left(\frac{ax\left(2a^2+b^2\right)}{b}-\frac{2a^4\int\frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}\right)}{b}}{2b}}{3b}$$

$$\Big\downarrow \text{ 3138}$$

$$-\frac{\frac{\frac{3\left(\frac{ax\left(2a^2+b^2\right)}{b}-\frac{4a^4\int\frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{b}\right)}{b}-\frac{2\left(3a^2+2b^2\right)\sinh(x)}{b}}{2b}-\frac{3a\sinh(x)\cosh(x)}{2b}}{3b}+\frac{\sinh(x)\cosh^2(x)}{3b}$$

$$\Big\downarrow \text{ 221}$$

3.54. $\int\frac{\cosh^4(x)}{a+b\cosh(x)}\,dx$

$$-\frac{\frac{3\left(\frac{ax(2a^2+b^2)}{b}-\frac{4a^4\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}\right)}{b}-\frac{2(3a^2+2b^2)\sinh(x)}{b}}{2b}-\frac{3a\sinh(x)\cosh(x)}{2b}}{3b}+\frac{\sinh(x)\cosh^2(x)}{3b}$$

input
```
Int[Cosh[x]^4/(a + b*Cosh[x]),x]
```

output
```
(Cosh[x]^2*Sinh[x])/(3*b) + ((-3*a*Cosh[x]*Sinh[x])/(2*b) - ((3*((a*(2*a^2
 + b^2)*x)/b - (4*a^4*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[
a - b]*b*Sqrt[a + b])))/b - (2*(3*a^2 + 2*b^2)*Sinh[x])/b)/(2*b))/(3*b)
```

3.54.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3272
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

3.54. $\int \frac{\cosh^4(x)}{a+b\cosh(x)}\,dx$

3.54.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(94) = 188$.

Time = 0.31 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.81

method	result
default	$\frac{2a^4 \operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^4\sqrt{(a+b)(a-b)}} - \frac{1}{3b\left(\tanh\left(\frac{x}{2}\right)-1\right)^3} - \frac{a+b}{2b^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^2} - \frac{2a^2+ab+2b^2}{2b^3\left(\tanh\left(\frac{x}{2}\right)-1\right)} + \frac{a(2a^2+b^2)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b^4} -$
risch	$-\frac{a^3x}{b^4} - \frac{ax}{2b^2} + \frac{e^{3x}}{24b} - \frac{a\,e^{2x}}{8b^2} + \frac{e^x a^2}{2b^3} + \frac{3\,e^x}{8b} - \frac{e^{-x}a^2}{2b^3} - \frac{3\,e^{-x}}{8b} + \frac{a\,e^{-2x}}{8b^2} - \frac{e^{-3x}}{24b} + \frac{a^4\ln\left(e^x + \frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,b^4} -$

input
```
int(cosh(x)^4/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
2*a^4/b^4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2
))-1/3/b/(tanh(1/2*x)-1)^3-1/2*(a+b)/b^2/(tanh(1/2*x)-1)^2-1/2*(2*a^2+a*b+
2*b^2)/b^3/(tanh(1/2*x)-1)+1/2*a*(2*a^2+b^2)/b^4*ln(tanh(1/2*x)-1)-1/3/b/(
tanh(1/2*x)+1)^3-1/2*(-a-b)/b^2/(tanh(1/2*x)+1)^2-1/2*(2*a^2+a*b+2*b^2)/b^
3/(tanh(1/2*x)+1)-1/2*a*(2*a^2+b^2)/b^4*ln(tanh(1/2*x)+1)
```

3.54.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 778 vs. $2(94) = 188$.

Time = 0.29 (sec) , antiderivative size = 1625, normalized size of antiderivative = 14.51

$$\int \frac{\cosh^4(x)}{a + b\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)^4/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[1/24*((a^2*b^3 - b^5)*cosh(x)^6 + (a^2*b^3 - b^5)*sinh(x)^6 - 3*(a^3*b^2
- a*b^4)*cosh(x)^5 - 3*(a^3*b^2 - a*b^4 - 2*(a^2*b^3 - b^5)*cosh(x))*sinh(
x)^5 - a^2*b^3 + b^5 - 12*(2*a^5 - a^3*b^2 - a*b^4)*x*cosh(x)^3 + 3*(4*a^4
*b - a^2*b^3 - 3*b^5)*cosh(x)^4 + 3*(4*a^4*b - a^2*b^3 - 3*b^5 + 5*(a^2*b^
3 - b^5)*cosh(x)^2 - 5*(a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^4 + 2*(10*(a^2*b
^3 - b^5)*cosh(x)^3 - 15*(a^3*b^2 - a*b^4)*cosh(x)^2 - 6*(2*a^5 - a^3*b^2
- a*b^4)*x + 6*(4*a^4*b - a^2*b^3 - 3*b^5)*cosh(x))*sinh(x)^3 - 3*(4*a^4*b
 - a^2*b^3 - 3*b^5)*cosh(x)^2 - 3*(4*a^4*b - a^2*b^3 - 3*b^5 - 5*(a^2*b^3
- b^5)*cosh(x)^4 + 10*(a^3*b^2 - a*b^4)*cosh(x)^3 + 12*(2*a^5 - a^3*b^2 -
a*b^4)*x*cosh(x) - 6*(4*a^4*b - a^2*b^3 - 3*b^5)*cosh(x)^2)*sinh(x)^2 + 24
*(a^4*cosh(x)^3 + 3*a^4*cosh(x)^2*sinh(x) + 3*a^4*cosh(x)*sinh(x)^2 + a^4*
sinh(x)^3)*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh
(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*c
osh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*c
osh(x) + a)*sinh(x) + b)) + 3*(a^3*b^2 - a*b^4)*cosh(x) + 3*(2*(a^2*b^3 -
b^5)*cosh(x)^5 + a^3*b^2 - a*b^4 - 5*(a^3*b^2 - a*b^4)*cosh(x)^4 - 12*(2*a
^5 - a^3*b^2 - a*b^4)*x*cosh(x)^2 + 4*(4*a^4*b - a^2*b^3 - 3*b^5)*cosh(x)^
3 - 2*(4*a^4*b - a^2*b^3 - 3*b^5)*cosh(x))*sinh(x))/((a^2*b^4 - b^6)*cosh(
x)^3 + 3*(a^2*b^4 - b^6)*cosh(x)^2*sinh(x) + 3*(a^2*b^4 - b^6)*cosh(x)*sin
h(x)^2 + (a^2*b^4 - b^6)*sinh(x)^3), 1/24*((a^2*b^3 - b^5)*cosh(x)^6 + ...
```

3.54.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^4(x)}{a+b\cosh(x)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)**4/(a+b*cosh(x)),x)
```

output
```
Timed out
```

3.54.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^4(x)}{a+b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(x)^4/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.54.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.19

$$\int \frac{\cosh^4(x)}{a+b\cosh(x)}\,dx = \frac{2\,a^4\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}b^4} + \frac{b^2e^{(3\,x)} - 3\,abe^{(2\,x)} + 12\,a^2e^x + 9\,b^2e^x}{24\,b^3} - \frac{(2\,a^3+ab^2)x}{2\,b^4} + \frac{\left(3\,ab^2e^x - b^3 - 3\,(4\,a^2b+3\,b^3)e^{(2\,x)}\right)e^{(-3\,x)}}{24\,b^4}$$

input
```
integrate(cosh(x)^4/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
2*a^4*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b^4) + 1/24*(
b^2*e^(3*x) - 3*a*b*e^(2*x) + 12*a^2*e^x + 9*b^2*e^x)/b^3 - 1/2*(2*a^3 + a
*b^2)*x/b^4 + 1/24*(3*a*b^2*e^x - b^3 - 3*(4*a^2*b + 3*b^3)*e^(2*x))*e^(-3
*x)/b^4
```

3.54.9 Mupad [B] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.87

$$\int \frac{\cosh^4(x)}{a+b\cosh(x)}\,dx = \frac{e^{3x}}{24\,b}-\frac{e^{-3x}}{24\,b}-\frac{x\,(2\,a^3+a\,b^2)}{2\,b^4}+\frac{e^{x}\,(4\,a^2+3\,b^2)}{8\,b^3} + \frac{a\,e^{-2x}}{8\,b^2}-\frac{a\,e^{2x}}{8\,b^2}-\frac{e^{-x}\,(4\,a^2+3\,b^2)}{8\,b^3} + \frac{a^4\,\ln\left(-\frac{2\,a^4\,e^{x}}{b^5}-\frac{2\,a^4\,(b+a\,e^{x})}{b^5\,\sqrt{a+b}\,\sqrt{a-b}}\right)}{b^4\,\sqrt{a+b}\,\sqrt{a-b}}-\frac{a^4\,\ln\left(\frac{2\,a^4\,(b+a\,e^{x})}{b^5\,\sqrt{a+b}\,\sqrt{a-b}}-\frac{2\,a^4\,e^{x}}{b^5}\right)}{b^4\,\sqrt{a+b}\,\sqrt{a-b}}$$

input
```
int(cosh(x)^4/(a + b*cosh(x)),x)
```

output
```
exp(3*x)/(24*b) - exp(-3*x)/(24*b) - (x*(a*b^2 + 2*a^3))/(2*b^4) + (exp(x)
*(4*a^2 + 3*b^2))/(8*b^3) + (a*exp(-2*x))/(8*b^2) - (a*exp(2*x))/(8*b^2) -
 (exp(-x)*(4*a^2 + 3*b^2))/(8*b^3) + (a^4*log(- (2*a^4*exp(x))/b^5 - (2*a^
4*(b + a*exp(x)))/(b^5*(a + b)^(1/2)*(a - b)^(1/2))))/(b^4*(a + b)^(1/2)*(
a - b)^(1/2)) - (a^4*log((2*a^4*(b + a*exp(x)))/(b^5*(a + b)^(1/2)*(a - b)
^(1/2)) - (2*a^4*exp(x))/b^5))/(b^4*(a + b)^(1/2)*(a - b)^(1/2))
```

3.55 $\int \frac{\cosh^3(x)}{a+b\cosh(x)}\,dx$

3.55.1 Optimal result

Integrand size = 13, antiderivative size = 85

$$\int \frac{\cosh^3(x)}{a+b\cosh(x)}\,dx=\frac{(2a^2+b^2)\,x}{2b^3}-\frac{2a^3\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^3\sqrt{a+b}}-\frac{a\sinh(x)}{b^2}+\frac{\cosh(x)\sinh(x)}{2b}$$

output
```
1/2*(2*a^2+b^2)*x/b^3-a*sinh(x)/b^2+1/2*cosh(x)*sinh(x)/b-2*a^3*arctanh((a
-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/b^3/(a-b)^(1/2)/(a+b)^(1/2)
```

3.55.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int \frac{\cosh^3(x)}{a+b\cosh(x)}\,dx=\frac{4a^2x+2b^2x+\frac{8a^3\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}-4ab\sinh(x)+b^2\sinh(2x)}{4b^3}$$

input
```
Integrate[Cosh[x]^3/(a + b*Cosh[x]),x]
```

output
```
(4*a^2*x + 2*b^2*x + (8*a^3*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/
Sqrt[-a^2 + b^2] - 4*a*b*Sinh[x] + b^2*Sinh[2*x])/(4*b^3)
```

3.55.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {3042, 3272, 3042, 3502, 3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)^3}{a+b\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3272}$$

$$\frac{\int \frac{-2a\cosh^2(x)+b\cosh(x)+a}{a+b\cosh(x)}dx}{2b}+\frac{\sinh(x)\cosh(x)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(x)\cosh(x)}{2b}+\frac{\int \frac{-2a\sin\left(ix+\frac{\pi}{2}\right)^2+b\sin\left(ix+\frac{\pi}{2}\right)+a}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{2b}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\frac{\int \frac{ab+\left(2a^2+b^2\right)\cosh(x)}{a+b\cosh(x)}dx}{b}-\frac{2a\sinh(x)}{b}}{2b}+\frac{\sinh(x)\cosh(x)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(x)\cosh(x)}{2b}+\frac{-\frac{2a\sinh(x)}{b}+\frac{\int \frac{ab+\left(2a^2+b^2\right)\sin\left(ix+\frac{\pi}{2}\right)}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}}{2b}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\frac{\frac{x\left(2a^2+b^2\right)}{b}-\frac{2a^3\int \frac{1}{a+b\cosh(x)}dx}{b}}{b}-\frac{2a\sinh(x)}{b}}{2b}+\frac{\sinh(x)\cosh(x)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(x)\cosh(x)}{2b}+\frac{-\frac{2a\sinh(x)}{b}+\frac{\frac{x\left(2a^2+b^2\right)}{b}-\frac{2a^3\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}}{b}}{2b}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\frac{\frac{x(2a^2+b^2)}{b}-\frac{4a^3\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{b}}{b}-\frac{2a\sinh(x)}{b}}{2b}+\frac{\sinh(x)\cosh(x)}{2b}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\frac{x(2a^2+b^2)}{b}-\frac{4a^3\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}}{b}-\frac{2a\sinh(x)}{b}}{2b}+\frac{\sinh(x)\cosh(x)}{2b}$$

input
```
Int[Cosh[x]^3/(a + b*Cosh[x]),x]
```

output
```
(Cosh[x]*Sinh[x])/(2*b) + ((((2*a^2 + b^2)*x)/b - (4*a^3*ArcTanh[(Sqrt[a -
 b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]))/b - (2*a*Sinh[x]
)/b)/(2*b)
```

3.55.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3272
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.55.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(71) = 142$.

Time = 0.18 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.80

method	result
default	$-\frac{2a^3 \operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^3\sqrt{(a+b)(a-b)}}+\frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}-\frac{-2a-b}{2b^2\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{\left(-2a^2-b^2\right)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b^3}-\frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}$
risch	$\frac{x\,a^2}{b^3}+\frac{x}{2b}+\frac{e^{2x}}{8b}-\frac{a\,e^{x}}{2b^2}+\frac{a\,e^{-x}}{2b^2}-\frac{e^{-2x}}{8b}+\frac{a^3\ln\left(e^x+\frac{a\sqrt{a^2-b^2}+a^2-b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,b^3}-\frac{a^3\ln\left(e^x+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,b^3}$

input
```
int(cosh(x)^3/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-2*a^3/b^3/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/
2))+1/2/b/(tanh(1/2*x)-1)^2-1/2*(-2*a-b)/b^2/(tanh(1/2*x)-1)+1/2/b^3*(-2*a
^2-b^2)*ln(tanh(1/2*x)-1)-1/2/b/(tanh(1/2*x)+1)^2-1/2*(-2*a-b)/b^2/(tanh(1
/2*x)+1)+1/2*(2*a^2+b^2)/b^3*ln(tanh(1/2*x)+1)
```

3.55.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 417 vs. $2(71) = 142$.

Time = 0.28 (sec) , antiderivative size = 903, normalized size of antiderivative = 10.62

$$\int \frac{\cosh^3(x)}{a + b\cosh(x)}\, dx = \text{Too large to display}$$

input
```
integrate(cosh(x)^3/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[1/8*((a^2*b^2 - b^4)*cosh(x)^4 + (a^2*b^2 - b^4)*sinh(x)^4 - a^2*b^2 + b^
4 + 4*(2*a^4 - a^2*b^2 - b^4)*x*cosh(x)^2 - 4*(a^3*b - a*b^3)*cosh(x)^3 -
4*(a^3*b - a*b^3 - (a^2*b^2 - b^4)*cosh(x))*sinh(x)^3 + 2*(3*(a^2*b^2 - b^
4)*cosh(x)^2 + 2*(2*a^4 - a^2*b^2 - b^4)*x - 6*(a^3*b - a*b^3)*cosh(x))*si
nh(x)^2 + 8*(a^3*cosh(x)^2 + 2*a^3*cosh(x)*sinh(x) + a^3*sinh(x)^2)*sqrt(a
^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2
 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x
) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(
x) + b)) + 4*(a^3*b - a*b^3)*cosh(x) + 4*(a^3*b - a*b^3 + (a^2*b^2 - b^4)*
cosh(x)^3 + 2*(2*a^4 - a^2*b^2 - b^4)*x*cosh(x) - 3*(a^3*b - a*b^3)*cosh(x
)^2)*sinh(x))/((a^2*b^3 - b^5)*cosh(x)^2 + 2*(a^2*b^3 - b^5)*cosh(x)*sinh(
x) + (a^2*b^3 - b^5)*sinh(x)^2), 1/8*((a^2*b^2 - b^4)*cosh(x)^4 + (a^2*b^2
 - b^4)*sinh(x)^4 - a^2*b^2 + b^4 + 4*(2*a^4 - a^2*b^2 - b^4)*x*cosh(x)^2
- 4*(a^3*b - a*b^3)*cosh(x)^3 - 4*(a^3*b - a*b^3 - (a^2*b^2 - b^4)*cosh(x)
)*sinh(x)^3 + 2*(3*(a^2*b^2 - b^4)*cosh(x)^2 + 2*(2*a^4 - a^2*b^2 - b^4)*x
 - 6*(a^3*b - a*b^3)*cosh(x))*sinh(x)^2 + 16*(a^3*cosh(x)^2 + 2*a^3*cosh(x
)*sinh(x) + a^3*sinh(x)^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*co
sh(x) + b*sinh(x) + a)/(a^2 - b^2)) + 4*(a^3*b - a*b^3)*cosh(x) + 4*(a^3*b
 - a*b^3 + (a^2*b^2 - b^4)*cosh(x)^3 + 2*(2*a^4 - a^2*b^2 - b^4)*x*cosh(x)
 - 3*(a^3*b - a*b^3)*cosh(x)^2)*sinh(x))/((a^2*b^3 - b^5)*cosh(x)^2 + 2...
```

3.55.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4559 vs. $2(73) = 146$.

Time = 157.92 (sec) , antiderivative size = 4559, normalized size of antiderivative = 53.64

$$\int \frac{\cosh^3(x)}{a + b\cosh(x)}\, dx = \text{Too large to display}$$

input
```
integrate(cosh(x)**3/(a+b*cosh(x)),x)
```

output
```
Piecewise((zoo*(-x*sinh(x)**2/2 + x*cosh(x)**2/2 + sinh(x)*cosh(x)/2), Eq(
a, 0) & Eq(b, 0)), (3*x*tanh(x/2)**4/(2*b*tanh(x/2)**4 - 4*b*tanh(x/2)**2
+ 2*b) - 6*x*tanh(x/2)**2/(2*b*tanh(x/2)**4 - 4*b*tanh(x/2)**2 + 2*b) + 3*
x/(2*b*tanh(x/2)**4 - 4*b*tanh(x/2)**2 + 2*b) - 2*tanh(x/2)**5/(2*b*tanh(x
/2)**4 - 4*b*tanh(x/2)**2 + 2*b) + 10*tanh(x/2)**3/(2*b*tanh(x/2)**4 - 4*b
*tanh(x/2)**2 + 2*b) - 4*tanh(x/2)/(2*b*tanh(x/2)**4 - 4*b*tanh(x/2)**2 +
2*b), Eq(a, b)), (3*x*tanh(x/2)**5/(2*b*tanh(x/2)**5 - 4*b*tanh(x/2)**3 +
2*b*tanh(x/2)) - 6*x*tanh(x/2)**3/(2*b*tanh(x/2)**5 - 4*b*tanh(x/2)**3 + 2
*b*tanh(x/2)) + 3*x*tanh(x/2)/(2*b*tanh(x/2)**5 - 4*b*tanh(x/2)**3 + 2*b*t
anh(x/2)) - 4*tanh(x/2)**4/(2*b*tanh(x/2)**5 - 4*b*tanh(x/2)**3 + 2*b*tanh
(x/2)) + 10*tanh(x/2)**2/(2*b*tanh(x/2)**5 - 4*b*tanh(x/2)**3 + 2*b*tanh(x
/2)) - 2/(2*b*tanh(x/2)**5 - 4*b*tanh(x/2)**3 + 2*b*tanh(x/2)), Eq(a, -b))
, ((-2*sinh(x)**3/3 + sinh(x)*cosh(x)**2)/a, Eq(b, 0)), (2*a**3*x*sqrt(a/(
a - b) + b/(a - b))*tanh(x/2)**4/(2*a*b**3*sqrt(a/(a - b) + b/(a - b))*tan
h(x/2)**4 - 4*a*b**3*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 + 2*a*b**3*s
qrt(a/(a - b) + b/(a - b)) - 2*b**4*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)*
*4 + 4*b**4*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 - 2*b**4*sqrt(a/(a -
b) + b/(a - b))) - 4*a**3*x*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2/(2*a*
b**3*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**4 - 4*a*b**3*sqrt(a/(a - b) +
b/(a - b))*tanh(x/2)**2 + 2*a*b**3*sqrt(a/(a - b) + b/(a - b)) - 2*b**4...
```

3.55.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^3(x)}{a + b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(x)^3/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.55.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.08

$$\int \frac{\cosh^3(x)}{a+b\cosh(x)}\,dx = -\frac{2\,a^3\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}b^3} + \frac{be^{(2\,x)}-4\,ae^x}{8\,b^2} + \frac{(2\,a^2+b^2)x}{2\,b^3} + \frac{(4\,abe^x-b^2)e^{(-2\,x)}}{8\,b^3}$$

input
```
integrate(cosh(x)^3/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
-2*a^3*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b^3) + 1/8*(
b*e^(2*x) - 4*a*e^x)/b^2 + 1/2*(2*a^2 + b^2)*x/b^3 + 1/8*(4*a*b*e^x - b^2)
*e^(-2*x)/b^3
```

3.55.9 Mupad [B] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.96

$$\int \frac{\cosh^3(x)}{a+b\cosh(x)}\,dx = \frac{e^{2\,x}}{8\,b} - \frac{e^{-2\,x}}{8\,b} - \frac{a\,e^x}{2\,b^2} + \frac{a\,e^{-x}}{2\,b^2} + \frac{x\,(2\,a^2+b^2)}{2\,b^3} + \frac{a^3\,\ln\left(\frac{2\,a^3\,e^x}{b^4} - \frac{2\,a^3\,(b+a\,e^x)}{b^4\,\sqrt{a+b}\,\sqrt{a-b}}\right)}{b^3\,\sqrt{a+b}\,\sqrt{a-b}} - \frac{a^3\,\ln\left(\frac{2\,a^3\,e^x}{b^4} + \frac{2\,a^3\,(b+a\,e^x)}{b^4\,\sqrt{a+b}\,\sqrt{a-b}}\right)}{b^3\,\sqrt{a+b}\,\sqrt{a-b}}$$

input
```
int(cosh(x)^3/(a + b*cosh(x)),x)
```

output
```
exp(2*x)/(8*b) - exp(-2*x)/(8*b) - (a*exp(x))/(2*b^2) + (a*exp(-x))/(2*b^2
) + (x*(2*a^2 + b^2))/(2*b^3) + (a^3*log((2*a^3*exp(x))/b^4 - (2*a^3*(b +
a*exp(x)))/(b^4*(a + b)^(1/2)*(a - b)^(1/2))))/(b^3*(a + b)^(1/2)*(a - b)^
(1/2)) - (a^3*log((2*a^3*exp(x))/b^4 + (2*a^3*(b + a*exp(x)))/(b^4*(a + b)
^(1/2)*(a - b)^(1/2))))/(b^3*(a + b)^(1/2)*(a - b)^(1/2))
```

3.56 $\int \frac{\cosh^2(x)}{a+b\cosh(x)}\,dx$

3.56.1 Optimal result

Integrand size = 13, antiderivative size = 62

$$\int \frac{\cosh^2(x)}{a+b\cosh(x)}\,dx=-\frac{ax}{b^2}+\frac{2a^2\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^2\sqrt{a+b}}+\frac{\sinh(x)}{b}$$

output
```
-a*x/b^2+sinh(x)/b+2*a^2*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/b^2/
(a-b)^(1/2)/(a+b)^(1/2)
```

3.56.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int \frac{\cosh^2(x)}{a+b\cosh(x)}\,dx=\frac{a\left(-x-\frac{2a\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}\right)+b\sinh(x)}{b^2}$$

input
```
Integrate[Cosh[x]^2/(a + b*Cosh[x]),x]
```

output
```
(a*(-x - (2*a*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^
2]) + b*Sinh[x])/b^2
```

3.56.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {3042, 3225, 25, 27, 3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)^2}{a+b\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3225}$$

$$\frac{\int -\frac{a\cosh(x)}{a+b\cosh(x)}dx}{b}+\frac{\sinh(x)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh(x)}{b}-\frac{\int \frac{a\cosh(x)}{a+b\cosh(x)}dx}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sinh(x)}{b}-\frac{a\int \frac{\cosh(x)}{a+b\cosh(x)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(x)}{b}-\frac{a\int \frac{\sin\left(ix+\frac{\pi}{2}\right)}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\sinh(x)}{b}-\frac{a\left(\frac{x}{b}-\frac{a\int \frac{1}{a+b\cosh(x)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(x)}{b}-\frac{a\left(\frac{x}{b}-\frac{a\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\sinh(x)}{b}-\frac{a\left(\frac{x}{b}-\frac{2a\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 221$$

$$\frac{\sinh(x)}{b}-\frac{a\left(\frac{x}{b}-\frac{2a\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}\right)}{b}$$

input
```
Int[Cosh[x]^2/(a + b*Cosh[x]),x]
```

output
```
-((a*(x/b - (2*a*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b
]*b*Sqrt[a + b])))/b) + Sinh[x]/b
```

3.56.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3225
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Simp[1/d
Int[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.56.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.52

method	result	size
default	$\frac{2a^{2} \operatorname{arctanh}\left(\frac{(a-b) \tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^{2} \sqrt{(a+b)(a-b)}}-\frac{1}{b\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{a \ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b^{2}}-\frac{1}{b\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{a \ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^{2}}$	94
risch	$-\frac{a x}{b^{2}}+\frac{{\mathrm e}^{x}}{2 b}-\frac{{\mathrm e}^{-x}}{2 b}+\frac{a^{2} \ln\left({\mathrm e}^{x}+\frac{a \sqrt{a^{2}-b^{2}}-a^{2}+b^{2}}{b \sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\, b^{2}}-\frac{a^{2} \ln\left({\mathrm e}^{x}+\frac{a \sqrt{a^{2}-b^{2}}+a^{2}-b^{2}}{b \sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\, b^{2}}$	144

input
```
int(cosh(x)^2/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
2*a^2/b^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2
))-1/b/(tanh(1/2*x)-1)+a/b^2*ln(tanh(1/2*x)-1)-1/b/(tanh(1/2*x)+1)-a/b^2*l
n(tanh(1/2*x)+1)
```

3.56.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(52) = 104$.

Time = 0.26 (sec) , antiderivative size = 449, normalized size of antiderivative = 7.24

$$\int \frac{\cosh^2(x)}{a + b\cosh(x)}\, dx$$
$$= \left[-\frac{a^2b - b^3 + 2\,(a^3 - ab^2)x\cosh(x) - (a^2b - b^3)\cosh(x)^2 - (a^2b - b^3)\sinh(x)^2 - 2\,(a^2\cosh(x) + a^2\sin\right.$$
$$-\frac{a^2b - b^3 + 2\,(a^3 - ab^2)x\cosh(x) - (a^2b - b^3)\cosh(x)^2 - (a^2b - b^3)\sinh(x)^2 + 4\,(a^2\cosh(x) + a^2\sinh}{2\,((a^2b^2 - b^4)\cosh(x) -}$$

input
```
integrate(cosh(x)^2/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[-1/2*(a^2*b - b^3 + 2*(a^3 - a*b^2)*x*cosh(x) - (a^2*b - b^3)*cosh(x)^2 -
 (a^2*b - b^3)*sinh(x)^2 - 2*(a^2*cosh(x) + a^2*sinh(x))*sqrt(a^2 - b^2)*l
og((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*c
osh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*
cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)) +
2*((a^3 - a*b^2)*x - (a^2*b - b^3)*cosh(x))*sinh(x))/((a^2*b^2 - b^4)*cosh
(x) + (a^2*b^2 - b^4)*sinh(x)), -1/2*(a^2*b - b^3 + 2*(a^3 - a*b^2)*x*cosh
(x) - (a^2*b - b^3)*cosh(x)^2 - (a^2*b - b^3)*sinh(x)^2 + 4*(a^2*cosh(x) +
 a^2*sinh(x))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sin
h(x) + a)/(a^2 - b^2)) + 2*((a^3 - a*b^2)*x - (a^2*b - b^3)*cosh(x))*sinh(
x))/((a^2*b^2 - b^4)*cosh(x) + (a^2*b^2 - b^4)*sinh(x))]
```

3.56.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1275 vs. $2(53) = 106$.

Time = 51.49 (sec) , antiderivative size = 1275, normalized size of antiderivative = 20.56

$$\int \frac{\cosh^2(x)}{a + b\cosh(x)}\, dx = \text{Too large to display}$$

input
```
integrate(cosh(x)**2/(a+b*cosh(x)),x)
```

output
```
Piecewise((zoo*sinh(x), Eq(a, 0) & Eq(b, 0)), (-x*tanh(x/2)**2/(b*tanh(x/2
)**2 - b) + x/(b*tanh(x/2)**2 - b) + tanh(x/2)**3/(b*tanh(x/2)**2 - b) - 3
*tanh(x/2)/(b*tanh(x/2)**2 - b), Eq(a, b)), (x*tanh(x/2)**3/(b*tanh(x/2)**
3 - b*tanh(x/2)) - x*tanh(x/2)/(b*tanh(x/2)**3 - b*tanh(x/2)) - 3*tanh(x/2
)**2/(b*tanh(x/2)**3 - b*tanh(x/2)) + 1/(b*tanh(x/2)**3 - b*tanh(x/2)), Eq
(a, -b)), ((-x*sinh(x)**2/2 + x*cosh(x)**2/2 + sinh(x)*cosh(x)/2)/a, Eq(b,
 0)), (-a**2*x*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2/(a*b**2*sqrt(a/(a
- b) + b/(a - b))*tanh(x/2)**2 - a*b**2*sqrt(a/(a - b) + b/(a - b)) - b**3
*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 + b**3*sqrt(a/(a - b) + b/(a - b
))) + a**2*x*sqrt(a/(a - b) + b/(a - b))/(a*b**2*sqrt(a/(a - b) + b/(a - b
))*tanh(x/2)**2 - a*b**2*sqrt(a/(a - b) + b/(a - b)) - b**3*sqrt(a/(a - b)
 + b/(a - b))*tanh(x/2)**2 + b**3*sqrt(a/(a - b) + b/(a - b))) - a**2*log(
-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))*tanh(x/2)**2/(a*b**2*sqrt(a/(a -
 b) + b/(a - b))*tanh(x/2)**2 - a*b**2*sqrt(a/(a - b) + b/(a - b)) - b**3*
sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 + b**3*sqrt(a/(a - b) + b/(a - b)
)) + a**2*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b**2*sqrt(a/(a
- b) + b/(a - b))*tanh(x/2)**2 - a*b**2*sqrt(a/(a - b) + b/(a - b)) - b**3
*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 + b**3*sqrt(a/(a - b) + b/(a - b
))) + a**2*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))*tanh(x/2)**2/(a*b*
*2*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 - a*b**2*sqrt(a/(a - b) + b...
```

3.56.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^2(x)}{a + b\cosh(x)}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(x)^2/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.56.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^2(x)}{a+b\cosh(x)}\,dx = \frac{2\,a^2\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}b^2} - \frac{ax}{b^2} - \frac{e^{(-x)}}{2\,b} + \frac{e^x}{2\,b}$$

input
```
integrate(cosh(x)^2/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
2*a^2*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b^2) - a*x/b^
2 - 1/2*e^(-x)/b + 1/2*e^x/b
```

3.56.9 Mupad [B] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.24

$$\int \frac{\cosh^2(x)}{a+b\cosh(x)}\,dx = \frac{e^x}{2\,b} - \frac{e^{-x}}{2\,b} - \frac{a\,x}{b^2} + \frac{a^2\,\ln\left(-\frac{2\,a^2\,e^x}{b^3} - \frac{2\,a^2\,(b+a\,e^x)}{b^3\,\sqrt{a+b}\,\sqrt{a-b}}\right)}{b^2\,\sqrt{a+b}\,\sqrt{a-b}} - \frac{a^2\,\ln\left(\frac{2\,a^2\,(b+a\,e^x)}{b^3\,\sqrt{a+b}\,\sqrt{a-b}} - \frac{2\,a^2\,e^x}{b^3}\right)}{b^2\,\sqrt{a+b}\,\sqrt{a-b}}$$

input
```
int(cosh(x)^2/(a + b*cosh(x)),x)
```

output
```
exp(x)/(2*b) - exp(-x)/(2*b) - (a*x)/b^2 + (a^2*log(- (2*a^2*exp(x))/b^3 -
 (2*a^2*(b + a*exp(x)))/(b^3*(a + b)^(1/2)*(a - b)^(1/2))))/(b^2*(a + b)^(
1/2)*(a - b)^(1/2)) - (a^2*log((2*a^2*(b + a*exp(x)))/(b^3*(a + b)^(1/2)*(
a - b)^(1/2)) - (2*a^2*exp(x))/b^3))/(b^2*(a + b)^(1/2)*(a - b)^(1/2))
```

3.57 $\int \frac{\cosh(x)}{a+b\cosh(x)}\,dx$

3.57.1 Optimal result

Integrand size = 11, antiderivative size = 52

$$\int \frac{\cosh(x)}{a+b\cosh(x)}\,dx = \frac{x}{b} - \frac{2a\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b\sqrt{a+b}}$$

output
```
x/b-2*a*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/b/(a-b)^(1/2)/(a+b)^(
1/2)
```

3.57.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

$$\int \frac{\cosh(x)}{a+b\cosh(x)}\,dx = \frac{x + \frac{2a\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}}{b}$$

input
```
Integrate[Cosh[x]/(a + b*Cosh[x]),x]
```

output
```
(x + (2*a*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2])/
b
```

3.57.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)}{a+b\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3214}$$

$$\frac{x}{b}-\frac{a\int \frac{1}{a+b\cosh(x)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x}{b}-\frac{a\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{3138}$$

$$\frac{x}{b}-\frac{2a\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{b}$$

$$\Big\downarrow \text{221}$$

$$\frac{x}{b}-\frac{2a\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}$$

input
```
Int[Cosh[x]/(a + b*Cosh[x]),x]
```

output
```
x/b - (2*a*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sq
rt[a + b])
```

3.57.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.57.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.23

method	result	size
default	$-\frac{2a\,\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b\sqrt{(a+b)(a-b)}}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b}$	64
risch	$\frac{x}{b}+\frac{a\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}-b^{2}}+a^{2}-b^{2}}{b\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,b}-\frac{a\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}-b^{2}}-a^{2}+b^{2}}{b\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,b}$	122

input
```
int(cosh(x)/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-2*a/b/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))-
1/b*ln(tanh(1/2*x)-1)+1/b*ln(tanh(1/2*x)+1)
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 218, normalized size of antiderivative = 4.19

$$\int \frac{\cosh(x)}{a+b\cosh(x)}\,dx$$
$$= \left[\frac{\sqrt{a^2-b^2}a\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2ab\cosh(x)+2a^2-b^2+2(b^2\cosh(x)+ab)\sinh(x)+2\sqrt{a^2-b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2a\cosh(x)+2(b\cosh(x)+a)\sinh(x)+b}\right)+(a}{a^2b-b^3}\right.$$

input
```
integrate(cosh(x)/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[(sqrt(a^2 - b^2)*a*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2
*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*cosh(x)
+ b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x)
+ a)*sinh(x) + b)) + (a^2 - b^2)*x)/(a^2*b - b^3), (2*sqrt(-a^2 + b^2)*a*a
rctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) + (a^2 -
b^2)*x)/(a^2*b - b^3)]
```

3.57.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(41) = 82$.

Time = 12.26 (sec) , antiderivative size = 241, normalized size of antiderivative = 4.63

$$\int \frac{\cosh(x)}{a+b\cosh(x)}\,dx$$
$$= \begin{cases} \tilde{\infty}x \\ \frac{x}{b}-\frac{\tanh\left(\frac{x}{2}\right)}{b} \\ \frac{x}{b}-\frac{1}{b\tanh\left(\frac{x}{2}\right)} \\ \frac{\sinh(x)}{a} \\ \frac{ax\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}+\frac{a\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}-\frac{a\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}-\frac{bx\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-}}} \end{cases}$$

input
```
integrate(cosh(x)/(a+b*cosh(x)),x)
```

output
```
Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x/b - tanh(x/2)/b, Eq(a, b)), (x/
b - 1/(b*tanh(x/2)), Eq(a, -b)), (sinh(x)/a, Eq(b, 0)), (a*x*sqrt(a/(a - b
) + b/(a - b))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/
(a - b))) + a*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a
 - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) - a*log(sqrt(a/(a -
 b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt
(a/(a - b) + b/(a - b))) - b*x*sqrt(a/(a - b) + b/(a - b))/(a*b*sqrt(a/(a
- b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))), True))
```

3.57.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh(x)}{a+b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(x)/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.57.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int \frac{\cosh(x)}{a+b\cosh(x)}\,dx = -\frac{2\,a\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}b}+\frac{x}{b}$$

input
```
integrate(cosh(x)/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
-2*a*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b) + x/b
```

3.57.9 Mupad [B] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.10

$$\int \frac{\cosh(x)}{a+b\cosh(x)}\,dx = \frac{x}{b}+\frac{a\,\ln\left(\frac{2\,a\,e^x}{b^2}-\frac{2\,a\,(b+a\,e^x)}{b^2\,\sqrt{a+b}\,\sqrt{a-b}}\right)}{b\,\sqrt{a+b}\,\sqrt{a-b}}-\frac{a\,\ln\left(\frac{2\,a\,e^x}{b^2}+\frac{2\,a\,(b+a\,e^x)}{b^2\,\sqrt{a+b}\,\sqrt{a-b}}\right)}{b\,\sqrt{a+b}\,\sqrt{a-b}}$$

input
```
int(cosh(x)/(a + b*cosh(x)),x)
```

output
```
x/b + (a*log((2*a*exp(x))/b^2 - (2*a*(b + a*exp(x)))/(b^2*(a + b)^(1/2)*(a
 - b)^(1/2))))/(b*(a + b)^(1/2)*(a - b)^(1/2)) - (a*log((2*a*exp(x))/b^2 +
 (2*a*(b + a*exp(x)))/(b^2*(a + b)^(1/2)*(a - b)^(1/2))))/(b*(a + b)^(1/2)
*(a - b)^(1/2))
```

3.58 $\int \frac{\text{sech}(x)}{a+b\cosh(x)}\,dx$

3.58.1 Optimal result

Integrand size = 11, antiderivative size = 54

$$\int \frac{\text{sech}(x)}{a+b\cosh(x)}\,dx=\frac{\arctan(\sinh(x))}{a}-\frac{2b\,\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}$$

output
```
arctan(sinh(x))/a-2*b*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/a/(a-b)
^(1/2)/(a+b)^(1/2)
```

3.58.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{\text{sech}(x)}{a+b\cosh(x)}\,dx=\frac{2\left(\arctan\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{b\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}\right)}{a}$$

input
```
Integrate[Sech[x]/(a + b*Cosh[x]),x]
```

output
```
(2*(ArcTan[Tanh[x/2]] + (b*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/S
qrt[-a^2 + b^2]))/a
```

3.58.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 3226, 3042, 3138, 221, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(\frac{\pi}{2}+ix\right)\left(a+b\sin\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{3226}$$

$$\frac{\int \text{sech}(x)dx}{a}-\frac{b\int \frac{1}{a+b\cosh(x)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}-\frac{b\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a}$$

$$\Big\downarrow \text{3138}$$

$$-\frac{2b\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a}+\frac{\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \text{221}$$

$$-\frac{2b\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}+\frac{\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\arctan(\sinh(x))}{a}-\frac{2b\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}$$

input
```
Int[Sech[x]/(a + b*Cosh[x]),x]
```

output
```
ArcTan[Sinh[x]]/a - (2*b*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a*
Sqrt[a - b]*Sqrt[a + b])
```

3.58.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3226
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]),
 x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.58.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{2b \,\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)(a-b)}}+\frac{2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	51
risch	$\frac{b\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}-b^{2}}+a^{2}-b^{2}}{b\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,a}-\frac{b\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}-b^{2}}-a^{2}+b^{2}}{b\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,a}+\frac{i\ln(\mathrm{e}^{x}+i)}{a}-\frac{i\ln(\mathrm{e}^{x}-i)}{a}$	141

input
```
int(sech(x)/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-2*b/a/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))+
2/a*arctan(tanh(1/2*x))
```

3.58.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 227, normalized size of antiderivative = 4.20

$$\int \frac{\operatorname{sech}(x)}{a+b\cosh(x)}\,dx = \left[\frac{\sqrt{a^2-b^2}b\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\,(b^2\cosh(x)+ab)\sinh(x)+2\,\sqrt{a^2-b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)+2\,(}{a^3-ab^2}\right.$$

input
```
integrate(sech(x)/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[(sqrt(a^2 - b^2)*b*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2
*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*cosh(x)
+ b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x)
+ a)*sinh(x) + b)) + 2*(a^2 - b^2)*arctan(cosh(x) + sinh(x)))/(a^3 - a*b^2
), 2*(sqrt(-a^2 + b^2)*b*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) +
 a)/(a^2 - b^2)) + (a^2 - b^2)*arctan(cosh(x) + sinh(x)))/(a^3 - a*b^2)]
```

3.58.6 Sympy [F]

$$\int \frac{\operatorname{sech}(x)}{a+b\cosh(x)}\,dx = \int \frac{\operatorname{sech}(x)}{a+b\cosh(x)}\,dx$$

input
```
integrate(sech(x)/(a+b*cosh(x)),x)
```

output
```
Integral(sech(x)/(a + b*cosh(x)), x)
```

3.58.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}(x)}{a+b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sech(x)/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.58.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{\operatorname{sech}(x)}{a+b\cosh(x)}\,dx = -\frac{2\,b\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}a}+\frac{2\,\arctan\left(e^x\right)}{a}$$

input
```
integrate(sech(x)/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
-2*b*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a) + 2*arctan(
e^x)/a
```

3.58.9 Mupad [B] (verification not implemented)

Time = 4.43 (sec) , antiderivative size = 286, normalized size of antiderivative = 5.30

$$\int \frac{\operatorname{sech}(x)}{a+b\cosh(x)}\,dx$$
$$= \frac{b\,\ln\left(64\,a^4\,b-64\,a^2\,b^3+128\,a^5\,e^x+32\,a\,b^3\,\sqrt{a^2-b^2}-64\,a^3\,b\,\sqrt{a^2-b^2}+32\,a\,b^4\,e^x-128\,a^4\,e^x\,\sqrt{a^2-b^2}\right.}{a\,\sqrt{a^2-b^2}}$$
$$-\frac{b\,\ln\left(64\,a^4\,b-64\,a^2\,b^3+128\,a^5\,e^x-32\,a\,b^3\,\sqrt{a^2-b^2}+64\,a^3\,b\,\sqrt{a^2-b^2}+32\,a\,b^4\,e^x+128\,a^4\,e^x\,\sqrt{a^2-b^2}\right.}{a\,\sqrt{a^2-b^2}}$$
$$-\frac{\ln\left(e^x-\mathrm{i}\right)\,1\mathrm{i}-\ln\left(e^x+1\mathrm{i}\right)\,1\mathrm{i}}{a}$$

input
```
int(1/(cosh(x)*(a + b*cosh(x))),x)
```

output
```
(b*log(64*a^4*b - 64*a^2*b^3 + 128*a^5*exp(x) + 32*a*b^3*(a^2 - b^2)^(1/2)
 - 64*a^3*b*(a^2 - b^2)^(1/2) + 32*a*b^4*exp(x) - 128*a^4*exp(x)*(a^2 - b^
2)^(1/2) - 160*a^3*b^2*exp(x) + 96*a^2*b^2*exp(x)*(a^2 - b^2)^(1/2)))/(a*(
a^2 - b^2)^(1/2)) - (b*log(64*a^4*b - 64*a^2*b^3 + 128*a^5*exp(x) - 32*a*b
^3*(a^2 - b^2)^(1/2) + 64*a^3*b*(a^2 - b^2)^(1/2) + 32*a*b^4*exp(x) + 128*
a^4*exp(x)*(a^2 - b^2)^(1/2) - 160*a^3*b^2*exp(x) - 96*a^2*b^2*exp(x)*(a^2
 - b^2)^(1/2)))/(a*(a^2 - b^2)^(1/2)) - (log(exp(x) - 1i)*1i - log(exp(x)
+ 1i)*1i)/a
```

3.58. $\int \frac{\operatorname{sech}(x)}{a+b\cosh(x)}\,dx$

3.59 $\int \frac{\operatorname{sech}^2(x)}{a+b\cosh(x)}\,dx$

3.59.1 Optimal result

Integrand size = 13, antiderivative size = 64

$$\int \frac{\operatorname{sech}^2(x)}{a+b\cosh(x)}\,dx = -\frac{b\arctan(\sinh(x))}{a^2}+\frac{2b^2\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}}+\frac{\tanh(x)}{a}$$

output
```
-b*arctan(sinh(x))/a^2+2*b^2*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/
a^2/(a-b)^(1/2)/(a+b)^(1/2)+tanh(x)/a
```

3.59.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int \frac{\operatorname{sech}^2(x)}{a+b\cosh(x)}\,dx = \frac{-2b\arctan\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{2b^2\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+a\tanh(x)}{a^2}$$

input
```
Integrate[Sech[x]^2/(a + b*Cosh[x]),x]
```

output
```
(-2*b*ArcTan[Tanh[x/2]] - (2*b^2*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^
2]])/Sqrt[-a^2 + b^2] + a*Tanh[x])/a^2
```

3.59.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.769, Rules used = {3042, 3281, 25, 27, 3042, 3226, 3042, 3138, 221, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^2(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(\frac{\pi}{2}+ix\right)^2\left(a+b\sin\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{3281}$$

$$\frac{\int -\frac{b\operatorname{sech}(x)}{a+b\cosh(x)}dx}{a}+\frac{\tanh(x)}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\tanh(x)}{a}-\frac{\int \frac{b\operatorname{sech}(x)}{a+b\cosh(x)}dx}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\tanh(x)}{a}-\frac{b\int \frac{\operatorname{sech}(x)}{a+b\cosh(x)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tanh(x)}{a}-\frac{b\int \frac{1}{\sin\left(ix+\frac{\pi}{2}\right)\left(a+b\sin\left(ix+\frac{\pi}{2}\right)\right)}dx}{a}$$

$$\Big\downarrow \text{3226}$$

$$\frac{\tanh(x)}{a}-\frac{b\left(\frac{\int \operatorname{sech}(x)dx}{a}-\frac{b\int \frac{1}{a+b\cosh(x)}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tanh(x)}{a}-\frac{b\left(\frac{\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}-\frac{b\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\tanh(x)}{a}-\frac{b\left(-\frac{2b\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a}+\frac{\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}\right)}{a}$$

$$\Big\downarrow \ 221$$

$$\frac{\tanh(x)}{a}-\frac{b\left(-\frac{2b\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}+\frac{\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}\right)}{a}$$

$$\Big\downarrow \ 4257$$

$$\frac{\tanh(x)}{a}-\frac{b\left(\frac{\arctan(\sinh(x))}{a}-\frac{2b\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}\right)}{a}$$

input
```
Int[Sech[x]^2/(a + b*Cosh[x]),x]
```

output
```
-((b*(ArcTan[Sinh[x]]/a - (2*b*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]
])/(a*Sqrt[a - b]*Sqrt[a + b])))/a) + Tanh[x]/a
```

3.59.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3226
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]),
 x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.59.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14

method	result	size
default	$\frac{2b^{2}\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^{2}\sqrt{(a+b)(a-b)}}-\frac{2\left(-\frac{a\tanh\left(\frac{x}{2}\right)}{1+\tanh\left(\frac{x}{2}\right)^{2}}+b\arctan\left(\tanh\left(\frac{x}{2}\right)\right)\right)}{a^{2}}$	73
risch	$-\frac{2}{a\left(1+{\mathrm e}^{2x}\right)}+\frac{ib\ln\left({\mathrm e}^{x}-i\right)}{a^{2}}-\frac{ib\ln\left({\mathrm e}^{x}+i\right)}{a^{2}}+\frac{b^{2}\ln\left({\mathrm e}^{x}+\frac{a\sqrt{a^{2}-b^{2}}-a^{2}+b^{2}}{b\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,a^{2}}-\frac{b^{2}\ln\left({\mathrm e}^{x}+\frac{a\sqrt{a^{2}-b^{2}}+a^{2}-b^{2}}{b\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,a^{2}}$	160

input
```
int(sech(x)^2/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
2*b^2/a^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2
))-2/a^2*(-a*tanh(1/2*x)/(1+tanh(1/2*x)^2)+b*arctan(tanh(1/2*x)))
```

3.59.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(54) = 108$.

Time = 0.28 (sec) , antiderivative size = 515, normalized size of antiderivative = 8.05

$$\int \frac{\operatorname{sech}^2(x)}{a+b\cosh(x)}\,dx$$
$$= \left[-\frac{2\,a^3-2\,ab^2-\left(b^2\cosh\left(x\right)^2+2\,b^2\cosh\left(x\right)\sinh\left(x\right)+b^2\sinh\left(x\right)^2+b^2\right)\sqrt{a^2-b^2}\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)}{\cdots}\right.}{a^4\cdots}\right.$$
$$-\frac{2\left(a^3-ab^2+\left(b^2\cosh\left(x\right)^2+2\,b^2\cosh\left(x\right)\sinh\left(x\right)+b^2\sinh\left(x\right)^2+b^2\right)\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos\cdots}{\cdots}\right.\right.}{a^4-a^2b^2+\left(a^4-a^2b^2\right)\cosh\cdots}$$

input
```
integrate(sech(x)^2/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[-(2*a^3 - 2*a*b^2 - (b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^
2 + b^2)*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x
) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cos
h(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cos
h(x) + a)*sinh(x) + b)) + 2*(a^2*b - b^3 + (a^2*b - b^3)*cosh(x)^2 + 2*(a^
2*b - b^3)*cosh(x)*sinh(x) + (a^2*b - b^3)*sinh(x)^2)*arctan(cosh(x) + sin
h(x)))/(a^4 - a^2*b^2 + (a^4 - a^2*b^2)*cosh(x)^2 + 2*(a^4 - a^2*b^2)*cosh
(x)*sinh(x) + (a^4 - a^2*b^2)*sinh(x)^2), -2*(a^3 - a*b^2 + (b^2*cosh(x)^2
 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2 + b^2)*sqrt(-a^2 + b^2)*arctan(-s
qrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) + (a^2*b - b^3 +
(a^2*b - b^3)*cosh(x)^2 + 2*(a^2*b - b^3)*cosh(x)*sinh(x) + (a^2*b - b^3)*
sinh(x)^2)*arctan(cosh(x) + sinh(x)))/(a^4 - a^2*b^2 + (a^4 - a^2*b^2)*cos
h(x)^2 + 2*(a^4 - a^2*b^2)*cosh(x)*sinh(x) + (a^4 - a^2*b^2)*sinh(x)^2)]
```

3.59.6 Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{a+b\cosh(x)}\,dx = \int \frac{\operatorname{sech}^2{(x)}}{a+b\cosh{(x)}}\,dx$$

input
```
integrate(sech(x)**2/(a+b*cosh(x)),x)
```

output
```
Integral(sech(x)**2/(a + b*cosh(x)), x)
```

3.59.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^2(x)}{a+b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sech(x)^2/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.59.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{sech}^2(x)}{a+b\cosh(x)}\,dx = \frac{2\,b^2\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}a^2} - \frac{2\,b\arctan\left(e^x\right)}{a^2} - \frac{2}{a(e^{(2\,x)}+1)}$$

input
```
integrate(sech(x)^2/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
2*b^2*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a^2) - 2*b*ar
ctan(e^x)/a^2 - 2/(a*(e^(2*x) + 1))
```

3.59.9 Mupad [B] (verification not implemented)

Time = 4.04 (sec) , antiderivative size = 294, normalized size of antiderivative = 4.59

$$\int \frac{\operatorname{sech}^2(x)}{a+b\cosh(x)}\,dx$$
$$= \frac{b^2\,\ln\left(64\,a\,b^3-64\,a^3\,b+32\,b^3\,\sqrt{a^2-b^2}-128\,a^4\,e^x-32\,b^4\,e^x-64\,a^2\,b\,\sqrt{a^2-b^2}-128\,a^3\,e^x\,\sqrt{a^2-b^2}+1\right.}{a^2\,\sqrt{a^2-b^2}}$$
$$+\frac{b\,(\ln(32\,e^x-32\mathrm{i})\,1\mathrm{i}-\ln(32\,e^x+32\mathrm{i})\,1\mathrm{i})}{a^2}$$
$$-\frac{b^2\,\ln\left(64\,a^3\,b-64\,a\,b^3+32\,b^3\,\sqrt{a^2-b^2}+128\,a^4\,e^x+32\,b^4\,e^x-64\,a^2\,b\,\sqrt{a^2-b^2}-128\,a^3\,e^x\,\sqrt{a^2-b^2}-\right.}{a^2\,\sqrt{a^2-b^2}}$$
$$-\frac{2}{a+a\,e^{2\,x}}$$

input
```
int(1/(cosh(x)^2*(a + b*cosh(x))),x)
```

output
```
(b*(log(32*exp(x) - 32i)*1i - log(32*exp(x) + 32i)*1i))/a^2 - 2/(a + a*exp
(2*x)) - (b^2*log(64*a^3*b - 64*a*b^3 + 32*b^3*(a^2 - b^2)^(1/2) + 128*a^4
*exp(x) + 32*b^4*exp(x) - 64*a^2*b*(a^2 - b^2)^(1/2) - 128*a^3*exp(x)*(a^2
 - b^2)^(1/2) - 160*a^2*b^2*exp(x) + 96*a*b^2*exp(x)*(a^2 - b^2)^(1/2)))/(
a^2*(a^2 - b^2)^(1/2)) + (b^2*log(64*a*b^3 - 64*a^3*b + 32*b^3*(a^2 - b^2)
^(1/2) - 128*a^4*exp(x) - 32*b^4*exp(x) - 64*a^2*b*(a^2 - b^2)^(1/2) - 128
*a^3*exp(x)*(a^2 - b^2)^(1/2) + 160*a^2*b^2*exp(x) + 96*a*b^2*exp(x)*(a^2
- b^2)^(1/2)))/(a^2*(a^2 - b^2)^(1/2))
```

3.60 $\int \frac{\text{sech}^3(x)}{a+b\cosh(x)}\,dx$

3.60.1 Optimal result

Integrand size = 13, antiderivative size = 87

$$\int \frac{\text{sech}^3(x)}{a+b\cosh(x)}\,dx=\frac{(a^2+2b^2)\arctan(\sinh(x))}{2a^3}-\frac{2b^3\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\sqrt{a+b}}-\frac{b\tanh(x)}{a^2}+\frac{\text{sech}(x)\tanh(x)}{2a}$$

output
```
1/2*(a^2+2*b^2)*arctan(sinh(x))/a^3-2*b^3*arctanh((a-b)^(1/2)*tanh(1/2*x)/
(a+b)^(1/2))/a^3/(a-b)^(1/2)/(a+b)^(1/2)-b*tanh(x)/a^2+1/2*sech(x)*tanh(x)
/a
```

3.60.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

$$\int \frac{\text{sech}^3(x)}{a+b\cosh(x)}\,dx=\frac{2(a^2+2b^2)\arctan\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{4b^3\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+a(-2b+a\,\text{sech}(x))\tanh(x)}{2a^3}$$

input
```
Integrate[Sech[x]^3/(a + b*Cosh[x]),x]
```

output
```
(2*(a^2 + 2*b^2)*ArcTan[Tanh[x/2]] + (4*b^3*ArcTan[((a - b)*Tanh[x/2])/Sqr
t[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + a*(-2*b + a*Sech[x])*Tanh[x])/(2*a^3)
```

3.60.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.923$, Rules used = {3042, 3281, 25, 3042, 3534, 25, 3042, 3480, 3042, 3138, 221, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^3(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(\frac{\pi}{2}+ix\right)^3\left(a+b\sin\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{3281}$$

$$\frac{\int -\frac{(-b\cosh^2(x)-a\cosh(x)+2b)\text{sech}^2(x)}{a+b\cosh(x)}dx}{2a}+\frac{\tanh(x)\text{sech}(x)}{2a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\tanh(x)\text{sech}(x)}{2a}-\frac{\int \frac{(-b\cosh^2(x)-a\cosh(x)+2b)\text{sech}^2(x)}{a+b\cosh(x)}dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tanh(x)\text{sech}(x)}{2a}-\frac{\int \frac{-b\sin\left(ix+\frac{\pi}{2}\right)^2-a\sin\left(ix+\frac{\pi}{2}\right)+2b}{\sin\left(ix+\frac{\pi}{2}\right)^2\left(a+b\sin\left(ix+\frac{\pi}{2}\right)\right)}dx}{2a}$$

$$\Big\downarrow \text{3534}$$

$$\frac{\tanh(x)\text{sech}(x)}{2a}-\frac{\frac{\int -\frac{(a^2+b\cosh(x)a+2b^2)\text{sech}(x)}{a+b\cosh(x)}dx}{a}+\frac{2b\tanh(x)}{a}}{2a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\tanh(x)\text{sech}(x)}{2a}-\frac{\frac{2b\tanh(x)}{a}-\frac{\int \frac{(a^2+b\cosh(x)a+2b^2)\text{sech}(x)}{a+b\cosh(x)}dx}{a}}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tanh(x)\text{sech}(x)}{2a}-\frac{\frac{2b\tanh(x)}{a}-\frac{\int \frac{a^2+b\sin\left(ix+\frac{\pi}{2}\right)a+2b^2}{\sin\left(ix+\frac{\pi}{2}\right)\left(a+b\sin\left(ix+\frac{\pi}{2}\right)\right)}dx}{a}}{2a}$$

$$\Big\downarrow \text{3480}$$

$$\frac{\tanh(x)\text{sech}(x)}{2a}-\frac{\frac{2b\tanh(x)}{a}-\frac{\frac{(a^2+2b^2)\int \text{sech}(x)dx}{a}-\frac{2b^3\int \frac{1}{a+b\cosh(x)}dx}{a}}{a}}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tanh(x)\text{sech}(x)}{2a}-\frac{\frac{2b\tanh(x)}{a}-\frac{\frac{(a^2+2b^2)\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}-\frac{2b^3\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a}}{a}}{2a}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\tanh(x)\text{sech}(x)}{2a}-\frac{\frac{2b\tanh(x)}{a}-\frac{-\frac{4b^3\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a}+\frac{(a^2+2b^2)\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}}{a}}{2a}$$

$$\Big\downarrow \text{221}$$

$$\frac{\tanh(x)\text{sech}(x)}{2a}-\frac{\frac{2b\tanh(x)}{a}-\frac{-\frac{4b^3\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}+\frac{(a^2+2b^2)\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}}{a}}{2a}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\tanh(x)\text{sech}(x)}{2a}-\frac{\frac{2b\tanh(x)}{a}-\frac{\frac{(a^2+2b^2)\arctan(\sinh(x))}{a}-\frac{4b^3\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}}{a}}{2a}$$

input
```
Int[Sech[x]^3/(a + b*Cosh[x]),x]
```

output
```
(Sech[x]*Tanh[x])/(2*a) - (-((((a^2 + 2*b^2)*ArcTan[Sinh[x]])/a - (4*b^3*A
rcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]))/
a) + (2*b*Tanh[x])/a)/(2*a)
```

3.60. $\displaystyle\int \frac{\text{sech}^3(x)}{a+b\cosh(x)}\,dx$

3.60.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.60. $\int \frac{\operatorname{sech}^3(x)}{a+b\cosh(x)}\,dx$

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.60.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.25

method	result
default	$-\frac{2b^3 \operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^3\sqrt{(a+b)(a-b)}}+\frac{\frac{2\left(\left(-\frac{1}{2}a^2-ab\right)\tanh\left(\frac{x}{2}\right)^3+\left(\frac{1}{2}a^2-ab\right)\tanh\left(\frac{x}{2}\right)\right)}{\left(1+\tanh\left(\frac{x}{2}\right)^2\right)^2}+\left(a^2+2b^2\right)\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a^3}$
risch	$\frac{a\,e^{3x}+2b\,e^{2x}-a\,e^{x}+2b}{\left(1+e^{2x}\right)^2a^2}+\frac{b^3\ln\left(e^x+\frac{a\sqrt{a^2-b^2}+a^2-b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,a^3}-\frac{b^3\ln\left(e^x+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,a^3}+\frac{i\ln(e^x+i)}{2a}+\frac{i\ln(e^x+i)b^2}{a^3}-\frac{i\ln(e$

input
```
int(sech(x)^3/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-2*b^3/a^3/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/
2))+2/a^3*(((-1/2*a^2-a*b)*tanh(1/2*x)^3+(1/2*a^2-a*b)*tanh(1/2*x))/(1+tan
h(1/2*x)^2)^2+1/2*(a^2+2*b^2)*arctan(tanh(1/2*x)))
```

3.60.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 651 vs. $2(73) = 146$.

Time = 0.33 (sec) , antiderivative size = 1370, normalized size of antiderivative = 15.75

$$\int \frac{\operatorname{sech}^3(x)}{a + b\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(x)^3/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[(2*a^3*b - 2*a*b^3 + (a^4 - a^2*b^2)*cosh(x)^3 + (a^4 - a^2*b^2)*sinh(x)^
3 + 2*(a^3*b - a*b^3)*cosh(x)^2 + (2*a^3*b - 2*a*b^3 + 3*(a^4 - a^2*b^2)*c
osh(x))*sinh(x)^2 + (b^3*cosh(x)^4 + 4*b^3*cosh(x)*sinh(x)^3 + b^3*sinh(x)
^4 + 2*b^3*cosh(x)^2 + b^3 + 2*(3*b^3*cosh(x)^2 + b^3)*sinh(x)^2 + 4*(b^3*
cosh(x)^3 + b^3*cosh(x))*sinh(x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2
*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) +
 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2
 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)) + ((a^4 + a^2*b^2 - 2*b^4
)*cosh(x)^4 + 4*(a^4 + a^2*b^2 - 2*b^4)*cosh(x)*sinh(x)^3 + (a^4 + a^2*b^2
 - 2*b^4)*sinh(x)^4 + a^4 + a^2*b^2 - 2*b^4 + 2*(a^4 + a^2*b^2 - 2*b^4)*co
sh(x)^2 + 2*(a^4 + a^2*b^2 - 2*b^4 + 3*(a^4 + a^2*b^2 - 2*b^4)*cosh(x)^2)*
sinh(x)^2 + 4*((a^4 + a^2*b^2 - 2*b^4)*cosh(x)^3 + (a^4 + a^2*b^2 - 2*b^4)
*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - (a^4 - a^2*b^2)*cosh(x) - (
a^4 - a^2*b^2 - 3*(a^4 - a^2*b^2)*cosh(x)^2 - 4*(a^3*b - a*b^3)*cosh(x))*s
inh(x))/(a^5 - a^3*b^2 + (a^5 - a^3*b^2)*cosh(x)^4 + 4*(a^5 - a^3*b^2)*cos
h(x)*sinh(x)^3 + (a^5 - a^3*b^2)*sinh(x)^4 + 2*(a^5 - a^3*b^2)*cosh(x)^2 +
 2*(a^5 - a^3*b^2 + 3*(a^5 - a^3*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^5 - a^3
*b^2)*cosh(x)^3 + (a^5 - a^3*b^2)*cosh(x))*sinh(x)), (2*a^3*b - 2*a*b^3 +
(a^4 - a^2*b^2)*cosh(x)^3 + (a^4 - a^2*b^2)*sinh(x)^3 + 2*(a^3*b - a*b^3)*
cosh(x)^2 + (2*a^3*b - 2*a*b^3 + 3*(a^4 - a^2*b^2)*cosh(x))*sinh(x)^2 +...
```

3.60.6 Sympy [F]

$$\int \frac{\operatorname{sech}^3(x)}{a + b\cosh(x)}\,dx = \int \frac{\operatorname{sech}^3(x)}{a + b\cosh(x)}\,dx$$

input
```
integrate(sech(x)**3/(a+b*cosh(x)),x)
```

output
```
Integral(sech(x)**3/(a + b*cosh(x)), x)
```

3.60.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^3(x)}{a + b\cosh(x)}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate(sech(x)^3/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.60.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

$$\int \frac{\operatorname{sech}^3(x)}{a + b\cosh(x)}\, dx = -\frac{2\, b^3 \arctan\left(\frac{be^x + a}{\sqrt{-a^2 + b^2}}\right)}{\sqrt{-a^2 + b^2}a^3} + \frac{(a^2 + 2\, b^2) \arctan\left(e^x\right)}{a^3} + \frac{ae^{(3\,x)} + 2\, be^{(2\,x)} - ae^x + 2\, b}{a^2 {\left(e^{(2\,x)} + 1\right)}^2}$$

input
```
integrate(sech(x)^3/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
-2*b^3*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a^3) + (a^2
+ 2*b^2)*arctan(e^x)/a^3 + (a*e^(3*x) + 2*b*e^(2*x) - a*e^x + 2*b)/(a^2*(e
^(2*x) + 1)^2)
```

3.60.9 Mupad [B] (verification not implemented)

Time = 5.09 (sec) , antiderivative size = 476, normalized size of antiderivative = 5.47

$$\int \frac{\operatorname{sech}^3(x)}{a+b\cosh(x)}\,dx = \frac{e^x}{a+a\,e^{2x}} - \frac{2\,e^x}{a+2\,a\,e^{2x}+a\,e^{4x}} + \frac{2\,b}{a^2\,e^{2x}+a^2}$$
$$-\frac{\ln\left(1+e^x\,1i\right)\,1i-\ln\left(e^x+1i\right)\,1i}{2\,a} - \frac{b^2\left(\ln\left(1+e^x\,1i\right)\,1i-\ln\left(e^x+1i\right)\,1i\right)}{a^3}$$
$$-\frac{b^3\,\ln\left(16\,a^5\,b-48\,a\,b^5-24\,b^5\,\sqrt{a^2-b^2}+32\,a^3\,b^3+32\,a^6\,e^x+24\,b^6\,e^x+16\,a^4\,b\,\sqrt{a^2-b^2}+40\,a^2\,b^3\,\sqrt{a}\right.}{a^3\,\sqrt{a^2-}}$$
$$+\frac{b^3\,\ln\left(16\,a^5\,b-48\,a\,b^5+24\,b^5\,\sqrt{a^2-b^2}+32\,a^3\,b^3+32\,a^6\,e^x+24\,b^6\,e^x-16\,a^4\,b\,\sqrt{a^2-b^2}-40\,a^2\,b^3\,\sqrt{a}\right.}{a^3\,\sqrt{a^2-}}$$

input
```
int(1/(cosh(x)^3*(a + b*cosh(x))),x)
```

output
```
exp(x)/(a + a*exp(2*x)) - (2*exp(x))/(a + 2*a*exp(2*x) + a*exp(4*x)) + (2*
b)/(a^2*exp(2*x) + a^2) - (log(exp(x)*1i + 1)*1i - log(exp(x) + 1i)*1i)/(2
*a) - (b^2*(log(exp(x)*1i + 1)*1i - log(exp(x) + 1i)*1i))/a^3 - (b^3*log(1
6*a^5*b - 48*a*b^5 - 24*b^5*(a^2 - b^2)^(1/2) + 32*a^3*b^3 + 32*a^6*exp(x)
 + 24*b^6*exp(x) + 16*a^4*b*(a^2 - b^2)^(1/2) + 40*a^2*b^3*(a^2 - b^2)^(1/
2) + 32*a^5*exp(x)*(a^2 - b^2)^(1/2) - 112*a^2*b^4*exp(x) + 56*a^4*b^2*exp
(x) + 72*a^3*b^2*exp(x)*(a^2 - b^2)^(1/2) - 72*a*b^4*exp(x)*(a^2 - b^2)^(1
/2)))/(a^3*(a^2 - b^2)^(1/2)) + (b^3*log(16*a^5*b - 48*a*b^5 + 24*b^5*(a^2
 - b^2)^(1/2) + 32*a^3*b^3 + 32*a^6*exp(x) + 24*b^6*exp(x) - 16*a^4*b*(a^2
 - b^2)^(1/2) - 40*a^2*b^3*(a^2 - b^2)^(1/2) - 32*a^5*exp(x)*(a^2 - b^2)^(
1/2) - 112*a^2*b^4*exp(x) + 56*a^4*b^2*exp(x) - 72*a^3*b^2*exp(x)*(a^2 - b
^2)^(1/2) + 72*a*b^4*exp(x)*(a^2 - b^2)^(1/2)))/(a^3*(a^2 - b^2)^(1/2))
```

3.61 $\int \frac{\text{sech}^4(x)}{a+b\cosh(x)}\,dx$

3.61.1 Optimal result

Integrand size = 13, antiderivative size = 114

$$\int \frac{\text{sech}^4(x)}{a+b\cosh(x)}\,dx=-\frac{b(a^2+2b^2)\arctan(\sinh(x))}{2a^4}+\frac{2b^4\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}}+\frac{(2a^2+3b^2)\tanh(x)}{3a^3}-\frac{b\,\text{sech}(x)\tanh(x)}{2a^2}+\frac{\text{sech}^2(x)\tanh(x)}{3a}$$

output
```
-1/2*b*(a^2+2*b^2)*arctan(sinh(x))/a^4+2*b^4*arctanh((a-b)^(1/2)*tanh(1/2*
x)/(a+b)^(1/2))/a^4/(a-b)^(1/2)/(a+b)^(1/2)+1/3*(2*a^2+3*b^2)*tanh(x)/a^3-
1/2*b*sech(x)*tanh(x)/a^2+1/3*sech(x)^2*tanh(x)/a
```

3.61.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.89

$$\int \frac{\text{sech}^4(x)}{a+b\cosh(x)}\,dx=\frac{-6b(a^2+2b^2)\arctan\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{12b^4\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+a\left(4a^2+6b^2-3ab\,\text{sech}(x)+2a^2\text{sech}^2(x)\right)\tan}{6a^4}$$

input
```
Integrate[Sech[x]^4/(a + b*Cosh[x]),x]
```

output
```
(-6*b*(a^2 + 2*b^2)*ArcTan[Tanh[x/2]] - (12*b^4*ArcTan[((a - b)*Tanh[x/2])
/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + a*(4*a^2 + 6*b^2 - 3*a*b*Sech[x] +
2*a^2*Sech[x]^2)*Tanh[x])/(6*a^4)
```

3.61.3 Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.15, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.154$, Rules used = {3042, 3281, 25, 3042, 3534, 25, 3042, 3534, 27, 3042, 3480, 3042, 3138, 221, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^4(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(\frac{\pi}{2}+ix\right)^4\left(a+b\sin\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{3281}$$

$$\frac{\int -\frac{\left(-2b\cosh^2(x)-2a\cosh(x)+3b\right)\text{sech}^3(x)}{a+b\cosh(x)}dx}{3a}+\frac{\tanh(x)\text{sech}^2(x)}{3a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\tanh(x)\text{sech}^2(x)}{3a}-\frac{\int \frac{\left(-2b\cosh^2(x)-2a\cosh(x)+3b\right)\text{sech}^3(x)}{a+b\cosh(x)}dx}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tanh(x)\text{sech}^2(x)}{3a}-\frac{\int \frac{-2b\sin\left(ix+\frac{\pi}{2}\right)^2-2a\sin\left(ix+\frac{\pi}{2}\right)+3b}{\sin\left(ix+\frac{\pi}{2}\right)^3\left(a+b\sin\left(ix+\frac{\pi}{2}\right)\right)}dx}{3a}$$

$$\Big\downarrow \text{3534}$$

$$\frac{\tanh(x)\text{sech}^2(x)}{3a}-\frac{\frac{\int -\frac{\left(-3b^2\cosh^2(x)+ab\cosh(x)+2\left(2a^2+3b^2\right)\right)\text{sech}^2(x)}{a+b\cosh(x)}dx}{2a}+\frac{3b\tanh(x)\text{sech}(x)}{2a}}{3a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\tanh(x)\text{sech}^2(x)}{3a}-\frac{\frac{3b\tanh(x)\text{sech}(x)}{2a}-\frac{\int \frac{\left(-3b^2\cosh^2(x)+ab\cosh(x)+2\left(2a^2+3b^2\right)\right)\text{sech}^2(x)}{a+b\cosh(x)}dx}{2a}}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tanh(x)\text{sech}^2(x)}{3a}-\frac{\frac{3b\tanh(x)\text{sech}(x)}{2a}-\frac{\int\frac{-3b^2\sin\left(ix+\frac{\pi}{2}\right)^2+ab\sin\left(ix+\frac{\pi}{2}\right)+2\left(2a^2+3b^2\right)}{\sin\left(ix+\frac{\pi}{2}\right)^2\left(a+b\sin\left(ix+\frac{\pi}{2}\right)\right)}dx}{2a}}{3a}$$

$$\Big\downarrow \text{3534}$$

$$\frac{\tanh(x)\text{sech}^2(x)}{3a}-\frac{\frac{3b\tanh(x)\text{sech}(x)}{2a}-\frac{\frac{\int-\frac{3\left(a\cosh(x)b^2+\left(a^2+2b^2\right)b\right)\text{sech}(x)}{a+b\cosh(x)}dx}{a}+\frac{2\left(2a^2+3b^2\right)\tanh(x)}{a}}{2a}}{3a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\tanh(x)\text{sech}^2(x)}{3a}-\frac{\frac{3b\tanh(x)\text{sech}(x)}{2a}-\frac{\frac{2\left(2a^2+3b^2\right)\tanh(x)}{a}-\frac{3\int\frac{\left(a\cosh(x)b^2+\left(a^2+2b^2\right)b\right)\text{sech}(x)}{a+b\cosh(x)}dx}{a}}{2a}}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tanh(x)\text{sech}^2(x)}{3a}-\frac{\frac{3b\tanh(x)\text{sech}(x)}{2a}-\frac{\frac{2\left(2a^2+3b^2\right)\tanh(x)}{a}-\frac{3\int\frac{a\sin\left(ix+\frac{\pi}{2}\right)b^2+\left(a^2+2b^2\right)b}{\sin\left(ix+\frac{\pi}{2}\right)\left(a+b\sin\left(ix+\frac{\pi}{2}\right)\right)}dx}{a}}{2a}}{3a}$$

$$\Big\downarrow \text{3480}$$

$$\frac{\tanh(x)\text{sech}^2(x)}{3a}-\frac{\frac{3b\tanh(x)\text{sech}(x)}{2a}-\frac{\frac{2\left(2a^2+3b^2\right)\tanh(x)}{a}-\frac{3\left(\frac{b\left(a^2+2b^2\right)\int\text{sech}(x)dx}{a}-\frac{2b^4\int\frac{1}{a+b\cosh(x)}dx}{a}\right)}{a}}{2a}}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tanh(x)\text{sech}^2(x)}{3a}-\frac{\frac{3b\tanh(x)\text{sech}(x)}{2a}-\frac{\frac{2\left(2a^2+3b^2\right)\tanh(x)}{a}-\frac{3\left(\frac{b\left(a^2+2b^2\right)\int\csc\left(ix+\frac{\pi}{2}\right)dx}{a}-\frac{2b^4\int\frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a}\right)}{a}}{2a}}{3a}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\tanh(x)\text{sech}^2(x)}{3a}-\frac{\frac{3b\tanh(x)\text{sech}(x)}{2a}-\frac{\frac{2\left(2a^2+3b^2\right)\tanh(x)}{a}-\frac{3\left(-\frac{4b^4\int\frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a}+\frac{b\left(a^2+2b^2\right)\int\csc\left(ix+\frac{\pi}{2}\right)dx}{a}\right)}{a}}{2a}}{3a}$$

$$\Big\downarrow \text{221}$$

3.61. $\int\frac{\text{sech}^4(x)}{a+b\cosh(x)}dx$

$$\frac{\tanh(x)\text{sech}^2(x)}{3a}-\frac{\frac{3b\tanh(x)\text{sech}(x)}{2a}-\frac{\frac{2(2a^2+3b^2)\tanh(x)}{a}-\frac{3\left(-\frac{4b^4\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}+\frac{b(a^2+2b^2)\int\csc\left(ix+\frac{\pi}{2}\right)dx}{a}\right)}{a}}{2a}}{3a}$$

$$\Big\downarrow \; 4257$$

$$\frac{\tanh(x)\text{sech}^2(x)}{3a}-\frac{\frac{3b\tanh(x)\text{sech}(x)}{2a}-\frac{\frac{2(2a^2+3b^2)\tanh(x)}{a}-\frac{3\left(\frac{b(a^2+2b^2)\arctan(\sinh(x))}{a}-\frac{4b^4\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}\right)}{a}}{2a}}{3a}$$

input
```
Int[Sech[x]^4/(a + b*Cosh[x]),x]
```

output
```
(Sech[x]^2*Tanh[x])/(3*a) - ((3*b*Sech[x]*Tanh[x])/(2*a) - ((-3*((b*(a^2 +
 2*b^2)*ArcTan[Sinh[x]])/a - (4*b^4*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a
 + b]])/(a*Sqrt[a - b]*Sqrt[a + b])))/a + (2*(2*a^2 + 3*b^2)*Tanh[x])/a)/(
2*a))/(3*a)
```

3.61.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.61. $\int \frac{\text{sech}^4(x)}{a+b\cosh(x)}\,dx$

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.61. $\int \frac{\operatorname{sech}^4(x)}{a+b\cosh(x)}\,dx$

3.61.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.27

method	result
default	$\frac{2b^4 \operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^4\sqrt{(a+b)(a-b)}} - \frac{2\left(\frac{\left(-a^3-\frac{1}{2}a^2b-a\,b^2\right)\tanh\left(\frac{x}{2}\right)^5+\left(-\frac{2}{3}a^3-2a\,b^2\right)\tanh\left(\frac{x}{2}\right)^3+\left(-a^3-a\,b^2+\frac{1}{2}a^2b\right)\tanh\left(\frac{x}{2}\right)}{\left(1+\tanh\left(\frac{x}{2}\right)^2\right)^3}+\frac{b\left(a^2+2b^2\right)}{}\right.}{a^4}$
risch	$-\frac{3ab\,\mathrm{e}^{5x}+6b^2\mathrm{e}^{4x}+12a^2\mathrm{e}^{2x}+12b^2\mathrm{e}^{2x}-3b\,\mathrm{e}^{x}a+4a^2+6b^2}{3a^3(1+\mathrm{e}^{2x})^3}+\frac{b^4\ln\left(\mathrm{e}^x+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,a^4}-\frac{b^4\ln\left(\mathrm{e}^x+\frac{a\sqrt{a^2-b^2}+a^2-b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,a^4}+\frac{ib}{}$

input
```
int(sech(x)^4/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
2*b^4/a^4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2
))-2/a^4*(((-a^3-1/2*a^2*b-a*b^2)*tanh(1/2*x)^5+(-2/3*a^3-2*a*b^2)*tanh(1/
2*x)^3+(-a^3-a*b^2+1/2*a^2*b)*tanh(1/2*x))/(1+tanh(1/2*x)^2)^3+1/2*b*(a^2+
2*b^2)*arctan(tanh(1/2*x)))
```

3.61.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1207 vs. 2(96) = 192.

Time = 0.34 (sec) , antiderivative size = 2483, normalized size of antiderivative = 21.78

$$\int \frac{\operatorname{sech}^4(x)}{a+b\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(x)^4/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[-1/3*(3*(a^4*b - a^2*b^3)*cosh(x)^5 + 3*(a^4*b - a^2*b^3)*sinh(x)^5 + 4*a
^5 + 2*a^3*b^2 - 6*a*b^4 + 6*(a^3*b^2 - a*b^4)*cosh(x)^4 + 3*(2*a^3*b^2 -
2*a*b^4 + 5*(a^4*b - a^2*b^3)*cosh(x))*sinh(x)^4 + 6*(5*(a^4*b - a^2*b^3)*
cosh(x)^2 + 4*(a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^3 + 12*(a^5 - a*b^4)*cosh
(x)^2 + 6*(2*a^5 - 2*a*b^4 + 5*(a^4*b - a^2*b^3)*cosh(x)^3 + 6*(a^3*b^2 -
a*b^4)*cosh(x)^2)*sinh(x)^2 - 3*(b^4*cosh(x)^6 + 6*b^4*cosh(x)*sinh(x)^5 +
 b^4*sinh(x)^6 + 3*b^4*cosh(x)^4 + 3*b^4*cosh(x)^2 + 3*(5*b^4*cosh(x)^2 +
b^4)*sinh(x)^4 + b^4 + 4*(5*b^4*cosh(x)^3 + 3*b^4*cosh(x))*sinh(x)^3 + 3*(
5*b^4*cosh(x)^4 + 6*b^4*cosh(x)^2 + b^4)*sinh(x)^2 + 6*(b^4*cosh(x)^5 + 2*
b^4*cosh(x)^3 + b^4*cosh(x))*sinh(x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 +
 b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(
x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(
x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)) + 3*((a^4*b + a^2*b^3
 - 2*b^5)*cosh(x)^6 + 6*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)*sinh(x)^5 + (a^4
*b + a^2*b^3 - 2*b^5)*sinh(x)^6 + a^4*b + a^2*b^3 - 2*b^5 + 3*(a^4*b + a^2
*b^3 - 2*b^5)*cosh(x)^4 + 3*(a^4*b + a^2*b^3 - 2*b^5 + 5*(a^4*b + a^2*b^3
- 2*b^5)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)^3 +
 3*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x))*sinh(x)^3 + 3*(a^4*b + a^2*b^3 - 2*b
^5)*cosh(x)^2 + 3*(a^4*b + a^2*b^3 - 2*b^5 + 5*(a^4*b + a^2*b^3 - 2*b^5)*c
osh(x)^4 + 6*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)^2)*sinh(x)^2 + 6*((a^4*b...
```

3.61.6 Sympy [F]

$$\int \frac{\operatorname{sech}^4(x)}{a+b\cosh(x)}\,dx=\int \frac{\operatorname{sech}^4{(x)}}{a+b\cosh{(x)}}\,dx$$

input
```
integrate(sech(x)**4/(a+b*cosh(x)),x)
```

output
```
Integral(sech(x)**4/(a + b*cosh(x)), x)
```

3.61.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^4(x)}{a+b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sech(x)^4/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.61.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.08

$$\int \frac{\operatorname{sech}^4(x)}{a+b\cosh(x)}\,dx = \frac{2\,b^4\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}a^4} - \frac{(a^2b+2\,b^3)\arctan\left(e^x\right)}{a^4} - \frac{3\,abe^{(5\,x)}+6\,b^2e^{(4\,x)}+12\,a^2e^{(2\,x)}+12\,b^2e^{(2\,x)}-3\,abe^x+4\,a^2+6\,b^2}{3\,a^3\left(e^{(2\,x)}+1\right)^3}$$

input
```
integrate(sech(x)^4/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
2*b^4*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a^4) - (a^2*b
 + 2*b^3)*arctan(e^x)/a^4 - 1/3*(3*a*b*e^(5*x) + 6*b^2*e^(4*x) + 12*a^2*e^
(2*x) + 12*b^2*e^(2*x) - 3*a*b*e^x + 4*a^2 + 6*b^2)/(a^3*(e^(2*x) + 1)^3)
```

3.61.9 Mupad [B] (verification not implemented)

Time = 5.64 (sec) , antiderivative size = 547, normalized size of antiderivative = 4.80

$$\int \frac{\operatorname{sech}^4(x)}{a+b\cosh(x)}\,dx = \frac{8}{3\,\left(a+3\,a\,\mathrm{e}^{2\,x}+3\,a\,\mathrm{e}^{4\,x}+a\,\mathrm{e}^{6\,x}\right)}-\frac{4}{a+2\,a\,\mathrm{e}^{2\,x}+a\,\mathrm{e}^{4\,x}}$$
$$-\frac{2\,b^2}{a^3\,\mathrm{e}^{2\,x}+a^3}+\frac{b^3\,\left(\ln\left(\mathrm{e}^x-\mathrm{i}\right)\,1\mathrm{i}-\ln\left(\mathrm{e}^x+1\mathrm{i}\right)\,1\mathrm{i}\right)}{a^4}-\frac{b\,\mathrm{e}^x}{a^2\,\mathrm{e}^{2\,x}+a^2}$$
$$+\frac{2\,b\,\mathrm{e}^x}{2\,a^2\,\mathrm{e}^{2\,x}+a^2\,\mathrm{e}^{4\,x}+a^2}+\frac{b\,\left(\ln\left(\mathrm{e}^x-\mathrm{i}\right)\,1\mathrm{i}-\ln\left(\mathrm{e}^x+1\mathrm{i}\right)\,1\mathrm{i}\right)}{2\,a^2}$$
$$+\frac{b^4\,\ln\left(32\,a^3\,b^4-24\,b^6\,\sqrt{a^2-b^2}-48\,a\,b^6+16\,a^5\,b^2+24\,b^7\,\mathrm{e}^x+32\,a^6\,b\,\mathrm{e}^x+40\,a^2\,b^4\,\sqrt{a^2-b^2}+16\,a^4\,b^2}{a^4\,\sqrt{a^2}}$$
$$-\frac{b^4\,\ln\left(24\,b^6\,\sqrt{a^2-b^2}-48\,a\,b^6+32\,a^3\,b^4+16\,a^5\,b^2+24\,b^7\,\mathrm{e}^x+32\,a^6\,b\,\mathrm{e}^x-40\,a^2\,b^4\,\sqrt{a^2-b^2}-16\,a^4\,b^2}{a^4\,\sqrt{a^2}}$$

input
```
int(1/(cosh(x)^4*(a + b*cosh(x))),x)
```

output
```
8/(3*(a + 3*a*exp(2*x) + 3*a*exp(4*x) + a*exp(6*x))) - 4/(a + 2*a*exp(2*x)
 + a*exp(4*x)) - (2*b^2)/(a^3*exp(2*x) + a^3) + (b^3*(log(exp(x) - 1i)*1i
- log(exp(x) + 1i)*1i))/a^4 - (b*exp(x))/(a^2*exp(2*x) + a^2) + (2*b*exp(x
))/(2*a^2*exp(2*x) + a^2*exp(4*x) + a^2) + (b*(log(exp(x) - 1i)*1i - log(e
xp(x) + 1i)*1i))/(2*a^2) + (b^4*log(32*a^3*b^4 - 24*b^6*(a^2 - b^2)^(1/2)
- 48*a*b^6 + 16*a^5*b^2 + 24*b^7*exp(x) + 32*a^6*b*exp(x) + 40*a^2*b^4*(a^
2 - b^2)^(1/2) + 16*a^4*b^2*(a^2 - b^2)^(1/2) - 112*a^2*b^5*exp(x) + 56*a^
4*b^3*exp(x) + 72*a^3*b^3*exp(x)*(a^2 - b^2)^(1/2) - 72*a*b^5*exp(x)*(a^2
- b^2)^(1/2) + 32*a^5*b*exp(x)*(a^2 - b^2)^(1/2)))/(a^4*(a^2 - b^2)^(1/2))
 - (b^4*log(24*b^6*(a^2 - b^2)^(1/2) - 48*a*b^6 + 32*a^3*b^4 + 16*a^5*b^2
+ 24*b^7*exp(x) + 32*a^6*b*exp(x) - 40*a^2*b^4*(a^2 - b^2)^(1/2) - 16*a^4*
b^2*(a^2 - b^2)^(1/2) - 112*a^2*b^5*exp(x) + 56*a^4*b^3*exp(x) - 72*a^3*b^
3*exp(x)*(a^2 - b^2)^(1/2) + 72*a*b^5*exp(x)*(a^2 - b^2)^(1/2) - 32*a^5*b*
exp(x)*(a^2 - b^2)^(1/2)))/(a^4*(a^2 - b^2)^(1/2))
```

3.62 $\int (a+b\cosh (c+dx))^5\,dx$

3.62.1 Optimal result

Integrand size = 12, antiderivative size = 183

$$\begin{aligned}\int (a+b\cosh (c+dx))^5\,dx =& \frac{1}{8}a\left(8a^4+40a^2b^2+15b^4\right)x \\ &+\frac{b(107a^4+192a^2b^2+16b^4)\sinh (c+dx)}{30d} \\ &+\frac{7ab^2(22a^2+23b^2)\cosh (c+dx)\sinh (c+dx)}{120d} \\ &+\frac{b(47a^2+16b^2)\,(a+b\cosh (c+dx))^2\sinh (c+dx)}{60d} \\ &+\frac{9ab(a+b\cosh (c+dx))^3\sinh (c+dx)}{20d} \\ &+\frac{b(a+b\cosh (c+dx))^4\sinh (c+dx)}{5d}\end{aligned}$$

output
```
1/8*a*(8*a^4+40*a^2*b^2+15*b^4)*x+1/30*b*(107*a^4+192*a^2*b^2+16*b^4)*sinh
(d*x+c)/d+7/120*a*b^2*(22*a^2+23*b^2)*cosh(d*x+c)*sinh(d*x+c)/d+1/60*b*(47
*a^2+16*b^2)*(a+b*cosh(d*x+c))^2*sinh(d*x+c)/d+9/20*a*b*(a+b*cosh(d*x+c))^
3*sinh(d*x+c)/d+1/5*b*(a+b*cosh(d*x+c))^4*sinh(d*x+c)/d
```

3.62.2 Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.73

$$\int (a+b\cosh(c+dx))^5\,dx$$
$$=\frac{60a(8a^4+40a^2b^2+15b^4)(c+dx)+300b(8a^4+12a^2b^2+b^4)\sinh(c+dx)+600ab^2(2a^2+b^2)\sinh(2(c+}{480d}$$

input
```
Integrate[(a + b*Cosh[c + d*x])^5,x]
```

output
```
(60*a*(8*a^4 + 40*a^2*b^2 + 15*b^4)*(c + d*x) + 300*b*(8*a^4 + 12*a^2*b^2
+ b^4)*Sinh[c + d*x] + 600*a*b^2*(2*a^2 + b^2)*Sinh[2*(c + d*x)] + 50*b^3*
(8*a^2 + b^2)*Sinh[3*(c + d*x)] + 75*a*b^4*Sinh[4*(c + d*x)] + 6*b^5*Sinh[
5*(c + d*x)])/(480*d)
```

3.62.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 3135, 3042, 3232, 3042, 3232, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cosh(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^5dx$$

$$\Big\downarrow \text{3135}$$

$$\frac{1}{5}\int (a+b\cosh(c+dx))^3\left(5a^2+9b\cosh(c+dx)a+4b^2\right)dx+\frac{b\sinh(c+dx)(a+b\cosh(c+dx))^4}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sinh(c+dx)(a+b\cosh(c+dx))^4}{5d}+$$
$$\frac{1}{5}\int \left(a+b\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^3\left(5a^2+9b\sin\left(ic+idx+\frac{\pi}{2}\right)a+4b^2\right)dx$$

$$\Big\downarrow \text{3232}$$

$$\frac{1}{5}\left(\frac{1}{4}\int (a+b\cosh(c+dx))^2\left(a\left(20a^2+43b^2\right)+b\left(47a^2+16b^2\right)\cosh(c+dx)\right)dx+\frac{9ab\sinh(c+dx)(a+b\cosh(c\right.$$
$$\frac{b\sinh(c+dx)(a+b\cosh(c+dx))^4}{5d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b\sinh(c+dx)(a+b\cosh(c+dx))^4}{5d}+$$
$$\frac{1}{5}\left(\frac{9ab\sinh(c+dx)(a+b\cosh(c+dx))^3}{4d}+\frac{1}{4}\int\left(a+b\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^2\left(a\left(20a^2+43b^2\right)+b\left(47a^2+16b^2\right)s\right.\right.$$

$$\Big\downarrow \text{ 3232}$$

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\int (a+b\cosh(c+dx))\left(60a^4+223b^2a^2+7b\left(22a^2+23b^2\right)\cosh(c+dx)a+32b^4\right)dx+\frac{b\left(47a^2+16b^2\right)\sin}{}\right.\right.$$
$$\frac{b\sinh(c+dx)(a+b\cosh(c+dx))^4}{5d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b\sinh(c+dx)(a+b\cosh(c+dx))^4}{5d}+$$
$$\frac{1}{5}\left(\frac{9ab\sinh(c+dx)(a+b\cosh(c+dx))^3}{4d}+\frac{1}{4}\left(\frac{b\left(47a^2+16b^2\right)\sinh(c+dx)(a+b\cosh(c+dx))^2}{3d}+\frac{1}{3}\int\left(a+b\sin\right.\right.\right.$$

$$\Big\downarrow \text{ 3213}$$

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{b\left(47a^2+16b^2\right)\sinh(c+dx)(a+b\cosh(c+dx))^2}{3d}+\frac{1}{3}\left(\frac{7ab^2\left(22a^2+23b^2\right)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{2b}{}\right.\right.\right.$$
$$\frac{b\sinh(c+dx)(a+b\cosh(c+dx))^4}{5d}$$

input
```
Int[(a + b*Cosh[c + d*x])^5,x]
```

output
```
(b*(a + b*Cosh[c + d*x])^4*Sinh[c + d*x])/(5*d) + ((9*a*b*(a + b*Cosh[c +
d*x])^3*Sinh[c + d*x])/(4*d) + ((b*(47*a^2 + 16*b^2)*(a + b*Cosh[c + d*x])
^2*Sinh[c + d*x])/(3*d) + ((15*a*(8*a^4 + 40*a^2*b^2 + 15*b^4)*x)/2 + (2*b
*(107*a^4 + 192*a^2*b^2 + 16*b^4)*Sinh[c + d*x])/d + (7*a*b^2*(22*a^2 + 23
*b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(2*d))/3)/4)/5
```

3.62. $\int (a+b\cosh(c+dx))^5\,dx$

3.62.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3135
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[1/n   Int[(a + b*
Sin[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x]
, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
 IntegerQ[2*n]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

3.62.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.72

method	result
parallelrisch	$\frac{600(2a^3b^2+ab^4)\sinh(2dx+2c)+50(8a^2b^3+b^5)\sinh(3dx+3c)+75ab^4\sinh(4dx+4c)+6b^5\sinh(5dx+5c)+300(8a^4b+12a}{480d}$
derivativedivides	$\frac{b^5\left(\frac{8}{15}+\frac{\cosh(dx+c)^4}{5}+\frac{4\cosh(dx+c)^2}{15}\right)\sinh(dx+c)+5ab^4\left(\left(\frac{\cosh(dx+c)^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+10a^2b^3}{d}$
default	$\frac{b^5\left(\frac{8}{15}+\frac{\cosh(dx+c)^4}{5}+\frac{4\cosh(dx+c)^2}{15}\right)\sinh(dx+c)+5ab^4\left(\left(\frac{\cosh(dx+c)^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+10a^2b^3}{d}$
parts	$a^5x+\frac{b^5\left(\frac{8}{15}+\frac{\cosh(dx+c)^4}{5}+\frac{4\cosh(dx+c)^2}{15}\right)\sinh(dx+c)}{d}+\frac{5a^4b\sinh(dx+c)}{d}+\frac{10a^3b^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\right.}{d}$
risch	$a^5x+5a^3b^2x+\frac{15ab^4x}{8}+\frac{b^5e^{5dx+5c}}{160d}+\frac{5ab^4e^{4dx+4c}}{64d}+\frac{5b^3e^{3dx+3c}a^2}{12d}+\frac{5b^5e^{3dx+3c}}{96d}+\frac{5a^3b^2e^{2dx+2c}}{4d}+$

input
```
int((a+b*cosh(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
1/480*(600*(2*a^3*b^2+a*b^4)*sinh(2*d*x+2*c)+50*(8*a^2*b^3+b^5)*sinh(3*d*x
+3*c)+75*a*b^4*sinh(4*d*x+4*c)+6*b^5*sinh(5*d*x+5*c)+300*(8*a^4*b+12*a^2*b
^3+b^5)*sinh(d*x+c)+480*d*(a^4+5*a^2*b^2+15/8*b^4)*x*a)/d
```

3.62.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.04

$$\int (a+b\cosh(c+dx))^5\,dx$$
$$= \frac{3\,b^5\sinh\left(dx+c\right)^5+5\left(6\,b^5\cosh\left(dx+c\right)^2+30\,ab^4\cosh\left(dx+c\right)+40\,a^2b^3+5\,b^5\right)\sinh\left(dx+c\right)^3+30\,(8\,a$$

input
```
integrate((a+b*cosh(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/240*(3*b^5*sinh(d*x + c)^5 + 5*(6*b^5*cosh(d*x + c)^2 + 30*a*b^4*cosh(d*
x + c) + 40*a^2*b^3 + 5*b^5)*sinh(d*x + c)^3 + 30*(8*a^5 + 40*a^3*b^2 + 15
*a*b^4)*d*x + 15*(b^5*cosh(d*x + c)^4 + 10*a*b^4*cosh(d*x + c)^3 + 80*a^4*
b + 120*a^2*b^3 + 10*b^5 + 5*(8*a^2*b^3 + b^5)*cosh(d*x + c)^2 + 40*(2*a^3
*b^2 + a*b^4)*cosh(d*x + c))*sinh(d*x + c))/d
```

3.62.6 Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.72

$$\int (a+b\cosh(c+dx))^5\,dx$$
$$= \begin{cases} a^5x+\frac{5a^4b\sinh\left(c+dx\right)}{d}-5a^3b^2x\sinh^2\left(c+dx\right)+5a^3b^2x\cosh^2\left(c+dx\right)+\frac{5a^3b^2\sinh\left(c+dx\right)\cosh\left(c+dx\right)}{d}-\frac{20a^2b^3\sin}{3} \\ x(a+b\cosh\left(c\right))^5 \end{cases}$$

input
```
integrate((a+b*cosh(d*x+c))**5,x)
```

output
```
Piecewise((a**5*x + 5*a**4*b*sinh(c + d*x)/d - 5*a**3*b**2*x*sinh(c + d*x)
**2 + 5*a**3*b**2*x*cosh(c + d*x)**2 + 5*a**3*b**2*sinh(c + d*x)*cosh(c +
d*x)/d - 20*a**2*b**3*sinh(c + d*x)**3/(3*d) + 10*a**2*b**3*sinh(c + d*x)*
cosh(c + d*x)**2/d + 15*a*b**4*x*sinh(c + d*x)**4/8 - 15*a*b**4*x*sinh(c +
 d*x)**2*cosh(c + d*x)**2/4 + 15*a*b**4*x*cosh(c + d*x)**4/8 - 15*a*b**4*s
inh(c + d*x)**3*cosh(c + d*x)/(8*d) + 25*a*b**4*sinh(c + d*x)*cosh(c + d*x
)**3/(8*d) + 8*b**5*sinh(c + d*x)**5/(15*d) - 4*b**5*sinh(c + d*x)**3*cosh
(c + d*x)**2/(3*d) + b**5*sinh(c + d*x)*cosh(c + d*x)**4/d, Ne(d, 0)), (x*
(a + b*cosh(c))**5, True))
```

3.62.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.49

$$\int (a + b\cosh(c + dx))^5\,dx$$
$$= \frac{5}{64}ab^4\left(24\,x + \frac{e^{(4\,dx+4\,c)}}{d} + \frac{8\,e^{(2\,dx+2\,c)}}{d} - \frac{8\,e^{(-2\,dx-2\,c)}}{d} - \frac{e^{(-4\,dx-4\,c)}}{d}\right)$$
$$+ \frac{5}{4}a^3b^2\left(4\,x + \frac{e^{(2\,dx+2\,c)}}{d} - \frac{e^{(-2\,dx-2\,c)}}{d}\right) + a^5x$$
$$+ \frac{1}{480}b^5\left(\frac{3\,e^{(5\,dx+5\,c)}}{d} + \frac{25\,e^{(3\,dx+3\,c)}}{d} + \frac{150\,e^{(dx+c)}}{d} - \frac{150\,e^{(-dx-c)}}{d} - \frac{25\,e^{(-3\,dx-3\,c)}}{d} - \frac{3\,e^{(-5\,dx-5\,c)}}{d}\right)$$
$$+ \frac{5}{12}a^2b^3\left(\frac{e^{(3\,dx+3\,c)}}{d} + \frac{9\,e^{(dx+c)}}{d} - \frac{9\,e^{(-dx-c)}}{d} - \frac{e^{(-3\,dx-3\,c)}}{d}\right) + \frac{5\,a^4b\sinh(dx + c)}{d}$$

input
```
integrate((a+b*cosh(d*x+c))^5,x, algorithm="maxima")
```

output
```
5/64*a*b^4*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x -
 2*c)/d - e^(-4*d*x - 4*c)/d) + 5/4*a^3*b^2*(4*x + e^(2*d*x + 2*c)/d - e^(
-2*d*x - 2*c)/d) + a^5*x + 1/480*b^5*(3*e^(5*d*x + 5*c)/d + 25*e^(3*d*x +
3*c)/d + 150*e^(d*x + c)/d - 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d -
3*e^(-5*d*x - 5*c)/d) + 5/12*a^2*b^3*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d
- 9*e^(-d*x - c)/d - e^(-3*d*x - 3*c)/d) + 5*a^4*b*sinh(d*x + c)/d
```

3.62.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.44

$$\int (a+b\cosh(c+dx))^5\,dx = \frac{b^5e^{(5\,dx+5\,c)}}{160\,d} + \frac{5\,ab^4e^{(4\,dx+4\,c)}}{64\,d} - \frac{5\,ab^4e^{(-4\,dx-4\,c)}}{64\,d} - \frac{b^5e^{(-5\,dx-5\,c)}}{160\,d} + \frac{1}{8}\left(8\,a^5+40\,a^3b^2+15\,ab^4\right)x + \frac{5\left(8\,a^2b^3+b^5\right)e^{(3\,dx+3\,c)}}{96\,d} + \frac{5\left(2\,a^3b^2+ab^4\right)e^{(2\,dx+2\,c)}}{8\,d} + \frac{5\left(8\,a^4b+12\,a^2b^3+b^5\right)e^{(dx+c)}}{16\,d} - \frac{5\left(8\,a^4b+12\,a^2b^3+b^5\right)e^{(-dx-c)}}{16\,d} - \frac{5\left(2\,a^3b^2+ab^4\right)e^{(-2\,dx-2\,c)}}{8\,d} - \frac{5\left(8\,a^2b^3+b^5\right)e^{(-3\,dx-3\,c)}}{96\,d}$$

input
```
integrate((a+b*cosh(d*x+c))^5,x, algorithm="giac")
```

output
```
1/160*b^5*e^(5*d*x + 5*c)/d + 5/64*a*b^4*e^(4*d*x + 4*c)/d - 5/64*a*b^4*e^
(-4*d*x - 4*c)/d - 1/160*b^5*e^(-5*d*x - 5*c)/d + 1/8*(8*a^5 + 40*a^3*b^2
+ 15*a*b^4)*x + 5/96*(8*a^2*b^3 + b^5)*e^(3*d*x + 3*c)/d + 5/8*(2*a^3*b^2
+ a*b^4)*e^(2*d*x + 2*c)/d + 5/16*(8*a^4*b + 12*a^2*b^3 + b^5)*e^(d*x + c)
/d - 5/16*(8*a^4*b + 12*a^2*b^3 + b^5)*e^(-d*x - c)/d - 5/8*(2*a^3*b^2 + a
*b^4)*e^(-2*d*x - 2*c)/d - 5/96*(8*a^2*b^3 + b^5)*e^(-3*d*x - 3*c)/d
```

3.62.9 Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.87

$$\int (a+b\cosh(c+dx))^5\,dx$$

$$= \frac{75\,b^5\sinh(c+d\,x) + \frac{25\,b^5\sinh(3\,c+3\,d\,x)}{2} + \frac{3\,b^5\sinh(5\,c+5\,d\,x)}{2} + 150\,a\,b^4\sinh(2\,c+2\,d\,x) + \frac{75\,a\,b^4\sinh(4\,c+4\,d\,x)}{4} + 90}{}$$

input
```
int((a + b*cosh(c + d*x))^5,x)
```

output
```
(75*b^5*sinh(c + d*x) + (25*b^5*sinh(3*c + 3*d*x))/2 + (3*b^5*sinh(5*c + 5
*d*x))/2 + 150*a*b^4*sinh(2*c + 2*d*x) + (75*a*b^4*sinh(4*c + 4*d*x))/4 +
900*a^2*b^3*sinh(c + d*x) + 300*a^3*b^2*sinh(2*c + 2*d*x) + 100*a^2*b^3*si
nh(3*c + 3*d*x) + 600*a^4*b*sinh(c + d*x) + 120*a^5*d*x + 225*a*b^4*d*x +
600*a^3*b^2*d*x)/(120*d)
```

3.63 $\int (a+b\cosh(c+dx))^4\,dx$

3.63.1 Optimal result

Integrand size = 12, antiderivative size = 137

$$\begin{aligned}\int (a+b\cosh(c+dx))^4\,dx &= \frac{1}{8}\left(8a^4+24a^2b^2+3b^4\right)x+\frac{ab(19a^2+16b^2)\sinh(c+dx)}{6d}\\ &+\frac{b^2(26a^2+9b^2)\cosh(c+dx)\sinh(c+dx)}{24d}\\ &+\frac{7ab(a+b\cosh(c+dx))^2\sinh(c+dx)}{12d}\\ &+\frac{b(a+b\cosh(c+dx))^3\sinh(c+dx)}{4d}\end{aligned}$$

output
```
1/8*(8*a^4+24*a^2*b^2+3*b^4)*x+1/6*a*b*(19*a^2+16*b^2)*sinh(d*x+c)/d+1/24*
b^2*(26*a^2+9*b^2)*cosh(d*x+c)*sinh(d*x+c)/d+7/12*a*b*(a+b*cosh(d*x+c))^2*
sinh(d*x+c)/d+1/4*b*(a+b*cosh(d*x+c))^3*sinh(d*x+c)/d
```

3.63.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.76

$$\int (a+b\cosh(c+dx))^4\,dx$$

$$=\frac{12\left(8a^4+24a^2b^2+3b^4\right)(c+dx)+96ab(4a^2+3b^2)\sinh(c+dx)+24b^2(6a^2+b^2)\sinh(2(c+dx))+32ab^3}{96d}$$

input
```
Integrate[(a + b*Cosh[c + d*x])^4,x]
```

output
```
(12*(8*a^4 + 24*a^2*b^2 + 3*b^4)*(c + d*x) + 96*a*b*(4*a^2 + 3*b^2)*Sinh[c
 + d*x] + 24*b^2*(6*a^2 + b^2)*Sinh[2*(c + d*x)] + 32*a*b^3*Sinh[3*(c + d*
x)] + 3*b^4*Sinh[4*(c + d*x)])/(96*d)
```

3.63.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3135, 3042, 3232, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cosh(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^4dx$$

$$\Big\downarrow \text{3135}$$

$$\frac{1}{4}\int (a+b\cosh(c+dx))^2\left(4a^2+7b\cosh(c+dx)a+3b^2\right)dx+\frac{b\sinh(c+dx)(a+b\cosh(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sinh(c+dx)(a+b\cosh(c+dx))^3}{4d}+\frac{1}{4}\int \left(a+b\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^2\left(4a^2+7b\sin\left(ic+idx+\frac{\pi}{2}\right)a+3b^2\right)dx$$

$$\Big\downarrow \text{3232}$$

$$\frac{1}{4}\left(\frac{1}{3}\int (a+b\cosh(c+dx))\left(a\left(12a^2+23b^2\right)+b\left(26a^2+9b^2\right)\cosh(c+dx)\right)dx+\frac{7ab\sinh(c+dx)(a+b\cosh(c+\right.}{3d}\ \frac{b\sinh(c+dx)(a+b\cosh(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sinh(c+dx)(a+b\cosh(c+dx))^3}{4d}+$$
$$\frac{1}{4}\left(\frac{7ab\sinh(c+dx)(a+b\cosh(c+dx))^2}{3d}+\frac{1}{3}\int\left(a+b\sin\left(ic+idx+\frac{\pi}{2}\right)\right)\left(a(12a^2+23b^2)+b(26a^2+9b^2)\sin\right.\right.$$

$\downarrow$ 3213

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{2ab(19a^2+16b^2)\sinh(c+dx)}{d}+\frac{b^2(26a^2+9b^2)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{3}{2}x(8a^4+24a^2b^2+3b^4)\right)+\right.$$
$$\frac{b\sinh(c+dx)(a+b\cosh(c+dx))^3}{4d}$$

input
```
Int[(a + b*Cosh[c + d*x])^4,x]
```

output
```
(b*(a + b*Cosh[c + d*x])^3*Sinh[c + d*x])/(4*d) + ((7*a*b*(a + b*Cosh[c +
d*x])^2*Sinh[c + d*x])/(3*d) + ((3*(8*a^4 + 24*a^2*b^2 + 3*b^4)*x)/2 + (2*
a*b*(19*a^2 + 16*b^2)*Sinh[c + d*x])/d + (b^2*(26*a^2 + 9*b^2)*Cosh[c + d*
x]*Sinh[c + d*x])/(2*d))/3)/4
```

3.63.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3135
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[1/n   Int[(a + b*
Sin[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x]
, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
 IntegerQ[2*n]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

3.63.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.74

method	result
parallelrisch	$\frac{24(6a^2b^2+b^4)\sinh(2dx+2c)+32a\,b^3\sinh(3dx+3c)+3b^4\sinh(4dx+4c)+96(4a^3b+3a\,b^3)\sinh(dx+c)+96dx(a^4+3a^2b^2+}{96d}$
derivativedivides	$\frac{b^4\left(\left(\frac{\cosh(dx+c)^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+4a\,b^3\left(\frac{2}{3}+\frac{\cosh(dx+c)^2}{3}\right)\sinh(dx+c)+6a^2b^2\left(\frac{\cosh(dx+c)\sinh(}{2}\right.}{d}$
default	$\frac{b^4\left(\left(\frac{\cosh(dx+c)^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+4a\,b^3\left(\frac{2}{3}+\frac{\cosh(dx+c)^2}{3}\right)\sinh(dx+c)+6a^2b^2\left(\frac{\cosh(dx+c)\sinh(}{2}\right.}{d}$
parts	$x\,a^4+\frac{b^4\left(\left(\frac{\cosh(dx+c)^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{4a^3b\sinh(dx+c)}{d}+\frac{6a^2b^2\left(\frac{\cosh(dx+c)\sinh(dx+c}{2}\right.}{d}$
risch	$x\,a^4+3x\,a^2b^2+\frac{3x\,b^4}{8}+\frac{b^4e^{4dx+4c}}{64d}+\frac{a\,b^3e^{3dx+3c}}{6d}+\frac{3b^2e^{2dx+2c}a^2}{4d}+\frac{b^4e^{2dx+2c}}{8d}+\frac{2a^3b\,e^{dx+c}}{d}+\frac{3a\,b^3e}{2c}$

input
```
int((a+b*cosh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/96*(24*(6*a^2*b^2+b^4)*sinh(2*d*x+2*c)+32*a*b^3*sinh(3*d*x+3*c)+3*b^4*si
nh(4*d*x+4*c)+96*(4*a^3*b+3*a*b^3)*sinh(d*x+c)+96*d*x*(a^4+3*a^2*b^2+3/8*b
^4))/d
```

3.63.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.90

$$\int (a+b\cosh(c+dx))^4\,dx$$
$$=\frac{(3\,b^4\cosh(dx+c)+8\,ab^3)\sinh(dx+c)^3+3\,(8\,a^4+24\,a^2b^2+3\,b^4)dx+3\left(b^4\cosh(dx+c)^3+8\,ab^3\cosh\right.}{24\,d}$$

input
```
integrate((a+b*cosh(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/24*((3*b^4*cosh(d*x + c) + 8*a*b^3)*sinh(d*x + c)^3 + 3*(8*a^4 + 24*a^2*
b^2 + 3*b^4)*d*x + 3*(b^4*cosh(d*x + c)^3 + 8*a*b^3*cosh(d*x + c)^2 + 32*a
^3*b + 24*a*b^3 + 4*(6*a^2*b^2 + b^4)*cosh(d*x + c))*sinh(d*x + c))/d
```

3.63.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.75

$$\int (a+b\cosh(c+dx))^4\,dx$$
$$=\begin{cases} a^4x+\frac{4a^3b\sinh{(c+dx)}}{d}-3a^2b^2x\sinh^2{(c+dx)}+3a^2b^2x\cosh^2{(c+dx)}+\frac{3a^2b^2\sinh{(c+dx)}\cosh{(c+dx)}}{d}-\frac{8ab^3\sinh^3}{3d} \\ x(a+b\cosh{(c)})^4 \end{cases}$$

input
```
integrate((a+b*cosh(d*x+c))**4,x)
```

output
```
Piecewise((a**4*x + 4*a**3*b*sinh(c + d*x)/d - 3*a**2*b**2*x*sinh(c + d*x)
**2 + 3*a**2*b**2*x*cosh(c + d*x)**2 + 3*a**2*b**2*sinh(c + d*x)*cosh(c +
d*x)/d - 8*a*b**3*sinh(c + d*x)**3/(3*d) + 4*a*b**3*sinh(c + d*x)*cosh(c +
 d*x)**2/d + 3*b**4*x*sinh(c + d*x)**4/8 - 3*b**4*x*sinh(c + d*x)**2*cosh(
c + d*x)**2/4 + 3*b**4*x*cosh(c + d*x)**4/8 - 3*b**4*sinh(c + d*x)**3*cosh
(c + d*x)/(8*d) + 5*b**4*sinh(c + d*x)*cosh(c + d*x)**3/(8*d), Ne(d, 0)),
(x*(a + b*cosh(c))**4, True))
```

3.63.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.34

$$\int (a+b\cosh(c+dx))^4\,dx$$
$$=\frac{1}{64}b^4\left(24x+\frac{e^{(4dx+4c)}}{d}+\frac{8e^{(2dx+2c)}}{d}-\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)$$
$$+\frac{3}{4}a^2b^2\left(4x+\frac{e^{(2dx+2c)}}{d}-\frac{e^{(-2dx-2c)}}{d}\right)+a^4x$$
$$+\frac{1}{6}ab^3\left(\frac{e^{(3dx+3c)}}{d}+\frac{9e^{(dx+c)}}{d}-\frac{9e^{(-dx-c)}}{d}-\frac{e^{(-3dx-3c)}}{d}\right)+\frac{4a^3b\sinh{(dx+c)}}{d}$$

input
```
integrate((a+b*cosh(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/64*b^4*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2
*c)/d - e^(-4*d*x - 4*c)/d) + 3/4*a^2*b^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2
*d*x - 2*c)/d) + a^4*x + 1/6*a*b^3*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d -
9*e^(-d*x - c)/d - e^(-3*d*x - 3*c)/d) + 4*a^3*b*sinh(d*x + c)/d
```

3.63.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.43

$$\begin{aligned}\int (a+b\cosh(c+dx))^4\,dx &= \frac{b^4e^{(4\,dx+4\,c)}}{64\,d}+\frac{ab^3e^{(3\,dx+3\,c)}}{6\,d}-\frac{ab^3e^{(-3\,dx-3\,c)}}{6\,d}\\ &\quad-\frac{b^4e^{(-4\,dx-4\,c)}}{64\,d}+\frac{1}{8}\left(8\,a^4+24\,a^2b^2+3\,b^4\right)x\\ &\quad+\frac{(6\,a^2b^2+b^4)e^{(2\,dx+2\,c)}}{8\,d}+\frac{(4\,a^3b+3\,ab^3)e^{(dx+c)}}{2\,d}\\ &\quad-\frac{(4\,a^3b+3\,ab^3)e^{(-dx-c)}}{2\,d}-\frac{(6\,a^2b^2+b^4)e^{(-2\,dx-2\,c)}}{8\,d}\end{aligned}$$

input
```
integrate((a+b*cosh(d*x+c))^4,x, algorithm="giac")
```

output
```
1/64*b^4*e^(4*d*x + 4*c)/d + 1/6*a*b^3*e^(3*d*x + 3*c)/d - 1/6*a*b^3*e^(-3
*d*x - 3*c)/d - 1/64*b^4*e^(-4*d*x - 4*c)/d + 1/8*(8*a^4 + 24*a^2*b^2 + 3*
b^4)*x + 1/8*(6*a^2*b^2 + b^4)*e^(2*d*x + 2*c)/d + 1/2*(4*a^3*b + 3*a*b^3)
*e^(d*x + c)/d - 1/2*(4*a^3*b + 3*a*b^3)*e^(-d*x - c)/d - 1/8*(6*a^2*b^2 +
 b^4)*e^(-2*d*x - 2*c)/d
```

3.63.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.83

$$\int (a+b\cosh(c+dx))^4\,dx$$
$$=\frac{6\,b^4\sinh(2\,c+2\,d\,x)+\frac{3\,b^4\sinh(4\,c+4\,d\,x)}{4}+8\,a\,b^3\sinh(3\,c+3\,d\,x)+36\,a^2\,b^2\sinh(2\,c+2\,d\,x)+72\,a\,b^3\sinh(c}{24\,d}$$

input
```
int((a + b*cosh(c + d*x))^4,x)
```

output
```
(6*b^4*sinh(2*c + 2*d*x) + (3*b^4*sinh(4*c + 4*d*x))/4 + 8*a*b^3*sinh(3*c
+ 3*d*x) + 36*a^2*b^2*sinh(2*c + 2*d*x) + 72*a*b^3*sinh(c + d*x) + 96*a^3*
b*sinh(c + d*x) + 24*a^4*d*x + 9*b^4*d*x + 72*a^2*b^2*d*x)/(24*d)
```

3.63. $\int (a + b\cosh(c + dx))^4 \, dx$

3.64 $\int (a+b\cosh(c+dx))^3\,dx$

3.64.1 Optimal result

Integrand size = 12, antiderivative size = 90

$$\int (a+b\cosh(c+dx))^3\,dx=\frac{1}{2}a\left(2a^2+3b^2\right)x+\frac{2b(4a^2+b^2)\sinh(c+dx)}{3d}+\frac{5ab^2\cosh(c+dx)\sinh(c+dx)}{6d}+\frac{b(a+b\cosh(c+dx))^2\sinh(c+dx)}{3d}$$

output
```
1/2*a*(2*a^2+3*b^2)*x+2/3*b*(4*a^2+b^2)*sinh(d*x+c)/d+5/6*a*b^2*cosh(d*x+c
)*sinh(d*x+c)/d+1/3*b*(a+b*cosh(d*x+c))^2*sinh(d*x+c)/d
```

3.64.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.89

$$\int (a+b\cosh(c+dx))^3\,dx = \frac{12a^3c+18ab^2c+12a^3dx+18ab^2dx+9b(4a^2+b^2)\sinh(c+dx)+9ab^2\sinh(2(c+dx))+b^3\sinh(3(c+d}{12d}$$

input
```
Integrate[(a + b*Cosh[c + d*x])^3,x]
```

output
```
(12*a^3*c + 18*a*b^2*c + 12*a^3*d*x + 18*a*b^2*d*x + 9*b*(4*a^2 + b^2)*Sin
h[c + d*x] + 9*a*b^2*Sinh[2*(c + d*x)] + b^3*Sinh[3*(c + d*x)])/(12*d)
```

3.64.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3135, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cosh(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{3135}$$

$$\frac{1}{3}\int (a+b\cosh(c+dx))\left(3a^2+5b\cosh(c+dx)a+2b^2\right)dx+\frac{b\sinh(c+dx)(a+b\cosh(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sinh(c+dx)(a+b\cosh(c+dx))^2}{3d}+\frac{1}{3}\int \left(a+b\sin\left(ic+idx+\frac{\pi}{2}\right)\right)\left(3a^2+5b\sin\left(ic+idx+\frac{\pi}{2}\right)a+2b^2\right)dx$$

$$\Big\downarrow \text{3213}$$

$$\frac{1}{3}\left(\frac{2b(4a^2+b^2)\sinh(c+dx)}{d}+\frac{3}{2}ax(2a^2+3b^2)+\frac{5ab^2\sinh(c+dx)\cosh(c+dx)}{2d}\right)+\frac{b\sinh(c+dx)(a+b\cosh(c+dx))^2}{3d}$$

input
```
Int[(a + b*Cosh[c + d*x])^3,x]
```

output
```
(b*(a + b*Cosh[c + d*x])^2*Sinh[c + d*x])/(3*d) + ((3*a*(2*a^2 + 3*b^2)*x)
/2 + (2*b*(4*a^2 + b^2)*Sinh[c + d*x])/d + (5*a*b^2*Cosh[c + d*x]*Sinh[c +
 d*x])/(2*d))/3
```

3.64.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3135
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[1/n   Int[(a + b*
Sin[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x]
, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
 IntegerQ[2*n]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.64.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

method	result
parallelrisch	$\frac{9 a \,b^{2} \sinh \left(2 d x +2 c \right)+b^{3} \sinh \left(3 d x +3 c \right)+9 \left(4 a^{2} b +b^{3}\right) \sinh \left(d x +c \right)+12 d \left(a^{2}+\frac{3 b^{2}}{2}\right) x a}{12 d}$
derivativedivides	$\frac{b^{3} \left(\frac{2}{3}+\frac{\cosh \left(d x +c \right)^{2}}{3}\right) \sinh \left(d x +c \right)+3 a \,b^{2} \left(\frac{\cosh \left(d x +c \right) \sinh \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)+3 a^{2} b \sinh \left(d x +c \right)+a^{3} \left(d x +c \right)}{d}$
default	$\frac{b^{3} \left(\frac{2}{3}+\frac{\cosh \left(d x +c \right)^{2}}{3}\right) \sinh \left(d x +c \right)+3 a \,b^{2} \left(\frac{\cosh \left(d x +c \right) \sinh \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)+3 a^{2} b \sinh \left(d x +c \right)+a^{3} \left(d x +c \right)}{d}$
parts	$a^{3} x +\frac{b^{3} \left(\frac{2}{3}+\frac{\cosh \left(d x +c \right)^{2}}{3}\right) \sinh \left(d x +c \right)}{d}+\frac{3 a^{2} b \sinh \left(d x +c \right)}{d}+\frac{3 a \,b^{2} \left(\frac{\cosh \left(d x +c \right) \sinh \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)}{d}$
risch	$a^{3} x +\frac{3 a \,b^{2} x}{2}+\frac{b^{3} {\mathrm e}^{3 d x +3 c}}{24 d}+\frac{3 a \,b^{2} {\mathrm e}^{2 d x +2 c}}{8 d}+\frac{3 b \,{\mathrm e}^{d x +c} a^{2}}{2 d}+\frac{3 b^{3} {\mathrm e}^{d x +c}}{8 d}-\frac{3 b \,{\mathrm e}^{-d x -c} a^{2}}{2 d}-\frac{3 b^{3} {\mathrm e}^{-d x -c}}{8 d}-$ 3a b

input
```
int((a+b*cosh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/12*(9*a*b^2*sinh(2*d*x+2*c)+b^3*sinh(3*d*x+3*c)+9*(4*a^2*b+b^3)*sinh(d*x
+c)+12*d*(a^2+3/2*b^2)*x*a)/d
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.87

$$\int (a+b\cosh(c+dx))^3\,dx$$
$$= \frac{b^3\sinh\left(dx+c\right)^3 + 6\left(2\,a^3+3\,ab^2\right)dx + 3\left(b^3\cosh\left(dx+c\right)^2 + 6\,ab^2\cosh\left(dx+c\right) + 12\,a^2b + 3\,b^3\right)\sinh\left(dx\right.}{12\,d}$$

input
```
integrate((a+b*cosh(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/12*(b^3*sinh(d*x + c)^3 + 6*(2*a^3 + 3*a*b^2)*d*x + 3*(b^3*cosh(d*x + c)
^2 + 6*a*b^2*cosh(d*x + c) + 12*a^2*b + 3*b^3)*sinh(d*x + c))/d
```

3.64.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.42

$$\int (a+b\cosh(c+dx))^3\,dx$$
$$= \begin{cases} a^3x + \frac{3a^2b\sinh(c+dx)}{d} - \frac{3ab^2x\sinh^2(c+dx)}{2} + \frac{3ab^2x\cosh^2(c+dx)}{2} + \frac{3ab^2\sinh(c+dx)\cosh(c+dx)}{2d} - \frac{2b^3\sinh^3(c+dx)}{3d} + \frac{b^3\sinh(\ldots}{} \\ x(a+b\cosh(c))^3 \end{cases}$$

input
```
integrate((a+b*cosh(d*x+c))**3,x)
```

output
```
Piecewise((a**3*x + 3*a**2*b*sinh(c + d*x)/d - 3*a*b**2*x*sinh(c + d*x)**2
/2 + 3*a*b**2*x*cosh(c + d*x)**2/2 + 3*a*b**2*sinh(c + d*x)*cosh(c + d*x)/
(2*d) - 2*b**3*sinh(c + d*x)**3/(3*d) + b**3*sinh(c + d*x)*cosh(c + d*x)**
2/d, Ne(d, 0)), (x*(a + b*cosh(c))**3, True))
```

3.64.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.29

$$\int (a+b\cosh(c+dx))^3\,dx = \frac{3}{8}\,ab^2\left(4\,x+\frac{e^{(2\,dx+2\,c)}}{d}-\frac{e^{(-2\,dx-2\,c)}}{d}\right)+a^3x + \frac{1}{24}\,b^3\left(\frac{e^{(3\,dx+3\,c)}}{d}+\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}-\frac{e^{(-3\,dx-3\,c)}}{d}\right) + \frac{3\,a^2b\sinh\left(dx+c\right)}{d}$$

input
```
integrate((a+b*cosh(d*x+c))^3,x, algorithm="maxima")
```

output
```
3/8*a*b^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) + a^3*x + 1/24*b^
3*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d - e^(-3*d*x - 3*
c)/d) + 3*a^2*b*sinh(d*x + c)/d
```

3.64.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.46

$$\int (a+b\cosh(c+dx))^3\,dx = \frac{b^3e^{(3\,dx+3\,c)}}{24\,d}+\frac{3\,ab^2e^{(2\,dx+2\,c)}}{8\,d}-\frac{3\,ab^2e^{(-2\,dx-2\,c)}}{8\,d} - \frac{b^3e^{(-3\,dx-3\,c)}}{24\,d}+\frac{1}{2}\left(2\,a^3+3\,ab^2\right)x + \frac{3\left(4\,a^2b+b^3\right)e^{(dx+c)}}{8\,d}-\frac{3\left(4\,a^2b+b^3\right)e^{(-dx-c)}}{8\,d}$$

input
```
integrate((a+b*cosh(d*x+c))^3,x, algorithm="giac")
```

output
```
1/24*b^3*e^(3*d*x + 3*c)/d + 3/8*a*b^2*e^(2*d*x + 2*c)/d - 3/8*a*b^2*e^(-2
*d*x - 2*c)/d - 1/24*b^3*e^(-3*d*x - 3*c)/d + 1/2*(2*a^3 + 3*a*b^2)*x + 3/
8*(4*a^2*b + b^3)*e^(d*x + c)/d - 3/8*(4*a^2*b + b^3)*e^(-d*x - c)/d
```

3.64.9 Mupad [B] (verification not implemented)

Time = 1.77 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.81

$$\int (a+b\cosh(c+dx))^3\,dx$$
$$=\frac{\frac{9\,b^3\sinh(c+d\,x)}{2}+\frac{b^3\sinh(3\,c+3\,d\,x)}{2}+\frac{9\,a\,b^2\sinh(2\,c+2\,d\,x)}{2}+18\,a^2\,b\sinh(c+d\,x)+6\,a^3\,d\,x+9\,a\,b^2\,d\,x}{6\,d}$$

input
```
int((a + b*cosh(c + d*x))^3,x)
```

output
```
((9*b^3*sinh(c + d*x))/2 + (b^3*sinh(3*c + 3*d*x))/2 + (9*a*b^2*sinh(2*c +
 2*d*x))/2 + 18*a^2*b*sinh(c + d*x) + 6*a^3*d*x + 9*a*b^2*d*x)/(6*d)
```

3.65 $\int (a+b\cosh(c+dx))^2\,dx$

3.65.1 Optimal result

Integrand size = 12, antiderivative size = 50

$$\int (a+b\cosh(c+dx))^2\,dx=\frac{1}{2}\left(2a^2+b^2\right)x+\frac{2ab\sinh(c+dx)}{d}+\frac{b^2\cosh(c+dx)\sinh(c+dx)}{2d}$$

output
```
1/2*(2*a^2+b^2)*x+2*a*b*sinh(d*x+c)/d+1/2*b^2*cosh(d*x+c)*sinh(d*x+c)/d
```

3.65.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int (a+b\cosh(c+dx))^2\,dx=\frac{2(2a^2+b^2)(c+dx)+8ab\sinh(c+dx)+b^2\sinh(2(c+dx))}{4d}$$

input
```
Integrate[(a + b*Cosh[c + d*x])^2,x]
```

output
```
(2*(2*a^2 + b^2)*(c + d*x) + 8*a*b*Sinh[c + d*x] + b^2*Sinh[2*(c + d*x)])/
(4*d)
```

3.65.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cosh(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{3123}$$

$$\frac{1}{2}x(2a^2+b^2)+\frac{2ab\sinh(c+dx)}{d}+\frac{b^2\sinh(c+dx)\cosh(c+dx)}{2d}$$

input
```
Int[(a + b*Cosh[c + d*x])^2,x]
```

output
```
((2*a^2 + b^2)*x)/2 + (2*a*b*Sinh[c + d*x])/d + (b^2*Cosh[c + d*x]*Sinh[c
+ d*x])/(2*d)
```

3.65.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3123
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

3.65.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

method	result	size
parallelrisch	$\frac{b^2 \sinh(2dx+2c)+8ab\sinh(dx+c)+4d\left(a^2+\frac{b^2}{2}\right)x}{4d}$	43
parts	$x\,a^2+\frac{b^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2ab\sinh(dx+c)}{d}$	49
derivativedivides	$\frac{b^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab\sinh(dx+c)+a^2(dx+c)}{d}$	51
default	$\frac{b^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab\sinh(dx+c)+a^2(dx+c)}{d}$	51
risch	$x\,a^2+\frac{b^2x}{2}+\frac{b^2 e^{2dx+2c}}{8d}+\frac{e^{dx+c}ab}{d}-\frac{e^{-dx-c}ab}{d}-\frac{b^2 e^{-2dx-2c}}{8d}$	75

input
```
int((a+b*cosh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(b^2*sinh(2*d*x+2*c)+8*a*b*sinh(d*x+c)+4*d*(a^2+1/2*b^2)*x)/d
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

$$\int (a+b\cosh(c+dx))^2\,dx = \frac{(2\,a^2+b^2)dx+(b^2\cosh{(dx+c)}+4\,ab)\sinh{(dx+c)}}{2\,d}$$

input
```
integrate((a+b*cosh(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*((2*a^2 + b^2)*d*x + (b^2*cosh(d*x + c) + 4*a*b)*sinh(d*x + c))/d
```

3.65.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.56

$$\int (a+b\cosh(c+dx))^2\,dx = \begin{cases} a^2x+\frac{2ab\sinh{(c+dx)}}{d}-\frac{b^2x\sinh^2{(c+dx)}}{2}+\frac{b^2x\cosh^2{(c+dx)}}{2}+\frac{b^2\sinh{(c+dx)}\cosh{(c+dx)}}{2d} & \text{for } d\neq 0 \\ x(a+b\cosh{(c)})^2 & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*cosh(d*x+c))**2,x)
```

output
```
Piecewise((a**2*x + 2*a*b*sinh(c + d*x)/d - b**2*x*sinh(c + d*x)**2/2 + b*
*2*x*cosh(c + d*x)**2/2 + b**2*sinh(c + d*x)*cosh(c + d*x)/(2*d), Ne(d, 0)
), (x*(a + b*cosh(c))**2, True))
```

3.65.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.10

$$\int (a+b\cosh(c+dx))^2\,dx = \frac{1}{8}\,b^2\left(4\,x+\frac{e^{(2\,dx+2\,c)}}{d}-\frac{e^{(-2\,dx-2\,c)}}{d}\right)+a^2x+\frac{2\,ab\sinh\left(dx+c\right)}{d}$$

input
```
integrate((a+b*cosh(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/8*b^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) + a^2*x + 2*a*b*sin
h(d*x + c)/d
```

3.65.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.50

$$\int (a+b\cosh(c+dx))^2\,dx = \frac{1}{2}\left(2\,a^2+b^2\right)x+\frac{b^2e^{(2\,dx+2\,c)}}{8\,d}+\frac{abe^{(dx+c)}}{d}-\frac{abe^{(-dx-c)}}{d}-\frac{b^2e^{(-2\,dx-2\,c)}}{8\,d}$$

input
```
integrate((a+b*cosh(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(2*a^2 + b^2)*x + 1/8*b^2*e^(2*d*x + 2*c)/d + a*b*e^(d*x + c)/d - a*b*
e^(-d*x - c)/d - 1/8*b^2*e^(-2*d*x - 2*c)/d
```

3.65.9 Mupad [B] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int (a + b\cosh(c + dx))^2 \, dx = \frac{\frac{\sinh(2\,c+2\,d\,x)\,b^2}{4} + 2\,a\sinh(c + d\,x)\;b}{d} + a^2\,x + \frac{b^2\,x}{2}$$

input
```
int((a + b*cosh(c + d*x))^2,x)
```

output
```
((b^2*sinh(2*c + 2*d*x))/4 + 2*a*b*sinh(c + d*x))/d + a^2*x + (b^2*x)/2
```

3.66 $\int (a+b\cosh (c+d x)) \, dx$

3.66.1 Optimal result

Integrand size = 10, antiderivative size = 15

$$\int (a+b\cosh (c+d x)) \, dx=a x+\frac{b \sinh (c+d x)}{d}$$

output
```
a*x+b*sinh(d*x+c)/d
```

3.66.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int (a+b\cosh (c+d x)) \, dx=a x+\frac{b \cosh (d x) \sinh (c)}{d}+\frac{b \cosh (c) \sinh (d x)}{d}$$

input
```
Integrate[a + b*Cosh[c + d*x],x]
```

output
```
a*x + (b*Cosh[d*x]*Sinh[c])/d + (b*Cosh[c]*Sinh[d*x])/d
```

3.66.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cosh(c+dx))\,dx$$

$$\Big\downarrow \text{ 2009}$$

$$ax+\frac{b\sinh(c+dx)}{d}$$

input
```
Int[a + b*Cosh[c + d*x],x]
```

output
```
a*x + (b*Sinh[c + d*x])/d
```

3.66.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.66.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
default	$ax+\frac{b\sinh(dx+c)}{d}$	16
parallelrisch	$ax+\frac{b\sinh(dx+c)}{d}$	16
parts	$ax+\frac{b\sinh(dx+c)}{d}$	16
derivativedivides	$\frac{a(dx+c)+b\sinh(dx+c)}{d}$	21
risch	$ax+\frac{e^{dx+c}b}{2d}-\frac{e^{-dx-c}b}{2d}$	32

input
```
int(a+b*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
a*x+b*sinh(d*x+c)/d
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (a+b\cosh(c+dx))\,dx=\frac{adx+b\sinh{(dx+c)}}{d}$$

input
```
integrate(a+b*cosh(d*x+c),x, algorithm="fricas")
```

output
```
(a*d*x + b*sinh(d*x + c))/d
```

3.66.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (a+b\cosh(c+dx))\,dx=ax+b\left(\begin{cases}\frac{\sinh{(c+dx)}}{d} & \text{for } d\neq 0\\ x\cosh{(c)} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(a+b*cosh(d*x+c),x)
```

output
```
a*x + b*Piecewise((sinh(c + d*x)/d, Ne(d, 0)), (x*cosh(c), True))
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (a+b\cosh(c+dx))\,dx=ax+\frac{b\sinh{(dx+c)}}{d}$$

input
```
integrate(a+b*cosh(d*x+c),x, algorithm="maxima")
```

output
```
a*x + b*sinh(d*x + c)/d
```

3.66.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(15) = 30$.

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.13

$$\int (a + b\cosh(c + dx))\,dx = ax + \frac{1}{2}b\left(\frac{e^{(dx+c)}}{d} - \frac{e^{(-dx-c)}}{d}\right)$$

input
```
integrate(a+b*cosh(d*x+c),x, algorithm="giac")
```

output
```
a*x + 1/2*b*(e^(d*x + c)/d - e^(-d*x - c)/d)
```

3.66.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (a + b\cosh(c + dx))\,dx = a\,x + \frac{b\sinh(c + d\,x)}{d}$$

input
```
int(a + b*cosh(c + d*x),x)
```

output
```
a*x + (b*sinh(c + d*x))/d
```

3.67 $\int \frac{1}{a+b\cosh(c+dx)}\,dx$

3.67.1 Optimal result

Integrand size = 12, antiderivative size = 49

$$\int \frac{1}{a+b\cosh(c+dx)}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}d}$$

output
```
2*arctanh((a-b)^(1/2)*tanh(1/2*d*x+1/2*c)/(a+b)^(1/2))/d/(a-b)^(1/2)/(a+b)
^(1/2)
```

3.67.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{1}{a+b\cosh(c+dx)}\,dx=-\frac{2\arctan\left(\frac{(a-b)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}d}$$

input
```
Integrate[(a + b*Cosh[c + d*x])^(-1),x]
```

output
```
(-2*ArcTan[((a - b)*Tanh[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(Sqrt[-a^2 + b^2
]*d)
```

3.67.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\cosh(c+dx)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \ 3138$$

$$-\frac{2i\int \frac{1}{-((a-b)\tanh^2\left(\frac{1}{2}(c+dx)\right))+a+b}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

$$\Big\downarrow \ 218$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}$$

input
```
Int[(a + b*Cosh[c + d*x])^(-1),x]
```

output
```
(2*ArcTanh[(Sqrt[a - b]*Tanh[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt
[a + b]*d)
```

3.67.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

3.67.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{2 \operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{d\sqrt{(a+b)(a-b)}}$	44
default	$\frac{2 \operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{d\sqrt{(a+b)(a-b)}}$	44
risch	$\frac{\ln\left(e^{dx+c}+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d}-\frac{\ln\left(e^{dx+c}+\frac{a\sqrt{a^2-b^2}+a^2-b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d}$	123

input
```
int(1/(a+b*cosh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/d/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1
/2))
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 237, normalized size of antiderivative = 4.84

$$\int \frac{1}{a+b\cosh(c+dx)}\,dx$$
$$= \left[\frac{\log\left(\frac{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)+2\,a^2-b^2+2\,(b^2\cosh(dx+c)+ab)\sinh(dx+c)-2\,\sqrt{a^2-b^2}(b\cosh(dx+c)+b\sinh(dx+c)}{b\cosh(dx+c)^2+b\sinh(dx+c)^2+2\,a\cosh(dx+c)+2\,(b\cosh(dx+c)+a)\sinh(dx+c)+b}\right.}{\sqrt{a^2-b^2}d}\right.$$
$$\left.-\frac{2\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cosh(dx+c)+b\sinh(dx+c)+a)}{a^2-b^2}\right)}{(a^2-b^2)d}\right]$$

input
```
integrate(1/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
[log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*
a^2 - b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 - b^2)*
(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x +
 c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) + b))/(s
qrt(a^2 - b^2)*d), -2*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(d*
x + c) + b*sinh(d*x + c) + a)/(a^2 - b^2))/((a^2 - b^2)*d)]
```

3.67.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(41) = 82$.

Time = 2.40 (sec) , antiderivative size = 163, normalized size of antiderivative = 3.33

$$\int \frac{1}{a+b\cosh(c+dx)}\,dx = \begin{cases} \frac{\tilde{\infty}x}{\cosh(c)} & \text{for } a=0 \wedge b=0 \wedge d=0 \\ \frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{bd} & \text{for } a=b \\ -\frac{1}{bd\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)} & \text{for } a=-b \\ \frac{x}{a+b\cosh(c)} & \text{for } d=0 \\ -\frac{\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{ad\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-bd\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} + \frac{\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{ad\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-bd\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*cosh(d*x+c)),x)
```

output
```
Piecewise((zoo*x/cosh(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (tanh(c/2 + d*x
/2)/(b*d), Eq(a, b)), (-1/(b*d*tanh(c/2 + d*x/2)), Eq(a, -b)), (x/(a + b*c
osh(c)), Eq(d, 0)), (-log(-sqrt(a/(a - b) + b/(a - b)) + tanh(c/2 + d*x/2)
)/(a*d*sqrt(a/(a - b) + b/(a - b)) - b*d*sqrt(a/(a - b) + b/(a - b))) + lo
g(sqrt(a/(a - b) + b/(a - b)) + tanh(c/2 + d*x/2))/(a*d*sqrt(a/(a - b) + b
/(a - b)) - b*d*sqrt(a/(a - b) + b/(a - b))), True))
```

3.67.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a + b\cosh(c + dx)}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.67.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{1}{a + b\cosh(c + dx)}\, dx = \frac{2\arctan\left(\frac{be^{(dx+c)}+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}d}$$

input
```
integrate(1/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

output
```
2*arctan((b*e^(d*x + c) + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*d)
```

3.67.9 Mupad [B] (verification not implemented)

Time = 2.00 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

$$\int \frac{1}{a + b\cosh(c + dx)}\, dx = \frac{2\,\mathrm{atan}\left(\frac{a\,d+b\,d\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}}{\sqrt{b^2\,d^2-a^2\,d^2}}\right)}{\sqrt{b^2\,d^2-a^2\,d^2}}$$

input
```
int(1/(a + b*cosh(c + d*x)),x)
```

output
```
(2*atan((a*d + b*d*exp(d*x)*exp(c))/(b^2*d^2 - a^2*d^2)^(1/2)))/(b^2*d^2 -
 a^2*d^2)^(1/2)
```

3.68 $\int \frac{1}{(a+b\cosh(c+dx))^2}\,dx$

3.68.1 Optimal result

Integrand size = 12, antiderivative size = 86

$$\int \frac{1}{(a+b\cosh(c+dx))^2}\,dx=\frac{2a\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d}-\frac{b\sinh(c+dx)}{(a^2-b^2)\,d(a+b\cosh(c+dx))}$$

output
```
2*a*arctanh((a-b)^(1/2)*tanh(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/(a+b)
^(3/2)/d-b*sinh(d*x+c)/(a^2-b^2)/d/(a+b*cosh(d*x+c))
```

3.68.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.98

$$\int \frac{1}{(a+b\cosh(c+dx))^2}\,dx=\frac{\frac{2a\arctan\left(\frac{(a-b)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}}-\frac{b\sinh(c+dx)}{(a-b)(a+b)(a+b\cosh(c+dx))}}{d}$$

input
```
Integrate[(a + b*Cosh[c + d*x])^(-2),x]
```

output
```
((2*a*ArcTan[((a - b)*Tanh[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(
3/2) - (b*Sinh[c + d*x])/((a - b)*(a + b)*(a + b*Cosh[c + d*x])))/d
```

3.68.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {3042, 3143, 25, 27, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\cosh(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3143}$$

$$-\frac{\int -\frac{a}{a+b\cosh(c+dx)}dx}{a^2-b^2}-\frac{b\sinh(c+dx)}{d\left(a^2-b^2\right)(a+b\cosh(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a}{a+b\cosh(c+dx)}dx}{a^2-b^2}-\frac{b\sinh(c+dx)}{d\left(a^2-b^2\right)(a+b\cosh(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{1}{a+b\cosh(c+dx)}dx}{a^2-b^2}-\frac{b\sinh(c+dx)}{d\left(a^2-b^2\right)(a+b\cosh(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\sinh(c+dx)}{d\left(a^2-b^2\right)(a+b\cosh(c+dx))}+\frac{a\int \frac{1}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3138}$$

$$-\frac{b\sinh(c+dx)}{d\left(a^2-b^2\right)(a+b\cosh(c+dx))}-\frac{2ia\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{1}{2}(c+dx)\right)\right)+a+b}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{218}$$

$$\frac{2a\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)}-\frac{b\sinh(c+dx)}{d\left(a^2-b^2\right)(a+b\cosh(c+dx))}$$

input
```
Int[(a + b*Cosh[c + d*x])^(-2),x]
```

output
```
(2*a*ArcTanh[(Sqrt[a - b]*Tanh[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sq
rt[a + b]*(a^2 - b^2)*d) - (b*Sinh[c + d*x])/((a^2 - b^2)*d*(a + b*Cosh[c
+ d*x]))
```

3.68.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.68.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.37

method	result	size
derivativedivides	$\frac{\frac{2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)}+\frac{2a\,\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}}{d}$	118
default	$\frac{\frac{2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)}+\frac{2a\,\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}}{d}$	118
risch	$\frac{2a\,e^{dx+c}+2b}{d(a^2-b^2)(b\,e^{2dx+2c}+2a\,e^{dx+c}+b)}+\frac{a\ln\left(e^{dx+c}+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)d}-\frac{a\ln\left(e^{dx+c}+\frac{a\sqrt{a^2-b^2}+a^2-b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)d}$	199

input
```
int(1/(a+b*cosh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*b/(a^2-b^2)*tanh(1/2*d*x+1/2*c)/(tanh(1/2*d*x+1/2*c)^2*a-tanh(1/2*d
*x+1/2*c)^2*b-a-b)+2*a/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(
1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

3.68.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(77) = 154.

Time = 0.26 (sec) , antiderivative size = 743, normalized size of antiderivative = 8.64

$$\int \frac{1}{(a+b\cosh(c+dx))^2}\,dx$$
$$= \left[\frac{2\,a^2b-2\,b^3-\left(ab\cosh\left(dx+c\right)^2+ab\sinh\left(dx+c\right)^2+2\,a^2\cosh\left(dx+c\right)+ab+2\left(ab\cosh\left(dx+c\right)+a^2\right.\right.}{(a^4b-2\,a^2b^3+b^5)d\cosh\left(dx+c\right)^2+(a^4b-2\,a^2b^3+b^5)d\sinh}\right.$$

input
```
integrate(1/(a+b*cosh(d*x+c))^2,x, algorithm="fricas")
```

output
```
[(2*a^2*b - 2*b^3 - (a*b*cosh(d*x + c)^2 + a*b*sinh(d*x + c)^2 + 2*a^2*cos
h(d*x + c) + a*b + 2*(a*b*cosh(d*x + c) + a^2)*sinh(d*x + c))*sqrt(a^2 - b
^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) +
 2*a^2 - b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 - b^
2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*
x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) + b))
 + 2*(a^3 - a*b^2)*cosh(d*x + c) + 2*(a^3 - a*b^2)*sinh(d*x + c))/((a^4*b
- 2*a^2*b^3 + b^5)*d*cosh(d*x + c)^2 + (a^4*b - 2*a^2*b^3 + b^5)*d*sinh(d*
x + c)^2 + 2*(a^5 - 2*a^3*b^2 + a*b^4)*d*cosh(d*x + c) + (a^4*b - 2*a^2*b^
3 + b^5)*d + 2*((a^4*b - 2*a^2*b^3 + b^5)*d*cosh(d*x + c) + (a^5 - 2*a^3*b
^2 + a*b^4)*d)*sinh(d*x + c)), 2*(a^2*b - b^3 - (a*b*cosh(d*x + c)^2 + a*b
*sinh(d*x + c)^2 + 2*a^2*cosh(d*x + c) + a*b + 2*(a*b*cosh(d*x + c) + a^2)
*sinh(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(d*x + c)
 + b*sinh(d*x + c) + a)/(a^2 - b^2)) + (a^3 - a*b^2)*cosh(d*x + c) + (a^3
- a*b^2)*sinh(d*x + c))/((a^4*b - 2*a^2*b^3 + b^5)*d*cosh(d*x + c)^2 + (a^
4*b - 2*a^2*b^3 + b^5)*d*sinh(d*x + c)^2 + 2*(a^5 - 2*a^3*b^2 + a*b^4)*d*c
osh(d*x + c) + (a^4*b - 2*a^2*b^3 + b^5)*d + 2*((a^4*b - 2*a^2*b^3 + b^5)*
d*cosh(d*x + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d)*sinh(d*x + c))]
```

3.68.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2332 vs. $2(70) = 140$.

Time = 40.23 (sec) , antiderivative size = 2332, normalized size of antiderivative = 27.12

$$\int \frac{1}{(a+b\cosh(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cosh(d*x+c))**2,x)
```

output
```
Piecewise((zoo*x/cosh(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (-tanh(c/2 +
 d*x/2)**3/(6*b**2*d) + tanh(c/2 + d*x/2)/(2*b**2*d), Eq(a, b)), (1/(2*b**
2*d*tanh(c/2 + d*x/2)) - 1/(6*b**2*d*tanh(c/2 + d*x/2)**3), Eq(a, -b)), (x
/(a + b*cosh(c))**2, Eq(d, 0)), (-a**2*log(-sqrt(a/(a - b) + b/(a - b)) +
tanh(c/2 + d*x/2))*tanh(c/2 + d*x/2)**2/(a**4*d*sqrt(a/(a - b) + b/(a - b)
)*tanh(c/2 + d*x/2)**2 - a**4*d*sqrt(a/(a - b) + b/(a - b)) - 2*a**3*b*d*s
qrt(a/(a - b) + b/(a - b))*tanh(c/2 + d*x/2)**2 + 2*a**2*b**2*d*sqrt(a/(a
- b) + b/(a - b)) + 2*a*b**3*d*sqrt(a/(a - b) + b/(a - b))*tanh(c/2 + d*x/
2)**2 - b**4*d*sqrt(a/(a - b) + b/(a - b))*tanh(c/2 + d*x/2)**2 - b**4*d*s
qrt(a/(a - b) + b/(a - b))) + a**2*log(-sqrt(a/(a - b) + b/(a - b)) + tanh
(c/2 + d*x/2))/(a**4*d*sqrt(a/(a - b) + b/(a - b))*tanh(c/2 + d*x/2)**2 -
a**4*d*sqrt(a/(a - b) + b/(a - b)) - 2*a**3*b*d*sqrt(a/(a - b) + b/(a - b)
)*tanh(c/2 + d*x/2)**2 + 2*a**2*b**2*d*sqrt(a/(a - b) + b/(a - b)) + 2*a*b
**3*d*sqrt(a/(a - b) + b/(a - b))*tanh(c/2 + d*x/2)**2 - b**4*d*sqrt(a/(a
- b) + b/(a - b))*tanh(c/2 + d*x/2)**2 - b**4*d*sqrt(a/(a - b) + b/(a - b)
)) + a**2*log(sqrt(a/(a - b) + b/(a - b)) + tanh(c/2 + d*x/2))*tanh(c/2 +
d*x/2)**2/(a**4*d*sqrt(a/(a - b) + b/(a - b))*tanh(c/2 + d*x/2)**2 - a**4*
d*sqrt(a/(a - b) + b/(a - b)) - 2*a**3*b*d*sqrt(a/(a - b) + b/(a - b))*tan
h(c/2 + d*x/2)**2 + 2*a**2*b**2*d*sqrt(a/(a - b) + b/(a - b)) + 2*a*b**3*d
*sqrt(a/(a - b) + b/(a - b))*tanh(c/2 + d*x/2)**2 - b**4*d*sqrt(a/(a - ...
```

3.68.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\cosh(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*cosh(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.68.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.15

$$\int \frac{1}{(a+b\cosh(c+dx))^2}\,dx = \frac{2\left(\frac{a\arctan\left(\frac{be^{(dx+c)}+a}{\sqrt{-a^2+b^2}}\right)}{(a^2-b^2)\sqrt{-a^2+b^2}} + \frac{ae^{(dx+c)}+b}{(a^2-b^2)\left(be^{(2\,dx+2\,c)}+2\,ae^{(dx+c)}+b\right)}\right)}{d}$$

input
```
integrate(1/(a+b*cosh(d*x+c))^2,x, algorithm="giac")
```

output
```
2*(a*arctan((b*e^(d*x + c) + a)/sqrt(-a^2 + b^2))/((a^2 - b^2)*sqrt(-a^2 +
 b^2)) + (a*e^(d*x + c) + b)/((a^2 - b^2)*(b*e^(2*d*x + 2*c) + 2*a*e^(d*x
+ c) + b)))/d
```

3.68.9 Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.50

$$\int \frac{1}{(a+b\cosh(c+dx))^2}\,dx = \frac{\frac{2\,b^2}{d\,(a^2\,b-b^3)}+\frac{2\,a\,b\,\mathrm{e}^{c+d\,x}}{d\,(a^2\,b-b^3)}}{b+2\,a\,\mathrm{e}^{c+d\,x}+b\,\mathrm{e}^{2\,c+2\,d\,x}} + \frac{a\,\ln\left(-\frac{2\,a\,\mathrm{e}^{c+d\,x}}{b\,(a^2-b^2)}-\frac{2\,a\,\left(b+a\,\mathrm{e}^{c+d\,x}\right)}{b\,{(a+b)}^{3/2}\,{(a-b)}^{3/2}}\right)}{d\,{(a+b)}^{3/2}\,{(a-b)}^{3/2}} - \frac{a\,\ln\left(\frac{2\,a\,\left(b+a\,\mathrm{e}^{c+d\,x}\right)}{b\,{(a+b)}^{3/2}\,{(a-b)}^{3/2}}-\frac{2\,a\,\mathrm{e}^{c+d\,x}}{b\,(a^2-b^2)}\right)}{d\,{(a+b)}^{3/2}\,{(a-b)}^{3/2}}$$

input
```
int(1/(a + b*cosh(c + d*x))^2,x)
```

output
```
((2*b^2)/(d*(a^2*b - b^3)) + (2*a*b*exp(c + d*x))/(d*(a^2*b - b^3)))/(b +
2*a*exp(c + d*x) + b*exp(2*c + 2*d*x)) + (a*log(- (2*a*exp(c + d*x))/(b*(a
^2 - b^2)) - (2*a*(b + a*exp(c + d*x)))/(b*(a + b)^(3/2)*(a - b)^(3/2))))/
(d*(a + b)^(3/2)*(a - b)^(3/2)) - (a*log((2*a*(b + a*exp(c + d*x)))/(b*(a
+ b)^(3/2)*(a - b)^(3/2)) - (2*a*exp(c + d*x))/(b*(a^2 - b^2))))/(d*(a + b
)^(3/2)*(a - b)^(3/2))
```

3.69 $\int \frac{1}{(a+b\cosh(c+dx))^3}\,dx$

3.69.1 Optimal result

Integrand size = 12, antiderivative size = 133

$$\int \frac{1}{(a+b\cosh(c+dx))^3}\,dx = \frac{(2a^2+b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{b\sinh(c+dx)}{2\left(a^2-b^2\right)d(a+b\cosh(c+dx))^2} - \frac{3ab\sinh(c+dx)}{2\left(a^2-b^2\right)^2d(a+b\cosh(c+dx))}$$

output
```
(2*a^2+b^2)*arctanh((a-b)^(1/2)*tanh(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/
2)/(a+b)^(5/2)/d-1/2*b*sinh(d*x+c)/(a^2-b^2)/d/(a+b*cosh(d*x+c))^2-3/2*a*b
*sinh(d*x+c)/(a^2-b^2)^2/d/(a+b*cosh(d*x+c))
```

3.69.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.85

$$\int \frac{1}{(a+b\cosh(c+dx))^3}\,dx = \frac{-\frac{2(2a^2+b^2)\arctan\left(\frac{(a-b)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}} + \frac{b(-4a^2+b^2-3ab\cosh(c+dx))\sinh(c+dx)}{(a-b)^2(a+b)^2(a+b\cosh(c+dx))^2}}{2d}$$

input
```
Integrate[(a + b*Cosh[c + d*x])^(-3),x]
```

output
```
((-2*(2*a^2 + b^2)*ArcTan[((a - b)*Tanh[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(
-a^2 + b^2)^(5/2) + (b*(-4*a^2 + b^2 - 3*a*b*Cosh[c + d*x])*Sinh[c + d*x])
/((a - b)^2*(a + b)^2*(a + b*Cosh[c + d*x])^2))/(2*d)
```

3.69.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.20, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {3042, 3143, 25, 3042, 3233, 25, 27, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\cosh(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3143}$$

$$-\frac{\int -\frac{2a-b\cosh(c+dx)}{(a+b\cosh(c+dx))^2}dx}{2\left(a^2-b^2\right)}-\frac{b\sinh(c+dx)}{2d\left(a^2-b^2\right)(a+b\cosh(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2a-b\cosh(c+dx)}{(a+b\cosh(c+dx))^2}dx}{2\left(a^2-b^2\right)}-\frac{b\sinh(c+dx)}{2d\left(a^2-b^2\right)(a+b\cosh(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\sinh(c+dx)}{2d\left(a^2-b^2\right)(a+b\cosh(c+dx))^2}+\frac{\int \frac{2a-b\sin\left(ic+idx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^2}dx}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3233}$$

$$\frac{-\frac{\int -\frac{2a^2+b^2}{a+b\cosh(c+dx)}dx}{a^2-b^2}-\frac{3ab\sinh(c+dx)}{d(a^2-b^2)(a+b\cosh(c+dx))}}{2\left(a^2-b^2\right)}-\frac{b\sinh(c+dx)}{2d\left(a^2-b^2\right)(a+b\cosh(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{2a^2+b^2}{a+b\cosh(c+dx)}dx}{a^2-b^2}-\frac{3ab\sinh(c+dx)}{d(a^2-b^2)(a+b\cosh(c+dx))}}{2(a^2-b^2)}-\frac{b\sinh(c+dx)}{2d(a^2-b^2)(a+b\cosh(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{(2a^2+b^2)\int \frac{1}{a+b\cosh(c+dx)}dx}{a^2-b^2}-\frac{3ab\sinh(c+dx)}{d(a^2-b^2)(a+b\cosh(c+dx))}}{2(a^2-b^2)}-\frac{b\sinh(c+dx)}{2d(a^2-b^2)(a+b\cosh(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\sinh(c+dx)}{2d(a^2-b^2)(a+b\cosh(c+dx))^2}+\frac{-\frac{3ab\sinh(c+dx)}{d(a^2-b^2)(a+b\cosh(c+dx))}+\frac{(2a^2+b^2)\int \frac{1}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{a^2-b^2}}{2(a^2-b^2)}$$

$$\Big\downarrow \text{3138}$$

$$-\frac{b\sinh(c+dx)}{2d(a^2-b^2)(a+b\cosh(c+dx))^2}+\frac{-\frac{3ab\sinh(c+dx)}{d(a^2-b^2)(a+b\cosh(c+dx))}-\frac{2i(2a^2+b^2)\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{1}{2}(c+dx)\right)\right)+a+b}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d(a^2-b^2)}}{2(a^2-b^2)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{2(2a^2+b^2)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}(a^2-b^2)}-\frac{3ab\sinh(c+dx)}{d(a^2-b^2)(a+b\cosh(c+dx))}}{2(a^2-b^2)}-\frac{b\sinh(c+dx)}{2d(a^2-b^2)(a+b\cosh(c+dx))^2}$$

input
```
Int[(a + b*Cosh[c + d*x])^(-3),x]
```

output
```
-1/2*(b*Sinh[c + d*x])/((a^2 - b^2)*d*(a + b*Cosh[c + d*x])^2) + ((2*(2*a^
2 + b^2)*ArcTanh[(Sqrt[a - b]*Tanh[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b
]*Sqrt[a + b]*(a^2 - b^2)*d) - (3*a*b*Sinh[c + d*x])/((a^2 - b^2)*d*(a + b
*Cosh[c + d*x])))/(2*(a^2 - b^2))
```

3.69. $\int \frac{1}{(a+b\cosh(c+dx))^3}\,dx$

3.69.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.69.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.40

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{(4a+b)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2(a-b)\left(a^{2}+2ab+b^{2}\right)}+\frac{(4a-b)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^{2}-2ab+b^{2}\right)}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b-a-b\right)^{2}}+\frac{\left(2a^{2}+b^{2}\right)\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\left(a^{4}-2a^{2}b^{2}+b^{4}\right)\sqrt{(a+b)(a-b)}}}{d}$
default	$\frac{-\frac{2\left(-\frac{(4a+b)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2(a-b)\left(a^{2}+2ab+b^{2}\right)}+\frac{(4a-b)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^{2}-2ab+b^{2}\right)}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b-a-b\right)^{2}}+\frac{\left(2a^{2}+b^{2}\right)\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\left(a^{4}-2a^{2}b^{2}+b^{4}\right)\sqrt{(a+b)(a-b)}}}{d}$
risch	$\frac{2a^{2}b\,\mathrm{e}^{3dx+3c}+b^{3}\mathrm{e}^{3dx+3c}+6a^{3}\mathrm{e}^{2dx+2c}+3\,\mathrm{e}^{2dx+2c}a\,b^{2}+10\,\mathrm{e}^{dx+c}a^{2}b-\mathrm{e}^{dx+c}b^{3}+3a\,b^{2}}{d(a^{2}-b^{2})^{2}\left(b\,\mathrm{e}^{2dx+2c}+2a\,\mathrm{e}^{dx+c}+b\right)^{2}}+\frac{\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^{2}-b^{2}}-a^{2}+b^{2}}{b\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,(a+b)^{2}(a-b)^{2}d}$

input
```
int(1/(a+b*cosh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(-1/2*(4*a+b)*b/(a-b)/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3+1/2*(4
*a-b)*b/(a+b)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^2*
a-tanh(1/2*d*x+1/2*c)^2*b-a-b)^2+(2*a^2+b^2)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a
-b))^(1/2)*arctanh((a-b)*tanh(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

3.69.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1239 vs. 2(120) = 240.

Time = 0.28 (sec) , antiderivative size = 2591, normalized size of antiderivative = 19.48

$$\int \frac{1}{(a+b\cosh(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cosh(d*x+c))^3,x, algorithm="fricas")
```

output
```
[1/2*(6*a^3*b^2 - 6*a*b^4 + 2*(2*a^4*b - a^2*b^3 - b^5)*cosh(d*x + c)^3 +
2*(2*a^4*b - a^2*b^3 - b^5)*sinh(d*x + c)^3 + 6*(2*a^5 - a^3*b^2 - a*b^4)*
cosh(d*x + c)^2 + 6*(2*a^5 - a^3*b^2 - a*b^4 + (2*a^4*b - a^2*b^3 - b^5)*c
osh(d*x + c))*sinh(d*x + c)^2 + ((2*a^2*b^2 + b^4)*cosh(d*x + c)^4 + (2*a^
2*b^2 + b^4)*sinh(d*x + c)^4 + 2*a^2*b^2 + b^4 + 4*(2*a^3*b + a*b^3)*cosh(
d*x + c)^3 + 4*(2*a^3*b + a*b^3 + (2*a^2*b^2 + b^4)*cosh(d*x + c))*sinh(d*
x + c)^3 + 2*(4*a^4 + 4*a^2*b^2 + b^4)*cosh(d*x + c)^2 + 2*(4*a^4 + 4*a^2*
b^2 + b^4 + 3*(2*a^2*b^2 + b^4)*cosh(d*x + c)^2 + 6*(2*a^3*b + a*b^3)*cosh
(d*x + c))*sinh(d*x + c)^2 + 4*(2*a^3*b + a*b^3)*cosh(d*x + c) + 4*(2*a^3*
b + a*b^3 + (2*a^2*b^2 + b^4)*cosh(d*x + c)^3 + 3*(2*a^3*b + a*b^3)*cosh(d
*x + c)^2 + (4*a^4 + 4*a^2*b^2 + b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a
^2 - b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x
+ c) + 2*a^2 - b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^
2 - b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*s
inh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c)
 + b)) + 2*(10*a^4*b - 11*a^2*b^3 + b^5)*cosh(d*x + c) + 2*(10*a^4*b - 11*
a^2*b^3 + b^5 + 3*(2*a^4*b - a^2*b^3 - b^5)*cosh(d*x + c)^2 + 6*(2*a^5 - a
^3*b^2 - a*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^6*b^2 - 3*a^4*b^4 + 3*a^
2*b^6 - b^8)*d*cosh(d*x + c)^4 + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d
*sinh(d*x + c)^4 + 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cosh(d*x...
```

3.69.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cosh(d*x+c))**3,x)
```

output
```
Timed out
```

3.69.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\cosh(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*cosh(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.69.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.47

$$\int \frac{1}{(a+b\cosh(c+dx))^3}\,dx = \frac{\frac{(2a^2+b^2)\arctan\left(\frac{be^{(dx+c)}+a}{\sqrt{-a^2+b^2}}\right)}{(a^4-2a^2b^2+b^4)\sqrt{-a^2+b^2}} + \frac{2a^2be^{(3dx+3c)}+b^3e^{(3dx+3c)}+6a^3e^{(2dx+2c)}+3ab^2e^{(2dx+2c)}+10a^2be^{(dx+c)}-b^3e^{(dx+c)}+3ab^2}{(a^4-2a^2b^2+b^4)\left(be^{(2dx+2c)}+2ae^{(dx+c)}+b\right)^2}}{d}$$

input
```
integrate(1/(a+b*cosh(d*x+c))^3,x, algorithm="giac")
```

output
```
((2*a^2 + b^2)*arctan((b*e^(d*x + c) + a)/sqrt(-a^2 + b^2))/((a^4 - 2*a^2*
b^2 + b^4)*sqrt(-a^2 + b^2)) + (2*a^2*b*e^(3*d*x + 3*c) + b^3*e^(3*d*x + 3
*c) + 6*a^3*e^(2*d*x + 2*c) + 3*a*b^2*e^(2*d*x + 2*c) + 10*a^2*b*e^(d*x +
c) - b^3*e^(d*x + c) + 3*a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*(b*e^(2*d*x + 2*c
) + 2*a*e^(d*x + c) + b)^2))/d
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(c+dx))^3}\,dx = \int \frac{1}{\left(a+b\cosh\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int(1/(a + b*cosh(c + d*x))^3,x)
```

output
```
int(1/(a + b*cosh(c + d*x))^3, x)
```

3.70 $\int \frac{1}{(a+b\cosh(c+dx))^4}\,dx$

3.70.1 Optimal result

Integrand size = 12, antiderivative size = 184

$$\int \frac{1}{(a+b\cosh(c+dx))^4}\,dx = \frac{a(2a^2+3b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{b\sinh(c+dx)}{3(a^2-b^2)\,d(a+b\cosh(c+dx))^3} - \frac{5ab\sinh(c+dx)}{6(a^2-b^2)^2\,d(a+b\cosh(c+dx))^2} - \frac{b(11a^2+4b^2)\sinh(c+dx)}{6(a^2-b^2)^3\,d(a+b\cosh(c+dx))}$$

output
```
a*(2*a^2+3*b^2)*arctanh((a-b)^(1/2)*tanh(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)
^(7/2)/(a+b)^(7/2)/d-1/3*b*sinh(d*x+c)/(a^2-b^2)/d/(a+b*cosh(d*x+c))^3-5/6
*a*b*sinh(d*x+c)/(a^2-b^2)^2/d/(a+b*cosh(d*x+c))^2-1/6*b*(11*a^2+4*b^2)*si
nh(d*x+c)/(a^2-b^2)^3/d/(a+b*cosh(d*x+c))
```

3.70.2 Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.87

$$\int \frac{1}{(a+b\cosh(c+dx))^4}\,dx = \frac{\frac{6a(2a^2+3b^2)\arctan\left(\frac{(a-b)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{7/2}} - \frac{b(36a^4+a^2b^2+8b^4+6ab(9a^2+b^2)\cosh(c+dx)+(11a^2b^2+4b^4)\cosh(2(c+dx)))\sinh(c+dx)}{2(a-b)^3(a+b)^3(a+b\cosh(c+dx))^3}}{6d}$$

input
```
Integrate[(a + b*Cosh[c + d*x])^(-4),x]
```

output
```
((6*a*(2*a^2 + 3*b^2)*ArcTan[((a - b)*Tanh[(c + d*x)/2])/Sqrt[-a^2 + b^2]]
)/(-a^2 + b^2)^(7/2) - (b*(36*a^4 + a^2*b^2 + 8*b^4 + 6*a*b*(9*a^2 + b^2)*
Cosh[c + d*x] + (11*a^2*b^2 + 4*b^4)*Cosh[2*(c + d*x)])*Sinh[c + d*x])/(2*
(a - b)^3*(a + b)^3*(a + b*Cosh[c + d*x])^3))/(6*d)
```

3.70.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.23, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 3143, 25, 3042, 3233, 25, 3042, 3233, 27, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\cosh(c+dx))^4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{3143}$$

$$-\frac{\int -\frac{3a-2b\cosh(c+dx)}{(a+b\cosh(c+dx))^3}dx}{3(a^2-b^2)} - \frac{b\sinh(c+dx)}{3d(a^2-b^2)(a+b\cosh(c+dx))^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3a-2b\cosh(c+dx)}{(a+b\cosh(c+dx))^3}dx}{3(a^2-b^2)} - \frac{b\sinh(c+dx)}{3d(a^2-b^2)(a+b\cosh(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\sinh(c+dx)}{3d\left(a^2-b^2\right)(a+b\cosh(c+dx))^3}+\frac{\int\frac{3a-2b\sin\left(ic+idx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^3}dx}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3233}$$

$$\frac{-\frac{\int-\frac{2\left(3a^2+2b^2\right)-5ab\cosh(c+dx)}{(a+b\cosh(c+dx))^2}dx}{2(a^2-b^2)}-\frac{5ab\sinh(c+dx)}{2d(a^2-b^2)(a+b\cosh(c+dx))^2}}{3\left(a^2-b^2\right)}-\frac{b\sinh(c+dx)}{3d\left(a^2-b^2\right)(a+b\cosh(c+dx))^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int\frac{2\left(3a^2+2b^2\right)-5ab\cosh(c+dx)}{(a+b\cosh(c+dx))^2}dx}{2(a^2-b^2)}-\frac{5ab\sinh(c+dx)}{2d(a^2-b^2)(a+b\cosh(c+dx))^2}}{3\left(a^2-b^2\right)}-\frac{b\sinh(c+dx)}{3d\left(a^2-b^2\right)(a+b\cosh(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\sinh(c+dx)}{3d\left(a^2-b^2\right)(a+b\cosh(c+dx))^3}+\frac{-\frac{5ab\sinh(c+dx)}{2d(a^2-b^2)(a+b\cosh(c+dx))^2}+\frac{\int\frac{2\left(3a^2+2b^2\right)-5ab\sin\left(ic+idx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^2}dx}{2(a^2-b^2)}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3233}$$

$$\frac{\frac{-\frac{\int-\frac{3a\left(2a^2+3b^2\right)}{a+b\cosh(c+dx)}dx}{a^2-b^2}-\frac{b\left(11a^2+4b^2\right)\sinh(c+dx)}{d\left(a^2-b^2\right)(a+b\cosh(c+dx))}}{2(a^2-b^2)}-\frac{5ab\sinh(c+dx)}{2d(a^2-b^2)(a+b\cosh(c+dx))^2}}{3\left(a^2-b^2\right)}-\frac{b\sinh(c+dx)}{3d\left(a^2-b^2\right)(a+b\cosh(c+dx))^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3a\left(2a^2+3b^2\right)\int\frac{1}{a+b\cosh(c+dx)}dx}{a^2-b^2}-\frac{b\left(11a^2+4b^2\right)\sinh(c+dx)}{d\left(a^2-b^2\right)(a+b\cosh(c+dx))}}{2(a^2-b^2)}-\frac{5ab\sinh(c+dx)}{2d(a^2-b^2)(a+b\cosh(c+dx))^2}}{3\left(a^2-b^2\right)}-\frac{b\sinh(c+dx)}{3d\left(a^2-b^2\right)(a+b\cosh(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\sinh(c+dx)}{3d\left(a^2-b^2\right)(a+b\cosh(c+dx))^3}+\frac{-\frac{5ab\sinh(c+dx)}{2d(a^2-b^2)(a+b\cosh(c+dx))^2}+\frac{-\frac{b\left(11a^2+4b^2\right)\sinh(c+dx)}{d\left(a^2-b^2\right)(a+b\cosh(c+dx))}+\frac{3a\left(2a^2+3b^2\right)\int\frac{1}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{a^2-b^2}}{2(a^2-b^2)}}{3\left(a^2-b^2\right)}$$

3.70. $\int\frac{1}{(a+b\cosh(c+dx))^4}\,dx$

$$\Big\downarrow \; 3138$$

$$-\frac{b\sinh(c+dx)}{3d(a^2-b^2)(a+b\cosh(c+dx))^3}+\frac{-\frac{5ab\sinh(c+dx)}{2d(a^2-b^2)(a+b\cosh(c+dx))^2}+\frac{-\frac{b(11a^2+4b^2)\sinh(c+dx)}{d(a^2-b^2)(a+b\cosh(c+dx))}-\frac{6ia(2a^2+3b^2)\int\frac{1}{-\left((a-b)\tanh^2\left(\frac{1}{2}(c+dx)\right)\right)+a+b}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d(a^2-b^2)}}{2(a^2-b^2)}}{3(a^2-b^2)}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{6a(2a^2+3b^2)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}(a^2-b^2)}-\frac{b(11a^2+4b^2)\sinh(c+dx)}{d(a^2-b^2)(a+b\cosh(c+dx))}}{2(a^2-b^2)}-\frac{5ab\sinh(c+dx)}{2d(a^2-b^2)(a+b\cosh(c+dx))^2}}{3(a^2-b^2)}-\frac{b\sinh(c+dx)}{3d(a^2-b^2)(a+b\cosh(c+dx))^3}$$

input
```
Int[(a + b*Cosh[c + d*x])^(-4),x]
```

output
```
-1/3*(b*Sinh[c + d*x])/((a^2 - b^2)*d*(a + b*Cosh[c + d*x])^3) + ((-5*a*b*
Sinh[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Cosh[c + d*x])^2) + ((6*a*(2*a^2 +
3*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*
Sqrt[a + b]*(a^2 - b^2)*d) - (b*(11*a^2 + 4*b^2)*Sinh[c + d*x])/((a^2 - b^
2)*d*(a + b*Cosh[c + d*x])))/(2*(a^2 - b^2)))/(3*(a^2 - b^2))
```

3.70.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.70.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.54

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{\left(6a^2+3ab+2b^2\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2(a-b)\left(a^3+3a^2b+3a\,b^2+b^3\right)}+\frac{2\left(9a^2+b^2\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3\left(a^2+2ab+b^2\right)\left(a^2-2ab+b^2\right)}-\frac{\left(6a^2-3ab+2b^2\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^3-3a^2b+3a\,b^2-b^3\right)}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^3}+\frac{a\left(2a^2+3b^2\right)\operatorname{arctanh}}{\left(a^6-3a^4b^2+3a^2\right.}}{d}$
default	$\frac{-\frac{2\left(-\frac{\left(6a^2+3ab+2b^2\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2(a-b)\left(a^3+3a^2b+3a\,b^2+b^3\right)}+\frac{2\left(9a^2+b^2\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3\left(a^2+2ab+b^2\right)\left(a^2-2ab+b^2\right)}-\frac{\left(6a^2-3ab+2b^2\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^3-3a^2b+3a\,b^2-b^3\right)}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^3}+\frac{a\left(2a^2+3b^2\right)\operatorname{arctanh}}{\left(a^6-3a^4b^2+3a^2\right.}}{d}$
risch	$\frac{6a^3b^2e^{5dx+5c}+9a\,b^4e^{5dx+5c}+30a^4b\,e^{4dx+4c}+45a^2b^3e^{4dx+4c}+44a^5e^{3dx+3c}+82a^3b^2e^{3dx+3c}+24a\,b^4e^{3dx+3c}+102a^4b\,e^{}}{3d(a^2-b^2)^3\left(b\,e^{2dx+2c}+2a\,e^{dx+c}+b\right)^3}$

input
```
int(1/(a+b*cosh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(-1/2*(6*a^2+3*a*b+2*b^2)*b/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tanh(1
/2*d*x+1/2*c)^5+2/3*(9*a^2+b^2)*b/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tanh(1/2
*d*x+1/2*c)^3-1/2*(6*a^2-3*a*b+2*b^2)*b/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*ta
nh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^2*a-tanh(1/2*d*x+1/2*c)^2*b-a-b)^3
+a*(2*a^2+3*b^2)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh
((a-b)*tanh(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

3.70.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2798 vs. $2(169) = 338$.

Time = 0.32 (sec) , antiderivative size = 5705, normalized size of antiderivative = 31.01

$$\int \frac{1}{(a+b\cosh(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cosh(d*x+c))^4,x, algorithm="fricas")
```

output
```
Too large to include
```

3.70.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cosh(d*x+c))**4,x)
```

output
```
Timed out
```

3.70.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\cosh(c+dx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*cosh(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.70.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.79

$$\int \frac{1}{(a+b\cosh(c+dx))^4}\,dx = \frac{\frac{3\,(2\,a^3+3\,ab^2)\arctan\left(\frac{be^{(dx+c)}+a}{\sqrt{-a^2+b^2}}\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{-a^2+b^2}} + \frac{6\,a^3b^2e^{(5\,dx+5\,c)}+9\,ab^4e^{(5\,dx+5\,c)}+30\,a^4be^{(4\,dx+4\,c)}+45\,a^2b^3e^{(4\,dx+4\,c)}+44\,a^5e^{(3\,dx+3\,c)}+82\,a^3b^2e^{(3\,dx \cdots}{(a^6-3\,a^4b^2+3\,a \cdots}}{3\,d}$$

input
```
integrate(1/(a+b*cosh(d*x+c))^4,x, algorithm="giac")
```

output
```
1/3*(3*(2*a^3 + 3*a*b^2)*arctan((b*e^(d*x + c) + a)/sqrt(-a^2 + b^2))/((a^
6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) + (6*a^3*b^2*e^(5*d*x +
 5*c) + 9*a*b^4*e^(5*d*x + 5*c) + 30*a^4*b*e^(4*d*x + 4*c) + 45*a^2*b^3*e^
(4*d*x + 4*c) + 44*a^5*e^(3*d*x + 3*c) + 82*a^3*b^2*e^(3*d*x + 3*c) + 24*a
*b^4*e^(3*d*x + 3*c) + 102*a^4*b*e^(2*d*x + 2*c) + 36*a^2*b^3*e^(2*d*x + 2
*c) + 12*b^5*e^(2*d*x + 2*c) + 60*a^3*b^2*e^(d*x + c) + 15*a*b^4*e^(d*x +
c) + 11*a^2*b^3 + 4*b^5)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(b*e^(2*d*x
+ 2*c) + 2*a*e^(d*x + c) + b)^3))/d
```

3.70.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(c+dx))^4}\,dx = \int \frac{1}{\left(a+b\cosh\left(c+d\,x\right)\right)^4}\,dx$$

input
```
int(1/(a + b*cosh(c + d*x))^4,x)
```

output
```
int(1/(a + b*cosh(c + d*x))^4, x)
```

3.71 $\int \frac{1}{3+5\cosh(c+dx)}\,dx$

3.71.1 Optimal result

Integrand size = 12, antiderivative size = 22

$$\int \frac{1}{3+5\cosh(c+dx)}\,dx = \frac{\arctan\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

output
```
1/2*arctan(1/2*tanh(1/2*d*x+1/2*c))/d
```

3.71.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{3+5\cosh(c+dx)}\,dx = -\frac{\arctan\left(2\coth\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

input
```
Integrate[(3 + 5*Cosh[c + d*x])^(-1),x]
```

output
```
-1/2*ArcTan[2*Coth[(c + d*x)/2]]/d
```

3.71.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3138, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{5\cosh(c+dx)+3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{3+5\sin\left(ic+idx+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \text{3138}$$
$$-\frac{2i\int \frac{1}{2\tanh^2\left(\frac{1}{2}(c+dx)\right)+8}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$
$$\Big\downarrow \text{219}$$
$$\frac{\arctan\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

input
```
Int[(3 + 5*Cosh[c + d*x])^(-1),x]
```

output
```
ArcTan[Tanh[(c + d*x)/2]/2]/(2*d)
```

3.71.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

3.71.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{2d}$	18
default	$\frac{\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{2d}$	18
risch	$\frac{i\ln\left(\mathrm{e}^{dx+c}+\frac{3}{5}+\frac{4i}{5}\right)}{4d}-\frac{i\ln\left(\mathrm{e}^{dx+c}+\frac{3}{5}-\frac{4i}{5}\right)}{4d}$	36
parallelrisch	$-\frac{i\left(\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2i\right)-\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2i\right)\right)}{4d}$	36

input
```
int(1/(3+5*cosh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(1/2*tanh(1/2*d*x+1/2*c))/d
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{3+5\cosh(c+dx)}\,dx = \frac{\arctan\left(\frac{5}{4}\cosh\left(dx+c\right)+\frac{5}{4}\sinh\left(dx+c\right)+\frac{3}{4}\right)}{2\,d}$$

input
```
integrate(1/(3+5*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*arctan(5/4*cosh(d*x + c) + 5/4*sinh(d*x + c) + 3/4)/d
```

3.71.6 Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{3+5\cosh(c+dx)}\,dx = \begin{cases} \frac{\operatorname{atan}\left(\frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}\right)}{2d} & \text{for } d \neq 0 \\ \frac{x}{5\cosh(c)+3} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(3+5*cosh(d*x+c)),x)
```

output
```
Piecewise((atan(tanh(c/2 + d*x/2)/2)/(2*d), Ne(d, 0)), (x/(5*cosh(c) + 3),
 True))
```

3.71.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{1}{3+5\cosh(c+dx)}\,dx = -\frac{\arctan\left(\frac{5}{4}e^{(-dx-c)}+\frac{3}{4}\right)}{2\,d}$$

input
```
integrate(1/(3+5*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*arctan(5/4*e^(-d*x - c) + 3/4)/d
```

3.71.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{1}{3+5\cosh(c+dx)}\,dx = \frac{\arctan\left(\frac{5}{4}e^{(dx+c)}+\frac{3}{4}\right)}{2\,d}$$

input
```
integrate(1/(3+5*cosh(d*x+c)),x, algorithm="giac")
```

output
```
1/2*arctan(5/4*e^(d*x + c) + 3/4)/d
```

3.71.9 Mupad [B] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \frac{1}{3+5\cosh(c+dx)}\,dx = \frac{\operatorname{atan}\left(\frac{3\sqrt{d^2}+5\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{d^2}}{4\,d}\right)}{2\sqrt{d^2}}$$

input
```
int(1/(5*cosh(c + d*x) + 3),x)
```

output
```
atan((3*(d^2)^(1/2) + 5*exp(d*x)*exp(c)*(d^2)^(1/2))/(4*d))/(2*(d^2)^(1/2)
)
```

3.72 $\int \frac{1}{(3+5\cosh(c+dx))^2}\,dx$

3.72.1 Optimal result

Integrand size = 12, antiderivative size = 48

$$\int \frac{1}{(3+5\cosh(c+dx))^2}\,dx = -\frac{3\arctan\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{32d} + \frac{5\sinh(c+dx)}{16d(3+5\cosh(c+dx))}$$

output
```
-3/32*arctan(1/2*tanh(1/2*d*x+1/2*c))/d+5/16*sinh(d*x+c)/d/(3+5*cosh(d*x+c
))
```

3.72.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \frac{1}{(3+5\cosh(c+dx))^2}\,dx = \frac{3\arctan\left(2\coth\left(\frac{1}{2}(c+dx)\right)\right) + \frac{10\sinh(c+dx)}{3+5\cosh(c+dx)}}{32d}$$

input
```
Integrate[(3 + 5*Cosh[c + d*x])^(-2),x]
```

output
```
(3*ArcTan[2*Coth[(c + d*x)/2]] + (10*Sinh[c + d*x])/(3 + 5*Cosh[c + d*x]))
/(32*d)
```

3.72.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3143, 27, 3042, 3138, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5\cosh(c+dx)+3)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(3+5\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{ 3143}$$

$$\frac{1}{16}\int -\frac{3}{5\cosh(c+dx)+3}dx+\frac{5\sinh(c+dx)}{16d(5\cosh(c+dx)+3)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{5\sinh(c+dx)}{16d(5\cosh(c+dx)+3)}-\frac{3}{16}\int \frac{1}{5\cosh(c+dx)+3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{5\sinh(c+dx)}{16d(5\cosh(c+dx)+3)}-\frac{3}{16}\int \frac{1}{5\sin\left(ic+idx+\frac{\pi}{2}\right)+3}dx$$

$$\Big\downarrow \text{ 3138}$$

$$\frac{5\sinh(c+dx)}{16d(5\cosh(c+dx)+3)}+\frac{3i\int \frac{1}{2\tanh^2\left(\frac{1}{2}(c+dx)\right)+8}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{8d}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{5\sinh(c+dx)}{16d(5\cosh(c+dx)+3)}-\frac{3\arctan\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{32d}$$

input
```
Int[(3 + 5*Cosh[c + d*x])^(-2),x]
```

output
```
(-3*ArcTan[Tanh[(c + d*x)/2]/2])/(32*d) + (5*Sinh[c + d*x])/(16*d*(3 + 5*C
osh[c + d*x]))
```

3.72.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.72.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{16\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+4\right)}-\frac{3\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{32}}{d}$	46
default	$\frac{\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{16\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+4\right)}-\frac{3\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{32}}{d}$	46
risch	$-\frac{3\,e^{dx+c}+5}{8d\left(5\,e^{2dx+2c}+6\,e^{dx+c}+5\right)}+\frac{3i\ln\left(e^{dx+c}+\frac{3}{5}-\frac{4i}{5}\right)}{64d}-\frac{3i\ln\left(e^{dx+c}+\frac{3}{5}+\frac{4i}{5}\right)}{64d}$	74
parallelrisch	$\frac{(15i\cosh(dx+c)+9i)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2i\right)+(-15i\cosh(dx+c)-9i)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2i\right)+20\sinh(dx+c)}{320\cosh(dx+c)d+192d}$	78

input
```
int(1/(3+5*cosh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(5/16*tanh(1/2*d*x+1/2*c)/(tanh(1/2*d*x+1/2*c)^2+4)-3/32*arctan(1/2*ta
nh(1/2*d*x+1/2*c)))
```

3.72.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(41) = 82$.

Time = 0.25 (sec) , antiderivative size = 147, normalized size of antiderivative = 3.06

$$\int \frac{1}{(3+5\cosh(c+dx))^2}\,dx = -\frac{3\left(5\cosh\left(dx+c\right)^{2}+2\left(5\cosh\left(dx+c\right)+3\right)\sinh\left(dx+c\right)+5\sinh\left(dx+c\right)^{2}+6\cosh\left(dx+c\right)+5\right)\text{ar}}{32\left(5\,d\cosh\left(dx+c\right)^{2}+5\,d\sinh\left(dx+c\right)^{2}+6\,d\cosh\left(dx+c\right)\right.}$$

input
```
integrate(1/(3+5*cosh(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/32*(3*(5*cosh(d*x + c)^2 + 2*(5*cosh(d*x + c) + 3)*sinh(d*x + c) + 5*si
nh(d*x + c)^2 + 6*cosh(d*x + c) + 5)*arctan(5/4*cosh(d*x + c) + 5/4*sinh(d
*x + c) + 3/4) + 12*cosh(d*x + c) + 12*sinh(d*x + c) + 20)/(5*d*cosh(d*x +
 c)^2 + 5*d*sinh(d*x + c)^2 + 6*d*cosh(d*x + c) + 2*(5*d*cosh(d*x + c) + 3
*d)*sinh(d*x + c) + 5*d)
```

3.72.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.10 (sec) , antiderivative size = 291, normalized size of antiderivative = 6.06

$$\int \frac{1}{(3+5\cosh(c+dx))^2}\,dx$$
$$= \begin{cases} -\frac{\log\left(-3e^{-dx}-4ie^{-dx}\right)}{25d\cosh^2\left(dx+\log\left(-3e^{-dx}-4ie^{-dx}\right)-\log(5)\right)+30d\cosh\left(dx+\log\left(-3e^{-dx}-4ie^{-dx}\right)-\log(5)\right)+9d} & \text{for } c=\log\left((-3-4i)\,e^{-\cdots}\right. \\ \frac{x}{25\cosh^2\left(dx+\log\left(-3e^{-dx}+4ie^{-dx}\right)-\log(5)\right)+30\cosh\left(dx+\log\left(-3e^{-dx}+4ie^{-dx}\right)-\log(5)\right)+9} & \text{for } c=\log\left((-3+4i)\,e^{-\cdots}\right. \\ \frac{x}{(5\cosh(c)+3)^2} & \text{for } d=0 \\ -\frac{3\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{atan}\left(\frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}\right)}{32d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)+128d}+\frac{10\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{32d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)+128d}-\frac{12\operatorname{atan}\left(\frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}\right)}{32d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)+128d} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(3+5*cosh(d*x+c))**2,x)
```

output
```
Piecewise((-log(-3*exp(-d*x) - 4*I*exp(-d*x))/(25*d*cosh(d*x + log(-3*exp(
-d*x) - 4*I*exp(-d*x)) - log(5))**2 + 30*d*cosh(d*x + log(-3*exp(-d*x) - 4
*I*exp(-d*x)) - log(5)) + 9*d), Eq(c, log((-3 - 4*I)*exp(-d*x)) - log(5)))
, (x/(25*cosh(d*x + log(-3*exp(-d*x) + 4*I*exp(-d*x)) - log(5))**2 + 30*co
sh(d*x + log(-3*exp(-d*x) + 4*I*exp(-d*x)) - log(5)) + 9), Eq(c, log((-3 +
 4*I)*exp(-d*x)) - log(5))), (x/(5*cosh(c) + 3)**2, Eq(d, 0)), (-3*tanh(c/
2 + d*x/2)**2*atan(tanh(c/2 + d*x/2)/2)/(32*d*tanh(c/2 + d*x/2)**2 + 128*d
) + 10*tanh(c/2 + d*x/2)/(32*d*tanh(c/2 + d*x/2)**2 + 128*d) - 12*atan(tan
h(c/2 + d*x/2)/2)/(32*d*tanh(c/2 + d*x/2)**2 + 128*d), True))
```

3.72.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.33

$$\int \frac{1}{(3+5\cosh(c+dx))^2}\,dx = \frac{3\arctan\left(\frac{5}{4}\,e^{(-dx-c)}+\frac{3}{4}\right)}{32\,d}+\frac{3\,e^{(-dx-c)}+5}{8\,d\left(6\,e^{(-dx-c)}+5\,e^{(-2\,dx-2\,c)}+5\right)}$$

input
```
integrate(1/(3+5*cosh(d*x+c))^2,x, algorithm="maxima")
```

output
```
3/32*arctan(5/4*e^(-d*x - c) + 3/4)/d + 1/8*(3*e^(-d*x - c) + 5)/(d*(6*e^(
-d*x - c) + 5*e^(-2*d*x - 2*c) + 5))
```

3.72.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

$$\int \frac{1}{(3+5\cosh(c+dx))^2}\,dx = -\frac{\frac{4\,(3\,e^{(dx+c)}+5)}{5\,e^{(2\,dx+2\,c)}+6\,e^{(dx+c)}+5} + 3\,\arctan\left(\frac{5}{4}\,e^{(dx+c)} + \frac{3}{4}\right)}{32\,d}$$

input
```
integrate(1/(3+5*cosh(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/32*(4*(3*e^(d*x + c) + 5)/(5*e^(2*d*x + 2*c) + 6*e^(d*x + c) + 5) + 3*a
rctan(5/4*e^(d*x + c) + 3/4))/d
```

3.72.9 Mupad [B] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.54

$$\int \frac{1}{(3+5\cosh(c+dx))^2}\,dx = -\frac{\frac{3\,e^{c+d\,x}}{8\,d}+\frac{5}{8\,d}}{6\,e^{c+d\,x}+5\,e^{2\,c+2\,d\,x}+5} - \frac{3\,\mathrm{atan}\left(\left(\frac{3}{4\,d}+\frac{5\,e^{d\,x}\,e^{c}}{4\,d}\right)\,\sqrt{d^2}\right)}{32\,\sqrt{d^2}}$$

input
```
int(1/(5*cosh(c + d*x) + 3)^2,x)
```

output
```
- ((3*exp(c + d*x))/(8*d) + 5/(8*d))/(6*exp(c + d*x) + 5*exp(2*c + 2*d*x)
+ 5) - (3*atan((3/(4*d) + (5*exp(d*x)*exp(c))/(4*d))*(d^2)^(1/2)))/(32*(d^
2)^(1/2))
```

3.73 $\int \frac{1}{(3+5\cosh(c+dx))^3}\,dx$

3.73.1 Optimal result

Integrand size = 12, antiderivative size = 73

$$\int \frac{1}{(3+5\cosh(c+dx))^3}\,dx = \frac{43\arctan\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{1024d} + \frac{5\sinh(c+dx)}{32d(3+5\cosh(c+dx))^2} - \frac{45\sinh(c+dx)}{512d(3+5\cosh(c+dx))}$$

output
```
43/1024*arctan(1/2*tanh(1/2*d*x+1/2*c))/d+5/32*sinh(d*x+c)/d/(3+5*cosh(d*x
+c))^2-45/512*sinh(d*x+c)/d/(3+5*cosh(d*x+c))
```

3.73.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.73

$$\int \frac{1}{(3+5\cosh(c+dx))^3}\,dx = -\frac{43\arctan\left(2\coth\left(\frac{1}{2}(c+dx)\right)\right) + \frac{10(11+45\cosh(c+dx))\sinh(c+dx)}{(3+5\cosh(c+dx))^2}}{1024d}$$

input
```
Integrate[(3 + 5*Cosh[c + d*x])^(-3),x]
```

output
```
-1/1024*(43*ArcTan[2*Coth[(c + d*x)/2]] + (10*(11 + 45*Cosh[c + d*x])*Sinh
[c + d*x])/(3 + 5*Cosh[c + d*x])^2)/d
```

3.73.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3143, 25, 3042, 3233, 27, 3042, 3138, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5\cosh(c+dx)+3)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(3+5\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3143}$$

$$\frac{1}{32}\int -\frac{6-5\cosh(c+dx)}{(5\cosh(c+dx)+3)^2}dx+\frac{5\sinh(c+dx)}{32d(5\cosh(c+dx)+3)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{5\sinh(c+dx)}{32d(5\cosh(c+dx)+3)^2}-\frac{1}{32}\int \frac{6-5\cosh(c+dx)}{(5\cosh(c+dx)+3)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\sinh(c+dx)}{32d(5\cosh(c+dx)+3)^2}-\frac{1}{32}\int \frac{6-5\sin\left(ic+idx+\frac{\pi}{2}\right)}{\left(5\sin\left(ic+idx+\frac{\pi}{2}\right)+3\right)^2}dx$$

$$\Big\downarrow \text{3233}$$

$$\frac{1}{32}\left(-\frac{1}{16}\int -\frac{43}{5\cosh(c+dx)+3}dx-\frac{45\sinh(c+dx)}{16d(5\cosh(c+dx)+3)}\right)+\frac{5\sinh(c+dx)}{32d(5\cosh(c+dx)+3)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32}\left(\frac{43}{16}\int \frac{1}{5\cosh(c+dx)+3}dx-\frac{45\sinh(c+dx)}{16d(5\cosh(c+dx)+3)}\right)+\frac{5\sinh(c+dx)}{32d(5\cosh(c+dx)+3)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\sinh(c+dx)}{32d(5\cosh(c+dx)+3)^2}+\frac{1}{32}\left(-\frac{45\sinh(c+dx)}{16d(5\cosh(c+dx)+3)}+\frac{43}{16}\int \frac{1}{5\sin\left(ic+idx+\frac{\pi}{2}\right)+3}dx\right)$$

$$\Big\downarrow \text{3138}$$

$$\frac{5\sinh(c+dx)}{32d(5\cosh(c+dx)+3)^2}+\frac{1}{32}\left(-\frac{45\sinh(c+dx)}{16d(5\cosh(c+dx)+3)}-\frac{43i\int\frac{1}{2\tanh^2\left(\frac{1}{2}(c+dx)\right)+8}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{8d}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{32}\left(\frac{43\arctan\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{32d}-\frac{45\sinh(c+dx)}{16d(5\cosh(c+dx)+3)}\right)+\frac{5\sinh(c+dx)}{32d(5\cosh(c+dx)+3)^2}$$

input
```
Int[(3 + 5*Cosh[c + d*x])^(-3),x]
```

output
```
(5*Sinh[c + d*x])/(32*d*(3 + 5*Cosh[c + d*x])^2) + ((43*ArcTan[Tanh[(c + d
*x)/2]/2])/(32*d) - (45*Sinh[c + d*x])/(16*d*(3 + 5*Cosh[c + d*x])))/32
```

3.73.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.73.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{\frac{-\frac{85\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{128}-\frac{35\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{32}}{4\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+4\right)^{2}}+\frac{43\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{1024}}{d}$
default	$\frac{\frac{-\frac{85\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{128}-\frac{35\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{32}}{4\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+4\right)^{2}}+\frac{43\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{1024}}{d}$
risch	$\frac{215\,e^{3dx+3c}+387\,e^{2dx+2c}+325\,e^{dx+c}+225}{256d\left(5\,e^{2dx+2c}+6\,e^{dx+c}+5\right)^{2}}+\frac{43i\ln\left(e^{dx+c}+\frac{3}{5}+\frac{4i}{5}\right)}{2048d}-\frac{43i\ln\left(e^{dx+c}+\frac{3}{5}-\frac{4i}{5}\right)}{2048d}$
parallelrisch	$\frac{43i(-43-25\cosh(2dx+2c)-60\cosh(dx+c))\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2i\right)+43i(43+25\cosh(2dx+2c)+60\cosh(dx+c))\ln\left(\tanh\right.}{2048d(43+25\cosh(2dx+2c)+60\cosh(dx+c))}$

input
```
int(1/(3+5*cosh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4*(-85/128*tanh(1/2*d*x+1/2*c)^3-35/32*tanh(1/2*d*x+1/2*c))/(tanh(1
/2*d*x+1/2*c)^2+4)^2+43/1024*arctan(1/2*tanh(1/2*d*x+1/2*c)))
```

3.73.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(64) = 128$.

Time = 0.25 (sec) , antiderivative size = 408, normalized size of antiderivative = 5.59

$$\int \frac{1}{(3+5\cosh(c+dx))^3}\,dx$$
$$= \frac{860\cosh(dx+c)^3 + 516(5\cosh(dx+c)+3)\sinh(dx+c)^2 + 860\sinh(dx+c)^3 + 43(25\cosh(dx+c)}{}$$

input
```
integrate(1/(3+5*cosh(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/1024*(860*cosh(d*x + c)^3 + 516*(5*cosh(d*x + c) + 3)*sinh(d*x + c)^2 +
860*sinh(d*x + c)^3 + 43*(25*cosh(d*x + c)^4 + 20*(5*cosh(d*x + c) + 3)*si
nh(d*x + c)^3 + 25*sinh(d*x + c)^4 + 60*cosh(d*x + c)^3 + 2*(75*cosh(d*x +
 c)^2 + 90*cosh(d*x + c) + 43)*sinh(d*x + c)^2 + 86*cosh(d*x + c)^2 + 4*(2
5*cosh(d*x + c)^3 + 45*cosh(d*x + c)^2 + 43*cosh(d*x + c) + 15)*sinh(d*x +
 c) + 60*cosh(d*x + c) + 25)*arctan(5/4*cosh(d*x + c) + 5/4*sinh(d*x + c)
+ 3/4) + 1548*cosh(d*x + c)^2 + 4*(645*cosh(d*x + c)^2 + 774*cosh(d*x + c)
 + 325)*sinh(d*x + c) + 1300*cosh(d*x + c) + 900)/(25*d*cosh(d*x + c)^4 +
25*d*sinh(d*x + c)^4 + 60*d*cosh(d*x + c)^3 + 20*(5*d*cosh(d*x + c) + 3*d)
*sinh(d*x + c)^3 + 86*d*cosh(d*x + c)^2 + 2*(75*d*cosh(d*x + c)^2 + 90*d*c
osh(d*x + c) + 43*d)*sinh(d*x + c)^2 + 60*d*cosh(d*x + c) + 4*(25*d*cosh(d
*x + c)^3 + 45*d*cosh(d*x + c)^2 + 43*d*cosh(d*x + c) + 15*d)*sinh(d*x + c
) + 25*d)
```

3.73.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.04 (sec) , antiderivative size = 507, normalized size of antiderivative = 6.95

$$\int \frac{1}{(3+5\cosh(c+dx))^3}\,dx$$
$$= \begin{cases} -\frac{\log(-3e^{-dx}-4ie^{-dx})}{125d\cosh^3(dx+\log(-3e^{-dx}-4ie^{-dx})-\log(5))+225d\cosh^2(dx+\log(-3e^{-dx}-4ie^{-dx})-\log(5))+135d\cosh(dx+\log(-3e^{-dx}-4ie^{-dx})-} \\ \frac{x}{125\cosh^3(dx+\log(-3e^{-dx}+4ie^{-dx})-\log(5))+225\cosh^2(dx+\log(-3e^{-dx}+4ie^{-dx})-\log(5))+135\cosh(dx+\log(-3e^{-dx}+4ie^{-dx})-\log(5} \\ \frac{x}{(5\cosh(c)+3)^3} \\ \frac{43\tanh^4\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{atan}\left(\frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}\right)}{1024d\tanh^4\left(\frac{c}{2}+\frac{dx}{2}\right)+8192d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)+16384d} - \frac{170\tanh^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{1024d\tanh^4\left(\frac{c}{2}+\frac{dx}{2}\right)+8192d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)+16384d} + \frac{344\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right.}{1024d\tanh^4\left(\frac{c}{2}+\frac{dx}{2}\right)+} \end{cases}$$

input
```
integrate(1/(3+5*cosh(d*x+c))**3,x)
```

output
```
Piecewise((-log(-3*exp(-d*x) - 4*I*exp(-d*x))/(125*d*cosh(d*x + log(-3*exp
(-d*x) - 4*I*exp(-d*x)) - log(5))**3 + 225*d*cosh(d*x + log(-3*exp(-d*x) -
 4*I*exp(-d*x)) - log(5))**2 + 135*d*cosh(d*x + log(-3*exp(-d*x) - 4*I*exp
(-d*x)) - log(5)) + 27*d), Eq(c, log((-3 - 4*I)*exp(-d*x)) - log(5))), (x/
(125*cosh(d*x + log(-3*exp(-d*x) + 4*I*exp(-d*x)) - log(5))**3 + 225*cosh(
d*x + log(-3*exp(-d*x) + 4*I*exp(-d*x)) - log(5))**2 + 135*cosh(d*x + log(
-3*exp(-d*x) + 4*I*exp(-d*x)) - log(5)) + 27), Eq(c, log((-3 + 4*I)*exp(-d
*x)) - log(5))), (x/(5*cosh(c) + 3)**3, Eq(d, 0)), (43*tanh(c/2 + d*x/2)**
4*atan(tanh(c/2 + d*x/2)/2)/(1024*d*tanh(c/2 + d*x/2)**4 + 8192*d*tanh(c/2
 + d*x/2)**2 + 16384*d) - 170*tanh(c/2 + d*x/2)**3/(1024*d*tanh(c/2 + d*x/
2)**4 + 8192*d*tanh(c/2 + d*x/2)**2 + 16384*d) + 344*tanh(c/2 + d*x/2)**2*
atan(tanh(c/2 + d*x/2)/2)/(1024*d*tanh(c/2 + d*x/2)**4 + 8192*d*tanh(c/2 +
 d*x/2)**2 + 16384*d) - 280*tanh(c/2 + d*x/2)/(1024*d*tanh(c/2 + d*x/2)**4
 + 8192*d*tanh(c/2 + d*x/2)**2 + 16384*d) + 688*atan(tanh(c/2 + d*x/2)/2)/
(1024*d*tanh(c/2 + d*x/2)**4 + 8192*d*tanh(c/2 + d*x/2)**2 + 16384*d), Tru
e))
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.48

$$\int \frac{1}{(3+5\cosh(c+dx))^3}\,dx = -\frac{43\arctan\left(\frac{5}{4}\,e^{(-dx-c)}+\frac{3}{4}\right)}{1024\,d} - \frac{325\,e^{(-dx-c)}+387\,e^{(-2\,dx-2\,c)}+215\,e^{(-3\,dx-3\,c)}+225}{256\,d(60\,e^{(-dx-c)}+86\,e^{(-2\,dx-2\,c)}+60\,e^{(-3\,dx-3\,c)}+25\,e^{(-4\,dx-4\,c)}+25)}$$

input
```
integrate(1/(3+5*cosh(d*x+c))^3,x, algorithm="maxima")
```

output
```
-43/1024*arctan(5/4*e^(-d*x - c) + 3/4)/d - 1/256*(325*e^(-d*x - c) + 387*
e^(-2*d*x - 2*c) + 215*e^(-3*d*x - 3*c) + 225)/(d*(60*e^(-d*x - c) + 86*e^
(-2*d*x - 2*c) + 60*e^(-3*d*x - 3*c) + 25*e^(-4*d*x - 4*c) + 25))
```

3.73.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04

$$\int \frac{1}{(3+5\cosh(c+dx))^3}\,dx = \frac{\frac{4\left(215\,e^{(3\,dx+3\,c)}+387\,e^{(2\,dx+2\,c)}+325\,e^{(dx+c)}+225\right)}{\left(5\,e^{(2\,dx+2\,c)}+6\,e^{(dx+c)}+5\right)^2}+43\,\arctan\left(\frac{5}{4}\,e^{(dx+c)}+\frac{3}{4}\right)}{1024\,d}$$

input
```
integrate(1/(3+5*cosh(d*x+c))^3,x, algorithm="giac")
```

output
```
1/1024*(4*(215*e^(3*d*x + 3*c) + 387*e^(2*d*x + 2*c) + 325*e^(d*x + c) + 2
25)/(5*e^(2*d*x + 2*c) + 6*e^(d*x + c) + 5)^2 + 43*arctan(5/4*e^(d*x + c)
+ 3/4))/d
```

3.73.9 Mupad [B] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.88

$$\int \frac{1}{(3+5\cosh(c+dx))^3}\,dx = \frac{\frac{43\,\mathrm{e}^{c+d\,x}}{256\,d}+\frac{129}{1280\,d}}{6\,\mathrm{e}^{c+d\,x}+5\,\mathrm{e}^{2\,c+2\,d\,x}+5} - \frac{\frac{7\,\mathrm{e}^{c+d\,x}}{40\,d}-\frac{3}{8\,d}}{60\,\mathrm{e}^{c+d\,x}+86\,\mathrm{e}^{2\,c+2\,d\,x}+60\,\mathrm{e}^{3\,c+3\,d\,x}+25\,\mathrm{e}^{4\,c+4\,d\,x}+25} + \frac{43\,\mathrm{atan}\left(\left(\frac{3}{4\,d}+\frac{5\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}}{4\,d}\right)\,\sqrt{d^2}\right)}{1024\,\sqrt{d^2}}$$

input
```
int(1/(5*cosh(c + d*x) + 3)^3,x)
```

output
```
((43*exp(c + d*x))/(256*d) + 129/(1280*d))/(6*exp(c + d*x) + 5*exp(2*c + 2
*d*x) + 5) - ((7*exp(c + d*x))/(40*d) - 3/(8*d))/(60*exp(c + d*x) + 86*exp
(2*c + 2*d*x) + 60*exp(3*c + 3*d*x) + 25*exp(4*c + 4*d*x) + 25) + (43*atan
((3/(4*d) + (5*exp(d*x)*exp(c))/(4*d))*(d^2)^(1/2)))/(1024*(d^2)^(1/2))
```

3.74 $\int \frac{1}{(3+5\cosh(c+dx))^4}\,dx$

3.74.1 Optimal result

Integrand size = 12, antiderivative size = 98

$$\int \frac{1}{(3+5\cosh(c+dx))^4}\,dx = -\frac{279\arctan\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{16384d} + \frac{5\sinh(c+dx)}{48d(3+5\cosh(c+dx))^3} - \frac{25\sinh(c+dx)}{512d(3+5\cosh(c+dx))^2} + \frac{995\sinh(c+dx)}{24576d(3+5\cosh(c+dx))}$$

output
```
-279/16384*arctan(1/2*tanh(1/2*d*x+1/2*c))/d+5/48*sinh(d*x+c)/d/(3+5*cosh(
d*x+c))^3-25/512*sinh(d*x+c)/d/(3+5*cosh(d*x+c))^2+995/24576*sinh(d*x+c)/d
/(3+5*cosh(d*x+c))
```

3.74.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.64

$$\int \frac{1}{(3+5\cosh(c+dx))^4}\,dx = \frac{837\arctan\left(2\coth\left(\frac{1}{2}(c+dx)\right)\right) + \frac{5(8141+9540\cosh(c+dx)+4975\cosh(2(c+dx)))\sinh(c+dx)}{(3+5\cosh(c+dx))^3}}{49152d}$$

input
```
Integrate[(3 + 5*Cosh[c + d*x])^(-4),x]
```

output
```
(837*ArcTan[2*Coth[(c + d*x)/2]] + (5*(8141 + 9540*Cosh[c + d*x] + 4975*Co
sh[2*(c + d*x)])*Sinh[c + d*x])/(3 + 5*Cosh[c + d*x])^3)/(49152*d)
```

3.74.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3143, 25, 3042, 3233, 25, 3042, 3233, 27, 3042, 3138, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5\cosh(c+dx)+3)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(3+5\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{3143}$$

$$\frac{1}{48}\int -\frac{9-10\cosh(c+dx)}{(5\cosh(c+dx)+3)^3}dx+\frac{5\sinh(c+dx)}{48d(5\cosh(c+dx)+3)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{5\sinh(c+dx)}{48d(5\cosh(c+dx)+3)^3}-\frac{1}{48}\int \frac{9-10\cosh(c+dx)}{(5\cosh(c+dx)+3)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\sinh(c+dx)}{48d(5\cosh(c+dx)+3)^3}-\frac{1}{48}\int \frac{9-10\sin\left(ic+idx+\frac{\pi}{2}\right)}{\left(5\sin\left(ic+idx+\frac{\pi}{2}\right)+3\right)^3}dx$$

$$\Big\downarrow \text{3233}$$

$$\frac{1}{48}\left(-\frac{1}{32}\int -\frac{154-75\cosh(c+dx)}{(5\cosh(c+dx)+3)^2}dx-\frac{75\sinh(c+dx)}{32d(5\cosh(c+dx)+3)^2}\right)+\frac{5\sinh(c+dx)}{48d(5\cosh(c+dx)+3)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{48}\left(\frac{1}{32}\int \frac{154-75\cosh(c+dx)}{(5\cosh(c+dx)+3)^2}dx-\frac{75\sinh(c+dx)}{32d(5\cosh(c+dx)+3)^2}\right)+\frac{5\sinh(c+dx)}{48d(5\cosh(c+dx)+3)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\sinh(c+dx)}{48d(5\cosh(c+dx)+3)^3}+\frac{1}{48}\left(-\frac{75\sinh(c+dx)}{32d(5\cosh(c+dx)+3)^2}+\frac{1}{32}\int \frac{154-75\sin\left(ic+idx+\frac{\pi}{2}\right)}{\left(5\sin\left(ic+idx+\frac{\pi}{2}\right)+3\right)^2}dx\right)$$

$$\Big\downarrow \text{3233}$$

$$\frac{1}{48}\left(\frac{1}{32}\left(\frac{1}{16}\int -\frac{837}{5\cosh(c+dx)+3}dx+\frac{995\sinh(c+dx)}{16d(5\cosh(c+dx)+3)}\right)-\frac{75\sinh(c+dx)}{32d(5\cosh(c+dx)+3)^2}\right)+\frac{5\sinh(c+dx)}{48d(5\cosh(c+dx)+3)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{48}\left(\frac{1}{32}\left(\frac{995\sinh(c+dx)}{16d(5\cosh(c+dx)+3)}-\frac{837}{16}\int \frac{1}{5\cosh(c+dx)+3}dx\right)-\frac{75\sinh(c+dx)}{32d(5\cosh(c+dx)+3)^2}\right)+\frac{5\sinh(c+dx)}{48d(5\cosh(c+dx)+3)^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{5\sinh(c+dx)}{48d(5\cosh(c+dx)+3)^3}+\frac{1}{48}\left(-\frac{75\sinh(c+dx)}{32d(5\cosh(c+dx)+3)^2}+\frac{1}{32}\left(\frac{995\sinh(c+dx)}{16d(5\cosh(c+dx)+3)}-\frac{837}{16}\int \frac{1}{5\sin\left(ic+idx+\frac{\pi}{2}\right)+3}dx\right)\right)$$

$$\Big\downarrow \; 3138$$

$$\frac{5\sinh(c+dx)}{48d(5\cosh(c+dx)+3)^3}+\frac{1}{48}\left(-\frac{75\sinh(c+dx)}{32d(5\cosh(c+dx)+3)^2}+\frac{1}{32}\left(\frac{995\sinh(c+dx)}{16d(5\cosh(c+dx)+3)}+\frac{837i\int \frac{1}{2\tanh^2\left(\frac{1}{2}(c+dx)\right)+8}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{8d}\right.\right.$$

$$\Big\downarrow \; 219$$

$$\frac{1}{48}\left(\frac{1}{32}\left(\frac{995\sinh(c+dx)}{16d(5\cosh(c+dx)+3)}-\frac{837\arctan\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{32d}\right)-\frac{75\sinh(c+dx)}{32d(5\cosh(c+dx)+3)^2}\right)+\frac{5\sinh(c+dx)}{48d(5\cosh(c+dx)+3)^3}$$

input
```
Int[(3 + 5*Cosh[c + d*x])^(-4),x]
```

output
```
(5*Sinh[c + d*x])/(48*d*(3 + 5*Cosh[c + d*x])^3) + ((-75*Sinh[c + d*x])/(3
2*d*(3 + 5*Cosh[c + d*x])^2) + ((-837*ArcTan[Tanh[(c + d*x)/2]/2])/(32*d)
+ (995*Sinh[c + d*x])/(16*d*(3 + 5*Cosh[c + d*x])))/32)/48
```

3.74. $\int \frac{1}{(3+5\cosh(c+dx))^4}\,dx$

3.74.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.74.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{-\frac{-\frac{745\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{1024}-\frac{265\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{96}-\frac{295\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{64}}{8\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+4\right)^{3}}-\frac{279\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{16384}}{d}$
default	$\frac{-\frac{-\frac{745\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{1024}-\frac{265\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{96}-\frac{295\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{64}}{8\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+4\right)^{3}}-\frac{279\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{16384}}{d}$
risch	$-\frac{20925\,e^{5dx+5c}+62775\,e^{4dx+4c}+111042\,e^{3dx+3c}+119310\,e^{2dx+2c}+68625\,e^{dx+c}+24875}{12288d\left(5\,e^{2dx+2c}+6\,e^{dx+c}+5\right)^{3}}+\frac{279i\ln\left(e^{dx+c}+\frac{3}{5}-\frac{4i}{5}\right)}{32768d}-$
parallelrisch	$\frac{837i(558+125\cosh(3dx+3c)+915\cosh(dx+c)+450\cosh(2dx+2c))\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2i\right)+837i(-558-125\cosh(3dx+}{98304d(558+125\cosh(3dx+3}$

input
```
int(1/(3+5*cosh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/8*(-745/1024*tanh(1/2*d*x+1/2*c)^5-265/96*tanh(1/2*d*x+1/2*c)^3-29
5/64*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^2+4)^3-279/16384*arctan(1/2
*tanh(1/2*d*x+1/2*c)))
```

3.74.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 793 vs. 2(87) = 174.

Time = 0.27 (sec) , antiderivative size = 793, normalized size of antiderivative = 8.09

$$\int \frac{1}{(3+5\cosh(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(3+5*cosh(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/49152*(83700*cosh(d*x + c)^5 + 83700*(5*cosh(d*x + c) + 3)*sinh(d*x + c
)^4 + 83700*sinh(d*x + c)^5 + 251100*cosh(d*x + c)^4 + 2232*(375*cosh(d*x
+ c)^2 + 450*cosh(d*x + c) + 199)*sinh(d*x + c)^3 + 444168*cosh(d*x + c)^3
 + 24*(34875*cosh(d*x + c)^3 + 62775*cosh(d*x + c)^2 + 55521*cosh(d*x + c)
 + 19885)*sinh(d*x + c)^2 + 837*(125*cosh(d*x + c)^6 + 150*(5*cosh(d*x + c
) + 3)*sinh(d*x + c)^5 + 125*sinh(d*x + c)^6 + 450*cosh(d*x + c)^5 + 15*(1
25*cosh(d*x + c)^2 + 150*cosh(d*x + c) + 61)*sinh(d*x + c)^4 + 915*cosh(d*
x + c)^4 + 4*(625*cosh(d*x + c)^3 + 1125*cosh(d*x + c)^2 + 915*cosh(d*x +
c) + 279)*sinh(d*x + c)^3 + 1116*cosh(d*x + c)^3 + 3*(625*cosh(d*x + c)^4
+ 1500*cosh(d*x + c)^3 + 1830*cosh(d*x + c)^2 + 1116*cosh(d*x + c) + 305)*
sinh(d*x + c)^2 + 915*cosh(d*x + c)^2 + 6*(125*cosh(d*x + c)^5 + 375*cosh(
d*x + c)^4 + 610*cosh(d*x + c)^3 + 558*cosh(d*x + c)^2 + 305*cosh(d*x + c)
 + 75)*sinh(d*x + c) + 450*cosh(d*x + c) + 125)*arctan(5/4*cosh(d*x + c) +
 5/4*sinh(d*x + c) + 3/4) + 477240*cosh(d*x + c)^2 + 12*(34875*cosh(d*x +
c)^4 + 83700*cosh(d*x + c)^3 + 111042*cosh(d*x + c)^2 + 79540*cosh(d*x + c
) + 22875)*sinh(d*x + c) + 274500*cosh(d*x + c) + 99500)/(125*d*cosh(d*x +
 c)^6 + 125*d*sinh(d*x + c)^6 + 450*d*cosh(d*x + c)^5 + 150*(5*d*cosh(d*x
+ c) + 3*d)*sinh(d*x + c)^5 + 915*d*cosh(d*x + c)^4 + 15*(125*d*cosh(d*x +
 c)^2 + 150*d*cosh(d*x + c) + 61*d)*sinh(d*x + c)^4 + 1116*d*cosh(d*x + c)
^3 + 4*(625*d*cosh(d*x + c)^3 + 1125*d*cosh(d*x + c)^2 + 915*d*cosh(d*x...
```

3.74.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.75 (sec) , antiderivative size = 784, normalized size of antiderivative = 8.00

$$\int \frac{1}{(3 + 5\cosh(c + dx))^4}\, dx = \text{Too large to display}$$

input
```
integrate(1/(3+5*cosh(d*x+c))**4,x)
```

output
```
Piecewise((-log(-3*exp(-d*x) - 4*I*exp(-d*x))/(625*d*cosh(d*x + log(-3*exp
(-d*x) - 4*I*exp(-d*x)) - log(5))**4 + 1500*d*cosh(d*x + log(-3*exp(-d*x)
- 4*I*exp(-d*x)) - log(5))**3 + 1350*d*cosh(d*x + log(-3*exp(-d*x) - 4*I*e
xp(-d*x)) - log(5))**2 + 540*d*cosh(d*x + log(-3*exp(-d*x) - 4*I*exp(-d*x)
) - log(5)) + 81*d), Eq(c, log((-3 - 4*I)*exp(-d*x)) - log(5))), (x/(625*c
osh(d*x + log(-3*exp(-d*x) + 4*I*exp(-d*x)) - log(5))**4 + 1500*cosh(d*x +
 log(-3*exp(-d*x) + 4*I*exp(-d*x)) - log(5))**3 + 1350*cosh(d*x + log(-3*e
xp(-d*x) + 4*I*exp(-d*x)) - log(5))**2 + 540*cosh(d*x + log(-3*exp(-d*x) +
 4*I*exp(-d*x)) - log(5)) + 81), Eq(c, log((-3 + 4*I)*exp(-d*x)) - log(5))
), (x/(5*cosh(c) + 3)**4, Eq(d, 0)), (-837*tanh(c/2 + d*x/2)**6*atan(tanh(
c/2 + d*x/2)/2)/(49152*d*tanh(c/2 + d*x/2)**6 + 589824*d*tanh(c/2 + d*x/2)
**4 + 2359296*d*tanh(c/2 + d*x/2)**2 + 3145728*d) + 4470*tanh(c/2 + d*x/2)
**5/(49152*d*tanh(c/2 + d*x/2)**6 + 589824*d*tanh(c/2 + d*x/2)**4 + 235929
6*d*tanh(c/2 + d*x/2)**2 + 3145728*d) - 10044*tanh(c/2 + d*x/2)**4*atan(ta
nh(c/2 + d*x/2)/2)/(49152*d*tanh(c/2 + d*x/2)**6 + 589824*d*tanh(c/2 + d*x
/2)**4 + 2359296*d*tanh(c/2 + d*x/2)**2 + 3145728*d) + 16960*tanh(c/2 + d*
x/2)**3/(49152*d*tanh(c/2 + d*x/2)**6 + 589824*d*tanh(c/2 + d*x/2)**4 + 23
59296*d*tanh(c/2 + d*x/2)**2 + 3145728*d) - 40176*tanh(c/2 + d*x/2)**2*ata
n(tanh(c/2 + d*x/2)/2)/(49152*d*tanh(c/2 + d*x/2)**6 + 589824*d*tanh(c/2 +
 d*x/2)**4 + 2359296*d*tanh(c/2 + d*x/2)**2 + 3145728*d) + 28320*tanh(c...
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.55

$$\int \frac{1}{(3+5\cosh(c+dx))^4}\,dx = \frac{279\arctan\left(\frac{5}{4}\,e^{(-dx-c)}+\frac{3}{4}\right)}{16384\,d} + \frac{68625\,e^{(-dx-c)}+119310\,e^{(-2\,dx-2\,c)}+111042\,e^{(-3\,dx-3\,c)}+62775\,e^{(-4\,dx-4\,c)}+20925\,e^{(-5\,dx-5\,c)}+2}{12288\,d(450\,e^{(-dx-c)}+915\,e^{(-2\,dx-2\,c)}+1116\,e^{(-3\,dx-3\,c)}+915\,e^{(-4\,dx-4\,c)}+450\,e^{(-5\,dx-5\,c)}+125\,e^{(-6\,dx-}}$$

input
```
integrate(1/(3+5*cosh(d*x+c))^4,x, algorithm="maxima")
```

output
```
279/16384*arctan(5/4*e^(-d*x - c) + 3/4)/d + 1/12288*(68625*e^(-d*x - c) +
 119310*e^(-2*d*x - 2*c) + 111042*e^(-3*d*x - 3*c) + 62775*e^(-4*d*x - 4*c
) + 20925*e^(-5*d*x - 5*c) + 24875)/(d*(450*e^(-d*x - c) + 915*e^(-2*d*x -
 2*c) + 1116*e^(-3*d*x - 3*c) + 915*e^(-4*d*x - 4*c) + 450*e^(-5*d*x - 5*c
) + 125*e^(-6*d*x - 6*c) + 125))
```

3.74.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int \frac{1}{(3+5\cosh(c+dx))^4}\,dx = -\frac{\frac{4\left(20925\,e^{(5\,dx+5\,c)}+62775\,e^{(4\,dx+4\,c)}+111042\,e^{(3\,dx+3\,c)}+119310\,e^{(2\,dx+2\,c)}+68625\,e^{(dx+c)}+24875\right)}{\left(5\,e^{(2\,dx+2\,c)}+6\,e^{(dx+c)}+5\right)^3}+837\,\arctan\left(\frac{5}{4}\,e^{(dx+c)}+\frac{3}{4}\right)}{49152\,d}$$

input
```
integrate(1/(3+5*cosh(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/49152*(4*(20925*e^(5*d*x + 5*c) + 62775*e^(4*d*x + 4*c) + 111042*e^(3*d
*x + 3*c) + 119310*e^(2*d*x + 2*c) + 68625*e^(d*x + c) + 24875)/(5*e^(2*d*
x + 2*c) + 6*e^(d*x + c) + 5)^3 + 837*arctan(5/4*e^(d*x + c) + 3/4))/d
```

3.74.9 Mupad [B] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.28

$$\int \frac{1}{(3+5\cosh(c+dx))^4}\,dx$$
$$= \frac{\frac{39\,e^{c+d\,x}}{50\,d}+\frac{7}{30\,d}}{450\,e^{c+d\,x}+915\,e^{2\,c+2\,d\,x}+1116\,e^{3\,c+3\,d\,x}+915\,e^{4\,c+4\,d\,x}+450\,e^{5\,c+5\,d\,x}+125\,e^{6\,c+6\,d\,x}+125} - \frac{\frac{93\,e^{c+d\,x}}{640\,d}+\frac{791}{3200\,d}}{60\,e^{c+d\,x}+86\,e^{2\,c+2\,d\,x}+60\,e^{3\,c+3\,d\,x}+25\,e^{4\,c+4\,d\,x}+25} - \frac{279\,\mathrm{atan}\left(\left(\frac{3}{4\,d}+\frac{5\,e^{d\,x}\,e^{c}}{4\,d}\right)\sqrt{d^2}\right)}{16384\,\sqrt{d^2}} - \frac{\frac{279\,e^{c+d\,x}}{4096\,d}+\frac{837}{20480\,d}}{6\,e^{c+d\,x}+5\,e^{2\,c+2\,d\,x}+5}$$

input
```
int(1/(5*cosh(c + d*x) + 3)^4,x)
```

output
```
((39*exp(c + d*x))/(50*d) + 7/(30*d))/(450*exp(c + d*x) + 915*exp(2*c + 2*
d*x) + 1116*exp(3*c + 3*d*x) + 915*exp(4*c + 4*d*x) + 450*exp(5*c + 5*d*x)
 + 125*exp(6*c + 6*d*x) + 125) - ((93*exp(c + d*x))/(640*d) + 791/(3200*d)
)/(60*exp(c + d*x) + 86*exp(2*c + 2*d*x) + 60*exp(3*c + 3*d*x) + 25*exp(4*
c + 4*d*x) + 25) - (279*atan((3/(4*d) + (5*exp(d*x)*exp(c))/(4*d))*(d^2)^(
1/2)))/(16384*(d^2)^(1/2)) - ((279*exp(c + d*x))/(4096*d) + 837/(20480*d))
/(6*exp(c + d*x) + 5*exp(2*c + 2*d*x) + 5)
```

3.75 $\int \frac{1}{5+3\cosh(c+dx)}\,dx$

3.75.1 Optimal result

Integrand size = 12, antiderivative size = 31

$$\int \frac{1}{5+3\cosh(c+dx)}\,dx = \frac{x}{4} - \frac{\operatorname{arctanh}\left(\frac{\sinh(c+dx)}{3+\cosh(c+dx)}\right)}{2d}$$

output
```
1/4*x-1/2*arctanh(sinh(d*x+c)/(3+cosh(d*x+c)))/d
```

3.75.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 65 vs. 2(31) = 62.

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.10

$$\int \frac{1}{5+3\cosh(c+dx)}\,dx = -\frac{\log\left(2\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{4d} + \frac{\log\left(2\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

input
```
Integrate[(5 + 3*Cosh[c + d*x])^(-1),x]
```

output
```
-1/4*Log[2*Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2]]/d + Log[2*Cosh[(c + d*x)
/2] + Sinh[(c + d*x)/2]]/(4*d)
```

3.75.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3\cosh(c+dx)+5}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{5+3\sin\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 3136$$

$$\frac{x}{4}-\frac{\operatorname{arctanh}\left(\frac{\sinh(c+dx)}{\cosh(c+dx)+3}\right)}{2d}$$

input
```
Int[(5 + 3*Cosh[c + d*x])^(-1),x]
```

output
```
x/4 - ArcTanh[Sinh[c + d*x]/(3 + Cosh[c + d*x])]/(2*d)
```

3.75.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

3.75.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

method	result	size
risch	$\frac{\ln\left(\frac{1}{3}+\mathrm{e}^{dx+c}\right)}{4d}-\frac{\ln\left(\mathrm{e}^{dx+c}+3\right)}{4d}$	30
parallelrisch	$\frac{-\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)+\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{4d}$	33
derivativedivides	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{4}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{4}}{d}$	34
default	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{4}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{4}}{d}$	34

input
```
int(1/(5+3*cosh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*ln(1/3+exp(d*x+c))-1/4/d*ln(exp(d*x+c)+3)
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int \frac{1}{5+3\cosh(c+dx)}\,dx = \frac{\log\left(3\,\cosh\left(dx+c\right)+3\,\sinh\left(dx+c\right)+1\right)-\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)+3\right)}{4\,d}$$

input
```
integrate(1/(5+3*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(log(3*cosh(d*x + c) + 3*sinh(d*x + c) + 1) - log(cosh(d*x + c) + sinh
(d*x + c) + 3))/d
```

3.75.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{1}{5+3\cosh(c+dx)}\,dx = \begin{cases} -\frac{\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-2\right)}{4d} + \frac{\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)+2\right)}{4d} & \text{for } d \neq 0 \\ \frac{x}{3\cosh(c)+5} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(5+3*cosh(d*x+c)),x)
```

output
```
Piecewise((-log(tanh(c/2 + d*x/2) - 2)/(4*d) + log(tanh(c/2 + d*x/2) + 2)/
(4*d), Ne(d, 0)), (x/(3*cosh(c) + 5), True))
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int \frac{1}{5+3\cosh(c+dx)}\,dx = -\frac{\log\left(3\,e^{(-dx-c)}+1\right)}{4\,d} + \frac{\log\left(e^{(-dx-c)}+3\right)}{4\,d}$$

input
```
integrate(1/(5+3*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/4*log(3*e^(-d*x - c) + 1)/d + 1/4*log(e^(-d*x - c) + 3)/d
```

3.75.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{1}{5+3\cosh(c+dx)}\,dx = \frac{\log\left(3\,e^{(dx+c)}+1\right) - \log\left(e^{(dx+c)}+3\right)}{4\,d}$$

input
```
integrate(1/(5+3*cosh(d*x+c)),x, algorithm="giac")
```

output
```
1/4*(log(3*e^(d*x + c) + 1) - log(e^(d*x + c) + 3))/d
```

3.75.9 Mupad [B] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \frac{1}{5+3\cosh(c+dx)}\,dx = -\frac{\operatorname{atan}\left(\frac{5\sqrt{-d^2}+3\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-d^2}}{4\,d}\right)}{2\sqrt{-d^2}}$$

input
```
int(1/(3*cosh(c + d*x) + 5),x)
```

output
```
-atan((5*(-d^2)^(1/2) + 3*exp(d*x)*exp(c)*(-d^2)^(1/2))/(4*d))/(2*(-d^2)^(
1/2))
```

3.76 $\int \frac{1}{(5+3\cosh(c+dx))^2}\,dx$

3.76.1 Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \frac{1}{(5+3\cosh(c+dx))^2}\,dx=\frac{5x}{64}-\frac{5\text{arctanh}\left(\frac{\sinh(c+dx)}{3+\cosh(c+dx)}\right)}{32d}-\frac{3\sinh(c+dx)}{16d(5+3\cosh(c+dx))}$$

output
```
5/64*x-5/32*arctanh(sinh(d*x+c)/(3+cosh(d*x+c)))/d-3/16*sinh(d*x+c)/d/(5+3
*cosh(d*x+c))
```

3.76.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 144 vs. 2(56) = 112.

Time = 0.11 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.57

$$\int \frac{1}{(5+3\cosh(c+dx))^2}\,dx$$
$$=\frac{-15\cosh(c+dx)\left(\log\left(2\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(2\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+dx\right.\right.\right.}{64d}$$

input
```
Integrate[(5 + 3*Cosh[c + d*x])^(-2),x]
```

output
```
(-15*Cosh[c + d*x]*(Log[2*Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2]] - Log[2*C
osh[(c + d*x)/2] + Sinh[(c + d*x)/2]]) + 25*(-Log[2*Cosh[(c + d*x)/2] - Si
nh[(c + d*x)/2]] + Log[2*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2]]) - 12*Sinh
[c + d*x])/(64*d*(5 + 3*Cosh[c + d*x]))
```

3.76.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3042, 3143, 27, 3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3\cosh(c+dx)+5)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(5+3\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{ 3143}$$

$$-\frac{1}{16}\int -\frac{5}{3\cosh(c+dx)+5}dx-\frac{3\sinh(c+dx)}{16d(3\cosh(c+dx)+5)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{5}{16}\int \frac{1}{3\cosh(c+dx)+5}dx-\frac{3\sinh(c+dx)}{16d(3\cosh(c+dx)+5)}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{3\sinh(c+dx)}{16d(3\cosh(c+dx)+5)}+\frac{5}{16}\int \frac{1}{3\sin\left(ic+idx+\frac{\pi}{2}\right)+5}dx$$

$$\Big\downarrow \text{ 3136}$$

$$\frac{5}{16}\left(\frac{x}{4}-\frac{\operatorname{arctanh}\left(\frac{\sinh(c+dx)}{\cosh(c+dx)+3}\right)}{2d}\right)-\frac{3\sinh(c+dx)}{16d(3\cosh(c+dx)+5)}$$

input
```
Int[(5 + 3*Cosh[c + d*x])^(-2),x]
```

output
```
(5*(x/4 - ArcTanh[Sinh[c + d*x]/(3 + Cosh[c + d*x])]/(2*d)))/16 - (3*Sinh[
c + d*x])/(16*d*(5 + 3*Cosh[c + d*x]))
```

3.76.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.76.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{\frac{3}{32\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}-\frac{5\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{64}+\frac{3}{32\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}+\frac{5\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{64}}{d}$
default	$\frac{\frac{3}{32\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}-\frac{5\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{64}+\frac{3}{32\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}+\frac{5\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{64}}{d}$
risch	$\frac{5\,e^{dx+c}+3}{8d\left(3\,e^{2dx+2c}+10\,e^{dx+c}+3\right)}+\frac{5\ln\left(\frac{1}{3}+e^{dx+c}\right)}{64d}-\frac{5\ln\left(e^{dx+c}+3\right)}{64d}$
parallelrisch	$\frac{-15\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)\cosh(dx+c)+15\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)\cosh(dx+c)-12\sinh(dx+c)-25\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{64d(5+3\cosh(dx+c))}$

input
```
int(1/(5+3*cosh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(3/32/(tanh(1/2*d*x+1/2*c)-2)-5/64*ln(tanh(1/2*d*x+1/2*c)-2)+3/32/(tan
h(1/2*d*x+1/2*c)+2)+5/64*ln(tanh(1/2*d*x+1/2*c)+2))
```

3.76.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(50) = 100$.

Time = 0.27 (sec) , antiderivative size = 212, normalized size of antiderivative = 3.79

$$\int \frac{1}{(5+3\cosh(c+dx))^2}\,dx$$
$$= \frac{5\left(3\,\cosh\left(dx+c\right)^2+2\left(3\,\cosh\left(dx+c\right)+5\right)\sinh\left(dx+c\right)+3\,\sinh\left(dx+c\right)^2+10\,\cosh\left(dx+c\right)+3\right)\log}{}$$

input
```
integrate(1/(5+3*cosh(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/64*(5*(3*cosh(d*x + c)^2 + 2*(3*cosh(d*x + c) + 5)*sinh(d*x + c) + 3*sin
h(d*x + c)^2 + 10*cosh(d*x + c) + 3)*log(3*cosh(d*x + c) + 3*sinh(d*x + c)
 + 1) - 5*(3*cosh(d*x + c)^2 + 2*(3*cosh(d*x + c) + 5)*sinh(d*x + c) + 3*s
inh(d*x + c)^2 + 10*cosh(d*x + c) + 3)*log(cosh(d*x + c) + sinh(d*x + c) +
 3) + 40*cosh(d*x + c) + 40*sinh(d*x + c) + 24)/(3*d*cosh(d*x + c)^2 + 3*d
*sinh(d*x + c)^2 + 10*d*cosh(d*x + c) + 2*(3*d*cosh(d*x + c) + 5*d)*sinh(d
*x + c) + 3*d)
```

3.76.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(48) = 96$.

Time = 0.73 (sec) , antiderivative size = 199, normalized size of antiderivative = 3.55

$$\int \frac{1}{(5+3\cosh(c+dx))^2}\,dx$$
$$= \begin{cases} -\frac{5\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-2\right)\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{64d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-256d} + \frac{20\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-2\right)}{64d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-256d} + \frac{5\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)+2\right)\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{64d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-256d} - \frac{20\log\left(\tanh\left(\frac{c}{2}+\right.\right.}{64d\tanh^2\left(\frac{c}{2}+\frac{d}{2}\right.} \\ \frac{x}{(3\cosh(c)+5)^2} \end{cases}$$

input
```
integrate(1/(5+3*cosh(d*x+c))**2,x)
```

output
```
Piecewise((-5*log(tanh(c/2 + d*x/2) - 2)*tanh(c/2 + d*x/2)**2/(64*d*tanh(c
/2 + d*x/2)**2 - 256*d) + 20*log(tanh(c/2 + d*x/2) - 2)/(64*d*tanh(c/2 + d
*x/2)**2 - 256*d) + 5*log(tanh(c/2 + d*x/2) + 2)*tanh(c/2 + d*x/2)**2/(64*
d*tanh(c/2 + d*x/2)**2 - 256*d) - 20*log(tanh(c/2 + d*x/2) + 2)/(64*d*tanh
(c/2 + d*x/2)**2 - 256*d) + 12*tanh(c/2 + d*x/2)/(64*d*tanh(c/2 + d*x/2)**
2 - 256*d), Ne(d, 0)), (x/(3*cosh(c) + 5)**2, True))
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.45

$$\int \frac{1}{(5+3\cosh(c+dx))^2}\,dx = -\frac{5\,\log\left(3\,e^{(-dx-c)}+1\right)}{64\,d} + \frac{5\,\log\left(e^{(-dx-c)}+3\right)}{64\,d} - \frac{5\,e^{(-dx-c)}+3}{8\,d(10\,e^{(-dx-c)}+3\,e^{(-2\,dx-2\,c)}+3)}$$

input
```
integrate(1/(5+3*cosh(d*x+c))^2,x, algorithm="maxima")
```

output
```
-5/64*log(3*e^(-d*x - c) + 1)/d + 5/64*log(e^(-d*x - c) + 3)/d - 1/8*(5*e^
(-d*x - c) + 3)/(d*(10*e^(-d*x - c) + 3*e^(-2*d*x - 2*c) + 3))
```

3.76.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.16

$$\int \frac{1}{(5+3\cosh(c+dx))^2}\,dx = \frac{\frac{8\,\left(5\,e^{(dx+c)}+3\right)}{3\,e^{(2\,dx+2\,c)}+10\,e^{(dx+c)}+3} + 5\,\log\left(3\,e^{(dx+c)}+1\right) - 5\,\log\left(e^{(dx+c)}+3\right)}{64\,d}$$

input
```
integrate(1/(5+3*cosh(d*x+c))^2,x, algorithm="giac")
```

output
```
1/64*(8*(5*e^(d*x + c) + 3)/(3*e^(2*d*x + 2*c) + 10*e^(d*x + c) + 3) + 5*l
og(3*e^(d*x + c) + 1) - 5*log(e^(d*x + c) + 3))/d
```

3.76.9 Mupad [B] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.38

$$\int \frac{1}{(5+3\cosh(c+dx))^2}\,dx = \frac{\frac{5\,e^{c+d\,x}}{8\,d}+\frac{3}{8\,d}}{10\,e^{c+d\,x}+3\,e^{2\,c+2\,d\,x}+3} - \frac{5\,\mathrm{atan}\left(\left(\frac{5}{4\,d}+\frac{3\,e^{d\,x}\,e^{c}}{4\,d}\right)\,\sqrt{-d^2}\right)}{32\,\sqrt{-d^2}}$$

input
```
int(1/(3*cosh(c + d*x) + 5)^2,x)
```

output
```
((5*exp(c + d*x))/(8*d) + 3/(8*d))/(10*exp(c + d*x) + 3*exp(2*c + 2*d*x) +
 3) - (5*atan((5/(4*d) + (3*exp(d*x)*exp(c))/(4*d))*(-d^2)^(1/2)))/(32*(-d
^2)^(1/2))
```

3.77 $\int \frac{1}{(5+3\cosh(c+dx))^3}\,dx$

3.77.1 Optimal result

Integrand size = 12, antiderivative size = 81

$$\int \frac{1}{(5+3\cosh(c+dx))^3}\,dx = \frac{59x}{2048} - \frac{59\text{arctanh}\left(\frac{\sinh(c+dx)}{3+\cosh(c+dx)}\right)}{1024d} - \frac{3\sinh(c+dx)}{32d(5+3\cosh(c+dx))^2} - \frac{45\sinh(c+dx)}{512d(5+3\cosh(c+dx))}$$

output
```
59/2048*x-59/1024*arctanh(sinh(d*x+c)/(3+cosh(d*x+c)))/d-3/32*sinh(d*x+c)/
d/(5+3*cosh(d*x+c))^2-45/512*sinh(d*x+c)/d/(5+3*cosh(d*x+c))
```

3.77.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 217 vs. $2(81) = 162$.

Time = 0.18 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.68

$$\int \frac{1}{(5+3\cosh(c+dx))^3}\,dx = -\frac{59\log\left(2\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} + \frac{59\log\left(2\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} - \frac{3}{512d\left(2\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{45\sinh\left(\frac{1}{2}(c+dx)\right)}{2048d\left(2\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{3}{512d\left(2\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{45\sinh\left(\frac{1}{2}(c+dx)\right)}{2048d\left(2\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+dx)\right)\right)}$$

input
```
Integrate[(5 + 3*Cosh[c + d*x])^(-3),x]
```

output
```
(-59*Log[2*Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2]])/(2048*d) + (59*Log[2*Co
sh[(c + d*x)/2] + Sinh[(c + d*x)/2]])/(2048*d) - 3/(512*d*(2*Cosh[(c + d*x
)/2] - Sinh[(c + d*x)/2])^2) - (45*Sinh[(c + d*x)/2])/(2048*d*(2*Cosh[(c +
 d*x)/2] - Sinh[(c + d*x)/2])) + 3/(512*d*(2*Cosh[(c + d*x)/2] + Sinh[(c +
 d*x)/2])^2) - (45*Sinh[(c + d*x)/2])/(2048*d*(2*Cosh[(c + d*x)/2] + Sinh[
(c + d*x)/2]))
```

3.77.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3143, 25, 3042, 3233, 27, 3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3\cosh(c+dx)+5)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(5+3\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3143}$$

$$-\frac{1}{32}\int -\frac{10-3\cosh(c+dx)}{(3\cosh(c+dx)+5)^2}dx-\frac{3\sinh(c+dx)}{32d(3\cosh(c+dx)+5)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{32}\int \frac{10-3\cosh(c+dx)}{(3\cosh(c+dx)+5)^2}dx-\frac{3\sinh(c+dx)}{32d(3\cosh(c+dx)+5)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\sinh(c+dx)}{32d(3\cosh(c+dx)+5)^2}+\frac{1}{32}\int \frac{10-3\sin\left(ic+idx+\frac{\pi}{2}\right)}{\left(3\sin\left(ic+idx+\frac{\pi}{2}\right)+5\right)^2}dx$$

$$\Big\downarrow \text{3233}$$

$$\frac{1}{32}\left(-\frac{1}{16}\int -\frac{59}{3\cosh(c+dx)+5}dx-\frac{45\sinh(c+dx)}{16d(3\cosh(c+dx)+5)}\right)-\frac{3\sinh(c+dx)}{32d(3\cosh(c+dx)+5)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32}\left(\frac{59}{16}\int \frac{1}{3\cosh(c+dx)+5}dx-\frac{45\sinh(c+dx)}{16d(3\cosh(c+dx)+5)}\right)-\frac{3\sinh(c+dx)}{32d(3\cosh(c+dx)+5)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\sinh(c+dx)}{32d(3\cosh(c+dx)+5)^2}+\frac{1}{32}\left(-\frac{45\sinh(c+dx)}{16d(3\cosh(c+dx)+5)}+\frac{59}{16}\int \frac{1}{3\sin\left(ic+idx+\frac{\pi}{2}\right)+5}dx\right)$$

$$\Big\downarrow \text{3136}$$

$$\frac{1}{32}\left(\frac{59}{16}\left(\frac{x}{4}-\frac{\text{arctanh}\left(\frac{\sinh(c+dx)}{\cosh(c+dx)+3}\right)}{2d}\right)-\frac{45\sinh(c+dx)}{16d(3\cosh(c+dx)+5)}\right)-\frac{3\sinh(c+dx)}{32d(3\cosh(c+dx)+5)^2}$$

input
```
Int[(5 + 3*Cosh[c + d*x])^(-3),x]
```

output
```
(-3*Sinh[c + d*x])/(32*d*(5 + 3*Cosh[c + d*x])^2) + ((59*(x/4 - ArcTanh[Si
nh[c + d*x]/(3 + Cosh[c + d*x])]/(2*d)))/16 - (45*Sinh[c + d*x])/(16*d*(5
+ 3*Cosh[c + d*x])))/32
```

3.77. $\int \frac{1}{(5+3\cosh(c+dx))^3}\,dx$

3.77.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.77.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.11

method	result
risch	$\frac{\frac{177\,\mathrm{e}^{3dx+3c}}{256}+\frac{885\,\mathrm{e}^{2dx+2c}}{256}+\frac{723\,\mathrm{e}^{dx+c}}{256}+\frac{135}{256}}{d\left(3\,\mathrm{e}^{2dx+2c}+10\,\mathrm{e}^{dx+c}+3\right)^2}-\frac{59\ln\left(\mathrm{e}^{dx+c}+3\right)}{2048d}+\frac{59\ln\left(\frac{1}{3}+\mathrm{e}^{dx+c}\right)}{2048d}$
derivativedivides	$\frac{\frac{9}{512\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^2}+\frac{69}{1024\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}-\frac{59\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{2048}-\frac{9}{512\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^2}+\frac{69}{1024\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}-}{d}$
default	$\frac{\frac{9}{512\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^2}+\frac{69}{1024\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}-\frac{59\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{2048}-\frac{9}{512\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^2}+\frac{69}{1024\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}-}{d}$
parallelrisch	$\frac{(-3540\cosh(dx+c)-531\cosh(2dx+2c)-3481)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)+(3540\cosh(dx+c)+531\cosh(2dx+2c)+3481)\ln\left(\right.}{2048d(59+9\cosh(2dx+2c)+60\cosh(dx+c))}$

input
```
int(1/(5+3*cosh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
3/256*(59*exp(3*d*x+3*c)+295*exp(2*d*x+2*c)+241*exp(d*x+c)+45)/d/(3*exp(2*
d*x+2*c)+10*exp(d*x+c)+3)^2-59/2048/d*ln(exp(d*x+c)+3)+59/2048/d*ln(1/3+ex
p(d*x+c))
```

3.77.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 563 vs. 2(73) = 146.

Time = 0.26 (sec) , antiderivative size = 563, normalized size of antiderivative = 6.95

$$\int \frac{1}{(5+3\cosh(c+dx))^3}\,dx$$
$$= \frac{1416\cosh(dx+c)^3+1416\,(3\cosh(dx+c)+5)\sinh(dx+c)^2+1416\sinh(dx+c)^3+7080\cosh(dx+}{}$$

input
```
integrate(1/(5+3*cosh(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2048*(1416*cosh(d*x + c)^3 + 1416*(3*cosh(d*x + c) + 5)*sinh(d*x + c)^2
+ 1416*sinh(d*x + c)^3 + 7080*cosh(d*x + c)^2 + 59*(9*cosh(d*x + c)^4 + 12
*(3*cosh(d*x + c) + 5)*sinh(d*x + c)^3 + 9*sinh(d*x + c)^4 + 60*cosh(d*x +
 c)^3 + 2*(27*cosh(d*x + c)^2 + 90*cosh(d*x + c) + 59)*sinh(d*x + c)^2 + 1
18*cosh(d*x + c)^2 + 4*(9*cosh(d*x + c)^3 + 45*cosh(d*x + c)^2 + 59*cosh(d
*x + c) + 15)*sinh(d*x + c) + 60*cosh(d*x + c) + 9)*log(3*cosh(d*x + c) +
3*sinh(d*x + c) + 1) - 59*(9*cosh(d*x + c)^4 + 12*(3*cosh(d*x + c) + 5)*si
nh(d*x + c)^3 + 9*sinh(d*x + c)^4 + 60*cosh(d*x + c)^3 + 2*(27*cosh(d*x +
c)^2 + 90*cosh(d*x + c) + 59)*sinh(d*x + c)^2 + 118*cosh(d*x + c)^2 + 4*(9
*cosh(d*x + c)^3 + 45*cosh(d*x + c)^2 + 59*cosh(d*x + c) + 15)*sinh(d*x +
c) + 60*cosh(d*x + c) + 9)*log(cosh(d*x + c) + sinh(d*x + c) + 3) + 24*(17
7*cosh(d*x + c)^2 + 590*cosh(d*x + c) + 241)*sinh(d*x + c) + 5784*cosh(d*x
 + c) + 1080)/(9*d*cosh(d*x + c)^4 + 9*d*sinh(d*x + c)^4 + 60*d*cosh(d*x +
 c)^3 + 12*(3*d*cosh(d*x + c) + 5*d)*sinh(d*x + c)^3 + 118*d*cosh(d*x + c)
^2 + 2*(27*d*cosh(d*x + c)^2 + 90*d*cosh(d*x + c) + 59*d)*sinh(d*x + c)^2
+ 60*d*cosh(d*x + c) + 4*(9*d*cosh(d*x + c)^3 + 45*d*cosh(d*x + c)^2 + 59*
d*cosh(d*x + c) + 15*d)*sinh(d*x + c) + 9*d)
```

3.77.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(71) = 142$.

Time = 1.38 (sec) , antiderivative size = 445, normalized size of antiderivative = 5.49

$$\int \frac{1}{(5+3\cosh(c+dx))^3}\,dx = \begin{cases} -\frac{59\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-2\right)\tanh^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{2048d\tanh^4\left(\frac{c}{2}+\frac{dx}{2}\right)-16384d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)+32768d} + \frac{472\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-2\right)\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2048d\tanh^4\left(\frac{c}{2}+\frac{dx}{2}\right)-16384d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)+32768d} - \frac{944\,\mathrm{l}}{2048d\tanh^4\left(\frac{c}{2}+\frac{d}{\ }\right.} \\ \frac{x}{(3\cosh(c)+5)^3} \end{cases}$$

input
```
integrate(1/(5+3*cosh(d*x+c))**3,x)
```

output
```
Piecewise((-59*log(tanh(c/2 + d*x/2) - 2)*tanh(c/2 + d*x/2)**4/(2048*d*tan
h(c/2 + d*x/2)**4 - 16384*d*tanh(c/2 + d*x/2)**2 + 32768*d) + 472*log(tanh
(c/2 + d*x/2) - 2)*tanh(c/2 + d*x/2)**2/(2048*d*tanh(c/2 + d*x/2)**4 - 163
84*d*tanh(c/2 + d*x/2)**2 + 32768*d) - 944*log(tanh(c/2 + d*x/2) - 2)/(204
8*d*tanh(c/2 + d*x/2)**4 - 16384*d*tanh(c/2 + d*x/2)**2 + 32768*d) + 59*lo
g(tanh(c/2 + d*x/2) + 2)*tanh(c/2 + d*x/2)**4/(2048*d*tanh(c/2 + d*x/2)**4
 - 16384*d*tanh(c/2 + d*x/2)**2 + 32768*d) - 472*log(tanh(c/2 + d*x/2) + 2
)*tanh(c/2 + d*x/2)**2/(2048*d*tanh(c/2 + d*x/2)**4 - 16384*d*tanh(c/2 + d
*x/2)**2 + 32768*d) + 944*log(tanh(c/2 + d*x/2) + 2)/(2048*d*tanh(c/2 + d*
x/2)**4 - 16384*d*tanh(c/2 + d*x/2)**2 + 32768*d) + 276*tanh(c/2 + d*x/2)*
*3/(2048*d*tanh(c/2 + d*x/2)**4 - 16384*d*tanh(c/2 + d*x/2)**2 + 32768*d)
- 816*tanh(c/2 + d*x/2)/(2048*d*tanh(c/2 + d*x/2)**4 - 16384*d*tanh(c/2 +
d*x/2)**2 + 32768*d), Ne(d, 0)), (x/(3*cosh(c) + 5)**3, True))
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.54

$$\int \frac{1}{(5+3\cosh(c+dx))^3}\,dx = -\frac{59\,\log\left(3\,e^{(-dx-c)}+1\right)}{2048\,d} + \frac{59\,\log\left(e^{(-dx-c)}+3\right)}{2048\,d} - \frac{3\left(241\,e^{(-dx-c)}+295\,e^{(-2\,dx-2\,c)}+59\,e^{(-3\,dx-3\,c)}+45\right)}{256\,d(60\,e^{(-dx-c)}+118\,e^{(-2\,dx-2\,c)}+60\,e^{(-3\,dx-3\,c)}+9\,e^{(-4\,dx-4\,c)}+9)}$$

input
```
integrate(1/(5+3*cosh(d*x+c))^3,x, algorithm="maxima")
```

output
```
-59/2048*log(3*e^(-d*x - c) + 1)/d + 59/2048*log(e^(-d*x - c) + 3)/d - 3/2
56*(241*e^(-d*x - c) + 295*e^(-2*d*x - 2*c) + 59*e^(-3*d*x - 3*c) + 45)/(d
*(60*e^(-d*x - c) + 118*e^(-2*d*x - 2*c) + 60*e^(-3*d*x - 3*c) + 9*e^(-4*d
*x - 4*c) + 9))
```

3.77.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.07

$$\int \frac{1}{(5+3\cosh(c+dx))^3}\,dx = \frac{\frac{24\left(59\,e^{(3\,dx+3\,c)}+295\,e^{(2\,dx+2\,c)}+241\,e^{(dx+c)}+45\right)}{\left(3\,e^{(2\,dx+2\,c)}+10\,e^{(dx+c)}+3\right)^2}+59\,\log\left(3\,e^{(dx+c)}+1\right)-59\,\log\left(e^{(dx+c)}+3\right)}{2048\,d}$$

input
```
integrate(1/(5+3*cosh(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2048*(24*(59*e^(3*d*x + 3*c) + 295*e^(2*d*x + 2*c) + 241*e^(d*x + c) + 4
5)/(3*e^(2*d*x + 2*c) + 10*e^(d*x + c) + 3)^2 + 59*log(3*e^(d*x + c) + 1)
- 59*log(e^(d*x + c) + 3))/d
```

3.77.9 Mupad [B] (verification not implemented)

Time = 1.82 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.74

$$\int \frac{1}{(5+3\cosh(c+dx))^3}\,dx = \frac{\frac{59\,\mathrm{e}^{c+d\,x}}{256\,d}+\frac{295}{768\,d}}{10\,\mathrm{e}^{c+d\,x}+3\,\mathrm{e}^{2\,c+2\,d\,x}+3} - \frac{59\,\mathrm{atan}\left(\left(\frac{5}{4\,d}+\frac{3\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}}{4\,d}\right)\sqrt{-d^2}\right)}{1024\,\sqrt{-d^2}} - \frac{\frac{41\,\mathrm{e}^{c+d\,x}}{24\,d}+\frac{5}{8\,d}}{60\,\mathrm{e}^{c+d\,x}+118\,\mathrm{e}^{2\,c+2\,d\,x}+60\,\mathrm{e}^{3\,c+3\,d\,x}+9\,\mathrm{e}^{4\,c+4\,d\,x}+9}$$

input
```
int(1/(3*cosh(c + d*x) + 5)^3,x)
```

output
```
((59*exp(c + d*x))/(256*d) + 295/(768*d))/(10*exp(c + d*x) + 3*exp(2*c + 2
*d*x) + 3) - (59*atan((5/(4*d) + (3*exp(d*x)*exp(c))/(4*d))*(-d^2)^(1/2)))
/(1024*(-d^2)^(1/2)) - ((41*exp(c + d*x))/(24*d) + 5/(8*d))/(60*exp(c + d*
x) + 118*exp(2*c + 2*d*x) + 60*exp(3*c + 3*d*x) + 9*exp(4*c + 4*d*x) + 9)
```

3.78 $\int \frac{1}{(5+3\cosh(c+dx))^4}\,dx$

3.78.1 Optimal result

Integrand size = 12, antiderivative size = 106

$$\int \frac{1}{(5+3\cosh(c+dx))^4}\,dx = \frac{385x}{32768} - \frac{385\text{arctanh}\left(\frac{\sinh(c+dx)}{3+\cosh(c+dx)}\right)}{16384d} - \frac{\sinh(c+dx)}{16d(5+3\cosh(c+dx))^3} - \frac{25\sinh(c+dx)}{512d(5+3\cosh(c+dx))^2} - \frac{311\sinh(c+dx)}{8192d(5+3\cosh(c+dx))}$$

output
```
385/32768*x-385/16384*arctanh(sinh(d*x+c)/(3+cosh(d*x+c)))/d-1/16*sinh(d*x
+c)/d/(5+3*cosh(d*x+c))^3-25/512*sinh(d*x+c)/d/(5+3*cosh(d*x+c))^2-311/819
2*sinh(d*x+c)/d/(5+3*cosh(d*x+c))
```

3.78.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 296 vs. 2(106) = 212.

Time = 0.27 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.79

$$\int \frac{1}{(5+3\cosh(c+dx))^4}\,dx = -\frac{296450\log\left(2\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)\right)+10395\cosh(3(c+dx))\log\left(2\cosh\left(\frac{1}{2}(c+dx)\right)-s\right.}{}$$

input
```
Integrate[(5 + 3*Cosh[c + d*x])^(-4),x]
```

output
```
-1/131072*(296450*Log[2*Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2]] + 10395*Cos
h[3*(c + d*x)]*Log[2*Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2]] + 377685*Cosh[
c + d*x]*(Log[2*Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2]] - Log[2*Cosh[(c + d
*x)/2] + Sinh[(c + d*x)/2]]) + 103950*Cosh[2*(c + d*x)]*(Log[2*Cosh[(c + d
*x)/2] - Sinh[(c + d*x)/2]] - Log[2*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2]]
) - 296450*Log[2*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2]] - 10395*Cosh[3*(c
+ d*x)]*Log[2*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2]] + 175788*Sinh[c + d*x
] + 84240*Sinh[2*(c + d*x)] + 11196*Sinh[3*(c + d*x)])/(d*(5 + 3*Cosh[c +
d*x])^3)
```

3.78.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {3042, 3143, 27, 3042, 3233, 25, 3042, 3233, 27, 3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3\cosh(c+dx)+5)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(5+3\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{3143}$$

$$-\frac{1}{48}\int -\frac{3(5-2\cosh(c+dx))}{(3\cosh(c+dx)+5)^3}dx-\frac{\sinh(c+dx)}{16d(3\cosh(c+dx)+5)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\int \frac{5-2\cosh(c+dx)}{(3\cosh(c+dx)+5)^3}dx-\frac{\sinh(c+dx)}{16d(3\cosh(c+dx)+5)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(c+dx)}{16d(3\cosh(c+dx)+5)^3}+\frac{1}{16}\int \frac{5-2\sin\left(ic+idx+\frac{\pi}{2}\right)}{\left(3\sin\left(ic+idx+\frac{\pi}{2}\right)+5\right)^3}dx$$

$$\Big\downarrow \text{3233}$$

$$\frac{1}{16}\left(-\frac{1}{32}\int -\frac{62-25\cosh(c+dx)}{(3\cosh(c+dx)+5)^2}dx-\frac{25\sinh(c+dx)}{32d(3\cosh(c+dx)+5)^2}\right)-\frac{\sinh(c+dx)}{16d(3\cosh(c+dx)+5)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{16}\left(\frac{1}{32}\int \frac{62-25\cosh(c+dx)}{(3\cosh(c+dx)+5)^2}dx-\frac{25\sinh(c+dx)}{32d(3\cosh(c+dx)+5)^2}\right)-\frac{\sinh(c+dx)}{16d(3\cosh(c+dx)+5)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(c+dx)}{16d(3\cosh(c+dx)+5)^3}+\frac{1}{16}\left(-\frac{25\sinh(c+dx)}{32d(3\cosh(c+dx)+5)^2}+\frac{1}{32}\int \frac{62-25\sin\left(ic+idx+\frac{\pi}{2}\right)}{\left(3\sin\left(ic+idx+\frac{\pi}{2}\right)+5\right)^2}dx\right)$$

$$\Big\downarrow \text{3233}$$

$$\frac{1}{16}\left(\frac{1}{32}\left(-\frac{1}{16}\int -\frac{385}{3\cosh(c+dx)+5}dx-\frac{311\sinh(c+dx)}{16d(3\cosh(c+dx)+5)}\right)-\frac{25\sinh(c+dx)}{32d(3\cosh(c+dx)+5)^2}\right)-\frac{\sinh(c+dx)}{16d(3\cosh(c+dx)+5)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\left(\frac{1}{32}\left(\frac{385}{16}\int \frac{1}{3\cosh(c+dx)+5}dx-\frac{311\sinh(c+dx)}{16d(3\cosh(c+dx)+5)}\right)-\frac{25\sinh(c+dx)}{32d(3\cosh(c+dx)+5)^2}\right)-\frac{\sinh(c+dx)}{16d(3\cosh(c+dx)+5)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(c+dx)}{16d(3\cosh(c+dx)+5)^3}+\frac{1}{16}\left(-\frac{25\sinh(c+dx)}{32d(3\cosh(c+dx)+5)^2}+\frac{1}{32}\left(-\frac{311\sinh(c+dx)}{16d(3\cosh(c+dx)+5)}+\frac{385}{16}\int \frac{1}{3\sin\left(ic+idx+\frac{\pi}{2}\right)+5}dx\right)\right)$$

$$\Big\downarrow \text{3136}$$

$$\frac{1}{16}\left(\frac{1}{32}\left(\frac{385}{16}\left(\frac{x}{4}-\frac{\operatorname{arctanh}\left(\frac{\sinh(c+dx)}{\cosh(c+dx)+3}\right)}{2d}\right)-\frac{311\sinh(c+dx)}{16d(3\cosh(c+dx)+5)}\right)-\frac{25\sinh(c+dx)}{32d(3\cosh(c+dx)+5)^2}\right)-\frac{\sinh(c+dx)}{16d(3\cosh(c+dx)+5)^3}$$

input
```
Int[(5 + 3*Cosh[c + d*x])^(-4),x]
```

3.78. $\int \frac{1}{(5+3\cosh(c+dx))^4}\,dx$

output
```
-1/16*Sinh[c + d*x]/(d*(5 + 3*Cosh[c + d*x])^3) + ((-25*Sinh[c + d*x])/(32
*d*(5 + 3*Cosh[c + d*x])^2) + ((385*(x/4 - ArcTanh[Sinh[c + d*x]/(3 + Cosh
[c + d*x])]/(2*d)))/16 - (311*Sinh[c + d*x])/(16*d*(5 + 3*Cosh[c + d*x])))
/32)/16
```

3.78.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.78.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.06

method	result
risch	$\frac{10395\,e^{5dx+5c}+86625\,e^{4dx+4c}+239470\,e^{3dx+3c}+218466\,e^{2dx+2c}+73575\,e^{dx+c}+8397}{12288d\left(3\,e^{2dx+2c}+10\,e^{dx+c}+3\right)^3}-\frac{385\ln\left(e^{dx+c}+3\right)}{32768d}+\frac{385\ln(}{32}$
derivativedivides	$\frac{\frac{9}{2048\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^3}+\frac{81}{4096\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^2}+\frac{639}{16384\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}-\frac{385\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{32768}+\frac{9}{2048\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}}{d}$
default	$\frac{\frac{9}{2048\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^3}+\frac{81}{4096\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^2}+\frac{639}{16384\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}-\frac{385\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{32768}+\frac{9}{2048\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}}{d}$
parallelrisch	$\frac{(-377685\cosh(dx+c)-103950\cosh(2dx+2c)-10395\cosh(3dx+3c)-296450)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)+(377685\cosh(dx+}{32768d(770+27\cosh(}$

input
```
int(1/(5+3*cosh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/12288*(10395*exp(5*d*x+5*c)+86625*exp(4*d*x+4*c)+239470*exp(3*d*x+3*c)+2
18466*exp(2*d*x+2*c)+73575*exp(d*x+c)+8397)/d/(3*exp(2*d*x+2*c)+10*exp(d*x
+c)+3)^3-385/32768/d*ln(exp(d*x+c)+3)+385/32768/d*ln(1/3+exp(d*x+c))
```

3.78.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1078 vs. 2(96) = 192.

Time = 0.25 (sec) , antiderivative size = 1078, normalized size of antiderivative = 10.17

$$\int \frac{1}{(5+3\cosh(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(5+3*cosh(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/98304*(83160*cosh(d*x + c)^5 + 138600*(3*cosh(d*x + c) + 5)*sinh(d*x + c
)^4 + 83160*sinh(d*x + c)^5 + 693000*cosh(d*x + c)^4 + 6160*(135*cosh(d*x
+ c)^2 + 450*cosh(d*x + c) + 311)*sinh(d*x + c)^3 + 1915760*cosh(d*x + c)^
3 + 48*(17325*cosh(d*x + c)^3 + 86625*cosh(d*x + c)^2 + 119735*cosh(d*x +
c) + 36411)*sinh(d*x + c)^2 + 1747728*cosh(d*x + c)^2 + 1155*(27*cosh(d*x
+ c)^6 + 54*(3*cosh(d*x + c) + 5)*sinh(d*x + c)^5 + 27*sinh(d*x + c)^6 + 2
70*cosh(d*x + c)^5 + 9*(45*cosh(d*x + c)^2 + 150*cosh(d*x + c) + 109)*sinh
(d*x + c)^4 + 981*cosh(d*x + c)^4 + 4*(135*cosh(d*x + c)^3 + 675*cosh(d*x
+ c)^2 + 981*cosh(d*x + c) + 385)*sinh(d*x + c)^3 + 1540*cosh(d*x + c)^3 +
 3*(135*cosh(d*x + c)^4 + 900*cosh(d*x + c)^3 + 1962*cosh(d*x + c)^2 + 154
0*cosh(d*x + c) + 327)*sinh(d*x + c)^2 + 981*cosh(d*x + c)^2 + 6*(27*cosh(
d*x + c)^5 + 225*cosh(d*x + c)^4 + 654*cosh(d*x + c)^3 + 770*cosh(d*x + c)
^2 + 327*cosh(d*x + c) + 45)*sinh(d*x + c) + 270*cosh(d*x + c) + 27)*log(3
*cosh(d*x + c) + 3*sinh(d*x + c) + 1) - 1155*(27*cosh(d*x + c)^6 + 54*(3*c
osh(d*x + c) + 5)*sinh(d*x + c)^5 + 27*sinh(d*x + c)^6 + 270*cosh(d*x + c)
^5 + 9*(45*cosh(d*x + c)^2 + 150*cosh(d*x + c) + 109)*sinh(d*x + c)^4 + 98
1*cosh(d*x + c)^4 + 4*(135*cosh(d*x + c)^3 + 675*cosh(d*x + c)^2 + 981*cos
h(d*x + c) + 385)*sinh(d*x + c)^3 + 1540*cosh(d*x + c)^3 + 3*(135*cosh(d*x
 + c)^4 + 900*cosh(d*x + c)^3 + 1962*cosh(d*x + c)^2 + 1540*cosh(d*x + c)
+ 327)*sinh(d*x + c)^2 + 981*cosh(d*x + c)^2 + 6*(27*cosh(d*x + c)^5 + ...
```

3.78.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 784 vs. $2(94) = 188$.

Time = 2.69 (sec) , antiderivative size = 784, normalized size of antiderivative = 7.40

$$\int \frac{1}{(5+3\cosh(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(5+3*cosh(d*x+c))**4,x)
```

output
```
Piecewise((-385*log(tanh(c/2 + d*x/2) - 2)*tanh(c/2 + d*x/2)**6/(32768*d*t
anh(c/2 + d*x/2)**6 - 393216*d*tanh(c/2 + d*x/2)**4 + 1572864*d*tanh(c/2 +
 d*x/2)**2 - 2097152*d) + 4620*log(tanh(c/2 + d*x/2) - 2)*tanh(c/2 + d*x/2
)**4/(32768*d*tanh(c/2 + d*x/2)**6 - 393216*d*tanh(c/2 + d*x/2)**4 + 15728
64*d*tanh(c/2 + d*x/2)**2 - 2097152*d) - 18480*log(tanh(c/2 + d*x/2) - 2)*
tanh(c/2 + d*x/2)**2/(32768*d*tanh(c/2 + d*x/2)**6 - 393216*d*tanh(c/2 + d
*x/2)**4 + 1572864*d*tanh(c/2 + d*x/2)**2 - 2097152*d) + 24640*log(tanh(c/
2 + d*x/2) - 2)/(32768*d*tanh(c/2 + d*x/2)**6 - 393216*d*tanh(c/2 + d*x/2)
**4 + 1572864*d*tanh(c/2 + d*x/2)**2 - 2097152*d) + 385*log(tanh(c/2 + d*x
/2) + 2)*tanh(c/2 + d*x/2)**6/(32768*d*tanh(c/2 + d*x/2)**6 - 393216*d*tan
h(c/2 + d*x/2)**4 + 1572864*d*tanh(c/2 + d*x/2)**2 - 2097152*d) - 4620*log
(tanh(c/2 + d*x/2) + 2)*tanh(c/2 + d*x/2)**4/(32768*d*tanh(c/2 + d*x/2)**6
 - 393216*d*tanh(c/2 + d*x/2)**4 + 1572864*d*tanh(c/2 + d*x/2)**2 - 209715
2*d) + 18480*log(tanh(c/2 + d*x/2) + 2)*tanh(c/2 + d*x/2)**2/(32768*d*tanh
(c/2 + d*x/2)**6 - 393216*d*tanh(c/2 + d*x/2)**4 + 1572864*d*tanh(c/2 + d*
x/2)**2 - 2097152*d) - 24640*log(tanh(c/2 + d*x/2) + 2)/(32768*d*tanh(c/2
+ d*x/2)**6 - 393216*d*tanh(c/2 + d*x/2)**4 + 1572864*d*tanh(c/2 + d*x/2)*
*2 - 2097152*d) + 2556*tanh(c/2 + d*x/2)**5/(32768*d*tanh(c/2 + d*x/2)**6
- 393216*d*tanh(c/2 + d*x/2)**4 + 1572864*d*tanh(c/2 + d*x/2)**2 - 2097152
*d) - 14976*tanh(c/2 + d*x/2)**3/(32768*d*tanh(c/2 + d*x/2)**6 - 393216...
```

3.78.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.59

$$\int \frac{1}{(5+3\cosh(c+dx))^4}\,dx = -\frac{385\,\log\left(3\,e^{(-dx-c)}+1\right)}{32768\,d} + \frac{385\,\log\left(e^{(-dx-c)}+3\right)}{32768\,d} - \frac{73575\,e^{(-dx-c)} + 218466\,e^{(-2\,dx-2\,c)} + 239470\,e^{(-3\,dx-3\,c)} + 86625\,e^{(-4\,dx-4\,c)} + 10395\,e^{(-5\,dx-5\,c)} + 8\cdots}{12288\,d(270\,e^{(-dx-c)} + 981\,e^{(-2\,dx-2\,c)} + 1540\,e^{(-3\,dx-3\,c)} + 981\,e^{(-4\,dx-4\,c)} + 270\,e^{(-5\,dx-5\,c)} + 27\,e^{(-6\,dx-6\cdots}}$$

input
```
integrate(1/(5+3*cosh(d*x+c))^4,x, algorithm="maxima")
```

output
```
-385/32768*log(3*e^(-d*x - c) + 1)/d + 385/32768*log(e^(-d*x - c) + 3)/d -
 1/12288*(73575*e^(-d*x - c) + 218466*e^(-2*d*x - 2*c) + 239470*e^(-3*d*x
- 3*c) + 86625*e^(-4*d*x - 4*c) + 10395*e^(-5*d*x - 5*c) + 8397)/(d*(270*e
^(-d*x - c) + 981*e^(-2*d*x - 2*c) + 1540*e^(-3*d*x - 3*c) + 981*e^(-4*d*x
 - 4*c) + 270*e^(-5*d*x - 5*c) + 27*e^(-6*d*x - 6*c) + 27))
```

3.78.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.03

$$\int \frac{1}{(5+3\cosh(c+dx))^4}\,dx = \frac{\frac{8\left(10395\,e^{(5\,dx+5\,c)}+86625\,e^{(4\,dx+4\,c)}+239470\,e^{(3\,dx+3\,c)}+218466\,e^{(2\,dx+2\,c)}+73575\,e^{(dx+c)}+8397\right)}{\left(3\,e^{(2\,dx+2\,c)}+10\,e^{(dx+c)}+3\right)^3}+1155\,\log\left(3\,e^{(dx+c)}+1\right)-115}{98304\,d}$$

input
```
integrate(1/(5+3*cosh(d*x+c))^4,x, algorithm="giac")
```

output
```
1/98304*(8*(10395*e^(5*d*x + 5*c) + 86625*e^(4*d*x + 4*c) + 239470*e^(3*d*
x + 3*c) + 218466*e^(2*d*x + 2*c) + 73575*e^(d*x + c) + 8397)/(3*e^(2*d*x
+ 2*c) + 10*e^(d*x + c) + 3)^3 + 1155*log(3*e^(d*x + c) + 1) - 1155*log(e^
(d*x + c) + 3))/d
```

3.78.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.13

$$\int \frac{1}{(5+3\cosh(c+dx))^4}\,dx = \frac{\frac{385\,e^{c+d\,x}}{4096\,d}+\frac{1925}{12288\,d}}{10\,e^{c+d\,x}+3\,e^{2\,c+2\,d\,x}+3} - \frac{385\,\mathrm{atan}\left(\left(\frac{5}{4\,d}+\frac{3\,e^{d\,x}\,e^{c}}{4\,d}\right)\sqrt{-d^2}\right)}{16384\,\sqrt{-d^2}} - \frac{\frac{385\,e^{c+d\,x}}{1152\,d}+\frac{3461}{3456\,d}}{60\,e^{c+d\,x}+118\,e^{2\,c+2\,d\,x}+60\,e^{3\,c+3\,d\,x}+9\,e^{4\,c+4\,d\,x}+9} + \frac{\frac{365\,e^{c+d\,x}}{54\,d}+\frac{41}{18\,d}}{270\,e^{c+d\,x}+981\,e^{2\,c+2\,d\,x}+1540\,e^{3\,c+3\,d\,x}+981\,e^{4\,c+4\,d\,x}+270\,e^{5\,c+5\,d\,x}+27\,e^{6\,c+6\,d\,x}+27}$$

input
```
int(1/(3*cosh(c + d*x) + 5)^4,x)
```

output
```
((385*exp(c + d*x))/(4096*d) + 1925/(12288*d))/(10*exp(c + d*x) + 3*exp(2*
c + 2*d*x) + 3) - (385*atan((5/(4*d) + (3*exp(d*x)*exp(c))/(4*d))*(-d^2)^(
1/2)))/(16384*(-d^2)^(1/2)) - ((385*exp(c + d*x))/(1152*d) + 3461/(3456*d)
)/(60*exp(c + d*x) + 118*exp(2*c + 2*d*x) + 60*exp(3*c + 3*d*x) + 9*exp(4*
c + 4*d*x) + 9) + ((365*exp(c + d*x))/(54*d) + 41/(18*d))/(270*exp(c + d*x
) + 981*exp(2*c + 2*d*x) + 1540*exp(3*c + 3*d*x) + 981*exp(4*c + 4*d*x) +
270*exp(5*c + 5*d*x) + 27*exp(6*c + 6*d*x) + 27)
```

3.79 $\int (a+b\cosh(x))^{5/2}\,dx$

3.79.1 Optimal result

Integrand size = 10, antiderivative size = 153

$$\int (a+b\cosh(x))^{5/2}\,dx = -\frac{2i(23a^2+9b^2)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{15\sqrt{\frac{a+b\cosh(x)}{a+b}}} + \frac{16ia(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{15\sqrt{a+b\cosh(x)}} + \frac{16}{15}ab\sqrt{a+b\cosh(x)}\sinh(x) + \frac{2}{5}b(a+b\cosh(x))^{3/2}\sinh(x)$$

output
```
2/5*b*(a+b*cosh(x))^(3/2)*sinh(x)+16/15*a*b*sinh(x)*(a+b*cosh(x))^(1/2)-2/
15*I*(23*a^2+9*b^2)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(1/2
*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cosh(x))^(1/2)/((a+b*cosh(x))/(a+b))^(1/
2)+16/15*I*a*(a^2-b^2)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(
1/2*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cosh(x))/(a+b))^(1/2)/(a+b*cosh(x))^
(1/2)
```

3.79.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.98

$$\int (a+b\cosh(x))^{5/2}\,dx = \frac{-2i(23a^3+23a^2b+9ab^2+9b^3)\sqrt{\frac{a+b\cosh(x)}{a+b}}E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)+16ia(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,\text{Ellip}}{15\sqrt{a+b\cosh(x)}}$$

input
```
Integrate[(a + b*Cosh[x])^(5/2),x]
```

output
```
((-2*I)*(23*a^3 + 23*a^2*b + 9*a*b^2 + 9*b^3)*Sqrt[(a + b*Cosh[x])/(a + b)
]*EllipticE[(I/2)*x, (2*b)/(a + b)] + (16*I)*a*(a^2 - b^2)*Sqrt[(a + b*Cos
h[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)] + b*(22*a^2 + 3*b^2 + 28*
a*b*Cosh[x] + 3*b^2*Cosh[2*x])*Sinh[x])/(15*Sqrt[a + b*Cosh[x]])
```

3.79.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.500$, Rules used = {3042, 3135, 27, 3042, 3232, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cosh(x))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(\frac{\pi}{2}+ix\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{3135}$$

$$\frac{2}{5}\int \frac{1}{2}\sqrt{a+b\cosh(x)}\left(5a^2+8b\cosh(x)a+3b^2\right)dx+\frac{2}{5}b\sinh(x)(a+b\cosh(x))^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \sqrt{a+b\cosh(x)}\left(5a^2+8b\cosh(x)a+3b^2\right)dx+\frac{2}{5}b\sinh(x)(a+b\cosh(x))^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}b\sinh(x)(a+b\cosh(x))^{3/2}+\frac{1}{5}\int\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}\left(5a^2+8b\sin\left(ix+\frac{\pi}{2}\right)a+3b^2\right)dx$$

$$\Big\downarrow \; 3232$$

$$\frac{1}{5}\left(\frac{2}{3}\int\frac{a\left(15a^2+17b^2\right)+b\left(23a^2+9b^2\right)\cosh(x)}{2\sqrt{a+b\cosh(x)}}dx+\frac{16}{3}ab\sinh(x)\sqrt{a+b\cosh(x)}\right)+\frac{2}{5}b\sinh(x)(a+b\cosh(x))^{3/2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{5}\left(\frac{1}{3}\int\frac{a\left(15a^2+17b^2\right)+b\left(23a^2+9b^2\right)\cosh(x)}{\sqrt{a+b\cosh(x)}}dx+\frac{16}{3}ab\sinh(x)\sqrt{a+b\cosh(x)}\right)+\frac{2}{5}b\sinh(x)(a+b\cosh(x))^{3/2}$$

$$\Big\downarrow \; 3042$$

$$\frac{2}{5}b\sinh(x)(a+b\cosh(x))^{3/2}+\frac{1}{5}\left(\frac{16}{3}ab\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\int\frac{a\left(15a^2+17b^2\right)+b\left(23a^2+9b^2\right)\sin\left(ix+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx\right)$$

$$\Big\downarrow \; 3231$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\left(23a^2+9b^2\right)\int\sqrt{a+b\cosh(x)}dx-8a\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\cosh(x)}}dx\right)+\frac{16}{3}ab\sinh(x)\sqrt{a+b\cosh(x)}\right)+\frac{2}{5}b\sinh(x)(a+b\cosh(x))^{3/2}$$

$$\Big\downarrow \; 3042$$

$$\frac{2}{5}b\sinh(x)(a+b\cosh(x))^{3/2}+\frac{1}{5}\left(\frac{16}{3}ab\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(\left(23a^2+9b^2\right)\int\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx-8a\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(ix+\right.}}\right.\right.$$

$$\Big\downarrow \; 3134$$

$$\frac{2}{5}b\sinh(x)(a+b\cosh(x))^{3/2}+\frac{1}{5}\left(\frac{16}{3}ab\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(\frac{\left(23a^2+9b^2\right)\sqrt{a+b\cosh(x)}\int\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}dx}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}-8a\left(a^2-b^2\right)\int\frac{}{\sqrt{a+}}\right.\right.$$

$$\Big\downarrow \; 3042$$

3.79. $\int(a+b\cosh(x))^{5/2}\,dx$

$$\frac{1}{5}\left(\frac{16}{3}ab\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(\frac{(23a^2+9b^2)\sqrt{a+b\cosh(x)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin(ix+\frac{\pi}{2})}{a+b}}dx}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}-8a(a^2-b^2)\int\frac{}{\sqrt{a}}\right.\right.\frac{2}{5}b\sinh(x)(a+b\cosh(x))^{3/2}+$$

$\downarrow$ 3132

$$\frac{1}{5}\left(\frac{16}{3}ab\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(-8a(a^2-b^2)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx-\frac{2i(23a^2+9b^2)\sqrt{a+b\cosh(x)}E}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}\right.\right.\frac{2}{5}b\sinh(x)(a+b\cosh(x))^{3/2}+$$

$\downarrow$ 3142

$$\frac{1}{5}\left(\frac{16}{3}ab\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(-\frac{8a(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}}dx}{\sqrt{a+b\cosh(x)}}-\frac{2i(23a^2+9b^2)\sqrt{a+b\cos}}{\sqrt{\frac{a+b\cosh(x}{a+b}}}\right.\right.\frac{2}{5}b\sinh(x)(a+b\cosh(x))^{3/2}+$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{16}{3}ab\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(-\frac{8a(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cosh(x)}}-\frac{2i(23a^2+9b^2)\sqrt{a+b\,c}}{\sqrt{\frac{a+b\cos}{a+b}}}\right.\right.\frac{2}{5}b\sinh(x)(a+b\cosh(x))^{3/2}+$$

$\downarrow$ 3140

$$\frac{1}{5}\left(\frac{16}{3}ab\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(\frac{16ia(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{\sqrt{a+b\cosh(x)}}-\frac{2i(23a^2+9b^2)\sqrt{a+b\,co}}{\sqrt{\frac{a+b\cosh(}{a+b}}}\right.\right.\frac{2}{5}b\sinh(x)(a+b\cosh(x))^{3/2}+$$

input
```
Int[(a + b*Cosh[x])^(5/2),x]
```

output
```
(2*b*(a + b*Cosh[x])^(3/2)*Sinh[x])/5 + ((((-2*I)*(23*a^2 + 9*b^2)*Sqrt[a
+ b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/Sqrt[(a + b*Cosh[x])/(a +
b)] + ((16*I)*a*(a^2 - b^2)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*
x, (2*b)/(a + b)])/Sqrt[a + b*Cosh[x]])/3 + (16*a*b*Sqrt[a + b*Cosh[x]]*Si
nh[x])/3)/5
```

3.79. $\int(a+b\cosh(x))^{5/2}\,dx$

3.79.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3135
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[1/n   Int[(a + b*
Sin[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x]
, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
 IntegerQ[2*n]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

3.79.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 684 vs. $2(169) = 338$.

Time = 6.49 (sec) , antiderivative size = 685, normalized size of antiderivative = 4.48

method	result
default	$\frac{2\left(24\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}}\,\sinh\left(\frac{x}{2}\right)^{6}b^{3}+\left(56\sqrt{-\frac{2b}{a-b}}\,a\,b^{2}+24\sqrt{-\frac{2b}{a-b}}\,b^{3}\right)\sinh\left(\frac{x}{2}\right)^{4}\cosh\left(\frac{x}{2}\right)+\left(22\sqrt{-\frac{2b}{a-b}}\,a^{2}b+28\sqrt{-\frac{2b}{a-b}}\,a\,b^{2}+6\sqrt{-\cdots}\right.\right.}{}$

input
```
int((a+b*cosh(x))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(24*cosh(1/2*x)*(-2*b/(a-b))^(1/2)*sinh(1/2*x)^6*b^3+(56*(-2*b/(a-b))
^(1/2)*a*b^2+24*(-2*b/(a-b))^(1/2)*b^3)*sinh(1/2*x)^4*cosh(1/2*x)+(22*(-2*
b/(a-b))^(1/2)*a^2*b+28*(-2*b/(a-b))^(1/2)*a*b^2+6*(-2*b/(a-b))^(1/2)*b^3)
*sinh(1/2*x)^2*cosh(1/2*x)+15*a^3*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1
/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-
2*(a-b)/b)^(1/2))+23*a^2*b*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-s
inh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)
/b)^(1/2))+17*a*b^2*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2
*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/
2))+9*b^3*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/
2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))-46*(2*
b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(
cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))*a^2*b-18*(2*b/(a-b)
*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(cosh(1/
2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))*b^3)*((2*cosh(1/2*x)^2*b+a
-b)*sinh(1/2*x)^2)^(1/2)/(-2*b/(a-b))^(1/2)/(2*sinh(1/2*x)^4*b+(a+b)*sinh(
1/2*x)^2)^(1/2)/sinh(1/2*x)/(2*sinh(1/2*x)^2*b+a+b)^(1/2)
```

3.79.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 464, normalized size of antiderivative = 3.03

$$\int (a + b\cosh(x))^{5/2}\,dx =$$

$$-\frac{4\left(\sqrt{2}(a^3 - 33\,ab^2)\cosh\left(x\right)^2 + 2\sqrt{2}(a^3 - 33\,ab^2)\cosh\left(x\right)\sinh\left(x\right) + \sqrt{2}(a^3 - 33\,ab^2)\sinh\left(x\right)^2\right)\sqrt{b}\mathrm{weierstr}}{}$$

input
```
integrate((a+b*cosh(x))^(5/2),x, algorithm="fricas")
```

output
```
-1/90*(4*(sqrt(2)*(a^3 - 33*a*b^2)*cosh(x)^2 + 2*sqrt(2)*(a^3 - 33*a*b^2)*
cosh(x)*sinh(x) + sqrt(2)*(a^3 - 33*a*b^2)*sinh(x)^2)*sqrt(b)*weierstrassP
Inverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos
h(x) + 3*b*sinh(x) + 2*a)/b) + 12*(sqrt(2)*(23*a^2*b + 9*b^3)*cosh(x)^2 +
2*sqrt(2)*(23*a^2*b + 9*b^3)*cosh(x)*sinh(x) + sqrt(2)*(23*a^2*b + 9*b^3)*
sinh(x)^2)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 -
 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 -
 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) - 3*(3*b^3*cosh(x
)^4 + 3*b^3*sinh(x)^4 + 22*a*b^2*cosh(x)^3 - 22*a*b^2*cosh(x) + 2*(6*b^3*c
osh(x) + 11*a*b^2)*sinh(x)^3 - 3*b^3 - 4*(23*a^2*b + 9*b^3)*cosh(x)^2 + 2*
(9*b^3*cosh(x)^2 + 33*a*b^2*cosh(x) - 46*a^2*b - 18*b^3)*sinh(x)^2 + 2*(6*
b^3*cosh(x)^3 + 33*a*b^2*cosh(x)^2 - 11*a*b^2 - 4*(23*a^2*b + 9*b^3)*cosh(
x))*sinh(x))*sqrt(b*cosh(x) + a))/(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*s
inh(x)^2)
```

3.79.6 Sympy [F]

$$\int (a + b\cosh(x))^{5/2}\,dx = \int \left(a + b\cosh\left(x\right)\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cosh(x))**(5/2),x)
```

output
```
Integral((a + b*cosh(x))**(5/2), x)
```

3.79.7 Maxima [F]

$$\int (a+b\cosh(x))^{5/2}\,dx = \int (b\cosh(x)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cosh(x))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cosh(x) + a)^(5/2), x)
```

3.79.8 Giac [F]

$$\int (a+b\cosh(x))^{5/2}\,dx = \int (b\cosh(x)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cosh(x))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cosh(x) + a)^(5/2), x)
```

3.79.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cosh(x))^{5/2}\,dx = \int (a+b\cosh(x))^{5/2}\,dx$$

input
```
int((a + b*cosh(x))^(5/2),x)
```

output
```
int((a + b*cosh(x))^(5/2), x)
```

3.80 $\int (a+b\cosh(x))^{3/2}\,dx$

3.80.1 Optimal result

Integrand size = 10, antiderivative size = 124

$$\int (a+b\cosh(x))^{3/2}\,dx = -\frac{8ia\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{3\sqrt{\frac{a+b\cosh(x)}{a+b}}} + \frac{2i(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{3\sqrt{a+b\cosh(x)}} + \frac{2}{3}b\sqrt{a+b\cosh(x)}\sinh(x)$$

output
```
2/3*b*sinh(x)*(a+b*cosh(x))^(1/2)-8/3*I*a*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x
)*EllipticE(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cosh(x))^(1/2)/((a
+b*cosh(x))/(a+b))^(1/2)+2/3*I*(a^2-b^2)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)
*EllipticF(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cosh(x))/(a+b))^(1
/2)/(a+b*cosh(x))^(1/2)
```

3.80.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90

$$\int (a+b\cosh(x))^{3/2}\,dx = \frac{-8ia(a+b)\sqrt{\frac{a+b\cosh(x)}{a+b}}E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right) + 2i(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right) + 2b(a+}{3\sqrt{a+b\cosh(x)}}$$

input
```
Integrate[(a + b*Cosh[x])^(3/2),x]
```

output
```
((-8*I)*a*(a + b)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticE[(I/2)*x, (2*b)/(
a + b)] + (2*I)*(a^2 - b^2)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*
x, (2*b)/(a + b)] + 2*b*(a + b*Cosh[x])*Sinh[x])/(3*Sqrt[a + b*Cosh[x]])
```

3.80.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.200$, Rules used = {3042, 3135, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cosh(x))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(\frac{\pi}{2}+ix\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{3135}$$

$$\frac{2}{3}\int \frac{3a^2+4b\cosh(x)a+b^2}{2\sqrt{a+b\cosh(x)}}dx+\frac{2}{3}b\sinh(x)\sqrt{a+b\cosh(x)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{3a^2+4b\cosh(x)a+b^2}{\sqrt{a+b\cosh(x)}}dx+\frac{2}{3}b\sinh(x)\sqrt{a+b\cosh(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}b\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\int \frac{3a^2+4b\sin\left(ix+\frac{\pi}{2}\right)a+b^2}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3231}$$

$$\frac{1}{3}\left(4a\int \sqrt{a+b\cosh(x)}dx-\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\cosh(x)}}dx\right)+\frac{2}{3}b\sinh(x)\sqrt{a+b\cosh(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}b\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(4a\int\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx-\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx\right)$$

$$\Big\downarrow \text{3134}$$

$$\frac{2}{3}b\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(\frac{4a\sqrt{a+b\cosh(x)}\int\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}dx}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}-\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}b\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(\frac{4a\sqrt{a+b\cosh(x)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}-\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx\right)$$

$$\Big\downarrow \text{3132}$$

$$\frac{2}{3}b\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(-\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx-\frac{8ia\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}\right)$$

$$\Big\downarrow \text{3142}$$

$$\frac{2}{3}b\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(-\frac{\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}}dx}{\sqrt{a+b\cosh(x)}}-\frac{8ia\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}b\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(-\frac{\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cosh(x)}}-\frac{8ia\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}\right)$$

$$\Big\downarrow \text{3140}$$

3.80. $\int(a+b\cosh(x))^{3/2}\,dx$

$$\frac{2}{3}b\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(\frac{2i(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{\sqrt{a+b\cosh(x)}}-\frac{8ia\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}\right)$$

input
```
Int[(a + b*Cosh[x])^(3/2),x]
```

output
```
(((-8*I)*a*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/Sqrt[(a
+ b*Cosh[x])/(a + b)] + ((2*I)*(a^2 - b^2)*Sqrt[(a + b*Cosh[x])/(a + b)]*E
llipticF[(I/2)*x, (2*b)/(a + b)])/Sqrt[a + b*Cosh[x]])/3 + (2*b*Sqrt[a + b
*Cosh[x]]*Sinh[x])/3
```

3.80.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3135
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[1/n   Int[(a + b*
Sin[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x]
, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
 IntegerQ[2*n]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

3.80.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(144) = 288$.

Time = 1.43 (sec) , antiderivative size = 466, normalized size of antiderivative = 3.76

method	result
default	$\frac{2\left(4\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}}\,\sinh\left(\frac{x}{2}\right)^{4}b^{2}+2\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}}\,\sinh\left(\frac{x}{2}\right)^{2}ab+2\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}}\,\sinh\left(\frac{x}{2}\right)^{2}b^{2}+3a^{2}\sqrt{\frac{2b\sinh\left(\frac{x}{2}\right)^{2}}{a-b}+\frac{a+b}{a-b}}\,\sqrt{-\sin\right.\right.}{}$

input
```
int((a+b*cosh(x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(4*cosh(1/2*x)*(-2*b/(a-b))^(1/2)*sinh(1/2*x)^4*b^2+2*cosh(1/2*x)*(-2*
b/(a-b))^(1/2)*sinh(1/2*x)^2*a*b+2*cosh(1/2*x)*(-2*b/(a-b))^(1/2)*sinh(1/2
*x)^2*b^2+3*a^2*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^
2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+
4*a*b*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*E
llipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+b^2*(2*b/(
a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cos
h(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))-8*(2*b/(a-b)*sinh(1/2*
x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x)*(-2*b
/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))*a*b)*((2*cosh(1/2*x)^2*b+a-b)*sinh(1
/2*x)^2)^(1/2)/(-2*b/(a-b))^(1/2)/(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^2)^
(1/2)/sinh(1/2*x)/(2*sinh(1/2*x)^2*b+a+b)^(1/2)
```

3.80.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.14

$$\int (a+b\cosh(x))^{3/2}\,dx = \frac{2\left(\sqrt{2}(a^2+3\,b^2)\cosh{(x)}+\sqrt{2}(a^2+3\,b^2)\sinh{(x)}\right)\sqrt{b}\text{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},\,-\right.}{}$$

input
```
integrate((a+b*cosh(x))^(3/2),x, algorithm="fricas")
```

output
```
1/9*(2*(sqrt(2)*(a^2 + 3*b^2)*cosh(x) + sqrt(2)*(a^2 + 3*b^2)*sinh(x))*sqr
t(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/
b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) - 24*(sqrt(2)*a*b*cosh(x) +
sqrt(2)*a*b*sinh(x))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/2
7*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/2
7*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) + 3*(b^
2*cosh(x)^2 + b^2*sinh(x)^2 - 8*a*b*cosh(x) - b^2 + 2*(b^2*cosh(x) - 4*a*b
)*sinh(x))*sqrt(b*cosh(x) + a))/(b*cosh(x) + b*sinh(x))
```

3.80.6 Sympy [F]

$$\int (a+b\cosh(x))^{3/2}\,dx = \int (a+b\cosh{(x)})^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cosh(x))**(3/2),x)
```

output
```
Integral((a + b*cosh(x))**(3/2), x)
```

3.80.7 Maxima [F]

$$\int (a+b\cosh(x))^{3/2}\,dx = \int (b\cosh{(x)}+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cosh(x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cosh(x) + a)^(3/2), x)
```

3.80.8 Giac [F]

$$\int (a+b\cosh(x))^{3/2}\,dx = \int (b\cosh{(x)}+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cosh(x))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cosh(x) + a)^(3/2), x)
```

3.80.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cosh(x))^{3/2}\,dx=\int (a+b\cosh(x))^{3/2}\,dx$$

input
```
int((a + b*cosh(x))^(3/2),x)
```

output
```
int((a + b*cosh(x))^(3/2), x)
```

3.81 $\int \sqrt{a+b\cosh(c+dx)}\,dx$

3.81.1 Optimal result

Integrand size = 14, antiderivative size = 61

$$\int \sqrt{a+b\cosh(c+dx)}\,dx=-\frac{2i\sqrt{a+b\cosh(c+dx)}E\left(\frac{1}{2}i(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cosh(c+dx)}{a+b}}}$$

output
```
-2*I*(cosh(1/2*d*x+1/2*c)^2)^(1/2)/cosh(1/2*d*x+1/2*c)*EllipticE(I*sinh(1/
2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cosh(d*x+c))^(1/2)/d/((a+b*cosh
(d*x+c))/(a+b))^(1/2)
```

3.81.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \sqrt{a+b\cosh(c+dx)}\,dx=-\frac{2i\sqrt{a+b\cosh(c+dx)}E\left(\frac{1}{2}i(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cosh(c+dx)}{a+b}}}$$

input
```
Integrate[Sqrt[a + b*Cosh[c + d*x]],x]
```

output
```
((-2*I)*Sqrt[a + b*Cosh[c + d*x]]*EllipticE[(I/2)*(c + d*x), (2*b)/(a + b)
])/(d*Sqrt[(a + b*Cosh[c + d*x])/(a + b)])
```

3.81.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3134, 3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\cosh(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3134}$$

$$\frac{\sqrt{a+b\cosh(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cosh(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cosh(c+dx)}{a+b}}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\cosh(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(ic+idx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cosh(c+dx)}{a+b}}}$$

$$\Big\downarrow \text{3132}$$

$$-\frac{2i\sqrt{a+b\cosh(c+dx)}E\left(\frac{1}{2}i(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cosh(c+dx)}{a+b}}}$$

input
```
Int[Sqrt[a + b*Cosh[c + d*x]],x]
```

output
```
((-2*I)*Sqrt[a + b*Cosh[c + d*x]]*EllipticE[(I/2)*(c + d*x), (2*b)/(a + b)
])/(d*Sqrt[(a + b*Cosh[c + d*x])/(a + b)])
```

3.81.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

3.81.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(86) = 172$.

Time = 1.46 (sec) , antiderivative size = 276, normalized size of antiderivative = 4.52

method	result
default	$\frac{2\left(a\,\mathrm{EllipticF}\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-\frac{2b}{a-b}},\frac{\sqrt{-\frac{2(a-b)}{b}}}{2}\right)+b\,\mathrm{EllipticF}\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-\frac{2b}{a-b}},\frac{\sqrt{-\frac{2(a-b)}{b}}}{2}\right)-2b\,\mathrm{EllipticE}\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\cdots\right.\right.}{\sqrt{-\frac{2b}{a-b}}\sqrt{2\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}b+(a+b)\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\sinh\cdots}}$
risch	$\frac{\sqrt{2}\sqrt{\left(b\,\mathrm{e}^{2dx+2c}+2a\,\mathrm{e}^{dx+c}+b\right)\mathrm{e}^{-dx-c}}}{d}+\left(\frac{2a\left(a+\sqrt{a^{2}-b^{2}}\right)\sqrt{\frac{\left(\mathrm{e}^{dx+c}+\frac{a+\sqrt{a^{2}-b^{2}}}{b}\right)b}{a+\sqrt{a^{2}-b^{2}}}}\sqrt{\frac{\mathrm{e}^{dx+c}-\frac{-a+\sqrt{a^{2}-b^{2}}}{b}}{-\frac{a+\sqrt{a^{2}-b^{2}}}{b}-\frac{-a+\sqrt{a^{2}-b^{2}}}{b}}}\sqrt{-\frac{b\,\mathrm{e}^{dx+c}}{a+\sqrt{a^{2}-b^{2}}}}\,\mathrm{E}\cdots}{b\sqrt{\mathrm{e}^{3dx+3c}b+2\,\mathrm{e}^{2dx+2c}a\cdots}}\right.$

input
```
int((a+b*cosh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(a*EllipticF(cosh(1/2*d*x+1/2*c)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/
2))+b*EllipticF(cosh(1/2*d*x+1/2*c)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1
/2))-2*b*EllipticE(cosh(1/2*d*x+1/2*c)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)
^(1/2)))*(-sinh(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cosh(1/2*d*x+1/2*c)^2+a-b)/(
a-b))^(1/2)*((2*b*cosh(1/2*d*x+1/2*c)^2+a-b)*sinh(1/2*d*x+1/2*c)^2)^(1/2)/
(-2*b/(a-b))^(1/2)/(2*sinh(1/2*d*x+1/2*c)^4*b+(a+b)*sinh(1/2*d*x+1/2*c)^2)
^(1/2)/sinh(1/2*d*x+1/2*c)/(2*sinh(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

3.81.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 196, normalized size of antiderivative = 3.21

$$\int \sqrt{a+b\cosh(c+dx)}\,dx$$
$$= \frac{2\left(\sqrt{2}a\sqrt{b}\text{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2}, -\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3}, \frac{3\,b\cosh(dx+c)+3\,b\sinh(dx+c)+2\,a}{3\,b}\right) - 3\,\sqrt{2}b^{\frac{3}{2}}\text{weierstras}\right.}{}$$

input
```
integrate((a+b*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2/3*(sqrt(2)*a*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*
(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(d*x + c) + 3*b*sinh(d*x + c) + 2*a)/b
) - 3*sqrt(2)*b^(3/2)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^
3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^
3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(d*x + c) + 3*b*sinh(d*x + c) + 2*a)/b)) -
3*sqrt(b*cosh(d*x + c) + a)*b)/(b*d)
```

3.81.6 Sympy [F]

$$\int \sqrt{a+b\cosh(c+dx)}\,dx = \int \sqrt{a+b\cosh{(c+dx)}}\,dx$$

input
```
integrate((a+b*cosh(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*cosh(c + d*x)), x)
```

3.81.7 Maxima [F]

$$\int \sqrt{a+b\cosh(c+dx)}\,dx = \int \sqrt{b\cosh\left(dx+c\right)+a}\,dx$$

input
```
integrate((a+b*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cosh(d*x + c) + a), x)
```

3.81.8 Giac [F]

$$\int \sqrt{a+b\cosh(c+dx)}\,dx = \int \sqrt{b\cosh\left(dx+c\right)+a}\,dx$$

input
```
integrate((a+b*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cosh(d*x + c) + a), x)
```

3.81.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cosh(c+dx)}\,dx = \int \sqrt{a+b\cosh\left(c+d\,x\right)}\,dx$$

input
```
int((a + b*cosh(c + d*x))^(1/2),x)
```

output
```
int((a + b*cosh(c + d*x))^(1/2), x)
```

3.82 $\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx$

3.82.1 Optimal result

Integrand size = 10, antiderivative size = 46

$$\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx = -\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{\sqrt{a+b\cosh(x)}}$$

output
```
-2*I*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/2)*(b/
(a+b))^(1/2))*((a+b*cosh(x))/(a+b))^(1/2)/(a+b*cosh(x))^(1/2)
```

3.82.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx = -\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{\sqrt{a+b\cosh(x)}}$$

input
```
Integrate[1/Sqrt[a + b*Cosh[x]],x]
```

output
```
((-2*I)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/S
qrt[a + b*Cosh[x]]
```

3.82.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\cosh(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+b\sin\left(\frac{\pi}{2}+ix\right)}}dx$$

$$\Big\downarrow \text{3142}$$

$$\frac{\sqrt{\frac{a+b\cosh(x)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}}dx}{\sqrt{a+b\cosh(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\frac{a+b\cosh(x)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cosh(x)}}$$

$$\Big\downarrow \text{3140}$$

$$-\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{\sqrt{a+b\cosh(x)}}$$

input
```
Int[1/Sqrt[a + b*Cosh[x]],x]
```

output
```
((-2*I)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/S
qrt[a + b*Cosh[x]]
```

3.82.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

3.82.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(60) = 120$.

Time = 0.20 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.17

method	result	size
default	$\frac{2\sqrt{\left(2\cosh\left(\frac{x}{2}\right)^{2} b+a-b\right)\sinh\left(\frac{x}{2}\right)^{2}}\,\sqrt{\frac{2\cosh\left(\frac{x}{2}\right)^{2} b+a-b}{a-b}}\,\sqrt{-\sinh\left(\frac{x}{2}\right)^{2}}\,\operatorname{EllipticF}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}}, \frac{\sqrt{-\frac{2(a-b)}{b}}}{2}\right)}{\sqrt{-\frac{2b}{a-b}}\,\sqrt{2\sinh\left(\frac{x}{2}\right)^{4} b+(a+b)\sinh\left(\frac{x}{2}\right)^{2}}\,\sinh\left(\frac{x}{2}\right)\sqrt{2\sinh\left(\frac{x}{2}\right)^{2} b+a+b}}$	146

input
```
int(1/(a+b*cosh(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*((2*cosh(1/2*x)^2*b+a-b)*sinh(1/2*x)^2)^(1/2)/(-2*b/(a-b))^(1/2)*((2*cos
h(1/2*x)^2*b+a-b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)/(2*sinh(1/2*x)^4*b+(
a+b)*sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2
*(a-b)/b)^(1/2))/sinh(1/2*x)/(2*sinh(1/2*x)^2*b+a+b)^(1/2)
```

3.82.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.33

$$\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx = \frac{2\sqrt{2}\text{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\frac{3\,b\cosh(x)+3\,b\sinh(x)+2\,a}{3\,b}\right)}{\sqrt{b}}$$

input
```
integrate(1/(a+b*cosh(x))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(2)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*
b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)/sqrt(b)
```

3.82.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx = \int \frac{1}{\sqrt{a+b\cosh{(x)}}}\,dx$$

input
```
integrate(1/(a+b*cosh(x))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*cosh(x)), x)
```

3.82.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx = \int \frac{1}{\sqrt{b\cosh{(x)}+a}}\,dx$$

input
```
integrate(1/(a+b*cosh(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*cosh(x) + a), x)
```

3.82.8 Giac [F]

$$\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx = \int \frac{1}{\sqrt{b\cosh(x)+a}}\,dx$$

input
```
integrate(1/(a+b*cosh(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*cosh(x) + a), x)
```

3.82.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx = \int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx$$

input
```
int(1/(a + b*cosh(x))^(1/2),x)
```

output
```
int(1/(a + b*cosh(x))^(1/2), x)
```

3.83 $\int \frac{1}{(a+b\cosh(x))^{3/2}}\,dx$

3.83.1 Optimal result

Integrand size = 10, antiderivative size = 84

$$\int \frac{1}{(a+b\cosh(x))^{3/2}}\,dx=-\frac{2i\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}}-\frac{2b\sinh(x)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}$$

output
```
-2*b*sinh(x)/(a^2-b^2)/(a+b*cosh(x))^(1/2)-2*I*(cosh(1/2*x)^2)^(1/2)/cosh(
1/2*x)*EllipticE(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cosh(x))^(1/2
)/(a^2-b^2)/((a+b*cosh(x))/(a+b))^(1/2)
```

3.83.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.81

$$\int \frac{1}{(a+b\cosh(x))^{3/2}}\,dx=-\frac{2\left(i(a+b)\sqrt{\frac{a+b\cosh(x)}{a+b}}E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)+b\sinh(x)\right)}{(a-b)(a+b)\sqrt{a+b\cosh(x)}}$$

input
```
Integrate[(a + b*Cosh[x])^(-3/2),x]
```

output
```
(-2*(I*(a + b)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticE[(I/2)*x, (2*b)/(a +
 b)] + b*Sinh[x]))/((a - b)*(a + b)*Sqrt[a + b*Cosh[x]])
```

3.83.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {3042, 3143, 27, 3042, 3134, 3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\cosh(x))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\sin\left(\frac{\pi}{2}+ix\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3143}$$

$$-\frac{2\int -\frac{1}{2}\sqrt{a+b\cosh(x)}dx}{a^2-b^2}-\frac{2b\sinh(x)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sqrt{a+b\cosh(x)}dx}{a^2-b^2}-\frac{2b\sinh(x)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b\sinh(x)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}+\frac{\int \sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3134}$$

$$\frac{\sqrt{a+b\cosh(x)}\int \sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}dx}{(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}}-\frac{2b\sinh(x)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b\sinh(x)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}+\frac{\sqrt{a+b\cosh(x)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}dx}{(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

$$\Big\downarrow \text{3132}$$

$$-\frac{2b\sinh(x)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}-\frac{2i\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

input
```
Int[(a + b*Cosh[x])^(-3/2),x]
```

output
```
((-2*I)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/((a^2 - b^2
)*Sqrt[(a + b*Cosh[x])/(a + b)]) - (2*b*Sinh[x])/((a^2 - b^2)*Sqrt[a + b*C
osh[x]])
```

3.83.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.83.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(96) = 192$.

Time = 1.18 (sec) , antiderivative size = 298, normalized size of antiderivative = 3.55

method	result
default	$-\frac{2\left(2\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}}\,\sinh\left(\frac{x}{2}\right)^{2}b-\sqrt{-\sinh\left(\frac{x}{2}\right)^{2}}\,\sqrt{\frac{2b\sinh\left(\frac{x}{2}\right)^{2}}{a-b}+\frac{a+b}{a-b}}\,\text{EllipticF}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}},\frac{\sqrt{-\frac{2(a-b)}{b}}}{2}\right)a-\sqrt{-\sinh\left(\frac{x}{2}\right)^{2}}\cdots}{\sqrt{-\frac{2b}{a-b}}\,(a-b)(\cdots}$

input
```
int(1/(a+b*cosh(x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(2*cosh(1/2*x)*(-2*b/(a-b))^(1/2)*sinh(1/2*x)^2*b-(-sinh(1/2*x)^2)^(1/2
)*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/
(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))*a-(-sinh(1/2*x)^2)^(1/2)*(2*b/(a-b)*s
inh(1/2*x)^2+(a+b)/(a-b))^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1
/2*(-2*(a-b)/b)^(1/2))*b+2*(-sinh(1/2*x)^2)^(1/2)*(2*b/(a-b)*sinh(1/2*x)^2
+(a+b)/(a-b))^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)
/b)^(1/2))*b)/(-2*b/(a-b))^(1/2)/(a-b)/(a+b)/sinh(1/2*x)/(2*sinh(1/2*x)^2*
b+a+b)^(1/2)
```

3.83.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 413, normalized size of antiderivative = 4.92

$$\int \frac{1}{(a+b\cosh(x))^{3/2}}\,dx = \frac{2\left(\left(\sqrt{2}ab\cosh\left(x\right)^{2}+\sqrt{2}ab\sinh\left(x\right)^{2}+2\sqrt{2}a^{2}\cosh\left(x\right)+\sqrt{2}ab+2\left(\sqrt{2}ab\cosh\left(\cdots\right.\right.\right.}{\cdots}$$

input
```
integrate(1/(a+b*cosh(x))^(3/2),x, algorithm="fricas")
```

output
```
2/3*((sqrt(2)*a*b*cosh(x)^2 + sqrt(2)*a*b*sinh(x)^2 + 2*sqrt(2)*a^2*cosh(x
) + sqrt(2)*a*b + 2*(sqrt(2)*a*b*cosh(x) + sqrt(2)*a^2)*sinh(x))*sqrt(b)*w
eierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1
/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) - 3*(sqrt(2)*b^2*cosh(x)^2 + sqrt(
2)*b^2*sinh(x)^2 + 2*sqrt(2)*a*b*cosh(x) + sqrt(2)*b^2 + 2*(sqrt(2)*b^2*co
sh(x) + sqrt(2)*a*b)*sinh(x))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/
b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/
b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)
) - 6*(b^2*cosh(x)^2 + b^2*sinh(x)^2 + a*b*cosh(x) + (2*b^2*cosh(x) + a*b)
*sinh(x))*sqrt(b*cosh(x) + a))/(a^2*b^2 - b^4 + (a^2*b^2 - b^4)*cosh(x)^2
+ (a^2*b^2 - b^4)*sinh(x)^2 + 2*(a^3*b - a*b^3)*cosh(x) + 2*(a^3*b - a*b^3
 + (a^2*b^2 - b^4)*cosh(x))*sinh(x))
```

3.83.6 Sympy [F]

$$\int \frac{1}{(a+b\cosh(x))^{3/2}}\,dx = \int \frac{1}{(a+b\cosh{(x)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cosh(x))**(3/2),x)
```

output
```
Integral((a + b*cosh(x))**(-3/2), x)
```

3.83.7 Maxima [F]

$$\int \frac{1}{(a+b\cosh(x))^{3/2}}\,dx = \int \frac{1}{(b\cosh{(x)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cosh(x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cosh(x) + a)^(-3/2), x)
```

3.83.8 Giac [F]

$$\int \frac{1}{(a+b\cosh(x))^{3/2}}\,dx = \int \frac{1}{(b\cosh(x)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cosh(x))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cosh(x) + a)^(-3/2), x)
```

3.83.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(x))^{3/2}}\,dx = \int \frac{1}{(a+b\cosh(x))^{3/2}}\,dx$$

input
```
int(1/(a + b*cosh(x))^(3/2),x)
```

output
```
int(1/(a + b*cosh(x))^(3/2), x)
```

3.84 $\int \frac{1}{(a+b\cosh(x))^{5/2}}\,dx$

3.84.1 Optimal result

Integrand size = 10, antiderivative size = 177

$$\begin{aligned}\int \frac{1}{(a+b\cosh(x))^{5/2}}\,dx &= -\frac{8ia\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{3(a^2-b^2)^2\sqrt{\frac{a+b\cosh(x)}{a+b}}} \\ &\quad+\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{3(a^2-b^2)\sqrt{a+b\cosh(x)}} \\ &\quad-\frac{2b\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}-\frac{8ab\sinh(x)}{3(a^2-b^2)^2\sqrt{a+b\cosh(x)}}\end{aligned}$$

output
```
-2/3*b*sinh(x)/(a^2-b^2)/(a+b*cosh(x))^(3/2)-8/3*a*b*sinh(x)/(a^2-b^2)^2/(
a+b*cosh(x))^(1/2)-8/3*I*a*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*s
inh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cosh(x))^(1/2)/(a^2-b^2)^2/((a+b*
cosh(x))/(a+b))^(1/2)+2/3*I*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*
sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cosh(x))/(a+b))^(1/2)/(a^2-b^2)
/(a+b*cosh(x))^(1/2)
```

3.84.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.76

$$\int \frac{1}{(a+b\cosh(x))^{5/2}}\,dx = \frac{-8ia(a+b)^2\left(\frac{a+b\cosh(x)}{a+b}\right)^{3/2}E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)+2i(a-b)(a+b)^2\left(\frac{a+b\cosh(x)}{a+b}\right)^{3/2}\text{Ellipti}}{3(a-b)^2(a+b)^2(a+b\cosh(x))}$$

input
```
Integrate[(a + b*Cosh[x])^(-5/2),x]
```

output
```
((-8*I)*a*(a + b)^2*((a + b*Cosh[x])/(a + b))^(3/2)*EllipticE[(I/2)*x, (2*
b)/(a + b)] + (2*I)*(a - b)*(a + b)^2*((a + b*Cosh[x])/(a + b))^(3/2)*Elli
pticF[(I/2)*x, (2*b)/(a + b)] + 2*b*(-5*a^2 + b^2 - 4*a*b*Cosh[x])*Sinh[x]
)/(3*(a - b)^2*(a + b)^2*(a + b*Cosh[x])^(3/2))
```

3.84.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.06, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {3042, 3143, 27, 3042, 3233, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\cosh(x))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\left(a+b\sin\left(\frac{\pi}{2}+ix\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 3143$$

$$-\frac{2\int -\frac{3a-b\cosh(x)}{2(a+b\cosh(x))^{3/2}}dx}{3\left(a^2-b^2\right)}-\frac{2b\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{3a-b\cosh(x)}{(a+b\cosh(x))^{3/2}}dx}{3\left(a^2-b^2\right)}-\frac{2b\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2b\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}+\frac{\int\frac{3a-b\sin\left(ix+\frac{\pi}{2}\right)}{\left(a+b\sin\left(ix+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3(a^2-b^2)}$$

$\big\downarrow$ 3233

$$\frac{-\frac{2\int-\frac{3a^2+4b\cosh(x)a+b^2}{2\sqrt{a+b\cosh(x)}}dx}{a^2-b^2}-\frac{8ab\sinh(x)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}}{3(a^2-b^2)}-\frac{2b\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}$$

$\big\downarrow$ 27

$$\frac{\frac{\int\frac{3a^2+4b\cosh(x)a+b^2}{\sqrt{a+b\cosh(x)}}dx}{a^2-b^2}-\frac{8ab\sinh(x)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}}{3(a^2-b^2)}-\frac{2b\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}$$

$\big\downarrow$ 3042

$$-\frac{2b\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}+\frac{-\frac{8ab\sinh(x)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}+\frac{\int\frac{3a^2+4b\sin\left(ix+\frac{\pi}{2}\right)a+b^2}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{a^2-b^2}}{3(a^2-b^2)}$$

$\big\downarrow$ 3231

$$\frac{\frac{4a\int\sqrt{a+b\cosh(x)}dx-(a^2-b^2)\int\frac{1}{\sqrt{a+b\cosh(x)}}dx}{a^2-b^2}-\frac{8ab\sinh(x)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}}{3(a^2-b^2)}-\frac{2b\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}$$

$\big\downarrow$ 3042

$$-\frac{2b\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}+\frac{-\frac{8ab\sinh(x)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}+\frac{4a\int\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx-(a^2-b^2)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{a^2-b^2}}{3(a^2-b^2)}$$

$\big\downarrow$ 3134

$$-\frac{2b\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}+\frac{-\frac{8ab\sinh(x)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}+\frac{\frac{4a\sqrt{a+b\cosh(x)}\int\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}dx}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}-(a^2-b^2)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{a^2-b^2}}{3(a^2-b^2)}$$

$\big\downarrow$ 3042

3.84. $\int\frac{1}{(a+b\cosh(x))^{5/2}}dx$

$$-\frac{2b\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}+\frac{-\frac{8ab\sinh(x)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}+\frac{\frac{4a\sqrt{a+b\cosh(x)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}-(a^2-b^2)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{a^2-b^2}}{3(a^2-b^2)}$$

$$\Big\downarrow \text{3132}$$

$$-\frac{2b\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}+\frac{-\frac{8ab\sinh(x)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}+\frac{-(a^2-b^2)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx-\frac{8ia\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}}{a^2-b^2}}{3(a^2-b^2)}$$

$$\Big\downarrow \text{3142}$$

$$-\frac{2b\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}+\frac{-\frac{8ab\sinh(x)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}+\frac{-\frac{(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}}dx}{\sqrt{a+b\cosh(x)}}-\frac{8ia\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}}{a^2-b^2}}{3(a^2-b^2)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}+\frac{-\frac{8ab\sinh(x)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}+\frac{-\frac{(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cosh(x)}}-\frac{8ia\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}}{a^2-b^2}}{3(a^2-b^2)}$$

$$\Big\downarrow \text{3140}$$

$$-\frac{2b\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}+\frac{-\frac{8ab\sinh(x)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}+\frac{\frac{2i(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\operatorname{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{\sqrt{a+b\cosh(x)}}-\frac{8ia\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}}{a^2-b^2}}{3(a^2-b^2)}$$

input
```
Int[(a + b*Cosh[x])^(-5/2),x]
```

3.84. $\int \frac{1}{(a+b\cosh(x))^{5/2}}\,dx$

output
```
(-2*b*Sinh[x])/(3*(a^2 - b^2)*(a + b*Cosh[x])^(3/2)) + ((((-8*I)*a*Sqrt[a
+ b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/Sqrt[(a + b*Cosh[x])/(a +
b)] + ((2*I)*(a^2 - b^2)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x,
(2*b)/(a + b)])/Sqrt[a + b*Cosh[x]])/(a^2 - b^2) - (8*a*b*Sinh[x])/((a^2 -
 b^2)*Sqrt[a + b*Cosh[x]]))/(3*(a^2 - b^2))
```

3.84.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.84.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(193) = 386$.

Time = 2.63 (sec) , antiderivative size = 459, normalized size of antiderivative = 2.59

method	result
default	$\frac{\sqrt{\left(2\cosh\left(\frac{x}{2}\right)^{2}b+a-b\right)\sinh\left(\frac{x}{2}\right)^{2}}\left(-\frac{\cosh\left(\frac{x}{2}\right)\sqrt{2\sinh\left(\frac{x}{2}\right)^{4}b+(a+b)\sinh\left(\frac{x}{2}\right)^{2}}}{3b(a-b)(a+b)\left(\cosh\left(\frac{x}{2}\right)^{2}+\frac{a-b}{2b}\right)^{2}}-\frac{16\sinh\left(\frac{x}{2}\right)^{2}b\cosh\left(\frac{x}{2}\right)a}{3(a-b)^{2}(a+b)^{2}\sqrt{\left(2\cosh\left(\frac{x}{2}\right)^{2}b+a-b\right)\sinh\left(\frac{x}{2}\right)^{2}}}+\frac{2(3a-b)}{}\right.}{}$

input
```
int(1/(a+b*cosh(x))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((2*cosh(1/2*x)^2*b+a-b)*sinh(1/2*x)^2)^(1/2)*(-1/3/b/(a-b)/(a+b)*cosh(1/2
*x)*(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^2)^(1/2)/(cosh(1/2*x)^2+1/2*(a-b)
/b)^2-16/3*sinh(1/2*x)^2*b/(a-b)^2/(a+b)^2*cosh(1/2*x)*a/((2*cosh(1/2*x)^2
*b+a-b)*sinh(1/2*x)^2)^(1/2)+2*(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)/(-2*b
/(a-b))^(1/2)*((2*cosh(1/2*x)^2*b+a-b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)
/(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b
/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))-32/3*a*b/(a+b)^2/(a-b)^2*(-a+b)/(-
2*b/(a-b))^(1/2)*((2*cosh(1/2*x)^2*b+a-b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1
/2)/(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^2)^(1/2)/(2*a-2*b)*(EllipticF(cos
h(1/2*x)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))-EllipticE(cosh(1/2*x
)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))))/sinh(1/2*x)/(2*sinh(1/2*x
)^2*b+a+b)^(1/2)
```

3.84.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 1281, normalized size of antiderivative = 7.24

$$\int \frac{1}{(a+b\cosh(x))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cosh(x))^(5/2),x, algorithm="fricas")
```

output
```
2/9*((sqrt(2)*(a^2*b^2 + 3*b^4)*cosh(x)^4 + sqrt(2)*(a^2*b^2 + 3*b^4)*sinh
(x)^4 + 4*sqrt(2)*(a^3*b + 3*a*b^3)*cosh(x)^3 + 4*(sqrt(2)*(a^2*b^2 + 3*b^
4)*cosh(x) + sqrt(2)*(a^3*b + 3*a*b^3))*sinh(x)^3 + 2*sqrt(2)*(2*a^4 + 7*a
^2*b^2 + 3*b^4)*cosh(x)^2 + 2*(3*sqrt(2)*(a^2*b^2 + 3*b^4)*cosh(x)^2 + 6*s
qrt(2)*(a^3*b + 3*a*b^3)*cosh(x) + sqrt(2)*(2*a^4 + 7*a^2*b^2 + 3*b^4))*si
nh(x)^2 + 4*sqrt(2)*(a^3*b + 3*a*b^3)*cosh(x) + 4*(sqrt(2)*(a^2*b^2 + 3*b^
4)*cosh(x)^3 + 3*sqrt(2)*(a^3*b + 3*a*b^3)*cosh(x)^2 + sqrt(2)*(2*a^4 + 7*
a^2*b^2 + 3*b^4)*cosh(x) + sqrt(2)*(a^3*b + 3*a*b^3))*sinh(x) + sqrt(2)*(a
^2*b^2 + 3*b^4))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/2
7*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) - 12*(sq
rt(2)*a*b^3*cosh(x)^4 + sqrt(2)*a*b^3*sinh(x)^4 + 4*sqrt(2)*a^2*b^2*cosh(x
)^3 + 4*sqrt(2)*a^2*b^2*cosh(x) + sqrt(2)*a*b^3 + 4*(sqrt(2)*a*b^3*cosh(x)
 + sqrt(2)*a^2*b^2)*sinh(x)^3 + 2*sqrt(2)*(2*a^3*b + a*b^3)*cosh(x)^2 + 2*
(3*sqrt(2)*a*b^3*cosh(x)^2 + 6*sqrt(2)*a^2*b^2*cosh(x) + sqrt(2)*(2*a^3*b
+ a*b^3))*sinh(x)^2 + 4*(sqrt(2)*a*b^3*cosh(x)^3 + 3*sqrt(2)*a^2*b^2*cosh(
x)^2 + sqrt(2)*a^2*b^2 + sqrt(2)*(2*a^3*b + a*b^3)*cosh(x))*sinh(x))*sqrt(
b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, w
eierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1
/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) - 6*(4*a*b^3*cosh(x)^4 + 4*a*b^3*
sinh(x)^4 + (13*a^2*b^2 - b^4)*cosh(x)^3 + (16*a*b^3*cosh(x) + 13*a^2*b...
```

3.84.6 Sympy [F]

$$\int \frac{1}{(a+b\cosh(x))^{5/2}}\,dx = \int \frac{1}{(a+b\cosh{(x)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cosh(x))**(5/2),x)
```

output
```
Integral((a + b*cosh(x))**(-5/2), x)
```

3.84.7 Maxima [F]

$$\int \frac{1}{(a+b\cosh(x))^{5/2}}\,dx = \int \frac{1}{(b\cosh(x)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cosh(x))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cosh(x) + a)^(-5/2), x)
```

3.84.8 Giac [F]

$$\int \frac{1}{(a+b\cosh(x))^{5/2}}\,dx = \int \frac{1}{(b\cosh(x)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cosh(x))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cosh(x) + a)^(-5/2), x)
```

3.84.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(x))^{5/2}}\,dx = \int \frac{1}{(a+b\cosh(x))^{5/2}}\,dx$$

input
```
int(1/(a + b*cosh(x))^(5/2),x)
```

output
```
int(1/(a + b*cosh(x))^(5/2), x)
```

3.85 $\int \frac{1}{(a+b\cosh(x))^{7/2}}\,dx$

3.85.1 Optimal result

Integrand size = 10, antiderivative size = 227

$$\int \frac{1}{(a+b\cosh(x))^{7/2}}\,dx = -\frac{2i(23a^2+9b^2)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{15(a^2-b^2)^3\sqrt{\frac{a+b\cosh(x)}{a+b}}} + \frac{16ia\sqrt{\frac{a+b\cosh(x)}{a+b}}\operatorname{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{15(a^2-b^2)^2\sqrt{a+b\cosh(x)}} - \frac{2b\sinh(x)}{5(a^2-b^2)(a+b\cosh(x))^{5/2}} - \frac{16ab\sinh(x)}{15(a^2-b^2)^2(a+b\cosh(x))^{3/2}} - \frac{2b(23a^2+9b^2)\sinh(x)}{15(a^2-b^2)^3\sqrt{a+b\cosh(x)}}$$

output
```
-2/5*b*sinh(x)/(a^2-b^2)/(a+b*cosh(x))^(5/2)-16/15*a*b*sinh(x)/(a^2-b^2)^2
/(a+b*cosh(x))^(3/2)-2/15*b*(23*a^2+9*b^2)*sinh(x)/(a^2-b^2)^3/(a+b*cosh(x
))^(1/2)-2/15*I*(23*a^2+9*b^2)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE
(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cosh(x))^(1/2)/(a^2-b^2)^3/((
a+b*cosh(x))/(a+b))^(1/2)+16/15*I*a*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*Elli
pticF(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cosh(x))/(a+b))^(1/2)/(
a^2-b^2)^2/(a+b*cosh(x))^(1/2)
```

3.85.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.73

$$\int \frac{1}{(a+b\cosh(x))^{7/2}}\,dx = \frac{2\left(-\frac{i\left(\frac{a+b\cosh(x)}{a+b}\right)^{5/2}\left((23a^2+9b^2)E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)+8a(-a+b)\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)\right)}{(a-b)^3}+\frac{b(34a^4-5a^2b^2+3b^4+2}{}\right.}{15(a+b\cosh(x))^{5/2}}$$

input
```
Integrate[(a + b*Cosh[x])^(-7/2),x]
```

output
```
(2*(((-I)*((a + b*Cosh[x])/(a + b))^(5/2)*((23*a^2 + 9*b^2)*EllipticE[(I/2
)*x, (2*b)/(a + b)] + 8*a*(-a + b)*EllipticF[(I/2)*x, (2*b)/(a + b)]))/(a
- b)^3 + (b*(34*a^4 - 5*a^2*b^2 + 3*b^4 + 2*a*b*(27*a^2 + 5*b^2)*Cosh[x] +
 b^2*(23*a^2 + 9*b^2)*Cosh[x]^2)*Sinh[x])/(-a^2 + b^2)^3))/(15*(a + b*Cosh
[x])^(5/2))
```

3.85.3 Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.11, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.800, Rules used = {3042, 3143, 27, 3042, 3233, 27, 3042, 3233, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\cosh(x))^{7/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\left(a+b\sin\left(\frac{\pi}{2}+ix\right)\right)^{7/2}}dx$$

$$\Big\downarrow \ 3143$$

$$-\frac{2\int -\frac{5a-3b\cosh(x)}{2(a+b\cosh(x))^{5/2}}dx}{5(a^2-b^2)}-\frac{2b\sinh(x)}{5(a^2-b^2)(a+b\cosh(x))^{5/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{5a-3b\cosh(x)}{(a+b\cosh(x))^{5/2}}dx}{5(a^2-b^2)}-\frac{2b\sinh(x)}{5(a^2-b^2)(a+b\cosh(x))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b\sinh(x)}{5\left(a^2-b^2\right)(a+b\cosh(x))^{5/2}}+\frac{\int \frac{5a-3b\sin\left(ix+\frac{\pi}{2}\right)}{\left(a+b\sin\left(ix+\frac{\pi}{2}\right)\right)^{5/2}}dx}{5\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3233}$$

$$\frac{-\frac{2\int -\frac{3\left(5a^2+3b^2\right)-8ab\cosh(x)}{2(a+b\cosh(x))^{3/2}}dx}{3(a^2-b^2)}-\frac{16ab\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}}{5\left(a^2-b^2\right)}-\frac{2b\sinh(x)}{5\left(a^2-b^2\right)(a+b\cosh(x))^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{3\left(5a^2+3b^2\right)-8ab\cosh(x)}{(a+b\cosh(x))^{3/2}}dx}{3(a^2-b^2)}-\frac{16ab\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}}{5\left(a^2-b^2\right)}-\frac{2b\sinh(x)}{5\left(a^2-b^2\right)(a+b\cosh(x))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b\sinh(x)}{5\left(a^2-b^2\right)(a+b\cosh(x))^{5/2}}+\frac{-\frac{16ab\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}+\frac{\int \frac{3\left(5a^2+3b^2\right)-8ab\sin\left(ix+\frac{\pi}{2}\right)}{\left(a+b\sin\left(ix+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3(a^2-b^2)}}{5\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3233}$$

$$\frac{\frac{-\frac{2\int -\frac{a\left(15a^2+17b^2\right)+b\left(23a^2+9b^2\right)\cosh(x)}{2\sqrt{a+b\cosh(x)}}dx}{a^2-b^2}-\frac{2b\left(23a^2+9b^2\right)\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}}{3(a^2-b^2)}-\frac{16ab\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}}{5\left(a^2-b^2\right)}-\frac{2b\sinh(x)}{5\left(a^2-b^2\right)(a+b\cosh(x))^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{\int \frac{a\left(15a^2+17b^2\right)+b\left(23a^2+9b^2\right)\cosh(x)}{\sqrt{a+b\cosh(x)}}dx}{a^2-b^2}-\frac{2b\left(23a^2+9b^2\right)\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}}{3(a^2-b^2)}-\frac{16ab\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}}{5\left(a^2-b^2\right)}-\frac{2b\sinh(x)}{5\left(a^2-b^2\right)(a+b\cosh(x))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

3.85. $\int \frac{1}{(a+b\cosh(x))^{7/2}}\,dx$

$$-\frac{2b\sinh(x)}{5\left(a^2-b^2\right)\left(a+b\cosh(x)\right)^{5/2}}+\frac{-\frac{16ab\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}+\frac{-\frac{2b\left(23a^2+9b^2\right)\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{\int\frac{a\left(15a^2+17b^2\right)+b\left(23a^2+9b^2\right)\sin\left(ix+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{a^2-b^2}}{3(a^2-b^2)}}{5\left(a^2-b^2\right)}$$

$\downarrow$ 3231

$$\frac{\frac{\frac{\left(23a^2+9b^2\right)\int\sqrt{a+b\cosh(x)}dx-8a\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\cosh(x)}}dx}{a^2-b^2}-\frac{2b\left(23a^2+9b^2\right)\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}}{3(a^2-b^2)}-\frac{16ab\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}}{5\left(a^2-b^2\right)}-\frac{2b\sinh(x)}{5\left(a^2-b^2\right)\left(a+b\cosh(x)\right)^{5/2}}$$

$\downarrow$ 3042

$$-\frac{2b\sinh(x)}{5\left(a^2-b^2\right)\left(a+b\cosh(x)\right)^{5/2}}+\frac{-\frac{16ab\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}+\frac{-\frac{2b\left(23a^2+9b^2\right)\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{\left(23a^2+9b^2\right)\int\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx-8a\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{a^2-b^2}}{3(a^2-b^2)}}{5\left(a^2-b^2\right)}$$

$\downarrow$ 3134

$$-\frac{2b\sinh(x)}{5\left(a^2-b^2\right)\left(a+b\cosh(x)\right)^{5/2}}+\frac{-\frac{16ab\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}+\frac{-\frac{2b\left(23a^2+9b^2\right)\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{\frac{\left(23a^2+9b^2\right)\sqrt{a+b\cosh(x)}\int\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}dx}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}-8a\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{a^2-b^2}}{3(a^2-b^2)}}{5\left(a^2-b^2\right)}$$

$\downarrow$ 3042

$$-\frac{2b\sinh(x)}{5\left(a^2-b^2\right)\left(a+b\cosh(x)\right)^{5/2}}+\frac{-\frac{16ab\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}+\frac{-\frac{2b\left(23a^2+9b^2\right)\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{\frac{\left(23a^2+9b^2\right)\sqrt{a+b\cosh(x)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}-8a\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{a^2-b^2}}{3(a^2-b^2)}}{5\left(a^2-b^2\right)}$$

$\downarrow$ 3132

3.85. $\int\frac{1}{(a+b\cosh(x))^{7/2}}dx$

$$-\frac{2b\sinh(x)}{5\left(a^2-b^2\right)(a+b\cosh(x))^{5/2}}+\frac{-\frac{16ab\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}+\frac{-\frac{2b\left(23a^2+9b^2\right)\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{-8a\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx-\frac{2i\left(23a^2+9b^2\right)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}}{a^2-b^2}}{3(a^2-b^2)}}{5\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 3142$$

$$-\frac{2b\sinh(x)}{5\left(a^2-b^2\right)(a+b\cosh(x))^{5/2}}+\frac{-\frac{16ab\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}+\frac{-\frac{2b\left(23a^2+9b^2\right)\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{-\frac{8a\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}}dx}{\sqrt{a+b\cosh(x)}}-\frac{2i\left(23a^2+9b^2\right)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}}{a^2-b^2}}{3(a^2-b^2)}}{5\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 3042$$

$$-\frac{2b\sinh(x)}{5\left(a^2-b^2\right)(a+b\cosh(x))^{5/2}}+\frac{-\frac{16ab\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}+\frac{-\frac{2b\left(23a^2+9b^2\right)\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{-\frac{8a\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cosh(x)}}-\frac{2i\left(23a^2+9b^2\right)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}}{a^2-b^2}}{3(a^2-b^2)}}{5\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 3140$$

$$-\frac{2b\sinh(x)}{5\left(a^2-b^2\right)(a+b\cosh(x))^{5/2}}+\frac{-\frac{16ab\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}+\frac{-\frac{2b\left(23a^2+9b^2\right)\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{\frac{16ia\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\operatorname{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{\sqrt{a+b\cosh(x)}}-\frac{2i\left(23a^2+9b^2\right)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}}{a^2-b^2}}{3(a^2-b^2)}}{5\left(a^2-b^2\right)}$$

input
```
Int[(a + b*Cosh[x])^(-7/2),x]
```

3.85. $\int\frac{1}{(a+b\cosh(x))^{7/2}}\,dx$

output
```
(-2*b*Sinh[x])/(5*(a^2 - b^2)*(a + b*Cosh[x])^(5/2)) + ((-16*a*b*Sinh[x])/
(3*(a^2 - b^2)*(a + b*Cosh[x])^(3/2)) + ((((-2*I)*(23*a^2 + 9*b^2)*Sqrt[a
+ b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/Sqrt[(a + b*Cosh[x])/(a +
b)] + ((16*I)*a*(a^2 - b^2)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*
x, (2*b)/(a + b)])/Sqrt[a + b*Cosh[x]])/(a^2 - b^2) - (2*b*(23*a^2 + 9*b^2
)*Sinh[x])/((a^2 - b^2)*Sqrt[a + b*Cosh[x]]))/(3*(a^2 - b^2)))/(5*(a^2 - b
^2))
```

3.85.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.85.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 565 vs. $2(239) = 478$.

Time = 2.96 (sec) , antiderivative size = 566, normalized size of antiderivative = 2.49

method	result
default	$\frac{\sqrt{\left(2\cosh\left(\frac{x}{2}\right)^{2}b+a-b\right)\sinh\left(\frac{x}{2}\right)^{2}}\left(-\frac{\cosh\left(\frac{x}{2}\right)\sqrt{2\sinh\left(\frac{x}{2}\right)^{4}b+(a+b)\sinh\left(\frac{x}{2}\right)^{2}}}{10b^{2}(a-b)(a+b)\left(\cosh\left(\frac{x}{2}\right)^{2}+\frac{a-b}{2b}\right)^{3}}-\frac{8a\cosh\left(\frac{x}{2}\right)\sqrt{2\sinh\left(\frac{x}{2}\right)^{4}b+(a+b)\sinh\left(\frac{x}{2}\right)^{2}}}{15b(a+b)^{2}(a-b)^{2}\left(\cosh\left(\frac{x}{2}\right)^{2}+\frac{a-b}{2b}\right)^{2}}-\frac{4\sinh}{15(a-b)^{3}(a+b}\right.}{}$

input
```
int(1/(a+b*cosh(x))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
((2*cosh(1/2*x)^2*b+a-b)*sinh(1/2*x)^2)^(1/2)*(-1/10/b^2/(a-b)/(a+b)*cosh(
1/2*x)*(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^2)^(1/2)/(cosh(1/2*x)^2+1/2*(a
-b)/b)^3-8/15*a/b/(a+b)^2/(a-b)^2*cosh(1/2*x)*(2*sinh(1/2*x)^4*b+(a+b)*sin
h(1/2*x)^2)^(1/2)/(cosh(1/2*x)^2+1/2*(a-b)/b)^2-4/15*sinh(1/2*x)^2*b/(a-b)
^3/(a+b)^3*cosh(1/2*x)*(23*a^2+9*b^2)/((2*cosh(1/2*x)^2*b+a-b)*sinh(1/2*x)
^2)^(1/2)+2*(15*a^2-8*a*b+9*b^2)/(15*a^5+15*a^4*b-30*a^3*b^2-30*a^2*b^3+15
*a*b^4+15*b^5)/(-2*b/(a-b))^(1/2)*((2*cosh(1/2*x)^2*b+a-b)/(a-b))^(1/2)*(-
sinh(1/2*x)^2)^(1/2)/(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^2)^(1/2)*Ellipti
cF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))-8/15*b*(23*a^2
+9*b^2)/(a+b)^3/(a-b)^3*(-a+b)/(-2*b/(a-b))^(1/2)*((2*cosh(1/2*x)^2*b+a-b)
/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)/(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^
2)^(1/2)/(2*a-2*b)*(EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*
b)/b)^(1/2))-EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(
1/2))))/sinh(1/2*x)/(2*sinh(1/2*x)^2*b+a+b)^(1/2)
```

3.85.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 3315, normalized size of antiderivative = 14.60

$$\int \frac{1}{(a+b\cosh(x))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cosh(x))^(7/2),x, algorithm="fricas")
```

output
```
-2/45*((sqrt(2)*(a^3*b^3 - 33*a*b^5)*cosh(x)^6 + sqrt(2)*(a^3*b^3 - 33*a*b
^5)*sinh(x)^6 + 6*sqrt(2)*(a^4*b^2 - 33*a^2*b^4)*cosh(x)^5 + 6*(sqrt(2)*(a
^3*b^3 - 33*a*b^5)*cosh(x) + sqrt(2)*(a^4*b^2 - 33*a^2*b^4))*sinh(x)^5 + 3
*sqrt(2)*(4*a^5*b - 131*a^3*b^3 - 33*a*b^5)*cosh(x)^4 + 3*(5*sqrt(2)*(a^3*
b^3 - 33*a*b^5)*cosh(x)^2 + 10*sqrt(2)*(a^4*b^2 - 33*a^2*b^4)*cosh(x) + sq
rt(2)*(4*a^5*b - 131*a^3*b^3 - 33*a*b^5))*sinh(x)^4 + 4*sqrt(2)*(2*a^6 - 6
3*a^4*b^2 - 99*a^2*b^4)*cosh(x)^3 + 4*(5*sqrt(2)*(a^3*b^3 - 33*a*b^5)*cosh
(x)^3 + 15*sqrt(2)*(a^4*b^2 - 33*a^2*b^4)*cosh(x)^2 + 3*sqrt(2)*(4*a^5*b -
 131*a^3*b^3 - 33*a*b^5)*cosh(x) + sqrt(2)*(2*a^6 - 63*a^4*b^2 - 99*a^2*b^
4))*sinh(x)^3 + 3*sqrt(2)*(4*a^5*b - 131*a^3*b^3 - 33*a*b^5)*cosh(x)^2 + 3
*(5*sqrt(2)*(a^3*b^3 - 33*a*b^5)*cosh(x)^4 + 20*sqrt(2)*(a^4*b^2 - 33*a^2*
b^4)*cosh(x)^3 + 6*sqrt(2)*(4*a^5*b - 131*a^3*b^3 - 33*a*b^5)*cosh(x)^2 +
4*sqrt(2)*(2*a^6 - 63*a^4*b^2 - 99*a^2*b^4)*cosh(x) + sqrt(2)*(4*a^5*b - 1
31*a^3*b^3 - 33*a*b^5))*sinh(x)^2 + 6*sqrt(2)*(a^4*b^2 - 33*a^2*b^4)*cosh(
x) + 6*(sqrt(2)*(a^3*b^3 - 33*a*b^5)*cosh(x)^5 + 5*sqrt(2)*(a^4*b^2 - 33*a
^2*b^4)*cosh(x)^4 + 2*sqrt(2)*(4*a^5*b - 131*a^3*b^3 - 33*a*b^5)*cosh(x)^3
 + 2*sqrt(2)*(2*a^6 - 63*a^4*b^2 - 99*a^2*b^4)*cosh(x)^2 + sqrt(2)*(4*a^5*
b - 131*a^3*b^3 - 33*a*b^5)*cosh(x) + sqrt(2)*(a^4*b^2 - 33*a^2*b^4))*sinh
(x) + sqrt(2)*(a^3*b^3 - 33*a*b^5))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2
 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh...
```

3.85.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(x))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cosh(x))**(7/2),x)
```

output
```
Timed out
```

3.85.7 Maxima [F]

$$\int \frac{1}{(a+b\cosh(x))^{7/2}}\,dx = \int \frac{1}{(b\cosh(x)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*cosh(x))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*cosh(x) + a)^(-7/2), x)
```

3.85.8 Giac [F]

$$\int \frac{1}{(a+b\cosh(x))^{7/2}}\,dx = \int \frac{1}{(b\cosh(x)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*cosh(x))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*cosh(x) + a)^(-7/2), x)
```

3.85.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(x))^{7/2}}\,dx = \int \frac{1}{(a+b\cosh(x))^{7/2}}\,dx$$

input
```
int(1/(a + b*cosh(x))^(7/2),x)
```

output
```
int(1/(a + b*cosh(x))^(7/2), x)
```

3.86 $\int \frac{\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx$

3.86.1 Optimal result

Integrand size = 13, antiderivative size = 100

$$\int \frac{\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx = -\frac{2i\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{2ia\sqrt{\frac{a+b\cosh(x)}{a+b}}\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}$$

output
```
-2*I*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(1/2*x),2^(1/2)*(b/
(a+b))^(1/2))*(a+b*cosh(x))^(1/2)/b/((a+b*cosh(x))/(a+b))^(1/2)+2*I*a*(cos
h(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1
/2))*((a+b*cosh(x))/(a+b))^(1/2)/b/(a+b*cosh(x))^(1/2)
```

3.86.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.73

$$\int \frac{\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx = -\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\left((a+b)E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)-a\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)\right)}{b\sqrt{a+b\cosh(x)}}$$

input
```
Integrate[Cosh[x]/Sqrt[a + b*Cosh[x]],x]
```

output
```
((-2*I)*Sqrt[(a + b*Cosh[x])/(a + b)]*((a + b)*EllipticE[(I/2)*x, (2*b)/(a
 + b)] - a*EllipticF[(I/2)*x, (2*b)/(a + b)]))/(b*Sqrt[a + b*Cosh[x]])
```

3.86.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(x)}{\sqrt{a+b\cosh(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)}{\sqrt{a+b\sin\left(\frac{\pi}{2}+ix\right)}}dx$$

$$\Big\downarrow \text{3231}$$

$$\frac{\int \sqrt{a+b\cosh(x)}dx}{b}-\frac{a\int \frac{1}{\sqrt{a+b\cosh(x)}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}-\frac{a\int \frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \text{3134}$$

$$\frac{\sqrt{a+b\cosh(x)}\int \sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}dx}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}-\frac{a\int \frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\cosh(x)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}-\frac{a\int \frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \text{3132}$$

$$-\frac{a\int \frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}-\frac{2i\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

$$\Big\downarrow \text{3142}$$

$$-\frac{a\sqrt{\frac{a+b\cosh(x)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}}dx}{b\sqrt{a+b\cosh(x)}}-\frac{2i\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\sqrt{\frac{a+b\cosh(x)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cosh(x)}}-\frac{2i\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{2ia\sqrt{\frac{a+b\cosh(x)}{a+b}}\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}-\frac{2i\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

input
```
Int[Cosh[x]/Sqrt[a + b*Cosh[x]],x]
```

output
```
((-2*I)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[(a
+ b*Cosh[x])/(a + b)]) + ((2*I)*a*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[
(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[a + b*Cosh[x]])
```

3.86.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

3.86.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.81

method	result
default	$\frac{2\left(\operatorname{EllipticF}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}}, \frac{\sqrt{-\frac{2(a-b)}{b}}}{2}\right)-2\operatorname{EllipticE}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}}, \frac{\sqrt{-\frac{2(a-b)}{b}}}{2}\right)\right)\sqrt{-\sinh\left(\frac{x}{2}\right)^{2}}\,\sqrt{\frac{2\cosh\left(\frac{x}{2}\right)^{2}b+a-b}{a-b}}\,\sqrt{\left(2\right.}}{\sqrt{-\frac{2b}{a-b}}\,\sqrt{2\sinh\left(\frac{x}{2}\right)^{4}b+(a+b)\sinh\left(\frac{x}{2}\right)^{2}}\,\sinh\left(\frac{x}{2}\right)\sqrt{2\sinh\left(\frac{x}{2}\right)^{2}b+a+b}}$
risch	$\frac{\left(b\,e^{2x}+2a\,e^{x}+b\right)\sqrt{2}\,e^{-x}}{b\sqrt{\left(b\,e^{2x}+2a\,e^{x}+b\right)e^{-x}}}+\frac{\left(-\frac{4\left(b\,e^{2x}+2a\,e^{x}+b\right)}{b\sqrt{\left(b\,e^{2x}+2a\,e^{x}+b\right)e^{x}}}+\frac{4\left(a+\sqrt{a^{2}-b^{2}}\right)\sqrt{\frac{\left(e^{x}+\frac{a+\sqrt{a^{2}-b^{2}}}{b}\right)b}{a+\sqrt{a^{2}-b^{2}}}}\,\sqrt{\frac{e^{x}-\frac{-a+\sqrt{a^{2}-b^{2}}}{b}}{-\frac{a+\sqrt{a^{2}-b^{2}}}{b}-\frac{-a+\sqrt{a^{2}-b^{2}}}{b}}}\,\sqrt{-\frac{e^{x}b}{a+\sqrt{a^{2}}}}}{}\right.}{}$

input
```
int(cosh(x)/(a+b*cosh(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))-2*Elli
pticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2)))*(-sinh(1/2*x
)^2)^(1/2)*((2*cosh(1/2*x)^2*b+a-b)/(a-b))^(1/2)*((2*cosh(1/2*x)^2*b+a-b)*
sinh(1/2*x)^2)^(1/2)/(-2*b/(a-b))^(1/2)/(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*
x)^2)^(1/2)/sinh(1/2*x)/(2*sinh(1/2*x)^2*b+a+b)^(1/2)
```

3.86.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.74

$$\int \frac{\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx = -\frac{2\left(2\sqrt{2}a\sqrt{b}\text{weierstrassPInverse}\left(\frac{4\left(4a^2-3b^2\right)}{3b^2},-\frac{8\left(8a^3-9ab^2\right)}{27b^3},\frac{3b\cosh(x)+3b\sinh(x)+2a}{3b}\right)+3\sqrt{2}b^{\frac{3}{2}}\text{weierstrassZ}\right.}{}$$

input
```
integrate(cosh(x)/(a+b*cosh(x))^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(2*sqrt(2)*a*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/
27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) + 3*sqr
t(2)*b^(3/2)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b
^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b
^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) + 3*sqrt(b*cosh(x) + a)
*b)/b^2
```

3.86.6 Sympy [F]

$$\int \frac{\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx = \int \frac{\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx$$

input
```
integrate(cosh(x)/(a+b*cosh(x))**(1/2),x)
```

output
```
Integral(cosh(x)/sqrt(a + b*cosh(x)), x)
```

3.86.7 Maxima [F]

$$\int \frac{\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx = \int \frac{\cosh\left(x\right)}{\sqrt{b\cosh\left(x\right)+a}}\,dx$$

input
```
integrate(cosh(x)/(a+b*cosh(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cosh(x)/sqrt(b*cosh(x) + a), x)
```

3.86.8 Giac [F]

$$\int \frac{\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx = \int \frac{\cosh\left(x\right)}{\sqrt{b\cosh\left(x\right)+a}}\,dx$$

input
```
integrate(cosh(x)/(a+b*cosh(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(cosh(x)/sqrt(b*cosh(x) + a), x)
```

3.86.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx = \int \frac{\cosh(x)}{\sqrt{a+b\cosh\left(x\right)}}\,dx$$

input
```
int(cosh(x)/(a + b*cosh(x))^(1/2),x)
```

output
```
int(cosh(x)/(a + b*cosh(x))^(1/2), x)
```

3.87 $\int (a+a\cosh(x))^{5/2}(A+B\cosh(x))\,dx$

3.87.1 Optimal result

Integrand size = 17, antiderivative size = 94

$$\int (a+a\cosh(x))^{5/2}(A+B\cosh(x))\,dx=\frac{64a^3(7A+5B)\sinh(x)}{105\sqrt{a+a\cosh(x)}}+\frac{16}{105}a^2(7A+5B)\sqrt{a+a\cosh(x)}\sinh(x)+\frac{2}{35}a(7A+5B)(a+a\cosh(x))^{3/2}\sinh(x)+\frac{2}{7}B(a+a\cosh(x))^{5/2}\sinh(x)$$

output
```
2/35*a*(7*A+5*B)*(a+a*cosh(x))^(3/2)*sinh(x)+2/7*B*(a+a*cosh(x))^(5/2)*sin
h(x)+64/105*a^3*(7*A+5*B)*sinh(x)/(a+a*cosh(x))^(1/2)+16/105*a^2*(7*A+5*B)
*sinh(x)*(a+a*cosh(x))^(1/2)
```

3.87.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.64

$$\int (a+a\cosh(x))^{5/2}(A+B\cosh(x))\,dx=\frac{1}{210}a^2\sqrt{a(1+\cosh(x))}(1246A+1040B+(392A+505B)\cosh(x)+6(7A+20B)\cosh(2x)+15B\cosh(3x))\tanh\left(\frac{x}{2}\right)$$

input
```
Integrate[(a + a*Cosh[x])^(5/2)*(A + B*Cosh[x]),x]
```

output
```
(a^2*Sqrt[a*(1 + Cosh[x])]*(1246*A + 1040*B + (392*A + 505*B)*Cosh[x] + 6*
(7*A + 20*B)*Cosh[2*x] + 15*B*Cosh[3*x])*Tanh[x/2])/210
```

3.87.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {3042, 3230, 3042, 3126, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cosh(x)+a)^{5/2}(A+B\cosh(x))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+a\sin\left(\frac{\pi}{2}+ix\right)\right)^{5/2}\left(A+B\sin\left(\frac{\pi}{2}+ix\right)\right)dx$$

$$\Big\downarrow \text{3230}$$

$$\frac{1}{7}(7A+5B)\int (\cosh(x)a+a)^{5/2}dx+\frac{2}{7}B\sinh(x)(a\cosh(x)+a)^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}B\sinh(x)(a\cosh(x)+a)^{5/2}+\frac{1}{7}(7A+5B)\int \left(\sin\left(ix+\frac{\pi}{2}\right)a+a\right)^{5/2}dx$$

$$\Big\downarrow \text{3126}$$

$$\frac{1}{7}(7A+5B)\left(\frac{8}{5}a\int (\cosh(x)a+a)^{3/2}dx+\frac{2}{5}a\sinh(x)(a\cosh(x)+a)^{3/2}\right)+\frac{2}{7}B\sinh(x)(a\cosh(x)+a)^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}B\sinh(x)(a\cosh(x)+a)^{5/2}+\frac{1}{7}(7A+5B)\left(\frac{2}{5}a\sinh(x)(a\cosh(x)+a)^{3/2}+\frac{8}{5}a\int \left(\sin\left(ix+\frac{\pi}{2}\right)a+a\right)^{3/2}dx\right)$$

$$\Big\downarrow \text{3126}$$

$$\frac{1}{7}(7A+5B)\left(\frac{8}{5}a\left(\frac{4}{3}a\int \sqrt{\cosh(x)a+a}dx+\frac{2}{3}a\sinh(x)\sqrt{a\cosh(x)+a}\right)+\frac{2}{5}a\sinh(x)(a\cosh(x)+a)^{3/2}\right)+\frac{2}{7}B\sinh(x)(a\cosh(x)+a)^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}B\sinh(x)(a\cosh(x)+a)^{5/2}+\frac{1}{7}(7A+5B)\left(\frac{2}{5}a\sinh(x)(a\cosh(x)+a)^{3/2}+\frac{8}{5}a\left(\frac{2}{3}a\sinh(x)\sqrt{a\cosh(x)+a}+\frac{4}{3}a\int\sqrt{\sin\left(ix+\frac{\pi}{2}\right)a+a}dx\right)\right)$$

$$\Big\downarrow \text{3125}$$

$$\frac{1}{7}(7A+5B)\left(\frac{8}{5}a\left(\frac{8a^2\sinh(x)}{3\sqrt{a\cosh(x)+a}}+\frac{2}{3}a\sinh(x)\sqrt{a\cosh(x)+a}\right)+\frac{2}{5}a\sinh(x)(a\cosh(x)+a)^{3/2}\right)+\frac{2}{7}B\sinh(x)(a\cosh(x)+a)^{5/2}$$

input
```
Int[(a + a*Cosh[x])^(5/2)*(A + B*Cosh[x]),x]
```

output
```
(2*B*(a + a*Cosh[x])^(5/2)*Sinh[x])/7 + ((7*A + 5*B)*((2*a*(a + a*Cosh[x])
^(3/2)*Sinh[x])/5 + (8*a*((8*a^2*Sinh[x])/(3*Sqrt[a + a*Cosh[x]]) + (2*a*S
qrt[a + a*Cosh[x]]*Sinh[x])/3))/5))/7
```

3.87.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

3.87.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.76

method	result
default	$\frac{8\cosh\left(\frac{x}{2}\right)a^{3}\sinh\left(\frac{x}{2}\right)\left(30B\sinh\left(\frac{x}{2}\right)^{6}+(21A+105B)\sinh\left(\frac{x}{2}\right)^{4}+(70A+140B)\sinh\left(\frac{x}{2}\right)^{2}+105A+105B\right)\sqrt{2}}{105\sqrt{a\cosh\left(\frac{x}{2}\right)^{2}}}$
parts	$\frac{8Aa^{3}\cosh\left(\frac{x}{2}\right)\sinh\left(\frac{x}{2}\right)\left(3\cosh\left(\frac{x}{2}\right)^{4}+4\cosh\left(\frac{x}{2}\right)^{2}+8\right)\sqrt{2}}{15\sqrt{a\cosh\left(\frac{x}{2}\right)^{2}}}+\frac{8B\cosh\left(\frac{x}{2}\right)a^{3}\sinh\left(\frac{x}{2}\right)\left(6\cosh\left(\frac{x}{2}\right)^{6}+3\cosh\left(\frac{x}{2}\right)^{4}+4\cosh\left(\frac{x}{2}\right)^{2}+8\right)\sqrt{2}}{21\sqrt{a\cosh\left(\frac{x}{2}\right)^{2}}}$

input
```
int((a+a*cosh(x))^(5/2)*(A+B*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
8/105*cosh(1/2*x)*a^3*sinh(1/2*x)*(30*B*sinh(1/2*x)^6+(21*A+105*B)*sinh(1/
2*x)^4+(70*A+140*B)*sinh(1/2*x)^2+105*A+105*B)*2^(1/2)/(a*cosh(1/2*x)^2)^(
1/2)
```

3.87.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 563 vs. 2(78) = 156.

Time = 0.26 (sec) , antiderivative size = 563, normalized size of antiderivative = 5.99

$$\int (a+a\cosh(x))^{5/2}(A+B\cosh(x))\,dx = \frac{\sqrt{\frac{1}{2}}\left(15\,Ba^{2}\cosh\left(x\right)^{7}+15\,Ba^{2}\sinh\left(x\right)^{7}+21\,(2\,A+5\,B)a^{2}\cosh\left(x\right)^{6}+35\,(10\,A+11\,B)a\right.}{}$$

input
```
integrate((a+a*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

output
```
1/420*sqrt(1/2)*(15*B*a^2*cosh(x)^7 + 15*B*a^2*sinh(x)^7 + 21*(2*A + 5*B)*
a^2*cosh(x)^6 + 35*(10*A + 11*B)*a^2*cosh(x)^5 + 525*(4*A + 3*B)*a^2*cosh(
x)^4 + 21*(5*B*a^2*cosh(x) + (2*A + 5*B)*a^2)*sinh(x)^6 - 525*(4*A + 3*B)*
a^2*cosh(x)^3 + 7*(45*B*a^2*cosh(x)^2 + 18*(2*A + 5*B)*a^2*cosh(x) + 5*(10
*A + 11*B)*a^2)*sinh(x)^5 - 35*(10*A + 11*B)*a^2*cosh(x)^2 + 35*(15*B*a^2*
cosh(x)^3 + 9*(2*A + 5*B)*a^2*cosh(x)^2 + 5*(10*A + 11*B)*a^2*cosh(x) + 15
*(4*A + 3*B)*a^2)*sinh(x)^4 - 21*(2*A + 5*B)*a^2*cosh(x) + 35*(15*B*a^2*co
sh(x)^4 + 12*(2*A + 5*B)*a^2*cosh(x)^3 + 10*(10*A + 11*B)*a^2*cosh(x)^2 +
60*(4*A + 3*B)*a^2*cosh(x) - 15*(4*A + 3*B)*a^2)*sinh(x)^3 - 15*B*a^2 + 35
*(9*B*a^2*cosh(x)^5 + 9*(2*A + 5*B)*a^2*cosh(x)^4 + 10*(10*A + 11*B)*a^2*c
osh(x)^3 + 90*(4*A + 3*B)*a^2*cosh(x)^2 - 45*(4*A + 3*B)*a^2*cosh(x) - (10
*A + 11*B)*a^2)*sinh(x)^2 + 7*(15*B*a^2*cosh(x)^6 + 18*(2*A + 5*B)*a^2*cos
h(x)^5 + 25*(10*A + 11*B)*a^2*cosh(x)^4 + 300*(4*A + 3*B)*a^2*cosh(x)^3 -
225*(4*A + 3*B)*a^2*cosh(x)^2 - 10*(10*A + 11*B)*a^2*cosh(x) - 3*(2*A + 5*
B)*a^2)*sinh(x))*sqrt(a/(cosh(x) + sinh(x)))/(cosh(x)^3 + 3*cosh(x)^2*sinh
(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)
```

3.87.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cosh(x))^{5/2}(A+B\cosh(x))\,dx=\text{Timed out}$$

input
```
integrate((a+a*cosh(x))**(5/2)*(A+B*cosh(x)),x)
```

output
```
Timed out
```

3.87.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(78) = 156.

Time = 0.30 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.52

$$\int (a+a\cosh(x))^{5/2}(A+B\cosh(x))\,dx=\frac{1}{60}\left(3\sqrt{2}a^{\frac{5}{2}}e^{\left(\frac{5}{2}x\right)}+25\sqrt{2}a^{\frac{5}{2}}e^{\left(\frac{3}{2}x\right)}+150\sqrt{2}a^{\frac{5}{2}}e^{\left(\frac{1}{2}x\right)}-150\sqrt{2}a^{\frac{5}{2}}e^{\left(-\frac{1}{2}x\right)}-25\sqrt{2}a^{\frac{5}{2}}e^{\left(-\frac{3}{2}x\right)}\right.$$
$$+\frac{1}{168}\left(\left(3\sqrt{2}a^{\frac{5}{2}}e^{(-x)}+21\sqrt{2}a^{\frac{5}{2}}e^{(-2x)}+70\sqrt{2}a^{\frac{5}{2}}e^{(-3x)}+210\sqrt{2}a^{\frac{5}{2}}e^{(-4x)}-105\sqrt{2}a^{\frac{5}{2}}e^{(-5x)}-7\sqrt{2}a^{\frac{5}{2}}e^{(-6x)}\right.\right.$$

input
```
integrate((a+a*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

output
```
1/60*(3*sqrt(2)*a^(5/2)*e^(5/2*x) + 25*sqrt(2)*a^(5/2)*e^(3/2*x) + 150*sqr
t(2)*a^(5/2)*e^(1/2*x) - 150*sqrt(2)*a^(5/2)*e^(-1/2*x) - 25*sqrt(2)*a^(5/
2)*e^(-3/2*x) - 3*sqrt(2)*a^(5/2)*e^(-5/2*x))*A + 1/168*((3*sqrt(2)*a^(5/2
)*e^(-x) + 21*sqrt(2)*a^(5/2)*e^(-2*x) + 70*sqrt(2)*a^(5/2)*e^(-3*x) + 210
*sqrt(2)*a^(5/2)*e^(-4*x) - 105*sqrt(2)*a^(5/2)*e^(-5*x) - 7*sqrt(2)*a^(5/
2)*e^(-6*x))*e^(9/2*x) + (7*sqrt(2)*a^(5/2)*e^(-x) + 105*sqrt(2)*a^(5/2)*e
^(-2*x) - 210*sqrt(2)*a^(5/2)*e^(-3*x) - 70*sqrt(2)*a^(5/2)*e^(-4*x) - 21*
sqrt(2)*a^(5/2)*e^(-5*x) - 3*sqrt(2)*a^(5/2)*e^(-6*x))*e^(5/2*x))*B
```

3.87.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.63

$$\int (a + a\cosh(x))^{5/2}(A + B\cosh(x))\,dx =$$

$$-\frac{1}{840}\sqrt{2}\left(\frac{(2100\,Aa^6e^{(3x)} + 1575\,Ba^6e^{(3x)} + 350\,Aa^6e^{(2x)} + 385\,Ba^6e^{(2x)} + 42\,Aa^6e^{x} + 105\,Ba^6e^{x} + 15\,Ba}{a^{\frac{7}{2}}}\right.$$

input
```
integrate((a+a*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="giac")
```

output
```
-1/840*sqrt(2)*((2100*A*a^6*e^(3*x) + 1575*B*a^6*e^(3*x) + 350*A*a^6*e^(2*
x) + 385*B*a^6*e^(2*x) + 42*A*a^6*e^x + 105*B*a^6*e^x + 15*B*a^6)*e^(-7/2*
x)/a^(7/2) - (15*B*a^(19/2)*e^(7/2*x) + 42*A*a^(19/2)*e^(5/2*x) + 105*B*a^
(19/2)*e^(5/2*x) + 350*A*a^(19/2)*e^(3/2*x) + 385*B*a^(19/2)*e^(3/2*x) + 2
100*A*a^(19/2)*e^(1/2*x) + 1575*B*a^(19/2)*e^(1/2*x))/a^7)
```

3.87.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\cosh(x))^{5/2}(A + B\cosh(x))\,dx = \int (A + B\cosh(x))\,\left(a + a\cosh(x)\right)^{5/2}\,dx$$

input
```
int((A + B*cosh(x))*(a + a*cosh(x))^(5/2),x)
```

output
```
int((A + B*cosh(x))*(a + a*cosh(x))^(5/2), x)
```

3.88 $\int (a + a\cosh(x))^{3/2}(A + B\cosh(x))\,dx$

3.88.1 Optimal result

Integrand size = 17, antiderivative size = 68

$$\int (a + a\cosh(x))^{3/2}(A + B\cosh(x))\,dx = \frac{8a^2(5A + 3B)\sinh(x)}{15\sqrt{a + a\cosh(x)}} + \frac{2}{15}a(5A + 3B)\sqrt{a + a\cosh(x)}\sinh(x) + \frac{2}{5}B(a + a\cosh(x))^{3/2}\sinh(x)$$

output
```
2/5*B*(a+a*cosh(x))^(3/2)*sinh(x)+8/15*a^2*(5*A+3*B)*sinh(x)/(a+a*cosh(x))
^(1/2)+2/15*a*(5*A+3*B)*sinh(x)*(a+a*cosh(x))^(1/2)
```

3.88.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.68

$$\int (a + a\cosh(x))^{3/2}(A + B\cosh(x))\,dx = \frac{1}{15}a\sqrt{a(1 + \cosh(x))}(50A + 39B + 2(5A + 9B)\cosh(x) + 3B\cosh(2x))\tanh\left(\frac{x}{2}\right)$$

input
```
Integrate[(a + a*Cosh[x])^(3/2)*(A + B*Cosh[x]),x]
```

output
```
(a*Sqrt[a*(1 + Cosh[x])]*(50*A + 39*B + 2*(5*A + 9*B)*Cosh[x] + 3*B*Cosh[2
*x])*Tanh[x/2])/15
```

3.88.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3042, 3230, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cosh(x)+a)^{3/2}(A+B\cosh(x))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+a\sin\left(\frac{\pi}{2}+ix\right)\right)^{3/2}\left(A+B\sin\left(\frac{\pi}{2}+ix\right)\right)dx$$

$$\Big\downarrow \text{3230}$$

$$\frac{1}{5}(5A+3B)\int (\cosh(x)a+a)^{3/2}dx+\frac{2}{5}B\sinh(x)(a\cosh(x)+a)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}B\sinh(x)(a\cosh(x)+a)^{3/2}+\frac{1}{5}(5A+3B)\int \left(\sin\left(ix+\frac{\pi}{2}\right)a+a\right)^{3/2}dx$$

$$\Big\downarrow \text{3126}$$

$$\frac{1}{5}(5A+3B)\left(\frac{4}{3}a\int \sqrt{\cosh(x)a+a}dx+\frac{2}{3}a\sinh(x)\sqrt{a\cosh(x)+a}\right)+\frac{2}{5}B\sinh(x)(a\cosh(x)+a)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}B\sinh(x)(a\cosh(x)+a)^{3/2}+\frac{1}{5}(5A+3B)\left(\frac{2}{3}a\sinh(x)\sqrt{a\cosh(x)+a}+\frac{4}{3}a\int \sqrt{\sin\left(ix+\frac{\pi}{2}\right)a+a}dx\right)$$

$$\Big\downarrow \text{3125}$$

$$\frac{1}{5}(5A+3B)\left(\frac{8a^2\sinh(x)}{3\sqrt{a\cosh(x)+a}}+\frac{2}{3}a\sinh(x)\sqrt{a\cosh(x)+a}\right)+\frac{2}{5}B\sinh(x)(a\cosh(x)+a)^{3/2}$$

input
```
Int[(a + a*Cosh[x])^(3/2)*(A + B*Cosh[x]),x]
```

output
```
(2*B*(a + a*Cosh[x])^(3/2)*Sinh[x])/5 + ((5*A + 3*B)*((8*a^2*Sinh[x])/(3*S
qrt[a + a*Cosh[x]]) + (2*a*Sqrt[a + a*Cosh[x]]*Sinh[x])/3))/5
```

3.88.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

3.88.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{4 \cosh \left(\frac{x}{2}\right) a^{2} \sinh \left(\frac{x}{2}\right) \left(6 B \sinh \left(\frac{x}{2}\right)^{4}+\left(5 A+15 B\right) \sinh \left(\frac{x}{2}\right)^{2}+15 A+15 B\right) \sqrt{2}}{15 \sqrt{a \cosh \left(\frac{x}{2}\right)^{2}}}$	57
parts	$\frac{4 A \,a^{2} \cosh \left(\frac{x}{2}\right) \sinh \left(\frac{x}{2}\right) \left(\cosh \left(\frac{x}{2}\right)^{2}+2\right) \sqrt{2}}{3 \sqrt{a \cosh \left(\frac{x}{2}\right)^{2}}}+\frac{4 B \cosh \left(\frac{x}{2}\right) a^{2} \sinh \left(\frac{x}{2}\right) \left(2 \cosh \left(\frac{x}{2}\right)^{4}+\cosh \left(\frac{x}{2}\right)^{2}+2\right) \sqrt{2}}{5 \sqrt{a \cosh \left(\frac{x}{2}\right)^{2}}}$	80

input
```
int((a+a*cosh(x))^(3/2)*(A+B*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
4/15*cosh(1/2*x)*a^2*sinh(1/2*x)*(6*B*sinh(1/2*x)^4+(5*A+15*B)*sinh(1/2*x)
^2+15*A+15*B)*2^(1/2)/(a*cosh(1/2*x)^2)^(1/2)
```

3.88.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(56) = 112$.

Time = 0.26 (sec) , antiderivative size = 279, normalized size of antiderivative = 4.10

$$\int (a + a\cosh(x))^{3/2}(A + B\cosh(x))\,dx = \frac{\sqrt{\frac{1}{2}}\left(3\,Ba\cosh\left(x\right)^{5} + 3\,Ba\sinh\left(x\right)^{5} + 5\,(2\,A + 3\,B)a\cosh\left(x\right)^{4} + 30\,(3\,A + 2\,B)a\cosh\left(x\right)\right.}{}$$

input
```
integrate((a+a*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

output
```
1/30*sqrt(1/2)*(3*B*a*cosh(x)^5 + 3*B*a*sinh(x)^5 + 5*(2*A + 3*B)*a*cosh(x
)^4 + 30*(3*A + 2*B)*a*cosh(x)^3 + 5*(3*B*a*cosh(x) + (2*A + 3*B)*a)*sinh(
x)^4 - 30*(3*A + 2*B)*a*cosh(x)^2 + 10*(3*B*a*cosh(x)^2 + 2*(2*A + 3*B)*a*
cosh(x) + 3*(3*A + 2*B)*a)*sinh(x)^3 - 5*(2*A + 3*B)*a*cosh(x) + 30*(B*a*c
osh(x)^3 + (2*A + 3*B)*a*cosh(x)^2 + 3*(3*A + 2*B)*a*cosh(x) - (3*A + 2*B)
*a)*sinh(x)^2 - 3*B*a + 5*(3*B*a*cosh(x)^4 + 4*(2*A + 3*B)*a*cosh(x)^3 + 1
8*(3*A + 2*B)*a*cosh(x)^2 - 12*(3*A + 2*B)*a*cosh(x) - (2*A + 3*B)*a)*sinh
(x))*sqrt(a/(cosh(x) + sinh(x)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2)
```

3.88.6 Sympy [F]

$$\int (a + a\cosh(x))^{3/2}(A + B\cosh(x))\,dx = \int \left(a(\cosh\left(x\right) + 1)\right)^{\frac{3}{2}}\left(A + B\cosh\left(x\right)\right)\,dx$$

input
```
integrate((a+a*cosh(x))**(3/2)*(A+B*cosh(x)),x)
```

output
```
Integral((a*(cosh(x) + 1))**(3/2)*(A + B*cosh(x)), x)
```

3.88.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(56) = 112$.

Time = 0.31 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.40

$$\int (a+a\cosh(x))^{3/2}(A+B\cosh(x))\,dx = \frac{1}{6}\left(\sqrt{2}a^{\frac{3}{2}}e^{\left(\frac{3}{2}x\right)}+9\sqrt{2}a^{\frac{3}{2}}e^{\left(\frac{1}{2}x\right)}-9\sqrt{2}a^{\frac{3}{2}}e^{\left(-\frac{1}{2}x\right)}-\sqrt{2}a^{\frac{3}{2}}e^{\left(-\frac{3}{2}x\right)}\right)A + \frac{1}{20}\left(\left(\sqrt{2}a^{\frac{3}{2}}e^{(-x)}+5\sqrt{2}a^{\frac{3}{2}}e^{(-2x)}+15\sqrt{2}a^{\frac{3}{2}}e^{(-3x)}-5\sqrt{2}a^{\frac{3}{2}}e^{(-4x)}\right)e^{\left(\frac{7}{2}x\right)}+\left(5\sqrt{2}a^{\frac{3}{2}}e^{(-x)}-15\sqrt{2}a^{\frac{3}{2}}e^{(-2x}\right.\right.$$

input
```
integrate((a+a*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

output
```
1/6*(sqrt(2)*a^(3/2)*e^(3/2*x) + 9*sqrt(2)*a^(3/2)*e^(1/2*x) - 9*sqrt(2)*a
^(3/2)*e^(-1/2*x) - sqrt(2)*a^(3/2)*e^(-3/2*x))*A + 1/20*((sqrt(2)*a^(3/2)
*e^(-x) + 5*sqrt(2)*a^(3/2)*e^(-2*x) + 15*sqrt(2)*a^(3/2)*e^(-3*x) - 5*sqr
t(2)*a^(3/2)*e^(-4*x))*e^(7/2*x) + (5*sqrt(2)*a^(3/2)*e^(-x) - 15*sqrt(2)*
a^(3/2)*e^(-2*x) - 5*sqrt(2)*a^(3/2)*e^(-3*x) - sqrt(2)*a^(3/2)*e^(-4*x))*
e^(3/2*x))*B
```

3.88.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(56) = 112$.

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.66

$$\int (a+a\cosh(x))^{3/2}(A+B\cosh(x))\,dx = -\frac{1}{60}\sqrt{2}\left(\frac{\left(90\,Aa^4e^{(2x)}+60\,Ba^4e^{(2x)}+10\,Aa^4e^x+15\,Ba^4e^x+3\,Ba^4\right)e^{\left(-\frac{5}{2}x\right)}}{a^{\frac{5}{2}}} - \frac{3\,Ba^{\frac{13}{2}}e^{\left(\frac{5}{2}x\right)}+10\,Aa^{\frac{13}{2}}e^{\left(\frac{3}{2}x\right.}}{}\right.$$

input
```
integrate((a+a*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="giac")
```

output
```
-1/60*sqrt(2)*((90*A*a^4*e^(2*x) + 60*B*a^4*e^(2*x) + 10*A*a^4*e^x + 15*B*
a^4*e^x + 3*B*a^4)*e^(-5/2*x)/a^(5/2) - (3*B*a^(13/2)*e^(5/2*x) + 10*A*a^(
13/2)*e^(3/2*x) + 15*B*a^(13/2)*e^(3/2*x) + 90*A*a^(13/2)*e^(1/2*x) + 60*B
*a^(13/2)*e^(1/2*x))/a^5)
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\cosh(x))^{3/2}(A + B\cosh(x))\,dx = \int (A + B\cosh(x))\,(a + a\cosh(x))^{3/2}\,dx$$

input
```
int((A + B*cosh(x))*(a + a*cosh(x))^(3/2),x)
```

output
```
int((A + B*cosh(x))*(a + a*cosh(x))^(3/2), x)
```

3.89 $\int \sqrt{a+a\cosh(x)}(A+B\cosh(x))\,dx$

3.89.1 Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \sqrt{a+a\cosh(x)}(A+B\cosh(x))\,dx=\frac{2a(3A+B)\sinh(x)}{3\sqrt{a+a\cosh(x)}}+\frac{2}{3}B\sqrt{a+a\cosh(x)}\sinh(x)$$

output
```
2/3*a*(3*A+B)*sinh(x)/(a+a*cosh(x))^(1/2)+2/3*B*sinh(x)*(a+a*cosh(x))^(1/2
)
```

3.89.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \sqrt{a+a\cosh(x)}(A+B\cosh(x))\,dx=\frac{2}{3}\sqrt{a(1+\cosh(x))}(3A+2B+B\cosh(x))\tanh\left(\frac{x}{2}\right)$$

input
```
Integrate[Sqrt[a + a*Cosh[x]]*(A + B*Cosh[x]),x]
```

output
```
(2*Sqrt[a*(1 + Cosh[x])]*(3*A + 2*B + B*Cosh[x])*Tanh[x/2])/3
```

3.89.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 3230, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\cosh(x)+a}(A+B\cosh(x))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{a+a\sin\left(\frac{\pi}{2}+ix\right)}\left(A+B\sin\left(\frac{\pi}{2}+ix\right)\right)dx$$
$$\Big\downarrow \text{3230}$$
$$\frac{1}{3}(3A+B)\int \sqrt{\cosh(x)a+a}dx+\frac{2}{3}B\sinh(x)\sqrt{a\cosh(x)+a}$$
$$\Big\downarrow \text{3042}$$
$$\frac{2}{3}B\sinh(x)\sqrt{a\cosh(x)+a}+\frac{1}{3}(3A+B)\int \sqrt{\sin\left(ix+\frac{\pi}{2}\right)a+a}dx$$
$$\Big\downarrow \text{3125}$$
$$\frac{2a(3A+B)\sinh(x)}{3\sqrt{a\cosh(x)+a}}+\frac{2}{3}B\sinh(x)\sqrt{a\cosh(x)+a}$$

input
```
Int[Sqrt[a + a*Cosh[x]]*(A + B*Cosh[x]),x]
```

output
```
(2*a*(3*A + B)*Sinh[x])/(3*Sqrt[a + a*Cosh[x]]) + (2*B*Sqrt[a + a*Cosh[x]]
*Sinh[x])/3
```

3.89.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

3.89.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{2\cosh\left(\frac{x}{2}\right)a\sinh\left(\frac{x}{2}\right)\left(2B\cosh\left(\frac{x}{2}\right)^{2}+3A+B\right)\sqrt{2}}{3\sqrt{a\cosh\left(\frac{x}{2}\right)^{2}}}$	39
parts	$\frac{2Aa\cosh\left(\frac{x}{2}\right)\sinh\left(\frac{x}{2}\right)\sqrt{2}}{\sqrt{a\cosh\left(\frac{x}{2}\right)^{2}}}+\frac{2B\cosh\left(\frac{x}{2}\right)a\sinh\left(\frac{x}{2}\right)\left(2\cosh\left(\frac{x}{2}\right)^{2}+1\right)\sqrt{2}}{3\sqrt{a\cosh\left(\frac{x}{2}\right)^{2}}}$	62

input
```
int((a+a*cosh(x))^(1/2)*(A+B*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
2/3*cosh(1/2*x)*a*sinh(1/2*x)*(2*B*cosh(1/2*x)^2+3*A+B)*2^(1/2)/(a*cosh(1/
2*x)^2)^(1/2)
```

3.89.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(32) = 64.

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.50

$$\int \sqrt{a+a\cosh(x)}(A+B\cosh(x))\,dx$$
$$=\frac{\sqrt{\frac{1}{2}}\left(B\cosh\left(x\right)^{3}+B\sinh\left(x\right)^{3}+3\left(2A+B\right)\cosh\left(x\right)^{2}+3\left(B\cosh\left(x\right)+2A+B\right)\sinh\left(x\right)^{2}-3\left(2A+B\right.}{3\left(\cosh\left(x\right)+\sinh\right.}$$

input
```
integrate((a+a*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

output
```
1/3*sqrt(1/2)*(B*cosh(x)^3 + B*sinh(x)^3 + 3*(2*A + B)*cosh(x)^2 + 3*(B*co
sh(x) + 2*A + B)*sinh(x)^2 - 3*(2*A + B)*cosh(x) + 3*(B*cosh(x)^2 + 2*(2*A
 + B)*cosh(x) - 2*A - B)*sinh(x) - B)*sqrt(a/(cosh(x) + sinh(x)))/(cosh(x)
 + sinh(x))
```

3.89.6 Sympy [F]

$$\int \sqrt{a + a\cosh(x)}(A + B\cosh(x))\,dx = \int \sqrt{a\left(\cosh\left(x\right) + 1\right)}\left(A + B\cosh\left(x\right)\right)\,dx$$

input
```
integrate((a+a*cosh(x))**(1/2)*(A+B*cosh(x)),x)
```

output
```
Integral(sqrt(a*(cosh(x) + 1))*(A + B*cosh(x)), x)
```

3.89.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(32) = 64$.

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.25

$$\int \sqrt{a + a\cosh(x)}(A + B\cosh(x))\,dx = \left(\sqrt{2}\sqrt{a}e^{\left(\frac{1}{2}x\right)} - \sqrt{2}\sqrt{a}e^{\left(-\frac{1}{2}x\right)}\right)A + \frac{1}{6}\left(\left(\sqrt{2}\sqrt{a}e^{(-x)} + 3\sqrt{2}\sqrt{a}e^{(-2x)}\right)e^{\left(\frac{5}{2}x\right)} - \left(3\sqrt{2}\sqrt{a}e^{(-x)} + \sqrt{2}\sqrt{a}e^{(-2x)}\right)e^{\left(\frac{1}{2}x\right)}\right)B$$

input
```
integrate((a+a*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

output
```
(sqrt(2)*sqrt(a)*e^(1/2*x) - sqrt(2)*sqrt(a)*e^(-1/2*x))*A + 1/6*((sqrt(2)
*sqrt(a)*e^(-x) + 3*sqrt(2)*sqrt(a)*e^(-2*x))*e^(5/2*x) - (3*sqrt(2)*sqrt(
a)*e^(-x) + sqrt(2)*sqrt(a)*e^(-2*x))*e^(1/2*x))*B
```

3.89.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(32) = 64$.

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.78

$$\int \sqrt{a + a\cosh(x)}(A + B\cosh(x))\,dx$$
$$= -\frac{1}{6}\sqrt{2}\left(\frac{(6\,Aa^2e^x + 3\,Ba^2e^x + Ba^2)e^{\left(-\frac{3}{2}\,x\right)}}{a^{\frac{3}{2}}} - \frac{Ba^{\frac{7}{2}}e^{\left(\frac{3}{2}\,x\right)} + 6\,Aa^{\frac{7}{2}}e^{\left(\frac{1}{2}\,x\right)} + 3\,Ba^{\frac{7}{2}}e^{\left(\frac{1}{2}\,x\right)}}{a^3}\right)$$

input
```
integrate((a+a*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="giac")
```

output
```
-1/6*sqrt(2)*((6*A*a^2*e^x + 3*B*a^2*e^x + B*a^2)*e^(-3/2*x)/a^(3/2) - (B*
a^(7/2)*e^(3/2*x) + 6*A*a^(7/2)*e^(1/2*x) + 3*B*a^(7/2)*e^(1/2*x))/a^3)
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + a\cosh(x)}(A + B\cosh(x))\,dx = \int (A + B\cosh(x))\,\sqrt{a + a\cosh(x)}\,dx$$

input
```
int((A + B*cosh(x))*(a + a*cosh(x))^(1/2),x)
```

output
```
int((A + B*cosh(x))*(a + a*cosh(x))^(1/2), x)
```

3.90 $\int (a-a\cosh(x))^{5/2}(A+B\cosh(x))\,dx$

3.90.1 Optimal result

Integrand size = 18, antiderivative size = 98

$$\int (a-a\cosh(x))^{5/2}(A+B\cosh(x))\,dx = -\frac{64a^3(7A-5B)\sinh(x)}{105\sqrt{a-a\cosh(x)}} - \frac{16}{105}a^2(7A-5B)\sqrt{a-a\cosh(x)}\sinh(x) - \frac{2}{35}a(7A-5B)(a-a\cosh(x))^{3/2}\sinh(x) + \frac{2}{7}B(a-a\cosh(x))^{5/2}\sinh(x)$$

output
```
-2/35*a*(7*A-5*B)*(a-a*cosh(x))^(3/2)*sinh(x)+2/7*B*(a-a*cosh(x))^(5/2)*si
nh(x)-64/105*a^3*(7*A-5*B)*sinh(x)/(a-a*cosh(x))^(1/2)-16/105*a^2*(7*A-5*B
)*sinh(x)*(a-a*cosh(x))^(1/2)
```

3.90.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.62

$$\int (a-a\cosh(x))^{5/2}(A+B\cosh(x))\,dx = \frac{1}{210}a^2\sqrt{a-a\cosh(x)}(1246A-1040B + (-392A+505B)\cosh(x) + 6(7A-20B)\cosh(2x) + 15B\cosh(3x))\coth\left(\frac{x}{2}\right)$$

input
```
Integrate[(a - a*Cosh[x])^(5/2)*(A + B*Cosh[x]),x]
```

output
```
(a^2*Sqrt[a - a*Cosh[x]]*(1246*A - 1040*B + (-392*A + 505*B)*Cosh[x] + 6*(
7*A - 20*B)*Cosh[2*x] + 15*B*Cosh[3*x])*Coth[x/2])/210
```

3.90.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3042, 3230, 3042, 3126, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-a\cosh(x))^{5/2}(A+B\cosh(x))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\sin\left(\frac{\pi}{2}+ix\right)\right)^{5/2}\left(A+B\sin\left(\frac{\pi}{2}+ix\right)\right)dx$$

$$\Big\downarrow \text{3230}$$

$$\frac{1}{7}(7A-5B)\int (a-a\cosh(x))^{5/2}dx+\frac{2}{7}B\sinh(x)(a-a\cosh(x))^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}B\sinh(x)(a-a\cosh(x))^{5/2}+\frac{1}{7}(7A-5B)\int \left(a-a\sin\left(ix+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{3126}$$

$$\frac{1}{7}(7A-5B)\left(\frac{8}{5}a\int (a-a\cosh(x))^{3/2}dx-\frac{2}{5}a\sinh(x)(a-a\cosh(x))^{3/2}\right)+\frac{2}{7}B\sinh(x)(a-a\cosh(x))^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}B\sinh(x)(a-a\cosh(x))^{5/2}+\frac{1}{7}(7A-5B)\left(-\frac{2}{5}a\sinh(x)(a-a\cosh(x))^{3/2}+\frac{8}{5}a\int \left(a-a\sin\left(ix+\frac{\pi}{2}\right)\right)^{3/2}dx\right)$$

$$\Big\downarrow \text{3126}$$

$$\frac{1}{7}(7A-5B)\left(\frac{8}{5}a\left(\frac{4}{3}a\int \sqrt{a-a\cosh(x)}dx-\frac{2}{3}a\sinh(x)\sqrt{a-a\cosh(x)}\right)-\frac{2}{5}a\sinh(x)(a-a\cosh(x))^{3/2}\right)+\frac{2}{7}B\sinh(x)(a-a\cosh(x))^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}B\sinh(x)(a-a\cosh(x))^{5/2}+\frac{1}{7}(7A-5B)\left(-\frac{2}{5}a\sinh(x)(a-a\cosh(x))^{3/2}+\frac{8}{5}a\left(-\frac{2}{3}a\sinh(x)\sqrt{a-a\cosh(x)}+\frac{4}{3}a\int\sqrt{a-a\sin\left(ix+\frac{\pi}{2}\right)}dx\right)\right)$$

$$\Big\downarrow \text{ 3125}$$

$$\frac{1}{7}(7A-5B)\left(\frac{8}{5}a\left(-\frac{8a^2\sinh(x)}{3\sqrt{a-a\cosh(x)}}-\frac{2}{3}a\sinh(x)\sqrt{a-a\cosh(x)}\right)-\frac{2}{5}a\sinh(x)(a-a\cosh(x))^{3/2}\right)+\frac{2}{7}B\sinh(x)(a-a\cosh(x))^{5/2}$$

input
```
Int[(a - a*Cosh[x])^(5/2)*(A + B*Cosh[x]),x]
```

output
```
(2*B*(a - a*Cosh[x])^(5/2)*Sinh[x])/7 + ((7*A - 5*B)*((-2*a*(a - a*Cosh[x]
)^(3/2)*Sinh[x])/5 + (8*a*((-8*a^2*Sinh[x])/(3*Sqrt[a - a*Cosh[x]]) - (2*a
*Sqrt[a - a*Cosh[x]]*Sinh[x])/3))/5))/7
```

3.90.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

3.90.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.70

method	result	s
default	$-\frac{16 \sinh\left(\frac{x}{2}\right) a^{3} \cosh\left(\frac{x}{2}\right) \left(30 B \sinh\left(\frac{x}{2}\right)^{6}+\left(21 A -15 B \right) \sinh\left(\frac{x}{2}\right)^{4}+\left(-28 A +20 B \right) \sinh\left(\frac{x}{2}\right)^{2}+56 A -40 B \right)}{105 \sqrt{-2 \sinh\left(\frac{x}{2}\right)^{2} a}}$	6
parts	$-\frac{16 A \sinh\left(\frac{x}{2}\right) a^{3} \cosh\left(\frac{x}{2}\right) \left(3 \sinh\left(\frac{x}{2}\right)^{4}-4 \sinh\left(\frac{x}{2}\right)^{2}+8\right)}{15 \sqrt{-2 \sinh\left(\frac{x}{2}\right)^{2} a}}-\frac{16 B \sinh\left(\frac{x}{2}\right) a^{3} \cosh\left(\frac{x}{2}\right) \left(6 \sinh\left(\frac{x}{2}\right)^{6}-3 \sinh\left(\frac{x}{2}\right)^{4}+4 \sinh\left(\frac{x}{2}\right)^{2}-8\right)}{21 \sqrt{-2 \sinh\left(\frac{x}{2}\right)^{2} a}}$	9

input
```
int((a-a*cosh(x))^(5/2)*(A+B*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-16/105*sinh(1/2*x)*a^3*cosh(1/2*x)*(30*B*sinh(1/2*x)^6+(21*A-15*B)*sinh(1
/2*x)^4+(-28*A+20*B)*sinh(1/2*x)^2+56*A-40*B)/(-2*sinh(1/2*x)^2*a)^(1/2)
```

3.90.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 564 vs. 2(82) = 164.

Time = 0.26 (sec) , antiderivative size = 564, normalized size of antiderivative = 5.76

$$\int (a-a\cosh(x))^{5/2}(A+B\cosh(x))\,dx = \frac{\sqrt{\frac{1}{2}}\left(15\,Ba^2\cosh(x)^7+15\,Ba^2\sinh(x)^7+21\,(2\,A-5\,B)a^2\cosh(x)^6-35\,(10\,A-11\,B)a\right.}{}$$

input
```
integrate((a-a*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

output
```
1/420*sqrt(1/2)*(15*B*a^2*cosh(x)^7 + 15*B*a^2*sinh(x)^7 + 21*(2*A - 5*B)*
a^2*cosh(x)^6 - 35*(10*A - 11*B)*a^2*cosh(x)^5 + 525*(4*A - 3*B)*a^2*cosh(
x)^4 + 21*(5*B*a^2*cosh(x) + (2*A - 5*B)*a^2)*sinh(x)^6 + 525*(4*A - 3*B)*
a^2*cosh(x)^3 + 7*(45*B*a^2*cosh(x)^2 + 18*(2*A - 5*B)*a^2*cosh(x) - 5*(10
*A - 11*B)*a^2)*sinh(x)^5 - 35*(10*A - 11*B)*a^2*cosh(x)^2 + 35*(15*B*a^2*
cosh(x)^3 + 9*(2*A - 5*B)*a^2*cosh(x)^2 - 5*(10*A - 11*B)*a^2*cosh(x) + 15
*(4*A - 3*B)*a^2)*sinh(x)^4 + 21*(2*A - 5*B)*a^2*cosh(x) + 35*(15*B*a^2*co
sh(x)^4 + 12*(2*A - 5*B)*a^2*cosh(x)^3 - 10*(10*A - 11*B)*a^2*cosh(x)^2 +
60*(4*A - 3*B)*a^2*cosh(x) + 15*(4*A - 3*B)*a^2)*sinh(x)^3 + 15*B*a^2 + 35
*(9*B*a^2*cosh(x)^5 + 9*(2*A - 5*B)*a^2*cosh(x)^4 - 10*(10*A - 11*B)*a^2*c
osh(x)^3 + 90*(4*A - 3*B)*a^2*cosh(x)^2 + 45*(4*A - 3*B)*a^2*cosh(x) - (10
*A - 11*B)*a^2)*sinh(x)^2 + 7*(15*B*a^2*cosh(x)^6 + 18*(2*A - 5*B)*a^2*cos
h(x)^5 - 25*(10*A - 11*B)*a^2*cosh(x)^4 + 300*(4*A - 3*B)*a^2*cosh(x)^3 +
225*(4*A - 3*B)*a^2*cosh(x)^2 - 10*(10*A - 11*B)*a^2*cosh(x) + 3*(2*A - 5*
B)*a^2)*sinh(x))*sqrt(-a/(cosh(x) + sinh(x)))/(cosh(x)^3 + 3*cosh(x)^2*sin
h(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)
```

3.90.6 Sympy [F(-1)]

Timed out.

$$\int (a - a\cosh(x))^{5/2}(A + B\cosh(x))\,dx = \text{Timed out}$$

input
```
integrate((a-a*cosh(x))**(5/2)*(A+B*cosh(x)),x)
```

output
```
Timed out
```

3.90.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(82) = 164$.

Time = 0.29 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.94

$$\int (a - a\cosh(x))^{5/2}(A + B\cosh(x))\,dx = \frac{1}{60}\left(\frac{25\sqrt{2}a^{\frac{5}{2}}e^{(-x)}}{(-e^{(-x)})^{\frac{5}{2}}} - \frac{150\sqrt{2}a^{\frac{5}{2}}e^{(-2x)}}{(-e^{(-x)})^{\frac{5}{2}}} - \frac{150\sqrt{2}a^{\frac{5}{2}}e^{(-3x)}}{(-e^{(-x)})^{\frac{5}{2}}} + \frac{25\sqrt{2}a^{\frac{5}{2}}e^{(-4x)}}{(-e^{(-x)})^{\frac{5}{2}}} - \frac{3\sqrt{2}a^{\frac{5}{2}}e^{(-5}}{(-e^{(-x)})^{\frac{5}{2}}} + \frac{1}{168}B\left(\frac{\left(21\sqrt{2}a^{\frac{5}{2}}e^{(-x)} - 70\sqrt{2}a^{\frac{5}{2}}e^{(-2x)} + 210\sqrt{2}a^{\frac{5}{2}}e^{(-3x)} + 105\sqrt{2}a^{\frac{5}{2}}e^{(-4x)} - 7\sqrt{2}a^{\frac{5}{2}}e^{(-5x)} - 3\sqrt{2}a^{\frac{5}{2}}\right)}{(-e^{(-x)})^{\frac{5}{2}}}\right.$$

input
```
integrate((a-a*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

output
```
1/60*(25*sqrt(2)*a^(5/2)*e^(-x)/(-e^(-x))^(5/2) - 150*sqrt(2)*a^(5/2)*e^(-
2*x)/(-e^(-x))^(5/2) - 150*sqrt(2)*a^(5/2)*e^(-3*x)/(-e^(-x))^(5/2) + 25*s
qrt(2)*a^(5/2)*e^(-4*x)/(-e^(-x))^(5/2) - 3*sqrt(2)*a^(5/2)*e^(-5*x)/(-e^(
-x))^(5/2) - 3*sqrt(2)*a^(5/2)/(-e^(-x))^(5/2))*A + 1/168*B*((21*sqrt(2)*a
^(5/2)*e^(-x) - 70*sqrt(2)*a^(5/2)*e^(-2*x) + 210*sqrt(2)*a^(5/2)*e^(-3*x)
 + 105*sqrt(2)*a^(5/2)*e^(-4*x) - 7*sqrt(2)*a^(5/2)*e^(-5*x) - 3*sqrt(2)*a
^(5/2))*e^x/(-e^(-x))^(5/2) - (7*sqrt(2)*a^(5/2)*e^(-x) - 105*sqrt(2)*a^(5
/2)*e^(-2*x) - 210*sqrt(2)*a^(5/2)*e^(-3*x) + 70*sqrt(2)*a^(5/2)*e^(-4*x)
- 21*sqrt(2)*a^(5/2)*e^(-5*x) + 3*sqrt(2)*a^(5/2)*e^(-6*x))/(-e^(-x))^(5/2
))
```

3.90.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(82) = 164$.

Time = 0.29 (sec) , antiderivative size = 295, normalized size of antiderivative = 3.01

$$\int (a-a\cosh(x))^{5/2}(A+B\cosh(x))\,dx = \frac{1}{840}\sqrt{2}\left(\frac{(2100\,Aa^6e^{(3\,x)}\mathrm{sgn}(-e^x+1) - 1575\,Ba^6e^{(3\,x)}\mathrm{sgn}(-e^x+1) - 350\,Aa^6e^{(2\,x)}\mathrm{sgn}(-\right.$$

input
```
integrate((a-a*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="giac")
```

output
```
1/840*sqrt(2)*((2100*A*a^6*e^(3*x)*sgn(-e^x + 1) - 1575*B*a^6*e^(3*x)*sgn(
-e^x + 1) - 350*A*a^6*e^(2*x)*sgn(-e^x + 1) + 385*B*a^6*e^(2*x)*sgn(-e^x +
 1) + 42*A*a^6*e^x*sgn(-e^x + 1) - 105*B*a^6*e^x*sgn(-e^x + 1) + 15*B*a^6*
sgn(-e^x + 1))*e^(-3*x)/(sqrt(-a*e^x)*a^3) - (15*sqrt(-a*e^x)*B*a^9*e^(3*x
)*sgn(-e^x + 1) + 42*sqrt(-a*e^x)*A*a^9*e^(2*x)*sgn(-e^x + 1) - 105*sqrt(-
a*e^x)*B*a^9*e^(2*x)*sgn(-e^x + 1) - 350*sqrt(-a*e^x)*A*a^9*e^x*sgn(-e^x +
 1) + 385*sqrt(-a*e^x)*B*a^9*e^x*sgn(-e^x + 1) + 2100*sqrt(-a*e^x)*A*a^9*s
gn(-e^x + 1) - 1575*sqrt(-a*e^x)*B*a^9*sgn(-e^x + 1))/a^7)
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int (a - a\cosh(x))^{5/2}(A + B\cosh(x))\,dx = \int (A + B\cosh(x))\,\left(a - a\cosh(x)\right)^{5/2}\,dx$$

input
```
int((A + B*cosh(x))*(a - a*cosh(x))^(5/2),x)
```

output
```
int((A + B*cosh(x))*(a - a*cosh(x))^(5/2), x)
```

3.91 $\int (a-a\cosh(x))^{3/2}(A+B\cosh(x))\,dx$

3.91.1 Optimal result

Integrand size = 18, antiderivative size = 71

$$\int (a-a\cosh(x))^{3/2}(A+B\cosh(x))\,dx = -\frac{8a^2(5A-3B)\sinh(x)}{15\sqrt{a-a\cosh(x)}} - \frac{2}{15}a(5A-3B)\sqrt{a-a\cosh(x)}\sinh(x) + \frac{2}{5}B(a-a\cosh(x))^{3/2}\sinh(x)$$

output
```
2/5*B*(a-a*cosh(x))^(3/2)*sinh(x)-8/15*a^2*(5*A-3*B)*sinh(x)/(a-a*cosh(x))
^(1/2)-2/15*a*(5*A-3*B)*sinh(x)*(a-a*cosh(x))^(1/2)
```

3.91.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.66

$$\int (a-a\cosh(x))^{3/2}(A+B\cosh(x))\,dx = -\frac{1}{15}a\sqrt{a-a\cosh(x)}(-50A+39B+2(5A-9B)\cosh(x)+3B\cosh(2x))\coth\left(\frac{x}{2}\right)$$

input
```
Integrate[(a - a*Cosh[x])^(3/2)*(A + B*Cosh[x]),x]
```

output
```
-1/15*(a*Sqrt[a - a*Cosh[x]]*(-50*A + 39*B + 2*(5*A - 9*B)*Cosh[x] + 3*B*C
osh[2*x])*Coth[x/2])
```

3.91.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3230, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-a\cosh(x))^{3/2}(A+B\cosh(x))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\sin\left(\frac{\pi}{2}+ix\right)\right)^{3/2}\left(A+B\sin\left(\frac{\pi}{2}+ix\right)\right)dx$$

$$\Big\downarrow \text{3230}$$

$$\frac{1}{5}(5A-3B)\int (a-a\cosh(x))^{3/2}dx+\frac{2}{5}B\sinh(x)(a-a\cosh(x))^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}B\sinh(x)(a-a\cosh(x))^{3/2}+\frac{1}{5}(5A-3B)\int \left(a-a\sin\left(ix+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{3126}$$

$$\frac{1}{5}(5A-3B)\left(\frac{4}{3}a\int \sqrt{a-a\cosh(x)}dx-\frac{2}{3}a\sinh(x)\sqrt{a-a\cosh(x)}\right)+\frac{2}{5}B\sinh(x)(a-a\cosh(x))^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}B\sinh(x)(a-a\cosh(x))^{3/2}+\frac{1}{5}(5A-3B)\left(-\frac{2}{3}a\sinh(x)\sqrt{a-a\cosh(x)}+\frac{4}{3}a\int \sqrt{a-a\sin\left(ix+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \text{3125}$$

$$\frac{1}{5}(5A-3B)\left(-\frac{8a^2\sinh(x)}{3\sqrt{a-a\cosh(x)}}-\frac{2}{3}a\sinh(x)\sqrt{a-a\cosh(x)}\right)+\frac{2}{5}B\sinh(x)(a-a\cosh(x))^{3/2}$$

input
```
Int[(a - a*Cosh[x])^(3/2)*(A + B*Cosh[x]),x]
```

output
```
(2*B*(a - a*Cosh[x])^(3/2)*Sinh[x])/5 + ((5*A - 3*B)*((-8*a^2*Sinh[x])/(3*
Sqrt[a - a*Cosh[x]]) - (2*a*Sqrt[a - a*Cosh[x]]*Sinh[x])/3))/5
```

3.91.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

3.91.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{8\sinh\left(\frac{x}{2}\right)a^{2}\cosh\left(\frac{x}{2}\right)\left(6B\sinh\left(\frac{x}{2}\right)^{4}+(5A-3B)\sinh\left(\frac{x}{2}\right)^{2}-10A+6B\right)}{15\sqrt{-2\sinh\left(\frac{x}{2}\right)^{2}a}}$	55
parts	$\frac{8A\sinh\left(\frac{x}{2}\right)a^{2}\cosh\left(\frac{x}{2}\right)\left(\cosh\left(\frac{x}{2}\right)^{2}-3\right)}{3\sqrt{-2\sinh\left(\frac{x}{2}\right)^{2}a}}+\frac{8B\sinh\left(\frac{x}{2}\right)a^{2}\cosh\left(\frac{x}{2}\right)\left(2\cosh\left(\frac{x}{2}\right)^{4}-5\cosh\left(\frac{x}{2}\right)^{2}+5\right)}{5\sqrt{-2\sinh\left(\frac{x}{2}\right)^{2}a}}$	78

input
```
int((a-a*cosh(x))^(3/2)*(A+B*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
8/15*sinh(1/2*x)*a^2*cosh(1/2*x)*(6*B*sinh(1/2*x)^4+(5*A-3*B)*sinh(1/2*x)^
2-10*A+6*B)/(-2*sinh(1/2*x)^2*a)^(1/2)
```

3.91.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(59) = 118$.

Time = 0.26 (sec) , antiderivative size = 279, normalized size of antiderivative = 3.93

$$\int (a - a\cosh(x))^{3/2}(A + B\cosh(x))\,dx =$$
$$-\frac{\sqrt{\frac{1}{2}}\left(3\,Ba\cosh\left(x\right)^{5} + 3\,Ba\sinh\left(x\right)^{5} + 5\,(2\,A - 3\,B)a\cosh\left(x\right)^{4} - 30\,(3\,A - 2\,B)a\cosh\left(x\right)^{3} + 5\,(3\,Ba\cosh\right.}{}$$

input
```
integrate((a-a*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

output
```
-1/30*sqrt(1/2)*(3*B*a*cosh(x)^5 + 3*B*a*sinh(x)^5 + 5*(2*A - 3*B)*a*cosh(
x)^4 - 30*(3*A - 2*B)*a*cosh(x)^3 + 5*(3*B*a*cosh(x) + (2*A - 3*B)*a)*sinh
(x)^4 - 30*(3*A - 2*B)*a*cosh(x)^2 + 10*(3*B*a*cosh(x)^2 + 2*(2*A - 3*B)*a
*cosh(x) - 3*(3*A - 2*B)*a)*sinh(x)^3 + 5*(2*A - 3*B)*a*cosh(x) + 30*(B*a*
cosh(x)^3 + (2*A - 3*B)*a*cosh(x)^2 - 3*(3*A - 2*B)*a*cosh(x) - (3*A - 2*B
)*a)*sinh(x)^2 + 3*B*a + 5*(3*B*a*cosh(x)^4 + 4*(2*A - 3*B)*a*cosh(x)^3 -
18*(3*A - 2*B)*a*cosh(x)^2 - 12*(3*A - 2*B)*a*cosh(x) + (2*A - 3*B)*a)*sin
h(x))*sqrt(-a/(cosh(x) + sinh(x)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x
)^2)
```

3.91.6 Sympy [F]

$$\int (a - a\cosh(x))^{3/2}(A + B\cosh(x))\,dx = \int \left(-a(\cosh\left(x\right) - 1)\right)^{\frac{3}{2}}\left(A + B\cosh\left(x\right)\right)\,dx$$

input
```
integrate((a-a*cosh(x))**(3/2)*(A+B*cosh(x)),x)
```

output
```
Integral((-a*(cosh(x) - 1))**(3/2)*(A + B*cosh(x)), x)
```

3.91.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(59) = 118$.

Time = 0.30 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.80

$$\int (a-a\cosh(x))^{3/2}(A+B\cosh(x))\,dx = \frac{1}{6}\left(\frac{9\sqrt{2}a^{\frac{3}{2}}e^{(-x)}}{(-e^{(-x)})^{\frac{3}{2}}}+\frac{9\sqrt{2}a^{\frac{3}{2}}e^{(-2x)}}{(-e^{(-x)})^{\frac{3}{2}}}-\frac{\sqrt{2}a^{\frac{3}{2}}e^{(-3x)}}{(-e^{(-x)})^{\frac{3}{2}}}-\frac{\sqrt{2}a^{\frac{3}{2}}}{(-e^{(-x)})^{\frac{3}{2}}}\right)A + \frac{1}{20}B\left(\frac{\left(5\sqrt{2}a^{\frac{3}{2}}e^{(-x)}-15\sqrt{2}a^{\frac{3}{2}}e^{(-2x)}-5\sqrt{2}a^{\frac{3}{2}}e^{(-3x)}-\sqrt{2}a^{\frac{3}{2}}\right)e^{x}}{(-e^{(-x)})^{\frac{3}{2}}}-\frac{5\sqrt{2}a^{\frac{3}{2}}e^{(-x)}+15\sqrt{2}a^{\frac{3}{2}}e^{(-2x)}-5\sqrt{}}{(-e^{(-x)})^{\frac{3}{2}}}\right.$$

input
```
integrate((a-a*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

output
```
1/6*(9*sqrt(2)*a^(3/2)*e^(-x)/(-e^(-x))^(3/2) + 9*sqrt(2)*a^(3/2)*e^(-2*x)
/(-e^(-x))^(3/2) - sqrt(2)*a^(3/2)*e^(-3*x)/(-e^(-x))^(3/2) - sqrt(2)*a^(3
/2)/(-e^(-x))^(3/2))*A + 1/20*B*((5*sqrt(2)*a^(3/2)*e^(-x) - 15*sqrt(2)*a^
(3/2)*e^(-2*x) - 5*sqrt(2)*a^(3/2)*e^(-3*x) - sqrt(2)*a^(3/2))*e^x/(-e^(-x
))^(3/2) - (5*sqrt(2)*a^(3/2)*e^(-x) + 15*sqrt(2)*a^(3/2)*e^(-2*x) - 5*sqr
t(2)*a^(3/2)*e^(-3*x) + sqrt(2)*a^(3/2)*e^(-4*x))/(-e^(-x))^(3/2))
```

3.91.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(59) = 118$.

Time = 0.27 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.99

$$\int (a-a\cosh(x))^{3/2}(A+B\cosh(x))\,dx = \frac{1}{60}\sqrt{2}\left(\frac{(90\,Aa^4e^{(2x)}\mathrm{sgn}(-e^x+1)-60\,Ba^4e^{(2x)}\mathrm{sgn}(-e^x+1)-10\,Aa^4e^x\mathrm{sgn}(-e^x+1)+}{\sqrt{-ae^x}a^2}\right.$$

input
```
integrate((a-a*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="giac")
```

output
```
1/60*sqrt(2)*((90*A*a^4*e^(2*x)*sgn(-e^x + 1) - 60*B*a^4*e^(2*x)*sgn(-e^x
+ 1) - 10*A*a^4*e^x*sgn(-e^x + 1) + 15*B*a^4*e^x*sgn(-e^x + 1) - 3*B*a^4*s
gn(-e^x + 1))*e^(-2*x)/(sqrt(-a*e^x)*a^2) + (3*sqrt(-a*e^x)*B*a^6*e^(2*x)*
sgn(-e^x + 1) + 10*sqrt(-a*e^x)*A*a^6*e^x*sgn(-e^x + 1) - 15*sqrt(-a*e^x)*
B*a^6*e^x*sgn(-e^x + 1) - 90*sqrt(-a*e^x)*A*a^6*sgn(-e^x + 1) + 60*sqrt(-a
*e^x)*B*a^6*sgn(-e^x + 1))/a^5)
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int (a-a\cosh(x))^{3/2}(A+B\cosh(x))\,dx=\int (A+B\cosh(x))\,\left(a-a\cosh(x)\right)^{3/2}\,dx$$

input
```
int((A + B*cosh(x))*(a - a*cosh(x))^(3/2),x)
```

output
```
int((A + B*cosh(x))*(a - a*cosh(x))^(3/2), x)
```

3.92 $\int \sqrt{a-a\cosh(x)}(A+B\cosh(x))\,dx$

3.92.1 Optimal result

Integrand size = 18, antiderivative size = 44

$$\int \sqrt{a-a\cosh(x)}(A+B\cosh(x))\,dx = -\frac{2a(3A-B)\sinh(x)}{3\sqrt{a-a\cosh(x)}}+\frac{2}{3}B\sqrt{a-a\cosh(x)}\sinh(x)$$

output
```
-2/3*a*(3*A-B)*sinh(x)/(a-a*cosh(x))^(1/2)+2/3*B*sinh(x)*(a-a*cosh(x))^(1/
2)
```

3.92.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int \sqrt{a-a\cosh(x)}(A+B\cosh(x))\,dx = \frac{2}{3}\sqrt{a-a\cosh(x)}(3A-2B+B\cosh(x))\coth\left(\frac{x}{2}\right)$$

input
```
Integrate[Sqrt[a - a*Cosh[x]]*(A + B*Cosh[x]),x]
```

output
```
(2*Sqrt[a - a*Cosh[x]]*(3*A - 2*B + B*Cosh[x])*Coth[x/2])/3
```

3.92.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3042, 3230, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-a\cosh(x)}(A+B\cosh(x))\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sqrt{a-a\sin\left(\frac{\pi}{2}+ix\right)}\left(A+B\sin\left(\frac{\pi}{2}+ix\right)\right)dx$$

$$\Big\downarrow \ 3230$$

$$\frac{1}{3}(3A-B)\int \sqrt{a-a\cosh(x)}dx+\frac{2}{3}B\sinh(x)\sqrt{a-a\cosh(x)}$$

$$\Big\downarrow \ 3042$$

$$\frac{2}{3}B\sinh(x)\sqrt{a-a\cosh(x)}+\frac{1}{3}(3A-B)\int \sqrt{a-a\sin\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \ 3125$$

$$\frac{2}{3}B\sinh(x)\sqrt{a-a\cosh(x)}-\frac{2a(3A-B)\sinh(x)}{3\sqrt{a-a\cosh(x)}}$$

input
```
Int[Sqrt[a - a*Cosh[x]]*(A + B*Cosh[x]),x]
```

output
```
(-2*a*(3*A - B)*Sinh[x])/(3*Sqrt[a - a*Cosh[x]]) + (2*B*Sqrt[a - a*Cosh[x]
]*Sinh[x])/3
```

3.92.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

3.92.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{4 \sinh\left(\frac{x}{2}\right) a \cosh\left(\frac{x}{2}\right) \left(2 B \cosh\left(\frac{x}{2}\right)^{2}+3 A-3 B\right)}{3 \sqrt{-2 \sinh\left(\frac{x}{2}\right)^{2} a}}$	39
parts	$-\frac{4 A \sinh\left(\frac{x}{2}\right) a \cosh\left(\frac{x}{2}\right)}{\sqrt{-2 \sinh\left(\frac{x}{2}\right)^{2} a}}-\frac{4 B \sinh\left(\frac{x}{2}\right) a \cosh\left(\frac{x}{2}\right) \left(2 \cosh\left(\frac{x}{2}\right)^{2}-3\right)}{3 \sqrt{-2 \sinh\left(\frac{x}{2}\right)^{2} a}}$	58

input
```
int((a-a*cosh(x))^(1/2)*(A+B*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-4/3*sinh(1/2*x)*a*cosh(1/2*x)*(2*B*cosh(1/2*x)^2+3*A-3*B)/(-2*sinh(1/2*x)
^2*a)^(1/2)
```

3.92.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(36) = 72.

Time = 0.26 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.43

$$\int \sqrt{a-a\cosh(x)}(A+B\cosh(x))\,dx$$
$$=\frac{\sqrt{\frac{1}{2}}\left(B\cosh\left(x\right)^{3}+B\sinh\left(x\right)^{3}+3\left(2\,A-B\right)\cosh\left(x\right)^{2}+3\left(B\cosh\left(x\right)+2\,A-B\right)\sinh\left(x\right)^{2}+3\left(2\,A-B\right.}{3\left(\cosh\left(x\right)+\sinh\right.}$$

input
```
integrate((a-a*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

output
```
1/3*sqrt(1/2)*(B*cosh(x)^3 + B*sinh(x)^3 + 3*(2*A - B)*cosh(x)^2 + 3*(B*co
sh(x) + 2*A - B)*sinh(x)^2 + 3*(2*A - B)*cosh(x) + 3*(B*cosh(x)^2 + 2*(2*A
 - B)*cosh(x) + 2*A - B)*sinh(x) + B)*sqrt(-a/(cosh(x) + sinh(x)))/(cosh(x
) + sinh(x))
```

3.92.6 Sympy [F]

$$\int \sqrt{a-a\cosh(x)}(A+B\cosh(x))\,dx=\int \sqrt{-a\left(\cosh\left(x\right)-1\right)}(A+B\cosh\left(x\right))\;dx$$

input
```
integrate((a-a*cosh(x))**(1/2)*(A+B*cosh(x)),x)
```

output
```
Integral(sqrt(-a*(cosh(x) - 1))*(A + B*cosh(x)), x)
```

3.92.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(36) = 72$.

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.48

$$\begin{aligned}&\int \sqrt{a-a\cosh(x)}(A+B\cosh(x))\,dx\\&=-\left(\frac{\sqrt{2}\sqrt{a}e^{(-x)}}{\sqrt{-e^{(-x)}}}+\frac{\sqrt{2}\sqrt{a}}{\sqrt{-e^{(-x)}}}\right)A\\&\quad+\frac{1}{6}\left(\frac{\left(3\sqrt{2}\sqrt{a}e^{(-x)}-\sqrt{2}\sqrt{a}\right)e^{x}}{\sqrt{-e^{(-x)}}}+\frac{3\sqrt{2}\sqrt{a}e^{(-x)}-\sqrt{2}\sqrt{a}e^{(-2x)}}{\sqrt{-e^{(-x)}}}\right)B\end{aligned}$$

input
```
integrate((a-a*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

output
```
-(sqrt(2)*sqrt(a)*e^(-x)/sqrt(-e^(-x)) + sqrt(2)*sqrt(a)/sqrt(-e^(-x)))*A
+ 1/6*((3*sqrt(2)*sqrt(a)*e^(-x) - sqrt(2)*sqrt(a))*e^x/sqrt(-e^(-x)) + (3
*sqrt(2)*sqrt(a)*e^(-x) - sqrt(2)*sqrt(a)*e^(-2*x))/sqrt(-e^(-x)))*B
```

3.92.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(36) = 72$.

Time = 0.26 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.98

$$\int \sqrt{a - a\cosh(x)}(A + B\cosh(x))\,dx = \frac{1}{6}\sqrt{2}\left(\frac{(6\,Aa^2e^x\mathrm{sgn}(-e^x+1) - 3\,Ba^2e^x\mathrm{sgn}(-e^x+1) + Ba^2\mathrm{sgn}(-e^x+1))e^{(-x)}}{\sqrt{-ae^x}a} - \frac{\sqrt{-ae^x}Ba^3e^x\mathrm{sgn}(-e^x$$

input
```
integrate((a-a*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="giac")
```

output
```
1/6*sqrt(2)*((6*A*a^2*e^x*sgn(-e^x + 1) - 3*B*a^2*e^x*sgn(-e^x + 1) + B*a^
2*sgn(-e^x + 1))*e^(-x)/(sqrt(-a*e^x)*a) - (sqrt(-a*e^x)*B*a^3*e^x*sgn(-e^
x + 1) + 6*sqrt(-a*e^x)*A*a^3*sgn(-e^x + 1) - 3*sqrt(-a*e^x)*B*a^3*sgn(-e^
x + 1))/a^3)
```

3.92.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a - a\cosh(x)}(A + B\cosh(x))\,dx = \int (A + B\cosh(x))\ \sqrt{a - a\cosh(x)}\,dx$$

input
```
int((A + B*cosh(x))*(a - a*cosh(x))^(1/2),x)
```

output
```
int((A + B*cosh(x))*(a - a*cosh(x))^(1/2), x)
```

3.93 $\int \frac{A+B\cosh(x)}{1+\cosh(x)}\,dx$

3.93.1 Optimal result

Integrand size = 13, antiderivative size = 18

$$\int \frac{A+B\cosh(x)}{1+\cosh(x)}\,dx = Bx + \frac{(A-B)\sinh(x)}{1+\cosh(x)}$$

output
```
B*x+(A-B)*sinh(x)/(1+cosh(x))
```

3.93.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int \frac{A+B\cosh(x)}{1+\cosh(x)}\,dx = \sinh(x)\left(\frac{A-B}{1+\cosh(x)} - \frac{B\arcsin(\cosh(x))}{\sqrt{-\sinh^2(x)}}\right)$$

input
```
Integrate[(A + B*Cosh[x])/(1 + Cosh[x]),x]
```

output
```
Sinh[x]*((A - B)/(1 + Cosh[x]) - (B*ArcSin[Cosh[x]])/Sqrt[-Sinh[x]^2])
```

3.93.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{\cosh(x)+1}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{1+\sin\left(\frac{\pi}{2}+ix\right)}dx$$
$$\Big\downarrow \text{3214}$$
$$(A-B)\int \frac{1}{\cosh(x)+1}dx+Bx$$
$$\Big\downarrow \text{3042}$$
$$Bx+(A-B)\int \frac{1}{\sin\left(ix+\frac{\pi}{2}\right)+1}dx$$
$$\Big\downarrow \text{3127}$$
$$\frac{(A-B)\sinh(x)}{\cosh(x)+1}+Bx$$

input
```
Int[(A + B*Cosh[x])/(1 + Cosh[x]),x]
```

output
```
B*x + ((A - B)*Sinh[x])/(1 + Cosh[x])
```

3.93.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.93.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
parallelrisch	$Bx + \tanh\left(\frac{x}{2}\right)(A - B)$	15
risch	$Bx - \frac{2A}{e^{x}+1} + \frac{2B}{e^{x}+1}$	23
default	$A\tanh\left(\frac{x}{2}\right) - B\tanh\left(\frac{x}{2}\right) - B\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right) + B\ln\left(\tanh\left(\frac{x}{2}\right) + 1\right)$	34

input
```
int((A+B*cosh(x))/(cosh(x)+1),x,method=_RETURNVERBOSE)
```

output
```
B*x+tanh(1/2*x)*(A-B)
```

3.93.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.61

$$\int \frac{A + B\cosh(x)}{1 + \cosh(x)}\,dx = \frac{Bx\cosh(x) + Bx\sinh(x) + Bx - 2A + 2B}{\cosh(x) + \sinh(x) + 1}$$

input
```
integrate((A+B*cosh(x))/(1+cosh(x)),x, algorithm="fricas")
```

output
```
(B*x*cosh(x) + B*x*sinh(x) + B*x - 2*A + 2*B)/(cosh(x) + sinh(x) + 1)
```

3.93.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{A + B\cosh(x)}{1 + \cosh(x)}\,dx = A\tanh\left(\frac{x}{2}\right) + Bx - B\tanh\left(\frac{x}{2}\right)$$

input
```
integrate((A+B*cosh(x))/(1+cosh(x)),x)
```

output
```
A*tanh(x/2) + B*x - B*tanh(x/2)
```

3.93.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44

$$\int \frac{A + B\cosh(x)}{1 + \cosh(x)}\,dx = B\left(x - \frac{2}{e^{(-x)} + 1}\right) + \frac{2A}{e^{(-x)} + 1}$$

input
```
integrate((A+B*cosh(x))/(1+cosh(x)),x, algorithm="maxima")
```

output
```
B*(x - 2/(e^(-x) + 1)) + 2*A/(e^(-x) + 1)
```

3.93.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{A + B\cosh(x)}{1 + \cosh(x)}\,dx = Bx - \frac{2(A - B)}{e^x + 1}$$

input
```
integrate((A+B*cosh(x))/(1+cosh(x)),x, algorithm="giac")
```

output
```
B*x - 2*(A - B)/(e^x + 1)
```

3.93.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{A+B\cosh(x)}{1+\cosh(x)}\,dx = B\,x - \frac{2\,A-2\,B}{\mathrm{e}^x+1}$$

input
```
int((A + B*cosh(x))/(cosh(x) + 1),x)
```

output
```
B*x - (2*A - 2*B)/(exp(x) + 1)
```

3.94 $\int \frac{A+B\cosh(x)}{(1+\cosh(x))^2}\,dx$

3.94.1 Optimal result

Integrand size = 13, antiderivative size = 35

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^2}\,dx = \frac{(A-B)\sinh(x)}{3(1+\cosh(x))^2} + \frac{(A+2B)\sinh(x)}{3(1+\cosh(x))}$$

output
```
1/3*(A-B)*sinh(x)/(1+cosh(x))^2+1/3*(A+2*B)*sinh(x)/(1+cosh(x))
```

3.94.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^2}\,dx = \frac{(2A+B+(A+2B)\cosh(x))\sinh(x)}{3(1+\cosh(x))^2}$$

input
```
Integrate[(A + B*Cosh[x])/(1 + Cosh[x])^2,x]
```

output
```
((2*A + B + (A + 2*B)*Cosh[x])*Sinh[x])/(3*(1 + Cosh[x])^2)
```

3.94.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3229, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{(\cosh(x)+1)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{\left(1+\sin\left(\frac{\pi}{2}+ix\right)\right)^2}dx$$

$$\Big\downarrow \text{3229}$$

$$\frac{1}{3}(A+2B)\int \frac{1}{\cosh(x)+1}dx+\frac{(A-B)\sinh(x)}{3(\cosh(x)+1)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-B)\sinh(x)}{3(\cosh(x)+1)^2}+\frac{1}{3}(A+2B)\int \frac{1}{\sin\left(ix+\frac{\pi}{2}\right)+1}dx$$

$$\Big\downarrow \text{3127}$$

$$\frac{(A+2B)\sinh(x)}{3(\cosh(x)+1)}+\frac{(A-B)\sinh(x)}{3(\cosh(x)+1)^2}$$

input
```
Int[(A + B*Cosh[x])/(1 + Cosh[x])^2,x]
```

output
```
((A - B)*Sinh[x])/(3*(1 + Cosh[x])^2) + ((A + 2*B)*Sinh[x])/(3*(1 + Cosh[x
]))
```

3.94.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

3.94.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

method	result	size
risch	$-\frac{2(3B\,e^{2x}+3A\,e^{x}+3B\,e^{x}+A+2B)}{3(e^{x}+1)^{3}}$	31
default	$-\frac{A\tanh\left(\frac{x}{2}\right)^{3}}{6}+\frac{B\tanh\left(\frac{x}{2}\right)^{3}}{6}+\frac{A\tanh\left(\frac{x}{2}\right)}{2}+\frac{B\tanh\left(\frac{x}{2}\right)}{2}$	34
parallelrisch	$\frac{\operatorname{csch}(x)^{3}\left(\frac{(A+2B)\cosh(3x)}{6}-B\cosh(2x)+\frac{(-3A+2B)\cosh(x)}{2}+\frac{4A}{3}-\frac{B}{3}\right)}{2}$	43

input
```
int((A+B*cosh(x))/(cosh(x)+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(3*B*exp(x)^2+3*A*exp(x)+3*B*exp(x)+A+2*B)/(exp(x)+1)^3
```

3.94.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.43

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^2}\,dx = -\frac{2\,((A+5\,B)\cosh{(x)}-(A-B)\sinh{(x)}+3\,A+3\,B)}{3\,\left(\cosh{(x)}^2+2\,(\cosh{(x)}+1)\sinh{(x)}+\sinh{(x)}^2+4\,\cosh{(x)}+3\right)}$$

input
```
integrate((A+B*cosh(x))/(1+cosh(x))^2,x, algorithm="fricas")
```

output
```
-2/3*((A + 5*B)*cosh(x) - (A - B)*sinh(x) + 3*A + 3*B)/(cosh(x)^2 + 2*(cos
h(x) + 1)*sinh(x) + sinh(x)^2 + 4*cosh(x) + 3)
```

3.94.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^2}\,dx = -\frac{A\tanh^3\left(\frac{x}{2}\right)}{6}+\frac{A\tanh\left(\frac{x}{2}\right)}{2}+\frac{B\tanh^3\left(\frac{x}{2}\right)}{6}+\frac{B\tanh\left(\frac{x}{2}\right)}{2}$$

input
```
integrate((A+B*cosh(x))/(1+cosh(x))**2,x)
```

output
```
-A*tanh(x/2)**3/6 + A*tanh(x/2)/2 + B*tanh(x/2)**3/6 + B*tanh(x/2)/2
```

3.94.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(31) = 62.

Time = 0.20 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.69

$$\begin{aligned}\int \frac{A+B\cosh(x)}{(1+\cosh(x))^2}\,dx &= \frac{2}{3}\,B\left(\frac{3\,e^{(-x)}}{3\,e^{(-x)}+3\,e^{(-2\,x)}+e^{(-3\,x)}+1}+\frac{3\,e^{(-2\,x)}}{3\,e^{(-x)}+3\,e^{(-2\,x)}+e^{(-3\,x)}+1}+\frac{2}{3\,e^{(-x)}+3\,e^{(-2\,x)}+e^{(-3\,x)}+1}\right)\\ &\quad+\frac{2}{3}\,A\left(\frac{3\,e^{(-x)}}{3\,e^{(-x)}+3\,e^{(-2\,x)}+e^{(-3\,x)}+1}+\frac{1}{3\,e^{(-x)}+3\,e^{(-2\,x)}+e^{(-3\,x)}+1}\right)\end{aligned}$$

input
```
integrate((A+B*cosh(x))/(1+cosh(x))^2,x, algorithm="maxima")
```

output
```
2/3*B*(3*e^(-x)/(3*e^(-x) + 3*e^(-2*x) + e^(-3*x) + 1) + 3*e^(-2*x)/(3*e^(
-x) + 3*e^(-2*x) + e^(-3*x) + 1) + 2/(3*e^(-x) + 3*e^(-2*x) + e^(-3*x) + 1
)) + 2/3*A*(3*e^(-x)/(3*e^(-x) + 3*e^(-2*x) + e^(-3*x) + 1) + 1/(3*e^(-x)
+ 3*e^(-2*x) + e^(-3*x) + 1))
```

3.94.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^2}\,dx = -\frac{2\left(3\,Be^{(2\,x)}+3\,Ae^{x}+3\,Be^{x}+A+2\,B\right)}{3\,(e^{x}+1)^{3}}$$

input
```
integrate((A+B*cosh(x))/(1+cosh(x))^2,x, algorithm="giac")
```

output
```
-2/3*(3*B*e^(2*x) + 3*A*e^x + 3*B*e^x + A + 2*B)/(e^x + 1)^3
```

3.94.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^2}\,dx = -\frac{2\,(A+2\,B+3\,A\,\mathrm{e}^{x}+3\,B\,\mathrm{e}^{x}+3\,B\,\mathrm{e}^{2\,x})}{3\,(\mathrm{e}^{x}+1)^{3}}$$

input
```
int((A + B*cosh(x))/(cosh(x) + 1)^2,x)
```

output
```
-(2*(A + 2*B + 3*A*exp(x) + 3*B*exp(x) + 3*B*exp(2*x)))/(3*(exp(x) + 1)^3)
```

3.95 $\int \frac{A+B\cosh(x)}{(1+\cosh(x))^3}\,dx$

3.95.1 Optimal result

Integrand size = 13, antiderivative size = 56

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^3}\,dx = \frac{(A-B)\sinh(x)}{5(1+\cosh(x))^3} + \frac{(2A+3B)\sinh(x)}{15(1+\cosh(x))^2} + \frac{(2A+3B)\sinh(x)}{15(1+\cosh(x))}$$

output
```
1/5*(A-B)*sinh(x)/(1+cosh(x))^3+1/15*(2*A+3*B)*sinh(x)/(1+cosh(x))^2+1/15*
(2*A+3*B)*sinh(x)/(1+cosh(x))
```

3.95.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.73

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^3}\,dx = \frac{(7A+3B+(6A+9B)\cosh(x)+(2A+3B)\cosh^2(x))\sinh(x)}{15(1+\cosh(x))^3}$$

input
```
Integrate[(A + B*Cosh[x])/(1 + Cosh[x])^3,x]
```

output
```
((7*A + 3*B + (6*A + 9*B)*Cosh[x] + (2*A + 3*B)*Cosh[x]^2)*Sinh[x])/(15*(1
 + Cosh[x])^3)
```

3.95.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3042, 3229, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{(\cosh(x)+1)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{\left(1+\sin\left(\frac{\pi}{2}+ix\right)\right)^3}dx$$

$$\Big\downarrow \text{3229}$$

$$\frac{1}{5}(2A+3B)\int \frac{1}{(\cosh(x)+1)^2}dx+\frac{(A-B)\sinh(x)}{5(\cosh(x)+1)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-B)\sinh(x)}{5(\cosh(x)+1)^3}+\frac{1}{5}(2A+3B)\int \frac{1}{\left(\sin\left(ix+\frac{\pi}{2}\right)+1\right)^2}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{1}{5}(2A+3B)\left(\frac{1}{3}\int \frac{1}{\cosh(x)+1}dx+\frac{\sinh(x)}{3(\cosh(x)+1)^2}\right)+\frac{(A-B)\sinh(x)}{5(\cosh(x)+1)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-B)\sinh(x)}{5(\cosh(x)+1)^3}+\frac{1}{5}(2A+3B)\left(\frac{\sinh(x)}{3(\cosh(x)+1)^2}+\frac{1}{3}\int \frac{1}{\sin\left(ix+\frac{\pi}{2}\right)+1}dx\right)$$

$$\Big\downarrow \text{3127}$$

$$\frac{(A-B)\sinh(x)}{5(\cosh(x)+1)^3}+\frac{1}{5}(2A+3B)\left(\frac{\sinh(x)}{3(\cosh(x)+1)}+\frac{\sinh(x)}{3(\cosh(x)+1)^2}\right)$$

input
```
Int[(A + B*Cosh[x])/(1 + Cosh[x])^3,x]
```

output
```
((A - B)*Sinh[x])/(5*(1 + Cosh[x])^3) + ((2*A + 3*B)*(Sinh[x]/(3*(1 + Cosh
[x])^2) + Sinh[x]/(3*(1 + Cosh[x]))))/5
```

3.95.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

3.95.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.62

method	result	size
parallelrisch	$\frac{\tanh\left(\frac{x}{2}\right)\left((A-B)\tanh\left(\frac{x}{2}\right)^{4}-\frac{10A\tanh\left(\frac{x}{2}\right)^{2}}{3}+5A+5B\right)}{20}$	35
default	$\frac{(A-B)\tanh\left(\frac{x}{2}\right)^{5}}{20}-\frac{A\tanh\left(\frac{x}{2}\right)^{3}}{6}+\frac{A\tanh\left(\frac{x}{2}\right)}{4}+\frac{B\tanh\left(\frac{x}{2}\right)}{4}$	38
risch	$-\frac{2\left(15B\,e^{3x}+20A\,e^{2x}+15B\,e^{2x}+10A\,e^{x}+15B\,e^{x}+2A+3B\right)}{15\left(e^{x}+1\right)^{5}}$	47

input
```
int((A+B*cosh(x))/(cosh(x)+1)^3,x,method=_RETURNVERBOSE)
```

output
```
1/20*tanh(1/2*x)*((A-B)*tanh(1/2*x)^4-10/3*A*tanh(1/2*x)^2+5*A+5*B)
```

3.95.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(50) = 100$.

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.27

$$\int \frac{A + B\cosh(x)}{(1+\cosh(x))^3}\, dx =$$
$$-\frac{2\left(15\,B\cosh\left(x\right)^{2}+15\,B\sinh\left(x\right)^{2}+2\left(11\,A+9\,B\right)\cosh\left(x\right.\cdots}{15\left(\cosh\left(x\right)^{4}+\left(4\,\cosh\left(x\right)+5\right)\sinh\left(x\right)^{3}+\sinh\left(x\right)^{4}+5\,\cosh\left(x\right)^{3}+\left(6\,\cosh\left(x\right)^{2}+15\,\cosh\left(x\right)+10\right)\cdots}$$

input
```
integrate((A+B*cosh(x))/(1+cosh(x))^3,x, algorithm="fricas")
```

output
```
-2/15*(15*B*cosh(x)^2 + 15*B*sinh(x)^2 + 2*(11*A + 9*B)*cosh(x) + 6*(5*B*c
osh(x) + 3*A + 2*B)*sinh(x) + 10*A + 15*B)/(cosh(x)^4 + (4*cosh(x) + 5)*si
nh(x)^3 + sinh(x)^4 + 5*cosh(x)^3 + (6*cosh(x)^2 + 15*cosh(x) + 10)*sinh(x
)^2 + 10*cosh(x)^2 + (4*cosh(x)^3 + 15*cosh(x)^2 + 20*cosh(x) + 9)*sinh(x)
 + 11*cosh(x) + 5)
```

3.95.6 Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \frac{A + B\cosh(x)}{(1+\cosh(x))^3}\, dx = \frac{A\tanh^{5}\left(\frac{x}{2}\right)}{20} - \frac{A\tanh^{3}\left(\frac{x}{2}\right)}{6} + \frac{A\tanh\left(\frac{x}{2}\right)}{4} - \frac{B\tanh^{5}\left(\frac{x}{2}\right)}{20} + \frac{B\tanh\left(\frac{x}{2}\right)}{4}$$

input
```
integrate((A+B*cosh(x))/(1+cosh(x))**3,x)
```

output
```
A*tanh(x/2)**5/20 - A*tanh(x/2)**3/6 + A*tanh(x/2)/4 - B*tanh(x/2)**5/20 +
 B*tanh(x/2)/4
```

3.95.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. 2(50) = 100.

Time = 0.19 (sec) , antiderivative size = 263, normalized size of antiderivative = 4.70

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^3}\,dx$$
$$= \frac{4}{15}A\left(\frac{5\,e^{(-x)}}{5\,e^{(-x)}+10\,e^{(-2\,x)}+10\,e^{(-3\,x)}+5\,e^{(-4\,x)}+e^{(-5\,x)}+1}+\frac{10\,e^{(-2\,x)}}{5\,e^{(-x)}+10\,e^{(-2\,x)}+10\,e^{(-3\,x)}+5\,e^{(-4\,x)}+\right.$$
$$+\frac{2}{5}B\left(\frac{5\,e^{(-x)}}{5\,e^{(-x)}+10\,e^{(-2\,x)}+10\,e^{(-3\,x)}+5\,e^{(-4\,x)}+e^{(-5\,x)}+1}+\frac{5\,e^{(-2\,x)}}{5\,e^{(-x)}+10\,e^{(-2\,x)}+10\,e^{(-3\,x)}+5\,e^{(-4\,x)}\right.$$

input
```
integrate((A+B*cosh(x))/(1+cosh(x))^3,x, algorithm="maxima")
```

output
```
4/15*A*(5*e^(-x)/(5*e^(-x) + 10*e^(-2*x) + 10*e^(-3*x) + 5*e^(-4*x) + e^(-
5*x) + 1) + 10*e^(-2*x)/(5*e^(-x) + 10*e^(-2*x) + 10*e^(-3*x) + 5*e^(-4*x)
 + e^(-5*x) + 1) + 1/(5*e^(-x) + 10*e^(-2*x) + 10*e^(-3*x) + 5*e^(-4*x) +
e^(-5*x) + 1)) + 2/5*B*(5*e^(-x)/(5*e^(-x) + 10*e^(-2*x) + 10*e^(-3*x) + 5
*e^(-4*x) + e^(-5*x) + 1) + 5*e^(-2*x)/(5*e^(-x) + 10*e^(-2*x) + 10*e^(-3*
x) + 5*e^(-4*x) + e^(-5*x) + 1) + 5*e^(-3*x)/(5*e^(-x) + 10*e^(-2*x) + 10*
e^(-3*x) + 5*e^(-4*x) + e^(-5*x) + 1) + 1/(5*e^(-x) + 10*e^(-2*x) + 10*e^(
-3*x) + 5*e^(-4*x) + e^(-5*x) + 1))
```

3.95.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^3}\,dx = -\frac{2\left(15\,Be^{(3\,x)}+20\,Ae^{(2\,x)}+15\,Be^{(2\,x)}+10\,Ae^{x}+15\,Be^{x}+2\,A+3\,B\right)}{15\,(e^{x}+1)^{5}}$$

input
```
integrate((A+B*cosh(x))/(1+cosh(x))^3,x, algorithm="giac")
```

output
```
-2/15*(15*B*e^(3*x) + 20*A*e^(2*x) + 15*B*e^(2*x) + 10*A*e^x + 15*B*e^x +
2*A + 3*B)/(e^x + 1)^5
```

3.95.9 Mupad [B] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.52

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^3}\,dx = -\frac{\frac{4\,B\,e^{x}}{5}+\frac{8\,A\,e^{2\,x}}{5}+\frac{4\,B\,e^{3\,x}}{5}}{10\,e^{2\,x}+10\,e^{3\,x}+5\,e^{4\,x}+e^{5\,x}+5\,e^{x}+1} - \frac{\frac{B}{5}+\frac{4\,A\,e^{x}}{5}+\frac{3\,B\,e^{2\,x}}{5}}{6\,e^{2\,x}+4\,e^{3\,x}+e^{4\,x}+4\,e^{x}+1} - \frac{\frac{4\,A}{15}+\frac{2\,B\,e^{x}}{5}}{3\,e^{2\,x}+e^{3\,x}+3\,e^{x}+1} - \frac{B}{5\,\left(e^{2\,x}+2\,e^{x}+1\right)}$$

input
```
int((A + B*cosh(x))/(cosh(x) + 1)^3,x)
```

output
```
- ((4*B*exp(x))/5 + (8*A*exp(2*x))/5 + (4*B*exp(3*x))/5)/(10*exp(2*x) + 10
*exp(3*x) + 5*exp(4*x) + exp(5*x) + 5*exp(x) + 1) - (B/5 + (4*A*exp(x))/5
+ (3*B*exp(2*x))/5)/(6*exp(2*x) + 4*exp(3*x) + exp(4*x) + 4*exp(x) + 1) -
((4*A)/15 + (2*B*exp(x))/5)/(3*exp(2*x) + exp(3*x) + 3*exp(x) + 1) - B/(5*
(exp(2*x) + 2*exp(x) + 1))
```

3.96 $\int \frac{A+B\cosh(x)}{(1+\cosh(x))^4}\,dx$

3.96.1 Optimal result

Integrand size = 13, antiderivative size = 75

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^4}\,dx = \frac{(A-B)\sinh(x)}{7(1+\cosh(x))^4}+\frac{(3A+4B)\sinh(x)}{35(1+\cosh(x))^3} + \frac{2(3A+4B)\sinh(x)}{105(1+\cosh(x))^2}+\frac{2(3A+4B)\sinh(x)}{105(1+\cosh(x))}$$

output
```
1/7*(A-B)*sinh(x)/(1+cosh(x))^4+1/35*(3*A+4*B)*sinh(x)/(1+cosh(x))^3+2/105
*(3*A+4*B)*sinh(x)/(1+cosh(x))^2+2/105*(3*A+4*B)*sinh(x)/(1+cosh(x))
```

3.96.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.73

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^4}\,dx = \frac{(36A+13B+13(3A+4B)\cosh(x)+8(3A+4B)\cosh^2(x)+(6A+8B)\cosh^3(x))\sinh(x)}{105(1+\cosh(x))^4}$$

input
```
Integrate[(A + B*Cosh[x])/(1 + Cosh[x])^4,x]
```

output
```
((36*A + 13*B + 13*(3*A + 4*B)*Cosh[x] + 8*(3*A + 4*B)*Cosh[x]^2 + (6*A +
8*B)*Cosh[x]^3)*Sinh[x])/(105*(1 + Cosh[x])^4)
```

3.96.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {3042, 3229, 3042, 3129, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{(\cosh(x)+1)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{\left(1+\sin\left(\frac{\pi}{2}+ix\right)\right)^4}dx$$

$$\Big\downarrow \text{3229}$$

$$\frac{1}{7}(3A+4B)\int \frac{1}{(\cosh(x)+1)^3}dx+\frac{(A-B)\sinh(x)}{7(\cosh(x)+1)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-B)\sinh(x)}{7(\cosh(x)+1)^4}+\frac{1}{7}(3A+4B)\int \frac{1}{\left(\sin\left(ix+\frac{\pi}{2}\right)+1\right)^3}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{1}{7}(3A+4B)\left(\frac{2}{5}\int \frac{1}{(\cosh(x)+1)^2}dx+\frac{\sinh(x)}{5(\cosh(x)+1)^3}\right)+\frac{(A-B)\sinh(x)}{7(\cosh(x)+1)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-B)\sinh(x)}{7(\cosh(x)+1)^4}+\frac{1}{7}(3A+4B)\left(\frac{\sinh(x)}{5(\cosh(x)+1)^3}+\frac{2}{5}\int \frac{1}{\left(\sin\left(ix+\frac{\pi}{2}\right)+1\right)^2}dx\right)$$

$$\Big\downarrow \text{3129}$$

$$\frac{1}{7}(3A+4B)\left(\frac{2}{5}\left(\frac{1}{3}\int \frac{1}{\cosh(x)+1}dx+\frac{\sinh(x)}{3(\cosh(x)+1)^2}\right)+\frac{\sinh(x)}{5(\cosh(x)+1)^3}\right)+\frac{(A-B)\sinh(x)}{7(\cosh(x)+1)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-B)\sinh(x)}{7(\cosh(x)+1)^4}+\frac{1}{7}(3A+4B)\left(\frac{\sinh(x)}{5(\cosh(x)+1)^3}+\frac{2}{5}\left(\frac{\sinh(x)}{3(\cosh(x)+1)^2}+\frac{1}{3}\int \frac{1}{\sin\left(ix+\frac{\pi}{2}\right)+1}dx\right)\right)$$

$$\Big\downarrow \text{3127}$$

$$\frac{(A-B)\sinh(x)}{7(\cosh(x)+1)^4}+\frac{1}{7}(3A+4B)\left(\frac{\sinh(x)}{5(\cosh(x)+1)^3}+\frac{2}{5}\left(\frac{\sinh(x)}{3(\cosh(x)+1)}+\frac{\sinh(x)}{3(\cosh(x)+1)^2}\right)\right)$$

input
```
Int[(A + B*Cosh[x])/(1 + Cosh[x])^4,x]
```

output
```
((A - B)*Sinh[x])/(7*(1 + Cosh[x])^4) + ((3*A + 4*B)*(Sinh[x]/(5*(1 + Cosh
[x])^3) + (2*(Sinh[x]/(3*(1 + Cosh[x])^2) + Sinh[x]/(3*(1 + Cosh[x]))))/5)
)/7
```

3.96.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

3.96.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.73

method	result
default	$-\frac{(A-B)\tanh\left(\frac{x}{2}\right)^{7}}{56}-\frac{(-3A+B)\tanh\left(\frac{x}{2}\right)^{5}}{40}-\frac{(3A+B)\tanh\left(\frac{x}{2}\right)^{3}}{24}+\frac{A\tanh\left(\frac{x}{2}\right)}{8}+\frac{B\tanh\left(\frac{x}{2}\right)}{8}$
risch	$-\frac{4(70B\,e^{4x}+105A\,e^{3x}+70B\,e^{3x}+63A\,e^{2x}+84B\,e^{2x}+21A\,e^{x}+28B\,e^{x}+3A+4B)}{105(e^{x}+1)^{7}}$
parallelrisch	$\frac{\left(\left(448A-\frac{2128B}{3}\right)\cosh(2x)+(-49A+308B)\cosh(3x)+\left(-7A-\frac{28B}{3}\right)\cosh(5x)+\left(A+\frac{4B}{3}\right)\cosh(7x)-\frac{140B\cosh(4x)}{3}+(-1225A+}{1120}$

input
```
int((A+B*cosh(x))/(cosh(x)+1)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/56*(A-B)*tanh(1/2*x)^7-1/40*(-3*A+B)*tanh(1/2*x)^5-1/24*(3*A+B)*tanh(1/
2*x)^3+1/8*A*tanh(1/2*x)+1/8*B*tanh(1/2*x)
```

3.96.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(67) = 134$.

Time = 0.24 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.33

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^4}\,dx =$$

$$-\frac{4\left((3A+74B)\cosh(x\right.}{105\left(\cosh(x)^5+(5\cosh(x)+7)\sinh(x)^4+\sinh(x)^5+7\cosh(x)^4+\left(10\cosh(x)^2+28\cosh(x)+21\right.\right.}$$

input
```
integrate((A+B*cosh(x))/(1+cosh(x))^4,x, algorithm="fricas")
```

output
```
-4/105*((3*A + 74*B)*cosh(x)^2 + (3*A + 74*B)*sinh(x)^2 + 14*(9*A + 7*B)*c
osh(x) - 6*((A - 22*B)*cosh(x) - 14*A - 7*B)*sinh(x) + 63*A + 84*B)/(cosh(
x)^5 + (5*cosh(x) + 7)*sinh(x)^4 + sinh(x)^5 + 7*cosh(x)^4 + (10*cosh(x)^2
 + 28*cosh(x) + 21)*sinh(x)^3 + 21*cosh(x)^3 + (10*cosh(x)^3 + 42*cosh(x)^
2 + 63*cosh(x) + 36)*sinh(x)^2 + 36*cosh(x)^2 + (5*cosh(x)^4 + 28*cosh(x)^
3 + 63*cosh(x)^2 + 68*cosh(x) + 28)*sinh(x) + 42*cosh(x) + 21)
```

3.96.6 Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.04

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^4}\,dx = -\frac{A\tanh^7\left(\frac{x}{2}\right)}{56} + \frac{3A\tanh^5\left(\frac{x}{2}\right)}{40} - \frac{A\tanh^3\left(\frac{x}{2}\right)}{8} + \frac{A\tanh\left(\frac{x}{2}\right)}{8} + \frac{B\tanh^7\left(\frac{x}{2}\right)}{56} - \frac{B\tanh^5\left(\frac{x}{2}\right)}{40} - \frac{B\tanh^3\left(\frac{x}{2}\right)}{24} + \frac{B\tanh\left(\frac{x}{2}\right)}{8}$$

input
```
integrate((A+B*cosh(x))/(1+cosh(x))**4,x)
```

output
```
-A*tanh(x/2)**7/56 + 3*A*tanh(x/2)**5/40 - A*tanh(x/2)**3/8 + A*tanh(x/2)/
8 + B*tanh(x/2)**7/56 - B*tanh(x/2)**5/40 - B*tanh(x/2)**3/24 + B*tanh(x/2
)/8
```

3.96.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. 2(67) = 134.

Time = 0.20 (sec) , antiderivative size = 449, normalized size of antiderivative = 5.99

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^4}\,dx$$
$$= \frac{8}{105}B\left(\frac{14\,e^{(-x)}}{7\,e^{(-x)} + 21\,e^{(-2\,x)} + 35\,e^{(-3\,x)} + 35\,e^{(-4\,x)} + 21\,e^{(-5\,x)} + 7\,e^{(-6\,x)} + e^{(-7\,x)} + 1} + \frac{}{7\,e^{(-x)} + 21\,e^{(-2\,x}}\right.$$
$$+ \frac{4}{35}A\left(\frac{7\,e^{(-x)}}{7\,e^{(-x)} + 21\,e^{(-2\,x)} + 35\,e^{(-3\,x)} + 35\,e^{(-4\,x)} + 21\,e^{(-5\,x)} + 7\,e^{(-6\,x)} + e^{(-7\,x)} + 1} + \frac{}{7\,e^{(-x)} + 21\,e^{(-2}}\right.$$

input
```
integrate((A+B*cosh(x))/(1+cosh(x))^4,x, algorithm="maxima")
```

output
```
8/105*B*(14*e^(-x)/(7*e^(-x) + 21*e^(-2*x) + 35*e^(-3*x) + 35*e^(-4*x) + 2
1*e^(-5*x) + 7*e^(-6*x) + e^(-7*x) + 1) + 42*e^(-2*x)/(7*e^(-x) + 21*e^(-2
*x) + 35*e^(-3*x) + 35*e^(-4*x) + 21*e^(-5*x) + 7*e^(-6*x) + e^(-7*x) + 1)
 + 35*e^(-3*x)/(7*e^(-x) + 21*e^(-2*x) + 35*e^(-3*x) + 35*e^(-4*x) + 21*e^
(-5*x) + 7*e^(-6*x) + e^(-7*x) + 1) + 35*e^(-4*x)/(7*e^(-x) + 21*e^(-2*x)
+ 35*e^(-3*x) + 35*e^(-4*x) + 21*e^(-5*x) + 7*e^(-6*x) + e^(-7*x) + 1) + 2
/(7*e^(-x) + 21*e^(-2*x) + 35*e^(-3*x) + 35*e^(-4*x) + 21*e^(-5*x) + 7*e^(
-6*x) + e^(-7*x) + 1)) + 4/35*A*(7*e^(-x)/(7*e^(-x) + 21*e^(-2*x) + 35*e^(
-3*x) + 35*e^(-4*x) + 21*e^(-5*x) + 7*e^(-6*x) + e^(-7*x) + 1) + 21*e^(-2*
x)/(7*e^(-x) + 21*e^(-2*x) + 35*e^(-3*x) + 35*e^(-4*x) + 21*e^(-5*x) + 7*e
^(-6*x) + e^(-7*x) + 1) + 35*e^(-3*x)/(7*e^(-x) + 21*e^(-2*x) + 35*e^(-3*x
) + 35*e^(-4*x) + 21*e^(-5*x) + 7*e^(-6*x) + e^(-7*x) + 1) + 1/(7*e^(-x) +
 21*e^(-2*x) + 35*e^(-3*x) + 35*e^(-4*x) + 21*e^(-5*x) + 7*e^(-6*x) + e^(-
7*x) + 1))
```

3.96.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

$$\int \frac{A + B\cosh(x)}{(1 + \cosh(x))^4} \, dx = -\frac{4\left(70\,Be^{(4\,x)} + 105\,Ae^{(3\,x)} + 70\,Be^{(3\,x)} + 63\,Ae^{(2\,x)} + 84\,Be^{(2\,x)} + 21\,Ae^{x} + 28\,Be^{x} + 3\,A + 4\,B\right)}{105\,(e^{x} + 1)^{7}}$$

input
```
integrate((A+B*cosh(x))/(1+cosh(x))^4,x, algorithm="giac")
```

output
```
-4/105*(70*B*e^(4*x) + 105*A*e^(3*x) + 70*B*e^(3*x) + 63*A*e^(2*x) + 84*B*
e^(2*x) + 21*A*e^x + 28*B*e^x + 3*A + 4*B)/(e^x + 1)^7
```

3.96.9 Mupad [B] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 231, normalized size of antiderivative = 3.08

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^4}\,dx = -\frac{\frac{4A}{35}+\frac{8B\,e^x}{35}}{6\,e^{2x}+4\,e^{3x}+e^{4x}+4\,e^x+1} - \frac{8B}{105\,\left(3\,e^{2x}+e^{3x}+3\,e^x+1\right)} - \frac{\frac{16A\,e^{3x}}{7}+\frac{8B\,e^{2x}}{7}+\frac{8B\,e^{4x}}{7}}{21\,e^{2x}+35\,e^{3x}+35\,e^{4x}+21\,e^{5x}+7\,e^{6x}+e^{7x}+7\,e^x+1} - \frac{\frac{8B\,e^x}{21}+\frac{8A\,e^{2x}}{7}+\frac{16B\,e^{3x}}{21}}{15\,e^{2x}+20\,e^{3x}+15\,e^{4x}+6\,e^{5x}+e^{6x}+6\,e^x+1} - \frac{\frac{8B}{105}+\frac{16A\,e^x}{35}+\frac{16B\,e^{2x}}{35}}{10\,e^{2x}+10\,e^{3x}+5\,e^{4x}+e^{5x}+5\,e^x+1}$$

input
```
int((A + B*cosh(x))/(cosh(x) + 1)^4,x)
```

output
```
- ((4*A)/35 + (8*B*exp(x))/35)/(6*exp(2*x) + 4*exp(3*x) + exp(4*x) + 4*exp
(x) + 1) - (8*B)/(105*(3*exp(2*x) + exp(3*x) + 3*exp(x) + 1)) - ((16*A*exp
(3*x))/7 + (8*B*exp(2*x))/7 + (8*B*exp(4*x))/7)/(21*exp(2*x) + 35*exp(3*x)
 + 35*exp(4*x) + 21*exp(5*x) + 7*exp(6*x) + exp(7*x) + 7*exp(x) + 1) - ((8
*B*exp(x))/21 + (8*A*exp(2*x))/7 + (16*B*exp(3*x))/21)/(15*exp(2*x) + 20*e
xp(3*x) + 15*exp(4*x) + 6*exp(5*x) + exp(6*x) + 6*exp(x) + 1) - ((8*B)/105
 + (16*A*exp(x))/35 + (16*B*exp(2*x))/35)/(10*exp(2*x) + 10*exp(3*x) + 5*e
xp(4*x) + exp(5*x) + 5*exp(x) + 1)
```

3.97 $\int \frac{A+B\cosh(x)}{1-\cosh(x)}\,dx$

3.97.1 Optimal result

Integrand size = 15, antiderivative size = 20

$$\int \frac{A+B\cosh(x)}{1-\cosh(x)}\,dx=-Bx-\frac{(A+B)\sinh(x)}{1-\cosh(x)}$$

output
```
-B*x-(A+B)*sinh(x)/(1-cosh(x))
```

3.97.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 42 vs. 2(20) = 40.

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.10

$$\int \frac{A+B\cosh(x)}{1-\cosh(x)}\,dx=\sinh(x)\left(\frac{A+B}{-1+\cosh(x)}-\frac{2B\arcsin\left(\sqrt{-\sinh^2\left(\frac{x}{2}\right)}\right)}{\sqrt{-\sinh^2(x)}}\right)$$

input
```
Integrate[(A + B*Cosh[x])/(1 - Cosh[x]),x]
```

output
```
Sinh[x]*((A + B)/(-1 + Cosh[x]) - (2*B*ArcSin[Sqrt[-Sinh[x/2]^2]])/Sqrt[-S
inh[x]^2])
```

3.97.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{1-\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{1-\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3214}$$

$$(A+B)\int \frac{1}{1-\cosh(x)}dx-Bx$$

$$\Big\downarrow \text{3042}$$

$$-Bx+(A+B)\int \frac{1}{1-\sin\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3127}$$

$$-\frac{(A+B)\sinh(x)}{1-\cosh(x)}-Bx$$

input
```
Int[(A + B*Cosh[x])/(1 - Cosh[x]),x]
```

output
```
-(B*x) - ((A + B)*Sinh[x])/(1 - Cosh[x])
```

3.97.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.97.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
parallelrisch	$\frac{-x\tanh\left(\frac{x}{2}\right)B+A+B}{\tanh\left(\frac{x}{2}\right)}$	19
risch	$-Bx+\frac{2A}{e^{x}-1}+\frac{2B}{e^{x}-1}$	24
default	$-\frac{-A-B}{\tanh\left(\frac{x}{2}\right)}+B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	36

input
```
int((A+B*cosh(x))/(1-cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
(-x*tanh(1/2*x)*B+A+B)/tanh(1/2*x)
```

3.97.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{A+B\cosh(x)}{1-\cosh(x)}\,dx = -\frac{Bx\cosh\left(x\right)+Bx\sinh\left(x\right)-Bx-2\,A-2\,B}{\cosh\left(x\right)+\sinh\left(x\right)-1}$$

input
```
integrate((A+B*cosh(x))/(1-cosh(x)),x, algorithm="fricas")
```

output
```
-(B*x*cosh(x) + B*x*sinh(x) - B*x - 2*A - 2*B)/(cosh(x) + sinh(x) - 1)
```

3.97.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{A+B\cosh(x)}{1-\cosh(x)}\,dx = \frac{A}{\tanh\left(\frac{x}{2}\right)} - Bx + \frac{B}{\tanh\left(\frac{x}{2}\right)}$$

input
```
integrate((A+B*cosh(x))/(1-cosh(x)),x)
```

output
```
A/tanh(x/2) - B*x + B/tanh(x/2)
```

3.97.7 Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{A+B\cosh(x)}{1-\cosh(x)}\,dx = -B\left(x + \frac{2}{e^{(-x)}-1}\right) - \frac{2A}{e^{(-x)}-1}$$

input
```
integrate((A+B*cosh(x))/(1-cosh(x)),x, algorithm="maxima")
```

output
```
-B*(x + 2/(e^(-x) - 1)) - 2*A/(e^(-x) - 1)
```

3.97.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{A+B\cosh(x)}{1-\cosh(x)}\,dx = -Bx + \frac{2(A+B)}{e^{x}-1}$$

input
```
integrate((A+B*cosh(x))/(1-cosh(x)),x, algorithm="giac")
```

output
```
-B*x + 2*(A + B)/(e^x - 1)
```

3.97.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{A+B\cosh(x)}{1-\cosh(x)}\,dx = \frac{2\,A+2\,B}{\mathrm{e}^x-1}-B\,x$$

input
```
int(-(A + B*cosh(x))/(cosh(x) - 1),x)
```

output
```
(2*A + 2*B)/(exp(x) - 1) - B*x
```

3.98 $\int \frac{A+B\cosh(x)}{(1-\cosh(x))^2}\,dx$

3.98.1 Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^2}\,dx = -\frac{(A+B)\sinh(x)}{3(1-\cosh(x))^2} - \frac{(A-2B)\sinh(x)}{3(1-\cosh(x))}$$

output
```
-1/3*(A+B)*sinh(x)/(1-cosh(x))^2-1/3*(A-2*B)*sinh(x)/(1-cosh(x))
```

3.98.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^2}\,dx = \frac{(-2A+B+(A-2B)\cosh(x))\sinh(x)}{3(-1+\cosh(x))^2}$$

input
```
Integrate[(A + B*Cosh[x])/(1 - Cosh[x])^2,x]
```

output
```
((-2*A + B + (A - 2*B)*Cosh[x])*Sinh[x])/(3*(-1 + Cosh[x])^2)
```

3.98.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3229, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{\left(1-\sin\left(\frac{\pi}{2}+ix\right)\right)^2}dx$$

$$\Big\downarrow \text{ 3229}$$

$$\frac{1}{3}(A-2B)\int \frac{1}{1-\cosh(x)}dx-\frac{(A+B)\sinh(x)}{3(1-\cosh(x))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{(A+B)\sinh(x)}{3(1-\cosh(x))^2}+\frac{1}{3}(A-2B)\int \frac{1}{1-\sin\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3127}$$

$$-\frac{(A-2B)\sinh(x)}{3(1-\cosh(x))}-\frac{(A+B)\sinh(x)}{3(1-\cosh(x))^2}$$

input
```
Int[(A + B*Cosh[x])/(1 - Cosh[x])^2,x]
```

output
```
-1/3*((A + B)*Sinh[x])/(1 - Cosh[x])^2 - ((A - 2*B)*Sinh[x])/(3*(1 - Cosh[
x]))
```

3.98.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

3.98.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

method	result	size
default	$-\frac{-A+B}{2\tanh\left(\frac{x}{2}\right)}-\frac{A+B}{6\tanh\left(\frac{x}{2}\right)^{3}}$	26
risch	$-\frac{2\left(3B\,{\mathrm e}^{2x}+3A\,{\mathrm e}^{x}-3B\,{\mathrm e}^{x}-A+2B\right)}{3\left({\mathrm e}^{x}-1\right)^{3}}$	33
parallelrisch	$\frac{(-A+2B)\sinh(2x)+4\sinh(x)\left(A-\frac{B}{2}\right)}{-3\cosh(2x)-9+12\cosh(x)}$	38

input
```
int((A+B*cosh(x))/(1-cosh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-A+B)/tanh(1/2*x)-1/6*(A+B)/tanh(1/2*x)^3
```

3.98.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.30

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^2}\,dx = \frac{2\left((A-5\,B)\cosh\left(x\right)-(A+B)\sinh\left(x\right)-3\,A+3\,B\right)}{3\left(\cosh\left(x\right)^2+2\left(\cosh\left(x\right)-1\right)\sinh\left(x\right)+\sinh\left(x\right)^2-4\,\cosh\left(x\right)+3\right)}$$

input
```
integrate((A+B*cosh(x))/(1-cosh(x))^2,x, algorithm="fricas")
```

output
```
2/3*((A - 5*B)*cosh(x) - (A + B)*sinh(x) - 3*A + 3*B)/(cosh(x)^2 + 2*(cosh
(x) - 1)*sinh(x) + sinh(x)^2 - 4*cosh(x) + 3)
```

3.98.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^2}\,dx = \frac{A}{2\tanh\left(\frac{x}{2}\right)} - \frac{A}{6\tanh^3\left(\frac{x}{2}\right)} - \frac{B}{2\tanh\left(\frac{x}{2}\right)} - \frac{B}{6\tanh^3\left(\frac{x}{2}\right)}$$

input
```
integrate((A+B*cosh(x))/(1-cosh(x))**2,x)
```

output
```
A/(2*tanh(x/2)) - A/(6*tanh(x/2)**3) - B/(2*tanh(x/2)) - B/(6*tanh(x/2)**3
)
```

3.98.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(29) = 58.

Time = 0.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.54

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^2}\,dx = -\frac{2}{3}\,B\left(\frac{3\,e^{(-x)}}{3\,e^{(-x)}-3\,e^{(-2\,x)}+e^{(-3\,x)}-1} - \frac{3\,e^{(-2\,x)}}{3\,e^{(-x)}-3\,e^{(-2\,x)}+e^{(-3\,x)}-1} - \frac{2}{3\,e^{(-x)}-3\,e^{(-2\,x)}+e^{(-3\,x)}-1}\right. + \frac{2}{3}\,A\left(\frac{3\,e^{(-x)}}{3\,e^{(-x)}-3\,e^{(-2\,x)}+e^{(-3\,x)}-1} - \frac{1}{3\,e^{(-x)}-3\,e^{(-2\,x)}+e^{(-3\,x)}-1}\right)$$

input
```
integrate((A+B*cosh(x))/(1-cosh(x))^2,x, algorithm="maxima")
```

output
```
-2/3*B*(3*e^(-x)/(3*e^(-x) - 3*e^(-2*x) + e^(-3*x) - 1) - 3*e^(-2*x)/(3*e^
(-x) - 3*e^(-2*x) + e^(-3*x) - 1) - 2/(3*e^(-x) - 3*e^(-2*x) + e^(-3*x) -
1)) + 2/3*A*(3*e^(-x)/(3*e^(-x) - 3*e^(-2*x) + e^(-3*x) - 1) - 1/(3*e^(-x)
 - 3*e^(-2*x) + e^(-3*x) - 1))
```

3.98.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^2}\,dx = -\frac{2\left(3\,Be^{(2\,x)}+3\,Ae^{x}-3\,Be^{x}-A+2\,B\right)}{3\,(e^{x}-1)^{3}}$$

input
```
integrate((A+B*cosh(x))/(1-cosh(x))^2,x, algorithm="giac")
```

output
```
-2/3*(3*B*e^(2*x) + 3*A*e^x - 3*B*e^x - A + 2*B)/(e^x - 1)^3
```

3.98.9 Mupad [B] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^2}\,dx = -\frac{2\,(2\,B-A+3\,A\,\mathrm{e}^{x}-3\,B\,\mathrm{e}^{x}+3\,B\,\mathrm{e}^{2\,x})}{3\,(\mathrm{e}^{x}-1)^{3}}$$

input
```
int((A + B*cosh(x))/(cosh(x) - 1)^2,x)
```

output
```
-(2*(2*B - A + 3*A*exp(x) - 3*B*exp(x) + 3*B*exp(2*x)))/(3*(exp(x) - 1)^3)
```

3.99 $\int \frac{A+B\cosh(x)}{(1-\cosh(x))^3}\,dx$

3.99.1 Optimal result

Integrand size = 15, antiderivative size = 60

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^3}\,dx = -\frac{(A+B)\sinh(x)}{5(1-\cosh(x))^3}-\frac{(2A-3B)\sinh(x)}{15(1-\cosh(x))^2}-\frac{(2A-3B)\sinh(x)}{15(1-\cosh(x))}$$

output
```
-1/5*(A+B)*sinh(x)/(1-cosh(x))^3-1/15*(2*A-3*B)*sinh(x)/(1-cosh(x))^2-1/15
*(2*A-3*B)*sinh(x)/(1-cosh(x))
```

3.99.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.68

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^3}\,dx = \frac{(7A-3B+(-6A+9B)\cosh(x)+(2A-3B)\cosh^2(x))\sinh(x)}{15(-1+\cosh(x))^3}$$

input
```
Integrate[(A + B*Cosh[x])/(1 - Cosh[x])^3,x]
```

output
```
((7*A - 3*B + (-6*A + 9*B)*Cosh[x] + (2*A - 3*B)*Cosh[x]^2)*Sinh[x])/(15*(
-1 + Cosh[x])^3)
```

3.99.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3229, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{\left(1-\sin\left(\frac{\pi}{2}+ix\right)\right)^3}dx$$

$$\Big\downarrow \text{3229}$$

$$\frac{1}{5}(2A-3B)\int \frac{1}{(1-\cosh(x))^2}dx-\frac{(A+B)\sinh(x)}{5(1-\cosh(x))^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(A+B)\sinh(x)}{5(1-\cosh(x))^3}+\frac{1}{5}(2A-3B)\int \frac{1}{\left(1-\sin\left(ix+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{1}{5}(2A-3B)\left(\frac{1}{3}\int \frac{1}{1-\cosh(x)}dx-\frac{\sinh(x)}{3(1-\cosh(x))^2}\right)-\frac{(A+B)\sinh(x)}{5(1-\cosh(x))^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(A+B)\sinh(x)}{5(1-\cosh(x))^3}+\frac{1}{5}(2A-3B)\left(-\frac{\sinh(x)}{3(1-\cosh(x))^2}+\frac{1}{3}\int \frac{1}{1-\sin\left(ix+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \text{3127}$$

$$\frac{1}{5}(2A-3B)\left(-\frac{\sinh(x)}{3(1-\cosh(x))}-\frac{\sinh(x)}{3(1-\cosh(x))^2}\right)-\frac{(A+B)\sinh(x)}{5(1-\cosh(x))^3}$$

input
```
Int[(A + B*Cosh[x])/(1 - Cosh[x])^3,x]
```

output
```
-1/5*((A + B)*Sinh[x])/(1 - Cosh[x])^3 + ((2*A - 3*B)*(-1/3*Sinh[x]/(1 - C
osh[x])^2 - Sinh[x]/(3*(1 - Cosh[x]))))/5
```

3.99.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

3.99.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.65

method	result	size
default	$-\frac{-A+B}{4\tanh\left(\frac{x}{2}\right)}-\frac{-A-B}{20\tanh\left(\frac{x}{2}\right)^{5}}-\frac{A}{6\tanh\left(\frac{x}{2}\right)^{3}}$	39
risch	$\frac{2B\,\mathrm{e}^{3x}+\frac{8A\,\mathrm{e}^{2x}}{3}-2B\,\mathrm{e}^{2x}-\frac{4A\,\mathrm{e}^{x}}{3}+2B\,\mathrm{e}^{x}+\frac{4A}{15}-\frac{2B}{5}}{\left(\mathrm{e}^{x}-1\right)^{5}}$	47
parallelrisch	$\frac{(-12A+18B)\sinh(2x)+(2A-3B)\sinh(3x)+30\sinh(x)\left(A-\frac{B}{2}\right)}{15\cosh(3x)+225\cosh(x)-90\cosh(2x)-150}$	56

input
```
int((A+B*cosh(x))/(1-cosh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-A+B)/tanh(1/2*x)-1/20*(-A-B)/tanh(1/2*x)^5-1/6*A/tanh(1/2*x)^3
```

3.99.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(48) = 96$.

Time = 0.25 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.12

$$\int \frac{A + B\cosh(x)}{(1 - \cosh(x))^3}\, dx$$
$$= \frac{2 \left(15\,B\cosh\left(x\right)^2 + 15\,B\sinh\left(x\right)^2 + 2\,(11\,A - 9\,B)\cosh\left(x\right) - \cdots\right.}{15\,\left(\cosh\left(x\right)^4 + \left(4\,\cosh\left(x\right) - 5\right)\sinh\left(x\right)^3 + \sinh\left(x\right)^4 - 5\,\cosh\left(x\right)^3 + \left(6\,\cosh\left(x\right)^2 - 15\,\cosh\left(x\right) + 10\right)\sin \cdots\right.}$$

input
```
integrate((A+B*cosh(x))/(1-cosh(x))^3,x, algorithm="fricas")
```

output
```
2/15*(15*B*cosh(x)^2 + 15*B*sinh(x)^2 + 2*(11*A - 9*B)*cosh(x) + 6*(5*B*co
sh(x) + 3*A - 2*B)*sinh(x) - 10*A + 15*B)/(cosh(x)^4 + (4*cosh(x) - 5)*sin
h(x)^3 + sinh(x)^4 - 5*cosh(x)^3 + (6*cosh(x)^2 - 15*cosh(x) + 10)*sinh(x)
^2 + 10*cosh(x)^2 + (4*cosh(x)^3 - 15*cosh(x)^2 + 20*cosh(x) - 9)*sinh(x)
- 11*cosh(x) + 5)
```

3.99.6 Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

$$\int \frac{A + B\cosh(x)}{(1 - \cosh(x))^3}\, dx = \frac{A}{4\tanh\left(\frac{x}{2}\right)} - \frac{A}{6\tanh^3\left(\frac{x}{2}\right)} + \frac{A}{20\tanh^5\left(\frac{x}{2}\right)} - \frac{B}{4\tanh\left(\frac{x}{2}\right)} + \frac{B}{20\tanh^5\left(\frac{x}{2}\right)}$$

input
```
integrate((A+B*cosh(x))/(1-cosh(x))**3,x)
```

output
```
A/(4*tanh(x/2)) - A/(6*tanh(x/2)**3) + A/(20*tanh(x/2)**5) - B/(4*tanh(x/2
)) + B/(20*tanh(x/2)**5)
```

3.99.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(48) = 96.

Time = 0.19 (sec) , antiderivative size = 267, normalized size of antiderivative = 4.45

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^3}\,dx = -\frac{2}{5}\,B\left(\frac{5\,e^{(-x)}}{5\,e^{(-x)}-10\,e^{(-2\,x)}+10\,e^{(-3\,x)}-5\,e^{(-4\,x)}+e^{(-5\,x)}-1}-\frac{5\,e^{(-2\,x)}}{5\,e^{(-x)}-10\,e^{(-2\,x)}+10\,e^{(-3\,x)}-5\,e^{(-4\,x)}\right. + \frac{4}{15}\,A\left(\frac{5\,e^{(-x)}}{5\,e^{(-x)}-10\,e^{(-2\,x)}+10\,e^{(-3\,x)}-5\,e^{(-4\,x)}+e^{(-5\,x)}-1}-\frac{10\,e^{(-2\,x)}}{5\,e^{(-x)}-10\,e^{(-2\,x)}+10\,e^{(-3\,x)}-5\,e^{(-4\,x}\right.$$

input
```
integrate((A+B*cosh(x))/(1-cosh(x))^3,x, algorithm="maxima")
```

output
```
-2/5*B*(5*e^(-x)/(5*e^(-x) - 10*e^(-2*x) + 10*e^(-3*x) - 5*e^(-4*x) + e^(-
5*x) - 1) - 5*e^(-2*x)/(5*e^(-x) - 10*e^(-2*x) + 10*e^(-3*x) - 5*e^(-4*x)
+ e^(-5*x) - 1) + 5*e^(-3*x)/(5*e^(-x) - 10*e^(-2*x) + 10*e^(-3*x) - 5*e^(
-4*x) + e^(-5*x) - 1) - 1/(5*e^(-x) - 10*e^(-2*x) + 10*e^(-3*x) - 5*e^(-4*
x) + e^(-5*x) - 1)) + 4/15*A*(5*e^(-x)/(5*e^(-x) - 10*e^(-2*x) + 10*e^(-3*
x) - 5*e^(-4*x) + e^(-5*x) - 1) - 10*e^(-2*x)/(5*e^(-x) - 10*e^(-2*x) + 10
*e^(-3*x) - 5*e^(-4*x) + e^(-5*x) - 1) - 1/(5*e^(-x) - 10*e^(-2*x) + 10*e^
(-3*x) - 5*e^(-4*x) + e^(-5*x) - 1))
```

3.99.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^3}\,dx = \frac{2\left(15\,Be^{(3\,x)}+20\,Ae^{(2\,x)}-15\,Be^{(2\,x)}-10\,Ae^{x}+15\,Be^{x}+2\,A-3\,B\right)}{15\,(e^{x}-1)^{5}}$$

input
```
integrate((A+B*cosh(x))/(1-cosh(x))^3,x, algorithm="giac")
```

output
```
2/15*(15*B*e^(3*x) + 20*A*e^(2*x) - 15*B*e^(2*x) - 10*A*e^x + 15*B*e^x + 2
*A - 3*B)/(e^x - 1)^5
```

3.99.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.38

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^3}\,dx = \frac{\frac{B}{5}+\frac{4\,A\,e^{x}}{5}+\frac{3\,B\,e^{2\,x}}{5}}{6\,e^{2\,x}-4\,e^{3\,x}+e^{4\,x}-4\,e^{x}+1}-\frac{\frac{4\,A}{15}+\frac{2\,B\,e^{x}}{5}}{3\,e^{2\,x}-e^{3\,x}-3\,e^{x}+1} - \frac{\frac{4\,B\,e^{x}}{5}+\frac{8\,A\,e^{2\,x}}{5}+\frac{4\,B\,e^{3\,x}}{5}}{10\,e^{2\,x}-10\,e^{3\,x}+5\,e^{4\,x}-e^{5\,x}-5\,e^{x}+1}+\frac{B}{5\,\left(e^{2\,x}-2\,e^{x}+1\right)}$$

input
```
int(-(A + B*cosh(x))/(cosh(x) - 1)^3,x)
```

output
```
(B/5 + (4*A*exp(x))/5 + (3*B*exp(2*x))/5)/(6*exp(2*x) - 4*exp(3*x) + exp(4
*x) - 4*exp(x) + 1) - ((4*A)/15 + (2*B*exp(x))/5)/(3*exp(2*x) - exp(3*x) -
 3*exp(x) + 1) - ((4*B*exp(x))/5 + (8*A*exp(2*x))/5 + (4*B*exp(3*x))/5)/(1
0*exp(2*x) - 10*exp(3*x) + 5*exp(4*x) - exp(5*x) - 5*exp(x) + 1) + B/(5*(e
xp(2*x) - 2*exp(x) + 1))
```

3.100 $\int \frac{A+B\cosh(x)}{(1-\cosh(x))^4}\,dx$

3.100.1 Optimal result

Integrand size = 15, antiderivative size = 81

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^4}\,dx = -\frac{(A+B)\sinh(x)}{7(1-\cosh(x))^4} - \frac{(3A-4B)\sinh(x)}{35(1-\cosh(x))^3} - \frac{2(3A-4B)\sinh(x)}{105(1-\cosh(x))^2} - \frac{2(3A-4B)\sinh(x)}{105(1-\cosh(x))}$$

output
```
-1/7*(A+B)*sinh(x)/(1-cosh(x))^4-1/35*(3*A-4*B)*sinh(x)/(1-cosh(x))^3-2/10
5*(3*A-4*B)*sinh(x)/(1-cosh(x))^2-2/105*(3*A-4*B)*sinh(x)/(1-cosh(x))
```

3.100.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.68

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^4}\,dx = \frac{(-36A+13B+13(3A-4B)\cosh(x)-8(3A-4B)\cosh^2(x)+(6A-8B)\cosh^3(x))\sinh(x)}{105(-1+\cosh(x))^4}$$

input
```
Integrate[(A + B*Cosh[x])/(1 - Cosh[x])^4,x]
```

output
```
((-36*A + 13*B + 13*(3*A - 4*B)*Cosh[x] - 8*(3*A - 4*B)*Cosh[x]^2 + (6*A -
 8*B)*Cosh[x]^3)*Sinh[x])/(105*(-1 + Cosh[x])^4)
```

3.100.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {3042, 3229, 3042, 3129, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{\left(1-\sin\left(\frac{\pi}{2}+ix\right)\right)^4}dx$$

$$\Big\downarrow \text{3229}$$

$$\frac{1}{7}(3A-4B)\int \frac{1}{(1-\cosh(x))^3}dx-\frac{(A+B)\sinh(x)}{7(1-\cosh(x))^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(A+B)\sinh(x)}{7(1-\cosh(x))^4}+\frac{1}{7}(3A-4B)\int \frac{1}{\left(1-\sin\left(ix+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{1}{7}(3A-4B)\left(\frac{2}{5}\int \frac{1}{(1-\cosh(x))^2}dx-\frac{\sinh(x)}{5(1-\cosh(x))^3}\right)-\frac{(A+B)\sinh(x)}{7(1-\cosh(x))^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(A+B)\sinh(x)}{7(1-\cosh(x))^4}+\frac{1}{7}(3A-4B)\left(-\frac{\sinh(x)}{5(1-\cosh(x))^3}+\frac{2}{5}\int \frac{1}{\left(1-\sin\left(ix+\frac{\pi}{2}\right)\right)^2}dx\right)$$

$$\Big\downarrow \text{3129}$$

$$\frac{1}{7}(3A-4B)\left(\frac{2}{5}\left(\frac{1}{3}\int \frac{1}{1-\cosh(x)}dx-\frac{\sinh(x)}{3(1-\cosh(x))^2}\right)-\frac{\sinh(x)}{5(1-\cosh(x))^3}\right)-\frac{(A+B)\sinh(x)}{7(1-\cosh(x))^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(A+B)\sinh(x)}{7(1-\cosh(x))^4}+\frac{1}{7}(3A-4B)\left(-\frac{\sinh(x)}{5(1-\cosh(x))^3}+\frac{2}{5}\left(-\frac{\sinh(x)}{3(1-\cosh(x))^2}+\frac{1}{3}\int \frac{1}{1-\sin\left(ix+\frac{\pi}{2}\right)}dx\right)\right)$$

$$\Big\downarrow \text{3127}$$

$$\frac{1}{7}(3A-4B)\left(\frac{2}{5}\left(-\frac{\sinh(x)}{3(1-\cosh(x))}-\frac{\sinh(x)}{3(1-\cosh(x))^2}\right)-\frac{\sinh(x)}{5(1-\cosh(x))^3}\right)-\frac{(A+B)\sinh(x)}{7(1-\cosh(x))^4}$$

input
```
Int[(A + B*Cosh[x])/(1 - Cosh[x])^4,x]
```

output
```
-1/7*((A + B)*Sinh[x])/(1 - Cosh[x])^4 + ((3*A - 4*B)*(-1/5*Sinh[x]/(1 - C
osh[x])^3 + (2*(-1/3*Sinh[x]/(1 - Cosh[x])^2 - Sinh[x]/(3*(1 - Cosh[x]))))
/5))/7
```

3.100.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

3.100.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.69

method	result	size
default	$-\frac{-A+B}{8\tanh\left(\frac{x}{2}\right)}-\frac{-3A-B}{40\tanh\left(\frac{x}{2}\right)^{5}}-\frac{3A-B}{24\tanh\left(\frac{x}{2}\right)^{3}}-\frac{A+B}{56\tanh\left(\frac{x}{2}\right)^{7}}$	56
risch	$-\frac{4\left(70B\,e^{4x}+105A\,e^{3x}-70B\,e^{3x}-63A\,e^{2x}+84B\,e^{2x}+21A\,e^{x}-28B\,e^{x}-3A+4B\right)}{105\left(e^{x}-1\right)^{7}}$	61
parallelrisch	$\frac{(-168A+224B)\sinh(2x)+(48A-64B)\sinh(3x)+(-6A+8B)\sinh(4x)+336\sinh(x)\left(A-\frac{B}{2}\right)}{-105\cosh(4x)-2940\cosh(2x)-3675+840\cosh(3x)+5880\cosh(x)}$	74

input
```
int((A+B*cosh(x))/(1-cosh(x))^4,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(-A+B)/tanh(1/2*x)-1/40*(-3*A-B)/tanh(1/2*x)^5-1/24*(3*A-B)/tanh(1/2*
x)^3-1/56*(A+B)/tanh(1/2*x)^7
```

3.100.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(65) = 130$.

Time = 0.25 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.16

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^4}\,dx$$
$$=\frac{4\left((3\,A-74\,B)\cosh\left(x\right)^{2}\right.}{105\left(\cosh\left(x\right)^{5}+\left(5\,\cosh\left(x\right)-7\right)\sinh\left(x\right)^{4}+\sinh\left(x\right)^{5}-7\,\cosh\left(x\right)^{4}+\left(10\,\cosh\left(x\right)^{2}-28\,\cosh\left(x\right)+21\right)\right.}$$

input
```
integrate((A+B*cosh(x))/(1-cosh(x))^4,x, algorithm="fricas")
```

output
```
4/105*((3*A - 74*B)*cosh(x)^2 + (3*A - 74*B)*sinh(x)^2 - 14*(9*A - 7*B)*co
sh(x) - 6*((A + 22*B)*cosh(x) + 14*A - 7*B)*sinh(x) + 63*A - 84*B)/(cosh(x
)^5 + (5*cosh(x) - 7)*sinh(x)^4 + sinh(x)^5 - 7*cosh(x)^4 + (10*cosh(x)^2
- 28*cosh(x) + 21)*sinh(x)^3 + 21*cosh(x)^3 + (10*cosh(x)^3 - 42*cosh(x)^2
 + 63*cosh(x) - 36)*sinh(x)^2 - 36*cosh(x)^2 + (5*cosh(x)^4 - 28*cosh(x)^3
 + 63*cosh(x)^2 - 68*cosh(x) + 28)*sinh(x) + 42*cosh(x) - 21)
```

3.100.6 Sympy [A] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^4}\,dx = \frac{A}{8\tanh\left(\frac{x}{2}\right)} - \frac{A}{8\tanh^3\left(\frac{x}{2}\right)} + \frac{3A}{40\tanh^5\left(\frac{x}{2}\right)} - \frac{A}{56\tanh^7\left(\frac{x}{2}\right)} - \frac{B}{8\tanh\left(\frac{x}{2}\right)} + \frac{B}{24\tanh^3\left(\frac{x}{2}\right)} + \frac{B}{40\tanh^5\left(\frac{x}{2}\right)} - \frac{B}{56\tanh^7\left(\frac{x}{2}\right)}$$

input
```
integrate((A+B*cosh(x))/(1-cosh(x))**4,x)
```

output
```
A/(8*tanh(x/2)) - A/(8*tanh(x/2)**3) + 3*A/(40*tanh(x/2)**5) - A/(56*tanh(
x/2)**7) - B/(8*tanh(x/2)) + B/(24*tanh(x/2)**3) + B/(40*tanh(x/2)**5) - B
/(56*tanh(x/2)**7)
```

3.100.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(65) = 130$.

Time = 0.19 (sec) , antiderivative size = 451, normalized size of antiderivative = 5.57

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^4}\,dx =$$

$$-\frac{8}{105}B\left(\frac{14\,e^{(-x)}}{7\,e^{(-x)} - 21\,e^{(-2\,x)} + 35\,e^{(-3\,x)} - 35\,e^{(-4\,x)} + 21\,e^{(-5\,x)} - 7\,e^{(-6\,x)} + e^{(-7\,x)} - 1} - \frac{}{7\,e^{(-x)} - 21\,e^{(-}}\right.$$

$$+\frac{4}{35}A\left(\frac{7\,e^{(-x)}}{7\,e^{(-x)} - 21\,e^{(-2\,x)} + 35\,e^{(-3\,x)} - 35\,e^{(-4\,x)} + 21\,e^{(-5\,x)} - 7\,e^{(-6\,x)} + e^{(-7\,x)} - 1} - \frac{}{7\,e^{(-x)} - 21\,e^{(-2}}\right.$$

input
```
integrate((A+B*cosh(x))/(1-cosh(x))^4,x, algorithm="maxima")
```

output
```
-8/105*B*(14*e^(-x)/(7*e^(-x) - 21*e^(-2*x) + 35*e^(-3*x) - 35*e^(-4*x) +
21*e^(-5*x) - 7*e^(-6*x) + e^(-7*x) - 1) - 42*e^(-2*x)/(7*e^(-x) - 21*e^(-
2*x) + 35*e^(-3*x) - 35*e^(-4*x) + 21*e^(-5*x) - 7*e^(-6*x) + e^(-7*x) - 1
) + 35*e^(-3*x)/(7*e^(-x) - 21*e^(-2*x) + 35*e^(-3*x) - 35*e^(-4*x) + 21*e
^(-5*x) - 7*e^(-6*x) + e^(-7*x) - 1) - 35*e^(-4*x)/(7*e^(-x) - 21*e^(-2*x)
 + 35*e^(-3*x) - 35*e^(-4*x) + 21*e^(-5*x) - 7*e^(-6*x) + e^(-7*x) - 1) -
2/(7*e^(-x) - 21*e^(-2*x) + 35*e^(-3*x) - 35*e^(-4*x) + 21*e^(-5*x) - 7*e^
(-6*x) + e^(-7*x) - 1)) + 4/35*A*(7*e^(-x)/(7*e^(-x) - 21*e^(-2*x) + 35*e^
(-3*x) - 35*e^(-4*x) + 21*e^(-5*x) - 7*e^(-6*x) + e^(-7*x) - 1) - 21*e^(-2
*x)/(7*e^(-x) - 21*e^(-2*x) + 35*e^(-3*x) - 35*e^(-4*x) + 21*e^(-5*x) - 7*
e^(-6*x) + e^(-7*x) - 1) + 35*e^(-3*x)/(7*e^(-x) - 21*e^(-2*x) + 35*e^(-3*
x) - 35*e^(-4*x) + 21*e^(-5*x) - 7*e^(-6*x) + e^(-7*x) - 1) - 1/(7*e^(-x)
- 21*e^(-2*x) + 35*e^(-3*x) - 35*e^(-4*x) + 21*e^(-5*x) - 7*e^(-6*x) + e^(
-7*x) - 1))
```

3.100.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.74

$$\int \frac{A + B\cosh(x)}{(1 - \cosh(x))^4}\,dx = -\frac{4\left(70\,Be^{(4\,x)} + 105\,Ae^{(3\,x)} - 70\,Be^{(3\,x)} - 63\,Ae^{(2\,x)} + 84\,Be^{(2\,x)} + 21\,Ae^{x} - 28\,Be^{x} - 3\,A + 4\,B\right)}{105\,(e^{x} - 1)^{7}}$$

input
```
integrate((A+B*cosh(x))/(1-cosh(x))^4,x, algorithm="giac")
```

output
```
-4/105*(70*B*e^(4*x) + 105*A*e^(3*x) - 70*B*e^(3*x) - 63*A*e^(2*x) + 84*B*
e^(2*x) + 21*A*e^x - 28*B*e^x - 3*A + 4*B)/(e^x - 1)^7
```

3.100.9 Mupad [B] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.88

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^4}\,dx = \frac{\frac{8\,B}{105}+\frac{16\,A\,e^x}{35}+\frac{16\,B\,e^{2\,x}}{35}}{10\,e^{2\,x}-10\,e^{3\,x}+5\,e^{4\,x}-e^{5\,x}-5\,e^x+1} - \frac{\frac{4\,A}{35}+\frac{8\,B\,e^x}{35}}{6\,e^{2\,x}-4\,e^{3\,x}+e^{4\,x}-4\,e^x+1} - \frac{\frac{8\,B\,e^x}{21}+\frac{8\,A\,e^{2\,x}}{7}+\frac{16\,B\,e^{3\,x}}{21}}{15\,e^{2\,x}-20\,e^{3\,x}+15\,e^{4\,x}-6\,e^{5\,x}+e^{6\,x}-6\,e^x+1} + \frac{8\,B}{105\,\left(3\,e^{2\,x}-e^{3\,x}-3\,e^x+1\right)} + \frac{\frac{16\,A\,e^{3\,x}}{7}+\frac{8\,B\,e^{2\,x}}{7}+\frac{8\,B\,e^{4\,x}}{7}}{21\,e^{2\,x}-35\,e^{3\,x}+35\,e^{4\,x}-21\,e^{5\,x}+7\,e^{6\,x}-e^{7\,x}-7\,e^x+1}$$

input
```
int((A + B*cosh(x))/(cosh(x) - 1)^4,x)
```

output
```
((8*B)/105 + (16*A*exp(x))/35 + (16*B*exp(2*x))/35)/(10*exp(2*x) - 10*exp(
3*x) + 5*exp(4*x) - exp(5*x) - 5*exp(x) + 1) - ((4*A)/35 + (8*B*exp(x))/35
)/(6*exp(2*x) - 4*exp(3*x) + exp(4*x) - 4*exp(x) + 1) - ((8*B*exp(x))/21 +
 (8*A*exp(2*x))/7 + (16*B*exp(3*x))/21)/(15*exp(2*x) - 20*exp(3*x) + 15*ex
p(4*x) - 6*exp(5*x) + exp(6*x) - 6*exp(x) + 1) + (8*B)/(105*(3*exp(2*x) -
exp(3*x) - 3*exp(x) + 1)) + ((16*A*exp(3*x))/7 + (8*B*exp(2*x))/7 + (8*B*e
xp(4*x))/7)/(21*exp(2*x) - 35*exp(3*x) + 35*exp(4*x) - 21*exp(5*x) + 7*exp
(6*x) - exp(7*x) - 7*exp(x) + 1)
```

3.101 $\int \frac{A+B\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx$

3.101.1 Optimal result

Integrand size = 17, antiderivative size = 56

$$\int \frac{A+B\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx=\frac{\sqrt{2}(A-B)\arctan\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a+a\cosh(x)}}\right)}{\sqrt{a}}+\frac{2B\sinh(x)}{\sqrt{a+a\cosh(x)}}$$

output
```
(A-B)*arctan(1/2*sinh(x)*a^(1/2)*2^(1/2)/(a+a*cosh(x))^(1/2))*2^(1/2)/a^(1
/2)+2*B*sinh(x)/(a+a*cosh(x))^(1/2)
```

3.101.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.73

$$\int \frac{A+B\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx=\frac{2\cosh\left(\frac{x}{2}\right)\left((A-B)\arctan\left(\sinh\left(\frac{x}{2}\right)\right)+2B\sinh\left(\frac{x}{2}\right)\right)}{\sqrt{a(1+\cosh(x))}}$$

input
```
Integrate[(A + B*Cosh[x])/Sqrt[a + a*Cosh[x]],x]
```

output
```
(2*Cosh[x/2]*((A - B)*ArcTan[Sinh[x/2]] + 2*B*Sinh[x/2]))/Sqrt[a*(1 + Cosh
[x])]
```

3.101.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{\sqrt{a\cosh(x)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{\sqrt{a+a\sin\left(\frac{\pi}{2}+ix\right)}}dx$$

$$\Big\downarrow \text{3230}$$

$$(A-B)\int \frac{1}{\sqrt{\cosh(x)a+a}}dx+\frac{2B\sinh(x)}{\sqrt{a\cosh(x)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2B\sinh(x)}{\sqrt{a\cosh(x)+a}}+(A-B)\int \frac{1}{\sqrt{\sin\left(ix+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{3128}$$

$$\frac{2B\sinh(x)}{\sqrt{a\cosh(x)+a}}+2i(A-B)\int \frac{1}{\frac{a^2\sinh^2(x)}{\cosh(x)a+a}+2a}d\left(-\frac{ia\sinh(x)}{\sqrt{\cosh(x)a+a}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{2}(A-B)\arctan\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a\cosh(x)+a}}\right)}{\sqrt{a}}+\frac{2B\sinh(x)}{\sqrt{a\cosh(x)+a}}$$

input
```
Int[(A + B*Cosh[x])/Sqrt[a + a*Cosh[x]],x]
```

output
```
(Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a + a*Cosh[x]])])/
Sqrt[a] + (2*B*Sinh[x])/Sqrt[a + a*Cosh[x]]
```

3.101.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

3.101.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(45) = 90$.

Time = 0.41 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.29

method	result
default	$-\frac{\cosh\left(\frac{x}{2}\right)\sqrt{\sinh\left(\frac{x}{2}\right)^{2}a}\left(\ln\left(\frac{2\sqrt{\sinh\left(\frac{x}{2}\right)^{2}a}\,\sqrt{-a}-2a}{\cosh\left(\frac{x}{2}\right)}\right)aA-2B\sqrt{\sinh\left(\frac{x}{2}\right)^{2}a}\,\sqrt{-a}-\ln\left(\frac{2\sqrt{\sinh\left(\frac{x}{2}\right)^{2}a}\,\sqrt{-a}-2a}{\cosh\left(\frac{x}{2}\right)}\right)aB\right)\sqrt{2}}{a\sqrt{-a}\,\sinh\left(\frac{x}{2}\right)\sqrt{a\cosh\left(\frac{x}{2}\right)^{2}}}$
parts	$-\frac{A\cosh\left(\frac{x}{2}\right)\sqrt{\sinh\left(\frac{x}{2}\right)^{2}a}\,\ln\left(\frac{2\sqrt{\sinh\left(\frac{x}{2}\right)^{2}a}\,\sqrt{-a}-2a}{\cosh\left(\frac{x}{2}\right)}\right)\sqrt{2}}{\sqrt{-a}\,\sinh\left(\frac{x}{2}\right)\sqrt{a\cosh\left(\frac{x}{2}\right)^{2}}}+\frac{B\cosh\left(\frac{x}{2}\right)\sqrt{\sinh\left(\frac{x}{2}\right)^{2}a}\left(2\sqrt{\sinh\left(\frac{x}{2}\right)^{2}a}\,\sqrt{-a}+\ln\left(\frac{2\sqrt{\sinh\left(\frac{x}{2}\right)^{2}a}\,\sqrt{-a}\right.\right.}{\cosh\left(\frac{x}{2}\right)}}{a\sqrt{-a}\,\sinh\left(\frac{x}{2}\right)\sqrt{a\cosh\left(\frac{x}{2}\right)^{2}}}$

input
```
int((A+B*cosh(x))/(a+a*cosh(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-cosh(1/2*x)*(sinh(1/2*x)^2*a)^(1/2)*(ln(2/cosh(1/2*x)*((sinh(1/2*x)^2*a)^
(1/2)*(-a)^(1/2)-a))*a*A-2*B*(sinh(1/2*x)^2*a)^(1/2)*(-a)^(1/2)-ln(2/cosh(
1/2*x)*((sinh(1/2*x)^2*a)^(1/2)*(-a)^(1/2)-a))*a*B)/a/(-a)^(1/2)/sinh(1/2*
x)*2^(1/2)/(a*cosh(1/2*x)^2)^(1/2)
```

3.101.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.29

$$\int \frac{A+B\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx$$
$$= \frac{2\left(\sqrt{2}(A-B)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{\cosh(x)+\sinh(x)}}(\cosh(x)+\sinh(x))}{\sqrt{a}}\right)+\sqrt{\frac{1}{2}}(B\cosh(x)+B\sinh(x)-B)\sqrt{\frac{\ }{\cosh(}}\right.}{a}$$

input
```
integrate((A+B*cosh(x))/(a+a*cosh(x))^(1/2),x, algorithm="fricas")
```

output
```
2*(sqrt(2)*(A - B)*sqrt(a)*arctan(sqrt(2)*sqrt(1/2)*sqrt(a/(cosh(x) + sinh
(x)))*(cosh(x) + sinh(x))/sqrt(a)) + sqrt(1/2)*(B*cosh(x) + B*sinh(x) - B)
*sqrt(a/(cosh(x) + sinh(x))))/a
```

3.101.6 Sympy [F]

$$\int \frac{A+B\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx = \int \frac{A+B\cosh(x)}{\sqrt{a(\cosh(x)+1)}}\,dx$$

input
```
integrate((A+B*cosh(x))/(a+a*cosh(x))**(1/2),x)
```

output
```
Integral((A + B*cosh(x))/sqrt(a*(cosh(x) + 1)), x)
```

3.101.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(45) = 90$.

Time = 0.37 (sec) , antiderivative size = 174, normalized size of antiderivative = 3.11

$$\int \frac{A+B\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx = 2\left(\sqrt{2}\left(\frac{\arctan\left(e^{\left(\frac{1}{2}x\right)}\right)}{\sqrt{a}}+\frac{e^{\left(\frac{1}{2}x\right)}}{\sqrt{a}e^{x}+\sqrt{a}}\right)-\frac{\sqrt{2}e^{\left(\frac{1}{2}x\right)}}{\sqrt{a}e^{x}+\sqrt{a}}\right)A - \frac{1}{3}\left(3\sqrt{2}\left(\frac{\arctan\left(e^{\left(\frac{1}{2}x\right)}\right)}{\sqrt{a}}-\frac{e^{\left(\frac{1}{2}x\right)}}{\sqrt{a}e^{x}+\sqrt{a}}\right)-\sqrt{2}\left(\frac{3\arctan\left(e^{\left(-\frac{1}{2}x\right)}\right)}{\sqrt{a}}-\frac{2e^{\left(-\frac{1}{2}x\right)}}{\sqrt{a}}-\frac{e^{\left(-\frac{1}{2}x\right)}}{\sqrt{a}e^{(-x)}+\sqrt{a}}\right)\cdots$$

input
```
integrate((A+B*cosh(x))/(a+a*cosh(x))^(1/2),x, algorithm="maxima")
```

output
```
2*(sqrt(2)*(arctan(e^(1/2*x))/sqrt(a) + e^(1/2*x)/(sqrt(a)*e^x + sqrt(a)))
 - sqrt(2)*e^(1/2*x)/(sqrt(a)*e^x + sqrt(a)))*A - 1/3*(3*sqrt(2)*(arctan(e
^(1/2*x))/sqrt(a) - e^(1/2*x)/(sqrt(a)*e^x + sqrt(a))) - sqrt(2)*(3*arctan
(e^(-1/2*x))/sqrt(a) - 2*e^(-1/2*x)/sqrt(a) - e^(-1/2*x)/(sqrt(a)*e^(-x) +
 sqrt(a))) - (3*sqrt(2)*sqrt(a)*e^(3/2*x) - sqrt(2)*sqrt(a)*e^(-1/2*x))/(a
*e^x + a))*B
```

3.101.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int \frac{A+B\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx = \frac{2\sqrt{2}(A-B)\arctan\left(e^{\left(\frac{1}{2}x\right)}\right)}{\sqrt{a}}+\frac{\sqrt{2}Be^{\left(\frac{1}{2}x\right)}}{\sqrt{a}}-\frac{\sqrt{2}Be^{\left(-\frac{1}{2}x\right)}}{\sqrt{a}}$$

input
```
integrate((A+B*cosh(x))/(a+a*cosh(x))^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(2)*(A - B)*arctan(e^(1/2*x))/sqrt(a) + sqrt(2)*B*e^(1/2*x)/sqrt(a)
- sqrt(2)*B*e^(-1/2*x)/sqrt(a)
```

3.101.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx = \int \frac{A+B\cosh(x)}{\sqrt{a+a\cosh\left(x\right)}}\,dx$$

input
```
int((A + B*cosh(x))/(a + a*cosh(x))^(1/2),x)
```

output
```
int((A + B*cosh(x))/(a + a*cosh(x))^(1/2), x)
```

3.102 $\int \frac{A+B\cosh(x)}{(a+a\cosh(x))^{3/2}}\,dx$

3.102.1 Optimal result

Integrand size = 17, antiderivative size = 65

$$\int \frac{A+B\cosh(x)}{(a+a\cosh(x))^{3/2}}\,dx = \frac{(A+3B)\arctan\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a+a\cosh(x)}}\right)}{2\sqrt{2}a^{3/2}} + \frac{(A-B)\sinh(x)}{2(a+a\cosh(x))^{3/2}}$$

output
```
1/2*(A-B)*sinh(x)/(a+a*cosh(x))^(3/2)+1/4*(A+3*B)*arctan(1/2*sinh(x)*a^(1/
2)*2^(1/2)/(a+a*cosh(x))^(1/2))/a^(3/2)*2^(1/2)
```

3.102.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.68

$$\int \frac{A+B\cosh(x)}{(a+a\cosh(x))^{3/2}}\,dx = \frac{(A+3B)\arctan\left(\sinh\left(\frac{x}{2}\right)\right)\cosh^3\left(\frac{x}{2}\right)+\frac{1}{2}(A-B)\sinh(x)}{(a(1+\cosh(x)))^{3/2}}$$

input
```
Integrate[(A + B*Cosh[x])/(a + a*Cosh[x])^(3/2),x]
```

output
```
((A + 3*B)*ArcTan[Sinh[x/2]]*Cosh[x/2]^3 + ((A - B)*Sinh[x])/2)/(a*(1 + Co
sh[x]))^(3/2)
```

3.102.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 3229, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{(a\cosh(x)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{\left(a+a\sin\left(\frac{\pi}{2}+ix\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3229}$$

$$\frac{(A+3B)\int \frac{1}{\sqrt{\cosh(x)a+a}}dx}{4a}+\frac{(A-B)\sinh(x)}{2(a\cosh(x)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-B)\sinh(x)}{2(a\cosh(x)+a)^{3/2}}+\frac{(A+3B)\int \frac{1}{\sqrt{\sin\left(ix+\frac{\pi}{2}\right)a+a}}dx}{4a}$$

$$\Big\downarrow \text{3128}$$

$$\frac{(A-B)\sinh(x)}{2(a\cosh(x)+a)^{3/2}}+\frac{i(A+3B)\int \frac{1}{\frac{a^2\sinh^2(x)}{\cosh(x)a+a}+2a}d\left(-\frac{ia\sinh(x)}{\sqrt{\cosh(x)a+a}}\right)}{2a}$$

$$\Big\downarrow \text{219}$$

$$\frac{(A+3B)\arctan\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a\cosh(x)+a}}\right)}{2\sqrt{2}a^{3/2}}+\frac{(A-B)\sinh(x)}{2(a\cosh(x)+a)^{3/2}}$$

input
```
Int[(A + B*Cosh[x])/(a + a*Cosh[x])^(3/2),x]
```

output
```
((A + 3*B)*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a + a*Cosh[x]])])/(2*Sqr
t[2]*a^(3/2)) + ((A - B)*Sinh[x])/(2*(a + a*Cosh[x])^(3/2))
```

3.102.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

3.102.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(50) = 100$.

Time = 0.45 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.45

method	result
default	$-\frac{\sqrt{\sinh\left(\frac{x}{2}\right)^{2} a}\,\left(A\ln\left(\frac{2\sqrt{\sinh\left(\frac{x}{2}\right)^{2} a}\,\sqrt{-a}-2a}{\cosh\left(\frac{x}{2}\right)}\right)\cosh\left(\frac{x}{2}\right)^{2} a+3B\ln\left(\frac{2\sqrt{\sinh\left(\frac{x}{2}\right)^{2} a}\,\sqrt{-a}-2a}{\cosh\left(\frac{x}{2}\right)}\right) a\cosh\left(\frac{x}{2}\right)^{2}-A\sqrt{-a}\,\sqrt{\sinh\left(\frac{x}{2}\right)^{2} a}+B\sqrt{}\right.}{4\cosh\left(\frac{x}{2}\right) a^{2}\sqrt{-a}\,\sinh\left(\frac{x}{2}\right)\sqrt{a\cosh\left(\frac{x}{2}\right)^{2}}}$
parts	$-\frac{A\sqrt{\sinh\left(\frac{x}{2}\right)^{2} a}\,\left(\ln\left(\frac{2\sqrt{\sinh\left(\frac{x}{2}\right)^{2} a}\,\sqrt{-a}-2a}{\cosh\left(\frac{x}{2}\right)}\right) a\cosh\left(\frac{x}{2}\right)^{2}-\sqrt{\sinh\left(\frac{x}{2}\right)^{2} a}\,\sqrt{-a}\right)\sqrt{2}}{4a^{2}\cosh\left(\frac{x}{2}\right)\sqrt{-a}\,\sinh\left(\frac{x}{2}\right)\sqrt{a\cosh\left(\frac{x}{2}\right)^{2}}}-\frac{B\sqrt{\sinh\left(\frac{x}{2}\right)^{2} a}\,\left(3\ln\left(\frac{2\sqrt{\sinh\left(\frac{x}{2}\right)^{2} a}\,\sqrt{-a}-}{\cosh\left(\frac{x}{2}\right)}\right.\right.}{4\cosh\left(\frac{x}{2}\right) a^{2}\sqrt{-a}\,\sin}$

input
```
int((A+B*cosh(x))/(a+a*cosh(x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(sinh(1/2*x)^2*a)^(1/2)*(A*ln(2/cosh(1/2*x)*((sinh(1/2*x)^2*a)^(1/2)*
(-a)^(1/2)-a))*cosh(1/2*x)^2*a+3*B*ln(2/cosh(1/2*x)*((sinh(1/2*x)^2*a)^(1/
2)*(-a)^(1/2)-a))*a*cosh(1/2*x)^2-A*(-a)^(1/2)*(sinh(1/2*x)^2*a)^(1/2)+B*(
sinh(1/2*x)^2*a)^(1/2)*(-a)^(1/2))/cosh(1/2*x)/a^2/(-a)^(1/2)/sinh(1/2*x)*
2^(1/2)/(a*cosh(1/2*x)^2)^(1/2)
```

3.102.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(50) = 100$.

Time = 0.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.91

$$\int \frac{A + B\cosh(x)}{(a + a\cosh(x))^{3/2}}\,dx =$$

$$-\frac{\sqrt{2}\left((A+3\,B)\cosh\left(x\right)^{2}+(A+3\,B)\sinh\left(x\right)^{2}+2\,(A+3\,B)\cosh\left(x\right)+2\,\left((A+3\,B)\cosh\left(x\right)+A+3\,B\right)s}{2\,(a}$$

input
```
integrate((A+B*cosh(x))/(a+a*cosh(x))^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(2)*((A + 3*B)*cosh(x)^2 + (A + 3*B)*sinh(x)^2 + 2*(A + 3*B)*cos
h(x) + 2*((A + 3*B)*cosh(x) + A + 3*B)*sinh(x) + A + 3*B)*sqrt(a)*arctan(s
qrt(2)*sqrt(1/2)*sqrt(a/(cosh(x) + sinh(x)))/sqrt(a)) - 2*sqrt(1/2)*((A -
B)*cosh(x)^2 + (A - B)*sinh(x)^2 - (A - B)*cosh(x) + (2*(A - B)*cosh(x) -
A + B)*sinh(x))*sqrt(a/(cosh(x) + sinh(x))))/(a^2*cosh(x)^2 + a^2*sinh(x)^
2 + 2*a^2*cosh(x) + a^2 + 2*(a^2*cosh(x) + a^2)*sinh(x))
```

3.102.6 Sympy [F]

$$\int \frac{A + B\cosh(x)}{(a + a\cosh(x))^{3/2}}\,dx = \int \frac{A + B\cosh\left(x\right)}{\left(a\left(\cosh\left(x\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cosh(x))/(a+a*cosh(x))**(3/2),x)
```

output
```
Integral((A + B*cosh(x))/(a*(cosh(x) + 1))**(3/2), x)
```

3.102.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(50) = 100$.

Time = 0.38 (sec) , antiderivative size = 300, normalized size of antiderivative = 4.62

$$\int \frac{A+B\cosh(x)}{(a+a\cosh(x))^{3/2}}\,dx = \frac{1}{6}\left(\sqrt{2}\left(\frac{3\,e^{\left(\frac{5}{2}\,x\right)}+8\,e^{\left(\frac{3}{2}\,x\right)}-3\,e^{\left(\frac{1}{2}\,x\right)}}{a^{\frac{3}{2}}e^{(3\,x)}+3\,a^{\frac{3}{2}}e^{(2\,x)}+3\,a^{\frac{3}{2}}e^{x}+a^{\frac{3}{2}}}+\frac{3\,\arctan\left(e^{\left(\frac{1}{2}\,x\right)}\right)}{a^{\frac{3}{2}}}\right)-\frac{}{a^{\frac{3}{2}}e^{(3\,x)}+3}\right.$$
$$+\frac{1}{20}\left(\sqrt{2}\left(\frac{15\,e^{\left(\frac{5}{2}\,x\right)}+40\,e^{\left(\frac{3}{2}\,x\right)}+33\,e^{\left(\frac{1}{2}\,x\right)}}{a^{\frac{3}{2}}e^{(3\,x)}+3\,a^{\frac{3}{2}}e^{(2\,x)}+3\,a^{\frac{3}{2}}e^{x}+a^{\frac{3}{2}}}+\frac{15\,\arctan\left(e^{\left(\frac{1}{2}\,x\right)}\right)}{a^{\frac{3}{2}}}\right)+5\,\sqrt{2}\left(\frac{3\,e^{\left(\frac{5}{2}\,x\right)}-8\,e^{\left(\frac{3}{2}\,x\right)}-3\,e^{\left(\frac{1}{2}\right.}}{a^{\frac{3}{2}}e^{(3\,x)}+3\,a^{\frac{3}{2}}e^{(2\,x)}+3\,a^{\frac{3}{2}}e^{x}}\right.\right.$$

input
```
integrate((A+B*cosh(x))/(a+a*cosh(x))^(3/2),x, algorithm="maxima")
```

output
```
1/6*(sqrt(2)*((3*e^(5/2*x) + 8*e^(3/2*x) - 3*e^(1/2*x))/(a^(3/2)*e^(3*x) +
 3*a^(3/2)*e^(2*x) + 3*a^(3/2)*e^x + a^(3/2)) + 3*arctan(e^(1/2*x))/a^(3/2
)) - 8*sqrt(2)*e^(3/2*x)/(a^(3/2)*e^(3*x) + 3*a^(3/2)*e^(2*x) + 3*a^(3/2)*
e^x + a^(3/2)))*A + 1/20*(sqrt(2)*((15*e^(5/2*x) + 40*e^(3/2*x) + 33*e^(1/
2*x))/(a^(3/2)*e^(3*x) + 3*a^(3/2)*e^(2*x) + 3*a^(3/2)*e^x + a^(3/2)) + 15
*arctan(e^(1/2*x))/a^(3/2)) + 5*sqrt(2)*((3*e^(5/2*x) - 8*e^(3/2*x) - 3*e^
(1/2*x))/(a^(3/2)*e^(3*x) + 3*a^(3/2)*e^(2*x) + 3*a^(3/2)*e^x + a^(3/2)) +
 3*arctan(e^(1/2*x))/a^(3/2)) - 8*(5*sqrt(2)*sqrt(a)*e^(5/2*x) + sqrt(2)*s
qrt(a)*e^(1/2*x))/(a^2*e^(3*x) + 3*a^2*e^(2*x) + 3*a^2*e^x + a^2))*B
```

3.102.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.20

$$\int \frac{A+B\cosh(x)}{(a+a\cosh(x))^{3/2}}\,dx = \frac{(\sqrt{2}A+3\,\sqrt{2}B)\arctan\left(e^{\left(\frac{1}{2}\,x\right)}\right)}{2\,a^{\frac{3}{2}}} + \frac{\sqrt{2}\left(Aa^{\frac{3}{2}}e^{\left(\frac{3}{2}\,x\right)}-Ba^{\frac{3}{2}}e^{\left(\frac{3}{2}\,x\right)}-Aa^{\frac{3}{2}}e^{\left(\frac{1}{2}\,x\right)}+Ba^{\frac{3}{2}}e^{\left(\frac{1}{2}\,x\right)}\right)}{2\,(ae^{x}+a)^{2}a}$$

input
```
integrate((A+B*cosh(x))/(a+a*cosh(x))^(3/2),x, algorithm="giac")
```

output
```
1/2*(sqrt(2)*A + 3*sqrt(2)*B)*arctan(e^(1/2*x))/a^(3/2) + 1/2*sqrt(2)*(A*a
^(3/2)*e^(3/2*x) - B*a^(3/2)*e^(3/2*x) - A*a^(3/2)*e^(1/2*x) + B*a^(3/2)*e
^(1/2*x))/((a*e^x + a)^2*a)
```

3.102.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)}{(a+a\cosh(x))^{3/2}}\,dx=\int \frac{A+B\cosh(x)}{\left(a+a\cosh\left(x\right)\right)^{3/2}}\,dx$$

input
```
int((A + B*cosh(x))/(a + a*cosh(x))^(3/2),x)
```

output
```
int((A + B*cosh(x))/(a + a*cosh(x))^(3/2), x)
```

3.103 $\int \frac{A+B\cosh(x)}{(a+a\cosh(x))^{5/2}}\,dx$

3.103.1 Optimal result

Integrand size = 17, antiderivative size = 93

$$\int \frac{A+B\cosh(x)}{(a+a\cosh(x))^{5/2}}\,dx = \frac{(3A+5B)\arctan\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a+a\cosh(x)}}\right)}{16\sqrt{2}a^{5/2}} + \frac{(A-B)\sinh(x)}{4(a+a\cosh(x))^{5/2}} + \frac{(3A+5B)\sinh(x)}{16a(a+a\cosh(x))^{3/2}}$$

output
```
1/4*(A-B)*sinh(x)/(a+a*cosh(x))^(5/2)+1/16*(3*A+5*B)*sinh(x)/a/(a+a*cosh(x
))^(3/2)+1/32*(3*A+5*B)*arctan(1/2*sinh(x)*a^(1/2)*2^(1/2)/(a+a*cosh(x))^(
1/2))/a^(5/2)*2^(1/2)
```

3.103.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.61

$$\int \frac{A+B\cosh(x)}{(a+a\cosh(x))^{5/2}}\,dx = \frac{4(3A+5B)\arctan\left(\sinh\left(\frac{x}{2}\right)\right)\cosh^5\left(\frac{x}{2}\right)+(7A+B+(3A+5B)\cosh(x))\sinh(x)}{16(a(1+\cosh(x)))^{5/2}}$$

input
```
Integrate[(A + B*Cosh[x])/(a + a*Cosh[x])^(5/2),x]
```

output
```
(4*(3*A + 5*B)*ArcTan[Sinh[x/2]]*Cosh[x/2]^5 + (7*A + B + (3*A + 5*B)*Cosh
[x])*Sinh[x])/(16*(a*(1 + Cosh[x]))^(5/2))
```

3.103.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3042, 3229, 3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{(a\cosh(x)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{\left(a+a\sin\left(\frac{\pi}{2}+ix\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3229}$$

$$\frac{(3A+5B)\int \frac{1}{(\cosh(x)a+a)^{3/2}}dx}{8a}+\frac{(A-B)\sinh(x)}{4(a\cosh(x)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-B)\sinh(x)}{4(a\cosh(x)+a)^{5/2}}+\frac{(3A+5B)\int \frac{1}{\left(\sin\left(ix+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a}$$

$$\Big\downarrow \text{3129}$$

$$\frac{(3A+5B)\left(\frac{\int \frac{1}{\sqrt{\cosh(x)a+a}}dx}{4a}+\frac{\sinh(x)}{2(a\cosh(x)+a)^{3/2}}\right)}{8a}+\frac{(A-B)\sinh(x)}{4(a\cosh(x)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-B)\sinh(x)}{4(a\cosh(x)+a)^{5/2}}+\frac{(3A+5B)\left(\frac{\sinh(x)}{2(a\cosh(x)+a)^{3/2}}+\frac{\int \frac{1}{\sqrt{\sin\left(ix+\frac{\pi}{2}\right)a+a}}dx}{4a}\right)}{8a}$$

$$\Big\downarrow \text{3128}$$

$$\frac{(A-B)\sinh(x)}{4(a\cosh(x)+a)^{5/2}}+\frac{(3A+5B)\left(\frac{\sinh(x)}{2(a\cosh(x)+a)^{3/2}}+\frac{i\int \frac{1}{\frac{a^2\sinh^2(x)}{\cosh(x)a+a}+2a}d\left(-\frac{ia\sinh(x)}{\sqrt{\cosh(x)a+a}}\right)}{2a}\right)}{8a}$$

$$\Big\downarrow \text{219}$$

$$\frac{(3A+5B)\left(\frac{\arctan\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a\cosh(x)+a}}\right)}{2\sqrt{2}a^{3/2}}+\frac{\sinh(x)}{2(a\cosh(x)+a)^{3/2}}\right)}{8a}+\frac{(A-B)\sinh(x)}{4(a\cosh(x)+a)^{5/2}}$$

input
```
Int[(A + B*Cosh[x])/(a + a*Cosh[x])^(5/2),x]
```

output
```
((A - B)*Sinh[x])/(4*(a + a*Cosh[x])^(5/2)) + ((3*A + 5*B)*(ArcTan[(Sqrt[a
]*Sinh[x])/(Sqrt[2]*Sqrt[a + a*Cosh[x]])]/(2*Sqrt[2]*a^(3/2)) + Sinh[x]/(2
*(a + a*Cosh[x])^(3/2))))/(8*a)
```

3.103.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

3.103.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. 2(74) = 148.

Time = 0.51 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.25

method	result
default	$-\frac{\sqrt{\sinh\left(\frac{x}{2}\right)^{2} a}\,\left(3 A \ln\left(\frac{2 \sqrt{\sinh\left(\frac{x}{2}\right)^{2} a}\,\sqrt{-a}-2 a}{\cosh\left(\frac{x}{2}\right)}\right) \cosh\left(\frac{x}{2}\right)^{4} a+5 B \ln\left(\frac{2 \sqrt{\sinh\left(\frac{x}{2}\right)^{2} a}\,\sqrt{-a}-2 a}{\cosh\left(\frac{x}{2}\right)}\right) \cosh\left(\frac{x}{2}\right)^{4} a-3 A \sqrt{\sinh\left(\frac{x}{2}\right)^{2} a}\,\cosh\left(\frac{x}{2}\right)}{32 \cosh\left(\frac{x}{2}\right)^{3} a^{3} \sqrt{-a}\,\sinh\left(\frac{x}{2}\right) \sqrt{a \cosh\left(\frac{x}{2}\right)}}$
parts	$-\frac{A \sqrt{\sinh\left(\frac{x}{2}\right)^{2} a}\,\left(3 \ln\left(\frac{2 \sqrt{\sinh\left(\frac{x}{2}\right)^{2} a}\,\sqrt{-a}-2 a}{\cosh\left(\frac{x}{2}\right)}\right) a \cosh\left(\frac{x}{2}\right)^{4}-3 \sqrt{\sinh\left(\frac{x}{2}\right)^{2} a}\,\cosh\left(\frac{x}{2}\right)^{2} \sqrt{-a}-2 \sqrt{\sinh\left(\frac{x}{2}\right)^{2} a}\,\sqrt{-a}\right) \sqrt{2}}{32 a^{3} \cosh\left(\frac{x}{2}\right)^{3} \sqrt{-a}\,\sinh\left(\frac{x}{2}\right) \sqrt{a \cosh\left(\frac{x}{2}\right)^{2}}}-\frac{B \sqrt{\sinh}}{}$

input
```
int((A+B*cosh(x))/(a+a*cosh(x))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32*(sinh(1/2*x)^2*a)^(1/2)*(3*A*ln(2/cosh(1/2*x)*((sinh(1/2*x)^2*a)^(1/
2)*(-a)^(1/2)-a))*cosh(1/2*x)^4*a+5*B*ln(2/cosh(1/2*x)*((sinh(1/2*x)^2*a)^
(1/2)*(-a)^(1/2)-a))*cosh(1/2*x)^4*a-3*A*(sinh(1/2*x)^2*a)^(1/2)*cosh(1/2*
x)^2*(-a)^(1/2)-5*B*(sinh(1/2*x)^2*a)^(1/2)*(-a)^(1/2)*cosh(1/2*x)^2-2*A*(
-a)^(1/2)*(sinh(1/2*x)^2*a)^(1/2)+2*B*(sinh(1/2*x)^2*a)^(1/2)*(-a)^(1/2))/
cosh(1/2*x)^3/a^3/(-a)^(1/2)/sinh(1/2*x)*2^(1/2)/(a*cosh(1/2*x)^2)^(1/2)
```

3.103.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 509 vs. 2(74) = 148.

Time = 0.28 (sec) , antiderivative size = 509, normalized size of antiderivative = 5.47

$$\int \frac{A+B\cosh(x)}{(a+a\cosh(x))^{5/2}}\,dx =$$

$$-\frac{\sqrt{2}\left((3\,A+5\,B)\cosh\left(x\right)^{4}+(3\,A+5\,B)\sinh\left(x\right)^{4}+4\,(3\,A+5\,B)\cosh\left(x\right)^{3}+4\,((3\,A+5\,B)\cosh\left(x\right)+3\,A\right.}{}$$

input
```
integrate((A+B*cosh(x))/(a+a*cosh(x))^(5/2),x, algorithm="fricas")
```

output
```
-1/16*(sqrt(2)*((3*A + 5*B)*cosh(x)^4 + (3*A + 5*B)*sinh(x)^4 + 4*(3*A + 5
*B)*cosh(x)^3 + 4*((3*A + 5*B)*cosh(x) + 3*A + 5*B)*sinh(x)^3 + 6*(3*A + 5
*B)*cosh(x)^2 + 6*((3*A + 5*B)*cosh(x)^2 + 2*(3*A + 5*B)*cosh(x) + 3*A + 5
*B)*sinh(x)^2 + 4*(3*A + 5*B)*cosh(x) + 4*((3*A + 5*B)*cosh(x)^3 + 3*(3*A
+ 5*B)*cosh(x)^2 + 3*(3*A + 5*B)*cosh(x) + 3*A + 5*B)*sinh(x) + 3*A + 5*B)
*sqrt(a)*arctan(sqrt(2)*sqrt(1/2)*sqrt(a/(cosh(x) + sinh(x)))/sqrt(a)) - 2
*sqrt(1/2)*((3*A + 5*B)*cosh(x)^4 + (3*A + 5*B)*sinh(x)^4 + (11*A - 3*B)*c
osh(x)^3 + (4*(3*A + 5*B)*cosh(x) + 11*A - 3*B)*sinh(x)^3 - (11*A - 3*B)*c
osh(x)^2 + (6*(3*A + 5*B)*cosh(x)^2 + 3*(11*A - 3*B)*cosh(x) - 11*A + 3*B)
*sinh(x)^2 - (3*A + 5*B)*cosh(x) + (4*(3*A + 5*B)*cosh(x)^3 + 3*(11*A - 3*
B)*cosh(x)^2 - 2*(11*A - 3*B)*cosh(x) - 3*A - 5*B)*sinh(x))*sqrt(a/(cosh(x
) + sinh(x))))/(a^3*cosh(x)^4 + a^3*sinh(x)^4 + 4*a^3*cosh(x)^3 + 6*a^3*co
sh(x)^2 + 4*a^3*cosh(x) + 4*(a^3*cosh(x) + a^3)*sinh(x)^3 + a^3 + 6*(a^3*c
osh(x)^2 + 2*a^3*cosh(x) + a^3)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + 3*a^3*cosh(
x)^2 + 3*a^3*cosh(x) + a^3)*sinh(x))
```

3.103.6 Sympy [F]

$$\int \frac{A+B\cosh(x)}{(a+a\cosh(x))^{5/2}}\,dx=\int \frac{A+B\cosh{(x)}}{(a(\cosh{(x)}+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cosh(x))/(a+a*cosh(x))**(5/2),x)
```

output
```
Integral((A + B*cosh(x))/(a*(cosh(x) + 1))**(5/2), x)
```

3.103.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(74) = 148$.

Time = 0.39 (sec) , antiderivative size = 427, normalized size of antiderivative = 4.59

$$\int \frac{A+B\cosh(x)}{(a+a\cosh(x))^{5/2}}\,dx=\frac{1}{80}\left(\sqrt{2}\left(\frac{15\,e^{\left(\frac{9}{2}\,x\right)}+70\,e^{\left(\frac{7}{2}\,x\right)}+128\,e^{\left(\frac{5}{2}\,x\right)}-70\,e^{\left(\frac{3}{2}\,x\right)}-15\,e^{\left(\frac{1}{2}\,x\right)}}{a^{\frac{5}{2}}e^{(5\,x)}+5\,a^{\frac{5}{2}}e^{(4\,x)}+10\,a^{\frac{5}{2}}e^{(3\,x)}+10\,a^{\frac{5}{2}}e^{(2\,x)}+5\,a^{\frac{5}{2}}e^{x}+a^{\frac{5}{2}}}+\frac{15\text{ arct}}{}\right.\right.$$

$$+\frac{1}{672}\left(\sqrt{2}\left(\frac{105\,e^{\left(\frac{9}{2}\,x\right)}+490\,e^{\left(\frac{7}{2}\,x\right)}+896\,e^{\left(\frac{5}{2}\,x\right)}+790\,e^{\left(\frac{3}{2}\,x\right)}-105\,e^{\left(\frac{1}{2}\,x\right)}}{a^{\frac{5}{2}}e^{(5\,x)}+5\,a^{\frac{5}{2}}e^{(4\,x)}+10\,a^{\frac{5}{2}}e^{(3\,x)}+10\,a^{\frac{5}{2}}e^{(2\,x)}+5\,a^{\frac{5}{2}}e^{x}+a^{\frac{5}{2}}}+\frac{105\arctan\left(e^{\left(\frac{1}{2}\,x\right)}\right)}{a^{\frac{5}{2}}}\right)+7\sqrt{2}\left(\right.\right.$$

input
```
integrate((A+B*cosh(x))/(a+a*cosh(x))^(5/2),x, algorithm="maxima")
```

output
```
1/80*(sqrt(2)*((15*e^(9/2*x) + 70*e^(7/2*x) + 128*e^(5/2*x) - 70*e^(3/2*x)
 - 15*e^(1/2*x))/(a^(5/2)*e^(5*x) + 5*a^(5/2)*e^(4*x) + 10*a^(5/2)*e^(3*x)
 + 10*a^(5/2)*e^(2*x) + 5*a^(5/2)*e^x + a^(5/2)) + 15*arctan(e^(1/2*x))/a^
(5/2)) - 128*sqrt(2)*e^(5/2*x)/(a^(5/2)*e^(5*x) + 5*a^(5/2)*e^(4*x) + 10*a
^(5/2)*e^(3*x) + 10*a^(5/2)*e^(2*x) + 5*a^(5/2)*e^x + a^(5/2)))*A + 1/672*
(sqrt(2)*((105*e^(9/2*x) + 490*e^(7/2*x) + 896*e^(5/2*x) + 790*e^(3/2*x) -
 105*e^(1/2*x))/(a^(5/2)*e^(5*x) + 5*a^(5/2)*e^(4*x) + 10*a^(5/2)*e^(3*x)
+ 10*a^(5/2)*e^(2*x) + 5*a^(5/2)*e^x + a^(5/2)) + 105*arctan(e^(1/2*x))/a^
(5/2)) + 7*sqrt(2)*((15*e^(9/2*x) + 70*e^(7/2*x) - 128*e^(5/2*x) - 70*e^(3
/2*x) - 15*e^(1/2*x))/(a^(5/2)*e^(5*x) + 5*a^(5/2)*e^(4*x) + 10*a^(5/2)*e^
(3*x) + 10*a^(5/2)*e^(2*x) + 5*a^(5/2)*e^x + a^(5/2)) + 15*arctan(e^(1/2*x
))/a^(5/2)) - 128*(7*sqrt(2)*sqrt(a)*e^(7/2*x) + 3*sqrt(2)*sqrt(a)*e^(3/2*
x))/(a^3*e^(5*x) + 5*a^3*e^(4*x) + 10*a^3*e^(3*x) + 10*a^3*e^(2*x) + 5*a^3
*e^x + a^3))*B
```

3.103.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.27

$$\int \frac{A+B\cosh(x)}{(a+a\cosh(x))^{5/2}}\,dx = \frac{\sqrt{2}(3A+5B)\arctan\left(e^{\left(\frac{1}{2}x\right)}\right)}{16\,a^{\frac{5}{2}}} + \frac{\sqrt{2}\left(3\,Aa^{\frac{7}{2}}e^{\left(\frac{7}{2}x\right)}+5\,Ba^{\frac{7}{2}}e^{\left(\frac{7}{2}x\right)}+11\,Aa^{\frac{7}{2}}e^{\left(\frac{5}{2}x\right)}-3\,Ba^{\frac{7}{2}}e^{\left(\frac{5}{2}x\right)}-11\,Aa^{\frac{7}{2}}e^{\left(\frac{3}{2}x\right)}+3\,Ba^{\frac{7}{2}}e^{\left(\frac{3}{2}x\right)}-3\,Aa^{\frac{7}{2}}e^{\left(\frac{1}{2}x\right)}-\right.}{16\,(ae^{x}+a)^{4}a^{2}}$$

input
```
integrate((A+B*cosh(x))/(a+a*cosh(x))^(5/2),x, algorithm="giac")
```

output
```
1/16*sqrt(2)*(3*A + 5*B)*arctan(e^(1/2*x))/a^(5/2) + 1/16*sqrt(2)*(3*A*a^(
7/2)*e^(7/2*x) + 5*B*a^(7/2)*e^(7/2*x) + 11*A*a^(7/2)*e^(5/2*x) - 3*B*a^(7
/2)*e^(5/2*x) - 11*A*a^(7/2)*e^(3/2*x) + 3*B*a^(7/2)*e^(3/2*x) - 3*A*a^(7/
2)*e^(1/2*x) - 5*B*a^(7/2)*e^(1/2*x))/((a*e^x + a)^4*a^2)
```

3.103.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)}{(a+a\cosh(x))^{5/2}}\,dx = \int \frac{A+B\cosh(x)}{\left(a+a\cosh\left(x\right)\right)^{5/2}}\,dx$$

input
```
int((A + B*cosh(x))/(a + a*cosh(x))^(5/2),x)
```

output
```
int((A + B*cosh(x))/(a + a*cosh(x))^(5/2), x)
```

3.104 $\int \frac{A+B\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx$

3.104.1 Optimal result

Integrand size = 18, antiderivative size = 57

$$\int \frac{A+B\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx = -\frac{\sqrt{2}(A+B)\arctan\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a-a\cosh(x)}}\right)}{\sqrt{a}} + \frac{2B\sinh(x)}{\sqrt{a-a\cosh(x)}}$$

output
```
-(A+B)*arctan(1/2*sinh(x)*a^(1/2)*2^(1/2)/(a-a*cosh(x))^(1/2))*2^(1/2)/a^(
1/2)+2*B*sinh(x)/(a-a*cosh(x))^(1/2)
```

3.104.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{A+B\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx = \frac{2\left(2B\cosh\left(\frac{x}{2}\right)-(A+B)\left(\log\left(\cosh\left(\frac{x}{4}\right)\right)-\log\left(\sinh\left(\frac{x}{4}\right)\right)\right)\right)\sinh\left(\frac{x}{2}\right)}{\sqrt{a-a\cosh(x)}}$$

input
```
Integrate[(A + B*Cosh[x])/Sqrt[a - a*Cosh[x]],x]
```

output
```
(2*(2*B*Cosh[x/2] - (A + B)*(Log[Cosh[x/4]] - Log[Sinh[x/4]]))*Sinh[x/2])/
Sqrt[a - a*Cosh[x]]
```

3.104.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{\sqrt{a-a\cosh(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{\sqrt{a-a\sin\left(\frac{\pi}{2}+ix\right)}}dx$$

$$\Big\downarrow \text{3230}$$

$$(A+B)\int \frac{1}{\sqrt{a-a\cosh(x)}}dx+\frac{2B\sinh(x)}{\sqrt{a-a\cosh(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2B\sinh(x)}{\sqrt{a-a\cosh(x)}}+(A+B)\int \frac{1}{\sqrt{a-a\sin\left(ix+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3128}$$

$$\frac{2B\sinh(x)}{\sqrt{a-a\cosh(x)}}+2i(A+B)\int \frac{1}{\frac{a^2\sinh^2(x)}{a-a\cosh(x)}+2a}d\frac{ia\sinh(x)}{\sqrt{a-a\cosh(x)}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2B\sinh(x)}{\sqrt{a-a\cosh(x)}}-\frac{\sqrt{2}(A+B)\arctan\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a-a\cosh(x)}}\right)}{\sqrt{a}}$$

input
```
Int[(A + B*Cosh[x])/Sqrt[a - a*Cosh[x]],x]
```

output
```
-((Sqrt[2]*(A + B)*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a - a*Cosh[x]])]
)/Sqrt[a]) + (2*B*Sinh[x])/Sqrt[a - a*Cosh[x]]
```

3.104.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

3.104.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.11

method	result	size
default	$\frac{\sinh\left(\frac{x}{2}\right)\left(\ln\left(\cosh\left(\frac{x}{2}\right)-1\right)A-\ln\left(\cosh\left(\frac{x}{2}\right)+1\right)A+\ln\left(\cosh\left(\frac{x}{2}\right)-1\right)B-\ln\left(\cosh\left(\frac{x}{2}\right)+1\right)B+4B\cosh\left(\frac{x}{2}\right)\right)}{\sqrt{-2\sinh\left(\frac{x}{2}\right)^{2}a}}$	63
parts	$-\frac{2A\sinh\left(\frac{x}{2}\right)\operatorname{arctanh}\left(\cosh\left(\frac{x}{2}\right)\right)}{\sqrt{-2\sinh\left(\frac{x}{2}\right)^{2}a}}+\frac{B\sinh\left(\frac{x}{2}\right)\left(4\cosh\left(\frac{x}{2}\right)+\ln\left(\cosh\left(\frac{x}{2}\right)-1\right)-\ln\left(\cosh\left(\frac{x}{2}\right)+1\right)\right)}{\sqrt{-2\sinh\left(\frac{x}{2}\right)^{2}a}}$	65

input
```
int((A+B*cosh(x))/(a-a*cosh(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
sinh(1/2*x)*(ln(cosh(1/2*x)-1)*A-ln(cosh(1/2*x)+1)*A+ln(cosh(1/2*x)-1)*B-l
n(cosh(1/2*x)+1)*B+4*B*cosh(1/2*x))/(-2*sinh(1/2*x)^2*a)^(1/2)
```

3.104.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(46) = 92$.

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.74

$$\int \frac{A+B\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx = \frac{\sqrt{2}(A+B)a\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a}{\cosh(x)+\sinh(x)}}\sqrt{-\frac{1}{a}}(\cosh(x)+\sinh(x))-\cosh(x)-\sinh(x)-1}{\cosh(x)+\sinh(x)-1}\right) - 2\sqrt{\frac{1}{2}}(B\cosh(x)+B}{a}$$

input
```
integrate((A+B*cosh(x))/(a-a*cosh(x))^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(2)*(A + B)*a*sqrt(-1/a)*log((2*sqrt(2)*sqrt(1/2)*sqrt(-a/(cosh(x) +
sinh(x)))*sqrt(-1/a)*(cosh(x) + sinh(x)) - cosh(x) - sinh(x) - 1)/(cosh(x)
 + sinh(x) - 1)) - 2*sqrt(1/2)*(B*cosh(x) + B*sinh(x) + B)*sqrt(-a/(cosh(x
) + sinh(x))))/a
```

3.104.6 Sympy [F]

$$\int \frac{A+B\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx = \int \frac{A+B\cosh{(x)}}{\sqrt{-a(\cosh{(x)}-1)}}\,dx$$

input
```
integrate((A+B*cosh(x))/(a-a*cosh(x))**(1/2),x)
```

output
```
Integral((A + B*cosh(x))/sqrt(-a*(cosh(x) - 1)), x)
```

3.104.7 Maxima [F]

$$\int \frac{A+B\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx = \int \frac{B\cosh{(x)}+A}{\sqrt{-a\cosh{(x)}+a}}\,dx$$

input
```
integrate((A+B*cosh(x))/(a-a*cosh(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*cosh(x) + A)/sqrt(-a*cosh(x) + a), x)
```

3.104.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.47

$$\int \frac{A+B\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx=-\frac{2\left(\sqrt{2}A+\sqrt{2}B\right)\arctan\left(\frac{\sqrt{-ae^x}}{\sqrt{a}}\right)}{\sqrt{a}\mathrm{sgn}\left(-e^x+1\right)}-\frac{\sqrt{2}B}{\sqrt{-ae^x}\mathrm{sgn}\left(-e^x+1\right)}+\frac{\sqrt{2}\sqrt{-ae^x}B}{a\mathrm{sgn}\left(-e^x+1\right)}$$

input
```
integrate((A+B*cosh(x))/(a-a*cosh(x))^(1/2),x, algorithm="giac")
```

output
```
-2*(sqrt(2)*A + sqrt(2)*B)*arctan(sqrt(-a*e^x)/sqrt(a))/(sqrt(a)*sgn(-e^x
+ 1)) - sqrt(2)*B/(sqrt(-a*e^x)*sgn(-e^x + 1)) + sqrt(2)*sqrt(-a*e^x)*B/(a
*sgn(-e^x + 1))
```

3.104.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx=\int \frac{A+B\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx$$

input
```
int((A + B*cosh(x))/(a - a*cosh(x))^(1/2),x)
```

output
```
int((A + B*cosh(x))/(a - a*cosh(x))^(1/2), x)
```

3.105 $\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{3/2}}\,dx$

3.105.1 Optimal result

Integrand size = 18, antiderivative size = 65

$$\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{3/2}}\,dx=-\frac{(A-3B)\arctan\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a-a\cosh(x)}}\right)}{2\sqrt{2}a^{3/2}}-\frac{(A+B)\sinh(x)}{2(a-a\cosh(x))^{3/2}}$$

output
```
-1/2*(A+B)*sinh(x)/(a-a*cosh(x))^(3/2)-1/4*(A-3*B)*arctan(1/2*sinh(x)*a^(1
/2)*2^(1/2)/(a-a*cosh(x))^(1/2))/a^(3/2)*2^(1/2)
```

3.105.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.25

$$\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{3/2}}\,dx=\frac{\left((A+B)\text{csch}^2\left(\frac{x}{4}\right)-4(A-3B)\left(\log\left(\cosh\left(\frac{x}{4}\right)\right)-\log\left(\sinh\left(\frac{x}{4}\right)\right)\right)+(A+B)\text{sec}}{4a(-1+\cosh(x))\sqrt{a-a\cosh(x)}}$$

input
```
Integrate[(A + B*Cosh[x])/(a - a*Cosh[x])^(3/2),x]
```

output
```
(((A + B)*Csch[x/4]^2 - 4*(A - 3*B)*(Log[Cosh[x/4]] - Log[Sinh[x/4]]) + (A
 + B)*Sech[x/4]^2)*Sinh[x/2]^3)/(4*a*(-1 + Cosh[x])*Sqrt[a - a*Cosh[x]])
```

3.105.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3042, 3229, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{\left(a-a\sin\left(\frac{\pi}{2}+ix\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3229}$$

$$\frac{(A-3B)\int \frac{1}{\sqrt{a-a\cosh(x)}}dx}{4a}-\frac{(A+B)\sinh(x)}{2(a-a\cosh(x))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(A+B)\sinh(x)}{2(a-a\cosh(x))^{3/2}}+\frac{(A-3B)\int \frac{1}{\sqrt{a-a\sin\left(ix+\frac{\pi}{2}\right)}}dx}{4a}$$

$$\Big\downarrow \text{3128}$$

$$-\frac{(A+B)\sinh(x)}{2(a-a\cosh(x))^{3/2}}+\frac{i(A-3B)\int \frac{1}{\frac{a^2\sinh^2(x)}{a-a\cosh(x)}+2a}d\frac{ia\sinh(x)}{\sqrt{a-a\cosh(x)}}}{2a}$$

$$\Big\downarrow \text{219}$$

$$-\frac{(A-3B)\arctan\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a-a\cosh(x)}}\right)}{2\sqrt{2}a^{3/2}}-\frac{(A+B)\sinh(x)}{2(a-a\cosh(x))^{3/2}}$$

input
```
Int[(A + B*Cosh[x])/(a - a*Cosh[x])^(3/2),x]
```

output
```
-1/2*((A - 3*B)*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a - a*Cosh[x]])])/(
Sqrt[2]*a^(3/2)) - ((A + B)*Sinh[x])/(2*(a - a*Cosh[x])^(3/2))
```

3.105.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

3.105.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.28

method	result
default	$\frac{\cosh\left(\frac{x}{2}\right)(2A+2B)+\left(\ln\left(\cosh\left(\frac{x}{2}\right)-1\right)A-\ln\left(\cosh\left(\frac{x}{2}\right)+1\right)A-3\ln\left(\cosh\left(\frac{x}{2}\right)-1\right)B+3\ln\left(\cosh\left(\frac{x}{2}\right)+1\right)B\right)\sinh\left(\frac{x}{2}\right)^{2}}{4a\sinh\left(\frac{x}{2}\right)\sqrt{-2\sinh\left(\frac{x}{2}\right)^{2}a}}$
parts	$\frac{A\left(2\cosh\left(\frac{x}{2}\right)+\left(\ln\left(\cosh\left(\frac{x}{2}\right)-1\right)-\ln\left(\cosh\left(\frac{x}{2}\right)+1\right)\right)\sinh\left(\frac{x}{2}\right)^{2}\right)}{4a\sinh\left(\frac{x}{2}\right)\sqrt{-2\sinh\left(\frac{x}{2}\right)^{2}a}}-\frac{B\left(-2\cosh\left(\frac{x}{2}\right)+\left(3\ln\left(\cosh\left(\frac{x}{2}\right)-1\right)-3\ln\left(\cosh\left(\frac{x}{2}\right)+1\right)\right)\sinh\left(\frac{x}{2}\right)^{2}\right)}{4a\sinh\left(\frac{x}{2}\right)\sqrt{-2\sinh\left(\frac{x}{2}\right)^{2}a}}$

input
```
int((A+B*cosh(x))/(a-a*cosh(x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/a*(cosh(1/2*x)*(2*A+2*B)+(ln(cosh(1/2*x)-1)*A-ln(cosh(1/2*x)+1)*A-3*ln
(cosh(1/2*x)-1)*B+3*ln(cosh(1/2*x)+1)*B)*sinh(1/2*x)^2)/sinh(1/2*x)/(-2*si
nh(1/2*x)^2*a)^(1/2)
```

3.105.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(50) = 100$.

Time = 0.26 (sec) , antiderivative size = 217, normalized size of antiderivative = 3.34

$$\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{3/2}}\,dx = \frac{\sqrt{2}\left((A-3\,B)\cosh\left(x\right)^{2}+(A-3\,B)\sinh\left(x\right)^{2}-2\,(A-3\,B)\cosh\left(x\right)+2\,((A-3}{}$$

input
```
integrate((A+B*cosh(x))/(a-a*cosh(x))^(3/2),x, algorithm="fricas")
```

output
```
1/4*(sqrt(2)*((A - 3*B)*cosh(x)^2 + (A - 3*B)*sinh(x)^2 - 2*(A - 3*B)*cosh
(x) + 2*((A - 3*B)*cosh(x) - A + 3*B)*sinh(x) + A - 3*B)*sqrt(-a)*log((2*s
qrt(2)*sqrt(1/2)*sqrt(-a)*sqrt(-a/(cosh(x) + sinh(x)))*(cosh(x) + sinh(x))
 - a*cosh(x) - a*sinh(x) - a)/(cosh(x) + sinh(x) - 1)) - 4*sqrt(1/2)*((A +
 B)*cosh(x)^2 + (A + B)*sinh(x)^2 + (A + B)*cosh(x) + (2*(A + B)*cosh(x) +
 A + B)*sinh(x))*sqrt(-a/(cosh(x) + sinh(x))))/(a^2*cosh(x)^2 + a^2*sinh(x
)^2 - 2*a^2*cosh(x) + a^2 + 2*(a^2*cosh(x) - a^2)*sinh(x))
```

3.105.6 Sympy [F]

$$\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{3/2}}\,dx = \int \frac{A+B\cosh\left(x\right)}{\left(-a\left(\cosh\left(x\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cosh(x))/(a-a*cosh(x))**(3/2),x)
```

output
```
Integral((A + B*cosh(x))/(-a*(cosh(x) - 1))**(3/2), x)
```

3.105.7 Maxima [F]

$$\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{3/2}}\,dx = \int \frac{B\cosh\left(x\right)+A}{\left(-a\cosh\left(x\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cosh(x))/(a-a*cosh(x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*cosh(x) + A)/(-a*cosh(x) + a)^(3/2), x)
```

3.105.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(50) = 100$.

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.71

$$\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{3/2}}\,dx = -\frac{\left(\sqrt{2}A-3\sqrt{2}B\right)\arctan\left(\frac{\sqrt{-ae^x}}{\sqrt{a}}\right)}{2\,a^{\frac{3}{2}}\mathrm{sgn}\left(-e^x+1\right)} + \frac{\sqrt{2}\left(\sqrt{-ae^x}Aae^x+\sqrt{-ae^x}Bae^x+\sqrt{-ae^x}Aa+\sqrt{-ae^x}Ba\right)}{2\left(ae^x-a\right)^2 a\mathrm{sgn}\left(-e^x+1\right)}$$

input
```
integrate((A+B*cosh(x))/(a-a*cosh(x))^(3/2),x, algorithm="giac")
```

output
```
-1/2*(sqrt(2)*A - 3*sqrt(2)*B)*arctan(sqrt(-a*e^x)/sqrt(a))/(a^(3/2)*sgn(-
e^x + 1)) + 1/2*sqrt(2)*(sqrt(-a*e^x)*A*a*e^x + sqrt(-a*e^x)*B*a*e^x + sqr
t(-a*e^x)*A*a + sqrt(-a*e^x)*B*a)/((a*e^x - a)^2*a*sgn(-e^x + 1))
```

3.105.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{3/2}}\,dx = \int \frac{A+B\cosh(x)}{\left(a-a\cosh\left(x\right)\right)^{3/2}}\,dx$$

input
```
int((A + B*cosh(x))/(a - a*cosh(x))^(3/2),x)
```

output
```
int((A + B*cosh(x))/(a - a*cosh(x))^(3/2), x)
```

3.106 $\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{5/2}}\,dx$

3.106.1 Optimal result

Integrand size = 18, antiderivative size = 94

$$\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{5/2}}\,dx = -\frac{(3A-5B)\arctan\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a-a\cosh(x)}}\right)}{16\sqrt{2}a^{5/2}} - \frac{(A+B)\sinh(x)}{4(a-a\cosh(x))^{5/2}} - \frac{(3A-5B)\sinh(x)}{16a(a-a\cosh(x))^{3/2}}$$

output
```
-1/4*(A+B)*sinh(x)/(a-a*cosh(x))^(5/2)-1/16*(3*A-5*B)*sinh(x)/a/(a-a*cosh(
x))^(3/2)-1/32*(3*A-5*B)*arctan(1/2*sinh(x)*a^(1/2)*2^(1/2)/(a-a*cosh(x))^
(1/2))/a^(5/2)*2^(1/2)
```

3.106.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.26

$$\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{5/2}}\,dx = \frac{\left(2(3A-5B)\text{csch}^2\left(\frac{x}{4}\right) - (A+B)\text{csch}^4\left(\frac{x}{4}\right) - 8(3A-5B)\left(\log\left(\cosh\left(\frac{x}{4}\right)\right) - \log\left(\right.\right.\right.}{32a^2(-1+\cosh(x))^2\sqrt{a-}}$$

input
```
Integrate[(A + B*Cosh[x])/(a - a*Cosh[x])^(5/2),x]
```

output
```
((2*(3*A - 5*B)*Csch[x/4]^2 - (A + B)*Csch[x/4]^4 - 8*(3*A - 5*B)*(Log[Cos
h[x/4]] - Log[Sinh[x/4]]) + 2*(3*A - 5*B)*Sech[x/4]^2 + (A + B)*Sech[x/4]^
4)*Sinh[x/2]^5)/(32*a^2*(-1 + Cosh[x])^2*Sqrt[a - a*Cosh[x]])
```

3.106.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 3229, 3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{\left(a-a\sin\left(\frac{\pi}{2}+ix\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3229}$$

$$\frac{(3A-5B)\int \frac{1}{(a-a\cosh(x))^{3/2}}dx}{8a}-\frac{(A+B)\sinh(x)}{4(a-a\cosh(x))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(A+B)\sinh(x)}{4(a-a\cosh(x))^{5/2}}+\frac{(3A-5B)\int \frac{1}{\left(a-a\sin\left(ix+\frac{\pi}{2}\right)\right)^{3/2}}dx}{8a}$$

$$\Big\downarrow \text{3129}$$

$$\frac{(3A-5B)\left(\frac{\int \frac{1}{\sqrt{a-a\cosh(x)}}dx}{4a}-\frac{\sinh(x)}{2(a-a\cosh(x))^{3/2}}\right)}{8a}-\frac{(A+B)\sinh(x)}{4(a-a\cosh(x))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(A+B)\sinh(x)}{4(a-a\cosh(x))^{5/2}}+\frac{(3A-5B)\left(-\frac{\sinh(x)}{2(a-a\cosh(x))^{3/2}}+\frac{\int \frac{1}{\sqrt{a-a\sin\left(ix+\frac{\pi}{2}\right)}}dx}{4a}\right)}{8a}$$

$$\Big\downarrow \text{3128}$$

$$-\frac{(A+B)\sinh(x)}{4(a-a\cosh(x))^{5/2}}+\frac{(3A-5B)\left(-\frac{\sinh(x)}{2(a-a\cosh(x))^{3/2}}+\frac{i\int \frac{1}{\frac{a^2\sinh^2(x)}{a-a\cosh(x)}+2a}d\frac{ia\sinh(x)}{\sqrt{a-a\cosh(x)}}}{2a}\right)}{8a}$$

$$\Big\downarrow \text{219}$$

$$\frac{(3A-5B)\left(-\frac{\arctan\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a-a\cosh(x)}}\right)}{2\sqrt{2}a^{3/2}}-\frac{\sinh(x)}{2(a-a\cosh(x))^{3/2}}\right)}{8a}-\frac{(A+B)\sinh(x)}{4(a-a\cosh(x))^{5/2}}$$

input
```
Int[(A + B*Cosh[x])/(a - a*Cosh[x])^(5/2),x]
```

output
```
-1/4*((A + B)*Sinh[x])/(a - a*Cosh[x])^(5/2) + ((3*A - 5*B)*(-1/2*ArcTan[(
Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a - a*Cosh[x]])]/(Sqrt[2]*a^(3/2)) - Sinh[x
]/(2*(a - a*Cosh[x])^(3/2))))/(8*a)
```

3.106.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

3.106.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.26

method	result
default	$\frac{(6A-10B)\cosh\left(\frac{x}{2}\right)\sinh\left(\frac{x}{2}\right)^{2}+(-4A-4B)\cosh\left(\frac{x}{2}\right)+\left(3\ln\left(\cosh\left(\frac{x}{2}\right)-1\right)A-3\ln\left(\cosh\left(\frac{x}{2}\right)+1\right)A-5\ln\left(\cosh\left(\frac{x}{2}\right)-1\right)B+5\ln\left(\cosh\left(\frac{x}{2}\right)+\right.\right.}{32a^{2}\left(\cosh\left(\frac{x}{2}\right)+1\right)\left(\cosh\left(\frac{x}{2}\right)-1\right)\sinh\left(\frac{x}{2}\right)\sqrt{-2\sinh\left(\frac{x}{2}\right)^{2}a}}$
parts	$\frac{A\left(6\sinh\left(\frac{x}{2}\right)^{2}\cosh\left(\frac{x}{2}\right)-4\cosh\left(\frac{x}{2}\right)+\left(3\ln\left(\cosh\left(\frac{x}{2}\right)-1\right)-3\ln\left(\cosh\left(\frac{x}{2}\right)+1\right)\right)\sinh\left(\frac{x}{2}\right)^{4}\right)}{32a^{2}\left(\cosh\left(\frac{x}{2}\right)+1\right)\left(\cosh\left(\frac{x}{2}\right)-1\right)\sinh\left(\frac{x}{2}\right)\sqrt{-2\sinh\left(\frac{x}{2}\right)^{2}a}}-\frac{B\left(10\sinh\left(\frac{x}{2}\right)^{2}\cosh\left(\frac{x}{2}\right)+4\cosh\left(\frac{x}{2}\right)+(5\ln\right.}{32a^{2}\left(\cosh\left(\frac{x}{2}\right)+1\right)\left(\cosh\left(\frac{x}{2}\right.\right.}$

input
```
int((A+B*cosh(x))/(a-a*cosh(x))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/a^2*((6*A-10*B)*cosh(1/2*x)*sinh(1/2*x)^2+(-4*A-4*B)*cosh(1/2*x)+(3*l
n(cosh(1/2*x)-1)*A-3*ln(cosh(1/2*x)+1)*A-5*ln(cosh(1/2*x)-1)*B+5*ln(cosh(1
/2*x)+1)*B)*sinh(1/2*x)^4)/(cosh(1/2*x)+1)/(cosh(1/2*x)-1)/sinh(1/2*x)/(-2
*sinh(1/2*x)^2*a)^(1/2)
```

3.106.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(75) = 150$.

Time = 0.27 (sec) , antiderivative size = 548, normalized size of antiderivative = 5.83

$$\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{5/2}}\,dx = \frac{\sqrt{2}\left((3\,A-5\,B)\cosh\left(x\right)^{4}+(3\,A-5\,B)\sinh\left(x\right)^{4}-4\,(3\,A-5\,B)\cosh\left(x\right)^{3}+4\,(\right.}{}$$

input
```
integrate((A+B*cosh(x))/(a-a*cosh(x))^(5/2),x, algorithm="fricas")
```

output
```
1/32*(sqrt(2)*((3*A - 5*B)*cosh(x)^4 + (3*A - 5*B)*sinh(x)^4 - 4*(3*A - 5*
B)*cosh(x)^3 + 4*((3*A - 5*B)*cosh(x) - 3*A + 5*B)*sinh(x)^3 + 6*(3*A - 5*
B)*cosh(x)^2 + 6*((3*A - 5*B)*cosh(x)^2 - 2*(3*A - 5*B)*cosh(x) + 3*A - 5*
B)*sinh(x)^2 - 4*(3*A - 5*B)*cosh(x) + 4*((3*A - 5*B)*cosh(x)^3 - 3*(3*A -
 5*B)*cosh(x)^2 + 3*(3*A - 5*B)*cosh(x) - 3*A + 5*B)*sinh(x) + 3*A - 5*B)*
sqrt(-a)*log((2*sqrt(2)*sqrt(1/2)*sqrt(-a)*sqrt(-a/(cosh(x) + sinh(x)))*(c
osh(x) + sinh(x)) - a*cosh(x) - a*sinh(x) - a)/(cosh(x) + sinh(x) - 1)) -
4*sqrt(1/2)*((3*A - 5*B)*cosh(x)^4 + (3*A - 5*B)*sinh(x)^4 - (11*A + 3*B)*
cosh(x)^3 + (4*(3*A - 5*B)*cosh(x) - 11*A - 3*B)*sinh(x)^3 - (11*A + 3*B)*
cosh(x)^2 + (6*(3*A - 5*B)*cosh(x)^2 - 3*(11*A + 3*B)*cosh(x) - 11*A - 3*B
)*sinh(x)^2 + (3*A - 5*B)*cosh(x) + (4*(3*A - 5*B)*cosh(x)^3 - 3*(11*A + 3
*B)*cosh(x)^2 - 2*(11*A + 3*B)*cosh(x) + 3*A - 5*B)*sinh(x))*sqrt(-a/(cosh
(x) + sinh(x))))/(a^3*cosh(x)^4 + a^3*sinh(x)^4 - 4*a^3*cosh(x)^3 + 6*a^3*
cosh(x)^2 - 4*a^3*cosh(x) + 4*(a^3*cosh(x) - a^3)*sinh(x)^3 + a^3 + 6*(a^3
*cosh(x)^2 - 2*a^3*cosh(x) + a^3)*sinh(x)^2 + 4*(a^3*cosh(x)^3 - 3*a^3*cos
h(x)^2 + 3*a^3*cosh(x) - a^3)*sinh(x))
```

3.106.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cosh(x))/(a-a*cosh(x))**(5/2),x)
```

output
```
Timed out
```

3.106.7 Maxima [F]

$$\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{5/2}}\,dx = \int \frac{B\cosh\left(x\right)+A}{\left(-a\cosh\left(x\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cosh(x))/(a-a*cosh(x))^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*cosh(x) + A)/(-a*cosh(x) + a)^(5/2), x)
```

3.106.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(75) = 150$.

Time = 0.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.01

$$\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{5/2}}\,dx = -\frac{\sqrt{2}(3\,A-5\,B)\arctan\left(\frac{\sqrt{-ae^x}}{\sqrt{a}}\right)}{16\,a^{\frac{5}{2}}\mathrm{sgn}\left(-e^x+1\right)} + \frac{\sqrt{2}(3\,\sqrt{-ae^x}Aa^3e^{(3\,x)} - 5\,\sqrt{-ae^x}Ba^3e^{(3\,x)} - 11\,\sqrt{-ae^x}Aa^3e^{(2\,x)} - 3\,\sqrt{-ae^x}Ba^3e^{(2\,x)} - 11\,\sqrt{-ae^x}Aa^3e^x -}{16\left(ae^x-a\right)^4a^2\mathrm{sgn}\left(-e^x+1\right)}$$

input
```
integrate((A+B*cosh(x))/(a-a*cosh(x))^(5/2),x, algorithm="giac")
```

output
```
-1/16*sqrt(2)*(3*A - 5*B)*arctan(sqrt(-a*e^x)/sqrt(a))/(a^(5/2)*sgn(-e^x +
 1)) + 1/16*sqrt(2)*(3*sqrt(-a*e^x)*A*a^3*e^(3*x) - 5*sqrt(-a*e^x)*B*a^3*e
^(3*x) - 11*sqrt(-a*e^x)*A*a^3*e^(2*x) - 3*sqrt(-a*e^x)*B*a^3*e^(2*x) - 11
*sqrt(-a*e^x)*A*a^3*e^x - 3*sqrt(-a*e^x)*B*a^3*e^x + 3*sqrt(-a*e^x)*A*a^3
- 5*sqrt(-a*e^x)*B*a^3)/((a*e^x - a)^4*a^2*sgn(-e^x + 1))
```

3.106.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{5/2}}\,dx = \int \frac{A+B\cosh(x)}{\left(a-a\cosh\left(x\right)\right)^{5/2}}\,dx$$

input
```
int((A + B*cosh(x))/(a - a*cosh(x))^(5/2),x)
```

output
```
int((A + B*cosh(x))/(a - a*cosh(x))^(5/2), x)
```

3.107 $\int (a+b\cosh(x))^{5/2}(A+B\cosh(x))\,dx$

3.107.1 Optimal result

Integrand size = 17, antiderivative size = 233

$$\begin{aligned}\int (a+b\cosh(x))^{5/2}(A+B\cosh(x))\,dx = & -\frac{2i(161a^2Ab+63Ab^3+15a^3B+145ab^2B)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{105b\sqrt{\frac{a+b\cosh(x)}{a+b}}} \\ & +\frac{2i(a^2-b^2)\left(56aAb+15a^2B+25b^2B\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\operatorname{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{105b\sqrt{a+b\cosh(x)}} \\ & +\frac{2}{105}\left(56aAb+15a^2B+25b^2B\right)\sqrt{a+b\cosh(x)}\sinh(x) \\ & +\frac{2}{35}(7Ab+5aB)(a+b\cosh(x))^{3/2}\sinh(x)+\frac{2}{7}B(a+b\cosh(x))^{5/2}\sinh(x)\end{aligned}$$

output
```
2/35*(7*A*b+5*B*a)*(a+b*cosh(x))^(3/2)*sinh(x)+2/7*B*(a+b*cosh(x))^(5/2)*s
inh(x)+2/105*(56*A*a*b+15*B*a^2+25*B*b^2)*sinh(x)*(a+b*cosh(x))^(1/2)-2/10
5*I*(161*A*a^2*b+63*A*b^3+15*B*a^3+145*B*a*b^2)*(cosh(1/2*x)^2)^(1/2)/cosh
(1/2*x)*EllipticE(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cosh(x))^(1/
2)/b/((a+b*cosh(x))/(a+b))^(1/2)+2/105*I*(a^2-b^2)*(56*A*a*b+15*B*a^2+25*B
*b^2)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/2)*(b
/(a+b))^(1/2))*((a+b*cosh(x))/(a+b))^(1/2)/b/(a+b*cosh(x))^(1/2)
```

3.107.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.87

$$\int (a+b\cosh(x))^{5/2}(A+B\cosh(x))\,dx = -\frac{\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\left(b(105a^3A+119aAb^2+135a^2bB+25b^3B)\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)+(161a^2Ab+63Ab^3+15a^3B+145ab^2B)\left((a\right.\right.}{b}}{}$$

input
```
Integrate[(a + b*Cosh[x])^(5/2)*(A + B*Cosh[x]),x]
```

output
```
(((-2*I)*Sqrt[(a + b*Cosh[x])/(a + b)]*(b*(105*a^3*A + 119*a*A*b^2 + 135*a
^2*b*B + 25*b^3*B)*EllipticF[(I/2)*x, (2*b)/(a + b)] + (161*a^2*A*b + 63*A
*b^3 + 15*a^3*B + 145*a*b^2*B)*((a + b)*EllipticE[(I/2)*x, (2*b)/(a + b)]
- a*EllipticF[(I/2)*x, (2*b)/(a + b)])))/b + (a + b*Cosh[x])*(154*a*A*b +
90*a^2*B + 65*b^2*B + 6*b*(7*A*b + 15*a*B)*Cosh[x] + 15*b^2*B*Cosh[2*x])*S
inh[x])/(105*Sqrt[a + b*Cosh[x]])
```

3.107.3 Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.05, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.059$, Rules used = {3042, 3232, 27, 3042, 3232, 27, 3042, 3232, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cosh(x))^{5/2}(A+B\cosh(x))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(\frac{\pi}{2}+ix\right)\right)^{5/2}\left(A+B\sin\left(\frac{\pi}{2}+ix\right)\right)dx$$

$$\Big\downarrow \text{3232}$$

$$\frac{2}{7}\int \frac{1}{2}(a+b\cosh(x))^{3/2}(7aA+5bB+(7Ab+5aB)\cosh(x))dx+\frac{2}{7}B\sinh(x)(a+b\cosh(x))^{5/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int (a+b\cosh(x))^{3/2}(7aA+5bB+(7Ab+5aB)\cosh(x))dx+\frac{2}{7}B\sinh(x)(a+b\cosh(x))^{5/2}$$

$\downarrow$ 3042

$$\frac{2}{7}B\sinh(x)(a+b\cosh(x))^{5/2}+\frac{1}{7}\int \left(a+b\sin\left(ix+\frac{\pi}{2}\right)\right)^{3/2}\left(7aA+5bB+(7Ab+5aB)\sin\left(ix+\frac{\pi}{2}\right)\right)dx$$

$\downarrow$ 3232

$$\frac{1}{7}\left(\frac{2}{5}\int \frac{1}{2}\sqrt{a+b\cosh(x)}\left(35Aa^2+40bBa+21Ab^2+\left(15Ba^2+56Aba+25b^2B\right)\cosh(x)\right)dx+\frac{2}{5}\sinh(x)(5aB+\right.$$
$$\frac{2}{7}B\sinh(x)(a+b\cosh(x))^{5/2}$$

$\downarrow$ 27

$$\frac{1}{7}\left(\frac{1}{5}\int \sqrt{a+b\cosh(x)}\left(35Aa^2+40bBa+21Ab^2+\left(15Ba^2+56Aba+25b^2B\right)\cosh(x)\right)dx+\frac{2}{5}\sinh(x)(5aB+7\right.$$
$$\frac{2}{7}B\sinh(x)(a+b\cosh(x))^{5/2}$$

$\downarrow$ 3042

$$\frac{2}{7}B\sinh(x)(a+b\cosh(x))^{5/2}+$$
$$\frac{1}{7}\left(\frac{2}{5}\sinh(x)(5aB+7Ab)(a+b\cosh(x))^{3/2}+\frac{1}{5}\int \sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}\left(35Aa^2+40bBa+21Ab^2+\left(15Ba^2+56\right.\right.\right.$$

$\downarrow$ 3232

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{2}{3}\int \frac{105Aa^3+135bBa^2+119Ab^2a+25b^3B+\left(15Ba^3+161Aba^2+145b^2Ba+63Ab^3\right)\cosh(x)}{2\sqrt{a+b\cosh(x)}}dx+\frac{2}{3}\sin\right.\right.$$
$$\frac{2}{7}B\sinh(x)(a+b\cosh(x))^{5/2}$$

$\downarrow$ 27

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{105Aa^3+135bBa^2+119Ab^2a+25b^3B+\left(15Ba^3+161Aba^2+145b^2Ba+63Ab^3\right)\cosh(x)}{\sqrt{a+b\cosh(x)}}dx+\frac{2}{3}\sin\right.\right.$$
$$\frac{2}{7}B\sinh(x)(a+b\cosh(x))^{5/2}$$

$\downarrow$ 3042

3.107. $\int (a+b\cosh(x))^{5/2}(A+B\cosh(x))\,dx$

$$\frac{2}{7}B\sinh(x)(a+b\cosh(x))^{5/2}+$$
$$\frac{1}{7}\left(\frac{2}{5}\sinh(x)(5aB+7Ab)(a+b\cosh(x))^{3/2}+\frac{1}{5}\left(\frac{2}{3}\sinh(x)\left(15a^2B+56aAb+25b^2B\right)\sqrt{a+b\cosh(x)}+\frac{1}{3}\int\frac{10}{}\right.\right.$$

$\downarrow$ 3231

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(15a^3B+161a^2Ab+145ab^2B+63Ab^3\right)\int\sqrt{a+b\cosh(x)}dx}{b}-\frac{\left(a^2-b^2\right)\left(15a^2B+56aAb+25b^2B\right)}{b}\right.\right.\right.$$
$$\frac{2}{7}B\sinh(x)(a+b\cosh(x))^{5/2}$$

$\downarrow$ 3042

$$\frac{2}{7}B\sinh(x)(a+b\cosh(x))^{5/2}+$$
$$\frac{1}{7}\left(\frac{2}{5}\sinh(x)(5aB+7Ab)(a+b\cosh(x))^{3/2}+\frac{1}{5}\left(\frac{2}{3}\sinh(x)\left(15a^2B+56aAb+25b^2B\right)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(\frac{(15}{}\right.\right.\right.$$

$\downarrow$ 3134

$$\frac{2}{7}B\sinh(x)(a+b\cosh(x))^{5/2}+$$
$$\frac{1}{7}\left(\frac{2}{5}\sinh(x)(5aB+7Ab)(a+b\cosh(x))^{3/2}+\frac{1}{5}\left(\frac{2}{3}\sinh(x)\left(15a^2B+56aAb+25b^2B\right)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(\frac{(15}{}\right.\right.\right.$$

$\downarrow$ 3042

$$\frac{2}{7}B\sinh(x)(a+b\cosh(x))^{5/2}+$$
$$\frac{1}{7}\left(\frac{2}{5}\sinh(x)(5aB+7Ab)(a+b\cosh(x))^{3/2}+\frac{1}{5}\left(\frac{2}{3}\sinh(x)\left(15a^2B+56aAb+25b^2B\right)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(\frac{(15}{}\right.\right.\right.$$

$\downarrow$ 3132

$$\frac{2}{7}B\sinh(x)(a+b\cosh(x))^{5/2}+$$
$$\frac{1}{7}\left(\frac{2}{5}\sinh(x)(5aB+7Ab)(a+b\cosh(x))^{3/2}+\frac{1}{5}\left(\frac{2}{3}\sinh(x)\left(15a^2B+56aAb+25b^2B\right)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(-\frac{(}{}\right.\right.\right.$$

$\downarrow$ 3142

3.107. $\int(a+b\cosh(x))^{5/2}(A+B\cosh(x))\,dx$

$$\frac{2}{7}B\sinh(x)(a+b\cosh(x))^{5/2}+$$
$$\frac{1}{7}\left(\frac{2}{5}\sinh(x)(5aB+7Ab)(a+b\cosh(x))^{3/2}+\frac{1}{5}\left(\frac{2}{3}\sinh(x)\left(15a^2B+56aAb+25b^2B\right)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(-\frac{(\ldots}{\ldots}\right.\right.\right.$$

$\big\downarrow$ 3042

$$\frac{2}{7}B\sinh(x)(a+b\cosh(x))^{5/2}+$$
$$\frac{1}{7}\left(\frac{2}{5}\sinh(x)(5aB+7Ab)(a+b\cosh(x))^{3/2}+\frac{1}{5}\left(\frac{2}{3}\sinh(x)\left(15a^2B+56aAb+25b^2B\right)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(-\frac{(\ldots}{\ldots}\right.\right.\right.$$

$\big\downarrow$ 3140

$$\frac{2}{7}B\sinh(x)(a+b\cosh(x))^{5/2}+$$
$$\frac{1}{7}\left(\frac{2}{5}\sinh(x)(5aB+7Ab)(a+b\cosh(x))^{3/2}+\frac{1}{5}\left(\frac{2}{3}\sinh(x)\left(15a^2B+56aAb+25b^2B\right)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(\frac{2i(\ldots}{\ldots}\right.\right.\right.$$

input
```
Int[(a + b*Cosh[x])^(5/2)*(A + B*Cosh[x]),x]
```

output
```
(2*B*(a + b*Cosh[x])^(5/2)*Sinh[x])/7 + ((2*(7*A*b + 5*a*B)*(a + b*Cosh[x]
)^(3/2)*Sinh[x])/5 + ((((-2*I)*(161*a^2*A*b + 63*A*b^3 + 15*a^3*B + 145*a*
b^2*B)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[(a +
 b*Cosh[x])/(a + b)]) + ((2*I)*(a^2 - b^2)*(56*a*A*b + 15*a^2*B + 25*b^2*B
)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt
[a + b*Cosh[x]]))/3 + (2*(56*a*A*b + 15*a^2*B + 25*b^2*B)*Sqrt[a + b*Cosh[
x]]*Sinh[x])/3)/5)/7
```

3.107.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

3.107. $\int (a+b\cosh(x))^{5/2}(A+B\cosh(x))\,dx$

3.107.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1364 vs. $2(245) = 490$.

Time = 6.79 (sec) , antiderivative size = 1365, normalized size of antiderivative = 5.86

method	result	size
default	Expression too large to display	1365
parts	Expression too large to display	1454

input
```
int((a+b*cosh(x))^(5/2)*(A+B*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
2/105*(240*B*cosh(1/2*x)*(-2*b/(a-b))^(1/2)*sinh(1/2*x)^8*b^3+(168*A*(-2*b
/(a-b))^(1/2)*b^3+480*B*(-2*b/(a-b))^(1/2)*a*b^2+360*B*(-2*b/(a-b))^(1/2)*
b^3)*sinh(1/2*x)^6*cosh(1/2*x)+(392*A*(-2*b/(a-b))^(1/2)*a*b^2+168*A*(-2*b
/(a-b))^(1/2)*b^3+360*B*(-2*b/(a-b))^(1/2)*a^2*b+480*B*(-2*b/(a-b))^(1/2)*
a*b^2+280*B*(-2*b/(a-b))^(1/2)*b^3)*sinh(1/2*x)^4*cosh(1/2*x)+(154*A*(-2*b
/(a-b))^(1/2)*a^2*b+196*A*(-2*b/(a-b))^(1/2)*a*b^2+42*A*(-2*b/(a-b))^(1/2)
*b^3+90*B*(-2*b/(a-b))^(1/2)*a^3+180*B*(-2*b/(a-b))^(1/2)*a^2*b+170*B*(-2*
b/(a-b))^(1/2)*a*b^2+80*B*(-2*b/(a-b))^(1/2)*b^3)*sinh(1/2*x)^2*cosh(1/2*x
)+105*A*a^3*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(
1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+161*
A*a^2*b*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)
*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+119*A*a*
b^2*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*Ell
ipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+63*A*b^3*(2*
b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(
cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))-322*A*(2*b/(a-b)*si
nh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x
)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))*a^2*b-126*A*(2*b/(a-b)*sinh(1
/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x)*(-
2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))*b^3+15*a^3*B*(2*b/(a-b)*sinh(1...
```

3.107.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 1141, normalized size of antiderivative = 4.90

$$\int (a+b\cosh(x))^{5/2}(A+B\cosh(x))\,dx = \text{Too large to display}$$

input
```
integrate((a+b*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

output
```
-1/1260*(8*(sqrt(2)*(30*B*a^4 + 7*A*a^3*b - 115*B*a^2*b^2 - 231*A*a*b^3 -
75*B*b^4)*cosh(x)^3 + 3*sqrt(2)*(30*B*a^4 + 7*A*a^3*b - 115*B*a^2*b^2 - 23
1*A*a*b^3 - 75*B*b^4)*cosh(x)^2*sinh(x) + 3*sqrt(2)*(30*B*a^4 + 7*A*a^3*b
- 115*B*a^2*b^2 - 231*A*a*b^3 - 75*B*b^4)*cosh(x)*sinh(x)^2 + sqrt(2)*(30*
B*a^4 + 7*A*a^3*b - 115*B*a^2*b^2 - 231*A*a*b^3 - 75*B*b^4)*sinh(x)^3)*sqr
t(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/
b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) + 24*(sqrt(2)*(15*B*a^3*b +
161*A*a^2*b^2 + 145*B*a*b^3 + 63*A*b^4)*cosh(x)^3 + 3*sqrt(2)*(15*B*a^3*b
+ 161*A*a^2*b^2 + 145*B*a*b^3 + 63*A*b^4)*cosh(x)^2*sinh(x) + 3*sqrt(2)*(1
5*B*a^3*b + 161*A*a^2*b^2 + 145*B*a*b^3 + 63*A*b^4)*cosh(x)*sinh(x)^2 + sq
rt(2)*(15*B*a^3*b + 161*A*a^2*b^2 + 145*B*a*b^3 + 63*A*b^4)*sinh(x)^3)*sqr
t(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3,
 weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3,
 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) - 3*(15*B*b^4*cosh(x)^6 + 15*B*
b^4*sinh(x)^6 + 6*(15*B*a*b^3 + 7*A*b^4)*cosh(x)^5 + 6*(15*B*b^4*cosh(x) +
 15*B*a*b^3 + 7*A*b^4)*sinh(x)^5 - 15*B*b^4 + (180*B*a^2*b^2 + 308*A*a*b^3
 + 115*B*b^4)*cosh(x)^4 + (225*B*b^4*cosh(x)^2 + 180*B*a^2*b^2 + 308*A*a*b
^3 + 115*B*b^4 + 30*(15*B*a*b^3 + 7*A*b^4)*cosh(x))*sinh(x)^4 - 8*(15*B*a^
3*b + 161*A*a^2*b^2 + 145*B*a*b^3 + 63*A*b^4)*cosh(x)^3 + 4*(75*B*b^4*cosh
(x)^3 - 30*B*a^3*b - 322*A*a^2*b^2 - 290*B*a*b^3 - 126*A*b^4 + 15*(15*B...
```

3.107.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cosh(x))^{5/2}(A+B\cosh(x))\,dx = \text{Timed out}$$

input
```
integrate((a+b*cosh(x))**(5/2)*(A+B*cosh(x)),x)
```

output
```
Timed out
```

3.107.7 Maxima [F]

$$\int (a+b\cosh(x))^{5/2}(A+B\cosh(x))\,dx = \int (B\cosh(x)+A)(b\cosh(x)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

output
```
integrate((B*cosh(x) + A)*(b*cosh(x) + a)^(5/2), x)
```

3.107.8 Giac [F]

$$\int (a+b\cosh(x))^{5/2}(A+B\cosh(x))\,dx = \int (B\cosh(x)+A)(b\cosh(x)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="giac")
```

output
```
integrate((B*cosh(x) + A)*(b*cosh(x) + a)^(5/2), x)
```

3.107.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cosh(x))^{5/2}(A+B\cosh(x))\,dx = \int (A+B\cosh(x))\,(a+b\cosh(x))^{5/2}\,dx$$

input
```
int((A + B*cosh(x))*(a + b*cosh(x))^(5/2),x)
```

output
```
int((A + B*cosh(x))*(a + b*cosh(x))^(5/2), x)
```

3.108 $\int (a+b\cosh(x))^{3/2}(A+B\cosh(x))\,dx$

3.108.1 Optimal result

Integrand size = 17, antiderivative size = 181

$$\int (a+b\cosh(x))^{3/2}(A+B\cosh(x))\,dx = -\frac{2i(20aAb+3a^2B+9b^2B)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{15b\sqrt{\frac{a+b\cosh(x)}{a+b}}} + \frac{2i(a^2-b^2)(5Ab+3aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}\operatorname{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{15b\sqrt{a+b\cosh(x)}} + \frac{2}{15}(5Ab+3aB)\sqrt{a+b\cosh(x)}\sinh(x) + \frac{2}{5}B(a+b\cosh(x))^{3/2}\sinh(x)$$

output
```
2/5*B*(a+b*cosh(x))^(3/2)*sinh(x)+2/15*(5*A*b+3*B*a)*sinh(x)*(a+b*cosh(x))
^(1/2)-2/15*I*(20*A*a*b+3*B*a^2+9*B*b^2)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)
*EllipticE(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cosh(x))^(1/2)/b/((
a+b*cosh(x))/(a+b))^(1/2)+2/15*I*(a^2-b^2)*(5*A*b+3*B*a)*(cosh(1/2*x)^2)^(
1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*co
sh(x))/(a+b))^(1/2)/b/(a+b*cosh(x))^(1/2)
```

3.108.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.69

$$\int (a+b\cosh(x))^{3/2}(A+B\cosh(x))\,dx = \frac{2}{15}\sqrt{a+b\cosh(x)}\left(-\frac{i\left((20aAb+3a^2B+9b^2B)\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)-(a-b)(5Ab+3aB)\,\text{Ellipti}\right.}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}} + (5Ab+6aB+3bB\cosh(x))\sinh(x)\right)$$

input
```
Integrate[(a + b*Cosh[x])^(3/2)*(A + B*Cosh[x]),x]
```

output
```
(2*Sqrt[a + b*Cosh[x]]*(((-I)*((20*a*A*b + 3*a^2*B + 9*b^2*B)*EllipticE[(I
/2)*x, (2*b)/(a + b)] - (a - b)*(5*A*b + 3*a*B)*EllipticF[(I/2)*x, (2*b)/(
a + b)]))/(b*Sqrt[(a + b*Cosh[x])/(a + b)]) + (5*A*b + 6*a*B + 3*b*B*Cosh[
x])*Sinh[x]))/15
```

3.108.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.882$, Rules used = {3042, 3232, 27, 3042, 3232, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cosh(x))^{3/2}(A+B\cosh(x))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(a+b\sin\left(\frac{\pi}{2}+ix\right)\right)^{3/2}\left(A+B\sin\left(\frac{\pi}{2}+ix\right)\right)dx$$

$$\Big\downarrow \; 3232$$

$$\frac{2}{5}\int \frac{1}{2}\sqrt{a+b\cosh(x)}(5aA+3bB+(5Ab+3aB)\cosh(x))dx+\frac{2}{5}B\sinh(x)(a+b\cosh(x))^{3/2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{5}\int \sqrt{a+b\cosh(x)}(5aA+3bB+(5Ab+3aB)\cosh(x))dx+\frac{2}{5}B\sinh(x)(a+b\cosh(x))^{3/2}$$

$\downarrow$ 3042

$$\frac{2}{5}B\sinh(x)(a+b\cosh(x))^{3/2}+$$
$$\frac{1}{5}\int \sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}\left(5aA+3bB+(5Ab+3aB)\sin\left(ix+\frac{\pi}{2}\right)\right)dx$$

$\downarrow$ 3232

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{15Aa^2+12bBa+5Ab^2+\left(3Ba^2+20Aba+9b^2B\right)\cosh(x)}{2\sqrt{a+b\cosh(x)}}dx+\frac{2}{3}\sinh(x)(3aB+5Ab)\sqrt{a+b\cosh(x)}\right)$$
$$\frac{2}{5}B\sinh(x)(a+b\cosh(x))^{3/2}$$

$\downarrow$ 27

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{15Aa^2+12bBa+5Ab^2+\left(3Ba^2+20Aba+9b^2B\right)\cosh(x)}{\sqrt{a+b\cosh(x)}}dx+\frac{2}{3}\sinh(x)(3aB+5Ab)\sqrt{a+b\cosh(x)}\right)$$
$$\frac{2}{5}B\sinh(x)(a+b\cosh(x))^{3/2}$$

$\downarrow$ 3042

$$\frac{2}{5}B\sinh(x)(a+b\cosh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{2}{3}\sinh(x)(3aB+5Ab)\sqrt{a+b\cosh(x)}+\frac{1}{3}\int \frac{15Aa^2+12bBa+5Ab^2+\left(3Ba^2+20Aba+9b^2B\right)\sin\left(ix+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}\right.$$

$\downarrow$ 3231

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(3a^2B+20aAb+9b^2B\right)\int \sqrt{a+b\cosh(x)}dx}{b}-\frac{\left(a^2-b^2\right)(3aB+5Ab)\int \frac{1}{\sqrt{a+b\cosh(x)}}dx}{b}\right)+\frac{2}{3}\sinh(x)(3a\right.$$
$$\frac{2}{5}B\sinh(x)(a+b\cosh(x))^{3/2}$$

$\downarrow$ 3042

$$\frac{2}{5}B\sinh(x)(a+b\cosh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{2}{3}\sinh(x)(3aB+5Ab)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(\frac{\left(3a^2B+20aAb+9b^2B\right)\int \sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}-\frac{\left(a^2-b^2\right)(3\right.\right.$$

$\downarrow$ 3134

3.108. $\int (a+b\cosh(x))^{3/2}(A+B\cosh(x))\,dx$

$$\frac{2}{5}B\sinh(x)(a+b\cosh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{2}{3}\sinh(x)(3aB+5Ab)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(\frac{\left(3a^2B+20aAb+9b^2B\right)\sqrt{a+b\cosh(x)}\int\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}dx}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}-\right.\right.$$

$\downarrow$ 3042

$$\frac{2}{5}B\sinh(x)(a+b\cosh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{2}{3}\sinh(x)(3aB+5Ab)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(\frac{\left(3a^2B+20aAb+9b^2B\right)\sqrt{a+b\cosh(x)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}\right.\right.$$

$\downarrow$ 3132

$$\frac{2}{5}B\sinh(x)(a+b\cosh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{2}{3}\sinh(x)(3aB+5Ab)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(-\frac{\left(a^2-b^2\right)(3aB+5Ab)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}-\frac{2i\left(3a^2B+20aAb\right.}{}\right.\right.$$

$\downarrow$ 3142

$$\frac{2}{5}B\sinh(x)(a+b\cosh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{2}{3}\sinh(x)(3aB+5Ab)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(-\frac{\left(a^2-b^2\right)(3aB+5Ab)\sqrt{\frac{a+b\cosh(x)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}}dx}{b\sqrt{a+b\cosh(x)}}-\frac{2i\left(3a^2\right.}{}\right.\right.$$

$\downarrow$ 3042

$$\frac{2}{5}B\sinh(x)(a+b\cosh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{2}{3}\sinh(x)(3aB+5Ab)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(-\frac{\left(a^2-b^2\right)(3aB+5Ab)\sqrt{\frac{a+b\cosh(x)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cosh(x)}}-\frac{2i(3}{}\right.\right.$$

$\downarrow$ 3140

$$\frac{2}{5}B\sinh(x)(a+b\cosh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{2}{3}\sinh(x)(3aB+5Ab)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(\frac{2i\left(a^2-b^2\right)(3aB+5Ab)\sqrt{\frac{a+b\cosh(x)}{a+b}}\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}-\frac{2i(3a}{}\right.\right.$$

3.108. $\int(a+b\cosh(x))^{3/2}(A+B\cosh(x))\,dx$

input
```
Int[(a + b*Cosh[x])^(3/2)*(A + B*Cosh[x]),x]
```

output
```
(2*B*(a + b*Cosh[x])^(3/2)*Sinh[x])/5 + ((((-2*I)*(20*a*A*b + 3*a^2*B + 9*
b^2*B)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[(a +
 b*Cosh[x])/(a + b)]) + ((2*I)*(a^2 - b^2)*(5*A*b + 3*a*B)*Sqrt[(a + b*Cos
h[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[a + b*Cosh[x]]))
/3 + (2*(5*A*b + 3*a*B)*Sqrt[a + b*Cosh[x]]*Sinh[x])/3)/5
```

3.108.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

3.108.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 972 vs. $2(197) = 394$.

Time = 5.22 (sec) , antiderivative size = 973, normalized size of antiderivative = 5.38

method	result	size
default	Expression too large to display	973
parts	Expression too large to display	1066

input
```
int((a+b*cosh(x))^(3/2)*(A+B*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
2/15*(24*B*cosh(1/2*x)*(-2*b/(a-b))^(1/2)*sinh(1/2*x)^6*b^2+(20*A*(-2*b/(a
-b))^(1/2)*b^2+36*B*(-2*b/(a-b))^(1/2)*a*b+24*B*(-2*b/(a-b))^(1/2)*b^2)*si
nh(1/2*x)^4*cosh(1/2*x)+(10*A*(-2*b/(a-b))^(1/2)*a*b+10*A*(-2*b/(a-b))^(1/
2)*b^2+12*B*(-2*b/(a-b))^(1/2)*a^2+18*B*(-2*b/(a-b))^(1/2)*a*b+6*B*(-2*b/(
a-b))^(1/2)*b^2)*sinh(1/2*x)^2*cosh(1/2*x)+15*A*a^2*(2*b/(a-b)*sinh(1/2*x)
^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(
a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+20*b*A*a*(2*b/(a-b)*sinh(1/2*x)^2+(a+b
)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(
1/2),1/2*(-2*(a-b)/b)^(1/2))+5*b^2*A*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))
^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2
*(-2*(a-b)/b)^(1/2))-40*A*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-si
nh(1/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/
b)^(1/2))*a*b+3*a^2*B*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1
/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(
1/2))+12*B*a*b*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2
)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+9
*B*b^2*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*
EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))-6*B*(2*b/
(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(co
sh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))*a^2-18*B*(2*b/(a-b...
```

3.108.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 635, normalized size of antiderivative = 3.51

$$\int (a+b\cosh(x))^{3/2}(A+B\cosh(x))\,dx =$$

$$-\frac{4\left(\sqrt{2}(6\,Ba^3-5\,Aa^2b-18\,Bab^2-15\,Ab^3)\cosh{(x)}^2+2\,\sqrt{2}(6\,Ba^3-5\,Aa^2b-18\,Bab^2-15\,Ab^3)\cosh{(x)}\right.}{}$$

input
```
integrate((a+b*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

output
```
-1/90*(4*(sqrt(2)*(6*B*a^3 - 5*A*a^2*b - 18*B*a*b^2 - 15*A*b^3)*cosh(x)^2
+ 2*sqrt(2)*(6*B*a^3 - 5*A*a^2*b - 18*B*a*b^2 - 15*A*b^3)*cosh(x)*sinh(x)
+ sqrt(2)*(6*B*a^3 - 5*A*a^2*b - 18*B*a*b^2 - 15*A*b^3)*sinh(x)^2)*sqrt(b)
*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3,
 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) + 12*(sqrt(2)*(3*B*a^2*b + 20*A*
a*b^2 + 9*B*b^3)*cosh(x)^2 + 2*sqrt(2)*(3*B*a^2*b + 20*A*a*b^2 + 9*B*b^3)*
cosh(x)*sinh(x) + sqrt(2)*(3*B*a^2*b + 20*A*a*b^2 + 9*B*b^3)*sinh(x)^2)*sq
rt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3
, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3
, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) - 3*(3*B*b^3*cosh(x)^4 + 3*B*b
^3*sinh(x)^4 - 3*B*b^3 + 2*(6*B*a*b^2 + 5*A*b^3)*cosh(x)^3 + 2*(6*B*b^3*co
sh(x) + 6*B*a*b^2 + 5*A*b^3)*sinh(x)^3 - 4*(3*B*a^2*b + 20*A*a*b^2 + 9*B*b
^3)*cosh(x)^2 + 2*(9*B*b^3*cosh(x)^2 - 6*B*a^2*b - 40*A*a*b^2 - 18*B*b^3 +
 3*(6*B*a*b^2 + 5*A*b^3)*cosh(x))*sinh(x)^2 - 2*(6*B*a*b^2 + 5*A*b^3)*cosh
(x) + 2*(6*B*b^3*cosh(x)^3 - 6*B*a*b^2 - 5*A*b^3 + 3*(6*B*a*b^2 + 5*A*b^3)
*cosh(x)^2 - 4*(3*B*a^2*b + 20*A*a*b^2 + 9*B*b^3)*cosh(x))*sinh(x))*sqrt(b
*cosh(x) + a))/(b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2)
```

3.108.6 Sympy [F]

$$\int (a+b\cosh(x))^{3/2}(A+B\cosh(x))\,dx=\int (A+B\cosh{(x)})\,(a+b\cosh{(x)})^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cosh(x))**(3/2)*(A+B*cosh(x)),x)
```

output
```
Integral((A + B*cosh(x))*(a + b*cosh(x))**(3/2), x)
```

3.108.7 Maxima [F]

$$\int (a+b\cosh(x))^{3/2}(A+B\cosh(x))\,dx=\int (B\cosh{(x)}+A)(b\cosh{(x)}+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

output
```
integrate((B*cosh(x) + A)*(b*cosh(x) + a)^(3/2), x)
```

3.108.8 Giac [F]

$$\int (a+b\cosh(x))^{3/2}(A+B\cosh(x))\,dx = \int (B\cosh(x)+A)(b\cosh(x)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="giac")
```

output
```
integrate((B*cosh(x) + A)*(b*cosh(x) + a)^(3/2), x)
```

3.108.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cosh(x))^{3/2}(A+B\cosh(x))\,dx = \int (A+B\cosh(x))\,\left(a+b\cosh(x)\right)^{3/2}\,dx$$

input
```
int((A + B*cosh(x))*(a + b*cosh(x))^(3/2),x)
```

output
```
int((A + B*cosh(x))*(a + b*cosh(x))^(3/2), x)
```

3.109 $\int \sqrt{a+b\cosh(x)}(A+B\cosh(x))\,dx$

3.109.1 Optimal result

Integrand size = 17, antiderivative size = 138

$$\int \sqrt{a+b\cosh(x)}(A+B\cosh(x))\,dx = -\frac{2i(3Ab+aB)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{3b\sqrt{\frac{a+b\cosh(x)}{a+b}}} + \frac{2i(a^2-b^2)\,B\sqrt{\frac{a+b\cosh(x)}{a+b}}\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{3b\sqrt{a+b\cosh(x)}} + \frac{2}{3}B\sqrt{a+b\cosh(x)}\sinh(x)$$

output
```
2/3*B*sinh(x)*(a+b*cosh(x))^(1/2)-2/3*I*(3*A*b+B*a)*(cosh(1/2*x)^2)^(1/2)/
cosh(1/2*x)*EllipticE(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cosh(x))
^(1/2)/b/((a+b*cosh(x))/(a+b))^(1/2)+2/3*I*(a^2-b^2)*B*(cosh(1/2*x)^2)^(1/
2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cosh
(x))/(a+b))^(1/2)/b/(a+b*cosh(x))^(1/2)
```

3.109.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.89

$$\int \sqrt{a+b\cosh(x)}(A+B\cosh(x))\,dx$$
$$=\frac{-2i(a+b)(3Ab+aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)+2i(a^2-b^2)\,B\sqrt{\frac{a+b\cosh(x)}{a+b}}\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)+2bB(a+b\,c}{3b\sqrt{a+b\cosh(x)}}$$

input
```
Integrate[Sqrt[a + b*Cosh[x]]*(A + B*Cosh[x]),x]
```

output
```
((-2*I)*(a + b)*(3*A*b + a*B)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticE[(I/2
)*x, (2*b)/(a + b)] + (2*I)*(a^2 - b^2)*B*Sqrt[(a + b*Cosh[x])/(a + b)]*El
lipticF[(I/2)*x, (2*b)/(a + b)] + 2*b*B*(a + b*Cosh[x])*Sinh[x])/(3*b*Sqrt
[a + b*Cosh[x]])
```

3.109.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {3042, 3232, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\cosh(x)}(A+B\cosh(x))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+b\sin\left(\frac{\pi}{2}+ix\right)}\left(A+B\sin\left(\frac{\pi}{2}+ix\right)\right)dx$$

$$\Big\downarrow \text{3232}$$

$$\frac{2}{3}\int \frac{3aA+bB+(3Ab+aB)\cosh(x)}{2\sqrt{a+b\cosh(x)}}dx+\frac{2}{3}B\sinh(x)\sqrt{a+b\cosh(x)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{3aA+bB+(3Ab+aB)\cosh(x)}{\sqrt{a+b\cosh(x)}}dx+\frac{2}{3}B\sinh(x)\sqrt{a+b\cosh(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}B\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\int\frac{3aA+bB+(3Ab+aB)\sin\left(ix+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3231}$$

$$\frac{1}{3}\left(\frac{(aB+3Ab)\int\sqrt{a+b\cosh(x)}dx}{b}-\frac{B(a^2-b^2)\int\frac{1}{\sqrt{a+b\cosh(x)}}dx}{b}\right)+\frac{2}{3}B\sinh(x)\sqrt{a+b\cosh(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}B\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(\frac{(aB+3Ab)\int\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}-\frac{B(a^2-b^2)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}\right)$$

$$\Big\downarrow \text{3134}$$

$$\frac{2}{3}B\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(\frac{(aB+3Ab)\sqrt{a+b\cosh(x)}\int\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}dx}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}-\frac{B(a^2-b^2)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}B\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(\frac{(aB+3Ab)\sqrt{a+b\cosh(x)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}-\frac{B(a^2-b^2)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}\right)$$

$$\Big\downarrow \text{3132}$$

$$\frac{2}{3}B\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(-\frac{B(a^2-b^2)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}-\frac{2i(aB+3Ab)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}\right)$$

$$\Big\downarrow \text{3142}$$

$$\frac{2}{3}B\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(-\frac{B(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}}dx}{b\sqrt{a+b\cosh(x)}}-\frac{2i(aB+3Ab)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}\right)$$

3.109. $\int\sqrt{a+b\cosh(x)}(A+B\cosh(x))\,dx$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}B\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(-\frac{B(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cosh(x)}}-\frac{2i(aB+3Ab)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}\right)$$

$$\Big\downarrow \text{3140}$$

$$\frac{2}{3}B\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(\frac{2iB(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}-\frac{2i(aB+3Ab)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}\right)$$

input
```
Int[Sqrt[a + b*Cosh[x]]*(A + B*Cosh[x]),x]
```

output
```
(((-2*I)*(3*A*b + a*B)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b
)])/(b*Sqrt[(a + b*Cosh[x])/(a + b)]) + ((2*I)*(a^2 - b^2)*B*Sqrt[(a + b*C
osh[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[a + b*Cosh[x]]
))/3 + (2*B*Sqrt[a + b*Cosh[x]]*Sinh[x])/3
```

3.109.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

3.109. $\int \sqrt{a+b\cosh(x)}(A+B\cosh(x))\,dx$

3.109.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 598 vs. $2(158) = 316$.

Time = 5.14 (sec) , antiderivative size = 599, normalized size of antiderivative = 4.34

method	result
parts	$\frac{2A\left(a\,\mathrm{EllipticF}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}},\frac{\sqrt{-\frac{2(a-b)}{b}}}{2}\right)+b\,\mathrm{EllipticF}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}},\frac{\sqrt{-\frac{2(a-b)}{b}}}{2}\right)-2b\,\mathrm{EllipticE}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}},\frac{\sqrt{-2}}{}\right.\right.}{\sqrt{-\frac{2b}{a-b}}\sqrt{2\sinh\left(\frac{x}{2}\right)^{4}b+(a+b)\sinh\left(\frac{x}{2}\right)^{2}}\,\sinh\left(\frac{x}{2}\right)\sqrt{2\sinh\left(\frac{x}{2}\right)^{2}}}$
default	$\frac{2\left(4B\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}}\,\sinh\left(\frac{x}{2}\right)^{4}b+2B\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}}\,\sinh\left(\frac{x}{2}\right)^{2}a+2B\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}}\,\sinh\left(\frac{x}{2}\right)^{2}b+3Aa\sqrt{\frac{2b\sinh\left(\frac{x}{2}\right)^{2}}{a-b}+\frac{a+b}{a-b}}\sqrt{-}\right.}{}$

input

```
int((a+b*cosh(x))^(1/2)*(A+B*cosh(x)),x,method=_RETURNVERBOSE)
```

output

```
2*A*(a*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+b*
EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))-2*b*Ellip
ticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2)))*(-sinh(1/2*x)
^2)^(1/2)*((2*cosh(1/2*x)^2*b+a-b)/(a-b))^(1/2)*((2*cosh(1/2*x)^2*b+a-b)*s
inh(1/2*x)^2)^(1/2)/(-2*b/(a-b))^(1/2)/(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x
)^2)^(1/2)/sinh(1/2*x)/(2*sinh(1/2*x)^2*b+a+b)^(1/2)+2/3*B*(4*cosh(1/2*x)*
(-2*b/(a-b))^(1/2)*sinh(1/2*x)^4*b+2*cosh(1/2*x)*(-2*b/(a-b))^(1/2)*sinh(1
/2*x)^2*a+2*cosh(1/2*x)*(-2*b/(a-b))^(1/2)*sinh(1/2*x)^2*b+(-sinh(1/2*x)^2
)^(1/2)*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*EllipticF(cosh(1/2*x)*
(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))*a+(-sinh(1/2*x)^2)^(1/2)*(2*b/(
a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(
1/2),1/2*(-2*(a-b)/b)^(1/2))*b-2*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/
2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2
*(a-b)/b)^(1/2))*a)*((2*cosh(1/2*x)^2*b+a-b)*sinh(1/2*x)^2)^(1/2)/(-2*b/(a
-b))^(1/2)/(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^2)^(1/2)/sinh(1/2*x)/(2*si
nh(1/2*x)^2*b+a+b)^(1/2)
```

3.109.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.36

$$\int \sqrt{a+b\cosh(x)}(A+B\cosh(x))\,dx =$$
$$-\frac{2\left(\sqrt{2}(2\,Ba^2-3\,Aab-3\,Bb^2)\cosh\left(x\right)+\sqrt{2}(2\,Ba^2-3\,Aab-3\,Bb^2)\sinh\left(x\right)\right)\sqrt{b}\text{weierstrassPInverse}\left(\right.}{}$$

input
```
integrate((a+b*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

output
```
-1/9*(2*(sqrt(2)*(2*B*a^2 - 3*A*a*b - 3*B*b^2)*cosh(x) + sqrt(2)*(2*B*a^2
- 3*A*a*b - 3*B*b^2)*sinh(x))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b
^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a
)/b) + 6*(sqrt(2)*(B*a*b + 3*A*b^2)*cosh(x) + sqrt(2)*(B*a*b + 3*A*b^2)*si
nh(x))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a
*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a
*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) - 3*(B*b^2*cosh(x)^2
+ B*b^2*sinh(x)^2 - B*b^2 - 2*(B*a*b + 3*A*b^2)*cosh(x) + 2*(B*b^2*cosh(x)
 - B*a*b - 3*A*b^2)*sinh(x))*sqrt(b*cosh(x) + a))/(b^2*cosh(x) + b^2*sinh(
x))
```

3.109.6 Sympy [F]

$$\int \sqrt{a+b\cosh(x)}(A+B\cosh(x))\,dx = \int \left(A+B\cosh\left(x\right)\right)\sqrt{a+b\cosh\left(x\right)}\,dx$$

input
```
integrate((a+b*cosh(x))**(1/2)*(A+B*cosh(x)),x)
```

output
```
Integral((A + B*cosh(x))*sqrt(a + b*cosh(x)), x)
```

3.109.7 Maxima [F]

$$\int \sqrt{a+b\cosh(x)}(A+B\cosh(x))\,dx = \int (B\cosh(x)+A)\sqrt{b\cosh(x)+a}\,dx$$

input
```
integrate((a+b*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

output
```
integrate((B*cosh(x) + A)*sqrt(b*cosh(x) + a), x)
```

3.109.8 Giac [F]

$$\int \sqrt{a+b\cosh(x)}(A+B\cosh(x))\,dx = \int (B\cosh(x)+A)\sqrt{b\cosh(x)+a}\,dx$$

input
```
integrate((a+b*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="giac")
```

output
```
integrate((B*cosh(x) + A)*sqrt(b*cosh(x) + a), x)
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cosh(x)}(A+B\cosh(x))\,dx = \int (A+B\cosh(x))\ \sqrt{a+b\cosh(x)}\,dx$$

input
```
int((A + B*cosh(x))*(a + b*cosh(x))^(1/2),x)
```

output
```
int((A + B*cosh(x))*(a + b*cosh(x))^(1/2), x)
```

3.110 $\int \frac{A+B\cosh(x)}{a+b\cosh(x)}\,dx$

3.110.1 Optimal result

Integrand size = 15, antiderivative size = 60

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)}\,dx = \frac{Bx}{b}+\frac{2(Ab-aB)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b\sqrt{a+b}}$$

output
```
B*x/b+2*(A*b-B*a)*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/b/(a-b)^(1/
2)/(a+b)^(1/2)
```

3.110.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)}\,dx = \frac{Bx}{b}+\frac{2(-Ab+aB)\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{b\sqrt{-a^2+b^2}}$$

input
```
Integrate[(A + B*Cosh[x])/(a + b*Cosh[x]),x]
```

output
```
(B*x)/b + (2*(-(A*b) + a*B)*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/
(b*Sqrt[-a^2 + b^2])
```

3.110.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{a+b\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3214}$$

$$\frac{(Ab-aB)\int \frac{1}{a+b\cosh(x)}dx}{b}+\frac{Bx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{Bx}{b}+\frac{(Ab-aB)\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{3138}$$

$$\frac{2(Ab-aB)\int \frac{1}{-((a-b)\tanh^2\left(\frac{x}{2}\right))+a+b}d\tanh\left(\frac{x}{2}\right)}{b}+\frac{Bx}{b}$$

$$\Big\downarrow \text{221}$$

$$\frac{2(Ab-aB)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}+\frac{Bx}{b}$$

input
```
Int[(A + B*Cosh[x])/(a + b*Cosh[x]),x]
```

output
```
(B*x)/b + (2*(A*b - a*B)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sq
rt[a - b]*b*Sqrt[a + b])
```

3.110.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.110.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.22

method	result
default	$-\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b}+\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b}-\frac{2(-bA+Ba)\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b\sqrt{(a+b)(a-b)}}$
risch	$\frac{Bx}{b}+\frac{\ln\left(e^{x}+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)A}{\sqrt{a^2-b^2}}-\frac{\ln\left(e^{x}+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)Ba}{\sqrt{a^2-b^2}\,b}-\frac{\ln\left(e^{x}+\frac{a\sqrt{a^2-b^2}+a^2-b^2}{b\sqrt{a^2-b^2}}\right)A}{\sqrt{a^2-b^2}}+\frac{\ln\left(e^{x}+\frac{a\sqrt{a^2-b^2}+a^2-b}{b\sqrt{a^2-b^2}}\right.}{\sqrt{a^2-b^2}\,b}$

input
```
int((A+B*cosh(x))/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-B/b*ln(tanh(1/2*x)-1)+B/b*ln(tanh(1/2*x)+1)-2/b*(-A*b+B*a)/((a+b)*(a-b))^
(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

3.110.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 240, normalized size of antiderivative = 4.00

$$\int \frac{A + B\cosh(x)}{a + b\cosh(x)}\,dx$$
$$= \left[-\frac{(Ba - Ab)\sqrt{a^2 - b^2}\log\left(\frac{b^2\cosh(x)^2 + b^2\sinh(x)^2 + 2\,ab\cosh(x) + 2\,a^2 - b^2 + 2\,(b^2\cosh(x) + ab)\sinh(x) - 2\,\sqrt{a^2 - b^2}(b\cosh(x) + b\sinh(}{b\cosh(x)^2 + b\sinh(x)^2 + 2\,a\cosh(x) + 2\,(b\cosh(x) + a)\sinh(x) + b}\right.}{a^2 b - b^3}\right.$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[-((B*a - A*b)*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*
cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*
(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*
(b*cosh(x) + a)*sinh(x) + b)) - (B*a^2 - B*b^2)*x)/(a^2*b - b^3), (2*(B*a
- A*b)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) +
a)/(a^2 - b^2)) + (B*a^2 - B*b^2)*x)/(a^2*b - b^3)]
```

3.110.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(49) = 98$.

Time = 12.97 (sec) , antiderivative size = 403, normalized size of antiderivative = 6.72

$$\int \frac{A + B\cosh(x)}{a + b\cosh(x)}\,dx$$
$$= \begin{cases} \tilde{\infty}\left(2A\,\operatorname{atan}\left(\tanh\left(\frac{x}{2}\right)\right) + Bx\right) \\ \frac{A\tanh\left(\frac{x}{2}\right)}{b} + \frac{Bx}{b} - \frac{B\tanh\left(\frac{x}{2}\right)}{b} \\ -\frac{A}{b\tanh\left(\frac{x}{2}\right)} + \frac{Bx}{b} - \frac{B}{b\tanh\left(\frac{x}{2}\right)} \\ \frac{Ax + B\sinh(x)}{a} \\ -\frac{Ab\log\left(-\sqrt{\frac{a}{a-b} + \frac{b}{a-b}} + \tanh\left(\frac{x}{2}\right)\right)}{ab\sqrt{\frac{a}{a-b} + \frac{b}{a-b}} - b^2\sqrt{\frac{a}{a-b} + \frac{b}{a-b}}} + \frac{Ab\log\left(\sqrt{\frac{a}{a-b} + \frac{b}{a-b}} + \tanh\left(\frac{x}{2}\right)\right)}{ab\sqrt{\frac{a}{a-b} + \frac{b}{a-b}} - b^2\sqrt{\frac{a}{a-b} + \frac{b}{a-b}}} + \frac{Bax\sqrt{\frac{a}{a-b} + \frac{b}{a-b}}}{ab\sqrt{\frac{a}{a-b} + \frac{b}{a-b}} - b^2\sqrt{\frac{a}{a-b} + \frac{b}{a-b}}} + \frac{Ba\log\left(-\sqrt{\frac{a}{a-b} + \frac{b}{a-b}} + \mathrm{t}\right.}{ab\sqrt{\frac{a}{a-b} + \frac{b}{a-b}} - b^2\sqrt{\frac{a}{a-}}} \end{cases}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x)),x)
```

output
```
Piecewise((zoo*(2*A*atan(tanh(x/2)) + B*x), Eq(a, 0) & Eq(b, 0)), (A*tanh(
x/2)/b + B*x/b - B*tanh(x/2)/b, Eq(a, b)), (-A/(b*tanh(x/2)) + B*x/b - B/(
b*tanh(x/2)), Eq(a, -b)), ((A*x + B*sinh(x))/a, Eq(b, 0)), (-A*b*log(-sqrt
(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b*
*2*sqrt(a/(a - b) + b/(a - b))) + A*b*log(sqrt(a/(a - b) + b/(a - b)) + ta
nh(x/2))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b
))) + B*a*x*sqrt(a/(a - b) + b/(a - b))/(a*b*sqrt(a/(a - b) + b/(a - b)) -
 b**2*sqrt(a/(a - b) + b/(a - b))) + B*a*log(-sqrt(a/(a - b) + b/(a - b))
+ tanh(x/2))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a
 - b))) - B*a*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a
- b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) - B*b*x*sqrt(a/(a -
b) + b/(a - b))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b
/(a - b))), True))
```

3.110.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.110.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)}\,dx = \frac{Bx}{b} - \frac{2\,(Ba-Ab)\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}b}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
B*x/b - 2*(B*a - A*b)*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^
2)*b)
```

3.110.9 Mupad [B] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 242, normalized size of antiderivative = 4.03

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{b^2\,\mathrm{e}^x\,\sqrt{b^4-a^2\,b^2}\,\left(\frac{2\,\left(A\,b\,\sqrt{b^4-a^2\,b^2}-B\,a\,\sqrt{b^4-a^2\,b^2}\right)}{b^4\,\sqrt{b^4-a^2\,b^2}\,\sqrt{(A\,b-B\,a)^2}}+\frac{2\,a^2\,\sqrt{A^2\,b^2-2\,A\,B\,a\,b+B^2\,a^2}}{b^2\,\left(b^4-a^2\,b^2\right)\,(A\,b-B\,a)}\right)}{2}+\frac{a\,b\,\sqrt{A^2\,b^2-2\,A\,B\,a\,b+B^2\,a^2}}{\sqrt{b^4-a^2\,b^2}\,(A\,b-B\,a)}\right)\,\sqrt{A^2\,b^2-}}{\sqrt{b^4-a^2\,b^2}} + \frac{B\,x}{b}$$

input
```
int((A + B*cosh(x))/(a + b*cosh(x)),x)
```

output
```
(2*atan((b^2*exp(x)*(b^4 - a^2*b^2)^(1/2)*((2*(A*b*(b^4 - a^2*b^2)^(1/2) -
 B*a*(b^4 - a^2*b^2)^(1/2)))/(b^4*(b^4 - a^2*b^2)^(1/2)*((A*b - B*a)^2)^(1
/2)) + (2*a^2*(A^2*b^2 + B^2*a^2 - 2*A*B*a*b)^(1/2))/(b^2*(b^4 - a^2*b^2)*
(A*b - B*a))))/2 + (a*b*(A^2*b^2 + B^2*a^2 - 2*A*B*a*b)^(1/2))/((b^4 - a^2
*b^2)^(1/2)*(A*b - B*a)))*(A^2*b^2 + B^2*a^2 - 2*A*B*a*b)^(1/2))/(b^4 - a^
2*b^2)^(1/2) + (B*x)/b
```

3.111 $\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^2}\,dx$

3.111.1 Optimal result

Integrand size = 15, antiderivative size = 82

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^2}\,dx=\frac{2(aA-bB)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}-\frac{(Ab-aB)\sinh(x)}{(a^2-b^2)\,(a+b\cosh(x))}$$

output
```
2*(A*a-B*b)*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(3/2)/(a+b)
^(3/2)-(A*b-B*a)*sinh(x)/(a^2-b^2)/(a+b*cosh(x))
```

3.111.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.99

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^2}\,dx=\frac{2(aA-bB)\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}}+\frac{(-Ab+aB)\sinh(x)}{(a-b)(a+b)(a+b\cosh(x))}$$

input
```
Integrate[(A + B*Cosh[x])/(a + b*Cosh[x])^2,x]
```

output
```
(2*(a*A - b*B)*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^
(3/2) + ((-(A*b) + a*B)*Sinh[x])/((a - b)*(a + b)*(a + b*Cosh[x]))
```

3.111.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3042, 3233, 25, 27, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{\left(a+b\sin\left(\frac{\pi}{2}+ix\right)\right)^2}dx$$

$$\Big\downarrow \text{3233}$$

$$-\frac{\int -\frac{aA-bB}{a+b\cosh(x)}dx}{a^2-b^2}-\frac{\sinh(x)(Ab-aB)}{(a^2-b^2)(a+b\cosh(x))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{aA-bB}{a+b\cosh(x)}dx}{a^2-b^2}-\frac{\sinh(x)(Ab-aB)}{(a^2-b^2)(a+b\cosh(x))}$$

$$\Big\downarrow \text{27}$$

$$\frac{(aA-bB)\int \frac{1}{a+b\cosh(x)}dx}{a^2-b^2}-\frac{\sinh(x)(Ab-aB)}{(a^2-b^2)(a+b\cosh(x))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(x)(Ab-aB)}{(a^2-b^2)(a+b\cosh(x))}+\frac{(aA-bB)\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3138}$$

$$\frac{2(aA-bB)\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a^2-b^2}-\frac{\sinh(x)(Ab-aB)}{(a^2-b^2)(a+b\cosh(x))}$$

$$\Big\downarrow \text{221}$$

$$\frac{2(aA-bB)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}\,(a^2-b^2)}-\frac{\sinh(x)(Ab-aB)}{(a^2-b^2)(a+b\cosh(x))}$$

input
```
Int[(A + B*Cosh[x])/(a + b*Cosh[x])^2,x]
```

output
```
(2*(a*A - b*B)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*
Sqrt[a + b]*(a^2 - b^2)) - ((A*b - a*B)*Sinh[x])/((a^2 - b^2)*(a + b*Cosh[
x]))
```

3.111.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.111.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.32

method	result
default	$\frac{2(bA-Ba)\tanh\left(\frac{x}{2}\right)}{(a^2-b^2)\left(\tanh\left(\frac{x}{2}\right)^2a-\tanh\left(\frac{x}{2}\right)^2b-a-b\right)}+\frac{2(Aa-Bb)\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}$
risch	$\frac{2(bA-Ba)(a\,\mathrm{e}^x+b)}{b(a^2-b^2)(b\,\mathrm{e}^{2x}+2a\,\mathrm{e}^x+b)}+\frac{\ln\left(\mathrm{e}^x+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)Aa}{\sqrt{a^2-b^2}\,(a+b)(a-b)}-\frac{\ln\left(\mathrm{e}^x+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)Bb}{\sqrt{a^2-b^2}\,(a+b)(a-b)}-\frac{\ln\left(\mathrm{e}^x+\frac{a\sqrt{a^2-b^2}+a^2-b^2}{b\sqrt{a^2-b^2}}\right)Aa}{\sqrt{a^2-b^2}\,(a+b)(a-b)}+$

input
```
int((A+B*cosh(x))/(a+b*cosh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
2*(A*b-B*a)/(a^2-b^2)*tanh(1/2*x)/(tanh(1/2*x)^2*a-tanh(1/2*x)^2*b-a-b)+2*
(A*a-B*b)/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)
*(a-b))^(1/2))
```

3.111.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(71) = 142$.

Time = 0.29 (sec) , antiderivative size = 828, normalized size of antiderivative = 10.10

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^2}\,dx$$
$$= \left[-\frac{2\,Ba^3b-2\,Aa^2b^2-2\,Bab^3+2\,Ab^4-\left(Aab^2-Bb^3+(Aab^2-Bb^3)\cosh{(x)}^2+(Aab^2-Bb^3)\sinh{(x)}\right.}{a^4b^2-}\right.$$
$$-\frac{2\left(Ba^3b-Aa^2b^2-Bab^3+Ab^4+\left(Aab^2-Bb^3+(Aab^2-Bb^3)\cosh{(x)}^2+(Aab^2-Bb^3)\sinh{(x)}^2+\right.\right.}{a^4b^2-2\,a^2b^4+b^6+(a^4b^2-2\,a^2b}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))^2,x, algorithm="fricas")
```

output
```
[-(2*B*a^3*b - 2*A*a^2*b^2 - 2*B*a*b^3 + 2*A*b^4 - (A*a*b^2 - B*b^3 + (A*a
*b^2 - B*b^3)*cosh(x)^2 + (A*a*b^2 - B*b^3)*sinh(x)^2 + 2*(A*a^2*b - B*a*b
^2)*cosh(x) + 2*(A*a^2*b - B*a*b^2 + (A*a*b^2 - B*b^3)*cosh(x))*sinh(x))*s
qrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2
- b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*s
inh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*
sinh(x) + b)) + 2*(B*a^4 - A*a^3*b - B*a^2*b^2 + A*a*b^3)*cosh(x) + 2*(B*a
^4 - A*a^3*b - B*a^2*b^2 + A*a*b^3)*sinh(x))/(a^4*b^2 - 2*a^2*b^4 + b^6 +
(a^4*b^2 - 2*a^2*b^4 + b^6)*cosh(x)^2 + (a^4*b^2 - 2*a^2*b^4 + b^6)*sinh(x
)^2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5)*cosh(x) + 2*(a^5*b - 2*a^3*b^3 + a*b^5
 + (a^4*b^2 - 2*a^2*b^4 + b^6)*cosh(x))*sinh(x)), -2*(B*a^3*b - A*a^2*b^2
- B*a*b^3 + A*b^4 + (A*a*b^2 - B*b^3 + (A*a*b^2 - B*b^3)*cosh(x)^2 + (A*a*
b^2 - B*b^3)*sinh(x)^2 + 2*(A*a^2*b - B*a*b^2)*cosh(x) + 2*(A*a^2*b - B*a*
b^2 + (A*a*b^2 - B*b^3)*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a
^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) + (B*a^4 - A*a^3*b - B*
a^2*b^2 + A*a*b^3)*cosh(x) + (B*a^4 - A*a^3*b - B*a^2*b^2 + A*a*b^3)*sinh(
x))/(a^4*b^2 - 2*a^2*b^4 + b^6 + (a^4*b^2 - 2*a^2*b^4 + b^6)*cosh(x)^2 + (
a^4*b^2 - 2*a^2*b^4 + b^6)*sinh(x)^2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5)*cosh(
x) + 2*(a^5*b - 2*a^3*b^3 + a*b^5 + (a^4*b^2 - 2*a^2*b^4 + b^6)*cosh(x))*s
inh(x))]
```

3.111.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3890 vs. $2(66) = 132$.

Time = 155.99 (sec) , antiderivative size = 3890, normalized size of antiderivative = 47.44

$$\int \frac{A + B\cosh(x)}{(a + b\cosh(x))^2}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))**2,x)
```

output
```
Piecewise((zoo*(2*A*tanh(x/2)/(tanh(x/2)**2 + 1) + 2*B*tanh(x/2)**2*atan(t
anh(x/2))/(tanh(x/2)**2 + 1) + 2*B*atan(tanh(x/2))/(tanh(x/2)**2 + 1)), Eq
(a, 0) & Eq(b, 0)), (-A*tanh(x/2)**3/(6*b**2) + A*tanh(x/2)/(2*b**2) + B*t
anh(x/2)**3/(6*b**2) + B*tanh(x/2)/(2*b**2), Eq(a, b)), (A/(2*b**2*tanh(x/
2)) - A/(6*b**2*tanh(x/2)**3) - B/(2*b**2*tanh(x/2)) - B/(6*b**2*tanh(x/2)
**3), Eq(a, -b)), (-A*a**2*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))*t
anh(x/2)**2/(a**4*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 - a**4*sqrt(a/(
a - b) + b/(a - b)) - 2*a**3*b*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 +
2*a**2*b**2*sqrt(a/(a - b) + b/(a - b)) + 2*a*b**3*sqrt(a/(a - b) + b/(a -
 b))*tanh(x/2)**2 - b**4*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 - b**4*s
qrt(a/(a - b) + b/(a - b))) + A*a**2*log(-sqrt(a/(a - b) + b/(a - b)) + ta
nh(x/2))/(a**4*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 - a**4*sqrt(a/(a -
 b) + b/(a - b)) - 2*a**3*b*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 + 2*a
**2*b**2*sqrt(a/(a - b) + b/(a - b)) + 2*a*b**3*sqrt(a/(a - b) + b/(a - b)
)*tanh(x/2)**2 - b**4*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 - b**4*sqrt
(a/(a - b) + b/(a - b))) + A*a**2*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x
/2))*tanh(x/2)**2/(a**4*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 - a**4*sq
rt(a/(a - b) + b/(a - b)) - 2*a**3*b*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)
**2 + 2*a**2*b**2*sqrt(a/(a - b) + b/(a - b)) + 2*a*b**3*sqrt(a/(a - b) +
b/(a - b))*tanh(x/2)**2 - b**4*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2...
```

3.111.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\cosh(x)}{(a + b\cosh(x))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.111.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.30

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^2}\,dx = \frac{2\,(Aa-Bb)\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{(a^2-b^2)\sqrt{-a^2+b^2}} - \frac{2\,(Ba^2e^x-Aabe^x+Bab-Ab^2)}{(a^2b-b^3)(be^{(2\,x)}+2\,ae^x+b)}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))^2,x, algorithm="giac")
```

output
```
2*(A*a - B*b)*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/((a^2 - b^2)*sqrt(-a^2
+ b^2)) - 2*(B*a^2*e^x - A*a*b*e^x + B*a*b - A*b^2)/((a^2*b - b^3)*(b*e^(2
*x) + 2*a*e^x + b))
```

3.111.9 Mupad [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 246, normalized size of antiderivative = 3.00

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^2}\,dx = \frac{\frac{2\,(A\,b^3-B\,a\,b^2)}{b\,(a^2\,b-b^3)} - \frac{2\,e^x\,(B\,a^2\,b^2-A\,a\,b^3)}{b^2\,(a^2\,b-b^3)}}{b+2\,a\,e^x+b\,e^{2\,x}} + \frac{\ln\left(-\frac{2\,e^x\,(A\,a-B\,b)}{b\,(a^2-b^2)} - \frac{2\,(b+a\,e^x)\,(A\,a-B\,b)}{b\,(a+b)^{3/2}\,(a-b)^{3/2}}\right)(A\,a-B\,b)}{(a+b)^{3/2}\,(a-b)^{3/2}} - \frac{\ln\left(\frac{2\,(b+a\,e^x)\,(A\,a-B\,b)}{b\,(a+b)^{3/2}\,(a-b)^{3/2}} - \frac{2\,e^x\,(A\,a-B\,b)}{b\,(a^2-b^2)}\right)(A\,a-B\,b)}{(a+b)^{3/2}\,(a-b)^{3/2}}$$

input
```
int((A + B*cosh(x))/(a + b*cosh(x))^2,x)
```

output
```
((2*(A*b^3 - B*a*b^2))/(b*(a^2*b - b^3)) - (2*exp(x)*(B*a^2*b^2 - A*a*b^3)
)/(b^2*(a^2*b - b^3)))/(b + 2*a*exp(x) + b*exp(2*x)) + (log(- (2*exp(x)*(A
*a - B*b))/(b*(a^2 - b^2)) - (2*(b + a*exp(x))*(A*a - B*b))/(b*(a + b)^(3/
2)*(a - b)^(3/2)))*(A*a - B*b))/((a + b)^(3/2)*(a - b)^(3/2)) - (log((2*(b
 + a*exp(x))*(A*a - B*b))/(b*(a + b)^(3/2)*(a - b)^(3/2)) - (2*exp(x)*(A*a
 - B*b))/(b*(a^2 - b^2)))*(A*a - B*b))/((a + b)^(3/2)*(a - b)^(3/2))
```

3.112 $\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^3}\,dx$

3.112.1 Optimal result

Integrand size = 15, antiderivative size = 135

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^3}\,dx = \frac{(2a^2A+Ab^2-3abB)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}} - \frac{(Ab-aB)\sinh(x)}{2\left(a^2-b^2\right)(a+b\cosh(x))^2} - \frac{(3aAb-a^2B-2b^2B)\sinh(x)}{2\left(a^2-b^2\right)^2(a+b\cosh(x))}$$

output
```
(2*A*a^2+A*b^2-3*B*a*b)*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)
^(5/2)/(a+b)^(5/2)-1/2*(A*b-B*a)*sinh(x)/(a^2-b^2)/(a+b*cosh(x))^2-1/2*(3*
A*a*b-B*a^2-2*B*b^2)*sinh(x)/(a^2-b^2)^2/(a+b*cosh(x))
```

3.112.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.99

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^3}\,dx = \frac{1}{2}\left(-\frac{2(2a^2A+Ab^2-3abB)\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}} + \frac{(-Ab+aB)\sinh(x)}{(a-b)(a+b)(a+b\cosh(x))^2} + \frac{(-3aAb+a^2B+2b^2B)\sinh(x)}{(a-b)^2(a+b)^2(a+b\cosh(x))}\right)$$

input
```
Integrate[(A + B*Cosh[x])/(a + b*Cosh[x])^3,x]
```

output
```
((-2*(2*a^2*A + A*b^2 - 3*a*b*B)*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^
2]])/(-a^2 + b^2)^(5/2) + ((-(A*b) + a*B)*Sinh[x])/((a - b)*(a + b)*(a + b
*Cosh[x])^2) + ((-3*a*A*b + a^2*B + 2*b^2*B)*Sinh[x])/((a - b)^2*(a + b)^2
*(a + b*Cosh[x])))/2
```

3.112.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.19, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3233, 25, 3042, 3233, 25, 27, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{\left(a+b\sin\left(\frac{\pi}{2}+ix\right)\right)^3}dx$$

$$\Big\downarrow \text{3233}$$

$$-\frac{\int -\frac{2(aA-bB)-(Ab-aB)\cosh(x)}{(a+b\cosh(x))^2}dx}{2(a^2-b^2)}-\frac{\sinh(x)(Ab-aB)}{2(a^2-b^2)(a+b\cosh(x))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2(aA-bB)-(Ab-aB)\cosh(x)}{(a+b\cosh(x))^2}dx}{2(a^2-b^2)}-\frac{\sinh(x)(Ab-aB)}{2(a^2-b^2)(a+b\cosh(x))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(x)(Ab-aB)}{2(a^2-b^2)(a+b\cosh(x))^2}+\frac{\int \frac{2(aA-bB)+(aB-Ab)\sin\left(ix+\frac{\pi}{2}\right)}{\left(a+b\sin\left(ix+\frac{\pi}{2}\right)\right)^2}dx}{2(a^2-b^2)}$$

$$\Big\downarrow \text{3233}$$

$$\frac{-\frac{\int -\frac{2Aa^2-3bBa+Ab^2}{a+b\cosh(x)}dx}{a^2-b^2}-\frac{\sinh(x)\left(a^2(-B)+3aAb-2b^2B\right)}{(a^2-b^2)(a+b\cosh(x))}}{2(a^2-b^2)}-\frac{\sinh(x)(Ab-aB)}{2(a^2-b^2)(a+b\cosh(x))^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{2Aa^2-3bBa+Ab^2}{a+b\cosh(x)}dx}{a^2-b^2}-\frac{\sinh(x)(a^2(-B)+3aAb-2b^2B)}{(a^2-b^2)(a+b\cosh(x))}}{2(a^2-b^2)}-\frac{\sinh(x)(Ab-aB)}{2(a^2-b^2)(a+b\cosh(x))^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{(2a^2A-3abB+Ab^2)\int \frac{1}{a+b\cosh(x)}dx}{a^2-b^2}-\frac{\sinh(x)(a^2(-B)+3aAb-2b^2B)}{(a^2-b^2)(a+b\cosh(x))}}{2(a^2-b^2)}-\frac{\sinh(x)(Ab-aB)}{2(a^2-b^2)(a+b\cosh(x))^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\sinh(x)(Ab-aB)}{2(a^2-b^2)(a+b\cosh(x))^2}+\frac{-\frac{\sinh(x)(a^2(-B)+3aAb-2b^2B)}{(a^2-b^2)(a+b\cosh(x))}+\frac{(2a^2A-3abB+Ab^2)\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}}{2(a^2-b^2)}$$

$$\Big\downarrow \; 3138$$

$$\frac{\frac{2(2a^2A-3abB+Ab^2)\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a^2-b^2}-\frac{\sinh(x)(a^2(-B)+3aAb-2b^2B)}{(a^2-b^2)(a+b\cosh(x))}}{2(a^2-b^2)}-\frac{\sinh(x)(Ab-aB)}{2(a^2-b^2)(a+b\cosh(x))^2}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{2(2a^2A-3abB+Ab^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}(a^2-b^2)}-\frac{\sinh(x)(a^2(-B)+3aAb-2b^2B)}{(a^2-b^2)(a+b\cosh(x))}}{2(a^2-b^2)}-\frac{\sinh(x)(Ab-aB)}{2(a^2-b^2)(a+b\cosh(x))^2}$$

input
```
Int[(A + B*Cosh[x])/(a + b*Cosh[x])^3,x]
```

output
```
-1/2*((A*b - a*B)*Sinh[x])/((a^2 - b^2)*(a + b*Cosh[x])^2) + ((2*(2*a^2*A
+ A*b^2 - 3*a*b*B)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a -
 b]*Sqrt[a + b]*(a^2 - b^2)) - ((3*a*A*b - a^2*B - 2*b^2*B)*Sinh[x])/((a^2
 - b^2)*(a + b*Cosh[x])))/(2*(a^2 - b^2))
```

3.112. $\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^3}\,dx$

3.112.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.112.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.53

method	result
default	$-\frac{2\left(-\frac{\left(4bAa+b^{2}A-2a^{2}B-Bab-2B\,b^{2}\right)\tanh\left(\frac{x}{2}\right)^{3}}{2(a-b)\left(a^{2}+2ab+b^{2}\right)}+\frac{\left(4bAa-b^{2}A-2a^{2}B+Bab-2B\,b^{2}\right)\tanh\left(\frac{x}{2}\right)}{2(a+b)\left(a^{2}-2ab+b^{2}\right)}\right)}{\left(\tanh\left(\frac{x}{2}\right)^{2}a-\tanh\left(\frac{x}{2}\right)^{2}b-a-b\right)^{2}}+\frac{\left(2A\,a^{2}+b^{2}A-3Bab\right)\operatorname{arctanh}\left(\frac{(a-b}{\sqrt{(a}}\right.}{\left(a^{4}-2a^{2}b^{2}+b^{4}\right)\sqrt{(a+b)(a-}}$
risch	$\frac{2A\,a^{2}b^{2}\mathrm{e}^{3x}+A\,b^{4}\mathrm{e}^{3x}-3Ba\,b^{3}\mathrm{e}^{3x}+6A\,a^{3}b\,\mathrm{e}^{2x}+3Aa\,b^{3}\mathrm{e}^{2x}-2B\,a^{4}\mathrm{e}^{2x}-5B\,a^{2}b^{2}\mathrm{e}^{2x}-2B\,b^{4}\mathrm{e}^{2x}+10A\,a^{2}b^{2}\mathrm{e}^{x}-A\,b^{4}\mathrm{e}^{x}-4B\,a^{3}b\,\mathrm{e}^{x}-5Ba}{b\left(a^{2}-b^{2}\right)^{2}\left(b\,\mathrm{e}^{2x}+2a\,\mathrm{e}^{x}+b\right)^{2}}$

input
```
int((A+B*cosh(x))/(a+b*cosh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-2*(-1/2*(4*A*a*b+A*b^2-2*B*a^2-B*a*b-2*B*b^2)/(a-b)/(a^2+2*a*b+b^2)*tanh(
1/2*x)^3+1/2*(4*A*a*b-A*b^2-2*B*a^2+B*a*b-2*B*b^2)/(a+b)/(a^2-2*a*b+b^2)*t
anh(1/2*x))/(tanh(1/2*x)^2*a-tanh(1/2*x)^2*b-a-b)^2+(2*A*a^2+A*b^2-3*B*a*b
)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)
*(a-b))^(1/2))
```

3.112.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1545 vs. $2(120) = 240$.

Time = 0.34 (sec) , antiderivative size = 3166, normalized size of antiderivative = 23.45

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))^3,x, algorithm="fricas")
```

output
```
[-1/2*(2*B*a^4*b^2 - 6*A*a^3*b^3 + 2*B*a^2*b^4 + 6*A*a*b^5 - 4*B*b^6 - 2*(
2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*cosh(x)^3 - 2*(
2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*sinh(x)^3 + 2*(
2*B*a^6 - 6*A*a^5*b + 3*B*a^4*b^2 + 3*A*a^3*b^3 - 3*B*a^2*b^4 + 3*A*a*b^5
- 2*B*b^6)*cosh(x)^2 + 2*(2*B*a^6 - 6*A*a^5*b + 3*B*a^4*b^2 + 3*A*a^3*b^3
- 3*B*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6 - 3*(2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2
*b^4 + 3*B*a*b^5 - A*b^6)*cosh(x))*sinh(x)^2 - (2*A*a^2*b^3 - 3*B*a*b^4 +
A*b^5 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(x)^4 + (2*A*a^2*b^3 - 3*B*a
*b^4 + A*b^5)*sinh(x)^4 + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*cosh(x)^
3 + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*
b^5)*cosh(x))*sinh(x)^3 + 2*(4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a
*b^4 + A*b^5)*cosh(x)^2 + 2*(4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a
*b^4 + A*b^5 + 3*(2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(x)^2 + 6*(2*A*a^3*
b^2 - 3*B*a^2*b^3 + A*a*b^4)*cosh(x))*sinh(x)^2 + 4*(2*A*a^3*b^2 - 3*B*a^2
*b^3 + A*a*b^4)*cosh(x) + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 + (2*A*a^
2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(x)^3 + 3*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*
b^4)*cosh(x)^2 + (4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^
5)*cosh(x))*sinh(x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 +
2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 -
 b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cos...
```

3.112.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))**3,x)
```

output
```
Timed out
```

3.112.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.112.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(120) = 240$.

Time = 0.26 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.84

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^3}\,dx = \frac{(2\,Aa^2-3\,Bab+Ab^2)\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{(a^4-2\,a^2b^2+b^4)\sqrt{-a^2+b^2}} + \frac{2\,Aa^2b^2e^{(3\,x)}-3\,Bab^3e^{(3\,x)}+Ab^4e^{(3\,x)}-2\,Ba^4e^{(2\,x)}+6\,Aa^3be^{(2\,x)}-5\,Ba^2b^2e^{(2\,x)}+3\,Aab^3e^{(2\,x)}-2\,Bb^4}{(a^4b-2\,a^2b^3+b^5)(be^{(2\,x)}+2\,ae}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))^3,x, algorithm="giac")
```

output
```
(2*A*a^2 - 3*B*a*b + A*b^2)*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/((a^4 - 2
*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) + (2*A*a^2*b^2*e^(3*x) - 3*B*a*b^3*e^(3*
x) + A*b^4*e^(3*x) - 2*B*a^4*e^(2*x) + 6*A*a^3*b*e^(2*x) - 5*B*a^2*b^2*e^(
2*x) + 3*A*a*b^3*e^(2*x) - 2*B*b^4*e^(2*x) - 4*B*a^3*b*e^x + 10*A*a^2*b^2*
e^x - 5*B*a*b^3*e^x - A*b^4*e^x - B*a^2*b^2 + 3*A*a*b^3 - 2*B*b^4)/((a^4*b
 - 2*a^2*b^3 + b^5)*(b*e^(2*x) + 2*a*e^x + b)^2)
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^3}\,dx = \int \frac{A+B\cosh(x)}{(a+b\cosh(x))^3}\,dx$$

input
```
int((A + B*cosh(x))/(a + b*cosh(x))^3,x)
```

output
```
int((A + B*cosh(x))/(a + b*cosh(x))^3, x)
```

3.113 $\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^4}\,dx$

3.113.1 Optimal result

Integrand size = 15, antiderivative size = 197

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^4}\,dx = \frac{(2a^3A+3aAb^2-4a^2bB-b^3B)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}} - \frac{(Ab-aB)\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^3} - \frac{(5aAb-2a^2B-3b^2B)\sinh(x)}{6(a^2-b^2)^2(a+b\cosh(x))^2} - \frac{(11a^2Ab+4Ab^3-2a^3B-13ab^2B)\sinh(x)}{6(a^2-b^2)^3(a+b\cosh(x))}$$

output
```
(2*A*a^3+3*A*a*b^2-4*B*a^2*b-B*b^3)*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^
(1/2))/(a-b)^(7/2)/(a+b)^(7/2)-1/3*(A*b-B*a)*sinh(x)/(a^2-b^2)/(a+b*cosh(x
))^3-1/6*(5*A*a*b-2*B*a^2-3*B*b^2)*sinh(x)/(a^2-b^2)^2/(a+b*cosh(x))^2-1/6
*(11*A*a^2*b+4*A*b^3-2*B*a^3-13*B*a*b^2)*sinh(x)/(a^2-b^2)^3/(a+b*cosh(x))
```

3.113.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.99

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^4}\,dx = \frac{1}{6}\left(\frac{6(2a^3A+3aAb^2-4a^2bB-b^3B)\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{7/2}} + \frac{2(-Ab+aB)\sinh(x)}{(a-b)(a+b)(a+b\cosh(x))^3} + \frac{(-5aAb+2a^2B+3b^2B)\sinh(x)}{(a-b)^2(a+b)^2(a+b\cosh(x))^2} + \frac{(-11a^2Ab-4Ab^3+2a^3B+13ab^2B)\sinh(x)}{(a-b)^3(a+b)^3(a+b\cosh(x))}\right)$$

input
```
Integrate[(A + B*Cosh[x])/(a + b*Cosh[x])^4,x]
```

output
```
((6*(2*a^3*A + 3*a*A*b^2 - 4*a^2*b*B - b^3*B)*ArcTan[((a - b)*Tanh[x/2])/S
qrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) + (2*(-(A*b) + a*B)*Sinh[x])/((a - b)
*(a + b)*(a + b*Cosh[x])^3) + ((-5*a*A*b + 2*a^2*B + 3*b^2*B)*Sinh[x])/((a
 - b)^2*(a + b)^2*(a + b*Cosh[x])^2) + ((-11*a^2*A*b - 4*A*b^3 + 2*a^3*B +
 13*a*b^2*B)*Sinh[x])/((a - b)^3*(a + b)^3*(a + b*Cosh[x])))/6
```

3.113.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.21, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 3233, 25, 3042, 3233, 25, 3042, 3233, 27, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^4}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{\left(a+b\sin\left(\frac{\pi}{2}+ix\right)\right)^4}dx$$

$$\Big\downarrow \; 3233$$

$$-\frac{\int -\frac{3(aA-bB)-2(Ab-aB)\cosh(x)}{(a+b\cosh(x))^3}dx}{3\left(a^2-b^2\right)}-\frac{\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^3}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{3(aA-bB)-2(Ab-aB)\cosh(x)}{(a+b\cosh(x))^3}dx}{3\left(a^2-b^2\right)}-\frac{\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^3}$$

$\Big\downarrow$ 3042

$$-\frac{\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^3}+\frac{\int \frac{3(aA-bB)-2(Ab-aB)\sin\left(ix+\frac{\pi}{2}\right)}{\left(a+b\sin\left(ix+\frac{\pi}{2}\right)\right)^3}dx}{3\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3233

$$\frac{-\frac{\int -\frac{2\left(3Aa^2-5bBa+2Ab^2\right)-\left(-2Ba^2+5Aba-3b^2B\right)\cosh(x)}{(a+b\cosh(x))^2}dx}{2(a^2-b^2)}-\frac{\sinh(x)\left(-2a^2B+5aAb-3b^2B\right)}{2(a^2-b^2)(a+b\cosh(x))^2}}{3\left(a^2-b^2\right)}-\frac{\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^3}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{2\left(3Aa^2-5bBa+2Ab^2\right)-\left(-2Ba^2+5Aba-3b^2B\right)\cosh(x)}{(a+b\cosh(x))^2}dx}{2(a^2-b^2)}-\frac{\sinh(x)\left(-2a^2B+5aAb-3b^2B\right)}{2(a^2-b^2)(a+b\cosh(x))^2}}{3\left(a^2-b^2\right)}-\frac{\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^3}$$

$\Big\downarrow$ 3042

$$-\frac{\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^3}+\frac{-\frac{\sinh(x)\left(-2a^2B+5aAb-3b^2B\right)}{2(a^2-b^2)(a+b\cosh(x))^2}+\frac{\int \frac{2\left(3Aa^2-5bBa+2Ab^2\right)+\left(2Ba^2-5Aba+3b^2B\right)\sin\left(ix+\frac{\pi}{2}\right)}{\left(a+b\sin\left(ix+\frac{\pi}{2}\right)\right)^2}dx}{2(a^2-b^2)}}{3\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3233

$$\frac{\frac{-\frac{\int -\frac{3\left(2Aa^3-4bBa^2+3Ab^2a-b^3B\right)}{a+b\cosh(x)}dx}{a^2-b^2}-\frac{\sinh(x)\left(-2a^3B+11a^2Ab-13ab^2B+4Ab^3\right)}{\left(a^2-b^2\right)(a+b\cosh(x))}}{2(a^2-b^2)}-\frac{\sinh(x)\left(-2a^2B+5aAb-3b^2B\right)}{2(a^2-b^2)(a+b\cosh(x))^2}}{3\left(a^2-b^2\right)}-\frac{\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^3}$$

$\Big\downarrow$ 27

3.113. $\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^4}\,dx$

$$\frac{\frac{\frac{3\left(2a^3A-4a^2bB+3aAb^2-b^3B\right)\int \frac{1}{a+b\cosh(x)}dx}{a^2-b^2}-\frac{\sinh(x)\left(-2a^3B+11a^2Ab-13ab^2B+4Ab^3\right)}{\left(a^2-b^2\right)(a+b\cosh(x))}}{2(a^2-b^2)}-\frac{\sinh(x)(-2a^2B+5aAb-3b^2B)}{2(a^2-b^2)(a+b\cosh(x))^2}}{3\left(a^2-b^2\right)}-\frac{\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^3}+\frac{-\frac{\sinh(x)(-2a^2B+5aAb-3b^2B)}{2(a^2-b^2)(a+b\cosh(x))^2}+\frac{-\frac{\sinh(x)\left(-2a^3B+11a^2Ab-13ab^2B+4Ab^3\right)}{\left(a^2-b^2\right)(a+b\cosh(x))}+\frac{3\left(2a^3A-4a^2bB+3aAb^2-b^3B\right)\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}}{2(a^2-b^2)}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3138$$

$$\frac{\frac{\frac{6\left(2a^3A-4a^2bB+3aAb^2-b^3B\right)\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a^2-b^2}-\frac{\sinh(x)\left(-2a^3B+11a^2Ab-13ab^2B+4Ab^3\right)}{\left(a^2-b^2\right)(a+b\cosh(x))}}{2(a^2-b^2)}-\frac{\sinh(x)(-2a^2B+5aAb-3b^2B)}{2(a^2-b^2)(a+b\cosh(x))^2}}{3\left(a^2-b^2\right)}-\frac{\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^3}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{6\left(2a^3A-4a^2bB+3aAb^2-b^3B\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)}-\frac{\sinh(x)\left(-2a^3B+11a^2Ab-13ab^2B+4Ab^3\right)}{\left(a^2-b^2\right)(a+b\cosh(x))}}{2(a^2-b^2)}-\frac{\sinh(x)(-2a^2B+5aAb-3b^2B)}{2(a^2-b^2)(a+b\cosh(x))^2}}{3\left(a^2-b^2\right)}-\frac{\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^3}$$

input
```
Int[(A + B*Cosh[x])/(a + b*Cosh[x])^4,x]
```

output
```
-1/3*((A*b - a*B)*Sinh[x])/((a^2 - b^2)*(a + b*Cosh[x])^3) + (-1/2*((5*a*A
*b - 2*a^2*B - 3*b^2*B)*Sinh[x])/((a^2 - b^2)*(a + b*Cosh[x])^2) + ((6*(2*
a^3*A + 3*a*A*b^2 - 4*a^2*b*B - b^3*B)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqr
t[a + b]])/(Sqrt[a - b]*Sqrt[a + b]*(a^2 - b^2)) - ((11*a^2*A*b + 4*A*b^3
- 2*a^3*B - 13*a*b^2*B)*Sinh[x])/((a^2 - b^2)*(a + b*Cosh[x])))/(2*(a^2 -
b^2)))/(3*(a^2 - b^2))
```

3.113. $\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^4}\,dx$

3.113.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.113.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.74

method	result
default	$-\frac{2\left(-\frac{\left(6A\,a^{2}b+3Aa\,b^{2}+2A\,b^{3}-2a^{3}B-2B\,a^{2}b-6Ba\,b^{2}-B\,b^{3}\right)\tanh\left(\frac{x}{2}\right)^{5}}{2(a-b)\left(a^{3}+3a^{2}b+3a\,b^{2}+b^{3}\right)}+\frac{2\left(9A\,a^{2}b+A\,b^{3}-3a^{3}B-7Ba\,b^{2}\right)\tanh\left(\frac{x}{2}\right)^{3}}{3\left(a^{2}+2ab+b^{2}\right)\left(a^{2}-2ab+b^{2}\right)}-\frac{\left(6A\,a^{2}b-3Aa\,b^{2}+2A\,b \cdots\right.}{2(a+ \cdots}\right.}{\left(\tanh\left(\frac{x}{2}\right)^{2}a-\tanh\left(\frac{x}{2}\right)^{2}b-a-b\right)^{3}}$
risch	$\frac{-15Ba\,b^{5}e^{4x}+44A\,a^{5}b\,e^{3x}+82A\,a^{3}b^{3}e^{3x}+4A\,b^{6}+30A\,a^{4}b^{2}e^{4x}-24B\,a^{5}b\,e^{2x}-102B\,a^{3}b^{3}e^{2x}-24Ba\,b^{5}e^{2x}-2B\,a^{3}b^{3}-13Ba\,b^{5}+11A\,a \cdots}{\cdots}$

input
```
int((A+B*cosh(x))/(a+b*cosh(x))^4,x,method=_RETURNVERBOSE)
```

output
```
-2*(-1/2*(6*A*a^2*b+3*A*a*b^2+2*A*b^3-2*B*a^3-2*B*a^2*b-6*B*a*b^2-B*b^3)/(
a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tanh(1/2*x)^5+2/3*(9*A*a^2*b+A*b^3-3*B*a^3-
7*B*a*b^2)/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tanh(1/2*x)^3-1/2*(6*A*a^2*b-3*
A*a*b^2+2*A*b^3-2*B*a^3+2*B*a^2*b-6*B*a*b^2+B*b^3)/(a+b)/(a^3-3*a^2*b+3*a*
b^2-b^3)*tanh(1/2*x))/(tanh(1/2*x)^2*a-tanh(1/2*x)^2*b-a-b)^3+(2*A*a^3+3*A
*a*b^2-4*B*a^2*b-B*b^3)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*
arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

3.113.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3767 vs. $2(181) = 362$.

Time = 0.48 (sec) , antiderivative size = 7603, normalized size of antiderivative = 38.59

$$\int \frac{A + B\cosh(x)}{(a + b\cosh(x))^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))^4,x, algorithm="fricas")
```

output
```
Too large to include
```

3.113.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cosh(x)}{(a + b\cosh(x))^4}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))**4,x)
```

output
```
Timed out
```

3.113.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.113.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 453 vs. $2(181) = 362$.

Time = 0.27 (sec) , antiderivative size = 453, normalized size of antiderivative = 2.30

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^4}\,dx = \frac{(2\,Aa^3 - 4\,Ba^2b + 3\,Aab^2 - Bb^3)\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{(a^6 - 3\,a^4b^2 + 3\,a^2b^4 - b^6)\sqrt{-a^2+b^2}} + \frac{6\,Aa^3b^3e^{(5\,x)} - 12\,Ba^2b^4e^{(5\,x)} + 9\,Aab^5e^{(5\,x)} - 3\,Bb^6e^{(5\,x)} + 30\,Aa^4b^2e^{(4\,x)} - 60\,Ba^3b^3e^{(4\,x)} + 45\,Aa^2b^4e^{(4}}{}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))^4,x, algorithm="giac")
```

output
```
(2*A*a^3 - 4*B*a^2*b + 3*A*a*b^2 - B*b^3)*arctan((b*e^x + a)/sqrt(-a^2 + b
^2))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) + 1/3*(6*A*a^3
*b^3*e^(5*x) - 12*B*a^2*b^4*e^(5*x) + 9*A*a*b^5*e^(5*x) - 3*B*b^6*e^(5*x)
+ 30*A*a^4*b^2*e^(4*x) - 60*B*a^3*b^3*e^(4*x) + 45*A*a^2*b^4*e^(4*x) - 15*
B*a*b^5*e^(4*x) - 8*B*a^6*e^(3*x) + 44*A*a^5*b*e^(3*x) - 64*B*a^4*b^2*e^(3
*x) + 82*A*a^3*b^3*e^(3*x) - 78*B*a^2*b^4*e^(3*x) + 24*A*a*b^5*e^(3*x) - 2
4*B*a^5*b*e^(2*x) + 102*A*a^4*b^2*e^(2*x) - 102*B*a^3*b^3*e^(2*x) + 36*A*a
^2*b^4*e^(2*x) - 24*B*a*b^5*e^(2*x) + 12*A*b^6*e^(2*x) - 12*B*a^4*b^2*e^x
+ 60*A*a^3*b^3*e^x - 66*B*a^2*b^4*e^x + 15*A*a*b^5*e^x + 3*B*b^6*e^x - 2*B
*a^3*b^3 + 11*A*a^2*b^4 - 13*B*a*b^5 + 4*A*b^6)/((a^6*b - 3*a^4*b^3 + 3*a^
2*b^5 - b^7)*(b*e^(2*x) + 2*a*e^x + b)^3)
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^4}\,dx=\int \frac{A+B\cosh(x)}{\left(a+b\cosh\left(x\right)\right)^4}\,dx$$

input
```
int((A + B*cosh(x))/(a + b*cosh(x))^4,x)
```

output
```
int((A + B*cosh(x))/(a + b*cosh(x))^4, x)
```

3.114 $\int \frac{\frac{bB}{a}+B\cosh(x)}{a+b\cosh(x)}\,dx$

3.114.1 Optimal result

Integrand size = 20, antiderivative size = 56

$$\int \frac{\frac{bB}{a}+B\cosh(x)}{a+b\cosh(x)}\,dx=\frac{Bx}{b}-\frac{2\sqrt{a-b}\sqrt{a+b}B\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab}$$

output
```
B*x/b-2*B*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))*(a-b)^(1/2)*(a+b)^(
1/2)/a/b
```

3.114.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{\frac{bB}{a}+B\cosh(x)}{a+b\cosh(x)}\,dx=\frac{B\left(\frac{ax}{b}+\frac{2\sqrt{-a^2+b^2}\arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{b}\right)}{a}$$

input
```
Integrate[((b*B)/a + B*Cosh[x])/(a + b*Cosh[x]),x]
```

output
```
(B*((a*x)/b + (2*Sqrt[-a^2 + b^2]*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[-a^2 +
b^2]])/b))/a
```

3.114.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\frac{bB}{a}+B\cosh(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\frac{bB}{a}+B\sin\left(\frac{\pi}{2}+ix\right)}{a+b\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3214}$$

$$\frac{Bx}{b}-\frac{B\left(a-\frac{b^2}{a}\right)\int\frac{1}{a+b\cosh(x)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{Bx}{b}-\frac{B\left(a-\frac{b^2}{a}\right)\int\frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{3138}$$

$$\frac{Bx}{b}-\frac{2B\left(a-\frac{b^2}{a}\right)\int\frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{b}$$

$$\Big\downarrow \text{221}$$

$$\frac{Bx}{b}-\frac{2B\left(a-\frac{b^2}{a}\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}$$

input
```
Int[((b*B)/a + B*Cosh[x])/(a + b*Cosh[x]),x]
```

output
```
(B*x)/b - (2*(a - b^2/a)*B*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(
Sqrt[a - b]*b*Sqrt[a + b])
```

3.114.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.114.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.45

method	result	size
default	$\frac{2B\left(-\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b}+\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2b}-\frac{\left(a^{2}-b^{2}\right)\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b\sqrt{(a+b)(a-b)}}\right)}{a}$	81
risch	$\frac{Bx}{b}+\frac{\sqrt{a^{2}-b^{2}}\,B\ln\left(\mathrm{e}^{x}+\frac{a+\sqrt{a^{2}-b^{2}}}{b}\right)}{ba}-\frac{\sqrt{a^{2}-b^{2}}\,B\ln\left(\mathrm{e}^{x}-\frac{-a+\sqrt{a^{2}-b^{2}}}{b}\right)}{ba}$	92

input
```
int((b*B/a+B*cosh(x))/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
2*B/a*(-1/2*a/b*ln(tanh(1/2*x)-1)+1/2*a/b*ln(tanh(1/2*x)+1)-(a^2-b^2)/b/((
a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2)))
```

3.114.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 190, normalized size of antiderivative = 3.39

$$\int \frac{\frac{bB}{a} + B\cosh(x)}{a + b\cosh(x)}\,dx$$
$$= \left[\frac{Bax + \sqrt{a^2-b^2}B\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\,(b^2\cosh(x)+ab)\sinh(x)+2\,\sqrt{a^2-b^2}(b\cosh(x)+b\sinh(x)+}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right.}{ab} \right.$$

input
```
integrate((b*B/a+B*cosh(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[(B*a*x + sqrt(a^2 - b^2)*B*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cos
h(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*
cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*
cosh(x) + a)*sinh(x) + b)))/(a*b), (B*a*x + 2*sqrt(-a^2 + b^2)*B*arctan(-s
qrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)))/(a*b)]
```

3.114.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(44) = 88.

Time = 13.46 (sec) , antiderivative size = 168, normalized size of antiderivative = 3.00

$$\int \frac{\frac{bB}{a} + B\cosh(x)}{a + b\cosh(x)}\,dx$$
$$= \begin{cases} \text{NaN} \\ \frac{B\sinh(x)}{a} \\ \frac{Bx}{b} \\ \frac{Bx}{b} + \frac{B\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{b\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} - \frac{B\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{b\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} + \frac{B\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{a\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} - \frac{B\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\right.}{a\sqrt{\frac{a}{a-b}+}} \end{cases}$$

input
```
integrate((b*B/a+B*cosh(x))/(a+b*cosh(x)),x)
```

output
```
Piecewise((nan, Eq(a, 0) & Eq(b, 0)), (B*sinh(x)/a, Eq(b, 0)), (B*x/b, Eq(
a, b) | Eq(a, -b)), (B*x/b + B*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2
))/(b*sqrt(a/(a - b) + b/(a - b))) - B*log(sqrt(a/(a - b) + b/(a - b)) + t
anh(x/2))/(b*sqrt(a/(a - b) + b/(a - b))) + B*log(-sqrt(a/(a - b) + b/(a -
 b)) + tanh(x/2))/(a*sqrt(a/(a - b) + b/(a - b))) - B*log(sqrt(a/(a - b) +
 b/(a - b)) + tanh(x/2))/(a*sqrt(a/(a - b) + b/(a - b))), True))
```

3.114.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\frac{bB}{a} + B\cosh(x)}{a + b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*B/a+B*cosh(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.114.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int \frac{\frac{bB}{a} + B\cosh(x)}{a + b\cosh(x)}\,dx = \frac{Bx}{b} - \frac{2\,(Ba^2 - Bb^2)\arctan\left(\frac{be^x + a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}ab}$$

input
```
integrate((b*B/a+B*cosh(x))/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
B*x/b - 2*(B*a^2 - B*b^2)*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2
+ b^2)*a*b)
```

3.114.9 Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 205, normalized size of antiderivative = 3.66

$$\int \frac{\frac{bB}{a} + B\cosh(x)}{a + b\cosh(x)}\,dx$$
$$= \frac{2\,\mathrm{atan}\left(\frac{b\sqrt{a^2\,b^2}\,\sqrt{B^2\,b^2-B^2\,a^2}}{B\,(b^4-a^2\,b^2)} + \frac{a\,b^2\,\mathrm{e}^x\,\left(\frac{2\sqrt{B^2\,b^2-B^2\,a^2}}{B\,b^2\,\left(b^4-a^2\,b^2\right)} - \frac{2\left(B\,a^2\,\sqrt{a^2\,b^2}-B\,b^2\,\sqrt{a^2\,b^2}\right)}{a^2\,b^4\,\sqrt{-B^2\,\left(a^2-b^2\right)}\,\sqrt{a^2\,b^2}}\right)\sqrt{a^2\,b^2}}{2}\right)\sqrt{B^2\,b^2-B^2\,a^2}}{\sqrt{a^2\,b^2}}$$
$$+ \frac{B\,x}{b}$$

input
```
int((B*cosh(x) + (B*b)/a)/(a + b*cosh(x)),x)
```

output
```
(2*atan((b*(a^2*b^2)^(1/2)*(B^2*b^2 - B^2*a^2)^(1/2))/(B*(b^4 - a^2*b^2))
+ (a*b^2*exp(x)*((2*(B^2*b^2 - B^2*a^2)^(1/2))/(B*b^2*(b^4 - a^2*b^2)) - (
2*(B*a^2*(a^2*b^2)^(1/2) - B*b^2*(a^2*b^2)^(1/2)))/(a^2*b^4*(-B^2*(a^2 - b
^2))^(1/2)*(a^2*b^2)^(1/2)))*(a^2*b^2)^(1/2))/2)*(B^2*b^2 - B^2*a^2)^(1/2)
)/(a^2*b^2)^(1/2) + (B*x)/b
```

3.115 $\int \frac{\frac{aB}{b}+B\cosh(x)}{a+b\cosh(x)}\,dx$

3.115.1 Optimal result

Integrand size = 20, antiderivative size = 6

$$\int \frac{\frac{aB}{b}+B\cosh(x)}{a+b\cosh(x)}\,dx=\frac{Bx}{b}$$

output
```
B*x/b
```

3.115.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\frac{aB}{b}+B\cosh(x)}{a+b\cosh(x)}\,dx=\frac{Bx}{b}$$

input
```
Integrate[((a*B)/b + B*Cosh[x])/(a + b*Cosh[x]),x]
```

output
```
(B*x)/b
```

3.115.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2011, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\frac{aB}{b}+B\cosh(x)}{a+b\cosh(x)}\,dx$$

$$\Big\downarrow \text{2011}$$

$$\frac{B\int 1dx}{b}$$

$$\Big\downarrow \text{24}$$

$$\frac{Bx}{b}$$

input
```
Int[((a*B)/b + B*Cosh[x])/(a + b*Cosh[x]),x]
```

output
```
(B*x)/b
```

3.115.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

3.115.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
default	$\frac{Bx}{b}$	7
risch	$\frac{Bx}{b}$	7

input
```
int((a*B/b+B*cosh(x))/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
B*x/b
```

3.115.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\frac{aB}{b} + B\cosh(x)}{a + b\cosh(x)}\, dx = \frac{Bx}{b}$$

input
```
integrate((a*B/b+B*cosh(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
B*x/b
```

3.115.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.50

$$\int \frac{\frac{aB}{b} + B\cosh(x)}{a + b\cosh(x)}\, dx = \frac{Bx}{b}$$

input
```
integrate((a*B/b+B*cosh(x))/(a+b*cosh(x)),x)
```

output
```
B*x/b
```

3.115.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\frac{aB}{b} + B\cosh(x)}{a + b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*B/b+B*cosh(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.115.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\frac{aB}{b} + B\cosh(x)}{a + b\cosh(x)}\,dx = \frac{Bx}{b}$$

input
```
integrate((a*B/b+B*cosh(x))/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
B*x/b
```

3.115.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\frac{aB}{b} + B\cosh(x)}{a + b\cosh(x)}\,dx = \frac{B\,x}{b}$$

input
```
int((B*cosh(x) + (B*a)/b)/(a + b*cosh(x)),x)
```

output
```
(B*x)/b
```

3.116 $\int \frac{a+b\cosh(x)}{(b+a\cosh(x))^2}\,dx$

3.116.1 Optimal result

Integrand size = 15, antiderivative size = 11

$$\int \frac{a+b\cosh(x)}{(b+a\cosh(x))^2}\,dx = \frac{\sinh(x)}{b+a\cosh(x)}$$

output
```
sinh(x)/(b+a*cosh(x))
```

3.116.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{a+b\cosh(x)}{(b+a\cosh(x))^2}\,dx = \frac{\sinh(x)}{b+a\cosh(x)}$$

input
```
Integrate[(a + b*Cosh[x])/(b + a*Cosh[x])^2,x]
```

output
```
Sinh[x]/(b + a*Cosh[x])
```

3.116.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3233, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\cosh(x)}{(a\cosh(x)+b)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sin\left(\frac{\pi}{2}+ix\right)}{\left(b+a\sin\left(\frac{\pi}{2}+ix\right)\right)^2}dx$$

$$\Big\downarrow \text{3233}$$

$$\frac{\int 0dx}{a^2-b^2}+\frac{\sinh(x)}{a\cosh(x)+b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sinh(x)}{a\cosh(x)+b}$$

input
```
Int[(a + b*Cosh[x])/(b + a*Cosh[x])^2,x]
```

output
```
Sinh[x]/(b + a*Cosh[x])
```

3.116.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.116.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
parallelrisch	$\frac{\sinh(x)}{b+a\cosh(x)}$	12
risch	$-\frac{2(\mathrm{e}^{x} b+a)}{a(a\,\mathrm{e}^{2x}+2\,\mathrm{e}^{x} b+a)}$	27
default	$\frac{2\tanh\left(\frac{x}{2}\right)}{\tanh\left(\frac{x}{2}\right)^{2} a-\tanh\left(\frac{x}{2}\right)^{2} b+a+b}$	29

input
```
int((a+b*cosh(x))/(b+a*cosh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
sinh(x)/(b+a*cosh(x))
```

3.116.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(11) = 22$.

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 4.91

$$\int \frac{a+b\cosh(x)}{(b+a\cosh(x))^2}\,dx$$
$$= -\frac{2\left(b\cosh\left(x\right)+b\sinh\left(x\right)+a\right)}{a^{2}\cosh\left(x\right)^{2}+a^{2}\sinh\left(x\right)^{2}+2\,ab\cosh\left(x\right)+a^{2}+2\left(a^{2}\cosh\left(x\right)+ab\right)\sinh\left(x\right)}$$

input
```
integrate((a+b*cosh(x))/(b+a*cosh(x))^2,x, algorithm="fricas")
```

output
```
-2*(b*cosh(x) + b*sinh(x) + a)/(a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh
(x) + a^2 + 2*(a^2*cosh(x) + a*b)*sinh(x))
```

3.116.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\cosh(x)}{(b + a\cosh(x))^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cosh(x))/(b+a*cosh(x))**2,x)
```

output
```
Timed out
```

3.116.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\cosh(x)}{(b + a\cosh(x))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*cosh(x))/(b+a*cosh(x))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.116.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(11) = 22$.

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.36

$$\int \frac{a + b\cosh(x)}{(b + a\cosh(x))^2}\,dx = -\frac{2\,(be^x + a)}{(ae^{(2\,x)} + 2\,be^x + a)a}$$

input
```
integrate((a+b*cosh(x))/(b+a*cosh(x))^2,x, algorithm="giac")
```

output
```
-2*(b*e^x + a)/((a*e^(2*x) + 2*b*e^x + a)*a)
```

3.116.9 Mupad [B] (verification not implemented)

Time = 1.78 (sec) , antiderivative size = 51, normalized size of antiderivative = 4.64

$$\int \frac{a+b\cosh(x)}{(b+a\cosh(x))^2}\,dx = -\frac{\frac{2\,\mathrm{e}^{x}\,\left(a\,b^3-a^3\,b\right)}{a\,\left(a\,b^2-a^3\right)}+2}{a+2\,b\,\mathrm{e}^{x}+a\,\mathrm{e}^{2\,x}}$$

input
```
int((a + b*cosh(x))/(b + a*cosh(x))^2,x)
```

output
```
-((2*exp(x)*(a*b^3 - a^3*b))/(a*(a*b^2 - a^3)) + 2)/(a + 2*b*exp(x) + a*ex
p(2*x))
```

3.117 $\int \frac{3+\cosh(x)}{2-\cosh(x)}\,dx$

3.117.1 Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \frac{3+\cosh(x)}{2-\cosh(x)}\,dx = -x+\frac{5x}{\sqrt{3}}+\frac{10\text{arctanh}\left(\frac{\sinh(x)}{2+\sqrt{3}-\cosh(x)}\right)}{\sqrt{3}}$$

output
```
-x+5/3*x*3^(1/2)+10/3*arctanh(sinh(x)/(2-cosh(x)+3^(1/2)))*3^(1/2)
```

3.117.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{3+\cosh(x)}{2-\cosh(x)}\,dx = -x+\frac{10\text{arctanh}\left(\sqrt{3}\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{3}}$$

input
```
Integrate[(3 + Cosh[x])/(2 - Cosh[x]),x]
```

output
```
-x + (10*ArcTanh[Sqrt[3]*Tanh[x/2]])/Sqrt[3]
```

3.117.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3214, 3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(x)+3}{2-\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{3+\sin\left(\frac{\pi}{2}+ix\right)}{2-\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3214}$$

$$5\int \frac{1}{2-\cosh(x)}dx-x$$

$$\Big\downarrow \text{3042}$$

$$-x+5\int \frac{1}{2-\sin\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3136}$$

$$5\left(\frac{2\text{arctanh}\left(\frac{\sinh(x)}{-\cosh(x)+\sqrt{3}+2}\right)}{\sqrt{3}}+\frac{x}{\sqrt{3}}\right)-x$$

input
```
Int[(3 + Cosh[x])/(2 - Cosh[x]),x]
```

output
```
-x + 5*(x/Sqrt[3] + (2*ArcTanh[Sinh[x]/(2 + Sqrt[3] - Cosh[x])])/Sqrt[3])
```

3.117.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.117.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{10\sqrt{3}\,\operatorname{arctanh}\left(\tanh\left(\frac{x}{2}\right)\sqrt{3}\right)}{3}+\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	32
risch	$-x+\frac{5\sqrt{3}\,\ln\left(e^{x}-2+\sqrt{3}\right)}{3}-\frac{5\sqrt{3}\,\ln\left(e^{x}-2-\sqrt{3}\right)}{3}$	33

input
```
int((3+cosh(x))/(2-cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
10/3*3^(1/2)*arctanh(tanh(1/2*x)*3^(1/2))+ln(tanh(1/2*x)-1)-ln(tanh(1/2*x)
+1)
```

3.117.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25

$$\int \frac{3+\cosh(x)}{2-\cosh(x)}\,dx=\frac{5}{3}\sqrt{3}\log\left(-\frac{2\left(\sqrt{3}-2\right)\cosh\left(x\right)-\left(2\sqrt{3}-3\right)\sinh\left(x\right)-\sqrt{3}+2}{\cosh\left(x\right)-2}\right)-x$$

input
```
integrate((3+cosh(x))/(2-cosh(x)),x, algorithm="fricas")
```

output
```
5/3*sqrt(3)*log(-(2*(sqrt(3) - 2)*cosh(x) - (2*sqrt(3) - 3)*sinh(x) - sqrt
(3) + 2)/(cosh(x) - 2)) - x
```

3.117.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \frac{3+\cosh(x)}{2-\cosh(x)}\,dx = -x - \frac{5\sqrt{3}\log\left(\tanh\left(\frac{x}{2}\right) - \frac{\sqrt{3}}{3}\right)}{3} + \frac{5\sqrt{3}\log\left(\tanh\left(\frac{x}{2}\right) + \frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate((3+cosh(x))/(2-cosh(x)),x)
```

output
```
-x - 5*sqrt(3)*log(tanh(x/2) - sqrt(3)/3)/3 + 5*sqrt(3)*log(tanh(x/2) + sq
rt(3)/3)/3
```

3.117.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{3+\cosh(x)}{2-\cosh(x)}\,dx = \frac{5}{3}\sqrt{3}\log\left(-\frac{\sqrt{3}-e^{(-x)}+2}{\sqrt{3}+e^{(-x)}-2}\right) - x$$

input
```
integrate((3+cosh(x))/(2-cosh(x)),x, algorithm="maxima")
```

output
```
5/3*sqrt(3)*log(-(sqrt(3) - e^(-x) + 2)/(sqrt(3) + e^(-x) - 2)) - x
```

3.117.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \frac{3+\cosh(x)}{2-\cosh(x)}\,dx = -\frac{5}{3}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+2e^{x}-4\right|}{\left|2\sqrt{3}+2e^{x}-4\right|}\right) - x$$

input
```
integrate((3+cosh(x))/(2-cosh(x)),x, algorithm="giac")
```

output
```
-5/3*sqrt(3)*log(abs(-2*sqrt(3) + 2*e^x - 4)/abs(2*sqrt(3) + 2*e^x - 4)) -
 x
```

3.117.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.33

$$\int \frac{3+\cosh(x)}{2-\cosh(x)}\,dx = \frac{5\sqrt{3}\,\ln\left(10\,e^x+\frac{5\sqrt{3}\,(4\,e^x-2)}{3}\right)}{3}-\frac{5\sqrt{3}\,\ln\left(10\,e^x-\frac{5\sqrt{3}\,(4\,e^x-2)}{3}\right)}{3}-x$$

input
```
int(-(cosh(x) + 3)/(cosh(x) - 2),x)
```

output
```
(5*3^(1/2)*log(10*exp(x) + (5*3^(1/2)*(4*exp(x) - 2))/3))/3 - (5*3^(1/2)*l
og(10*exp(x) - (5*3^(1/2)*(4*exp(x) - 2))/3))/3 - x
```

3.118 $\int \frac{A+B\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx$

3.118.1 Optimal result

Integrand size = 17, antiderivative size = 108

$$\int \frac{A+B\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx = -\frac{2iB\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}} - \frac{2i(Ab-aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}$$

output
```
-2*I*B*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(1/2*x),2^(1/2)*(
b/(a+b))^(1/2))*(a+b*cosh(x))^(1/2)/b/((a+b*cosh(x))/(a+b))^(1/2)-2*I*(A*b
-B*a)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/2)*(b
/(a+b))^(1/2))*((a+b*cosh(x))/(a+b))^(1/2)/b/(a+b*cosh(x))^(1/2)
```

3.118.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.74

$$\int \frac{A+B\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx = -\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\left((a+b)BE\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)+(Ab-aB)\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)\right)}{b\sqrt{a+b\cosh(x)}}$$

input
```
Integrate[(A + B*Cosh[x])/Sqrt[a + b*Cosh[x]],x]
```

output
```
((-2*I)*Sqrt[(a + b*Cosh[x])/(a + b)]*((a + b)*B*EllipticE[(I/2)*x, (2*b)/
(a + b)] + (A*b - a*B)*EllipticF[(I/2)*x, (2*b)/(a + b)]))/(b*Sqrt[a + b*C
osh[x]])
```

3.118.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{\sqrt{a+b\cosh(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{\sqrt{a+b\sin\left(\frac{\pi}{2}+ix\right)}}dx$$

$$\Big\downarrow \text{3231}$$

$$\frac{(Ab-aB)\int \frac{1}{\sqrt{a+b\cosh(x)}}dx}{b}+\frac{B\int \sqrt{a+b\cosh(x)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(Ab-aB)\int \frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}+\frac{B\int \sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{3134}$$

$$\frac{B\sqrt{a+b\cosh(x)}\int \sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}dx}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{(Ab-aB)\int \frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(Ab-aB)\int \frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}+\frac{B\sqrt{a+b\cosh(x)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

$$\Big\downarrow \ 3132$$

$$\frac{(Ab-aB)\int \frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}-\frac{2iB\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

$$\Big\downarrow \ 3142$$

$$\frac{(Ab-aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}}dx}{b\sqrt{a+b\cosh(x)}}-\frac{2iB\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

$$\Big\downarrow \ 3042$$

$$\frac{(Ab-aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cosh(x)}}-\frac{2iB\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

$$\Big\downarrow \ 3140$$

$$-\frac{2i(Ab-aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}\operatorname{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}-\frac{2iB\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

input
```
Int[(A + B*Cosh[x])/Sqrt[a + b*Cosh[x]],x]
```

output
```
((-2*I)*B*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[(
a + b*Cosh[x])/(a + b)]) - ((2*I)*(A*b - a*B)*Sqrt[(a + b*Cosh[x])/(a + b)
]*EllipticF[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[a + b*Cosh[x]])
```

3.118.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

3.118.4 Maple [A] (verified)

Time = 1.69 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.02

method	result
default	$\frac{2\left(A\,\mathrm{EllipticF}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}},\frac{\sqrt{-\frac{2(a-b)}{b}}}{2}\right)+B\,\mathrm{EllipticF}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}},\frac{\sqrt{-\frac{2(a-b)}{b}}}{2}\right)-2B\,\mathrm{EllipticE}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}},\sqrt{-2\right.\right.}{\sqrt{-\frac{2b}{a-b}}\,\sqrt{2\sinh\left(\frac{x}{2}\right)^{4}b+(a+b)\sinh\left(\frac{x}{2}\right)^{2}}\,\sinh\left(\frac{x}{2}\right)\sqrt{2\sinh\left(\frac{x}{2}\right)^{2}}}$
parts	$\frac{2A\sqrt{\left(2\cosh\left(\frac{x}{2}\right)^{2}b+a-b\right)\sinh\left(\frac{x}{2}\right)^{2}}\,\sqrt{\frac{2\cosh\left(\frac{x}{2}\right)^{2}b+a-b}{a-b}}\,\sqrt{-\sinh\left(\frac{x}{2}\right)^{2}}\,\mathrm{EllipticF}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}},\frac{\sqrt{-\frac{2(a-b)}{b}}}{2}\right)}{\sqrt{-\frac{2b}{a-b}}\,\sqrt{2\sinh\left(\frac{x}{2}\right)^{4}b+(a+b)\sinh\left(\frac{x}{2}\right)^{2}}\,\sinh\left(\frac{x}{2}\right)\sqrt{2\sinh\left(\frac{x}{2}\right)^{2}b+a+b}}+\frac{2B\left(\mathrm{EllipticF}\right.}{}$
risch	$\frac{B\left(b\,e^{2x}+2a\,e^{x}+b\right)\sqrt{2}\,e^{-x}}{b\sqrt{\left(b\,e^{2x}+2a\,e^{x}+b\right)e^{-x}}}+\frac{\left(\frac{4A\left(a+\sqrt{a^{2}-b^{2}}\right)\sqrt{\frac{\left(e^{x}+\frac{a+\sqrt{a^{2}-b^{2}}}{b}\right)b}{a+\sqrt{a^{2}-b^{2}}}}\,\sqrt{\frac{e^{x}-\frac{-a+\sqrt{a^{2}-b^{2}}}{b}}{-\frac{a+\sqrt{a^{2}-b^{2}}}{b}-\frac{-a+\sqrt{a^{2}-b^{2}}}{b}}}\,\sqrt{-\frac{e^{x}b}{a+\sqrt{a^{2}-b^{2}}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(e^{x}+\right.}{}}\right.}{b\sqrt{e^{3x}b+2a\,e^{2x}+e^{x}b}}\right.}{}$

input
```
int((A+B*cosh(x))/(a+b*cosh(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(A*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+B*El
lipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))-2*B*Ellipti
cE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2)))*(-sinh(1/2*x)^2
)^(1/2)*((2*cosh(1/2*x)^2*b+a-b)/(a-b))^(1/2)*((2*cosh(1/2*x)^2*b+a-b)*sin
h(1/2*x)^2)^(1/2)/(-2*b/(a-b))^(1/2)/(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^
2)^(1/2)/sinh(1/2*x)/(2*sinh(1/2*x)^2*b+a+b)^(1/2)
```

3.118.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.69

$$\int \frac{A+B\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx =$$

$$-\frac{2\left(3\sqrt{2}Bb^{\frac{3}{2}}\,\mathrm{weierstrassZeta}\left(\frac{4\left(4a^{2}-3b^{2}\right)}{3b^{2}},-\frac{8\left(8a^{3}-9ab^{2}\right)}{27b^{3}},\mathrm{weierstrassPInverse}\left(\frac{4\left(4a^{2}-3b^{2}\right)}{3b^{2}},-\frac{8\left(8a^{3}-9ab^{2}\right)}{27b^{3}},3b\right.\right.\right.}{}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(3*sqrt(2)*B*b^(3/2)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(
8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(
8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) + sqrt(2)*
(2*B*a - 3*A*b)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27
*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) + 3*sqrt(
b*cosh(x) + a)*B*b)/b^2
```

3.118.6 Sympy [F]

$$\int \frac{A+B\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx = \int \frac{A+B\cosh{(x)}}{\sqrt{a+b\cosh{(x)}}}\,dx$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))**(1/2),x)
```

output
```
Integral((A + B*cosh(x))/sqrt(a + b*cosh(x)), x)
```

3.118.7 Maxima [F]

$$\int \frac{A+B\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx = \int \frac{B\cosh{(x)}+A}{\sqrt{b\cosh{(x)}+a}}\,dx$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*cosh(x) + A)/sqrt(b*cosh(x) + a), x)
```

3.118.8 Giac [F]

$$\int \frac{A+B\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx = \int \frac{B\cosh{(x)}+A}{\sqrt{b\cosh{(x)}+a}}\,dx$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))^(1/2),x, algorithm="giac")
```

output
```
integrate((B*cosh(x) + A)/sqrt(b*cosh(x) + a), x)
```

3.118.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx = \int \frac{A+B\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx$$

input
```
int((A + B*cosh(x))/(a + b*cosh(x))^(1/2),x)
```

output
```
int((A + B*cosh(x))/(a + b*cosh(x))^(1/2), x)
```

3.119 $\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{3/2}}\,dx$

3.119.1 Optimal result

Integrand size = 17, antiderivative size = 152

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{3/2}}\,dx=-\frac{2i(Ab-aB)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{b\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}}-\frac{2iB\sqrt{\frac{a+b\cosh(x)}{a+b}}\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}-\frac{2(Ab-aB)\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}$$

output
```
-2*(A*b-B*a)*sinh(x)/(a^2-b^2)/(a+b*cosh(x))^(1/2)-2*I*(A*b-B*a)*(cosh(1/2
*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*
(a+b*cosh(x))^(1/2)/b/(a^2-b^2)/((a+b*cosh(x))/(a+b))^(1/2)-2*I*B*(cosh(1/
2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))
*((a+b*cosh(x))/(a+b))^(1/2)/b/(a+b*cosh(x))^(1/2)
```

3.119.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.88

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{3/2}}\,dx=\frac{2i(a+b)(-Ab+aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)-2i\left(a^2-b^2\right)B\sqrt{\frac{a+b\cosh(x)}{a+b}}\,\text{EllipticF}}{(a-b)b(a+b)\sqrt{a+b\cosh(x)}}$$

input
```
Integrate[(A + B*Cosh[x])/(a + b*Cosh[x])^(3/2),x]
```

output
```
((2*I)*(a + b)*(-(A*b) + a*B)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticE[(I/2
)*x, (2*b)/(a + b)] - (2*I)*(a^2 - b^2)*B*Sqrt[(a + b*Cosh[x])/(a + b)]*El
lipticF[(I/2)*x, (2*b)/(a + b)] + 2*b*(-(A*b) + a*B)*Sinh[x])/((a - b)*b*(
a + b)*Sqrt[a + b*Cosh[x]])
```

3.119.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {3042, 3233, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{\left(a+b\sin\left(\frac{\pi}{2}+ix\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3233}$$

$$-\frac{2\int -\frac{aA-bB+(Ab-aB)\cosh(x)}{2\sqrt{a+b\cosh(x)}}dx}{a^2-b^2}-\frac{2\sinh(x)(Ab-aB)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{aA-bB+(Ab-aB)\cosh(x)}{\sqrt{a+b\cosh(x)}}dx}{a^2-b^2}-\frac{2\sinh(x)(Ab-aB)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sinh(x)(Ab-aB)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}+\frac{\int \frac{aA-bB+(Ab-aB)\sin\left(ix+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3231}$$

$$\frac{\frac{B(a^2-b^2)\int \frac{1}{\sqrt{a+b\cosh(x)}}dx}{b}+\frac{(Ab-aB)\int \sqrt{a+b\cosh(x)}dx}{b}}{a^2-b^2}-\frac{2\sinh(x)(Ab-aB)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sinh(x)(Ab-aB)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}+\frac{\frac{B(a^2-b^2)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}+\frac{(Ab-aB)\int\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}}{a^2-b^2}$$

$$\Big\downarrow \; 3134$$

$$-\frac{2\sinh(x)(Ab-aB)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}+\frac{\frac{(Ab-aB)\sqrt{a+b\cosh(x)}\int\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}dx}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{B(a^2-b^2)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}}{a^2-b^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2\sinh(x)(Ab-aB)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}+\frac{\frac{B(a^2-b^2)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}+\frac{(Ab-aB)\sqrt{a+b\cosh(x)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}}{a^2-b^2}$$

$$\Big\downarrow \; 3132$$

$$-\frac{2\sinh(x)(Ab-aB)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}+\frac{\frac{B(a^2-b^2)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}-\frac{2i(Ab-aB)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}}{a^2-b^2}$$

$$\Big\downarrow \; 3142$$

$$-\frac{2\sinh(x)(Ab-aB)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}+\frac{\frac{B(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}}dx}{b\sqrt{a+b\cosh(x)}}-\frac{2i(Ab-aB)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}}{a^2-b^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2\sinh(x)(Ab-aB)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}+\frac{\frac{B(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cosh(x)}}-\frac{2i(Ab-aB)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}}{a^2-b^2}$$

$$\Big\downarrow \; 3140$$

$$-\frac{2\sinh(x)(Ab-aB)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}+\frac{-\frac{2iB(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}-\frac{2i(Ab-aB)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}}{a^2-b^2}$$

3.119. $\int\frac{A+B\cosh(x)}{(a+b\cosh(x))^{3/2}}dx$

input
```
Int[(A + B*Cosh[x])/(a + b*Cosh[x])^(3/2),x]
```

output
```
(((-2*I)*(A*b - a*B)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)]
)/(b*Sqrt[(a + b*Cosh[x])/(a + b)]) - ((2*I)*(a^2 - b^2)*B*Sqrt[(a + b*Cos
h[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[a + b*Cosh[x]]))
/(a^2 - b^2) - (2*(A*b - a*B)*Sinh[x])/((a^2 - b^2)*Sqrt[a + b*Cosh[x]])
```

3.119.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.119.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 484 vs. $2(178) = 356$.

Time = 2.76 (sec) , antiderivative size = 485, normalized size of antiderivative = 3.19

method	result
default	$\frac{\sqrt{\left(2\cosh\left(\frac{x}{2}\right)^{2} b+a-b\right)\sinh\left(\frac{x}{2}\right)^{2}}\left(\frac{2B\sqrt{\frac{2\cosh\left(\frac{x}{2}\right)^{2} b+a-b}{a-b}}\sqrt{-\sinh\left(\frac{x}{2}\right)^{2}}\,\operatorname{EllipticF}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}},\frac{\sqrt{\frac{-2a+2b}{b}}}{2}\right)}{b\sqrt{-\frac{2b}{a-b}}\sqrt{2\sinh\left(\frac{x}{2}\right)^{4} b+(a+b)\sinh\left(\frac{x}{2}\right)^{2}}}-\frac{2(bA-Ba)\sqrt{2\sinh\left(\frac{x}{2}\right)\cdots}}{\cdots}\cdots\right.}{\cdots}$
parts	$-\frac{2A\left(2\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}}\sinh\left(\frac{x}{2}\right)^{2} b-\sqrt{-\sinh\left(\frac{x}{2}\right)^{2}}\sqrt{\frac{2b\sinh\left(\frac{x}{2}\right)^{2}}{a-b}+\frac{a+b}{a-b}}\,\operatorname{EllipticF}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}},\frac{\sqrt{-\frac{2(a-b)}{b}}}{2}\right)a-\sqrt{-\sinh\left(\frac{x}{2}\right)\cdots}\cdots\right.}{\sqrt{-\frac{2b}{a-b}}\,(a-b)\cdots}$

input
```
int((A+B*cosh(x))/(a+b*cosh(x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((2*cosh(1/2*x)^2*b+a-b)*sinh(1/2*x)^2)^(1/2)*(2*B/b/(-2*b/(a-b))^(1/2)*((
2*cosh(1/2*x)^2*b+a-b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)/(2*sinh(1/2*x)^
4*b+(a+b)*sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/
2*((-2*a+2*b)/b)^(1/2))-2*(A*b-B*a)/b/sinh(1/2*x)^2/(2*sinh(1/2*x)^2*b+a+b
)/(-2*b/(a-b))^(1/2)/(a^2-b^2)*(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^2)^(1/
2)*(2*cosh(1/2*x)*(-2*b/(a-b))^(1/2)*sinh(1/2*x)^2*b-(-sinh(1/2*x)^2)^(1/2
)*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/
(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))*a-(-sinh(1/2*x)^2)^(1/2)*(2*b/(a-b)
*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2)
,1/2*((-2*a+2*b)/b)^(1/2))*b+2*(-sinh(1/2*x)^2)^(1/2)*(2*b/(a-b)*sinh(1/2*
x)^2+(a+b)/(a-b))^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*((-2*
a+2*b)/b)^(1/2))*b))/sinh(1/2*x)/(2*sinh(1/2*x)^2*b+a+b)^(1/2)
```

3.119.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 639, normalized size of antiderivative = 4.20

$$\int \frac{A + B\cosh(x)}{(a + b\cosh(x))^{3/2}}\, dx = \frac{2\left(\left(\sqrt{2}(2\,Ba^2b + Aab^2 - 3\,Bb^3)\cosh(x)^2 + \sqrt{2}(2\,Ba^2b + Aab^2 - 3\,Bb^3)\sinh(x)\right.\right.}{}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))^(3/2),x, algorithm="fricas")
```

output
```
2/3*((sqrt(2)*(2*B*a^2*b + A*a*b^2 - 3*B*b^3)*cosh(x)^2 + sqrt(2)*(2*B*a^2
*b + A*a*b^2 - 3*B*b^3)*sinh(x)^2 + 2*sqrt(2)*(2*B*a^3 + A*a^2*b - 3*B*a*b
^2)*cosh(x) + 2*(sqrt(2)*(2*B*a^2*b + A*a*b^2 - 3*B*b^3)*cosh(x) + sqrt(2)
*(2*B*a^3 + A*a^2*b - 3*B*a*b^2))*sinh(x) + sqrt(2)*(2*B*a^2*b + A*a*b^2 -
 3*B*b^3))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a
^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) + 3*(sqrt(2)*(
B*a*b^2 - A*b^3)*cosh(x)^2 + sqrt(2)*(B*a*b^2 - A*b^3)*sinh(x)^2 + 2*sqrt(
2)*(B*a^2*b - A*a*b^2)*cosh(x) + 2*(sqrt(2)*(B*a*b^2 - A*b^3)*cosh(x) + sq
rt(2)*(B*a^2*b - A*a*b^2))*sinh(x) + sqrt(2)*(B*a*b^2 - A*b^3))*sqrt(b)*we
ierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weiers
trassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3
*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) + 6*((B*a*b^2 - A*b^3)*cosh(x)^2 + (B*
a*b^2 - A*b^3)*sinh(x)^2 + (B*a^2*b - A*a*b^2)*cosh(x) + (B*a^2*b - A*a*b^
2 + 2*(B*a*b^2 - A*b^3)*cosh(x))*sinh(x))*sqrt(b*cosh(x) + a))/(a^2*b^3 -
b^5 + (a^2*b^3 - b^5)*cosh(x)^2 + (a^2*b^3 - b^5)*sinh(x)^2 + 2*(a^3*b^2 -
 a*b^4)*cosh(x) + 2*(a^3*b^2 - a*b^4 + (a^2*b^3 - b^5)*cosh(x))*sinh(x))
```

3.119.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cosh(x)}{(a + b\cosh(x))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))**(3/2),x)
```

output
```
Timed out
```

3.119.7 Maxima [F]

$$\int \frac{A + B\cosh(x)}{(a + b\cosh(x))^{3/2}}\,dx = \int \frac{B\cosh(x) + A}{(b\cosh(x) + a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*cosh(x) + A)/(b*cosh(x) + a)^(3/2), x)
```

3.119.8 Giac [F]

$$\int \frac{A + B\cosh(x)}{(a + b\cosh(x))^{3/2}}\,dx = \int \frac{B\cosh(x) + A}{(b\cosh(x) + a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))^(3/2),x, algorithm="giac")
```

output
```
integrate((B*cosh(x) + A)/(b*cosh(x) + a)^(3/2), x)
```

3.119.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{3/2}}\,dx = \int \frac{A+B\cosh(x)}{\left(a+b\cosh\left(x\right)\right)^{3/2}}\,dx$$

input
```
int((A + B*cosh(x))/(a + b*cosh(x))^(3/2),x)
```

output
```
int((A + B*cosh(x))/(a + b*cosh(x))^(3/2), x)
```

3.120 $\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{5/2}}\,dx$

3.120.1 Optimal result

Integrand size = 17, antiderivative size = 231

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{5/2}}\,dx = -\frac{2i(4aAb-a^2B-3b^2B)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{3b\,(a^2-b^2)^2\sqrt{\frac{a+b\cosh(x)}{a+b}}} + \frac{2i(Ab-aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{3b\,(a^2-b^2)\sqrt{a+b\cosh(x)}} - \frac{2(Ab-aB)\sinh(x)}{3\,(a^2-b^2)\,(a+b\cosh(x))^{3/2}} - \frac{2(4aAb-a^2B-3b^2B)\sinh(x)}{3\,(a^2-b^2)^2\sqrt{a+b\cosh(x)}}$$

output
```
-2/3*(A*b-B*a)*sinh(x)/(a^2-b^2)/(a+b*cosh(x))^(3/2)-2/3*(4*A*a*b-B*a^2-3*
B*b^2)*sinh(x)/(a^2-b^2)^2/(a+b*cosh(x))^(1/2)-2/3*I*(4*A*a*b-B*a^2-3*B*b^
2)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(1/2*x),2^(1/2)*(b/(a
+b))^(1/2))*(a+b*cosh(x))^(1/2)/b/(a^2-b^2)^2/((a+b*cosh(x))/(a+b))^(1/2)+
2/3*I*(A*b-B*a)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),
2^(1/2)*(b/(a+b))^(1/2))*((a+b*cosh(x))/(a+b))^(1/2)/b/(a^2-b^2)/(a+b*cosh
(x))^(1/2)
```

3.120.2 Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.74

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{5/2}}\,dx = \frac{2\left(\frac{i\left(\frac{a+b\cosh(x)}{a+b}\right)^{3/2}\left((-4aAb+a^2B+3b^2B)E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)-(a-b)(-Ab+aB)\operatorname{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)\right)}{(a-b)^2b}+\frac{(-5a^2A\right.}{}}{3(a+b\cosh(x))^{3/2}}$$

input
```
Integrate[(A + B*Cosh[x])/(a + b*Cosh[x])^(5/2),x]
```

output
```
(2*((I*((a + b*Cosh[x])/(a + b))^(3/2)*((-4*a*A*b + a^2*B + 3*b^2*B)*Ellip
ticE[(I/2)*x, (2*b)/(a + b)] - (a - b)*(-(A*b) + a*B)*EllipticF[(I/2)*x, (
2*b)/(a + b)]))/((a - b)^2*b) + ((-5*a^2*A*b + A*b^3 + 2*a^3*B + 2*a*b^2*B
 + b*(-4*a*A*b + a^2*B + 3*b^2*B)*Cosh[x])*Sinh[x])/(a^2 - b^2)^2))/(3*(a
+ b*Cosh[x])^(3/2))
```

3.120.3 Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.882, Rules used = {3042, 3233, 27, 3042, 3233, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{\left(a+b\sin\left(\frac{\pi}{2}+ix\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3233}$$

$$-\frac{2\int -\frac{3(aA-bB)-(Ab-aB)\cosh(x)}{2(a+b\cosh(x))^{3/2}}dx}{3(a^2-b^2)}-\frac{2\sinh(x)(Ab-aB)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3(aA-bB)-(Ab-aB)\cosh(x)}{(a+b\cosh(x))^{3/2}}dx}{3(a^2-b^2)}-\frac{2\sinh(x)(Ab-aB)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}+\frac{\int \frac{3(aA-bB)+(aB-Ab)\sin\left(ix+\frac{\pi}{2}\right)}{\left(a+b\sin\left(ix+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3233$$

$$\frac{-\frac{2\int -\frac{3Aa^2-4bBa+Ab^2+\left(-Ba^2+4Aba-3b^2B\right)\cosh(x)}{2\sqrt{a+b\cosh(x)}}dx}{a^2-b^2}-\frac{2\sinh(x)\left(a^2(-B)+4aAb-3b^2B\right)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}}{3\left(a^2-b^2\right)}-\frac{2\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{3Aa^2-4bBa+Ab^2+\left(-Ba^2+4Aba-3b^2B\right)\cosh(x)}{\sqrt{a+b\cosh(x)}}dx}{a^2-b^2}-\frac{2\sinh(x)\left(a^2(-B)+4aAb-3b^2B\right)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}}{3\left(a^2-b^2\right)}-\frac{2\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}+\frac{-\frac{2\sinh(x)\left(a^2(-B)+4aAb-3b^2B\right)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{\int \frac{3Aa^2-4bBa+Ab^2+\left(-Ba^2+4Aba-3b^2B\right)\sin\left(ix+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{a^2-b^2}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3231$$

$$\frac{\frac{\frac{\left(a^2(-B)+4aAb-3b^2B\right)\int\sqrt{a+b\cosh(x)}dx}{b}-\frac{\left(a^2-b^2\right)(Ab-aB)\int\frac{1}{\sqrt{a+b\cosh(x)}}dx}{b}}{a^2-b^2}-\frac{2\sinh(x)\left(a^2(-B)+4aAb-3b^2B\right)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}}{3\left(a^2-b^2\right)}-\frac{2\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}+\frac{-\frac{2\sinh(x)\left(a^2(-B)+4aAb-3b^2B\right)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{\frac{\left(a^2(-B)+4aAb-3b^2B\right)\int\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}-\frac{\left(a^2-b^2\right)(Ab-aB)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}}{a^2-b^2}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3134$$

3.120. $\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{5/2}}\,dx$

$$-\frac{2\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}+\frac{-\frac{2\sinh(x)\left(a^2(-B)+4aAb-3b^2B\right)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{\frac{\left(a^2(-B)+4aAb-3b^2B\right)\sqrt{a+b\cosh(x)}\int\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}dx}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}-\frac{\left(a^2-b^2\right)(Ab-aB)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}}{a^2-b^2}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}+\frac{-\frac{2\sinh(x)\left(a^2(-B)+4aAb-3b^2B\right)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{\frac{\left(a^2(-B)+4aAb-3b^2B\right)\sqrt{a+b\cosh(x)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}-\frac{\left(a^2-b^2\right)(Ab-aB)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}}{a^2-b^2}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3132$$

$$-\frac{2\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}+\frac{-\frac{2\sinh(x)\left(a^2(-B)+4aAb-3b^2B\right)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{-\frac{\left(a^2-b^2\right)(Ab-aB)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}-\frac{2i\left(a^2(-B)+4aAb-3b^2B\right)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}}{a^2-b^2}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3142$$

$$-\frac{2\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}+\frac{-\frac{2\sinh(x)\left(a^2(-B)+4aAb-3b^2B\right)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{-\frac{\left(a^2-b^2\right)(Ab-aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}}dx}{b\sqrt{a+b\cosh(x)}}-\frac{2i\left(a^2(-B)+4aAb-3b^2B\right)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}}{a^2-b^2}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}+\frac{-\frac{2\sinh(x)\left(a^2(-B)+4aAb-3b^2B\right)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{-\frac{\left(a^2-b^2\right)(Ab-aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cosh(x)}}-\frac{2i\left(a^2(-B)+4aAb-3b^2B\right)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}}{a^2-b^2}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3140$$

3.120. $\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{5/2}}\,dx$

$$-\frac{2\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}+\frac{-\frac{2\sinh(x)\left(a^2(-B)+4aAb-3b^2B\right)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{\frac{2i\left(a^2-b^2\right)(Ab-aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}\operatorname{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}-\frac{2i\left(a^2(-B)+4aAb-3b^2B\right)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}}{a^2-b^2}}{3\left(a^2-b^2\right)}$$

input
```
Int[(A + B*Cosh[x])/(a + b*Cosh[x])^(5/2),x]
```

output
```
(-2*(A*b - a*B)*Sinh[x])/(3*(a^2 - b^2)*(a + b*Cosh[x])^(3/2)) + ((((-2*I)
*(4*a*A*b - a^2*B - 3*b^2*B)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/
(a + b)])/(b*Sqrt[(a + b*Cosh[x])/(a + b)]) + ((2*I)*(a^2 - b^2)*(A*b - a*
B)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/(b*Sqr
t[a + b*Cosh[x]]))/(a^2 - b^2) - (2*(4*a*A*b - a^2*B - 3*b^2*B)*Sinh[x])/(
(a^2 - b^2)*Sqrt[a + b*Cosh[x]]))/(3*(a^2 - b^2))
```

3.120.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.120.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 796 vs. $2(247) = 494$.

Time = 4.98 (sec) , antiderivative size = 797, normalized size of antiderivative = 3.45

method	result
default	$\sqrt{\left(2\cosh\left(\frac{x}{2}\right)^2 b+a-b\right)\sinh\left(\frac{x}{2}\right)^2}\left(-\frac{2B\sqrt{2\sinh\left(\frac{x}{2}\right)^4 b+(a+b)\sinh\left(\frac{x}{2}\right)^2}\left(2\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}}\sinh\left(\frac{x}{2}\right)^2 b-\sqrt{-\sinh\left(\frac{x}{2}\right)^2}\sqrt{\frac{2b\sinh\left(\frac{x}{2}\right)^2}{a-b}+\frac{a+}{a-}}\right.}{}\right.$
parts	Expression too large to display

input
```
int((A+B*cosh(x))/(a+b*cosh(x))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((2*cosh(1/2*x)^2*b+a-b)*sinh(1/2*x)^2)^(1/2)*(-2*B/b/sinh(1/2*x)^2/(2*sin
h(1/2*x)^2*b+a+b)/(-2*b/(a-b))^(1/2)/(a^2-b^2)*(2*sinh(1/2*x)^4*b+(a+b)*si
nh(1/2*x)^2)^(1/2)*(2*cosh(1/2*x)*(-2*b/(a-b))^(1/2)*sinh(1/2*x)^2*b-(-sin
h(1/2*x)^2)^(1/2)*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*EllipticF(co
sh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))*a-(-sinh(1/2*x)^2)^
(1/2)*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*EllipticF(cosh(1/2*x)*(-
2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))*b+2*(-sinh(1/2*x)^2)^(1/2)*(2*b
/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-b))
^(1/2),1/2*((-2*a+2*b)/b)^(1/2))*b)+2*(A*b-B*a)/b*(-1/6/b/(a-b)/(a+b)*cosh
(1/2*x)*(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^2)^(1/2)/(cosh(1/2*x)^2+1/2*(
a-b)/b)^2-8/3*sinh(1/2*x)^2*b/(a-b)^2/(a+b)^2*cosh(1/2*x)*a/((2*cosh(1/2*x
)^2*b+a-b)*sinh(1/2*x)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)/(-2*
b/(a-b))^(1/2)*((2*cosh(1/2*x)^2*b+a-b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2
)/(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*
b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))-16/3*a*b/(a+b)^2/(a-b)^2*(-a+b)/(
-2*b/(a-b))^(1/2)*((2*cosh(1/2*x)^2*b+a-b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(
1/2)/(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^2)^(1/2)/(2*a-2*b)*(EllipticF(co
sh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))-EllipticE(cosh(1/2*
x)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2)))))/sinh(1/2*x)/(2*sinh(1/2
*x)^2*b+a+b)^(1/2)
```

3.120.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 2153, normalized size of antiderivative = 9.32

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))^(5/2),x, algorithm="fricas")
```

output
```
2/9*((sqrt(2)*(2*B*a^3*b^2 + A*a^2*b^3 - 6*B*a*b^4 + 3*A*b^5)*cosh(x)^4 +
sqrt(2)*(2*B*a^3*b^2 + A*a^2*b^3 - 6*B*a*b^4 + 3*A*b^5)*sinh(x)^4 + 4*sqrt
(2)*(2*B*a^4*b + A*a^3*b^2 - 6*B*a^2*b^3 + 3*A*a*b^4)*cosh(x)^3 + 4*(sqrt(
2)*(2*B*a^3*b^2 + A*a^2*b^3 - 6*B*a*b^4 + 3*A*b^5)*cosh(x) + sqrt(2)*(2*B*
a^4*b + A*a^3*b^2 - 6*B*a^2*b^3 + 3*A*a*b^4))*sinh(x)^3 + 2*sqrt(2)*(4*B*a
^5 + 2*A*a^4*b - 10*B*a^3*b^2 + 7*A*a^2*b^3 - 6*B*a*b^4 + 3*A*b^5)*cosh(x)
^2 + 2*(3*sqrt(2)*(2*B*a^3*b^2 + A*a^2*b^3 - 6*B*a*b^4 + 3*A*b^5)*cosh(x)^
2 + 6*sqrt(2)*(2*B*a^4*b + A*a^3*b^2 - 6*B*a^2*b^3 + 3*A*a*b^4)*cosh(x) +
sqrt(2)*(4*B*a^5 + 2*A*a^4*b - 10*B*a^3*b^2 + 7*A*a^2*b^3 - 6*B*a*b^4 + 3*
A*b^5))*sinh(x)^2 + 4*sqrt(2)*(2*B*a^4*b + A*a^3*b^2 - 6*B*a^2*b^3 + 3*A*a
*b^4)*cosh(x) + 4*(sqrt(2)*(2*B*a^3*b^2 + A*a^2*b^3 - 6*B*a*b^4 + 3*A*b^5)
*cosh(x)^3 + 3*sqrt(2)*(2*B*a^4*b + A*a^3*b^2 - 6*B*a^2*b^3 + 3*A*a*b^4)*c
osh(x)^2 + sqrt(2)*(4*B*a^5 + 2*A*a^4*b - 10*B*a^3*b^2 + 7*A*a^2*b^3 - 6*B
*a*b^4 + 3*A*b^5)*cosh(x) + sqrt(2)*(2*B*a^4*b + A*a^3*b^2 - 6*B*a^2*b^3 +
 3*A*a*b^4))*sinh(x) + sqrt(2)*(2*B*a^3*b^2 + A*a^2*b^3 - 6*B*a*b^4 + 3*A*
b^5))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 -
9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) + 3*(sqrt(2)*(B*a^2
*b^3 - 4*A*a*b^4 + 3*B*b^5)*cosh(x)^4 + sqrt(2)*(B*a^2*b^3 - 4*A*a*b^4 + 3
*B*b^5)*sinh(x)^4 + 4*sqrt(2)*(B*a^3*b^2 - 4*A*a^2*b^3 + 3*B*a*b^4)*cosh(x
)^3 + 4*(sqrt(2)*(B*a^2*b^3 - 4*A*a*b^4 + 3*B*b^5)*cosh(x) + sqrt(2)*(B...
```

3.120.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))**(5/2),x)
```

output
```
Timed out
```

3.120.7 Maxima [F]

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{5/2}}\,dx = \int \frac{B\cosh\left(x\right)+A}{\left(b\cosh\left(x\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*cosh(x) + A)/(b*cosh(x) + a)^(5/2), x)
```

3.120.8 Giac [F]

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{5/2}}\,dx = \int \frac{B\cosh\left(x\right)+A}{\left(b\cosh\left(x\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))^(5/2),x, algorithm="giac")
```

output
```
integrate((B*cosh(x) + A)/(b*cosh(x) + a)^(5/2), x)
```

3.120.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{5/2}}\,dx = \int \frac{A+B\cosh(x)}{\left(a+b\cosh\left(x\right)\right)^{5/2}}\,dx$$

input
```
int((A + B*cosh(x))/(a + b*cosh(x))^(5/2),x)
```

output
```
int((A + B*cosh(x))/(a + b*cosh(x))^(5/2), x)
```

3.121 $\int \left(a\cosh^2(x)\right)^{7/2}\,dx$

3.121.1 Optimal result

Integrand size = 10, antiderivative size = 72

$$\int \left(a\cosh^2(x)\right)^{7/2}\,dx=\frac{16}{35}a^3\sqrt{a\cosh^2(x)}\tanh(x)+\frac{8}{35}a^2\left(a\cosh^2(x)\right)^{3/2}\tanh(x)+\frac{6}{35}a\left(a\cosh^2(x)\right)^{5/2}\tanh(x)+\frac{1}{7}\left(a\cosh^2(x)\right)^{7/2}\tanh(x)$$

output
```
8/35*a^2*(a*cosh(x)^2)^(3/2)*tanh(x)+6/35*a*(a*cosh(x)^2)^(5/2)*tanh(x)+1/
7*(a*cosh(x)^2)^(7/2)*tanh(x)+16/35*a^3*(a*cosh(x)^2)^(1/2)*tanh(x)
```

3.121.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.54

$$\int \left(a\cosh^2(x)\right)^{7/2}\,dx=\frac{1}{35}a^3\sqrt{a\cosh^2(x)}\left(35+35\sinh^2(x)+21\sinh^4(x)+5\sinh^6(x)\right)\tanh(x)$$

input
```
Integrate[(a*Cosh[x]^2)^(7/2),x]
```

output
```
(a^3*Sqrt[a*Cosh[x]^2]*(35 + 35*Sinh[x]^2 + 21*Sinh[x]^4 + 5*Sinh[x]^6)*Ta
nh[x])/35
```

3.121.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3682, 3042, 3682, 3042, 3682, 3042, 3686, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a \cosh^2(x)\right)^{7/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a \sin\left(\frac{\pi}{2}+ix\right)^2\right)^{7/2} dx$$

$$\Big\downarrow \text{3682}$$

$$\frac{6}{7}a\int \left(a \cosh^2(x)\right)^{5/2} dx+\frac{1}{7}\tanh(x) \left(a \cosh^2(x)\right)^{7/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\tanh(x) \left(a \cosh^2(x)\right)^{7/2}+\frac{6}{7}a\int \left(a \sin\left(ix+\frac{\pi}{2}\right)^2\right)^{5/2} dx$$

$$\Big\downarrow \text{3682}$$

$$\frac{6}{7}a\left(\frac{4}{5}a\int \left(a \cosh^2(x)\right)^{3/2} dx+\frac{1}{5}\tanh(x) \left(a \cosh^2(x)\right)^{5/2}\right)+\frac{1}{7}\tanh(x) \left(a \cosh^2(x)\right)^{7/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\tanh(x) \left(a \cosh^2(x)\right)^{7/2}+\frac{6}{7}a\left(\frac{1}{5}\tanh(x) \left(a \cosh^2(x)\right)^{5/2}+\frac{4}{5}a\int \left(a \sin\left(ix+\frac{\pi}{2}\right)^2\right)^{3/2} dx\right)$$

$$\Big\downarrow \text{3682}$$

$$\frac{6}{7}a\left(\frac{4}{5}a\left(\frac{2}{3}a\int \sqrt{a \cosh^2(x)}dx+\frac{1}{3}\tanh(x) \left(a \cosh^2(x)\right)^{3/2}\right)+\frac{1}{5}\tanh(x) \left(a \cosh^2(x)\right)^{5/2}\right)+\frac{1}{7}\tanh(x) \left(a \cosh^2(x)\right)^{7/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\tanh(x) \left(a \cosh^2(x)\right)^{7/2}+\frac{6}{7}a\left(\frac{1}{5}\tanh(x) \left(a \cosh^2(x)\right)^{5/2}+\frac{4}{5}a\left(\frac{1}{3}\tanh(x) \left(a \cosh^2(x)\right)^{3/2}+\frac{2}{3}a\int \sqrt{a \sin\left(ix+\frac{\pi}{2}\right)^2}dx\right)\right)$$

$$\Big\downarrow \ 3686$$

$$\frac{6}{7}a\left(\frac{4}{5}a\left(\frac{2}{3}a\text{sech}(x)\sqrt{a\cosh^2(x)}\int\cosh(x)dx+\frac{1}{3}\tanh(x)\left(a\cosh^2(x)\right)^{3/2}\right)+\frac{1}{5}\tanh(x)\left(a\cosh^2(x)\right)^{5/2}\right)+\frac{1}{7}\tanh(x)\left(a\cosh^2(x)\right)^{7/2}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{7}\tanh(x)\left(a\cosh^2(x)\right)^{7/2}+\frac{6}{7}a\left(\frac{1}{5}\tanh(x)\left(a\cosh^2(x)\right)^{5/2}+\frac{4}{5}a\left(\frac{1}{3}\tanh(x)\left(a\cosh^2(x)\right)^{3/2}+\frac{2}{3}a\text{sech}(x)\sqrt{a\cosh^2(x)}\int\sin\left(ix+\frac{\pi}{2}\right)dx\right)\right)$$

$$\Big\downarrow \ 3117$$

$$\frac{1}{7}\tanh(x)\left(a\cosh^2(x)\right)^{7/2}+\frac{6}{7}a\left(\frac{1}{5}\tanh(x)\left(a\cosh^2(x)\right)^{5/2}+\frac{4}{5}a\left(\frac{1}{3}\tanh(x)\left(a\cosh^2(x)\right)^{3/2}+\frac{2}{3}a\tanh(x)\sqrt{a\cosh^2(x)}\right)\right)$$

input
```
Int[(a*Cosh[x]^2)^(7/2),x]
```

output
```
((a*Cosh[x]^2)^(7/2)*Tanh[x])/7 + (6*a*(((a*Cosh[x]^2)^(5/2)*Tanh[x])/5 +
(4*a*((2*a*Sqrt[a*Cosh[x]^2]*Tanh[x])/3 + ((a*Cosh[x]^2)^(3/2)*Tanh[x])/3)
)/5))/7
```

3.121.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3682
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Simp[b*((2*p - 1)/(2*p))   Int[(b*S
in[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && G
tQ[p, 1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.121.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.53

method	result
default	$\frac{a^4 \cosh(x) \sinh(x)\left(5\cosh(x)^6+6\cosh(x)^4+8\cosh(x)^2+16\right)}{35\sqrt{a\cosh(x)^2}}$
risch	$\frac{a^3 e^{8x}\sqrt{a(1+e^{2x})^2 e^{-2x}}}{896+896\,e^{2x}}+\frac{7a^3 e^{6x}\sqrt{a(1+e^{2x})^2 e^{-2x}}}{640(1+e^{2x})}+\frac{7a^3 e^{4x}\sqrt{a(1+e^{2x})^2 e^{-2x}}}{128(1+e^{2x})}+\frac{35a^3 e^{2x}\sqrt{a(1+e^{2x})^2 e^{-2x}}}{128(1+e^{2x})}-\frac{35\sqrt{a(1+e^{2x}}}{128(1+e}$

input
```
int((a*cosh(x)^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*a^4*cosh(x)*sinh(x)*(5*cosh(x)^6+6*cosh(x)^4+8*cosh(x)^2+16)/(a*cosh(
x)^2)^(1/2)
```

3.121.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 817 vs. 2(56) = 112.

Time = 0.28 (sec) , antiderivative size = 817, normalized size of antiderivative = 11.35

$$\int \left(a\cosh^2(x)\right)^{7/2}\,dx = \text{Too large to display}$$

input
```
integrate((a*cosh(x)^2)^(7/2),x, algorithm="fricas")
```

output
```
1/4480*(70*a^3*cosh(x)*e^x*sinh(x)^13 + 5*a^3*e^x*sinh(x)^14 + 7*(65*a^3*c
osh(x)^2 + 7*a^3)*e^x*sinh(x)^12 + 28*(65*a^3*cosh(x)^3 + 21*a^3*cosh(x))*
e^x*sinh(x)^11 + 7*(715*a^3*cosh(x)^4 + 462*a^3*cosh(x)^2 + 35*a^3)*e^x*si
nh(x)^10 + 70*(143*a^3*cosh(x)^5 + 154*a^3*cosh(x)^3 + 35*a^3*cosh(x))*e^x
*sinh(x)^9 + 35*(429*a^3*cosh(x)^6 + 693*a^3*cosh(x)^4 + 315*a^3*cosh(x)^2
 + 35*a^3)*e^x*sinh(x)^8 + 8*(2145*a^3*cosh(x)^7 + 4851*a^3*cosh(x)^5 + 36
75*a^3*cosh(x)^3 + 1225*a^3*cosh(x))*e^x*sinh(x)^7 + 7*(2145*a^3*cosh(x)^8
 + 6468*a^3*cosh(x)^6 + 7350*a^3*cosh(x)^4 + 4900*a^3*cosh(x)^2 - 175*a^3)
*e^x*sinh(x)^6 + 14*(715*a^3*cosh(x)^9 + 2772*a^3*cosh(x)^7 + 4410*a^3*cos
h(x)^5 + 4900*a^3*cosh(x)^3 - 525*a^3*cosh(x))*e^x*sinh(x)^5 + 35*(143*a^3
*cosh(x)^10 + 693*a^3*cosh(x)^8 + 1470*a^3*cosh(x)^6 + 2450*a^3*cosh(x)^4
- 525*a^3*cosh(x)^2 - 7*a^3)*e^x*sinh(x)^4 + 140*(13*a^3*cosh(x)^11 + 77*a
^3*cosh(x)^9 + 210*a^3*cosh(x)^7 + 490*a^3*cosh(x)^5 - 175*a^3*cosh(x)^3 -
 7*a^3*cosh(x))*e^x*sinh(x)^3 + 7*(65*a^3*cosh(x)^12 + 462*a^3*cosh(x)^10
+ 1575*a^3*cosh(x)^8 + 4900*a^3*cosh(x)^6 - 2625*a^3*cosh(x)^4 - 210*a^3*c
osh(x)^2 - 7*a^3)*e^x*sinh(x)^2 + 14*(5*a^3*cosh(x)^13 + 42*a^3*cosh(x)^11
 + 175*a^3*cosh(x)^9 + 700*a^3*cosh(x)^7 - 525*a^3*cosh(x)^5 - 70*a^3*cosh
(x)^3 - 7*a^3*cosh(x))*e^x*sinh(x) + (5*a^3*cosh(x)^14 + 49*a^3*cosh(x)^12
 + 245*a^3*cosh(x)^10 + 1225*a^3*cosh(x)^8 - 1225*a^3*cosh(x)^6 - 245*a^3*
cosh(x)^4 - 49*a^3*cosh(x)^2 - 5*a^3)*e^x)*sqrt(a*e^(4*x) + 2*a*e^(2*x)...
```

3.121.6 Sympy [F(-1)]

Timed out.

$$\int \left(a\cosh^2(x)\right)^{7/2} dx = \text{Timed out}$$

input
```
integrate((a*cosh(x)**2)**(7/2),x)
```

output
```
Timed out
```

3.121.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

$$\int \left(a\cosh^2(x)\right)^{7/2}\,dx = \frac{1}{896}\,a^{\frac{7}{2}}e^{(7\,x)} + \frac{7}{640}\,a^{\frac{7}{2}}e^{(5\,x)} + \frac{7}{128}\,a^{\frac{7}{2}}e^{(3\,x)} - \frac{35}{128}\,a^{\frac{7}{2}}e^{(-x)} - \frac{7}{128}\,a^{\frac{7}{2}}e^{(-3\,x)} - \frac{7}{640}\,a^{\frac{7}{2}}e^{(-5\,x)} - \frac{1}{896}\,a^{\frac{7}{2}}e^{(-7\,x)} + \frac{35}{128}\,a^{\frac{7}{2}}e^{x}$$

input
```
integrate((a*cosh(x)^2)^(7/2),x, algorithm="maxima")
```

output
```
1/896*a^(7/2)*e^(7*x) + 7/640*a^(7/2)*e^(5*x) + 7/128*a^(7/2)*e^(3*x) - 35
/128*a^(7/2)*e^(-x) - 7/128*a^(7/2)*e^(-3*x) - 7/640*a^(7/2)*e^(-5*x) - 1/
896*a^(7/2)*e^(-7*x) + 35/128*a^(7/2)*e^x
```

3.121.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.10

$$\int \left(a\cosh^2(x)\right)^{7/2}\,dx = \frac{1}{4480}\left(5\,a^3e^{(7\,x)} + 49\,a^3e^{(5\,x)} + 245\,a^3e^{(3\,x)} + 1225\,a^3e^{x} - \left(1225\,a^3e^{(6\,x)} + 245\,a^3e^{(4\,x)}\right.\right.$$

input
```
integrate((a*cosh(x)^2)^(7/2),x, algorithm="giac")
```

output
```
1/4480*(5*a^3*e^(7*x) + 49*a^3*e^(5*x) + 245*a^3*e^(3*x) + 1225*a^3*e^x -
(1225*a^3*e^(6*x) + 245*a^3*e^(4*x) + 49*a^3*e^(2*x) + 5*a^3)*e^(-7*x))*sq
rt(a)
```

3.121.9 Mupad [F(-1)]

Timed out.

$$\int \left(a\cosh^2(x)\right)^{7/2}\,dx = \int \left(a\cosh(x)^2\right)^{7/2}\,dx$$

input
```
int((a*cosh(x)^2)^(7/2),x)
```

output
```
int((a*cosh(x)^2)^(7/2), x)
```

3.122 $\int \left(a\cosh^2(x)\right)^{5/2}\,dx$

3.122.1 Optimal result

Integrand size = 10, antiderivative size = 53

$$\int \left(a\cosh^2(x)\right)^{5/2}\,dx=\frac{8}{15}a^2\sqrt{a\cosh^2(x)}\tanh(x)+\frac{4}{15}a\left(a\cosh^2(x)\right)^{3/2}\tanh(x)+\frac{1}{5}\left(a\cosh^2(x)\right)^{5/2}\tanh(x)$$

output
```
4/15*a*(a*cosh(x)^2)^(3/2)*tanh(x)+1/5*(a*cosh(x)^2)^(5/2)*tanh(x)+8/15*a^
2*(a*cosh(x)^2)^(1/2)*tanh(x)
```

3.122.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.62

$$\int \left(a\cosh^2(x)\right)^{5/2}\,dx=\frac{1}{15}a^2\sqrt{a\cosh^2(x)}\left(15+10\sinh^2(x)+3\sinh^4(x)\right)\tanh(x)$$

input
```
Integrate[(a*Cosh[x]^2)^(5/2),x]
```

output
```
(a^2*Sqrt[a*Cosh[x]^2]*(15 + 10*Sinh[x]^2 + 3*Sinh[x]^4)*Tanh[x])/15
```

3.122.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 3682, 3042, 3682, 3042, 3686, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\cosh^2(x)\right)^{5/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(\frac{\pi}{2}+ix\right)^2\right)^{5/2} dx$$

$$\Big\downarrow \text{3682}$$

$$\frac{4}{5}a\int \left(a\cosh^2(x)\right)^{3/2} dx+\frac{1}{5}\tanh(x)\left(a\cosh^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\tanh(x)\left(a\cosh^2(x)\right)^{5/2}+\frac{4}{5}a\int \left(a\sin\left(ix+\frac{\pi}{2}\right)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{3682}$$

$$\frac{4}{5}a\left(\frac{2}{3}a\int \sqrt{a\cosh^2(x)}dx+\frac{1}{3}\tanh(x)\left(a\cosh^2(x)\right)^{3/2}\right)+\frac{1}{5}\tanh(x)\left(a\cosh^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\tanh(x)\left(a\cosh^2(x)\right)^{5/2}+\frac{4}{5}a\left(\frac{1}{3}\tanh(x)\left(a\cosh^2(x)\right)^{3/2}+\frac{2}{3}a\int \sqrt{a\sin\left(ix+\frac{\pi}{2}\right)^2}dx\right)$$

$$\Big\downarrow \text{3686}$$

$$\frac{4}{5}a\left(\frac{2}{3}a\text{sech}(x)\sqrt{a\cosh^2(x)}\int \cosh(x)dx+\frac{1}{3}\tanh(x)\left(a\cosh^2(x)\right)^{3/2}\right)+\frac{1}{5}\tanh(x)\left(a\cosh^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\tanh(x)\left(a\cosh^2(x)\right)^{5/2}+$$
$$\frac{4}{5}a\left(\frac{1}{3}\tanh(x)\left(a\cosh^2(x)\right)^{3/2}+\frac{2}{3}a\text{sech}(x)\sqrt{a\cosh^2(x)}\int \sin\left(ix+\frac{\pi}{2}\right)dx\right)$$

$$\Big\downarrow \text{3117}$$

$$\frac{1}{5}\tanh(x)\left(a\cosh^2(x)\right)^{5/2}+\frac{4}{5}a\left(\frac{1}{3}\tanh(x)\left(a\cosh^2(x)\right)^{3/2}+\frac{2}{3}a\tanh(x)\sqrt{a\cosh^2(x)}\right)$$

input
```
Int[(a*Cosh[x]^2)^(5/2),x]
```

output
```
((a*Cosh[x]^2)^(5/2)*Tanh[x])/5 + (4*a*((2*a*Sqrt[a*Cosh[x]^2]*Tanh[x])/3
+ ((a*Cosh[x]^2)^(3/2)*Tanh[x])/3))/5
```

3.122.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3682
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Simp[b*((2*p - 1)/(2*p))   Int[(b*S
in[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && G
tQ[p, 1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.122.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.60

method	result
default	$\frac{a^{3}\cosh(x)\sinh(x)\left(3\cosh(x)^{4}+4\cosh(x)^{2}+8\right)}{15\sqrt{a\cosh(x)^{2}}}$
risch	$\frac{a^{2}e^{6x}\sqrt{a(1+e^{2x})^{2}e^{-2x}}}{160+160\,e^{2x}}+\frac{5a^{2}e^{4x}\sqrt{a(1+e^{2x})^{2}e^{-2x}}}{96(1+e^{2x})}+\frac{5a^{2}e^{2x}\sqrt{a(1+e^{2x})^{2}e^{-2x}}}{16(1+e^{2x})}-\frac{5\sqrt{a(1+e^{2x})^{2}e^{-2x}}\,a^{2}}{16(1+e^{2x})}-\frac{5a^{2}e^{-2x}\sqrt{a(1+e^{2x})^{2}e^{-2x}}}{96(1+e^{2x})}$

input
```
int((a*cosh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*a^3*cosh(x)*sinh(x)*(3*cosh(x)^4+4*cosh(x)^2+8)/(a*cosh(x)^2)^(1/2)
```

3.122.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(41) = 82$.

Time = 0.26 (sec) , antiderivative size = 501, normalized size of antiderivative = 9.45

$$\int \left(a\cosh^{2}(x)\right)^{5/2}\,dx=\frac{\left(30\,a^{2}\cosh\left(x\right)e^{x}\sinh\left(x\right)^{9}+3\,a^{2}e^{x}\sinh\left(x\right)^{10}+5\left(27\,a^{2}\cosh\left(x\right)^{2}+5\,a^{2}\right)e^{x}\sinh\left(x\right)\right.}{}$$

input
```
integrate((a*cosh(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/480*(30*a^2*cosh(x)*e^x*sinh(x)^9 + 3*a^2*e^x*sinh(x)^10 + 5*(27*a^2*cos
h(x)^2 + 5*a^2)*e^x*sinh(x)^8 + 40*(9*a^2*cosh(x)^3 + 5*a^2*cosh(x))*e^x*s
inh(x)^7 + 10*(63*a^2*cosh(x)^4 + 70*a^2*cosh(x)^2 + 15*a^2)*e^x*sinh(x)^6
 + 4*(189*a^2*cosh(x)^5 + 350*a^2*cosh(x)^3 + 225*a^2*cosh(x))*e^x*sinh(x)
^5 + 10*(63*a^2*cosh(x)^6 + 175*a^2*cosh(x)^4 + 225*a^2*cosh(x)^2 - 15*a^2
)*e^x*sinh(x)^4 + 40*(9*a^2*cosh(x)^7 + 35*a^2*cosh(x)^5 + 75*a^2*cosh(x)^
3 - 15*a^2*cosh(x))*e^x*sinh(x)^3 + 5*(27*a^2*cosh(x)^8 + 140*a^2*cosh(x)^
6 + 450*a^2*cosh(x)^4 - 180*a^2*cosh(x)^2 - 5*a^2)*e^x*sinh(x)^2 + 10*(3*a
^2*cosh(x)^9 + 20*a^2*cosh(x)^7 + 90*a^2*cosh(x)^5 - 60*a^2*cosh(x)^3 - 5*
a^2*cosh(x))*e^x*sinh(x) + (3*a^2*cosh(x)^10 + 25*a^2*cosh(x)^8 + 150*a^2*
cosh(x)^6 - 150*a^2*cosh(x)^4 - 25*a^2*cosh(x)^2 - 3*a^2)*e^x)*sqrt(a*e^(4
*x) + 2*a*e^(2*x) + a)*e^(-x)/(cosh(x)^5*e^(2*x) + (e^(2*x) + 1)*sinh(x)^5
 + cosh(x)^5 + 5*(cosh(x)*e^(2*x) + cosh(x))*sinh(x)^4 + 10*(cosh(x)^2*e^(
2*x) + cosh(x)^2)*sinh(x)^3 + 10*(cosh(x)^3*e^(2*x) + cosh(x)^3)*sinh(x)^2
 + 5*(cosh(x)^4*e^(2*x) + cosh(x)^4)*sinh(x))
```

3.122.6 Sympy [F(-1)]

Timed out.

$$\int \left(a\cosh^2(x)\right)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a*cosh(x)**2)**(5/2),x)
```

output
```
Timed out
```

3.122.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \left(a\cosh^2(x)\right)^{5/2}\,dx = \frac{1}{160}\,a^{\frac{5}{2}}e^{(5\,x)} + \frac{5}{96}\,a^{\frac{5}{2}}e^{(3\,x)} - \frac{5}{16}\,a^{\frac{5}{2}}e^{(-x)} - \frac{5}{96}\,a^{\frac{5}{2}}e^{(-3\,x)} - \frac{1}{160}\,a^{\frac{5}{2}}e^{(-5\,x)} + \frac{5}{16}\,a^{\frac{5}{2}}e^{x}$$

input
```
integrate((a*cosh(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/160*a^(5/2)*e^(5*x) + 5/96*a^(5/2)*e^(3*x) - 5/16*a^(5/2)*e^(-x) - 5/96*
a^(5/2)*e^(-3*x) - 1/160*a^(5/2)*e^(-5*x) + 5/16*a^(5/2)*e^x
```

3.122.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15

$$\int \left(a\cosh^2(x)\right)^{5/2}\,dx = \frac{1}{480}\left(3\,a^2e^{(5\,x)} + 25\,a^2e^{(3\,x)} + 150\,a^2e^{x} - \left(150\,a^2e^{(4\,x)} + 25\,a^2e^{(2\,x)} + 3\,a^2\right)e^{(-5\,x)}\right)\sqrt{a}$$

input
```
integrate((a*cosh(x)^2)^(5/2),x, algorithm="giac")
```

output
```
1/480*(3*a^2*e^(5*x) + 25*a^2*e^(3*x) + 150*a^2*e^x - (150*a^2*e^(4*x) + 2
5*a^2*e^(2*x) + 3*a^2)*e^(-5*x))*sqrt(a)
```

3.122.9 Mupad [F(-1)]

Timed out.

$$\int \left(a\cosh^2(x)\right)^{5/2}\,dx=\int \left(a\cosh(x)^2\right)^{5/2}\,dx$$

input
```
int((a*cosh(x)^2)^(5/2),x)
```

output
```
int((a*cosh(x)^2)^(5/2), x)
```

3.123 $\int \left(a \cosh^2(x)\right)^{3/2} dx$

3.123.1 Optimal result

Integrand size = 10, antiderivative size = 34

$$\int \left(a \cosh^2(x)\right)^{3/2} dx = \frac{2}{3} a \sqrt{a \cosh^2(x)} \tanh(x) + \frac{1}{3} \left(a \cosh^2(x)\right)^{3/2} \tanh(x)$$

output
```
1/3*(a*cosh(x)^2)^(3/2)*tanh(x)+2/3*a*(a*cosh(x)^2)^(1/2)*tanh(x)
```

3.123.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int \left(a \cosh^2(x)\right)^{3/2} dx = \frac{1}{3} a \sqrt{a \cosh^2(x)} \left(3 + \sinh^2(x)\right) \tanh(x)$$

input
```
Integrate[(a*Cosh[x]^2)^(3/2),x]
```

output
```
(a*Sqrt[a*Cosh[x]^2]*(3 + Sinh[x]^2)*Tanh[x])/3
```

3.123.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3682, 3042, 3686, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\cosh^2(x)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(\frac{\pi}{2}+ix\right)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{3682}$$

$$\frac{2}{3}a\int \sqrt{a\cosh^2(x)}dx+\frac{1}{3}\tanh(x)\left(a\cosh^2(x)\right)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\tanh(x)\left(a\cosh^2(x)\right)^{3/2}+\frac{2}{3}a\int \sqrt{a\sin\left(ix+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{2}{3}a\text{sech}(x)\sqrt{a\cosh^2(x)}\int \cosh(x)dx+\frac{1}{3}\tanh(x)\left(a\cosh^2(x)\right)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\tanh(x)\left(a\cosh^2(x)\right)^{3/2}+\frac{2}{3}a\text{sech}(x)\sqrt{a\cosh^2(x)}\int \sin\left(ix+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3117}$$

$$\frac{1}{3}\tanh(x)\left(a\cosh^2(x)\right)^{3/2}+\frac{2}{3}a\tanh(x)\sqrt{a\cosh^2(x)}$$

input
```
Int[(a*Cosh[x]^2)^(3/2),x]
```

output
```
(2*a*Sqrt[a*Cosh[x]^2]*Tanh[x])/3 + ((a*Cosh[x]^2)^(3/2)*Tanh[x])/3
```

3.123.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3682
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Simp[b*((2*p - 1)/(2*p))   Int[(b*S
in[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && G
tQ[p, 1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.123.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

method	result	size
default	$\frac{a^{2} \cosh \left(x \right) \sinh \left(x \right) \left(\cosh \left(x \right)^{2}+2\right)}{3 \sqrt{a \cosh \left(x \right)^{2}}}$	24
risch	$\frac{a \,{\mathrm e}^{4 x} \sqrt{a \left(1+{\mathrm e}^{2 x}\right)^{2} {\mathrm e}^{-2 x}}}{24+24 \,{\mathrm e}^{2 x}}+\frac{3 a \,{\mathrm e}^{2 x} \sqrt{a \left(1+{\mathrm e}^{2 x}\right)^{2} {\mathrm e}^{-2 x}}}{8 \left(1+{\mathrm e}^{2 x}\right)}-\frac{3 \sqrt{a \left(1+{\mathrm e}^{2 x}\right)^{2} {\mathrm e}^{-2 x}}\, a}{8 \left(1+{\mathrm e}^{2 x}\right)}-\frac{a \,{\mathrm e}^{-2 x} \sqrt{a \left(1+{\mathrm e}^{2 x}\right)^{2} {\mathrm e}^{-2 x}}}{24 \left(1+{\mathrm e}^{2 x}\right)}$	122

input
```
int((a*cosh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2*cosh(x)*sinh(x)*(cosh(x)^2+2)/(a*cosh(x)^2)^(1/2)
```

3.123.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(26) = 52.

Time = 0.26 (sec) , antiderivative size = 222, normalized size of antiderivative = 6.53

$$\int \left(a\cosh^2(x)\right)^{3/2}\,dx = \frac{\left(6\,a\cosh\left(x\right)e^{x}\sinh\left(x\right)^{5} + ae^{x}\sinh\left(x\right)^{6} + 3\left(5\,a\cosh\left(x\right)^{2} + 3\,a\right)e^{x}\sinh\left(x\right)^{4} + 4\left(5\,a\right.}{}$$

input
```
integrate((a*cosh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/24*(6*a*cosh(x)*e^x*sinh(x)^5 + a*e^x*sinh(x)^6 + 3*(5*a*cosh(x)^2 + 3*a
)*e^x*sinh(x)^4 + 4*(5*a*cosh(x)^3 + 9*a*cosh(x))*e^x*sinh(x)^3 + 3*(5*a*c
osh(x)^4 + 18*a*cosh(x)^2 - 3*a)*e^x*sinh(x)^2 + 6*(a*cosh(x)^5 + 6*a*cosh
(x)^3 - 3*a*cosh(x))*e^x*sinh(x) + (a*cosh(x)^6 + 9*a*cosh(x)^4 - 9*a*cosh
(x)^2 - a)*e^x)*sqrt(a*e^(4*x) + 2*a*e^(2*x) + a)*e^(-x)/(cosh(x)^3*e^(2*x
) + (e^(2*x) + 1)*sinh(x)^3 + cosh(x)^3 + 3*(cosh(x)*e^(2*x) + cosh(x))*si
nh(x)^2 + 3*(cosh(x)^2*e^(2*x) + cosh(x)^2)*sinh(x))
```

3.123.6 Sympy [F(-1)]

Timed out.

$$\int \left(a\cosh^2(x)\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((a*cosh(x)**2)**(3/2),x)
```

output
```
Timed out
```

3.123.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \left(a\cosh^2(x)\right)^{3/2}\,dx = \frac{1}{24}\,a^{\frac{3}{2}}e^{(3\,x)} - \frac{3}{8}\,a^{\frac{3}{2}}e^{(-x)} - \frac{1}{24}\,a^{\frac{3}{2}}e^{(-3\,x)} + \frac{3}{8}\,a^{\frac{3}{2}}e^{x}$$

input
```
integrate((a*cosh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/24*a^(3/2)*e^(3*x) - 3/8*a^(3/2)*e^(-x) - 1/24*a^(3/2)*e^(-3*x) + 3/8*a^
(3/2)*e^x
```

3.123.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \left(a\cosh^2(x)\right)^{3/2} dx = -\frac{1}{24}\left(\left(9\,e^{(2\,x)}+1\right)e^{(-3\,x)}-e^{(3\,x)}-9\,e^{x}\right)a^{\frac{3}{2}}$$

input
```
integrate((a*cosh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/24*((9*e^(2*x) + 1)*e^(-3*x) - e^(3*x) - 9*e^x)*a^(3/2)
```

3.123.9 Mupad [F(-1)]

Timed out.

$$\int \left(a\cosh^2(x)\right)^{3/2} dx = \int \left(a\cosh(x)^2\right)^{3/2} dx$$

input
```
int((a*cosh(x)^2)^(3/2),x)
```

output
```
int((a*cosh(x)^2)^(3/2), x)
```

3.124 $\int \sqrt{a \cosh^2(x)}\,dx$

3.124.1 Optimal result

Integrand size = 10, antiderivative size = 13

$$\int \sqrt{a \cosh^2(x)}\,dx = \sqrt{a \cosh^2(x)}\tanh(x)$$

output
```
(a*cosh(x)^2)^(1/2)*tanh(x)
```

3.124.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sqrt{a \cosh^2(x)}\,dx = \sqrt{a \cosh^2(x)}\tanh(x)$$

input
```
Integrate[Sqrt[a*Cosh[x]^2],x]
```

output
```
Sqrt[a*Cosh[x]^2]*Tanh[x]
```

3.124.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3686, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\cosh^2(x)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sqrt{a\sin\left(\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \ 3686$$

$$\text{sech}(x)\sqrt{a\cosh^2(x)}\int \cosh(x)dx$$

$$\Big\downarrow \ 3042$$

$$\text{sech}(x)\sqrt{a\cosh^2(x)}\int \sin\left(ix+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \ 3117$$

$$\tanh(x)\sqrt{a\cosh^2(x)}$$

input
```
Int[Sqrt[a*Cosh[x]^2],x]
```

output
```
Sqrt[a*Cosh[x]^2]*Tanh[x]
```

3.124.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.124.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{a\cosh(x)\sinh(x)}{\sqrt{a\cosh(x)^2}}$	15
risch	$\frac{\sqrt{a(1+e^{2x})^2 e^{-2x}}\, e^{2x}}{2+2\,e^{2x}} - \frac{\sqrt{a(1+e^{2x})^2 e^{-2x}}}{2(1+e^{2x})}$	58

input
```
int((a*cosh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(a*cosh(x)^2)^(1/2)*a*cosh(x)*sinh(x)
```

3.124.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(11) = 22.

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 5.31

$$\int \sqrt{a\cosh^2(x)}\,dx = \frac{\left(2\cosh(x)\,e^x\sinh(x) + e^x\sinh(x)^2 + \left(\cosh(x)^2 - 1\right)e^x\right)\sqrt{ae^{(4x)} + 2\,ae^{(2x)} + a}\,e^{(-x)}}{2\left(\cosh(x)\,e^{(2x)} + \left(e^{(2x)} + 1\right)\sinh(x) + \cosh(x)\right)}$$

input
```
integrate((a*cosh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*cosh(x)*e^x*sinh(x) + e^x*sinh(x)^2 + (cosh(x)^2 - 1)*e^x)*sqrt(a*e
^(4*x) + 2*a*e^(2*x) + a)*e^(-x)/(cosh(x)*e^(2*x) + (e^(2*x) + 1)*sinh(x)
+ cosh(x))
```

3.124.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \sqrt{a\cosh^2(x)}\,dx = \frac{\sqrt{a\cosh^2{(x)}}\sinh{(x)}}{\cosh{(x)}}$$

input
```
integrate((a*cosh(x)**2)**(1/2),x)
```

output
```
sqrt(a*cosh(x)**2)*sinh(x)/cosh(x)
```

3.124.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \sqrt{a\cosh^2(x)}\,dx = -\frac{1}{2}\sqrt{a}e^{(-x)} + \frac{1}{2}\sqrt{a}e^x$$

input
```
integrate((a*cosh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(a)*e^(-x) + 1/2*sqrt(a)*e^x
```

3.124.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \sqrt{a\cosh^2(x)}\,dx = -\frac{1}{2}\sqrt{a}\left(e^{(-x)} - e^x\right)$$

input
```
integrate((a*cosh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(a)*(e^(-x) - e^x)
```

3.124.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \sqrt{a \cosh^2(x)}\, dx = \sqrt{a} \tanh(x) \left(\frac{e^{-x}}{2} + \frac{e^x}{2}\right)$$

input
```
int((a*cosh(x)^2)^(1/2),x)
```

output
```
a^(1/2)*tanh(x)*(exp(-x)/2 + exp(x)/2)
```

3.125 $\int \frac{1}{\sqrt{a \cosh^2(x)}} \, dx$

3.125.1 Optimal result

Integrand size = 10, antiderivative size = 16

$$\int \frac{1}{\sqrt{a \cosh^2(x)}} \, dx = \frac{\arctan(\sinh(x)) \cosh(x)}{\sqrt{a \cosh^2(x)}}$$

output
```
arctan(sinh(x))*cosh(x)/(a*cosh(x)^2)^(1/2)
```

3.125.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a \cosh^2(x)}} \, dx = \frac{\arctan(\sinh(x)) \cosh(x)}{\sqrt{a \cosh^2(x)}}$$

input
```
Integrate[1/Sqrt[a*Cosh[x]^2],x]
```

output
```
(ArcTan[Sinh[x]]*Cosh[x])/Sqrt[a*Cosh[x]^2]
```

3.125.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3686, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\cosh^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\sin\left(\frac{\pi}{2}+ix\right)^2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\cosh(x)\int \text{sech}(x)dx}{\sqrt{a\cosh^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh(x)\int \csc\left(ix+\frac{\pi}{2}\right)dx}{\sqrt{a\cosh^2(x)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\cosh(x)\arctan(\sinh(x))}{\sqrt{a\cosh^2(x)}}$$

input
```
Int[1/Sqrt[a*Cosh[x]^2],x]
```

output
```
(ArcTan[Sinh[x]]*Cosh[x])/Sqrt[a*Cosh[x]^2]
```

3.125.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.125.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(14) = 28$.

Time = 0.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 3.44

method	result	size
default	$-\frac{\cosh(x)\sqrt{a\sinh(x)^2}\,\ln\left(\frac{2\sqrt{-a}\,\sqrt{a\sinh(x)^2}-2a}{\cosh(x)}\right)}{\sqrt{-a}\,\sinh(x)\sqrt{a\cosh(x)^2}}$	55
risch	$\frac{i\mathrm{e}^{-x}(1+\mathrm{e}^{2x})\ln(\mathrm{e}^{x}+i)}{\sqrt{a(1+\mathrm{e}^{2x})^2\mathrm{e}^{-2x}}}-\frac{i\mathrm{e}^{-x}(1+\mathrm{e}^{2x})\ln(\mathrm{e}^{x}-i)}{\sqrt{a(1+\mathrm{e}^{2x})^2\mathrm{e}^{-2x}}}$	72

input
```
int(1/(a*cosh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-cosh(x)*(a*sinh(x)^2)^(1/2)/(-a)^(1/2)*ln(2*((-a)^(1/2)*(a*sinh(x)^2)^(1/
2)-a)/cosh(x))/sinh(x)/(a*cosh(x)^2)^(1/2)
```

3.125.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(14) = 28$.

Time = 0.26 (sec) , antiderivative size = 186, normalized size of antiderivative = 11.62

$$\int \frac{1}{\sqrt{a\cosh^2(x)}}\,dx$$
$$= \left[-\frac{\sqrt{-a}\log\left(\frac{a\cosh(x)^2-2\sqrt{ae^{(4x)}+2ae^{(2x)}+a}(\cosh(x)e^x+e^x\sinh(x))\sqrt{-a}e^{(-x)}+(ae^{(2x)}+a)\sinh(x)^2+\left(a\cosh(x)^2-a\right)e^{(2x)}+2(a\cosh}{(e^{(2x)}+1)\sinh(x)^2+\cosh(x)^2+\left(\cosh(x)^2+1\right)e^{(2x)}+2(\cosh(x)e^{(2x)}+\cosh(x))\sinh(x)+1}\right.}{a}\right.$$

input
```
integrate(1/(a*cosh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-sqrt(-a)*log((a*cosh(x)^2 - 2*sqrt(a*e^(4*x) + 2*a*e^(2*x) + a)*(cosh(x)
*e^x + e^x*sinh(x))*sqrt(-a)*e^(-x) + (a*e^(2*x) + a)*sinh(x)^2 + (a*cosh(
x)^2 - a)*e^(2*x) + 2*(a*cosh(x)*e^(2*x) + a*cosh(x))*sinh(x) - a)/((e^(2*
x) + 1)*sinh(x)^2 + cosh(x)^2 + (cosh(x)^2 + 1)*e^(2*x) + 2*(cosh(x)*e^(2*
x) + cosh(x))*sinh(x) + 1))/a, 2*sqrt(a*e^(4*x) + 2*a*e^(2*x) + a)*arctan(
cosh(x) + sinh(x))/(a*e^(2*x) + a)]
```

3.125.6 Sympy [F]

$$\int \frac{1}{\sqrt{a\cosh^2(x)}}\,dx = \int \frac{1}{\sqrt{a\cosh^2(x)}}\,dx$$

input
```
integrate(1/(a*cosh(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a*cosh(x)**2), x)
```

3.125.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{a\cosh^2(x)}}\,dx = \frac{2\arctan(e^x)}{\sqrt{a}}$$

input
```
integrate(1/(a*cosh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
2*arctan(e^x)/sqrt(a)
```

3.125.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a\cosh^2(x)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a*cosh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.125.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\cosh^2(x)}}\,dx = \int \frac{1}{\sqrt{a\cosh(x)^2}}\,dx$$

input
```
int(1/(a*cosh(x)^2)^(1/2),x)
```

output
```
int(1/(a*cosh(x)^2)^(1/2), x)
```

3.126 $\int \frac{1}{(a\cosh^2(x))^{3/2}}\,dx$

3.126.1 Optimal result

Integrand size = 10, antiderivative size = 42

$$\int \frac{1}{(a\cosh^2(x))^{3/2}}\,dx=\frac{\arctan(\sinh(x))\cosh(x)}{2a\sqrt{a\cosh^2(x)}}+\frac{\tanh(x)}{2a\sqrt{a\cosh^2(x)}}$$

output
```
1/2*arctan(sinh(x))*cosh(x)/a/(a*cosh(x)^2)^(1/2)+1/2*tanh(x)/a/(a*cosh(x)
^2)^(1/2)
```

3.126.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int \frac{1}{(a\cosh^2(x))^{3/2}}\,dx=\frac{\arctan(\sinh(x))\cosh(x)+\tanh(x)}{2a\sqrt{a\cosh^2(x)}}$$

input
```
Integrate[(a*Cosh[x]^2)^(-3/2),x]
```

output
```
(ArcTan[Sinh[x]]*Cosh[x] + Tanh[x])/(2*a*Sqrt[a*Cosh[x]^2])
```

3.126.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3683, 3042, 3686, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\cosh^2(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sin\left(\frac{\pi}{2}+ix\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3683}$$

$$\frac{\int \frac{1}{\sqrt{a\cosh^2(x)}}dx}{2a}+\frac{\tanh(x)}{2a\sqrt{a\cosh^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tanh(x)}{2a\sqrt{a\cosh^2(x)}}+\frac{\int \frac{1}{\sqrt{a\sin\left(ix+\frac{\pi}{2}\right)^2}}dx}{2a}$$

$$\Big\downarrow \text{3686}$$

$$\frac{\cosh(x)\int \operatorname{sech}(x)dx}{2a\sqrt{a\cosh^2(x)}}+\frac{\tanh(x)}{2a\sqrt{a\cosh^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tanh(x)}{2a\sqrt{a\cosh^2(x)}}+\frac{\cosh(x)\int \csc\left(ix+\frac{\pi}{2}\right)dx}{2a\sqrt{a\cosh^2(x)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\cosh(x)\arctan(\sinh(x))}{2a\sqrt{a\cosh^2(x)}}+\frac{\tanh(x)}{2a\sqrt{a\cosh^2(x)}}$$

input
```
Int[(a*Cosh[x]^2)^(-3/2),x]
```

output
```
(ArcTan[Sinh[x]]*Cosh[x])/(2*a*Sqrt[a*Cosh[x]^2]) + Tanh[x]/(2*a*Sqrt[a*Co
sh[x]^2])
```

3.126.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3683
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[Cot[e + f*x]*
((b*Sin[e + f*x]^2)^(p + 1)/(b*f*(2*p + 1))), x] + Simp[2*((p + 1)/(b*(2*p
+ 1)))   Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&
!IntegerQ[p] && LtQ[p, -1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.126.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(34) = 68$.

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.95

method	result	size
default	$\frac{\sqrt{a\sinh(x)^2}\left(-\ln\left(\frac{2\sqrt{-a}\,\sqrt{a\sinh(x)^2}-2a}{\cosh(x)}\right)a\cosh(x)^2+\sqrt{-a}\,\sqrt{a\sinh(x)^2}\right)}{2a^2\cosh(x)\sqrt{-a}\,\sinh(x)\sqrt{a\cosh(x)^2}}$	82
risch	$\frac{e^{2x}-1}{a(1+e^{2x})\sqrt{a(1+e^{2x})^2 e^{-2x}}}+\frac{i(1+e^{2x})e^{-x}\ln(e^x+i)}{2a\sqrt{a(1+e^{2x})^2 e^{-2x}}}-\frac{i(1+e^{2x})e^{-x}\ln(e^x-i)}{2a\sqrt{a(1+e^{2x})^2 e^{-2x}}}$	112

input
```
int(1/(a*cosh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a^2/cosh(x)*(a*sinh(x)^2)^(1/2)*(-ln(2*((-a)^(1/2)*(a*sinh(x)^2)^(1/2)
-a)/cosh(x))*a*cosh(x)^2+(-a)^(1/2)*(a*sinh(x)^2)^(1/2))/(-a)^(1/2)/sinh(x
)/(a*cosh(x)^2)^(1/2)
```

3.126.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(34) = 68.

Time = 0.26 (sec) , antiderivative size = 299, normalized size of antiderivative = 7.12

$$\int \frac{1}{\left(a\cosh^2(x)\right)^{3/2}}\,dx = \frac{\left(3\cosh(x)\,e^x\sinh(x)^2 + e^x\sinh(x)^3 + \left(3\cosh(x)^2-1\right)e^x\sinh(x) + \left(4\cosh(x)\,e \cdots\right.\right.}{a^2\cosh(x)^4 + \left(a^2e^{(2x)}+a^2\right)\sinh(x)^4 + 2\,a^2\cosh(x)^2 + 4\left(a^2\cosh(x)\,e^{(2x)} + \cdots\right.}$$

input
```
integrate(1/(a*cosh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
(3*cosh(x)*e^x*sinh(x)^2 + e^x*sinh(x)^3 + (3*cosh(x)^2 - 1)*e^x*sinh(x) +
 (4*cosh(x)*e^x*sinh(x)^3 + e^x*sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*e^x*sinh(x
)^2 + 4*(cosh(x)^3 + cosh(x))*e^x*sinh(x) + (cosh(x)^4 + 2*cosh(x)^2 + 1)*
e^x)*arctan(cosh(x) + sinh(x)) + (cosh(x)^3 - cosh(x))*e^x)*sqrt(a*e^(4*x)
 + 2*a*e^(2*x) + a)*e^(-x)/(a^2*cosh(x)^4 + (a^2*e^(2*x) + a^2)*sinh(x)^4
+ 2*a^2*cosh(x)^2 + 4*(a^2*cosh(x)*e^(2*x) + a^2*cosh(x))*sinh(x)^3 + 2*(3
*a^2*cosh(x)^2 + a^2 + (3*a^2*cosh(x)^2 + a^2)*e^(2*x))*sinh(x)^2 + a^2 +
(a^2*cosh(x)^4 + 2*a^2*cosh(x)^2 + a^2)*e^(2*x) + 4*(a^2*cosh(x)^3 + a^2*c
osh(x) + (a^2*cosh(x)^3 + a^2*cosh(x))*e^(2*x))*sinh(x))
```

3.126.6 Sympy [F]

$$\int \frac{1}{\left(a\cosh^2(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\cosh^2\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*cosh(x)**2)**(3/2),x)
```

output
```
Integral((a*cosh(x)**2)**(-3/2), x)
```

3.126.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(a\cosh^2(x)\right)^{3/2}}\,dx = \frac{e^{(3x)} - e^x}{a^{\frac{3}{2}}e^{(4x)} + 2\,a^{\frac{3}{2}}e^{(2x)} + a^{\frac{3}{2}}} + \frac{\arctan\left(e^x\right)}{a^{\frac{3}{2}}}$$

input
```
integrate(1/(a*cosh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
(e^(3*x) - e^x)/(a^(3/2)*e^(4*x) + 2*a^(3/2)*e^(2*x) + a^(3/2)) + arctan(e
^x)/a^(3/2)
```

3.126.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.33

$$\int \frac{1}{\left(a\cosh^2(x)\right)^{3/2}}\,dx = \frac{\frac{\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}\right)}{\sqrt{a}} - \frac{4\left(e^{(-x)}-e^x\right)}{\left(\left(e^{(-x)}-e^x\right)^2+4\right)\sqrt{a}}}{4\,a}$$

input
```
integrate(1/(a*cosh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
1/4*((pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))/sqrt(a) - 4*(e^(-x) - e^x)/
(((e^(-x) - e^x)^2 + 4)*sqrt(a)))/a
```

3.126.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\cosh^2(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\cosh\left(x\right)^2\right)^{3/2}}\,dx$$

input
```
int(1/(a*cosh(x)^2)^(3/2),x)
```

output
```
int(1/(a*cosh(x)^2)^(3/2), x)
```

3.127 $\int \frac{1}{\left(a\cosh^2(x)\right)^{5/2}}\,dx$

3.127.1 Optimal result

Integrand size = 10, antiderivative size = 61

$$\int \frac{1}{\left(a\cosh^2(x)\right)^{5/2}}\,dx=\frac{3\arctan(\sinh(x))\cosh(x)}{8a^2\sqrt{a\cosh^2(x)}}+\frac{\tanh(x)}{4a\left(a\cosh^2(x)\right)^{3/2}}+\frac{3\tanh(x)}{8a^2\sqrt{a\cosh^2(x)}}$$

output
```
3/8*arctan(sinh(x))*cosh(x)/a^2/(a*cosh(x)^2)^(1/2)+1/4*tanh(x)/a/(a*cosh(
x)^2)^(3/2)+3/8*tanh(x)/a^2/(a*cosh(x)^2)^(1/2)
```

3.127.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.59

$$\int \frac{1}{\left(a\cosh^2(x)\right)^{5/2}}\,dx=\frac{3\arctan(\sinh(x))\cosh(x)+\left(3+2\text{sech}^2(x)\right)\tanh(x)}{8a^2\sqrt{a\cosh^2(x)}}$$

input
```
Integrate[(a*Cosh[x]^2)^(-5/2),x]
```

output
```
(3*ArcTan[Sinh[x]]*Cosh[x] + (3 + 2*Sech[x]^2)*Tanh[x])/(8*a^2*Sqrt[a*Cosh
[x]^2])
```

3.127.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 3683, 3042, 3683, 3042, 3686, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\cosh^2(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sin\left(\frac{\pi}{2}+ix\right)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{3683}$$

$$\frac{3\int \frac{1}{\left(a\cosh^2(x)\right)^{3/2}}dx}{4a}+\frac{\tanh(x)}{4a\left(a\cosh^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tanh(x)}{4a\left(a\cosh^2(x)\right)^{3/2}}+\frac{3\int \frac{1}{\left(a\sin\left(ix+\frac{\pi}{2}\right)^2\right)^{3/2}}dx}{4a}$$

$$\Big\downarrow \text{3683}$$

$$\frac{3\left(\frac{\int \frac{1}{\sqrt{a\cosh^2(x)}}dx}{2a}+\frac{\tanh(x)}{2a\sqrt{a\cosh^2(x)}}\right)}{4a}+\frac{\tanh(x)}{4a\left(a\cosh^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tanh(x)}{4a\left(a\cosh^2(x)\right)^{3/2}}+\frac{3\left(\frac{\tanh(x)}{2a\sqrt{a\cosh^2(x)}}+\frac{\int \frac{1}{\sqrt{a\sin\left(ix+\frac{\pi}{2}\right)^2}}dx}{2a}\right)}{4a}$$

$$\Big\downarrow \text{3686}$$

$$\frac{3\left(\frac{\cosh(x)\int \text{sech}(x)dx}{2a\sqrt{a\cosh^2(x)}}+\frac{\tanh(x)}{2a\sqrt{a\cosh^2(x)}}\right)}{4a}+\frac{\tanh(x)}{4a\left(a\cosh^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tanh(x)}{4a\left(a\cosh^2(x)\right)^{3/2}}+\frac{3\left(\frac{\tanh(x)}{2a\sqrt{a\cosh^2(x)}}+\frac{\cosh(x)\int\csc\left(ix+\frac{\pi}{2}\right)dx}{2a\sqrt{a\cosh^2(x)}}\right)}{4a}$$

$$\Big\downarrow \text{4257}$$

$$\frac{3\left(\frac{\cosh(x)\arctan(\sinh(x))}{2a\sqrt{a\cosh^2(x)}}+\frac{\tanh(x)}{2a\sqrt{a\cosh^2(x)}}\right)}{4a}+\frac{\tanh(x)}{4a\left(a\cosh^2(x)\right)^{3/2}}$$

input
```
Int[(a*Cosh[x]^2)^(-5/2),x]
```

output
```
Tanh[x]/(4*a*(a*Cosh[x]^2)^(3/2)) + (3*((ArcTan[Sinh[x]]*Cosh[x])/(2*a*Sqr
t[a*Cosh[x]^2]) + Tanh[x]/(2*a*Sqrt[a*Cosh[x]^2])))/(4*a)
```

3.127.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3683
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[Cot[e + f*x]*
((b*Sin[e + f*x]^2)^(p + 1)/(b*f*(2*p + 1))), x] + Simp[2*((p + 1)/(b*(2*p
+ 1)))   Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&
!IntegerQ[p] && LtQ[p, -1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.127. $\int \frac{1}{\left(a\cosh^2(x)\right)^{5/2}}\,dx$

3.127.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(49) = 98$.

Time = 0.19 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.67

method	result	size
default	$\frac{\sqrt{a \sinh(x)^{2}}\, \left(-3 \ln\left(\frac{2 \sqrt{-a}\, \sqrt{a \sinh(x)^{2}}-2 a}{\cosh(x)}\right) a \cosh(x)^{4}+3 \cosh(x)^{2} \sqrt{a \sinh(x)^{2}}\, \sqrt{-a}+2 \sqrt{-a}\, \sqrt{a \sinh(x)^{2}}\right)}{8 a^{3} \cosh(x)^{3} \sqrt{-a}\, \sinh(x) \sqrt{a \cosh(x)^{2}}}$	102
risch	$\frac{3 \,{\mathrm e}^{6 x}+11 \,{\mathrm e}^{4 x}-11 \,{\mathrm e}^{2 x}-3}{4 a^{2} \left(1+{\mathrm e}^{2 x}\right)^{3} \sqrt{a \left(1+{\mathrm e}^{2 x}\right)^{2} {\mathrm e}^{-2 x}}}+\frac{3 i \left(1+{\mathrm e}^{2 x}\right) {\mathrm e}^{-x} \ln\left({\mathrm e}^{x}+i\right)}{8 a^{2} \sqrt{a \left(1+{\mathrm e}^{2 x}\right)^{2} {\mathrm e}^{-2 x}}}-\frac{3 i \left(1+{\mathrm e}^{2 x}\right) {\mathrm e}^{-x} \ln\left({\mathrm e}^{x}-i\right)}{8 a^{2} \sqrt{a \left(1+{\mathrm e}^{2 x}\right)^{2} {\mathrm e}^{-2 x}}}$	127

input
```
int(1/(a*cosh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/a^3/cosh(x)^3*(a*sinh(x)^2)^(1/2)*(-3*ln(2*((-a)^(1/2)*(a*sinh(x)^2)^(
1/2)-a)/cosh(x))*a*cosh(x)^4+3*cosh(x)^2*(a*sinh(x)^2)^(1/2)*(-a)^(1/2)+2*
(-a)^(1/2)*(a*sinh(x)^2)^(1/2))/(-a)^(1/2)/sinh(x)/(a*cosh(x)^2)^(1/2)
```

3.127.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 837 vs. $2(49) = 98$.

Time = 0.27 (sec) , antiderivative size = 837, normalized size of antiderivative = 13.72

$$\int \frac{1}{\left(a \cosh^{2}(x)\right)^{5/2}}\, dx = \text{Too large to display}$$

input
```
integrate(1/(a*cosh(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/4*(21*cosh(x)*e^x*sinh(x)^6 + 3*e^x*sinh(x)^7 + (63*cosh(x)^2 + 11)*e^x*
sinh(x)^5 + 5*(21*cosh(x)^3 + 11*cosh(x))*e^x*sinh(x)^4 + (105*cosh(x)^4 +
 110*cosh(x)^2 - 11)*e^x*sinh(x)^3 + (63*cosh(x)^5 + 110*cosh(x)^3 - 33*co
sh(x))*e^x*sinh(x)^2 + (21*cosh(x)^6 + 55*cosh(x)^4 - 33*cosh(x)^2 - 3)*e^
x*sinh(x) + 3*(8*cosh(x)*e^x*sinh(x)^7 + e^x*sinh(x)^8 + 4*(7*cosh(x)^2 +
1)*e^x*sinh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*e^x*sinh(x)^5 + 2*(35*cosh(
x)^4 + 30*cosh(x)^2 + 3)*e^x*sinh(x)^4 + 8*(7*cosh(x)^5 + 10*cosh(x)^3 + 3
*cosh(x))*e^x*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)^4 + 9*cosh(x)^2 + 1)
*e^x*sinh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5 + 3*cosh(x)^3 + cosh(x))*e^x*s
inh(x) + (cosh(x)^8 + 4*cosh(x)^6 + 6*cosh(x)^4 + 4*cosh(x)^2 + 1)*e^x)*ar
ctan(cosh(x) + sinh(x)) + (3*cosh(x)^7 + 11*cosh(x)^5 - 11*cosh(x)^3 - 3*c
osh(x))*e^x)*sqrt(a*e^(4*x) + 2*a*e^(2*x) + a)*e^(-x)/(a^3*cosh(x)^8 + 4*a
^3*cosh(x)^6 + (a^3*e^(2*x) + a^3)*sinh(x)^8 + 8*(a^3*cosh(x)*e^(2*x) + a^
3*cosh(x))*sinh(x)^7 + 6*a^3*cosh(x)^4 + 4*(7*a^3*cosh(x)^2 + a^3 + (7*a^3
*cosh(x)^2 + a^3)*e^(2*x))*sinh(x)^6 + 8*(7*a^3*cosh(x)^3 + 3*a^3*cosh(x)
+ (7*a^3*cosh(x)^3 + 3*a^3*cosh(x))*e^(2*x))*sinh(x)^5 + 4*a^3*cosh(x)^2 +
 2*(35*a^3*cosh(x)^4 + 30*a^3*cosh(x)^2 + 3*a^3 + (35*a^3*cosh(x)^4 + 30*a
^3*cosh(x)^2 + 3*a^3)*e^(2*x))*sinh(x)^4 + 8*(7*a^3*cosh(x)^5 + 10*a^3*cos
h(x)^3 + 3*a^3*cosh(x) + (7*a^3*cosh(x)^5 + 10*a^3*cosh(x)^3 + 3*a^3*cosh(
x))*e^(2*x))*sinh(x)^3 + a^3 + 4*(7*a^3*cosh(x)^6 + 15*a^3*cosh(x)^4 + ...
```

3.127.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\cosh^2(x)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*cosh(x)**2)**(5/2),x)
```

output
```
Timed out
```

3.127.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.23

$$\int \frac{1}{\left(a\cosh^2(x)\right)^{5/2}}\,dx = \frac{3\,e^{(7\,x)} + 11\,e^{(5\,x)} - 11\,e^{(3\,x)} - 3\,e^{x}}{4\left(a^{\frac{5}{2}}e^{(8\,x)} + 4\,a^{\frac{5}{2}}e^{(6\,x)} + 6\,a^{\frac{5}{2}}e^{(4\,x)} + 4\,a^{\frac{5}{2}}e^{(2\,x)} + a^{\frac{5}{2}}\right)} + \frac{3\,\arctan\left(e^{x}\right)}{4\,a^{\frac{5}{2}}}$$

input
```
integrate(1/(a*cosh(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/4*(3*e^(7*x) + 11*e^(5*x) - 11*e^(3*x) - 3*e^x)/(a^(5/2)*e^(8*x) + 4*a^(
5/2)*e^(6*x) + 6*a^(5/2)*e^(4*x) + 4*a^(5/2)*e^(2*x) + a^(5/2)) + 3/4*arct
an(e^x)/a^(5/2)
```

3.127.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10

$$\int \frac{1}{\left(a\cosh^2(x)\right)^{5/2}}\,dx = \frac{3\left(\pi + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)\right)}{16\,a^{\frac{5}{2}}} - \frac{3\left(e^{(-x)} - e^{x}\right)^{3} + 20\,e^{(-x)} - 20\,e^{x}}{4\left(\left(e^{(-x)} - e^{x}\right)^{2} + 4\right)^{2}a^{\frac{5}{2}}}$$

input
```
integrate(1/(a*cosh(x)^2)^(5/2),x, algorithm="giac")
```

output
```
3/16*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))/a^(5/2) - 1/4*(3*(e^(-x) -
e^x)^3 + 20*e^(-x) - 20*e^x)/(((e^(-x) - e^x)^2 + 4)^2*a^(5/2))
```

3.127.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\cosh^2(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\cosh\left(x\right)^2\right)^{5/2}}\,dx$$

input
```
int(1/(a*cosh(x)^2)^(5/2),x)
```

output
```
int(1/(a*cosh(x)^2)^(5/2), x)
```

3.128 $\int \left(a\cosh^3(x)\right)^{5/2}\,dx$

3.128.1 Optimal result

Integrand size = 10, antiderivative size = 121

$$\int \left(a\cosh^3(x)\right)^{5/2}\,dx = -\frac{26ia^2\sqrt{a\cosh^3(x)}\,\text{EllipticF}\left(\frac{ix}{2},2\right)}{77\cosh^{\frac{3}{2}}(x)} + \frac{78}{385}a^2\cosh(x)\sqrt{a\cosh^3(x)}\sinh(x) + \frac{26}{165}a^2\cosh^3(x)\sqrt{a\cosh^3(x)}\sinh(x) + \frac{2}{15}a^2\cosh^5(x)\sqrt{a\cosh^3(x)}\sinh(x) + \frac{26}{77}a^2\sqrt{a\cosh^3(x)}\tanh(x)$$

output
```
-26/77*I*a^2*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(
1/2))*(a*cosh(x)^3)^(1/2)/cosh(x)^(3/2)+78/385*a^2*cosh(x)*sinh(x)*(a*cosh
(x)^3)^(1/2)+26/165*a^2*cosh(x)^3*sinh(x)*(a*cosh(x)^3)^(1/2)+2/15*a^2*cos
h(x)^5*sinh(x)*(a*cosh(x)^3)^(1/2)+26/77*a^2*(a*cosh(x)^3)^(1/2)*tanh(x)
```

3.128.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.54

$$\int \left(a\cosh^3(x)\right)^{5/2}\,dx = \frac{a\left(a\cosh^3(x)\right)^{3/2}\left(-12480i\,\text{EllipticF}\left(\frac{ix}{2},2\right) + \sqrt{\cosh(x)}(15465\sinh(x) + 3657\sinh(\right.}{36960\cosh^{\frac{9}{2}}(x)}$$

input
```
Integrate[(a*Cosh[x]^3)^(5/2),x]
```

output
```
(a*(a*Cosh[x]^3)^(3/2)*((-12480*I)*EllipticF[(I/2)*x, 2] + Sqrt[Cosh[x]]*(
15465*Sinh[x] + 3657*Sinh[3*x] + 749*Sinh[5*x] + 77*Sinh[7*x]))/(36960*Co
sh[x]^(9/2))
```

3.128.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.82, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.200, Rules used = {3042, 3686, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\cosh^3(x)\right)^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(\frac{\pi}{2}+ix\right)^3\right)^{5/2}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{a^2\sqrt{a\cosh^3(x)}\int \cosh^{\frac{15}{2}}(x)dx}{\cosh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\sqrt{a\cosh^3(x)}\int \sin\left(ix+\frac{\pi}{2}\right)^{15/2}dx}{\cosh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3115}$$

$$\frac{a^2\sqrt{a\cosh^3(x)}\left(\frac{13}{15}\int \cosh^{\frac{11}{2}}(x)dx+\frac{2}{15}\sinh(x)\cosh^{\frac{13}{2}}(x)\right)}{\cosh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\sqrt{a\cosh^3(x)}\left(\frac{2}{15}\sinh(x)\cosh^{\frac{13}{2}}(x)+\frac{13}{15}\int \sin\left(ix+\frac{\pi}{2}\right)^{11/2}dx\right)}{\cosh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3115}$$

$$\frac{a^2\sqrt{a\cosh^3(x)}\left(\frac{13}{15}\left(\frac{9}{11}\int\cosh^{\frac{7}{2}}(x)dx+\frac{2}{11}\sinh(x)\cosh^{\frac{9}{2}}(x)\right)+\frac{2}{15}\sinh(x)\cosh^{\frac{13}{2}}(x)\right)}{\cosh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\sqrt{a\cosh^3(x)}\left(\frac{2}{15}\sinh(x)\cosh^{\frac{13}{2}}(x)+\frac{13}{15}\left(\frac{2}{11}\sinh(x)\cosh^{\frac{9}{2}}(x)+\frac{9}{11}\int\sin\left(ix+\frac{\pi}{2}\right)^{7/2}dx\right)\right)}{\cosh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3115}$$

$$\frac{a^2\sqrt{a\cosh^3(x)}\left(\frac{13}{15}\left(\frac{9}{11}\left(\frac{5}{7}\int\cosh^{\frac{3}{2}}(x)dx+\frac{2}{7}\sinh(x)\cosh^{\frac{5}{2}}(x)\right)+\frac{2}{11}\sinh(x)\cosh^{\frac{9}{2}}(x)\right)+\frac{2}{15}\sinh(x)\cosh^{\frac{13}{2}}(x)\right)}{\cosh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\sqrt{a\cosh^3(x)}\left(\frac{2}{15}\sinh(x)\cosh^{\frac{13}{2}}(x)+\frac{13}{15}\left(\frac{2}{11}\sinh(x)\cosh^{\frac{9}{2}}(x)+\frac{9}{11}\left(\frac{2}{7}\sinh(x)\cosh^{\frac{5}{2}}(x)+\frac{5}{7}\int\sin\left(ix+\frac{\pi}{2}\right)^{3/2}dx\right.\right.\right.}{\cosh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3115}$$

$$\frac{a^2\sqrt{a\cosh^3(x)}\left(\frac{13}{15}\left(\frac{9}{11}\left(\frac{5}{7}\left(\frac{1}{3}\int\frac{1}{\sqrt{\cosh(x)}}dx+\frac{2}{3}\sinh(x)\sqrt{\cosh(x)}\right)+\frac{2}{7}\sinh(x)\cosh^{\frac{5}{2}}(x)\right)+\frac{2}{11}\sinh(x)\cosh^{\frac{9}{2}}(x)\right)+\right.}{\cosh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\sqrt{a\cosh^3(x)}\left(\frac{2}{15}\sinh(x)\cosh^{\frac{13}{2}}(x)+\frac{13}{15}\left(\frac{2}{11}\sinh(x)\cosh^{\frac{9}{2}}(x)+\frac{9}{11}\left(\frac{2}{7}\sinh(x)\cosh^{\frac{5}{2}}(x)+\frac{5}{7}\left(\frac{2}{3}\sinh(x)\sqrt{\cosh(x}\right.\right.\right.\right.}{\cosh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{a^2\sqrt{a\cosh^3(x)}\left(\frac{2}{15}\sinh(x)\cosh^{\frac{13}{2}}(x)+\frac{13}{15}\left(\frac{2}{11}\sinh(x)\cosh^{\frac{9}{2}}(x)+\frac{9}{11}\left(\frac{2}{7}\sinh(x)\cosh^{\frac{5}{2}}(x)+\frac{5}{7}\left(\frac{2}{3}\sinh(x)\sqrt{\cosh(x)}\right.\right.\right.\right.}{\cosh^{\frac{3}{2}}(x)}$$

input
```
Int[(a*Cosh[x]^3)^(5/2),x]
```

3.128. $\int\left(a\cosh^3(x)\right)^{5/2}dx$

output
```
(a^2*Sqrt[a*Cosh[x]^3]*((2*Cosh[x]^(13/2)*Sinh[x])/15 + (13*((2*Cosh[x]^(9
/2)*Sinh[x])/11 + (9*((2*Cosh[x]^(5/2)*Sinh[x])/7 + (5*(((-2*I)/3)*Ellipti
cF[(I/2)*x, 2] + (2*Sqrt[Cosh[x]]*Sinh[x])/3))/7))/11))/15))/Cosh[x]^(3/2)
```

3.128.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.128.4 Maple [F]

$$\int \left(a \cosh \left(x \right)^{3}\right)^{\frac{5}{2}}d x$$

input
```
int((a*cosh(x)^3)^(5/2),x)
```

output
```
int((a*cosh(x)^3)^(5/2),x)
```

3.128.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 823, normalized size of antiderivative = 6.80

$$\int \left(a\cosh^3(x)\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((a*cosh(x)^3)^(5/2),x, algorithm="fricas")
```

output
```
1/73920*(24960*(sqrt(2)*a^2*cosh(x)^7 + 7*sqrt(2)*a^2*cosh(x)^6*sinh(x) +
21*sqrt(2)*a^2*cosh(x)^5*sinh(x)^2 + 35*sqrt(2)*a^2*cosh(x)^4*sinh(x)^3 +
35*sqrt(2)*a^2*cosh(x)^3*sinh(x)^4 + 21*sqrt(2)*a^2*cosh(x)^2*sinh(x)^5 +
7*sqrt(2)*a^2*cosh(x)*sinh(x)^6 + sqrt(2)*a^2*sinh(x)^7)*sqrt(a)*weierstra
ssPInverse(-4, 0, cosh(x) + sinh(x)) + (77*a^2*cosh(x)^14 + 1078*a^2*cosh(
x)*sinh(x)^13 + 77*a^2*sinh(x)^14 + 749*a^2*cosh(x)^12 + 7*(1001*a^2*cosh(
x)^2 + 107*a^2)*sinh(x)^12 + 3657*a^2*cosh(x)^10 + 28*(1001*a^2*cosh(x)^3
+ 321*a^2*cosh(x))*sinh(x)^11 + (77077*a^2*cosh(x)^4 + 49434*a^2*cosh(x)^2
 + 3657*a^2)*sinh(x)^10 + 15465*a^2*cosh(x)^8 + 2*(77077*a^2*cosh(x)^5 + 8
2390*a^2*cosh(x)^3 + 18285*a^2*cosh(x))*sinh(x)^9 + 3*(77077*a^2*cosh(x)^6
 + 123585*a^2*cosh(x)^4 + 54855*a^2*cosh(x)^2 + 5155*a^2)*sinh(x)^8 - 1546
5*a^2*cosh(x)^6 + 24*(11011*a^2*cosh(x)^7 + 24717*a^2*cosh(x)^5 + 18285*a^
2*cosh(x)^3 + 5155*a^2*cosh(x))*sinh(x)^7 + 3*(77077*a^2*cosh(x)^8 + 23069
2*a^2*cosh(x)^6 + 255990*a^2*cosh(x)^4 + 144340*a^2*cosh(x)^2 - 5155*a^2)*
sinh(x)^6 - 3657*a^2*cosh(x)^4 + 2*(77077*a^2*cosh(x)^9 + 296604*a^2*cosh(
x)^7 + 460782*a^2*cosh(x)^5 + 433020*a^2*cosh(x)^3 - 46395*a^2*cosh(x))*si
nh(x)^5 + (77077*a^2*cosh(x)^10 + 370755*a^2*cosh(x)^8 + 767970*a^2*cosh(x
)^6 + 1082550*a^2*cosh(x)^4 - 231975*a^2*cosh(x)^2 - 3657*a^2)*sinh(x)^4 -
 749*a^2*cosh(x)^2 + 4*(7007*a^2*cosh(x)^11 + 41195*a^2*cosh(x)^9 + 109710
*a^2*cosh(x)^7 + 216510*a^2*cosh(x)^5 - 77325*a^2*cosh(x)^3 - 3657*a^2*...
```

3.128.6 Sympy [F(-1)]

Timed out.

$$\int \left(a\cosh^3(x)\right)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a*cosh(x)**3)**(5/2),x)
```

output
```
Timed out
```

3.128.7 Maxima [F]

$$\int \left(a\cosh^3(x)\right)^{5/2} dx = \int \left(a\cosh\left(x\right)^3\right)^{\frac{5}{2}} dx$$

input
```
integrate((a*cosh(x)^3)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*cosh(x)^3)^(5/2), x)
```

3.128.8 Giac [F]

$$\int \left(a\cosh^3(x)\right)^{5/2} dx = \int \left(a\cosh\left(x\right)^3\right)^{\frac{5}{2}} dx$$

input
```
integrate((a*cosh(x)^3)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*cosh(x)^3)^(5/2), x)
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int \left(a\cosh^3(x)\right)^{5/2} dx = \int \left(a\cosh(x)^3\right)^{5/2} dx$$

input
```
int((a*cosh(x)^3)^(5/2),x)
```

output
```
int((a*cosh(x)^3)^(5/2), x)
```

3.129 $\int \left(a\cosh^3(x)\right)^{3/2}\,dx$

3.129.1 Optimal result

Integrand size = 10, antiderivative size = 71

$$\int \left(a\cosh^3(x)\right)^{3/2}\,dx = -\frac{14ia\sqrt{a\cosh^3(x)}\,E\left(\frac{ix}{2}\middle|2\right)}{15\cosh^{\frac{3}{2}}(x)} + \frac{14}{45}a\sqrt{a\cosh^3(x)}\sinh(x) + \frac{2}{9}a\cosh^2(x)\sqrt{a\cosh^3(x)}\sinh(x)$$

output
```
-14/15*I*a*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(1/2*x),2^(1/
2))*(a*cosh(x)^3)^(1/2)/cosh(x)^(3/2)+14/45*a*sinh(x)*(a*cosh(x)^3)^(1/2)+
2/9*a*cosh(x)^2*sinh(x)*(a*cosh(x)^3)^(1/2)
```

3.129.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

$$\int \left(a\cosh^3(x)\right)^{3/2}\,dx = \frac{\left(a\cosh^3(x)\right)^{3/2}\left(-168iE\left(\frac{ix}{2}\middle|2\right)+\sqrt{\cosh(x)}(38\sinh(2x)+5\sinh(4x))\right)}{180\cosh^{\frac{9}{2}}(x)}$$

input
```
Integrate[(a*Cosh[x]^3)^(3/2),x]
```

output
```
((a*Cosh[x]^3)^(3/2)*((-168*I)*EllipticE[(I/2)*x, 2] + Sqrt[Cosh[x]]*(38*S
inh[2*x] + 5*Sinh[4*x])))/(180*Cosh[x]^(9/2))
```

3.129.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 3686, 3042, 3115, 3042, 3115, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\cosh^3(x)\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(\frac{\pi}{2}+ix\right)^3\right)^{3/2} dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{a\sqrt{a\cosh^3(x)}\int \cosh^{\frac{9}{2}}(x)dx}{\cosh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sqrt{a\cosh^3(x)}\int \sin\left(ix+\frac{\pi}{2}\right)^{9/2}dx}{\cosh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3115}$$

$$\frac{a\sqrt{a\cosh^3(x)}\left(\frac{7}{9}\int \cosh^{\frac{5}{2}}(x)dx+\frac{2}{9}\sinh(x)\cosh^{\frac{7}{2}}(x)\right)}{\cosh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sqrt{a\cosh^3(x)}\left(\frac{2}{9}\sinh(x)\cosh^{\frac{7}{2}}(x)+\frac{7}{9}\int \sin\left(ix+\frac{\pi}{2}\right)^{5/2}dx\right)}{\cosh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3115}$$

$$\frac{a\sqrt{a\cosh^3(x)}\left(\frac{7}{9}\left(\frac{3}{5}\int \sqrt{\cosh(x)}dx+\frac{2}{5}\sinh(x)\cosh^{\frac{3}{2}}(x)\right)+\frac{2}{9}\sinh(x)\cosh^{\frac{7}{2}}(x)\right)}{\cosh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sqrt{a\cosh^3(x)}\left(\frac{2}{9}\sinh(x)\cosh^{\frac{7}{2}}(x)+\frac{7}{9}\left(\frac{2}{5}\sinh(x)\cosh^{\frac{3}{2}}(x)+\frac{3}{5}\int \sqrt{\sin\left(ix+\frac{\pi}{2}\right)}dx\right)\right)}{\cosh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{a\sqrt{a\cosh^3(x)}\left(\frac{2}{9}\sinh(x)\cosh^{\frac{7}{2}}(x)+\frac{7}{9}\left(\frac{2}{5}\sinh(x)\cosh^{\frac{3}{2}}(x)-\frac{6}{5}iE\left(\frac{ix}{2}\middle|2\right)\right)\right)}{\cosh^{\frac{3}{2}}(x)}$$

input
```
Int[(a*Cosh[x]^3)^(3/2),x]
```

output
```
(a*Sqrt[a*Cosh[x]^3]*((2*Cosh[x]^(7/2)*Sinh[x])/9 + (7*(((-6*I)/5)*Ellipti
cE[(I/2)*x, 2] + (2*Cosh[x]^(3/2)*Sinh[x])/5))/9))/Cosh[x]^(3/2)
```

3.129.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.129.4 Maple [F]

$$\int \left(a\cosh\left(x\right)^{3}\right)^{\frac{3}{2}}dx$$

input
```
int((a*cosh(x)^3)^(3/2),x)
```

output
```
int((a*cosh(x)^3)^(3/2),x)
```

3.129.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 317, normalized size of antiderivative = 4.46

$$\int \left(a\cosh^3(x)\right)^{3/2} dx =$$
$$-\frac{336\left(\sqrt{2}a\cosh\left(x\right)^{4}+4\sqrt{2}a\cosh\left(x\right)^{3}\sinh\left(x\right)+6\sqrt{2}a\cosh\left(x\right)^{2}\sinh\left(x\right)^{2}+4\sqrt{2}a\cosh\left(x\right)\sinh\left(x\right)^{3}+\sqrt{2}\ldots\right.}{}$$

input
```
integrate((a*cosh(x)^3)^(3/2),x, algorithm="fricas")
```

output
```
-1/360*(336*(sqrt(2)*a*cosh(x)^4 + 4*sqrt(2)*a*cosh(x)^3*sinh(x) + 6*sqrt(
2)*a*cosh(x)^2*sinh(x)^2 + 4*sqrt(2)*a*cosh(x)*sinh(x)^3 + sqrt(2)*a*sinh(
x)^4)*sqrt(a)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(x) +
sinh(x))) - (5*a*cosh(x)^8 + 40*a*cosh(x)*sinh(x)^7 + 5*a*sinh(x)^8 + 38*a
*cosh(x)^6 + 2*(70*a*cosh(x)^2 + 19*a)*sinh(x)^6 + 4*(70*a*cosh(x)^3 + 57*
a*cosh(x))*sinh(x)^5 - 336*a*cosh(x)^4 + 2*(175*a*cosh(x)^4 + 285*a*cosh(x
)^2 - 168*a)*sinh(x)^4 + 8*(35*a*cosh(x)^5 + 95*a*cosh(x)^3 - 168*a*cosh(x
))*sinh(x)^3 - 38*a*cosh(x)^2 + 2*(70*a*cosh(x)^6 + 285*a*cosh(x)^4 - 1008
*a*cosh(x)^2 - 19*a)*sinh(x)^2 + 4*(10*a*cosh(x)^7 + 57*a*cosh(x)^5 - 336*
a*cosh(x)^3 - 19*a*cosh(x))*sinh(x) - 5*a)*sqrt(a*cosh(x)))/(cosh(x)^4 + 4
*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)
^4)
```

3.129.6 Sympy [F(-1)]

Timed out.

$$\int \left(a\cosh^3(x)\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((a*cosh(x)**3)**(3/2),x)
```

output
```
Timed out
```

3.129.7 Maxima [F]

$$\int \left(a\cosh^3(x)\right)^{3/2}\,dx = \int \left(a\cosh\left(x\right)^3\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a*cosh(x)^3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*cosh(x)^3)^(3/2), x)
```

3.129.8 Giac [F]

$$\int \left(a\cosh^3(x)\right)^{3/2}\,dx = \int \left(a\cosh\left(x\right)^3\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a*cosh(x)^3)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*cosh(x)^3)^(3/2), x)
```

3.129.9 Mupad [F(-1)]

Timed out.

$$\int \left(a\cosh^3(x)\right)^{3/2}\,dx=\int \left(a\cosh(x)^3\right)^{3/2}\,dx$$

input
```
int((a*cosh(x)^3)^(3/2),x)
```

output
```
int((a*cosh(x)^3)^(3/2), x)
```

3.130 $\int \sqrt{a \cosh^3(x)}\, dx$

3.130.1 Optimal result

Integrand size = 10, antiderivative size = 48

$$\int \sqrt{a \cosh^3(x)}\, dx = -\frac{2i\sqrt{a \cosh^3(x)}\,\text{EllipticF}\left(\frac{ix}{2},2\right)}{3\cosh^{\frac{3}{2}}(x)} + \frac{2}{3}\sqrt{a\cosh^3(x)}\tanh(x)$$

output
```
-2/3*I*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/2))*
(a*cosh(x)^3)^(1/2)/cosh(x)^(3/2)+2/3*(a*cosh(x)^3)^(1/2)*tanh(x)
```

3.130.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.23

$$\int \sqrt{a \cosh^3(x)}\, dx = \frac{2}{3}\sqrt{a\cosh^3(x)}\left(\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\cosh(2x) - \sinh(2x)\right)\operatorname{sech}^2(x)\sqrt{1+\cosh(2x)+\sinh(2x)}+\tanh(x)\right)$$

input
```
Integrate[Sqrt[a*Cosh[x]^3],x]
```

output
```
(2*Sqrt[a*Cosh[x]^3]*(Hypergeometric2F1[1/4, 1/2, 5/4, -Cosh[2*x] - Sinh[2
*x]]*Sech[x]^2*Sqrt[1 + Cosh[2*x] + Sinh[2*x]] + Tanh[x]))/3
```

3.130.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3686, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\cosh^3(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\sin\left(\frac{\pi}{2}+ix\right)^3}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\sqrt{a\cosh^3(x)}\int \cosh^{\frac{3}{2}}(x)dx}{\cosh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a\cosh^3(x)}\int \sin\left(ix+\frac{\pi}{2}\right)^{3/2}dx}{\cosh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\sqrt{a\cosh^3(x)}\left(\frac{1}{3}\int \frac{1}{\sqrt{\cosh(x)}}dx+\frac{2}{3}\sinh(x)\sqrt{\cosh(x)}\right)}{\cosh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a\cosh^3(x)}\left(\frac{2}{3}\sinh(x)\sqrt{\cosh(x)}+\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(ix+\frac{\pi}{2}\right)}}dx\right)}{\cosh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\sqrt{a\cosh^3(x)}\left(\frac{2}{3}\sinh(x)\sqrt{\cosh(x)}-\frac{2}{3}i\,\text{EllipticF}\left(\frac{ix}{2},2\right)\right)}{\cosh^{\frac{3}{2}}(x)}$$

input
```
Int[Sqrt[a*Cosh[x]^3],x]
```

output
```
(Sqrt[a*Cosh[x]^3]*(((-2*I)/3)*EllipticF[(I/2)*x, 2] + (2*Sqrt[Cosh[x]]*Si
nh[x])/3))/Cosh[x]^(3/2)
```

3.130.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.130.4 Maple [F]

$$\int \sqrt{a\cosh(x)^3}dx$$

input
```
int((a*cosh(x)^3)^(1/2),x)
```

output
```
int((a*cosh(x)^3)^(1/2),x)
```

3.130.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.23

$$\int \sqrt{a\cosh^3(x)}\,dx$$
$$= \frac{2\left(\sqrt{2}\cosh\left(x\right)+\sqrt{2}\sinh\left(x\right)\right)\sqrt{a}\text{weierstrassPInverse}(-4,0,\cosh\left(x\right)+\sinh\left(x\right))+\sqrt{a\cosh\left(x\right)}\left(\cosh\left(x\right)^2\right.}{3\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

input
```
integrate((a*cosh(x)^3)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(2*(sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(a)*weierstrassPInverse(-4,
 0, cosh(x) + sinh(x)) + sqrt(a*cosh(x))*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
sinh(x)^2 - 1))/(cosh(x) + sinh(x))
```

3.130.6 Sympy [F]

$$\int \sqrt{a\cosh^3(x)}\,dx = \int \sqrt{a\cosh^3{\left(x\right)}}\,dx$$

input
```
integrate((a*cosh(x)**3)**(1/2),x)
```

output
```
Integral(sqrt(a*cosh(x)**3), x)
```

3.130.7 Maxima [F]

$$\int \sqrt{a\cosh^3(x)}\,dx = \int \sqrt{a\cosh\left(x\right)^3}\,dx$$

input
```
integrate((a*cosh(x)^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*cosh(x)^3), x)
```

3.130.8 Giac [F]

$$\int \sqrt{a\cosh^3(x)}\,dx = \int \sqrt{a\cosh(x)^3}\,dx$$

input
```
integrate((a*cosh(x)^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*cosh(x)^3), x)
```

3.130.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a\cosh^3(x)}\,dx = \int \sqrt{a\cosh(x)^3}\,dx$$

input
```
int((a*cosh(x)^3)^(1/2),x)
```

output
```
int((a*cosh(x)^3)^(1/2), x)
```

3.131 $\int \frac{1}{\sqrt{a\cosh^3(x)}}\,dx$

3.131.1 Optimal result

Integrand size = 10, antiderivative size = 46

$$\int \frac{1}{\sqrt{a\cosh^3(x)}}\,dx=\frac{2i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)}{\sqrt{a\cosh^3(x)}}+\frac{2\cosh(x)\sinh(x)}{\sqrt{a\cosh^3(x)}}$$

output
```
2*I*cosh(x)^(3/2)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(1/2*x
),2^(1/2))/(a*cosh(x)^3)^(1/2)+2*cosh(x)*sinh(x)/(a*cosh(x)^3)^(1/2)
```

3.131.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{a\cosh^3(x)}}\,dx=\frac{2\cosh(x)\left(i\sqrt{\cosh(x)}E\left(\frac{ix}{2}\middle|2\right)+\sinh(x)\right)}{\sqrt{a\cosh^3(x)}}$$

input
```
Integrate[1/Sqrt[a*Cosh[x]^3],x]
```

output
```
(2*Cosh[x]*(I*Sqrt[Cosh[x]]*EllipticE[(I/2)*x, 2] + Sinh[x]))/Sqrt[a*Cosh[
x]^3]
```

3.131.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3686, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\cosh^3(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\sin\left(\frac{\pi}{2}+ix\right)^3}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\int \frac{1}{\cosh^{\frac{3}{2}}(x)}dx}{\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\int \frac{1}{\sin\left(ix+\frac{\pi}{2}\right)^{3/2}}dx}{\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\left(\frac{2\sinh(x)}{\sqrt{\cosh(x)}}-\int \sqrt{\cosh(x)}dx\right)}{\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\left(\frac{2\sinh(x)}{\sqrt{\cosh(x)}}-\int \sqrt{\sin\left(ix+\frac{\pi}{2}\right)}dx\right)}{\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\left(\frac{2\sinh(x)}{\sqrt{\cosh(x)}}+2iE\left(\frac{ix}{2}\middle|2\right)\right)}{\sqrt{a\cosh^3(x)}}$$

input
```
Int[1/Sqrt[a*Cosh[x]^3],x]
```

output
```
(Cosh[x]^(3/2)*((2*I)*EllipticE[(I/2)*x, 2] + (2*Sinh[x])/Sqrt[Cosh[x]]))/
Sqrt[a*Cosh[x]^3]
```

3.131.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.131.4 Maple [F]

$$\int \frac{1}{\sqrt{a \cosh(x)^3}} dx$$

input
```
int(1/(a*cosh(x)^3)^(1/2),x)
```

output
```
int(1/(a*cosh(x)^3)^(1/2),x)
```

3.131.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.02

$$\int \frac{1}{\sqrt{a\cosh^3(x)}}\,dx$$
$$= \frac{2\left(\left(\sqrt{2}\cosh{(x)}^2 + 2\sqrt{2}\cosh{(x)}\sinh{(x)} + \sqrt{2}\sinh{(x)}^2 + \sqrt{2}\right)\sqrt{a}\text{weierstrassZeta}(-4, 0, \text{weierstrassPIn}}{a\cosh{(x)}^2 + 2a\cosh{(x)}\sinh{(}}$$

input
```
integrate(1/(a*cosh(x)^3)^(1/2),x, algorithm="fricas")
```

output
```
2*((sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 + sq
rt(2))*sqrt(a)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(x) +
 sinh(x))) + 2*sqrt(a*cosh(x))*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
)/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)
```

3.131.6 Sympy [F]

$$\int \frac{1}{\sqrt{a\cosh^3(x)}}\,dx = \int \frac{1}{\sqrt{a\cosh^3{(x)}}}\,dx$$

input
```
integrate(1/(a*cosh(x)**3)**(1/2),x)
```

output
```
Integral(1/sqrt(a*cosh(x)**3), x)
```

3.131.7 Maxima [F]

$$\int \frac{1}{\sqrt{a\cosh^3(x)}}\,dx = \int \frac{1}{\sqrt{a\cosh{(x)}^3}}\,dx$$

input
```
integrate(1/(a*cosh(x)^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(a*cosh(x)^3), x)
```

3.131.8 Giac [F]

$$\int \frac{1}{\sqrt{a\cosh^3(x)}}\,dx = \int \frac{1}{\sqrt{a\cosh(x)^3}}\,dx$$

input
```
integrate(1/(a*cosh(x)^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(a*cosh(x)^3), x)
```

3.131.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\cosh^3(x)}}\,dx = \int \frac{1}{\sqrt{a\cosh(x)^3}}\,dx$$

input
```
int(1/(a*cosh(x)^3)^(1/2),x)
```

output
```
int(1/(a*cosh(x)^3)^(1/2), x)
```

3.132 $\int \frac{1}{(a\cosh^3(x))^{3/2}}\,dx$

3.132.1 Optimal result

Integrand size = 10, antiderivative size = 75

$$\int \frac{1}{(a\cosh^3(x))^{3/2}}\,dx=-\frac{10i\cosh^{\frac{3}{2}}(x)\,\text{EllipticF}\left(\frac{ix}{2},2\right)}{21a\sqrt{a\cosh^3(x)}}+\frac{10\sinh(x)}{21a\sqrt{a\cosh^3(x)}}+\frac{2\text{sech}(x)\tanh(x)}{7a\sqrt{a\cosh^3(x)}}$$

output
```
-10/21*I*cosh(x)^(3/2)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(
1/2*x),2^(1/2))/a/(a*cosh(x)^3)^(1/2)+10/21*sinh(x)/a/(a*cosh(x)^3)^(1/2)+
2/7*sech(x)*tanh(x)/a/(a*cosh(x)^3)^(1/2)
```

3.132.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.64

$$\int \frac{1}{(a\cosh^3(x))^{3/2}}\,dx=\frac{2\cosh^2(x)\left(-5i\cosh^{\frac{5}{2}}(x)\,\text{EllipticF}\left(\frac{ix}{2},2\right)+5\cosh(x)\sinh(x)+3\tanh(x)\right)}{21\left(a\cosh^3(x)\right)^{3/2}}$$

input
```
Integrate[(a*Cosh[x]^3)^(-3/2),x]
```

output
```
(2*Cosh[x]^2*((-5*I)*Cosh[x]^(5/2)*EllipticF[(I/2)*x, 2] + 5*Cosh[x]*Sinh[
x] + 3*Tanh[x]))/(21*(a*Cosh[x]^3)^(3/2))
```

3.132.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.87, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 3686, 3042, 3116, 3042, 3116, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\cosh^3(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sin\left(\frac{\pi}{2}+ix\right)^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\int \frac{1}{\cosh^{\frac{9}{2}}(x)}dx}{a\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\int \frac{1}{\sin\left(ix+\frac{\pi}{2}\right)^{9/2}}dx}{a\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\left(\frac{5}{7}\int \frac{1}{\cosh^{\frac{5}{2}}(x)}dx+\frac{2\sinh(x)}{7\cosh^{\frac{7}{2}}(x)}\right)}{a\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\left(\frac{2\sinh(x)}{7\cosh^{\frac{7}{2}}(x)}+\frac{5}{7}\int \frac{1}{\sin\left(ix+\frac{\pi}{2}\right)^{5/2}}dx\right)}{a\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\cosh(x)}}dx+\frac{2\sinh(x)}{3\cosh^{\frac{3}{2}}(x)}\right)+\frac{2\sinh(x)}{7\cosh^{\frac{7}{2}}(x)}\right)}{a\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \color{blue}{3042}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\left(\frac{2\sinh(x)}{7\cosh^{\frac{7}{2}}(x)}+\frac{5}{7}\left(\frac{2\sinh(x)}{3\cosh^{\frac{3}{2}}(x)}+\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(ix+\frac{\pi}{2}\right)}}dx\right)\right)}{a\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \color{blue}{3120}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\left(\frac{2\sinh(x)}{7\cosh^{\frac{7}{2}}(x)}+\frac{5}{7}\left(\frac{2\sinh(x)}{3\cosh^{\frac{3}{2}}(x)}-\frac{2}{3}i\,\text{EllipticF}\left(\frac{ix}{2},2\right)\right)\right)}{a\sqrt{a\cosh^3(x)}}$$

input
```
Int[(a*Cosh[x]^3)^(-3/2),x]
```

output
```
(Cosh[x]^(3/2)*((2*Sinh[x])/(7*Cosh[x]^(7/2)) + (5*(((-2*I)/3)*EllipticF[(
I/2)*x, 2] + (2*Sinh[x])/(3*Cosh[x]^(3/2))))/7))/(a*Sqrt[a*Cosh[x]^3])
```

3.132.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.132. $\int \frac{1}{\left(a\cosh^3(x)\right)^{3/2}}\,dx$

3.132.4 Maple [F]

$$\int \frac{1}{\left(a \cosh \left(x \right)^{3}\right)^{\frac{3}{2}}}d x$$

input
```
int(1/(a*cosh(x)^3)^(3/2),x)
```

output
```
int(1/(a*cosh(x)^3)^(3/2),x)
```

3.132.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 629, normalized size of antiderivative = 8.39

$$\int \frac{1}{\left(a \cosh^3(x)\right)^{3/2}}\, dx = \text{Too large to display}$$

input
```
integrate(1/(a*cosh(x)^3)^(3/2),x, algorithm="fricas")
```

output
```
2/21*(5*(sqrt(2)*cosh(x)^8 + 8*sqrt(2)*cosh(x)*sinh(x)^7 + sqrt(2)*sinh(x)
^8 + 4*(7*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^6 + 4*sqrt(2)*cosh(x)^6 + 8
*(7*sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x))*sinh(x)^5 + 2*(35*sqrt(2)*cosh(
x)^4 + 30*sqrt(2)*cosh(x)^2 + 3*sqrt(2))*sinh(x)^4 + 6*sqrt(2)*cosh(x)^4 +
 8*(7*sqrt(2)*cosh(x)^5 + 10*sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x))*sinh(x
)^3 + 4*(7*sqrt(2)*cosh(x)^6 + 15*sqrt(2)*cosh(x)^4 + 9*sqrt(2)*cosh(x)^2
+ sqrt(2))*sinh(x)^2 + 4*sqrt(2)*cosh(x)^2 + 8*(sqrt(2)*cosh(x)^7 + 3*sqrt
(2)*cosh(x)^5 + 3*sqrt(2)*cosh(x)^3 + sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*
sqrt(a)*weierstrassPInverse(-4, 0, cosh(x) + sinh(x)) + 2*(5*cosh(x)^7 + 3
5*cosh(x)*sinh(x)^6 + 5*sinh(x)^7 + (105*cosh(x)^2 + 17)*sinh(x)^5 + 17*co
sh(x)^5 + 5*(35*cosh(x)^3 + 17*cosh(x))*sinh(x)^4 + (175*cosh(x)^4 + 170*c
osh(x)^2 - 17)*sinh(x)^3 - 17*cosh(x)^3 + (105*cosh(x)^5 + 170*cosh(x)^3 -
 51*cosh(x))*sinh(x)^2 + (35*cosh(x)^6 + 85*cosh(x)^4 - 51*cosh(x)^2 - 5)*
sinh(x) - 5*cosh(x))*sqrt(a*cosh(x)))/(a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(
x)^7 + a^2*sinh(x)^8 + 4*a^2*cosh(x)^6 + 4*(7*a^2*cosh(x)^2 + a^2)*sinh(x)
^6 + 6*a^2*cosh(x)^4 + 8*(7*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^5 + 2*(
35*a^2*cosh(x)^4 + 30*a^2*cosh(x)^2 + 3*a^2)*sinh(x)^4 + 4*a^2*cosh(x)^2 +
 8*(7*a^2*cosh(x)^5 + 10*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + 4*(7*a
^2*cosh(x)^6 + 15*a^2*cosh(x)^4 + 9*a^2*cosh(x)^2 + a^2)*sinh(x)^2 + a^2 +
 8*(a^2*cosh(x)^7 + 3*a^2*cosh(x)^5 + 3*a^2*cosh(x)^3 + a^2*cosh(x))*si...
```

3.132.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\cosh^3(x)\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*cosh(x)**3)**(3/2),x)
```

output
```
Timed out
```

3.132.7 Maxima [F]

$$\int \frac{1}{\left(a\cosh^3(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\cosh\left(x\right)^3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*cosh(x)^3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*cosh(x)^3)^(-3/2), x)
```

3.132.8 Giac [F]

$$\int \frac{1}{\left(a\cosh^3(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\cosh\left(x\right)^3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*cosh(x)^3)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*cosh(x)^3)^(-3/2), x)
```

3.132.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\cosh^3(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\cosh\left(x\right)^3\right)^{3/2}}\,dx$$

input
```
int(1/(a*cosh(x)^3)^(3/2),x)
```

output
```
int(1/(a*cosh(x)^3)^(3/2), x)
```

3.133 $\int \frac{1}{\left(a\cosh^3(x)\right)^{5/2}}\,dx$

3.133.1 Optimal result

Integrand size = 10, antiderivative size = 121

$$\int \frac{1}{\left(a\cosh^3(x)\right)^{5/2}}\,dx = \frac{154i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)}{195a^2\sqrt{a\cosh^3(x)}}+\frac{154\cosh(x)\sinh(x)}{195a^2\sqrt{a\cosh^3(x)}}+\frac{154\tanh(x)}{585a^2\sqrt{a\cosh^3(x)}}+\frac{22\operatorname{sech}^2(x)\tanh(x)}{117a^2\sqrt{a\cosh^3(x)}}+\frac{2\operatorname{sech}^4(x)\tanh(x)}{13a^2\sqrt{a\cosh^3(x)}}$$

output
```
154/195*I*cosh(x)^(3/2)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh
(1/2*x),2^(1/2))/a^2/(a*cosh(x)^3)^(1/2)+154/195*cosh(x)*sinh(x)/a^2/(a*co
sh(x)^3)^(1/2)+154/585*tanh(x)/a^2/(a*cosh(x)^3)^(1/2)+22/117*sech(x)^2*ta
nh(x)/a^2/(a*cosh(x)^3)^(1/2)+2/13*sech(x)^4*tanh(x)/a^2/(a*cosh(x)^3)^(1/
2)
```

3.133.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.50

$$\int \frac{1}{\left(a\cosh^3(x)\right)^{5/2}}\,dx = \frac{462i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)+462\cosh(x)\sinh(x)+2\left(77+55\operatorname{sech}^2(x)+45\operatorname{sech}^4(x)\right)\operatorname{ta}}{585a^2\sqrt{a\cosh^3(x)}}$$

input
```
Integrate[(a*Cosh[x]^3)^(-5/2),x]
```

output
```
((462*I)*Cosh[x]^(3/2)*EllipticE[(I/2)*x, 2] + 462*Cosh[x]*Sinh[x] + 2*(77
 + 55*Sech[x]^2 + 45*Sech[x]^4)*Tanh[x])/(585*a^2*Sqrt[a*Cosh[x]^3])
```

3.133.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.79, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.200$, Rules used = {3042, 3686, 3042, 3116, 3042, 3116, 3042, 3116, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cosh^3(x))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sin\left(\frac{\pi}{2}+ix\right)^3\right)^{5/2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\int \frac{1}{\cosh^{\frac{15}{2}}(x)}dx}{a^2\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\int \frac{1}{\sin\left(ix+\frac{\pi}{2}\right)^{15/2}}dx}{a^2\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\left(\frac{11}{13}\int \frac{1}{\cosh^{\frac{11}{2}}(x)}dx+\frac{2\sinh(x)}{13\cosh^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\left(\frac{2\sinh(x)}{13\cosh^{\frac{13}{2}}(x)}+\frac{11}{13}\int \frac{1}{\sin\left(ix+\frac{\pi}{2}\right)^{11/2}}dx\right)}{a^2\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\left(\frac{11}{13}\left(\frac{7}{9}\int\frac{1}{\cosh^{\frac{7}{2}}(x)}dx+\frac{2\sinh(x)}{9\cosh^{\frac{9}{2}}(x)}\right)+\frac{2\sinh(x)}{13\cosh^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\left(\frac{2\sinh(x)}{13\cosh^{\frac{13}{2}}(x)}+\frac{11}{13}\left(\frac{2\sinh(x)}{9\cosh^{\frac{9}{2}}(x)}+\frac{7}{9}\int\frac{1}{\sin\left(ix+\frac{\pi}{2}\right)^{7/2}}dx\right)\right)}{a^2\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\left(\frac{11}{13}\left(\frac{7}{9}\left(\frac{3}{5}\int\frac{1}{\cosh^{\frac{3}{2}}(x)}dx+\frac{2\sinh(x)}{5\cosh^{\frac{5}{2}}(x)}\right)+\frac{2\sinh(x)}{9\cosh^{\frac{9}{2}}(x)}\right)+\frac{2\sinh(x)}{13\cosh^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\left(\frac{2\sinh(x)}{13\cosh^{\frac{13}{2}}(x)}+\frac{11}{13}\left(\frac{2\sinh(x)}{9\cosh^{\frac{9}{2}}(x)}+\frac{7}{9}\left(\frac{2\sinh(x)}{5\cosh^{\frac{5}{2}}(x)}+\frac{3}{5}\int\frac{1}{\sin\left(ix+\frac{\pi}{2}\right)^{3/2}}dx\right)\right)\right)}{a^2\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\left(\frac{11}{13}\left(\frac{7}{9}\left(\frac{3}{5}\left(\frac{2\sinh(x)}{\sqrt{\cosh(x)}}-\int\sqrt{\cosh(x)}dx\right)+\frac{2\sinh(x)}{5\cosh^{\frac{5}{2}}(x)}\right)+\frac{2\sinh(x)}{9\cosh^{\frac{9}{2}}(x)}\right)+\frac{2\sinh(x)}{13\cosh^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\left(\frac{2\sinh(x)}{13\cosh^{\frac{13}{2}}(x)}+\frac{11}{13}\left(\frac{2\sinh(x)}{9\cosh^{\frac{9}{2}}(x)}+\frac{7}{9}\left(\frac{2\sinh(x)}{5\cosh^{\frac{5}{2}}(x)}+\frac{3}{5}\left(\frac{2\sinh(x)}{\sqrt{\cosh(x)}}-\int\sqrt{\sin\left(ix+\frac{\pi}{2}\right)}dx\right)\right)\right)\right)}{a^2\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\left(\frac{2\sinh(x)}{13\cosh^{\frac{13}{2}}(x)}+\frac{11}{13}\left(\frac{2\sinh(x)}{9\cosh^{\frac{9}{2}}(x)}+\frac{7}{9}\left(\frac{2\sinh(x)}{5\cosh^{\frac{5}{2}}(x)}+\frac{3}{5}\left(\frac{2\sinh(x)}{\sqrt{\cosh(x)}}+2iE\left(\frac{ix}{2}\middle|2\right)\right)\right)\right)\right)}{a^2\sqrt{a\cosh^3(x)}}$$

input
```
Int[(a*Cosh[x]^3)^(-5/2),x]
```

3.133. $\int \frac{1}{\left(a\cosh^3(x)\right)^{5/2}}\,dx$

output
```
(Cosh[x]^(3/2)*((2*Sinh[x])/(13*Cosh[x]^(13/2)) + (11*((2*Sinh[x])/(9*Cosh
[x]^(9/2)) + (7*((2*Sinh[x])/(5*Cosh[x]^(5/2)) + (3*((2*I)*EllipticE[(I/2)
*x, 2] + (2*Sinh[x])/Sqrt[Cosh[x]]))/5))/9))/13))/(a^2*Sqrt[a*Cosh[x]^3])
```

3.133.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.133.4 Maple [F]

$$\int \frac{1}{\left(a \cosh \left(x\right)^{3}\right)^{\frac{5}{2}}}dx$$

input
```
int(1/(a*cosh(x)^3)^(5/2),x)
```

output
```
int(1/(a*cosh(x)^3)^(5/2),x)
```

3.133.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 1668, normalized size of antiderivative = 13.79

$$\int \frac{1}{\left(a\cosh^3(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*cosh(x)^3)^(5/2),x, algorithm="fricas")
```

output
```
2/585*(231*(sqrt(2)*cosh(x)^14 + 14*sqrt(2)*cosh(x)*sinh(x)^13 + sqrt(2)*s
inh(x)^14 + 7*(13*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^12 + 7*sqrt(2)*cosh
(x)^12 + 28*(13*sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x))*sinh(x)^11 + 7*(143
*sqrt(2)*cosh(x)^4 + 66*sqrt(2)*cosh(x)^2 + 3*sqrt(2))*sinh(x)^10 + 21*sqr
t(2)*cosh(x)^10 + 14*(143*sqrt(2)*cosh(x)^5 + 110*sqrt(2)*cosh(x)^3 + 15*s
qrt(2)*cosh(x))*sinh(x)^9 + 7*(429*sqrt(2)*cosh(x)^6 + 495*sqrt(2)*cosh(x)
^4 + 135*sqrt(2)*cosh(x)^2 + 5*sqrt(2))*sinh(x)^8 + 35*sqrt(2)*cosh(x)^8 +
 8*(429*sqrt(2)*cosh(x)^7 + 693*sqrt(2)*cosh(x)^5 + 315*sqrt(2)*cosh(x)^3
+ 35*sqrt(2)*cosh(x))*sinh(x)^7 + 7*(429*sqrt(2)*cosh(x)^8 + 924*sqrt(2)*c
osh(x)^6 + 630*sqrt(2)*cosh(x)^4 + 140*sqrt(2)*cosh(x)^2 + 5*sqrt(2))*sinh
(x)^6 + 35*sqrt(2)*cosh(x)^6 + 14*(143*sqrt(2)*cosh(x)^9 + 396*sqrt(2)*cos
h(x)^7 + 378*sqrt(2)*cosh(x)^5 + 140*sqrt(2)*cosh(x)^3 + 15*sqrt(2)*cosh(x
))*sinh(x)^5 + 7*(143*sqrt(2)*cosh(x)^10 + 495*sqrt(2)*cosh(x)^8 + 630*sqr
t(2)*cosh(x)^6 + 350*sqrt(2)*cosh(x)^4 + 75*sqrt(2)*cosh(x)^2 + 3*sqrt(2))
*sinh(x)^4 + 21*sqrt(2)*cosh(x)^4 + 28*(13*sqrt(2)*cosh(x)^11 + 55*sqrt(2)
*cosh(x)^9 + 90*sqrt(2)*cosh(x)^7 + 70*sqrt(2)*cosh(x)^5 + 25*sqrt(2)*cosh
(x)^3 + 3*sqrt(2)*cosh(x))*sinh(x)^3 + 7*(13*sqrt(2)*cosh(x)^12 + 66*sqrt(
2)*cosh(x)^10 + 135*sqrt(2)*cosh(x)^8 + 140*sqrt(2)*cosh(x)^6 + 75*sqrt(2)
*cosh(x)^4 + 18*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^2 + 7*sqrt(2)*cosh(x)
^2 + 14*(sqrt(2)*cosh(x)^13 + 6*sqrt(2)*cosh(x)^11 + 15*sqrt(2)*cosh(x)...
```

3.133.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\cosh^3(x)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*cosh(x)**3)**(5/2),x)
```

output
```
Timed out
```

3.133.7 Maxima [F]

$$\int \frac{1}{(a\cosh^3(x))^{5/2}}\,dx = \int \frac{1}{(a\cosh(x)^3)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*cosh(x)^3)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*cosh(x)^3)^(-5/2), x)
```

3.133.8 Giac [F]

$$\int \frac{1}{(a\cosh^3(x))^{5/2}}\,dx = \int \frac{1}{(a\cosh(x)^3)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*cosh(x)^3)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*cosh(x)^3)^(-5/2), x)
```

3.133.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\cosh^3(x))^{5/2}}\,dx = \int \frac{1}{(a\cosh(x)^3)^{5/2}}\,dx$$

input
```
int(1/(a*cosh(x)^3)^(5/2),x)
```

output
```
int(1/(a*cosh(x)^3)^(5/2), x)
```

3.134 $\int \left(a\cosh^4(x)\right)^{5/2}\,dx$

3.134.1 Optimal result

Integrand size = 10, antiderivative size = 132

$$\begin{aligned}\int \left(a\cosh^4(x)\right)^{5/2}\,dx &= \frac{63}{256}a^2x\sqrt{a\cosh^4(x)}\operatorname{sech}^2(x)\\ &+\frac{21}{128}a^2\cosh(x)\sqrt{a\cosh^4(x)}\sinh(x)\\ &+\frac{21}{160}a^2\cosh^3(x)\sqrt{a\cosh^4(x)}\sinh(x)+\frac{9}{80}a^2\cosh^5(x)\sqrt{a\cosh^4(x)}\sinh(x)\\ &+\frac{1}{10}a^2\cosh^7(x)\sqrt{a\cosh^4(x)}\sinh(x)+\frac{63}{256}a^2\sqrt{a\cosh^4(x)}\tanh(x)\end{aligned}$$

output
```
63/256*a^2*x*sech(x)^2*(a*cosh(x)^4)^(1/2)+21/128*a^2*cosh(x)*sinh(x)*(a*c
osh(x)^4)^(1/2)+21/160*a^2*cosh(x)^3*sinh(x)*(a*cosh(x)^4)^(1/2)+9/80*a^2*
cosh(x)^5*sinh(x)*(a*cosh(x)^4)^(1/2)+1/10*a^2*cosh(x)^7*sinh(x)*(a*cosh(x
)^4)^(1/2)+63/256*a^2*(a*cosh(x)^4)^(1/2)*tanh(x)
```

3.134.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.40

$$\int \left(a\cosh^4(x)\right)^{5/2}\,dx=\frac{a\left(a\cosh^4(x)\right)^{3/2}\operatorname{sech}^6(x)(2520x+2100\sinh(2x)+600\sinh(4x)+150\sinh(6x)+2}{10240}$$

input
```
Integrate[(a*Cosh[x]^4)^(5/2),x]
```

output
```
(a*(a*Cosh[x]^4)^(3/2)*Sech[x]^6*(2520*x + 2100*Sinh[2*x] + 600*Sinh[4*x]
+ 150*Sinh[6*x] + 25*Sinh[8*x] + 2*Sinh[10*x]))/10240
```

3.134.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.70, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.300, Rules used = {3042, 3686, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a \cosh^4(x)\right)^{5/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a \sin\left(\frac{\pi}{2}+ix\right)^4\right)^{5/2} dx$$

$$\Big\downarrow \text{3686}$$

$$a^2\text{sech}^2(x)\sqrt{a\cosh^4(x)}\int \cosh^{10}(x)dx$$

$$\Big\downarrow \text{3042}$$

$$a^2\text{sech}^2(x)\sqrt{a\cosh^4(x)}\int \sin\left(ix+\frac{\pi}{2}\right)^{10} dx$$

$$\Big\downarrow \text{3115}$$

$$a^2\text{sech}^2(x)\sqrt{a\cosh^4(x)}\left(\frac{9}{10}\int \cosh^8(x)dx+\frac{1}{10}\sinh(x)\cosh^9(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2\text{sech}^2(x)\sqrt{a\cosh^4(x)}\left(\frac{1}{10}\sinh(x)\cosh^9(x)+\frac{9}{10}\int \sin\left(ix+\frac{\pi}{2}\right)^8 dx\right)$$

$$\Big\downarrow \text{3115}$$

$$a^2\text{sech}^2(x)\sqrt{a\cosh^4(x)}\left(\frac{9}{10}\left(\frac{7}{8}\int \cosh^6(x)dx+\frac{1}{8}\sinh(x)\cosh^7(x)\right)+\frac{1}{10}\sinh(x)\cosh^9(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2\text{sech}^2(x)\sqrt{a\cosh^4(x)}\left(\frac{1}{10}\sinh(x)\cosh^9(x)+\frac{9}{10}\left(\frac{1}{8}\sinh(x)\cosh^7(x)+\frac{7}{8}\int \sin\left(ix+\frac{\pi}{2}\right)^6 dx\right)\right)$$

$$\downarrow \text{3115}$$

$$a^2\text{sech}^2(x)\sqrt{a\cosh^4(x)}\left(\frac{9}{10}\left(\frac{7}{8}\left(\frac{5}{6}\int\cosh^4(x)dx+\frac{1}{6}\sinh(x)\cosh^5(x)\right)+\frac{1}{8}\sinh(x)\cosh^7(x)\right)+\frac{1}{10}\sinh(x)\cosh$$

$$\downarrow \text{3042}$$

$$a^2\text{sech}^2(x)\sqrt{a\cosh^4(x)}\left(\frac{1}{10}\sinh(x)\cosh^9(x)+\frac{9}{10}\left(\frac{1}{8}\sinh(x)\cosh^7(x)+\frac{7}{8}\left(\frac{1}{6}\sinh(x)\cosh^5(x)+\frac{5}{6}\int\sin\left(ix+\right.\right.\right.\right.$$

$$\downarrow \text{3115}$$

$$a^2\text{sech}^2(x)\sqrt{a\cosh^4(x)}\left(\frac{9}{10}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int\cosh^2(x)dx+\frac{1}{4}\sinh(x)\cosh^3(x)\right)+\frac{1}{6}\sinh(x)\cosh^5(x)\right)+\frac{1}{8}\sinh(x)\text{co}\right.\right.$$

$$\downarrow \text{3042}$$

$$a^2\text{sech}^2(x)\sqrt{a\cosh^4(x)}\left(\frac{1}{10}\sinh(x)\cosh^9(x)+\frac{9}{10}\left(\frac{1}{8}\sinh(x)\cosh^7(x)+\frac{7}{8}\left(\frac{1}{6}\sinh(x)\cosh^5(x)+\frac{5}{6}\left(\frac{1}{4}\sinh(x)\text{co}\right.\right.\right.\right.$$

$$\downarrow \text{3115}$$

$$a^2\text{sech}^2(x)\sqrt{a\cosh^4(x)}\left(\frac{9}{10}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)+\frac{1}{4}\sinh(x)\cosh^3(x)\right)+\frac{1}{6}\sinh(x)\cosh^5(x)\right)\right.\right.$$

$$\downarrow \text{24}$$

$$a^2\text{sech}^2(x)\sqrt{a\cosh^4(x)}\left(\frac{1}{10}\sinh(x)\cosh^9(x)+\frac{9}{10}\left(\frac{1}{8}\sinh(x)\cosh^7(x)+\frac{7}{8}\left(\frac{1}{6}\sinh(x)\cosh^5(x)+\frac{5}{6}\left(\frac{1}{4}\sinh(x)\text{co}\right.\right.\right.\right.$$

input
```
Int[(a*Cosh[x]^4)^(5/2),x]
```

output
```
a^2*Sqrt[a*Cosh[x]^4]*Sech[x]^2*((Cosh[x]^9*Sinh[x])/10 + (9*((Cosh[x]^7*S
inh[x])/8 + (7*((Cosh[x]^5*Sinh[x])/6 + (5*((Cosh[x]^3*Sinh[x])/4 + (3*(x/
2 + (Cosh[x]*Sinh[x])/2))/4))/6))/8))/10)
```

3.134. $\int \left(a\cosh^4(x)\right)^{5/2}\,dx$

3.134.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.134.4 Maple [A] (verified)

Time = 8.67 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.34

method	result
default	$\frac{\sqrt{8}\,\sqrt{2}\,a^{\frac{3}{2}}\left(1+\cosh(2x)\right)\sqrt{a(-1+\cosh(2x))(1+\cosh(2x))}\left(8\sqrt{a\sinh(2x)^{2}}\,\sqrt{a}\,\sinh(2x)^{4}+50\sqrt{a\sinh(2x)^{2}}\,\sqrt{a}\,\cosh(2x)\sinh(2x)^{2}+1\right.}{10240\sinh(2x)\sqrt{(1}}$
risch	$\frac{63a^{2}\mathrm{e}^{2x}\sqrt{a(1+\mathrm{e}^{2x})^{4}\mathrm{e}^{-4x}}\,x}{256(1+\mathrm{e}^{2x})^{2}}+\frac{a^{2}\mathrm{e}^{12x}\sqrt{a(1+\mathrm{e}^{2x})^{4}\mathrm{e}^{-4x}}}{10240(1+\mathrm{e}^{2x})^{2}}+\frac{5a^{2}\mathrm{e}^{10x}\sqrt{a(1+\mathrm{e}^{2x})^{4}\mathrm{e}^{-4x}}}{4096(1+\mathrm{e}^{2x})^{2}}+\frac{15a^{2}\mathrm{e}^{8x}\sqrt{a(1+\mathrm{e}^{2x})^{4}\mathrm{e}^{-4x}}}{2048(1+\mathrm{e}^{2x})^{2}}+\frac{15a^{2}\mathrm{e}^{6x}}{51}$

input
```
int((a*cosh(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/10240*8^(1/2)*2^(1/2)*a^(3/2)*(1+cosh(2*x))*(a*(-1+cosh(2*x))*(1+cosh(2*
x)))^(1/2)*(8*(a*sinh(2*x)^2)^(1/2)*a^(1/2)*sinh(2*x)^4+50*(a*sinh(2*x)^2)
^(1/2)*a^(1/2)*cosh(2*x)*sinh(2*x)^2+160*(a*sinh(2*x)^2)^(1/2)*a^(1/2)*sin
h(2*x)^2+325*cosh(2*x)*(a*sinh(2*x)^2)^(1/2)*a^(1/2)+640*(a*sinh(2*x)^2)^(
1/2)*a^(1/2)+315*ln(a^(1/2)*cosh(2*x)+(a*sinh(2*x)^2)^(1/2))*a)/sinh(2*x)/
((1+cosh(2*x))^2*a)^(1/2)
```

3.134.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1597 vs. $2(108) = 216$.

Time = 0.32 (sec) , antiderivative size = 1597, normalized size of antiderivative = 12.10

$$\int \left(a\cosh^4(x)\right)^{5/2} dx = \text{Too large to display}$$

input
```
integrate((a*cosh(x)^4)^(5/2),x, algorithm="fricas")
```

output
```
1/20480*(40*a^2*cosh(x)*e^(2*x)*sinh(x)^19 + 2*a^2*e^(2*x)*sinh(x)^20 + 5*
(76*a^2*cosh(x)^2 + 5*a^2)*e^(2*x)*sinh(x)^18 + 30*(76*a^2*cosh(x)^3 + 15*
a^2*cosh(x))*e^(2*x)*sinh(x)^17 + 15*(646*a^2*cosh(x)^4 + 255*a^2*cosh(x)^
2 + 10*a^2)*e^(2*x)*sinh(x)^16 + 48*(646*a^2*cosh(x)^5 + 425*a^2*cosh(x)^3
 + 50*a^2*cosh(x))*e^(2*x)*sinh(x)^15 + 60*(1292*a^2*cosh(x)^6 + 1275*a^2*
cosh(x)^4 + 300*a^2*cosh(x)^2 + 10*a^2)*e^(2*x)*sinh(x)^14 + 120*(1292*a^2
*cosh(x)^7 + 1785*a^2*cosh(x)^5 + 700*a^2*cosh(x)^3 + 70*a^2*cosh(x))*e^(2
*x)*sinh(x)^13 + 60*(4199*a^2*cosh(x)^8 + 7735*a^2*cosh(x)^6 + 4550*a^2*co
sh(x)^4 + 910*a^2*cosh(x)^2 + 35*a^2)*e^(2*x)*sinh(x)^12 + 80*(4199*a^2*co
sh(x)^9 + 9945*a^2*cosh(x)^7 + 8190*a^2*cosh(x)^5 + 2730*a^2*cosh(x)^3 + 3
15*a^2*cosh(x))*e^(2*x)*sinh(x)^11 + 2*(184756*a^2*cosh(x)^10 + 546975*a^2
*cosh(x)^8 + 600600*a^2*cosh(x)^6 + 300300*a^2*cosh(x)^4 + 69300*a^2*cosh(
x)^2 + 2520*a^2*x)*e^(2*x)*sinh(x)^10 + 20*(16796*a^2*cosh(x)^11 + 60775*a
^2*cosh(x)^9 + 85800*a^2*cosh(x)^7 + 60060*a^2*cosh(x)^5 + 23100*a^2*cosh(
x)^3 + 2520*a^2*x*cosh(x))*e^(2*x)*sinh(x)^9 + 30*(8398*a^2*cosh(x)^12 + 3
6465*a^2*cosh(x)^10 + 64350*a^2*cosh(x)^8 + 60060*a^2*cosh(x)^6 + 34650*a^
2*cosh(x)^4 + 7560*a^2*x*cosh(x)^2 - 70*a^2)*e^(2*x)*sinh(x)^8 + 240*(646*
a^2*cosh(x)^13 + 3315*a^2*cosh(x)^11 + 7150*a^2*cosh(x)^9 + 8580*a^2*cosh(
x)^7 + 6930*a^2*cosh(x)^5 + 2520*a^2*x*cosh(x)^3 - 70*a^2*cosh(x))*e^(2*x)
*sinh(x)^7 + 60*(1292*a^2*cosh(x)^14 + 7735*a^2*cosh(x)^12 + 20020*a^2*...
```

3.134.6 Sympy [F(-1)]

Timed out.

$$\int \left(a\cosh^4(x)\right)^{5/2} dx = \text{Timed out}$$

input
```
integrate((a*cosh(x)**4)**(5/2),x)
```

output
```
Timed out
```

3.134.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.76

$$\int \left(a\cosh^4(x)\right)^{5/2} dx = \frac{63}{256}a^{\frac{5}{2}}x + \frac{1}{20480}\left(25\,a^{\frac{5}{2}}e^{(-2\,x)} + 150\,a^{\frac{5}{2}}e^{(-4\,x)} + 600\,a^{\frac{5}{2}}e^{(-6\,x)} + 2100\,a^{\frac{5}{2}}e^{(-8\,x)} - 2100\,a^{\frac{5}{2}}e^{(-12\,x)} - 600\,a^{\frac{5}{2}}e^{(-14\,x)} - 15$$

input
```
integrate((a*cosh(x)^4)^(5/2),x, algorithm="maxima")
```

output
```
63/256*a^(5/2)*x + 1/20480*(25*a^(5/2)*e^(-2*x) + 150*a^(5/2)*e^(-4*x) + 6
00*a^(5/2)*e^(-6*x) + 2100*a^(5/2)*e^(-8*x) - 2100*a^(5/2)*e^(-12*x) - 600
*a^(5/2)*e^(-14*x) - 150*a^(5/2)*e^(-16*x) - 25*a^(5/2)*e^(-18*x) - 2*a^(5
/2)*e^(-20*x) + 2*a^(5/2))*e^(10*x)
```

3.134.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.86

$$\int \left(a\cosh^4(x)\right)^{5/2} dx = \frac{1}{20480}\left(5040\,a^2x + 2\,a^2e^{(10\,x)} + 25\,a^2e^{(8\,x)} + 150\,a^2e^{(6\,x)} + 600\,a^2e^{(4\,x)} + 2100\,a^2e^{(2\,x)}\right.$$

input
```
integrate((a*cosh(x)^4)^(5/2),x, algorithm="giac")
```

output
```
1/20480*(5040*a^2*x + 2*a^2*e^(10*x) + 25*a^2*e^(8*x) + 150*a^2*e^(6*x) +
600*a^2*e^(4*x) + 2100*a^2*e^(2*x) - (5754*a^2*e^(10*x) + 2100*a^2*e^(8*x)
 + 600*a^2*e^(6*x) + 150*a^2*e^(4*x) + 25*a^2*e^(2*x) + 2*a^2)*e^(-10*x))*
sqrt(a)
```

3.134.9 Mupad [F(-1)]

Timed out.

$$\int \left(a\cosh^4(x)\right)^{5/2}\,dx=\int \left(a\cosh(x)^4\right)^{5/2}\,dx$$

input
```
int((a*cosh(x)^4)^(5/2),x)
```

output
```
int((a*cosh(x)^4)^(5/2), x)
```

3.135 $\int \left(a\cosh^4(x)\right)^{3/2} dx$

3.135.1 Optimal result

Integrand size = 10, antiderivative size = 78

$$\int \left(a\cosh^4(x)\right)^{3/2} dx = \frac{5}{16}ax\sqrt{a\cosh^4(x)}\operatorname{sech}^2(x) + \frac{5}{24}a\cosh(x)\sqrt{a\cosh^4(x)}\sinh(x) + \frac{1}{6}a\cosh^3(x)\sqrt{a\cosh^4(x)}\sinh(x) + \frac{5}{16}a\sqrt{a\cosh^4(x)}\tanh(x)$$

output
```
5/16*a*x*sech(x)^2*(a*cosh(x)^4)^(1/2)+5/24*a*cosh(x)*sinh(x)*(a*cosh(x)^4
)^(1/2)+1/6*a*cosh(x)^3*sinh(x)*(a*cosh(x)^4)^(1/2)+5/16*a*(a*cosh(x)^4)^(
1/2)*tanh(x)
```

3.135.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.49

$$\int \left(a\cosh^4(x)\right)^{3/2} dx = \frac{1}{192}\left(a\cosh^4(x)\right)^{3/2}\operatorname{sech}^6(x)(60x+45\sinh(2x)+9\sinh(4x)+\sinh(6x))$$

input
```
Integrate[(a*Cosh[x]^4)^(3/2),x]
```

output
```
((a*Cosh[x]^4)^(3/2)*Sech[x]^6*(60*x + 45*Sinh[2*x] + 9*Sinh[4*x] + Sinh[6
*x]))/192
```

3.135.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.77, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {3042, 3686, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a \cosh^4(x)\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a \sin\left(\frac{\pi}{2}+ix\right)^4\right)^{3/2} dx$$

$$\Big\downarrow \text{3686}$$

$$a\text{sech}^2(x)\sqrt{a \cosh^4(x)}\int \cosh^6(x)dx$$

$$\Big\downarrow \text{3042}$$

$$a\text{sech}^2(x)\sqrt{a \cosh^4(x)}\int \sin\left(ix+\frac{\pi}{2}\right)^6 dx$$

$$\Big\downarrow \text{3115}$$

$$a\text{sech}^2(x)\sqrt{a \cosh^4(x)}\left(\frac{5}{6}\int \cosh^4(x)dx+\frac{1}{6}\sinh(x)\cosh^5(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$a\text{sech}^2(x)\sqrt{a \cosh^4(x)}\left(\frac{1}{6}\sinh(x)\cosh^5(x)+\frac{5}{6}\int \sin\left(ix+\frac{\pi}{2}\right)^4 dx\right)$$

$$\Big\downarrow \text{3115}$$

$$a\text{sech}^2(x)\sqrt{a \cosh^4(x)}\left(\frac{5}{6}\left(\frac{3}{4}\int \cosh^2(x)dx+\frac{1}{4}\sinh(x)\cosh^3(x)\right)+\frac{1}{6}\sinh(x)\cosh^5(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$a\text{sech}^2(x)\sqrt{a \cosh^4(x)}\left(\frac{1}{6}\sinh(x)\cosh^5(x)+\frac{5}{6}\left(\frac{1}{4}\sinh(x)\cosh^3(x)+\frac{3}{4}\int \sin\left(ix+\frac{\pi}{2}\right)^2 dx\right)\right)$$

$$\Big\downarrow \text{3115}$$

$$a\text{sech}^2(x)\sqrt{a \cosh^4(x)}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)+\frac{1}{4}\sinh(x)\cosh^3(x)\right)+\frac{1}{6}\sinh(x)\cosh^5(x)\right)$$

$$\Big\downarrow \; 24$$

$$a\,\text{sech}^2(x)\sqrt{a\cosh^4(x)}\left(\frac{1}{6}\sinh(x)\cosh^5(x)+\frac{5}{6}\left(\frac{1}{4}\sinh(x)\cosh^3(x)+\frac{3}{4}\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)\right)\right)$$

input
```
Int[(a*Cosh[x]^4)^(3/2),x]
```

output
```
a*Sqrt[a*Cosh[x]^4]*Sech[x]^2*((Cosh[x]^5*Sinh[x])/6 + (5*((Cosh[x]^3*Sinh
[x])/4 + (3*(x/2 + (Cosh[x]*Sinh[x])/2))/4))/6)
```

3.135.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.135. $\int \left(a\cosh^4(x)\right)^{3/2}\,dx$

3.135.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(62) = 124.

Time = 0.26 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.68

method	result
default	$\frac{\sqrt{8}\,\sqrt{2}\,\sqrt{a}\,(1+\cosh(2x))\sqrt{a(-1+\cosh(2x))(1+\cosh(2x))}\left(2\sqrt{a\sinh(2x)^{2}}\,\sqrt{a}\,\sinh(2x)^{2}+9\cosh(2x)\sqrt{a\sinh(2x)^{2}}\,\sqrt{a}+24\sqrt{a\sinh(2\right.}{384\sinh(2x)\sqrt{(1+\cosh(2x))^{2}a}}$
risch	$\frac{5a\,e^{2x}\sqrt{a(1+e^{2x})^{4}e^{-4x}}\,x}{16(1+e^{2x})^{2}}+\frac{a\,e^{8x}\sqrt{a(1+e^{2x})^{4}e^{-4x}}}{384(1+e^{2x})^{2}}+\frac{3a\,e^{6x}\sqrt{a(1+e^{2x})^{4}e^{-4x}}}{128(1+e^{2x})^{2}}+\frac{15a\,e^{4x}\sqrt{a(1+e^{2x})^{4}e^{-4x}}}{128(1+e^{2x})^{2}}-\frac{15\sqrt{a(1+e^{2x})^{4}}}{128(1+e^{2x}}$

input
```
int((a*cosh(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/384*8^(1/2)*2^(1/2)*a^(1/2)*(1+cosh(2*x))*(a*(-1+cosh(2*x))*(1+cosh(2*x)
))^(1/2)*(2*(a*sinh(2*x)^2)^(1/2)*a^(1/2)*sinh(2*x)^2+9*cosh(2*x)*(a*sinh(
2*x)^2)^(1/2)*a^(1/2)+24*(a*sinh(2*x)^2)^(1/2)*a^(1/2)+15*ln(a^(1/2)*cosh(
2*x)+(a*sinh(2*x)^2)^(1/2))*a)/sinh(2*x)/((1+cosh(2*x))^2*a)^(1/2)
```

3.135.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 659 vs. 2(62) = 124.

Time = 0.29 (sec) , antiderivative size = 659, normalized size of antiderivative = 8.45

$$\int \left(a\cosh^{4}(x)\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate((a*cosh(x)^4)^(3/2),x, algorithm="fricas")
```

output
```
1/384*(12*a*cosh(x)*e^(2*x)*sinh(x)^11 + a*e^(2*x)*sinh(x)^12 + 3*(22*a*co
sh(x)^2 + 3*a)*e^(2*x)*sinh(x)^10 + 10*(22*a*cosh(x)^3 + 9*a*cosh(x))*e^(2
*x)*sinh(x)^9 + 45*(11*a*cosh(x)^4 + 9*a*cosh(x)^2 + a)*e^(2*x)*sinh(x)^8
+ 72*(11*a*cosh(x)^5 + 15*a*cosh(x)^3 + 5*a*cosh(x))*e^(2*x)*sinh(x)^7 + 6
*(154*a*cosh(x)^6 + 315*a*cosh(x)^4 + 210*a*cosh(x)^2 + 20*a*x)*e^(2*x)*si
nh(x)^6 + 36*(22*a*cosh(x)^7 + 63*a*cosh(x)^5 + 70*a*cosh(x)^3 + 20*a*x*co
sh(x))*e^(2*x)*sinh(x)^5 + 45*(11*a*cosh(x)^8 + 42*a*cosh(x)^6 + 70*a*cosh
(x)^4 + 40*a*x*cosh(x)^2 - a)*e^(2*x)*sinh(x)^4 + 20*(11*a*cosh(x)^9 + 54*
a*cosh(x)^7 + 126*a*cosh(x)^5 + 120*a*x*cosh(x)^3 - 9*a*cosh(x))*e^(2*x)*s
inh(x)^3 + 3*(22*a*cosh(x)^10 + 135*a*cosh(x)^8 + 420*a*cosh(x)^6 + 600*a*
x*cosh(x)^4 - 90*a*cosh(x)^2 - 3*a)*e^(2*x)*sinh(x)^2 + 6*(2*a*cosh(x)^11
+ 15*a*cosh(x)^9 + 60*a*cosh(x)^7 + 120*a*x*cosh(x)^5 - 30*a*cosh(x)^3 - 3
*a*cosh(x))*e^(2*x)*sinh(x) + (a*cosh(x)^12 + 9*a*cosh(x)^10 + 45*a*cosh(x
)^8 + 120*a*x*cosh(x)^6 - 45*a*cosh(x)^4 - 9*a*cosh(x)^2 - a)*e^(2*x))*sqr
t(a*e^(8*x) + 4*a*e^(6*x) + 6*a*e^(4*x) + 4*a*e^(2*x) + a)*e^(-2*x)/(cosh(
x)^6*e^(4*x) + 2*cosh(x)^6*e^(2*x) + (e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^6 +
 cosh(x)^6 + 6*(cosh(x)*e^(4*x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^5 +
 15*(cosh(x)^2*e^(4*x) + 2*cosh(x)^2*e^(2*x) + cosh(x)^2)*sinh(x)^4 + 20*(
cosh(x)^3*e^(4*x) + 2*cosh(x)^3*e^(2*x) + cosh(x)^3)*sinh(x)^3 + 15*(cosh(
x)^4*e^(4*x) + 2*cosh(x)^4*e^(2*x) + cosh(x)^4)*sinh(x)^2 + 6*(cosh(x)^...
```

3.135.6 Sympy [F(-1)]

Timed out.

$$\int \left(a\cosh^4(x)\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate((a*cosh(x)**4)**(3/2),x)
```

output
```
Timed out
```

3.135.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.79

$$\int \left(a\cosh^4(x)\right)^{3/2}\,dx = \frac{5}{16}\,a^{\frac{3}{2}}x + \frac{1}{384}\left(9\,a^{\frac{3}{2}}e^{(-2\,x)} + 45\,a^{\frac{3}{2}}e^{(-4\,x)} - 45\,a^{\frac{3}{2}}e^{(-8\,x)} - 9\,a^{\frac{3}{2}}e^{(-10\,x)} - a^{\frac{3}{2}}e^{(-12\,x)} + a^{\frac{3}{2}}\right)e^{(6\,x)}$$

input
```
integrate((a*cosh(x)^4)^(3/2),x, algorithm="maxima")
```

output
```
5/16*a^(3/2)*x + 1/384*(9*a^(3/2)*e^(-2*x) + 45*a^(3/2)*e^(-4*x) - 45*a^(3
/2)*e^(-8*x) - 9*a^(3/2)*e^(-10*x) - a^(3/2)*e^(-12*x) + a^(3/2))*e^(6*x)
```

3.135.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.67

$$\int \left(a\cosh^4(x)\right)^{3/2}\,dx = -\frac{1}{384}\left(\left(110\,e^{(6\,x)} + 45\,e^{(4\,x)} + 9\,e^{(2\,x)} + 1\right)e^{(-6\,x)} - 120\,x - e^{(6\,x)} - 9\,e^{(4\,x)} - 45\,e^{(2\,x)}\right)a^{\frac{3}{2}}$$

input
```
integrate((a*cosh(x)^4)^(3/2),x, algorithm="giac")
```

output
```
-1/384*((110*e^(6*x) + 45*e^(4*x) + 9*e^(2*x) + 1)*e^(-6*x) - 120*x - e^(6
*x) - 9*e^(4*x) - 45*e^(2*x))*a^(3/2)
```

3.135.9 Mupad [F(-1)]

Timed out.

$$\int \left(a\cosh^4(x)\right)^{3/2}\,dx = \int \left(a\cosh(x)^4\right)^{3/2}\,dx$$

input
```
int((a*cosh(x)^4)^(3/2),x)
```

output
```
int((a*cosh(x)^4)^(3/2), x)
```

3.136 $\int \sqrt{a \cosh^4(x)} \, dx$

3.136.1 Optimal result

Integrand size = 10, antiderivative size = 36

$$\int \sqrt{a \cosh^4(x)} \, dx = \frac{1}{2} x \sqrt{a \cosh^4(x)} \operatorname{sech}^2(x) + \frac{1}{2} \sqrt{a \cosh^4(x)} \tanh(x)$$

output
```
1/2*x*sech(x)^2*(a*cosh(x)^4)^(1/2)+1/2*(a*cosh(x)^4)^(1/2)*tanh(x)
```

3.136.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int \sqrt{a \cosh^4(x)} \, dx = \frac{1}{2} \sqrt{a \cosh^4(x)} \operatorname{sech}^2(x) (x + \cosh(x) \sinh(x))$$

input
```
Integrate[Sqrt[a*Cosh[x]^4],x]
```

output
```
(Sqrt[a*Cosh[x]^4]*Sech[x]^2*(x + Cosh[x]*Sinh[x]))/2
```

3.136.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3686, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a \cosh^4(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{a \sin\left(\frac{\pi}{2}+ix\right)^4}dx$$

$$\Big\downarrow \; 3686$$

$$\text{sech}^2(x)\sqrt{a \cosh^4(x)}\int \cosh^2(x)dx$$

$$\Big\downarrow \; 3042$$

$$\text{sech}^2(x)\sqrt{a \cosh^4(x)}\int \sin\left(ix+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \; 3115$$

$$\text{sech}^2(x)\sqrt{a \cosh^4(x)}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)$$

$$\Big\downarrow \; 24$$

$$\text{sech}^2(x)\sqrt{a \cosh^4(x)}\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)$$

input
```
Int[Sqrt[a*Cosh[x]^4],x]
```

output
```
Sqrt[a*Cosh[x]^4]*Sech[x]^2*(x/2 + (Cosh[x]*Sinh[x])/2)
```

3.136.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.136.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(28) = 56$.

Time = 0.22 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.47

method	result	size
default	$\frac{\sqrt{8}\,\sqrt{2}\,(1+\cosh(2x))\sqrt{a(-1+\cosh(2x))(1+\cosh(2x))}\left(\ln\left(\sqrt{a}\,\cosh(2x)+\sqrt{a\sinh(2x)^{2}}\right)a+\sqrt{a\sinh(2x)^{2}}\,\sqrt{a}\right)}{16\sqrt{a}\,\sinh(2x)\sqrt{(1+\cosh(2x))^{2}a}}$	89
risch	$\frac{\sqrt{a(1+e^{2x})^{4}e^{-4x}}\,e^{2x}x}{2(1+e^{2x})^{2}}+\frac{\sqrt{a(1+e^{2x})^{4}e^{-4x}}\,e^{4x}}{8(1+e^{2x})^{2}}-\frac{\sqrt{a(1+e^{2x})^{4}e^{-4x}}}{8(1+e^{2x})^{2}}$	89

input
```
int((a*cosh(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16*8^(1/2)*2^(1/2)*(1+cosh(2*x))*(a*(-1+cosh(2*x))*(1+cosh(2*x)))^(1/2)*
(ln(a^(1/2)*cosh(2*x)+(a*sinh(2*x)^2)^(1/2))*a+(a*sinh(2*x)^2)^(1/2)*a^(1/
2))/a^(1/2)/sinh(2*x)/((1+cosh(2*x))^2*a)^(1/2)
```

3.136. $\int \sqrt{a\cosh^4(x)}\,dx$

3.136.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(28) = 56$.

Time = 0.25 (sec) , antiderivative size = 180, normalized size of antiderivative = 5.00

$$\int \sqrt{a\cosh^4(x)}\,dx$$
$$= \frac{\left(4\,\cosh\left(x\right)e^{\left(2\,x\right)}\sinh\left(x\right)^{3} + e^{\left(2\,x\right)}\sinh\left(x\right)^{4} + 2\,\left(3\,\cosh\left(x\right)^{2} + 2\,x\right)e^{\left(2\,x\right)}\sinh\left(x\right)^{2} + 4\,\left(\cosh\left(x\right)^{3} + 2\,x\cosh\right.\right.}{8\,\left(\cosh\left(x\right)^{2}e^{\left(4\,x\right)} + 2\,\cosh\left(x\right)^{2}e^{\left(2\,x\right)} + \left(e^{\left(4\,x\right)} + 2\,e^{\left(2\,x\right)} + 1\right)\sinh\left(x\right)^{2}\right.}$$

input
```
integrate((a*cosh(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
1/8*(4*cosh(x)*e^(2*x)*sinh(x)^3 + e^(2*x)*sinh(x)^4 + 2*(3*cosh(x)^2 + 2*
x)*e^(2*x)*sinh(x)^2 + 4*(cosh(x)^3 + 2*x*cosh(x))*e^(2*x)*sinh(x) + (cosh
(x)^4 + 4*x*cosh(x)^2 - 1)*e^(2*x))*sqrt(a*e^(8*x) + 4*a*e^(6*x) + 6*a*e^(
4*x) + 4*a*e^(2*x) + a)*e^(-2*x)/(cosh(x)^2*e^(4*x) + 2*cosh(x)^2*e^(2*x)
+ (e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + 2*(cosh(x)*e^(4*x) + 2
*cosh(x)*e^(2*x) + cosh(x))*sinh(x))
```

3.136.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{a\cosh^4(x)}\,dx = \text{Timed out}$$

input
```
integrate((a*cosh(x)**4)**(1/2),x)
```

output
```
Timed out
```

3.136.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int \sqrt{a\cosh^4(x)}\,dx = -\frac{1}{8}\,\left(\sqrt{a}e^{\left(-4\,x\right)} - \sqrt{a}\right)e^{\left(2\,x\right)} + \frac{1}{2}\,\sqrt{a}x$$

input
```
integrate((a*cosh(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
-1/8*(sqrt(a)*e^(-4*x) - sqrt(a))*e^(2*x) + 1/2*sqrt(a)*x
```

3.136.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \sqrt{a\cosh^4(x)}\,dx = -\frac{1}{8}\left((2\,e^{(2\,x)}+1)e^{(-2\,x)} - 4\,x - e^{(2\,x)}\right)\sqrt{a}$$

input
```
integrate((a*cosh(x)^4)^(1/2),x, algorithm="giac")
```

output
```
-1/8*((2*e^(2*x) + 1)*e^(-2*x) - 4*x - e^(2*x))*sqrt(a)
```

3.136.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a\cosh^4(x)}\,dx = \int \sqrt{a\cosh(x)^4}\,dx$$

input
```
int((a*cosh(x)^4)^(1/2),x)
```

output
```
int((a*cosh(x)^4)^(1/2), x)
```

3.137 $\int \frac{1}{\sqrt{a \cosh^4(x)}} \, dx$

3.137.1 Optimal result

Integrand size = 10, antiderivative size = 15

$$\int \frac{1}{\sqrt{a \cosh^4(x)}} \, dx = \frac{\cosh(x) \sinh(x)}{\sqrt{a \cosh^4(x)}}$$

output
```
cosh(x)*sinh(x)/(a*cosh(x)^4)^(1/2)
```

3.137.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a \cosh^4(x)}} \, dx = \frac{\cosh(x) \sinh(x)}{\sqrt{a \cosh^4(x)}}$$

input
```
Integrate[1/Sqrt[a*Cosh[x]^4],x]
```

output
```
(Cosh[x]*Sinh[x])/Sqrt[a*Cosh[x]^4]
```

3.137.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3686, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\cosh^4(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\sin\left(\frac{\pi}{2}+ix\right)^4}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\cosh^2(x)\int \operatorname{sech}^2(x)dx}{\sqrt{a\cosh^4(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh^2(x)\int \csc\left(ix+\frac{\pi}{2}\right)^2dx}{\sqrt{a\cosh^4(x)}}$$

$$\Big\downarrow \text{4254}$$

$$\frac{i\cosh^2(x)\int 1d(-i\tanh(x))}{\sqrt{a\cosh^4(x)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sinh(x)\cosh(x)}{\sqrt{a\cosh^4(x)}}$$

input
```
Int[1/Sqrt[a*Cosh[x]^4],x]
```

output
```
(Cosh[x]*Sinh[x])/Sqrt[a*Cosh[x]^4]
```

3.137.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.137.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(13) = 26$.

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

method	result	size
risch	$-\frac{2\,\mathrm{e}^{-2x}\left(1+\mathrm{e}^{2x}\right)}{\sqrt{a\left(1+\mathrm{e}^{2x}\right)^{4}\mathrm{e}^{-4x}}}$	29
default	$\frac{\sqrt{8}\,\sqrt{2}\,\sqrt{a(-1+\cosh(2x))(1+\cosh(2x))}\,\sqrt{a\sinh(2x)^{2}}}{4a\sinh(2x)\sqrt{(1+\cosh(2x))^{2}a}}$	56

input
```
int(1/(a*cosh(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/(a*(1+exp(2*x))^4*exp(-4*x))^(1/2)*exp(-2*x)*(1+exp(2*x))
```

3.137.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(13) = 26$.

Time = 0.25 (sec) , antiderivative size = 116, normalized size of antiderivative = 7.73

$$\int \frac{1}{\sqrt{a\cosh^4(x)}}\,dx = -\frac{2\sqrt{ae^{(8x)} + 4ae^{(6x)} + 6ae^{(4x)} + 4ae^{(2x)} + a}}{a\cosh{(x)}^2 + (ae^{(4x)} + 2ae^{(2x)} + a)\sinh{(x)}^2 + (a\cosh{(x)}^2 + a)e^{(4x)} + 2(a\cosh{(x)}^2 + a)e^{(2x)} + 2(a}$$

input
```
integrate(1/(a*cosh(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(a*e^(8*x) + 4*a*e^(6*x) + 6*a*e^(4*x) + 4*a*e^(2*x) + a)/(a*cosh(x
)^2 + (a*e^(4*x) + 2*a*e^(2*x) + a)*sinh(x)^2 + (a*cosh(x)^2 + a)*e^(4*x)
+ 2*(a*cosh(x)^2 + a)*e^(2*x) + 2*(a*cosh(x)*e^(4*x) + 2*a*cosh(x)*e^(2*x)
 + a*cosh(x))*sinh(x) + a)
```

3.137.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\cosh^4(x)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*cosh(x)**4)**(1/2),x)
```

output
```
Timed out
```

3.137.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt{a\cosh^4(x)}}\,dx = \frac{2}{\sqrt{a}e^{(-2x)} + \sqrt{a}}$$

input
```
integrate(1/(a*cosh(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
2/(sqrt(a)*e^(-2*x) + sqrt(a))
```

3.137.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{a\cosh^4(x)}}\,dx = -\frac{2}{\sqrt{a}(e^{(2x)}+1)}$$

input
```
integrate(1/(a*cosh(x)^4)^(1/2),x, algorithm="giac")
```

output
```
-2/(sqrt(a)*(e^(2*x) + 1))
```

3.137.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.60

$$\int \frac{1}{\sqrt{a\cosh^4(x)}}\,dx = -\frac{\mathrm{e}^{-x}\sqrt{a\left(\frac{\mathrm{e}^{-x}}{2}+\frac{\mathrm{e}^{x}}{2}\right)^4}}{a\left(\frac{\mathrm{e}^{-x}}{2}+\frac{\mathrm{e}^{x}}{2}\right)^3}$$

input
```
int(1/(a*cosh(x)^4)^(1/2),x)
```

output
```
-(exp(-x)*(a*(exp(-x)/2 + exp(x)/2)^4)^(1/2))/(a*(exp(-x)/2 + exp(x)/2)^3)
```

3.138 $\int \frac{1}{\left(a\cosh^4(x)\right)^{3/2}}\,dx$

3.138.1 Optimal result

Integrand size = 10, antiderivative size = 67

$$\int \frac{1}{\left(a\cosh^4(x)\right)^{3/2}}\,dx=\frac{\cosh(x)\sinh(x)}{a\sqrt{a\cosh^4(x)}}-\frac{2\sinh^2(x)\tanh(x)}{3a\sqrt{a\cosh^4(x)}}+\frac{\sinh^2(x)\tanh^3(x)}{5a\sqrt{a\cosh^4(x)}}$$

output
```
cosh(x)*sinh(x)/a/(a*cosh(x)^4)^(1/2)-2/3*sinh(x)^2*tanh(x)/a/(a*cosh(x)^4
)^(1/2)+1/5*sinh(x)^2*tanh(x)^3/a/(a*cosh(x)^4)^(1/2)
```

3.138.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.45

$$\int \frac{1}{\left(a\cosh^4(x)\right)^{3/2}}\,dx=\frac{\cosh(x)(8+6\cosh(2x)+\cosh(4x))\sinh(x)}{15\left(a\cosh^4(x)\right)^{3/2}}$$

input
```
Integrate[(a*Cosh[x]^4)^(-3/2),x]
```

output
```
(Cosh[x]*(8 + 6*Cosh[2*x] + Cosh[4*x])*Sinh[x])/(15*(a*Cosh[x]^4)^(3/2))
```

3.138.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3686, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\cosh^4(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sin\left(\frac{\pi}{2}+ix\right)^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\cosh^2(x)\int \operatorname{sech}^6(x)dx}{a\sqrt{a\cosh^4(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh^2(x)\int \csc\left(ix+\frac{\pi}{2}\right)^6dx}{a\sqrt{a\cosh^4(x)}}$$

$$\Big\downarrow \text{4254}$$

$$\frac{i\cosh^2(x)\int \left(\tanh^4(x)-2\tanh^2(x)+1\right)d(-i\tanh(x))}{a\sqrt{a\cosh^4(x)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\cosh^2(x)\left(-\frac{1}{5}i\tanh^5(x)+\frac{2}{3}i\tanh^3(x)-i\tanh(x)\right)}{a\sqrt{a\cosh^4(x)}}$$

input
```
Int[(a*Cosh[x]^4)^(-3/2),x]
```

output
```
(I*Cosh[x]^2*((-I)*Tanh[x] + ((2*I)/3)*Tanh[x]^3 - (I/5)*Tanh[x]^5))/(a*Sq
rt[a*Cosh[x]^4])
```

3.138.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.138.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

method	result	size
risch	$-\frac{16\,\mathrm{e}^{-2x}\left(10\,\mathrm{e}^{4x}+5\,\mathrm{e}^{2x}+1\right)}{15a\left(1+\mathrm{e}^{2x}\right)^{3}\sqrt{a\left(1+\mathrm{e}^{2x}\right)^{4}\mathrm{e}^{-4x}}}$	48
default	$\frac{\sqrt{8}\,\sqrt{2}\,\left(2\cosh\left(2x\right)^{2}+6\cosh\left(2x\right)+7\right)\sqrt{a\sinh\left(2x\right)^{2}}\,\sqrt{a\left(-1+\cosh\left(2x\right)\right)\left(1+\cosh\left(2x\right)\right)}}{15a^{2}\left(1+\cosh\left(2x\right)\right)^{2}\sinh\left(2x\right)\sqrt{\left(1+\cosh\left(2x\right)\right)^{2}a}}$	80

input
```
int(1/(a*cosh(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-16/15/a/(1+exp(2*x))^3*exp(-2*x)/(a*(1+exp(2*x))^4*exp(-4*x))^(1/2)*(10*e
xp(4*x)+5*exp(2*x)+1)
```

3.138.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1137 vs. 2(57) = 114.

Time = 0.27 (sec) , antiderivative size = 1137, normalized size of antiderivative = 16.97

$$\int \frac{1}{\left(a\cosh^4(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*cosh(x)^4)^(3/2),x, algorithm="fricas")
```

output
```
-16/15*(40*cosh(x)*e^(2*x)*sinh(x)^3 + 10*e^(2*x)*sinh(x)^4 + 5*(12*cosh(x
)^2 + 1)*e^(2*x)*sinh(x)^2 + 10*(4*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x) +
(10*cosh(x)^4 + 5*cosh(x)^2 + 1)*e^(2*x))*sqrt(a*e^(8*x) + 4*a*e^(6*x) + 6
*a*e^(4*x) + 4*a*e^(2*x) + a)*e^(-2*x)/(a^2*cosh(x)^10 + (a^2*e^(4*x) + 2*
a^2*e^(2*x) + a^2)*sinh(x)^10 + 5*a^2*cosh(x)^8 + 10*(a^2*cosh(x)*e^(4*x)
+ 2*a^2*cosh(x)*e^(2*x) + a^2*cosh(x))*sinh(x)^9 + 5*(9*a^2*cosh(x)^2 + a^
2 + (9*a^2*cosh(x)^2 + a^2)*e^(4*x) + 2*(9*a^2*cosh(x)^2 + a^2)*e^(2*x))*s
inh(x)^8 + 10*a^2*cosh(x)^6 + 40*(3*a^2*cosh(x)^3 + a^2*cosh(x) + (3*a^2*c
osh(x)^3 + a^2*cosh(x))*e^(4*x) + 2*(3*a^2*cosh(x)^3 + a^2*cosh(x))*e^(2*x
))*sinh(x)^7 + 10*(21*a^2*cosh(x)^4 + 14*a^2*cosh(x)^2 + a^2 + (21*a^2*cos
h(x)^4 + 14*a^2*cosh(x)^2 + a^2)*e^(4*x) + 2*(21*a^2*cosh(x)^4 + 14*a^2*co
sh(x)^2 + a^2)*e^(2*x))*sinh(x)^6 + 10*a^2*cosh(x)^4 + 4*(63*a^2*cosh(x)^5
 + 70*a^2*cosh(x)^3 + 15*a^2*cosh(x) + (63*a^2*cosh(x)^5 + 70*a^2*cosh(x)^
3 + 15*a^2*cosh(x))*e^(4*x) + 2*(63*a^2*cosh(x)^5 + 70*a^2*cosh(x)^3 + 15*
a^2*cosh(x))*e^(2*x))*sinh(x)^5 + 10*(21*a^2*cosh(x)^6 + 35*a^2*cosh(x)^4
+ 15*a^2*cosh(x)^2 + a^2 + (21*a^2*cosh(x)^6 + 35*a^2*cosh(x)^4 + 15*a^2*c
osh(x)^2 + a^2)*e^(4*x) + 2*(21*a^2*cosh(x)^6 + 35*a^2*cosh(x)^4 + 15*a^2*
cosh(x)^2 + a^2)*e^(2*x))*sinh(x)^4 + 5*a^2*cosh(x)^2 + 40*(3*a^2*cosh(x)^
7 + 7*a^2*cosh(x)^5 + 5*a^2*cosh(x)^3 + a^2*cosh(x) + (3*a^2*cosh(x)^7 + 7
*a^2*cosh(x)^5 + 5*a^2*cosh(x)^3 + a^2*cosh(x))*e^(4*x) + 2*(3*a^2*cosh...
```

3.138.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\cosh^4(x)\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*cosh(x)**4)**(3/2),x)
```

output
```
Timed out
```

3.138.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(57) = 114.

Time = 0.27 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.46

$$\int \frac{1}{\left(a\cosh^4(x)\right)^{3/2}}\,dx = \frac{16\,e^{(-2\,x)}}{3\left(5\,a^{\frac{3}{2}}e^{(-2\,x)}+10\,a^{\frac{3}{2}}e^{(-4\,x)}+10\,a^{\frac{3}{2}}e^{(-6\,x)}+5\,a^{\frac{3}{2}}e^{(-8\,x)}+a^{\frac{3}{2}}e^{(-10\,x)}+a^{\frac{3}{2}}\right)} + \frac{32\,e^{(-4\,x)}}{3\left(5\,a^{\frac{3}{2}}e^{(-2\,x)}+10\,a^{\frac{3}{2}}e^{(-4\,x)}+10\,a^{\frac{3}{2}}e^{(-6\,x)}+5\,a^{\frac{3}{2}}e^{(-8\,x)}+a^{\frac{3}{2}}e^{(-10\,x)}+a^{\frac{3}{2}}\right)} + \frac{16}{15\left(5\,a^{\frac{3}{2}}e^{(-2\,x)}+10\,a^{\frac{3}{2}}e^{(-4\,x)}+10\,a^{\frac{3}{2}}e^{(-6\,x)}+5\,a^{\frac{3}{2}}e^{(-8\,x)}+a^{\frac{3}{2}}e^{(-10\,x)}+a^{\frac{3}{2}}\right)}$$

input
```
integrate(1/(a*cosh(x)^4)^(3/2),x, algorithm="maxima")
```

output
```
16/3*e^(-2*x)/(5*a^(3/2)*e^(-2*x) + 10*a^(3/2)*e^(-4*x) + 10*a^(3/2)*e^(-6
*x) + 5*a^(3/2)*e^(-8*x) + a^(3/2)*e^(-10*x) + a^(3/2)) + 32/3*e^(-4*x)/(5
*a^(3/2)*e^(-2*x) + 10*a^(3/2)*e^(-4*x) + 10*a^(3/2)*e^(-6*x) + 5*a^(3/2)*
e^(-8*x) + a^(3/2)*e^(-10*x) + a^(3/2)) + 16/15/(5*a^(3/2)*e^(-2*x) + 10*a
^(3/2)*e^(-4*x) + 10*a^(3/2)*e^(-6*x) + 5*a^(3/2)*e^(-8*x) + a^(3/2)*e^(-1
0*x) + a^(3/2))
```

3.138.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.40

$$\int \frac{1}{\left(a\cosh^4(x)\right)^{3/2}}\,dx = -\frac{16\left(10\,e^{(4\,x)}+5\,e^{(2\,x)}+1\right)}{15\,a^{\frac{3}{2}}\left(e^{(2\,x)}+1\right)^5}$$

input
```
integrate(1/(a*cosh(x)^4)^(3/2),x, algorithm="giac")
```

output
```
-16/15*(10*e^(4*x) + 5*e^(2*x) + 1)/(a^(3/2)*(e^(2*x) + 1)^5)
```

3.138.9 Mupad [B] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

$$\int \frac{1}{\left(a\cosh^4(x)\right)^{3/2}}\,dx = -\frac{64\,\mathrm{e}^{2\,x}\,\sqrt{a\,{\left(\frac{\mathrm{e}^{-x}}{2}+\frac{\mathrm{e}^{x}}{2}\right)}^4}\,\left(5\,\mathrm{e}^{2\,x}+10\,\mathrm{e}^{4\,x}+1\right)}{15\,a^2\,{\left(\mathrm{e}^{2\,x}+1\right)}^7}$$

input
```
int(1/(a*cosh(x)^4)^(3/2),x)
```

output
```
-(64*exp(2*x)*(a*(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(5*exp(2*x) + 10*exp(4*x)
 + 1))/(15*a^2*(exp(2*x) + 1)^7)
```

3.139 $\int \frac{1}{(a\cosh^4(x))^{5/2}}\,dx$

3.139.1 Optimal result

Integrand size = 10, antiderivative size = 117

$$\int \frac{1}{(a\cosh^4(x))^{5/2}}\,dx = \frac{\cosh(x)\sinh(x)}{a^2\sqrt{a\cosh^4(x)}} - \frac{4\sinh^2(x)\tanh(x)}{3a^2\sqrt{a\cosh^4(x)}} + \frac{6\sinh^2(x)\tanh^3(x)}{5a^2\sqrt{a\cosh^4(x)}} - \frac{4\sinh^2(x)\tanh^5(x)}{7a^2\sqrt{a\cosh^4(x)}} + \frac{\sinh^2(x)\tanh^7(x)}{9a^2\sqrt{a\cosh^4(x)}}$$

output
```
cosh(x)*sinh(x)/a^2/(a*cosh(x)^4)^(1/2)-4/3*sinh(x)^2*tanh(x)/a^2/(a*cosh(
x)^4)^(1/2)+6/5*sinh(x)^2*tanh(x)^3/a^2/(a*cosh(x)^4)^(1/2)-4/7*sinh(x)^2*
tanh(x)^5/a^2/(a*cosh(x)^4)^(1/2)+1/9*sinh(x)^2*tanh(x)^7/a^2/(a*cosh(x)^4
)^(1/2)
```

3.139.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.40

$$\int \frac{1}{(a\cosh^4(x))^{5/2}}\,dx = \frac{(128+130\cosh(2x)+46\cosh(4x)+10\cosh(6x)+\cosh(8x))\text{sech}^6(x)\tanh(x)}{315a^2\sqrt{a\cosh^4(x)}}$$

input
```
Integrate[(a*Cosh[x]^4)^(-5/2),x]
```

output
```
((128 + 130*Cosh[2*x] + 46*Cosh[4*x] + 10*Cosh[6*x] + Cosh[8*x])*Sech[x]^6
*Tanh[x])/(315*a^2*Sqrt[a*Cosh[x]^4])
```

3.139.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.58, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3686, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cosh^4(x))^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\left(a\sin\left(\frac{\pi}{2}+ix\right)^4\right)^{5/2}}dx$$

$$\Big\downarrow \; 3686$$

$$\frac{\cosh^2(x)\int \operatorname{sech}^{10}(x)dx}{a^2\sqrt{a\cosh^4(x)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\cosh^2(x)\int \csc\left(ix+\frac{\pi}{2}\right)^{10}dx}{a^2\sqrt{a\cosh^4(x)}}$$

$$\Big\downarrow \; 4254$$

$$\frac{i\cosh^2(x)\int \left(\tanh^8(x)-4\tanh^6(x)+6\tanh^4(x)-4\tanh^2(x)+1\right)d(-i\tanh(x))}{a^2\sqrt{a\cosh^4(x)}}$$

$$\Big\downarrow \; 2009$$

$$\frac{i\cosh^2(x)\left(-\frac{1}{9}i\tanh^9(x)+\frac{4}{7}i\tanh^7(x)-\frac{6}{5}i\tanh^5(x)+\frac{4}{3}i\tanh^3(x)-i\tanh(x)\right)}{a^2\sqrt{a\cosh^4(x)}}$$

input
```
Int[(a*Cosh[x]^4)^(-5/2),x]
```

output
```
(I*Cosh[x]^2*((-I)*Tanh[x] + ((4*I)/3)*Tanh[x]^3 - ((6*I)/5)*Tanh[x]^5 + (
(4*I)/7)*Tanh[x]^7 - (I/9)*Tanh[x]^9))/(a^2*Sqrt[a*Cosh[x]^4])
```

3.139.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.139.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.51

method	result	size
risch	$-\frac{256\,{\mathrm e}^{-2x} \left(126\,{\mathrm e}^{8x}+84\,{\mathrm e}^{6x}+36\,{\mathrm e}^{4x}+9\,{\mathrm e}^{2x}+1\right)}{315 a^{2} \left(1+{\mathrm e}^{2x}\right)^{7} \sqrt{a \left(1+{\mathrm e}^{2x}\right)^{4} {\mathrm e}^{-4x}}}$	60
default	$\frac{4\sqrt{8}\,\sqrt{2}\,\left(8\cosh\left(2x\right)^{4}+40\cosh\left(2x\right)^{3}+84\cosh\left(2x\right)^{2}+100\cosh\left(2x\right)+83\right)\sqrt{a\sinh\left(2x\right)^{2}}\,\sqrt{a\left(-1+\cosh\left(2x\right)\right)\left(1+\cosh\left(2x\right)\right)}}{315 a^{3}\left(1+\cosh\left(2x\right)\right)^{4}\sinh\left(2x\right)\sqrt{\left(1+\cosh\left(2x\right)\right)^{2} a}}$	96

input
```
int(1/(a*cosh(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-256/315/a^2/(1+exp(2*x))^7*exp(-2*x)/(a*(1+exp(2*x))^4*exp(-4*x))^(1/2)*(
126*exp(8*x)+84*exp(6*x)+36*exp(4*x)+9*exp(2*x)+1)
```

3.139.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3065 vs. $2(99) = 198$.

Time = 0.35 (sec) , antiderivative size = 3065, normalized size of antiderivative = 26.20

$$\int \frac{1}{\left(a\cosh^4(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*cosh(x)^4)^(5/2),x, algorithm="fricas")
```

output
```
-256/315*(1008*cosh(x)*e^(2*x)*sinh(x)^7 + 126*e^(2*x)*sinh(x)^8 + 84*(42*
cosh(x)^2 + 1)*e^(2*x)*sinh(x)^6 + 504*(14*cosh(x)^3 + cosh(x))*e^(2*x)*si
nh(x)^5 + 36*(245*cosh(x)^4 + 35*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^4 + 48*(14
7*cosh(x)^5 + 35*cosh(x)^3 + 3*cosh(x))*e^(2*x)*sinh(x)^3 + 9*(392*cosh(x)
^6 + 140*cosh(x)^4 + 24*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^2 + 18*(56*cosh(x)^
7 + 28*cosh(x)^5 + 8*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x) + (126*cosh(x)^8
 + 84*cosh(x)^6 + 36*cosh(x)^4 + 9*cosh(x)^2 + 1)*e^(2*x))*sqrt(a*e^(8*x)
+ 4*a*e^(6*x) + 6*a*e^(4*x) + 4*a*e^(2*x) + a)*e^(-2*x)/(a^3*cosh(x)^18 +
9*a^3*cosh(x)^16 + (a^3*e^(4*x) + 2*a^3*e^(2*x) + a^3)*sinh(x)^18 + 18*(a^
3*cosh(x)*e^(4*x) + 2*a^3*cosh(x)*e^(2*x) + a^3*cosh(x))*sinh(x)^17 + 36*a
^3*cosh(x)^14 + 9*(17*a^3*cosh(x)^2 + a^3 + (17*a^3*cosh(x)^2 + a^3)*e^(4*
x) + 2*(17*a^3*cosh(x)^2 + a^3)*e^(2*x))*sinh(x)^16 + 48*(17*a^3*cosh(x)^3
 + 3*a^3*cosh(x) + (17*a^3*cosh(x)^3 + 3*a^3*cosh(x))*e^(4*x) + 2*(17*a^3*
cosh(x)^3 + 3*a^3*cosh(x))*e^(2*x))*sinh(x)^15 + 84*a^3*cosh(x)^12 + 36*(8
5*a^3*cosh(x)^4 + 30*a^3*cosh(x)^2 + a^3 + (85*a^3*cosh(x)^4 + 30*a^3*cosh
(x)^2 + a^3)*e^(4*x) + 2*(85*a^3*cosh(x)^4 + 30*a^3*cosh(x)^2 + a^3)*e^(2*
x))*sinh(x)^14 + 504*(17*a^3*cosh(x)^5 + 10*a^3*cosh(x)^3 + a^3*cosh(x) +
(17*a^3*cosh(x)^5 + 10*a^3*cosh(x)^3 + a^3*cosh(x))*e^(4*x) + 2*(17*a^3*co
sh(x)^5 + 10*a^3*cosh(x)^3 + a^3*cosh(x))*e^(2*x))*sinh(x)^13 + 126*a^3*co
sh(x)^10 + 84*(221*a^3*cosh(x)^6 + 195*a^3*cosh(x)^4 + 39*a^3*cosh(x)^2...
```

3.139.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\cosh^4(x)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*cosh(x)**4)**(5/2),x)
```

output
```
Timed out
```

3.139.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. 2(99) = 198.

Time = 0.27 (sec) , antiderivative size = 457, normalized size of antiderivative = 3.91

$$\int \frac{1}{\left(a\cosh^4(x)\right)^{5/2}}\,dx = \frac{256\,e^{(-2\,x)}}{35\left(9\,a^{\frac{5}{2}}e^{(-2\,x)} + 36\,a^{\frac{5}{2}}e^{(-4\,x)} + 84\,a^{\frac{5}{2}}e^{(-6\,x)} + 126\,a^{\frac{5}{2}}e^{(-8\,x)} + 126\,a^{\frac{5}{2}}e^{(-10\,x)} + 84\,a^{\frac{5}{2}}e\right.}$$
$$+ \frac{1024\,e^{(-4\,x)}}{35\left(9\,a^{\frac{5}{2}}e^{(-2\,x)} + 36\,a^{\frac{5}{2}}e^{(-4\,x)} + 84\,a^{\frac{5}{2}}e^{(-6\,x)} + 126\,a^{\frac{5}{2}}e^{(-8\,x)} + 126\,a^{\frac{5}{2}}e^{(-10\,x)} + 84\,a^{\frac{5}{2}}e^{(-12\,x)} + 36\,a^{\frac{5}{2}}e^{(-14\,x)} +\right.}$$
$$+ \frac{1024\,e^{(-6\,x)}}{15\left(9\,a^{\frac{5}{2}}e^{(-2\,x)} + 36\,a^{\frac{5}{2}}e^{(-4\,x)} + 84\,a^{\frac{5}{2}}e^{(-6\,x)} + 126\,a^{\frac{5}{2}}e^{(-8\,x)} + 126\,a^{\frac{5}{2}}e^{(-10\,x)} + 84\,a^{\frac{5}{2}}e^{(-12\,x)} + 36\,a^{\frac{5}{2}}e^{(-14\,x)} +\right.}$$
$$+ \frac{512\,e^{(-8\,x)}}{5\left(9\,a^{\frac{5}{2}}e^{(-2\,x)} + 36\,a^{\frac{5}{2}}e^{(-4\,x)} + 84\,a^{\frac{5}{2}}e^{(-6\,x)} + 126\,a^{\frac{5}{2}}e^{(-8\,x)} + 126\,a^{\frac{5}{2}}e^{(-10\,x)} + 84\,a^{\frac{5}{2}}e^{(-12\,x)} + 36\,a^{\frac{5}{2}}e^{(-14\,x)} +\right.}$$
$$+ \frac{256}{315\left(9\,a^{\frac{5}{2}}e^{(-2\,x)} + 36\,a^{\frac{5}{2}}e^{(-4\,x)} + 84\,a^{\frac{5}{2}}e^{(-6\,x)} + 126\,a^{\frac{5}{2}}e^{(-8\,x)} + 126\,a^{\frac{5}{2}}e^{(-10\,x)} + 84\,a^{\frac{5}{2}}e^{(-12\,x)} + 36\,a^{\frac{5}{2}}e^{(-14\,x)}\right.}$$

input
```
integrate(1/(a*cosh(x)^4)^(5/2),x, algorithm="maxima")
```

output
```
256/35*e^(-2*x)/(9*a^(5/2)*e^(-2*x) + 36*a^(5/2)*e^(-4*x) + 84*a^(5/2)*e^(
-6*x) + 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) + 84*a^(5/2)*e^(-12*x
) + 36*a^(5/2)*e^(-14*x) + 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x) + a^(5/
2)) + 1024/35*e^(-4*x)/(9*a^(5/2)*e^(-2*x) + 36*a^(5/2)*e^(-4*x) + 84*a^(5
/2)*e^(-6*x) + 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) + 84*a^(5/2)*e
^(-12*x) + 36*a^(5/2)*e^(-14*x) + 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x)
+ a^(5/2)) + 1024/15*e^(-6*x)/(9*a^(5/2)*e^(-2*x) + 36*a^(5/2)*e^(-4*x) +
84*a^(5/2)*e^(-6*x) + 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) + 84*a^
(5/2)*e^(-12*x) + 36*a^(5/2)*e^(-14*x) + 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(
-18*x) + a^(5/2)) + 512/5*e^(-8*x)/(9*a^(5/2)*e^(-2*x) + 36*a^(5/2)*e^(-4*
x) + 84*a^(5/2)*e^(-6*x) + 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) +
84*a^(5/2)*e^(-12*x) + 36*a^(5/2)*e^(-14*x) + 9*a^(5/2)*e^(-16*x) + a^(5/2
)*e^(-18*x) + a^(5/2)) + 256/315/(9*a^(5/2)*e^(-2*x) + 36*a^(5/2)*e^(-4*x)
 + 84*a^(5/2)*e^(-6*x) + 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) + 84
*a^(5/2)*e^(-12*x) + 36*a^(5/2)*e^(-14*x) + 9*a^(5/2)*e^(-16*x) + a^(5/2)*
e^(-18*x) + a^(5/2))
```

3.139.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.33

$$\int \frac{1}{\left(a \cosh^4(x)\right)^{5/2}}\,dx = -\frac{256 \left(126 e^{(8 x)} + 84 e^{(6 x)} + 36 e^{(4 x)} + 9 e^{(2 x)} + 1\right)}{315 a^{\frac{5}{2}} \left(e^{(2 x)} + 1\right)^9}$$

input
```
integrate(1/(a*cosh(x)^4)^(5/2),x, algorithm="giac")
```

output
```
-256/315*(126*e^(8*x) + 84*e^(6*x) + 36*e^(4*x) + 9*e^(2*x) + 1)/(a^(5/2)*
(e^(2*x) + 1)^9)
```

3.139.9 Mupad [B] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.19

$$\int \frac{1}{\left(a \cosh^4(x)\right)^{5/2}}\,dx = \frac{4096\, e^{4x} \sqrt{a \left(\frac{e^{-x}}{2} + \frac{e^x}{2}\right)^4}}{3 a^3 \left(e^{2x} + 1\right)^6 \left(e^{2x} + 2 e^{4x} + e^{6x}\right)} - \frac{2048\, e^{4x} \sqrt{a \left(\frac{e^{-x}}{2} + \frac{e^x}{2}\right)^4}}{5 a^3 \left(e^{2x} + 1\right)^5 \left(e^{2x} + 2 e^{4x} + e^{6x}\right)} - \frac{12288\, e^{4x} \sqrt{a \left(\frac{e^{-x}}{2} + \frac{e^x}{2}\right)^4}}{7 a^3 \left(e^{2x} + 1\right)^7 \left(e^{2x} + 2 e^{4x} + e^{6x}\right)} + \frac{1024\, e^{4x} \sqrt{a \left(\frac{e^{-x}}{2} + \frac{e^x}{2}\right)^4}}{a^3 \left(e^{2x} + 1\right)^8 \left(e^{2x} + 2 e^{4x} + e^{6x}\right)} - \frac{2048\, e^{4x} \sqrt{a \left(\frac{e^{-x}}{2} + \frac{e^x}{2}\right)^4}}{9 a^3 \left(e^{2x} + 1\right)^9 \left(e^{2x} + 2 e^{4x} + e^{6x}\right)}$$

input
```
int(1/(a*cosh(x)^4)^(5/2),x)
```

output
```
(4096*exp(4*x)*(a*(exp(-x)/2 + exp(x)/2)^4)^(1/2))/(3*a^3*(exp(2*x) + 1)^6
*(exp(2*x) + 2*exp(4*x) + exp(6*x))) - (2048*exp(4*x)*(a*(exp(-x)/2 + exp(
x)/2)^4)^(1/2))/(5*a^3*(exp(2*x) + 1)^5*(exp(2*x) + 2*exp(4*x) + exp(6*x))
) - (12288*exp(4*x)*(a*(exp(-x)/2 + exp(x)/2)^4)^(1/2))/(7*a^3*(exp(2*x) +
 1)^7*(exp(2*x) + 2*exp(4*x) + exp(6*x))) + (1024*exp(4*x)*(a*(exp(-x)/2 +
 exp(x)/2)^4)^(1/2))/(a^3*(exp(2*x) + 1)^8*(exp(2*x) + 2*exp(4*x) + exp(6*
x))) - (2048*exp(4*x)*(a*(exp(-x)/2 + exp(x)/2)^4)^(1/2))/(9*a^3*(exp(2*x)
 + 1)^9*(exp(2*x) + 2*exp(4*x) + exp(6*x)))
```

3.140 $\int \frac{\sinh(x)}{(1+\cosh(x))^2} dx$

3.140.1 Optimal result

Integrand size = 9, antiderivative size = 8

$$\int \frac{\sinh(x)}{(1+\cosh(x))^2} dx = -\frac{1}{1+\cosh(x)}$$

output
```
-1/(1+cosh(x))
```

3.140.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.50

$$\int \frac{\sinh(x)}{(1+\cosh(x))^2} dx = -\frac{1}{2}\text{sech}^2\left(\frac{x}{2}\right)$$

input
```
Integrate[Sinh[x]/(1 + Cosh[x])^2,x]
```

output
```
-1/2*Sech[x/2]^2
```

3.140.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3042, 26, 3146, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)}{(\cosh(x)+1)^2}dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{i\cos\left(-\frac{\pi}{2}+ix\right)}{\left(1-\sin\left(-\frac{\pi}{2}+ix\right)\right)^2}dx$$

$$\Big\downarrow \; 26$$

$$-i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)}{\left(1-\sin\left(ix-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \; 3146$$

$$\int \frac{1}{(\cosh(x)+1)^2}d\cosh(x)$$

$$\Big\downarrow \; 17$$

$$-\frac{1}{\cosh(x)+1}$$

input
```
Int[Sinh[x]/(1 + Cosh[x])^2,x]
```

output
```
-(1 + Cosh[x])^(-1)
```

3.140.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

3.140.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
derivativedivides	$-\frac{1}{\cosh(x)+1}$	9
default	$-\frac{1}{\cosh(x)+1}$	9
risch	$-\frac{2\,\mathrm{e}^{x}}{(\mathrm{e}^{x}+1)^{2}}$	11

input
```
int(sinh(x)/(cosh(x)+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(cosh(x)+1)
```

3.140.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(8) = 16.

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 3.88

$$\int \frac{\sinh(x)}{(1+\cosh(x))^2}\,dx = -\frac{2\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}{\cosh\left(x\right)^2+2\left(\cosh\left(x\right)+1\right)\sinh\left(x\right)+\sinh\left(x\right)^2+2\,\cosh\left(x\right)+1}$$

input
```
integrate(sinh(x)/(1+cosh(x))^2,x, algorithm="fricas")
```

output
```
-2*(cosh(x) + sinh(x))/(cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 +
2*cosh(x) + 1)
```

3.140.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{\sinh(x)}{(1+\cosh(x))^2}\,dx = -\frac{1}{\cosh{(x)}+1}$$

input
```
integrate(sinh(x)/(1+cosh(x))**2,x)
```

output
```
-1/(cosh(x) + 1)
```

3.140.7 Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(x)}{(1+\cosh(x))^2}\,dx = -\frac{1}{\cosh{(x)}+1}$$

input
```
integrate(sinh(x)/(1+cosh(x))^2,x, algorithm="maxima")
```

output
```
-1/(cosh(x) + 1)
```

3.140.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{\sinh(x)}{(1+\cosh(x))^2}\,dx = -\frac{2\,e^x}{(e^x+1)^2}$$

input
```
integrate(sinh(x)/(1+cosh(x))^2,x, algorithm="giac")
```

output
```
-2*e^x/(e^x + 1)^2
```

3.140.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(x)}{(1+\cosh(x))^2}\,dx = -\frac{1}{\cosh(x)+1}$$

input
```
int(sinh(x)/(cosh(x) + 1)^2,x)
```

output
```
-1/(cosh(x) + 1)
```

3.141 $\int \frac{\sinh(x)}{(1-\cosh(x))^2} dx$

3.141.1 Optimal result

Integrand size = 11, antiderivative size = 8

$$\int \frac{\sinh(x)}{(1-\cosh(x))^2} dx = \frac{1}{1-\cosh(x)}$$

output
```
1/(1-cosh(x))
```

3.141.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.50

$$\int \frac{\sinh(x)}{(1-\cosh(x))^2} dx = -\frac{1}{2}\operatorname{csch}^2\left(\frac{x}{2}\right)$$

input
```
Integrate[Sinh[x]/(1 - Cosh[x])^2,x]
```

output
```
-1/2*Csch[x/2]^2
```

3.141.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 26, 3146, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)}{(1-\cosh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\cos\left(-\frac{\pi}{2}+ix\right)}{\left(1+\sin\left(-\frac{\pi}{2}+ix\right)\right)^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)}{\left(\sin\left(ix-\frac{\pi}{2}\right)+1\right)^2}dx$$

$$\Big\downarrow \text{3146}$$

$$-\int \frac{1}{(1-\cosh(x))^2}d(-\cosh(x))$$

$$\Big\downarrow \text{17}$$

$$\frac{1}{1-\cosh(x)}$$

input
```
Int[Sinh[x]/(1 - Cosh[x])^2,x]
```

output
```
(1 - Cosh[x])^(-1)
```

3.141.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

3.141.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
derivativedivides	$\frac{1}{1-\cosh(x)}$	9
default	$\frac{1}{1-\cosh(x)}$	9
risch	$-\frac{2\,e^{x}}{\left(e^{x}-1\right)^{2}}$	11

input
```
int(sinh(x)/(1-cosh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/(1-cosh(x))
```

3.141.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(8) = 16.

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 3.88

$$\int \frac{\sinh(x)}{(1-\cosh(x))^2}\,dx = -\frac{2\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}{\cosh\left(x\right)^{2}+2\left(\cosh\left(x\right)-1\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}-2\,\cosh\left(x\right)+1}$$

input
```
integrate(sinh(x)/(1-cosh(x))^2,x, algorithm="fricas")
```

output
```
-2*(cosh(x) + sinh(x))/(cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + sinh(x)^2 -
2*cosh(x) + 1)
```

3.141.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{\sinh(x)}{(1-\cosh(x))^2}\,dx = -\frac{1}{\cosh(x)-1}$$

input
```
integrate(sinh(x)/(1-cosh(x))**2,x)
```

output
```
-1/(cosh(x) - 1)
```

3.141.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(x)}{(1-\cosh(x))^2}\,dx = -\frac{1}{\cosh(x)-1}$$

input
```
integrate(sinh(x)/(1-cosh(x))^2,x, algorithm="maxima")
```

output
```
-1/(cosh(x) - 1)
```

3.141.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{\sinh(x)}{(1-\cosh(x))^2}\,dx = -\frac{2\,e^x}{(e^x-1)^2}$$

input
```
integrate(sinh(x)/(1-cosh(x))^2,x, algorithm="giac")
```

output
```
-2*e^x/(e^x - 1)^2
```

3.141.9 Mupad [B] (verification not implemented)

Time = 1.63 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(x)}{(1-\cosh(x))^2}\,dx = -\frac{1}{\cosh(x)-1}$$

input
```
int(sinh(x)/(cosh(x) - 1)^2,x)
```

output
```
-1/(cosh(x) - 1)
```

3.142 $\int \frac{\sinh^2(x)}{(1+\cosh(x))^2}\,dx$

3.142.1 Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{\sinh^2(x)}{(1+\cosh(x))^2}\,dx = x-\frac{2\sinh(x)}{1+\cosh(x)}$$

output
```
x-2*sinh(x)/(1+cosh(x))
```

3.142.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{\sinh^2(x)}{(1+\cosh(x))^2}\,dx = 2\text{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)$$

input
```
Integrate[Sinh[x]^2/(1 + Cosh[x])^2,x]
```

output
```
2*ArcTanh[Tanh[x/2]] - 2*Tanh[x/2]
```

3.142.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 25, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)}{(\cosh(x)+1)^2}dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{\cos\left(-\frac{\pi}{2}+ix\right)^2}{\left(1-\sin\left(-\frac{\pi}{2}+ix\right)\right)^2}dx$$

$$\Big\downarrow \ 25$$

$$-\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^2}{\left(1-\sin\left(ix-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \ 3159$$

$$\int 1dx-\frac{2\sinh(x)}{\cosh(x)+1}$$

$$\Big\downarrow \ 24$$

$$x-\frac{2\sinh(x)}{\cosh(x)+1}$$

input
```
Int[Sinh[x]^2/(1 + Cosh[x])^2,x]
```

output
```
x - (2*Sinh[x])/(1 + Cosh[x])
```

3.142.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

3.142.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
risch	$x+\frac{4}{{\mathrm e}^{x}+1}$	11
default	$-2\tanh\left(\frac{x}{2}\right)-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	24

input
```
int(sinh(x)^2/(cosh(x)+1)^2,x,method=_RETURNVERBOSE)
```

output
```
x+4/(exp(x)+1)
```

3.142.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\sinh^2(x)}{(1+\cosh(x))^2}\,dx=\frac{x\cosh\left(x\right)+x\sinh\left(x\right)+x+4}{\cosh\left(x\right)+\sinh\left(x\right)+1}$$

input
```
integrate(sinh(x)^2/(1+cosh(x))^2,x, algorithm="fricas")
```

output
```
(x*cosh(x) + x*sinh(x) + x + 4)/(cosh(x) + sinh(x) + 1)
```

3.142.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{\sinh^2(x)}{(1+\cosh(x))^2} \, dx = x - 2 \tanh\left(\frac{x}{2}\right)$$

input
```
integrate(sinh(x)**2/(1+cosh(x))**2,x)
```

output
```
x - 2*tanh(x/2)
```

3.142.7 Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^2(x)}{(1+\cosh(x))^2} \, dx = x - \frac{4}{e^{(-x)}+1}$$

input
```
integrate(sinh(x)^2/(1+cosh(x))^2,x, algorithm="maxima")
```

output
```
x - 4/(e^(-x) + 1)
```

3.142.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\sinh^2(x)}{(1+\cosh(x))^2} \, dx = x + \frac{4}{e^x+1}$$

input
```
integrate(sinh(x)^2/(1+cosh(x))^2,x, algorithm="giac")
```

output
```
x + 4/(e^x + 1)
```

3.142.9 Mupad [B] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\sinh^2(x)}{(1+\cosh(x))^2}\,dx = x + \frac{4}{e^x+1}$$

input
```
int(sinh(x)^2/(cosh(x) + 1)^2,x)
```

output
```
x + 4/(exp(x) + 1)
```

3.143 $\int \frac{\sinh^2(x)}{(1-\cosh(x))^2} dx$

3.143.1 Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^2} dx = x + \frac{2\sinh(x)}{1-\cosh(x)}$$

output
```
x+2*sinh(x)/(1-cosh(x))
```

3.143.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^2} dx = -2\coth\left(\frac{x}{2}\right) \text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\tanh^2\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Sinh[x]^2/(1 - Cosh[x])^2,x]
```

output
```
-2*Coth[x/2]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[x/2]^2]
```

3.143.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 25, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\cos\left(-\frac{\pi}{2}+ix\right)^2}{\left(1+\sin\left(-\frac{\pi}{2}+ix\right)\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^2}{\left(\sin\left(ix-\frac{\pi}{2}\right)+1\right)^2}dx$$

$$\Big\downarrow \text{3159}$$

$$\int 1dx+\frac{2\sinh(x)}{1-\cosh(x)}$$

$$\Big\downarrow \text{24}$$

$$x+\frac{2\sinh(x)}{1-\cosh(x)}$$

input
```
Int[Sinh[x]^2/(1 - Cosh[x])^2,x]
```

output
```
x + (2*Sinh[x])/(1 - Cosh[x])
```

3.143.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

3.143.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
risch	$x-\frac{4}{{\mathrm e}^{x}-1}$	11
default	$-\frac{2}{\tanh\left(\frac{x}{2}\right)}-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	26

input
```
int(sinh(x)^2/(1-cosh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
x-4/(exp(x)-1)
```

3.143.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^2}\,dx = \frac{x\cosh\left(x\right)+x\sinh\left(x\right)-x-4}{\cosh\left(x\right)+\sinh\left(x\right)-1}$$

input
```
integrate(sinh(x)^2/(1-cosh(x))^2,x, algorithm="fricas")
```

output
```
(x*cosh(x) + x*sinh(x) - x - 4)/(cosh(x) + sinh(x) - 1)
```

3.143.6 Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.50

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^2}\,dx = x - \frac{2}{\tanh\left(\frac{x}{2}\right)}$$

input
```
integrate(sinh(x)**2/(1-cosh(x))**2,x)
```

output
```
x - 2/tanh(x/2)
```

3.143.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^2}\,dx = x + \frac{4}{e^{(-x)} - 1}$$

input
```
integrate(sinh(x)^2/(1-cosh(x))^2,x, algorithm="maxima")
```

output
```
x + 4/(e^(-x) - 1)
```

3.143.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^2}\,dx = x - \frac{4}{e^x - 1}$$

input
```
integrate(sinh(x)^2/(1-cosh(x))^2,x, algorithm="giac")
```

output
```
x - 4/(e^x - 1)
```

3.143.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^2}\,dx = x - \frac{4}{e^x - 1}$$

input
```
int(sinh(x)^2/(cosh(x) - 1)^2,x)
```

output
```
x - 4/(exp(x) - 1)
```

3.144 $\int \frac{\sinh^3(x)}{(1+\cosh(x))^2} \, dx$

3.144.1 Optimal result

Integrand size = 11, antiderivative size = 10

$$\int \frac{\sinh^3(x)}{(1+\cosh(x))^2} \, dx = \cosh(x) - 2\log(1+\cosh(x))$$

output
```
cosh(x)-2*ln(1+cosh(x))
```

3.144.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.30

$$\int \frac{\sinh^3(x)}{(1+\cosh(x))^2} \, dx = -1 + \cosh(x) - 4\log\left(\cosh\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Sinh[x]^3/(1 + Cosh[x])^2,x]
```

output
```
-1 + Cosh[x] - 4*Log[Cosh[x/2]]
```

3.144.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 26, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(x)}{(\cosh(x)+1)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\cos\left(-\frac{\pi}{2}+ix\right)^3}{\left(1-\sin\left(-\frac{\pi}{2}+ix\right)\right)^2}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^3}{\left(1-\sin\left(ix-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3146}$$

$$-\int \frac{1-\cosh(x)}{\cosh(x)+1}d\cosh(x)$$

$$\Big\downarrow \text{49}$$

$$-\int \left(\frac{2}{\cosh(x)+1}-1\right)d\cosh(x)$$

$$\Big\downarrow \text{2009}$$

$$\cosh(x)-2\log(\cosh(x)+1)$$

input
```
Int[Sinh[x]^3/(1 + Cosh[x])^2,x]
```

output
```
Cosh[x] - 2*Log[1 + Cosh[x]]
```

3.144.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

3.144.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\cosh(x) - 2\ln(\cosh(x) + 1)$	11
default	$\cosh(x) - 2\ln(\cosh(x) + 1)$	11
risch	$2x + \frac{e^{x}}{2} + \frac{e^{-x}}{2} - 4\ln(e^{x} + 1)$	22

input
```
int(sinh(x)^3/(cosh(x)+1)^2,x,method=_RETURNVERBOSE)
```

output
```
cosh(x)-2*ln(cosh(x)+1)
```

3.144.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(10) = 20$.

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 4.80

$$\int \frac{\sinh^3(x)}{(1+\cosh(x))^2}\,dx = \frac{4\,x\cosh\left(x\right)+\cosh\left(x\right)^2-8\left(\cosh\left(x\right)+\sinh\left(x\right)\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)+2\left(2\,x+\cosh\left(x\right)\right)\sinh\left(x\right)}{2\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

input
```
integrate(sinh(x)^3/(1+cosh(x))^2,x, algorithm="fricas")
```

output
```
1/2*(4*x*cosh(x) + cosh(x)^2 - 8*(cosh(x) + sinh(x))*log(cosh(x) + sinh(x)
 + 1) + 2*(2*x + cosh(x))*sinh(x) + sinh(x)^2 + 1)/(cosh(x) + sinh(x))
```

3.144.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(10) = 20$.

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 5.80

$$\int \frac{\sinh^3(x)}{(1+\cosh(x))^2}\,dx = -\frac{2\log\left(\cosh\left(x\right)+1\right)\cosh\left(x\right)}{\cosh\left(x\right)+1} - \frac{2\log\left(\cosh\left(x\right)+1\right)}{\cosh\left(x\right)+1} - \frac{\sinh^2\left(x\right)}{\cosh\left(x\right)+1} + \frac{2\cosh^2\left(x\right)}{\cosh\left(x\right)+1} - \frac{2}{\cosh\left(x\right)+1}$$

input
```
integrate(sinh(x)**3/(1+cosh(x))**2,x)
```

output
```
-2*log(cosh(x) + 1)*cosh(x)/(cosh(x) + 1) - 2*log(cosh(x) + 1)/(cosh(x) +
1) - sinh(x)**2/(cosh(x) + 1) + 2*cosh(x)**2/(cosh(x) + 1) - 2/(cosh(x) +
1)
```

3.144.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(10) = 20$.

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.30

$$\int \frac{\sinh^3(x)}{(1+\cosh(x))^2}\,dx = -2\,x + \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^x - 4\,\log\left(e^{(-x)} + 1\right)$$

input
```
integrate(sinh(x)^3/(1+cosh(x))^2,x, algorithm="maxima")
```

output
```
-2*x + 1/2*e^(-x) + 1/2*e^x - 4*log(e^(-x) + 1)
```

3.144.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(10) = 20$.

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.10

$$\int \frac{\sinh^3(x)}{(1+\cosh(x))^2}\,dx = 2\,x + \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^x - 4\,\log\left(e^x + 1\right)$$

input
```
integrate(sinh(x)^3/(1+cosh(x))^2,x, algorithm="giac")
```

output
```
2*x + 1/2*e^(-x) + 1/2*e^x - 4*log(e^x + 1)
```

3.144.9 Mupad [B] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^3(x)}{(1+\cosh(x))^2}\,dx = \cosh(x) - 2\,\ln\left(\cosh(x) + 1\right)$$

input
```
int(sinh(x)^3/(cosh(x) + 1)^2,x)
```

output
```
cosh(x) - 2*log(cosh(x) + 1)
```

3.145 $\int \frac{\sinh^3(x)}{(1-\cosh(x))^2} dx$

3.145.1 Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{\sinh^3(x)}{(1-\cosh(x))^2} dx = \cosh(x) + 2\log(1-\cosh(x))$$

output
```
cosh(x)+2*ln(1-cosh(x))
```

3.145.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{\sinh^3(x)}{(1-\cosh(x))^2} dx = -1 + \cosh(x) + 4\log\left(\sinh\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Sinh[x]^3/(1 - Cosh[x])^2,x]
```

output
```
-1 + Cosh[x] + 4*Log[Sinh[x/2]]
```

3.145.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 26, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(x)}{(1-\cosh(x))^2} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i \cos\left(-\frac{\pi}{2}+ix\right)^3}{\left(1+\sin\left(-\frac{\pi}{2}+ix\right)\right)^2}dx$$

$$\Big\downarrow \text{26}$$

$$i \int \frac{\cos\left(ix-\frac{\pi}{2}\right)^3}{\left(\sin\left(ix-\frac{\pi}{2}\right)+1\right)^2}dx$$

$$\Big\downarrow \text{3146}$$

$$\int \frac{\cosh(x)+1}{1-\cosh(x)}d(-\cosh(x))$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{2}{1-\cosh(x)}-1\right)d(-\cosh(x))$$

$$\Big\downarrow \text{2009}$$

$$\cosh(x)+2\log(1-\cosh(x))$$

input
```
Int[Sinh[x]^3/(1 - Cosh[x])^2,x]
```

output
```
Cosh[x] + 2*Log[1 - Cosh[x]]
```

3.145.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

3.145.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\cosh(x) + 2\ln(\cosh(x) - 1)$	11
default	$\cosh(x) + 2\ln(\cosh(x) - 1)$	11
risch	$-2x + \frac{e^{x}}{2} + \frac{e^{-x}}{2} + 4\ln(e^{x} - 1)$	22

input
```
int(sinh(x)^3/(1-cosh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
cosh(x)+2*ln(cosh(x)-1)
```

3.145.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(12) = 24$.

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 4.50

$$\int \frac{\sinh^3(x)}{(1-\cosh(x))^2}\,dx = -\frac{4\,x\cosh\left(x\right)-\cosh\left(x\right)^2-8\left(\cosh\left(x\right)+\sinh\left(x\right)\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)+2\left(2\,x-\cosh\left(x\right)\right)\sinh}{2\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

input
```
integrate(sinh(x)^3/(1-cosh(x))^2,x, algorithm="fricas")
```

output
```
-1/2*(4*x*cosh(x) - cosh(x)^2 - 8*(cosh(x) + sinh(x))*log(cosh(x) + sinh(x
) - 1) + 2*(2*x - cosh(x))*sinh(x) - sinh(x)^2 - 1)/(cosh(x) + sinh(x))
```

3.145.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(10) = 20$.

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 4.83

$$\int \frac{\sinh^3(x)}{(1-\cosh(x))^2}\,dx = \frac{2\log\left(\cosh\left(x\right)-1\right)\cosh\left(x\right)}{\cosh\left(x\right)-1}-\frac{2\log\left(\cosh\left(x\right)-1\right)}{\cosh\left(x\right)-1} - \frac{\sinh^2\left(x\right)}{\cosh\left(x\right)-1}+\frac{2\cosh^2\left(x\right)}{\cosh\left(x\right)-1}-\frac{2}{\cosh\left(x\right)-1}$$

input
```
integrate(sinh(x)**3/(1-cosh(x))**2,x)
```

output
```
2*log(cosh(x) - 1)*cosh(x)/(cosh(x) - 1) - 2*log(cosh(x) - 1)/(cosh(x) - 1
) - sinh(x)**2/(cosh(x) - 1) + 2*cosh(x)**2/(cosh(x) - 1) - 2/(cosh(x) - 1
)
```

3.145.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.92

$$\int \frac{\sinh^3(x)}{(1-\cosh(x))^2}\,dx = 2\,x + \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x} + 4\,\log\left(e^{(-x)} - 1\right)$$

input
```
integrate(sinh(x)^3/(1-cosh(x))^2,x, algorithm="maxima")
```

output
```
2*x + 1/2*e^(-x) + 1/2*e^x + 4*log(e^(-x) - 1)
```

3.145.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\sinh^3(x)}{(1-\cosh(x))^2}\,dx = -2\,x + \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x} + 4\,\log\left(|e^{x} - 1|\right)$$

input
```
integrate(sinh(x)^3/(1-cosh(x))^2,x, algorithm="giac")
```

output
```
-2*x + 1/2*e^(-x) + 1/2*e^x + 4*log(abs(e^x - 1))
```

3.145.9 Mupad [B] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\sinh^3(x)}{(1-\cosh(x))^2}\,dx = 2\,\ln\left(\cosh(x) - 1\right) + \cosh(x)$$

input
```
int(sinh(x)^3/(cosh(x) - 1)^2,x)
```

output
```
2*log(cosh(x) - 1) + cosh(x)
```

3.146 $\int \frac{\sinh(x)}{(1+\cosh(x))^3} dx$

3.146.1 Optimal result

Integrand size = 9, antiderivative size = 10

$$\int \frac{\sinh(x)}{(1+\cosh(x))^3} dx = -\frac{1}{2(1+\cosh(x))^2}$$

output
```
-1/2/(1+cosh(x))^2
```

3.146.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\sinh(x)}{(1+\cosh(x))^3} dx = -\frac{1}{8}\operatorname{sech}^4\left(\frac{x}{2}\right)$$

input
```
Integrate[Sinh[x]/(1 + Cosh[x])^3,x]
```

output
```
-1/8*Sech[x/2]^4
```

3.146.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3042, 26, 3146, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)}{(\cosh(x)+1)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{i\cos\left(-\frac{\pi}{2}+ix\right)}{\left(1-\sin\left(-\frac{\pi}{2}+ix\right)\right)^3}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)}{\left(1-\sin\left(ix-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{ 3146}$$

$$\int \frac{1}{(\cosh(x)+1)^3}d\cosh(x)$$

$$\Big\downarrow \text{ 17}$$

$$-\frac{1}{2(\cosh(x)+1)^2}$$

input
```
Int[Sinh[x]/(1 + Cosh[x])^3,x]
```

output
```
-1/2*1/(1 + Cosh[x])^2
```

3.146.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

3.146.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$-\frac{1}{2(\cosh(x)+1)^2}$	9
default	$-\frac{1}{2(\cosh(x)+1)^2}$	9
risch	$-\frac{2\,\mathrm{e}^{2x}}{(\mathrm{e}^{x}+1)^4}$	13

input
```
int(sinh(x)/(cosh(x)+1)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/(cosh(x)+1)^2
```

3.146.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(8) = 16.

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 5.50

$$\int \frac{\sinh(x)}{(1+\cosh(x))^3}\,dx = -\frac{2\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}{\cosh\left(x\right)^3+\left(3\,\cosh\left(x\right)+4\right)\sinh\left(x\right)^2+\sinh\left(x\right)^3+4\,\cosh\left(x\right)^2+\left(3\,\cosh\left(x\right)^2+8\,\cosh\left(x\right)+5\right)\sinh\left(x\right.}$$

input
```
integrate(sinh(x)/(1+cosh(x))^3,x, algorithm="fricas")
```

output
```
-2*(cosh(x) + sinh(x))/(cosh(x)^3 + (3*cosh(x) + 4)*sinh(x)^2 + sinh(x)^3
+ 4*cosh(x)^2 + (3*cosh(x)^2 + 8*cosh(x) + 5)*sinh(x) + 7*cosh(x) + 4)
```

3.146.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

$$\int \frac{\sinh(x)}{(1+\cosh(x))^3}\,dx = -\frac{1}{2\cosh^2(x)+4\cosh(x)+2}$$

input
```
integrate(sinh(x)/(1+cosh(x))**3,x)
```

output
```
-1/(2*cosh(x)**2 + 4*cosh(x) + 2)
```

3.146.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\sinh(x)}{(1+\cosh(x))^3}\,dx = -\frac{1}{2(\cosh(x)+1)^2}$$

input
```
integrate(sinh(x)/(1+cosh(x))^3,x, algorithm="maxima")
```

output
```
-1/2/(cosh(x) + 1)^2
```

3.146.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\sinh(x)}{(1+\cosh(x))^3}\,dx = -\frac{2e^{(2x)}}{(e^x+1)^4}$$

input
```
integrate(sinh(x)/(1+cosh(x))^3,x, algorithm="giac")
```

output
```
-2*e^(2*x)/(e^x + 1)^4
```

3.146.9 Mupad [B] (verification not implemented)

Time = 1.78 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\sinh(x)}{(1+\cosh(x))^3}\,dx = -\frac{1}{2\left(\cosh\left(x\right)+1\right)^2}$$

input
```
int(sinh(x)/(cosh(x) + 1)^3,x)
```

output
```
-1/(2*(cosh(x) + 1)^2)
```

3.147 $\int \frac{\sinh(x)}{(1-\cosh(x))^3} dx$

3.147.1 Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{\sinh(x)}{(1-\cosh(x))^3} dx = \frac{1}{2(1-\cosh(x))^2}$$

output
```
1/2/(1-cosh(x))^2
```

3.147.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(x)}{(1-\cosh(x))^3} dx = \frac{1}{8}\text{csch}^4\left(\frac{x}{2}\right)$$

input
```
Integrate[Sinh[x]/(1 - Cosh[x])^3,x]
```

output
```
Csch[x/2]^4/8
```

3.147.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 26, 3146, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)}{(1-\cosh(x))^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{i\cos\left(-\frac{\pi}{2}+ix\right)}{\left(1+\sin\left(-\frac{\pi}{2}+ix\right)\right)^3}dx$$
$$\Big\downarrow \text{26}$$
$$-i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)}{\left(\sin\left(ix-\frac{\pi}{2}\right)+1\right)^3}dx$$
$$\Big\downarrow \text{3146}$$
$$-\int \frac{1}{(1-\cosh(x))^3}d(-\cosh(x))$$
$$\Big\downarrow \text{17}$$
$$\frac{1}{2(1-\cosh(x))^2}$$

input
```
Int[Sinh[x]/(1 - Cosh[x])^3,x]
```

output
```
1/(2*(1 - Cosh[x])^2)
```

3.147.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

3.147.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{1}{2(1-\cosh(x))^2}$	11
default	$\frac{1}{2(1-\cosh(x))^2}$	11
risch	$\frac{2\,\mathrm{e}^{2x}}{(\mathrm{e}^{x}-1)^4}$	13

input
```
int(sinh(x)/(1-cosh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/2/(1-cosh(x))^2
```

3.147.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(8) = 16.

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 4.58

$$\int \frac{\sinh(x)}{(1-\cosh(x))^3}\,dx$$
$$= \frac{2\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}{\cosh\left(x\right)^{3}+\left(3\,\cosh\left(x\right)-4\right)\sinh\left(x\right)^{2}+\sinh\left(x\right)^{3}-4\,\cosh\left(x\right)^{2}+\left(3\,\cosh\left(x\right)^{2}-8\,\cosh\left(x\right)+5\right)\sinh\left(x\right)\cdots}$$

input
```
integrate(sinh(x)/(1-cosh(x))^3,x, algorithm="fricas")
```

output
```
2*(cosh(x) + sinh(x))/(cosh(x)^3 + (3*cosh(x) - 4)*sinh(x)^2 + sinh(x)^3 -
 4*cosh(x)^2 + (3*cosh(x)^2 - 8*cosh(x) + 5)*sinh(x) + 7*cosh(x) - 4)
```

3.147.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sinh(x)}{(1-\cosh(x))^3}\,dx = \frac{1}{2\cosh^2(x) - 4\cosh(x) + 2}$$

input
```
integrate(sinh(x)/(1-cosh(x))**3,x)
```

output
```
1/(2*cosh(x)**2 - 4*cosh(x) + 2)
```

3.147.7 Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{\sinh(x)}{(1-\cosh(x))^3}\,dx = \frac{1}{2(\cosh(x)-1)^2}$$

input
```
integrate(sinh(x)/(1-cosh(x))^3,x, algorithm="maxima")
```

output
```
1/2/(cosh(x) - 1)^2
```

3.147.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(x)}{(1-\cosh(x))^3}\,dx = \frac{2e^{(2x)}}{(e^x-1)^4}$$

input
```
integrate(sinh(x)/(1-cosh(x))^3,x, algorithm="giac")
```

output
```
2*e^(2*x)/(e^x - 1)^4
```

3.147.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{\sinh(x)}{(1-\cosh(x))^3}\,dx = \frac{1}{2\left(\cosh\left(x\right)-1\right)^2}$$

input
```
int(-sinh(x)/(cosh(x) - 1)^3,x)
```

output
```
1/(2*(cosh(x) - 1)^2)
```

3.148 $\int \frac{\sinh^2(x)}{(1+\cosh(x))^3}\,dx$

3.148.1 Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{\sinh^2(x)}{(1+\cosh(x))^3}\,dx = \frac{\sinh^3(x)}{3(1+\cosh(x))^3}$$

output
```
1/3*sinh(x)^3/(1+cosh(x))^3
```

3.148.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\sinh^2(x)}{(1+\cosh(x))^3}\,dx = \frac{1}{3}\tanh^3\left(\frac{x}{2}\right)$$

input
```
Integrate[Sinh[x]^2/(1 + Cosh[x])^3,x]
```

output
```
Tanh[x/2]^3/3
```

3.148.3 Rubi [A] (verified)

Time = 0.20 (sec), antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 25, 3150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)}{(\cosh(x)+1)^3} \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{\cos\left(-\frac{\pi}{2}+ix\right)^2}{\left(1-\sin\left(-\frac{\pi}{2}+ix\right)\right)^3}dx$$
$$\Big\downarrow \text{25}$$
$$-\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^2}{\left(1-\sin\left(ix-\frac{\pi}{2}\right)\right)^3}dx$$
$$\Big\downarrow \text{3150}$$
$$\frac{\sinh^3(x)}{3(\cosh(x)+1)^3}$$

input
```
Int[Sinh[x]^2/(1 + Cosh[x])^3,x]
```

output
```
Sinh[x]^3/(3*(1 + Cosh[x])^3)
```

3.148.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3150
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

3.148.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{\tanh\left(\frac{x}{2}\right)^{3}}{3}$	9
risch	$-\frac{2\left(3\,{\mathrm e}^{2x}+1\right)}{3\left({\mathrm e}^{x}+1\right)^{3}}$	17

input
```
int(sinh(x)^2/(cosh(x)+1)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*tanh(1/2*x)^3
```

3.148.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(12) = 24.

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.36

$$\int \frac{\sinh^2(x)}{(1+\cosh(x))^3}\,dx = -\frac{4\,(2\,\cosh(x)+\sinh(x))}{3\,\left(\cosh(x)^2+2\,(\cosh(x)+1)\sinh(x)+\sinh(x)^2+4\,\cosh(x)+3\right)}$$

input
```
integrate(sinh(x)^2/(1+cosh(x))^3,x, algorithm="fricas")
```

output
```
-4/3*(2*cosh(x) + sinh(x))/(cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^
2 + 4*cosh(x) + 3)
```

3.148.6 Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.50

$$\int \frac{\sinh^2(x)}{(1+\cosh(x))^3}\,dx = \frac{\tanh^3\left(\frac{x}{2}\right)}{3}$$

input
```
integrate(sinh(x)**2/(1+cosh(x))**3,x)
```

output
```
tanh(x/2)**3/3
```

3.148.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(12) = 24.

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 3.50

$$\int \frac{\sinh^2(x)}{(1+\cosh(x))^3}\,dx = \frac{2\,e^{(-2\,x)}}{3\,e^{(-x)}+3\,e^{(-2\,x)}+e^{(-3\,x)}+1} + \frac{2}{3\,(3\,e^{(-x)}+3\,e^{(-2\,x)}+e^{(-3\,x)}+1)}$$

input
```
integrate(sinh(x)^2/(1+cosh(x))^3,x, algorithm="maxima")
```

output
```
2*e^(-2*x)/(3*e^(-x) + 3*e^(-2*x) + e^(-3*x) + 1) + 2/3/(3*e^(-x) + 3*e^(-
2*x) + e^(-3*x) + 1)
```

3.148.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\sinh^2(x)}{(1+\cosh(x))^3}\,dx = -\frac{2\left(3\,e^{(2\,x)}+1\right)}{3\,(e^x+1)^3}$$

input
```
integrate(sinh(x)^2/(1+cosh(x))^3,x, algorithm="giac")
```

output
```
-2/3*(3*e^(2*x) + 1)/(e^x + 1)^3
```

3.148.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\sinh^2(x)}{(1+\cosh(x))^3}\,dx = -\frac{2\,(3\,e^{2\,x}+1)}{3\,{(e^x+1)}^3}$$

input
```
int(sinh(x)^2/(cosh(x) + 1)^3,x)
```

output
```
-(2*(3*exp(2*x) + 1))/(3*(exp(x) + 1)^3)
```

3.149 $\int \frac{\sinh^2(x)}{(1-\cosh(x))^3} \, dx$

3.149.1 Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^3} \, dx = -\frac{\sinh^3(x)}{3(1-\cosh(x))^3}$$

output
```
-1/3*sinh(x)^3/(1-cosh(x))^3
```

3.149.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^3} \, dx = \frac{1}{3}\coth^3\left(\frac{x}{2}\right)$$

input
```
Integrate[Sinh[x]^2/(1 - Cosh[x])^3,x]
```

output
```
Coth[x/2]^3/3
```

3.149.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 25, 3150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^3} \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{\cos\left(-\frac{\pi}{2}+ix\right)^2}{\left(1+\sin\left(-\frac{\pi}{2}+ix\right)\right)^3}dx$$
$$\Big\downarrow \text{25}$$
$$-\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^2}{\left(\sin\left(ix-\frac{\pi}{2}\right)+1\right)^3}dx$$
$$\Big\downarrow \text{3150}$$
$$-\frac{\sinh^3(x)}{3(1-\cosh(x))^3}$$

input
```
Int[Sinh[x]^2/(1 - Cosh[x])^3,x]
```

output
```
-1/3*Sinh[x]^3/(1 - Cosh[x])^3
```

3.149.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3150
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]
```

3.149.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

method	result	size
default	$\frac{1}{3\tanh\left(\frac{x}{2}\right)^{3}}$	9
risch	$\frac{2\,{\rm e}^{2x}+\frac{2}{3}}{\left({\rm e}^{x}-1\right)^{3}}$	17

input
```
int(sinh(x)^2/(1-cosh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/3/tanh(1/2*x)^3
```

3.149.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(12) = 24.

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.06

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^3}\,dx = \frac{4\,(2\,\cosh\left(x\right)+\sinh\left(x\right))}{3\,\left(\cosh\left(x\right)^{2}+2\,(\cosh\left(x\right)-1)\sinh\left(x\right)+\sinh\left(x\right)^{2}-4\,\cosh\left(x\right)+3\right)}$$

input
```
integrate(sinh(x)^2/(1-cosh(x))^3,x, algorithm="fricas")
```

output
```
4/3*(2*cosh(x) + sinh(x))/(cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + sinh(x)^2
 - 4*cosh(x) + 3)
```

3.149.6 Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.50

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^3}\,dx = \frac{1}{3\tanh^3\left(\frac{x}{2}\right)}$$

input
```
integrate(sinh(x)**2/(1-cosh(x))**3,x)
```

output
```
1/(3*tanh(x/2)**3)
```

3.149.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(12) = 24.

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 3.06

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^3}\,dx = -\frac{2\,e^{(-2\,x)}}{3\,e^{(-x)}-3\,e^{(-2\,x)}+e^{(-3\,x)}-1} - \frac{2}{3\,(3\,e^{(-x)}-3\,e^{(-2\,x)}+e^{(-3\,x)}-1)}$$

input
```
integrate(sinh(x)^2/(1-cosh(x))^3,x, algorithm="maxima")
```

output
```
-2*e^(-2*x)/(3*e^(-x) - 3*e^(-2*x) + e^(-3*x) - 1) - 2/3/(3*e^(-x) - 3*e^(
-2*x) + e^(-3*x) - 1)
```

3.149.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^3}\,dx = \frac{2\,(3\,e^{(2\,x)}+1)}{3\,(e^x-1)^3}$$

input
```
integrate(sinh(x)^2/(1-cosh(x))^3,x, algorithm="giac")
```

output
```
2/3*(3*e^(2*x) + 1)/(e^x - 1)^3
```

3.149.9 Mupad [B] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^3}\,dx = \frac{2\,(3\,e^{2\,x}+1)}{3\,(e^x-1)^3}$$

input
```
int(-sinh(x)^2/(cosh(x) - 1)^3,x)
```

output
```
(2*(3*exp(2*x) + 1))/(3*(exp(x) - 1)^3)
```

3.150 $\int \frac{\sinh^3(x)}{(1+\cosh(x))^3} dx$

3.150.1 Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{\sinh^3(x)}{(1+\cosh(x))^3} dx = \frac{2}{1+\cosh(x)} + \log(1+\cosh(x))$$

output
```
2/(1+cosh(x))+ln(1+cosh(x))
```

3.150.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{\sinh^3(x)}{(1+\cosh(x))^3} dx = 2\log\left(\cosh\left(\frac{x}{2}\right)\right) - \tanh^2\left(\frac{x}{2}\right)$$

input
```
Integrate[Sinh[x]^3/(1 + Cosh[x])^3,x]
```

output
```
2*Log[Cosh[x/2]] - Tanh[x/2]^2
```

3.150.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 26, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(x)}{(\cosh(x)+1)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i\cos\left(-\frac{\pi}{2}+ix\right)^3}{\left(1-\sin\left(-\frac{\pi}{2}+ix\right)\right)^3}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^3}{\left(1-\sin\left(ix-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{ 3146}$$

$$-\int \frac{1-\cosh(x)}{(\cosh(x)+1)^2}d\cosh(x)$$

$$\Big\downarrow \text{ 49}$$

$$-\int \left(\frac{2}{(\cosh(x)+1)^2}+\frac{1}{-\cosh(x)-1}\right)d\cosh(x)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{\cosh(x)+1}+\log(\cosh(x)+1)$$

input
```
Int[Sinh[x]^3/(1 + Cosh[x])^3,x]
```

output
```
2/(1 + Cosh[x]) + Log[1 + Cosh[x]]
```

3.150.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

3.150.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$\frac{2}{\cosh(x)+1}+\ln\left(\cosh\left(x\right)+1\right)$	15
default	$\frac{2}{\cosh(x)+1}+\ln\left(\cosh\left(x\right)+1\right)$	15
risch	$-x+\frac{4\,{\rm e}^{x}}{\left({\rm e}^{x}+1\right)^{2}}+2\ln\left({\rm e}^{x}+1\right)$	22

input
```
int(sinh(x)^3/(cosh(x)+1)^3,x,method=_RETURNVERBOSE)
```

output
```
2/(cosh(x)+1)+ln(cosh(x)+1)
```

3.150.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(14) = 28$.

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 6.36

$$\int \frac{\sinh^3(x)}{(1+\cosh(x))^3}\,dx = -\frac{x\cosh(x)^2 + x\sinh(x)^2 + 2(x-2)\cosh(x) - 2\left(\cosh(x)^2 + 2(\cosh(x)+1)\sinh(x) + \sinh(x)^2 + 2\,\mathrm{c}\right.}{\cosh(x)^2 + 2(\cosh(x)+1)\sinh(x) + \sinh(}$$

input
```
integrate(sinh(x)^3/(1+cosh(x))^3,x, algorithm="fricas")
```

output
```
-(x*cosh(x)^2 + x*sinh(x)^2 + 2*(x - 2)*cosh(x) - 2*(cosh(x)^2 + 2*(cosh(x
) + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) + 1) + 2
*(x*cosh(x) + x - 2)*sinh(x) + x)/(cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + s
inh(x)^2 + 2*cosh(x) + 1)
```

3.150.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(12) = 24$.

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 9.00

$$\begin{aligned}\int \frac{\sinh^3(x)}{(1+\cosh(x))^3}\,dx &= \frac{2\log(\cosh(x)+1)\cosh^2(x)}{2\cosh^2(x)+4\cosh(x)+2} + \frac{4\log(\cosh(x)+1)\cosh(x)}{2\cosh^2(x)+4\cosh(x)+2} \\ &\quad + \frac{2\log(\cosh(x)+1)}{2\cosh^2(x)+4\cosh(x)+2} - \frac{\sinh^2(x)}{2\cosh^2(x)+4\cosh(x)+2} \\ &\quad + \frac{2\cosh(x)}{2\cosh^2(x)+4\cosh(x)+2} + \frac{2}{2\cosh^2(x)+4\cosh(x)+2}\end{aligned}$$

input
```
integrate(sinh(x)**3/(1+cosh(x))**3,x)
```

output
```
2*log(cosh(x) + 1)*cosh(x)**2/(2*cosh(x)**2 + 4*cosh(x) + 2) + 4*log(cosh(
x) + 1)*cosh(x)/(2*cosh(x)**2 + 4*cosh(x) + 2) + 2*log(cosh(x) + 1)/(2*cos
h(x)**2 + 4*cosh(x) + 2) - sinh(x)**2/(2*cosh(x)**2 + 4*cosh(x) + 2) + 2*c
osh(x)/(2*cosh(x)**2 + 4*cosh(x) + 2) + 2/(2*cosh(x)**2 + 4*cosh(x) + 2)
```

3.150.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(14) = 28.

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.21

$$\int \frac{\sinh^3(x)}{(1+\cosh(x))^3}\,dx = x + \frac{4\,e^{(-x)}}{2\,e^{(-x)} + e^{(-2\,x)} + 1} + 2\,\log\left(e^{(-x)} + 1\right)$$

input
```
integrate(sinh(x)^3/(1+cosh(x))^3,x, algorithm="maxima")
```

output
```
x + 4*e^(-x)/(2*e^(-x) + e^(-2*x) + 1) + 2*log(e^(-x) + 1)
```

3.150.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int \frac{\sinh^3(x)}{(1+\cosh(x))^3}\,dx = -x + \frac{4\,e^{x}}{(e^{x}+1)^2} + 2\,\log\left(e^{x} + 1\right)$$

input
```
integrate(sinh(x)^3/(1+cosh(x))^3,x, algorithm="giac")
```

output
```
-x + 4*e^x/(e^x + 1)^2 + 2*log(e^x + 1)
```

3.150.9 Mupad [B] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^3(x)}{(1+\cosh(x))^3}\,dx = \ln\left(\cosh(x) + 1\right) + \frac{2}{\cosh\left(x\right) + 1}$$

input
```
int(sinh(x)^3/(cosh(x) + 1)^3,x)
```

output
```
log(cosh(x) + 1) + 2/(cosh(x) + 1)
```

3.151 $\int \frac{\sinh^3(x)}{(1-\cosh(x))^3} dx$

3.151.1 Optimal result

Integrand size = 13, antiderivative size = 20

$$\int \frac{\sinh^3(x)}{(1-\cosh(x))^3} dx = -\frac{2}{1-\cosh(x)} - \log(1-\cosh(x))$$

output
```
-2/(1-cosh(x))-ln(1-cosh(x))
```

3.151.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{\sinh^3(x)}{(1-\cosh(x))^3} dx = \coth^2\left(\frac{x}{2}\right) - 2\log\left(\cosh\left(\frac{x}{2}\right)\right) - 2\log\left(\tanh\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Sinh[x]^3/(1 - Cosh[x])^3,x]
```

output
```
Coth[x/2]^2 - 2*Log[Cosh[x/2]] - 2*Log[Tanh[x/2]]
```

3.151.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 26, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(x)}{(1-\cosh(x))^3} \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{i \cos \left(-\frac{\pi}{2}+ix\right)^3}{\left(1+\sin \left(-\frac{\pi}{2}+ix\right)\right)^3}dx$$

$$\Big\downarrow \; 26$$

$$i \int \frac{\cos \left(ix-\frac{\pi}{2}\right)^3}{\left(\sin \left(ix-\frac{\pi}{2}\right)+1\right)^3}dx$$

$$\Big\downarrow \; 3146$$

$$\int \frac{\cosh(x)+1}{(1-\cosh(x))^2}d(-\cosh(x))$$

$$\Big\downarrow \; 49$$

$$\int \left(\frac{2}{(1-\cosh(x))^2}+\frac{1}{\cosh(x)-1}\right)d(-\cosh(x))$$

$$\Big\downarrow \; 2009$$

$$-\frac{2}{1-\cosh(x)}-\log(1-\cosh(x))$$

input
```
Int[Sinh[x]^3/(1 - Cosh[x])^3,x]
```

output
```
-2/(1 - Cosh[x]) - Log[1 - Cosh[x]]
```

3.151.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

3.151.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{2}{\cosh(x)-1}-\ln\left(\cosh\left(x\right)-1\right)$	17
default	$\frac{2}{\cosh(x)-1}-\ln\left(\cosh\left(x\right)-1\right)$	17
risch	$x+\frac{4\,\mathrm{e}^{x}}{\left(\mathrm{e}^{x}-1\right)^{2}}-2\ln\left(\mathrm{e}^{x}-1\right)$	20

input
```
int(sinh(x)^3/(1-cosh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
2/(cosh(x)-1)-ln(cosh(x)-1)
```

3.151.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(18) = 36$.

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 4.50

$$\int \frac{\sinh^3(x)}{(1-\cosh(x))^3}\,dx = \frac{x\cosh(x)^2 + x\sinh(x)^2 - 2(x-2)\cosh(x) - 2\left(\cosh(x)^2 + 2(\cosh(x)-1)\sinh(x) + \sinh(x)^2 - 2\cos\right.}{\cosh(x)^2 + 2(\cosh(x)-1)\sinh(x) + \sinh(x)}$$

input
```
integrate(sinh(x)^3/(1-cosh(x))^3,x, algorithm="fricas")
```

output
```
(x*cosh(x)^2 + x*sinh(x)^2 - 2*(x - 2)*cosh(x) - 2*(cosh(x)^2 + 2*(cosh(x)
 - 1)*sinh(x) + sinh(x)^2 - 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*
(x*cosh(x) - x + 2)*sinh(x) + x)/(cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + si
nh(x)^2 - 2*cosh(x) + 1)
```

3.151.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(14) = 28$.

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 6.30

$$\begin{aligned}\int \frac{\sinh^3(x)}{(1-\cosh(x))^3}\,dx = &-\frac{2\log(\cosh(x)-1)\cosh^2(x)}{2\cosh^2(x)-4\cosh(x)+2} + \frac{4\log(\cosh(x)-1)\cosh(x)}{2\cosh^2(x)-4\cosh(x)+2} \\ &- \frac{2\log(\cosh(x)-1)}{2\cosh^2(x)-4\cosh(x)+2} + \frac{\sinh^2(x)}{2\cosh^2(x)-4\cosh(x)+2} \\ &+ \frac{2\cosh(x)}{2\cosh^2(x)-4\cosh(x)+2} - \frac{2}{2\cosh^2(x)-4\cosh(x)+2}\end{aligned}$$

input
```
integrate(sinh(x)**3/(1-cosh(x))**3,x)
```

output
```
-2*log(cosh(x) - 1)*cosh(x)**2/(2*cosh(x)**2 - 4*cosh(x) + 2) + 4*log(cosh
(x) - 1)*cosh(x)/(2*cosh(x)**2 - 4*cosh(x) + 2) - 2*log(cosh(x) - 1)/(2*co
sh(x)**2 - 4*cosh(x) + 2) + sinh(x)**2/(2*cosh(x)**2 - 4*cosh(x) + 2) + 2*
cosh(x)/(2*cosh(x)**2 - 4*cosh(x) + 2) - 2/(2*cosh(x)**2 - 4*cosh(x) + 2)
```

3.151.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.75

$$\int \frac{\sinh^3(x)}{(1-\cosh(x))^3}\,dx = -x - \frac{4\,e^{(-x)}}{2\,e^{(-x)} - e^{(-2\,x)} - 1} - 2\,\log\left(e^{(-x)} - 1\right)$$

input
```
integrate(sinh(x)^3/(1-cosh(x))^3,x, algorithm="maxima")
```

output
```
-x - 4*e^(-x)/(2*e^(-x) - e^(-2*x) - 1) - 2*log(e^(-x) - 1)
```

3.151.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^3(x)}{(1-\cosh(x))^3}\,dx = x + \frac{4\,e^x}{(e^x-1)^2} - 2\,\log\left(|e^x - 1|\right)$$

input
```
integrate(sinh(x)^3/(1-cosh(x))^3,x, algorithm="giac")
```

output
```
x + 4*e^x/(e^x - 1)^2 - 2*log(abs(e^x - 1))
```

3.151.9 Mupad [B] (verification not implemented)

Time = 1.67 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\sinh^3(x)}{(1-\cosh(x))^3}\,dx = \frac{2}{\cosh(x) - 1} - \ln\left(\cosh(x) - 1\right)$$

input
```
int(-sinh(x)^3/(cosh(x) - 1)^3,x)
```

output
```
2/(cosh(x) - 1) - log(cosh(x) - 1)
```

3.152 $\int \frac{\sinh^8(x)}{a+a\cosh(x)}\,dx$

3.152.1 Optimal result

Integrand size = 13, antiderivative size = 57

$$\int \frac{\sinh^8(x)}{a+a\cosh(x)}\,dx = \frac{5x}{16a} - \frac{5\cosh(x)\sinh(x)}{16a} + \frac{5\cosh(x)\sinh^3(x)}{24a} - \frac{\cosh(x)\sinh^5(x)}{6a} + \frac{\sinh^7(x)}{7a}$$

output
```
5/16*x/a-5/16*cosh(x)*sinh(x)/a+5/24*cosh(x)*sinh(x)^3/a-1/6*cosh(x)*sinh(
x)^5/a+1/7*sinh(x)^7/a
```

3.152.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{\sinh^8(x)}{a+a\cosh(x)}\,dx = \frac{420x - 105\sinh(x) - 315\sinh(2x) + 63\sinh(3x) + 63\sinh(4x) - 21\sinh(5x) - 7\sinh(6x) + 3\sinh(7x)}{1344a}$$

input
```
Integrate[Sinh[x]^8/(a + a*Cosh[x]),x]
```

output
```
(420*x - 105*Sinh[x] - 315*Sinh[2*x] + 63*Sinh[3*x] + 63*Sinh[4*x] - 21*Si
nh[5*x] - 7*Sinh[6*x] + 3*Sinh[7*x])/(1344*a)
```

3.152.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3161, 25, 3042, 25, 3115, 3042, 3115, 25, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^8(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(-\frac{\pi}{2}+ix\right)^8}{a-a\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3161}$$

$$\frac{\int -\sinh^6(x)dx}{a}+\frac{\sinh^7(x)}{7a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh^7(x)}{7a}-\frac{\int \sinh^6(x)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^7(x)}{7a}-\frac{\int -\sin(ix)^6dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh^7(x)}{7a}+\frac{\int \sin(ix)^6dx}{a}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{5}{6}\int \sinh^4(x)dx-\frac{1}{6}\sinh^5(x)\cosh(x)}{a}+\frac{\sinh^7(x)}{7a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^7(x)}{7a}+\frac{-\frac{1}{6}\sinh^5(x)\cosh(x)+\frac{5}{6}\int \sin(ix)^4dx}{a}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{5}{6}\left(\frac{3}{4}\int -\sinh^2(x)dx+\frac{1}{4}\sinh^3(x)\cosh(x)\right)-\frac{1}{6}\sinh^5(x)\cosh(x)}{a}+\frac{\sinh^7(x)}{7a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{5}{6}\left(\frac{1}{4}\sinh^3(x)\cosh(x)-\frac{3}{4}\int\sinh^2(x)dx\right)-\frac{1}{6}\sinh^5(x)\cosh(x)}{a}+\frac{\sinh^7(x)}{7a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^7(x)}{7a}+\frac{-\frac{1}{6}\sinh^5(x)\cosh(x)+\frac{5}{6}\left(\frac{1}{4}\sinh^3(x)\cosh(x)-\frac{3}{4}\int-\sin(ix)^2dx\right)}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh^7(x)}{7a}+\frac{-\frac{1}{6}\sinh^5(x)\cosh(x)+\frac{5}{6}\left(\frac{1}{4}\sinh^3(x)\cosh(x)+\frac{3}{4}\int\sin(ix)^2dx\right)}{a}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)+\frac{1}{4}\sinh^3(x)\cosh(x)\right)-\frac{1}{6}\sinh^5(x)\cosh(x)}{a}+\frac{\sinh^7(x)}{7a}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sinh^7(x)}{7a}+\frac{\frac{5}{6}\left(\frac{1}{4}\sinh^3(x)\cosh(x)+\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)\right)-\frac{1}{6}\sinh^5(x)\cosh(x)}{a}$$

input
```
Int[Sinh[x]^8/(a + a*Cosh[x]),x]
```

output
```
Sinh[x]^7/(7*a) + (-1/6*(Cosh[x]*Sinh[x]^5) + (5*((Cosh[x]*Sinh[x]^3)/4 +
(3*(x/2 - (Cosh[x]*Sinh[x])/2))/4))/6)/a
```

3.152.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.152. $\int \frac{\sinh^8(x)}{a+a\cosh(x)}\,dx$

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

3.152.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(47) = 94$.

Time = 304.86 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.32

method	result
risch	$\frac{5x}{16a} + \frac{e^{7x}}{896a} - \frac{e^{6x}}{384a} - \frac{e^{5x}}{128a} + \frac{3e^{4x}}{128a} + \frac{3e^{3x}}{128a} - \frac{15e^{2x}}{128a} - \frac{5e^{x}}{128a} + \frac{5e^{-x}}{128a} + \frac{15e^{-2x}}{128a} - \frac{3e^{-3x}}{128a} - \frac{3e^{-4x}}{128a} + \frac{e^{-5x}}{128a} + \dots$
default	$\frac{-\frac{1}{7\left(\tanh\left(\frac{x}{2}\right)-1\right)^{7}} - \frac{2}{3\left(\tanh\left(\frac{x}{2}\right)-1\right)^{6}} - \frac{1}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{5}} - \frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^{4}} + \frac{11}{24\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}} - \frac{1}{8\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}} - \frac{5}{16\left(\tanh\left(\frac{x}{2}\right)-1\right)} - \frac{5\ln(\dots}{\dots}}{\dots}$

input
```
int(sinh(x)^8/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
5/16*x/a+1/896/a*exp(7*x)-1/384/a*exp(6*x)-1/128/a*exp(5*x)+3/128/a*exp(4*
x)+3/128/a*exp(3*x)-15/128/a*exp(2*x)-5/128/a*exp(x)+5/128/a*exp(-x)+15/12
8/a*exp(-2*x)-3/128/a*exp(-3*x)-3/128/a*exp(-4*x)+1/128/a*exp(-5*x)+1/384/
a*exp(-6*x)-1/896/a*exp(-7*x)
```

3.152.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(47) = 94$.

Time = 0.26 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.77

$$\int \frac{\sinh^8(x)}{a + a\cosh(x)}\,dx = \frac{3\sinh(x)^7 + 21\left(3\cosh(x)^2 - 2\cosh(x) - 1\right)\sinh(x)^5 + 7\left(15\cosh(x)^4 - 20\cosh(x)^3 - 30\cosh(x)^2 \dots\right.}{\dots}$$

input
```
integrate(sinh(x)^8/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
1/1344*(3*sinh(x)^7 + 21*(3*cosh(x)^2 - 2*cosh(x) - 1)*sinh(x)^5 + 7*(15*c
osh(x)^4 - 20*cosh(x)^3 - 30*cosh(x)^2 + 36*cosh(x) + 9)*sinh(x)^3 + 21*(c
osh(x)^6 - 2*cosh(x)^5 - 5*cosh(x)^4 + 12*cosh(x)^3 + 9*cosh(x)^2 - 30*cos
h(x) - 5)*sinh(x) + 420*x)/a
```

3.152.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1253 vs. $2(51) = 102$.

Time = 2.83 (sec) , antiderivative size = 1253, normalized size of antiderivative = 21.98

$$\int \frac{\sinh^8(x)}{a + a\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(x)**8/(a+a*cosh(x)),x)
```

output
```
105*x*tanh(x/2)**14/(336*a*tanh(x/2)**14 - 2352*a*tanh(x/2)**12 + 7056*a*t
anh(x/2)**10 - 11760*a*tanh(x/2)**8 + 11760*a*tanh(x/2)**6 - 7056*a*tanh(x
/2)**4 + 2352*a*tanh(x/2)**2 - 336*a) - 735*x*tanh(x/2)**12/(336*a*tanh(x/
2)**14 - 2352*a*tanh(x/2)**12 + 7056*a*tanh(x/2)**10 - 11760*a*tanh(x/2)**
8 + 11760*a*tanh(x/2)**6 - 7056*a*tanh(x/2)**4 + 2352*a*tanh(x/2)**2 - 336
*a) + 2205*x*tanh(x/2)**10/(336*a*tanh(x/2)**14 - 2352*a*tanh(x/2)**12 + 7
056*a*tanh(x/2)**10 - 11760*a*tanh(x/2)**8 + 11760*a*tanh(x/2)**6 - 7056*a
*tanh(x/2)**4 + 2352*a*tanh(x/2)**2 - 336*a) - 3675*x*tanh(x/2)**8/(336*a*
tanh(x/2)**14 - 2352*a*tanh(x/2)**12 + 7056*a*tanh(x/2)**10 - 11760*a*tanh
(x/2)**8 + 11760*a*tanh(x/2)**6 - 7056*a*tanh(x/2)**4 + 2352*a*tanh(x/2)**
2 - 336*a) + 3675*x*tanh(x/2)**6/(336*a*tanh(x/2)**14 - 2352*a*tanh(x/2)**
12 + 7056*a*tanh(x/2)**10 - 11760*a*tanh(x/2)**8 + 11760*a*tanh(x/2)**6 -
7056*a*tanh(x/2)**4 + 2352*a*tanh(x/2)**2 - 336*a) - 2205*x*tanh(x/2)**4/(
336*a*tanh(x/2)**14 - 2352*a*tanh(x/2)**12 + 7056*a*tanh(x/2)**10 - 11760*
a*tanh(x/2)**8 + 11760*a*tanh(x/2)**6 - 7056*a*tanh(x/2)**4 + 2352*a*tanh(
x/2)**2 - 336*a) + 735*x*tanh(x/2)**2/(336*a*tanh(x/2)**14 - 2352*a*tanh(x
/2)**12 + 7056*a*tanh(x/2)**10 - 11760*a*tanh(x/2)**8 + 11760*a*tanh(x/2)*
*6 - 7056*a*tanh(x/2)**4 + 2352*a*tanh(x/2)**2 - 336*a) - 105*x/(336*a*tan
h(x/2)**14 - 2352*a*tanh(x/2)**12 + 7056*a*tanh(x/2)**10 - 11760*a*tanh(x/
2)**8 + 11760*a*tanh(x/2)**6 - 7056*a*tanh(x/2)**4 + 2352*a*tanh(x/2)**...
```

3.152.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(47) = 94$.

Time = 0.18 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.79

$$\int \frac{\sinh^8(x)}{a+a\cosh(x)}\,dx$$
$$= -\frac{\left(7\,e^{(-x)}+21\,e^{(-2\,x)}-63\,e^{(-3\,x)}-63\,e^{(-4\,x)}+315\,e^{(-5\,x)}+105\,e^{(-6\,x)}-3\right)e^{(7\,x)}}{2688\,a}+\frac{5\,x}{16\,a}$$
$$+\frac{105\,e^{(-x)}+315\,e^{(-2\,x)}-63\,e^{(-3\,x)}-63\,e^{(-4\,x)}+21\,e^{(-5\,x)}+7\,e^{(-6\,x)}-3\,e^{(-7\,x)}}{2688\,a}$$

input
```
integrate(sinh(x)^8/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-1/2688*(7*e^(-x) + 21*e^(-2*x) - 63*e^(-3*x) - 63*e^(-4*x) + 315*e^(-5*x)
 + 105*e^(-6*x) - 3)*e^(7*x)/a + 5/16*x/a + 1/2688*(105*e^(-x) + 315*e^(-2
*x) - 63*e^(-3*x) - 63*e^(-4*x) + 21*e^(-5*x) + 7*e^(-6*x) - 3*e^(-7*x))/a
```

3.152.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.58

$$\int \frac{\sinh^8(x)}{a+a\cosh(x)}\,dx$$
$$= \frac{\left(105\,e^{(6\,x)}+315\,e^{(5\,x)}-63\,e^{(4\,x)}-63\,e^{(3\,x)}+21\,e^{(2\,x)}+7\,e^{x}-3\right)e^{(-7\,x)}+840\,x+3\,e^{(7\,x)}-7\,e^{(6\,x)}-21\,e^{(5}}{2688\,a}$$

input
```
integrate(sinh(x)^8/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
1/2688*((105*e^(6*x) + 315*e^(5*x) - 63*e^(4*x) - 63*e^(3*x) + 21*e^(2*x)
+ 7*e^x - 3)*e^(-7*x) + 840*x + 3*e^(7*x) - 7*e^(6*x) - 21*e^(5*x) + 63*e^
(4*x) + 63*e^(3*x) - 315*e^(2*x) - 105*e^x)/a
```

3.152.9 Mupad [B] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.30

$$\int \frac{\sinh^8(x)}{a+a\cosh(x)}\,dx = \frac{5\,e^{-x}}{128\,a}+\frac{15\,e^{-2\,x}}{128\,a}-\frac{15\,e^{2\,x}}{128\,a}-\frac{3\,e^{-3\,x}}{128\,a}+\frac{3\,e^{3\,x}}{128\,a}-\frac{3\,e^{-4\,x}}{128\,a}+\frac{3\,e^{4\,x}}{128\,a}+\frac{e^{-5\,x}}{128\,a}-\frac{e^{5\,x}}{128\,a}+\frac{e^{-6\,x}}{384\,a}-\frac{e^{6\,x}}{384\,a}-\frac{e^{-7\,x}}{896\,a}+\frac{e^{7\,x}}{896\,a}+\frac{5\,x}{16\,a}-\frac{5\,e^{x}}{128\,a}$$

input
```
int(sinh(x)^8/(a + a*cosh(x)),x)
```

output
```
(5*exp(-x))/(128*a) + (15*exp(-2*x))/(128*a) - (15*exp(2*x))/(128*a) - (3*
exp(-3*x))/(128*a) + (3*exp(3*x))/(128*a) - (3*exp(-4*x))/(128*a) + (3*exp
(4*x))/(128*a) + exp(-5*x)/(128*a) - exp(5*x)/(128*a) + exp(-6*x)/(384*a)
- exp(6*x)/(384*a) - exp(-7*x)/(896*a) + exp(7*x)/(896*a) + (5*x)/(16*a) -
 (5*exp(x))/(128*a)
```

3.153 $\int \frac{\sinh^7(x)}{a+a\cosh(x)}\,dx$

3.153.1 Optimal result

Integrand size = 13, antiderivative size = 46

$$\int \frac{\sinh^7(x)}{a+a\cosh(x)}\,dx=\frac{(a-a\cosh(x))^4}{a^5}-\frac{4(a-a\cosh(x))^5}{5a^6}+\frac{(a-a\cosh(x))^6}{6a^7}$$

output
```
(a-a*cosh(x))^4/a^5-4/5*(a-a*cosh(x))^5/a^6+1/6*(a-a*cosh(x))^6/a^7
```

3.153.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.59

$$\int \frac{\sinh^7(x)}{a+a\cosh(x)}\,dx=\frac{4(27+28\cosh(x)+5\cosh(2x))\sinh^8\left(\frac{x}{2}\right)}{15a}$$

input
```
Integrate[Sinh[x]^7/(a + a*Cosh[x]),x]
```

output
```
(4*(27 + 28*Cosh[x] + 5*Cosh[2*x])*Sinh[x/2]^8)/(15*a)
```

3.153.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 26, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^7(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\cos\left(-\frac{\pi}{2}+ix\right)^7}{a-a\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^7}{a-a\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3146}$$

$$-\frac{\int (a-a\cosh(x))^3(\cosh(x)a+a)^2d(a\cosh(x))}{a^7}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\int \left((a-a\cosh(x))^5-4a(a-a\cosh(x))^4+4a^2(a-a\cosh(x))^3\right)d(a\cosh(x))}{a^7}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-a^2(a-a\cosh(x))^4-\frac{1}{6}(a-a\cosh(x))^6+\frac{4}{5}a(a-a\cosh(x))^5}{a^7}$$

input
```
Int[Sinh[x]^7/(a + a*Cosh[x]),x]
```

output
```
-((-(a^2*(a - a*Cosh[x])^4) + (4*a*(a - a*Cosh[x])^5)/5 - (a - a*Cosh[x])^
6/6)/a^7)
```

3.153.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

3.153.4 Maple [A] (verified)

Time = 120.53 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

method	result	siz
derivativedivides	$\frac{\frac{\cosh(x)^6}{6}-\frac{\cosh(x)^5}{5}-\frac{\cosh(x)^4}{2}+\frac{2\cosh(x)^3}{3}+\frac{\cosh(x)^2}{2}-\cosh(x)}{a}$	40
default	$\frac{\frac{\cosh(x)^6}{6}-\frac{\cosh(x)^5}{5}-\frac{\cosh(x)^4}{2}+\frac{2\cosh(x)^3}{3}+\frac{\cosh(x)^2}{2}-\cosh(x)}{a}$	40
risch	$\frac{e^{6x}}{384a}-\frac{e^{5x}}{160a}-\frac{e^{4x}}{64a}+\frac{5\,e^{3x}}{96a}+\frac{5\,e^{2x}}{128a}-\frac{5\,e^{x}}{16a}-\frac{5\,e^{-x}}{16a}+\frac{5\,e^{-2x}}{128a}+\frac{5\,e^{-3x}}{96a}-\frac{e^{-4x}}{64a}-\frac{e^{-5x}}{160a}+\frac{e^{-6x}}{384a}$	10

input
```
int(sinh(x)^7/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/a*(1/6*cosh(x)^6-1/5*cosh(x)^5-1/2*cosh(x)^4+2/3*cosh(x)^3+1/2*cosh(x)^2
-cosh(x))
```

3.153.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(45) = 90.

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.04

$$\int \frac{\sinh^7(x)}{a + a\cosh(x)}\,dx$$
$$= \frac{5\,\cosh(x)^6 + 5\,\sinh(x)^6 - 12\,\cosh(x)^5 + 15\left(5\,\cosh(x)^2 - 4\,\cosh(x) - 2\right)\sinh(x)^4 - 30\,\cosh(x)^4 + 10}{}$$

input
```
integrate(sinh(x)^7/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
1/960*(5*cosh(x)^6 + 5*sinh(x)^6 - 12*cosh(x)^5 + 15*(5*cosh(x)^2 - 4*cosh
(x) - 2)*sinh(x)^4 - 30*cosh(x)^4 + 100*cosh(x)^3 + 15*(5*cosh(x)^4 - 8*co
sh(x)^3 - 12*cosh(x)^2 + 20*cosh(x) + 5)*sinh(x)^2 + 75*cosh(x)^2 - 600*co
sh(x))/a
```

3.153.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. 2(39) = 78.

Time = 1.83 (sec) , antiderivative size = 284, normalized size of antiderivative = 6.17

$$\int \frac{\sinh^7(x)}{a + a\cosh(x)}\,dx$$
$$= \frac{320\tanh^6\left(\frac{x}{2}\right)}{15a\tanh^{12}\left(\frac{x}{2}\right) - 90a\tanh^{10}\left(\frac{x}{2}\right) + 225a\tanh^8\left(\frac{x}{2}\right) - 300a\tanh^6\left(\frac{x}{2}\right) + 225a\tanh^4\left(\frac{x}{2}\right) - 90a\tanh^2\left(\frac{x}{2}\right) +}$$
$$- \frac{240\tanh^4\left(\frac{x}{2}\right)}{15a\tanh^{12}\left(\frac{x}{2}\right) - 90a\tanh^{10}\left(\frac{x}{2}\right) + 225a\tanh^8\left(\frac{x}{2}\right) - 300a\tanh^6\left(\frac{x}{2}\right) + 225a\tanh^4\left(\frac{x}{2}\right) - 90a\tanh^2\left(\frac{x}{2}\right)}$$
$$+ \frac{96\tanh^2\left(\frac{x}{2}\right)}{15a\tanh^{12}\left(\frac{x}{2}\right) - 90a\tanh^{10}\left(\frac{x}{2}\right) + 225a\tanh^8\left(\frac{x}{2}\right) - 300a\tanh^6\left(\frac{x}{2}\right) + 225a\tanh^4\left(\frac{x}{2}\right) - 90a\tanh^2\left(\frac{x}{2}\right)}$$
$$- \frac{16}{15a\tanh^{12}\left(\frac{x}{2}\right) - 90a\tanh^{10}\left(\frac{x}{2}\right) + 225a\tanh^8\left(\frac{x}{2}\right) - 300a\tanh^6\left(\frac{x}{2}\right) + 225a\tanh^4\left(\frac{x}{2}\right) - 90a\tanh^2\left(\frac{x}{2}\right)}$$

input
```
integrate(sinh(x)**7/(a+a*cosh(x)),x)
```

output
```
320*tanh(x/2)**6/(15*a*tanh(x/2)**12 - 90*a*tanh(x/2)**10 + 225*a*tanh(x/2
)**8 - 300*a*tanh(x/2)**6 + 225*a*tanh(x/2)**4 - 90*a*tanh(x/2)**2 + 15*a)
 - 240*tanh(x/2)**4/(15*a*tanh(x/2)**12 - 90*a*tanh(x/2)**10 + 225*a*tanh(
x/2)**8 - 300*a*tanh(x/2)**6 + 225*a*tanh(x/2)**4 - 90*a*tanh(x/2)**2 + 15
*a) + 96*tanh(x/2)**2/(15*a*tanh(x/2)**12 - 90*a*tanh(x/2)**10 + 225*a*tan
h(x/2)**8 - 300*a*tanh(x/2)**6 + 225*a*tanh(x/2)**4 - 90*a*tanh(x/2)**2 +
15*a) - 16/(15*a*tanh(x/2)**12 - 90*a*tanh(x/2)**10 + 225*a*tanh(x/2)**8 -
 300*a*tanh(x/2)**6 + 225*a*tanh(x/2)**4 - 90*a*tanh(x/2)**2 + 15*a)
```

3.153.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.83

$$\int \frac{\sinh^7(x)}{a+a\cosh(x)}\,dx = -\frac{\left(12\,e^{(-x)}+30\,e^{(-2\,x)}-100\,e^{(-3\,x)}-75\,e^{(-4\,x)}+600\,e^{(-5\,x)}-5\right)e^{(6\,x)}}{1920\,a} - \frac{600\,e^{(-x)}-75\,e^{(-2\,x)}-100\,e^{(-3\,x)}+30\,e^{(-4\,x)}+12\,e^{(-5\,x)}-5\,e^{(-6\,x)}}{1920\,a}$$

input
```
integrate(sinh(x)^7/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-1/1920*(12*e^(-x) + 30*e^(-2*x) - 100*e^(-3*x) - 75*e^(-4*x) + 600*e^(-5*
x) - 5)*e^(6*x)/a - 1/1920*(600*e^(-x) - 75*e^(-2*x) - 100*e^(-3*x) + 30*e
^(-4*x) + 12*e^(-5*x) - 5*e^(-6*x))/a
```

3.153.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.63

$$\int \frac{\sinh^7(x)}{a+a\cosh(x)}\,dx = -\frac{\left(600\,e^{(5\,x)}-75\,e^{(4\,x)}-100\,e^{(3\,x)}+30\,e^{(2\,x)}+12\,e^{x}-5\right)e^{(-6\,x)}-5\,e^{(6\,x)}+12\,e^{(5\,x)}+30\,e^{(4\,x)}-100\,e^{(3\,x)}}{1920\,a}$$

input
```
integrate(sinh(x)^7/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
-1/1920*((600*e^(5*x) - 75*e^(4*x) - 100*e^(3*x) + 30*e^(2*x) + 12*e^x - 5
)*e^(-6*x) - 5*e^(6*x) + 12*e^(5*x) + 30*e^(4*x) - 100*e^(3*x) - 75*e^(2*x
) + 600*e^x)/a
```

3.153.9 Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.33

$$\int \frac{\sinh^7(x)}{a+a\cosh(x)}\,dx = \frac{5\,e^{-2\,x}}{128\,a}-\frac{5\,e^{-x}}{16\,a}+\frac{5\,e^{2\,x}}{128\,a}+\frac{5\,e^{-3\,x}}{96\,a}+\frac{5\,e^{3\,x}}{96\,a}-\frac{e^{-4\,x}}{64\,a}-\frac{e^{4\,x}}{64\,a}-\frac{e^{-5\,x}}{160\,a}-\frac{e^{5\,x}}{160\,a}+\frac{e^{-6\,x}}{384\,a}+\frac{e^{6\,x}}{384\,a}-\frac{5\,e^{x}}{16\,a}$$

input
```
int(sinh(x)^7/(a + a*cosh(x)),x)
```

output
```
(5*exp(-2*x))/(128*a) - (5*exp(-x))/(16*a) + (5*exp(2*x))/(128*a) + (5*exp
(-3*x))/(96*a) + (5*exp(3*x))/(96*a) - exp(-4*x)/(64*a) - exp(4*x)/(64*a)
- exp(-5*x)/(160*a) - exp(5*x)/(160*a) + exp(-6*x)/(384*a) + exp(6*x)/(384
*a) - (5*exp(x))/(16*a)
```

3.154 $\int \frac{\sinh^6(x)}{a+a\cosh(x)}\,dx$

3.154.1 Optimal result

Integrand size = 13, antiderivative size = 44

$$\int \frac{\sinh^6(x)}{a+a\cosh(x)}\,dx=-\frac{3x}{8a}+\frac{3\cosh(x)\sinh(x)}{8a}-\frac{\cosh(x)\sinh^3(x)}{4a}+\frac{\sinh^5(x)}{5a}$$

output
```
-3/8*x/a+3/8*cosh(x)*sinh(x)/a-1/4*cosh(x)*sinh(x)^3/a+1/5*sinh(x)^5/a
```

3.154.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \frac{\sinh^6(x)}{a+a\cosh(x)}\,dx = \frac{-60x+20\sinh(x)+40\sinh(2x)-10\sinh(3x)-5\sinh(4x)+2\sinh(5x)}{160a}$$

input
```
Integrate[Sinh[x]^6/(a + a*Cosh[x]),x]
```

output
```
(-60*x + 20*Sinh[x] + 40*Sinh[2*x] - 10*Sinh[3*x] - 5*Sinh[4*x] + 2*Sinh[5
*x])/(160*a)
```

3.154.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {3042, 25, 3161, 3042, 3115, 25, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^6(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\cos\left(-\frac{\pi}{2}+ix\right)^6}{a-a\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^6}{a-a\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3161}$$

$$\frac{\sinh^5(x)}{5a}-\frac{\int \sinh^4(x)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^5(x)}{5a}-\frac{\int \sin(ix)^4dx}{a}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\sinh^5(x)}{5a}-\frac{\frac{3}{4}\int -\sinh^2(x)dx+\frac{1}{4}\sinh^3(x)\cosh(x)}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh^5(x)}{5a}-\frac{\frac{1}{4}\sinh^3(x)\cosh(x)-\frac{3}{4}\int \sinh^2(x)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^5(x)}{5a}-\frac{\frac{1}{4}\sinh^3(x)\cosh(x)-\frac{3}{4}\int -\sin(ix)^2dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh^5(x)}{5a}-\frac{\frac{1}{4}\sinh^3(x)\cosh(x)+\frac{3}{4}\int \sin(ix)^2dx}{a}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\sinh^5(x)}{5a} - \frac{\frac{3}{4}\left(\frac{\int 1dx}{2} - \frac{1}{2}\sinh(x)\cosh(x)\right) + \frac{1}{4}\sinh^3(x)\cosh(x)}{a}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sinh^5(x)}{5a} - \frac{\frac{1}{4}\sinh^3(x)\cosh(x) + \frac{3}{4}\left(\frac{x}{2} - \frac{1}{2}\sinh(x)\cosh(x)\right)}{a}$$

input
```
Int[Sinh[x]^6/(a + a*Cosh[x]),x]
```

output
```
Sinh[x]^5/(5*a) - ((Cosh[x]*Sinh[x]^3)/4 + (3*(x/2 - (Cosh[x]*Sinh[x])/2))
/4)/a
```

3.154.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

3.154. $\int \frac{\sinh^6(x)}{a+a\cosh(x)}\,dx$

3.154.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(36) = 72$.

Time = 42.99 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.18

method	result
risch	$-\frac{3x}{8a}+\frac{e^{5x}}{160a}-\frac{e^{4x}}{64a}-\frac{e^{3x}}{32a}+\frac{e^{2x}}{8a}+\frac{e^{x}}{16a}-\frac{e^{-x}}{16a}-\frac{e^{-2x}}{8a}+\frac{e^{-3x}}{32a}+\frac{e^{-4x}}{64a}-\frac{e^{-5x}}{160a}$
default	$\frac{-\frac{1}{5\left(\tanh\left(\frac{x}{2}\right)-1\right)^{5}}-\frac{3}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^{4}}-\frac{3}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}+\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{3}{8\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{8}-\frac{1}{5\left(\tanh\left(\frac{x}{2}\right)+1\right)^{5}}+\frac{}{4\left(\tanh\right.}}{a}$

input
```
int(sinh(x)^6/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-3/8*x/a+1/160/a*exp(5*x)-1/64/a*exp(4*x)-1/32/a*exp(3*x)+1/8/a*exp(2*x)+1
/16/a*exp(x)-1/16/a*exp(-x)-1/8/a*exp(-2*x)+1/32/a*exp(-3*x)+1/64/a*exp(-4
*x)-1/160/a*exp(-5*x)
```

3.154.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.30

$$\int \frac{\sinh^6(x)}{a+a\cosh(x)}\,dx$$
$$=\frac{\sinh\left(x\right)^{5}+5\left(2\cosh\left(x\right)^{2}-2\cosh\left(x\right)-1\right)\sinh\left(x\right)^{3}+5\left(\cosh\left(x\right)^{4}-2\cosh\left(x\right)^{3}-3\cosh\left(x\right)^{2}+8\cosh\right.}{80\,a}$$

input
```
integrate(sinh(x)^6/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
1/80*(sinh(x)^5 + 5*(2*cosh(x)^2 - 2*cosh(x) - 1)*sinh(x)^3 + 5*(cosh(x)^4
 - 2*cosh(x)^3 - 3*cosh(x)^2 + 8*cosh(x) + 2)*sinh(x) - 30*x)/a
```

3.154.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 692 vs. $2(37) = 74$.

Time = 1.15 (sec) , antiderivative size = 692, normalized size of antiderivative = 15.73

$$\int \frac{\sinh^6(x)}{a + a\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(x)**6/(a+a*cosh(x)),x)
```

output
```
-15*x*tanh(x/2)**10/(40*a*tanh(x/2)**10 - 200*a*tanh(x/2)**8 + 400*a*tanh(
x/2)**6 - 400*a*tanh(x/2)**4 + 200*a*tanh(x/2)**2 - 40*a) + 75*x*tanh(x/2)
**8/(40*a*tanh(x/2)**10 - 200*a*tanh(x/2)**8 + 400*a*tanh(x/2)**6 - 400*a*
tanh(x/2)**4 + 200*a*tanh(x/2)**2 - 40*a) - 150*x*tanh(x/2)**6/(40*a*tanh(
x/2)**10 - 200*a*tanh(x/2)**8 + 400*a*tanh(x/2)**6 - 400*a*tanh(x/2)**4 +
200*a*tanh(x/2)**2 - 40*a) + 150*x*tanh(x/2)**4/(40*a*tanh(x/2)**10 - 200*
a*tanh(x/2)**8 + 400*a*tanh(x/2)**6 - 400*a*tanh(x/2)**4 + 200*a*tanh(x/2)
**2 - 40*a) - 75*x*tanh(x/2)**2/(40*a*tanh(x/2)**10 - 200*a*tanh(x/2)**8 +
 400*a*tanh(x/2)**6 - 400*a*tanh(x/2)**4 + 200*a*tanh(x/2)**2 - 40*a) + 15
*x/(40*a*tanh(x/2)**10 - 200*a*tanh(x/2)**8 + 400*a*tanh(x/2)**6 - 400*a*t
anh(x/2)**4 + 200*a*tanh(x/2)**2 - 40*a) + 30*tanh(x/2)**9/(40*a*tanh(x/2)
**10 - 200*a*tanh(x/2)**8 + 400*a*tanh(x/2)**6 - 400*a*tanh(x/2)**4 + 200*
a*tanh(x/2)**2 - 40*a) - 140*tanh(x/2)**7/(40*a*tanh(x/2)**10 - 200*a*tanh
(x/2)**8 + 400*a*tanh(x/2)**6 - 400*a*tanh(x/2)**4 + 200*a*tanh(x/2)**2 -
40*a) - 256*tanh(x/2)**5/(40*a*tanh(x/2)**10 - 200*a*tanh(x/2)**8 + 400*a*
tanh(x/2)**6 - 400*a*tanh(x/2)**4 + 200*a*tanh(x/2)**2 - 40*a) + 140*tanh(
x/2)**3/(40*a*tanh(x/2)**10 - 200*a*tanh(x/2)**8 + 400*a*tanh(x/2)**6 - 40
0*a*tanh(x/2)**4 + 200*a*tanh(x/2)**2 - 40*a) - 30*tanh(x/2)/(40*a*tanh(x/
2)**10 - 200*a*tanh(x/2)**8 + 400*a*tanh(x/2)**6 - 400*a*tanh(x/2)**4 + 20
0*a*tanh(x/2)**2 - 40*a)
```

3.154.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(36) = 72$.

Time = 0.18 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.77

$$\int \frac{\sinh^6(x)}{a + a\cosh(x)}\,dx = -\frac{\left(5\,e^{(-x)} + 10\,e^{(-2\,x)} - 40\,e^{(-3\,x)} - 20\,e^{(-4\,x)} - 2\right)e^{(5\,x)}}{320\,a} - \frac{3\,x}{8\,a} - \frac{20\,e^{(-x)} + 40\,e^{(-2\,x)} - 10\,e^{(-3\,x)} - 5\,e^{(-4\,x)} + 2\,e^{(-5\,x)}}{320\,a}$$

input
```
integrate(sinh(x)^6/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-1/320*(5*e^(-x) + 10*e^(-2*x) - 40*e^(-3*x) - 20*e^(-4*x) - 2)*e^(5*x)/a
- 3/8*x/a - 1/320*(20*e^(-x) + 40*e^(-2*x) - 10*e^(-3*x) - 5*e^(-4*x) + 2*
e^(-5*x))/a
```

3.154.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.50

$$\int \frac{\sinh^6(x)}{a+a\cosh(x)}\,dx = -\frac{\left(20\,e^{(4\,x)}+40\,e^{(3\,x)}-10\,e^{(2\,x)}-5\,e^{x}+2\right)e^{(-5\,x)}+120\,x-2\,e^{(5\,x)}+5\,e^{(4\,x)}+10\,e^{(3\,x)}-40\,e^{(2\,x)}-20\,e^{x}}{320\,a}$$

input
```
integrate(sinh(x)^6/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
-1/320*((20*e^(4*x) + 40*e^(3*x) - 10*e^(2*x) - 5*e^x + 2)*e^(-5*x) + 120*
x - 2*e^(5*x) + 5*e^(4*x) + 10*e^(3*x) - 40*e^(2*x) - 20*e^x)/a
```

3.154.9 Mupad [B] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.16

$$\int \frac{\sinh^6(x)}{a+a\cosh(x)}\,dx = \frac{e^{2\,x}}{8\,a}-\frac{e^{-2\,x}}{8\,a}-\frac{e^{-x}}{16\,a}+\frac{e^{-3\,x}}{32\,a}-\frac{e^{3\,x}}{32\,a}+\frac{e^{-4\,x}}{64\,a}-\frac{e^{4\,x}}{64\,a}-\frac{e^{-5\,x}}{160\,a}+\frac{e^{5\,x}}{160\,a}-\frac{3\,x}{8\,a}+\frac{e^{x}}{16\,a}$$

input
```
int(sinh(x)^6/(a + a*cosh(x)),x)
```

output
```
exp(2*x)/(8*a) - exp(-2*x)/(8*a) - exp(-x)/(16*a) + exp(-3*x)/(32*a) - exp
(3*x)/(32*a) + exp(-4*x)/(64*a) - exp(4*x)/(64*a) - exp(-5*x)/(160*a) + ex
p(5*x)/(160*a) - (3*x)/(8*a) + exp(x)/(16*a)
```

3.155 $\int \frac{\sinh^5(x)}{a+a\cosh(x)}\,dx$

3.155.1 Optimal result

Integrand size = 13, antiderivative size = 33

$$\int \frac{\sinh^5(x)}{a+a\cosh(x)}\,dx=-\frac{2(a-a\cosh(x))^3}{3a^4}+\frac{(a-a\cosh(x))^4}{4a^5}$$

output
```
-2/3*(a-a*cosh(x))^3/a^4+1/4*(a-a*cosh(x))^4/a^5
```

3.155.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.64

$$\int \frac{\sinh^5(x)}{a+a\cosh(x)}\,dx=\frac{2(5+3\cosh(x))\sinh^6\left(\frac{x}{2}\right)}{3a}$$

input
```
Integrate[Sinh[x]^5/(a + a*Cosh[x]),x]
```

output
```
(2*(5 + 3*Cosh[x])*Sinh[x/2]^6)/(3*a)
```

3.155.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 26, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^5(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\cos\left(-\frac{\pi}{2}+ix\right)^5}{a-a\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^5}{a-a\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3146}$$

$$\frac{\int (a-a\cosh(x))^2(\cosh(x)a+a)d(a\cosh(x))}{a^5}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(2a(a-a\cosh(x))^2-(a-a\cosh(x))^3\right)d(a\cosh(x))}{a^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{4}(a-a\cosh(x))^4-\frac{2}{3}a(a-a\cosh(x))^3}{a^5}$$

input
```
Int[Sinh[x]^5/(a + a*Cosh[x]),x]
```

output
```
((-2*a*(a - a*Cosh[x])^3)/3 + (a - a*Cosh[x])^4/4)/a^5
```

3.155.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

3.155.4 Maple [A] (verified)

Time = 14.75 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{\frac{\cosh(x)^4}{4}-\frac{\cosh(x)^3}{3}-\frac{\cosh(x)^2}{2}+\cosh(x)}{a}$	26
default	$\frac{\frac{\cosh(x)^4}{4}-\frac{\cosh(x)^3}{3}-\frac{\cosh(x)^2}{2}+\cosh(x)}{a}$	26
risch	$\frac{e^{4x}}{64a}-\frac{e^{3x}}{24a}-\frac{e^{2x}}{16a}+\frac{3\,e^{x}}{8a}+\frac{3\,e^{-x}}{8a}-\frac{e^{-2x}}{16a}-\frac{e^{-3x}}{24a}+\frac{e^{-4x}}{64a}$	72

input
```
int(sinh(x)^5/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/a*(1/4*cosh(x)^4-1/3*cosh(x)^3-1/2*cosh(x)^2+cosh(x))
```

3.155.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.58

$$\int \frac{\sinh^5(x)}{a+a\cosh(x)}\,dx$$
$$=\frac{3\,\cosh\left(x\right)^{4}+3\,\sinh\left(x\right)^{4}-8\,\cosh\left(x\right)^{3}+6\,\left(3\,\cosh\left(x\right)^{2}-4\,\cosh\left(x\right)-2\right)\sinh\left(x\right)^{2}-12\,\cosh\left(x\right)^{2}+72\,\mathrm{c}}{96\,a}$$

input
```
integrate(sinh(x)^5/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
1/96*(3*cosh(x)^4 + 3*sinh(x)^4 - 8*cosh(x)^3 + 6*(3*cosh(x)^2 - 4*cosh(x)
 - 2)*sinh(x)^2 - 12*cosh(x)^2 + 72*cosh(x))/a
```

3.155.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. 2(27) = 54.

Time = 0.76 (sec) , antiderivative size = 150, normalized size of antiderivative = 4.55

$$\int \frac{\sinh^5(x)}{a+a\cosh(x)}\,dx=\frac{24\tanh^4\left(\frac{x}{2}\right)}{3a\tanh^8\left(\frac{x}{2}\right)-12a\tanh^6\left(\frac{x}{2}\right)+18a\tanh^4\left(\frac{x}{2}\right)-12a\tanh^2\left(\frac{x}{2}\right)+3a}-\frac{16\tanh^2\left(\frac{x}{2}\right)}{3a\tanh^8\left(\frac{x}{2}\right)-12a\tanh^6\left(\frac{x}{2}\right)+18a\tanh^4\left(\frac{x}{2}\right)-12a\tanh^2\left(\frac{x}{2}\right)+3a}+\frac{4}{3a\tanh^8\left(\frac{x}{2}\right)-12a\tanh^6\left(\frac{x}{2}\right)+18a\tanh^4\left(\frac{x}{2}\right)-12a\tanh^2\left(\frac{x}{2}\right)+3a}$$

input
```
integrate(sinh(x)**5/(a+a*cosh(x)),x)
```

output
```
24*tanh(x/2)**4/(3*a*tanh(x/2)**8 - 12*a*tanh(x/2)**6 + 18*a*tanh(x/2)**4
- 12*a*tanh(x/2)**2 + 3*a) - 16*tanh(x/2)**2/(3*a*tanh(x/2)**8 - 12*a*tanh
(x/2)**6 + 18*a*tanh(x/2)**4 - 12*a*tanh(x/2)**2 + 3*a) + 4/(3*a*tanh(x/2)
**8 - 12*a*tanh(x/2)**6 + 18*a*tanh(x/2)**4 - 12*a*tanh(x/2)**2 + 3*a)
```

3.155.7 Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.82

$$\int \frac{\sinh^5(x)}{a + a\cosh(x)}\,dx = -\frac{\left(8\,e^{(-x)} + 12\,e^{(-2\,x)} - 72\,e^{(-3\,x)} - 3\right)e^{(4\,x)}}{192\,a} + \frac{72\,e^{(-x)} - 12\,e^{(-2\,x)} - 8\,e^{(-3\,x)} + 3\,e^{(-4\,x)}}{192\,a}$$

input
```
integrate(sinh(x)^5/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-1/192*(8*e^(-x) + 12*e^(-2*x) - 72*e^(-3*x) - 3)*e^(4*x)/a + 1/192*(72*e^
(-x) - 12*e^(-2*x) - 8*e^(-3*x) + 3*e^(-4*x))/a
```

3.155.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.55

$$\int \frac{\sinh^5(x)}{a + a\cosh(x)}\,dx = \frac{\left(72\,e^{(3\,x)} - 12\,e^{(2\,x)} - 8\,e^{x} + 3\right)e^{(-4\,x)} + 3\,e^{(4\,x)} - 8\,e^{(3\,x)} - 12\,e^{(2\,x)} + 72\,e^{x}}{192\,a}$$

input
```
integrate(sinh(x)^5/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
1/192*((72*e^(3*x) - 12*e^(2*x) - 8*e^x + 3)*e^(-4*x) + 3*e^(4*x) - 8*e^(3
*x) - 12*e^(2*x) + 72*e^x)/a
```

3.155.9 Mupad [B] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.15

$$\int \frac{\sinh^5(x)}{a + a\cosh(x)}\,dx = \frac{3\,\mathrm{e}^{-x}}{8\,a} - \frac{\mathrm{e}^{-2\,x}}{16\,a} - \frac{\mathrm{e}^{2\,x}}{16\,a} - \frac{\mathrm{e}^{-3\,x}}{24\,a} - \frac{\mathrm{e}^{3\,x}}{24\,a} + \frac{\mathrm{e}^{-4\,x}}{64\,a} + \frac{\mathrm{e}^{4\,x}}{64\,a} + \frac{3\,\mathrm{e}^{x}}{8\,a}$$

input
```
int(sinh(x)^5/(a + a*cosh(x)),x)
```

output
```
(3*exp(-x))/(8*a) - exp(-2*x)/(16*a) - exp(2*x)/(16*a) - exp(-3*x)/(24*a)
- exp(3*x)/(24*a) + exp(-4*x)/(64*a) + exp(4*x)/(64*a) + (3*exp(x))/(8*a)
```

3.156 $\int \frac{\sinh^4(x)}{a+a\cosh(x)}\,dx$

3.156.1 Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{\sinh^4(x)}{a+a\cosh(x)}\,dx = \frac{x}{2a} - \frac{\cosh(x)\sinh(x)}{2a} + \frac{\sinh^3(x)}{3a}$$

output
```
1/2*x/a-1/2*cosh(x)*sinh(x)/a+1/3*sinh(x)^3/a
```

3.156.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{\sinh^4(x)}{a+a\cosh(x)}\,dx = \frac{6x-3\sinh(x)-3\sinh(2x)+\sinh(3x)}{12a}$$

input
```
Integrate[Sinh[x]^4/(a + a*Cosh[x]),x]
```

output
```
(6*x - 3*Sinh[x] - 3*Sinh[2*x] + Sinh[3*x])/(12*a)
```

3.156.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3042, 3161, 25, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^4(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(-\frac{\pi}{2}+ix\right)^4}{a-a\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3161}$$

$$\frac{\int -\sinh^2(x)dx}{a}+\frac{\sinh^3(x)}{3a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh^3(x)}{3a}-\frac{\int \sinh^2(x)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^3(x)}{3a}-\frac{\int -\sin(ix)^2dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh^3(x)}{3a}+\frac{\int \sin(ix)^2dx}{a}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{\int 1dx}{2}-\frac{1}{2}\sinh(x)\cosh(x)}{a}+\frac{\sinh^3(x)}{3a}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sinh^3(x)}{3a}+\frac{\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)}{a}$$

input
```
Int[Sinh[x]^4/(a + a*Cosh[x]),x]
```

output
```
Sinh[x]^3/(3*a) + (x/2 - (Cosh[x]*Sinh[x])/2)/a
```

3.156.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

3.156.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(25) = 50$.

Time = 3.72 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.94

method	result
risch	$\frac{x}{2a}+\frac{e^{3x}}{24a}-\frac{e^{2x}}{8a}-\frac{e^{x}}{8a}+\frac{e^{-x}}{8a}+\frac{e^{-2x}}{8a}-\frac{e^{-3x}}{24a}$
default	$\frac{-\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}-\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right.\right.}{2}}{a}$

input
```
int(sinh(x)^4/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x/a+1/24/a*exp(3*x)-1/8/a*exp(2*x)-1/8/a*exp(x)+1/8/a*exp(-x)+1/8/a*ex
p(-2*x)-1/24/a*exp(-3*x)
```

3.156.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{\sinh^4(x)}{a+a\cosh(x)}\,dx = \frac{\sinh(x)^3 + 3\left(\cosh(x)^2 - 2\cosh(x) - 1\right)\sinh(x) + 6x}{12a}$$

input
```
integrate(sinh(x)^4/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
1/12*(sinh(x)^3 + 3*(cosh(x)^2 - 2*cosh(x) - 1)*sinh(x) + 6*x)/a
```

3.156.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(22) = 44.

Time = 0.43 (sec) , antiderivative size = 294, normalized size of antiderivative = 9.48

$$\begin{aligned}\int \frac{\sinh^4(x)}{a+a\cosh(x)}\,dx &= \frac{3x\tanh^6\left(\frac{x}{2}\right)}{6a\tanh^6\left(\frac{x}{2}\right) - 18a\tanh^4\left(\frac{x}{2}\right) + 18a\tanh^2\left(\frac{x}{2}\right) - 6a} \\ &- \frac{9x\tanh^4\left(\frac{x}{2}\right)}{6a\tanh^6\left(\frac{x}{2}\right) - 18a\tanh^4\left(\frac{x}{2}\right) + 18a\tanh^2\left(\frac{x}{2}\right) - 6a} \\ &+ \frac{9x\tanh^2\left(\frac{x}{2}\right)}{6a\tanh^6\left(\frac{x}{2}\right) - 18a\tanh^4\left(\frac{x}{2}\right) + 18a\tanh^2\left(\frac{x}{2}\right) - 6a} \\ &- \frac{3x}{6a\tanh^6\left(\frac{x}{2}\right) - 18a\tanh^4\left(\frac{x}{2}\right) + 18a\tanh^2\left(\frac{x}{2}\right) - 6a} \\ &- \frac{6\tanh^5\left(\frac{x}{2}\right)}{6a\tanh^6\left(\frac{x}{2}\right) - 18a\tanh^4\left(\frac{x}{2}\right) + 18a\tanh^2\left(\frac{x}{2}\right) - 6a} \\ &- \frac{16\tanh^3\left(\frac{x}{2}\right)}{6a\tanh^6\left(\frac{x}{2}\right) - 18a\tanh^4\left(\frac{x}{2}\right) + 18a\tanh^2\left(\frac{x}{2}\right) - 6a} \\ &+ \frac{6\tanh\left(\frac{x}{2}\right)}{6a\tanh^6\left(\frac{x}{2}\right) - 18a\tanh^4\left(\frac{x}{2}\right) + 18a\tanh^2\left(\frac{x}{2}\right) - 6a}\end{aligned}$$

input
```
integrate(sinh(x)**4/(a+a*cosh(x)),x)
```

output
```
3*x*tanh(x/2)**6/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4 + 18*a*tanh(x/2)**2
 - 6*a) - 9*x*tanh(x/2)**4/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4 + 18*a*ta
nh(x/2)**2 - 6*a) + 9*x*tanh(x/2)**2/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4
 + 18*a*tanh(x/2)**2 - 6*a) - 3*x/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4 +
18*a*tanh(x/2)**2 - 6*a) - 6*tanh(x/2)**5/(6*a*tanh(x/2)**6 - 18*a*tanh(x/
2)**4 + 18*a*tanh(x/2)**2 - 6*a) - 16*tanh(x/2)**3/(6*a*tanh(x/2)**6 - 18*
a*tanh(x/2)**4 + 18*a*tanh(x/2)**2 - 6*a) + 6*tanh(x/2)/(6*a*tanh(x/2)**6
- 18*a*tanh(x/2)**4 + 18*a*tanh(x/2)**2 - 6*a)
```

3.156.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(25) = 50.

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.74

$$\int \frac{\sinh^4(x)}{a + a\cosh(x)}\,dx = -\frac{\left(3\,e^{(-x)} + 3\,e^{(-2\,x)} - 1\right)e^{(3\,x)}}{24\,a} + \frac{x}{2\,a} + \frac{3\,e^{(-x)} + 3\,e^{(-2\,x)} - e^{(-3\,x)}}{24\,a}$$

input
```
integrate(sinh(x)^4/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-1/24*(3*e^(-x) + 3*e^(-2*x) - 1)*e^(3*x)/a + 1/2*x/a + 1/24*(3*e^(-x) + 3
*e^(-2*x) - e^(-3*x))/a
```

3.156.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \frac{\sinh^4(x)}{a + a\cosh(x)}\,dx = \frac{\left(3\,e^{(2\,x)} + 3\,e^{x} - 1\right)e^{(-3\,x)} + 12\,x + e^{(3\,x)} - 3\,e^{(2\,x)} - 3\,e^{x}}{24\,a}$$

input
```
integrate(sinh(x)^4/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
1/24*((3*e^(2*x) + 3*e^x - 1)*e^(-3*x) + 12*x + e^(3*x) - 3*e^(2*x) - 3*e^
x)/a
```

3.156.9 Mupad [B] (verification not implemented)

Time = 1.61 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.90

$$\int \frac{\sinh^4(x)}{a+a\cosh(x)}\,dx = \frac{e^{-x}}{8\,a}+\frac{e^{-2\,x}}{8\,a}-\frac{e^{2\,x}}{8\,a}-\frac{e^{-3\,x}}{24\,a}+\frac{e^{3\,x}}{24\,a}+\frac{x}{2\,a}-\frac{e^{x}}{8\,a}$$

input
```
int(sinh(x)^4/(a + a*cosh(x)),x)
```

output
```
exp(-x)/(8*a) + exp(-2*x)/(8*a) - exp(2*x)/(8*a) - exp(-3*x)/(24*a) + exp(
3*x)/(24*a) + x/(2*a) - exp(x)/(8*a)
```

3.157 $\int \frac{\sinh^3(x)}{a+a\cosh(x)}\,dx$

3.157.1 Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{\sinh^3(x)}{a+a\cosh(x)}\,dx=-\frac{\cosh(x)}{a}+\frac{\cosh^2(x)}{2a}$$

output
```
-cosh(x)/a+1/2*cosh(x)^2/a
```

3.157.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{\sinh^3(x)}{a+a\cosh(x)}\,dx=\frac{2\sinh^4\left(\frac{x}{2}\right)}{a}$$

input
```
Integrate[Sinh[x]^3/(a + a*Cosh[x]),x]
```

output
```
(2*Sinh[x/2]^4)/a
```

3.157.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 26, 3146, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i\cos\left(-\frac{\pi}{2}+ix\right)^3}{a-a\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^3}{a-a\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3146}$$

$$-\frac{\int (a-a\cosh(x))d(a\cosh(x))}{a^3}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{(a-a\cosh(x))^2}{2a^3}$$

input
```
Int[Sinh[x]^3/(a + a*Cosh[x]),x]
```

output
```
(a - a*Cosh[x])^2/(2*a^3)
```

3.157.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])    I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

3.157.4 Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{-\cosh(x)+\frac{\cosh(x)^2}{2}}{a}$	16
default	$\frac{-\cosh(x)+\frac{\cosh(x)^2}{2}}{a}$	16
risch	$\frac{e^{2x}}{8a}-\frac{e^{x}}{2a}-\frac{e^{-x}}{2a}+\frac{e^{-2x}}{8a}$	36

input
```
int(sinh(x)^3/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/a*(-cosh(x)+1/2*cosh(x)^2)
```

3.157.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\sinh^3(x)}{a+a\cosh(x)}\,dx=\frac{\cosh\left(x\right)^2+\sinh\left(x\right)^2-4\,\cosh\left(x\right)}{4\,a}$$

input
```
integrate(sinh(x)^3/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
1/4*(cosh(x)^2 + sinh(x)^2 - 4*cosh(x))/a
```

3.157.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(12) = 24$.

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.58

$$\int \frac{\sinh^3(x)}{a+a\cosh(x)}\,dx = \frac{4\tanh^2\left(\frac{x}{2}\right)}{a\tanh^4\left(\frac{x}{2}\right)-2a\tanh^2\left(\frac{x}{2}\right)+a} - \frac{2}{a\tanh^4\left(\frac{x}{2}\right)-2a\tanh^2\left(\frac{x}{2}\right)+a}$$

input
```
integrate(sinh(x)**3/(a+a*cosh(x)),x)
```

output
```
4*tanh(x/2)**2/(a*tanh(x/2)**4 - 2*a*tanh(x/2)**2 + a) - 2/(a*tanh(x/2)**4
 - 2*a*tanh(x/2)**2 + a)
```

3.157.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{\sinh^3(x)}{a+a\cosh(x)}\,dx = -\frac{\left(4\,e^{(-x)}-1\right)e^{(2\,x)}}{8\,a} - \frac{4\,e^{(-x)}-e^{(-2\,x)}}{8\,a}$$

input
```
integrate(sinh(x)^3/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-1/8*(4*e^(-x) - 1)*e^(2*x)/a - 1/8*(4*e^(-x) - e^(-2*x))/a
```

3.157.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int \frac{\sinh^3(x)}{a+a\cosh(x)}\,dx = -\frac{(4\,e^x-1)e^{(-2\,x)}-e^{(2\,x)}+4\,e^x}{8\,a}$$

input
```
integrate(sinh(x)^3/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
-1/8*((4*e^x - 1)*e^(-2*x) - e^(2*x) + 4*e^x)/a
```

3.157.9 Mupad [B] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.84

$$\int \frac{\sinh^3(x)}{a + a\cosh(x)}\,dx = \frac{e^{-2x}}{8a} - \frac{e^{-x}}{2a} + \frac{e^{2x}}{8a} - \frac{e^{x}}{2a}$$

input
```
int(sinh(x)^3/(a + a*cosh(x)),x)
```

output
```
exp(-2*x)/(8*a) - exp(-x)/(2*a) + exp(2*x)/(8*a) - exp(x)/(2*a)
```

3.158 $\int \frac{\sinh^2(x)}{a+a\cosh(x)}\,dx$

3.158.1 Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{\sinh^2(x)}{a+a\cosh(x)}\,dx=-\frac{x}{a}+\frac{\sinh(x)}{a}$$

output
```
-x/a+sinh(x)/a
```

3.158.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{\sinh^2(x)}{a+a\cosh(x)}\,dx=\frac{2\left(-\frac{x}{2}+\frac{\sinh(x)}{2}\right)}{a}$$

input
```
Integrate[Sinh[x]^2/(a + a*Cosh[x]),x]
```

output
```
(2*(-1/2*x + Sinh[x]/2))/a
```

3.158.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 25, 3161, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{\cos\left(-\frac{\pi}{2}+ix\right)^2}{a-a\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \ 25$$

$$-\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^2}{a-a\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \ 3161$$

$$\frac{\sinh(x)}{a}-\frac{\int 1dx}{a}$$

$$\Big\downarrow \ 24$$

$$\frac{\sinh(x)}{a}-\frac{x}{a}$$

input
```
Int[Sinh[x]^2/(a + a*Cosh[x]),x]
```

output
```
-(x/a) + Sinh[x]/a
```

3.158.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

3.158.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.85

method	result	size
risch	$-\frac{x}{a}+\frac{{\mathrm e}^{x}}{2 a}-\frac{{\mathrm e}^{-x}}{2 a}$	24
default	$\frac{-\frac{1}{\tanh \left(\frac{x}{2}\right)-1}+\ln \left(\tanh \left(\frac{x}{2}\right)-1\right)-\frac{1}{\tanh \left(\frac{x}{2}\right)+1}-\ln \left(\tanh \left(\frac{x}{2}\right)+1\right)}{a}$	45

input
```
int(sinh(x)^2/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-x/a+1/2/a*exp(x)-1/2/a*exp(-x)
```

3.158.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\sinh^2(x)}{a+a\cosh(x)}\,dx=-\frac{x-\sinh\left(x\right)}{a}$$

input
```
integrate(sinh(x)^2/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
-(x - sinh(x))/a
```

3.158.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(7) = 14$.

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 3.54

$$\int \frac{\sinh^2(x)}{a + a\cosh(x)}\,dx = -\frac{x\tanh^2\left(\frac{x}{2}\right)}{a\tanh^2\left(\frac{x}{2}\right) - a} + \frac{x}{a\tanh^2\left(\frac{x}{2}\right) - a} - \frac{2\tanh\left(\frac{x}{2}\right)}{a\tanh^2\left(\frac{x}{2}\right) - a}$$

input
```
integrate(sinh(x)**2/(a+a*cosh(x)),x)
```

output
```
-x*tanh(x/2)**2/(a*tanh(x/2)**2 - a) + x/(a*tanh(x/2)**2 - a) - 2*tanh(x/2
)/(a*tanh(x/2)**2 - a)
```

3.158.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int \frac{\sinh^2(x)}{a + a\cosh(x)}\,dx = -\frac{x}{a} - \frac{e^{(-x)}}{2\,a} + \frac{e^x}{2\,a}$$

input
```
integrate(sinh(x)^2/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-x/a - 1/2*e^(-x)/a + 1/2*e^x/a
```

3.158.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{\sinh^2(x)}{a + a\cosh(x)}\,dx = -\frac{2\,x + e^{(-x)} - e^x}{2\,a}$$

input
```
integrate(sinh(x)^2/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
-1/2*(2*x + e^(-x) - e^x)/a
```

3.158.9 Mupad [B] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int \frac{\sinh^2(x)}{a + a\cosh(x)}\,dx = \frac{e^x}{2\,a} - \frac{x}{a} - \frac{e^{-x}}{2\,a}$$

input
```
int(sinh(x)^2/(a + a*cosh(x)),x)
```

output
```
exp(x)/(2*a) - x/a - exp(-x)/(2*a)
```

3.159 $\int \frac{\sinh(x)}{a+a\cosh(x)}\,dx$

3.159.1 Optimal result

Integrand size = 11, antiderivative size = 9

$$\int \frac{\sinh(x)}{a+a\cosh(x)}\,dx = \frac{\log(1+\cosh(x))}{a}$$

output
```
ln(1+cosh(x))/a
```

3.159.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.33

$$\int \frac{\sinh(x)}{a+a\cosh(x)}\,dx = \frac{2\log\left(\cosh\left(\frac{x}{2}\right)\right)}{a}$$

input
```
Integrate[Sinh[x]/(a + a*Cosh[x]),x]
```

output
```
(2*Log[Cosh[x/2]])/a
```

3.159.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 26, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{i\cos\left(-\frac{\pi}{2}+ix\right)}{a-a\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \; 26$$

$$-i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)}{a-a\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 3146$$

$$\frac{\int \frac{1}{\cosh(x)a+a}d(a\cosh(x))}{a}$$

$$\Big\downarrow \; 16$$

$$\frac{\log(a\cosh(x)+a)}{a}$$

input
```
Int[Sinh[x]/(a + a*Cosh[x]),x]
```

output
```
Log[a + a*Cosh[x]]/a
```

3.159.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

3.159.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$\frac{\ln(a+a\cosh(x))}{a}$	12
default	$\frac{\ln(a+a\cosh(x))}{a}$	12
risch	$-\frac{x}{a}+\frac{2\ln(\mathrm{e}^{x}+1)}{a}$	18

input
```
int(sinh(x)/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(a+a*cosh(x))/a
```

3.159.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.78

$$\int \frac{\sinh(x)}{a+a\cosh(x)}\,dx=-\frac{x-2\,\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)}{a}$$

input
```
integrate(sinh(x)/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
-(x - 2*log(cosh(x) + sinh(x) + 1))/a
```

3.159.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{\sinh(x)}{a + a\cosh(x)}\,dx = \frac{\log\left(\cosh\left(x\right) + 1\right)}{a}$$

input
```
integrate(sinh(x)/(a+a*cosh(x)),x)
```

output
```
log(cosh(x) + 1)/a
```

3.159.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{\sinh(x)}{a + a\cosh(x)}\,dx = \frac{\log\left(a\cosh\left(x\right) + a\right)}{a}$$

input
```
integrate(sinh(x)/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
log(a*cosh(x) + a)/a
```

3.159.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.89

$$\int \frac{\sinh(x)}{a + a\cosh(x)}\,dx = -\frac{x}{a} + \frac{2\,\log\left(e^{x} + 1\right)}{a}$$

input
```
integrate(sinh(x)/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
-x/a + 2*log(e^x + 1)/a
```

3.159.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(x)}{a + a\cosh(x)} \, dx = \frac{\ln(\cosh(x) + 1)}{a}$$

input
```
int(sinh(x)/(a + a*cosh(x)),x)
```

output
```
log(cosh(x) + 1)/a
```

3.160 $\int \frac{\operatorname{csch}(x)}{a+a\cosh(x)}\,dx$

3.160.1 Optimal result

Integrand size = 11, antiderivative size = 23

$$\int \frac{\operatorname{csch}(x)}{a+a\cosh(x)}\,dx=-\frac{\operatorname{arctanh}(\cosh(x))}{2a}+\frac{1}{2(a+a\cosh(x))}$$

output
```
-1/2*arctanh(cosh(x))/a+1/2/(a+a*cosh(x))
```

3.160.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{\operatorname{csch}(x)}{a+a\cosh(x)}\,dx=\frac{1-2\cosh^2\left(\frac{x}{2}\right)\left(\log\left(\cosh\left(\frac{x}{2}\right)\right)-\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)}{2a(1+\cosh(x))}$$

input
```
Integrate[Csch[x]/(a + a*Cosh[x]),x]
```

output
```
(1 - 2*Cosh[x/2]^2*(Log[Cosh[x/2]] - Log[Sinh[x/2]]))/(2*a*(1 + Cosh[x]))
```

3.160.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 26, 3146, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i}{\cos\left(-\frac{\pi}{2}+ix\right)\left(a-a\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{1}{\cos\left(ix-\frac{\pi}{2}\right)\left(a-a\sin\left(ix-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{ 3146}$$

$$-a\int \frac{1}{(a-a\cosh(x))(\cosh(x)a+a)^2}d(a\cosh(x))$$

$$\Big\downarrow \text{ 54}$$

$$-a\int \left(\frac{1}{2\left(a^2-a^2\cosh^2(x)\right)a}+\frac{1}{2(\cosh(x)a+a)^2a}\right)d(a\cosh(x))$$

$$\Big\downarrow \text{ 2009}$$

$$-a\left(\frac{\text{arctanh}(\cosh(x))}{2a^2}-\frac{1}{2a(a\cosh(x)+a)}\right)$$

input
```
Int[Csch[x]/(a + a*Cosh[x]),x]
```

output
```
-(a*(ArcTanh[Cosh[x]]/(2*a^2) - 1/(2*a*(a + a*Cosh[x]))))
```

3.160.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

3.160.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{-\frac{\tanh\left(\frac{x}{2}\right)^{2}}{2}+\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2a}$	20
risch	$\frac{e^{x}}{\left(e^{x}+1\right)^{2} a}+\frac{\ln\left(e^{x}-1\right)}{2a}-\frac{\ln\left(e^{x}+1\right)}{2a}$	34

input
```
int(csch(x)/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2/a*(-1/2*tanh(1/2*x)^2+ln(tanh(1/2*x)))
```

3.160.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(19) = 38$.

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 4.48

$$\int \frac{\operatorname{csch}(x)}{a + a\cosh(x)}\,dx = -\frac{\left(\cosh(x)^2 + 2\left(\cosh(x)+1\right)\sinh(x) + \sinh(x)^2 + 2\cosh(x) + 1\right)\log\left(\cosh(x)+\sinh(x)+1\right) - (\cos}{2\left(a\cosh(x)^2 + a\sinh(x)^2 + 2\right.}$$

input
```
integrate(csch(x)/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
-1/2*((cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*lo
g(cosh(x) + sinh(x) + 1) - (cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^
2 + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) - 2*cosh(x) - 2*sinh(x))/(a*
cosh(x)^2 + a*sinh(x)^2 + 2*a*cosh(x) + 2*(a*cosh(x) + a)*sinh(x) + a)
```

3.160.6 Sympy [F]

$$\int \frac{\operatorname{csch}(x)}{a + a\cosh(x)}\,dx = \frac{\int \frac{\operatorname{csch}(x)}{\cosh(x)+1}\,dx}{a}$$

input
```
integrate(csch(x)/(a+a*cosh(x)),x)
```

output
```
Integral(csch(x)/(cosh(x) + 1), x)/a
```

3.160.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(19) = 38$.

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.04

$$\int \frac{\operatorname{csch}(x)}{a + a\cosh(x)}\,dx = \frac{e^{(-x)}}{2\,ae^{(-x)} + ae^{(-2\,x)} + a} - \frac{\log\left(e^{(-x)}+1\right)}{2\,a} + \frac{\log\left(e^{(-x)}-1\right)}{2\,a}$$

input
```
integrate(csch(x)/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
e^(-x)/(2*a*e^(-x) + a*e^(-2*x) + a) - 1/2*log(e^(-x) + 1)/a + 1/2*log(e^(
-x) - 1)/a
```

3.160.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(19) = 38$.

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.26

$$\int \frac{\operatorname{csch}(x)}{a + a\cosh(x)}\,dx = -\frac{\log\left(e^{(-x)} + e^x + 2\right)}{4\,a} + \frac{\log\left(e^{(-x)} + e^x - 2\right)}{4\,a} + \frac{e^{(-x)} + e^x + 6}{4\,a(e^{(-x)} + e^x + 2)}$$

input
```
integrate(csch(x)/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
-1/4*log(e^(-x) + e^x + 2)/a + 1/4*log(e^(-x) + e^x - 2)/a + 1/4*(e^(-x) +
 e^x + 6)/(a*(e^(-x) + e^x + 2))
```

3.160.9 Mupad [B] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.22

$$\int \frac{\operatorname{csch}(x)}{a + a\cosh(x)}\,dx = \frac{1}{a\,\left(e^x + 1\right)} - \frac{1}{a\,\left(e^{2\,x} + 2\,e^x + 1\right)} - \frac{\operatorname{atan}\left(\frac{e^x\,\sqrt{-a^2}}{a}\right)}{\sqrt{-a^2}}$$

input
```
int(1/(sinh(x)*(a + a*cosh(x))),x)
```

output
```
1/(a*(exp(x) + 1)) - 1/(a*(exp(2*x) + 2*exp(x) + 1)) - atan((exp(x)*(-a^2)
^(1/2))/a)/(-a^2)^(1/2)
```

3.161 $\int \frac{\operatorname{csch}^2(x)}{a+a\cosh(x)}\,dx$

3.161.1 Optimal result

Integrand size = 13, antiderivative size = 24

$$\int \frac{\operatorname{csch}^2(x)}{a+a\cosh(x)}\,dx=-\frac{2\coth(x)}{3a}+\frac{\operatorname{csch}(x)}{3(a+a\cosh(x))}$$

output
```
-2/3*coth(x)/a+1/3*csch(x)/(a+a*cosh(x))
```

3.161.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

$$\int \frac{\operatorname{csch}^2(x)}{a+a\cosh(x)}\,dx=-\frac{(2\cosh(x)+\cosh(2x))\operatorname{csch}\left(\frac{x}{2}\right)\operatorname{sech}^3\left(\frac{x}{2}\right)}{12a}$$

input
```
Integrate[Csch[x]^2/(a + a*Cosh[x]),x]
```

output
```
-1/12*((2*Cosh[x] + Cosh[2*x])*Csch[x/2]*Sech[x/2]^3)/a
```

3.161.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {3042, 25, 3151, 25, 3042, 25, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{1}{\cos\left(-\frac{\pi}{2}+ix\right)^2\left(a-a\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \ 25$$

$$-\int \frac{1}{\cos\left(ix-\frac{\pi}{2}\right)^2\left(a-a\sin\left(ix-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \ 3151$$

$$\frac{\operatorname{csch}(x)}{3(a\cosh(x)+a)}-\frac{2\int -\operatorname{csch}^2(x)dx}{3a}$$

$$\Big\downarrow \ 25$$

$$\frac{2\int \operatorname{csch}^2(x)dx}{3a}+\frac{\operatorname{csch}(x)}{3(a\cosh(x)+a)}$$

$$\Big\downarrow \ 3042$$

$$\frac{\operatorname{csch}(x)}{3(a\cosh(x)+a)}+\frac{2\int -\csc(ix)^2dx}{3a}$$

$$\Big\downarrow \ 25$$

$$\frac{\operatorname{csch}(x)}{3(a\cosh(x)+a)}-\frac{2\int \csc(ix)^2dx}{3a}$$

$$\Big\downarrow \ 4254$$

$$\frac{\operatorname{csch}(x)}{3(a\cosh(x)+a)}-\frac{2i\int 1d(-i\coth(x))}{3a}$$

$$\Big\downarrow \ 24$$

$$\frac{\operatorname{csch}(x)}{3(a\cosh(x)+a)}-\frac{2\coth(x)}{3a}$$

input
```
Int[Csch[x]^2/(a + a*Cosh[x]),x]
```

output
```
(-2*Coth[x])/(3*a) + Csch[x]/(3*(a + a*Cosh[x]))
```

3.161.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.161.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

method	result	size
risch	$-\frac{4(2\,\mathrm{e}^{x}+1)}{3(\mathrm{e}^{x}-1)a(\mathrm{e}^{x}+1)^{3}}$	24
default	$\frac{\frac{\tanh\left(\frac{x}{2}\right)^{3}}{3}-2\tanh\left(\frac{x}{2}\right)-\frac{1}{\tanh\left(\frac{x}{2}\right)}}{4a}$	29

input
```
int(csch(x)^2/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-4/3*(2*exp(x)+1)/(exp(x)-1)/a/(exp(x)+1)^3
```

3.161.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(20) = 40$.

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 3.92

$$\int \frac{\operatorname{csch}^2(x)}{a+a\cosh(x)}\,dx =$$
$$-\frac{4\,(2\,\cosh{(x)}+2\,\sinh{(x)}+1}{3\,\left(a\cosh{(x)}^4+a\sinh{(x)}^4+2\,a\cosh{(x)}^3+2\,(2\,a\cosh{(x)}+a)\sinh{(x)}^3+6\,\left(a\cosh{(x)}^2+a\cosh{(x)}\right)}$$

input
```
integrate(csch(x)^2/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
-4/3*(2*cosh(x) + 2*sinh(x) + 1)/(a*cosh(x)^4 + a*sinh(x)^4 + 2*a*cosh(x)^
3 + 2*(2*a*cosh(x) + a)*sinh(x)^3 + 6*(a*cosh(x)^2 + a*cosh(x))*sinh(x)^2
- 2*a*cosh(x) + 2*(2*a*cosh(x)^3 + 3*a*cosh(x)^2 - a)*sinh(x) - a)
```

3.161.6 Sympy [F]

$$\int \frac{\operatorname{csch}^2(x)}{a+a\cosh(x)}\,dx = \frac{\int \frac{\operatorname{csch}^2{(x)}}{\cosh{(x)}+1}\,dx}{a}$$

input
```
integrate(csch(x)**2/(a+a*cosh(x)),x)
```

output
```
Integral(csch(x)**2/(cosh(x) + 1), x)/a
```

3.161.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(20) = 40.

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.46

$$\int \frac{\operatorname{csch}^2(x)}{a + a\cosh(x)}\,dx = -\frac{8\,e^{(-x)}}{3\,(2\,ae^{(-x)} - 2\,ae^{(-3\,x)} - ae^{(-4\,x)} + a)} - \frac{4}{3\,(2\,ae^{(-x)} - 2\,ae^{(-3\,x)} - ae^{(-4\,x)} + a)}$$

input
```
integrate(csch(x)^2/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-8/3*e^(-x)/(2*a*e^(-x) - 2*a*e^(-3*x) - a*e^(-4*x) + a) - 4/3/(2*a*e^(-x)
 - 2*a*e^(-3*x) - a*e^(-4*x) + a)
```

3.161.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{\operatorname{csch}^2(x)}{a + a\cosh(x)}\,dx = -\frac{1}{2\,a(e^x - 1)} + \frac{3\,e^{(2\,x)} + 12\,e^x + 5}{6\,a(e^x + 1)^3}$$

input
```
integrate(csch(x)^2/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
-1/2/(a*(e^x - 1)) + 1/6*(3*e^(2*x) + 12*e^x + 5)/(a*(e^x + 1)^3)
```

3.161.9 Mupad [B] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 89, normalized size of antiderivative = 3.71

$$\int \frac{\operatorname{csch}^2(x)}{a + a\cosh(x)}\,dx = \frac{\frac{e^{2\,x}}{6\,a} + \frac{1}{6\,a} + \frac{e^x}{a}}{3\,e^{2\,x} + e^{3\,x} + 3\,e^x + 1} + \frac{\frac{1}{2\,a} + \frac{e^x}{6\,a}}{e^{2\,x} + 2\,e^x + 1} - \frac{1}{2\,a\,(e^x - 1)} + \frac{1}{6\,a\,(e^x + 1)}$$

input
```
int(1/(sinh(x)^2*(a + a*cosh(x))),x)
```

output
```
(exp(2*x)/(6*a) + 1/(6*a) + exp(x)/a)/(3*exp(2*x) + exp(3*x) + 3*exp(x) +
1) + (1/(2*a) + exp(x)/(6*a))/(exp(2*x) + 2*exp(x) + 1) - 1/(2*a*(exp(x) -
 1)) + 1/(6*a*(exp(x) + 1))
```

3.162 $\int \frac{\operatorname{csch}^3(x)}{a+a\cosh(x)}\,dx$

3.162.1 Optimal result

Integrand size = 13, antiderivative size = 49

$$\int \frac{\operatorname{csch}^3(x)}{a+a\cosh(x)}\,dx = \frac{3\operatorname{arctanh}(\cosh(x))}{8a} + \frac{1}{8(a-a\cosh(x))} - \frac{a}{8(a+a\cosh(x))^2} - \frac{1}{4(a+a\cosh(x))}$$

output
```
3/8*arctanh(cosh(x))/a+1/8/(a-a*cosh(x))-1/8*a/(a+a*cosh(x))^2-1/4/(a+a*co
sh(x))
```

3.162.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{csch}^3(x)}{a+a\cosh(x)}\,dx = -\frac{4+2\coth^2\left(\frac{x}{2}\right)-12\cosh^2\left(\frac{x}{2}\right)\left(\log\left(\cosh\left(\frac{x}{2}\right)\right)-\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)+\operatorname{sech}^2\left(\frac{x}{2}\right)}{16a(1+\cosh(x))}$$

input
```
Integrate[Csch[x]^3/(a + a*Cosh[x]),x]
```

output
```
-1/16*(4 + 2*Coth[x/2]^2 - 12*Cosh[x/2]^2*(Log[Cosh[x/2]] - Log[Sinh[x/2]]
) + Sech[x/2]^2)/(a*(1 + Cosh[x]))
```

3.162.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.24, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 26, 3146, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\cos\left(-\frac{\pi}{2}+ix\right)^3\left(a-a\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\cos\left(ix-\frac{\pi}{2}\right)^3\left(a-a\sin\left(ix-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3146}$$

$$a^3\int \frac{1}{(a-a\cosh(x))^2(\cosh(x)a+a)^3}d(a\cosh(x))$$

$$\Big\downarrow \text{54}$$

$$a^3\int \left(\frac{1}{8a^3(a-a\cosh(x))^2}+\frac{1}{4a^3(\cosh(x)a+a)^2}+\frac{1}{4a^2(\cosh(x)a+a)^3}+\frac{3}{8a^3\left(a^2-a^2\cosh^2(x)\right)}\right)d(a\cosh(x))$$

$$\Big\downarrow \text{2009}$$

$$a^3\left(\frac{3\text{arctanh}(\cosh(x))}{8a^4}+\frac{1}{8a^3(a-a\cosh(x))}-\frac{1}{4a^3(a\cosh(x)+a)}-\frac{1}{8a^2(a\cosh(x)+a)^2}\right)$$

input
```
Int[Csch[x]^3/(a + a*Cosh[x]),x]
```

output
```
a^3*((3*ArcTanh[Cosh[x]])/(8*a^4) + 1/(8*a^3*(a - a*Cosh[x])) - 1/(8*a^2*(
a + a*Cosh[x])^2) - 1/(4*a^3*(a + a*Cosh[x])))
```

3.162.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

3.162.4 Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{-\frac{\tanh\left(\frac{x}{2}\right)^{4}}{4}+\frac{3\tanh\left(\frac{x}{2}\right)^{2}}{2}-3\ln\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{1}{2\tanh\left(\frac{x}{2}\right)^{2}}}{8a}$	38
risch	$-\frac{e^{x}\left(3\,e^{4x}+6\,e^{3x}-2\,e^{2x}+6\,e^{x}+3\right)}{4\left(e^{x}-1\right)^{2}a\left(e^{x}+1\right)^{4}}+\frac{3\ln\left(e^{x}+1\right)}{8a}-\frac{3\ln\left(e^{x}-1\right)}{8a}$	65

input
```
int(csch(x)^3/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/8/a*(-1/4*tanh(1/2*x)^4+3/2*tanh(1/2*x)^2-3*ln(tanh(1/2*x))-1/2/tanh(1/2
*x)^2)
```

3.162.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(42) = 84$.

Time = 0.26 (sec) , antiderivative size = 631, normalized size of antiderivative = 12.88

$$\int \frac{\operatorname{csch}^3(x)}{a + a\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(x)^3/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
-1/8*(6*cosh(x)^5 + 6*(5*cosh(x) + 2)*sinh(x)^4 + 6*sinh(x)^5 + 12*cosh(x)
^4 + 4*(15*cosh(x)^2 + 12*cosh(x) - 1)*sinh(x)^3 - 4*cosh(x)^3 + 12*(5*cos
h(x)^3 + 6*cosh(x)^2 - cosh(x) + 1)*sinh(x)^2 + 12*cosh(x)^2 - 3*(cosh(x)^
6 + 2*(3*cosh(x) + 1)*sinh(x)^5 + sinh(x)^6 + 2*cosh(x)^5 + (15*cosh(x)^2
+ 10*cosh(x) - 1)*sinh(x)^4 - cosh(x)^4 + 4*(5*cosh(x)^3 + 5*cosh(x)^2 - c
osh(x) - 1)*sinh(x)^3 - 4*cosh(x)^3 + (15*cosh(x)^4 + 20*cosh(x)^3 - 6*cos
h(x)^2 - 12*cosh(x) - 1)*sinh(x)^2 - cosh(x)^2 + 2*(3*cosh(x)^5 + 5*cosh(x
)^4 - 2*cosh(x)^3 - 6*cosh(x)^2 - cosh(x) + 1)*sinh(x) + 2*cosh(x) + 1)*lo
g(cosh(x) + sinh(x) + 1) + 3*(cosh(x)^6 + 2*(3*cosh(x) + 1)*sinh(x)^5 + si
nh(x)^6 + 2*cosh(x)^5 + (15*cosh(x)^2 + 10*cosh(x) - 1)*sinh(x)^4 - cosh(x
)^4 + 4*(5*cosh(x)^3 + 5*cosh(x)^2 - cosh(x) - 1)*sinh(x)^3 - 4*cosh(x)^3
+ (15*cosh(x)^4 + 20*cosh(x)^3 - 6*cosh(x)^2 - 12*cosh(x) - 1)*sinh(x)^2 -
 cosh(x)^2 + 2*(3*cosh(x)^5 + 5*cosh(x)^4 - 2*cosh(x)^3 - 6*cosh(x)^2 - co
sh(x) + 1)*sinh(x) + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 6*(5*cosh
(x)^4 + 8*cosh(x)^3 - 2*cosh(x)^2 + 4*cosh(x) + 1)*sinh(x) + 6*cosh(x))/(a
*cosh(x)^6 + a*sinh(x)^6 + 2*a*cosh(x)^5 + 2*(3*a*cosh(x) + a)*sinh(x)^5 -
 a*cosh(x)^4 + (15*a*cosh(x)^2 + 10*a*cosh(x) - a)*sinh(x)^4 - 4*a*cosh(x)
^3 + 4*(5*a*cosh(x)^3 + 5*a*cosh(x)^2 - a*cosh(x) - a)*sinh(x)^3 - a*cosh(
x)^2 + (15*a*cosh(x)^4 + 20*a*cosh(x)^3 - 6*a*cosh(x)^2 - 12*a*cosh(x) - a
)*sinh(x)^2 + 2*a*cosh(x) + 2*(3*a*cosh(x)^5 + 5*a*cosh(x)^4 - 2*a*cosh...
```

3.162.6 Sympy [F]

$$\int \frac{\operatorname{csch}^3(x)}{a + a\cosh(x)}\,dx = \frac{\int \frac{\operatorname{csch}^3(x)}{\cosh(x)+1}\,dx}{a}$$

input
```
integrate(csch(x)**3/(a+a*cosh(x)),x)
```

output
```
Integral(csch(x)**3/(cosh(x) + 1), x)/a
```

3.162.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(42) = 84$.

Time = 0.19 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.10

$$\int \frac{\operatorname{csch}^3(x)}{a + a\cosh(x)}\,dx = -\frac{3\,e^{(-x)} + 6\,e^{(-2\,x)} - 2\,e^{(-3\,x)} + 6\,e^{(-4\,x)} + 3\,e^{(-5\,x)}}{4\,(2\,ae^{(-x)} - ae^{(-2\,x)} - 4\,ae^{(-3\,x)} - ae^{(-4\,x)} + 2\,ae^{(-5\,x)} + ae^{(-6\,x)} + a)} + \frac{3\,\log\left(e^{(-x)} + 1\right)}{8\,a} - \frac{3\,\log\left(e^{(-x)} - 1\right)}{8\,a}$$

input
```
integrate(csch(x)^3/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-1/4*(3*e^(-x) + 6*e^(-2*x) - 2*e^(-3*x) + 6*e^(-4*x) + 3*e^(-5*x))/(2*a*e
^(-x) - a*e^(-2*x) - 4*a*e^(-3*x) - a*e^(-4*x) + 2*a*e^(-5*x) + a*e^(-6*x)
 + a) + 3/8*log(e^(-x) + 1)/a - 3/8*log(e^(-x) - 1)/a
```

3.162.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(42) = 84$.

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.92

$$\int \frac{\operatorname{csch}^3(x)}{a + a\cosh(x)}\,dx = \frac{3\,\log\left(e^{(-x)} + e^x + 2\right)}{16\,a} - \frac{3\,\log\left(e^{(-x)} + e^x - 2\right)}{16\,a} + \frac{3\,e^{(-x)} + 3\,e^x - 10}{16\,a(e^{(-x)} + e^x - 2)} - \frac{9\,\left(e^{(-x)} + e^x\right)^2 + 52\,e^{(-x)} + 52\,e^x + 84}{32\,a(e^{(-x)} + e^x + 2)^2}$$

input
```
integrate(csch(x)^3/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
3/16*log(e^(-x) + e^x + 2)/a - 3/16*log(e^(-x) + e^x - 2)/a + 1/16*(3*e^(-
x) + 3*e^x - 10)/(a*(e^(-x) + e^x - 2)) - 1/32*(9*(e^(-x) + e^x)^2 + 52*e^
(-x) + 52*e^x + 84)/(a*(e^(-x) + e^x + 2)^2)
```

3.162.9 Mupad [B] (verification not implemented)

Time = 1.67 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.33

$$\int \frac{\operatorname{csch}^3(x)}{a+a\cosh(x)}\,dx = \frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^x\,\sqrt{-a^2}}{a}\right)}{4\,\sqrt{-a^2}} - \frac{1}{2\,a\,\left(6\,\mathrm{e}^{2\,x}+4\,\mathrm{e}^{3\,x}+\mathrm{e}^{4\,x}+4\,\mathrm{e}^x+1\right)} - \frac{1}{4\,a\,\left(\mathrm{e}^x-1\right)} - \frac{1}{2\,a\,\left(\mathrm{e}^x+1\right)} - \frac{1}{4\,a\,\left(\mathrm{e}^{2\,x}-2\,\mathrm{e}^x+1\right)} + \frac{1}{a\,\left(3\,\mathrm{e}^{2\,x}+\mathrm{e}^{3\,x}+3\,\mathrm{e}^x+1\right)}$$

input

```
int(1/(sinh(x)^3*(a + a*cosh(x))),x)
```

output

```
(3*atan((exp(x)*(-a^2)^(1/2))/a))/(4*(-a^2)^(1/2)) - 1/(2*a*(6*exp(2*x) +
4*exp(3*x) + exp(4*x) + 4*exp(x) + 1)) - 1/(4*a*(exp(x) - 1)) - 1/(2*a*(ex
p(x) + 1)) - 1/(4*a*(exp(2*x) - 2*exp(x) + 1)) + 1/(a*(3*exp(2*x) + exp(3*
x) + 3*exp(x) + 1))
```

3.163 $\int \frac{\operatorname{csch}^4(x)}{a+a\cosh(x)}\,dx$

3.163.1 Optimal result

Integrand size = 13, antiderivative size = 37

$$\int \frac{\operatorname{csch}^4(x)}{a+a\cosh(x)}\,dx = \frac{4\coth(x)}{5a} - \frac{4\coth^3(x)}{15a} + \frac{\operatorname{csch}^3(x)}{5(a+a\cosh(x))}$$

output
```
4/5*coth(x)/a-4/15*coth(x)^3/a+1/5*csch(x)^3/(a+a*cosh(x))
```

3.163.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{csch}^4(x)}{a+a\cosh(x)}\,dx = \frac{(-6\cosh(x)-2\cosh(2x)+2\cosh(3x)+\cosh(4x))\operatorname{csch}^3(x)}{15a(1+\cosh(x))}$$

input
```
Integrate[Csch[x]^4/(a + a*Cosh[x]),x]
```

output
```
((-6*Cosh[x] - 2*Cosh[2*x] + 2*Cosh[3*x] + Cosh[4*x])*Csch[x]^3)/(15*a*(1
+ Cosh[x]))
```

3.163.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 3151, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^4(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos\left(-\frac{\pi}{2}+ix\right)^4\left(a-a\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{3151}$$

$$\frac{4\int \operatorname{csch}^4(x)dx}{5a}+\frac{\operatorname{csch}^3(x)}{5(a\cosh(x)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\operatorname{csch}^3(x)}{5(a\cosh(x)+a)}+\frac{4\int \csc(ix)^4dx}{5a}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\operatorname{csch}^3(x)}{5(a\cosh(x)+a)}+\frac{4i\int \left(1-\coth^2(x)\right)d(-i\coth(x))}{5a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\operatorname{csch}^3(x)}{5(a\cosh(x)+a)}+\frac{4i\left(\frac{1}{3}i\coth^3(x)-i\coth(x)\right)}{5a}$$

input
```
Int[Csch[x]^4/(a + a*Cosh[x]),x]
```

output
```
(((4*I)/5)*((-I)*Coth[x] + (I/3)*Coth[x]^3))/a + Csch[x]^3/(5*(a + a*Cosh[
x]))
```

3.163.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.163.4 Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

method	result	size
risch	$-\frac{16\left(6\,{\mathrm e}^{3x}+2\,{\mathrm e}^{2x}-2\,{\mathrm e}^{x}-1\right)}{15a\left({\mathrm e}^{x}+1\right)^{5}\left({\mathrm e}^{x}-1\right)^{3}}$	36
default	$\frac{\frac{\tanh\left(\frac{x}{2}\right)^{5}}{5}-\frac{4\tanh\left(\frac{x}{2}\right)^{3}}{3}+6\tanh\left(\frac{x}{2}\right)+\frac{4}{\tanh\left(\frac{x}{2}\right)}-\frac{1}{3\tanh\left(\frac{x}{2}\right)^{3}}}{16a}$	45

input
```
int(csch(x)^4/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-16/15*(6*exp(3*x)+2*exp(2*x)-2*exp(x)-1)/a/(exp(x)+1)^5/(exp(x)-1)^3
```

3.163.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(31) = 62$.

Time = 0.25 (sec) , antiderivative size = 250, normalized size of antiderivative = 6.76

$$\int \frac{\operatorname{csch}^4(x)}{a + a\cosh(x)}\,dx =$$

$$-\frac{}{15\left(a\cosh\left(x\right)^{7} + a\sinh\left(x\right)^{7} + 2\,a\cosh\left(x\right)^{6} + \left(7\,a\cosh\left(x\right) + 2\,a\right)\sinh\left(x\right)^{6} - 2\,a\cosh\left(x\right)^{5} + \left(21\,a\cosh\right.\right.}$$

input
```
integrate(csch(x)^4/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
-16/15*(6*cosh(x)^2 + 3*(4*cosh(x) + 1)*sinh(x) + 6*sinh(x)^2 + cosh(x) -
2)/(a*cosh(x)^7 + a*sinh(x)^7 + 2*a*cosh(x)^6 + (7*a*cosh(x) + 2*a)*sinh(x
)^6 - 2*a*cosh(x)^5 + (21*a*cosh(x)^2 + 12*a*cosh(x) - 2*a)*sinh(x)^5 - 6*
a*cosh(x)^4 + (35*a*cosh(x)^3 + 30*a*cosh(x)^2 - 10*a*cosh(x) - 6*a)*sinh(
x)^4 + (35*a*cosh(x)^4 + 40*a*cosh(x)^3 - 20*a*cosh(x)^2 - 24*a*cosh(x))*s
inh(x)^3 + 6*a*cosh(x)^2 + (21*a*cosh(x)^5 + 30*a*cosh(x)^4 - 20*a*cosh(x)
^3 - 36*a*cosh(x)^2 + 6*a)*sinh(x)^2 + a*cosh(x) + (7*a*cosh(x)^6 + 12*a*c
osh(x)^5 - 10*a*cosh(x)^4 - 24*a*cosh(x)^3 + 12*a*cosh(x) + 3*a)*sinh(x) -
 2*a)
```

3.163.6 Sympy [F]

$$\int \frac{\operatorname{csch}^4(x)}{a + a\cosh(x)}\,dx = \frac{\int \frac{\operatorname{csch}^4(x)}{\cosh(x)+1}\,dx}{a}$$

input
```
integrate(csch(x)**4/(a+a*cosh(x)),x)
```

output
```
Integral(csch(x)**4/(cosh(x) + 1), x)/a
```

3.163.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(31) = 62$.

Time = 0.19 (sec) , antiderivative size = 233, normalized size of antiderivative = 6.30

$$\int \frac{\operatorname{csch}^4(x)}{a+a\cosh(x)}\,dx = \frac{32\,e^{(-x)}}{15\,(2\,ae^{(-x)}-2\,ae^{(-2\,x)}-6\,ae^{(-3\,x)}+6\,ae^{(-5\,x)}+2\,ae^{(-6\,x)}-2\,ae^{(-7\,x)}-ae^{(-8\,x)}+a)} - \frac{32\,e^{(-2\,x)}}{15\,(2\,ae^{(-x)}-2\,ae^{(-2\,x)}-6\,ae^{(-3\,x)}+6\,ae^{(-5\,x)}+2\,ae^{(-6\,x)}-2\,ae^{(-7\,x)}-ae^{(-8\,x)}+a)} - \frac{32\,e^{(-3\,x)}}{5\,(2\,ae^{(-x)}-2\,ae^{(-2\,x)}-6\,ae^{(-3\,x)}+6\,ae^{(-5\,x)}+2\,ae^{(-6\,x)}-2\,ae^{(-7\,x)}-ae^{(-8\,x)}+a)} + \frac{16}{15\,(2\,ae^{(-x)}-2\,ae^{(-2\,x)}-6\,ae^{(-3\,x)}+6\,ae^{(-5\,x)}+2\,ae^{(-6\,x)}-2\,ae^{(-7\,x)}-ae^{(-8\,x)}+a)}$$

input
```
integrate(csch(x)^4/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
32/15*e^(-x)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*
a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a) - 32/15*e^(-2*x)/(2*a*e^(-x) -
 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x)
- a*e^(-8*x) + a) - 32/5*e^(-3*x)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x
) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a) + 16/15/(
2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2
*a*e^(-7*x) - a*e^(-8*x) + a)
```

3.163.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.59

$$\int \frac{\operatorname{csch}^4(x)}{a+a\cosh(x)}\,dx = \frac{9\,e^{(2\,x)}-24\,e^{x}+11}{24\,a(e^{x}-1)^3} - \frac{45\,e^{(4\,x)}+240\,e^{(3\,x)}+490\,e^{(2\,x)}+320\,e^{x}+73}{120\,a(e^{x}+1)^5}$$

input
```
integrate(csch(x)^4/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
1/24*(9*e^(2*x) - 24*e^x + 11)/(a*(e^x - 1)^3) - 1/120*(45*e^(4*x) + 240*e
^(3*x) + 490*e^(2*x) + 320*e^x + 73)/(a*(e^x + 1)^5)
```

3.163.9 Mupad [B] (verification not implemented)

Time = 1.67 (sec) , antiderivative size = 263, normalized size of antiderivative = 7.11

$$\int \frac{\operatorname{csch}^4(x)}{a+a\cosh(x)}\,dx = \frac{1}{6\,a\,\left(3\,{\mathrm{e}}^{2\,x}-{\mathrm{e}}^{3\,x}-3\,{\mathrm{e}}^{x}+1\right)}-\frac{\frac{3\,{\mathrm{e}}^{2\,x}}{8\,a}+\frac{3\,{\mathrm{e}}^{3\,x}}{40\,a}+\frac{1}{8\,a}+\frac{5\,{\mathrm{e}}^{x}}{8\,a}}{6\,{\mathrm{e}}^{2\,x}+4\,{\mathrm{e}}^{3\,x}+{\mathrm{e}}^{4\,x}+4\,{\mathrm{e}}^{x}+1}-\frac{\frac{3\,{\mathrm{e}}^{2\,x}}{40\,a}+\frac{5}{24\,a}+\frac{{\mathrm{e}}^{x}}{4\,a}}{3\,{\mathrm{e}}^{2\,x}+{\mathrm{e}}^{3\,x}+3\,{\mathrm{e}}^{x}+1}-\frac{\frac{1}{8\,a}+\frac{3\,{\mathrm{e}}^{x}}{40\,a}}{{\mathrm{e}}^{2\,x}+2\,{\mathrm{e}}^{x}+1}-\frac{\frac{5\,{\mathrm{e}}^{2\,x}}{4\,a}+\frac{{\mathrm{e}}^{3\,x}}{2\,a}+\frac{3\,{\mathrm{e}}^{4\,x}}{40\,a}+\frac{3}{40\,a}+\frac{{\mathrm{e}}^{x}}{2\,a}}{10\,{\mathrm{e}}^{2\,x}+10\,{\mathrm{e}}^{3\,x}+5\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{5\,x}+5\,{\mathrm{e}}^{x}+1}-\frac{1}{4\,a\,\left({\mathrm{e}}^{2\,x}-2\,{\mathrm{e}}^{x}+1\right)}+\frac{3}{8\,a\,\left({\mathrm{e}}^{x}-1\right)}-\frac{3}{40\,a\,\left({\mathrm{e}}^{x}+1\right)}$$

input
```
int(1/(sinh(x)^4*(a + a*cosh(x))),x)
```

output
```
1/(6*a*(3*exp(2*x) - exp(3*x) - 3*exp(x) + 1)) - ((3*exp(2*x))/(8*a) + (3*
exp(3*x))/(40*a) + 1/(8*a) + (5*exp(x))/(8*a))/(6*exp(2*x) + 4*exp(3*x) +
exp(4*x) + 4*exp(x) + 1) - ((3*exp(2*x))/(40*a) + 5/(24*a) + exp(x)/(4*a))
/(3*exp(2*x) + exp(3*x) + 3*exp(x) + 1) - (1/(8*a) + (3*exp(x))/(40*a))/(e
xp(2*x) + 2*exp(x) + 1) - ((5*exp(2*x))/(4*a) + exp(3*x)/(2*a) + (3*exp(4*
x))/(40*a) + 3/(40*a) + exp(x)/(2*a))/(10*exp(2*x) + 10*exp(3*x) + 5*exp(4
*x) + exp(5*x) + 5*exp(x) + 1) - 1/(4*a*(exp(2*x) - 2*exp(x) + 1)) + 3/(8*
a*(exp(x) - 1)) - 3/(40*a*(exp(x) + 1))
```

3.164 $\int \frac{\operatorname{csch}^5(x)}{a+a\cosh(x)}\,dx$

3.164.1 Optimal result

Integrand size = 13, antiderivative size = 78

$$\int \frac{\operatorname{csch}^5(x)}{a+a\cosh(x)}\,dx = -\frac{5\operatorname{arctanh}(\cosh(x))}{16a} - \frac{a}{32(a-a\cosh(x))^2} - \frac{1}{8(a-a\cosh(x))} + \frac{a^2}{24(a+a\cosh(x))^3} + \frac{3a}{32(a+a\cosh(x))^2} + \frac{3}{16(a+a\cosh(x))}$$

output
```
-5/16*arctanh(cosh(x))/a-1/32*a/(a-a*cosh(x))^2-1/8/(a-a*cosh(x))+1/24*a^2
/(a+a*cosh(x))^3+3/32*a/(a+a*cosh(x))^2+3/16/(a+a*cosh(x))
```

3.164.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{csch}^5(x)}{a+a\cosh(x)}\,dx = \frac{\cosh^2\left(\frac{x}{2}\right)\left(24\operatorname{csch}^2\left(\frac{x}{2}\right) - 3\operatorname{csch}^4\left(\frac{x}{2}\right) - 120\log\left(\cosh\left(\frac{x}{2}\right)\right) + 120\log\left(\sinh\left(\frac{x}{2}\right)\right) + 36\operatorname{sech}^2\left(\frac{x}{2}\right) + 9\operatorname{sech}^4\left(\frac{x}{2}\right)\right.}{192(a+a\cosh(x))}$$

input
```
Integrate[Csch[x]^5/(a + a*Cosh[x]),x]
```

output
```
(Cosh[x/2]^2*(24*Csch[x/2]^2 - 3*Csch[x/2]^4 - 120*Log[Cosh[x/2]] + 120*Lo
g[Sinh[x/2]] + 36*Sech[x/2]^2 + 9*Sech[x/2]^4 + 2*Sech[x/2]^6))/(192*(a +
a*Cosh[x]))
```

3.164.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 26, 3146, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^5(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\cos\left(-\frac{\pi}{2}+ix\right)^5\left(a-a\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\cos\left(ix-\frac{\pi}{2}\right)^5\left(a-a\sin\left(ix-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3146}$$

$$-a^5\int \frac{1}{(a-a\cosh(x))^3(\cosh(x)a+a)^4}d(a\cosh(x))$$

$$\Big\downarrow \text{54}$$

$$-a^5\int \left(\frac{1}{8a^5(a-a\cosh(x))^2}+\frac{3}{16a^5(\cosh(x)a+a)^2}+\frac{1}{16a^4(a-a\cosh(x))^3}+\frac{3}{16a^4(\cosh(x)a+a)^3}+\frac{}{8a^3(\cosh(\right.$$

$$\Big\downarrow \text{2009}$$

$$-a^5\left(\frac{5\text{arctanh}(\cosh(x))}{16a^6}+\frac{1}{8a^5(a-a\cosh(x))}-\frac{3}{16a^5(a\cosh(x)+a)}+\frac{1}{32a^4(a-a\cosh(x))^2}-\frac{3}{32a^4(a\cosh(x)\right.$$

input
```
Int[Csch[x]^5/(a + a*Cosh[x]),x]
```

output
```
-(a^5*((5*ArcTanh[Cosh[x]])/(16*a^6) + 1/(32*a^4*(a - a*Cosh[x])^2) + 1/(8
*a^5*(a - a*Cosh[x])) - 1/(24*a^3*(a + a*Cosh[x])^3) - 3/(32*a^4*(a + a*Co
sh[x])^2) - 3/(16*a^5*(a + a*Cosh[x]))))
```

3.164.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

3.164.4 Maple [A] (verified)

Time = 5.89 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{-\frac{\tanh\left(\frac{x}{2}\right)^{6}}{6}+\frac{5\tanh\left(\frac{x}{2}\right)^{4}}{4}-5\tanh\left(\frac{x}{2}\right)^{2}+10\ln\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{1}{4\tanh\left(\frac{x}{2}\right)^{4}}+\frac{5}{2\tanh\left(\frac{x}{2}\right)^{2}}}{32a}$	54
risch	$\frac{e^{x}\left(15\,e^{8x}+30\,e^{7x}-40\,e^{6x}-110\,e^{5x}+18\,e^{4x}-110\,e^{3x}-40\,e^{2x}+30\,e^{x}+15\right)}{24\left(e^{x}+1\right)^{6}a\left(e^{x}-1\right)^{4}}+\frac{5\ln\left(e^{x}-1\right)}{16a}-\frac{5\ln\left(e^{x}+1\right)}{16a}$	89

input
```
int(csch(x)^5/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/32/a*(-1/6*tanh(1/2*x)^6+5/4*tanh(1/2*x)^4-5*tanh(1/2*x)^2+10*ln(tanh(1/
2*x))-1/4/tanh(1/2*x)^4+5/2/tanh(1/2*x)^2)
```

3.164.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1551 vs. $2(68) = 136$.

Time = 0.27 (sec) , antiderivative size = 1551, normalized size of antiderivative = 19.88

$$\int \frac{\operatorname{csch}^5(x)}{a + a\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(x)^5/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
1/48*(30*cosh(x)^9 + 30*(9*cosh(x) + 2)*sinh(x)^8 + 30*sinh(x)^9 + 60*cosh
(x)^8 + 40*(27*cosh(x)^2 + 12*cosh(x) - 2)*sinh(x)^7 - 80*cosh(x)^7 + 20*(
126*cosh(x)^3 + 84*cosh(x)^2 - 28*cosh(x) - 11)*sinh(x)^6 - 220*cosh(x)^6
+ 12*(315*cosh(x)^4 + 280*cosh(x)^3 - 140*cosh(x)^2 - 110*cosh(x) + 3)*sin
h(x)^5 + 36*cosh(x)^5 + 20*(189*cosh(x)^5 + 210*cosh(x)^4 - 140*cosh(x)^3
- 165*cosh(x)^2 + 9*cosh(x) - 11)*sinh(x)^4 - 220*cosh(x)^4 + 40*(63*cosh(
x)^6 + 84*cosh(x)^5 - 70*cosh(x)^4 - 110*cosh(x)^3 + 9*cosh(x)^2 - 22*cosh
(x) - 2)*sinh(x)^3 - 80*cosh(x)^3 + 60*(18*cosh(x)^7 + 28*cosh(x)^6 - 28*c
osh(x)^5 - 55*cosh(x)^4 + 6*cosh(x)^3 - 22*cosh(x)^2 - 4*cosh(x) + 1)*sinh
(x)^2 + 60*cosh(x)^2 - 15*(cosh(x)^10 + 2*(5*cosh(x) + 1)*sinh(x)^9 + sinh
(x)^10 + 2*cosh(x)^9 + 3*(15*cosh(x)^2 + 6*cosh(x) - 1)*sinh(x)^8 - 3*cosh
(x)^8 + 8*(15*cosh(x)^3 + 9*cosh(x)^2 - 3*cosh(x) - 1)*sinh(x)^7 - 8*cosh(
x)^7 + 2*(105*cosh(x)^4 + 84*cosh(x)^3 - 42*cosh(x)^2 - 28*cosh(x) + 1)*si
nh(x)^6 + 2*cosh(x)^6 + 12*(21*cosh(x)^5 + 21*cosh(x)^4 - 14*cosh(x)^3 - 1
4*cosh(x)^2 + cosh(x) + 1)*sinh(x)^5 + 12*cosh(x)^5 + 2*(105*cosh(x)^6 + 1
26*cosh(x)^5 - 105*cosh(x)^4 - 140*cosh(x)^3 + 15*cosh(x)^2 + 30*cosh(x) +
 1)*sinh(x)^4 + 2*cosh(x)^4 + 8*(15*cosh(x)^7 + 21*cosh(x)^6 - 21*cosh(x)^
5 - 35*cosh(x)^4 + 5*cosh(x)^3 + 15*cosh(x)^2 + cosh(x) - 1)*sinh(x)^3 - 8
*cosh(x)^3 + 3*(15*cosh(x)^8 + 24*cosh(x)^7 - 28*cosh(x)^6 - 56*cosh(x)^5
+ 10*cosh(x)^4 + 40*cosh(x)^3 + 4*cosh(x)^2 - 8*cosh(x) - 1)*sinh(x)^2 ...
```

3.164.6 Sympy [F]

$$\int \frac{\operatorname{csch}^5(x)}{a + a\cosh(x)}\,dx = \frac{\int \frac{\operatorname{csch}^5(x)}{\cosh(x)+1}\,dx}{a}$$

input
```
integrate(csch(x)**5/(a+a*cosh(x)),x)
```

output
```
Integral(csch(x)**5/(cosh(x) + 1), x)/a
```

3.164.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(68) = 136$.

Time = 0.19 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.99

$$\int \frac{\operatorname{csch}^5(x)}{a + a\cosh(x)}\,dx$$
$$= \frac{15\,e^{(-x)} + 30\,e^{(-2\,x)} - 40\,e^{(-3\,x)} - 110\,e^{(-4\,x)} + 18\,e^{(-5\,x)} - 110\,e^{(-6\,x)} - 40\,e^{(-7\,x)} + 30\,e^{(-8\,x)} + 1}{24\,(2\,ae^{(-x)} - 3\,ae^{(-2\,x)} - 8\,ae^{(-3\,x)} + 2\,ae^{(-4\,x)} + 12\,ae^{(-5\,x)} + 2\,ae^{(-6\,x)} - 8\,ae^{(-7\,x)} - 3\,ae^{(-8\,x)} + 2\,ae^{(-9\,x}}$$
$$- \frac{5\,\log\left(e^{(-x)} + 1\right)}{16\,a} + \frac{5\,\log\left(e^{(-x)} - 1\right)}{16\,a}$$

input
```
integrate(csch(x)^5/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
1/24*(15*e^(-x) + 30*e^(-2*x) - 40*e^(-3*x) - 110*e^(-4*x) + 18*e^(-5*x) -
 110*e^(-6*x) - 40*e^(-7*x) + 30*e^(-8*x) + 15*e^(-9*x))/(2*a*e^(-x) - 3*a
*e^(-2*x) - 8*a*e^(-3*x) + 2*a*e^(-4*x) + 12*a*e^(-5*x) + 2*a*e^(-6*x) - 8
*a*e^(-7*x) - 3*a*e^(-8*x) + 2*a*e^(-9*x) + a*e^(-10*x) + a) - 5/16*log(e^
(-x) + 1)/a + 5/16*log(e^(-x) - 1)/a
```

3.164.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.49

$$\int \frac{\operatorname{csch}^5(x)}{a + a\cosh(x)}\,dx = -\frac{5\,\log\left(e^{(-x)} + e^x + 2\right)}{32\,a} + \frac{5\,\log\left(e^{(-x)} + e^x - 2\right)}{32\,a}$$
$$- \frac{15\,\left(e^{(-x)} + e^x\right)^2 - 76\,e^{(-x)} - 76\,e^x + 100}{64\,a(e^{(-x)} + e^x - 2)^2}$$
$$+ \frac{55\,\left(e^{(-x)} + e^x\right)^3 + 402\,\left(e^{(-x)} + e^x\right)^2 + 1020\,e^{(-x)} + 1020\,e^x + 936}{192\,a(e^{(-x)} + e^x + 2)^3}$$

input
```
integrate(csch(x)^5/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
-5/32*log(e^(-x) + e^x + 2)/a + 5/32*log(e^(-x) + e^x - 2)/a - 1/64*(15*(e
^(-x) + e^x)^2 - 76*e^(-x) - 76*e^x + 100)/(a*(e^(-x) + e^x - 2)^2) + 1/19
2*(55*(e^(-x) + e^x)^3 + 402*(e^(-x) + e^x)^2 + 1020*e^(-x) + 1020*e^x + 9
36)/(a*(e^(-x) + e^x + 2)^3)
```

3.164.9 Mupad [B] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 244, normalized size of antiderivative = 3.13

$$\int \frac{\operatorname{csch}^5(x)}{a+a\cosh(x)}\,dx = \frac{1}{a\,(10\,\mathrm{e}^{2\,x}+10\,\mathrm{e}^{3\,x}+5\,\mathrm{e}^{4\,x}+\mathrm{e}^{5\,x}+5\,\mathrm{e}^{x}+1)} + \frac{1}{4\,a\,(3\,\mathrm{e}^{2\,x}-\mathrm{e}^{3\,x}-3\,\mathrm{e}^{x}+1)} + \frac{1}{8\,a\,(\mathrm{e}^{2\,x}-2\,\mathrm{e}^{x}+1)} - \frac{1}{8\,a\,(6\,\mathrm{e}^{2\,x}-4\,\mathrm{e}^{3\,x}+\mathrm{e}^{4\,x}-4\,\mathrm{e}^{x}+1)} - \frac{5}{8\,a\,(6\,\mathrm{e}^{2\,x}+4\,\mathrm{e}^{3\,x}+\mathrm{e}^{4\,x}+4\,\mathrm{e}^{x}+1)} + \frac{1}{4\,a\,(\mathrm{e}^{x}-1)} + \frac{3}{8\,a\,(\mathrm{e}^{x}+1)} - \frac{5\,\mathrm{atan}\left(\frac{\mathrm{e}^{x}\,\sqrt{-a^2}}{a}\right)}{8\,\sqrt{-a^2}} - \frac{1}{3\,a\,(15\,\mathrm{e}^{2\,x}+20\,\mathrm{e}^{3\,x}+15\,\mathrm{e}^{4\,x}+6\,\mathrm{e}^{5\,x}+\mathrm{e}^{6\,x}+6\,\mathrm{e}^{x}+1)} - \frac{5}{12\,a\,(3\,\mathrm{e}^{2\,x}+\mathrm{e}^{3\,x}+3\,\mathrm{e}^{x}+1)}$$

input
```
int(1/(sinh(x)^5*(a + a*cosh(x))),x)
```

output
```
1/(a*(10*exp(2*x) + 10*exp(3*x) + 5*exp(4*x) + exp(5*x) + 5*exp(x) + 1)) +
 1/(4*a*(3*exp(2*x) - exp(3*x) - 3*exp(x) + 1)) + 1/(8*a*(exp(2*x) - 2*exp
(x) + 1)) - 1/(8*a*(6*exp(2*x) - 4*exp(3*x) + exp(4*x) - 4*exp(x) + 1)) -
5/(8*a*(6*exp(2*x) + 4*exp(3*x) + exp(4*x) + 4*exp(x) + 1)) + 1/(4*a*(exp(
x) - 1)) + 3/(8*a*(exp(x) + 1)) - (5*atan((exp(x)*(-a^2)^(1/2))/a))/(8*(-a
^2)^(1/2)) - 1/(3*a*(15*exp(2*x) + 20*exp(3*x) + 15*exp(4*x) + 6*exp(5*x)
+ exp(6*x) + 6*exp(x) + 1)) - 5/(12*a*(3*exp(2*x) + exp(3*x) + 3*exp(x) +
1))
```

3.165 $\int \frac{\sinh^7(x)}{a+b\cosh(x)}\,dx$

3.165.1 Optimal result

Integrand size = 13, antiderivative size = 140

$$\int \frac{\sinh^7(x)}{a+b\cosh(x)}\,dx = -\frac{a(a^4-3a^2b^2+3b^4)\cosh(x)}{b^6} + \frac{(a^4-3a^2b^2+3b^4)\cosh^2(x)}{2b^5} - \frac{a(a^2-3b^2)\cosh^3(x)}{3b^4} + \frac{(a^2-3b^2)\cosh^4(x)}{4b^3} - \frac{a\cosh^5(x)}{5b^2} + \frac{\cosh^6(x)}{6b} + \frac{(a^2-b^2)^3\log(a+b\cosh(x))}{b^7}$$

output
```
-a*(a^4-3*a^2*b^2+3*b^4)*cosh(x)/b^6+1/2*(a^4-3*a^2*b^2+3*b^4)*cosh(x)^2/b
^5-1/3*a*(a^2-3*b^2)*cosh(x)^3/b^4+1/4*(a^2-3*b^2)*cosh(x)^4/b^3-1/5*a*cos
h(x)^5/b^2+1/6*cosh(x)^6/b+(a^2-b^2)^3*ln(a+b*cosh(x))/b^7
```

3.165.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.03

$$\int \frac{\sinh^7(x)}{a+b\cosh(x)}\,dx = \frac{-120ab(8a^4-22a^2b^2+19b^4)\cosh(x)+15b^2(16a^4-40a^2b^2+29b^4)\cosh(2x)-20a(2a-3b)b^3(2a+3b)}{960}$$

input
```
Integrate[Sinh[x]^7/(a + b*Cosh[x]),x]
```

output
```
(-120*a*b*(8*a^4 - 22*a^2*b^2 + 19*b^4)*Cosh[x] + 15*b^2*(16*a^4 - 40*a^2*
b^2 + 29*b^4)*Cosh[2*x] - 20*a*(2*a - 3*b)*b^3*(2*a + 3*b)*Cosh[3*x] - 30*
b^4*(-a^2 + 2*b^2)*Cosh[4*x] - 12*a*b^5*Cosh[5*x] + 5*b^6*Cosh[6*x] + 960*
(a^2 - b^2)^3*Log[a + b*Cosh[x]])/(960*b^7)
```

3.165.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 26, 3147, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^7(x)}{a+b\cosh(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i\cos\left(-\frac{\pi}{2}+ix\right)^7}{a-b\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^7}{a-b\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3147}$$

$$-\frac{\int \frac{\left(b^2-b^2\cosh^2(x)\right)^3}{a+b\cosh(x)}d(b\cosh(x))}{b^7}$$

$$\Big\downarrow \text{ 476}$$

$$-\frac{\int \left(\left(\frac{3(b^2-a^2)b^2}{a^4}+1\right)a^5+b^4\cosh^4(x)a+b^2\left(a^2-3b^2\right)\cosh^2(x)a-b^5\cosh^5(x)-b^3\left(a^2-3b^2\right)\cosh^3(x)-b(a^4-\right.}{b^7}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{-\left(a^2-b^2\right)^3\log(a+b\cosh(x))-\frac{1}{4}b^4\left(a^2-3b^2\right)\cosh^4(x)+\frac{1}{3}ab^3\left(a^2-3b^2\right)\cosh^3(x)-\frac{1}{2}b^2\left(a^4-3a^2b^2+3b^4\right)\text{co}}{b^7}$$

input
```
Int[Sinh[x]^7/(a + b*Cosh[x]),x]
```

3.165. $\int \frac{\sinh^7(x)}{a+b\cosh(x)}\,dx$

output
```
-((a*b*(a^4 - 3*a^2*b^2 + 3*b^4)*Cosh[x] - (b^2*(a^4 - 3*a^2*b^2 + 3*b^4)*
Cosh[x]^2)/2 + (a*b^3*(a^2 - 3*b^2)*Cosh[x]^3)/3 - (b^4*(a^2 - 3*b^2)*Cosh
[x]^4)/4 + (a*b^5*Cosh[x]^5)/5 - (b^6*Cosh[x]^6)/6 - (a^2 - b^2)^3*Log[a +
 b*Cosh[x]])/b^7)
```

3.165.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.165.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.06

$$\frac{\frac{\cosh(x)^6 b^5}{6} - \frac{a\cosh(x)^5 b^4}{5} + \frac{b(a^2b^2-3b^4)\cosh(x)^4}{4} - \frac{a(a^2b^2-3b^4)\cosh(x)^3}{3} + \frac{(a^4-3a^2b^2+3b^4)\cosh(x)^2 b}{2} - a(a^4 - 3a^2b^2 + 3b^4)\,\mathrm{c}}{b^6}$$

input
```
int(sinh(x)^7/(a+b*cosh(x)),x)
```

output
```
1/b^6*(1/6*cosh(x)^6*b^5-1/5*a*cosh(x)^5*b^4+1/4*b*(a^2*b^2-3*b^4)*cosh(x)
^4-1/3*a*(a^2*b^2-3*b^4)*cosh(x)^3+1/2*(a^4-3*a^2*b^2+3*b^4)*cosh(x)^2*b-a
*(a^4-3*a^2*b^2+3*b^4)*cosh(x))+(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/b^7*ln(a+b*c
osh(x))
```

3.165.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2134 vs. $2(130) = 260$.

Time = 0.28 (sec) , antiderivative size = 2134, normalized size of antiderivative = 15.24

$$\int \frac{\sinh^7(x)}{a+b\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(x)^7/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
1/1920*(5*b^6*cosh(x)^12 + 5*b^6*sinh(x)^12 - 12*a*b^5*cosh(x)^11 + 12*(5*
b^6*cosh(x) - a*b^5)*sinh(x)^11 + 30*(a^2*b^4 - 2*b^6)*cosh(x)^10 + 6*(55*
b^6*cosh(x)^2 - 22*a*b^5*cosh(x) + 5*a^2*b^4 - 10*b^6)*sinh(x)^10 - 20*(4*
a^3*b^3 - 9*a*b^5)*cosh(x)^9 + 20*(55*b^6*cosh(x)^3 - 33*a*b^5*cosh(x)^2 -
 4*a^3*b^3 + 9*a*b^5 + 15*(a^2*b^4 - 2*b^6)*cosh(x))*sinh(x)^9 + 15*(16*a^
4*b^2 - 40*a^2*b^4 + 29*b^6)*cosh(x)^8 + 15*(165*b^6*cosh(x)^4 - 132*a*b^5
*cosh(x)^3 + 16*a^4*b^2 - 40*a^2*b^4 + 29*b^6 + 90*(a^2*b^4 - 2*b^6)*cosh(
x)^2 - 12*(4*a^3*b^3 - 9*a*b^5)*cosh(x))*sinh(x)^8 - 1920*(a^6 - 3*a^4*b^2
 + 3*a^2*b^4 - b^6)*x*cosh(x)^6 - 120*(8*a^5*b - 22*a^3*b^3 + 19*a*b^5)*co
sh(x)^7 + 120*(33*b^6*cosh(x)^5 - 33*a*b^5*cosh(x)^4 - 8*a^5*b + 22*a^3*b^
3 - 19*a*b^5 + 30*(a^2*b^4 - 2*b^6)*cosh(x)^3 - 6*(4*a^3*b^3 - 9*a*b^5)*co
sh(x)^2 + (16*a^4*b^2 - 40*a^2*b^4 + 29*b^6)*cosh(x))*sinh(x)^7 - 12*a*b^5
*cosh(x) + 12*(385*b^6*cosh(x)^6 - 462*a*b^5*cosh(x)^5 + 525*(a^2*b^4 - 2*
b^6)*cosh(x)^4 - 140*(4*a^3*b^3 - 9*a*b^5)*cosh(x)^3 + 35*(16*a^4*b^2 - 40
*a^2*b^4 + 29*b^6)*cosh(x)^2 - 160*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x -
 70*(8*a^5*b - 22*a^3*b^3 + 19*a*b^5)*cosh(x))*sinh(x)^6 + 5*b^6 - 120*(8*
a^5*b - 22*a^3*b^3 + 19*a*b^5)*cosh(x)^5 + 24*(165*b^6*cosh(x)^7 - 231*a*b
^5*cosh(x)^6 - 40*a^5*b + 110*a^3*b^3 - 95*a*b^5 + 315*(a^2*b^4 - 2*b^6)*c
osh(x)^5 - 105*(4*a^3*b^3 - 9*a*b^5)*cosh(x)^4 + 35*(16*a^4*b^2 - 40*a^2*b
^4 + 29*b^6)*cosh(x)^3 - 480*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh...
```

3.165.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^7(x)}{a + b\cosh(x)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(x)**7/(a+b*cosh(x)),x)
```

output
```
Timed out
```

3.165.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(130) = 260$.

Time = 0.20 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.21

$$\int \frac{\sinh^7(x)}{a + b\cosh(x)}\,dx = -\frac{(12\,ab^4e^{(-x)} - 5\,b^5 - 30\,(a^2b^3 - 2\,b^5)e^{(-2\,x)} + 20\,(4\,a^3b^2 - 9\,ab^4)e^{(-3\,x)} - 15\,(16\,a^4b - 40\,a^2b^3 + 29\,b^5)e^{(\cdots}}{1920\,b^6} - \frac{12\,ab^4e^{(-5\,x)} - 5\,b^5e^{(-6\,x)} + 120\,(8\,a^5 - 22\,a^3b^2 + 19\,ab^4)e^{(-x)} - 15\,(16\,a^4b - 40\,a^2b^3 + 29\,b^5)e^{(-2\,x)} + 20\cdots}{1920\,b^6} + \frac{(a^6 - 3\,a^4b^2 + 3\,a^2b^4 - b^6)x}{b^7} + \frac{(a^6 - 3\,a^4b^2 + 3\,a^2b^4 - b^6)\log\left(2\,ae^{(-x)} + be^{(-2\,x)} + b\right)}{b^7}$$

input
```
integrate(sinh(x)^7/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
-1/1920*(12*a*b^4*e^(-x) - 5*b^5 - 30*(a^2*b^3 - 2*b^5)*e^(-2*x) + 20*(4*a
^3*b^2 - 9*a*b^4)*e^(-3*x) - 15*(16*a^4*b - 40*a^2*b^3 + 29*b^5)*e^(-4*x)
+ 120*(8*a^5 - 22*a^3*b^2 + 19*a*b^4)*e^(-5*x))*e^(6*x)/b^6 - 1/1920*(12*a
*b^4*e^(-5*x) - 5*b^5*e^(-6*x) + 120*(8*a^5 - 22*a^3*b^2 + 19*a*b^4)*e^(-x
) - 15*(16*a^4*b - 40*a^2*b^3 + 29*b^5)*e^(-2*x) + 20*(4*a^3*b^2 - 9*a*b^4
)*e^(-3*x) - 30*(a^2*b^3 - 2*b^5)*e^(-4*x))/b^6 + (a^6 - 3*a^4*b^2 + 3*a^2
*b^4 - b^6)*x/b^7 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*log(2*a*e^(-x) + b
*e^(-2*x) + b)/b^7
```

3.165.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.64

$$\int \frac{\sinh^7(x)}{a+b\cosh(x)}\,dx$$
$$= \frac{5\,b^5\left(e^{(-x)}+e^x\right)^6-12\,ab^4\left(e^{(-x)}+e^x\right)^5+30\,a^2b^3\left(e^{(-x)}+e^x\right)^4-90\,b^5\left(e^{(-x)}+e^x\right)^4-80\,a^3b^2\left(e^{(-x)}+e^x\right)^3}{}$$
$$+\frac{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\log\left(\left|b\left(e^{(-x)}+e^x\right)+2\,a\right|\right)}{b^7}$$

input
```
integrate(sinh(x)^7/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
1/1920*(5*b^5*(e^(-x) + e^x)^6 - 12*a*b^4*(e^(-x) + e^x)^5 + 30*a^2*b^3*(e
^(-x) + e^x)^4 - 90*b^5*(e^(-x) + e^x)^4 - 80*a^3*b^2*(e^(-x) + e^x)^3 + 2
40*a*b^4*(e^(-x) + e^x)^3 + 240*a^4*b*(e^(-x) + e^x)^2 - 720*a^2*b^3*(e^(-
x) + e^x)^2 + 720*b^5*(e^(-x) + e^x)^2 - 960*a^5*(e^(-x) + e^x) + 2880*a^3
*b^2*(e^(-x) + e^x) - 2880*a*b^4*(e^(-x) + e^x))/b^6 + (a^6 - 3*a^4*b^2 +
3*a^2*b^4 - b^6)*log(abs(b*(e^(-x) + e^x) + 2*a))/b^7
```

3.165.9 Mupad [B] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.06

$$\int \frac{\sinh^7(x)}{a+b\cosh(x)}\,dx = \frac{e^{-6x}}{384\,b}+\frac{e^{6x}}{384\,b}-\frac{x\,(a^2-b^2)^3}{b^7}-\frac{e^{-x}\,(8\,a^5-22\,a^3\,b^2+19\,a\,b^4)}{16\,b^6}$$
$$+\frac{e^{-3x}\,(9\,a\,b^2-4\,a^3)}{96\,b^4}+\frac{e^{3x}\,(9\,a\,b^2-4\,a^3)}{96\,b^4}+\frac{e^{-4x}\,(a^2-2\,b^2)}{64\,b^3}$$
$$+\frac{e^{4x}\,(a^2-2\,b^2)}{64\,b^3}-\frac{a\,e^{-5x}}{160\,b^2}-\frac{a\,e^{5x}}{160\,b^2}+\frac{e^{-2x}\,(16\,a^4-40\,a^2\,b^2+29\,b^4)}{128\,b^5}$$
$$+\frac{e^{2x}\,(16\,a^4-40\,a^2\,b^2+29\,b^4)}{128\,b^5}-\frac{e^{x}\,(8\,a^5-22\,a^3\,b^2+19\,a\,b^4)}{16\,b^6}$$
$$+\frac{\ln\left(b+2\,a\,e^{x}+b\,e^{2x}\right)\,(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6)}{b^7}$$

input
```
int(sinh(x)^7/(a + b*cosh(x)),x)
```

output

```
exp(-6*x)/(384*b) + exp(6*x)/(384*b) - (x*(a^2 - b^2)^3)/b^7 - (exp(-x)*(1
9*a*b^4 + 8*a^5 - 22*a^3*b^2))/(16*b^6) + (exp(-3*x)*(9*a*b^2 - 4*a^3))/(9
6*b^4) + (exp(3*x)*(9*a*b^2 - 4*a^3))/(96*b^4) + (exp(-4*x)*(a^2 - 2*b^2))
/(64*b^3) + (exp(4*x)*(a^2 - 2*b^2))/(64*b^3) - (a*exp(-5*x))/(160*b^2) -
(a*exp(5*x))/(160*b^2) + (exp(-2*x)*(16*a^4 + 29*b^4 - 40*a^2*b^2))/(128*b
^5) + (exp(2*x)*(16*a^4 + 29*b^4 - 40*a^2*b^2))/(128*b^5) - (exp(x)*(19*a*
b^4 + 8*a^5 - 22*a^3*b^2))/(16*b^6) + (log(b + 2*a*exp(x) + b*exp(2*x))*(a
^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/b^7
```

3.165. $\int \frac{\sinh^7(x)}{a+b\cosh(x)}\,dx$

3.166 $\int \frac{\sinh^6(x)}{a+b\cosh(x)}\,dx$

3.166.1 Optimal result

Integrand size = 13, antiderivative size = 154

$$\int \frac{\sinh^6(x)}{a+b\cosh(x)}\,dx = -\frac{a(8a^4-20a^2b^2+15b^4)\,x}{8b^6} + \frac{2(a-b)^{5/2}(a+b)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^6} + \frac{\left(8(a^2-b^2)^2-ab(4a^2-7b^2)\cosh(x)\right)\sinh(x)}{8b^5} + \frac{(4(a^2-b^2)-3ab\cosh(x))\sinh^3(x)}{12b^3} + \frac{\sinh^5(x)}{5b}$$

output
```
-1/8*a*(8*a^4-20*a^2*b^2+15*b^4)*x/b^6+2*(a-b)^(5/2)*(a+b)^(5/2)*arctanh((
a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/b^6+1/8*(8*(a^2-b^2)^2-a*b*(4*a^2-7*b^
2)*cosh(x))*sinh(x)/b^5+1/12*(4*a^2-4*b^2-3*a*b*cosh(x))*sinh(x)^3/b^3+1/5
*sinh(x)^5/b
```

3.166.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^6(x)}{a+b\cosh(x)}\,dx$$
$$= \frac{-60a(8a^4-20a^2b^2+15b^4)\,x+960(-a^2+b^2)^{5/2}\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)+60b(8a^4-18a^2b^2+11b^4)\sinh(x}{480b^6}$$

input
```
Integrate[Sinh[x]^6/(a + b*Cosh[x]),x]
```

output
```
(-60*a*(8*a^4 - 20*a^2*b^2 + 15*b^4)*x + 960*(-a^2 + b^2)^(5/2)*ArcTan[((a
 - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]] + 60*b*(8*a^4 - 18*a^2*b^2 + 11*b^4)*Si
nh[x] - 120*a*b^2*(a^2 - 2*b^2)*Sinh[2*x] - 10*b^3*(-4*a^2 + 7*b^2)*Sinh[3
*x] - 15*a*b^4*Sinh[4*x] + 6*b^5*Sinh[5*x])/(480*b^6)
```

3.166.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.19, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.154, Rules used = {3042, 25, 3174, 25, 3042, 3344, 3042, 25, 3344, 25, 3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^6(x)}{a+b\cosh(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\cos\left(-\frac{\pi}{2}+ix\right)^6}{a-b\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^6}{a-b\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3174}$$

$$\frac{\int -\frac{(b+a\cosh(x))\sinh^4(x)}{a+b\cosh(x)}dx}{b}+\frac{\sinh^5(x)}{5b}$$

$$\Big\downarrow \; 25$$

$$\frac{\sinh^5(x)}{5b}-\frac{\int \frac{(b+a\cosh(x))\sinh^4(x)}{a+b\cosh(x)}dx}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sinh^5(x)}{5b}-\frac{\int \frac{\cos\left(ix+\frac{\pi}{2}\right)^4\left(b+a\sin\left(ix+\frac{\pi}{2}\right)\right)}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \; 3344$$

$$\frac{\sinh^5(x)}{5b}-\frac{\frac{\int \frac{\left(b\left(a^2-4b^2\right)+a\left(4a^2-7b^2\right)\cosh(x)\right)\sinh^2(x)}{a+b\cosh(x)}dx}{4b^2}-\frac{\sinh^3(x)\left(4\left(a^2-b^2\right)-3ab\cosh(x)\right)}{12b^2}}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sinh^5(x)}{5b}-\frac{-\frac{\sinh^3(x)\left(4\left(a^2-b^2\right)-3ab\cosh(x)\right)}{12b^2}+\frac{\int -\frac{\cos\left(ix+\frac{\pi}{2}\right)^2\left(b\left(a^2-4b^2\right)+a\left(4a^2-7b^2\right)\sin\left(ix+\frac{\pi}{2}\right)\right)}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{4b^2}}{b}$$

$$\Big\downarrow \; 25$$

$$\frac{\sinh^5(x)}{5b}-\frac{-\frac{\sinh^3(x)\left(4\left(a^2-b^2\right)-3ab\cosh(x)\right)}{12b^2}-\frac{\int \frac{\cos\left(ix+\frac{\pi}{2}\right)^2\left(b\left(a^2-4b^2\right)+a\left(4a^2-7b^2\right)\sin\left(ix+\frac{\pi}{2}\right)\right)}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{4b^2}}{b}$$

$$\Big\downarrow \; 3344$$

$$\frac{\sinh^5(x)}{5b}-\frac{-\frac{\frac{\int -\frac{b\left(4a^4-9b^2a^2+8b^4\right)+a\left(8a^4-20b^2a^2+15b^4\right)\cosh(x)}{a+b\cosh(x)}dx}{2b^2}+\frac{\sinh(x)\left(8\left(a^2-b^2\right)^2-ab\left(4a^2-7b^2\right)\cosh(x)\right)}{2b^2}}{4b^2}-\frac{\sinh^3(x)\left(4\left(a^2-b^2\right)-3ab\cosh(x)\right)}{12b^2}}{b}$$

$$\Big\downarrow \; 25$$

$$\frac{\sinh^5(x)}{5b}-\frac{-\frac{\frac{\sinh(x)\left(8\left(a^2-b^2\right)^2-ab\left(4a^2-7b^2\right)\cosh(x)\right)}{2b^2}-\frac{\int \frac{b\left(4a^4-9b^2a^2+8b^4\right)+a\left(8a^4-20b^2a^2+15b^4\right)\cosh(x)}{a+b\cosh(x)}dx}{2b^2}}{4b^2}-\frac{\sinh^3(x)\left(4\left(a^2-b^2\right)-3ab\cosh(x)\right)}{12b^2}}{b}$$

$$\Big\downarrow \; 3042$$

3.166. $\int \frac{\sinh^6(x)}{a+b\cosh(x)}\,dx$

$$\frac{\sinh^5(x)}{5b}-\frac{-\frac{\sinh^3(x)\left(4\left(a^2-b^2\right)-3ab\cosh(x)\right)}{12b^2}-\frac{\frac{\sinh(x)\left(8\left(a^2-b^2\right)^2-ab\left(4a^2-7b^2\right)\cosh(x)\right)}{2b^2}-\frac{\int\frac{b\left(4a^4-9b^2a^2+8b^4\right)+a\left(8a^4-20b^2a^2+15b^4\right)\sin\left(ix+\frac{\pi}{2}\right)}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{2b^2}}{4b^2}}{b}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\sinh^5(x)}{5b}-\frac{-\frac{\frac{\sinh(x)\left(8\left(a^2-b^2\right)^2-ab\left(4a^2-7b^2\right)\cosh(x)\right)}{2b^2}-\frac{\frac{ax\left(8a^4-20a^2b^2+15b^4\right)}{b}-\frac{8\left(a^2-b^2\right)^3\int\frac{1}{a+b\cosh(x)}dx}{b}}{2b^2}}{4b^2}-\frac{\sinh^3(x)\left(4\left(a^2-b^2\right)-3ab\cosh(x)\right)}{12b^2}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^5(x)}{5b}-\frac{-\frac{\sinh^3(x)\left(4\left(a^2-b^2\right)-3ab\cosh(x)\right)}{12b^2}-\frac{\frac{\sinh(x)\left(8\left(a^2-b^2\right)^2-ab\left(4a^2-7b^2\right)\cosh(x)\right)}{2b^2}-\frac{\frac{ax\left(8a^4-20a^2b^2+15b^4\right)}{b}-\frac{8\left(a^2-b^2\right)^3\int\frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}}{2b^2}}{4b^2}}{b}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\sinh^5(x)}{5b}-\frac{-\frac{\frac{\sinh(x)\left(8\left(a^2-b^2\right)^2-ab\left(4a^2-7b^2\right)\cosh(x)\right)}{2b^2}-\frac{\frac{ax\left(8a^4-20a^2b^2+15b^4\right)}{b}-\frac{16\left(a^2-b^2\right)^3\int\frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{b}}{2b^2}}{4b^2}-\frac{\sinh^3(x)\left(4\left(a^2-b^2\right)-3a\right.}{12b^2}}{b}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sinh^5(x)}{5b}-\frac{-\frac{\sinh^3(x)\left(4\left(a^2-b^2\right)-3ab\cosh(x)\right)}{12b^2}-\frac{\frac{\sinh(x)\left(8\left(a^2-b^2\right)^2-ab\left(4a^2-7b^2\right)\cosh(x)\right)}{2b^2}-\frac{\frac{ax\left(8a^4-20a^2b^2+15b^4\right)}{b}-\frac{16\left(a^2-b^2\right)^3\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right.}{b\sqrt{a-b}\sqrt{a+b}}}{2b^2}}{4b^2}}{b}$$

input
```
Int[Sinh[x]^6/(a + b*Cosh[x]),x]
```

3.166. $\int \frac{\sinh^6(x)}{a+b\cosh(x)}\,dx$

output
```
Sinh[x]^5/(5*b) - (-1/12*((4*(a^2 - b^2) - 3*a*b*Cosh[x])*Sinh[x]^3)/b^2 -
 (-1/2*((a*(8*a^4 - 20*a^2*b^2 + 15*b^4)*x)/b - (16*(a^2 - b^2)^3*ArcTanh[
(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]))/b^2 + (
(8*(a^2 - b^2)^2 - a*b*(4*a^2 - 7*b^2)*Cosh[x])*Sinh[x])/(2*b^2))/(4*b^2))
/b
```

3.166.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3174
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(b*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
 0] && IntegersQ[2*m, 2*p]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.166. $\int \frac{\sinh^6(x)}{a+b\cosh(x)}\,dx$

rule 3344
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Simp[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Si
n[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^
2*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1
, 0] && IntegerQ[2*m]
```

3.166.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(135) = 270$.

Time = 0.05 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.66

$$-\frac{1}{5b\left(\tanh\left(\frac{x}{2}\right)-1\right)^{5}}-\frac{a+2b}{4b^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{4}}-\frac{4a^{2}+6ab-b^{2}}{12b^{3}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}-\frac{4a^{3}+4a^{2}b-5a\,b^{2}-5b^{3}}{8b^{4}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{8a^{4}+4a^{3}b-}{8b^{5}\left(\right.}$$

input
```
int(sinh(x)^6/(a+b*cosh(x)),x)
```

output
```
-1/5/b/(tanh(1/2*x)-1)^5-1/4*(a+2*b)/b^2/(tanh(1/2*x)-1)^4-1/12*(4*a^2+6*a
*b-b^2)/b^3/(tanh(1/2*x)-1)^3-1/8*(4*a^3+4*a^2*b-5*a*b^2-5*b^3)/b^4/(tanh(
1/2*x)-1)^2-1/8*(8*a^4+4*a^3*b-16*a^2*b^2-7*a*b^3+8*b^4)/b^5/(tanh(1/2*x)-
1)+1/8*a*(8*a^4-20*a^2*b^2+15*b^4)/b^6*ln(tanh(1/2*x)-1)-2/b^6*(-a^6+3*a^4
*b^2-3*a^2*b^4+b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(
a-b))^(1/2))-1/5/b/(tanh(1/2*x)+1)^5-1/4*(-a-2*b)/b^2/(tanh(1/2*x)+1)^4-1/
12*(4*a^2+6*a*b-b^2)/b^3/(tanh(1/2*x)+1)^3-1/8*(-4*a^3-4*a^2*b+5*a*b^2+5*b
^3)/b^4/(tanh(1/2*x)+1)^2-1/8*(8*a^4+4*a^3*b-16*a^2*b^2-7*a*b^3+8*b^4)/b^5
/(tanh(1/2*x)+1)-1/8*a*(8*a^4-20*a^2*b^2+15*b^4)/b^6*ln(tanh(1/2*x)+1)
```

3.166.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1422 vs. $2(134) = 268$.

Time = 0.30 (sec) , antiderivative size = 2913, normalized size of antiderivative = 18.92

$$\int \frac{\sinh^6(x)}{a + b\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(x)^6/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[1/960*(6*b^5*cosh(x)^10 + 6*b^5*sinh(x)^10 - 15*a*b^4*cosh(x)^9 + 15*(4*b
^5*cosh(x) - a*b^4)*sinh(x)^9 + 10*(4*a^2*b^3 - 7*b^5)*cosh(x)^8 + 5*(54*b
^5*cosh(x)^2 - 27*a*b^4*cosh(x) + 8*a^2*b^3 - 14*b^5)*sinh(x)^8 - 120*(a^3
*b^2 - 2*a*b^4)*cosh(x)^7 + 20*(36*b^5*cosh(x)^3 - 27*a*b^4*cosh(x)^2 - 6*
a^3*b^2 + 12*a*b^4 + 4*(4*a^2*b^3 - 7*b^5)*cosh(x))*sinh(x)^7 - 120*(8*a^5
 - 20*a^3*b^2 + 15*a*b^4)*x*cosh(x)^5 + 60*(8*a^4*b - 18*a^2*b^3 + 11*b^5)
*cosh(x)^6 + 20*(63*b^5*cosh(x)^4 - 63*a*b^4*cosh(x)^3 + 24*a^4*b - 54*a^2
*b^3 + 33*b^5 + 14*(4*a^2*b^3 - 7*b^5)*cosh(x)^2 - 42*(a^3*b^2 - 2*a*b^4)*
cosh(x))*sinh(x)^6 + 15*a*b^4*cosh(x) + 2*(756*b^5*cosh(x)^5 - 945*a*b^4*c
osh(x)^4 + 280*(4*a^2*b^3 - 7*b^5)*cosh(x)^3 - 1260*(a^3*b^2 - 2*a*b^4)*co
sh(x)^2 - 60*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*x + 180*(8*a^4*b - 18*a^2*b^3
 + 11*b^5)*cosh(x))*sinh(x)^5 - 6*b^5 - 60*(8*a^4*b - 18*a^2*b^3 + 11*b^5)
*cosh(x)^4 + 10*(126*b^5*cosh(x)^6 - 189*a*b^4*cosh(x)^5 - 48*a^4*b + 108*
a^2*b^3 - 66*b^5 + 70*(4*a^2*b^3 - 7*b^5)*cosh(x)^4 - 420*(a^3*b^2 - 2*a*b
^4)*cosh(x)^3 - 60*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*x*cosh(x) + 90*(8*a^4*b
 - 18*a^2*b^3 + 11*b^5)*cosh(x)^2)*sinh(x)^4 + 120*(a^3*b^2 - 2*a*b^4)*cos
h(x)^3 + 20*(36*b^5*cosh(x)^7 - 63*a*b^4*cosh(x)^6 + 28*(4*a^2*b^3 - 7*b^5
)*cosh(x)^5 + 6*a^3*b^2 - 12*a*b^4 - 210*(a^3*b^2 - 2*a*b^4)*cosh(x)^4 - 6
0*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*x*cosh(x)^2 + 60*(8*a^4*b - 18*a^2*b^3 +
 11*b^5)*cosh(x)^3 - 12*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x))*sinh(x...
```

3.166.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^6(x)}{a + b\cosh(x)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(x)**6/(a+b*cosh(x)),x)
```

output
```
Timed out
```

3.166.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh^6(x)}{a+b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sinh(x)^6/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.166.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.73

$$\int \frac{\sinh^6(x)}{a+b\cosh(x)}\,dx$$
$$= \frac{6\,b^4e^{(5\,x)} - 15\,ab^3e^{(4\,x)} + 40\,a^2b^2e^{(3\,x)} - 70\,b^4e^{(3\,x)} - 120\,a^3be^{(2\,x)} + 240\,ab^3e^{(2\,x)} + 480\,a^4e^x - 1080\,a^2b^2e^x +}{960\,b^5}$$
$$- \frac{(8\,a^5 - 20\,a^3b^2 + 15\,ab^4)x}{8\,b^6}$$
$$+ \frac{(15\,ab^4e^x - 6\,b^5 - 60\,(8\,a^4b - 18\,a^2b^3 + 11\,b^5)e^{(4\,x)} + 120\,(a^3b^2 - 2\,ab^4)e^{(3\,x)} - 10\,(4\,a^2b^3 - 7\,b^5)e^{(2\,x)})e^{(}}{960\,b^6}$$
$$+ \frac{2\,(a^6 - 3\,a^4b^2 + 3\,a^2b^4 - b^6)\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}b^6}$$

input
```
integrate(sinh(x)^6/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
1/960*(6*b^4*e^(5*x) - 15*a*b^3*e^(4*x) + 40*a^2*b^2*e^(3*x) - 70*b^4*e^(3
*x) - 120*a^3*b*e^(2*x) + 240*a*b^3*e^(2*x) + 480*a^4*e^x - 1080*a^2*b^2*e
^x + 660*b^4*e^x)/b^5 - 1/8*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*x/b^6 + 1/960*
(15*a*b^4*e^x - 6*b^5 - 60*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*e^(4*x) + 120*(
a^3*b^2 - 2*a*b^4)*e^(3*x) - 10*(4*a^2*b^3 - 7*b^5)*e^(2*x))*e^(-5*x)/b^6
+ 2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*arctan((b*e^x + a)/sqrt(-a^2 + b^2
))/(sqrt(-a^2 + b^2)*b^6)
```

3.166.9 Mupad [B] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.26

$$\int \frac{\sinh^6(x)}{a+b\cosh(x)}\,dx$$
$$= \frac{e^{5x}}{160b} - \frac{e^{-5x}}{160b} - \frac{e^{-2x}(2ab^2-a^3)}{8b^4} + \frac{e^{2x}(2ab^2-a^3)}{8b^4} - \frac{x(8a^5-20a^3b^2+15ab^4)}{8b^6} + \frac{e^{x}(8a^4-18a^2b^2+11b^4)}{16b^5} + \frac{a\,e^{-4x}}{64b^2} - \frac{a\,e^{4x}}{64b^2} - \frac{e^{-x}(8a^4-18a^2b^2+11b^4)}{16b^5} - \frac{e^{-3x}(4a^2-7b^2)}{96b^3} + \frac{e^{3x}(4a^2-7b^2)}{96b^3} + \frac{\ln\left(-\frac{2e^{x}(a^6-3a^4b^2+3a^2b^4-b^6)}{b^7} - \frac{2(a+b)^{5/2}(b+a\,e^{x})(a-b)^{5/2}}{b^7}\right)(a+b)^{5/2}(a-b)^{5/2}}{b^6} - \frac{\ln\left(\frac{2(a+b)^{5/2}(b+a\,e^{x})(a-b)^{5/2}}{b^7} - \frac{2e^{x}(a^6-3a^4b^2+3a^2b^4-b^6)}{b^7}\right)(a+b)^{5/2}(a-b)^{5/2}}{b^6}$$

input
```
int(sinh(x)^6/(a + b*cosh(x)),x)
```

output
```
exp(5*x)/(160*b) - exp(-5*x)/(160*b) - (exp(-2*x)*(2*a*b^2 - a^3))/(8*b^4)
 + (exp(2*x)*(2*a*b^2 - a^3))/(8*b^4) - (x*(15*a*b^4 + 8*a^5 - 20*a^3*b^2)
)/(8*b^6) + (exp(x)*(8*a^4 + 11*b^4 - 18*a^2*b^2))/(16*b^5) + (a*exp(-4*x)
)/(64*b^2) - (a*exp(4*x))/(64*b^2) - (exp(-x)*(8*a^4 + 11*b^4 - 18*a^2*b^2
))/(16*b^5) - (exp(-3*x)*(4*a^2 - 7*b^2))/(96*b^3) + (exp(3*x)*(4*a^2 - 7*
b^2))/(96*b^3) + (log(- (2*exp(x)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/b^7
 - (2*(a + b)^(5/2)*(b + a*exp(x))*(a - b)^(5/2))/b^7)*(a + b)^(5/2)*(a -
b)^(5/2))/b^6 - (log((2*(a + b)^(5/2)*(b + a*exp(x))*(a - b)^(5/2))/b^7 -
(2*exp(x)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/b^7)*(a + b)^(5/2)*(a - b)^
(5/2))/b^6
```

3.167 $\int \frac{\sinh^5(x)}{a+b\cosh(x)}\,dx$

3.167.1 Optimal result

Integrand size = 13, antiderivative size = 83

$$\int \frac{\sinh^5(x)}{a+b\cosh(x)}\,dx = -\frac{a(a^2-2b^2)\cosh(x)}{b^4}+\frac{(a^2-2b^2)\cosh^2(x)}{2b^3} -\frac{a\cosh^3(x)}{3b^2}+\frac{\cosh^4(x)}{4b}+\frac{(a^2-b^2)^2\log(a+b\cosh(x))}{b^5}$$

output
```
-a*(a^2-2*b^2)*cosh(x)/b^4+1/2*(a^2-2*b^2)*cosh(x)^2/b^3-1/3*a*cosh(x)^3/b
^2+1/4*cosh(x)^4/b+(a^2-b^2)^2*ln(a+b*cosh(x))/b^5
```

3.167.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.01

$$\int \frac{\sinh^5(x)}{a+b\cosh(x)}\,dx = \frac{-24ab(4a^2-7b^2)\cosh(x)-12b^2(-2a^2+3b^2)\cosh(2x)-8ab^3\cosh(3x)+3b^4\cosh(4x)+96(a^2-b^2)^2\mathrm{lo}}{96b^5}$$

input
```
Integrate[Sinh[x]^5/(a + b*Cosh[x]),x]
```

output
```
(-24*a*b*(4*a^2 - 7*b^2)*Cosh[x] - 12*b^2*(-2*a^2 + 3*b^2)*Cosh[2*x] - 8*a
*b^3*Cosh[3*x] + 3*b^4*Cosh[4*x] + 96*(a^2 - b^2)^2*Log[a + b*Cosh[x]])/(9
6*b^5)
```

3.167.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 26, 3147, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^5(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\cos\left(-\frac{\pi}{2}+ix\right)^5}{a-b\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^5}{a-b\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3147}$$

$$\frac{\int \frac{(b^2-b^2\cosh^2(x))^2}{a+b\cosh(x)}d(b\cosh(x))}{b^5}$$

$$\Big\downarrow \text{476}$$

$$\frac{\int \left(-\left(\left(1-\frac{2b^2}{a^2}\right)a^3\right)-b^2\cosh^2(x)a+b^3\cosh^3(x)+b(a^2-2b^2)\cosh(x)+\frac{(a^2-b^2)^2}{a+b\cosh(x)}\right)d(b\cosh(x))}{b^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}b^2(a^2-2b^2)\cosh^2(x)-ab(a^2-2b^2)\cosh(x)+(a^2-b^2)^2\log(a+b\cosh(x))-\frac{1}{3}ab^3\cosh^3(x)+\frac{1}{4}b^4\cosh^4(x)}{b^5}$$

input
```
Int[Sinh[x]^5/(a + b*Cosh[x]),x]
```

output
```
(-(a*b*(a^2 - 2*b^2)*Cosh[x]) + (b^2*(a^2 - 2*b^2)*Cosh[x]^2)/2 - (a*b^3*C
osh[x]^3)/3 + (b^4*Cosh[x]^4)/4 + (a^2 - b^2)^2*Log[a + b*Cosh[x]])/b^5
```

3.167.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.167.4 Maple [A] (verified)

Time = 102.34 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99

method	result
derivativedivides	$-\frac{-\frac{\cosh(x)^4 b^3}{4}+\frac{a\cosh(x)^3 b^2}{3}-\frac{(a^2-2b^2)\cosh(x)^2 b}{2}+a(a^2-2b^2)\cosh(x)}{b^4}+\frac{(a^4-2a^2b^2+b^4)\ln(a+b\cosh(x))}{b^5}$
default	$-\frac{-\frac{\cosh(x)^4 b^3}{4}+\frac{a\cosh(x)^3 b^2}{3}-\frac{(a^2-2b^2)\cosh(x)^2 b}{2}+a(a^2-2b^2)\cosh(x)}{b^4}+\frac{(a^4-2a^2b^2+b^4)\ln(a+b\cosh(x))}{b^5}$
risch	$-\frac{x a^4}{b^5}+\frac{2x a^2}{b^3}-\frac{x}{b}+\frac{e^{4x}}{64b}-\frac{a e^{3x}}{24b^2}+\frac{e^{2x}a^2}{8b^3}-\frac{3e^{2x}}{16b}-\frac{a^3 e^{x}}{2b^4}+\frac{7a e^{x}}{8b^2}-\frac{a^3 e^{-x}}{2b^4}+\frac{7a e^{-x}}{8b^2}+\frac{e^{-2x}a^2}{8b^3}-$

input
```
int(sinh(x)^5/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/b^4*(-1/4*cosh(x)^4*b^3+1/3*a*cosh(x)^3*b^2-1/2*(a^2-2*b^2)*cosh(x)^2*b
+a*(a^2-2*b^2)*cosh(x))+(a^4-2*a^2*b^2+b^4)/b^5*ln(a+b*cosh(x))
```

3.167.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 866 vs. $2(77) = 154$.

Time = 0.27 (sec) , antiderivative size = 866, normalized size of antiderivative = 10.43

$$\int \frac{\sinh^5(x)}{a + b\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(x)^5/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
1/192*(3*b^4*cosh(x)^8 + 3*b^4*sinh(x)^8 - 8*a*b^3*cosh(x)^7 + 8*(3*b^4*co
sh(x) - a*b^3)*sinh(x)^7 + 12*(2*a^2*b^2 - 3*b^4)*cosh(x)^6 + 4*(21*b^4*co
sh(x)^2 - 14*a*b^3*cosh(x) + 6*a^2*b^2 - 9*b^4)*sinh(x)^6 - 192*(a^4 - 2*a
^2*b^2 + b^4)*x*cosh(x)^4 - 24*(4*a^3*b - 7*a*b^3)*cosh(x)^5 + 24*(7*b^4*c
osh(x)^3 - 7*a*b^3*cosh(x)^2 - 4*a^3*b + 7*a*b^3 + 3*(2*a^2*b^2 - 3*b^4)*c
osh(x))*sinh(x)^5 - 8*a*b^3*cosh(x) + 2*(105*b^4*cosh(x)^4 - 140*a*b^3*cos
h(x)^3 + 90*(2*a^2*b^2 - 3*b^4)*cosh(x)^2 - 96*(a^4 - 2*a^2*b^2 + b^4)*x -
 60*(4*a^3*b - 7*a*b^3)*cosh(x))*sinh(x)^4 + 3*b^4 - 24*(4*a^3*b - 7*a*b^3
)*cosh(x)^3 + 8*(21*b^4*cosh(x)^5 - 35*a*b^3*cosh(x)^4 - 12*a^3*b + 21*a*b
^3 + 30*(2*a^2*b^2 - 3*b^4)*cosh(x)^3 - 96*(a^4 - 2*a^2*b^2 + b^4)*x*cosh(
x) - 30*(4*a^3*b - 7*a*b^3)*cosh(x)^2)*sinh(x)^3 + 12*(2*a^2*b^2 - 3*b^4)*
cosh(x)^2 + 12*(7*b^4*cosh(x)^6 - 14*a*b^3*cosh(x)^5 + 15*(2*a^2*b^2 - 3*b
^4)*cosh(x)^4 + 2*a^2*b^2 - 3*b^4 - 96*(a^4 - 2*a^2*b^2 + b^4)*x*cosh(x)^2
 - 20*(4*a^3*b - 7*a*b^3)*cosh(x)^3 - 6*(4*a^3*b - 7*a*b^3)*cosh(x))*sinh(
x)^2 + 192*((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 4*(a^4 - 2*a^2*b^2 + b^4)*
cosh(x)^3*sinh(x) + 6*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2*sinh(x)^2 + 4*(a^4
 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^3 + (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^4)
*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) + 8*(3*b^4*cosh(x)^7 - 7*a*b^3
*cosh(x)^6 + 9*(2*a^2*b^2 - 3*b^4)*cosh(x)^5 - 96*(a^4 - 2*a^2*b^2 + b^4)*
x*cosh(x)^3 - 15*(4*a^3*b - 7*a*b^3)*cosh(x)^4 - a*b^3 - 9*(4*a^3*b - 7...
```

3.167.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^5(x)}{a + b\cosh(x)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(x)**5/(a+b*cosh(x)),x)
```

output
```
Timed out
```

3.167.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(77) = 154$.

Time = 0.19 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.14

$$\int \frac{\sinh^5(x)}{a+b\cosh(x)}\,dx$$
$$= -\frac{\left(8\,ab^2e^{(-x)} - 3\,b^3 - 12\,(2\,a^2b - 3\,b^3)e^{(-2\,x)} + 24\,(4\,a^3 - 7\,ab^2)e^{(-3\,x)}\right)e^{(4\,x)}}{192\,b^4}$$
$$- \frac{8\,ab^2e^{(-3\,x)} - 3\,b^3e^{(-4\,x)} + 24\,(4\,a^3 - 7\,ab^2)e^{(-x)} - 12\,(2\,a^2b - 3\,b^3)e^{(-2\,x)}}{192\,b^4}$$
$$+ \frac{(a^4 - 2\,a^2b^2 + b^4)x}{b^5} + \frac{(a^4 - 2\,a^2b^2 + b^4)\log\left(2\,ae^{(-x)} + be^{(-2\,x)} + b\right)}{b^5}$$

input
```
integrate(sinh(x)^5/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
-1/192*(8*a*b^2*e^(-x) - 3*b^3 - 12*(2*a^2*b - 3*b^3)*e^(-2*x) + 24*(4*a^3
 - 7*a*b^2)*e^(-3*x))*e^(4*x)/b^4 - 1/192*(8*a*b^2*e^(-3*x) - 3*b^3*e^(-4*
x) + 24*(4*a^3 - 7*a*b^2)*e^(-x) - 12*(2*a^2*b - 3*b^3)*e^(-2*x))/b^4 + (a
^4 - 2*a^2*b^2 + b^4)*x/b^5 + (a^4 - 2*a^2*b^2 + b^4)*log(2*a*e^(-x) + b*e
^(-2*x) + b)/b^5
```

3.167.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.49

$$\int \frac{\sinh^5(x)}{a+b\cosh(x)}\,dx$$
$$= \frac{3\,b^3\left(e^{(-x)} + e^x\right)^4 - 8\,ab^2\left(e^{(-x)} + e^x\right)^3 + 24\,a^2b\left(e^{(-x)} + e^x\right)^2 - 48\,b^3\left(e^{(-x)} + e^x\right)^2 - 96\,a^3\left(e^{(-x)} + e^x\right) + 19}{192\,b^4}$$
$$+ \frac{(a^4 - 2\,a^2b^2 + b^4)\log\left(\left|b\left(e^{(-x)} + e^x\right) + 2\,a\right|\right)}{b^5}$$

input
```
integrate(sinh(x)^5/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
1/192*(3*b^3*(e^(-x) + e^x)^4 - 8*a*b^2*(e^(-x) + e^x)^3 + 24*a^2*b*(e^(-x
) + e^x)^2 - 48*b^3*(e^(-x) + e^x)^2 - 96*a^3*(e^(-x) + e^x) + 192*a*b^2*(
e^(-x) + e^x))/b^4 + (a^4 - 2*a^2*b^2 + b^4)*log(abs(b*(e^(-x) + e^x) + 2*
a))/b^5
```

3.167.9 Mupad [B] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.04

$$\int \frac{\sinh^5(x)}{a+b\cosh(x)}\,dx = \frac{e^{-4x}}{64\,b}+\frac{e^{4x}}{64\,b}-\frac{x\,{\left(a^2-b^2\right)}^2}{b^5}+\frac{e^{-x}\,\left(7\,a\,b^2-4\,a^3\right)}{8\,b^4}+\frac{\ln\left(b+2\,a\,e^x+b\,e^{2\,x}\right)\,\left(a^4-2\,a^2\,b^2+b^4\right)}{b^5}-\frac{a\,e^{-3\,x}}{24\,b^2}-\frac{a\,e^{3\,x}}{24\,b^2}+\frac{e^{-2\,x}\,\left(2\,a^2-3\,b^2\right)}{16\,b^3}+\frac{e^{2\,x}\,\left(2\,a^2-3\,b^2\right)}{16\,b^3}+\frac{e^x\,\left(7\,a\,b^2-4\,a^3\right)}{8\,b^4}$$

input
```
int(sinh(x)^5/(a + b*cosh(x)),x)
```

output
```
exp(-4*x)/(64*b) + exp(4*x)/(64*b) - (x*(a^2 - b^2)^2)/b^5 + (exp(-x)*(7*a
*b^2 - 4*a^3))/(8*b^4) + (log(b + 2*a*exp(x) + b*exp(2*x))*(a^4 + b^4 - 2*
a^2*b^2))/b^5 - (a*exp(-3*x))/(24*b^2) - (a*exp(3*x))/(24*b^2) + (exp(-2*x
)*(2*a^2 - 3*b^2))/(16*b^3) + (exp(2*x)*(2*a^2 - 3*b^2))/(16*b^3) + (exp(x
)*(7*a*b^2 - 4*a^3))/(8*b^4)
```

3.168 $\int \frac{\sinh^4(x)}{a+b\cosh(x)}\,dx$

3.168.1 Optimal result

Integrand size = 13, antiderivative size = 104

$$\int \frac{\sinh^4(x)}{a+b\cosh(x)}\,dx = -\frac{a(2a^2-3b^2)\,x}{2b^4}+\frac{2(a-b)^{3/2}(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^4} + \frac{(2(a^2-b^2)-ab\cosh(x))\sinh(x)}{2b^3}+\frac{\sinh^3(x)}{3b}$$

output
```
-1/2*a*(2*a^2-3*b^2)*x/b^4+2*(a-b)^(3/2)*(a+b)^(3/2)*arctanh((a-b)^(1/2)*t
anh(1/2*x)/(a+b)^(1/2))/b^4+1/2*(2*a^2-2*b^2-a*b*cosh(x))*sinh(x)/b^3+1/3*
sinh(x)^3/b
```

3.168.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.91

$$\int \frac{\sinh^4(x)}{a+b\cosh(x)}\,dx = \frac{-12a^3x+18ab^2x-24(-a^2+b^2)^{3/2}\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)+12a^2b\sinh(x)-15b^3\sinh(x)-3ab^2\sinh(2x}{12b^4}$$

input
```
Integrate[Sinh[x]^4/(a + b*Cosh[x]),x]
```

output
```
(-12*a^3*x + 18*a*b^2*x - 24*(-a^2 + b^2)^(3/2)*ArcTan[((a - b)*Tanh[x/2])
/Sqrt[-a^2 + b^2]] + 12*a^2*b*Sinh[x] - 15*b^3*Sinh[x] - 3*a*b^2*Sinh[2*x]
 + b^3*Sinh[3*x])/(12*b^4)
```

3.168.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.20, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {3042, 3174, 3042, 25, 3344, 25, 3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^4(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(-\frac{\pi}{2}+ix\right)^4}{a-b\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3174}$$

$$\frac{\sinh^3(x)}{3b}-\frac{\int \frac{(b+a\cosh(x))\sinh^2(x)}{a+b\cosh(x)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^3(x)}{3b}-\frac{\int -\frac{\cos\left(ix+\frac{\pi}{2}\right)^2\left(b+a\sin\left(ix+\frac{\pi}{2}\right)\right)}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh^3(x)}{3b}+\frac{\int \frac{\cos\left(ix+\frac{\pi}{2}\right)^2\left(b+a\sin\left(ix+\frac{\pi}{2}\right)\right)}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{3344}$$

$$\frac{\frac{\int -\frac{b\left(a^2-2b^2\right)+a\left(2a^2-3b^2\right)\cosh(x)}{a+b\cosh(x)}dx}{2b^2}+\frac{\sinh(x)\left(2\left(a^2-b^2\right)-ab\cosh(x)\right)}{2b^2}}{b}+\frac{\sinh^3(x)}{3b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\sinh(x)\left(2\left(a^2-b^2\right)-ab\cosh(x)\right)}{2b^2}-\frac{\int \frac{b\left(a^2-2b^2\right)+a\left(2a^2-3b^2\right)\cosh(x)}{a+b\cosh(x)}dx}{2b^2}}{b}+\frac{\sinh^3(x)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^3(x)}{3b}+\frac{\frac{\sinh(x)(2(a^2-b^2)-ab\cosh(x))}{2b^2}-\frac{\int\frac{b(a^2-2b^2)+a(2a^2-3b^2)\sin\left(ix+\frac{\pi}{2}\right)}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{2b^2}}{b}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\frac{\sinh(x)(2(a^2-b^2)-ab\cosh(x))}{2b^2}-\frac{\frac{ax(2a^2-3b^2)}{b}-\frac{2(a^2-b^2)^2\int\frac{1}{a+b\cosh(x)}dx}{b}}{2b^2}}{b}+\frac{\sinh^3(x)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^3(x)}{3b}+\frac{\frac{\sinh(x)(2(a^2-b^2)-ab\cosh(x))}{2b^2}-\frac{\frac{ax(2a^2-3b^2)}{b}-\frac{2(a^2-b^2)^2\int\frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}}{2b^2}}{b}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\frac{\sinh(x)(2(a^2-b^2)-ab\cosh(x))}{2b^2}-\frac{\frac{ax(2a^2-3b^2)}{b}-\frac{4(a^2-b^2)^2\int\frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{b}}{2b^2}}{b}+\frac{\sinh^3(x)}{3b}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\sinh(x)(2(a^2-b^2)-ab\cosh(x))}{2b^2}-\frac{\frac{ax(2a^2-3b^2)}{b}-\frac{4(a^2-b^2)^2\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}}{2b^2}}{b}+\frac{\sinh^3(x)}{3b}$$

input
```
Int[Sinh[x]^4/(a + b*Cosh[x]),x]
```

output
```
Sinh[x]^3/(3*b) + (-1/2*((a*(2*a^2 - 3*b^2)*x)/b - (4*(a^2 - b^2)^2*ArcTan
h[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]))/b^2 +
 ((2*(a^2 - b^2) - a*b*Cosh[x])*Sinh[x])/(2*b^2))/b
```

3.168.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.168. $\int\frac{\sinh^4(x)}{a+b\cosh(x)}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3174
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(b*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
 0] && IntegersQ[2*m, 2*p]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3344
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Simp[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Si
n[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^
2*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1
, 0] && IntegerQ[2*m]
```

3.168.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(87) = 174$.

Time = 24.02 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.14

method	result
default	$-\frac{1}{3b\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}-\frac{a+b}{2b^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}-\frac{2a^2+ab-2b^2}{2b^3\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{a(2a^2-3b^2)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b^4}-\frac{2(-a^4+2a^2b^2-b^4)\,\mathrm{arctanh}\left(\right.}{b^4\sqrt{(a+b)(a-}}$
risch	$-\frac{a^3x}{b^4}+\frac{3ax}{2b^2}+\frac{e^{3x}}{24b}-\frac{a\,e^{2x}}{8b^2}+\frac{e^{x}a^2}{2b^3}-\frac{5\,e^{x}}{8b}-\frac{e^{-x}a^2}{2b^3}+\frac{5\,e^{-x}}{8b}+\frac{a\,e^{-2x}}{8b^2}-\frac{e^{-3x}}{24b}+\frac{\sqrt{a^2-b^2}\,\ln\left(e^{x}+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right.}{b^4}$

input
```
int(sinh(x)^4/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/3/b/(tanh(1/2*x)-1)^3-1/2*(a+b)/b^2/(tanh(1/2*x)-1)^2-1/2*(2*a^2+a*b-2*
b^2)/b^3/(tanh(1/2*x)-1)+1/2*a*(2*a^2-3*b^2)/b^4*ln(tanh(1/2*x)-1)-2/b^4*(
-a^4+2*a^2*b^2-b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(
a-b))^(1/2))-1/3/b/(tanh(1/2*x)+1)^3-1/2*(-a-b)/b^2/(tanh(1/2*x)+1)^2-1/2*
(2*a^2+a*b-2*b^2)/b^3/(tanh(1/2*x)+1)-1/2*a*(2*a^2-3*b^2)/b^4*ln(tanh(1/2*
x)+1)
```

3.168.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 515 vs. $2(86) = 172$.

Time = 0.28 (sec) , antiderivative size = 1099, normalized size of antiderivative = 10.57

$$\int \frac{\sinh^4(x)}{a+b\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(x)^4/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[1/24*(b^3*cosh(x)^6 + b^3*sinh(x)^6 - 3*a*b^2*cosh(x)^5 + 3*(2*b^3*cosh(x
) - a*b^2)*sinh(x)^5 - 12*(2*a^3 - 3*a*b^2)*x*cosh(x)^3 + 3*(4*a^2*b - 5*b
^3)*cosh(x)^4 + 3*(5*b^3*cosh(x)^2 - 5*a*b^2*cosh(x) + 4*a^2*b - 5*b^3)*si
nh(x)^4 + 3*a*b^2*cosh(x) + 2*(10*b^3*cosh(x)^3 - 15*a*b^2*cosh(x)^2 - 6*(
2*a^3 - 3*a*b^2)*x + 6*(4*a^2*b - 5*b^3)*cosh(x))*sinh(x)^3 - b^3 - 3*(4*a
^2*b - 5*b^3)*cosh(x)^2 + 3*(5*b^3*cosh(x)^4 - 10*a*b^2*cosh(x)^3 - 4*a^2*
b + 5*b^3 - 12*(2*a^3 - 3*a*b^2)*x*cosh(x) + 6*(4*a^2*b - 5*b^3)*cosh(x)^2
)*sinh(x)^2 - 24*((a^2 - b^2)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x)^2*sinh(x)
+ 3*(a^2 - b^2)*cosh(x)*sinh(x)^2 + (a^2 - b^2)*sinh(x)^3)*sqrt(a^2 - b^2)
*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2
*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(
b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b))
+ 3*(2*b^3*cosh(x)^5 - 5*a*b^2*cosh(x)^4 - 12*(2*a^3 - 3*a*b^2)*x*cosh(x)^
2 + 4*(4*a^2*b - 5*b^3)*cosh(x)^3 + a*b^2 - 2*(4*a^2*b - 5*b^3)*cosh(x))*s
inh(x))/(b^4*cosh(x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x)^2
 + b^4*sinh(x)^3), 1/24*(b^3*cosh(x)^6 + b^3*sinh(x)^6 - 3*a*b^2*cosh(x)^5
 + 3*(2*b^3*cosh(x) - a*b^2)*sinh(x)^5 - 12*(2*a^3 - 3*a*b^2)*x*cosh(x)^3
+ 3*(4*a^2*b - 5*b^3)*cosh(x)^4 + 3*(5*b^3*cosh(x)^2 - 5*a*b^2*cosh(x) + 4
*a^2*b - 5*b^3)*sinh(x)^4 + 3*a*b^2*cosh(x) + 2*(10*b^3*cosh(x)^3 - 15*a*b
^2*cosh(x)^2 - 6*(2*a^3 - 3*a*b^2)*x + 6*(4*a^2*b - 5*b^3)*cosh(x))*sin...
```

3.168.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^4(x)}{a + b\cosh(x)}\, dx = \text{Timed out}$$

input
```
integrate(sinh(x)**4/(a+b*cosh(x)),x)
```

output
```
Timed out
```

3.168.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh^4(x)}{a+b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sinh(x)^4/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.168.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.40

$$\int \frac{\sinh^4(x)}{a+b\cosh(x)}\,dx = \frac{b^2e^{(3x)} - 3\,abe^{(2x)} + 12\,a^2e^x - 15\,b^2e^x}{24\,b^3} - \frac{(2\,a^3 - 3\,ab^2)x}{2\,b^4} + \frac{\left(3\,ab^2e^x - b^3 - 3\,(4\,a^2b - 5\,b^3)e^{(2x)}\right)e^{(-3x)}}{24\,b^4} + \frac{2\,(a^4 - 2\,a^2b^2 + b^4)\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}b^4}$$

input
```
integrate(sinh(x)^4/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
1/24*(b^2*e^(3*x) - 3*a*b*e^(2*x) + 12*a^2*e^x - 15*b^2*e^x)/b^3 - 1/2*(2*
a^3 - 3*a*b^2)*x/b^4 + 1/24*(3*a*b^2*e^x - b^3 - 3*(4*a^2*b - 5*b^3)*e^(2*
x))*e^(-3*x)/b^4 + 2*(a^4 - 2*a^2*b^2 + b^4)*arctan((b*e^x + a)/sqrt(-a^2
+ b^2))/(sqrt(-a^2 + b^2)*b^4)
```

3.168.9 Mupad [B] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.13

$$\int \frac{\sinh^4(x)}{a+b\cosh(x)}\,dx = \frac{e^{3x}}{24b} - \frac{e^{-3x}}{24b} + \frac{x\,(3\,a\,b^2-2\,a^3)}{2\,b^4} + \frac{e^x\,(4\,a^2-5\,b^2)}{8\,b^3} + \frac{a\,e^{-2x}}{8\,b^2} - \frac{a\,e^{2x}}{8\,b^2} - \frac{e^{-x}\,(4\,a^2-5\,b^2)}{8\,b^3} + \frac{\ln\left(-\frac{2\,e^x\,(a^4-2\,a^2\,b^2+b^4)}{b^5} - \frac{2\,(a+b)^{3/2}\,(b+a\,e^x)\,(a-b)^{3/2}}{b^5}\right)(a+b)^{3/2}\,(a-b)^{3/2}}{b^4} - \frac{\ln\left(\frac{2\,(a+b)^{3/2}\,(b+a\,e^x)\,(a-b)^{3/2}}{b^5} - \frac{2\,e^x\,(a^4-2\,a^2\,b^2+b^4)}{b^5}\right)(a+b)^{3/2}\,(a-b)^{3/2}}{b^4}$$

input

```
int(sinh(x)^4/(a + b*cosh(x)),x)
```

output

```
exp(3*x)/(24*b) - exp(-3*x)/(24*b) + (x*(3*a*b^2 - 2*a^3))/(2*b^4) + (exp(
x)*(4*a^2 - 5*b^2))/(8*b^3) + (a*exp(-2*x))/(8*b^2) - (a*exp(2*x))/(8*b^2)
 - (exp(-x)*(4*a^2 - 5*b^2))/(8*b^3) + (log(- (2*exp(x)*(a^4 + b^4 - 2*a^2
*b^2))/b^5 - (2*(a + b)^(3/2)*(b + a*exp(x))*(a - b)^(3/2))/b^5)*(a + b)^(
3/2)*(a - b)^(3/2))/b^4 - (log((2*(a + b)^(3/2)*(b + a*exp(x))*(a - b)^(3/
2))/b^5 - (2*exp(x)*(a^4 + b^4 - 2*a^2*b^2))/b^5)*(a + b)^(3/2)*(a - b)^(3
/2))/b^4
```

3.169 $\int \frac{\sinh^3(x)}{a+b\cosh(x)}\,dx$

3.169.1 Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{\sinh^3(x)}{a+b\cosh(x)}\,dx = -\frac{a\cosh(x)}{b^2}+\frac{\cosh^2(x)}{2b}+\frac{(a^2-b^2)\log(a+b\cosh(x))}{b^3}$$

output
```
-a*cosh(x)/b^2+1/2*cosh(x)^2/b+(a^2-b^2)*ln(a+b*cosh(x))/b^3
```

3.169.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^3(x)}{a+b\cosh(x)}\,dx = -\frac{a\cosh(x)}{b^2}+\frac{\cosh(2x)}{4b}+\frac{(a^2-b^2)\log(a+b\cosh(x))}{b^3}$$

input
```
Integrate[Sinh[x]^3/(a + b*Cosh[x]),x]
```

output
```
-((a*Cosh[x])/b^2) + Cosh[2*x]/(4*b) + ((a^2 - b^2)*Log[a + b*Cosh[x]])/b^
3
```

3.169.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 26, 3147, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\cos\left(-\frac{\pi}{2}+ix\right)^3}{a-b\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^3}{a-b\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3147}$$

$$-\frac{\int \frac{b^2-b^2\cosh^2(x)}{a+b\cosh(x)}d(b\cosh(x))}{b^3}$$

$$\Big\downarrow \text{476}$$

$$-\frac{\int \left(a-b\cosh(x)+\frac{b^2-a^2}{a+b\cosh(x)}\right)d(b\cosh(x))}{b^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-(a^2-b^2)\log(a+b\cosh(x))+ab\cosh(x)-\frac{1}{2}b^2\cosh^2(x)}{b^3}$$

input
```
Int[Sinh[x]^3/(a + b*Cosh[x]),x]
```

output
```
-((a*b*Cosh[x] - (b^2*Cosh[x]^2)/2 - (a^2 - b^2)*Log[a + b*Cosh[x]])/b^3)
```

3.169.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.169.4 Maple [A] (verified)

Time = 3.60 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

method	result	size
derivativedivides	$-\frac{-\frac{b\cosh(x)^2}{2}+a\cosh(x)}{b^2}+\frac{(a^2-b^2)\ln(a+b\cosh(x))}{b^3}$	39
default	$-\frac{-\frac{b\cosh(x)^2}{2}+a\cosh(x)}{b^2}+\frac{(a^2-b^2)\ln(a+b\cosh(x))}{b^3}$	39
risch	$-\frac{x\,a^2}{b^3}+\frac{x}{b}+\frac{e^{2x}}{8b}-\frac{a\,e^{x}}{2b^2}-\frac{a\,e^{-x}}{2b^2}+\frac{e^{-2x}}{8b}+\frac{\ln\left(e^{2x}+\frac{2a\,e^{x}}{b}+1\right)a^2}{b^3}-\frac{\ln\left(e^{2x}+\frac{2a\,e^{x}}{b}+1\right)}{b}$	94

input
```
int(sinh(x)^3/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/b^2*(-1/2*b*cosh(x)^2+a*cosh(x))+(a^2-b^2)*ln(a+b*cosh(x))/b^3
```

3.169.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. 2(38) = 76.

Time = 0.28 (sec) , antiderivative size = 234, normalized size of antiderivative = 5.85

$$\int \frac{\sinh^3(x)}{a+b\cosh(x)}\,dx = \frac{b^2\cosh(x)^4 + b^2\sinh(x)^4 - 4\,ab\cosh(x)^3 - 8\,(a^2-b^2)x\cosh(x)^2 + 4\,(b^2\cosh(x)-ab)\sinh(x)^3 - 4\,ab\,c}{}$$

input
```
integrate(sinh(x)^3/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
1/8*(b^2*cosh(x)^4 + b^2*sinh(x)^4 - 4*a*b*cosh(x)^3 - 8*(a^2 - b^2)*x*cos
h(x)^2 + 4*(b^2*cosh(x) - a*b)*sinh(x)^3 - 4*a*b*cosh(x) + 2*(3*b^2*cosh(x
)^2 - 6*a*b*cosh(x) - 4*(a^2 - b^2)*x)*sinh(x)^2 + b^2 + 8*((a^2 - b^2)*co
sh(x)^2 + 2*(a^2 - b^2)*cosh(x)*sinh(x) + (a^2 - b^2)*sinh(x)^2)*log(2*(b*
cosh(x) + a)/(cosh(x) - sinh(x))) + 4*(b^2*cosh(x)^3 - 3*a*b*cosh(x)^2 - 4
*(a^2 - b^2)*x*cosh(x) - a*b)*sinh(x))/(b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh
(x) + b^3*sinh(x)^2)
```

3.169.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1591 vs. 2(34) = 68.

Time = 154.33 (sec) , antiderivative size = 1591, normalized size of antiderivative = 39.78

$$\int \frac{\sinh^3(x)}{a+b\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(x)**3/(a+b*cosh(x)),x)
```

output
```
Piecewise((zoo*(-x*tanh(x/2)**4/(tanh(x/2)**4 - 2*tanh(x/2)**2 + 1) + 2*x*
tanh(x/2)**2/(tanh(x/2)**4 - 2*tanh(x/2)**2 + 1) - x/(tanh(x/2)**4 - 2*tan
h(x/2)**2 + 1) + 2*log(tanh(x/2) + 1)*tanh(x/2)**4/(tanh(x/2)**4 - 2*tanh(
x/2)**2 + 1) - 4*log(tanh(x/2) + 1)*tanh(x/2)**2/(tanh(x/2)**4 - 2*tanh(x/
2)**2 + 1) + 2*log(tanh(x/2) + 1)/(tanh(x/2)**4 - 2*tanh(x/2)**2 + 1) - lo
g(tanh(x/2)**2 + 1)*tanh(x/2)**4/(tanh(x/2)**4 - 2*tanh(x/2)**2 + 1) + 2*l
og(tanh(x/2)**2 + 1)*tanh(x/2)**2/(tanh(x/2)**4 - 2*tanh(x/2)**2 + 1) - lo
g(tanh(x/2)**2 + 1)/(tanh(x/2)**4 - 2*tanh(x/2)**2 + 1) + 2*tanh(x/2)**2/(
tanh(x/2)**4 - 2*tanh(x/2)**2 + 1)), Eq(a, 0) & Eq(b, 0)), (4*tanh(x/2)**2
/(b*tanh(x/2)**4 - 2*b*tanh(x/2)**2 + b) - 2/(b*tanh(x/2)**4 - 2*b*tanh(x/
2)**2 + b), Eq(a, b)), ((sinh(x)**2*cosh(x) - 2*cosh(x)**3/3)/a, Eq(b, 0))
, (a**2*x*tanh(x/2)**4/(b**3*tanh(x/2)**4 - 2*b**3*tanh(x/2)**2 + b**3) -
2*a**2*x*tanh(x/2)**2/(b**3*tanh(x/2)**4 - 2*b**3*tanh(x/2)**2 + b**3) + a
**2*x/(b**3*tanh(x/2)**4 - 2*b**3*tanh(x/2)**2 + b**3) + a**2*log(-sqrt(a/
(a - b) + b/(a - b)) + tanh(x/2))*tanh(x/2)**4/(b**3*tanh(x/2)**4 - 2*b**3
*tanh(x/2)**2 + b**3) - 2*a**2*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2
))*tanh(x/2)**2/(b**3*tanh(x/2)**4 - 2*b**3*tanh(x/2)**2 + b**3) + a**2*lo
g(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(b**3*tanh(x/2)**4 - 2*b**3*ta
nh(x/2)**2 + b**3) + a**2*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))*tan
h(x/2)**4/(b**3*tanh(x/2)**4 - 2*b**3*tanh(x/2)**2 + b**3) - 2*a**2*log...
```

3.169.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(38) = 76$.

Time = 0.20 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.10

$$\int \frac{\sinh^3(x)}{a+b\cosh(x)}\,dx = -\frac{\left(4\,ae^{(-x)}-b\right)e^{(2\,x)}}{8\,b^2} - \frac{4\,ae^{(-x)}-be^{(-2\,x)}}{8\,b^2} + \frac{(a^2-b^2)x}{b^3} + \frac{(a^2-b^2)\log\left(2\,ae^{(-x)}+be^{(-2\,x)}+b\right)}{b^3}$$

input
```
integrate(sinh(x)^3/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
-1/8*(4*a*e^(-x) - b)*e^(2*x)/b^2 - 1/8*(4*a*e^(-x) - b*e^(-2*x))/b^2 + (a
^2 - b^2)*x/b^3 + (a^2 - b^2)*log(2*a*e^(-x) + b*e^(-2*x) + b)/b^3
```

3.169.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.40

$$\int \frac{\sinh^3(x)}{a+b\cosh(x)}\,dx = \frac{b\left(e^{(-x)}+e^x\right)^2-4\,a\left(e^{(-x)}+e^x\right)}{8\,b^2}+\frac{(a^2-b^2)\log\left(\left|b\left(e^{(-x)}+e^x\right)+2\,a\right|\right)}{b^3}$$

input
```
integrate(sinh(x)^3/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
1/8*(b*(e^(-x) + e^x)^2 - 4*a*(e^(-x) + e^x))/b^2 + (a^2 - b^2)*log(abs(b*
(e^(-x) + e^x) + 2*a))/b^3
```

3.169.9 Mupad [B] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.98

$$\int \frac{\sinh^3(x)}{a+b\cosh(x)}\,dx = \frac{\mathrm{e}^{-2\,x}}{8\,b}+\frac{\mathrm{e}^{2\,x}}{8\,b}+\frac{\ln\left(b+2\,a\,\mathrm{e}^{x}+b\,\mathrm{e}^{2\,x}\right)\,\left(a^2-b^2\right)}{b^3}-\frac{a\,\mathrm{e}^{x}}{2\,b^2}-\frac{a\,\mathrm{e}^{-x}}{2\,b^2}-\frac{x\,\left(a^2-b^2\right)}{b^3}$$

input
```
int(sinh(x)^3/(a + b*cosh(x)),x)
```

output
```
exp(-2*x)/(8*b) + exp(2*x)/(8*b) + (log(b + 2*a*exp(x) + b*exp(2*x))*(a^2
- b^2))/b^3 - (a*exp(x))/(2*b^2) - (a*exp(-x))/(2*b^2) - (x*(a^2 - b^2))/b
^3
```

3.170 $\int \frac{\sinh^2(x)}{a+b\cosh(x)}\,dx$

3.170.1 Optimal result

Integrand size = 13, antiderivative size = 59

$$\int \frac{\sinh^2(x)}{a+b\cosh(x)}\,dx = -\frac{ax}{b^2} + \frac{2\sqrt{a-b}\sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^2} + \frac{\sinh(x)}{b}$$

output
```
-a*x/b^2+sinh(x)/b+2*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))*(a-b)^(1
/2)*(a+b)^(1/2)/b^2
```

3.170.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \frac{\sinh^2(x)}{a+b\cosh(x)}\,dx = \frac{-ax + 2\sqrt{-a^2+b^2}\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right) + b\sinh(x)}{b^2}$$

input
```
Integrate[Sinh[x]^2/(a + b*Cosh[x]),x]
```

output
```
(-(a*x) + 2*Sqrt[-a^2 + b^2]*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]]
+ b*Sinh[x])/b^2
```

3.170.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {3042, 25, 3174, 25, 3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)}{a+b\cosh(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\cos\left(-\frac{\pi}{2}+ix\right)^2}{a-b\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^2}{a-b\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3174}$$

$$\frac{\int -\frac{b+a\cosh(x)}{a+b\cosh(x)}dx}{b}+\frac{\sinh(x)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh(x)}{b}-\frac{\int \frac{b+a\cosh(x)}{a+b\cosh(x)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(x)}{b}-\frac{\int \frac{b+a\sin\left(ix+\frac{\pi}{2}\right)}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\sinh(x)}{b}-\frac{\frac{ax}{b}-\frac{(a^2-b^2)\int \frac{1}{a+b\cosh(x)}dx}{b}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(x)}{b}-\frac{\frac{ax}{b}-\frac{(a^2-b^2)\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}}{b}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\sinh(x)}{b}-\frac{\frac{ax}{b}-\frac{2(a^2-b^2)\int\frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{b}}{b}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sinh(x)}{b}-\frac{\frac{ax}{b}-\frac{2(a^2-b^2)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}}{b}$$

input
```
Int[Sinh[x]^2/(a + b*Cosh[x]),x]
```

output
```
-(((a*x)/b - (2*(a^2 - b^2)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/
(Sqrt[a - b]*b*Sqrt[a + b]))/b) + Sinh[x]/b
```

3.170.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3174
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(b*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
 0] && IntegersQ[2*m, 2*p]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.170.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(49) = 98$.

Time = 0.67 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.69

method	result	size
default	$-\frac{1}{b(\tanh(\frac{x}{2})-1)}+\frac{a\ln(\tanh(\frac{x}{2})-1)}{b^2}-\frac{2(-a^2+b^2)\operatorname{arctanh}\left(\frac{(a-b)\tanh(\frac{x}{2})}{\sqrt{(a+b)(a-b)}}\right)}{b^2\sqrt{(a+b)(a-b)}}-\frac{1}{b(\tanh(\frac{x}{2})+1)}-\frac{a\ln(\tanh(\frac{x}{2})+1)}{b^2}$	100
risch	$-\frac{ax}{b^2}+\frac{e^x}{2b}-\frac{e^{-x}}{2b}+\frac{\sqrt{a^2-b^2}\ln\left(e^x-\frac{-a+\sqrt{a^2-b^2}}{b}\right)}{b^2}-\frac{\sqrt{a^2-b^2}\ln\left(e^x+\frac{a+\sqrt{a^2-b^2}}{b}\right)}{b^2}$	101

input
```
int(sinh(x)^2/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/b/(tanh(1/2*x)-1)+a/b^2*ln(tanh(1/2*x)-1)-2/b^2*(-a^2+b^2)/((a+b)*(a-b)
)^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))-1/b/(tanh(1/2*x)+1)
-a/b^2*ln(tanh(1/2*x)+1)
```

3.170.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(49) = 98$.

Time = 0.28 (sec) , antiderivative size = 279, normalized size of antiderivative = 4.73

$$\int \frac{\sinh^2(x)}{a+b\cosh(x)}\,dx$$
$$= \left[-\frac{2\,ax\cosh(x)-b\cosh(x)^2-b\sinh(x)^2-2\sqrt{a^2-b^2}(\cosh(x)+\sinh(x))\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\,c\cdots}{b\cosh(\cdots}\right.}{2\left(b^2\cosh(x)+b^2\cdots\right.}\right.$$
$$-\frac{2\,ax\cosh(x)-b\cosh(x)^2-b\sinh(x)^2+4\sqrt{-a^2+b^2}(\cosh(x)+\sinh(x))\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cosh(x)+\cdots}{a^2-b^2}\right.}{2\left(b^2\cosh(x)+b^2\sinh(x)\right)}$$

input
```
integrate(sinh(x)^2/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[-1/2*(2*a*x*cosh(x) - b*cosh(x)^2 - b*sinh(x)^2 - 2*sqrt(a^2 - b^2)*(cosh
(x) + sinh(x))*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2
- b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*s
inh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*
sinh(x) + b)) + 2*(a*x - b*cosh(x))*sinh(x) + b)/(b^2*cosh(x) + b^2*sinh(x
)), -1/2*(2*a*x*cosh(x) - b*cosh(x)^2 - b*sinh(x)^2 + 4*sqrt(-a^2 + b^2)*(
cosh(x) + sinh(x))*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a
^2 - b^2)) + 2*(a*x - b*cosh(x))*sinh(x) + b)/(b^2*cosh(x) + b^2*sinh(x))]
```

3.170.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 892 vs. $2(49) = 98$.

Time = 51.50 (sec) , antiderivative size = 892, normalized size of antiderivative = 15.12

$$\int \frac{\sinh^2(x)}{a + b\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(x)**2/(a+b*cosh(x)),x)
```

output
```
Piecewise((zoo*(-2*tanh(x/2)**2*atan(tanh(x/2))/(tanh(x/2)**2 - 1) - 2*tan
h(x/2)/(tanh(x/2)**2 - 1) + 2*atan(tanh(x/2))/(tanh(x/2)**2 - 1)), Eq(a, 0
) & Eq(b, 0)), (-x*tanh(x/2)**2/(b*tanh(x/2)**2 - b) + x/(b*tanh(x/2)**2 -
 b) - 2*tanh(x/2)/(b*tanh(x/2)**2 - b), Eq(a, b)), (x*tanh(x/2)**2/(b*tanh
(x/2)**2 - b) - x/(b*tanh(x/2)**2 - b) - 2*tanh(x/2)/(b*tanh(x/2)**2 - b),
 Eq(a, -b)), ((x*sinh(x)**2/2 - x*cosh(x)**2/2 + sinh(x)*cosh(x)/2)/a, Eq(
b, 0)), (-a*x*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2/(b**2*sqrt(a/(a - b
) + b/(a - b))*tanh(x/2)**2 - b**2*sqrt(a/(a - b) + b/(a - b))) + a*x*sqrt
(a/(a - b) + b/(a - b))/(b**2*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 - b
**2*sqrt(a/(a - b) + b/(a - b))) - a*log(-sqrt(a/(a - b) + b/(a - b)) + ta
nh(x/2))*tanh(x/2)**2/(b**2*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 - b**
2*sqrt(a/(a - b) + b/(a - b))) + a*log(-sqrt(a/(a - b) + b/(a - b)) + tanh
(x/2))/(b**2*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 - b**2*sqrt(a/(a - b
) + b/(a - b))) + a*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))*tanh(x/2)
**2/(b**2*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 - b**2*sqrt(a/(a - b) +
 b/(a - b))) - a*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(b**2*sqrt(a
/(a - b) + b/(a - b))*tanh(x/2)**2 - b**2*sqrt(a/(a - b) + b/(a - b))) - 2
*b*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)/(b**2*sqrt(a/(a - b) + b/(a - b))
*tanh(x/2)**2 - b**2*sqrt(a/(a - b) + b/(a - b))) - b*log(-sqrt(a/(a - b)
+ b/(a - b)) + tanh(x/2))*tanh(x/2)**2/(b**2*sqrt(a/(a - b) + b/(a - b)...
```

3.170.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh^2(x)}{a+b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sinh(x)^2/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.170.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.15

$$\int \frac{\sinh^2(x)}{a+b\cosh(x)}\,dx = -\frac{ax}{b^2} - \frac{e^{(-x)}}{2\,b} + \frac{e^x}{2\,b} + \frac{2\,(a^2-b^2)\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}b^2}$$

input
```
integrate(sinh(x)^2/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
-a*x/b^2 - 1/2*e^(-x)/b + 1/2*e^x/b + 2*(a^2 - b^2)*arctan((b*e^x + a)/sqr
t(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b^2)
```

3.170.9 Mupad [B] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.36

$$\int \frac{\sinh^2(x)}{a+b\cosh(x)}\,dx = \frac{e^x}{2\,b} - \frac{e^{-x}}{2\,b} - \frac{a\,x}{b^2} + \frac{\ln\left(-\frac{2\,e^x\,(a^2-b^2)}{b^3} - \frac{2\,\sqrt{a+b}\,(b+a\,e^x)\,\sqrt{a-b}}{b^3}\right)\sqrt{a+b}\,\sqrt{a-b}}{b^2} - \frac{\ln\left(\frac{2\,\sqrt{a+b}\,(b+a\,e^x)\,\sqrt{a-b}}{b^3} - \frac{2\,e^x\,(a^2-b^2)}{b^3}\right)\sqrt{a+b}\,\sqrt{a-b}}{b^2}$$

input
```
int(sinh(x)^2/(a + b*cosh(x)),x)
```

output
```
exp(x)/(2*b) - exp(-x)/(2*b) - (a*x)/b^2 + (log(- (2*exp(x)*(a^2 - b^2))/b
^3 - (2*(a + b)^(1/2)*(b + a*exp(x))*(a - b)^(1/2))/b^3)*(a + b)^(1/2)*(a
- b)^(1/2))/b^2 - (log((2*(a + b)^(1/2)*(b + a*exp(x))*(a - b)^(1/2))/b^3
- (2*exp(x)*(a^2 - b^2))/b^3)*(a + b)^(1/2)*(a - b)^(1/2))/b^2
```

3.170. $\int \frac{\sinh^2(x)}{a+b\cosh(x)}\,dx$

3.171 $\int \frac{\sinh(x)}{a+b\cosh(x)}\,dx$

3.171.1 Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{\sinh(x)}{a+b\cosh(x)}\,dx = \frac{\log(a+b\cosh(x))}{b}$$

output
```
ln(a+b*cosh(x))/b
```

3.171.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(x)}{a+b\cosh(x)}\,dx = \frac{\log(a+b\cosh(x))}{b}$$

input
```
Integrate[Sinh[x]/(a + b*Cosh[x]),x]
```

output
```
Log[a + b*Cosh[x]]/b
```

3.171.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 26, 3147, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\cos\left(-\frac{\pi}{2}+ix\right)}{a-b\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)}{a-b\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3147}$$

$$\frac{\int \frac{1}{a+b\cosh(x)}d(b\cosh(x))}{b}$$

$$\Big\downarrow \text{16}$$

$$\frac{\log(a+b\cosh(x))}{b}$$

input
```
Int[Sinh[x]/(a + b*Cosh[x]),x]
```

output
```
Log[a + b*Cosh[x]]/b
```

3.171.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.171.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{\ln(a+b\cosh(x))}{b}$	12
default	$\frac{\ln(a+b\cosh(x))}{b}$	12
risch	$-\frac{x}{b}+\frac{\ln\left(e^{2x}+\frac{2a\,e^{x}}{b}+1\right)}{b}$	27

input
```
int(sinh(x)/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(a+b*cosh(x))/b
```

3.171.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(11) = 22.

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.45

$$\int \frac{\sinh(x)}{a+b\cosh(x)}\,dx = -\frac{x-\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right)}{b}$$

input
```
integrate(sinh(x)/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
-(x - log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))))/b
```

3.171.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \frac{\sinh(x)}{a+b\cosh(x)}\,dx = \begin{cases} \frac{\log\left(\frac{a}{b}+\cosh(x)\right)}{b} & \text{for } b\neq 0 \\ \frac{\cosh(x)}{a} & \text{otherwise} \end{cases}$$

input
```
integrate(sinh(x)/(a+b*cosh(x)),x)
```

output
```
Piecewise((log(a/b + cosh(x))/b, Ne(b, 0)), (cosh(x)/a, True))
```

3.171.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(x)}{a+b\cosh(x)}\,dx = \frac{\log(b\cosh(x)+a)}{b}$$

input
```
integrate(sinh(x)/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
log(b*cosh(x) + a)/b
```

3.171.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int \frac{\sinh(x)}{a+b\cosh(x)}\,dx = \frac{\log\left(\left|b\left(e^{(-x)}+e^{x}\right)+2a\right|\right)}{b}$$

input
```
integrate(sinh(x)/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
log(abs(b*(e^(-x) + e^x) + 2*a))/b
```

3.171.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(x)}{a + b\cosh(x)}\,dx = \frac{\ln\left(a + b\cosh(x)\right)}{b}$$

input
```
int(sinh(x)/(a + b*cosh(x)),x)
```

output
```
log(a + b*cosh(x))/b
```

3.172 $\int \frac{\operatorname{csch}(x)}{a+b\cosh(x)}\,dx$

3.172.1 Optimal result

Integrand size = 11, antiderivative size = 53

$$\int \frac{\operatorname{csch}(x)}{a+b\cosh(x)}\,dx=\frac{\log(1-\cosh(x))}{2(a+b)}-\frac{\log(1+\cosh(x))}{2(a-b)}+\frac{b\log(a+b\cosh(x))}{a^2-b^2}$$

output
```
1/2*ln(1-cosh(x))/(a+b)-1/2*ln(1+cosh(x))/(a-b)+b*ln(a+b*cosh(x))/(a^2-b^2
)
```

3.172.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \frac{\operatorname{csch}(x)}{a+b\cosh(x)}\,dx=\frac{\log\left(\cosh\left(\frac{x}{2}\right)\right)}{-a+b}+\frac{b\log(a+b\cosh(x))}{a^2-b^2}+\frac{\log\left(\sinh\left(\frac{x}{2}\right)\right)}{a+b}$$

input
```
Integrate[Csch[x]/(a + b*Cosh[x]),x]
```

output
```
Log[Cosh[x/2]]/(-a + b) + (b*Log[a + b*Cosh[x]])/(a^2 - b^2) + Log[Sinh[x/
2]]/(a + b)
```

3.172.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 26, 3147, 477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}(x)}{a+b\cosh(x)}dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{i}{\cos\left(-\frac{\pi}{2}+ix\right)\left(a-b\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$
$$\Big\downarrow \ 26$$
$$i\int \frac{1}{\cos\left(ix-\frac{\pi}{2}\right)\left(a-b\sin\left(ix-\frac{\pi}{2}\right)\right)}dx$$
$$\Big\downarrow \ 3147$$
$$-b\int \frac{1}{(a+b\cosh(x))\left(b^2-b^2\cosh^2(x)\right)}d(b\cosh(x))$$
$$\Big\downarrow \ 477$$
$$-\frac{\int \left(-\frac{b^2}{(a^2-b^2)(a+b\cosh(x))}+\frac{b}{2(a+b)(b-b\cosh(x))}+\frac{b}{2(a-b)(\cosh(x)b+b)}\right)d(b\cosh(x))}{b}$$
$$\Big\downarrow \ 2009$$
$$-\frac{-\frac{b^2\log(a+b\cosh(x))}{a^2-b^2}-\frac{b\log(b-b\cosh(x))}{2(a+b)}+\frac{b\log(b\cosh(x)+b)}{2(a-b)}}{b}$$

input
```
Int[Csch[x]/(a + b*Cosh[x]),x]
```

output
```
-((-1/2*(b*Log[b - b*Cosh[x]])/(a + b) - (b^2*Log[a + b*Cosh[x]])/(a^2 - b
^2) + (b*Log[b + b*Cosh[x]])/(2*(a - b)))/b)
```

3.172.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.172.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a+b}+\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)^{2}a-\tanh\left(\frac{x}{2}\right)^{2}b-a-b\right)}{(a+b)(a-b)}$	52
risch	$-\frac{x}{a+b}+\frac{x}{a-b}-\frac{2xb}{a^{2}-b^{2}}+\frac{\ln(e^{x}-1)}{a+b}-\frac{\ln(e^{x}+1)}{a-b}+\frac{b\ln\left(e^{2x}+\frac{2a\,e^{x}}{b}+1\right)}{a^{2}-b^{2}}$	87

input
```
int(csch(x)/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/(a+b)*ln(tanh(1/2*x))+b/(a+b)/(a-b)*ln(tanh(1/2*x)^2*a-tanh(1/2*x)^2*b-a
-b)
```

3.172.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

$$\int \frac{\operatorname{csch}(x)}{a+b\cosh(x)}\,dx = \frac{b\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right)-(a+b)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)+(a-b)\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)}{a^2-b^2}$$

input
```
integrate(csch(x)/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
(b*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) - (a + b)*log(cosh(x) + sinh
(x) + 1) + (a - b)*log(cosh(x) + sinh(x) - 1))/(a^2 - b^2)
```

3.172.6 Sympy [F]

$$\int \frac{\operatorname{csch}(x)}{a+b\cosh(x)}\,dx = \int \frac{\operatorname{csch}\left(x\right)}{a+b\cosh\left(x\right)}\,dx$$

input
```
integrate(csch(x)/(a+b*cosh(x)),x)
```

output
```
Integral(csch(x)/(a + b*cosh(x)), x)
```

3.172.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}(x)}{a+b\cosh(x)}\,dx = \frac{b\log\left(2\,ae^{(-x)}+be^{(-2\,x)}+b\right)}{a^2-b^2}-\frac{\log\left(e^{(-x)}+1\right)}{a-b}+\frac{\log\left(e^{(-x)}-1\right)}{a+b}$$

input
```
integrate(csch(x)/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
b*log(2*a*e^(-x) + b*e^(-2*x) + b)/(a^2 - b^2) - log(e^(-x) + 1)/(a - b) +
 log(e^(-x) - 1)/(a + b)
```

3.172.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.26

$$\int \frac{\operatorname{csch}(x)}{a+b\cosh(x)}\,dx = \frac{b^2\log\left(\left|b\left(e^{(-x)}+e^x\right)+2\,a\right|\right)}{a^2b-b^3} - \frac{\log\left(e^{(-x)}+e^x+2\right)}{2\,(a-b)} + \frac{\log\left(e^{(-x)}+e^x-2\right)}{2\,(a+b)}$$

input
```
integrate(csch(x)/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
b^2*log(abs(b*(e^(-x) + e^x) + 2*a))/(a^2*b - b^3) - 1/2*log(e^(-x) + e^x
+ 2)/(a - b) + 1/2*log(e^(-x) + e^x - 2)/(a + b)
```

3.172.9 Mupad [B] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 160, normalized size of antiderivative = 3.02

$$\int \frac{\operatorname{csch}(x)}{a+b\cosh(x)}\,dx = \frac{\ln\left(128\,a\,b^2-128\,a^2\,b+32\,a^3-32\,a^3\,\mathrm{e}^x-128\,a\,b^2\,\mathrm{e}^x+128\,a^2\,b\,\mathrm{e}^x\right)}{a+b} - \frac{\ln\left(128\,a\,b^2+128\,a^2\,b+32\,a^3+32\,a^3\,\mathrm{e}^x+128\,a\,b^2\,\mathrm{e}^x+128\,a^2\,b\,\mathrm{e}^x\right)}{a-b} + \frac{b\,\ln\left(16\,b^3\,\mathrm{e}^{2\,x}-4\,a^2\,b+16\,b^3-8\,a^3\,\mathrm{e}^x+32\,a\,b^2\,\mathrm{e}^x-4\,a^2\,b\,\mathrm{e}^{2\,x}\right)}{a^2-b^2}$$

input
```
int(1/(sinh(x)*(a + b*cosh(x))),x)
```

output
```
log(128*a*b^2 - 128*a^2*b + 32*a^3 - 32*a^3*exp(x) - 128*a*b^2*exp(x) + 12
8*a^2*b*exp(x))/(a + b) - log(128*a*b^2 + 128*a^2*b + 32*a^3 + 32*a^3*exp(
x) + 128*a*b^2*exp(x) + 128*a^2*b*exp(x))/(a - b) + (b*log(16*b^3*exp(2*x)
 - 4*a^2*b + 16*b^3 - 8*a^3*exp(x) + 32*a*b^2*exp(x) - 4*a^2*b*exp(2*x)))/
(a^2 - b^2)
```

3.173 $\int \frac{\operatorname{csch}^2(x)}{a+b\cosh(x)}\,dx$

3.173.1 Optimal result

Integrand size = 13, antiderivative size = 67

$$\int \frac{\operatorname{csch}^2(x)}{a+b\cosh(x)}\,dx=\frac{2b^2\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}+\frac{(b-a\cosh(x))\operatorname{csch}(x)}{a^2-b^2}$$

output
```
2*b^2*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(3/2)/(a+b)^(3/2)
+(b-a*cosh(x))*csch(x)/(a^2-b^2)
```

3.173.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.15

$$\int \frac{\operatorname{csch}^2(x)}{a+b\cosh(x)}\,dx=\frac{2b^2\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}}-\frac{\coth\left(\frac{x}{2}\right)}{2(a+b)}-\frac{\tanh\left(\frac{x}{2}\right)}{2(a-b)}$$

input
```
Integrate[Csch[x]^2/(a + b*Cosh[x]),x]
```

output
```
(2*b^2*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) -
Coth[x/2]/(2*(a + b)) - Tanh[x/2]/(2*(a - b))
```

3.173.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3042, 25, 3175, 27, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\cos\left(-\frac{\pi}{2}+ix\right)^2\left(a-b\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\cos\left(ix-\frac{\pi}{2}\right)^2\left(a-b\sin\left(ix-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3175}$$

$$\frac{\int \frac{b^2}{a+b\cosh(x)}dx}{a^2-b^2}+\frac{\operatorname{csch}(x)(b-a\cosh(x))}{a^2-b^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^2\int \frac{1}{a+b\cosh(x)}dx}{a^2-b^2}+\frac{\operatorname{csch}(x)(b-a\cosh(x))}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\operatorname{csch}(x)(b-a\cosh(x))}{a^2-b^2}+\frac{b^2\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3138}$$

$$\frac{2b^2\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a^2-b^2}+\frac{\operatorname{csch}(x)(b-a\cosh(x))}{a^2-b^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{2b^2\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)}+\frac{\operatorname{csch}(x)(b-a\cosh(x))}{a^2-b^2}$$

input
```
Int[Csch[x]^2/(a + b*Cosh[x]),x]
```

3.173. $\int \frac{\operatorname{csch}^2(x)}{a+b\cosh(x)}dx$

output
```
(2*b^2*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a +
 b]*(a^2 - b^2)) + ((b - a*Cosh[x])*Csch[x])/(a^2 - b^2)
```

3.173.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3175
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*
(a^2 - b^2)*(p + 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*
(a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ
[2*m, 2*p]
```

3.173. $\int \frac{\operatorname{csch}^2(x)}{a+b\cosh(x)}\,dx$

3.173.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.16

method	result	size
default	$-\frac{\tanh\left(\frac{x}{2}\right)}{2(a-b)}-\frac{1}{2(a+b)\tanh\left(\frac{x}{2}\right)}+\frac{2b^2\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}$	78
risch	$-\frac{2(-\mathrm{e}^{x}b+a)}{(\mathrm{e}^{2x}-1)(a^2-b^2)}+\frac{b^2\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)}-\frac{b^2\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^2-b^2}+a^2-b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)}$	167

input
```
int(csch(x)^2/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a-b)*tanh(1/2*x)-1/2/(a+b)/tanh(1/2*x)+2/(a+b)/(a-b)*b^2/((a+b)*(a-b
))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

3.173.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(59) = 118.

Time = 0.28 (sec) , antiderivative size = 470, normalized size of antiderivative = 7.01

$$\int \frac{\operatorname{csch}^2(x)}{a+b\cosh(x)}\,dx$$
$$= \left[\frac{2\,a^3-2\,ab^2+\left(b^2\cosh\left(x\right)^2+2\,b^2\cosh\left(x\right)\sinh\left(x\right)+b^2\sinh\left(x\right)^2-b^2\right)\sqrt{a^2-b^2}\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+}{b}\right.}{a^4-2\,a^2b^2+b^4-\left(a^4-2\,a^2b^2+b^4\right)\cosh\left(x\right)^2-2\left(a^4\right.}\right.$$

input
```
integrate(csch(x)^2/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[(2*a^3 - 2*a*b^2 + (b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2
 - b^2)*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x)
 + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*cosh
(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh
(x) + a)*sinh(x) + b)) - 2*(a^2*b - b^3)*cosh(x) - 2*(a^2*b - b^3)*sinh(x)
)/(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*
a^2*b^2 + b^4)*cosh(x)*sinh(x) - (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^2), 2*(a^
3 - a*b^2 + (b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2 - b^2)*
sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2
 - b^2)) - (a^2*b - b^3)*cosh(x) - (a^2*b - b^3)*sinh(x))/(a^4 - 2*a^2*b^2
 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*b^2 + b^4)*cos
h(x)*sinh(x) - (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^2)]
```

3.173.6 Sympy [F]

$$\int \frac{\operatorname{csch}^2(x)}{a+b\cosh(x)}\,dx=\int \frac{\operatorname{csch}^2(x)}{a+b\cosh(x)}\,dx$$

input
```
integrate(csch(x)**2/(a+b*cosh(x)),x)
```

output
```
Integral(csch(x)**2/(a + b*cosh(x)), x)
```

3.173.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}^2(x)}{a+b\cosh(x)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(csch(x)^2/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.173.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{csch}^2(x)}{a+b\cosh(x)}\,dx = \frac{2\,b^2\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{(a^2-b^2)\sqrt{-a^2+b^2}} + \frac{2\,(be^x-a)}{(a^2-b^2)(e^{(2\,x)}-1)}$$

input
```
integrate(csch(x)^2/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
2*b^2*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/((a^2 - b^2)*sqrt(-a^2 + b^2))
+ 2*(b*e^x - a)/((a^2 - b^2)*(e^(2*x) - 1))
```

3.173.9 Mupad [B] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 327, normalized size of antiderivative = 4.88

$$\int \frac{\operatorname{csch}^2(x)}{a+b\cosh(x)}\,dx = -\frac{\frac{2\,a}{a^2-b^2}-\frac{2\,b\,e^x}{a^2-b^2}}{e^{2\,x}-1} - \frac{2\,\mathrm{atan}\left(\left(\left(e^x\left(\frac{2}{(a^2-b^2)^2\sqrt{b^4}}+\frac{2\,a\left(a^3\sqrt{b^4}-a\,b^2\sqrt{b^4}\right)}{b^4\,(a^2-b^2)\sqrt{-(a^2-b^2)^3}\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}}\right)-\frac{2\,a\left(b^3\sqrt{b^4}-a^2\,b\sqrt{b^4}\right)}{b^4\,(a^2-b^2)\sqrt{-(a^2-b^2)^3}\sqrt{-a^6+3\,a^4\,b^2-3\,a}}\right.\right.\right.}{\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}}$$

input
```
int(1/(sinh(x)^2*(a + b*cosh(x))),x)
```

output
```
- ((2*a)/(a^2 - b^2) - (2*b*exp(x))/(a^2 - b^2))/(exp(2*x) - 1) - (2*atan(
(exp(x)*(2/((a^2 - b^2)^2*(b^4)^(1/2)) + (2*a*(a^3*(b^4)^(1/2) - a*b^2*(b^
4)^(1/2)))/(b^4*(a^2 - b^2)*(-(a^2 - b^2)^3)^(1/2)*(b^6 - a^6 - 3*a^2*b^4
+ 3*a^4*b^2)^(1/2))) - (2*a*(b^3*(b^4)^(1/2) - a^2*b*(b^4)^(1/2)))/(b^4*(a
^2 - b^2)*(-(a^2 - b^2)^3)^(1/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)
))*((b^3*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/2 - (a^2*b*(b^6 - a^6
- 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/2))*(b^4)^(1/2))/(b^6 - a^6 - 3*a^2*b^4 +
3*a^4*b^2)^(1/2)
```

3.174 $\int \frac{\operatorname{csch}^3(x)}{a+b\cosh(x)}\,dx$

3.174.1 Optimal result

Integrand size = 13, antiderivative size = 91

$$\int \frac{\operatorname{csch}^3(x)}{a+b\cosh(x)}\,dx = \frac{(b-a\cosh(x))\operatorname{csch}^2(x)}{2(a^2-b^2)} - \frac{(a+2b)\log(1-\cosh(x))}{4(a+b)^2} + \frac{(a-2b)\log(1+\cosh(x))}{4(a-b)^2} + \frac{b^3\log(a+b\cosh(x))}{(a^2-b^2)^2}$$

output
```
1/2*(b-a*cosh(x))*csch(x)^2/(a^2-b^2)-1/4*(a+2*b)*ln(1-cosh(x))/(a+b)^2+1/
4*(a-2*b)*ln(1+cosh(x))/(a-b)^2+b^3*ln(a+b*cosh(x))/(a^2-b^2)^2
```

3.174.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^3(x)}{a+b\cosh(x)}\,dx = \frac{1}{8}\left(-\frac{\operatorname{csch}^2\left(\frac{x}{2}\right)}{a+b} + \frac{4(a-2b)\log\left(\cosh\left(\frac{x}{2}\right)\right)}{(a-b)^2} + \frac{8b^3\log(a+b\cosh(x))}{(a^2-b^2)^2} - \frac{4(a+2b)\log\left(\sinh\left(\frac{x}{2}\right)\right)}{(a+b)^2} - \frac{\operatorname{sech}^2\left(\frac{x}{2}\right)}{a-b}\right)$$

input
```
Integrate[Csch[x]^3/(a + b*Cosh[x]),x]
```

output
```
(-(Csch[x/2]^2/(a + b)) + (4*(a - 2*b)*Log[Cosh[x/2]])/(a - b)^2 + (8*b^3*
Log[a + b*Cosh[x]])/(a^2 - b^2)^2 - (4*(a + 2*b)*Log[Sinh[x/2]])/(a + b)^2
 - Sech[x/2]^2/(a - b))/8
```

3.174.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.29, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 26, 3147, 477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^3(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\cos\left(-\frac{\pi}{2}+ix\right)^3\left(a-b\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\cos\left(ix-\frac{\pi}{2}\right)^3\left(a-b\sin\left(ix-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3147}$$

$$b^3\int \frac{1}{(a+b\cosh(x))\left(b^2-b^2\cosh^2(x)\right)^2}d(b\cosh(x))$$

$$\Big\downarrow \text{477}$$

$$\frac{\int \left(\frac{b^4}{(a^2-b^2)^2(a+b\cosh(x))}+\frac{b^2}{4(a+b)(b-b\cosh(x))^2}+\frac{b^2}{4(a-b)(\cosh(x)b+b)^2}+\frac{(a+2b)b}{4(a+b)^2(b-b\cosh(x))}+\frac{(a-2b)b}{4(a-b)^2(\cosh(x)b+b)}\right)d(b\cosh(x))}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b^4\log(a+b\cosh(x))}{(a^2-b^2)^2}+\frac{b^2}{4(a+b)(b-b\cosh(x))}-\frac{b^2}{4(a-b)(b\cosh(x)+b)}-\frac{b(a+2b)\log(b-b\cosh(x))}{4(a+b)^2}+\frac{b(a-2b)\log(b\cosh(x)+b)}{4(a-b)^2}}{b}$$

input
```
Int[Csch[x]^3/(a + b*Cosh[x]),x]
```

output
```
(b^2/(4*(a + b)*(b - b*Cosh[x])) - b^2/(4*(a - b)*(b + b*Cosh[x])) - (b*(a
 + 2*b)*Log[b - b*Cosh[x]])/(4*(a + b)^2) + (b^4*Log[a + b*Cosh[x]])/(a^2
- b^2)^2 + ((a - 2*b)*b*Log[b + b*Cosh[x]])/(4*(a - b)^2))/b
```

3.174.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.174.4 Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

method	result
default	$\frac{\tanh\left(\frac{x}{2}\right)^{2}}{8a-8b}-\frac{1}{8(a+b)\tanh\left(\frac{x}{2}\right)^{2}}+\frac{(-2a-4b)\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{4(a+b)^{2}}+\frac{b^{3}\ln\left(\tanh\left(\frac{x}{2}\right)^{2}a-\tanh\left(\frac{x}{2}\right)^{2}b-a-b\right)}{(a+b)^{2}(a-b)^{2}}$
risch	$\frac{ax}{2a^{2}+4ab+2b^{2}}+\frac{xb}{a^{2}+2ab+b^{2}}-\frac{xa}{2(a^{2}-2ab+b^{2})}+\frac{xb}{a^{2}-2ab+b^{2}}-\frac{2x\,b^{3}}{a^{4}-2a^{2}b^{2}+b^{4}}-\frac{e^{x}\left(a\,e^{2x}-2\,e^{x}b+a\right)}{\left(e^{2x}-1\right)^{2}\left(a^{2}-b^{2}\right)}-\frac{\ln\left(e^{x}-1\right)a}{2(a^{2}+2ab+b^{2})}-$

input
```
int(csch(x)^3/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/8*tanh(1/2*x)^2/(a-b)-1/8/(a+b)/tanh(1/2*x)^2+1/4/(a+b)^2*(-2*a-4*b)*ln(
tanh(1/2*x))+b^3/(a+b)^2/(a-b)^2*ln(tanh(1/2*x)^2*a-tanh(1/2*x)^2*b-a-b)
```

3.174.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 818 vs. $2(86) = 172$.

Time = 0.29 (sec) , antiderivative size = 818, normalized size of antiderivative = 8.99

$$\int \frac{\operatorname{csch}^3(x)}{a + b\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(x)^3/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
-1/2*(2*(a^3 - a*b^2)*cosh(x)^3 + 2*(a^3 - a*b^2)*sinh(x)^3 - 4*(a^2*b - b
^3)*cosh(x)^2 - 2*(2*a^2*b - 2*b^3 - 3*(a^3 - a*b^2)*cosh(x))*sinh(x)^2 +
2*(a^3 - a*b^2)*cosh(x) - 2*(b^3*cosh(x)^4 + 4*b^3*cosh(x)*sinh(x)^3 + b^3
*sinh(x)^4 - 2*b^3*cosh(x)^2 + b^3 + 2*(3*b^3*cosh(x)^2 - b^3)*sinh(x)^2 +
 4*(b^3*cosh(x)^3 - b^3*cosh(x))*sinh(x))*log(2*(b*cosh(x) + a)/(cosh(x) -
 sinh(x))) - ((a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^4 + 4*(a^3 - 3*a*b^2 - 2*b^3
)*cosh(x)*sinh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*sinh(x)^4 + a^3 - 3*a*b^2 -
2*b^3 - 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 - 2*(a^3 - 3*a*b^2 - 2*b^3 - 3
*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 - 3*a*b^2 - 2*b^3)
*cosh(x)^3 - (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(
x) + 1) + ((a^3 - 3*a*b^2 + 2*b^3)*cosh(x)^4 + 4*(a^3 - 3*a*b^2 + 2*b^3)*c
osh(x)*sinh(x)^3 + (a^3 - 3*a*b^2 + 2*b^3)*sinh(x)^4 + a^3 - 3*a*b^2 + 2*b
^3 - 2*(a^3 - 3*a*b^2 + 2*b^3)*cosh(x)^2 - 2*(a^3 - 3*a*b^2 + 2*b^3 - 3*(a
^3 - 3*a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 - 3*a*b^2 + 2*b^3)*co
sh(x)^3 - (a^3 - 3*a*b^2 + 2*b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x)
- 1) + 2*(a^3 - a*b^2 + 3*(a^3 - a*b^2)*cosh(x)^2 - 4*(a^2*b - b^3)*cosh(x
))*sinh(x))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 4*(a^4 - 2*a^2*b^2 + b^4)
*cosh(x)*sinh(x)^3 + (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^4 + a^4 - 2*a^2*b^2 +
 b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*b^2 + b^4 - 3*
(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^2 + 4*((a^4 - 2*a^2*b^2 + b^...
```

3.174.6 Sympy [F]

$$\int \frac{\operatorname{csch}^3(x)}{a+b\cosh(x)}\,dx=\int \frac{\operatorname{csch}^3{(x)}}{a+b\cosh{(x)}}\,dx$$

input
```
integrate(csch(x)**3/(a+b*cosh(x)),x)
```

output
```
Integral(csch(x)**3/(a + b*cosh(x)), x)
```

3.174.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.69

$$\int \frac{\operatorname{csch}^3(x)}{a+b\cosh(x)}\,dx=\frac{b^3\log\left(2\,ae^{(-x)}+be^{(-2\,x)}+b\right)}{a^4-2\,a^2b^2+b^4}+\frac{(a-2\,b)\log\left(e^{(-x)}+1\right)}{2\,(a^2-2\,ab+b^2)}-\frac{(a+2\,b)\log\left(e^{(-x)}-1\right)}{2\,(a^2+2\,ab+b^2)}-\frac{ae^{(-x)}-2\,be^{(-2\,x)}+ae^{(-3\,x)}}{a^2-b^2-2\,(a^2-b^2)e^{(-2\,x)}+(a^2-b^2)e^{(-4\,x)}}$$

input
```
integrate(csch(x)^3/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
b^3*log(2*a*e^(-x) + b*e^(-2*x) + b)/(a^4 - 2*a^2*b^2 + b^4) + 1/2*(a - 2*
b)*log(e^(-x) + 1)/(a^2 - 2*a*b + b^2) - 1/2*(a + 2*b)*log(e^(-x) - 1)/(a^
2 + 2*a*b + b^2) - (a*e^(-x) - 2*b*e^(-2*x) + a*e^(-3*x))/(a^2 - b^2 - 2*(
a^2 - b^2)*e^(-2*x) + (a^2 - b^2)*e^(-4*x))
```

3.174.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(86) = 172.

Time = 0.25 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.97

$$\int \frac{\operatorname{csch}^3(x)}{a+b\cosh(x)}\,dx=\frac{b^4\log\left(\left|b\left(e^{(-x)}+e^x\right)+2\,a\right|\right)}{a^4b-2\,a^2b^3+b^5}+\frac{(a-2\,b)\log\left(e^{(-x)}+e^x+2\right)}{4\,(a^2-2\,ab+b^2)}-\frac{(a+2\,b)\log\left(e^{(-x)}+e^x-2\right)}{4\,(a^2+2\,ab+b^2)}+\frac{b^3\left(e^{(-x)}+e^x\right)^2-2\,a^3\left(e^{(-x)}+e^x\right)+2\,ab^2\left(e^{(-x)}+e^x\right)+4\,a^2b-8\,b^3}{2\,(a^4-2\,a^2b^2+b^4)\left(\left(e^{(-x)}+e^x\right)^2-4\right)}$$

input
```
integrate(csch(x)^3/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
b^4*log(abs(b*(e^(-x) + e^x) + 2*a))/(a^4*b - 2*a^2*b^3 + b^5) + 1/4*(a -
2*b)*log(e^(-x) + e^x + 2)/(a^2 - 2*a*b + b^2) - 1/4*(a + 2*b)*log(e^(-x)
+ e^x - 2)/(a^2 + 2*a*b + b^2) + 1/2*(b^3*(e^(-x) + e^x)^2 - 2*a^3*(e^(-x)
 + e^x) + 2*a*b^2*(e^(-x) + e^x) + 4*a^2*b - 8*b^3)/((a^4 - 2*a^2*b^2 + b^
4)*((e^(-x) + e^x)^2 - 4))
```

3.174.9 Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 291, normalized size of antiderivative = 3.20

$$\int \frac{\operatorname{csch}^3(x)}{a+b\cosh(x)}\,dx = \frac{\frac{2\left(a^2 b-b^3\right)}{\left(a^2-b^2\right)^2}+\frac{\mathrm{e}^{x}\left(a\,b^2-a^3\right)}{\left(a^2-b^2\right)^2}}{\mathrm{e}^{2\,x}-1}+\frac{\frac{2\,b}{a^2-b^2}-\frac{2\,a\,\mathrm{e}^{x}}{a^2-b^2}}{\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1}$$
$$+\frac{b^3\,\ln\left(16\,b^7\,\mathrm{e}^{2\,x}-a^6\,b+16\,b^7-9\,a^2\,b^5+6\,a^4\,b^3-2\,a^7\,\mathrm{e}^{x}-9\,a^2\,b^5\,\mathrm{e}^{2\,x}+6\,a^4\,b^3\,\mathrm{e}^{2\,x}+32\,a\,b^6\,\mathrm{e}^{x}-a^6\,b\,\mathrm{e}^{2}\right.}{a^4-2\,a^2\,b^2+b^4}$$
$$-\frac{\ln\left(\mathrm{e}^{x}-1\right)\,\left(a+2\,b\right)}{2\,a^2+4\,a\,b+2\,b^2}+\frac{\ln\left(\mathrm{e}^{x}+1\right)\,\left(a-2\,b\right)}{2\,a^2-4\,a\,b+2\,b^2}$$

input
```
int(1/(sinh(x)^3*(a + b*cosh(x))),x)
```

output
```
((2*(a^2*b - b^3))/(a^2 - b^2)^2 + (exp(x)*(a*b^2 - a^3))/(a^2 - b^2)^2)/(
exp(2*x) - 1) + ((2*b)/(a^2 - b^2) - (2*a*exp(x))/(a^2 - b^2))/(exp(4*x) -
 2*exp(2*x) + 1) + (b^3*log(16*b^7*exp(2*x) - a^6*b + 16*b^7 - 9*a^2*b^5 +
 6*a^4*b^3 - 2*a^7*exp(x) - 9*a^2*b^5*exp(2*x) + 6*a^4*b^3*exp(2*x) + 32*a
*b^6*exp(x) - a^6*b*exp(2*x) - 18*a^3*b^4*exp(x) + 12*a^5*b^2*exp(x)))/(a^
4 + b^4 - 2*a^2*b^2) - (log(exp(x) - 1)*(a + 2*b))/(4*a*b + 2*a^2 + 2*b^2)
 + (log(exp(x) + 1)*(a - 2*b))/(2*a^2 - 4*a*b + 2*b^2)
```

3.175 $\int \frac{\operatorname{csch}^4(x)}{a+b\cosh(x)}\,dx$

3.175.1 Optimal result

Integrand size = 13, antiderivative size = 110

$$\int \frac{\operatorname{csch}^4(x)}{a+b\cosh(x)}\,dx = \frac{2b^4\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}} + \frac{\left(3b^3+a\left(2a^2-5b^2\right)\cosh(x)\right)\operatorname{csch}(x)}{3\left(a^2-b^2\right)^2} + \frac{(b-a\cosh(x))\operatorname{csch}^3(x)}{3\left(a^2-b^2\right)}$$

output
```
2*b^4*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(5/2)/(a+b)^(5/2)
+1/3*(3*b^3+a*(2*a^2-5*b^2)*cosh(x))*csch(x)/(a^2-b^2)^2+1/3*(b-a*cosh(x))
*csch(x)^3/(a^2-b^2)
```

3.175.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.28

$$\int \frac{\operatorname{csch}^4(x)}{a+b\cosh(x)}\,dx = \frac{1}{24}\left(-\frac{48b^4\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\left(-a^2+b^2\right)^{5/2}} + \frac{2(4a+7b)\coth\left(\frac{x}{2}\right)}{(a+b)^2} + \frac{8\operatorname{csch}^3(x)\sinh^4\left(\frac{x}{2}\right)}{a-b} - \frac{\operatorname{csch}^4\left(\frac{x}{2}\right)\sinh(x)}{2(a+b)} + \frac{8a\tanh\left(\frac{x}{2}\right)}{(a-b)^2} - \frac{14b\tanh\left(\frac{x}{2}\right)}{(a-b)^2}\right)$$

input
```
Integrate[Csch[x]^4/(a + b*Cosh[x]),x]
```

output
```
((-48*b^4*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2)
 + (2*(4*a + 7*b)*Coth[x/2])/(a + b)^2 + (8*Csch[x]^3*Sinh[x/2]^4)/(a - b)
 - (Csch[x/2]^4*Sinh[x])/(2*(a + b)) + (8*a*Tanh[x/2])/(a - b)^2 - (14*b*T
anh[x/2])/(a - b)^2)/24
```

3.175.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.22, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {3042, 3175, 3042, 25, 3345, 27, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^4(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos\left(-\frac{\pi}{2}+ix\right)^4\left(a-b\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{3175}$$

$$\frac{\text{csch}^3(x)(b-a\cosh(x))}{3\left(a^2-b^2\right)}-\frac{\int \frac{\left(2a^2+2b\cosh(x)a-3b^2\right)\text{csch}^2(x)}{a+b\cosh(x)}dx}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\text{csch}^3(x)(b-a\cosh(x))}{3\left(a^2-b^2\right)}-\frac{\int -\frac{2a^2-2b\sin\left(ix-\frac{\pi}{2}\right)a-3b^2}{\cos\left(ix-\frac{\pi}{2}\right)^2\left(a-b\sin\left(ix-\frac{\pi}{2}\right)\right)}dx}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\text{csch}^3(x)(b-a\cosh(x))}{3\left(a^2-b^2\right)}+\frac{\int \frac{2a^2-2b\sin\left(ix-\frac{\pi}{2}\right)a-3b^2}{\cos\left(ix-\frac{\pi}{2}\right)^2\left(a-b\sin\left(ix-\frac{\pi}{2}\right)\right)}dx}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3345}$$

$$\frac{\frac{\text{csch}(x)\left(a\left(2a^2-5b^2\right)\cosh(x)+3b^3\right)}{a^2-b^2}-\frac{\int -\frac{3b^4}{a+b\cosh(x)}dx}{a^2-b^2}}{3\left(a^2-b^2\right)}+\frac{\text{csch}^3(x)(b-a\cosh(x))}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{3b^4\int \frac{1}{a+b\cosh(x)}dx}{a^2-b^2}+\frac{\operatorname{csch}(x)\left(a\left(2a^2-5b^2\right)\cosh(x)+3b^3\right)}{a^2-b^2}}{3\left(a^2-b^2\right)}+\frac{\operatorname{csch}^3(x)(b-a\cosh(x))}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 3042$$

$$\frac{\operatorname{csch}^3(x)(b-a\cosh(x))}{3\left(a^2-b^2\right)}+\frac{\frac{\operatorname{csch}(x)\left(a\left(2a^2-5b^2\right)\cosh(x)+3b^3\right)}{a^2-b^2}+\frac{3b^4\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 3138$$

$$\frac{\frac{6b^4\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a^2-b^2}+\frac{\operatorname{csch}(x)\left(a\left(2a^2-5b^2\right)\cosh(x)+3b^3\right)}{a^2-b^2}}{3\left(a^2-b^2\right)}+\frac{\operatorname{csch}^3(x)(b-a\cosh(x))}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{6b^4\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)}+\frac{\operatorname{csch}(x)\left(a\left(2a^2-5b^2\right)\cosh(x)+3b^3\right)}{a^2-b^2}}{3\left(a^2-b^2\right)}+\frac{\operatorname{csch}^3(x)(b-a\cosh(x))}{3\left(a^2-b^2\right)}$$

input
```
Int[Csch[x]^4/(a + b*Cosh[x]),x]
```

output
```
((b - a*Cosh[x])*Csch[x]^3)/(3*(a^2 - b^2)) + ((6*b^4*ArcTanh[(Sqrt[a - b]
*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]*(a^2 - b^2)) + ((3*b^3
+ a*(2*a^2 - 5*b^2)*Cosh[x])*Csch[x])/(a^2 - b^2))/(3*(a^2 - b^2))
```

3.175.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.175. $\int \frac{\operatorname{csch}^4(x)}{a+b\cosh(x)}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3175
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*
(a^2 - b^2)*(p + 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*
(a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ
[2*m, 2*p]
```

rule 3345
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*(a^2 - b^2)*(p +
 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[p, -1] && IntegerQ[2*m]
```

3.175.4 Maple [A] (verified)

Time = 3.52 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.15

method	result
default	$-\frac{\frac{a \tanh\left(\frac{x}{2}\right)^{3}}{3}-\frac{b \tanh\left(\frac{x}{2}\right)^{3}}{3}-3 a \tanh\left(\frac{x}{2}\right)+5 b \tanh\left(\frac{x}{2}\right)}{8 (a-b)^{2}}-\frac{1}{24 (a+b) \tanh\left(\frac{x}{2}\right)^{3}}-\frac{-3 a-5 b}{8 (a+b)^{2} \tanh\left(\frac{x}{2}\right)}+\frac{2 b^{4} \operatorname{arctanh}\left(\frac{(a-b) \tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a-b)^{2} (a+b)^{2} \sqrt{(a+b)(a-b)}}$
risch	$-\frac{2\left(-3 b^{3} e^{5 x}+3 a\, b^{2} e^{4 x}-4 a^{2} b\, e^{3 x}+10 b^{3} e^{3 x}+6 a^{3} e^{2 x}-12 a\, b^{2} e^{2 x}-3 b^{3} e^{x}-2 a^{3}+5 a\, b^{2}\right)}{3\left(a^{4}-2 a^{2} b^{2}+b^{4}\right)\left(e^{2 x}-1\right)^{3}}+\frac{b^{4} \ln\left(e^{x}+\frac{a \sqrt{a^{2}-b^{2}}-a^{2}+b^{2}}{b \sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\, (a+b)^{2} (a-b)^{2}}-\frac{b^{4} \ln\left(e^{x} \ldots\right.}{\sqrt{a^{2}-\ldots}}$

input
```
int(csch(x)^4/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/8/(a-b)^2*(1/3*a*tanh(1/2*x)^3-1/3*b*tanh(1/2*x)^3-3*a*tanh(1/2*x)+5*b*
tanh(1/2*x))-1/24/(a+b)/tanh(1/2*x)^3-1/8/(a+b)^2*(-3*a-5*b)/tanh(1/2*x)+2
/(a-b)^2/(a+b)^2*b^4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*
(a-b))^(1/2))
```

3.175.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1135 vs. $2(97) = 194$.

Time = 0.30 (sec) , antiderivative size = 2339, normalized size of antiderivative = 21.26

$$\int \frac{\operatorname{csch}^4(x)}{a + b\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(x)^4/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[1/3*(6*(a^2*b^3 - b^5)*cosh(x)^5 + 6*(a^2*b^3 - b^5)*sinh(x)^5 + 4*a^5 -
14*a^3*b^2 + 10*a*b^4 - 6*(a^3*b^2 - a*b^4)*cosh(x)^4 - 6*(a^3*b^2 - a*b^4
 - 5*(a^2*b^3 - b^5)*cosh(x))*sinh(x)^4 + 4*(2*a^4*b - 7*a^2*b^3 + 5*b^5)*
cosh(x)^3 + 4*(2*a^4*b - 7*a^2*b^3 + 5*b^5 + 15*(a^2*b^3 - b^5)*cosh(x)^2
- 6*(a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^3 - 12*(a^5 - 3*a^3*b^2 + 2*a*b^4)*
cosh(x)^2 - 12*(a^5 - 3*a^3*b^2 + 2*a*b^4 - 5*(a^2*b^3 - b^5)*cosh(x)^3 +
3*(a^3*b^2 - a*b^4)*cosh(x)^2 - (2*a^4*b - 7*a^2*b^3 + 5*b^5)*cosh(x))*sin
h(x)^2 + 3*(b^4*cosh(x)^6 + 6*b^4*cosh(x)*sinh(x)^5 + b^4*sinh(x)^6 - 3*b^
4*cosh(x)^4 + 3*b^4*cosh(x)^2 + 3*(5*b^4*cosh(x)^2 - b^4)*sinh(x)^4 - b^4
+ 4*(5*b^4*cosh(x)^3 - 3*b^4*cosh(x))*sinh(x)^3 + 3*(5*b^4*cosh(x)^4 - 6*b
^4*cosh(x)^2 + b^4)*sinh(x)^2 + 6*(b^4*cosh(x)^5 - 2*b^4*cosh(x)^3 + b^4*c
osh(x))*sinh(x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*
b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2
)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) +
2*(b*cosh(x) + a)*sinh(x) + b)) + 6*(a^2*b^3 - b^5)*cosh(x) + 6*(a^2*b^3 -
 b^5 + 5*(a^2*b^3 - b^5)*cosh(x)^4 - 4*(a^3*b^2 - a*b^4)*cosh(x)^3 + 2*(2*
a^4*b - 7*a^2*b^3 + 5*b^5)*cosh(x)^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*cosh(
x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^6 + 6*(a^6 - 3*a
^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^5 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4
 - b^6)*sinh(x)^6 - a^6 + 3*a^4*b^2 - 3*a^2*b^4 + b^6 - 3*(a^6 - 3*a^4*...
```

3.175.6 Sympy [F]

$$\int \frac{\operatorname{csch}^4(x)}{a+b\cosh(x)}\,dx = \int \frac{\operatorname{csch}^4(x)}{a+b\cosh(x)}\,dx$$

input
```
integrate(csch(x)**4/(a+b*cosh(x)),x)
```

output
```
Integral(csch(x)**4/(a + b*cosh(x)), x)
```

3.175.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}^4(x)}{a+b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(csch(x)^4/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.175.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.42

$$\int \frac{\operatorname{csch}^4(x)}{a+b\cosh(x)}\,dx = \frac{2\,b^4\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{(a^4-2\,a^2b^2+b^4)\sqrt{-a^2+b^2}} + \frac{2\left(3\,b^3e^{(5\,x)}-3\,ab^2e^{(4\,x)}+4\,a^2be^{(3\,x)}-10\,b^3e^{(3\,x)}-6\,a^3e^{(2\,x)}+12\,ab^2e^{(2\,x)}+3\,b^3e^x+2\,a^3-5\,ab^2\right)}{3\,(a^4-2\,a^2b^2+b^4)(e^{(2\,x)}-1)^3}$$

input
```
integrate(csch(x)^4/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
2*b^4*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(-
a^2 + b^2)) + 2/3*(3*b^3*e^(5*x) - 3*a*b^2*e^(4*x) + 4*a^2*b*e^(3*x) - 10*
b^3*e^(3*x) - 6*a^3*e^(2*x) + 12*a*b^2*e^(2*x) + 3*b^3*e^x + 2*a^3 - 5*a*b
^2)/((a^4 - 2*a^2*b^2 + b^4)*(e^(2*x) - 1)^3)
```

3.175.9 Mupad [B] (verification not implemented)

Time = 2.76 (sec) , antiderivative size = 642, normalized size of antiderivative = 5.84

$$\int \frac{\operatorname{csch}^4(x)}{a+b\cosh(x)}\,dx = \frac{\frac{4\,(a\,b^2-a^3)}{(a^2-b^2)^2}+\frac{8\,e^x\,(a^2\,b-b^3)}{3\,(a^2-b^2)^2}}{e^{4\,x}-2\,e^{2\,x}+1}-\frac{\frac{2\,a\,b^2}{(a^2-b^2)^2}-\frac{2\,b^3\,e^x}{(a^2-b^2)^2}}{e^{2\,x}-1}-\frac{\frac{8\,a}{3\,(a^2-b^2)}-\frac{8\,b\,e^x}{3\,(a^2-b^2)}}{3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1} + \frac{2\,\mathrm{atan}\left(\left(e^x\left(\frac{2\,b^2}{(a^2-b^2)^2\,\sqrt{b^8}\,(a^4-2\,a^2\,b^2+b^4)}+\frac{2\,a\,\left(a^5\,\sqrt{b^8}-2\,a^3\,b^2\,\sqrt{b^8}+a\,b^4\,\sqrt{b^8}\right)}{b^6\,\sqrt{-(a^2-b^2)^5}\,(a^4-2\,a^2\,b^2+b^4)\,\sqrt{-a^{10}+5\,a^8\,b^2-10\,a^6\,b^4+10\,a^4\,b^6-5\,a^2\,b^8+b^{10}}}\right)+\right.\right.}{}$$

input
```
int(1/(sinh(x)^4*(a + b*cosh(x))),x)
```

output
```
((4*(a*b^2 - a^3))/(a^2 - b^2)^2 + (8*exp(x)*(a^2*b - b^3))/(3*(a^2 - b^2)
^2))/(exp(4*x) - 2*exp(2*x) + 1) - ((2*a*b^2)/(a^2 - b^2)^2 - (2*b^3*exp(x
))/(a^2 - b^2)^2)/(exp(2*x) - 1) - ((8*a)/(3*(a^2 - b^2)) - (8*b*exp(x))/(
3*(a^2 - b^2)))/(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1) + (2*atan((exp(x)
*((2*b^2)/((a^2 - b^2)^2*(b^8)^(1/2)*(a^4 + b^4 - 2*a^2*b^2)) + (2*a*(a^5*
(b^8)^(1/2) - 2*a^3*b^2*(b^8)^(1/2) + a*b^4*(b^8)^(1/2)))/(b^6*(-(a^2 - b^
2)^5)^(1/2)*(a^4 + b^4 - 2*a^2*b^2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6
- 10*a^6*b^4 + 5*a^8*b^2)^(1/2))) + (2*a*(b^5*(b^8)^(1/2) - 2*a^2*b^3*(b^8
)^(1/2) + a^4*b*(b^8)^(1/2)))/(b^6*(-(a^2 - b^2)^5)^(1/2)*(a^4 + b^4 - 2*a
^2*b^2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1
/2)))*((b^5*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2
)^(1/2))/2 - a^2*b^3*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 +
5*a^8*b^2)^(1/2) + (a^4*b*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b
^4 + 5*a^8*b^2)^(1/2))/2))*(b^8)^(1/2))/(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*
b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)
```

3.176 $\int \frac{\operatorname{csch}^5(x)}{a+b\cosh(x)}\,dx$

3.176.1 Optimal result

Integrand size = 13, antiderivative size = 151

$$\int \frac{\operatorname{csch}^5(x)}{a+b\cosh(x)}\,dx = \frac{(4b^3+a(3a^2-7b^2)\cosh(x))\operatorname{csch}^2(x)}{8(a^2-b^2)^2} + \frac{(b-a\cosh(x))\operatorname{csch}^4(x)}{4(a^2-b^2)} + \frac{(3a^2+9ab+8b^2)\log(1-\cosh(x))}{16(a+b)^3} - \frac{(3a^2-9ab+8b^2)\log(1+\cosh(x))}{16(a-b)^3} + \frac{b^5\log(a+b\cosh(x))}{(a^2-b^2)^3}$$

output
```
1/8*(4*b^3+a*(3*a^2-7*b^2)*cosh(x))*csch(x)^2/(a^2-b^2)^2+1/4*(b-a*cosh(x)
)*csch(x)^4/(a^2-b^2)+1/16*(3*a^2+9*a*b+8*b^2)*ln(1-cosh(x))/(a+b)^3-1/16*
(3*a^2-9*a*b+8*b^2)*ln(1+cosh(x))/(a-b)^3+b^5*ln(a+b*cosh(x))/(a^2-b^2)^3
```

3.176.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.09

$$\int \frac{\operatorname{csch}^5(x)}{a+b\cosh(x)}\,dx = \frac{1}{64}\left(\frac{2(3a+5b)\operatorname{csch}^2\left(\frac{x}{2}\right)}{(a+b)^2} - \frac{\operatorname{csch}^4\left(\frac{x}{2}\right)}{a+b} - \frac{8(3a^2-9ab+8b^2)\log\left(\cosh\left(\frac{x}{2}\right)\right)}{(a-b)^3} + \frac{64b^5\log(a+b\cosh(x))}{(a^2-b^2)^3} + \frac{8(3a^2+9ab+8b^2)\log\left(\sinh\left(\frac{x}{2}\right)\right)}{(a+b)^3} + \frac{2(3a-5b)\operatorname{sech}^2\left(\frac{x}{2}\right)}{(a-b)^2} + \frac{\operatorname{sech}^4\left(\frac{x}{2}\right)}{a-b}\right)$$

input
```
Integrate[Csch[x]^5/(a + b*Cosh[x]),x]
```

output
```
((2*(3*a + 5*b)*Csch[x/2]^2)/(a + b)^2 - Csch[x/2]^4/(a + b) - (8*(3*a^2 -
 9*a*b + 8*b^2)*Log[Cosh[x/2]])/(a - b)^3 + (64*b^5*Log[a + b*Cosh[x]])/(a
^2 - b^2)^3 + (8*(3*a^2 + 9*a*b + 8*b^2)*Log[Sinh[x/2]])/(a + b)^3 + (2*(3
*a - 5*b)*Sech[x/2]^2)/(a - b)^2 + Sech[x/2]^4/(a - b))/64
```

3.176.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.30, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 26, 3147, 477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^5(x)}{a+b\cosh(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{i}{\cos\left(-\frac{\pi}{2}+ix\right)^5\left(a-b\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \; 26$$

$$i\int \frac{1}{\cos\left(ix-\frac{\pi}{2}\right)^5\left(a-b\sin\left(ix-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \; 3147$$

$$-b^5\int \frac{1}{(a+b\cosh(x))\left(b^2-b^2\cosh^2(x)\right)^3}d(b\cosh(x))$$

$$\Big\downarrow \; 477$$

$$-\frac{\int \left(-\frac{b^6}{(a^2-b^2)^3(a+b\cosh(x))}+\frac{b^3}{8(a+b)(b-b\cosh(x))^3}+\frac{b^3}{8(a-b)(\cosh(x)b+b)^3}+\frac{(3a+5b)b^2}{16(a+b)^2(b-b\cosh(x))^2}+\frac{(3a-5b)b^2}{16(a-b)^2(\cosh(x)b+b)^2}+\right.}{b}$$

$$\Big\downarrow \; 2009$$

$$-\frac{-\frac{b(3a^2+9ab+8b^2)\log(b-b\cosh(x))}{16(a+b)^3}+\frac{b(3a^2-9ab+8b^2)\log(b\cosh(x)+b)}{16(a-b)^3}-\frac{b^6\log(a+b\cosh(x))}{(a^2-b^2)^3}+\frac{b^3}{16(a+b)(b-b\cosh(x))^2}-\frac{b^3}{16(a-b)(b\,\mathrm{co}}}{b}$$

input
```
Int[Csch[x]^5/(a + b*Cosh[x]),x]
```

output
```
-((b^3/(16*(a + b)*(b - b*Cosh[x])^2) + (b^2*(3*a + 5*b))/(16*(a + b)^2*(b
 - b*Cosh[x])) - b^3/(16*(a - b)*(b + b*Cosh[x])^2) - ((3*a - 5*b)*b^2)/(1
6*(a - b)^2*(b + b*Cosh[x])) - (b*(3*a^2 + 9*a*b + 8*b^2)*Log[b - b*Cosh[x
]])/(16*(a + b)^3) - (b^6*Log[a + b*Cosh[x]])/(a^2 - b^2)^3 + (b*(3*a^2 -
9*a*b + 8*b^2)*Log[b + b*Cosh[x]])/(16*(a - b)^3))/b)
```

3.176.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.176. $\int \frac{\operatorname{csch}^5(x)}{a+b\cosh(x)}\,dx$

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.176.4 Maple [A] (verified)

Time = 8.07 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.91

method	result
default	$\frac{\left(\tanh\left(\frac{x}{2}\right)^2 a-\tanh\left(\frac{x}{2}\right)^2 b-4a+6b\right)^2}{64(a-b)^3}-\frac{1}{64(a+b)\tanh\left(\frac{x}{2}\right)^4}-\frac{-4a-6b}{32(a+b)^2\tanh\left(\frac{x}{2}\right)^2}+\frac{(6a^2+18ab+16b^2)\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{16(a+b)^3}+\frac{b^5\ln\left(\mathrm{ta}\right.}{}$
risch	$-\frac{3x\,a^2}{8(a^3+3a^2b+3a\,b^2+b^3)}-\frac{9xab}{8(a^3+3a^2b+3a\,b^2+b^3)}-\frac{x\,b^2}{a^3+3a^2b+3a\,b^2+b^3}+\frac{3x\,a^2}{8(a^3-3a^2b+3a\,b^2-b^3)}-\frac{9xab}{8(a^3-3a^2b+3a\,b^2-b^3)}$

input
```
int(csch(x)^5/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/64*(tanh(1/2*x)^2*a-tanh(1/2*x)^2*b-4*a+6*b)^2/(a-b)^3-1/64/(a+b)/tanh(1
/2*x)^4-1/32*(-4*a-6*b)/(a+b)^2/tanh(1/2*x)^2+1/16/(a+b)^3*(6*a^2+18*a*b+1
6*b^2)*ln(tanh(1/2*x))+1/(a-b)^3*b^5/(a+b)^3*ln(tanh(1/2*x)^2*a-tanh(1/2*x
)^2*b-a-b)
```

3.176.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3450 vs. 2(144) = 288.

Time = 0.32 (sec) , antiderivative size = 3450, normalized size of antiderivative = 22.85

$$\int \frac{\operatorname{csch}^5(x)}{a+b\cosh(x)}\,dx=\text{Too large to display}$$

input
```
integrate(csch(x)^5/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
1/8*(2*(3*a^5 - 10*a^3*b^2 + 7*a*b^4)*cosh(x)^7 + 2*(3*a^5 - 10*a^3*b^2 +
7*a*b^4)*sinh(x)^7 + 16*(a^2*b^3 - b^5)*cosh(x)^6 + 2*(8*a^2*b^3 - 8*b^5 +
 7*(3*a^5 - 10*a^3*b^2 + 7*a*b^4)*cosh(x))*sinh(x)^6 - 2*(11*a^5 - 26*a^3*
b^2 + 15*a*b^4)*cosh(x)^5 - 2*(11*a^5 - 26*a^3*b^2 + 15*a*b^4 - 21*(3*a^5
- 10*a^3*b^2 + 7*a*b^4)*cosh(x)^2 - 48*(a^2*b^3 - b^5)*cosh(x))*sinh(x)^5
+ 32*(a^4*b - 3*a^2*b^3 + 2*b^5)*cosh(x)^4 + 2*(16*a^4*b - 48*a^2*b^3 + 32
*b^5 + 35*(3*a^5 - 10*a^3*b^2 + 7*a*b^4)*cosh(x)^3 + 120*(a^2*b^3 - b^5)*c
osh(x)^2 - 5*(11*a^5 - 26*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^4 - 2*(11*a
^5 - 26*a^3*b^2 + 15*a*b^4)*cosh(x)^3 - 2*(11*a^5 - 26*a^3*b^2 + 15*a*b^4
- 35*(3*a^5 - 10*a^3*b^2 + 7*a*b^4)*cosh(x)^4 - 160*(a^2*b^3 - b^5)*cosh(x
)^3 + 10*(11*a^5 - 26*a^3*b^2 + 15*a*b^4)*cosh(x)^2 - 64*(a^4*b - 3*a^2*b^
3 + 2*b^5)*cosh(x))*sinh(x)^3 + 16*(a^2*b^3 - b^5)*cosh(x)^2 + 2*(21*(3*a^
5 - 10*a^3*b^2 + 7*a*b^4)*cosh(x)^5 + 8*a^2*b^3 - 8*b^5 + 120*(a^2*b^3 - b
^5)*cosh(x)^4 - 10*(11*a^5 - 26*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + 96*(a^4*b
- 3*a^2*b^3 + 2*b^5)*cosh(x)^2 - 3*(11*a^5 - 26*a^3*b^2 + 15*a*b^4)*cosh(x
))*sinh(x)^2 + 2*(3*a^5 - 10*a^3*b^2 + 7*a*b^4)*cosh(x) + 8*(b^5*cosh(x)^8
 + 8*b^5*cosh(x)*sinh(x)^7 + b^5*sinh(x)^8 - 4*b^5*cosh(x)^6 + 6*b^5*cosh(
x)^4 - 4*b^5*cosh(x)^2 + 4*(7*b^5*cosh(x)^2 - b^5)*sinh(x)^6 + 8*(7*b^5*co
sh(x)^3 - 3*b^5*cosh(x))*sinh(x)^5 + b^5 + 2*(35*b^5*cosh(x)^4 - 30*b^5*co
sh(x)^2 + 3*b^5)*sinh(x)^4 + 8*(7*b^5*cosh(x)^5 - 10*b^5*cosh(x)^3 + 3*...
```

3.176.6 Sympy [F]

$$\int \frac{\operatorname{csch}^5(x)}{a+b\cosh(x)}\,dx=\int \frac{\operatorname{csch}^5(x)}{a+b\cosh(x)}\,dx$$

input
```
integrate(csch(x)**5/(a+b*cosh(x)),x)
```

output
```
Integral(csch(x)**5/(a + b*cosh(x)), x)
```

3.176.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(144) = 288.

Time = 0.24 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.30

$$\int \frac{\operatorname{csch}^5(x)}{a+b\cosh(x)}\,dx = \frac{b^5\log\left(2\,ae^{(-x)}+be^{(-2\,x)}+b\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6} - \frac{(3\,a^2-9\,ab+8\,b^2)\log\left(e^{(-x)}+1\right)}{8\,(a^3-3\,a^2b+3\,ab^2-b^3)} + \frac{(3\,a^2+9\,ab+8\,b^2)\log\left(e^{(-x)}-1\right)}{8\,(a^3+3\,a^2b+3\,ab^2+b^3)} + \frac{8\,b^3e^{(-2\,x)}+8\,b^3e^{(-6\,x)}+(3\,a^3-7\,ab^2)e^{(-x)}-(11\,a^3-15\,ab^2)e^{(-3\,x)}+16\,(a^2b-2\,b^3)e^{(-4\,x)}-(11\,a^3-}{4\,(a^4-2\,a^2b^2+b^4-4\,(a^4-2\,a^2b^2+b^4)e^{(-2\,x)}+6\,(a^4-2\,a^2b^2+b^4)e^{(-4\,x)}-4\,(a^4-2\,a^2b^2+b^4)e^{(-}}$$

input
```
integrate(csch(x)^5/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
b^5*log(2*a*e^(-x) + b*e^(-2*x) + b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) -
 1/8*(3*a^2 - 9*a*b + 8*b^2)*log(e^(-x) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^
3) + 1/8*(3*a^2 + 9*a*b + 8*b^2)*log(e^(-x) - 1)/(a^3 + 3*a^2*b + 3*a*b^2
+ b^3) + 1/4*(8*b^3*e^(-2*x) + 8*b^3*e^(-6*x) + (3*a^3 - 7*a*b^2)*e^(-x) -
 (11*a^3 - 15*a*b^2)*e^(-3*x) + 16*(a^2*b - 2*b^3)*e^(-4*x) - (11*a^3 - 15
*a*b^2)*e^(-5*x) + (3*a^3 - 7*a*b^2)*e^(-7*x))/(a^4 - 2*a^2*b^2 + b^4 - 4*
(a^4 - 2*a^2*b^2 + b^4)*e^(-2*x) + 6*(a^4 - 2*a^2*b^2 + b^4)*e^(-4*x) - 4*
(a^4 - 2*a^2*b^2 + b^4)*e^(-6*x) + (a^4 - 2*a^2*b^2 + b^4)*e^(-8*x))
```

3.176.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. 2(144) = 288.

Time = 0.27 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.24

$$\int \frac{\operatorname{csch}^5(x)}{a+b\cosh(x)}\,dx = \frac{b^6\log\left(\left|b\left(e^{(-x)}+e^x\right)+2\,a\right|\right)}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7} - \frac{(3\,a^2-9\,ab+8\,b^2)\log\left(e^{(-x)}+e^x+2\right)}{16\,(a^3-3\,a^2b+3\,ab^2-b^3)} + \frac{(3\,a^2+9\,ab+8\,b^2)\log\left(e^{(-x)}+e^x-2\right)}{16\,(a^3+3\,a^2b+3\,ab^2+b^3)} + \frac{3\,b^5\left(e^{(-x)}+e^x\right)^4+3\,a^5\left(e^{(-x)}+e^x\right)^3-10\,a^3b^2\left(e^{(-x)}+e^x\right)^3+7\,ab^4\left(e^{(-x)}+e^x\right)^3+8\,a^2b^3\left(e^{(-x)}+e^x\right)^2-}{4\,(a^6-3\,a^4b^2+3\,a^2b}$$

input
```
integrate(csch(x)^5/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
b^6*log(abs(b*(e^(-x) + e^x) + 2*a))/(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)
 - 1/16*(3*a^2 - 9*a*b + 8*b^2)*log(e^(-x) + e^x + 2)/(a^3 - 3*a^2*b + 3*a
*b^2 - b^3) + 1/16*(3*a^2 + 9*a*b + 8*b^2)*log(e^(-x) + e^x - 2)/(a^3 + 3*
a^2*b + 3*a*b^2 + b^3) + 1/4*(3*b^5*(e^(-x) + e^x)^4 + 3*a^5*(e^(-x) + e^x
)^3 - 10*a^3*b^2*(e^(-x) + e^x)^3 + 7*a*b^4*(e^(-x) + e^x)^3 + 8*a^2*b^3*(
e^(-x) + e^x)^2 - 32*b^5*(e^(-x) + e^x)^2 - 20*a^5*(e^(-x) + e^x) + 56*a^3
*b^2*(e^(-x) + e^x) - 36*a*b^4*(e^(-x) + e^x) + 16*a^4*b - 64*a^2*b^3 + 96
*b^5)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*((e^(-x) + e^x)^2 - 4)^2)
```

3.176.9 Mupad [B] (verification not implemented)

Time = 2.55 (sec) , antiderivative size = 559, normalized size of antiderivative = 3.70

$$\int \frac{\operatorname{csch}^5(x)}{a+b\cosh(x)}\,dx = \frac{\frac{4\,b}{a^2-b^2}-\frac{4\,a\,e^x}{a^2-b^2}}{6\,e^{4\,x}-4\,e^{2\,x}-4\,e^{6\,x}+e^{8\,x}+1}-\frac{\frac{2\,(b^5-a^2\,b^3)}{(a^2-b^2)^3}-\frac{e^x\,(3\,a^5-10\,a^3\,b^2+7\,a\,b^4)}{4\,(a^2-b^2)^3}}{e^{2\,x}-1}$$
$$+\frac{\frac{8\,(a^2\,b-b^3)}{(a^2-b^2)^2}+\frac{6\,e^x\,(a\,b^2-a^3)}{(a^2-b^2)^2}}{3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1}+\frac{\frac{2\,(2\,a^2\,b-b^3)}{(a^2-b^2)^2}-\frac{e^x\,(a^3+3\,a\,b^2)}{2\,(a^2-b^2)^2}}{e^{4\,x}-2\,e^{2\,x}+1}$$
$$+\frac{b^5\,\ln\left(256\,b^{11}\,e^{2\,x}-9\,a^{10}\,b+256\,b^{11}-225\,a^2\,b^9+300\,a^4\,b^7-190\,a^6\,b^5+60\,a^8\,b^3-18\,a^{11}\,e^x-225\,a^2\,b^9\right.}{}$$
$$+\frac{\ln(e^x-1)\,(3\,a^2+9\,a\,b+8\,b^2)}{8\,a^3+24\,a^2\,b+24\,a\,b^2+8\,b^3}-\frac{\ln(e^x+1)\,(3\,a^2-9\,a\,b+8\,b^2)}{8\,a^3-24\,a^2\,b+24\,a\,b^2-8\,b^3}$$

input
```
int(1/(sinh(x)^5*(a + b*cosh(x))),x)
```

output
```
((4*b)/(a^2 - b^2) - (4*a*exp(x))/(a^2 - b^2))/(6*exp(4*x) - 4*exp(2*x) -
4*exp(6*x) + exp(8*x) + 1) - ((2*(b^5 - a^2*b^3))/(a^2 - b^2)^3 - (exp(x)*
(7*a*b^4 + 3*a^5 - 10*a^3*b^2))/(4*(a^2 - b^2)^3))/(exp(2*x) - 1) + ((8*(a
^2*b - b^3))/(a^2 - b^2)^2 + (6*exp(x)*(a*b^2 - a^3))/(a^2 - b^2)^2)/(3*ex
p(2*x) - 3*exp(4*x) + exp(6*x) - 1) + ((2*(2*a^2*b - b^3))/(a^2 - b^2)^2 -
 (exp(x)*(3*a*b^2 + a^3))/(2*(a^2 - b^2)^2))/(exp(4*x) - 2*exp(2*x) + 1) +
 (b^5*log(256*b^11*exp(2*x) - 9*a^10*b + 256*b^11 - 225*a^2*b^9 + 300*a^4*
b^7 - 190*a^6*b^5 + 60*a^8*b^3 - 18*a^11*exp(x) - 225*a^2*b^9*exp(2*x) + 3
00*a^4*b^7*exp(2*x) - 190*a^6*b^5*exp(2*x) + 60*a^8*b^3*exp(2*x) + 512*a*b
^10*exp(x) - 9*a^10*b*exp(2*x) - 450*a^3*b^8*exp(x) + 600*a^5*b^6*exp(x) -
 380*a^7*b^4*exp(x) + 120*a^9*b^2*exp(x)))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*
b^2) + (log(exp(x) - 1)*(9*a*b + 3*a^2 + 8*b^2))/(24*a*b^2 + 24*a^2*b + 8*
a^3 + 8*b^3) - (log(exp(x) + 1)*(3*a^2 - 9*a*b + 8*b^2))/(24*a*b^2 - 24*a^
2*b + 8*a^3 - 8*b^3)
```

3.177 $\int \frac{\text{csch}^6(x)}{a+b\cosh(x)}\,dx$

3.177.1 Optimal result

Integrand size = 13, antiderivative size = 159

$$\int \frac{\text{csch}^6(x)}{a+b\cosh(x)}\,dx = \frac{2b^6\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}} + \frac{(15b^5-a(8a^4-26a^2b^2+33b^4)\cosh(x))\,\text{csch}(x)}{15\,(a^2-b^2)^3} + \frac{(5b^3+a(4a^2-9b^2)\cosh(x))\,\text{csch}^3(x)}{15\,(a^2-b^2)^2} + \frac{(b-a\cosh(x))\text{csch}^5(x)}{5\,(a^2-b^2)}$$

output
```
2*b^6*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(7/2)/(a+b)^(7/2)
+1/15*(15*b^5-a*(8*a^4-26*a^2*b^2+33*b^4)*cosh(x))*csch(x)/(a^2-b^2)^3+1/1
5*(5*b^3+a*(4*a^2-9*b^2)*cosh(x))*csch(x)^3/(a^2-b^2)^2+1/5*(b-a*cosh(x))*
csch(x)^5/(a^2-b^2)
```

3.177.2 Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.26

$$\int \frac{\operatorname{csch}^6(x)}{a+b\cosh(x)}\,dx = \frac{1}{480}\left(\frac{960b^6\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{7/2}} - \frac{2(64a^2+183ab+149b^2)\coth\left(\frac{x}{2}\right)}{(a+b)^3} - \frac{8(19a-29b)\operatorname{csch}^3(x)\sinh^4\left(\frac{x}{2}\right)}{(a-b)^2} - \frac{96\operatorname{csch}^5(x)\sinh^6\left(\frac{x}{2}\right)}{a-b} + \frac{(19a+29b)\operatorname{csch}^4\left(\frac{x}{2}\right)\sinh(x)}{2(a+b)^2} - \frac{3\operatorname{csch}^6\left(\frac{x}{2}\right)\sinh(x)}{2(a+b)} - \frac{2(64a^2-183ab+149b^2)\tanh\left(\frac{x}{2}\right)}{(a-b)^3}\right)$$

input
```
Integrate[Csch[x]^6/(a + b*Cosh[x]),x]
```

output
```
((960*b^6*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2)
 - (2*(64*a^2 + 183*a*b + 149*b^2)*Coth[x/2])/(a + b)^3 - (8*(19*a - 29*b)
*Csch[x]^3*Sinh[x/2]^4)/(a - b)^2 - (96*Csch[x]^5*Sinh[x/2]^6)/(a - b) + (
(19*a + 29*b)*Csch[x/2]^4*Sinh[x])/(2*(a + b)^2) - (3*Csch[x/2]^6*Sinh[x])
/(2*(a + b)) - (2*(64*a^2 - 183*a*b + 149*b^2)*Tanh[x/2])/(a - b)^3)/480
```

3.177.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.26, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 25, 3175, 25, 3042, 3345, 3042, 25, 3345, 27, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^6(x)}{a+b\cosh(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\cos\left(-\frac{\pi}{2}+ix\right)^6\left(a-b\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\cos\left(ix-\frac{\pi}{2}\right)^6\left(a-b\sin\left(ix-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \; 3175$$

$$\frac{\int -\frac{\left(4a^2+4b\cosh(x)a-5b^2\right)\operatorname{csch}^4(x)}{a+b\cosh(x)}dx}{5\left(a^2-b^2\right)}+\frac{\operatorname{csch}^5(x)(b-a\cosh(x))}{5\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\operatorname{csch}^5(x)(b-a\cosh(x))}{5\left(a^2-b^2\right)}-\frac{\int \frac{\left(4a^2+4b\cosh(x)a-5b^2\right)\operatorname{csch}^4(x)}{a+b\cosh(x)}dx}{5\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\operatorname{csch}^5(x)(b-a\cosh(x))}{5\left(a^2-b^2\right)}-\frac{\int \frac{4a^2-4b\sin\left(ix-\frac{\pi}{2}\right)a-5b^2}{\cos\left(ix-\frac{\pi}{2}\right)^4\left(a-b\sin\left(ix-\frac{\pi}{2}\right)\right)}dx}{5\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3345$$

$$\frac{\operatorname{csch}^5(x)(b-a\cosh(x))}{5\left(a^2-b^2\right)}-\frac{-\frac{\int \frac{\left(8a^4-18b^2a^2+2b\left(4a^2-9b^2\right)\cosh(x)a+15b^4\right)\operatorname{csch}^2(x)}{a+b\cosh(x)}dx}{3\left(a^2-b^2\right)}-\frac{\operatorname{csch}^3(x)\left(a\left(4a^2-9b^2\right)\cosh(x)+5b^3\right)}{3\left(a^2-b^2\right)}}{5\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\operatorname{csch}^5(x)(b-a\cosh(x))}{5\left(a^2-b^2\right)}-\frac{-\frac{\operatorname{csch}^3(x)\left(a\left(4a^2-9b^2\right)\cosh(x)+5b^3\right)}{3\left(a^2-b^2\right)}-\frac{\int -\frac{8a^4-18b^2a^2-2b\left(4a^2-9b^2\right)\sin\left(ix-\frac{\pi}{2}\right)a+15b^4}{\cos\left(ix-\frac{\pi}{2}\right)^2\left(a-b\sin\left(ix-\frac{\pi}{2}\right)\right)}dx}{3\left(a^2-b^2\right)}}{5\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\operatorname{csch}^5(x)(b-a\cosh(x))}{5\left(a^2-b^2\right)}-\frac{-\frac{\operatorname{csch}^3(x)\left(a\left(4a^2-9b^2\right)\cosh(x)+5b^3\right)}{3\left(a^2-b^2\right)}+\frac{\int \frac{8a^4-18b^2a^2-2b\left(4a^2-9b^2\right)\sin\left(ix-\frac{\pi}{2}\right)a+15b^4}{\cos\left(ix-\frac{\pi}{2}\right)^2\left(a-b\sin\left(ix-\frac{\pi}{2}\right)\right)}dx}{3\left(a^2-b^2\right)}}{5\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3345$$

$$\frac{\operatorname{csch}^5(x)(b-a\cosh(x))}{5\left(a^2-b^2\right)}-\frac{\frac{-\frac{\int \frac{15b^6}{a+b\cosh(x)}dx}{a^2-b^2}-\frac{\operatorname{csch}(x)\left(15b^5-a\left(8a^4-26a^2b^2+33b^4\right)\cosh(x)\right)}{a^2-b^2}}{3\left(a^2-b^2\right)}-\frac{\operatorname{csch}^3(x)\left(a\left(4a^2-9b^2\right)\cosh(x)+5b^3\right)}{3\left(a^2-b^2\right)}}{5\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 27$$

3.177. $\int \frac{\operatorname{csch}^6(x)}{a+b\cosh(x)}\,dx$

$$\frac{\operatorname{csch}^5(x)(b-a\cosh(x))}{5(a^2-b^2)}-\frac{\frac{-\frac{15b^6\int\frac{1}{a+b\cosh(x)}dx}{a^2-b^2}-\frac{\operatorname{csch}(x)\left(15b^5-a\left(8a^4-26a^2b^2+33b^4\right)\cosh(x)\right)}{a^2-b^2}}{3(a^2-b^2)}-\frac{\operatorname{csch}^3(x)\left(a\left(4a^2-9b^2\right)\cosh(x)+5b^3\right)}{3(a^2-b^2)}}{5(a^2-b^2)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\operatorname{csch}^5(x)(b-a\cosh(x))}{5(a^2-b^2)}-\frac{-\frac{\operatorname{csch}^3(x)\left(a\left(4a^2-9b^2\right)\cosh(x)+5b^3\right)}{3(a^2-b^2)}+\frac{-\frac{\operatorname{csch}(x)\left(15b^5-a\left(8a^4-26a^2b^2+33b^4\right)\cosh(x)\right)}{a^2-b^2}-\frac{15b^6\int\frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}}{3(a^2-b^2)}}{5(a^2-b^2)}$$

$$\Big\downarrow \; 3138$$

$$\frac{\operatorname{csch}^5(x)(b-a\cosh(x))}{5(a^2-b^2)}-\frac{\frac{-\frac{30b^6\int\frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a^2-b^2}-\frac{\operatorname{csch}(x)\left(15b^5-a\left(8a^4-26a^2b^2+33b^4\right)\cosh(x)\right)}{a^2-b^2}}{3(a^2-b^2)}-\frac{\operatorname{csch}^3(x)\left(a\left(4a^2-9b^2\right)\cosh(x)+5b^3\right)}{3(a^2-b^2)}}{5(a^2-b^2)}$$

$$\Big\downarrow \; 221$$

$$\frac{\operatorname{csch}^5(x)(b-a\cosh(x))}{5(a^2-b^2)}-\frac{\frac{-\frac{30b^6\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)}-\frac{\operatorname{csch}(x)\left(15b^5-a\left(8a^4-26a^2b^2+33b^4\right)\cosh(x)\right)}{a^2-b^2}}{3(a^2-b^2)}-\frac{\operatorname{csch}^3(x)\left(a\left(4a^2-9b^2\right)\cosh(x)+5b^3\right)}{3(a^2-b^2)}}{5(a^2-b^2)}$$

input
```
Int[Csch[x]^6/(a + b*Cosh[x]),x]
```

output
```
((b - a*Cosh[x])*Csch[x]^5)/(5*(a^2 - b^2)) - (-1/3*((5*b^3 + a*(4*a^2 - 9
*b^2)*Cosh[x])*Csch[x]^3)/(a^2 - b^2) + ((-30*b^6*ArcTanh[(Sqrt[a - b]*Tan
h[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]*(a^2 - b^2)) - ((15*b^5 - a
*(8*a^4 - 26*a^2*b^2 + 33*b^4)*Cosh[x])*Csch[x])/(a^2 - b^2))/(3*(a^2 - b^
2)))/(5*(a^2 - b^2))
```

3.177. $\int \frac{\operatorname{csch}^6(x)}{a+b\cosh(x)}\,dx$

3.177.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3175
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*
(a^2 - b^2)*(p + 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*
(a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ
[2*m, 2*p]
```

rule 3345
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*(a^2 - b^2)*(p +
 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[p, -1] && IntegerQ[2*m]
```

3.177. $\int \frac{\operatorname{csch}^6(x)}{a+b\cosh(x)}\,dx$

3.177.4 Maple [A] (verified)

Time = 18.16 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.34

method	result
default	$-\frac{\frac{a^2\tanh\left(\frac{x}{2}\right)^5}{5}-\frac{2ab\tanh\left(\frac{x}{2}\right)^5}{5}+\frac{b^2\tanh\left(\frac{x}{2}\right)^5}{5}-\frac{5a^2\tanh\left(\frac{x}{2}\right)^3}{3}+4ab\tanh\left(\frac{x}{2}\right)^3-\frac{7b^2\tanh\left(\frac{x}{2}\right)^3}{3}+10a^2\tanh\left(\frac{x}{2}\right)-28ab\tanh\left(\frac{x}{2}\right)+22b^2\tan}{32(a-b)^3}$
risch	$-\frac{2\left(-15b^5e^{9x}+15a\,b^4e^{8x}-20a^2b^3e^{7x}+80b^5e^{7x}+30a^3b^2e^{6x}-90a\,b^4e^{6x}-48a^4b\,e^{5x}+136a^2b^3e^{5x}-178b^5e^{5x}+80a^5e^{4x}-230a^3b^2e^{4x}+\right.}{15(a^4-2a^2b^2+b^4)(e^{2x}-1)^5(a^2-}$

input
```
int(csch(x)^6/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/32/(a-b)^3*(1/5*a^2*tanh(1/2*x)^5-2/5*a*b*tanh(1/2*x)^5+1/5*b^2*tanh(1/
2*x)^5-5/3*a^2*tanh(1/2*x)^3+4*a*b*tanh(1/2*x)^3-7/3*b^2*tanh(1/2*x)^3+10*
a^2*tanh(1/2*x)-28*a*b*tanh(1/2*x)+22*b^2*tanh(1/2*x))-1/160/(a+b)/tanh(1/
2*x)^5-1/96*(-5*a-7*b)/(a+b)^2/tanh(1/2*x)^3-1/32/(a+b)^3*(10*a^2+28*a*b+2
2*b^2)/tanh(1/2*x)+2/(a-b)^3/(a+b)^3*b^6/((a+b)*(a-b))^(1/2)*arctanh((a-b)
*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

3.177.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3156 vs. 2(144) = 288.

Time = 0.33 (sec) , antiderivative size = 6381, normalized size of antiderivative = 40.13

$$\int \frac{\operatorname{csch}^6(x)}{a+b\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(x)^6/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
Too large to include
```

3.177.6 Sympy [F]

$$\int \frac{\operatorname{csch}^6(x)}{a+b\cosh(x)}\,dx = \int \frac{\operatorname{csch}^6(x)}{a+b\cosh(x)}\,dx$$

input
```
integrate(csch(x)**6/(a+b*cosh(x)),x)
```

output
```
Integral(csch(x)**6/(a + b*cosh(x)), x)
```

3.177.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}^6(x)}{a+b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(csch(x)^6/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.177.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(144) = 288.

Time = 0.27 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.91

$$\int \frac{\operatorname{csch}^6(x)}{a+b\cosh(x)}\,dx = \frac{2\,b^6\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{-a^2+b^2}} + \frac{2\left(15\,b^5e^{(9\,x)}-15\,ab^4e^{(8\,x)}+20\,a^2b^3e^{(7\,x)}-80\,b^5e^{(7\,x)}-30\,a^3b^2e^{(6\,x)}+90\,ab^4e^{(6\,x)}+48\,a^4be^{(5\,x)}-136\,a^2\right.}{}$$

input
```
integrate(csch(x)^6/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
2*b^6*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 -
 b^6)*sqrt(-a^2 + b^2)) + 2/15*(15*b^5*e^(9*x) - 15*a*b^4*e^(8*x) + 20*a^2
*b^3*e^(7*x) - 80*b^5*e^(7*x) - 30*a^3*b^2*e^(6*x) + 90*a*b^4*e^(6*x) + 48
*a^4*b*e^(5*x) - 136*a^2*b^3*e^(5*x) + 178*b^5*e^(5*x) - 80*a^5*e^(4*x) +
230*a^3*b^2*e^(4*x) - 240*a*b^4*e^(4*x) + 20*a^2*b^3*e^(3*x) - 80*b^5*e^(3
*x) + 40*a^5*e^(2*x) - 130*a^3*b^2*e^(2*x) + 150*a*b^4*e^(2*x) + 15*b^5*e^
x - 8*a^5 + 26*a^3*b^2 - 33*a*b^4)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(e
^(2*x) - 1)^5)
```

3.177.9 Mupad [B] (verification not implemented)

Time = 3.39 (sec) , antiderivative size = 1031, normalized size of antiderivative = 6.48

$$\int \frac{\operatorname{csch}^6(x)}{a+b\cosh(x)}\,dx = \frac{\frac{16\,(a\,b^2-a^3)}{(a^2-b^2)^2}+\frac{64\,e^x\,(a^2\,b-b^3)}{5\,(a^2-b^2)^2}}{6\,e^{4\,x}-4\,e^{2\,x}-4\,e^{6\,x}+e^{8\,x}+1} - \frac{\frac{2\,a\,b^4}{(a^2-b^2)^3}-\frac{2\,b^5\,e^x}{(a^2-b^2)^3}}{e^{2\,x}-1} - \frac{\frac{32\,a}{5\,(a^2-b^2)}-\frac{32\,b\,e^x}{5\,(a^2-b^2)}}{5\,e^{2\,x}-10\,e^{4\,x}+10\,e^{6\,x}-5\,e^{8\,x}+e^{10\,x}-1} + \frac{\frac{8\,(3\,a\,b^2-4\,a^3)}{3\,(a^2-b^2)^2}+\frac{8\,e^x\,(12\,a^2\,b-7\,b^3)}{15\,(a^2-b^2)^2}}{3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1} + \frac{\frac{4\,(a\,b^4-a^3\,b^2)}{(a^2-b^2)^3}-\frac{8\,e^x\,(b^5-a^2\,b^3)}{3\,(a^2-b^2)^3}}{e^{4\,x}-2\,e^{2\,x}+1} - \frac{2\,\mathrm{atan}\left(\left(\left(e^x\left(\frac{2\,b^4}{(a^2-b^2)^3\,\sqrt{b^{12}}\,(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6)}+\frac{2\,a\,\left(a^7\,\sqrt{b^{12}}+3\,a^3\,b^4\,\sqrt{b^{12}}-3\,a^5\,b^2\,\sqrt{b^{12}}-a\,b^6\right.}{b^8\,\sqrt{-(a^2-b^2)^7}\,(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6)\,\sqrt{-a^{14}+7\,a^{12}\,b^2-21\,a^{10}\,b^4+35\,a^8\,b}}\right.\right.\right.\right.}{}$$

input
```
int(1/(sinh(x)^6*(a + b*cosh(x))),x)
```

output
```
((16*(a*b^2 - a^3))/(a^2 - b^2)^2 + (64*exp(x)*(a^2*b - b^3))/(5*(a^2 - b^
2)^2))/(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1) - ((2*a*b^4)/
(a^2 - b^2)^3 - (2*b^5*exp(x))/(a^2 - b^2)^3)/(exp(2*x) - 1) - ((32*a)/(5*
(a^2 - b^2)) - (32*b*exp(x))/(5*(a^2 - b^2)))/(5*exp(2*x) - 10*exp(4*x) +
10*exp(6*x) - 5*exp(8*x) + exp(10*x) - 1) + ((8*(3*a*b^2 - 4*a^3))/(3*(a^2
 - b^2)^2) + (8*exp(x)*(12*a^2*b - 7*b^3))/(15*(a^2 - b^2)^2))/(3*exp(2*x)
 - 3*exp(4*x) + exp(6*x) - 1) + ((4*(a*b^4 - a^3*b^2))/(a^2 - b^2)^3 - (8*
exp(x)*(b^5 - a^2*b^3))/(3*(a^2 - b^2)^3))/(exp(4*x) - 2*exp(2*x) + 1) - (
2*atan((exp(x)*((2*b^4)/((a^2 - b^2)^3*(b^12)^(1/2)*(a^6 - b^6 + 3*a^2*b^4
 - 3*a^4*b^2)) + (2*a*(a^7*(b^12)^(1/2) + 3*a^3*b^4*(b^12)^(1/2) - 3*a^5*b
^2*(b^12)^(1/2) - a*b^6*(b^12)^(1/2)))/(b^8*(-(a^2 - b^2)^7)^(1/2)*(a^6 -
b^6 + 3*a^2*b^4 - 3*a^4*b^2)*(b^14 - a^14 - 7*a^2*b^12 + 21*a^4*b^10 - 35*
a^6*b^8 + 35*a^8*b^6 - 21*a^10*b^4 + 7*a^12*b^2)^(1/2))) - (2*a*(b^7*(b^12
)^(1/2) - 3*a^2*b^5*(b^12)^(1/2) + 3*a^4*b^3*(b^12)^(1/2) - a^6*b*(b^12)^(
1/2)))/(b^8*(-(a^2 - b^2)^7)^(1/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)*(b^
14 - a^14 - 7*a^2*b^12 + 21*a^4*b^10 - 35*a^6*b^8 + 35*a^8*b^6 - 21*a^10*b
^4 + 7*a^12*b^2)^(1/2)))*((b^7*(b^14 - a^14 - 7*a^2*b^12 + 21*a^4*b^10 - 3
5*a^6*b^8 + 35*a^8*b^6 - 21*a^10*b^4 + 7*a^12*b^2)^(1/2))/2 - (a^6*b*(b^14
 - a^14 - 7*a^2*b^12 + 21*a^4*b^10 - 35*a^6*b^8 + 35*a^8*b^6 - 21*a^10*b^4
 + 7*a^12*b^2)^(1/2))/2 - (3*a^2*b^5*(b^14 - a^14 - 7*a^2*b^12 + 21*a^4...
```

3.177. $\int \frac{\operatorname{csch}^6(x)}{a+b\cosh(x)}\,dx$

3.178 $\int \frac{\sinh^2(x)}{(a+b\cosh(x))^2}\,dx$

3.178.1 Optimal result

Integrand size = 13, antiderivative size = 67

$$\int \frac{\sinh^2(x)}{(a+b\cosh(x))^2}\,dx = \frac{x}{b^2} - \frac{2a\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^2\sqrt{a+b}} - \frac{\sinh(x)}{b(a+b\cosh(x))}$$

output
```
x/b^2-sinh(x)/b/(a+b*cosh(x))-2*a*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1
/2))/b^2/(a-b)^(1/2)/(a+b)^(1/2)
```

3.178.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \frac{\sinh^2(x)}{(a+b\cosh(x))^2}\,dx = \frac{x + \frac{2a\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} - \frac{b\sinh(x)}{a+b\cosh(x)}}{b^2}$$

input
```
Integrate[Sinh[x]^2/(a + b*Cosh[x])^2,x]
```

output
```
(x + (2*a*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] -
 (b*Sinh[x])/(a + b*Cosh[x]))/b^2
```

3.178.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {3042, 25, 3172, 25, 3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)}{(a+b\cosh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\cos\left(-\frac{\pi}{2}+ix\right)^2}{\left(a-b\sin\left(-\frac{\pi}{2}+ix\right)\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^2}{\left(a-b\sin\left(ix-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3172}$$

$$-\frac{\int -\frac{\cosh(x)}{a+b\cosh(x)}dx}{b}-\frac{\sinh(x)}{b(a+b\cosh(x))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\cosh(x)}{a+b\cosh(x)}dx}{b}-\frac{\sinh(x)}{b(a+b\cosh(x))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(x)}{b(a+b\cosh(x))}+\frac{\int \frac{\sin\left(ix+\frac{\pi}{2}\right)}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\frac{x}{b}-\frac{a\int \frac{1}{a+b\cosh(x)}dx}{b}}{b}-\frac{\sinh(x)}{b(a+b\cosh(x))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(x)}{b(a+b\cosh(x))}+\frac{\frac{x}{b}-\frac{a\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}}{b}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\frac{x}{b}-\frac{2a\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{b}}{b}-\frac{\sinh(x)}{b(a+b\cosh(x))}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\frac{x}{b}-\frac{2a\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}}{b}-\frac{\sinh(x)}{b(a+b\cosh(x))}$$

input
```
Int[Sinh[x]^2/(a + b*Cosh[x])^2,x]
```

output
```
(x/b - (2*a*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*S
qrt[a + b]))/b - Sinh[x]/(b*(a + b*Cosh[x]))
```

3.178.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3172
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Simp[g^2*((p - 1)/(b*(m + 1)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Fre
eQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && I
ntegersQ[2*m, 2*p]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.178.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.48

method	result	size
default	$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b^2}+\frac{\frac{2b\tanh\left(\frac{x}{2}\right)}{\tanh\left(\frac{x}{2}\right)^2 a-\tanh\left(\frac{x}{2}\right)^2 b-a-b}-\frac{2a\,\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}}{b^2}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^2}$	99
risch	$\frac{x}{b^2}+\frac{2a\,\mathrm{e}^x+2b}{b^2\left(b\,\mathrm{e}^{2x}+2a\,\mathrm{e}^x+b\right)}+\frac{a\ln\left(\mathrm{e}^x+\frac{a\sqrt{a^2-b^2}+a^2-b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,b^2}-\frac{a\ln\left(\mathrm{e}^x+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,b^2}$	148

input
```
int(sinh(x)^2/(a+b*cosh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b^2*ln(tanh(1/2*x)-1)+2/b^2*(b*tanh(1/2*x)/(tanh(1/2*x)^2*a-tanh(1/2*x)
^2*b-a-b)-a/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1
/2)))+1/b^2*ln(tanh(1/2*x)+1)
```

3.178.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(57) = 114.

Time = 0.28 (sec) , antiderivative size = 700, normalized size of antiderivative = 10.45

$$\int \frac{\sinh^2(x)}{(a+b\cosh(x))^2}\,dx$$
$$= \left[\frac{(a^2b-b^3)x\cosh(x)^2+(a^2b-b^3)x\sinh(x)^2+2\,a^2b-2\,b^3+\left(ab\cosh(x)^2+ab\sinh(x)^2+2\,a^2\cosh(x)\right.}{}\right.$$

input
```
integrate(sinh(x)^2/(a+b*cosh(x))^2,x, algorithm="fricas")
```

output
```
[((a^2*b - b^3)*x*cosh(x)^2 + (a^2*b - b^3)*x*sinh(x)^2 + 2*a^2*b - 2*b^3
+ (a*b*cosh(x)^2 + a*b*sinh(x)^2 + 2*a^2*cosh(x) + a*b + 2*(a*b*cosh(x) +
a^2)*sinh(x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*c
osh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(
b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(
b*cosh(x) + a)*sinh(x) + b)) + (a^2*b - b^3)*x + 2*(a^3 - a*b^2 + (a^3 - a
*b^2)*x)*cosh(x) + 2*(a^3 - a*b^2 + (a^2*b - b^3)*x*cosh(x) + (a^3 - a*b^2
)*x)*sinh(x))/(a^2*b^3 - b^5 + (a^2*b^3 - b^5)*cosh(x)^2 + (a^2*b^3 - b^5)
*sinh(x)^2 + 2*(a^3*b^2 - a*b^4)*cosh(x) + 2*(a^3*b^2 - a*b^4 + (a^2*b^3 -
 b^5)*cosh(x))*sinh(x)), ((a^2*b - b^3)*x*cosh(x)^2 + (a^2*b - b^3)*x*sinh
(x)^2 + 2*a^2*b - 2*b^3 + 2*(a*b*cosh(x)^2 + a*b*sinh(x)^2 + 2*a^2*cosh(x)
 + a*b + 2*(a*b*cosh(x) + a^2)*sinh(x))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2
 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) + (a^2*b - b^3)*x + 2*(a^
3 - a*b^2 + (a^3 - a*b^2)*x)*cosh(x) + 2*(a^3 - a*b^2 + (a^2*b - b^3)*x*co
sh(x) + (a^3 - a*b^2)*x)*sinh(x))/(a^2*b^3 - b^5 + (a^2*b^3 - b^5)*cosh(x)
^2 + (a^2*b^3 - b^5)*sinh(x)^2 + 2*(a^3*b^2 - a*b^4)*cosh(x) + 2*(a^3*b^2
- a*b^4 + (a^2*b^3 - b^5)*cosh(x))*sinh(x))]
```

3.178.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^2(x)}{(a+b\cosh(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(sinh(x)**2/(a+b*cosh(x))**2,x)
```

output
```
Timed out
```

3.178.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh^2(x)}{(a+b\cosh(x))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sinh(x)^2/(a+b*cosh(x))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.178.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int \frac{\sinh^2(x)}{(a+b\cosh(x))^2}\,dx = -\frac{2\,a\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}b^2}+\frac{x}{b^2}+\frac{2\,(ae^x+b)}{(be^{(2\,x)}+2\,ae^x+b)b^2}$$

input
```
integrate(sinh(x)^2/(a+b*cosh(x))^2,x, algorithm="giac")
```

output
```
-2*a*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b^2) + x/b^2 +
 2*(a*e^x + b)/((b*e^(2*x) + 2*a*e^x + b)*b^2)
```

3.178.9 Mupad [B] (verification not implemented)

Time = 1.90 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.07

$$\int \frac{\sinh^2(x)}{(a+b\cosh(x))^2}\,dx = \frac{x}{b^2}+\frac{\frac{2}{b}+\frac{2\,a\,e^x}{b^2}}{b+2\,a\,e^x+b\,e^{2\,x}}+\frac{a\,\ln\left(\frac{2\,a\,e^x}{b^3}-\frac{2\,a\,(b+a\,e^x)}{b^3\,\sqrt{a+b}\,\sqrt{a-b}}\right)}{b^2\,\sqrt{a+b}\,\sqrt{a-b}} - \frac{a\,\ln\left(\frac{2\,a\,e^x}{b^3}+\frac{2\,a\,(b+a\,e^x)}{b^3\,\sqrt{a+b}\,\sqrt{a-b}}\right)}{b^2\,\sqrt{a+b}\,\sqrt{a-b}}$$

input
```
int(sinh(x)^2/(a + b*cosh(x))^2,x)
```

output
```
x/b^2 + (2/b + (2*a*exp(x))/b^2)/(b + 2*a*exp(x) + b*exp(2*x)) + (a*log((2
*a*exp(x))/b^3 - (2*a*(b + a*exp(x)))/(b^3*(a + b)^(1/2)*(a - b)^(1/2))))/
(b^2*(a + b)^(1/2)*(a - b)^(1/2)) - (a*log((2*a*exp(x))/b^3 + (2*a*(b + a*
exp(x)))/(b^3*(a + b)^(1/2)*(a - b)^(1/2))))/(b^2*(a + b)^(1/2)*(a - b)^(1
/2))
```

3.179 $\int \frac{\tanh^4(x)}{a+b\cosh(x)}\,dx$

3.179.1 Optimal result

Integrand size = 13, antiderivative size = 113

$$\int \frac{\tanh^4(x)}{a+b\cosh(x)}\,dx = \frac{b(3a^2-2b^2)\arctan(\sinh(x))}{2a^4} + \frac{2(a-b)^{3/2}(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4} - \frac{(4a^2-3b^2)\tanh(x)}{3a^3} - \frac{b\text{sech}(x)\tanh(x)}{2a^2} + \frac{\text{sech}^2(x)\tanh(x)}{3a}$$

output
```
1/2*b*(3*a^2-2*b^2)*arctan(sinh(x))/a^4+2*(a-b)^(3/2)*(a+b)^(3/2)*arctanh(
(a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/a^4-1/3*(4*a^2-3*b^2)*tanh(x)/a^3-1/2
*b*sech(x)*tanh(x)/a^2+1/3*sech(x)^2*tanh(x)/a
```

3.179.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

$$\int \frac{\tanh^4(x)}{a+b\cosh(x)}\,dx = \frac{6b(3a^2-2b^2)\arctan\left(\tanh\left(\frac{x}{2}\right)\right) - 12(-a^2+b^2)^{3/2}\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right) + a\left(-8a^2+6b^2-3ab\text{sech}(x)+\right.}{6a^4}$$

input
```
Integrate[Tanh[x]^4/(a + b*Cosh[x]),x]
```

output
```
(6*b*(3*a^2 - 2*b^2)*ArcTan[Tanh[x/2]] - 12*(-a^2 + b^2)^(3/2)*ArcTan[((a
- b)*Tanh[x/2])/Sqrt[-a^2 + b^2]] + a*(-8*a^2 + 6*b^2 - 3*a*b*Sech[x] + 2*
a^2*Sech[x]^2)*Tanh[x])/(6*a^4)
```

3.179.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.18, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.846, Rules used = {3042, 3204, 3042, 3534, 27, 3042, 3480, 3042, 3138, 221, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^4(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan\left(-\frac{\pi}{2}+ix\right)^4\left(a-b\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{3204}$$

$$-\frac{\int \frac{\left(-3\left(2a^2-b^2\right)\cosh^2(x)-ab\cosh(x)+2\left(4a^2-3b^2\right)\right)\text{sech}^2(x)}{a+b\cosh(x)}dx}{6a^2}-\frac{b\tanh(x)\text{sech}(x)}{2a^2}+\frac{\tanh(x)\text{sech}^2(x)}{3a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{-3\left(2a^2-b^2\right)\sin\left(ix+\frac{\pi}{2}\right)^2-ab\sin\left(ix+\frac{\pi}{2}\right)+2\left(4a^2-3b^2\right)}{\sin\left(ix+\frac{\pi}{2}\right)^2\left(a+b\sin\left(ix+\frac{\pi}{2}\right)\right)}dx}{6a^2}-\frac{b\tanh(x)\text{sech}(x)}{2a^2}+\frac{\tanh(x)\text{sech}^2(x)}{3a}$$

$$\Big\downarrow \text{3534}$$

$$-\frac{\frac{\int -\frac{3\left(b\left(3a^2-2b^2\right)+a\left(2a^2-b^2\right)\cosh(x)\right)\text{sech}(x)}{a+b\cosh(x)}dx}{a}+\frac{2\left(4a^2-3b^2\right)\tanh(x)}{a}}{6a^2}-\frac{b\tanh(x)\text{sech}(x)}{2a^2}+\frac{\tanh(x)\text{sech}^2(x)}{3a}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{2\left(4a^2-3b^2\right)\tanh(x)}{a}-\frac{3\int \frac{\left(b\left(3a^2-2b^2\right)+a\left(2a^2-b^2\right)\cosh(x)\right)\text{sech}(x)}{a+b\cosh(x)}dx}{a}}{6a^2}-\frac{b\tanh(x)\text{sech}(x)}{2a^2}+\frac{\tanh(x)\text{sech}^2(x)}{3a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{2(4a^2-3b^2)\tanh(x)}{a}-\frac{3\int \frac{b\left(3a^2-2b^2\right)+a\left(2a^2-b^2\right)\sin\left(ix+\frac{\pi}{2}\right)}{\sin\left(ix+\frac{\pi}{2}\right)\left(a+b\sin\left(ix+\frac{\pi}{2}\right)\right)}dx}{a}}{6a^2}-\frac{b\tanh(x)\text{sech}(x)}{2a^2}+\frac{\tanh(x)\text{sech}^2(x)}{3a}$$

$$\Big\downarrow \; 3480$$

$$-\frac{\frac{2(4a^2-3b^2)\tanh(x)}{a}-\frac{3\left(\frac{2\left(a^2-b^2\right)^2\int \frac{1}{a+b\cosh(x)}dx}{a}+\frac{b\left(3a^2-2b^2\right)\int \text{sech}(x)dx}{a}\right)}{a}}{6a^2}-\frac{b\tanh(x)\text{sech}(x)}{2a^2}+\frac{\tanh(x)\text{sech}^2(x)}{3a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{2(4a^2-3b^2)\tanh(x)}{a}-\frac{3\left(\frac{2\left(a^2-b^2\right)^2\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a}+\frac{b\left(3a^2-2b^2\right)\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}\right)}{a}}{6a^2}-\frac{b\tanh(x)\text{sech}(x)}{2a^2}+\frac{\tanh(x)\text{sech}^2(x)}{3a}$$

$$\Big\downarrow \; 3138$$

$$-\frac{\frac{2(4a^2-3b^2)\tanh(x)}{a}-\frac{3\left(\frac{4\left(a^2-b^2\right)^2\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a}+\frac{b\left(3a^2-2b^2\right)\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}\right)}{a}}{6a^2}-\frac{b\tanh(x)\text{sech}(x)}{2a^2}+\frac{\tanh(x)\text{sech}^2(x)}{3a}$$

$$\Big\downarrow \; 221$$

$$-\frac{\frac{2(4a^2-3b^2)\tanh(x)}{a}-\frac{3\left(\frac{4\left(a^2-b^2\right)^2\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}+\frac{b\left(3a^2-2b^2\right)\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}\right)}{a}}{6a^2}-\frac{b\tanh(x)\text{sech}(x)}{2a^2}+\frac{\tanh(x)\text{sech}^2(x)}{3a}$$

$$\Big\downarrow \; 4257$$

$$-\frac{\frac{2(4a^2-3b^2)\tanh(x)}{a}-\frac{3\left(\frac{b\left(3a^2-2b^2\right)\arctan(\sinh(x))}{a}+\frac{4\left(a^2-b^2\right)^2\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}\right)}{a}}{6a^2}-\frac{b\tanh(x)\text{sech}(x)}{2a^2}+\frac{\tanh(x)\text{sech}^2(x)}{3a}$$

3.179. $\int \frac{\tanh^4(x)}{a+b\cosh(x)}\,dx$

input
```
Int[Tanh[x]^4/(a + b*Cosh[x]),x]
```

output
```
-1/2*(b*Sech[x]*Tanh[x])/a^2 + (Sech[x]^2*Tanh[x])/(3*a) - ((-3*((b*(3*a^2
 - 2*b^2)*ArcTan[Sinh[x]])/a + (4*(a^2 - b^2)^2*ArcTanh[(Sqrt[a - b]*Tanh[
x/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b])))/a + (2*(4*a^2 - 3*b^2)*T
anh[x])/a)/(6*a^2)
```

3.179.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3204
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(3*a*f*Sin[
e + f*x]^3)), x] + (-Simp[b*(m - 2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m +
 1)/(6*a^2*f*Sin[e + f*x]^2)), x] - Simp[1/(6*a^2)   Int[((a + b*Sin[e + f*
x])^m/Sin[e + f*x]^2)*Simp[8*a^2 - b^2*(m - 1)*(m - 2) + a*b*m*Sin[e + f*x]
 - (6*a^2 - b^2*m*(m - 2))*Sin[e + f*x]^2, x], x], x]) /; FreeQ[{a, b, e, f
, m}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1] && IntegerQ[2*m]
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.179.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.36

method	result
default	$\frac{\frac{2\left(\left(-a^{3}+\frac{1}{2}a^{2}b+a\,b^{2}\right)\tanh\left(\frac{x}{2}\right)^{5}+\left(-\frac{10}{3}a^{3}+2a\,b^{2}\right)\tanh\left(\frac{x}{2}\right)^{3}+\left(-a^{3}+a\,b^{2}-\frac{1}{2}a^{2}b\right)\tanh\left(\frac{x}{2}\right)\right)}{\left(1+\tanh\left(\frac{x}{2}\right)^{2}\right)^{3}}+b\left(3a^{2}-2b^{2}\right)\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a^{4}}+\frac{2(a+b)^{2}(\cdots}{\cdots}$
risch	$\frac{-3ab\,\mathrm{e}^{5x}+12a^{2}\mathrm{e}^{4x}-6b^{2}\mathrm{e}^{4x}+12a^{2}\mathrm{e}^{2x}-12b^{2}\mathrm{e}^{2x}+3b\,\mathrm{e}^{x}a+8a^{2}-6b^{2}}{3a^{3}\left(1+\mathrm{e}^{2x}\right)^{3}}+\frac{\sqrt{a^{2}-b^{2}}\,\ln\left(\mathrm{e}^{x}-\frac{-a+\sqrt{a^{2}-b^{2}}}{b}\right)}{a^{2}}-\frac{\sqrt{a^{2}-b^{2}}\,\ln\left(\mathrm{e}^{x}-\frac{-a+\sqrt{a^{2}\cdots}}{b}\right.}{a^{4}}$

input
```
int(tanh(x)^4/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
2/a^4*(((-a^3+1/2*a^2*b+a*b^2)*tanh(1/2*x)^5+(-10/3*a^3+2*a*b^2)*tanh(1/2*
x)^3+(-a^3+a*b^2-1/2*a^2*b)*tanh(1/2*x))/(1+tanh(1/2*x)^2)^3+1/2*b*(3*a^2-
2*b^2)*arctan(tanh(1/2*x)))+2*(a+b)^2*(a-b)^2/a^4/((a+b)*(a-b))^(1/2)*arct
anh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

3.179.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 967 vs. $2(95) = 190$.

Time = 0.35 (sec) , antiderivative size = 2003, normalized size of antiderivative = 17.73

$$\int \frac{\tanh^4(x)}{a+b\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^4/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[-1/3*(3*a^2*b*cosh(x)^5 + 3*a^2*b*sinh(x)^5 - 6*(2*a^3 - a*b^2)*cosh(x)^4
 + 3*(5*a^2*b*cosh(x) - 4*a^3 + 2*a*b^2)*sinh(x)^4 - 3*a^2*b*cosh(x) + 6*(
5*a^2*b*cosh(x)^2 - 4*(2*a^3 - a*b^2)*cosh(x))*sinh(x)^3 - 8*a^3 + 6*a*b^2
 - 12*(a^3 - a*b^2)*cosh(x)^2 + 6*(5*a^2*b*cosh(x)^3 - 2*a^3 + 2*a*b^2 - 6
*(2*a^3 - a*b^2)*cosh(x)^2)*sinh(x)^2 + 3*((a^2 - b^2)*cosh(x)^6 + 6*(a^2
- b^2)*cosh(x)*sinh(x)^5 + (a^2 - b^2)*sinh(x)^6 + 3*(a^2 - b^2)*cosh(x)^4
 + 3*(5*(a^2 - b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^4 + 4*(5*(a^2 - b^2)*co
sh(x)^3 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^3 + 3*(a^2 - b^2)*cosh(x)^2 + 3*(
5*(a^2 - b^2)*cosh(x)^4 + 6*(a^2 - b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 +
 a^2 - b^2 + 6*((a^2 - b^2)*cosh(x)^5 + 2*(a^2 - b^2)*cosh(x)^3 + (a^2 - b
^2)*cosh(x))*sinh(x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 +
 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2
- b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(
x) + 2*(b*cosh(x) + a)*sinh(x) + b)) - 3*((3*a^2*b - 2*b^3)*cosh(x)^6 + 6*
(3*a^2*b - 2*b^3)*cosh(x)*sinh(x)^5 + (3*a^2*b - 2*b^3)*sinh(x)^6 + 3*(3*a
^2*b - 2*b^3)*cosh(x)^4 + 3*(3*a^2*b - 2*b^3 + 5*(3*a^2*b - 2*b^3)*cosh(x)
^2)*sinh(x)^4 + 4*(5*(3*a^2*b - 2*b^3)*cosh(x)^3 + 3*(3*a^2*b - 2*b^3)*cos
h(x))*sinh(x)^3 + 3*a^2*b - 2*b^3 + 3*(3*a^2*b - 2*b^3)*cosh(x)^2 + 3*(5*(
3*a^2*b - 2*b^3)*cosh(x)^4 + 3*a^2*b - 2*b^3 + 6*(3*a^2*b - 2*b^3)*cosh(x)
^2)*sinh(x)^2 + 6*((3*a^2*b - 2*b^3)*cosh(x)^5 + 2*(3*a^2*b - 2*b^3)*co...
```

3.179.6 Sympy [F]

$$\int \frac{\tanh^4(x)}{a+b\cosh(x)}\,dx = \int \frac{\tanh^4(x)}{a+b\cosh(x)}\,dx$$

input
```
integrate(tanh(x)**4/(a+b*cosh(x)),x)
```

output
```
Integral(tanh(x)**4/(a + b*cosh(x)), x)
```

3.179.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\tanh^4(x)}{a+b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(tanh(x)^4/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.179.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.27

$$\int \frac{\tanh^4(x)}{a+b\cosh(x)}\,dx = \frac{(3\,a^2b-2\,b^3)\arctan(e^x)}{a^4} + \frac{2\,(a^4-2\,a^2b^2+b^4)\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}a^4} - \frac{3\,abe^{(5\,x)}-12\,a^2e^{(4\,x)}+6\,b^2e^{(4\,x)}-12\,a^2e^{(2\,x)}+12\,b^2e^{(2\,x)}-3\,abe^x-8\,a^2+6\,b^2}{3\,a^3(e^{(2\,x)}+1)^3}$$

input
```
integrate(tanh(x)^4/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
(3*a^2*b - 2*b^3)*arctan(e^x)/a^4 + 2*(a^4 - 2*a^2*b^2 + b^4)*arctan((b*e^
x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a^4) - 1/3*(3*a*b*e^(5*x) - 12*
a^2*e^(4*x) + 6*b^2*e^(4*x) - 12*a^2*e^(2*x) + 12*b^2*e^(2*x) - 3*a*b*e^x
- 8*a^2 + 6*b^2)/(a^3*(e^(2*x) + 1)^3)
```

3.179.9 Mupad [B] (verification not implemented)

Time = 7.04 (sec) , antiderivative size = 722, normalized size of antiderivative = 6.39

$$\int \frac{\tanh^4(x)}{a+b\cosh(x)}\,dx = \frac{8}{3\,a\,\left(3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1\right)} - \frac{\frac{4}{a}-\frac{2\,b\,e^x}{a^2}}{2\,e^{2\,x}+e^{4\,x}+1} + \frac{\frac{2\,\left(2\,a^2-b^2\right)}{a^3}-\frac{b\,e^x}{a^2}}{e^{2\,x}+1}$$

$$+ \frac{\ln\left(\frac{\left(\frac{32\,a^8-288\,e^x\,a^7\,b-272\,a^6\,b^2+600\,e^x\,a^5\,b^3+456\,a^4\,b^4-416\,e^x\,a^3\,b^5-288\,a^2\,b^6+96\,e^x\,a\,b^7+64\,b^8}{a^6\,b^4} - \frac{\left(\frac{32\,\sqrt{(a+b)^3\,(a-b)^3}\,\left(4\,e^x\,a^3+3\,a^2\,b-3\,e^x\,a\,b^2-2 \cdots\right.}{a^2\,b^5} \cdots\right.}{\cdots}\right)}{a^4} \cdots\right)}{\cdots}$$

$$- \frac{\ln\left(-\frac{\left(\frac{32\,a^8-288\,e^x\,a^7\,b-272\,a^6\,b^2+600\,e^x\,a^5\,b^3+456\,a^4\,b^4-416\,e^x\,a^3\,b^5-288\,a^2\,b^6+96\,e^x\,a\,b^7+64\,b^8}{a^6\,b^4} - \frac{\left(\frac{32\,\sqrt{(a+b)^3\,(a-b)^3}\,\left(4\,e^x\,a^3+3\,a^2\,b-3\,e^x\,a\,b^2 \cdots\right.}{a^2\,b^5} \cdots\right.}{\cdots}\right)}{a^4} \cdots\right)}{\cdots}$$

$$- \frac{b\,\ln\left(e^x-i\right)\,\left(3\,a^2-2\,b^2\right)\,1i}{2\,a^4} + \frac{b\,\ln\left(e^x+1i\right)\,\left(3\,a^2-2\,b^2\right)\,1i}{2\,a^4}$$

input
```
int(tanh(x)^4/(a + b*cosh(x)),x)
```

output
```
8/(3*a*(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1)) - (4/a - (2*b*exp(x))/a^2
)/(2*exp(2*x) + exp(4*x) + 1) + ((2*(2*a^2 - b^2))/a^3 - (b*exp(x))/a^2)/(
exp(2*x) + 1) + (log((((32*a^8 + 64*b^8 - 288*a^2*b^6 + 456*a^4*b^4 - 272*
a^6*b^2 + 96*a*b^7*exp(x) - 288*a^7*b*exp(x) - 416*a^3*b^5*exp(x) + 600*a^
5*b^3*exp(x))/(a^6*b^4) - (((32*((a + b)^3*(a - b)^3)^(1/2)*(3*a^2*b - 2*b
^3 + 4*a^3*exp(x) - 3*a*b^2*exp(x)))/(a^2*b^5) + (16*(a^2 - b^2)*(4*a^2*b
- 4*b^3 + 8*a^3*exp(x) - 7*a*b^2*exp(x)))/(a*b^5))*((a + b)^3*(a - b)^3)^(
1/2))/a^4)*((a + b)^3*(a - b)^3)^(1/2))/a^4 - (8*(a^2 - b^2)^2*(3*a^2 - 2*
b^2)*(6*a^2*b - 4*b^3 + 10*a^3*exp(x) - 7*a*b^2*exp(x)))/(a^9*b^3))*((a +
b)^3*(a - b)^3)^(1/2))/a^4 - (log(- (((32*a^8 + 64*b^8 - 288*a^2*b^6 + 456
*a^4*b^4 - 272*a^6*b^2 + 96*a*b^7*exp(x) - 288*a^7*b*exp(x) - 416*a^3*b^5*
exp(x) + 600*a^5*b^3*exp(x))/(a^6*b^4) - (((32*((a + b)^3*(a - b)^3)^(1/2)
*(3*a^2*b - 2*b^3 + 4*a^3*exp(x) - 3*a*b^2*exp(x)))/(a^2*b^5) - (16*(a^2 -
 b^2)*(4*a^2*b - 4*b^3 + 8*a^3*exp(x) - 7*a*b^2*exp(x)))/(a*b^5))*((a + b)
^3*(a - b)^3)^(1/2))/a^4)*((a + b)^3*(a - b)^3)^(1/2))/a^4 - (8*(a^2 - b^2
)^2*(3*a^2 - 2*b^2)*(6*a^2*b - 4*b^3 + 10*a^3*exp(x) - 7*a*b^2*exp(x)))/(a
^9*b^3))*((a + b)^3*(a - b)^3)^(1/2))/a^4 - (b*log(exp(x) - 1i)*(3*a^2 - 2
*b^2)*1i)/(2*a^4) + (b*log(exp(x) + 1i)*(3*a^2 - 2*b^2)*1i)/(2*a^4)
```

3.179. $\int \frac{\tanh^4(x)}{a+b\cosh(x)}\,dx$

3.180 $\int \frac{\tanh^3(x)}{a+b\cosh(x)}\,dx$

3.180.1 Optimal result

Integrand size = 13, antiderivative size = 57

$$\int \frac{\tanh^3(x)}{a+b\cosh(x)}\,dx = \frac{(a^2-b^2)\log(\cosh(x))}{a^3} - \frac{(a^2-b^2)\log(a+b\cosh(x))}{a^3} - \frac{b\text{sech}(x)}{a^2} + \frac{\text{sech}^2(x)}{2a}$$

output
```
(a^2-b^2)*ln(cosh(x))/a^3-(a^2-b^2)*ln(a+b*cosh(x))/a^3-b*sech(x)/a^2+1/2*
sech(x)^2/a
```

3.180.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int \frac{\tanh^3(x)}{a+b\cosh(x)}\,dx = \frac{2(a^2-b^2)(\log(\cosh(x))-\log(a+b\cosh(x)))-2ab\text{sech}(x)+a^2\text{sech}^2(x)}{2a^3}$$

input
```
Integrate[Tanh[x]^3/(a + b*Cosh[x]),x]
```

output
```
(2*(a^2 - b^2)*(Log[Cosh[x]] - Log[a + b*Cosh[x]]) - 2*a*b*Sech[x] + a^2*S
ech[x]^2)/(2*a^3)
```

3.180.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 26, 3200, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\tan\left(-\frac{\pi}{2}+ix\right)^3\left(a-b\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\left(a-b\sin\left(ix-\frac{\pi}{2}\right)\right)\tan\left(ix-\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{3200}$$

$$-\int \frac{\left(b^2-b^2\cosh^2(x)\right)\operatorname{sech}^3(x)}{b^3(a+b\cosh(x))}d(b\cosh(x))$$

$$\Big\downarrow \text{522}$$

$$-\int \left(\frac{\operatorname{sech}^3(x)}{ab}-\frac{\operatorname{sech}^2(x)}{a^2}+\frac{\left(b^2-a^2\right)\operatorname{sech}(x)}{a^3b}+\frac{a^2-b^2}{a^3(a+b\cosh(x))}\right)d(b\cosh(x))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\operatorname{sech}(x)}{a^2}+\frac{\left(a^2-b^2\right)\log(b\cosh(x))}{a^3}-\frac{\left(a^2-b^2\right)\log(a+b\cosh(x))}{a^3}+\frac{\operatorname{sech}^2(x)}{2a}$$

input
```
Int[Tanh[x]^3/(a + b*Cosh[x]),x]
```

output
```
((a^2 - b^2)*Log[b*Cosh[x]])/a^3 - ((a^2 - b^2)*Log[a + b*Cosh[x]])/a^3 -
(b*Sech[x])/a^2 + Sech[x]^2/(2*a)
```

3.180.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.180.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.67

method	result	size
default	$\frac{\frac{2a^2}{\left(1+\tanh\left(\frac{x}{2}\right)^2\right)^2}+(a^2-b^2)\ln\left(1+\tanh\left(\frac{x}{2}\right)^2\right)-\frac{2a(a+b)}{1+\tanh\left(\frac{x}{2}\right)^2}}{a^3}-\frac{(a-b)(a+b)\ln\left(\tanh\left(\frac{x}{2}\right)^2 a-\tanh\left(\frac{x}{2}\right)^2 b-a-b\right)}{a^3}$	95
risch	$\frac{2\,\mathrm{e}^{x}\left(-b\,\mathrm{e}^{2x}+a\,\mathrm{e}^{x}-b\right)}{\left(1+\mathrm{e}^{2x}\right)^2 a^2}+\frac{\ln(1+\mathrm{e}^{2x})}{a}-\frac{\ln(1+\mathrm{e}^{2x})b^2}{a^3}-\frac{\ln\left(\mathrm{e}^{2x}+\frac{2a\,\mathrm{e}^{x}}{b}+1\right)}{a}+\frac{\ln\left(\mathrm{e}^{2x}+\frac{2a\,\mathrm{e}^{x}}{b}+1\right)b^2}{a^3}$	100

input
```
int(tanh(x)^3/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(2*a^2/(1+tanh(1/2*x)^2)^2+(a^2-b^2)*ln(1+tanh(1/2*x)^2)-2*a*(a+b)/(
1+tanh(1/2*x)^2))-(a-b)*(a+b)/a^3*ln(tanh(1/2*x)^2*a-tanh(1/2*x)^2*b-a-b)
```

3.180.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(55) = 110$.

Time = 0.27 (sec) , antiderivative size = 450, normalized size of antiderivative = 7.89

$$\int \frac{\tanh^3(x)}{a+b\cosh(x)}\,dx = -\frac{2\,ab\cosh\left(x\right)^3+2\,ab\sinh\left(x\right)^3-2\,a^2\cosh\left(x\right)^2+2\,ab\cosh\left(x\right)+2\left(3\,ab\cosh\left(x\right)-a^2\right)\sinh\left(x\right)^2+\left(\left(a^2-\right.\right.}{}$$

input
```
integrate(tanh(x)^3/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
-(2*a*b*cosh(x)^3 + 2*a*b*sinh(x)^3 - 2*a^2*cosh(x)^2 + 2*a*b*cosh(x) + 2*
(3*a*b*cosh(x) - a^2)*sinh(x)^2 + ((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*c
osh(x)*sinh(x)^3 + (a^2 - b^2)*sinh(x)^4 + 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*
(a^2 - b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*
cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))*log(2*(b*cosh(x) + a)/(cosh(x) -
 sinh(x))) - ((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cosh(x)*sinh(x)^3 + (a
^2 - b^2)*sinh(x)^4 + 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 - b^2)*cosh(x)^2
 + a^2 - b^2)*sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(x)^3 + (a^2 - b^
2)*cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) + 2*(3*a*b*cosh(x)
^2 - 2*a^2*cosh(x) + a*b)*sinh(x))/(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^
3 + a^3*sinh(x)^4 + 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh(x)^2 + a^3)*sinh
(x)^2 + 4*(a^3*cosh(x)^3 + a^3*cosh(x))*sinh(x))
```

3.180.6 Sympy [F]

$$\int \frac{\tanh^3(x)}{a+b\cosh(x)}\,dx = \int \frac{\tanh^3\left(x\right)}{a+b\cosh\left(x\right)}\,dx$$

input
```
integrate(tanh(x)**3/(a+b*cosh(x)),x)
```

output
```
Integral(tanh(x)**3/(a + b*cosh(x)), x)
```

3.180.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.68

$$\int \frac{\tanh^3(x)}{a+b\cosh(x)}\,dx = -\frac{2\left(be^{(-x)}-ae^{(-2x)}+be^{(-3x)}\right)}{2a^2e^{(-2x)}+a^2e^{(-4x)}+a^2} - \frac{(a^2-b^2)\log\left(2ae^{(-x)}+be^{(-2x)}+b\right)}{a^3} + \frac{(a^2-b^2)\log\left(e^{(-2x)}+1\right)}{a^3}$$

input
```
integrate(tanh(x)^3/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
-2*(b*e^(-x) - a*e^(-2*x) + b*e^(-3*x))/(2*a^2*e^(-2*x) + a^2*e^(-4*x) + a
^2) - (a^2 - b^2)*log(2*a*e^(-x) + b*e^(-2*x) + b)/a^3 + (a^2 - b^2)*log(e
^(-2*x) + 1)/a^3
```

3.180.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(55) = 110.

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.02

$$\int \frac{\tanh^3(x)}{a+b\cosh(x)}\,dx = \frac{(a^2-b^2)\log\left(e^{(-x)}+e^x\right)}{a^3} - \frac{(a^2b-b^3)\log\left(\left|b\left(e^{(-x)}+e^x\right)+2a\right|\right)}{a^3b} - \frac{3a^2\left(e^{(-x)}+e^x\right)^2-3b^2\left(e^{(-x)}+e^x\right)^2+4ab\left(e^{(-x)}+e^x\right)-4a^2}{2a^3\left(e^{(-x)}+e^x\right)^2}$$

input
```
integrate(tanh(x)^3/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
(a^2 - b^2)*log(e^(-x) + e^x)/a^3 - (a^2*b - b^3)*log(abs(b*(e^(-x) + e^x)
 + 2*a))/(a^3*b) - 1/2*(3*a^2*(e^(-x) + e^x)^2 - 3*b^2*(e^(-x) + e^x)^2 +
4*a*b*(e^(-x) + e^x) - 4*a^2)/(a^3*(e^(-x) + e^x)^2)
```

3.180.9 Mupad [B] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 1221, normalized size of antiderivative = 21.42

$$\int \frac{\tanh^3(x)}{a+b\cosh(x)}\,dx = \frac{\frac{2}{a}-\frac{2\,b\,\mathrm{e}^x}{a^2}}{\mathrm{e}^{2\,x}+1} - \frac{2}{a\,\left(2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1\right)} + \frac{\left(2\,\mathrm{atan}\left(\left(4\,a^4\,b^3\,\left(a^2-b^2\right)^2\sqrt{-a^6}-4\,a^6\,b\,\left(a^2-b^2\right)^2\sqrt{-a^6}\right)\left(\mathrm{e}^x\left(\frac{1}{16\,a^4\,b^2\,\left(a^2-b^2\right)^3\sqrt{\left(a^2-b^2\right)^2}}-\frac{(a^2-}{16\,a^8\,b^2\,(a^2-b}\right.\right.\right.\right.}{}$$

input
```
int(tanh(x)^3/(a + b*cosh(x)),x)
```

output
```
(2/a - (2*b*exp(x))/a^2)/(exp(2*x) + 1) - 2/(a*(2*exp(2*x) + exp(4*x) + 1)
) + ((2*atan((4*a^4*b^3*(a^2 - b^2)^2*(-a^6)^(1/2) - 4*a^6*b*(a^2 - b^2)^2
*(-a^6)^(1/2))*(exp(x)*(1/(16*a^4*b^2*(a^2 - b^2)^3*((a^2 - b^2)^2)^(1/2))
 - (a^2 - 2*b^2)^2/(16*a^8*b^2*(a^2 - b^2)^3*((a^2 - b^2)^2)^(1/2))) + 1/(
8*a^5*b*(a^2 - b^2)^3*((a^2 - b^2)^2)^(1/2)) + (a^2 - 2*b^2)/(8*a^7*b*(a^2
 - b^2)^3*((a^2 - b^2)^2)^(1/2)))) + 2*atan((a^2*(-a^6)^(1/2)*(a^4 + b^4 -
 2*a^2*b^2)^(1/2) - 2*b^2*(-a^6)^(1/2)*(a^4 + b^4 - 2*a^2*b^2)^(1/2))/(2*a
^3*(a^2 - b^2)^2) + ((a^7 - a^5*b^2)*(-a^6)^(1/2))/(2*a^6*(a^2 - b^2)*((a^
2 - b^2)^2)^(1/2)) + (a^6*b^2*exp(3*x)*((2*(a^7 - a^5*b^2)*(a^4 + b^4 - 2*
a^2*b^2)^(1/2))/(a^11*b^3*(a^2 - b^2)*((a^2 - b^2)^2)^(1/2)) - (2*(a^2 - 2
*b^2)*(a^2*(-a^6)^(1/2)*(a^4 + b^4 - 2*a^2*b^2)^(1/2) - 2*b^2*(-a^6)^(1/2)
*(a^4 + b^4 - 2*a^2*b^2)^(1/2))*(a^4 + b^4 - 2*a^2*b^2)^(1/2))/(a^10*b^3*(
a^2 - b^2)^2*(-a^6)^(1/2)))*(-a^6)^(1/2))/(8*(a^4 + b^4 - 2*a^2*b^2)^(1/2)
) - (a^6*b^2*exp(x)*(-a^6)^(1/2)*((8*(a^4 + b^4 - 2*a^2*b^2))/(a^8*b*(a^2
- b^2)^2) - (4*(2*a^6*b - 2*a^4*b^3)*(a^4 + b^4 - 2*a^2*b^2)^(1/2))/(a^12*
b^2*(a^2 - b^2)*((a^2 - b^2)^2)^(1/2)) - (2*(a^7 - a^5*b^2)*(a^4 + b^4 - 2
*a^2*b^2)^(1/2))/(a^11*b^3*(a^2 - b^2)*((a^2 - b^2)^2)^(1/2)) + (2*(a^2 -
2*b^2)*(a^2*(-a^6)^(1/2)*(a^4 + b^4 - 2*a^2*b^2)^(1/2) - 2*b^2*(-a^6)^(1/2
)*(a^4 + b^4 - 2*a^2*b^2)^(1/2))*(a^4 + b^4 - 2*a^2*b^2)^(1/2))/(a^10*b^3*
(a^2 - b^2)^2*(-a^6)^(1/2))))/(8*(a^4 + b^4 - 2*a^2*b^2)^(1/2)) + (a^6*...
```

3.181 $\int \frac{\tanh^2(x)}{a+b\cosh(x)}\,dx$

3.181.1 Optimal result

Integrand size = 13, antiderivative size = 61

$$\int \frac{\tanh^2(x)}{a+b\cosh(x)}\,dx=\frac{b\arctan(\sinh(x))}{a^2}+\frac{2\sqrt{a-b}\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2}-\frac{\tanh(x)}{a}$$

output
```
b*arctan(sinh(x))/a^2+2*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))*(a-b)
^(1/2)*(a+b)^(1/2)/a^2-tanh(x)/a
```

3.181.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{\tanh^2(x)}{a+b\cosh(x)}\,dx=\frac{2b\arctan\left(\tanh\left(\frac{x}{2}\right)\right)+2\sqrt{-a^2+b^2}\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)-a\tanh(x)}{a^2}$$

input
```
Integrate[Tanh[x]^2/(a + b*Cosh[x]),x]
```

output
```
(2*b*ArcTan[Tanh[x/2]] + 2*Sqrt[-a^2 + b^2]*ArcTan[((a - b)*Tanh[x/2])/Sqr
t[-a^2 + b^2]] - a*Tanh[x])/a^2
```

3.181.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.23, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.923, Rules used = {3042, 25, 3202, 3042, 3535, 25, 3042, 3480, 3042, 3138, 221, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\tan\left(-\frac{\pi}{2}+ix\right)^2\left(a-b\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\left(a-b\sin\left(ix-\frac{\pi}{2}\right)\right)\tan\left(ix-\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3202}$$

$$-\int \frac{\left(1-\cosh^2(x)\right)\text{sech}^2(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{1-\sin\left(ix+\frac{\pi}{2}\right)^2}{\sin\left(ix+\frac{\pi}{2}\right)^2\left(a+b\sin\left(ix+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3535}$$

$$-\frac{\int -\frac{(b+a\cosh(x))\text{sech}(x)}{a+b\cosh(x)}dx}{a}-\frac{\tanh(x)}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(b+a\cosh(x))\text{sech}(x)}{a+b\cosh(x)}dx}{a}-\frac{\tanh(x)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tanh(x)}{a}+\frac{\int \frac{b+a\sin\left(ix+\frac{\pi}{2}\right)}{\sin\left(ix+\frac{\pi}{2}\right)\left(a+b\sin\left(ix+\frac{\pi}{2}\right)\right)}dx}{a}$$

$$\Big\downarrow \text{3480}$$

$$\frac{\frac{(a^2-b^2)\int \frac{1}{a+b\cosh(x)}dx}{a}+\frac{b\int \text{sech}(x)dx}{a}}{a}-\frac{\tanh(x)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tanh(x)}{a}+\frac{\frac{(a^2-b^2)\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a}+\frac{b\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}}{a}$$

$$\Big\downarrow \text{3138}$$

$$-\frac{\tanh(x)}{a}+\frac{\frac{2(a^2-b^2)\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a}+\frac{b\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}}{a}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\tanh(x)}{a}+\frac{\frac{2(a^2-b^2)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}+\frac{b\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}}{a}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\frac{2(a^2-b^2)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}+\frac{b\arctan(\sinh(x))}{a}}{a}-\frac{\tanh(x)}{a}$$

input
```
Int[Tanh[x]^2/(a + b*Cosh[x]),x]
```

output
```
((b*ArcTan[Sinh[x]])/a + (2*(a^2 - b^2)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sq
rt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]))/a - Tanh[x]/a
```

3.181.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3202
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((1 - Sin[e + f*x]^2)/Sin[e + f*x]^
2), x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3535
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n
+ 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d
*(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
 !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.181. $\int \frac{\tanh^2(x)}{a+b\cosh(x)}\,dx$

3.181.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.28

method	result	size
default	$\frac{-\frac{2a\tanh\left(\frac{x}{2}\right)}{1+\tanh\left(\frac{x}{2}\right)^{2}}+2b\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a^{2}}+\frac{2(a+b)(a-b)\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^{2}\sqrt{(a+b)(a-b)}}$	78
risch	$\frac{2}{a(1+\mathrm{e}^{2x})}+\frac{ib\ln(\mathrm{e}^{x}+i)}{a^{2}}-\frac{ib\ln(\mathrm{e}^{x}-i)}{a^{2}}+\frac{\sqrt{a^{2}-b^{2}}\,\ln\left(\mathrm{e}^{x}-\frac{-a+\sqrt{a^{2}-b^{2}}}{b}\right)}{a^{2}}-\frac{\sqrt{a^{2}-b^{2}}\,\ln\left(\mathrm{e}^{x}+\frac{a+\sqrt{a^{2}-b^{2}}}{b}\right)}{a^{2}}$	117

input
```
int(tanh(x)^2/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
2/a^2*(-a*tanh(1/2*x)/(1+tanh(1/2*x)^2)+b*arctan(tanh(1/2*x)))+2*(a+b)*(a-
b)/a^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

3.181.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(51) = 102.

Time = 0.29 (sec) , antiderivative size = 326, normalized size of antiderivative = 5.34

$$\int \frac{\tanh^2(x)}{a+b\cosh(x)}\,dx$$

$$= \left[\frac{\sqrt{a^2-b^2}\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+1\right)\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\,(b^2 c}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh}\right.}{a^2\cosh(x)^2 -}\right.$$

$$-\frac{2\left(\sqrt{-a^2+b^2}\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+1\right)\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cosh(x)+b\sinh(x)+a)}{a^2-b^2}\right)-\right.}{a^2\cosh(x)^2+2\,a^2\cosh(x)\sinh(x)}$$

input
```
integrate(tanh(x)^2/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[(sqrt(a^2 - b^2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log((b^2
*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x)
+ a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)
^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)) + 2*(b*co
sh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*arctan(cosh(x) + sinh(x))
 + 2*a)/(a^2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 + a^2), -2*
(sqrt(-a^2 + b^2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*arctan(-
sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) - (b*cosh(x)^2 +
 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*arctan(cosh(x) + sinh(x)) - a)/(a^
2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 + a^2)]
```

3.181.6 Sympy [F]

$$\int \frac{\tanh^2(x)}{a + b\cosh(x)}\, dx = \int \frac{\tanh^2{(x)}}{a + b\cosh{(x)}}\, dx$$

input
```
integrate(tanh(x)**2/(a+b*cosh(x)),x)
```

output
```
Integral(tanh(x)**2/(a + b*cosh(x)), x)
```

3.181.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\tanh^2(x)}{a + b\cosh(x)}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate(tanh(x)^2/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.181.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10

$$\int \frac{\tanh^2(x)}{a+b\cosh(x)}\,dx = \frac{2\,b\arctan\left(e^x\right)}{a^2} + \frac{2\left(a^2-b^2\right)\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}a^2} + \frac{2}{a(e^{(2x)}+1)}$$

input
```
integrate(tanh(x)^2/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
2*b*arctan(e^x)/a^2 + 2*(a^2 - b^2)*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(
sqrt(-a^2 + b^2)*a^2) + 2/(a*(e^(2*x) + 1))
```

3.181.9 Mupad [B] (verification not implemented)

Time = 4.45 (sec) , antiderivative size = 285, normalized size of antiderivative = 4.67

$$\int \frac{\tanh^2(x)}{a+b\cosh(x)}\,dx = \frac{2}{a+a\,e^{2x}} + \frac{\ln\left(64\,a^3\,b-64\,a\,b^3-32\,b^3\,\sqrt{a^2-b^2}+128\,a^4\,e^x+32\,b^4\,e^x+64\,a^2\,b\,\sqrt{a^2-b^2}+128\,a^3\,e^x\,\sqrt{a^2-b^2}-16\right.}{a^2} - \frac{\ln\left(64\,a^3\,b-64\,a\,b^3+32\,b^3\,\sqrt{a^2-b^2}+128\,a^4\,e^x+32\,b^4\,e^x-64\,a^2\,b\,\sqrt{a^2-b^2}-128\,a^3\,e^x\,\sqrt{a^2-b^2}-16\right.}{a^2} - \frac{b\left(\ln\left(e^x-i\right)\,1i-\ln\left(e^x+1i\right)\,1i\right)}{a^2}$$

input
```
int(tanh(x)^2/(a + b*cosh(x)),x)
```

output
```
2/(a + a*exp(2*x)) + (log(64*a^3*b - 64*a*b^3 - 32*b^3*(a^2 - b^2)^(1/2) +
 128*a^4*exp(x) + 32*b^4*exp(x) + 64*a^2*b*(a^2 - b^2)^(1/2) + 128*a^3*exp
(x)*(a^2 - b^2)^(1/2) - 160*a^2*b^2*exp(x) - 96*a*b^2*exp(x)*(a^2 - b^2)^(
1/2))*(a^2 - b^2)^(1/2))/a^2 - (log(64*a^3*b - 64*a*b^3 + 32*b^3*(a^2 - b^
2)^(1/2) + 128*a^4*exp(x) + 32*b^4*exp(x) - 64*a^2*b*(a^2 - b^2)^(1/2) - 1
28*a^3*exp(x)*(a^2 - b^2)^(1/2) - 160*a^2*b^2*exp(x) + 96*a*b^2*exp(x)*(a^
2 - b^2)^(1/2))*(a^2 - b^2)^(1/2))/a^2 - (b*(log(exp(x) - 1i)*1i - log(exp
(x) + 1i)*1i))/a^2
```

3.182 $\int \frac{\tanh(x)}{a+b\cosh(x)}\,dx$

3.182.1 Optimal result

Integrand size = 11, antiderivative size = 20

$$\int \frac{\tanh(x)}{a+b\cosh(x)}\,dx = \frac{\log(\cosh(x))}{a} - \frac{\log(a+b\cosh(x))}{a}$$

output
```
ln(cosh(x))/a-ln(a+b*cosh(x))/a
```

3.182.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\tanh(x)}{a+b\cosh(x)}\,dx = \frac{\log(\cosh(x))}{a} - \frac{\log(a+b\cosh(x))}{a}$$

input
```
Integrate[Tanh[x]/(a + b*Cosh[x]),x]
```

output
```
Log[Cosh[x]]/a - Log[a + b*Cosh[x]]/a
```

3.182.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 26, 3200, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\tan\left(-\frac{\pi}{2}+ix\right)\left(a-b\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\left(a-b\sin\left(ix-\frac{\pi}{2}\right)\right)\tan\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3200}$$

$$\int \frac{\operatorname{sech}(x)}{b(a+b\cosh(x))}d(b\cosh(x))$$

$$\Big\downarrow \text{47}$$

$$\frac{\int \frac{\operatorname{sech}(x)}{b}d(b\cosh(x))}{a}-\frac{\int \frac{1}{a+b\cosh(x)}d(b\cosh(x))}{a}$$

$$\Big\downarrow \text{14}$$

$$\frac{\log(b\cosh(x))}{a}-\frac{\int \frac{1}{a+b\cosh(x)}d(b\cosh(x))}{a}$$

$$\Big\downarrow \text{16}$$

$$\frac{\log(b\cosh(x))}{a}-\frac{\log(a+b\cosh(x))}{a}$$

input
```
Int[Tanh[x]/(a + b*Cosh[x]),x]
```

output
```
Log[b*Cosh[x]]/a - Log[a + b*Cosh[x]]/a
```

3.182.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.182.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

method	result	size
risch	$\frac{\ln\left(1+{\mathrm e}^{2 x}\right)}{a}-\frac{\ln\left({\mathrm e}^{2 x}+\frac{2 a \,{\mathrm e}^{x}}{b}+1\right)}{a}$	33
default	$\frac{\ln\left(1+\tanh\left(\frac{x}{2}\right)^{2}\right)}{a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)^{2} a-\tanh\left(\frac{x}{2}\right)^{2} b-a-b\right)}{a}$	45

input
```
int(tanh(x)/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/a*ln(1+exp(2*x))-1/a*ln(exp(2*x)+2*a/b*exp(x)+1)
```

3.182.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.00

$$\int \frac{\tanh(x)}{a+b\cosh(x)}\,dx = -\frac{\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right) - \log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)}{a}$$

input
```
integrate(tanh(x)/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
-(log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) - log(2*cosh(x)/(cosh(x) - si
nh(x))))/a
```

3.182.6 Sympy [F]

$$\int \frac{\tanh(x)}{a+b\cosh(x)}\,dx = \int \frac{\tanh(x)}{a+b\cosh(x)}\,dx$$

input
```
integrate(tanh(x)/(a+b*cosh(x)),x)
```

output
```
Integral(tanh(x)/(a + b*cosh(x)), x)
```

3.182.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \frac{\tanh(x)}{a+b\cosh(x)}\,dx = -\frac{\log\left(2\,ae^{(-x)} + be^{(-2\,x)} + b\right)}{a} + \frac{\log\left(e^{(-2\,x)} + 1\right)}{a}$$

input
```
integrate(tanh(x)/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
-log(2*a*e^(-x) + b*e^(-2*x) + b)/a + log(e^(-2*x) + 1)/a
```

3.182.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \frac{\tanh(x)}{a+b\cosh(x)}\,dx = \frac{\log\left(e^{(-x)}+e^x\right)}{a} - \frac{\log\left(\left|b\left(e^{(-x)}+e^x\right)+2\,a\right|\right)}{a}$$

input
```
integrate(tanh(x)/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
log(e^(-x) + e^x)/a - log(abs(b*(e^(-x) + e^x) + 2*a))/a
```

3.182.9 Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 201, normalized size of antiderivative = 10.05

$$\int \frac{\tanh(x)}{a+b\cosh(x)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{a\sqrt{-a^2}+b\,\mathrm{e}^x\sqrt{-a^2}+2\,a\,\mathrm{e}^{2\,x}\sqrt{-a^2}+b\,\mathrm{e}^{3\,x}\sqrt{-a^2}}{a^2}\right)}{\sqrt{-a^2}} - \frac{2\,\mathrm{atan}\left(\left(4\,a^4\,b\sqrt{-a^2}-4\,a^2\,b^3\sqrt{-a^2}\right)\left(\mathrm{e}^x\left(\frac{1}{16\,b^2\,(a^2-b^2)^2}-\frac{(a^2-2\,b^2)^2}{16\,a^4\,b^2\,(a^2-b^2)^2}\right)+\frac{1}{8\,a\,b\,(a^2-b^2)^2}+\frac{a^2-2\,b^2}{8\,a^3\,b\,(a^2-b^2)^2}\right)\right)}{\sqrt{-a^2}}$$

input
```
int(tanh(x)/(a + b*cosh(x)),x)
```

output
```
(2*atan((a*(-a^2)^(1/2) + b*exp(x)*(-a^2)^(1/2) + 2*a*exp(2*x)*(-a^2)^(1/2
) + b*exp(3*x)*(-a^2)^(1/2))/a^2))/(-a^2)^(1/2) - (2*atan((4*a^4*b*(-a^2)^
(1/2) - 4*a^2*b^3*(-a^2)^(1/2))*(exp(x)*(1/(16*b^2*(a^2 - b^2)^2) - (a^2 -
 2*b^2)^2/(16*a^4*b^2*(a^2 - b^2)^2)) + 1/(8*a*b*(a^2 - b^2)^2) + (a^2 - 2
*b^2)/(8*a^3*b*(a^2 - b^2)^2))))/(-a^2)^(1/2)
```

3.183 $\int \frac{\coth(x)}{a+b\cosh(x)}\,dx$

3.183.1 Optimal result

Integrand size = 11, antiderivative size = 54

$$\int \frac{\coth(x)}{a+b\cosh(x)}\,dx=\frac{\log(1-\cosh(x))}{2(a+b)}+\frac{\log(1+\cosh(x))}{2(a-b)}-\frac{a\log(a+b\cosh(x))}{a^2-b^2}$$

output
```
1/2*ln(1-cosh(x))/(a+b)+1/2*ln(1+cosh(x))/(a-b)-a*ln(a+b*cosh(x))/(a^2-b^2
)
```

3.183.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{\coth(x)}{a+b\cosh(x)}\,dx=\frac{\log\left(\cosh\left(\frac{x}{2}\right)\right)}{a-b}-\frac{a\log(a+b\cosh(x))}{a^2-b^2}+\frac{\log\left(\sinh\left(\frac{x}{2}\right)\right)}{a+b}$$

input
```
Integrate[Coth[x]/(a + b*Cosh[x]),x]
```

output
```
Log[Cosh[x/2]]/(a - b) - (a*Log[a + b*Cosh[x]])/(a^2 - b^2) + Log[Sinh[x/2
]]/(a + b)
```

3.183.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {3042, 26, 3200, 587, 16, 452, 219, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan\left(-\frac{\pi}{2}+ix\right)}{a-b\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan\left(ix-\frac{\pi}{2}\right)}{a-b\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3200}$$

$$-\int \frac{b\cosh(x)}{(a+b\cosh(x))\left(b^2-b^2\cosh^2(x)\right)}d(b\cosh(x))$$

$$\Big\downarrow \text{587}$$

$$\frac{\int \frac{b^2-ab\cosh(x)}{b^2-b^2\cosh^2(x)}d(b\cosh(x))}{a^2-b^2}-\frac{a\int \frac{1}{a+b\cosh(x)}d(b\cosh(x))}{a^2-b^2}$$

$$\Big\downarrow \text{16}$$

$$\frac{\int \frac{b^2-ab\cosh(x)}{b^2-b^2\cosh^2(x)}d(b\cosh(x))}{a^2-b^2}-\frac{a\log(a+b\cosh(x))}{a^2-b^2}$$

$$\Big\downarrow \text{452}$$

$$\frac{b^2\int \frac{1}{b^2-b^2\cosh^2(x)}d(b\cosh(x))-a\int \frac{b\cosh(x)}{b^2-b^2\cosh^2(x)}d(b\cosh(x))}{a^2-b^2}-\frac{a\log(a+b\cosh(x))}{a^2-b^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{b\,\text{arctanh}(\cosh(x))-a\int \frac{b\cosh(x)}{b^2-b^2\cosh^2(x)}d(b\cosh(x))}{a^2-b^2}-\frac{a\log(a+b\cosh(x))}{a^2-b^2}$$

$$\Big\downarrow \text{240}$$

$$\frac{\frac{1}{2}a\log\left(b^2-b^2\cosh^2(x)\right)+b\,\text{arctanh}(\cosh(x))}{a^2-b^2}-\frac{a\log(a+b\cosh(x))}{a^2-b^2}$$

input
```
Int[Coth[x]/(a + b*Cosh[x]),x]
```

output
```
-((a*Log[a + b*Cosh[x]])/(a^2 - b^2)) + (b*ArcTanh[Cosh[x]] + (a*Log[b^2 -
 b^2*Cosh[x]^2])/2)/(a^2 - b^2)
```

3.183.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 587
```
Int[(x_.)/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[(-
c)*(d/(b*c^2 + a*d^2))   Int[1/(c + d*x), x], x] + Simp[1/(b*c^2 + a*d^2)
 Int[(a*d + b*c*x)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
^2 + a*d^2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.183.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a+b}-\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)^{2}a-\tanh\left(\frac{x}{2}\right)^{2}b-a-b\right)}{(a+b)(a-b)}$	53
risch	$-\frac{x}{a+b}-\frac{x}{a-b}+\frac{2xa}{a^{2}-b^{2}}+\frac{\ln(e^{x}-1)}{a+b}+\frac{\ln(e^{x}+1)}{a-b}-\frac{a\ln\left(e^{2x}+\frac{2a\,e^{x}}{b}+1\right)}{a^{2}-b^{2}}$	88

input
```
int(coth(x)/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/(a+b)*ln(tanh(1/2*x))-a/(a+b)/(a-b)*ln(tanh(1/2*x)^2*a-tanh(1/2*x)^2*b-a
-b)
```

3.183.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \frac{\coth(x)}{a+b\cosh(x)}\,dx = -\frac{a\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right)-(a+b)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)-(a-b)\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)}{a^{2}-b^{2}}$$

input
```
integrate(coth(x)/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
-(a*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) - (a + b)*log(cosh(x) + sin
h(x) + 1) - (a - b)*log(cosh(x) + sinh(x) - 1))/(a^2 - b^2)
```

3.183.6 Sympy [F]

$$\int \frac{\coth(x)}{a+b\cosh(x)}\,dx=\int \frac{\coth(x)}{a+b\cosh(x)}\,dx$$

input
```
integrate(coth(x)/(a+b*cosh(x)),x)
```

output
```
Integral(coth(x)/(a + b*cosh(x)), x)
```

3.183.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

$$\int \frac{\coth(x)}{a+b\cosh(x)}\,dx=-\frac{a\log\left(2\,ae^{(-x)}+be^{(-2\,x)}+b\right)}{a^2-b^2}+\frac{\log\left(e^{(-x)}+1\right)}{a-b}+\frac{\log\left(e^{(-x)}-1\right)}{a+b}$$

input
```
integrate(coth(x)/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
-a*log(2*a*e^(-x) + b*e^(-2*x) + b)/(a^2 - b^2) + log(e^(-x) + 1)/(a - b)
+ log(e^(-x) - 1)/(a + b)
```

3.183.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.24

$$\int \frac{\coth(x)}{a+b\cosh(x)}\,dx=-\frac{ab\log\left(\left|b\left(e^{(-x)}+e^x\right)+2\,a\right|\right)}{a^2b-b^3}+\frac{\log\left(e^{(-x)}+e^x+2\right)}{2\,(a-b)}+\frac{\log\left(e^{(-x)}+e^x-2\right)}{2\,(a+b)}$$

input
```
integrate(coth(x)/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
-a*b*log(abs(b*(e^(-x) + e^x) + 2*a))/(a^2*b - b^3) + 1/2*log(e^(-x) + e^x
 + 2)/(a - b) + 1/2*log(e^(-x) + e^x - 2)/(a + b)
```

3.183.9 Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.74

$$\int \frac{\coth(x)}{a+b\cosh(x)}\,dx = \frac{\ln\left(128\,a\,b-128\,a^2-32\,b^2+128\,a^2\,\mathrm{e}^x+32\,b^2\,\mathrm{e}^x-128\,a\,b\,\mathrm{e}^x\right)}{a+b} + \frac{\ln\left(-128\,a\,b-128\,a^2-32\,b^2-128\,a^2\,\mathrm{e}^x-32\,b^2\,\mathrm{e}^x-128\,a\,b\,\mathrm{e}^x\right)}{a-b} - \frac{a\,\ln\left(16\,a^2\,b-4\,b^3\,\mathrm{e}^{2\,x}-4\,b^3+32\,a^3\,\mathrm{e}^x-8\,a\,b^2\,\mathrm{e}^x+16\,a^2\,b\,\mathrm{e}^{2\,x}\right)}{a^2-b^2}$$

input
```
int(coth(x)/(a + b*cosh(x)),x)
```

output
```
log(128*a*b - 128*a^2 - 32*b^2 + 128*a^2*exp(x) + 32*b^2*exp(x) - 128*a*b*
exp(x))/(a + b) + log(- 128*a*b - 128*a^2 - 32*b^2 - 128*a^2*exp(x) - 32*b
^2*exp(x) - 128*a*b*exp(x))/(a - b) - (a*log(16*a^2*b - 4*b^3*exp(2*x) - 4
*b^3 + 32*a^3*exp(x) - 8*a*b^2*exp(x) + 16*a^2*b*exp(2*x)))/(a^2 - b^2)
```

3.184 $\int \frac{\coth^2(x)}{a+b\cosh(x)}\,dx$

3.184.1 Optimal result

Integrand size = 13, antiderivative size = 77

$$\int \frac{\coth^2(x)}{a+b\cosh(x)}\,dx=\frac{2a^2\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}-\frac{a\coth(x)}{a^2-b^2}+\frac{b\text{csch}(x)}{a^2-b^2}$$

output
```
2*a^2*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(3/2)/(a+b)^(3/2)
-a*coth(x)/(a^2-b^2)+b*csch(x)/(a^2-b^2)
```

3.184.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \frac{\coth^2(x)}{a+b\cosh(x)}\,dx=\frac{2a^2\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}}-\frac{\coth\left(\frac{x}{2}\right)}{2(a+b)}-\frac{\tanh\left(\frac{x}{2}\right)}{2(a-b)}$$

input
```
Integrate[Coth[x]^2/(a + b*Cosh[x]),x]
```

output
```
(2*a^2*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) -
Coth[x/2]/(2*(a + b)) - Tanh[x/2]/(2*(a - b))
```

3.184.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.14, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.923, Rules used = {3042, 25, 3206, 25, 3042, 25, 3086, 24, 3138, 221, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan\left(-\frac{\pi}{2}+ix\right)^2}{a-b\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan\left(ix-\frac{\pi}{2}\right)^2}{a-b\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3206}$$

$$\frac{a^2\int \frac{1}{a+b\cosh(x)}dx}{a^2-b^2}-\frac{a\int -\text{csch}^2(x)dx}{a^2-b^2}-\frac{b\int \coth(x)\text{csch}(x)dx}{a^2-b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{a^2\int \frac{1}{a+b\cosh(x)}dx}{a^2-b^2}+\frac{a\int \text{csch}^2(x)dx}{a^2-b^2}-\frac{b\int \coth(x)\text{csch}(x)dx}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}+\frac{a\int -\csc(ix)^2dx}{a^2-b^2}-\frac{b\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)dx}{a^2-b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{a^2\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{a\int \csc(ix)^2dx}{a^2-b^2}-\frac{b\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)dx}{a^2-b^2}$$

$$\Big\downarrow \text{3086}$$

$$\frac{a^2\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{a\int \csc(ix)^2dx}{a^2-b^2}+\frac{ib\int 1d(-i\text{csch}(x))}{a^2-b^2}$$

$$\Big\downarrow \text{24}$$

$$\frac{a^2 \int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2} - \frac{a\int \csc(ix)^2 dx}{a^2-b^2} + \frac{b\text{csch}(x)}{a^2-b^2}$$

$\big\downarrow$ 3138

$$-\frac{a\int \csc(ix)^2 dx}{a^2-b^2} + \frac{2a^2 \int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a^2-b^2} + \frac{b\text{csch}(x)}{a^2-b^2}$$

$\big\downarrow$ 221

$$-\frac{a\int \csc(ix)^2 dx}{a^2-b^2} + \frac{2a^2\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}\,(a^2-b^2)} + \frac{b\text{csch}(x)}{a^2-b^2}$$

$\big\downarrow$ 4254

$$-\frac{ia\int 1d(-i\coth(x))}{a^2-b^2} + \frac{2a^2\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}\,(a^2-b^2)} + \frac{b\text{csch}(x)}{a^2-b^2}$$

$\big\downarrow$ 24

$$\frac{2a^2\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}\,(a^2-b^2)} - \frac{a\coth(x)}{a^2-b^2} + \frac{b\text{csch}(x)}{a^2-b^2}$$

input
```
Int[Coth[x]^2/(a + b*Cosh[x]),x]
```

output
```
(2*a^2*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a +
 b]*(a^2 - b^2)) - (a*Coth[x])/(a^2 - b^2) + (b*Csch[x])/(a^2 - b^2)
```

3.184.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3206
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[a/(a^2 - b^2)   Int[(g*Tan[e + f*x])^p/Sin[e + f*x]
^2, x], x] + (-Simp[b*(g/(a^2 - b^2))   Int[(g*Tan[e + f*x])^(p - 1)/Cos[e
+ f*x], x], x] - Simp[a^2*(g^2/(a^2 - b^2))   Int[(g*Tan[e + f*x])^(p - 2)/
(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2
, 0] && IntegersQ[2*p] && GtQ[p, 1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.184.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

method	result	size
default	$-\frac{\tanh\left(\frac{x}{2}\right)}{2(a-b)}-\frac{1}{2(a+b)\tanh\left(\frac{x}{2}\right)}+\frac{2a^2\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}$	78
risch	$-\frac{2(-e^x b+a)}{(e^{2x}-1)(a^2-b^2)}+\frac{a^2\ln\left(e^x+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)}-\frac{a^2\ln\left(e^x+\frac{a\sqrt{a^2-b^2}+a^2-b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)}$	167

input
```
int(coth(x)^2/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a-b)*tanh(1/2*x)-1/2/(a+b)/tanh(1/2*x)+2/(a+b)/(a-b)*a^2/((a+b)*(a-b
))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

3.184.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(67) = 134$.

Time = 0.28 (sec) , antiderivative size = 470, normalized size of antiderivative = 6.10

$$\int \frac{\coth^2(x)}{a + b\cosh(x)}\,dx$$
$$= \left[\frac{2\,a^3 - 2\,ab^2 + \left(a^2\cosh\left(x\right)^2 + 2\,a^2\cosh\left(x\right)\sinh\left(x\right) + a^2\sinh\left(x\right)^2 - a^2\right)\sqrt{a^2 - b^2}\log\left(\frac{b^2\cosh(x)^2 + b^2\sinh(x)^2 \ldots}{\ldots}\right.}{a^4 - 2\,a^2b^2 + b^4 - \left(a^4 - 2\,a^2b^2 + b^4\right)\cosh\left(x\right)^2 - 2\left(a^4 \ldots\right.}\right.$$

input
```
integrate(coth(x)^2/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[(2*a^3 - 2*a*b^2 + (a^2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2
 - a^2)*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x)
 + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*cosh
(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh
(x) + a)*sinh(x) + b)) - 2*(a^2*b - b^3)*cosh(x) - 2*(a^2*b - b^3)*sinh(x)
)/(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*
a^2*b^2 + b^4)*cosh(x)*sinh(x) - (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^2), 2*(a^
3 - a*b^2 + (a^2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 - a^2)*
sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2
 - b^2)) - (a^2*b - b^3)*cosh(x) - (a^2*b - b^3)*sinh(x))/(a^4 - 2*a^2*b^2
 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*b^2 + b^4)*cos
h(x)*sinh(x) - (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^2)]
```

3.184.6 Sympy [F]

$$\int \frac{\coth^2(x)}{a + b\cosh(x)}\,dx = \int \frac{\coth^2\left(x\right)}{a + b\cosh\left(x\right)}\,dx$$

input
```
integrate(coth(x)**2/(a+b*cosh(x)),x)
```

output
```
Integral(coth(x)**2/(a + b*cosh(x)), x)
```

3.184.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\coth^2(x)}{a + b\cosh(x)}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate(coth(x)^2/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.184.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int \frac{\coth^2(x)}{a + b\cosh(x)}\, dx = \frac{2\,a^2 \arctan\left(\frac{be^x + a}{\sqrt{-a^2+b^2}}\right)}{(a^2 - b^2)\sqrt{-a^2 + b^2}} + \frac{2\,(be^x - a)}{(a^2 - b^2)(e^{(2\,x)} - 1)}$$

input
```
integrate(coth(x)^2/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
2*a^2*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/((a^2 - b^2)*sqrt(-a^2 + b^2))
+ 2*(b*e^x - a)/((a^2 - b^2)*(e^(2*x) - 1))
```

3.184.9 Mupad [B] (verification not implemented)

Time = 2.11 (sec) , antiderivative size = 337, normalized size of antiderivative = 4.38

$$\int \frac{\coth^2(x)}{a + b\cosh(x)}\, dx = -\frac{\frac{2\,a}{a^2-b^2} - \frac{2\,b\,e^x}{a^2-b^2}}{e^{2\,x} - 1} - \frac{2\,\mathrm{atan}\left(\left(\left(e^x\left(\frac{2\,a^2}{b^2\,(a^2-b^2)^2\,\sqrt{a^4}} + \frac{2\,\left(a^3\,\sqrt{a^4} - a\,b^2\,\sqrt{a^4}\right)}{a\,b^2\,(a^2-b^2)\,\sqrt{-(a^2-b^2)^3}\,\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}}\right) - \frac{2\,\left(b^3\,\sqrt{a^4} - a^2\,b\,\sqrt{a^4}\right)}{a\,b^2\,(a^2-b^2)\,\sqrt{-(a^2-b^2)^3}\,\sqrt{-a^6+3\,a^4\,b}}\right.\right.\right.}{\sqrt{-a^6 + 3\,a^4\,b^2 - 3\,a^2\,b^4 + b^6}}$$

input
```
int(coth(x)^2/(a + b*cosh(x)),x)
```

output
```
- ((2*a)/(a^2 - b^2) - (2*b*exp(x))/(a^2 - b^2))/(exp(2*x) - 1) - (2*atan(
(exp(x)*((2*a^2)/(b^2*(a^2 - b^2)^2*(a^4)^(1/2)) + (2*(a^3*(a^4)^(1/2) - a
*b^2*(a^4)^(1/2)))/(a*b^2*(a^2 - b^2)*(-(a^2 - b^2)^3)^(1/2)*(b^6 - a^6 -
3*a^2*b^4 + 3*a^4*b^2)^(1/2))) - (2*(b^3*(a^4)^(1/2) - a^2*b*(a^4)^(1/2)))
/(a*b^2*(a^2 - b^2)*(-(a^2 - b^2)^3)^(1/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*
b^2)^(1/2)))*((b^3*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/2 - (a^2*b*(
b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/2))*(a^4)^(1/2))/(b^6 - a^6 - 3*
a^2*b^4 + 3*a^4*b^2)^(1/2)
```

3.184. $\int \frac{\coth^2(x)}{a+b\cosh(x)}\,dx$

3.185 $\int \frac{\coth^3(x)}{a+b\cosh(x)}\,dx$

3.185.1 Optimal result

Integrand size = 13, antiderivative size = 94

$$\int \frac{\coth^3(x)}{a+b\cosh(x)}\,dx=-\frac{(a-b\cosh(x))\operatorname{csch}^2(x)}{2(a^2-b^2)}+\frac{(2a+b)\log(1-\cosh(x))}{4(a+b)^2}+\frac{(2a-b)\log(1+\cosh(x))}{4(a-b)^2}-\frac{a^3\log(a+b\cosh(x))}{(a^2-b^2)^2}$$

output
```
-1/2*(a-b*cosh(x))*csch(x)^2/(a^2-b^2)+1/4*(2*a+b)*ln(1-cosh(x))/(a+b)^2+1
/4*(2*a-b)*ln(1+cosh(x))/(a-b)^2-a^3*ln(a+b*cosh(x))/(a^2-b^2)^2
```

3.185.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.07

$$\int \frac{\coth^3(x)}{a+b\cosh(x)}\,dx=\frac{1}{8}\left(-\frac{\operatorname{csch}^2\left(\frac{x}{2}\right)}{a+b}+\frac{4(2a-b)\log\left(\cosh\left(\frac{x}{2}\right)\right)}{(a-b)^2}-\frac{8a^3\log(a+b\cosh(x))}{(a^2-b^2)^2}+\frac{4(2a+b)\log\left(\sinh\left(\frac{x}{2}\right)\right)}{(a+b)^2}+\frac{\operatorname{sech}^2\left(\frac{x}{2}\right)}{a-b}\right)$$

input
```
Integrate[Coth[x]^3/(a + b*Cosh[x]),x]
```

output
```
(-(Csch[x/2]^2/(a + b)) + (4*(2*a - b)*Log[Cosh[x/2]])/(a - b)^2 - (8*a^3*
Log[a + b*Cosh[x]])/(a^2 - b^2)^2 + (4*(2*a + b)*Log[Sinh[x/2]])/(a + b)^2
 + Sech[x/2]^2/(a - b))/8
```

3.185.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.44, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {3042, 26, 3200, 601, 25, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{i\tan\left(-\frac{\pi}{2}+ix\right)^3}{a-b\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \; 26$$

$$i\int \frac{\tan\left(ix-\frac{\pi}{2}\right)^3}{a-b\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 3200$$

$$\int \frac{b^3\cosh^3(x)}{\left(b^2-b^2\cosh^2(x)\right)^2(a+b\cosh(x))}d(b\cosh(x))$$

$$\Big\downarrow \; 601$$

$$\frac{b^2(a-b\cosh(x))}{2\left(a^2-b^2\right)\left(b^2-b^2\cosh^2(x)\right)}-\frac{\int -\frac{b^2\left(\frac{ab^2}{a^2-b^2}-\frac{b\left(2a^2-b^2\right)\cosh(x)}{a^2-b^2}\right)}{(a+b\cosh(x))\left(b^2-b^2\cosh^2(x)\right)}d(b\cosh(x))}{2b^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{b^2\left(ab^2-b\left(2a^2-b^2\right)\cosh(x)\right)}{\left(a^2-b^2\right)(a+b\cosh(x))\left(b^2-b^2\cosh^2(x)\right)}d(b\cosh(x))}{2b^2}+\frac{b^2(a-b\cosh(x))}{2\left(a^2-b^2\right)\left(b^2-b^2\cosh^2(x)\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{ab^2-b(2a^2-b^2)\cosh(x)}{(a+b\cosh(x))(b^2-b^2\cosh^2(x))}d(b\cosh(x))}{2(a^2-b^2)}+\frac{b^2(a-b\cosh(x))}{2(a^2-b^2)(b^2-b^2\cosh^2(x))}$$

$$\Big\downarrow \text{657}$$

$$\frac{\int \left(-\frac{2a^3}{(a-b)(a+b)(a+b\cosh(x))}+\frac{-2a^2+ba+b^2}{2(a+b)(b-b\cosh(x))}+\frac{(2a-b)(a+b)}{2(a-b)(\cosh(x)b+b)}\right)d(b\cosh(x))}{2(a^2-b^2)}+\frac{b^2(a-b\cosh(x))}{2(a^2-b^2)(b^2-b^2\cosh^2(x))}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2(a-b\cosh(x))}{2(a^2-b^2)(b^2-b^2\cosh^2(x))}+\frac{-\frac{2a^3\log(a+b\cosh(x))}{a^2-b^2}+\frac{(a-b)(2a+b)\log(b-b\cosh(x))}{2(a+b)}+\frac{(2a-b)(a+b)\log(b\cosh(x)+b)}{2(a-b)}}{2(a^2-b^2)}$$

input
```
Int[Coth[x]^3/(a + b*Cosh[x]),x]
```

output
```
(b^2*(a - b*Cosh[x]))/(2*(a^2 - b^2)*(b^2 - b^2*Cosh[x]^2)) + (((a - b)*(2
*a + b)*Log[b - b*Cosh[x]])/(2*(a + b)) - (2*a^3*Log[a + b*Cosh[x]])/(a^2
- b^2) + ((2*a - b)*(a + b)*Log[b + b*Cosh[x]])/(2*(a - b)))/(2*(a^2 - b^2
))
```

3.185.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.185.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.97

method	result
default	$-\frac{\tanh\left(\frac{x}{2}\right)^{2}}{8(a-b)}-\frac{1}{8(a+b)\tanh\left(\frac{x}{2}\right)^{2}}+\frac{(4a+2b)\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{4(a+b)^{2}}-\frac{a^{3}\ln\left(\tanh\left(\frac{x}{2}\right)^{2}a-\tanh\left(\frac{x}{2}\right)^{2}b-a-b\right)}{(a+b)^{2}(a-b)^{2}}$
risch	$-\frac{xa}{a^{2}-2ab+b^{2}}+\frac{bx}{2a^{2}-4ab+2b^{2}}-\frac{xa}{a^{2}+2ab+b^{2}}-\frac{xb}{2(a^{2}+2ab+b^{2})}+\frac{2x\,a^{3}}{a^{4}-2a^{2}b^{2}+b^{4}}-\frac{e^{x}\left(-b\,e^{2x}+2a\,e^{x}-b\right)}{\left(e^{2x}-1\right)^{2}(a^{2}-b^{2})}+\frac{\ln(e^{x}+1)a}{a^{2}-2ab+b^{2}}-$

input
```
int(coth(x)^3/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/8*tanh(1/2*x)^2/(a-b)-1/8/(a+b)/tanh(1/2*x)^2+1/4/(a+b)^2*(4*a+2*b)*ln(
tanh(1/2*x))-a^3/(a+b)^2/(a-b)^2*ln(tanh(1/2*x)^2*a-tanh(1/2*x)^2*b-a-b)
```

3.185.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 839 vs. $2(89) = 178$.

Time = 0.28 (sec) , antiderivative size = 839, normalized size of antiderivative = 8.93

$$\int \frac{\coth^3(x)}{a + b\cosh(x)}\, dx = \text{Too large to display}$$

input
```
integrate(coth(x)^3/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
1/2*(2*(a^2*b - b^3)*cosh(x)^3 + 2*(a^2*b - b^3)*sinh(x)^3 - 4*(a^3 - a*b^
2)*cosh(x)^2 - 2*(2*a^3 - 2*a*b^2 - 3*(a^2*b - b^3)*cosh(x))*sinh(x)^2 + 2
*(a^2*b - b^3)*cosh(x) - 2*(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*
sinh(x)^4 - 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh(x)^2 - a^3)*sinh(x)^2 +
4*(a^3*cosh(x)^3 - a^3*cosh(x))*sinh(x))*log(2*(b*cosh(x) + a)/(cosh(x) -
sinh(x))) + ((2*a^3 + 3*a^2*b - b^3)*cosh(x)^4 + 4*(2*a^3 + 3*a^2*b - b^3)
*cosh(x)*sinh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*sinh(x)^4 + 2*a^3 + 3*a^2*b -
 b^3 - 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 - 2*(2*a^3 + 3*a^2*b - b^3 - 3*
(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^3 + 3*a^2*b - b^3)*
cosh(x)^3 - (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x
) + 1) + ((2*a^3 - 3*a^2*b + b^3)*cosh(x)^4 + 4*(2*a^3 - 3*a^2*b + b^3)*co
sh(x)*sinh(x)^3 + (2*a^3 - 3*a^2*b + b^3)*sinh(x)^4 + 2*a^3 - 3*a^2*b + b^
3 - 2*(2*a^3 - 3*a^2*b + b^3)*cosh(x)^2 - 2*(2*a^3 - 3*a^2*b + b^3 - 3*(2*
a^3 - 3*a^2*b + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^3 - 3*a^2*b + b^3)*cos
h(x)^3 - (2*a^3 - 3*a^2*b + b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) -
 1) + 2*(a^2*b - b^3 + 3*(a^2*b - b^3)*cosh(x)^2 - 4*(a^3 - a*b^2)*cosh(x)
)*sinh(x))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 4*(a^4 - 2*a^2*b^2 + b^4)*
cosh(x)*sinh(x)^3 + (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^4 + a^4 - 2*a^2*b^2 +
b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*b^2 + b^4 - 3*(
a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^2 + 4*((a^4 - 2*a^2*b^2 + b^4...
```

3.185.6 Sympy [F]

$$\int \frac{\coth^3(x)}{a+b\cosh(x)}\,dx = \int \frac{\coth^3(x)}{a+b\cosh(x)}\,dx$$

input
```
integrate(coth(x)**3/(a+b*cosh(x)),x)
```

output
```
Integral(coth(x)**3/(a + b*cosh(x)), x)
```

3.185.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.66

$$\begin{aligned}\int \frac{\coth^3(x)}{a+b\cosh(x)}\,dx = &-\frac{a^3\log\left(2\,ae^{(-x)}+be^{(-2\,x)}+b\right)}{a^4-2\,a^2b^2+b^4} \\ &+\frac{(2\,a-b)\log\left(e^{(-x)}+1\right)}{2\,(a^2-2\,ab+b^2)}+\frac{(2\,a+b)\log\left(e^{(-x)}-1\right)}{2\,(a^2+2\,ab+b^2)} \\ &+\frac{be^{(-x)}-2\,ae^{(-2\,x)}+be^{(-3\,x)}}{a^2-b^2-2\,(a^2-b^2)e^{(-2\,x)}+(a^2-b^2)e^{(-4\,x)}}\end{aligned}$$

input
```
integrate(coth(x)^3/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
-a^3*log(2*a*e^(-x) + b*e^(-2*x) + b)/(a^4 - 2*a^2*b^2 + b^4) + 1/2*(2*a -
 b)*log(e^(-x) + 1)/(a^2 - 2*a*b + b^2) + 1/2*(2*a + b)*log(e^(-x) - 1)/(a
^2 + 2*a*b + b^2) + (b*e^(-x) - 2*a*e^(-2*x) + b*e^(-3*x))/(a^2 - b^2 - 2*
(a^2 - b^2)*e^(-2*x) + (a^2 - b^2)*e^(-4*x))
```

3.185.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.89

$$\begin{aligned}\int \frac{\coth^3(x)}{a+b\cosh(x)}\,dx = &-\frac{a^3b\log\left(\left|b\left(e^{(-x)}+e^x\right)+2\,a\right|\right)}{a^4b-2\,a^2b^3+b^5} \\ &+\frac{(2\,a-b)\log\left(e^{(-x)}+e^x+2\right)}{4\,(a^2-2\,ab+b^2)}+\frac{(2\,a+b)\log\left(e^{(-x)}+e^x-2\right)}{4\,(a^2+2\,ab+b^2)} \\ &-\frac{a^3\left(e^{(-x)}+e^x\right)^2-2\,a^2b\left(e^{(-x)}+e^x\right)+2\,b^3\left(e^{(-x)}+e^x\right)-4\,ab^2}{2\,(a^4-2\,a^2b^2+b^4)\left(\left(e^{(-x)}+e^x\right)^2-4\right)}\end{aligned}$$

input
```
integrate(coth(x)^3/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
-a^3*b*log(abs(b*(e^(-x) + e^x) + 2*a))/(a^4*b - 2*a^2*b^3 + b^5) + 1/4*(2
*a - b)*log(e^(-x) + e^x + 2)/(a^2 - 2*a*b + b^2) + 1/4*(2*a + b)*log(e^(-
x) + e^x - 2)/(a^2 + 2*a*b + b^2) - 1/2*(a^3*(e^(-x) + e^x)^2 - 2*a^2*b*(e
^(-x) + e^x) + 2*b^3*(e^(-x) + e^x) - 4*a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*((
e^(-x) + e^x)^2 - 4))
```

3.185.9 Mupad [B] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 291, normalized size of antiderivative = 3.10

$$\int \frac{\coth^3(x)}{a+b\cosh(x)}\,dx = \frac{\frac{2\,(a\,b^2-a^3)}{(a^2-b^2)^2}+\frac{e^x\,(a^2\,b-b^3)}{(a^2-b^2)^2}}{e^{2\,x}-1} - \frac{\frac{2\,a}{a^2-b^2}-\frac{2\,b\,e^x}{a^2-b^2}}{e^{4\,x}-2\,e^{2\,x}+1} + \frac{\ln\left(e^x+1\right)\,\left(2\,a-b\right)}{2\,a^2-4\,a\,b+2\,b^2} - \frac{a^3\,\ln\left(b^7\,e^{2\,x}-16\,a^6\,b+b^7-6\,a^2\,b^5+9\,a^4\,b^3-32\,a^7\,e^x-6\,a^2\,b^5\,e^{2\,x}+9\,a^4\,b^3\,e^{2\,x}+2\,a\,b^6\,e^x-16\,a^6\,b\,e^{2\,x}\right.}{a^4-2\,a^2\,b^2+b^4} + \frac{\ln\left(e^x-1\right)\,\left(2\,a+b\right)}{2\,a^2+4\,a\,b+2\,b^2}$$

input
```
int(coth(x)^3/(a + b*cosh(x)),x)
```

output
```
((2*(a*b^2 - a^3))/(a^2 - b^2)^2 + (exp(x)*(a^2*b - b^3))/(a^2 - b^2)^2)/(
exp(2*x) - 1) - ((2*a)/(a^2 - b^2) - (2*b*exp(x))/(a^2 - b^2))/(exp(4*x) -
 2*exp(2*x) + 1) + (log(exp(x) + 1)*(2*a - b))/(2*a^2 - 4*a*b + 2*b^2) - (
a^3*log(b^7*exp(2*x) - 16*a^6*b + b^7 - 6*a^2*b^5 + 9*a^4*b^3 - 32*a^7*exp
(x) - 6*a^2*b^5*exp(2*x) + 9*a^4*b^3*exp(2*x) + 2*a*b^6*exp(x) - 16*a^6*b*
exp(2*x) - 12*a^3*b^4*exp(x) + 18*a^5*b^2*exp(x)))/(a^4 + b^4 - 2*a^2*b^2)
 + (log(exp(x) - 1)*(2*a + b))/(4*a*b + 2*a^2 + 2*b^2)
```

3.186 $\int \frac{\coth^4(x)}{a+b\cosh(x)}\,dx$

3.186.1 Optimal result

Integrand size = 13, antiderivative size = 137

$$\int \frac{\coth^4(x)}{a+b\cosh(x)}\,dx = \frac{2a^4\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}} - \frac{a^3\coth(x)}{(a^2-b^2)^2} - \frac{a\coth^3(x)}{3(a^2-b^2)} + \frac{a^2b\operatorname{csch}(x)}{(a^2-b^2)^2} + \frac{b\operatorname{csch}(x)}{a^2-b^2} + \frac{b\operatorname{csch}^3(x)}{3(a^2-b^2)}$$

output
```
2*a^4*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(5/2)/(a+b)^(5/2)
-a^3*coth(x)/(a^2-b^2)^2-1/3*a*coth(x)^3/(a^2-b^2)+a^2*b*csch(x)/(a^2-b^2)
^2+b*csch(x)/(a^2-b^2)+1/3*b*csch(x)^3/(a^2-b^2)
```

3.186.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96

$$\int \frac{\coth^4(x)}{a+b\cosh(x)}\,dx = \frac{1}{24}\left(-\frac{48a^4\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}} - \frac{2(8a+5b)\coth\left(\frac{x}{2}\right)}{(a+b)^2} + \frac{8\operatorname{csch}^3(x)\sinh^4\left(\frac{x}{2}\right)}{a-b} - \frac{\operatorname{csch}^4\left(\frac{x}{2}\right)\sinh(x)}{2(a+b)} + \frac{2(-8a+5b)\tanh\left(\frac{x}{2}\right)}{(a-b)^2}\right)$$

input
```
Integrate[Coth[x]^4/(a + b*Cosh[x]),x]
```

output
```
((-48*a^4*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2)
 - (2*(8*a + 5*b)*Coth[x/2])/(a + b)^2 + (8*Csch[x]^3*Sinh[x/2]^4)/(a - b)
 - (Csch[x/2]^4*Sinh[x])/(2*(a + b)) + (2*(-8*a + 5*b)*Tanh[x/2])/(a - b)^
2)/24
```

3.186.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.15, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.462, Rules used = {3042, 3206, 25, 3042, 25, 3086, 2009, 3087, 15, 3206, 25, 3042, 25, 3086, 24, 3138, 221, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^4(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan\left(-\frac{\pi}{2}+ix\right)^4}{a-b\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3206}$$

$$-\frac{a^2\int -\frac{\coth^2(x)}{a+b\cosh(x)}dx}{a^2-b^2}+\frac{b\int -\coth^3(x)\text{csch}(x)dx}{a^2-b^2}+\frac{a\int \coth^2(x)\text{csch}^2(x)dx}{a^2-b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{a^2\int \frac{\coth^2(x)}{a+b\cosh(x)}dx}{a^2-b^2}-\frac{b\int \coth^3(x)\text{csch}(x)dx}{a^2-b^2}+\frac{a\int \coth^2(x)\text{csch}^2(x)dx}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\int -\frac{\tan\left(ix-\frac{\pi}{2}\right)^2}{a-b\sin\left(ix-\frac{\pi}{2}\right)}dx}{a^2-b^2}+\frac{a\int \sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a^2-b^2}-\frac{b\int -\sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^3dx}{a^2-b^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^2\int \frac{\tan\left(ix-\frac{\pi}{2}\right)^2}{a-b\sin\left(ix-\frac{\pi}{2}\right)}dx}{a^2-b^2}+\frac{a\int \sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a^2-b^2}+\frac{b\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^3dx}{a^2-b^2}$$

$$\Big\downarrow \text{3086}$$

$$-\frac{ib\int\left(-\text{csch}^2(x)-1\right)d(-i\text{csch}(x))}{a^2-b^2}-\frac{a^2\int \frac{\tan\left(ix-\frac{\pi}{2}\right)^2}{a-b\sin\left(ix-\frac{\pi}{2}\right)}dx}{a^2-b^2}+\frac{a\int \sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a^2-b^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2\int \frac{\tan\left(ix-\frac{\pi}{2}\right)^2}{a-b\sin\left(ix-\frac{\pi}{2}\right)}dx}{a^2-b^2}+\frac{a\int \sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a^2-b^2}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{3087}$$

$$-\frac{ia\int -\coth^2(x)d(i\coth(x))}{a^2-b^2}-\frac{a^2\int \frac{\tan\left(ix-\frac{\pi}{2}\right)^2}{a-b\sin\left(ix-\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{15}$$

$$-\frac{a^2\int \frac{\tan\left(ix-\frac{\pi}{2}\right)^2}{a-b\sin\left(ix-\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{a\coth^3(x)}{3\left(a^2-b^2\right)}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{3206}$$

$$-\frac{a^2\left(-\frac{a^2\int\frac{1}{a+b\cosh(x)}dx}{a^2-b^2}+\frac{a\int -\text{csch}^2(x)dx}{a^2-b^2}+\frac{b\int \coth(x)\text{csch}(x)dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\coth^3(x)}{3\left(a^2-b^2\right)}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^2\left(-\frac{a^2\int\frac{1}{a+b\cosh(x)}dx}{a^2-b^2}-\frac{a\int \text{csch}^2(x)dx}{a^2-b^2}+\frac{b\int \coth(x)\text{csch}(x)dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\coth^3(x)}{3\left(a^2-b^2\right)}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a^2\left(-\frac{a^2\int\frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{a\int -\csc(ix)^2dx}{a^2-b^2}+\frac{b\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\coth^3(x)}{3\left(a^2-b^2\right)}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{25}$$

3.186. $\int \frac{\coth^4(x)}{a+b\cosh(x)}\,dx$

$$-\frac{a^2\left(-\frac{a^2\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}+\frac{a\int \csc(ix)^2dx}{a^2-b^2}+\frac{b\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\coth^3(x)}{3(a^2-b^2)}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{ 3086}$$

$$-\frac{a^2\left(-\frac{a^2\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}+\frac{a\int \csc(ix)^2dx}{a^2-b^2}-\frac{ib\int 1d(-i\text{csch}(x))}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\coth^3(x)}{3(a^2-b^2)}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{ 24}$$

$$-\frac{a^2\left(-\frac{a^2\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}+\frac{a\int \csc(ix)^2dx}{a^2-b^2}-\frac{b\text{csch}(x)}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\coth^3(x)}{3(a^2-b^2)}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{ 3138}$$

$$-\frac{a^2\left(\frac{a\int \csc(ix)^2dx}{a^2-b^2}-\frac{2a^2\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a^2-b^2}-\frac{b\text{csch}(x)}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\coth^3(x)}{3(a^2-b^2)}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{a^2\left(\frac{a\int \csc(ix)^2dx}{a^2-b^2}-\frac{2a^2\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}(a^2-b^2)}-\frac{b\text{csch}(x)}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\coth^3(x)}{3(a^2-b^2)}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{ 4254}$$

$$-\frac{a^2\left(\frac{ia\int 1d(-i\coth(x))}{a^2-b^2}-\frac{2a^2\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}(a^2-b^2)}-\frac{b\text{csch}(x)}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\coth^3(x)}{3(a^2-b^2)}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{ 24}$$

3.186. $\int \frac{\coth^4(x)}{a+b\cosh(x)}\,dx$

$$-\frac{a^2\left(-\frac{2a^2\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}(a^2-b^2)}+\frac{a\coth(x)}{a^2-b^2}-\frac{b\text{csch}(x)}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\coth^3(x)}{3\left(a^2-b^2\right)}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

input
```
Int[Coth[x]^4/(a + b*Cosh[x]),x]
```

output
```
-1/3*(a*Coth[x]^3)/(a^2 - b^2) - (a^2*((-2*a^2*ArcTanh[(Sqrt[a - b]*Tanh[x
/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]*(a^2 - b^2)) + (a*Coth[x])/(a^
2 - b^2) - (b*Csch[x])/(a^2 - b^2)))/(a^2 - b^2) - (I*b*(I*Csch[x] + (I/3)
*Csch[x]^3))/(a^2 - b^2)
```

3.186.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3206
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[a/(a^2 - b^2)   Int[(g*Tan[e + f*x])^p/Sin[e + f*x]
^2, x], x] + (-Simp[b*(g/(a^2 - b^2))   Int[(g*Tan[e + f*x])^(p - 1)/Cos[e
+ f*x], x], x] - Simp[a^2*(g^2/(a^2 - b^2))   Int[(g*Tan[e + f*x])^(p - 2)/
(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2
, 0] && IntegersQ[2*p] && GtQ[p, 1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.186.4 Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.93

method	result
default	$-\frac{\frac{a \tanh\left(\frac{x}{2}\right)^{3}}{3}-\frac{b \tanh\left(\frac{x}{2}\right)^{3}}{3}+5 a \tanh\left(\frac{x}{2}\right)-3 b \tanh\left(\frac{x}{2}\right)}{8 (a-b)^{2}}-\frac{1}{24 (a+b) \tanh\left(\frac{x}{2}\right)^{3}}-\frac{5 a+3 b}{8 (a+b)^{2} \tanh\left(\frac{x}{2}\right)}+\frac{2 a^{4} \operatorname{arctanh}\left(\frac{(a-b) \tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right.}{(a-b)^{2} (a+b)^{2} \sqrt{(a+b)(a-b}}$
risch	$-\frac{2\left(-6 a^{2} b\,\mathrm{e}^{5 x}+3 b^{3} \mathrm{e}^{5 x}+6 a^{3} \mathrm{e}^{4 x}-3 a\,b^{2} \mathrm{e}^{4 x}+8 a^{2} b\,\mathrm{e}^{3 x}-2 b^{3} \mathrm{e}^{3 x}-6 a^{3} \mathrm{e}^{2 x}-6 a^{2} b\,\mathrm{e}^{x}+3 b^{3} \mathrm{e}^{x}+4 a^{3}-a\,b^{2}\right)}{3 (a^{4}-2 a^{2} b^{2}+b^{4}) (\mathrm{e}^{2 x}-1)^{3}}+\frac{a^{4} \ln\left(\mathrm{e}^{x}+\frac{a \sqrt{a^{2}-b^{2}}-a^{2}+b^{2}}{b \sqrt{a^{2}-b^{2}}}\right.}{\sqrt{a^{2}-b^{2}}\, (a+b)^{2} (a-b)^{2}}$

input
```
int(coth(x)^4/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/8/(a-b)^2*(1/3*a*tanh(1/2*x)^3-1/3*b*tanh(1/2*x)^3+5*a*tanh(1/2*x)-3*b*
tanh(1/2*x))-1/24/(a+b)/tanh(1/2*x)^3-1/8*(5*a+3*b)/(a+b)^2/tanh(1/2*x)+2/
(a-b)^2/(a+b)^2*a^4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(
a-b))^(1/2))
```

3.186.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1174 vs. $2(123) = 246$.

Time = 0.27 (sec) , antiderivative size = 2417, normalized size of antiderivative = 17.64

$$\int \frac{\coth^4(x)}{a+b\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^4/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[1/3*(6*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x)^5 + 6*(2*a^4*b - 3*a^2*b^3 + b
^5)*sinh(x)^5 - 8*a^5 + 10*a^3*b^2 - 2*a*b^4 - 6*(2*a^5 - 3*a^3*b^2 + a*b^
4)*cosh(x)^4 - 6*(2*a^5 - 3*a^3*b^2 + a*b^4 - 5*(2*a^4*b - 3*a^2*b^3 + b^5
)*cosh(x))*sinh(x)^4 - 4*(4*a^4*b - 5*a^2*b^3 + b^5)*cosh(x)^3 - 4*(4*a^4*
b - 5*a^2*b^3 + b^5 - 15*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x)^2 + 6*(2*a^5
- 3*a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^3 + 12*(a^5 - a^3*b^2)*cosh(x)^2 + 1
2*(a^5 - a^3*b^2 + 5*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x)^3 - 3*(2*a^5 - 3*
a^3*b^2 + a*b^4)*cosh(x)^2 - (4*a^4*b - 5*a^2*b^3 + b^5)*cosh(x))*sinh(x)^
2 + 3*(a^4*cosh(x)^6 + 6*a^4*cosh(x)*sinh(x)^5 + a^4*sinh(x)^6 - 3*a^4*cos
h(x)^4 + 3*a^4*cosh(x)^2 + 3*(5*a^4*cosh(x)^2 - a^4)*sinh(x)^4 - a^4 + 4*(
5*a^4*cosh(x)^3 - 3*a^4*cosh(x))*sinh(x)^3 + 3*(5*a^4*cosh(x)^4 - 6*a^4*co
sh(x)^2 + a^4)*sinh(x)^2 + 6*(a^4*cosh(x)^5 - 2*a^4*cosh(x)^3 + a^4*cosh(x
))*sinh(x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cos
h(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*
cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*
cosh(x) + a)*sinh(x) + b)) + 6*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x) + 6*(2*
a^4*b - 3*a^2*b^3 + b^5 + 5*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x)^4 - 4*(2*a
^5 - 3*a^3*b^2 + a*b^4)*cosh(x)^3 - 2*(4*a^4*b - 5*a^2*b^3 + b^5)*cosh(x)^
2 + 4*(a^5 - a^3*b^2)*cosh(x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^
6)*cosh(x)^6 + 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^5 ...
```

3.186.6 Sympy [F]

$$\int \frac{\coth^4(x)}{a+b\cosh(x)}\,dx = \int \frac{\coth^4{(x)}}{a+b\cosh{(x)}}\,dx$$

input
```
integrate(coth(x)**4/(a+b*cosh(x)),x)
```

output
```
Integral(coth(x)**4/(a + b*cosh(x)), x)
```

3.186.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\coth^4(x)}{a+b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(coth(x)^4/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.186.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.26

$$\int \frac{\coth^4(x)}{a+b\cosh(x)}\,dx = \frac{2\,a^4\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{(a^4-2\,a^2b^2+b^4)\sqrt{-a^2+b^2}} + \frac{2\,(6\,a^2be^{(5\,x)}-3\,b^3e^{(5\,x)}-6\,a^3e^{(4\,x)}+3\,ab^2e^{(4\,x)}-8\,a^2be^{(3\,x)}+2\,b^3e^{(3\,x)}+6\,a^3e^{(2\,x)}+6\,a^2be^x-3\,b^3e^x-}{3\,(a^4-2\,a^2b^2+b^4)(e^{(2\,x)}-1)^3}$$

input
```
integrate(coth(x)^4/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
2*a^4*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(-
a^2 + b^2)) + 2/3*(6*a^2*b*e^(5*x) - 3*b^3*e^(5*x) - 6*a^3*e^(4*x) + 3*a*b
^2*e^(4*x) - 8*a^2*b*e^(3*x) + 2*b^3*e^(3*x) + 6*a^3*e^(2*x) + 6*a^2*b*e^x
 - 3*b^3*e^x - 4*a^3 + a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*(e^(2*x) - 1)^3)
```

3.186.9 Mupad [B] (verification not implemented)

Time = 2.55 (sec) , antiderivative size = 666, normalized size of antiderivative = 4.86

$$\int \frac{\coth^4(x)}{a+b\cosh(x)}\,dx = \frac{\frac{4\,(a\,b^2-a^3)}{(a^2-b^2)^2}+\frac{8\,\mathrm{e}^x\,(a^2\,b-b^3)}{3\,(a^2-b^2)^2}}{\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1}-\frac{\frac{8\,a}{3\,(a^2-b^2)}-\frac{8\,b\,\mathrm{e}^x}{3\,(a^2-b^2)}}{3\,\mathrm{e}^{2\,x}-3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}-1}-\frac{\frac{2\,a\,(2\,a^2-b^2)}{(a^2-b^2)^2}-\frac{2\,b\,\mathrm{e}^x\,(2\,a^2-b^2)}{(a^2-b^2)^2}}{\mathrm{e}^{2\,x}-1} + \frac{2\,\mathrm{atan}\left(\left(\mathrm{e}^x\left(\frac{2\,a^4}{b^2\,(a^2-b^2)^2\,\sqrt{a^8}\,(a^4-2\,a^2\,b^2+b^4)}+\frac{2\left(a^5\,\sqrt{a^8}-2\,a^3\,b^2\,\sqrt{a^8}+a\,b^4\,\sqrt{a^8}\right)}{a^3\,b^2\,\sqrt{-(a^2-b^2)^5}\,(a^4-2\,a^2\,b^2+b^4)\,\sqrt{-a^{10}+5\,a^8\,b^2-10\,a^6\,b^4+10\,a^4\,b^6-5\,a^2\,b^8+b^{10}}}\right.\right.\right.}{}$$

input
```
int(coth(x)^4/(a + b*cosh(x)),x)
```

output
```
((4*(a*b^2 - a^3))/(a^2 - b^2)^2 + (8*exp(x)*(a^2*b - b^3))/(3*(a^2 - b^2)
^2))/(exp(4*x) - 2*exp(2*x) + 1) - ((8*a)/(3*(a^2 - b^2)) - (8*b*exp(x))/(
3*(a^2 - b^2)))/(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1) - ((2*a*(2*a^2 -
b^2))/(a^2 - b^2)^2 - (2*b*exp(x)*(2*a^2 - b^2))/(a^2 - b^2)^2)/(exp(2*x)
- 1) + (2*atan((exp(x)*((2*a^4)/(b^2*(a^2 - b^2)^2*(a^8)^(1/2)*(a^4 + b^4
- 2*a^2*b^2)) + (2*(a^5*(a^8)^(1/2) - 2*a^3*b^2*(a^8)^(1/2) + a*b^4*(a^8)^
(1/2)))/(a^3*b^2*(-(a^2 - b^2)^5)^(1/2)*(a^4 + b^4 - 2*a^2*b^2)*(b^10 - a^
10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2))) + (2*(b^5*(a
^8)^(1/2) - 2*a^2*b^3*(a^8)^(1/2) + a^4*b*(a^8)^(1/2)))/(a^3*b^2*(-(a^2 -
b^2)^5)^(1/2)*(a^4 + b^4 - 2*a^2*b^2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^
6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)))*((b^5*(b^10 - a^10 - 5*a^2*b^8 + 10*a^
4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2))/2 - a^2*b^3*(b^10 - a^10 - 5*a^2*b^
8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) + (a^4*b*(b^10 - a^10 - 5*a
^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2))/2))*(a^8)^(1/2))/(b^1
0 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)
```

3.187 $\int \frac{\tanh^6(x)}{a+a\cosh(x)}\,dx$

3.187.1 Optimal result

Integrand size = 13, antiderivative size = 46

$$\int \frac{\tanh^6(x)}{a+a\cosh(x)}\,dx=\frac{3\arctan(\sinh(x))}{8a}-\frac{3\text{sech}(x)\tanh(x)}{8a}-\frac{\text{sech}(x)\tanh^3(x)}{4a}-\frac{\tanh^5(x)}{5a}$$

output
```
3/8*arctan(sinh(x))/a-3/8*sech(x)*tanh(x)/a-1/4*sech(x)*tanh(x)^3/a-1/5*ta
nh(x)^5/a
```

3.187.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \frac{\tanh^6(x)}{a+a\cosh(x)}\,dx =\frac{\cosh^2\left(\frac{x}{2}\right)\left(30\arctan\left(\tanh\left(\frac{x}{2}\right)\right)+\left(-8-25\text{sech}(x)+16\text{sech}^2(x)+10\text{sech}^3(x)-8\text{sech}^4(x)\right)\tanh(x)\right)}{20a(1+\cosh(x))}$$

input
```
Integrate[Tanh[x]^6/(a + a*Cosh[x]),x]
```

output
```
(Cosh[x/2]^2*(30*ArcTan[Tanh[x/2]] + (-8 - 25*Sech[x] + 16*Sech[x]^2 + 10*
Sech[x]^3 - 8*Sech[x]^4)*Tanh[x]))/(20*a*(1 + Cosh[x]))
```

3.187.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.077, Rules used = {3042, 25, 3185, 25, 3042, 3087, 15, 3091, 25, 3042, 25, 3091, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^6(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\tan\left(-\frac{\pi}{2}+ix\right)^6\left(a-a\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\left(a-a\sin\left(ix-\frac{\pi}{2}\right)\right)\tan\left(ix-\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \text{3185}$$

$$-\frac{\int \text{sech}^2(x)\tanh^4(x)dx}{a}-\frac{\int -\text{sech}(x)\tanh^4(x)dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \text{sech}(x)\tanh^4(x)dx}{a}-\frac{\int \text{sech}^2(x)\tanh^4(x)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sec(ix)\tan(ix)^4dx}{a}-\frac{\int \sec(ix)^2\tan(ix)^4dx}{a}$$

$$\Big\downarrow \text{3087}$$

$$\frac{i\int \tanh^4(x)d(i\tanh(x))}{a}+\frac{\int \sec(ix)\tan(ix)^4dx}{a}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\tanh^5(x)}{5a}+\frac{\int \sec(ix)\tan(ix)^4dx}{a}$$

$$\Big\downarrow \text{3091}$$

$$\frac{-\frac{3}{4}\int -\text{sech}(x)\tanh^2(x)dx-\frac{1}{4}\tanh^3(x)\text{sech}(x)}{a}-\frac{\tanh^5(x)}{5a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{3}{4}\int \text{sech}(x)\tanh^2(x)dx - \frac{1}{4}\tanh^3(x)\text{sech}(x)}{a} - \frac{\tanh^5(x)}{5a}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{\tanh^5(x)}{5a} + \frac{-\frac{1}{4}\tanh^3(x)\text{sech}(x) + \frac{3}{4}\int -\sec(ix)\tan(ix)^2dx}{a}$$

$$\Big\downarrow \textbf{25}$$

$$-\frac{\tanh^5(x)}{5a} + \frac{-\frac{1}{4}\tanh^3(x)\text{sech}(x) - \frac{3}{4}\int \sec(ix)\tan(ix)^2dx}{a}$$

$$\Big\downarrow \textbf{3091}$$

$$\frac{-\frac{3}{4}\left(\frac{1}{2}\tanh(x)\text{sech}(x) - \frac{\int \text{sech}(x)dx}{2}\right) - \frac{1}{4}\tanh^3(x)\text{sech}(x)}{a} - \frac{\tanh^5(x)}{5a}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{\tanh^5(x)}{5a} + \frac{-\frac{1}{4}\tanh^3(x)\text{sech}(x) - \frac{3}{4}\left(\frac{1}{2}\tanh(x)\text{sech}(x) - \frac{1}{2}\int \csc\left(ix + \frac{\pi}{2}\right)dx\right)}{a}$$

$$\Big\downarrow \textbf{4257}$$

$$\frac{-\frac{3}{4}\left(\frac{1}{2}\tanh(x)\text{sech}(x) - \frac{1}{2}\arctan(\sinh(x))\right) - \frac{1}{4}\tanh^3(x)\text{sech}(x)}{a} - \frac{\tanh^5(x)}{5a}$$

input
```
Int[Tanh[x]^6/(a + a*Cosh[x]),x]
```

output
```
-1/5*Tanh[x]^5/a + (-1/4*(Sech[x]*Tanh[x]^3) - (3*(-1/2*ArcTan[Sinh[x]] +
(Sech[x]*Tanh[x])/2))/4)/a
```

3.187.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.187.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.39

method	result	size
default	$\frac{\frac{64\left(\frac{3\tanh\left(\frac{x}{2}\right)^{9}}{256}+\frac{7\tanh\left(\frac{x}{2}\right)^{7}}{128}-\frac{\tanh\left(\frac{x}{2}\right)^{5}}{10}-\frac{7\tanh\left(\frac{x}{2}\right)^{3}}{128}-\frac{3\tanh\left(\frac{x}{2}\right)}{256}\right)}{\left(1+\tanh\left(\frac{x}{2}\right)^{2}\right)^{5}}+\frac{3\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{4}}{a}$	64
risch	$-\frac{25\,e^{9x}-40\,e^{8x}+10\,e^{7x}-80\,e^{4x}-10\,e^{3x}-25\,e^{x}-8}{20(1+e^{2x})^{5}a}+\frac{3i\ln(e^{x}+i)}{8a}-\frac{3i\ln(e^{x}-i)}{8a}$	75

input
```
int(tanh(x)^6/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
64/a*((3/256*tanh(1/2*x)^9+7/128*tanh(1/2*x)^7-1/10*tanh(1/2*x)^5-7/128*ta
nh(1/2*x)^3-3/256*tanh(1/2*x))/(1+tanh(1/2*x)^2)^5+3/256*arctan(tanh(1/2*x
)))
```

3.187.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 750 vs. $2(38) = 76$.

Time = 0.25 (sec) , antiderivative size = 750, normalized size of antiderivative = 16.30

$$\int \frac{\tanh^6(x)}{a + a\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^6/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
-1/20*(25*cosh(x)^9 + 5*(45*cosh(x) - 8)*sinh(x)^8 + 25*sinh(x)^9 - 40*cos
h(x)^8 + 10*(90*cosh(x)^2 - 32*cosh(x) + 1)*sinh(x)^7 + 10*cosh(x)^7 + 70*
(30*cosh(x)^3 - 16*cosh(x)^2 + cosh(x))*sinh(x)^6 + 70*(45*cosh(x)^4 - 32*
cosh(x)^3 + 3*cosh(x)^2)*sinh(x)^5 + 10*(315*cosh(x)^5 - 280*cosh(x)^4 + 3
5*cosh(x)^3 - 8)*sinh(x)^4 - 80*cosh(x)^4 + 10*(210*cosh(x)^6 - 224*cosh(x
)^5 + 35*cosh(x)^4 - 32*cosh(x) - 1)*sinh(x)^3 - 10*cosh(x)^3 + 10*(90*cos
h(x)^7 - 112*cosh(x)^6 + 21*cosh(x)^5 - 48*cosh(x)^2 - 3*cosh(x))*sinh(x)^
2 - 15*(cosh(x)^10 + 10*cosh(x)*sinh(x)^9 + sinh(x)^10 + 5*(9*cosh(x)^2 +
1)*sinh(x)^8 + 5*cosh(x)^8 + 40*(3*cosh(x)^3 + cosh(x))*sinh(x)^7 + 10*(21
*cosh(x)^4 + 14*cosh(x)^2 + 1)*sinh(x)^6 + 10*cosh(x)^6 + 4*(63*cosh(x)^5
+ 70*cosh(x)^3 + 15*cosh(x))*sinh(x)^5 + 10*(21*cosh(x)^6 + 35*cosh(x)^4 +
 15*cosh(x)^2 + 1)*sinh(x)^4 + 10*cosh(x)^4 + 40*(3*cosh(x)^7 + 7*cosh(x)^
5 + 5*cosh(x)^3 + cosh(x))*sinh(x)^3 + 5*(9*cosh(x)^8 + 28*cosh(x)^6 + 30*
cosh(x)^4 + 12*cosh(x)^2 + 1)*sinh(x)^2 + 5*cosh(x)^2 + 10*(cosh(x)^9 + 4*
cosh(x)^7 + 6*cosh(x)^5 + 4*cosh(x)^3 + cosh(x))*sinh(x) + 1)*arctan(cosh(
x) + sinh(x)) + 5*(45*cosh(x)^8 - 64*cosh(x)^7 + 14*cosh(x)^6 - 64*cosh(x)
^3 - 6*cosh(x)^2 - 5)*sinh(x) - 25*cosh(x) - 8)/(a*cosh(x)^10 + 10*a*cosh(
x)*sinh(x)^9 + a*sinh(x)^10 + 5*a*cosh(x)^8 + 5*(9*a*cosh(x)^2 + a)*sinh(x
)^8 + 40*(3*a*cosh(x)^3 + a*cosh(x))*sinh(x)^7 + 10*a*cosh(x)^6 + 10*(21*a
*cosh(x)^4 + 14*a*cosh(x)^2 + a)*sinh(x)^6 + 4*(63*a*cosh(x)^5 + 70*a*c...
```

3.187.6 Sympy [F]

$$\int \frac{\tanh^6(x)}{a + a\cosh(x)}\,dx = \frac{\int \frac{\tanh^6(x)}{\cosh(x)+1}\,dx}{a}$$

input
```
integrate(tanh(x)**6/(a+a*cosh(x)),x)
```

output
```
Integral(tanh(x)**6/(cosh(x) + 1), x)/a
```

3.187.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(38) = 76$.

Time = 0.29 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.93

$$\int \frac{\tanh^6(x)}{a + a\cosh(x)}\,dx = -\frac{25\,e^{(-x)} + 10\,e^{(-3\,x)} + 80\,e^{(-4\,x)} - 10\,e^{(-7\,x)} + 40\,e^{(-8\,x)} - 25\,e^{(-9\,x)} + 8}{20\,(5\,ae^{(-2\,x)} + 10\,ae^{(-4\,x)} + 10\,ae^{(-6\,x)} + 5\,ae^{(-8\,x)} + ae^{(-10\,x)} + a)} - \frac{3\,\arctan\left(e^{(-x)}\right)}{4\,a}$$

input
```
integrate(tanh(x)^6/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-1/20*(25*e^(-x) + 10*e^(-3*x) + 80*e^(-4*x) - 10*e^(-7*x) + 40*e^(-8*x) -
 25*e^(-9*x) + 8)/(5*a*e^(-2*x) + 10*a*e^(-4*x) + 10*a*e^(-6*x) + 5*a*e^(-
8*x) + a*e^(-10*x) + a) - 3/4*arctan(e^(-x))/a
```

3.187.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \frac{\tanh^6(x)}{a + a\cosh(x)}\,dx = \frac{3\,\arctan\left(e^{x}\right)}{4\,a} - \frac{25\,e^{(9\,x)} - 40\,e^{(8\,x)} + 10\,e^{(7\,x)} - 80\,e^{(4\,x)} - 10\,e^{(3\,x)} - 25\,e^{x} - 8}{20\,a(e^{(2\,x)} + 1)^5}$$

input
```
integrate(tanh(x)^6/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
3/4*arctan(e^x)/a - 1/20*(25*e^(9*x) - 40*e^(8*x) + 10*e^(7*x) - 80*e^(4*x
) - 10*e^(3*x) - 25*e^x - 8)/(a*(e^(2*x) + 1)^5)
```

3.187.9 Mupad [B] (verification not implemented)

Time = 1.77 (sec) , antiderivative size = 183, normalized size of antiderivative = 3.98

$$\int \frac{\tanh^6(x)}{a + a\cosh(x)}\,dx = \frac{\frac{16}{a} - \frac{6\,e^x}{a}}{3\,e^{2\,x} + 3\,e^{4\,x} + e^{6\,x} + 1} - \frac{\frac{8}{a} - \frac{9\,e^x}{2\,a}}{2\,e^{2\,x} + e^{4\,x} + 1} + \frac{32}{5\,a\,\left(5\,e^{2\,x} + 10\,e^{4\,x} + 10\,e^{6\,x} + 5\,e^{8\,x} + e^{10\,x} + 1\right)} - \frac{\frac{16}{a} - \frac{4\,e^x}{a}}{4\,e^{2\,x} + 6\,e^{4\,x} + 4\,e^{6\,x} + e^{8\,x} + 1} + \frac{\frac{2}{a} - \frac{5\,e^x}{4\,a}}{e^{2\,x} + 1} + \frac{3\,\mathrm{atan}\left(\frac{e^x\,\sqrt{a^2}}{a}\right)}{4\,\sqrt{a^2}}$$

input

```
int(tanh(x)^6/(a + a*cosh(x)),x)
```

output

```
(16/a - (6*exp(x))/a)/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - (8/a - (9
*exp(x))/(2*a))/(2*exp(2*x) + exp(4*x) + 1) + 32/(5*a*(5*exp(2*x) + 10*exp
(4*x) + 10*exp(6*x) + 5*exp(8*x) + exp(10*x) + 1)) - (16/a - (4*exp(x))/a)
/(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1) + (2/a - (5*exp(x))
/(4*a))/(exp(2*x) + 1) + (3*atan((exp(x)*(a^2)^(1/2))/a))/(4*(a^2)^(1/2))
```

3.188 $\int \frac{\tanh^5(x)}{a+a\cosh(x)}\,dx$

3.188.1 Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{\tanh^5(x)}{a+a\cosh(x)}\,dx=-\frac{\operatorname{sech}(x)}{a}+\frac{\operatorname{sech}^3(x)}{3a}-\frac{\tanh^4(x)}{4a}$$

output
```
-sech(x)/a+1/3*sech(x)^3/a-1/4*tanh(x)^4/a
```

3.188.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{\tanh^5(x)}{a+a\cosh(x)}\,dx=\frac{2(3+5\cosh(x))\operatorname{sech}^4(x)\sinh^6\left(\frac{x}{2}\right)}{3a}$$

input
```
Integrate[Tanh[x]^5/(a + a*Cosh[x]),x]
```

output
```
(2*(3 + 5*Cosh[x])*Sech[x]^4*Sinh[x/2]^6)/(3*a)
```

3.188.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.27, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.769, Rules used = {3042, 26, 3185, 26, 3042, 26, 3086, 2009, 3087, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^5(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\tan\left(-\frac{\pi}{2}+ix\right)^5\left(a-a\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\left(a-a\sin\left(ix-\frac{\pi}{2}\right)\right)\tan\left(ix-\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{3185}$$

$$i\left(\frac{\int i\text{sech}^2(x)\tanh^3(x)dx}{a}+\frac{\int -i\text{sech}(x)\tanh^3(x)dx}{a}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{i\int \text{sech}^2(x)\tanh^3(x)dx}{a}-\frac{i\int \text{sech}(x)\tanh^3(x)dx}{a}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{i\int i\sec(ix)^2\tan(ix)^3dx}{a}-\frac{i\int i\sec(ix)\tan(ix)^3dx}{a}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{\int \sec(ix)\tan(ix)^3dx}{a}-\frac{\int \sec(ix)^2\tan(ix)^3dx}{a}\right)$$

$$\Big\downarrow \text{3086}$$

$$i\left(-\frac{i\int \left(\text{sech}^2(x)-1\right)d\text{sech}(x)}{a}-\frac{\int \sec(ix)^2\tan(ix)^3dx}{a}\right)$$

$$\Big\downarrow \text{2009}$$

$$i\left(-\frac{\int \sec(ix)^2\tan(ix)^3dx}{a}-\frac{i\left(\frac{\text{sech}^3(x)}{3}-\text{sech}(x)\right)}{a}\right)$$

$$\Big\downarrow \ 3087$$

$$i\left(\frac{i\int -i\tanh^3(x)d(i\tanh(x))}{a}-\frac{i\left(\frac{\text{sech}^3(x)}{3}-\text{sech}(x)\right)}{a}\right)$$

$$\Big\downarrow \ 15$$

$$i\left(\frac{i\tanh^4(x)}{4a}-\frac{i\left(\frac{\text{sech}^3(x)}{3}-\text{sech}(x)\right)}{a}\right)$$

input
```
Int[Tanh[x]^5/(a + a*Cosh[x]),x]
```

output
```
I*(((-I)*(-Sech[x] + Sech[x]^3/3))/a + ((I/4)*Tanh[x]^4)/a)
```

3.188.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.188. $\int \frac{\tanh^5(x)}{a+a\cosh(x)}\,dx$

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

3.188.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

method	result	size
default	$\frac{\frac{32}{3\left(1+\tanh\left(\frac{x}{2}\right)^{2}\right)^{3}}-\frac{8}{\left(1+\tanh\left(\frac{x}{2}\right)^{2}\right)^{2}}-\frac{4}{\left(1+\tanh\left(\frac{x}{2}\right)^{2}\right)^{4}}}{a}$	43
risch	$-\frac{2\,\mathrm{e}^{x}\left(3\,\mathrm{e}^{6x}-3\,\mathrm{e}^{5x}+5\,\mathrm{e}^{4x}+5\,\mathrm{e}^{2x}-3\,\mathrm{e}^{x}+3\right)}{3\left(1+\mathrm{e}^{2x}\right)^{4}a}$	46

input
```
int(tanh(x)^5/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
32/a*(1/3/(1+tanh(1/2*x)^2)^3-1/4/(1+tanh(1/2*x)^2)^2-1/8/(1+tanh(1/2*x)^2
)^4)
```

3.188.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(26) = 52$.

Time = 0.26 (sec) , antiderivative size = 174, normalized size of antiderivative = 5.80

$$\int \frac{\tanh^5(x)}{a + a\cosh(x)}\,dx =$$
$$-\frac{2\left(3\cosh{(x)}^4 + 3\,(4\cosh{(x)} - 1)\sinh{(x)}^3 + 3\sinh{(x)}^4 - 3\cosh{(x)}^3 + \left(18\cosh{(x)}^2 - 9\cosh{(x)} + \right.\right.}{3\left(a\cosh{(x)}^5 + 5\,a\cosh{(x)}\sinh{(x)}^4 + a\sinh{(x)}^5 + 5\,a\cosh{(x)}^3 + \left(10\,a\cosh{(x)}^2 + 3\,a\right)\sinh{(x)}^3 + 5\right.}$$

input
```
integrate(tanh(x)^5/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
-2/3*(3*cosh(x)^4 + 3*(4*cosh(x) - 1)*sinh(x)^3 + 3*sinh(x)^4 - 3*cosh(x)^
3 + (18*cosh(x)^2 - 9*cosh(x) + 8)*sinh(x)^2 + 8*cosh(x)^2 + (12*cosh(x)^3
 - 9*cosh(x)^2 + 4*cosh(x) + 3)*sinh(x) - 3*cosh(x) + 5)/(a*cosh(x)^5 + 5*
a*cosh(x)*sinh(x)^4 + a*sinh(x)^5 + 5*a*cosh(x)^3 + (10*a*cosh(x)^2 + 3*a)
*sinh(x)^3 + 5*(2*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^2 + 10*a*cosh(x) + (5
*a*cosh(x)^4 + 9*a*cosh(x)^2 + 2*a)*sinh(x))
```

3.188.6 Sympy [F]

$$\int \frac{\tanh^5(x)}{a + a\cosh(x)}\,dx = \frac{\int \frac{\tanh^5{(x)}}{\cosh{(x)}+1}\,dx}{a}$$

input
```
integrate(tanh(x)**5/(a+a*cosh(x)),x)
```

output
```
Integral(tanh(x)**5/(cosh(x) + 1), x)/a
```

3.188.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(26) = 52$.

Time = 0.19 (sec) , antiderivative size = 223, normalized size of antiderivative = 7.43

$$\int \frac{\tanh^5(x)}{a+a\cosh(x)}\,dx = -\frac{2\,e^{(-x)}}{4\,ae^{(-2\,x)}+6\,ae^{(-4\,x)}+4\,ae^{(-6\,x)}+ae^{(-8\,x)}+a} + \frac{2\,e^{(-2\,x)}}{4\,ae^{(-2\,x)}+6\,ae^{(-4\,x)}+4\,ae^{(-6\,x)}+ae^{(-8\,x)}+a} - \frac{10\,e^{(-3\,x)}}{3\,(4\,ae^{(-2\,x)}+6\,ae^{(-4\,x)}+4\,ae^{(-6\,x)}+ae^{(-8\,x)}+a)} - \frac{10\,e^{(-5\,x)}}{3\,(4\,ae^{(-2\,x)}+6\,ae^{(-4\,x)}+4\,ae^{(-6\,x)}+ae^{(-8\,x)}+a)} + \frac{2\,e^{(-6\,x)}}{4\,ae^{(-2\,x)}+6\,ae^{(-4\,x)}+4\,ae^{(-6\,x)}+ae^{(-8\,x)}+a} - \frac{2\,e^{(-7\,x)}}{4\,ae^{(-2\,x)}+6\,ae^{(-4\,x)}+4\,ae^{(-6\,x)}+ae^{(-8\,x)}+a}$$

input
```
integrate(tanh(x)^5/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-2*e^(-x)/(4*a*e^(-2*x) + 6*a*e^(-4*x) + 4*a*e^(-6*x) + a*e^(-8*x) + a) +
2*e^(-2*x)/(4*a*e^(-2*x) + 6*a*e^(-4*x) + 4*a*e^(-6*x) + a*e^(-8*x) + a) -
 10/3*e^(-3*x)/(4*a*e^(-2*x) + 6*a*e^(-4*x) + 4*a*e^(-6*x) + a*e^(-8*x) +
a) - 10/3*e^(-5*x)/(4*a*e^(-2*x) + 6*a*e^(-4*x) + 4*a*e^(-6*x) + a*e^(-8*x
) + a) + 2*e^(-6*x)/(4*a*e^(-2*x) + 6*a*e^(-4*x) + 4*a*e^(-6*x) + a*e^(-8*
x) + a) - 2*e^(-7*x)/(4*a*e^(-2*x) + 6*a*e^(-4*x) + 4*a*e^(-6*x) + a*e^(-8
*x) + a)
```

3.188.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.60

$$\int \frac{\tanh^5(x)}{a+a\cosh(x)}\,dx = -\frac{2\left(3\left(e^{(-x)}+e^{x}\right)^3-3\left(e^{(-x)}+e^{x}\right)^2-4\,e^{(-x)}-4\,e^{x}+6\right)}{3\,a\left(e^{(-x)}+e^{x}\right)^4}$$

input
```
integrate(tanh(x)^5/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
-2/3*(3*(e^(-x) + e^x)^3 - 3*(e^(-x) + e^x)^2 - 4*e^(-x) - 4*e^x + 6)/(a*(
e^(-x) + e^x)^4)
```

3.188.9 Mupad [B] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 117, normalized size of antiderivative = 3.90

$$\int \frac{\tanh^5(x)}{a+a\cosh(x)}\,dx = \frac{\frac{8}{a}-\frac{8\,e^x}{3\,a}}{3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1}-\frac{\frac{6}{a}-\frac{8\,e^x}{3\,a}}{2\,e^{2\,x}+e^{4\,x}+1}+\frac{\frac{2}{a}-\frac{2\,e^x}{a}}{e^{2\,x}+1}-\frac{4}{a\,\left(4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1\right)}$$

input
```
int(tanh(x)^5/(a + a*cosh(x)),x)
```

output
```
(8/a - (8*exp(x))/(3*a))/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - (6/a -
 (8*exp(x))/(3*a))/(2*exp(2*x) + exp(4*x) + 1) + (2/a - (2*exp(x))/a)/(exp
(2*x) + 1) - 4/(a*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1))
```

3.189 $\int \frac{\tanh^4(x)}{a+a\cosh(x)}\,dx$

3.189.1 Optimal result

Integrand size = 13, antiderivative size = 33

$$\int \frac{\tanh^4(x)}{a+a\cosh(x)}\,dx=\frac{\arctan(\sinh(x))}{2a}-\frac{\operatorname{sech}(x)\tanh(x)}{2a}-\frac{\tanh^3(x)}{3a}$$

output
```
1/2*arctan(sinh(x))/a-1/2*sech(x)*tanh(x)/a-1/3*tanh(x)^3/a
```

3.189.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39

$$\int \frac{\tanh^4(x)}{a+a\cosh(x)}\,dx =\frac{\cosh^2\left(\frac{x}{2}\right)\left(6\arctan\left(\tanh\left(\frac{x}{2}\right)\right)+\left(-2-3\operatorname{sech}(x)+2\operatorname{sech}^2(x)\right)\tanh(x)\right)}{3a(1+\cosh(x))}$$

input
```
Integrate[Tanh[x]^4/(a + a*Cosh[x]),x]
```

output
```
(Cosh[x/2]^2*(6*ArcTan[Tanh[x/2]] + (-2 - 3*Sech[x] + 2*Sech[x]^2)*Tanh[x]
))/(3*a*(1 + Cosh[x]))
```

3.189.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.769, Rules used = {3042, 3185, 25, 3042, 25, 3087, 15, 3091, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^4(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan\left(-\frac{\pi}{2}+ix\right)^4\left(a-a\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{3185}$$

$$\frac{\int -\text{sech}^2(x)\tanh^2(x)dx}{a}+\frac{\int \text{sech}(x)\tanh^2(x)dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \text{sech}(x)\tanh^2(x)dx}{a}-\frac{\int \text{sech}^2(x)\tanh^2(x)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -\sec(ix)\tan(ix)^2dx}{a}-\frac{\int -\sec(ix)^2\tan(ix)^2dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \sec(ix)^2\tan(ix)^2dx}{a}-\frac{\int \sec(ix)\tan(ix)^2dx}{a}$$

$$\Big\downarrow \text{3087}$$

$$-\frac{i\int -\tanh^2(x)d(i\tanh(x))}{a}-\frac{\int \sec(ix)\tan(ix)^2dx}{a}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\tanh^3(x)}{3a}-\frac{\int \sec(ix)\tan(ix)^2dx}{a}$$

$$\Big\downarrow \text{3091}$$

$$-\frac{\frac{1}{2}\tanh(x)\text{sech}(x)-\frac{\int \text{sech}(x)dx}{2}}{a}-\frac{\tanh^3(x)}{3a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tanh^3(x)}{3a}-\frac{\frac{1}{2}\tanh(x)\text{sech}(x)-\frac{1}{2}\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\frac{1}{2}\tanh(x)\text{sech}(x)-\frac{1}{2}\arctan(\sinh(x))}{a}-\frac{\tanh^3(x)}{3a}$$

input
```
Int[Tanh[x]^4/(a + a*Cosh[x]),x]
```

output
```
-1/3*Tanh[x]^3/a - (-1/2*ArcTan[Sinh[x]] + (Sech[x]*Tanh[x])/2)/a
```

3.189.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

3.189. $\int \frac{\tanh^4(x)}{a+a\cosh(x)}\,dx$

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.189.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.45

method	result	size
default	$\frac{\frac{16\left(\frac{\tanh\left(\frac{x}{2}\right)^{5}}{16}-\frac{\tanh\left(\frac{x}{2}\right)^{3}}{6}-\frac{\tanh\left(\frac{x}{2}\right)}{16}\right)}{\left(1+\tanh\left(\frac{x}{2}\right)^{2}\right)^{3}}+\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	48
risch	$-\frac{3\,\mathrm{e}^{5x}-6\,\mathrm{e}^{4x}-3\,\mathrm{e}^{x}-2}{3(1+\mathrm{e}^{2x})^{3}a}+\frac{i\ln(\mathrm{e}^{x}+i)}{2a}-\frac{i\ln(\mathrm{e}^{x}-i)}{2a}$	57

input
```
int(tanh(x)^4/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
16/a*((1/16*tanh(1/2*x)^5-1/6*tanh(1/2*x)^3-1/16*tanh(1/2*x))/(1+tanh(1/2*
x)^2)^3+1/16*arctan(tanh(1/2*x)))
```

3.189.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. 2(27) = 54.

Time = 0.29 (sec) , antiderivative size = 315, normalized size of antiderivative = 9.55

$$\int \frac{\tanh^4(x)}{a+a\cosh(x)}\,dx =$$
$$-\frac{3\,\cosh\left(x\right)^{5}+3\left(5\,\cosh\left(x\right)-2\right)\sinh\left(x\right)^{4}+3\,\sinh\left(x\right)^{5}-6\,\cosh\left(x\right)^{4}+6\left(5\,\cosh\left(x\right)^{2}-4\,\cosh\left(x\right)\right)\sinh$$

input
```
integrate(tanh(x)^4/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
-1/3*(3*cosh(x)^5 + 3*(5*cosh(x) - 2)*sinh(x)^4 + 3*sinh(x)^5 - 6*cosh(x)^
4 + 6*(5*cosh(x)^2 - 4*cosh(x))*sinh(x)^3 + 6*(5*cosh(x)^3 - 6*cosh(x)^2)*
sinh(x)^2 - 3*(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^
2 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 3
*(5*cosh(x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 +
2*cosh(x)^3 + cosh(x))*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + 3*(5*cosh(
x)^4 - 8*cosh(x)^3 - 1)*sinh(x) - 3*cosh(x) - 2)/(a*cosh(x)^6 + 6*a*cosh(x
)*sinh(x)^5 + a*sinh(x)^6 + 3*a*cosh(x)^4 + 3*(5*a*cosh(x)^2 + a)*sinh(x)^
4 + 4*(5*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^3 + 3*a*cosh(x)^2 + 3*(5*a*cos
h(x)^4 + 6*a*cosh(x)^2 + a)*sinh(x)^2 + 6*(a*cosh(x)^5 + 2*a*cosh(x)^3 + a
*cosh(x))*sinh(x) + a)
```

3.189.6 Sympy [F]

$$\int \frac{\tanh^4(x)}{a + a\cosh(x)}\,dx = \frac{\int \frac{\tanh^4(x)}{\cosh(x)+1}\,dx}{a}$$

input
```
integrate(tanh(x)**4/(a+a*cosh(x)),x)
```

output
```
Integral(tanh(x)**4/(cosh(x) + 1), x)/a
```

3.189.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(27) = 54$.

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.73

$$\int \frac{\tanh^4(x)}{a + a\cosh(x)}\,dx = -\frac{3\,e^{(-x)} + 6\,e^{(-4\,x)} - 3\,e^{(-5\,x)} + 2}{3\,(3\,ae^{(-2\,x)} + 3\,ae^{(-4\,x)} + ae^{(-6\,x)} + a)} - \frac{\arctan\left(e^{(-x)}\right)}{a}$$

input
```
integrate(tanh(x)^4/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-1/3*(3*e^(-x) + 6*e^(-4*x) - 3*e^(-5*x) + 2)/(3*a*e^(-2*x) + 3*a*e^(-4*x)
 + a*e^(-6*x) + a) - arctan(e^(-x))/a
```

3.189.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int \frac{\tanh^4(x)}{a + a\cosh(x)}\,dx = \frac{\arctan(e^x)}{a} - \frac{3\,e^{(5\,x)} - 6\,e^{(4\,x)} - 3\,e^x - 2}{3\,a\left(e^{(2\,x)} + 1\right)^3}$$

input
```
integrate(tanh(x)^4/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
arctan(e^x)/a - 1/3*(3*e^(5*x) - 6*e^(4*x) - 3*e^x - 2)/(a*(e^(2*x) + 1)^3
)
```

3.189.9 Mupad [B] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.88

$$\int \frac{\tanh^4(x)}{a + a\cosh(x)}\,dx$$
$$= \frac{8}{3\,a\,\left(3\,\mathrm{e}^{2\,x} + 3\,\mathrm{e}^{4\,x} + \mathrm{e}^{6\,x} + 1\right)} - \frac{\frac{4}{a} - \frac{2\,\mathrm{e}^x}{a}}{2\,\mathrm{e}^{2\,x} + \mathrm{e}^{4\,x} + 1} + \frac{\frac{2}{a} - \frac{\mathrm{e}^x}{a}}{\mathrm{e}^{2\,x} + 1} + \frac{\mathrm{atan}\left(\frac{\mathrm{e}^x\,\sqrt{a^2}}{a}\right)}{\sqrt{a^2}}$$

input
```
int(tanh(x)^4/(a + a*cosh(x)),x)
```

output
```
8/(3*a*(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1)) - (4/a - (2*exp(x))/a)/(2
*exp(2*x) + exp(4*x) + 1) + (2/a - exp(x)/a)/(exp(2*x) + 1) + atan((exp(x)
*(a^2)^(1/2))/a)/(a^2)^(1/2)
```

3.190 $\int \frac{\tanh^3(x)}{a+a\cosh(x)}\,dx$

3.190.1 Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{\tanh^3(x)}{a+a\cosh(x)}\,dx=-\frac{\operatorname{sech}(x)}{a}+\frac{\operatorname{sech}^2(x)}{2a}$$

output
```
-sech(x)/a+1/2*sech(x)^2/a
```

3.190.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\tanh^3(x)}{a+a\cosh(x)}\,dx=\frac{2\operatorname{sech}^2(x)\sinh^4\left(\frac{x}{2}\right)}{a}$$

input
```
Integrate[Tanh[x]^3/(a + a*Cosh[x]),x]
```

output
```
(2*Sech[x]^2*Sinh[x/2]^4)/a
```

3.190.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {3042, 26, 3185, 26, 3042, 26, 3086, 15, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\tan\left(-\frac{\pi}{2}+ix\right)^3\left(a-a\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\left(a-a\sin\left(ix-\frac{\pi}{2}\right)\right)\tan\left(ix-\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{3185}$$

$$-i\left(\frac{\int -i\text{sech}^2(x)\tanh(x)dx}{a}+\frac{\int i\text{sech}(x)\tanh(x)dx}{a}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i\int \text{sech}(x)\tanh(x)dx}{a}-\frac{i\int \text{sech}^2(x)\tanh(x)dx}{a}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i\int -i\sec(ix)\tan(ix)dx}{a}-\frac{i\int -i\sec(ix)^2\tan(ix)dx}{a}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{\int \sec(ix)\tan(ix)dx}{a}-\frac{\int \sec(ix)^2\tan(ix)dx}{a}\right)$$

$$\Big\downarrow \text{3086}$$

$$-i\left(\frac{i\int \text{sech}(x)d\text{sech}(x)}{a}-\frac{i\int 1d\text{sech}(x)}{a}\right)$$

$$\Big\downarrow \text{15}$$

$$-i\left(\frac{i\text{sech}^2(x)}{2a}-\frac{i\int 1d\text{sech}(x)}{a}\right)$$
$$\Big\downarrow \text{24}$$
$$-i\left(\frac{i\text{sech}^2(x)}{2a}-\frac{i\text{sech}(x)}{a}\right)$$

input
```
Int[Tanh[x]^3/(a + a*Cosh[x]),x]
```

output
```
(-I)*(((-I)*Sech[x])/a + ((I/2)*Sech[x]^2)/a)
```

3.190.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

3.190. $\int \frac{\tanh^3(x)}{a+a\cosh(x)}\,dx$

3.190.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

method	result	size
risch	$-\frac{2\,\mathrm{e}^{x}\left(\mathrm{e}^{2 x}-\mathrm{e}^{x}+1\right)}{\left(1+\mathrm{e}^{2 x}\right)^{2} a}$	26
default	$\frac{\frac{2}{\left(1+\tanh\left(\frac{x}{2}\right)^{2}\right)^{2}}-\frac{4}{1+\tanh\left(\frac{x}{2}\right)^{2}}}{a}$	31

input
```
int(tanh(x)^3/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-2*exp(x)*(exp(2*x)-exp(x)+1)/(1+exp(2*x))^2/a
```

3.190.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(17) = 34.

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.47

$$\int \frac{\tanh^3(x)}{a+a\cosh(x)}\,dx = -\frac{2\left(\cosh\left(x\right)^{2}+\left(2\,\cosh\left(x\right)-1\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}-\cosh\left(x\right)+1\right)}{a\cosh\left(x\right)^{3}+3\,a\cosh\left(x\right)\sinh\left(x\right)^{2}+a\sinh\left(x\right)^{3}+3\,a\cosh\left(x\right)+\left(3\,a\cosh\left(x\right)^{2}+a\right)\sinh\left(x\right)}$$

input
```
integrate(tanh(x)^3/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
-2*(cosh(x)^2 + (2*cosh(x) - 1)*sinh(x) + sinh(x)^2 - cosh(x) + 1)/(a*cosh
(x)^3 + 3*a*cosh(x)*sinh(x)^2 + a*sinh(x)^3 + 3*a*cosh(x) + (3*a*cosh(x)^2
 + a)*sinh(x))
```

3.190.6 Sympy [F]

$$\int \frac{\tanh^3(x)}{a + a\cosh(x)}\,dx = \frac{\int \frac{\tanh^3(x)}{\cosh(x)+1}\,dx}{a}$$

input
```
integrate(tanh(x)**3/(a+a*cosh(x)),x)
```

output
```
Integral(tanh(x)**3/(cosh(x) + 1), x)/a
```

3.190.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(17) = 34.

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.68

$$\int \frac{\tanh^3(x)}{a + a\cosh(x)}\,dx = -\frac{2\,e^{(-x)}}{2\,ae^{(-2\,x)} + ae^{(-4\,x)} + a} + \frac{2\,e^{(-2\,x)}}{2\,ae^{(-2\,x)} + ae^{(-4\,x)} + a} - \frac{2\,e^{(-3\,x)}}{2\,ae^{(-2\,x)} + ae^{(-4\,x)} + a}$$

input
```
integrate(tanh(x)^3/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-2*e^(-x)/(2*a*e^(-2*x) + a*e^(-4*x) + a) + 2*e^(-2*x)/(2*a*e^(-2*x) + a*e
^(-4*x) + a) - 2*e^(-3*x)/(2*a*e^(-2*x) + a*e^(-4*x) + a)
```

3.190.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{\tanh^3(x)}{a + a\cosh(x)}\,dx = -\frac{2\left(e^{(-x)} + e^x - 1\right)}{a\left(e^{(-x)} + e^x\right)^2}$$

input
```
integrate(tanh(x)^3/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
-2*(e^(-x) + e^x - 1)/(a*(e^(-x) + e^x)^2)
```

3.190.9 Mupad [B] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

$$\int \frac{\tanh^3(x)}{a + a\cosh(x)}\,dx = -\frac{2\,e^x\,(e^{2\,x} - e^x + 1)}{a\,(e^{2\,x} + 1)^2}$$

input
```
int(tanh(x)^3/(a + a*cosh(x)),x)
```

output
```
-(2*exp(x)*(exp(2*x) - exp(x) + 1))/(a*(exp(2*x) + 1)^2)
```

3.191 $\int \frac{\tanh^2(x)}{a+a\cosh(x)}\,dx$

3.191.1 Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{\tanh^2(x)}{a+a\cosh(x)}\,dx=\frac{\arctan(\sinh(x))}{a}-\frac{\tanh(x)}{a}$$

output
```
arctan(sinh(x))/a-tanh(x)/a
```

3.191.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \frac{\tanh^2(x)}{a+a\cosh(x)}\,dx=\frac{2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)-\tanh(x)}{a}$$

input
```
Integrate[Tanh[x]^2/(a + a*Cosh[x]),x]
```

output
```
(2*ArcTan[Tanh[x/2]] - Tanh[x])/a
```

3.191.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {3042, 25, 3185, 25, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(x)}{a\cosh(x)+a}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{1}{\tan\left(-\frac{\pi}{2}+ix\right)^2\left(a-a\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{1}{\left(a-a\sin\left(ix-\frac{\pi}{2}\right)\right)\tan\left(ix-\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \; 3185$$

$$-\frac{\int \operatorname{sech}^2(x)dx}{a}-\frac{\int -\operatorname{sech}(x)dx}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \operatorname{sech}(x)dx}{a}-\frac{\int \operatorname{sech}^2(x)dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}-\frac{\int \csc\left(ix+\frac{\pi}{2}\right)^2dx}{a}$$

$$\Big\downarrow \; 4254$$

$$\frac{\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}-\frac{i\int 1d(-i\tanh(x))}{a}$$

$$\Big\downarrow \; 24$$

$$-\frac{\tanh(x)}{a}+\frac{\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \; 4257$$

$$\frac{\arctan(\sinh(x))}{a}-\frac{\tanh(x)}{a}$$

input
```
Int[Tanh[x]^2/(a + a*Cosh[x]),x]
```

output
```
ArcTan[Sinh[x]]/a - Tanh[x]/a
```

3.191.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.191.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.00

method	result	size
default	$\frac{-\frac{2\tanh\left(\frac{x}{2}\right)}{1+\tanh\left(\frac{x}{2}\right)^{2}}+2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	30
risch	$\frac{2}{a(1+\mathrm{e}^{2x})}+\frac{i\ln(\mathrm{e}^{x}+i)}{a}-\frac{i\ln(\mathrm{e}^{x}-i)}{a}$	39

input
```
int(tanh(x)^2/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
4/a*(-1/2*tanh(1/2*x)/(1+tanh(1/2*x)^2)+1/2*arctan(tanh(1/2*x)))
```

3.191.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(15) = 30$.

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 3.33

$$\int \frac{\tanh^2(x)}{a + a\cosh(x)}\,dx = \frac{2\left(\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1\right)\arctan(\cosh(x) + \sinh(x)) + 1\right)}{a\cosh(x)^2 + 2a\cosh(x)\sinh(x) + a\sinh(x)^2 + a}$$

input
```
integrate(tanh(x)^2/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
2*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*arctan(cosh(x) + sinh(x
)) + 1)/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)
```

3.191.6 Sympy [F]

$$\int \frac{\tanh^2(x)}{a + a\cosh(x)}\,dx = \frac{\int \frac{\tanh^2(x)}{\cosh(x)+1}\,dx}{a}$$

input
```
integrate(tanh(x)**2/(a+a*cosh(x)),x)
```

output
```
Integral(tanh(x)**2/(cosh(x) + 1), x)/a
```

3.191.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int \frac{\tanh^2(x)}{a+a\cosh(x)}\,dx = -\frac{2\arctan\left(e^{(-x)}\right)}{a} - \frac{2}{ae^{(-2x)}+a}$$

input
```
integrate(tanh(x)^2/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-2*arctan(e^(-x))/a - 2/(a*e^(-2*x) + a)
```

3.191.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \frac{\tanh^2(x)}{a+a\cosh(x)}\,dx = \frac{2\arctan\left(e^{x}\right)}{a} + \frac{2}{a(e^{(2x)}+1)}$$

input
```
integrate(tanh(x)^2/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
2*arctan(e^x)/a + 2/(a*(e^(2*x) + 1))
```

3.191.9 Mupad [B] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.20

$$\int \frac{\tanh^2(x)}{a+a\cosh(x)}\,dx = \frac{2}{a\,(\mathrm{e}^{2x}+1)} + \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{x}\sqrt{a^2}}{a}\right)}{\sqrt{a^2}}$$

input
```
int(tanh(x)^2/(a + a*cosh(x)),x)
```

output
```
2/(a*(exp(2*x) + 1)) + (2*atan((exp(x)*(a^2)^(1/2))/a))/(a^2)^(1/2)
```

3.192 $\int \frac{\tanh(x)}{a+a\cosh(x)}\,dx$

3.192.1 Optimal result

Integrand size = 11, antiderivative size = 18

$$\int \frac{\tanh(x)}{a+a\cosh(x)}\,dx = \frac{\log(\cosh(x))}{a} - \frac{\log(1+\cosh(x))}{a}$$

output
```
ln(cosh(x))/a-ln(1+cosh(x))/a
```

3.192.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{\tanh(x)}{a+a\cosh(x)}\,dx = -\frac{2\text{arctanh}(1+2\cosh(x))}{a}$$

input
```
Integrate[Tanh[x]/(a + a*Cosh[x]),x]
```

output
```
(-2*ArcTanh[1 + 2*Cosh[x]])/a
```

3.192.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 26, 3186, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\tan\left(-\frac{\pi}{2}+ix\right)\left(a-a\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\left(a-a\sin\left(ix-\frac{\pi}{2}\right)\right)\tan\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3186}$$

$$\int \frac{\text{sech}(x)}{a(a\cosh(x)+a)}d(a\cosh(x))$$

$$\Big\downarrow \text{47}$$

$$\frac{\int \frac{\text{sech}(x)}{a}d(a\cosh(x))}{a}-\frac{\int \frac{1}{\cosh(x)a+a}d(a\cosh(x))}{a}$$

$$\Big\downarrow \text{14}$$

$$\frac{\log(a\cosh(x))}{a}-\frac{\int \frac{1}{\cosh(x)a+a}d(a\cosh(x))}{a}$$

$$\Big\downarrow \text{16}$$

$$\frac{\log(a\cosh(x))}{a}-\frac{\log(a\cosh(x)+a)}{a}$$

input
```
Int[Tanh[x]/(a + a*Cosh[x]),x]
```

output
```
Log[a*Cosh[x]]/a - Log[a + a*Cosh[x]]/a
```

3.192.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.192.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{\ln\left(1+\tanh\left(\frac{x}{2}\right)^{2}\right)}{a}$	14
risch	$-\frac{2\ln(\mathrm{e}^{x}+1)}{a}+\frac{\ln(1+\mathrm{e}^{2x})}{a}$	23

input
```
int(tanh(x)/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/a*ln(1+tanh(1/2*x)^2)
```

3.192.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.56

$$\int \frac{\tanh(x)}{a + a\cosh(x)}\, dx = \frac{\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right) - 2\log\left(\cosh\left(x\right) + \sinh\left(x\right) + 1\right)}{a}$$

input
```
integrate(tanh(x)/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
(log(2*cosh(x)/(cosh(x) - sinh(x))) - 2*log(cosh(x) + sinh(x) + 1))/a
```

3.192.6 Sympy [F]

$$\int \frac{\tanh(x)}{a + a\cosh(x)}\, dx = \frac{\int \frac{\tanh\left(x\right)}{\cosh\left(x\right)+1}\, dx}{a}$$

input
```
integrate(tanh(x)/(a+a*cosh(x)),x)
```

output
```
Integral(tanh(x)/(cosh(x) + 1), x)/a
```

3.192.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int \frac{\tanh(x)}{a + a\cosh(x)}\, dx = -\frac{2\log\left(e^{(-x)} + 1\right)}{a} + \frac{\log\left(e^{(-2x)} + 1\right)}{a}$$

input
```
integrate(tanh(x)/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-2*log(e^(-x) + 1)/a + log(e^(-2*x) + 1)/a
```

3.192.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\tanh(x)}{a + a\cosh(x)}\,dx = \frac{\log\left(e^{(2x)} + 1\right)}{a} - \frac{2\,\log\left(e^{x} + 1\right)}{a}$$

input
```
integrate(tanh(x)/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
log(e^(2*x) + 1)/a - 2*log(e^x + 1)/a
```

3.192.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44

$$\int \frac{\tanh(x)}{a + a\cosh(x)}\,dx = -\frac{2\,\ln\left(36\,e^{x} + 36\right) - \ln\left(3\,e^{2\,x} + 3\right)}{a}$$

input
```
int(tanh(x)/(a + a*cosh(x)),x)
```

output
```
-(2*log(36*exp(x) + 36) - log(3*exp(2*x) + 3))/a
```

3.193 $\int \frac{\coth(x)}{a+a\cosh(x)}\,dx$

3.193.1 Optimal result

Integrand size = 11, antiderivative size = 33

$$\int \frac{\coth(x)}{a+a\cosh(x)}\,dx = -\frac{\operatorname{arctanh}(\cosh(x))}{2a}-\frac{\coth(x)\operatorname{csch}(x)}{2a}+\frac{\operatorname{csch}^2(x)}{2a}$$

output
```
-1/2*arctanh(cosh(x))/a-1/2*coth(x)*csch(x)/a+1/2*csch(x)^2/a
```

3.193.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

$$\int \frac{\coth(x)}{a+a\cosh(x)}\,dx = -\frac{1+2\cosh^2\left(\frac{x}{2}\right)\left(\log\left(\cosh\left(\frac{x}{2}\right)\right)-\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)}{2a(1+\cosh(x))}$$

input
```
Integrate[Coth[x]/(a + a*Cosh[x]),x]
```

output
```
-1/2*(1 + 2*Cosh[x/2]^2*(Log[Cosh[x/2]] - Log[Sinh[x/2]]))/(a*(1 + Cosh[x]
))
```

3.193.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.182, Rules used = {3042, 26, 3185, 26, 3042, 26, 3086, 15, 3091, 26, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan\left(-\frac{\pi}{2}+ix\right)}{a-a\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan\left(ix-\frac{\pi}{2}\right)}{a-a\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3185}$$

$$-i\left(\frac{\int i\coth^2(x)\text{csch}(x)dx}{a}+\frac{\int -i\coth(x)\text{csch}^2(x)dx}{a}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i\int \coth^2(x)\text{csch}(x)dx}{a}-\frac{i\int \coth(x)\text{csch}^2(x)dx}{a}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i\int -i\sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a}-\frac{i\int i\sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)dx}{a}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{\int \sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)dx}{a}+\frac{\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a}\right)$$

$$\Big\downarrow \text{3086}$$

$$-i\left(\frac{\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a}-\frac{i\int -i\text{csch}(x)d(-i\text{csch}(x))}{a}\right)$$

$$\Big\downarrow \text{15}$$

$$-i\left(\frac{\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a}+\frac{i\text{csch}^2(x)}{2a}\right)$$

$$\Big\downarrow \text{3091}$$

$$-i\left(\frac{-\frac{1}{2}\int -i\text{csch}(x)dx-\frac{1}{2}i\coth(x)\text{csch}(x)}{a}+\frac{i\text{csch}^2(x)}{2a}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{\frac{1}{2}i\int \text{csch}(x)dx-\frac{1}{2}i\coth(x)\text{csch}(x)}{a}+\frac{i\text{csch}^2(x)}{2a}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{\frac{1}{2}i\int i\csc(ix)dx-\frac{1}{2}i\coth(x)\text{csch}(x)}{a}+\frac{i\text{csch}^2(x)}{2a}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{-\frac{1}{2}\int \csc(ix)dx-\frac{1}{2}i\coth(x)\text{csch}(x)}{a}+\frac{i\text{csch}^2(x)}{2a}\right)$$

$$\Big\downarrow \text{4257}$$

$$-i\left(\frac{-\frac{1}{2}i\text{arctanh}(\cosh(x))-\frac{1}{2}i\coth(x)\text{csch}(x)}{a}+\frac{i\text{csch}^2(x)}{2a}\right)$$

input
```
Int[Coth[x]/(a + a*Cosh[x]),x]
```

output
```
(-I)*(((I/2)*Csch[x]^2)/a + ((-1/2*I)*ArcTanh[Cosh[x]] - (I/2)*Coth[x]*Csc
h[x])/a)
```

3.193.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

3.193. $\int \frac{\coth(x)}{a+a\cosh(x)}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.193.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{\frac{\tanh\left(\frac{x}{2}\right)^{2}}{2}+\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2a}$	20
risch	$-\frac{e^{x}}{\left(e^{x}+1\right)^{2}a}+\frac{\ln\left(e^{x}-1\right)}{2a}-\frac{\ln\left(e^{x}+1\right)}{2a}$	35

input
```
int(coth(x)/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2/a*(1/2*tanh(1/2*x)^2+ln(tanh(1/2*x)))
```

3.193. $\int \frac{\coth(x)}{a+a\cosh(x)}\,dx$

3.193.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(27) = 54.

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 3.12

$$\int \frac{\coth(x)}{a + a\cosh(x)}\,dx = -\frac{\left(\cosh\left(x\right)^{2} + 2\left(\cosh\left(x\right) + 1\right)\sinh\left(x\right) + \sinh\left(x\right)^{2} + 2\,\cosh\left(x\right) + 1\right)\log\left(\cosh\left(x\right) + \sinh\left(x\right) + 1\right) - (\cos}{2\left(a\cosh\left(x\right)^{2} + a\sinh\left(x\right)^{2} + 2}$$

input
```
integrate(coth(x)/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
-1/2*((cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*lo
g(cosh(x) + sinh(x) + 1) - (cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^
2 + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*cosh(x) + 2*sinh(x))/(a*
cosh(x)^2 + a*sinh(x)^2 + 2*a*cosh(x) + 2*(a*cosh(x) + a)*sinh(x) + a)
```

3.193.6 Sympy [F]

$$\int \frac{\coth(x)}{a + a\cosh(x)}\,dx = \frac{\int \frac{\coth\left(x\right)}{\cosh\left(x\right)+1}\,dx}{a}$$

input
```
integrate(coth(x)/(a+a*cosh(x)),x)
```

output
```
Integral(coth(x)/(cosh(x) + 1), x)/a
```

3.193.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.45

$$\int \frac{\coth(x)}{a + a\cosh(x)}\,dx = -\frac{e^{(-x)}}{2\,ae^{(-x)} + ae^{(-2\,x)} + a} - \frac{\log\left(e^{(-x)} + 1\right)}{2\,a} + \frac{\log\left(e^{(-x)} - 1\right)}{2\,a}$$

input
```
integrate(coth(x)/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-e^(-x)/(2*a*e^(-x) + a*e^(-2*x) + a) - 1/2*log(e^(-x) + 1)/a + 1/2*log(e^
(-x) - 1)/a
```

3.193.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.58

$$\int \frac{\coth(x)}{a + a\cosh(x)}\,dx = -\frac{\log\left(e^{(-x)} + e^{x} + 2\right)}{4\,a} + \frac{\log\left(e^{(-x)} + e^{x} - 2\right)}{4\,a} + \frac{e^{(-x)} + e^{x} - 2}{4\,a(e^{(-x)} + e^{x} + 2)}$$

input
```
integrate(coth(x)/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
-1/4*log(e^(-x) + e^x + 2)/a + 1/4*log(e^(-x) + e^x - 2)/a + 1/4*(e^(-x) +
 e^x - 2)/(a*(e^(-x) + e^x + 2))
```

3.193.9 Mupad [B] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.55

$$\int \frac{\coth(x)}{a + a\cosh(x)}\,dx = \frac{1}{a\,\left(\mathrm{e}^{2\,x} + 2\,\mathrm{e}^{x} + 1\right)} - \frac{1}{a\,\left(\mathrm{e}^{x} + 1\right)} - \frac{\mathrm{atan}\left(\frac{\mathrm{e}^{x}\,\sqrt{-a^2}}{a}\right)}{\sqrt{-a^2}}$$

input
```
int(coth(x)/(a + a*cosh(x)),x)
```

output
```
1/(a*(exp(2*x) + 2*exp(x) + 1)) - 1/(a*(exp(x) + 1)) - atan((exp(x)*(-a^2)
^(1/2))/a)/(-a^2)^(1/2)
```

3.194 $\int \frac{\coth^2(x)}{a+a\cosh(x)}\,dx$

3.194.1 Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{\coth^2(x)}{a+a\cosh(x)}\,dx=\frac{\coth^3(x)}{3a}-\frac{\operatorname{csch}(x)}{a}-\frac{\operatorname{csch}^3(x)}{3a}$$

output
```
1/3*coth(x)^3/a-csch(x)/a-1/3*csch(x)^3/a
```

3.194.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{\coth^2(x)}{a+a\cosh(x)}\,dx=\frac{(-3-4\cosh(x)+\cosh(2x))\operatorname{csch}(x)}{6a(1+\cosh(x))}$$

input
```
Integrate[Coth[x]^2/(a + a*Cosh[x]),x]
```

output
```
((-3 - 4*Cosh[x] + Cosh[2*x])*Csch[x])/(6*a*(1 + Cosh[x]))
```

3.194.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {3042, 25, 3185, 25, 3042, 25, 3086, 2009, 3087, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan\left(-\frac{\pi}{2}+ix\right)^2}{a-a\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan\left(ix-\frac{\pi}{2}\right)^2}{a-a\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3185}$$

$$-\frac{\int -\coth^3(x)\text{csch}(x)dx}{a}-\frac{\int \coth^2(x)\text{csch}^2(x)dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \coth^3(x)\text{csch}(x)dx}{a}-\frac{\int \coth^2(x)\text{csch}^2(x)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -\sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^3dx}{a}-\frac{\int \sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a}-\frac{\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^3dx}{a}$$

$$\Big\downarrow \text{3086}$$

$$\frac{i\int \left(-\text{csch}^2(x)-1\right)d(-i\text{csch}(x))}{a}-\frac{\int \sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a}-\frac{\int \sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a}$$

$$\Big\downarrow \text{3087}$$

$$\frac{i\int -\coth^2(x)d(i\coth(x))}{a}+\frac{i\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a}$$

$$\Big\downarrow \text{15}$$

$$\frac{\coth^3(x)}{3a}+\frac{i\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a}$$

input
```
Int[Coth[x]^2/(a + a*Cosh[x]),x]
```

output
```
Coth[x]^3/(3*a) + (I*(I*Csch[x] + (I/3)*Csch[x]^3))/a
```

3.194.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

3.194.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{\frac{\tanh\left(\frac{x}{2}\right)^3}{3}+2\tanh\left(\frac{x}{2}\right)-\frac{1}{\tanh\left(\frac{x}{2}\right)}}{4a}$	29
risch	$-\frac{2(3\,\mathrm{e}^{3x}+3\,\mathrm{e}^{2x}+\mathrm{e}^{x}-1)}{3(\mathrm{e}^{x}+1)^3 a(\mathrm{e}^{x}-1)}$	34

input
```
int(coth(x)^2/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/4/a*(1/3*tanh(1/2*x)^3+2*tanh(1/2*x)-1/tanh(1/2*x))
```

3.194.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(26) = 52.

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 3.03

$$\int \frac{\coth^2(x)}{a+a\cosh(x)}\,dx =$$
$$-\frac{2\left(3\cosh(x)^2+2\left(3\cosh(x)+2\right)\sinh(x)+3\sinh(x)^2+2\cosh(x)+\right.}{3\left(a\cosh(x)^3+a\sinh(x)^3+2a\cosh(x)^2+(3a\cosh(x)+2a)\sinh(x)^2-a\cosh(x)+\left(3a\cosh(x)^2\right.\right.}$$

input
```
integrate(coth(x)^2/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
-2/3*(3*cosh(x)^2 + 2*(3*cosh(x) + 2)*sinh(x) + 3*sinh(x)^2 + 2*cosh(x) +
1)/(a*cosh(x)^3 + a*sinh(x)^3 + 2*a*cosh(x)^2 + (3*a*cosh(x) + 2*a)*sinh(x
)^2 - a*cosh(x) + (3*a*cosh(x)^2 + 4*a*cosh(x) + a)*sinh(x) - 2*a)
```

3.194.6 Sympy [F]

$$\int \frac{\coth^2(x)}{a + a\cosh(x)}\,dx = \frac{\int \frac{\coth^2(x)}{\cosh(x)+1}\,dx}{a}$$

input
```
integrate(coth(x)**2/(a+a*cosh(x)),x)
```

output
```
Integral(coth(x)**2/(cosh(x) + 1), x)/a
```

3.194.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(26) = 52.

Time = 0.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 4.03

$$\begin{aligned}\int \frac{\coth^2(x)}{a + a\cosh(x)}\,dx = &-\frac{2\,e^{(-x)}}{3\,(2\,ae^{(-x)} - 2\,ae^{(-3\,x)} - ae^{(-4\,x)} + a)} \\ &- \frac{2\,e^{(-2\,x)}}{2\,ae^{(-x)} - 2\,ae^{(-3\,x)} - ae^{(-4\,x)} + a} \\ &- \frac{2\,e^{(-3\,x)}}{2\,ae^{(-x)} - 2\,ae^{(-3\,x)} - ae^{(-4\,x)} + a} \\ &+ \frac{2}{3\,(2\,ae^{(-x)} - 2\,ae^{(-3\,x)} - ae^{(-4\,x)} + a)}\end{aligned}$$

input
```
integrate(coth(x)^2/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-2/3*e^(-x)/(2*a*e^(-x) - 2*a*e^(-3*x) - a*e^(-4*x) + a) - 2*e^(-2*x)/(2*a
*e^(-x) - 2*a*e^(-3*x) - a*e^(-4*x) + a) - 2*e^(-3*x)/(2*a*e^(-x) - 2*a*e^
(-3*x) - a*e^(-4*x) + a) + 2/3/(2*a*e^(-x) - 2*a*e^(-3*x) - a*e^(-4*x) + a
)
```

3.194.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17

$$\int \frac{\coth^2(x)}{a + a\cosh(x)}\,dx = -\frac{1}{2\,a(e^x - 1)} - \frac{9\,e^{(2\,x)} + 12\,e^x + 7}{6\,a(e^x + 1)^3}$$

input
```
integrate(coth(x)^2/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
-1/2/(a*(e^x - 1)) - 1/6*(9*e^(2*x) + 12*e^x + 7)/(a*(e^x + 1)^3)
```

3.194.9 Mupad [B] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 92, normalized size of antiderivative = 3.07

$$\int \frac{\coth^2(x)}{a + a\cosh(x)}\,dx = -\frac{\frac{e^{2\,x}}{2\,a} + \frac{1}{2\,a} + \frac{e^x}{3\,a}}{3\,e^{2\,x} + e^{3\,x} + 3\,e^x + 1} - \frac{\frac{1}{6\,a} + \frac{e^x}{2\,a}}{e^{2\,x} + 2\,e^x + 1} - \frac{1}{2\,a\,\left(e^x - 1\right)} - \frac{1}{2\,a\,\left(e^x + 1\right)}$$

input
```
int(coth(x)^2/(a + a*cosh(x)),x)
```

output
```
- (exp(2*x)/(2*a) + 1/(2*a) + exp(x)/(3*a))/(3*exp(2*x) + exp(3*x) + 3*exp
(x) + 1) - (1/(6*a) + exp(x)/(2*a))/(exp(2*x) + 2*exp(x) + 1) - 1/(2*a*(ex
p(x) - 1)) - 1/(2*a*(exp(x) + 1))
```

3.195 $\int \frac{\coth^3(x)}{a+a\cosh(x)}\,dx$

3.195.1 Optimal result

Integrand size = 13, antiderivative size = 46

$$\int \frac{\coth^3(x)}{a+a\cosh(x)}\,dx=-\frac{3\text{arctanh}(\cosh(x))}{8a}+\frac{\coth^4(x)}{4a}-\frac{3\coth(x)\text{csch}(x)}{8a}-\frac{\coth^3(x)\text{csch}(x)}{4a}$$

output
```
-3/8*arctanh(cosh(x))/a+1/4*coth(x)^4/a-3/8*coth(x)*csch(x)/a-1/4*coth(x)^
3*csch(x)/a
```

3.195.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int \frac{\coth^3(x)}{a+a\cosh(x)}\,dx =\frac{-8-2\coth^2\left(\frac{x}{2}\right)-12\cosh^2\left(\frac{x}{2}\right)\left(\log\left(\cosh\left(\frac{x}{2}\right)\right)-\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)+\text{sech}^2\left(\frac{x}{2}\right)}{16a(1+\cosh(x))}$$

input
```
Integrate[Coth[x]^3/(a + a*Cosh[x]),x]
```

output
```
(-8 - 2*Coth[x/2]^2 - 12*Cosh[x/2]^2*(Log[Cosh[x/2]] - Log[Sinh[x/2]]) + S
ech[x/2]^2)/(16*a*(1 + Cosh[x]))
```

3.195.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.28, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 1.308$, Rules used = {3042, 26, 3185, 26, 3042, 26, 3087, 15, 3091, 26, 3042, 26, 3091, 26, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\tan\left(-\frac{\pi}{2}+ix\right)^3}{a-a\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\tan\left(ix-\frac{\pi}{2}\right)^3}{a-a\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3185}$$

$$i\left(\frac{\int -i\coth^4(x)\text{csch}(x)dx}{a}+\frac{\int i\coth^3(x)\text{csch}^2(x)dx}{a}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{i\int \coth^3(x)\text{csch}^2(x)dx}{a}-\frac{i\int \coth^4(x)\text{csch}(x)dx}{a}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{i\int -i\sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^3dx}{a}-\frac{i\int i\sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^4dx}{a}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{\int \sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^3dx}{a}+\frac{\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^4dx}{a}\right)$$

$$\Big\downarrow \text{3087}$$

$$i\left(\frac{\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^4dx}{a}-\frac{i\int -i\coth^3(x)d(i\coth(x))}{a}\right)$$

$$\Big\downarrow \text{15}$$

$$i\left(\frac{\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^4dx}{a}-\frac{i\coth^4(x)}{4a}\right)$$

$$\Big\downarrow \; 3091$$

$$i\left(\frac{\frac{1}{4}i\coth^3(x)\text{csch}(x)-\frac{3}{4}\int i\coth^2(x)\text{csch}(x)dx}{a}-\frac{i\coth^4(x)}{4a}\right)$$

$$\Big\downarrow \; 26$$

$$i\left(\frac{\frac{1}{4}i\coth^3(x)\text{csch}(x)-\frac{3}{4}i\int \coth^2(x)\text{csch}(x)dx}{a}-\frac{i\coth^4(x)}{4a}\right)$$

$$\Big\downarrow \; 3042$$

$$i\left(\frac{\frac{1}{4}i\coth^3(x)\text{csch}(x)-\frac{3}{4}i\int -i\sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a}-\frac{i\coth^4(x)}{4a}\right)$$

$$\Big\downarrow \; 26$$

$$i\left(\frac{\frac{1}{4}i\coth^3(x)\text{csch}(x)-\frac{3}{4}\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a}-\frac{i\coth^4(x)}{4a}\right)$$

$$\Big\downarrow \; 3091$$

$$i\left(\frac{\frac{1}{4}i\coth^3(x)\text{csch}(x)-\frac{3}{4}\left(-\frac{1}{2}\int -i\text{csch}(x)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)}{a}-\frac{i\coth^4(x)}{4a}\right)$$

$$\Big\downarrow \; 26$$

$$i\left(\frac{\frac{1}{4}i\coth^3(x)\text{csch}(x)-\frac{3}{4}\left(\frac{1}{2}i\int \text{csch}(x)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)}{a}-\frac{i\coth^4(x)}{4a}\right)$$

$$\Big\downarrow \; 3042$$

$$i\left(\frac{\frac{1}{4}i\coth^3(x)\text{csch}(x)-\frac{3}{4}\left(\frac{1}{2}i\int i\csc(ix)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)}{a}-\frac{i\coth^4(x)}{4a}\right)$$

$$\Big\downarrow \; 26$$

$$i\left(\frac{\frac{1}{4}i\coth^3(x)\text{csch}(x)-\frac{3}{4}\left(-\frac{1}{2}\int \csc(ix)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)}{a}-\frac{i\coth^4(x)}{4a}\right)$$

$$\Big\downarrow \; 4257$$

$$i\left(\frac{\frac{1}{4}i\coth^3(x)\text{csch}(x)-\frac{3}{4}\left(-\frac{1}{2}i\text{arctanh}(\cosh(x))-\frac{1}{2}i\coth(x)\text{csch}(x)\right)}{a}-\frac{i\coth^4(x)}{4a}\right)$$

3.195. $\int \frac{\coth^3(x)}{a+a\cosh(x)}dx$

input
```
Int[Coth[x]^3/(a + a*Cosh[x]),x]
```

output
```
I*(((-1/4*I)*Coth[x]^4)/a + ((I/4)*Coth[x]^3*Csch[x] - (3*((-1/2*I)*ArcTan
h[Cosh[x]] - (I/2)*Coth[x]*Csch[x]))/4)/a)
```

3.195.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.195. $\int \frac{\coth^3(x)}{a+a\cosh(x)}\,dx$

3.195.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\frac{\tanh\left(\frac{x}{2}\right)^{4}}{4}+\frac{3\tanh\left(\frac{x}{2}\right)^{2}}{2}+3\ln\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{1}{2\tanh\left(\frac{x}{2}\right)^{2}}}{8a}$	38
risch	$-\frac{e^{x}\left(5\,e^{4x}+2\,e^{3x}+2\,e^{2x}+2\,e^{x}+5\right)}{4\left(e^{x}+1\right)^{4}a\left(e^{x}-1\right)^{2}}-\frac{3\ln\left(e^{x}+1\right)}{8a}+\frac{3\ln\left(e^{x}-1\right)}{8a}$	65

input
```
int(coth(x)^3/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/8/a*(1/4*tanh(1/2*x)^4+3/2*tanh(1/2*x)^2+3*ln(tanh(1/2*x))-1/2/tanh(1/2*
x)^2)
```

3.195.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 631 vs. 2(38) = 76.

Time = 0.26 (sec) , antiderivative size = 631, normalized size of antiderivative = 13.72

$$\int \frac{\coth^3(x)}{a+a\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^3/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
-1/8*(10*cosh(x)^5 + 2*(25*cosh(x) + 2)*sinh(x)^4 + 10*sinh(x)^5 + 4*cosh(
x)^4 + 4*(25*cosh(x)^2 + 4*cosh(x) + 1)*sinh(x)^3 + 4*cosh(x)^3 + 4*(25*co
sh(x)^3 + 6*cosh(x)^2 + 3*cosh(x) + 1)*sinh(x)^2 + 4*cosh(x)^2 + 3*(cosh(x
)^6 + 2*(3*cosh(x) + 1)*sinh(x)^5 + sinh(x)^6 + 2*cosh(x)^5 + (15*cosh(x)^
2 + 10*cosh(x) - 1)*sinh(x)^4 - cosh(x)^4 + 4*(5*cosh(x)^3 + 5*cosh(x)^2 -
 cosh(x) - 1)*sinh(x)^3 - 4*cosh(x)^3 + (15*cosh(x)^4 + 20*cosh(x)^3 - 6*c
osh(x)^2 - 12*cosh(x) - 1)*sinh(x)^2 - cosh(x)^2 + 2*(3*cosh(x)^5 + 5*cosh
(x)^4 - 2*cosh(x)^3 - 6*cosh(x)^2 - cosh(x) + 1)*sinh(x) + 2*cosh(x) + 1)*
log(cosh(x) + sinh(x) + 1) - 3*(cosh(x)^6 + 2*(3*cosh(x) + 1)*sinh(x)^5 +
sinh(x)^6 + 2*cosh(x)^5 + (15*cosh(x)^2 + 10*cosh(x) - 1)*sinh(x)^4 - cosh
(x)^4 + 4*(5*cosh(x)^3 + 5*cosh(x)^2 - cosh(x) - 1)*sinh(x)^3 - 4*cosh(x)^
3 + (15*cosh(x)^4 + 20*cosh(x)^3 - 6*cosh(x)^2 - 12*cosh(x) - 1)*sinh(x)^2
 - cosh(x)^2 + 2*(3*cosh(x)^5 + 5*cosh(x)^4 - 2*cosh(x)^3 - 6*cosh(x)^2 -
cosh(x) + 1)*sinh(x) + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*(25*c
osh(x)^4 + 8*cosh(x)^3 + 6*cosh(x)^2 + 4*cosh(x) + 5)*sinh(x) + 10*cosh(x)
)/(a*cosh(x)^6 + a*sinh(x)^6 + 2*a*cosh(x)^5 + 2*(3*a*cosh(x) + a)*sinh(x)
^5 - a*cosh(x)^4 + (15*a*cosh(x)^2 + 10*a*cosh(x) - a)*sinh(x)^4 - 4*a*cos
h(x)^3 + 4*(5*a*cosh(x)^3 + 5*a*cosh(x)^2 - a*cosh(x) - a)*sinh(x)^3 - a*c
osh(x)^2 + (15*a*cosh(x)^4 + 20*a*cosh(x)^3 - 6*a*cosh(x)^2 - 12*a*cosh(x)
 - a)*sinh(x)^2 + 2*a*cosh(x) + 2*(3*a*cosh(x)^5 + 5*a*cosh(x)^4 - 2*a*...
```

3.195.6 Sympy [F]

$$\int \frac{\coth^3(x)}{a + a\cosh(x)}\,dx = \frac{\int \frac{\coth^3(x)}{\cosh(x)+1}\,dx}{a}$$

input
```
integrate(coth(x)**3/(a+a*cosh(x)),x)
```

output
```
Integral(coth(x)**3/(cosh(x) + 1), x)/a
```

3.195.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(38) = 76.

Time = 0.20 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.24

$$\int \frac{\coth^3(x)}{a + a\cosh(x)}\,dx = -\frac{5\,e^{(-x)} + 2\,e^{(-2\,x)} + 2\,e^{(-3\,x)} + 2\,e^{(-4\,x)} + 5\,e^{(-5\,x)}}{4\,(2\,ae^{(-x)} - ae^{(-2\,x)} - 4\,ae^{(-3\,x)} - ae^{(-4\,x)} + 2\,ae^{(-5\,x)} + ae^{(-6\,x)} + a)} - \frac{3\,\log\left(e^{(-x)} + 1\right)}{8\,a} + \frac{3\,\log\left(e^{(-x)} - 1\right)}{8\,a}$$

input
```
integrate(coth(x)^3/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-1/4*(5*e^(-x) + 2*e^(-2*x) + 2*e^(-3*x) + 2*e^(-4*x) + 5*e^(-5*x))/(2*a*e
^(-x) - a*e^(-2*x) - 4*a*e^(-3*x) - a*e^(-4*x) + 2*a*e^(-5*x) + a*e^(-6*x)
 + a) - 3/8*log(e^(-x) + 1)/a + 3/8*log(e^(-x) - 1)/a
```

3.195.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(38) = 76.

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.04

$$\int \frac{\coth^3(x)}{a + a\cosh(x)}\,dx = -\frac{3\,\log\left(e^{(-x)} + e^{x} + 2\right)}{16\,a} + \frac{3\,\log\left(e^{(-x)} + e^{x} - 2\right)}{16\,a} - \frac{3\,e^{(-x)} + 3\,e^{x} - 2}{16\,a(e^{(-x)} + e^{x} - 2)} + \frac{9\,\left(e^{(-x)} + e^{x}\right)^2 + 4\,e^{(-x)} + 4\,e^{x} - 12}{32\,a(e^{(-x)} + e^{x} + 2)^2}$$

input
```
integrate(coth(x)^3/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
-3/16*log(e^(-x) + e^x + 2)/a + 3/16*log(e^(-x) + e^x - 2)/a - 1/16*(3*e^(
-x) + 3*e^x - 2)/(a*(e^(-x) + e^x - 2)) + 1/32*(9*(e^(-x) + e^x)^2 + 4*e^(
-x) + 4*e^x - 12)/(a*(e^(-x) + e^x + 2)^2)
```

3.195.9 Mupad [B] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.87

$$\int \frac{\coth^3(x)}{a+a\cosh(x)}\,dx = \frac{3}{2\,a\,\left(\mathrm{e}^{2\,x}+2\,\mathrm{e}^{x}+1\right)}-\frac{1}{4\,a\,\left(\mathrm{e}^{2\,x}-2\,\mathrm{e}^{x}+1\right)}+\frac{1}{2\,a\,\left(6\,\mathrm{e}^{2\,x}+4\,\mathrm{e}^{3\,x}+\mathrm{e}^{4\,x}+4\,\mathrm{e}^{x}+1\right)}-\frac{1}{4\,a\,\left(\mathrm{e}^{x}-1\right)}-\frac{1}{a\,\left(\mathrm{e}^{x}+1\right)}-\frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{x}\,\sqrt{-a^2}}{a}\right)}{4\,\sqrt{-a^2}}-\frac{1}{a\,\left(3\,\mathrm{e}^{2\,x}+\mathrm{e}^{3\,x}+3\,\mathrm{e}^{x}+1\right)}$$

input
```
int(coth(x)^3/(a + a*cosh(x)),x)
```

output
```
3/(2*a*(exp(2*x) + 2*exp(x) + 1)) - 1/(4*a*(exp(2*x) - 2*exp(x) + 1)) + 1/
(2*a*(6*exp(2*x) + 4*exp(3*x) + exp(4*x) + 4*exp(x) + 1)) - 1/(4*a*(exp(x)
 - 1)) - 1/(a*(exp(x) + 1)) - (3*atan((exp(x)*(-a^2)^(1/2))/a))/(4*(-a^2)^
(1/2)) - 1/(a*(3*exp(2*x) + exp(3*x) + 3*exp(x) + 1))
```

3.196 $\int \frac{\coth^4(x)}{a+a\cosh(x)}\,dx$

3.196.1 Optimal result

Integrand size = 13, antiderivative size = 41

$$\int \frac{\coth^4(x)}{a+a\cosh(x)}\,dx = \frac{\coth^5(x)}{5a} - \frac{\operatorname{csch}(x)}{a} - \frac{2\operatorname{csch}^3(x)}{3a} - \frac{\operatorname{csch}^5(x)}{5a}$$

output
```
1/5*coth(x)^5/a-csch(x)/a-2/3*csch(x)^3/a-1/5*csch(x)^5/a
```

3.196.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{\coth^4(x)}{a+a\cosh(x)}\,dx = -\frac{(-25+8\cosh(x)+36\cosh(2x)+24\cosh(3x)-3\cosh(4x))\operatorname{csch}^3(x)}{120a(1+\cosh(x))}$$

input
```
Integrate[Coth[x]^4/(a + a*Cosh[x]),x]
```

output
```
-1/120*((-25 + 8*Cosh[x] + 36*Cosh[2*x] + 24*Cosh[3*x] - 3*Cosh[4*x])*Csch
[x]^3)/(a*(1 + Cosh[x]))
```

3.196.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {3042, 3185, 25, 3042, 25, 3086, 210, 2009, 3087, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^4(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan\left(-\frac{\pi}{2}+ix\right)^4}{a-a\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3185}$$

$$\frac{\int \coth^5(x)\text{csch}(x)dx}{a}+\frac{\int -\coth^4(x)\text{csch}^2(x)dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \coth^5(x)\text{csch}(x)dx}{a}-\frac{\int \coth^4(x)\text{csch}^2(x)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^5dx}{a}-\frac{\int -\sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^4dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^4dx}{a}+\frac{\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^5dx}{a}$$

$$\Big\downarrow \text{3086}$$

$$\frac{\int \sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^4dx}{a}-\frac{i\int \left(-\text{csch}^2(x)-1\right)^2d(-i\text{csch}(x))}{a}$$

$$\Big\downarrow \text{210}$$

$$\frac{\int \sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^4dx}{a}-\frac{i\int \left(\text{csch}^4(x)+2\text{csch}^2(x)+1\right)d(-i\text{csch}(x))}{a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\int \sec\left(ix-\frac{\pi}{2}\right)^2 \tan\left(ix-\frac{\pi}{2}\right)^4 dx}{a}-\frac{i\left(-\frac{1}{5}i\text{csch}^5(x)-\frac{2}{3}i\text{csch}^3(x)-i\text{csch}(x)\right)}{a}$$

$$\Big\downarrow \text{3087}$$

$$-\frac{i\int \coth^4(x)d(i\coth(x))}{a}-\frac{i\left(-\frac{1}{5}i\text{csch}^5(x)-\frac{2}{3}i\text{csch}^3(x)-i\text{csch}(x)\right)}{a}$$

$$\Big\downarrow \text{15}$$

$$\frac{\coth^5(x)}{5a}-\frac{i\left(-\frac{1}{5}i\text{csch}^5(x)-\frac{2}{3}i\text{csch}^3(x)-i\text{csch}(x)\right)}{a}$$

input
```
Int[Coth[x]^4/(a + a*Cosh[x]),x]
```

output
```
Coth[x]^5/(5*a) - (I*((-I)*Csch[x] - ((2*I)/3)*Csch[x]^3 - (I/5)*Csch[x]^5
))/a
```

3.196.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

3.196.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{\frac{\tanh\left(\frac{x}{2}\right)^{5}}{5}+\frac{4\tanh\left(\frac{x}{2}\right)^{3}}{3}+6\tanh\left(\frac{x}{2}\right)-\frac{4}{\tanh\left(\frac{x}{2}\right)}-\frac{1}{3\tanh\left(\frac{x}{2}\right)^{3}}}{16a}$	45
risch	$-\frac{2\left(15\,\mathrm{e}^{7x}+15\,\mathrm{e}^{6x}-5\,\mathrm{e}^{5x}-25\,\mathrm{e}^{4x}+13\,\mathrm{e}^{3x}+21\,\mathrm{e}^{2x}+9\,\mathrm{e}^{x}-3\right)}{15\left(\mathrm{e}^{x}+1\right)^{5}a\left(\mathrm{e}^{x}-1\right)^{3}}$	60

input
```
int(coth(x)^4/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/16/a*(1/5*tanh(1/2*x)^5+4/3*tanh(1/2*x)^3+6*tanh(1/2*x)-4/tanh(1/2*x)-1/
3/tanh(1/2*x)^3)
```

3.196.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(35) = 70.

Time = 0.25 (sec) , antiderivative size = 224, normalized size of antiderivative = 5.46

$$\int \frac{\coth^4(x)}{a+a\cosh(x)}\,dx =$$

$$-\frac{2\left(15\cosh\left(x\right)^{4}+6\left(10\cosh\left(x\right)+3\right)\sinh\left(x\right)^{3}+15\sinh}{15\left(a\cosh\left(x\right)^{5}+a\sinh\left(x\right)^{5}+2\,a\cosh\left(x\right)^{4}+\left(5\,a\cosh\left(x\right)+2\,a\right)\sinh\left(x\right)^{4}-3\,a\cosh\left(x\right)^{3}+\left(10\,a\cosh\right.}$$

input
```
integrate(coth(x)^4/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
-2/15*(15*cosh(x)^4 + 6*(10*cosh(x) + 3)*sinh(x)^3 + 15*sinh(x)^4 + 12*cos
h(x)^3 + 2*(45*cosh(x)^2 + 18*cosh(x) + 2)*sinh(x)^2 + 4*cosh(x)^2 + 2*(30
*cosh(x)^3 + 27*cosh(x)^2 - 14*cosh(x) - 23)*sinh(x) - 4*cosh(x) + 13)/(a*
cosh(x)^5 + a*sinh(x)^5 + 2*a*cosh(x)^4 + (5*a*cosh(x) + 2*a)*sinh(x)^4 -
3*a*cosh(x)^3 + (10*a*cosh(x)^2 + 8*a*cosh(x) - a)*sinh(x)^3 - 8*a*cosh(x)
^2 + (10*a*cosh(x)^3 + 12*a*cosh(x)^2 - 9*a*cosh(x) - 8*a)*sinh(x)^2 + 2*a
*cosh(x) + (5*a*cosh(x)^4 + 8*a*cosh(x)^3 - 3*a*cosh(x)^2 - 8*a*cosh(x) -
2*a)*sinh(x) + 6*a)
```

3.196.6 Sympy [F]

$$\int \frac{\coth^4(x)}{a + a\cosh(x)}\,dx = \frac{\int \frac{\coth^4(x)}{\cosh(x)+1}\,dx}{a}$$

input
```
integrate(coth(x)**4/(a+a*cosh(x)),x)
```

output
```
Integral(coth(x)**4/(cosh(x) + 1), x)/a
```

3.196.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(35) = 70$.

Time = 0.20 (sec) , antiderivative size = 469, normalized size of antiderivative = 11.44

$$\int \frac{\coth^4(x)}{a + a\cosh(x)}\,dx = -\frac{6\,e^{(-x)}}{5\,(2\,ae^{(-x)} - 2\,ae^{(-2\,x)} - 6\,ae^{(-3\,x)} + 6\,ae^{(-5\,x)} + 2\,ae^{(-6\,x)} - 2\,ae^{(-7\,x)} - ae^{(-8\,x)} + a)} - \frac{14\,e^{(-2\,x)}}{5\,(2\,ae^{(-x)} - 2\,ae^{(-2\,x)} - 6\,ae^{(-3\,x)} + 6\,ae^{(-5\,x)} + 2\,ae^{(-6\,x)} - 2\,ae^{(-7\,x)} - ae^{(-8\,x)} + a)} - \frac{26\,e^{(-3\,x)}}{15\,(2\,ae^{(-x)} - 2\,ae^{(-2\,x)} - 6\,ae^{(-3\,x)} + 6\,ae^{(-5\,x)} + 2\,ae^{(-6\,x)} - 2\,ae^{(-7\,x)} - ae^{(-8\,x)} + a)} + \frac{10\,e^{(-4\,x)}}{3\,(2\,ae^{(-x)} - 2\,ae^{(-2\,x)} - 6\,ae^{(-3\,x)} + 6\,ae^{(-5\,x)} + 2\,ae^{(-6\,x)} - 2\,ae^{(-7\,x)} - ae^{(-8\,x)} + a)} + \frac{2\,e^{(-5\,x)}}{3\,(2\,ae^{(-x)} - 2\,ae^{(-2\,x)} - 6\,ae^{(-3\,x)} + 6\,ae^{(-5\,x)} + 2\,ae^{(-6\,x)} - 2\,ae^{(-7\,x)} - ae^{(-8\,x)} + a)} - \frac{2\,e^{(-6\,x)}}{2\,ae^{(-x)} - 2\,ae^{(-2\,x)} - 6\,ae^{(-3\,x)} + 6\,ae^{(-5\,x)} + 2\,ae^{(-6\,x)} - 2\,ae^{(-7\,x)} - ae^{(-8\,x)} + a} - \frac{2\,e^{(-7\,x)}}{2\,ae^{(-x)} - 2\,ae^{(-2\,x)} - 6\,ae^{(-3\,x)} + 6\,ae^{(-5\,x)} + 2\,ae^{(-6\,x)} - 2\,ae^{(-7\,x)} - ae^{(-8\,x)} + a} + \frac{2}{5\,(2\,ae^{(-x)} - 2\,ae^{(-2\,x)} - 6\,ae^{(-3\,x)} + 6\,ae^{(-5\,x)} + 2\,ae^{(-6\,x)} - 2\,ae^{(-7\,x)} - ae^{(-8\,x)} + a)}$$

input
```
integrate(coth(x)^4/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-6/5*e^(-x)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a
*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a) - 14/5*e^(-2*x)/(2*a*e^(-x) - 2
*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) -
a*e^(-8*x) + a) - 26/15*e^(-3*x)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x)
 + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a) + 10/3*e^(
-4*x)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6
*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a) + 2/3*e^(-5*x)/(2*a*e^(-x) - 2*a*e^(-
2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8
*x) + a) - 2*e^(-6*x)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-
5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a) - 2*e^(-7*x)/(2*a*e^(
-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-
7*x) - a*e^(-8*x) + a) + 2/5/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6
*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a)
```

3.196. $\int \frac{\coth^4(x)}{a+a\cosh(x)}\,dx$

3.196.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.44

$$\int \frac{\coth^4(x)}{a + a\cosh(x)}\,dx = -\frac{15\,e^{(2\,x)} - 24\,e^x + 13}{24\,a(e^x - 1)^3} - \frac{165\,e^{(4\,x)} + 480\,e^{(3\,x)} + 650\,e^{(2\,x)} + 400\,e^x + 113}{120\,a(e^x + 1)^5}$$

input
```
integrate(coth(x)^4/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
-1/24*(15*e^(2*x) - 24*e^x + 13)/(a*(e^x - 1)^3) - 1/120*(165*e^(4*x) + 48
0*e^(3*x) + 650*e^(2*x) + 400*e^x + 113)/(a*(e^x + 1)^5)
```

3.196.9 Mupad [B] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 263, normalized size of antiderivative = 6.41

$$\begin{aligned}\int \frac{\coth^4(x)}{a + a\cosh(x)}\,dx &= \frac{1}{6\,a\,\left(3\,e^{2\,x} - e^{3\,x} - 3\,e^x + 1\right)} - \frac{\frac{3\,e^{2\,x}}{8\,a} + \frac{11\,e^{3\,x}}{40\,a} + \frac{1}{8\,a} + \frac{17\,e^x}{40\,a}}{6\,e^{2\,x} + 4\,e^{3\,x} + e^{4\,x} + 4\,e^x + 1} \\ &\quad - \frac{\frac{11\,e^{2\,x}}{40\,a} + \frac{17}{120\,a} + \frac{e^x}{4\,a}}{3\,e^{2\,x} + e^{3\,x} + 3\,e^x + 1} - \frac{\frac{1}{8\,a} + \frac{11\,e^x}{40\,a}}{e^{2\,x} + 2\,e^x + 1} \\ &\quad - \frac{\frac{17\,e^{2\,x}}{20\,a} + \frac{e^{3\,x}}{2\,a} + \frac{11\,e^{4\,x}}{40\,a} + \frac{11}{40\,a} + \frac{e^x}{2\,a}}{10\,e^{2\,x} + 10\,e^{3\,x} + 5\,e^{4\,x} + e^{5\,x} + 5\,e^x + 1} \\ &\quad - \frac{1}{4\,a\,\left(e^{2\,x} - 2\,e^x + 1\right)} - \frac{5}{8\,a\,\left(e^x - 1\right)} - \frac{11}{40\,a\,\left(e^x + 1\right)}\end{aligned}$$

input
```
int(coth(x)^4/(a + a*cosh(x)),x)
```

output
```
1/(6*a*(3*exp(2*x) - exp(3*x) - 3*exp(x) + 1)) - ((3*exp(2*x))/(8*a) + (11
*exp(3*x))/(40*a) + 1/(8*a) + (17*exp(x))/(40*a))/(6*exp(2*x) + 4*exp(3*x)
 + exp(4*x) + 4*exp(x) + 1) - ((11*exp(2*x))/(40*a) + 17/(120*a) + exp(x)/
(4*a))/(3*exp(2*x) + exp(3*x) + 3*exp(x) + 1) - (1/(8*a) + (11*exp(x))/(40
*a))/(exp(2*x) + 2*exp(x) + 1) - ((17*exp(2*x))/(20*a) + exp(3*x)/(2*a) +
(11*exp(4*x))/(40*a) + 11/(40*a) + exp(x)/(2*a))/(10*exp(2*x) + 10*exp(3*x
) + 5*exp(4*x) + exp(5*x) + 5*exp(x) + 1) - 1/(4*a*(exp(2*x) - 2*exp(x) +
1)) - 5/(8*a*(exp(x) - 1)) - 11/(40*a*(exp(x) + 1))
```

3.197 $\int \sqrt{a+b\cosh(x)}\tanh(x)\,dx$

3.197.1 Optimal result

Integrand size = 13, antiderivative size = 37

$$\int \sqrt{a+b\cosh(x)}\tanh(x)\,dx = -2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)+2\sqrt{a+b\cosh(x)}$$

output
```
-2*arctanh((a+b*cosh(x))^(1/2)/a^(1/2))*a^(1/2)+2*(a+b*cosh(x))^(1/2)
```

3.197.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \sqrt{a+b\cosh(x)}\tanh(x)\,dx = -2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)+2\sqrt{a+b\cosh(x)}$$

input
```
Integrate[Sqrt[a + b*Cosh[x]]*Tanh[x],x]
```

output
```
-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Cosh[x]]/Sqrt[a]] + 2*Sqrt[a + b*Cosh[x]]
```

3.197.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3042, 26, 3200, 60, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(x)\sqrt{a+b\cosh(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sqrt{a-b\sin\left(-\frac{\pi}{2}+ix\right)}}{\tan\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sqrt{a-b\sin\left(ix-\frac{\pi}{2}\right)}}{\tan\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3200}$$

$$\int \frac{\operatorname{sech}(x)\sqrt{a+b\cosh(x)}}{b}d(b\cosh(x))$$

$$\Big\downarrow \text{60}$$

$$a\int \frac{\operatorname{sech}(x)}{b\sqrt{a+b\cosh(x)}}d(b\cosh(x))+2\sqrt{a+b\cosh(x)}$$

$$\Big\downarrow \text{73}$$

$$2a\int \frac{1}{b^2\cosh^2(x)-a}d\sqrt{a+b\cosh(x)}+2\sqrt{a+b\cosh(x)}$$

$$\Big\downarrow \text{220}$$

$$2\sqrt{a+b\cosh(x)}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)$$

input
```
Int[Sqrt[a + b*Cosh[x]]*Tanh[x],x]
```

output
```
-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Cosh[x]]/Sqrt[a]] + 2*Sqrt[a + b*Cosh[x]]
```

3.197.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.197. $\int \sqrt{a+b\cosh(x)}\tanh(x)\,dx$

3.197.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(29) = 58.

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 3.38

method	result
default	$2\sqrt{2\sinh\left(\frac{x}{2}\right)^{2} b+a+b}-\sqrt{a}\,\ln\left(\frac{4\cosh\left(\frac{x}{2}\right) b\sqrt{2}+4\sqrt{a}\,\sqrt{2\sinh\left(\frac{x}{2}\right)^{2} b+a+b}+4a-4b}{2\cosh\left(\frac{x}{2}\right)-\sqrt{2}}\right)-\sqrt{a}\,\ln\left(-\frac{4\left(\cosh\left(\frac{x}{2}\right) b\right.}{}\right.$

input
```
int((a+b*cosh(x))^(1/2)*tanh(x),x,method=_RETURNVERBOSE)
```

output
```
2*(2*sinh(1/2*x)^2*b+a+b)^(1/2)-a^(1/2)*ln(4/(2*cosh(1/2*x)-2^(1/2))*(cosh
(1/2*x)*b*2^(1/2)+a^(1/2)*(2*sinh(1/2*x)^2*b+a+b)^(1/2)+a-b))-a^(1/2)*ln(-
4/(2*cosh(1/2*x)+2^(1/2))*(cosh(1/2*x)*b*2^(1/2)-a^(1/2)*(2*sinh(1/2*x)^2*
b+a+b)^(1/2)-a+b))
```

3.197.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(29) = 58.

Time = 0.43 (sec) , antiderivative size = 376, normalized size of antiderivative = 10.16

$$\int \sqrt{a+b\cosh(x)}\tanh(x)\,dx$$

$$=\left[\frac{1}{2}\sqrt{a}\log\left(-\frac{b^{2}\cosh(x)^{4}+b^{2}\sinh(x)^{4}+16\,ab\cosh(x)^{3}+4\left(b^{2}\cosh(x)+4\,ab\right)\sinh(x)^{3}+16\,ab\cosh(x\cdots}{\cdots}\right.\right.$$

$$+2\sqrt{b\cosh(x)+a},\sqrt{-a}\arctan\left(\frac{\left(b\cosh(x)^{2}+b\sinh(x)^{2}+4\,a\cosh(x)+2\left(b\cosh(x)+2\,a\right)\sinh(x)\cdots}{2\left(ab\cosh(x)^{2}+ab\sinh(x)^{2}+2\,a^{2}\cosh(x)+ab+2\left(ab\cosh\cdots\right.\right.}\right.$$

$$\left.+2\sqrt{b\cosh(x)+a}\right]$$

input
```
integrate((a+b*cosh(x))^(1/2)*tanh(x),x, algorithm="fricas")
```

output
```
[1/2*sqrt(a)*log(-(b^2*cosh(x)^4 + b^2*sinh(x)^4 + 16*a*b*cosh(x)^3 + 4*(b
^2*cosh(x) + 4*a*b)*sinh(x)^3 + 16*a*b*cosh(x) + 2*(16*a^2 + b^2)*cosh(x)^
2 + 2*(3*b^2*cosh(x)^2 + 24*a*b*cosh(x) + 16*a^2 + b^2)*sinh(x)^2 - 8*(b*c
osh(x)^3 + b*sinh(x)^3 + 4*a*cosh(x)^2 + (3*b*cosh(x) + 4*a)*sinh(x)^2 + b
*cosh(x) + (3*b*cosh(x)^2 + 8*a*cosh(x) + b)*sinh(x))*sqrt(b*cosh(x) + a)*
sqrt(a) + b^2 + 4*(b^2*cosh(x)^3 + 12*a*b*cosh(x)^2 + 4*a*b + (16*a^2 + b^
2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*c
osh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) +
1)) + 2*sqrt(b*cosh(x) + a), sqrt(-a)*arctan(1/2*(b*cosh(x)^2 + b*sinh(x)^
2 + 4*a*cosh(x) + 2*(b*cosh(x) + 2*a)*sinh(x) + b)*sqrt(b*cosh(x) + a)*sqr
t(-a)/(a*b*cosh(x)^2 + a*b*sinh(x)^2 + 2*a^2*cosh(x) + a*b + 2*(a*b*cosh(x
) + a^2)*sinh(x))) + 2*sqrt(b*cosh(x) + a)]
```

3.197.6 Sympy [F]

$$\int \sqrt{a+b\cosh(x)}\tanh(x)\,dx = \int \sqrt{a+b\cosh{(x)}}\tanh{(x)}\,dx$$

input
```
integrate((a+b*cosh(x))**(1/2)*tanh(x),x)
```

output
```
Integral(sqrt(a + b*cosh(x))*tanh(x), x)
```

3.197.7 Maxima [F]

$$\int \sqrt{a+b\cosh(x)}\tanh(x)\,dx = \int \sqrt{b\cosh{(x)}+a}\tanh{(x)}\,dx$$

input
```
integrate((a+b*cosh(x))^(1/2)*tanh(x),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cosh(x) + a)*tanh(x), x)
```

3.197.8 Giac [F]

$$\int \sqrt{a+b\cosh(x)}\tanh(x)\,dx=\int \sqrt{b\cosh\left(x\right)+a}\tanh\left(x\right)\,dx$$

input
```
integrate((a+b*cosh(x))^(1/2)*tanh(x),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cosh(x) + a)*tanh(x), x)
```

3.197.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cosh(x)}\tanh(x)\,dx=\int \tanh(x)\,\sqrt{a+b\cosh\left(x\right)}\,dx$$

input
```
int(tanh(x)*(a + b*cosh(x))^(1/2),x)
```

output
```
int(tanh(x)*(a + b*cosh(x))^(1/2), x)
```

3.198 $\int \frac{\tanh(x)}{\sqrt{a+b\cosh(x)}}\,dx$

3.198.1 Optimal result

Integrand size = 13, antiderivative size = 24

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh(x)}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
-2*arctanh((a+b*cosh(x))^(1/2)/a^(1/2))/a^(1/2)
```

3.198.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh(x)}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[Tanh[x]/Sqrt[a + b*Cosh[x]],x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*Cosh[x]]/Sqrt[a]])/Sqrt[a]
```

3.198.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 26, 3200, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh(x)}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i}{\tan\left(-\frac{\pi}{2}+ix\right)\sqrt{a-b\sin\left(-\frac{\pi}{2}+ix\right)}}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{1}{\sqrt{a-b\sin\left(ix-\frac{\pi}{2}\right)}\tan\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3200}$$

$$\int \frac{\text{sech}(x)}{b\sqrt{a+b\cosh(x)}}d(b\cosh(x))$$

$$\Big\downarrow \text{ 73}$$

$$2\int \frac{1}{b^2\cosh^2(x)-a}d\sqrt{a+b\cosh(x)}$$

$$\Big\downarrow \text{ 220}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Int[Tanh[x]/Sqrt[a + b*Cosh[x]],x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*Cosh[x]]/Sqrt[a]])/Sqrt[a]
```

3.198.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.198.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(18) = 36$.

Time = 0.21 (sec) , antiderivative size = 104, normalized size of antiderivative = 4.33

method	result	size
default	$-\frac{\ln\left(\frac{4\cosh\left(\frac{x}{2}\right)b\sqrt{2}+4\sqrt{a}\,\sqrt{2\sinh\left(\frac{x}{2}\right)^{2}b+a+b}+4a-4b}{2\cosh\left(\frac{x}{2}\right)-\sqrt{2}}\right)+\ln\left(-\frac{4\left(\cosh\left(\frac{x}{2}\right)b\sqrt{2}-\sqrt{a}\,\sqrt{2\sinh\left(\frac{x}{2}\right)^{2}b+a+b}-a+b\right)}{2\cosh\left(\frac{x}{2}\right)+\sqrt{2}}\right)}{\sqrt{a}}$	104

input
```
int(tanh(x)/(a+b*cosh(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a^(1/2)*(ln(4/(2*cosh(1/2*x)-2^(1/2))*(cosh(1/2*x)*b*2^(1/2)+a^(1/2)*(2
*sinh(1/2*x)^2*b+a+b)^(1/2)+a-b))+ln(-4/(2*cosh(1/2*x)+2^(1/2))*(cosh(1/2*
x)*b*2^(1/2)-a^(1/2)*(2*sinh(1/2*x)^2*b+a+b)^(1/2)-a+b)))
```

3.198.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(18) = 36$.

Time = 0.29 (sec) , antiderivative size = 356, normalized size of antiderivative = 14.83

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh(x)}}\,dx$$

$$= \left[\frac{\log\left(\frac{b^2\cosh(x)^4+b^2\sinh(x)^4+16\,ab\cosh(x)^3+4\,(b^2\cosh(x)+4\,ab)\sinh(x)^3+16\,ab\cosh(x)+2\,(16\,a^2+b^2)\cosh(x)^2+2\,\left(3\,b^2\cosh(x)^2+24\,ab\,c\right.}{\cosh(x}\right.}{}\right.$$

input
```
integrate(tanh(x)/(a+b*cosh(x))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log((b^2*cosh(x)^4 + b^2*sinh(x)^4 + 16*a*b*cosh(x)^3 + 4*(b^2*cosh(x
) + 4*a*b)*sinh(x)^3 + 16*a*b*cosh(x) + 2*(16*a^2 + b^2)*cosh(x)^2 + 2*(3*
b^2*cosh(x)^2 + 24*a*b*cosh(x) + 16*a^2 + b^2)*sinh(x)^2 - 8*(b*cosh(x)^3
+ b*sinh(x)^3 + 4*a*cosh(x)^2 + (3*b*cosh(x) + 4*a)*sinh(x)^2 + b*cosh(x)
+ (3*b*cosh(x)^2 + 8*a*cosh(x) + b)*sinh(x))*sqrt(b*cosh(x) + a)*sqrt(a) +
 b^2 + 4*(b^2*cosh(x)^3 + 12*a*b*cosh(x)^2 + 4*a*b + (16*a^2 + b^2)*cosh(x
))*sinh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2
+ 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1))/sqrt(
a), sqrt(-a)*arctan(1/2*(b*cosh(x)^2 + b*sinh(x)^2 + 4*a*cosh(x) + 2*(b*co
sh(x) + 2*a)*sinh(x) + b)*sqrt(b*cosh(x) + a)*sqrt(-a)/(a*b*cosh(x)^2 + a*
b*sinh(x)^2 + 2*a^2*cosh(x) + a*b + 2*(a*b*cosh(x) + a^2)*sinh(x)))/a]
```

3.198.6 Sympy [F]

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh(x)}}\,dx = \int \frac{\tanh{(x)}}{\sqrt{a+b\cosh{(x)}}}\,dx$$

input
```
integrate(tanh(x)/(a+b*cosh(x))**(1/2),x)
```

output
```
Integral(tanh(x)/sqrt(a + b*cosh(x)), x)
```

3.198.7 Maxima [F]

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh(x)}}\,dx = \int \frac{\tanh{(x)}}{\sqrt{b\cosh{(x)}+a}}\,dx$$

input
```
integrate(tanh(x)/(a+b*cosh(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)/sqrt(b*cosh(x) + a), x)
```

3.198.8 Giac [F]

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh(x)}}\,dx = \int \frac{\tanh{(x)}}{\sqrt{b\cosh{(x)}+a}}\,dx$$

input
```
integrate(tanh(x)/(a+b*cosh(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(tanh(x)/sqrt(b*cosh(x) + a), x)
```

3.198.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh(x)}{\sqrt{a + b\cosh(x)}}\, dx = \int \frac{\tanh(x)}{\sqrt{a + b\cosh{(x)}}}\, dx$$

input
```
int(tanh(x)/(a + b*cosh(x))^(1/2),x)
```

output
```
int(tanh(x)/(a + b*cosh(x))^(1/2), x)
```

3.199 $\int \frac{A+B\sinh(x)}{a+b\cosh(x)}\,dx$

3.199.1 Optimal result

Integrand size = 15, antiderivative size = 56

$$\int \frac{A+B\sinh(x)}{a+b\cosh(x)}\,dx=\frac{2A\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}+\frac{B\log(a+b\cosh(x))}{b}$$

output
```
B*ln(a+b*cosh(x))/b+2*A*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)
^(1/2)/(a+b)^(1/2)
```

3.199.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

$$\int \frac{A+B\sinh(x)}{a+b\cosh(x)}\,dx=-\frac{2A\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+\frac{B\log(a+b\cosh(x))}{b}$$

input
```
Integrate[(A + B*Sinh[x])/(a + b*Cosh[x]),x]
```

output
```
(-2*A*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + (B*
Log[a + b*Cosh[x]])/b
```

3.199.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sinh(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{A-iB\sin(ix)}{a+b\cos(ix)}dx$$

$$\Big\downarrow \ 4901$$

$$\int \left(\frac{A}{a+b\cosh(x)}+\frac{B\sinh(x)}{a+b\cosh(x)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2A\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}+\frac{B\log(a+b\cosh(x))}{b}$$

input
```
Int[(A + B*Sinh[x])/(a + b*Cosh[x]),x]
```

output
```
(2*A*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b
]) + (B*Log[a + b*Cosh[x]])/b
```

3.199.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

3.199.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(46) = 92$.

Time = 0.21 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.00

method	result
default	$-\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b}+\frac{\frac{2(Ba-Bb)\ln\left(\tanh\left(\frac{x}{2}\right)^{2}a-\tanh\left(\frac{x}{2}\right)^{2}b-a-b\right)}{2a-2b}+\frac{2bA\,\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}}{b}-\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b}$
risch	$\frac{Bx}{b}+\frac{2xB\,a^{2}b}{-a^{2}b^{2}+b^{4}}-\frac{2xB\,b^{3}}{-a^{2}b^{2}+b^{4}}+\frac{\ln\left(\mathrm{e}^{x}+\frac{bAa-\sqrt{A^{2}a^{2}b^{2}-A^{2}b^{4}}}{b^{2}A}\right)a^{2}B}{(a^{2}-b^{2})b}-\frac{b\ln\left(\mathrm{e}^{x}+\frac{bAa-\sqrt{A^{2}a^{2}b^{2}-A^{2}b^{4}}}{b^{2}A}\right)B}{a^{2}-b^{2}}+\frac{\ln\left(\mathrm{e}^{x}+\frac{bAa-\sqrt{}}{}\right.}{}$

input
```
int((A+B*sinh(x))/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-B/b*ln(tanh(1/2*x)-1)+2/b*(1/2*(B*a-B*b)/(a-b)*ln(tanh(1/2*x)^2*a-tanh(1/
2*x)^2*b-a-b)+b*A/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-
b))^(1/2)))-B/b*ln(tanh(1/2*x)+1)
```

3.199.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(46) = 92$.

Time = 0.27 (sec) , antiderivative size = 291, normalized size of antiderivative = 5.20

$$\int \frac{A+B\sinh(x)}{a+b\cosh(x)}\,dx$$
$$=\left[\frac{\sqrt{a^2-b^2}Ab\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\,(b^2\cosh(x)+ab)\sinh(x)-2\,\sqrt{a^2-b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)-(}{a^2b-b^3}\right.$$
$$-\frac{2\,\sqrt{-a^2+b^2}Ab\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cosh(x)+b\sinh(x)+a)}{a^2-b^2}\right)+(Ba^2-Bb^2)x-(Ba^2-Bb^2)\log\left(\frac{2\,(b\cosh(x)+a}{\cosh(x)-\sinh(}\right.}{a^2b-b^3}$$

input
```
integrate((A+B*sinh(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[(sqrt(a^2 - b^2)*A*b*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x
) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x
) + a)*sinh(x) + b)) - (B*a^2 - B*b^2)*x + (B*a^2 - B*b^2)*log(2*(b*cosh(x
) + a)/(cosh(x) - sinh(x))))/(a^2*b - b^3), -(2*sqrt(-a^2 + b^2)*A*b*arcta
n(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) + (B*a^2 - B*
b^2)*x - (B*a^2 - B*b^2)*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))))/(a^2*
b - b^3)]
```

3.199.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 741 vs. $2(48) = 96$.

Time = 12.98 (sec) , antiderivative size = 741, normalized size of antiderivative = 13.23

$$\int \frac{A + B\sinh(x)}{a + b\cosh(x)}\,dx$$
$$= \begin{cases} \tilde{\infty}\left(2A\operatorname{atan}\left(\tanh\left(\frac{x}{2}\right)\right) + Bx - 2B\log\left(\tanh\left(\frac{x}{2}\right)+1\right) + B\log\left(\tanh^2\left(\frac{x}{2}\right)+1\right)\right) \\ \frac{A\tanh\left(\frac{x}{2}\right)}{b} + \frac{Bx}{b} - \frac{2B\log\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b} \\ -\frac{A}{b\tanh\left(\frac{x}{2}\right)} + \frac{Bx}{b} - \frac{2B\log\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b} + \frac{2B\log\left(\tanh\left(\frac{x}{2}\right)\right)}{b} \\ \frac{Ax+B\cosh(x)}{a} \\ -\frac{Ab\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} + \frac{Ab\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} + \frac{Bax\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} + \frac{Ba\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\log\left(-\sqrt{\cdots}\right.}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-\cdots} \end{cases}$$

input
```
integrate((A+B*sinh(x))/(a+b*cosh(x)),x)
```

output
```
Piecewise((zoo*(2*A*atan(tanh(x/2)) + B*x - 2*B*log(tanh(x/2) + 1) + B*log
(tanh(x/2)**2 + 1)), Eq(a, 0) & Eq(b, 0)), (A*tanh(x/2)/b + B*x/b - 2*B*lo
g(tanh(x/2) + 1)/b, Eq(a, b)), (-A/(b*tanh(x/2)) + B*x/b - 2*B*log(tanh(x/
2) + 1)/b + 2*B*log(tanh(x/2))/b, Eq(a, -b)), ((A*x + B*cosh(x))/a, Eq(b,
0)), (-A*b*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a -
b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) + A*b*log(sqrt(a/(a -
b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(
a/(a - b) + b/(a - b))) + B*a*x*sqrt(a/(a - b) + b/(a - b))/(a*b*sqrt(a/(a
 - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) + B*a*sqrt(a/(a - b
) + b/(a - b))*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(
a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) + B*a*sqrt(a/(a -
b) + b/(a - b))*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(
a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) - 2*B*a*sqrt(a/(a
- b) + b/(a - b))*log(tanh(x/2) + 1)/(a*b*sqrt(a/(a - b) + b/(a - b)) - b*
*2*sqrt(a/(a - b) + b/(a - b))) - B*b*x*sqrt(a/(a - b) + b/(a - b))/(a*b*s
qrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) - B*b*sqrt(
a/(a - b) + b/(a - b))*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*
sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) - B*b*sqrt
(a/(a - b) + b/(a - b))*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*
sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) + 2*B*b...
```

3.199.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\sinh(x)}{a + b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*sinh(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.199.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.07

$$\int \frac{A+B\sinh(x)}{a+b\cosh(x)}\,dx = \frac{2\,A\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} - \frac{Bx}{b} + \frac{B\log\left(be^{(2\,x)}+2\,ae^x+b\right)}{b}$$

input
```
integrate((A+B*sinh(x))/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
2*A*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2) - B*x/b + B*log(
b*e^(2*x) + 2*a*e^x + b)/b
```

3.199.9 Mupad [B] (verification not implemented)

Time = 3.86 (sec) , antiderivative size = 197, normalized size of antiderivative = 3.52

$$\int \frac{A+B\sinh(x)}{a+b\cosh(x)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{A^2\,b^2\,e^x\,\sqrt{b^2-a^2}}{(A\,b^3-A\,a^2\,b)\,\sqrt{A^2}} + \frac{A^2\,a\,b\,\sqrt{b^2-a^2}}{(A\,b^3-A\,a^2\,b)\,\sqrt{A^2}}\right)\sqrt{A^2}}{\sqrt{b^2-a^2}} - \frac{B\,x}{b} + \frac{B\,b^3\,\ln\left(4\,A^2\,b+8\,A^2\,a\,e^x+4\,A^2\,b\,e^{2\,x}\right)}{b^4-a^2\,b^2} - \frac{B\,a^2\,b\,\ln\left(4\,A^2\,b+8\,A^2\,a\,e^x+4\,A^2\,b\,e^{2\,x}\right)}{b^4-a^2\,b^2}$$

input
```
int((A + B*sinh(x))/(a + b*cosh(x)),x)
```

output
```
(2*atan((A^2*b^2*exp(x)*(b^2 - a^2)^(1/2))/((A*b^3 - A*a^2*b)*(A^2)^(1/2))
 + (A^2*a*b*(b^2 - a^2)^(1/2))/((A*b^3 - A*a^2*b)*(A^2)^(1/2)))*(A^2)^(1/2
))/(b^2 - a^2)^(1/2) - (B*x)/b + (B*b^3*log(4*A^2*b + 8*A^2*a*exp(x) + 4*A
^2*b*exp(2*x)))/(b^4 - a^2*b^2) - (B*a^2*b*log(4*A^2*b + 8*A^2*a*exp(x) +
4*A^2*b*exp(2*x)))/(b^4 - a^2*b^2)
```

3.200 $\int \frac{A+B\sinh(x)}{1+\cosh(x)}\,dx$

3.200.1 Optimal result

Integrand size = 13, antiderivative size = 18

$$\int \frac{A+B\sinh(x)}{1+\cosh(x)}\,dx = B\log(1+\cosh(x)) + \frac{A\sinh(x)}{1+\cosh(x)}$$

output
```
B*ln(1+cosh(x))+A*sinh(x)/(1+cosh(x))
```

3.200.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{A+B\sinh(x)}{1+\cosh(x)}\,dx = 2B\log\left(\cosh\left(\frac{x}{2}\right)\right) + A\tanh\left(\frac{x}{2}\right)$$

input
```
Integrate[(A + B*Sinh[x])/(1 + Cosh[x]),x]
```

output
```
2*B*Log[Cosh[x/2]] + A*Tanh[x/2]
```

3.200.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sinh(x)}{\cosh(x)+1}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A-iB\sin(ix)}{1+\cos(ix)}dx$$
$$\Big\downarrow \text{4901}$$
$$\int \left(\frac{A}{\cosh(x)+1}+\frac{B\sinh(x)}{\cosh(x)+1}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{A\sinh(x)}{\cosh(x)+1}+B\log(\cosh(x)+1)$$

input
```
Int[(A + B*Sinh[x])/(1 + Cosh[x]),x]
```

output
```
B*Log[1 + Cosh[x]] + (A*Sinh[x])/(1 + Cosh[x])
```

3.200.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

3.200.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

method	result	size
risch	$-Bx-\frac{2A}{{\mathrm e}^{x}+1}+2B\ln\left({\mathrm e}^{x}+1\right)$	23
default	$A\tanh\left(\frac{x}{2}\right)-B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	28

input
```
int((A+B*sinh(x))/(cosh(x)+1),x,method=_RETURNVERBOSE)
```

output
```
-B*x-2/(exp(x)+1)*A+2*B*ln(exp(x)+1)
```

3.200.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(18) = 36.

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.56

$$\int\frac{A+B\sinh(x)}{1+\cosh(x)}\,dx = -\frac{Bx\cosh\left(x\right)+Bx\sinh\left(x\right)+Bx-2\left(B\cosh\left(x\right)+B\sinh\left(x\right)+B\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)+2A}{\cosh\left(x\right)+\sinh\left(x\right)+1}$$

input
```
integrate((A+B*sinh(x))/(1+cosh(x)),x, algorithm="fricas")
```

output
```
-(B*x*cosh(x) + B*x*sinh(x) + B*x - 2*(B*cosh(x) + B*sinh(x) + B)*log(cosh
(x) + sinh(x) + 1) + 2*A)/(cosh(x) + sinh(x) + 1)
```

3.200.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int\frac{A+B\sinh(x)}{1+\cosh(x)}\,dx = A\tanh\left(\frac{x}{2}\right)+Bx-2B\log\left(\tanh\left(\frac{x}{2}\right)+1\right)$$

input
```
integrate((A+B*sinh(x))/(1+cosh(x)),x)
```

output
```
A*tanh(x/2) + B*x - 2*B*log(tanh(x/2) + 1)
```

3.200.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{A+B\sinh(x)}{1+\cosh(x)}\,dx = B\log\left(\cosh\left(x\right)+1\right)+\frac{2\,A}{e^{(-x)}+1}$$

input
```
integrate((A+B*sinh(x))/(1+cosh(x)),x, algorithm="maxima")
```

output
```
B*log(cosh(x) + 1) + 2*A/(e^(-x) + 1)
```

3.200.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{A+B\sinh(x)}{1+\cosh(x)}\,dx = -Bx+2\,B\log\left(e^{x}+1\right)-\frac{2\,A}{e^{x}+1}$$

input
```
integrate((A+B*sinh(x))/(1+cosh(x)),x, algorithm="giac")
```

output
```
-B*x + 2*B*log(e^x + 1) - 2*A/(e^x + 1)
```

3.200.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{A+B\sinh(x)}{1+\cosh(x)}\,dx = 2\,B\,\ln\left(\mathrm{e}^{x}+1\right)-\frac{2\,A}{\mathrm{e}^{x}+1}-B\,x$$

input
```
int((A + B*sinh(x))/(cosh(x) + 1),x)
```

output
```
2*B*log(exp(x) + 1) - (2*A)/(exp(x) + 1) - B*x
```

3.201 $\int \frac{A+B\sinh(x)}{1-\cosh(x)}\,dx$

3.201.1 Optimal result

Integrand size = 15, antiderivative size = 24

$$\int \frac{A+B\sinh(x)}{1-\cosh(x)}\,dx=-B\log(1-\cosh(x))-\frac{A\sinh(x)}{1-\cosh(x)}$$

output
```
-B*ln(1-cosh(x))-A*sinh(x)/(1-cosh(x))
```

3.201.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{A+B\sinh(x)}{1-\cosh(x)}\,dx=A\coth\left(\frac{x}{2}\right)-2B\log\left(\sinh\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[(A + B*Sinh[x])/(1 - Cosh[x]),x]
```

output
```
A*Coth[x/2] - 2*B*Log[Sinh[x/2]]
```

3.201.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sinh(x)}{1-\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A-iB\sin(ix)}{1-\cos(ix)}dx$$

$$\Big\downarrow \text{4901}$$

$$\int \left(-\frac{A}{\cosh(x)-1}-\frac{B\sinh(x)}{\cosh(x)-1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{A\sinh(x)}{1-\cosh(x)}-B\log(1-\cosh(x))$$

input
```
Int[(A + B*Sinh[x])/(1 - Cosh[x]),x]
```

output
```
-(B*Log[1 - Cosh[x]]) - (A*Sinh[x])/(1 - Cosh[x])
```

3.201.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

3.201.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

method	result	size
risch	$Bx+\frac{2A}{e^{x}-1}-2B\ln\left(e^{x}-1\right)$	22
default	$\frac{A}{\tanh\left(\frac{x}{2}\right)}-2B\ln\left(\tanh\left(\frac{x}{2}\right)\right)+B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	36

input
```
int((A+B*sinh(x))/(1-cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
B*x+2/(exp(x)-1)*A-2*B*ln(exp(x)-1)
```

3.201.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(21) = 42.

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{A+B\sinh(x)}{1-\cosh(x)}\,dx = \frac{Bx\cosh\left(x\right)+Bx\sinh\left(x\right)-Bx-2\left(B\cosh\left(x\right)+B\sinh\left(x\right)-B\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)+2\,A}{\cosh\left(x\right)+\sinh\left(x\right)-1}$$

input
```
integrate((A+B*sinh(x))/(1-cosh(x)),x, algorithm="fricas")
```

output
```
(B*x*cosh(x) + B*x*sinh(x) - B*x - 2*(B*cosh(x) + B*sinh(x) - B)*log(cosh(
x) + sinh(x) - 1) + 2*A)/(cosh(x) + sinh(x) - 1)
```

3.201.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int \frac{A+B\sinh(x)}{1-\cosh(x)}\,dx = \frac{A}{\tanh\left(\frac{x}{2}\right)}-Bx+2B\log\left(\tanh\left(\frac{x}{2}\right)+1\right)-2B\log\left(\tanh\left(\frac{x}{2}\right)\right)$$

input
```
integrate((A+B*sinh(x))/(1-cosh(x)),x)
```

output
```
A/tanh(x/2) - B*x + 2*B*log(tanh(x/2) + 1) - 2*B*log(tanh(x/2))
```

3.201.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{A + B\sinh(x)}{1 - \cosh(x)}\,dx = -B\log\left(\cosh\left(x\right) - 1\right) - \frac{2\,A}{e^{(-x)} - 1}$$

input
```
integrate((A+B*sinh(x))/(1-cosh(x)),x, algorithm="maxima")
```

output
```
-B*log(cosh(x) - 1) - 2*A/(e^(-x) - 1)
```

3.201.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{A + B\sinh(x)}{1 - \cosh(x)}\,dx = Bx - 2\,B\log\left(|e^{x} - 1|\right) + \frac{2\,A}{e^{x} - 1}$$

input
```
integrate((A+B*sinh(x))/(1-cosh(x)),x, algorithm="giac")
```

output
```
B*x - 2*B*log(abs(e^x - 1)) + 2*A/(e^x - 1)
```

3.201.9 Mupad [B] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int \frac{A + B\sinh(x)}{1 - \cosh(x)}\,dx = B\,x + \frac{2\,A}{\mathrm{e}^{x} - 1} - 2\,B\,\ln\left(\mathrm{e}^{x} - 1\right)$$

input
```
int(-(A + B*sinh(x))/(cosh(x) - 1),x)
```

output
```
B*x + (2*A)/(exp(x) - 1) - 2*B*log(exp(x) - 1)
```

3.202 $\int \frac{A+B\tanh(x)}{a+b\cosh(x)}\,dx$

3.202.1 Optimal result

Integrand size = 15, antiderivative size = 65

$$\int \frac{A+B\tanh(x)}{a+b\cosh(x)}\,dx = \frac{2A\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}} + \frac{B\log(\cosh(x))}{a} - \frac{B\log(a+b\cosh(x))}{a}$$

output
```
B*ln(cosh(x))/a-B*ln(a+b*cosh(x))/a+2*A*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a
+b)^(1/2))/(a-b)^(1/2)/(a+b)^(1/2)
```

3.202.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \frac{A+B\tanh(x)}{a+b\cosh(x)}\,dx = -\frac{2A\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} + \frac{B(\log(\cosh(x))-\log(a+b\cosh(x)))}{a}$$

input
```
Integrate[(A + B*Tanh[x])/(a + b*Cosh[x]),x]
```

output
```
(-2*A*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + (B*
(Log[Cosh[x]] - Log[a + b*Cosh[x]]))/a
```

3.202.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tanh(x)}{a+b\cosh(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A-iB\tan(ix)}{a+b\cos(ix)}dx$$
$$\Big\downarrow \text{4901}$$
$$\int \left(\frac{A}{a+b\cosh(x)}+\frac{B\tanh(x)}{a+b\cosh(x)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{2A\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}-\frac{B\log(a+b\cosh(x))}{a}+\frac{B\log(\cosh(x))}{a}$$

input
```
Int[(A + B*Tanh[x])/(a + b*Cosh[x]),x]
```

output
```
(2*A*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b
]) + (B*Log[Cosh[x]])/a - (B*Log[a + b*Cosh[x]])/a
```

3.202.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

3.202.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.54

method	result
default	$\frac{B\ln\left(1+\tanh\left(\frac{x}{2}\right)^{2}\right)}{a}+\frac{\frac{2(-Ba+Bb)\ln\left(\tanh\left(\frac{x}{2}\right)^{2}a-\tanh\left(\frac{x}{2}\right)^{2}b-a-b\right)}{2a-2b}+\frac{2Aa\ \operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}}{a}$
risch	$-\frac{2xB}{a}-\frac{2x\,a^{3}B}{-a^{4}+a^{2}b^{2}}+\frac{2xBa\,b^{2}}{-a^{4}+a^{2}b^{2}}+\frac{B\ln\left(1+e^{2x}\right)}{a}-\frac{a\ln\left(e^{x}+\frac{A\,a^{2}-\sqrt{A^{2}a^{4}-A^{2}a^{2}b^{2}}}{bAa}\right)B}{a^{2}-b^{2}}+\frac{\ln\left(e^{x}+\frac{A\,a^{2}-\sqrt{A^{2}a^{4}-A^{2}a^{2}b^{2}}}{bAa}\right)B}{(a^{2}-b^{2})a}$

input
```
int((A+B*tanh(x))/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
B/a*ln(1+tanh(1/2*x)^2)+2/a*(1/2*(-B*a+B*b)/(a-b)*ln(tanh(1/2*x)^2*a-tanh(
1/2*x)^2*b-a-b)+A*a/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(
a-b))^(1/2)))
```

3.202.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(55) = 110.

Time = 0.29 (sec) , antiderivative size = 315, normalized size of antiderivative = 4.85

$$\int \frac{A+B\tanh(x)}{a+b\cosh(x)}\,dx$$
$$=\left[\frac{\sqrt{a^{2}-b^{2}}Aa\log\left(\frac{b^{2}\cosh(x)^{2}+b^{2}\sinh(x)^{2}+2\,ab\cosh(x)+2\,a^{2}-b^{2}+2\,(b^{2}\cosh(x)+ab)\sinh(x)-2\,\sqrt{a^{2}-b^{2}}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^{2}+b\sinh(x)^{2}+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)-}{a^{3}-ab^{2}}\right.$$
$$-\frac{2\sqrt{-a^{2}+b^{2}}Aa\arctan\left(-\frac{\sqrt{-a^{2}+b^{2}}(b\cosh(x)+b\sinh(x)+a)}{a^{2}-b^{2}}\right)+(Ba^{2}-Bb^{2})\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right)-(Ba^{2}-Bb}{a^{3}-ab^{2}}$$

input
```
integrate((A+B*tanh(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[(sqrt(a^2 - b^2)*A*a*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x
) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x
) + a)*sinh(x) + b)) - (B*a^2 - B*b^2)*log(2*(b*cosh(x) + a)/(cosh(x) - si
nh(x))) + (B*a^2 - B*b^2)*log(2*cosh(x)/(cosh(x) - sinh(x))))/(a^3 - a*b^2
), -(2*sqrt(-a^2 + b^2)*A*a*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x
) + a)/(a^2 - b^2)) + (B*a^2 - B*b^2)*log(2*(b*cosh(x) + a)/(cosh(x) - sin
h(x))) - (B*a^2 - B*b^2)*log(2*cosh(x)/(cosh(x) - sinh(x))))/(a^3 - a*b^2)
]
```

3.202.6 Sympy [F]

$$\int \frac{A+B\tanh(x)}{a+b\cosh(x)}\,dx = \int \frac{A+B\tanh{(x)}}{a+b\cosh{(x)}}\,dx$$

input
```
integrate((A+B*tanh(x))/(a+b*cosh(x)),x)
```

output
```
Integral((A + B*tanh(x))/(a + b*cosh(x)), x)
```

3.202.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tanh(x)}{a+b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*tanh(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.202.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

$$\int \frac{A+B\tanh(x)}{a+b\cosh(x)}\,dx = \frac{2\,A\arctan\left(\frac{be^{x}+a}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}} - \frac{B\log\left(be^{(2\,x)}+2\,ae^{x}+b\right)}{a} + \frac{B\log\left(e^{(2\,x)}+1\right)}{a}$$

input
```
integrate((A+B*tanh(x))/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
2*A*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2) - B*log(b*e^(2*x
) + 2*a*e^x + b)/a + B*log(e^(2*x) + 1)/a
```

3.202.9 Mupad [B] (verification not implemented)

Time = 13.45 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.46

$$\int \frac{A+B\tanh(x)}{a+b\cosh(x)}\,dx = \frac{B\,\ln\left(16\,B^{2}\,b^{2}-16\,B^{2}\,a^{2}-16\,B^{2}\,a^{2}\,e^{2\,x}+16\,B^{2}\,b^{2}\,e^{2\,x}\right)}{a} - \frac{B\,\ln\left(16\,B^{2}\,b+32\,B^{2}\,a\,e^{x}+16\,B^{2}\,b\,e^{2\,x}\right)}{a} - \frac{2\,\mathrm{atan}\left(\frac{A^{2}\,b^{2}\,e^{x}\,\sqrt{b^{2}-a^{2}}+A^{2}\,a\,b\,\sqrt{b^{2}-a^{2}}}{A\,b\,(a^{2}-b^{2})\,\sqrt{A^{2}}}\right)\,\sqrt{A^{2}}}{\sqrt{b^{2}-a^{2}}}$$

input
```
int((A + B*tanh(x))/(a + b*cosh(x)),x)
```

output
```
(B*log(16*B^2*b^2 - 16*B^2*a^2 - 16*B^2*a^2*exp(2*x) + 16*B^2*b^2*exp(2*x)
))/a - (B*log(16*B^2*b + 32*B^2*a*exp(x) + 16*B^2*b*exp(2*x)))/a - (2*atan
((A^2*b^2*exp(x)*(b^2 - a^2)^(1/2) + A^2*a*b*(b^2 - a^2)^(1/2))/(A*b*(a^2
- b^2)*(A^2)^(1/2)))*(A^2)^(1/2))/(b^2 - a^2)^(1/2)
```

3.203 $\int \frac{A+B\coth(x)}{a+b\cosh(x)}\,dx$

3.203.1 Optimal result

Integrand size = 15, antiderivative size = 100

$$\int \frac{A+B\coth(x)}{a+b\cosh(x)}\,dx = \frac{2A\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}} + \frac{B\log(1-\cosh(x))}{2(a+b)} + \frac{B\log(1+\cosh(x))}{2(a-b)} - \frac{aB\log(a+b\cosh(x))}{a^2-b^2}$$

output
```
1/2*B*ln(1-cosh(x))/(a+b)+1/2*B*ln(1+cosh(x))/(a-b)-a*B*ln(a+b*cosh(x))/(a
^2-b^2)+2*A*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(1/2)/(a+b)
^(1/2)
```

3.203.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.34

$$\int \frac{A+B\coth(x)}{a+b\cosh(x)}\,dx = \frac{(A+B\coth(x))\left(-2A(a^2-b^2)\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)+\sqrt{-a^2+b^2}\,B\left((a+b)\log\left(\cosh\left(\frac{x}{2}\right)\right)-a\log(a+\cdots\right.\right.}{(a-b)(a+b)\sqrt{-a^2+b^2}(B\cosh(x)+A\sinh(x))}$$

input
```
Integrate[(A + B*Coth[x])/(a + b*Cosh[x]),x]
```

output
```
((A + B*Coth[x])*(-2*A*(a^2 - b^2)*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 +
b^2]] + Sqrt[-a^2 + b^2]*B*((a + b)*Log[Cosh[x/2]] - a*Log[a + b*Cosh[x]]
+ (a - b)*Log[Sinh[x/2]]))*Sinh[x])/((a - b)*(a + b)*Sqrt[-a^2 + b^2]*(B*C
osh[x] + A*Sinh[x]))
```

3.203.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\coth(x)}{a+b\cosh(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+iB\cot(ix)}{a+b\cos(ix)}dx$$

$$\Big\downarrow \text{4901}$$

$$\int \left(\frac{A}{a+b\cosh(x)}+\frac{B\coth(x)}{a+b\cosh(x)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{bB\text{arctanh}(\cosh(x))}{a^2-b^2}+\frac{aB\log(\sinh(x))}{a^2-b^2}-\frac{aB\log(a+b\cosh(x))}{a^2-b^2}+\frac{2A\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}$$

input
```
Int[(A + B*Coth[x])/(a + b*Cosh[x]),x]
```

output
```
(b*B*ArcTanh[Cosh[x]])/(a^2 - b^2) + (2*A*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/
Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]) - (a*B*Log[a + b*Cosh[x]])/(a^2 -
b^2) + (a*B*Log[Sinh[x]])/(a^2 - b^2)
```

3.203.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

3.203.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.01

method	result
default	$\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a+b}+\frac{-\frac{Ba\ln\left(\tanh\left(\frac{x}{2}\right)^{2}a-\tanh\left(\frac{x}{2}\right)^{2}b-a-b\right)}{a-b}-\frac{(-2Aa-2bA)\,\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}}{a+b}$
risch	$-\frac{xB}{a-b}-\frac{xB}{a+b}-\frac{2x\,a^{3}B}{-a^{4}+2a^{2}b^{2}-b^{4}}+\frac{2xBa\,b^{2}}{-a^{4}+2a^{2}b^{2}-b^{4}}+\frac{B\ln(\mathrm{e}^{x}+1)}{a-b}+\frac{B\ln(\mathrm{e}^{x}-1)}{a+b}-\frac{\ln\left(\mathrm{e}^{x}+\frac{Aa-\sqrt{A^{2}a^{2}-A^{2}b^{2}}}{bA}\right)Ba}{(a+b)(a-b)}+\frac{\ln\left(\right.}{}$

input
```
int((A+B*coth(x))/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
B/(a+b)*ln(tanh(1/2*x))+1/(a+b)*(-B*a/(a-b)*ln(tanh(1/2*x)^2*a-tanh(1/2*x)
^2*b-a-b)-(-2*A*a-2*A*b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a
+b)*(a-b))^(1/2)))
```

3.203.5 Fricas [A] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 303, normalized size of antiderivative = 3.03

$$\int \frac{A+B\coth(x)}{a+b\cosh(x)}\,dx$$
$$= \left[-\frac{Ba\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right) - \sqrt{a^2-b^2}A\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\,(b^2\cosh(x)+ab)\sinh(x)-2\,\sqrt{a}}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(}\right.}{a}\right.$$
$$-\frac{Ba\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right)+2\,\sqrt{-a^2+b^2}A\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cosh(x)+b\sinh(x)+a)}{a^2-b^2}\right)-(Ba+Bb)\log(\cosh(}{a^2-b^2}$$

input
```
integrate((A+B*coth(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[-(B*a*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) - sqrt(a^2 - b^2)*A*log(
(b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh
(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cos
h(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)) - (B*
a + B*b)*log(cosh(x) + sinh(x) + 1) - (B*a - B*b)*log(cosh(x) + sinh(x) -
1))/(a^2 - b^2), -(B*a*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) + 2*sqrt
(-a^2 + b^2)*A*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 -
 b^2)) - (B*a + B*b)*log(cosh(x) + sinh(x) + 1) - (B*a - B*b)*log(cosh(x)
+ sinh(x) - 1))/(a^2 - b^2)]
```

3.203.6 Sympy [F]

$$\int \frac{A+B\coth(x)}{a+b\cosh(x)}\,dx = \int \frac{A+B\coth{(x)}}{a+b\cosh{(x)}}\,dx$$

input
```
integrate((A+B*coth(x))/(a+b*cosh(x)),x)
```

output
```
Integral((A + B*coth(x))/(a + b*cosh(x)), x)
```

3.203.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\coth(x)}{a+b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*coth(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.203.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.90

$$\int \frac{A+B\coth(x)}{a+b\cosh(x)}\,dx = -\frac{Ba\log\left(be^{(2\,x)}+2\,ae^{x}+b\right)}{a^2-b^2} + \frac{2\,A\arctan\left(\frac{be^{x}+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} + \frac{B\log\left(e^{x}+1\right)}{a-b} + \frac{B\log\left(\left|e^{x}-1\right|\right)}{a+b}$$

input
```
integrate((A+B*coth(x))/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
-B*a*log(b*e^(2*x) + 2*a*e^x + b)/(a^2 - b^2) + 2*A*arctan((b*e^x + a)/sqr
t(-a^2 + b^2))/sqrt(-a^2 + b^2) + B*log(e^x + 1)/(a - b) + B*log(abs(e^x -
 1))/(a + b)
```

3.203.9 Mupad [B] (verification not implemented)

Time = 4.65 (sec) , antiderivative size = 974, normalized size of antiderivative = 9.74

$$\int \frac{A+B\coth(x)}{a+b\cosh(x)}\,dx = \frac{B\,\ln\left(\mathrm{e}^x+1\right)}{a-b}$$

$$+\ \ln\left(\left(\frac{32\,\left(A^2\,a^2\,b+2\,\mathrm{e}^x\,A^2\,a\,b^2+A^2\,b^3+8\,\mathrm{e}^x\,A\,B\,a^3+4\,A\,B\,a^2\,b-2\,\mathrm{e}^x\,A\,B\,a\,b^2+4\,\mathrm{e}^x\,B^2\,a^3+3\,B^2\,a^2\,b+5\,\mathrm{e}^x\,B^2\,a\,b^2+B^2\,b^3\right)}{b^5}+\frac{\left(A\,\sqrt{(a+b)^3\,(a-b)^3}-B\,a^3+\cdots\right.}{\cdots}\right.\right.\cdots$$

$$-\ \ln\left(-\frac{32\,B\,\left(A^2\,a\,b+\mathrm{e}^x\,A^2\,b^2+4\,\mathrm{e}^x\,A\,B\,a^2+2\,A\,B\,a\,b-\mathrm{e}^x\,A\,B\,b^2+4\,\mathrm{e}^x\,B^2\,a^2+B^2\,a\,b\right)}{b^5}-\frac{\left(\frac{32\,\left(A^2\,a^2\,b+2\,\mathrm{e}^x\,A^2\,a\,b^2+A^2\,b^3+8\,\mathrm{e}^x\,A\,B\,a^3+4\,A\,B\,\cdots\right.}{\cdots}\right.}{\cdots}\right.\cdots$$

$$+\ \frac{B\,\ln\left(\mathrm{e}^x-1\right)}{a+b}$$

input
```
int((A + B*coth(x))/(a + b*cosh(x)),x)
```

output
```
(B*log(exp(x) + 1))/(a - b) + (log((((32*(A^2*b^3 + B^2*b^3 + A^2*a^2*b +
3*B^2*a^2*b + 4*B^2*a^3*exp(x) + 5*B^2*a*b^2*exp(x) + 4*A*B*a^2*b + 8*A*B*
a^3*exp(x) + 2*A^2*a*b^2*exp(x) - 2*A*B*a*b^2*exp(x)))/b^5 + ((A*((a + b)^
3*(a - b)^3)^(1/2) - B*a^3 + B*a*b^2)*(128*exp(x)*(a^2 - b^2)^3*(A - 2*B)
+ a*b^5*(64*A - 128*B) + a^5*b*(64*A - 128*B) + 96*b^6*exp(x)*(A - 3*B) -
a^3*b^3*(128*A - 256*B) - 192*a^2*b^4*exp(x)*(A - 3*B) + 96*a^4*b^2*exp(x)
*(A - 3*B) + 128*A*a^3*exp(x)*((a^2 - b^2)^3)^(1/2) + 96*A*a^2*b*((a^2 - b
^2)^3)^(1/2) - 32*A*a*b^2*exp(x)*((a^2 - b^2)^3)^(1/2)))/((b^7 - a^2*b^5)*
(a^2 - b^2)^2))*(A*((a + b)^3*(a - b)^3)^(1/2) - B*a^3 + B*a*b^2))/(a^2 -
b^2)^2 - (32*B*(A^2*b^2*exp(x) + 4*B^2*a^2*exp(x) + A^2*a*b + B^2*a*b + 4*
A*B*a^2*exp(x) - A*B*b^2*exp(x) + 2*A*B*a*b))/b^5)*(A*((a + b)^3*(a - b)^3
)^(1/2) - B*a^3 + B*a*b^2))/(a^4 + b^4 - 2*a^2*b^2) - (log(- (32*B*(A^2*b^
2*exp(x) + 4*B^2*a^2*exp(x) + A^2*a*b + B^2*a*b + 4*A*B*a^2*exp(x) - A*B*b
^2*exp(x) + 2*A*B*a*b))/b^5 - (((32*(A^2*b^3 + B^2*b^3 + A^2*a^2*b + 3*B^2
*a^2*b + 4*B^2*a^3*exp(x) + 5*B^2*a*b^2*exp(x) + 4*A*B*a^2*b + 8*A*B*a^3*e
xp(x) + 2*A^2*a*b^2*exp(x) - 2*A*B*a*b^2*exp(x)))/b^5 - ((B*a^3 + A*((a +
b)^3*(a - b)^3)^(1/2) - B*a*b^2)*(128*exp(x)*(a^2 - b^2)^3*(A - 2*B) + a*b
^5*(64*A - 128*B) + a^5*b*(64*A - 128*B) + 96*b^6*exp(x)*(A - 3*B) - a^3*b
^3*(128*A - 256*B) - 192*a^2*b^4*exp(x)*(A - 3*B) + 96*a^4*b^2*exp(x)*(A -
 3*B) - 128*A*a^3*exp(x)*((a^2 - b^2)^3)^(1/2) - 96*A*a^2*b*((a^2 - b^2...
```

3.203. $\int \frac{A+B\coth(x)}{a+b\cosh(x)}\,dx$

3.204 $\int \frac{A+B\text{sech}(x)}{a+b\cosh(x)}\,dx$

3.204.1 Optimal result

Integrand size = 15, antiderivative size = 62

$$\int \frac{A+B\text{sech}(x)}{a+b\cosh(x)}\,dx=\frac{B\arctan(\sinh(x))}{a}+\frac{2(aA-bB)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}$$

output
```
B*arctan(sinh(x))/a+2*(A*a-B*b)*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2
))/a/(a-b)^(1/2)/(a+b)^(1/2)
```

3.204.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02

$$\int \frac{A+B\text{sech}(x)}{a+b\cosh(x)}\,dx=\frac{2\left(B\arctan\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{(-aA+bB)\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}\right)}{a}$$

input
```
Integrate[(A + B*Sech[x])/(a + b*Cosh[x]),x]
```

output
```
(2*(B*ArcTan[Tanh[x/2]] + ((-(a*A) + b*B)*ArcTan[((a - b)*Tanh[x/2])/Sqrt[
-a^2 + b^2]])/Sqrt[-a^2 + b^2]))/a
```

3.204.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3042, 3307, 3042, 3480, 3042, 3138, 221, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\text{sech}(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\csc\left(\frac{\pi}{2}+ix\right)}{a+b\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3307}$$

$$\int \frac{\text{sech}(x)(A\cosh(x)+B)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B+A\sin\left(\frac{\pi}{2}+ix\right)}{\sin\left(\frac{\pi}{2}+ix\right)\left(a+b\sin\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{3480}$$

$$\frac{(aA-bB)\int \frac{1}{a+b\cosh(x)}dx}{a}+\frac{B\int \text{sech}(x)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(aA-bB)\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a}+\frac{B\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \text{3138}$$

$$\frac{2(aA-bB)\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a}+\frac{B\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \text{221}$$

$$\frac{2(aA-bB)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}+\frac{B\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \text{4257}$$

$$\frac{2(aA-bB)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}+\frac{B\arctan(\sinh(x))}{a}$$

input
```
Int[(A + B*Sech[x])/(a + b*Cosh[x]),x]
```

output
```
(B*ArcTan[Sinh[x]])/a + (2*(a*A - b*B)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqr
t[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b])
```

3.204.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3307
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((d + c*Sin[e +
 f*x])^n/Sin[e + f*x]^n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.204.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

method	result
default	$\frac{2B\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}-\frac{2(-Aa+Bb)\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)(a-b)}}$
risch	$\frac{iB\ln(\mathrm{e}^{x}+i)}{a}-\frac{iB\ln(\mathrm{e}^{x}-i)}{a}+\frac{\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}-b^{2}}-a^{2}+b^{2}}{b\sqrt{a^{2}-b^{2}}}\right)A}{\sqrt{a^{2}-b^{2}}}-\frac{\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}-b^{2}}-a^{2}+b^{2}}{b\sqrt{a^{2}-b^{2}}}\right)Bb}{\sqrt{a^{2}-b^{2}}\,a}-\frac{\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}-b^{2}}+a^{2}-b^{2}}{b\sqrt{a^{2}-b^{2}}}\right)A}{\sqrt{a^{2}-b^{2}}}+$

input
```
int((A+B*sech(x))/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
2*B/a*arctan(tanh(1/2*x))-2*(-A*a+B*b)/a/((a+b)*(a-b))^(1/2)*arctanh((a-b)
*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

3.204.5 Fricas [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 249, normalized size of antiderivative = 4.02

$$\int \frac{A+B\mathrm{sech}(x)}{a+b\cosh(x)}\,dx = \left[-\frac{(Aa-Bb)\sqrt{a^2-b^2}\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\,(b^2\cosh(x)+ab)\sinh(x)+2\,\sqrt{a^2-b^2}(b\cosh(x)+b\sinh(\right.}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}}{a^3-ab^2}\right.$$
$$-\frac{2\left((Aa-Bb)\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cosh(x)+b\sinh(x)+a)}{a^2-b^2}\right)-(Ba^2-Bb^2)\arctan\left(\cosh\left(x\right)+\sinh\right.\right.}{a^3-ab^2}$$

input
```
integrate((A+B*sech(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[-((A*a - B*b)*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*
cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*
(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*
(b*cosh(x) + a)*sinh(x) + b)) - 2*(B*a^2 - B*b^2)*arctan(cosh(x) + sinh(x)
))/(a^3 - a*b^2), -2*((A*a - B*b)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2
)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) - (B*a^2 - B*b^2)*arctan(cosh(x
) + sinh(x)))/(a^3 - a*b^2)]
```

3.204.6 Sympy [F]

$$\int \frac{A+B\text{sech}(x)}{a+b\cosh(x)}\,dx = \int \frac{A+B\,\text{sech}\,(x)}{a+b\cosh\,(x)}\,dx$$

input
```
integrate((A+B*sech(x))/(a+b*cosh(x)),x)
```

output
```
Integral((A + B*sech(x))/(a + b*cosh(x)), x)
```

3.204.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\text{sech}(x)}{a+b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*sech(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.204.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85

$$\int \frac{A+B\text{sech}(x)}{a+b\cosh(x)}\,dx = \frac{2\,B\arctan\,(e^x)}{a} + \frac{2\,(Aa-Bb)\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}a}$$

input
```
integrate((A+B*sech(x))/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
2*B*arctan(e^x)/a + 2*(A*a - B*b)*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sq
rt(-a^2 + b^2)*a)
```

3.204.9 Mupad [B] (verification not implemented)

Time = 7.26 (sec) , antiderivative size = 636, normalized size of antiderivative = 10.26

$$\int \frac{A+B\mathrm{sech}(x)}{a+b\cosh(x)}\,dx$$

$$= \frac{\ln\left(\frac{\sqrt{(a+b)\,(a-b)}\,(A\,a-B\,b)\left(\frac{32\left(A^2\,a^2\,b-2\,A\,B\,a\,b^2-4\,\mathrm{e}^x\,B^2\,a^3-2\,B^2\,a^2\,b+3\,\mathrm{e}^x\,B^2\,a\,b^2+2\,B^2\,b^3\right)}{b^5}+\frac{\sqrt{(a+b)\,(a-b)}\,(A\,a-B\,b)\left(\frac{32\,a^2\left(2\,B\,b^2-4\,A\,a^2\,\mathrm{e}^x\right.}{}\right.}{}\right.}{a\,b^2-a^3}\right)}{}$$

$$-\frac{\ln\left(-\frac{\sqrt{(a+b)\,(a-b)}\,(A\,a-B\,b)\left(\frac{32\left(A^2\,a^2\,b-2\,A\,B\,a\,b^2-4\,\mathrm{e}^x\,B^2\,a^3-2\,B^2\,a^2\,b+3\,\mathrm{e}^x\,B^2\,a\,b^2+2\,B^2\,b^3\right)}{b^5}-\frac{\sqrt{(a+b)\,(a-b)}\,(A\,a-B\,b)\left(\frac{32\,a^2\left(2\,B\,b^2-4\right.}{}\right.}{}\right.}{a\,b^2-a^3}\right)}{}$$

$$-\frac{B\,\ln\left(\mathrm{e}^x-\mathrm{i}\right)\,1\mathrm{i}}{a}+\frac{B\,\ln\left(\mathrm{e}^x+1\mathrm{i}\right)\,1\mathrm{i}}{a}$$

input
```
int((A + B/cosh(x))/(a + b*cosh(x)),x)
```

output
```
(B*log(exp(x) + 1i)*1i)/a - (B*log(exp(x) - 1i)*1i)/a + (log((((a + b)*(a
- b))^(1/2)*(A*a - B*b)*((32*(2*B^2*b^3 + A^2*a^2*b - 2*B^2*a^2*b - 4*B^2*
a^3*exp(x) + 3*B^2*a*b^2*exp(x) - 2*A*B*a*b^2))/b^5 + (((a + b)*(a - b))^(
1/2)*(A*a - B*b)*((32*a^2*(2*B*b^2 - 4*A*a^2*exp(x) + A*b^2*exp(x) - 2*A*a
*b + 3*B*a*b*exp(x)))/b^5 - (32*a^2*((a + b)*(a - b))^(1/2)*(A*a - B*b)*(3
*a^2*b - 2*b^3 + 4*a^3*exp(x) - 3*a*b^2*exp(x)))/(b^5*(a*b^2 - a^3))))/(a*
b^2 - a^3)))/(a*b^2 - a^3) - (32*B*(A*a - B*b)*(2*B*b - A*b*exp(x) + 4*B*a
*exp(x)))/b^5)*((a + b)*(a - b))^(1/2)*(A*a - B*b))/(a*b^2 - a^3) - (log(-
 (((a + b)*(a - b))^(1/2)*(A*a - B*b)*((32*(2*B^2*b^3 + A^2*a^2*b - 2*B^2*
a^2*b - 4*B^2*a^3*exp(x) + 3*B^2*a*b^2*exp(x) - 2*A*B*a*b^2))/b^5 - (((a +
 b)*(a - b))^(1/2)*(A*a - B*b)*((32*a^2*(2*B*b^2 - 4*A*a^2*exp(x) + A*b^2*
exp(x) - 2*A*a*b + 3*B*a*b*exp(x)))/b^5 + (32*a^2*((a + b)*(a - b))^(1/2)*
(A*a - B*b)*(3*a^2*b - 2*b^3 + 4*a^3*exp(x) - 3*a*b^2*exp(x)))/(b^5*(a*b^2
 - a^3))))/(a*b^2 - a^3)))/(a*b^2 - a^3) - (32*B*(A*a - B*b)*(2*B*b - A*b*
exp(x) + 4*B*a*exp(x)))/b^5)*((a + b)*(a - b))^(1/2)*(A*a - B*b))/(a*b^2 -
 a^3)
```

3.205 $\int \frac{A+B\operatorname{csch}(x)}{a+b\cosh(x)}\,dx$

3.205.1 Optimal result

Integrand size = 15, antiderivative size = 99

$$\int \frac{A+B\operatorname{csch}(x)}{a+b\cosh(x)}\,dx = \frac{2A\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}} + \frac{B\log(1-\cosh(x))}{2(a+b)} - \frac{B\log(1+\cosh(x))}{2(a-b)} + \frac{bB\log(a+b\cosh(x))}{a^2-b^2}$$

output
```
1/2*B*ln(1-cosh(x))/(a+b)-1/2*B*ln(1+cosh(x))/(a-b)+b*B*ln(a+b*cosh(x))/(a
^2-b^2)+2*A*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(1/2)/(a+b)
^(1/2)
```

3.205.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.17

$$\int \frac{A+B\operatorname{csch}(x)}{a+b\cosh(x)}\,dx = \frac{-2A(a^2-b^2)\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right) - \sqrt{-a^2+b^2}B\left((a+b)\log\left(\cosh\left(\frac{x}{2}\right)\right) - b\log(a+b\cosh(x)) + (-a+\right.}{(a-b)(a+b)\sqrt{-a^2+b^2}}$$

input
```
Integrate[(A + B*Csch[x])/(a + b*Cosh[x]),x]
```

output
```
(-2*A*(a^2 - b^2)*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]] - Sqrt[-a^2
 + b^2]*B*((a + b)*Log[Cosh[x/2]] - b*Log[a + b*Cosh[x]] + (-a + b)*Log[Si
nh[x/2]]))/((a - b)*(a + b)*Sqrt[-a^2 + b^2])
```

3.205.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3042, 4713, 26, 26, 3042, 26, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\text{csch}(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+iB\csc(ix)}{a+b\cos(ix)}dx$$

$$\Big\downarrow \text{4713}$$

$$\int -\frac{i\text{csch}(x)(iA\sinh(x)+iB)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{i\text{csch}(x)(B+A\sinh(x))}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\text{csch}(x)(A\sinh(x)+B)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i(B-iA\sin(ix))}{\sin(ix)(a+b\cos(ix))}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{B-iA\sin(ix)}{(a+b\cos(ix))\sin(ix)}dx$$

$$\Big\downarrow \text{4901}$$

$$i\int \left(-\frac{iA}{a+b\cosh(x)}-\frac{iB\text{csch}(x)}{a+b\cosh(x)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$i\left(-\frac{ibB\log(a+b\cosh(x))}{a^2-b^2}-\frac{2iA\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}-\frac{iB\log(1-\cosh(x))}{2(a+b)}+\frac{iB\log(\cosh(x)+1)}{2(a-b)}\right)$$

input
```
Int[(A + B*Csch[x])/(a + b*Cosh[x]),x]
```

output
```
I*(((-2*I)*A*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sq
rt[a + b]) - ((I/2)*B*Log[1 - Cosh[x]])/(a + b) + ((I/2)*B*Log[1 + Cosh[x]
])/(a - b) - (I*b*B*Log[a + b*Cosh[x]])/(a^2 - b^2))
```

3.205.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4713
```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

3.205. $\int \frac{A+B\text{csch}(x)}{a+b\cosh(x)}\,dx$

3.205.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01

method	result
default	$\frac{B\ln(\tanh(\frac{x}{2}))}{a+b}+\frac{\frac{Bb\ln\left(\tanh\left(\frac{x}{2}\right)^2a-\tanh\left(\frac{x}{2}\right)^2b-a-b\right)}{a-b}-\frac{(-2Aa-2bA)\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}}{a+b}$
risch	$\frac{xB}{a-b}-\frac{xB}{a+b}+\frac{2xB\,a^2b}{-a^4+2a^2b^2-b^4}-\frac{2xB\,b^3}{-a^4+2a^2b^2-b^4}-\frac{B\ln(e^x+1)}{a-b}+\frac{B\ln(e^x-1)}{a+b}+\frac{\ln\left(e^x+\frac{Aa-\sqrt{A^2a^2-A^2b^2}}{bA}\right)Bb}{(a+b)(a-b)}+\frac{\ln\left(e^x\right.}{}$

input
```
int((A+B*csch(x))/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
B/(a+b)*ln(tanh(1/2*x))+1/(a+b)*(B*b/(a-b)*ln(tanh(1/2*x)^2*a-tanh(1/2*x)^
2*b-a-b)-(-2*A*a-2*A*b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+
b)*(a-b))^(1/2)))
```

3.205.5 Fricas [A] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 298, normalized size of antiderivative = 3.01

$$\int \frac{A+B\text{csch}(x)}{a+b\cosh(x)}\,dx$$
$$=\left[\frac{Bb\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right)+\sqrt{a^2-b^2}A\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\,(b^2\cosh(x)+ab)\sinh(x)-2\,\sqrt{a^2-}}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+}\right.}{a^2}\right.$$

input
```
integrate((A+B*csch(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[(B*b*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) + sqrt(a^2 - b^2)*A*log((
b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(
x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh
(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)) - (B*a
 + B*b)*log(cosh(x) + sinh(x) + 1) + (B*a - B*b)*log(cosh(x) + sinh(x) - 1
))/(a^2 - b^2), (B*b*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) - 2*sqrt(-
a^2 + b^2)*A*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b
^2)) - (B*a + B*b)*log(cosh(x) + sinh(x) + 1) + (B*a - B*b)*log(cosh(x) +
sinh(x) - 1))/(a^2 - b^2)]
```

3.205.6 Sympy [F]

$$\int \frac{A + B\text{csch}(x)}{a + b\cosh(x)}\,dx = \int \frac{A + B\,\text{csch}\,(x)}{a + b\cosh\,(x)}\,dx$$

input
```
integrate((A+B*csch(x))/(a+b*cosh(x)),x)
```

output
```
Integral((A + B*csch(x))/(a + b*cosh(x)), x)
```

3.205.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\text{csch}(x)}{a + b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*csch(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.205.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.91

$$\int \frac{A + B\text{csch}(x)}{a + b\cosh(x)}\,dx = \frac{Bb\log\left(be^{(2\,x)} + 2\,ae^{x} + b\right)}{a^2 - b^2} + \frac{2\,A\arctan\left(\frac{be^{x}+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2 + b^2}} - \frac{B\log\left(e^{x} + 1\right)}{a - b} + \frac{B\log\left(|e^{x} - 1|\right)}{a + b}$$

input
```
integrate((A+B*csch(x))/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
B*b*log(b*e^(2*x) + 2*a*e^x + b)/(a^2 - b^2) + 2*A*arctan((b*e^x + a)/sqrt
(-a^2 + b^2))/sqrt(-a^2 + b^2) - B*log(e^x + 1)/(a - b) + B*log(abs(e^x -
1))/(a + b)
```

3.205.9 Mupad [B] (verification not implemented)

Time = 4.06 (sec) , antiderivative size = 983, normalized size of antiderivative = 9.93

$$\int \frac{A + B\text{csch}(x)}{a + b\cosh(x)}\,dx$$

$$= \frac{\ln\left(\frac{\left(\frac{32\left(A^2\,a^2\,b+2\,e^x\,A^2\,a\,b^2+A^2\,b^3-8\,e^x\,A\,B\,a^2\,b-4\,A\,B\,a\,b^2+2\,e^x\,A\,B\,b^3+4\,e^x\,B^2\,a^3+2\,B^2\,a^2\,b+3\,e^x\,B^2\,a\,b^2\right)}{b^5}+\frac{\left(\frac{32\left(2\,B\,b^4+B\,a^2\,b^2-4\,A\,a^4\,e^x-A\,b^4\,e^{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$$-\frac{B\,\ln\left(e^x+1\right)}{a-b}$$

$$-\frac{\ln\left(-\frac{32\left(e^x\,A^2\,B\,a\,b+A^2\,B\,b^2-4\,e^x\,A\,B^2\,a^2-2\,A\,B^2\,a\,b+e^x\,A\,B^2\,b^2+4\,e^x\,B^3\,a\,b+2\,B^3\,b^2\right)}{b^5}-\frac{\left(\frac{32\left(A^2\,a^2\,b+2\,e^x\,A^2\,a\,b^2+A^2\,b^3-8\,e^x\,A\,B\,a^2\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$$+\frac{B\,\ln\left(e^x-1\right)}{a+b}$$

input
```
int((A + B/sinh(x))/(a + b*cosh(x)),x)
```

output
```
(log((((32*(A^2*b^3 + A^2*a^2*b + 2*B^2*a^2*b + 4*B^2*a^3*exp(x) + 3*B^2*a
*b^2*exp(x) - 4*A*B*a*b^2 + 2*A*B*b^3*exp(x) + 2*A^2*a*b^2*exp(x) - 8*A*B*
a^2*b*exp(x)))/b^5 + (((32*(2*B*b^4 + B*a^2*b^2 - 4*A*a^4*exp(x) - A*b^4*e
xp(x) + 2*A*a*b^3 - 2*A*a^3*b + 6*B*a*b^3*exp(x) - 3*B*a^3*b*exp(x) + 5*A*
a^2*b^2*exp(x)))/b^5 - (32*(A*((a + b)^3*(a - b)^3)^(1/2) - B*b^3 + B*a^2*
b)*(3*a^4*b - 3*a^2*b^3 + 4*a^5*exp(x) + a*b^4*exp(x) - 5*a^3*b^2*exp(x)))
/(b^5*(a^4 + b^4 - 2*a^2*b^2)))*(A*((a + b)^3*(a - b)^3)^(1/2) - B*b^3 + B
*a^2*b))/(a^4 + b^4 - 2*a^2*b^2))*(A*((a + b)^3*(a - b)^3)^(1/2) - B*b^3 +
 B*a^2*b))/(a^4 + b^4 - 2*a^2*b^2) - (32*(2*B^3*b^2 + A^2*B*b^2 - 2*A*B^2*
a*b + 4*B^3*a*b*exp(x) - 4*A*B^2*a^2*exp(x) + A*B^2*b^2*exp(x) + A^2*B*a*b
*exp(x)))/b^5)*(A*((a + b)^3*(a - b)^3)^(1/2) - B*b^3 + B*a^2*b))/(a^4 + b
^4 - 2*a^2*b^2) - (B*log(exp(x) + 1))/(a - b) - (log(- (32*(2*B^3*b^2 + A^
2*B*b^2 - 2*A*B^2*a*b + 4*B^3*a*b*exp(x) - 4*A*B^2*a^2*exp(x) + A*B^2*b^2*
exp(x) + A^2*B*a*b*exp(x)))/b^5 - (((32*(A^2*b^3 + A^2*a^2*b + 2*B^2*a^2*b
 + 4*B^2*a^3*exp(x) + 3*B^2*a*b^2*exp(x) - 4*A*B*a*b^2 + 2*A*B*b^3*exp(x)
+ 2*A^2*a*b^2*exp(x) - 8*A*B*a^2*b*exp(x)))/b^5 - (((32*(2*B*b^4 + B*a^2*b
^2 - 4*A*a^4*exp(x) - A*b^4*exp(x) + 2*A*a*b^3 - 2*A*a^3*b + 6*B*a*b^3*exp
(x) - 3*B*a^3*b*exp(x) + 5*A*a^2*b^2*exp(x)))/b^5 + (32*(B*b^3 + A*((a + b
)^3*(a - b)^3)^(1/2) - B*a^2*b)*(3*a^4*b - 3*a^2*b^3 + 4*a^5*exp(x) + a*b^
4*exp(x) - 5*a^3*b^2*exp(x)))/(b^5*(a^4 + b^4 - 2*a^2*b^2)))*(B*b^3 + A...
```

3.205. $\int \frac{A+B\text{csch}(x)}{a+b\cosh(x)}\,dx$

3.206 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+b\cosh(d+ex)}\,dx$

3.206.1 Optimal result

Integrand size = 31, antiderivative size = 86

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+b\cosh(d+ex)}\,dx = \frac{Bx}{b}+\frac{2(Ab-aB)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b\sqrt{a+b}e} + \frac{C\log(a+b\cosh(d+ex))}{be}$$

output
```
B*x/b+C*ln(a+b*cosh(e*x+d))/b/e+2*(A*b-B*a)*arctanh((a-b)^(1/2)*tanh(1/2*e
*x+1/2*d)/(a+b)^(1/2))/b/e/(a-b)^(1/2)/(a+b)^(1/2)
```

3.206.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+b\cosh(d+ex)}\,dx = \frac{B(d+ex)+\frac{2(-Ab+aB)\arctan\left(\frac{(a-b)\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+C\log(a+b\cosh(d+ex))}{be}$$

input
```
Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x]),x]
```

output
```
(B*(d + e*x) + (2*(-(A*b) + a*B)*ArcTan[((a - b)*Tanh[(d + e*x)/2])/Sqrt[-
a^2 + b^2]])/Sqrt[-a^2 + b^2] + C*Log[a + b*Cosh[d + e*x]])/(b*e)
```

3.206.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {3042, 4877, 26, 3042, 26, 3147, 16, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+b\cosh(d+ex)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\cos(id+iex)-iC\sin(id+iex)}{a+b\cos(id+iex)}dx$$

$$\Big\downarrow \text{4877}$$

$$\int \frac{A+B\cosh(d+ex)}{a+b\cosh(d+ex)}dx-iC\int \frac{i\sinh(d+ex)}{a+b\cosh(d+ex)}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{A+B\cosh(d+ex)}{a+b\cosh(d+ex)}dx+C\int \frac{\sinh(d+ex)}{a+b\cosh(d+ex)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(id+iex+\frac{\pi}{2}\right)}{a+b\sin\left(id+iex+\frac{\pi}{2}\right)}dx+C\int -\frac{i\cos\left(id+iex-\frac{\pi}{2}\right)}{a-b\sin\left(id+iex-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{A+B\sin\left(id+iex+\frac{\pi}{2}\right)}{a+b\sin\left(id+iex+\frac{\pi}{2}\right)}dx-iC\int \frac{\cos\left(\frac{1}{2}(2id-\pi)+iex\right)}{a-b\sin\left(\frac{1}{2}(2id-\pi)+iex\right)}dx$$

$$\Big\downarrow \text{3147}$$

$$\frac{C\int \frac{1}{a+b\cosh(d+ex)}d(b\cosh(d+ex))}{be}+\int \frac{A+B\sin\left(id+iex+\frac{\pi}{2}\right)}{a+b\sin\left(id+iex+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{16}$$

$$\frac{C\log(a+b\cosh(d+ex))}{be}+\int \frac{A+B\sin\left(id+iex+\frac{\pi}{2}\right)}{a+b\sin\left(id+iex+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3214}$$

$$\frac{(Ab-aB)\int \frac{1}{a+b\cosh(d+ex)}dx}{b}+\frac{C\log(a+b\cosh(d+ex))}{be}+\frac{Bx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(Ab-aB)\int \frac{1}{a+b\sin\left(id+iex+\frac{\pi}{2}\right)}dx}{b}+\frac{C\log(a+b\cosh(d+ex))}{be}+\frac{Bx}{b}$$

$$\Big\downarrow \text{3138}$$

$$-\frac{2i(Ab-aB)\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{1}{2}(d+ex)\right)\right)+a+b}d\left(i\tanh\left(\frac{1}{2}(d+ex)\right)\right)}{be}+\frac{C\log(a+b\cosh(d+ex))}{be}+\frac{Bx}{b}$$

$$\Big\downarrow \text{218}$$

$$\frac{2(Ab-aB)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{be\sqrt{a-b}\sqrt{a+b}}+\frac{C\log(a+b\cosh(d+ex))}{be}+\frac{Bx}{b}$$

input
```
Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x]),x]
```

output
```
(B*x)/b + (2*(A*b - a*B)*ArcTanh[(Sqrt[a - b]*Tanh[(d + e*x)/2])/Sqrt[a +
b]])/(Sqrt[a - b]*b*Sqrt[a + b]*e) + (C*Log[a + b*Cosh[d + e*x]])/(b*e)
```

3.206.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 4877
```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Simp[d   Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[
c*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Intege
rQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.206.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(77) = 154$.

Time = 0.41 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.81

method	result
derivativedivides	$\frac{\frac{(-B-C)\ln\left(\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-1\right)}{b}+\frac{(B-C)\ln\left(\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)+1\right)}{b}+\frac{\frac{2(aC-bC)\ln\left(a\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}-b\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}-a-b\right)}{2a-2b}-\frac{2(-bA+}{}}{b}}{e}$
default	$\frac{\frac{(-B-C)\ln\left(\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-1\right)}{b}+\frac{(B-C)\ln\left(\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)+1\right)}{b}+\frac{\frac{2(aC-bC)\ln\left(a\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}-b\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}-a-b\right)}{2a-2b}-\frac{2(-bA+}{}}{b}}{e}$
risch	$\frac{Bx}{b}+\frac{xC}{b}+\frac{2C\,a^{2}b\,e^{2}x}{-a^{2}b^{2}e^{2}+b^{4}e^{2}}-\frac{2C\,b^{3}e^{2}x}{-a^{2}b^{2}e^{2}+b^{4}e^{2}}+\frac{2C\,a^{2}bde}{-a^{2}b^{2}e^{2}+b^{4}e^{2}}-\frac{2C\,b^{3}de}{-a^{2}b^{2}e^{2}+b^{4}e^{2}}+\frac{\ln\left(e^{ex+d}+\frac{bAa-a^{2}B-\sqrt{}}{}\right.}{}$

input
```
int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d)),x,method=_RETURNVERB
OSE)
```

output
```
1/e*((-B-C)/b*ln(tanh(1/2*e*x+1/2*d)-1)+(B-C)/b*ln(tanh(1/2*e*x+1/2*d)+1)+
2/b*(1/2*(C*a-C*b)/(a-b)*ln(a*tanh(1/2*e*x+1/2*d)^2-b*tanh(1/2*e*x+1/2*d)^
2-a-b)-(-A*b+B*a)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*e*x+1/2*d)/((
a+b)*(a-b))^(1/2))))
```

3.206.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 405, normalized size of antiderivative = 4.71

$$\int \frac{A + B\cosh(d+ex) + C\sinh(d+ex)}{a + b\cosh(d+ex)}\,dx$$
$$= \left[\frac{((B-C)a^2 - (B-C)b^2)ex - (Ba - Ab)\sqrt{a^2-b^2}\log\left(\frac{b^2\cosh(ex+d)^2 + b^2\sinh(ex+d)^2 + 2\,ab\cosh(ex+d) + 2\,a^2 - b^2 + 2\,(}{b\cosh(ex+d)^2 + b\sinh(ex+d)^2 + 2\,a\,}\right.}{(a^2b - b^3}\right.$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d)),x, algorithm="
fricas")
```

output
```
[(((B - C)*a^2 - (B - C)*b^2)*e*x - (B*a - A*b)*sqrt(a^2 - b^2)*log((b^2*c
osh(e*x + d)^2 + b^2*sinh(e*x + d)^2 + 2*a*b*cosh(e*x + d) + 2*a^2 - b^2 +
 2*(b^2*cosh(e*x + d) + a*b)*sinh(e*x + d) - 2*sqrt(a^2 - b^2)*(b*cosh(e*x
 + d) + b*sinh(e*x + d) + a))/(b*cosh(e*x + d)^2 + b*sinh(e*x + d)^2 + 2*a
*cosh(e*x + d) + 2*(b*cosh(e*x + d) + a)*sinh(e*x + d) + b)) + (C*a^2 - C*
b^2)*log(2*(b*cosh(e*x + d) + a)/(cosh(e*x + d) - sinh(e*x + d))))/((a^2*b
 - b^3)*e), (((B - C)*a^2 - (B - C)*b^2)*e*x + 2*(B*a - A*b)*sqrt(-a^2 + b
^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(e*x + d) + b*sinh(e*x + d) + a)/(a^2
- b^2)) + (C*a^2 - C*b^2)*log(2*(b*cosh(e*x + d) + a)/(cosh(e*x + d) - sin
h(e*x + d))))/((a^2*b - b^3)*e)]
```

3.206.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 695 vs. $2(73) = 146$.

Time = 15.33 (sec) , antiderivative size = 695, normalized size of antiderivative = 8.08

$$\int \frac{A + B\cosh(d + ex) + C\sinh(d + ex)}{a + b\cosh(d + ex)}\,dx$$

$$= \begin{cases} \frac{\tilde{\infty}x(A+B\cosh(d)+C\sinh(d))}{\cosh(d)} \\ \frac{A\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)}{be} + \frac{Bx}{b} - \frac{B\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)}{be} + \frac{Cx}{b} - \frac{2C\log\left(\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)}{be} \\ -\frac{A}{be\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)} + \frac{Bx}{b} - \frac{B}{be\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)} + \frac{Cx}{b} - \frac{2C\log\left(\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)}{be} + \frac{2C\log\left(\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{be} \\ \frac{Ax+\frac{B\sinh(d+ex)}{e}+\frac{C\cosh(d+ex)}{e}}{a} \\ \frac{x(A+B\cosh(d)+C\sinh(d))}{a+b\cosh(d)} \\ -\frac{Ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{abe+b^2e} + \frac{Ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{abe+b^2e} + \frac{Baex}{abe+b^2e} + \frac{Ba\sqrt{\frac{a}{a-b}+\frac{a}{}}}{} \end{cases}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d)),x)
```

output
```
Piecewise(((zoo*x*(A + B*cosh(d) + C*sinh(d))/cosh(d), Eq(a, 0) & Eq(b, 0)
& Eq(e, 0)), (A*tanh(d/2 + e*x/2)/(b*e) + B*x/b - B*tanh(d/2 + e*x/2)/(b*e
) + C*x/b - 2*C*log(tanh(d/2 + e*x/2) + 1)/(b*e), Eq(a, b)), (-A/(b*e*tanh
(d/2 + e*x/2)) + B*x/b - B/(b*e*tanh(d/2 + e*x/2)) + C*x/b - 2*C*log(tanh(
d/2 + e*x/2) + 1)/(b*e) + 2*C*log(tanh(d/2 + e*x/2))/(b*e), Eq(a, -b)), ((
A*x + B*sinh(d + e*x)/e + C*cosh(d + e*x)/e)/a, Eq(b, 0)), (x*(A + B*cosh(
d) + C*sinh(d))/(a + b*cosh(d)), Eq(e, 0)), (-A*b*sqrt(a/(a - b) + b/(a -
b))*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(d/2 + e*x/2))/(a*b*e + b**2*e)
 + A*b*sqrt(a/(a - b) + b/(a - b))*log(sqrt(a/(a - b) + b/(a - b)) + tanh(
d/2 + e*x/2))/(a*b*e + b**2*e) + B*a*e*x/(a*b*e + b**2*e) + B*a*sqrt(a/(a
- b) + b/(a - b))*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(d/2 + e*x/2))/(a
*b*e + b**2*e) - B*a*sqrt(a/(a - b) + b/(a - b))*log(sqrt(a/(a - b) + b/(a
 - b)) + tanh(d/2 + e*x/2))/(a*b*e + b**2*e) + B*b*e*x/(a*b*e + b**2*e) +
C*a*e*x/(a*b*e + b**2*e) + C*a*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(d/2
 + e*x/2))/(a*b*e + b**2*e) + C*a*log(sqrt(a/(a - b) + b/(a - b)) + tanh(d
/2 + e*x/2))/(a*b*e + b**2*e) - 2*C*a*log(tanh(d/2 + e*x/2) + 1)/(a*b*e +
b**2*e) + C*b*e*x/(a*b*e + b**2*e) + C*b*log(-sqrt(a/(a - b) + b/(a - b))
+ tanh(d/2 + e*x/2))/(a*b*e + b**2*e) + C*b*log(sqrt(a/(a - b) + b/(a - b)
) + tanh(d/2 + e*x/2))/(a*b*e + b**2*e) - 2*C*b*log(tanh(d/2 + e*x/2) + 1)
/(a*b*e + b**2*e), True))
```

3.206.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+b\cosh(d+ex)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d)),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.206.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.09

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+b\cosh(d+ex)}\,dx = \frac{\frac{(ex+d)(B-C)}{b}+\frac{C\log\left(be^{(2ex+2d)}+2ae^{(ex+d)}+b\right)}{b}-\frac{2(Ba-Ab)\arctan\left(\frac{be^{(ex+d)}+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,b}}{e}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d)),x, algorithm="
giac")
```

output
```
((e*x + d)*(B - C)/b + C*log(b*e^(2*e*x + 2*d) + 2*a*e^(e*x + d) + b)/b -
2*(B*a - A*b)*arctan((b*e^(e*x + d) + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^
2)*b))/e
```

3.206.9 Mupad [B] (verification not implemented)

Time = 2.94 (sec) , antiderivative size = 653, normalized size of antiderivative = 7.59

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+b\cosh(d+ex)}\,dx$$

$$= \frac{2\,\mathrm{atan}\left(\frac{a\sqrt{b^4\,e^2-a^2\,b^2\,e^2}\,\sqrt{A^2\,b^2-2\,A\,B\,a\,b+B^2\,a^2}}{B\,e\,a^3\,b-A\,e\,a^2\,b^2-B\,e\,a\,b^3+A\,e\,b^4}+\frac{a^2\,b^2\,\mathrm{e}^{e\,x}\,\mathrm{e}^{d}\,\sqrt{b^4\,e^2-a^2\,b^2\,e^2}\,\sqrt{A^2\,b^2-2\,A\,B\,a\,b+B^2\,a^2}}{B\,e\,a^3\,b^4-A\,e\,a^2\,b^5-B\,e\,a\,b^6+A\,e\,b^7}+\frac{A\,\mathrm{e}^{e\,x}\,\mathrm{e}^{d}\,\sqrt{b^4\,e^2-a^2\,b^2\,e^2}}{b\,e\,\sqrt{A^2\,b^2-2\,A\,B\,a\,b+B^2\,a^2}}\right.}{\sqrt{b^4\,e^2-a^2\,b^2\,e^2}}$$

$$+\frac{B\,x}{b}-\frac{C\,x}{b}$$

$$+\frac{C\,b^3\,e\,\ln\left(4\,A^2\,b^3+4\,B^2\,a^2\,b-8\,A\,B\,a\,b^2+8\,B^2\,a^3\,\mathrm{e}^{e\,x}\,\mathrm{e}^{d}+4\,A^2\,b^3\,\mathrm{e}^{2\,d}\,\mathrm{e}^{2\,e\,x}+8\,A^2\,a\,b^2\,\mathrm{e}^{e\,x}\,\mathrm{e}^{d}+4\,B^2\,a^2\,b\right.}{b^4\,e^2-a^2\,b^2\,e^2}$$

$$-\frac{C\,a^2\,b\,e\,\ln\left(4\,A^2\,b^3+4\,B^2\,a^2\,b-8\,A\,B\,a\,b^2+8\,B^2\,a^3\,\mathrm{e}^{e\,x}\,\mathrm{e}^{d}+4\,A^2\,b^3\,\mathrm{e}^{2\,d}\,\mathrm{e}^{2\,e\,x}+8\,A^2\,a\,b^2\,\mathrm{e}^{e\,x}\,\mathrm{e}^{d}+4\,B^2\,a^2\right.}{b^4\,e^2-a^2\,b^2\,e^2}$$

input
```
int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + b*cosh(d + e*x)),x)
```

output
```
(2*atan((a*(b^4*e^2 - a^2*b^2*e^2)^(1/2)*(A^2*b^2 + B^2*a^2 - 2*A*B*a*b)^(
1/2))/(A*b^4*e - B*a*b^3*e + B*a^3*b*e - A*a^2*b^2*e) + (a^2*b^2*exp(e*x)*
exp(d)*(b^4*e^2 - a^2*b^2*e^2)^(1/2)*(A^2*b^2 + B^2*a^2 - 2*A*B*a*b)^(1/2)
)/(A*b^7*e - B*a*b^6*e - A*a^2*b^5*e + B*a^3*b^4*e) + (A*exp(e*x)*exp(d)*(
b^4*e^2 - a^2*b^2*e^2)^(1/2))/(b*e*(A^2*b^2 + B^2*a^2 - 2*A*B*a*b)^(1/2))
- (B*a*exp(e*x)*exp(d)*(b^4*e^2 - a^2*b^2*e^2)^(1/2))/(b^2*e*(A^2*b^2 + B^
2*a^2 - 2*A*B*a*b)^(1/2)))*(A^2*b^2 + B^2*a^2 - 2*A*B*a*b)^(1/2))/(b^4*e^2
 - a^2*b^2*e^2)^(1/2) + (B*x)/b - (C*x)/b + (C*b^3*e*log(4*A^2*b^3 + 4*B^2
*a^2*b - 8*A*B*a*b^2 + 8*B^2*a^3*exp(e*x)*exp(d) + 4*A^2*b^3*exp(2*d)*exp(
2*e*x) + 8*A^2*a*b^2*exp(e*x)*exp(d) + 4*B^2*a^2*b*exp(2*d)*exp(2*e*x) - 1
6*A*B*a^2*b*exp(e*x)*exp(d) - 8*A*B*a*b^2*exp(2*d)*exp(2*e*x)))/(b^4*e^2 -
 a^2*b^2*e^2) - (C*a^2*b*e*log(4*A^2*b^3 + 4*B^2*a^2*b - 8*A*B*a*b^2 + 8*B
^2*a^3*exp(e*x)*exp(d) + 4*A^2*b^3*exp(2*d)*exp(2*e*x) + 8*A^2*a*b^2*exp(e
*x)*exp(d) + 4*B^2*a^2*b*exp(2*d)*exp(2*e*x) - 16*A*B*a^2*b*exp(e*x)*exp(d
) - 8*A*B*a*b^2*exp(2*d)*exp(2*e*x)))/(b^4*e^2 - a^2*b^2*e^2)
```

3.207 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^2}\,dx$

3.207.1 Optimal result

Integrand size = 31, antiderivative size = 121

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^2}\,dx = \frac{2(aA-bB)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}e} - \frac{C}{be(a+b\cosh(d+ex))} - \frac{(Ab-aB)\sinh(d+ex)}{(a^2-b^2)\,e(a+b\cosh(d+ex))}$$

output
```
2*(A*a-B*b)*arctanh((a-b)^(1/2)*tanh(1/2*e*x+1/2*d)/(a+b)^(1/2))/(a-b)^(3/
2)/(a+b)^(3/2)/e-C/b/e/(a+b*cosh(e*x+d))-(A*b-B*a)*sinh(e*x+d)/(a^2-b^2)/e
/(a+b*cosh(e*x+d))
```

3.207.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.95

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^2}\,dx = \frac{\frac{2(aA-bB)\arctan\left(\frac{(a-b)\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}} + \frac{(-a^2+b^2)C-b(Ab-aB)\sinh(d+ex)}{(a-b)b(a+b)(a+b\cosh(d+ex))}}{e}$$

input
```
Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x])^2,
x]
```

output
```
((2*(a*A - b*B)*ArcTan[((a - b)*Tanh[(d + e*x)/2])/Sqrt[-a^2 + b^2]])/(-a^
2 + b^2)^(3/2) + ((-a^2 + b^2)*C - b*(A*b - a*B)*Sinh[d + e*x])/((a - b)*b
*(a + b)*(a + b*Cosh[d + e*x])))/e
```

3.207.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.09, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.419, Rules used = {3042, 4877, 26, 3042, 26, 3147, 17, 3233, 25, 27, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^2}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{A+B\cos(id+iex)-iC\sin(id+iex)}{(a+b\cos(id+iex))^2}dx$$

$$\Big\downarrow \ 4877$$

$$\int \frac{A+B\cosh(d+ex)}{(a+b\cosh(d+ex))^2}dx-iC\int \frac{i\sinh(d+ex)}{(a+b\cosh(d+ex))^2}dx$$

$$\Big\downarrow \ 26$$

$$\int \frac{A+B\cosh(d+ex)}{(a+b\cosh(d+ex))^2}dx+C\int \frac{\sinh(d+ex)}{(a+b\cosh(d+ex))^2}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{A+B\sin\left(id+iex+\frac{\pi}{2}\right)}{\left(a+b\sin\left(id+iex+\frac{\pi}{2}\right)\right)^2}dx+C\int -\frac{i\cos\left(id+iex-\frac{\pi}{2}\right)}{\left(a-b\sin\left(id+iex-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \ 26$$

$$\int \frac{A+B\sin\left(id+iex+\frac{\pi}{2}\right)}{\left(a+b\sin\left(id+iex+\frac{\pi}{2}\right)\right)^2}dx-iC\int \frac{\cos\left(\frac{1}{2}(2id-\pi)+iex\right)}{\left(a-b\sin\left(\frac{1}{2}(2id-\pi)+iex\right)\right)^2}dx$$

$$\Big\downarrow \ 3147$$

$$\frac{C\int \frac{1}{(a+b\cosh(d+ex))^2}d(b\cosh(d+ex))}{be}+\int \frac{A+B\sin\left(id+iex+\frac{\pi}{2}\right)}{\left(a+b\sin\left(id+iex+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{17}$$

$$-\frac{C}{be(a+b\cosh(d+ex))}+\int \frac{A+B\sin\left(id+iex+\frac{\pi}{2}\right)}{\left(a+b\sin\left(id+iex+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3233}$$

$$-\frac{\int -\frac{aA-bB}{a+b\cosh(d+ex)}dx}{a^2-b^2}-\frac{(Ab-aB)\sinh(d+ex)}{e\left(a^2-b^2\right)(a+b\cosh(d+ex))}-\frac{C}{be(a+b\cosh(d+ex))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{aA-bB}{a+b\cosh(d+ex)}dx}{a^2-b^2}-\frac{(Ab-aB)\sinh(d+ex)}{e\left(a^2-b^2\right)(a+b\cosh(d+ex))}-\frac{C}{be(a+b\cosh(d+ex))}$$

$$\Big\downarrow \text{27}$$

$$\frac{(aA-bB)\int \frac{1}{a+b\cosh(d+ex)}dx}{a^2-b^2}-\frac{(Ab-aB)\sinh(d+ex)}{e\left(a^2-b^2\right)(a+b\cosh(d+ex))}-\frac{C}{be(a+b\cosh(d+ex))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(aA-bB)\int \frac{1}{a+b\sin\left(id+iex+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{(Ab-aB)\sinh(d+ex)}{e\left(a^2-b^2\right)(a+b\cosh(d+ex))}-\frac{C}{be(a+b\cosh(d+ex))}$$

$$\Big\downarrow \text{3138}$$

$$-\frac{2i(aA-bB)\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{1}{2}(d+ex)\right)\right)+a+b}d\left(i\tanh\left(\frac{1}{2}(d+ex)\right)\right)}{e\left(a^2-b^2\right)}-\frac{(Ab-aB)\sinh(d+ex)}{e\left(a^2-b^2\right)(a+b\cosh(d+ex))}-\frac{C}{be(a+b\cosh(d+ex))}$$

$$\Big\downarrow \text{218}$$

$$\frac{2(aA-bB)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{e\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)}-\frac{(Ab-aB)\sinh(d+ex)}{e\left(a^2-b^2\right)(a+b\cosh(d+ex))}-\frac{C}{be(a+b\cosh(d+ex))}$$

input
```
Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x])^2,x]
```

output
```
(2*(a*A - b*B)*ArcTanh[(Sqrt[a - b]*Tanh[(d + e*x)/2])/Sqrt[a + b]])/(Sqrt
[a - b]*Sqrt[a + b]*(a^2 - b^2)*e) - C/(b*e*(a + b*Cosh[d + e*x])) - ((A*b
 - a*B)*Sinh[d + e*x])/((a^2 - b^2)*e*(a + b*Cosh[d + e*x]))
```

3.207. $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^2}\,dx$

3.207.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 4877
```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Simp[d   Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[
c*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Intege
rQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.207.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{(bA-Ba)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)}{a^2-b^2}+\frac{C}{a-b}\right)}{a\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^2-b\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^2-a-b}+\frac{2(Aa-Bb)\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}}{e}$
default	$\frac{-\frac{2\left(-\frac{(bA-Ba)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)}{a^2-b^2}+\frac{C}{a-b}\right)}{a\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^2-b\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^2-a-b}+\frac{2(Aa-Bb)\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}}{e}$
risch	$\frac{2Aab\,\mathrm{e}^{ex+d}-2B\,a^2\mathrm{e}^{ex+d}-2C\,a^2\mathrm{e}^{ex+d}+2C\,b^2\mathrm{e}^{ex+d}+2b^2A-2Bab}{be(a^2-b^2)\left(b\,\mathrm{e}^{2ex+2d}+2a\,\mathrm{e}^{ex+d}+b\right)}+\frac{\ln\left(\mathrm{e}^{ex+d}+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)Aa}{\sqrt{a^2-b^2}\,(a+b)(a-b)e}-\frac{\ln\left(\mathrm{e}^{ex+d}\cdots\right.}{\sqrt{a^2\cdots}}$

input
```
int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^2,x,method=_RETURNVE
RBOSE)
```

output
```
1/e*(-2*(-(A*b-B*a)/(a^2-b^2)*tanh(1/2*e*x+1/2*d)+C/(a-b))/(a*tanh(1/2*e*x
+1/2*d)^2-b*tanh(1/2*e*x+1/2*d)^2-a-b)+2*(A*a-B*b)/(a+b)/(a-b)/((a+b)*(a-b
))^(1/2)*arctanh((a-b)*tanh(1/2*e*x+1/2*d)/((a+b)*(a-b))^(1/2)))
```

3.207.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. 2(111) = 222.

Time = 0.28 (sec) , antiderivative size = 1044, normalized size of antiderivative = 8.63

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^2,x, algorithm
="fricas")
```

output
```
[-(2*B*a^3*b - 2*A*a^2*b^2 - 2*B*a*b^3 + 2*A*b^4 - (A*a*b^2 - B*b^3 + (A*a
*b^2 - B*b^3)*cosh(e*x + d)^2 + (A*a*b^2 - B*b^3)*sinh(e*x + d)^2 + 2*(A*a
^2*b - B*a*b^2)*cosh(e*x + d) + 2*(A*a^2*b - B*a*b^2 + (A*a*b^2 - B*b^3)*c
osh(e*x + d))*sinh(e*x + d))*sqrt(a^2 - b^2)*log((b^2*cosh(e*x + d)^2 + b^
2*sinh(e*x + d)^2 + 2*a*b*cosh(e*x + d) + 2*a^2 - b^2 + 2*(b^2*cosh(e*x +
d) + a*b)*sinh(e*x + d) - 2*sqrt(a^2 - b^2)*(b*cosh(e*x + d) + b*sinh(e*x
+ d) + a))/(b*cosh(e*x + d)^2 + b*sinh(e*x + d)^2 + 2*a*cosh(e*x + d) + 2*
(b*cosh(e*x + d) + a)*sinh(e*x + d) + b)) + 2*((B + C)*a^4 - A*a^3*b - (B
+ 2*C)*a^2*b^2 + A*a*b^3 + C*b^4)*cosh(e*x + d) + 2*((B + C)*a^4 - A*a^3*b
 - (B + 2*C)*a^2*b^2 + A*a*b^3 + C*b^4)*sinh(e*x + d))/((a^4*b^2 - 2*a^2*b
^4 + b^6)*e*cosh(e*x + d)^2 + (a^4*b^2 - 2*a^2*b^4 + b^6)*e*sinh(e*x + d)^
2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5)*e*cosh(e*x + d) + (a^4*b^2 - 2*a^2*b^4 +
 b^6)*e + 2*((a^4*b^2 - 2*a^2*b^4 + b^6)*e*cosh(e*x + d) + (a^5*b - 2*a^3*
b^3 + a*b^5)*e)*sinh(e*x + d)), -2*(B*a^3*b - A*a^2*b^2 - B*a*b^3 + A*b^4
+ (A*a*b^2 - B*b^3 + (A*a*b^2 - B*b^3)*cosh(e*x + d)^2 + (A*a*b^2 - B*b^3)
*sinh(e*x + d)^2 + 2*(A*a^2*b - B*a*b^2)*cosh(e*x + d) + 2*(A*a^2*b - B*a*
b^2 + (A*a*b^2 - B*b^3)*cosh(e*x + d))*sinh(e*x + d))*sqrt(-a^2 + b^2)*arc
tan(-sqrt(-a^2 + b^2)*(b*cosh(e*x + d) + b*sinh(e*x + d) + a)/(a^2 - b^2))
 + ((B + C)*a^4 - A*a^3*b - (B + 2*C)*a^2*b^2 + A*a*b^3 + C*b^4)*cosh(e*x
+ d) + ((B + C)*a^4 - A*a^3*b - (B + 2*C)*a^2*b^2 + A*a*b^3 + C*b^4)*si...
```

3.207.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5175 vs. $2(97) = 194$.

Time = 174.73 (sec) , antiderivative size = 5175, normalized size of antiderivative = 42.77

$$\int \frac{A + B\cosh(d + ex) + C\sinh(d + ex)}{(a + b\cosh(d + ex))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))**2,x)
```

output
```
Piecewise((zoo*x*(A + B*cosh(d) + C*sinh(d))/cosh(d)**2, Eq(a, 0) & Eq(b,
0) & Eq(e, 0)), (-A*tanh(d/2 + e*x/2)**3/(6*b**2*e) + A*tanh(d/2 + e*x/2)/
(2*b**2*e) + B*tanh(d/2 + e*x/2)**3/(6*b**2*e) + B*tanh(d/2 + e*x/2)/(2*b*
*2*e) + C*tanh(d/2 + e*x/2)**2/(2*b**2*e), Eq(a, b)), (A/(2*b**2*e*tanh(d/
2 + e*x/2)) - A/(6*b**2*e*tanh(d/2 + e*x/2)**3) - B/(2*b**2*e*tanh(d/2 + e
*x/2)) - B/(6*b**2*e*tanh(d/2 + e*x/2)**3) - C/(2*b**2*e*tanh(d/2 + e*x/2)
**2), Eq(a, -b)), (x*(A + B*cosh(d) + C*sinh(d))/(a + b*cosh(d))**2, Eq(e,
 0)), (-A*a**2*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(d/2 + e*x/2))*tanh(
d/2 + e*x/2)**2/(a**4*e*sqrt(a/(a - b) + b/(a - b))*tanh(d/2 + e*x/2)**2 -
 a**4*e*sqrt(a/(a - b) + b/(a - b)) - 2*a**3*b*e*sqrt(a/(a - b) + b/(a - b
))*tanh(d/2 + e*x/2)**2 + 2*a**2*b**2*e*sqrt(a/(a - b) + b/(a - b)) + 2*a*
b**3*e*sqrt(a/(a - b) + b/(a - b))*tanh(d/2 + e*x/2)**2 - b**4*e*sqrt(a/(a
 - b) + b/(a - b))*tanh(d/2 + e*x/2)**2 - b**4*e*sqrt(a/(a - b) + b/(a - b
))) + A*a**2*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(d/2 + e*x/2))/(a**4*e
*sqrt(a/(a - b) + b/(a - b))*tanh(d/2 + e*x/2)**2 - a**4*e*sqrt(a/(a - b)
+ b/(a - b)) - 2*a**3*b*e*sqrt(a/(a - b) + b/(a - b))*tanh(d/2 + e*x/2)**2
 + 2*a**2*b**2*e*sqrt(a/(a - b) + b/(a - b)) + 2*a*b**3*e*sqrt(a/(a - b) +
 b/(a - b))*tanh(d/2 + e*x/2)**2 - b**4*e*sqrt(a/(a - b) + b/(a - b))*tanh
(d/2 + e*x/2)**2 - b**4*e*sqrt(a/(a - b) + b/(a - b))) + A*a**2*log(sqrt(a
/(a - b) + b/(a - b)) + tanh(d/2 + e*x/2))*tanh(d/2 + e*x/2)**2/(a**4*e...
```

3.207.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\cosh(d + ex) + C\sinh(d + ex)}{(a + b\cosh(d + ex))^2}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^2,x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.207.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.28

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^2}\,dx = \frac{2\left(\frac{(Aa-Bb)\arctan\left(\frac{be^{(ex+d)}+a}{\sqrt{-a^2+b^2}}\right)}{(a^2-b^2)\sqrt{-a^2+b^2}} - \frac{Ba^2e^{(ex+d)}+Ca^2e^{(ex+d)}-Aabe^{(ex+d)}-Cb^2e^{(ex+d)}+Bab-Ab^2}{(a^2b-b^3)\left(be^{(2\,ex+2\,d)}+2\,ae^{(ex+d)}+b\right)}\right)}{e}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^2,x, algorithm
="giac")
```

output
```
2*((A*a - B*b)*arctan((b*e^(e*x + d) + a)/sqrt(-a^2 + b^2))/((a^2 - b^2)*s
qrt(-a^2 + b^2)) - (B*a^2*e^(e*x + d) + C*a^2*e^(e*x + d) - A*a*b*e^(e*x +
 d) - C*b^2*e^(e*x + d) + B*a*b - A*b^2)/((a^2*b - b^3)*(b*e^(2*e*x + 2*d)
 + 2*a*e^(e*x + d) + b)))/e
```

3.207.9 Mupad [B] (verification not implemented)

Time = 2.35 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.49

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^2}\,dx = \frac{\frac{2\,(A\,b^3-B\,a\,b^2)}{b\,e\,(a^2\,b-b^3)}+\frac{2\,\mathrm{e}^{d+e\,x}\,(C\,b^4-B\,a^2\,b^2-C\,a^2\,b^2+A\,a\,b^3)}{b^2\,e\,(a^2\,b-b^3)}}{b+2\,a\,\mathrm{e}^{d+e\,x}+b\,\mathrm{e}^{2\,d+2\,e\,x}} + \frac{\ln\left(-\frac{2\,\mathrm{e}^{d+e\,x}\,(A\,a-B\,b)}{b\,(a^2-b^2)}-\frac{2\,(A\,a-B\,b)\,(b+a\,\mathrm{e}^{d+e\,x})}{b\,(a+b)^{3/2}\,(a-b)^{3/2}}\right)\,(A\,a-B\,b)}{e\,(a+b)^{3/2}\,(a-b)^{3/2}} - \frac{\ln\left(\frac{2\,(A\,a-B\,b)\,(b+a\,\mathrm{e}^{d+e\,x})}{b\,(a+b)^{3/2}\,(a-b)^{3/2}}-\frac{2\,\mathrm{e}^{d+e\,x}\,(A\,a-B\,b)}{b\,(a^2-b^2)}\right)\,(A\,a-B\,b)}{e\,(a+b)^{3/2}\,(a-b)^{3/2}}$$

input
```
int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + b*cosh(d + e*x))^2,x)
```

output
```
((2*(A*b^3 - B*a*b^2))/(b*e*(a^2*b - b^3)) + (2*exp(d + e*x)*(C*b^4 - B*a^
2*b^2 - C*a^2*b^2 + A*a*b^3))/(b^2*e*(a^2*b - b^3)))/(b + 2*a*exp(d + e*x)
 + b*exp(2*d + 2*e*x)) + (log(- (2*exp(d + e*x)*(A*a - B*b))/(b*(a^2 - b^2
)) - (2*(A*a - B*b)*(b + a*exp(d + e*x)))/(b*(a + b)^(3/2)*(a - b)^(3/2)))
*(A*a - B*b))/(e*(a + b)^(3/2)*(a - b)^(3/2)) - (log((2*(A*a - B*b)*(b + a
*exp(d + e*x)))/(b*(a + b)^(3/2)*(a - b)^(3/2)) - (2*exp(d + e*x)*(A*a - B
*b))/(b*(a^2 - b^2)))*(A*a - B*b))/(e*(a + b)^(3/2)*(a - b)^(3/2))
```

3.207. $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^2}\,dx$

3.208 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^3}\,dx$

3.208.1 Optimal result

Integrand size = 31, antiderivative size = 187

$$\begin{aligned}\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^3}\,dx &= \frac{(2a^2A+Ab^2-3abB)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}e}-\frac{C}{2be(a+b\cosh(d+ex))^2}\\&\quad-\frac{(Ab-aB)\sinh(d+ex)}{2(a^2-b^2)e(a+b\cosh(d+ex))^2}-\frac{(3aAb-a^2B-2b^2B)\sinh(d+ex)}{2(a^2-b^2)^2e(a+b\cosh(d+ex))}\end{aligned}$$

output
```
(2*A*a^2+A*b^2-3*B*a*b)*arctanh((a-b)^(1/2)*tanh(1/2*e*x+1/2*d)/(a+b)^(1/2
))/(a-b)^(5/2)/(a+b)^(5/2)/e-1/2*C/b/e/(a+b*cosh(e*x+d))^2-1/2*(A*b-B*a)*s
inh(e*x+d)/(a^2-b^2)/e/(a+b*cosh(e*x+d))^2-1/2*(3*A*a*b-B*a^2-2*B*b^2)*sin
h(e*x+d)/(a^2-b^2)^2/e/(a+b*cosh(e*x+d))
```

3.208.2 Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.94

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^3}\,dx = \frac{-\frac{2(2a^2A+Ab^2-3abB)\arctan\left(\frac{(a-b)\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}}+\frac{(-3aAb+a^2B+2b^2B)\sinh(d+ex)}{(a-b)^2(a+b)^2(a+b\cosh(d+ex))}+\frac{(-a^2+b^2)C-b(Ab-aB)\sinh(d+ex)}{(a-b)b(a+b)(a+b\cosh(d+ex))^2}}{2e}$$

input
```
Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x])^3,
x]
```

output
```
((-2*(2*a^2*A + A*b^2 - 3*a*b*B)*ArcTan[((a - b)*Tanh[(d + e*x)/2])/Sqrt[-
a^2 + b^2]])/(-a^2 + b^2)^(5/2) + ((-3*a*A*b + a^2*B + 2*b^2*B)*Sinh[d + e
*x])/((a - b)^2*(a + b)^2*(a + b*Cosh[d + e*x])) + ((-a^2 + b^2)*C - b*(A*
b - a*B)*Sinh[d + e*x])/((a - b)*b*(a + b)*(a + b*Cosh[d + e*x])^2))/(2*e)
```

3.208.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.14, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.516$, Rules used = {3042, 4877, 26, 3042, 26, 3147, 17, 3233, 25, 3042, 3233, 25, 27, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\cos(id+iex)-iC\sin(id+iex)}{(a+b\cos(id+iex))^3}dx$$

$$\Big\downarrow \text{4877}$$

$$\int \frac{A+B\cosh(d+ex)}{(a+b\cosh(d+ex))^3}dx-iC\int \frac{i\sinh(d+ex)}{(a+b\cosh(d+ex))^3}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{A+B\cosh(d+ex)}{(a+b\cosh(d+ex))^3}dx+C\int \frac{\sinh(d+ex)}{(a+b\cosh(d+ex))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(id+iex+\frac{\pi}{2}\right)}{\left(a+b\sin\left(id+iex+\frac{\pi}{2}\right)\right)^3}dx+C\int -\frac{i\cos\left(id+iex-\frac{\pi}{2}\right)}{\left(a-b\sin\left(id+iex-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{A+B\sin\left(id+iex+\frac{\pi}{2}\right)}{\left(a+b\sin\left(id+iex+\frac{\pi}{2}\right)\right)^3}dx-iC\int \frac{\cos\left(\frac{1}{2}(2id-\pi)+iex\right)}{\left(a-b\sin\left(\frac{1}{2}(2id-\pi)+iex\right)\right)^3}dx$$

$$\Big\downarrow \text{3147}$$

$$\frac{C\int \frac{1}{(a+b\cosh(d+ex))^3}d(b\cosh(d+ex))}{be}+\int \frac{A+B\sin\left(id+iex+\frac{\pi}{2}\right)}{\left(a+b\sin\left(id+iex+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{17}$$

$$-\frac{C}{2be(a+b\cosh(d+ex))^2}+\int \frac{A+B\sin\left(id+iex+\frac{\pi}{2}\right)}{\left(a+b\sin\left(id+iex+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3233}$$

$$-\frac{\int -\frac{2(aA-bB)-(Ab-aB)\cosh(d+ex)}{(a+b\cosh(d+ex))^2}dx}{2\left(a^2-b^2\right)}-\frac{(Ab-aB)\sinh(d+ex)}{2e\left(a^2-b^2\right)(a+b\cosh(d+ex))^2}-\frac{C}{2be(a+b\cosh(d+ex))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2(aA-bB)-(Ab-aB)\cosh(d+ex)}{(a+b\cosh(d+ex))^2}dx}{2\left(a^2-b^2\right)}-\frac{(Ab-aB)\sinh(d+ex)}{2e\left(a^2-b^2\right)(a+b\cosh(d+ex))^2}-\frac{C}{2be(a+b\cosh(d+ex))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2(aA-bB)+(aB-Ab)\sin\left(id+iex+\frac{\pi}{2}\right)}{\left(a+b\sin\left(id+iex+\frac{\pi}{2}\right)\right)^2}dx}{2\left(a^2-b^2\right)}-\frac{(Ab-aB)\sinh(d+ex)}{2e\left(a^2-b^2\right)(a+b\cosh(d+ex))^2}-\frac{C}{2be(a+b\cosh(d+ex))^2}$$

$$\Big\downarrow \text{3233}$$

$$\frac{-\frac{\int -\frac{2Aa^2-3bBa+Ab^2}{a+b\cosh(d+ex)}dx}{a^2-b^2}-\frac{\left(a^2(-B)+3aAb-2b^2B\right)\sinh(d+ex)}{e\left(a^2-b^2\right)(a+b\cosh(d+ex))}}{2\left(a^2-b^2\right)}-\frac{(Ab-aB)\sinh(d+ex)}{2e\left(a^2-b^2\right)(a+b\cosh(d+ex))^2}-\frac{C}{2be(a+b\cosh(d+ex))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{2Aa^2-3bBa+Ab^2}{a+b\cosh(d+ex)}dx}{a^2-b^2}-\frac{\left(a^2(-B)+3aAb-2b^2B\right)\sinh(d+ex)}{e\left(a^2-b^2\right)(a+b\cosh(d+ex))}}{2\left(a^2-b^2\right)}-\frac{(Ab-aB)\sinh(d+ex)}{2e\left(a^2-b^2\right)(a+b\cosh(d+ex))^2}-\frac{C}{2be(a+b\cosh(d+ex))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\left(2a^2A-3abB+Ab^2\right)\int \frac{1}{a+b\cosh(d+ex)}dx}{a^2-b^2}-\frac{\left(a^2(-B)+3aAb-2b^2B\right)\sinh(d+ex)}{e\left(a^2-b^2\right)(a+b\cosh(d+ex))}}{2\left(a^2-b^2\right)}-\frac{(Ab-aB)\sinh(d+ex)}{2e\left(a^2-b^2\right)(a+b\cosh(d+ex))^2}-\frac{C}{2be(a+b\cosh(d+ex))^2}$$

$$\Big\downarrow \text{3042}$$

3.208. $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^3}dx$

$$\frac{-\frac{(a^2(-B)+3aAb-2b^2B)\sinh(d+ex)}{e(a^2-b^2)(a+b\cosh(d+ex))}+\frac{(2a^2A-3abB+Ab^2)\int\frac{1}{a+b\sin\left(id+iex+\frac{\pi}{2}\right)}dx}{a^2-b^2}}{2(a^2-b^2)}-\frac{(Ab-aB)\sinh(d+ex)}{2e(a^2-b^2)(a+b\cosh(d+ex))^2}-\frac{C}{2be(a+b\cosh(d+ex))^2}$$

$$\Big\downarrow \text{3138}$$

$$\frac{-\frac{(a^2(-B)+3aAb-2b^2B)\sinh(d+ex)}{e(a^2-b^2)(a+b\cosh(d+ex))}-\frac{2i(2a^2A-3abB+Ab^2)\int\frac{1}{-\left((a-b)\tanh^2\left(\frac{1}{2}(d+ex)\right)\right)+a+b}d\left(i\tanh\left(\frac{1}{2}(d+ex)\right)\right)}{e(a^2-b^2)}}{2(a^2-b^2)}-\frac{(Ab-aB)\sinh(d+ex)}{2e(a^2-b^2)(a+b\cosh(d+ex))^2}-\frac{C}{2be(a+b\cosh(d+ex))^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{2(2a^2A-3abB+Ab^2)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{e\sqrt{a-b}\sqrt{a+b}(a^2-b^2)}-\frac{(a^2(-B)+3aAb-2b^2B)\sinh(d+ex)}{e(a^2-b^2)(a+b\cosh(d+ex))}}{2(a^2-b^2)}-\frac{(Ab-aB)\sinh(d+ex)}{2e(a^2-b^2)(a+b\cosh(d+ex))^2}-\frac{C}{2be(a+b\cosh(d+ex))^2}$$

input
```
Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x])^3,x]
```

output
```
-1/2*C/(b*e*(a + b*Cosh[d + e*x])^2) - ((A*b - a*B)*Sinh[d + e*x])/(2*(a^2
 - b^2)*e*(a + b*Cosh[d + e*x])^2) + ((2*(2*a^2*A + A*b^2 - 3*a*b*B)*ArcTa
nh[(Sqrt[a - b]*Tanh[(d + e*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]*
(a^2 - b^2)*e) - ((3*a*A*b - a^2*B - 2*b^2*B)*Sinh[d + e*x])/((a^2 - b^2)*
e*(a + b*Cosh[d + e*x])))/(2*(a^2 - b^2))
```

3.208.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 4877
```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Simp[d   Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[
c*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Intege
rQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.208. $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^3}\,dx$

3.208.4 Maple [A] (verified)

Time = 3.28 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.46

method	result
derivativedivides	$-\frac{\frac{2\left(-\frac{\left(4bAa+b^2A-2a^2B-Bab-2B\,b^2\right)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^3}{2(a-b)\left(a^2+2ab+b^2\right)}+\frac{C\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^2}{a-b}+\frac{\left(4bAa-b^2A-2a^2B+Bab-2B\,b^2\right)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}-\frac{a^2}{}}{\left(a\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^2-b\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^2-a-b\right)^2}}{e}$
default	$-\frac{\frac{2\left(-\frac{\left(4bAa+b^2A-2a^2B-Bab-2B\,b^2\right)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^3}{2(a-b)\left(a^2+2ab+b^2\right)}+\frac{C\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^2}{a-b}+\frac{\left(4bAa-b^2A-2a^2B+Bab-2B\,b^2\right)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}-\frac{a^2}{}}{\left(a\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^2-b\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^2-a-b\right)^2}}{e}$
risch	$\frac{2A\,a^2b^2\mathrm{e}^{3ex+3d}+A\,b^4\mathrm{e}^{3ex+3d}-3Ba\,b^3\mathrm{e}^{3ex+3d}+6A\,a^3b\,\mathrm{e}^{2ex+2d}+3Aa\,b^3\mathrm{e}^{2ex+2d}-2B\,a^4\mathrm{e}^{2ex+2d}-5B\,a^2b^2\mathrm{e}^{2ex+2d}-2B}{be(a^2-}$

input
```
int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^3,x,method=_RETURNVE
RBOSE)
```

output
```
1/e*(-2*(-1/2*(4*A*a*b+A*b^2-2*B*a^2-B*a*b-2*B*b^2)/(a-b)/(a^2+2*a*b+b^2)*
tanh(1/2*e*x+1/2*d)^3+C/(a-b)*tanh(1/2*e*x+1/2*d)^2+1/2*(4*A*a*b-A*b^2-2*B
*a^2+B*a*b-2*B*b^2)/(a+b)/(a^2-2*a*b+b^2)*tanh(1/2*e*x+1/2*d)-a*C/(a^2-2*a
*b+b^2))/(a*tanh(1/2*e*x+1/2*d)^2-b*tanh(1/2*e*x+1/2*d)^2-a-b)^2+(2*A*a^2+
A*b^2-3*B*a*b)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(
1/2*e*x+1/2*d)/((a+b)*(a-b))^(1/2)))
```

3.208.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1760 vs. 2(171) = 342.

Time = 0.34 (sec) , antiderivative size = 3636, normalized size of antiderivative = 19.44

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^3,x, algorithm
="fricas")
```

output
```
[-1/2*(2*B*a^4*b^2 - 6*A*a^3*b^3 + 2*B*a^2*b^4 + 6*A*a*b^5 - 4*B*b^6 - 2*(
2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*cosh(e*x + d)^3
 - 2*(2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*sinh(e*x
+ d)^3 + 2*(2*(B + C)*a^6 - 6*A*a^5*b + 3*(B - 2*C)*a^4*b^2 + 3*A*a^3*b^3
- 3*(B - 2*C)*a^2*b^4 + 3*A*a*b^5 - 2*(B + C)*b^6)*cosh(e*x + d)^2 + 2*(2*
(B + C)*a^6 - 6*A*a^5*b + 3*(B - 2*C)*a^4*b^2 + 3*A*a^3*b^3 - 3*(B - 2*C)*
a^2*b^4 + 3*A*a*b^5 - 2*(B + C)*b^6 - 3*(2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2
*b^4 + 3*B*a*b^5 - A*b^6)*cosh(e*x + d))*sinh(e*x + d)^2 - (2*A*a^2*b^3 -
3*B*a*b^4 + A*b^5 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(e*x + d)^4 + (2
*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*sinh(e*x + d)^4 + 4*(2*A*a^3*b^2 - 3*B*a^2
*b^3 + A*a*b^4)*cosh(e*x + d)^3 + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 +
 (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(e*x + d))*sinh(e*x + d)^3 + 2*(4*A
*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(e*x + d)^2 +
2*(4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 + 3*(2*A*a^2*
b^3 - 3*B*a*b^4 + A*b^5)*cosh(e*x + d)^2 + 6*(2*A*a^3*b^2 - 3*B*a^2*b^3 +
A*a*b^4)*cosh(e*x + d))*sinh(e*x + d)^2 + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A
*a*b^4)*cosh(e*x + d) + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 + (2*A*a^2*
b^3 - 3*B*a*b^4 + A*b^5)*cosh(e*x + d)^3 + 3*(2*A*a^3*b^2 - 3*B*a^2*b^3 +
A*a*b^4)*cosh(e*x + d)^2 + (4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*
b^4 + A*b^5)*cosh(e*x + d))*sinh(e*x + d))*sqrt(a^2 - b^2)*log((b^2*cos...
```

3.208.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cosh(d+ex) + C\sinh(d+ex)}{(a+b\cosh(d+ex))^3}\, dx = \text{Timed out}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))**3,x)
```

output
```
Timed out
```

3.208.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^3,x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.208.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(171) = 342$.

Time = 0.30 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.98

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^3}\,dx$$
$$= \frac{\frac{(2Aa^2-3Bab+Ab^2)\arctan\left(\frac{be^{(ex+d)}+a}{\sqrt{-a^2+b^2}}\right)}{(a^4-2a^2b^2+b^4)\sqrt{-a^2+b^2}} + \frac{2Aa^2b^2e^{(3ex+3d)}-3Bab^3e^{(3ex+3d)}+Ab^4e^{(3ex+3d)}-2Ba^4e^{(2ex+2d)}-2Ca^4e^{(2ex+2d)}+6Aa^3}{}}{}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^3,x, algorithm
="giac")
```

output
```
((2*A*a^2 - 3*B*a*b + A*b^2)*arctan((b*e^(e*x + d) + a)/sqrt(-a^2 + b^2))/
((a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) + (2*A*a^2*b^2*e^(3*e*x + 3*d)
- 3*B*a*b^3*e^(3*e*x + 3*d) + A*b^4*e^(3*e*x + 3*d) - 2*B*a^4*e^(2*e*x + 2
*d) - 2*C*a^4*e^(2*e*x + 2*d) + 6*A*a^3*b*e^(2*e*x + 2*d) - 5*B*a^2*b^2*e^
(2*e*x + 2*d) + 4*C*a^2*b^2*e^(2*e*x + 2*d) + 3*A*a*b^3*e^(2*e*x + 2*d) -
2*B*b^4*e^(2*e*x + 2*d) - 2*C*b^4*e^(2*e*x + 2*d) - 4*B*a^3*b*e^(e*x + d)
+ 10*A*a^2*b^2*e^(e*x + d) - 5*B*a*b^3*e^(e*x + d) - A*b^4*e^(e*x + d) - B
*a^2*b^2 + 3*A*a*b^3 - 2*B*b^4)/((a^4*b - 2*a^2*b^3 + b^5)*(b*e^(2*e*x + 2
*d) + 2*a*e^(e*x + d) + b)^2))/e
```

3.208.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^3}\,dx = \int \frac{A+B\cosh(d+e\,x)+C\sinh(d+e\,x)}{\left(a+b\cosh\left(d+e\,x\right)\right)^3}\,dx$$

input
```
int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + b*cosh(d + e*x))^3,x)
```

output
```
int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + b*cosh(d + e*x))^3, x)
```

3.209 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^4}\,dx$

3.209.1 Optimal result

Integrand size = 31, antiderivative size = 260

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^4}\,dx = \frac{(2a^3A+3aAb^2-4a^2bB-b^3B)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}e} - \frac{C}{3be(a+b\cosh(d+ex))^3} - \frac{(Ab-aB)\sinh(d+ex)}{3(a^2-b^2)e(a+b\cosh(d+ex))^3} - \frac{(5aAb-2a^2B-3b^2B)\sinh(d+ex)}{6(a^2-b^2)^2e(a+b\cosh(d+ex))^2} - \frac{(11a^2Ab+4Ab^3-2a^3B-13ab^2B)\sinh(d+ex)}{6(a^2-b^2)^3e(a+b\cosh(d+ex))}$$

output
```
(2*A*a^3+3*A*a*b^2-4*B*a^2*b-B*b^3)*arctanh((a-b)^(1/2)*tanh(1/2*e*x+1/2*d
)/(a+b)^(1/2))/(a-b)^(7/2)/(a+b)^(7/2)/e-1/3*C/b/e/(a+b*cosh(e*x+d))^3-1/3
*(A*b-B*a)*sinh(e*x+d)/(a^2-b^2)/e/(a+b*cosh(e*x+d))^3-1/6*(5*A*a*b-2*B*a^
2-3*B*b^2)*sinh(e*x+d)/(a^2-b^2)^2/e/(a+b*cosh(e*x+d))^2-1/6*(11*A*a^2*b+4
*A*b^3-2*B*a^3-13*B*a*b^2)*sinh(e*x+d)/(a^2-b^2)^3/e/(a+b*cosh(e*x+d))
```

3.209.2 Mathematica [A] (verified)

Time = 1.97 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.94

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^4}\,dx$$
$$= \frac{\frac{6(2a^3A+3aAb^2-4a^2bB-b^3B)\arctan\left(\frac{(a-b)\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{7/2}}+\frac{(-5aAb+2a^2B+3b^2B)\sinh(d+ex)}{(a-b)^2(a+b)^2(a+b\cosh(d+ex))^2}+\frac{(-11a^2Ab-4Ab^3+2a^3B+13ab^2B)\sinh(d+ex)}{(a-b)^3(a+b)^3(a+b\cosh(d+ex))}}{6e}$$

input
```
Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x])^4,
x]
```

output
```
((6*(2*a^3*A + 3*a*A*b^2 - 4*a^2*b*B - b^3*B)*ArcTan[((a - b)*Tanh[(d + e*
x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) + ((-5*a*A*b + 2*a^2*B + 3*b^
2*B)*Sinh[d + e*x])/((a - b)^2*(a + b)^2*(a + b*Cosh[d + e*x])^2) + ((-11*
a^2*A*b - 4*A*b^3 + 2*a^3*B + 13*a*b^2*B)*Sinh[d + e*x])/((a - b)^3*(a + b
)^3*(a + b*Cosh[d + e*x])) + (2*(-a^2 + b^2)*C - 2*b*(A*b - a*B)*Sinh[d +
e*x])/((a - b)*b*(a + b)*(a + b*Cosh[d + e*x])^3))/(6*e)
```

3.209.3 Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.16, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.581$, Rules used = {3042, 4877, 26, 3042, 26, 3147, 17, 3233, 25, 3042, 3233, 25, 3042, 3233, 27, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^4}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\cos(id+iex)-iC\sin(id+iex)}{(a+b\cos(id+iex))^4}dx$$
$$\Big\downarrow \text{4877}$$
$$\int \frac{A+B\cosh(d+ex)}{(a+b\cosh(d+ex))^4}dx-iC\int \frac{i\sinh(d+ex)}{(a+b\cosh(d+ex))^4}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{A+B\cosh(d+ex)}{(a+b\cosh(d+ex))^4}dx+C\int \frac{\sinh(d+ex)}{(a+b\cosh(d+ex))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(id+iex+\frac{\pi}{2}\right)}{\left(a+b\sin\left(id+iex+\frac{\pi}{2}\right)\right)^4}dx+C\int -\frac{i\cos\left(id+iex-\frac{\pi}{2}\right)}{\left(a-b\sin\left(id+iex-\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{A+B\sin\left(id+iex+\frac{\pi}{2}\right)}{\left(a+b\sin\left(id+iex+\frac{\pi}{2}\right)\right)^4}dx-iC\int \frac{\cos\left(\frac{1}{2}(2id-\pi)+iex\right)}{\left(a-b\sin\left(\frac{1}{2}(2id-\pi)+iex\right)\right)^4}dx$$

$$\Big\downarrow \text{3147}$$

$$\frac{C\int \frac{1}{(a+b\cosh(d+ex))^4}d(b\cosh(d+ex))}{be}+\int \frac{A+B\sin\left(id+iex+\frac{\pi}{2}\right)}{\left(a+b\sin\left(id+iex+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{17}$$

$$-\frac{C}{3be(a+b\cosh(d+ex))^3}+\int \frac{A+B\sin\left(id+iex+\frac{\pi}{2}\right)}{\left(a+b\sin\left(id+iex+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{3233}$$

$$-\frac{\int -\frac{3(aA-bB)-2(Ab-aB)\cosh(d+ex)}{(a+b\cosh(d+ex))^3}dx}{3\left(a^2-b^2\right)}-\frac{(Ab-aB)\sinh(d+ex)}{3e\left(a^2-b^2\right)(a+b\cosh(d+ex))^3}-\frac{C}{3be(a+b\cosh(d+ex))^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3(aA-bB)-2(Ab-aB)\cosh(d+ex)}{(a+b\cosh(d+ex))^3}dx}{3\left(a^2-b^2\right)}-\frac{(Ab-aB)\sinh(d+ex)}{3e\left(a^2-b^2\right)(a+b\cosh(d+ex))^3}-\frac{C}{3be(a+b\cosh(d+ex))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3(aA-bB)-2(Ab-aB)\sin\left(id+iex+\frac{\pi}{2}\right)}{\left(a+b\sin\left(id+iex+\frac{\pi}{2}\right)\right)^3}dx}{3\left(a^2-b^2\right)}-\frac{(Ab-aB)\sinh(d+ex)}{3e\left(a^2-b^2\right)(a+b\cosh(d+ex))^3}-\frac{C}{3be(a+b\cosh(d+ex))^3}$$

$$\Big\downarrow \text{3233}$$

$$\frac{-\frac{\int -\frac{2\left(3Aa^2-5bBa+2Ab^2\right)-\left(-2Ba^2+5Aba-3b^2B\right)\cosh(d+ex)}{(a+b\cosh(d+ex))^2}dx}{2(a^2-b^2)}-\frac{\left(-2a^2B+5aAb-3b^2B\right)\sinh(d+ex)}{2e(a^2-b^2)(a+b\cosh(d+ex))^2}}{3\left(a^2-b^2\right)}-\frac{(Ab-aB)\sinh(d+ex)}{3e\left(a^2-b^2\right)(a+b\cosh(d+ex))^3}-\frac{C}{3be(a+b\cosh(d+ex))^3}$$

$$\Big\downarrow \text{25}$$

3.209. $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^4}\,dx$

$$\frac{\frac{\int \frac{2\left(3Aa^2-5bBa+2Ab^2\right)-\left(-2Ba^2+5Aba-3b^2B\right)\cosh(d+ex)}{(a+b\cosh(d+ex))^2}dx}{2(a^2-b^2)}-\frac{\left(-2a^2B+5aAb-3b^2B\right)\sinh(d+ex)}{2e(a^2-b^2)(a+b\cosh(d+ex))^2}}{3\left(a^2-b^2\right)}-\frac{(Ab-aB)\sinh(d+ex)}{3e\left(a^2-b^2\right)(a+b\cosh(d+ex))^3}-\frac{C}{3be(a+b\cosh(d+ex))^3}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{\left(-2a^2B+5aAb-3b^2B\right)\sinh(d+ex)}{2e(a^2-b^2)(a+b\cosh(d+ex))^2}+\frac{\int \frac{2\left(3Aa^2-5bBa+2Ab^2\right)+\left(2Ba^2-5Aba+3b^2B\right)\sin\left(id+iex+\frac{\pi}{2}\right)}{\left(a+b\sin\left(id+iex+\frac{\pi}{2}\right)\right)^2}dx}{2(a^2-b^2)}}{3\left(a^2-b^2\right)}-\frac{(Ab-aB)\sinh(d+ex)}{3e\left(a^2-b^2\right)(a+b\cosh(d+ex))^3}-\frac{C}{3be(a+b\cosh(d+ex))^3}$$

$\Big\downarrow$ 3233

$$\frac{\frac{-\frac{\int -\frac{3\left(2Aa^3-4bBa^2+3Ab^2a-b^3B\right)}{a+b\cosh(d+ex)}dx}{a^2-b^2}-\frac{\left(-2a^3B+11a^2Ab-13ab^2B+4Ab^3\right)\sinh(d+ex)}{e\left(a^2-b^2\right)(a+b\cosh(d+ex))}}{2(a^2-b^2)}-\frac{\left(-2a^2B+5aAb-3b^2B\right)\sinh(d+ex)}{2e(a^2-b^2)(a+b\cosh(d+ex))^2}}{3\left(a^2-b^2\right)}-\frac{(Ab-aB)\sinh(d+ex)}{3e\left(a^2-b^2\right)(a+b\cosh(d+ex))^3}-\frac{C}{3be(a+b\cosh(d+ex))^3}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{3\left(2a^3A-4a^2bB+3aAb^2-b^3B\right)\int \frac{1}{a+b\cosh(d+ex)}dx}{a^2-b^2}-\frac{\left(-2a^3B+11a^2Ab-13ab^2B+4Ab^3\right)\sinh(d+ex)}{e\left(a^2-b^2\right)(a+b\cosh(d+ex))}}{2(a^2-b^2)}-\frac{\left(-2a^2B+5aAb-3b^2B\right)\sinh(d+ex)}{2e(a^2-b^2)(a+b\cosh(d+ex))^2}}{3\left(a^2-b^2\right)}-\frac{(Ab-aB)\sinh(d+ex)}{3e\left(a^2-b^2\right)(a+b\cosh(d+ex))^3}-\frac{C}{3be(a+b\cosh(d+ex))^3}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{\left(-2a^2B+5aAb-3b^2B\right)\sinh(d+ex)}{2e(a^2-b^2)(a+b\cosh(d+ex))^2}+\frac{-\frac{\left(-2a^3B+11a^2Ab-13ab^2B+4Ab^3\right)\sinh(d+ex)}{e\left(a^2-b^2\right)(a+b\cosh(d+ex))}+\frac{3\left(2a^3A-4a^2bB+3aAb^2-b^3B\right)\int \frac{1}{a+b\sin\left(id+iex+\frac{\pi}{2}\right)}dx}{a^2-b^2}}{2(a^2-b^2)}}{3\left(a^2-b^2\right)}-\frac{(Ab-aB)\sinh(d+ex)}{3e\left(a^2-b^2\right)(a+b\cosh(d+ex))^3}-\frac{C}{3be(a+b\cosh(d+ex))^3}$$

$\Big\downarrow$ 3138

3.209. $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^4}\,dx$

$$\frac{-\frac{\left(-2a^2B+5aAb-3b^2B\right)\sinh(d+ex)}{2e(a^2-b^2)(a+b\cosh(d+ex))^2}+\frac{-\frac{\left(-2a^3B+11a^2Ab-13ab^2B+4Ab^3\right)\sinh(d+ex)}{e\left(a^2-b^2\right)(a+b\cosh(d+ex))}-\frac{6i\left(2a^3A-4a^2bB+3aAb^2-b^3B\right)\int\frac{1}{-\left((a-b)\tanh^2\left(\frac{1}{2}(d+ex)\right)\right)-\dots}}{e\left(a^2-b^2\right)}}{2(a^2-b^2)}}{3\left(a^2-b^2\right)}$$

$$\frac{(Ab-aB)\sinh(d+ex)}{3e\left(a^2-b^2\right)(a+b\cosh(d+ex))^3}-\frac{C}{3be(a+b\cosh(d+ex))^3}$$

$\downarrow$ 218

$$-\frac{(Ab-aB)\sinh(d+ex)}{3e\left(a^2-b^2\right)(a+b\cosh(d+ex))^3}+\frac{\frac{\frac{6\left(2a^3A-4a^2bB+3aAb^2-b^3B\right)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{e\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)}-\frac{\left(-2a^3B+11a^2Ab-13ab^2B+4Ab^3\right)\sinh(d+ex)}{e\left(a^2-b^2\right)(a+b\cosh(d+ex))}}{2(a^2-b^2)}-\frac{\left(-2a^2B+5aAb-3b^2B\right)\sinh(d+ex)}{2e(a^2-b^2)(a+b\cosh(d+ex))^2}}{3\left(a^2-b^2\right)}$$

$$\frac{C}{3be(a+b\cosh(d+ex))^3}$$

input
```
Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x])^4,x]
```

output
```
-1/3*C/(b*e*(a + b*Cosh[d + e*x])^3) - ((A*b - a*B)*Sinh[d + e*x])/(3*(a^2
 - b^2)*e*(a + b*Cosh[d + e*x])^3) + (-1/2*((5*a*A*b - 2*a^2*B - 3*b^2*B)*
Sinh[d + e*x])/((a^2 - b^2)*e*(a + b*Cosh[d + e*x])^2) + ((6*(2*a^3*A + 3*
a*A*b^2 - 4*a^2*b*B - b^3*B)*ArcTanh[(Sqrt[a - b]*Tanh[(d + e*x)/2])/Sqrt[
a + b]])/(Sqrt[a - b]*Sqrt[a + b]*(a^2 - b^2)*e) - ((11*a^2*A*b + 4*A*b^3
- 2*a^3*B - 13*a*b^2*B)*Sinh[d + e*x])/((a^2 - b^2)*e*(a + b*Cosh[d + e*x]
)))/(2*(a^2 - b^2)))/(3*(a^2 - b^2))
```

3.209.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

3.209. $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^4}\,dx$

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 4877
```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Simp[d   Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[
c*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Intege
rQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.209. $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^4}\,dx$

3.209.4 Maple [A] (verified)

Time = 11.28 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.77

method	result
derivativedivides	$-\frac{2\left(-\frac{\left(6A\,a^{2}b+3Aa\,b^{2}+2A\,b^{3}-2a^{3}B-2B\,a^{2}b-6Ba\,b^{2}-B\,b^{3}\right)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^{5}}{2(a-b)\left(a^{3}+3a^{2}b+3a\,b^{2}+b^{3}\right)}+\frac{C\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^{4}}{a-b}+\frac{2\left(9A\,a^{2}b+A\,b^{3}-3a^{3}B-7Ba\,b^{2}\cdots\right.}{3\left(a^{2}+2ab+b^{2}\right)\left(a^{2}-2\cdots\right.}\cdots\right.}{\left(a\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}\cdots\right.}$
default	$-\frac{2\left(-\frac{\left(6A\,a^{2}b+3Aa\,b^{2}+2A\,b^{3}-2a^{3}B-2B\,a^{2}b-6Ba\,b^{2}-B\,b^{3}\right)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^{5}}{2(a-b)\left(a^{3}+3a^{2}b+3a\,b^{2}+b^{3}\right)}+\frac{C\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^{4}}{a-b}+\frac{2\left(9A\,a^{2}b+A\,b^{3}-3a^{3}B-7Ba\,b^{2}\cdots\right.}{3\left(a^{2}+2ab+b^{2}\right)\left(a^{2}-2\cdots\right.}\cdots\right.}{\left(a\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}\cdots\right.}$
risch	Expression too large to display

input
```
int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^4,x,method=_RETURNVE
RBOSE)
```

output
```
1/e*(-2*(-1/2*(6*A*a^2*b+3*A*a*b^2+2*A*b^3-2*B*a^3-2*B*a^2*b-6*B*a*b^2-B*b
^3)/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tanh(1/2*e*x+1/2*d)^5+C/(a-b)*tanh(1/2
*e*x+1/2*d)^4+2/3*(9*A*a^2*b+A*b^3-3*B*a^3-7*B*a*b^2)/(a^2+2*a*b+b^2)/(a^2
-2*a*b+b^2)*tanh(1/2*e*x+1/2*d)^3-2*a*C/(a^2-2*a*b+b^2)*tanh(1/2*e*x+1/2*d
)^2-1/2*(6*A*a^2*b-3*A*a*b^2+2*A*b^3-2*B*a^3+2*B*a^2*b-6*B*a*b^2+B*b^3)/(a
+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tanh(1/2*e*x+1/2*d)+1/3*C*(3*a^2+b^2)/(a^3-3
*a^2*b+3*a*b^2-b^3))/(a*tanh(1/2*e*x+1/2*d)^2-b*tanh(1/2*e*x+1/2*d)^2-a-b)
^3+(2*A*a^3+3*A*a*b^2-4*B*a^2*b-B*b^3)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b
)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*e*x+1/2*d)/((a+b)*(a-b))^(1/2)))
```

3.209.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4211 vs. $2(243) = 486$.

Time = 0.52 (sec) , antiderivative size = 8531, normalized size of antiderivative = 32.81

$$\int \frac{A + B\cosh(d + ex) + C\sinh(d + ex)}{(a + b\cosh(d + ex))^4}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^4,x, algorithm
="fricas")
```

output
```
Too large to include
```

3.209.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^4}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))**4,x)
```

output
```
Timed out
```

3.209.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^4,x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.209.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 657 vs. $2(243) = 486$.

Time = 0.30 (sec) , antiderivative size = 657, normalized size of antiderivative = 2.53

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^4}\,dx$$
$$= \frac{\frac{3\,(2\,Aa^3-4\,Ba^2b+3\,Aab^2-Bb^3)\arctan\left(\frac{be^{(ex+d)}+a}{\sqrt{-a^2+b^2}}\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{-a^2+b^2}} + \frac{6\,Aa^3b^3e^{(5\,ex+5\,d)}-12\,Ba^2b^4e^{(5\,ex+5\,d)}+9\,Aab^5e^{(5\,ex+5\,d)}-3\,Bb^6e^{(5\,ex+5\,d)}+30\,Aa^4}{}}{}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^4,x, algorithm
="giac")
```

output
```
1/3*(3*(2*A*a^3 - 4*B*a^2*b + 3*A*a*b^2 - B*b^3)*arctan((b*e^(e*x + d) + a
)/sqrt(-a^2 + b^2))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2))
 + (6*A*a^3*b^3*e^(5*e*x + 5*d) - 12*B*a^2*b^4*e^(5*e*x + 5*d) + 9*A*a*b^5
*e^(5*e*x + 5*d) - 3*B*b^6*e^(5*e*x + 5*d) + 30*A*a^4*b^2*e^(4*e*x + 4*d)
- 60*B*a^3*b^3*e^(4*e*x + 4*d) + 45*A*a^2*b^4*e^(4*e*x + 4*d) - 15*B*a*b^5
*e^(4*e*x + 4*d) - 8*B*a^6*e^(3*e*x + 3*d) - 8*C*a^6*e^(3*e*x + 3*d) + 44*
A*a^5*b*e^(3*e*x + 3*d) - 64*B*a^4*b^2*e^(3*e*x + 3*d) + 24*C*a^4*b^2*e^(3
*e*x + 3*d) + 82*A*a^3*b^3*e^(3*e*x + 3*d) - 78*B*a^2*b^4*e^(3*e*x + 3*d)
- 24*C*a^2*b^4*e^(3*e*x + 3*d) + 24*A*a*b^5*e^(3*e*x + 3*d) + 8*C*b^6*e^(3
*e*x + 3*d) - 24*B*a^5*b*e^(2*e*x + 2*d) + 102*A*a^4*b^2*e^(2*e*x + 2*d) -
 102*B*a^3*b^3*e^(2*e*x + 2*d) + 36*A*a^2*b^4*e^(2*e*x + 2*d) - 24*B*a*b^5
*e^(2*e*x + 2*d) + 12*A*b^6*e^(2*e*x + 2*d) - 12*B*a^4*b^2*e^(e*x + d) + 6
0*A*a^3*b^3*e^(e*x + d) - 66*B*a^2*b^4*e^(e*x + d) + 15*A*a*b^5*e^(e*x + d
) + 3*B*b^6*e^(e*x + d) - 2*B*a^3*b^3 + 11*A*a^2*b^4 - 13*B*a*b^5 + 4*A*b^
6)/((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*(b*e^(2*e*x + 2*d) + 2*a*e^(e*x
+ d) + b)^3))/e
```

3.209.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^4}\,dx=\int \frac{A+B\cosh(d+e\,x)+C\sinh(d+e\,x)}{\left(a+b\cosh\left(d+e\,x\right)\right)^4}\,dx$$

input
```
int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + b*cosh(d + e*x))^4,x)
```

output
```
int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + b*cosh(d + e*x))^4, x)
```

3.210 $\int \frac{x}{a+b\cosh^2(x)}\,dx$

3.210.1 Optimal result

Integrand size = 12, antiderivative size = 191

$$\int \frac{x}{a+b\cosh^2(x)}\,dx=\frac{x\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}}-\frac{x\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}}+\frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{4\sqrt{a}\sqrt{a+b}}-\frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{4\sqrt{a}\sqrt{a+b}}$$

output
```
1/2*x*ln(1+b*exp(2*x)/(2*a+b-2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b)^(1/2)-1
/2*x*ln(1+b*exp(2*x)/(2*a+b+2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b)^(1/2)+1/
4*polylog(2,-b*exp(2*x)/(2*a+b-2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b)^(1/2)
-1/4*polylog(2,-b*exp(2*x)/(2*a+b+2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b)^(1
/2)
```

3.210.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.49

$$\int \frac{x}{a+b\cosh^2(x)}\,dx = \frac{x\log\left(1-\frac{e^x}{\sqrt{-\frac{2a+b-2\sqrt{a(a+b)}}{b}}}\right)+x\log\left(1+\frac{e^x}{\sqrt{-\frac{2a+b-2\sqrt{a(a+b)}}{b}}}\right)-x\log\left(1-\frac{e^x}{\sqrt{-\frac{2a+b+2\sqrt{a(a+b)}}{b}}}\right)-x\log\left(1+\cdots\right.}{\cdots}$$

input
```
Integrate[x/(a + b*Cosh[x]^2),x]
```

output
```
(x*Log[1 - E^x/Sqrt[-((2*a + b - 2*Sqrt[a*(a + b)])/b)]] + x*Log[1 + E^x/S
qrt[-((2*a + b - 2*Sqrt[a*(a + b)])/b)]] - x*Log[1 - E^x/Sqrt[-((2*a + b +
 2*Sqrt[a*(a + b)])/b)]] - x*Log[1 + E^x/Sqrt[-((2*a + b + 2*Sqrt[a*(a + b
)])/b)]] + PolyLog[2, -(E^x/Sqrt[-((2*a + b - 2*Sqrt[a*(a + b)])/b)])] + P
olyLog[2, E^x/Sqrt[-((2*a + b - 2*Sqrt[a*(a + b)])/b)]] - PolyLog[2, -(E^x
/Sqrt[-((2*a + b + 2*Sqrt[a*(a + b)])/b)])] - PolyLog[2, E^x/Sqrt[-((2*a +
 b + 2*Sqrt[a*(a + b)])/b)]])/(2*Sqrt[a*(a + b)])
```

3.210.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6164, 3042, 3801, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+b\cosh^2(x)}dx$$

$$\Big\downarrow \text{ 6164}$$

$$2\int \frac{x}{2a+b+b\cosh(2x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$2\int \frac{x}{2a+b+b\sin\left(2ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3801}$$

$$4\int \frac{e^{2x}x}{e^{4x}b+b+2(2a+b)e^{2x}}dx$$

$$\Big\downarrow \text{ 2694}$$

$$4\left(\frac{b\int \frac{e^{2x}x}{2(2a-2\sqrt{a+b}\sqrt{a}+be^{2x}+b)}dx}{2\sqrt{a}\sqrt{a+b}}-\frac{b\int \frac{e^{2x}x}{2(2a+2\sqrt{a+b}\sqrt{a}+be^{2x}+b)}dx}{2\sqrt{a}\sqrt{a+b}}\right)$$

$$\Big\downarrow \text{ 27}$$

$$4 \left(\frac{b \int \frac{e^{2x} x}{2a-2\sqrt{a+b}\sqrt{a+be^{2x}+b}}dx}{4\sqrt{a}\sqrt{a+b}}-\frac{b \int \frac{e^{2x} x}{2a+2\sqrt{a+b}\sqrt{a+be^{2x}+b}}dx}{4\sqrt{a}\sqrt{a+b}}\right)$$

$$\Big\downarrow \text{ 2620}$$

$$4 \left(\frac{b \left(\frac{x \log \left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2b}-\frac{\int \log \left(\frac{e^{2x} b}{2a-2\sqrt{a+b}\sqrt{a+b}}+1\right)dx}{2b}\right)}{4\sqrt{a}\sqrt{a+b}}-\frac{b \left(\frac{x \log \left(\frac{be^{2x}}{2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2b}-\frac{\int \log \left(\frac{e^{2x} b}{2a+2\sqrt{a+b}\sqrt{a+b}}+1\right)dx}{2b}\right)}{4\sqrt{a}\sqrt{a+b}}\right)$$

$$\Big\downarrow \text{ 2715}$$

$$4 \left(\frac{b \left(\frac{x \log \left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2b}-\frac{\int e^{-2x} \log \left(\frac{e^{2x} b}{2a-2\sqrt{a+b}\sqrt{a+b}}+1\right)de^{2x}}{4b}\right)}{4\sqrt{a}\sqrt{a+b}}-\frac{b \left(\frac{x \log \left(\frac{be^{2x}}{2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2b}-\frac{\int e^{-2x} \log \left(\frac{e^{2x} b}{2a+2\sqrt{a+b}\sqrt{a+b}}\right.}{4b}\right.}{4\sqrt{a}\sqrt{a+b}}\right.$$

$$\Big\downarrow \text{ 2838}$$

$$4 \left(\frac{b \left(\frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a+b}}\right)}{4b}+\frac{x \log \left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2b}\right)}{4\sqrt{a}\sqrt{a+b}}-\frac{b \left(\frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{2a+2\sqrt{a+b}\sqrt{a+b}}\right)}{4b}+\frac{x \log \left(\frac{be^{2x}}{2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2b}\right)}{4\sqrt{a}\sqrt{a+b}}\right.$$

input
```
Int[x/(a + b*Cosh[x]^2),x]
```

output
```
4*((b*((x*Log[1 + (b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b])])/(2*b) +
PolyLog[2, -((b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b]))]/(4*b)))/(4*Sq
rt[a]*Sqrt[a + b]) - (b*((x*Log[1 + (b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt[
a + b])])/(2*b) + PolyLog[2, -((b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt[a + b
]))]/(4*b)))/(4*Sqrt[a]*Sqrt[a + b]))
```

3.210. $\int \frac{x}{a+b\cosh^2(x)}\,dx$

3.210.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3801
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[((c + d*x)^m*(E^((-I)*e
+ f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)
*e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c
, d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6164
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^2*(b_.) + (a_))^(n_)*(x_)^(m_.), x_Symbol] :>
 Simp[1/2^n   Int[x^m*(2*a + b + b*Cosh[2*c + 2*d*x])^n, x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[a - b, 0] && IGtQ[m, 0] && ILtQ[n, 0] && (EqQ[n, -1]
|| (EqQ[m, 1] && EqQ[n, -2]))
```

3.210.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(147) = 294$.

Time = 0.16 (sec) , antiderivative size = 487, normalized size of antiderivative = 2.55

method	result
risch	$\frac{x\ln\left(1-\frac{b\,\mathrm{e}^{2x}}{2\sqrt{a(a+b)}-2a-b}\right)}{2\sqrt{a(a+b)}}-\frac{x^2}{2\sqrt{a(a+b)}}+\frac{\operatorname{polylog}\left(2,\frac{b\,\mathrm{e}^{2x}}{2\sqrt{a(a+b)}-2a-b}\right)}{4\sqrt{a(a+b)}}+\frac{\ln\left(1-\frac{b\,\mathrm{e}^{2x}}{-2\sqrt{a(a+b)}-2a-b}\right)x}{-2\sqrt{a(a+b)}-2a-b}-\frac{x^2}{-2\sqrt{a(a+b)}-2a-b}$

input
```
int(x/(a+b*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(a*(a+b))^(1/2)*x*ln(1-b*exp(2*x)/(2*(a*(a+b))^(1/2)-2*a-b))-1/2/(a*(a
+b))^(1/2)*x^2+1/4/(a*(a+b))^(1/2)*polylog(2,b*exp(2*x)/(2*(a*(a+b))^(1/2)
-2*a-b))+1/(-2*(a*(a+b))^(1/2)-2*a-b)*ln(1-b*exp(2*x)/(-2*(a*(a+b))^(1/2)-
2*a-b))*x-1/(-2*(a*(a+b))^(1/2)-2*a-b)*x^2+1/(a*(a+b))^(1/2)/(-2*(a*(a+b))
^(1/2)-2*a-b)*ln(1-b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*a*x+1/2/(a*(a+b)
)^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*ln(1-b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a
-b))*b*x-1/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*a*x^2-1/2/(a*(a+b))^
(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*b*x^2+1/2/(-2*(a*(a+b))^(1/2)-2*a-b)*poly
log(2,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))+1/2/(a*(a+b))^(1/2)/(-2*(a*(a
+b))^(1/2)-2*a-b)*polylog(2,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*a+1/4/(
a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*polylog(2,b*exp(2*x)/(-2*(a*(a+b
))^(1/2)-2*a-b))*b
```

3.210.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 780 vs. $2(149) = 298$.

Time = 0.32 (sec) , antiderivative size = 780, normalized size of antiderivative = 4.08

$$\int \frac{x}{a+b\cosh^2(x)}\,dx =$$

$$-\frac{bx\sqrt{\frac{a^2+ab}{b^2}}\log\left(\frac{\left((2a+b)\cosh(x)+(2a+b)\sinh(x)-2\left(b\cosh(x)+b\sinh(x)\right)\sqrt{\frac{a^2+ab}{b^2}}\right)\sqrt{-\frac{2b\sqrt{\frac{a^2+ab}{b^2}}+2a+b}{b}}+b}{b}\right)+bx\sqrt{\frac{a^2+ab}{b^2}}\log\cdots}{\cdots}$$

input
```
integrate(x/(a+b*cosh(x)^2),x, algorithm="fricas")
```

output
```
-1/2*(b*x*sqrt((a^2 + a*b)/b^2)*log((((2*a + b)*cosh(x) + (2*a + b)*sinh(x
) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2
+ a*b)/b^2) + 2*a + b)/b) + b)/b) + b*x*sqrt((a^2 + a*b)/b^2)*log(-(((2*a
+ b)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a
*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b) - b)/b) - b*x*sqr
t((a^2 + a*b)/b^2)*log((((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh
(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) -
2*a - b)/b) + b)/b) - b*x*sqrt((a^2 + a*b)/b^2)*log(-(((2*a + b)*cosh(x) +
 (2*a + b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt
((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b) - b)/b) + b*sqrt((a^2 + a*b)/b^2
)*dilog(-(((2*a + b)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x
))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b) +
 b)/b + 1) + b*sqrt((a^2 + a*b)/b^2)*dilog((((2*a + b)*cosh(x) + (2*a + b)
*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqr
t((a^2 + a*b)/b^2) + 2*a + b)/b) - b)/b + 1) - b*sqrt((a^2 + a*b)/b^2)*dil
og(-(((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sq
rt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b) + b)/b
+ 1) - b*sqrt((a^2 + a*b)/b^2)*dilog((((2*a + b)*cosh(x) + (2*a + b)*sinh(
x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2
+ a*b)/b^2) - 2*a - b)/b) - b)/b + 1))/(a^2 + a*b)
```

3.210.6 Sympy [F]

$$\int \frac{x}{a+b\cosh^2(x)}\,dx = \int \frac{x}{a+b\cosh^2{(x)}}\,dx$$

input
```
integrate(x/(a+b*cosh(x)**2),x)
```

output
```
Integral(x/(a + b*cosh(x)**2), x)
```

3.210.7 Maxima [F]

$$\int \frac{x}{a+b\cosh^2(x)}\,dx = \int \frac{x}{b\cosh{(x)}^2+a}\,dx$$

input
```
integrate(x/(a+b*cosh(x)^2),x, algorithm="maxima")
```

output
```
integrate(x/(b*cosh(x)^2 + a), x)
```

3.210.8 Giac [F]

$$\int \frac{x}{a+b\cosh^2(x)}\,dx = \int \frac{x}{b\cosh{(x)}^2+a}\,dx$$

input
```
integrate(x/(a+b*cosh(x)^2),x, algorithm="giac")
```

output
```
integrate(x/(b*cosh(x)^2 + a), x)
```

3.210.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{a + b\cosh^2(x)}\,dx = \int \frac{x}{b\cosh{(x)}^2 + a}\,dx$$

input
```
int(x/(a + b*cosh(x)^2),x)
```

output
```
int(x/(a + b*cosh(x)^2), x)
```

3.211 $\int \frac{x^2}{a+b\cosh^2(x)}\,dx$

3.211.1 Optimal result

Integrand size = 14, antiderivative size = 291

$$\begin{aligned}\int \frac{x^2}{a+b\cosh^2(x)}\,dx &= \frac{x^2\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}}-\frac{x^2\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}}\\&\quad+\frac{x\,\mathrm{PolyLog}\left(2,-\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}}-\frac{x\,\mathrm{PolyLog}\left(2,-\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}}\\&\quad-\frac{\mathrm{PolyLog}\left(3,-\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{4\sqrt{a}\sqrt{a+b}}+\frac{\mathrm{PolyLog}\left(3,-\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{4\sqrt{a}\sqrt{a+b}}\end{aligned}$$

output
```
1/2*x^2*ln(1+b*exp(2*x)/(2*a+b-2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b)^(1/2)
-1/2*x^2*ln(1+b*exp(2*x)/(2*a+b+2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b)^(1/2
)+1/2*x*polylog(2,-b*exp(2*x)/(2*a+b-2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b)
^(1/2)-1/2*x*polylog(2,-b*exp(2*x)/(2*a+b+2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/
(a+b)^(1/2)-1/4*polylog(3,-b*exp(2*x)/(2*a+b-2*a^(1/2)*(a+b)^(1/2)))/a^(1/
2)/(a+b)^(1/2)+1/4*polylog(3,-b*exp(2*x)/(2*a+b+2*a^(1/2)*(a+b)^(1/2)))/a^
(1/2)/(a+b)^(1/2)
```

3.211.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.49

$$\int \frac{x^2}{a+b\cosh^2(x)}\,dx$$
$$= \frac{x^2\log\left(1-\frac{e^x}{\sqrt{-\frac{2a+b-2\sqrt{a(a+b)}}{b}}}\right)+x^2\log\left(1+\frac{e^x}{\sqrt{-\frac{2a+b-2\sqrt{a(a+b)}}{b}}}\right)-x^2\log\left(1-\frac{e^x}{\sqrt{-\frac{2a+b+2\sqrt{a(a+b)}}{b}}}\right)-x^2\log\left(1\right.}{}$$

input
```
Integrate[x^2/(a + b*Cosh[x]^2),x]
```

output
```
(x^2*Log[1 - E^x/Sqrt[-((2*a + b - 2*Sqrt[a*(a + b)])/b)]] + x^2*Log[1 + E
^x/Sqrt[-((2*a + b - 2*Sqrt[a*(a + b)])/b)]] - x^2*Log[1 - E^x/Sqrt[-((2*a
 + b + 2*Sqrt[a*(a + b)])/b)]] - x^2*Log[1 + E^x/Sqrt[-((2*a + b + 2*Sqrt[
a*(a + b)])/b)]] + 2*x*PolyLog[2, -(E^x/Sqrt[-((2*a + b - 2*Sqrt[a*(a + b)
])/b)])] + 2*x*PolyLog[2, E^x/Sqrt[-((2*a + b - 2*Sqrt[a*(a + b)])/b)]] -
2*x*PolyLog[2, -(E^x/Sqrt[-((2*a + b + 2*Sqrt[a*(a + b)])/b)])] - 2*x*Poly
Log[2, E^x/Sqrt[-((2*a + b + 2*Sqrt[a*(a + b)])/b)]] - 2*PolyLog[3, -(E^x/
Sqrt[-((2*a + b - 2*Sqrt[a*(a + b)])/b)])] - 2*PolyLog[3, E^x/Sqrt[-((2*a
+ b - 2*Sqrt[a*(a + b)])/b)]] + 2*PolyLog[3, -(E^x/Sqrt[-((2*a + b + 2*Sqr
t[a*(a + b)])/b)])] + 2*PolyLog[3, E^x/Sqrt[-((2*a + b + 2*Sqrt[a*(a + b)]
)/b)]])/(2*Sqrt[a*(a + b)])
```

3.211.3 Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {6164, 3042, 3801, 2694, 27, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+b\cosh^2(x)}\,dx$$
$$\Big\downarrow \text{ 6164}$$
$$2\int \frac{x^2}{2a+b+b\cosh(2x)}dx$$
$$\Big\downarrow \text{ 3042}$$

$$2\int \frac{x^2}{2a+b+b\sin\left(2ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3801}$$

$$4\int \frac{e^{2x}x^2}{e^{4x}b+b+2(2a+b)e^{2x}}dx$$

$$\Big\downarrow \text{ 2694}$$

$$4\left(\frac{b\int \frac{e^{2x}x^2}{2\left(2a-2\sqrt{a+b}\sqrt{a}+be^{2x}+b\right)}dx}{2\sqrt{a}\sqrt{a+b}}-\frac{b\int \frac{e^{2x}x^2}{2\left(2a+2\sqrt{a+b}\sqrt{a}+be^{2x}+b\right)}dx}{2\sqrt{a}\sqrt{a+b}}\right)$$

$$\Big\downarrow \text{ 27}$$

$$4\left(\frac{b\int \frac{e^{2x}x^2}{2a-2\sqrt{a+b}\sqrt{a}+be^{2x}+b}dx}{4\sqrt{a}\sqrt{a+b}}-\frac{b\int \frac{e^{2x}x^2}{2a+2\sqrt{a+b}\sqrt{a}+be^{2x}+b}dx}{4\sqrt{a}\sqrt{a+b}}\right)$$

$$\Big\downarrow \text{ 2620}$$

$$4\left(\frac{b\left(\frac{x^2\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2b}-\frac{\int x\log\left(\frac{e^{2x}b}{2a-2\sqrt{a+b}\sqrt{a}+b}+1\right)dx}{b}\right)}{4\sqrt{a}\sqrt{a+b}}-\frac{b\left(\frac{x^2\log\left(\frac{be^{2x}}{2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2b}-\frac{\int x\log\left(\frac{e^{2x}b}{2a+2\sqrt{a+b}\sqrt{a}+b}+1\right)dx}{b}\right.}{4\sqrt{a}\sqrt{a+b}}\right.$$

$$\Big\downarrow \text{ 3011}$$

$$4\left(\frac{b\left(\frac{x^2\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2b}-\frac{\frac{1}{2}\int \text{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a}+b}\right)dx-\frac{1}{2}x\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a}+b}\right)}{b}\right)}{4\sqrt{a}\sqrt{a+b}}-\frac{b\left(\frac{x^2\log\left(\frac{be^{2x}}{2\sqrt{a}\sqrt{a+b}+2a}\right.}{2b}\right.}{}\right.$$

$$\Big\downarrow \text{ 2720}$$

$$4\left(\frac{b\left(\frac{x^2\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2b}-\frac{\frac{1}{4}\int e^{-2x}\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a}+b}\right)de^{2x}-\frac{1}{2}x\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a}+b}\right)}{b}\right)}{4\sqrt{a}\sqrt{a+b}}-\frac{b\left(\frac{x^2\log\left(\frac{}{2\sqrt{a}\sqrt{}}\right.}{}\right.}{}\right.$$

$$\Big\downarrow \text{ 7143}$$

3.211. $\int \frac{x^2}{a+b\cosh^2(x)}\,dx$

$$4\left(\frac{b\left(\frac{x^2\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2b}-\frac{\frac{1}{4}\operatorname{PolyLog}\left(3,-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a}+b}\right)-\frac{1}{2}x\operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a}+b}\right)}{b}\right)}{4\sqrt{a}\sqrt{a+b}}-\frac{b\left(\frac{x^2\log\left(\frac{be^{2x}}{2\sqrt{a}\sqrt{a+b}+2a+b}+\right.}{2b}\right.}{}\right.$$

input
```
Int[x^2/(a + b*Cosh[x]^2),x]
```

output
```
4*((b*((x^2*Log[1 + (b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b])])/(2*b)
- (-1/2*(x*PolyLog[2, -((b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b]))]) +
 PolyLog[3, -((b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b]))]/4)/b))/(4*Sq
rt[a]*Sqrt[a + b]) - (b*((x^2*Log[1 + (b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqr
t[a + b])])/(2*b) - (-1/2*(x*PolyLog[2, -((b*E^(2*x))/(2*a + b + 2*Sqrt[a]
*Sqrt[a + b]))]) + PolyLog[3, -((b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt[a +
b]))]/4)/b))/(4*Sqrt[a]*Sqrt[a + b]))
```

3.211.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3801
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[((c + d*x)^m*(E^((-I)*e
+ f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)
*e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c
, d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6164
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^2*(b_.) + (a_))^(n_)*(x_)^(m_.), x_Symbol] :>
 Simp[1/2^n   Int[x^m*(2*a + b + b*Cosh[2*c + 2*d*x])^n, x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[a - b, 0] && IGtQ[m, 0] && ILtQ[n, 0] && (EqQ[n, -1]
|| (EqQ[m, 1] && EqQ[n, -2]))
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.211. $\int \frac{x^2}{a+b\cosh^2(x)}\,dx$

3.211.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 685 vs. $2(225) = 450$.

Time = 0.16 (sec) , antiderivative size = 686, normalized size of antiderivative = 2.36

method	result
risch	$-\frac{2x^3}{3\left(-2\sqrt{a(a+b)}-2a-b\right)}+\frac{x^2\ln\left(1-\frac{b\,e^{2x}}{-2\sqrt{a(a+b)}-2a-b}\right)}{-2\sqrt{a(a+b)}-2a-b}+\frac{x\,\mathrm{polylog}\left(2,\frac{b\,e^{2x}}{-2\sqrt{a(a+b)}-2a-b}\right)}{-2\sqrt{a(a+b)}-2a-b}-\frac{\mathrm{polylog}\left(3,\frac{b\,e^{2x}}{-2\sqrt{a(a+b)}-2a-b}\right)}{2\left(-2\sqrt{a(a+b)}-2a-b\right)}$

input
```
int(x^2/(a+b*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/(-2*(a*(a+b))^(1/2)-2*a-b)*x^3+1/(-2*(a*(a+b))^(1/2)-2*a-b)*x^2*ln(1-
b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))+1/(-2*(a*(a+b))^(1/2)-2*a-b)*x*poly
log(2,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))-1/2/(-2*(a*(a+b))^(1/2)-2*a-b
)*polylog(3,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))-2/3/(a*(a+b))^(1/2)/(-2
*(a*(a+b))^(1/2)-2*a-b)*a*x^3+1/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)
*a*x^2*ln(1-b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))+1/(a*(a+b))^(1/2)/(-2*(
a*(a+b))^(1/2)-2*a-b)*a*x*polylog(2,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))
-1/2/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*a*polylog(3,b*exp(2*x)/(-2
*(a*(a+b))^(1/2)-2*a-b))-1/3/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*b*
x^3+1/2/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*b*x^2*ln(1-b*exp(2*x)/(
-2*(a*(a+b))^(1/2)-2*a-b))+1/2/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*
b*x*polylog(2,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))-1/4/(a*(a+b))^(1/2)/(
-2*(a*(a+b))^(1/2)-2*a-b)*b*polylog(3,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b
))-1/3/(a*(a+b))^(1/2)*x^3+1/2/(a*(a+b))^(1/2)*x^2*ln(1-b*exp(2*x)/(2*(a*(
a+b))^(1/2)-2*a-b))+1/2/(a*(a+b))^(1/2)*x*polylog(2,b*exp(2*x)/(2*(a*(a+b)
)^(1/2)-2*a-b))-1/4/(a*(a+b))^(1/2)*polylog(3,b*exp(2*x)/(2*(a*(a+b))^(1/2
)-2*a-b))
```

3.211.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1162 vs. $2(228) = 456$.

Time = 0.30 (sec) , antiderivative size = 1162, normalized size of antiderivative = 3.99

$$\int \frac{x^2}{a+b\cosh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(a+b*cosh(x)^2),x, algorithm="fricas")
```

output
```
-1/2*(b*x^2*sqrt((a^2 + a*b)/b^2)*log((((2*a + b)*cosh(x) + (2*a + b)*sinh
(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^
2 + a*b)/b^2) + 2*a + b)/b) + b)/b) + b*x^2*sqrt((a^2 + a*b)/b^2)*log(-(((
2*a + b)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2
 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b) - b)/b) - b*x
^2*sqrt((a^2 + a*b)/b^2)*log((((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(
b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b
^2) - 2*a - b)/b) + b)/b) - b*x^2*sqrt((a^2 + a*b)/b^2)*log(-(((2*a + b)*c
osh(x) + (2*a + b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^
2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b) - b)/b) + 2*b*x*sqrt((a^
2 + a*b)/b^2)*dilog(-(((2*a + b)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x
) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2
*a + b)/b) + b)/b + 1) + 2*b*x*sqrt((a^2 + a*b)/b^2)*dilog((((2*a + b)*cos
h(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2)
)*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b) - b)/b + 1) - 2*b*x*sqrt(
(a^2 + a*b)/b^2)*dilog(-(((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cos
h(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) -
 2*a - b)/b) + b)/b + 1) - 2*b*x*sqrt((a^2 + a*b)/b^2)*dilog((((2*a + b)*c
osh(x) + (2*a + b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^
2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b) - b)/b + 1) - 2*b*sqr...
```

3.211.6 Sympy [F]

$$\int \frac{x^2}{a + b\cosh^2(x)}\,dx = \int \frac{x^2}{a + b\cosh^2{(x)}}\,dx$$

input
```
integrate(x**2/(a+b*cosh(x)**2),x)
```

output
```
Integral(x**2/(a + b*cosh(x)**2), x)
```

3.211.7 Maxima [F]

$$\int \frac{x^2}{a + b\cosh^2(x)}\,dx = \int \frac{x^2}{b\cosh(x)^2 + a}\,dx$$

input
```
integrate(x^2/(a+b*cosh(x)^2),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*cosh(x)^2 + a), x)
```

3.211.8 Giac [F]

$$\int \frac{x^2}{a + b\cosh^2(x)}\,dx = \int \frac{x^2}{b\cosh(x)^2 + a}\,dx$$

input
```
integrate(x^2/(a+b*cosh(x)^2),x, algorithm="giac")
```

output
```
integrate(x^2/(b*cosh(x)^2 + a), x)
```

3.211.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{a + b\cosh^2(x)}\,dx = \int \frac{x^2}{b\cosh(x)^2 + a}\,dx$$

input
```
int(x^2/(a + b*cosh(x)^2),x)
```

output
```
int(x^2/(a + b*cosh(x)^2), x)
```

3.212 $\int \frac{x^3}{a+b\cosh^2(x)}\,dx$

3.212.1 Optimal result

Integrand size = 14, antiderivative size = 391

$$\begin{aligned}\int \frac{x^3}{a+b\cosh^2(x)}\,dx &= \frac{x^3\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}}-\frac{x^3\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}}\\
&+\frac{3x^2\operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{4\sqrt{a}\sqrt{a+b}}\\
&-\frac{3x^2\operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{4\sqrt{a}\sqrt{a+b}}\\
&-\frac{3x\operatorname{PolyLog}\left(3,-\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{4\sqrt{a}\sqrt{a+b}}+\frac{3x\operatorname{PolyLog}\left(3,-\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{4\sqrt{a}\sqrt{a+b}}\\
&+\frac{3\operatorname{PolyLog}\left(4,-\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{8\sqrt{a}\sqrt{a+b}}-\frac{3\operatorname{PolyLog}\left(4,-\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{8\sqrt{a}\sqrt{a+b}}\end{aligned}$$

output
```
1/2*x^3*ln(1+b*exp(2*x)/(2*a+b-2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b)^(1/2)
-1/2*x^3*ln(1+b*exp(2*x)/(2*a+b+2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b)^(1/2
)+3/4*x^2*polylog(2,-b*exp(2*x)/(2*a+b-2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+
b)^(1/2)-3/4*x^2*polylog(2,-b*exp(2*x)/(2*a+b+2*a^(1/2)*(a+b)^(1/2)))/a^(1
/2)/(a+b)^(1/2)-3/4*x*polylog(3,-b*exp(2*x)/(2*a+b-2*a^(1/2)*(a+b)^(1/2)))
/a^(1/2)/(a+b)^(1/2)+3/4*x*polylog(3,-b*exp(2*x)/(2*a+b+2*a^(1/2)*(a+b)^(1
/2)))/a^(1/2)/(a+b)^(1/2)+3/8*polylog(4,-b*exp(2*x)/(2*a+b-2*a^(1/2)*(a+b)
^(1/2)))/a^(1/2)/(a+b)^(1/2)-3/8*polylog(4,-b*exp(2*x)/(2*a+b+2*a^(1/2)*(a
+b)^(1/2)))/a^(1/2)/(a+b)^(1/2)
```

3.212.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.48

$$\int \frac{x^3}{a+b\cosh^2(x)}\,dx$$
$$= \frac{x^3\log\left(1-\frac{e^x}{\sqrt{-\frac{2a+b-2\sqrt{a(a+b)}}{b}}}\right)+x^3\log\left(1+\frac{e^x}{\sqrt{-\frac{2a+b-2\sqrt{a(a+b)}}{b}}}\right)-x^3\log\left(1-\frac{e^x}{\sqrt{-\frac{2a+b+2\sqrt{a(a+b)}}{b}}}\right)-x^3\log\left(1\right.}{}$$

input
```
Integrate[x^3/(a + b*Cosh[x]^2),x]
```

output
```
(x^3*Log[1 - E^x/Sqrt[-((2*a + b - 2*Sqrt[a*(a + b)])/b)]] + x^3*Log[1 + E
^x/Sqrt[-((2*a + b - 2*Sqrt[a*(a + b)])/b)]] - x^3*Log[1 - E^x/Sqrt[-((2*a
 + b + 2*Sqrt[a*(a + b)])/b)]] - x^3*Log[1 + E^x/Sqrt[-((2*a + b + 2*Sqrt[
a*(a + b)])/b)]] + 3*x^2*PolyLog[2, -(E^x/Sqrt[-((2*a + b - 2*Sqrt[a*(a +
b)])/b)])] + 3*x^2*PolyLog[2, E^x/Sqrt[-((2*a + b - 2*Sqrt[a*(a + b)])/b)]
] - 3*x^2*PolyLog[2, -(E^x/Sqrt[-((2*a + b + 2*Sqrt[a*(a + b)])/b)])] - 3*
x^2*PolyLog[2, E^x/Sqrt[-((2*a + b + 2*Sqrt[a*(a + b)])/b)]] - 6*x*PolyLog
[3, -(E^x/Sqrt[-((2*a + b - 2*Sqrt[a*(a + b)])/b)])] - 6*x*PolyLog[3, E^x/
Sqrt[-((2*a + b - 2*Sqrt[a*(a + b)])/b)]] + 6*x*PolyLog[3, -(E^x/Sqrt[-((2
*a + b + 2*Sqrt[a*(a + b)])/b)])] + 6*x*PolyLog[3, E^x/Sqrt[-((2*a + b + 2
*Sqrt[a*(a + b)])/b)]] + 6*PolyLog[4, -(E^x/Sqrt[-((2*a + b - 2*Sqrt[a*(a
+ b)])/b)])] + 6*PolyLog[4, E^x/Sqrt[-((2*a + b - 2*Sqrt[a*(a + b)])/b)]]
- 6*PolyLog[4, -(E^x/Sqrt[-((2*a + b + 2*Sqrt[a*(a + b)])/b)])] - 6*PolyLo
g[4, E^x/Sqrt[-((2*a + b + 2*Sqrt[a*(a + b)])/b)]])/(2*Sqrt[a*(a + b)])
```

3.212.3 Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.91, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {6164, 3042, 3801, 2694, 27, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a+b\cosh^2(x)}\,dx$$
$$\Big\downarrow \; 6164$$

$$2\int \frac{x^3}{2a+b+b\cosh(2x)}dx$$

$\big\downarrow$ 3042

$$2\int \frac{x^3}{2a+b+b\sin\left(2ix+\frac{\pi}{2}\right)}dx$$

$\big\downarrow$ 3801

$$4\int \frac{e^{2x}x^3}{e^{4x}b+b+2(2a+b)e^{2x}}dx$$

$\big\downarrow$ 2694

$$4\left(\frac{b\int \frac{e^{2x}x^3}{2\left(2a-2\sqrt{a+b}\sqrt{a}+be^{2x}+b\right)}dx}{2\sqrt{a}\sqrt{a+b}}-\frac{b\int \frac{e^{2x}x^3}{2\left(2a+2\sqrt{a+b}\sqrt{a}+be^{2x}+b\right)}dx}{2\sqrt{a}\sqrt{a+b}}\right)$$

$\big\downarrow$ 27

$$4\left(\frac{b\int \frac{e^{2x}x^3}{2a-2\sqrt{a+b}\sqrt{a}+be^{2x}+b}dx}{4\sqrt{a}\sqrt{a+b}}-\frac{b\int \frac{e^{2x}x^3}{2a+2\sqrt{a+b}\sqrt{a}+be^{2x}+b}dx}{4\sqrt{a}\sqrt{a+b}}\right)$$

$\big\downarrow$ 2620

$$4\left(\frac{b\left(\frac{x^3\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2b}-\frac{3\int x^2\log\left(\frac{e^{2x}b}{2a-2\sqrt{a+b}\sqrt{a}+b}+1\right)dx}{2b}\right)}{4\sqrt{a}\sqrt{a+b}}-\frac{b\left(\frac{x^3\log\left(\frac{be^{2x}}{2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2b}-\frac{3\int x^2\log\left(\frac{e^{2x}b}{2a+2\sqrt{a+b}\sqrt{a}+b}+\right.}{2b}\right.}{4\sqrt{a}\sqrt{a+b}}\right.$$

$\big\downarrow$ 3011

$$4\left(\frac{b\left(\frac{x^3\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2b}-\frac{3\left(\int x\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a}+b}\right)dx-\frac{1}{2}x^2\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a}+b}\right)\right)}{2b}\right)}{4\sqrt{a}\sqrt{a+b}}-\frac{b\left(\frac{x^3\log\left(\frac{be}{2\sqrt{a}\sqrt{a+}}\right.}{2b}\right.}{}\right.$$

$\big\downarrow$ 7163

$$4\left(\frac{b\left(\frac{x^3\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2b}-\frac{3\left(-\frac{1}{2}\int \text{PolyLog}\left(3,-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a}+b}\right)dx-\frac{1}{2}x^2\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a}+b}\right)+\frac{1}{2}x\,\text{PolyLog}\left(3,-\frac{b}{2a-2\sqrt{}}\right.\right.}{2b}\right.}{4\sqrt{a}\sqrt{a+b}}\right.$$

3.212. $\int \frac{x^3}{a+b\cosh^2(x)}\,dx$

$$\Big\downarrow \text{2720}$$

$$4\left(\frac{b\left(\frac{x^3\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2b}-\frac{3\left(-\frac{1}{4}\int e^{-2x}\operatorname{PolyLog}\left(3,-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a+b}}\right)de^{2x}-\frac{1}{2}x^2\operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a+b}}\right)+\frac{1}{2}x\operatorname{PolyLog}\left(3,-\right.\right.}{2b}\right.}{4\sqrt{a}\sqrt{a+b}}\right.$$

$$\Big\downarrow \text{7143}$$

$$4\left(\frac{b\left(\frac{x^3\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2b}-\frac{3\left(-\frac{1}{2}x^2\operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a+b}}\right)+\frac{1}{2}x\operatorname{PolyLog}\left(3,-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a+b}}\right)-\frac{1}{4}\operatorname{PolyLog}\left(4,-\frac{be^{2x}}{2a-2\sqrt{a+b}\right.\right.}{2b}\right.}{4\sqrt{a}\sqrt{a+b}}\right.$$

input
```
Int[x^3/(a + b*Cosh[x]^2),x]
```

output
```
4*((b*((x^3*Log[1 + (b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b])])/(2*b)
- (3*(-1/2*(x^2*PolyLog[2, -((b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b])
)]) + (x*PolyLog[3, -((b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b]))])/2 -
 PolyLog[4, -((b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b]))]/4))/(2*b)))/
(4*Sqrt[a]*Sqrt[a + b]) - (b*((x^3*Log[1 + (b*E^(2*x))/(2*a + b + 2*Sqrt[a
]*Sqrt[a + b])])/(2*b) - (3*(-1/2*(x^2*PolyLog[2, -((b*E^(2*x))/(2*a + b +
 2*Sqrt[a]*Sqrt[a + b]))]) + (x*PolyLog[3, -((b*E^(2*x))/(2*a + b + 2*Sqrt
[a]*Sqrt[a + b]))])/2 - PolyLog[4, -((b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt
[a + b]))]/4))/(2*b)))/(4*Sqrt[a]*Sqrt[a + b]))
```

3.212.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3801
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[((c + d*x)^m*(E^((-I)*e
+ f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)
*e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c
, d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6164
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^2*(b_.) + (a_))^(n_)*(x_)^(m_.), x_Symbol] :>
 Simp[1/2^n   Int[x^m*(2*a + b + b*Cosh[2*c + 2*d*x])^n, x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[a - b, 0] && IGtQ[m, 0] && ILtQ[n, 0] && (EqQ[n, -1]
|| (EqQ[m, 1] && EqQ[n, -2]))
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.212. $\int \frac{x^3}{a+b\cosh^2(x)}\,dx$

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.212.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 888 vs. $2(303) = 606$.

Time = 0.19 (sec) , antiderivative size = 889, normalized size of antiderivative = 2.27

method	result	size
risch	Expression too large to display	889

input
```
int(x^3/(a+b*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(a*(a+b))^(1/2)*x^3*ln(1-b*exp(2*x)/(2*(a*(a+b))^(1/2)-2*a-b))-1/4/(a*
(a+b))^(1/2)*x^4+3/4/(a*(a+b))^(1/2)*x^2*polylog(2,b*exp(2*x)/(2*(a*(a+b))
^(1/2)-2*a-b))-3/4/(a*(a+b))^(1/2)*x*polylog(3,b*exp(2*x)/(2*(a*(a+b))^(1/
2)-2*a-b))+3/8/(a*(a+b))^(1/2)*polylog(4,b*exp(2*x)/(2*(a*(a+b))^(1/2)-2*a
-b))+1/(-2*(a*(a+b))^(1/2)-2*a-b)*ln(1-b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-
b))*x^3+1/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*ln(1-b*exp(2*x)/(-2*(
a*(a+b))^(1/2)-2*a-b))*a*x^3+1/2/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b
)*ln(1-b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*b*x^3-1/2/(-2*(a*(a+b))^(1/2
)-2*a-b)*x^4-1/2/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*a*x^4-1/4/(a*(
a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*b*x^4+3/2/(-2*(a*(a+b))^(1/2)-2*a-b
)*polylog(2,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*x^2+3/2/(a*(a+b))^(1/2)
/(-2*(a*(a+b))^(1/2)-2*a-b)*polylog(2,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b
))*a*x^2+3/4/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*polylog(2,b*exp(2*
x)/(-2*(a*(a+b))^(1/2)-2*a-b))*b*x^2-3/2/(-2*(a*(a+b))^(1/2)-2*a-b)*polylo
g(3,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*x-3/2/(a*(a+b))^(1/2)/(-2*(a*(a
+b))^(1/2)-2*a-b)*polylog(3,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*a*x-3/4
/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*polylog(3,b*exp(2*x)/(-2*(a*(a
+b))^(1/2)-2*a-b))*b*x+3/4/(-2*(a*(a+b))^(1/2)-2*a-b)*polylog(4,b*exp(2*x)
/(-2*(a*(a+b))^(1/2)-2*a-b))+3/4/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b
)*polylog(4,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*a+3/8/(a*(a+b))^(1/2...
```

3.212.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1542 vs. $2(307) = 614$.

Time = 0.31 (sec) , antiderivative size = 1542, normalized size of antiderivative = 3.94

$$\int \frac{x^3}{a+b\cosh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(a+b*cosh(x)^2),x, algorithm="fricas")
```

output
```
-1/2*(b*x^3*sqrt((a^2 + a*b)/b^2)*log((((2*a + b)*cosh(x) + (2*a + b)*sinh
(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^
2 + a*b)/b^2) + 2*a + b)/b) + b)/b) + b*x^3*sqrt((a^2 + a*b)/b^2)*log(-(((
2*a + b)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2
 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b) - b)/b) - b*x
^3*sqrt((a^2 + a*b)/b^2)*log((((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(
b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b
^2) - 2*a - b)/b) + b)/b) - b*x^3*sqrt((a^2 + a*b)/b^2)*log(-(((2*a + b)*c
osh(x) + (2*a + b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^
2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b) - b)/b) + 3*b*x^2*sqrt((
a^2 + a*b)/b^2)*dilog(-(((2*a + b)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh
(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) +
 2*a + b)/b) + b)/b + 1) + 3*b*x^2*sqrt((a^2 + a*b)/b^2)*dilog((((2*a + b)
*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/
b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b) - b)/b + 1) - 3*b*x^2
*sqrt((a^2 + a*b)/b^2)*dilog(-(((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*
(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/
b^2) - 2*a - b)/b) + b)/b + 1) - 3*b*x^2*sqrt((a^2 + a*b)/b^2)*dilog((((2*
a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 +
 a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b) - b)/b + 1) -...
```

3.212.6 Sympy [F]

$$\int \frac{x^3}{a+b\cosh^2(x)}\,dx = \int \frac{x^3}{a+b\cosh^2(x)}\,dx$$

input
```
integrate(x**3/(a+b*cosh(x)**2),x)
```

output
```
Integral(x**3/(a + b*cosh(x)**2), x)
```

3.212.7 Maxima [F]

$$\int \frac{x^3}{a+b\cosh^2(x)}\,dx = \int \frac{x^3}{b\cosh{(x)}^2+a}\,dx$$

input
```
integrate(x^3/(a+b*cosh(x)^2),x, algorithm="maxima")
```

output
```
integrate(x^3/(b*cosh(x)^2 + a), x)
```

3.212.8 Giac [F]

$$\int \frac{x^3}{a+b\cosh^2(x)}\,dx = \int \frac{x^3}{b\cosh{(x)}^2+a}\,dx$$

input
```
integrate(x^3/(a+b*cosh(x)^2),x, algorithm="giac")
```

output
```
integrate(x^3/(b*cosh(x)^2 + a), x)
```

3.212.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{a+b\cosh^2(x)}\,dx = \int \frac{x^3}{b\cosh{(x)}^2+a}\,dx$$

input
```
int(x^3/(a + b*cosh(x)^2),x)
```

output
```
int(x^3/(a + b*cosh(x)^2), x)
```

3.213 $\int \frac{\cosh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

3.213.1 Optimal result

Integrand size = 36, antiderivative size = 58

$$\int \frac{\cosh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{3\text{Chi}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a} - \frac{\text{Chi}\left(\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}$$

output
```
-3/4*Chi((-a*x+1)^(1/2)/(a*x+1)^(1/2))/a-1/4*Chi(3*(-a*x+1)^(1/2)/(a*x+1)^
(1/2))/a
```

3.213.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \frac{\cosh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \frac{-3\text{Chi}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) - \text{Chi}\left(\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}$$

input
```
Integrate[Cosh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2),x]
```

output
```
(-3*CoshIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]] - CoshIntegral[(3*Sqrt[1 - a
*x])/Sqrt[1 + a*x]])/(4*a)
```

3.213.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7232, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{1-a^2x^2}dx$$

$$\Big\downarrow \text{7232}$$

$$-\frac{\int \frac{\sqrt{ax+1}\cosh^3\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{ax+1}\sin\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\pi}{2}\right)^3}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{\int \left(\frac{3\sqrt{ax+1}\cosh\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4\sqrt{1-ax}}+\frac{\sqrt{ax+1}\cosh\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4\sqrt{1-ax}}\right)d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{3}{4}\text{Chi}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{1}{4}\text{Chi}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

input
```
Int[Cosh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2),x]
```

output
```
-(((3*CoshIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]])/4 + CoshIntegral[(3*Sqrt[
1 - a*x])/Sqrt[1 + a*x]]/4)/a)
```

3.213.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

3.213.4 Maple [F]

$$\int \frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^{3}}{-a^{2}x^{2}+1}dx$$

input
```
int(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)
```

output
```
int(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)
```

3.213.5 Fricas [F]

$$\int \frac{\cosh^{3}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^{2}x^{2}}\,dx = \int -\frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^{3}}{a^{2}x^{2}-1}\,dx$$

input
```
integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="
fricas")
```

output
```
integral(-cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

3.213.6 Sympy [F]

$$\int \frac{\cosh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\cosh^3\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

input
```
integrate(cosh((-a*x+1)**(1/2)/(a*x+1)**(1/2))**3/(-a**2*x**2+1),x)
```

output
```
-Integral(cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))**3/(a**2*x**2 - 1), x)
```

3.213.7 Maxima [F]

$$\int \frac{\cosh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1}\,dx$$

input
```
integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="
maxima")
```

output
```
-integrate(cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

3.213.8 Giac [F]

$$\int \frac{\cosh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1}\,dx$$

input
```
integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="
giac")
```

output
```
integrate(-cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

3.213.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\cosh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)^3}{a^2\,x^2-1}\,dx$$

input
```
int(-cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^3/(a^2*x^2 - 1),x)
```

output
```
-int(cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^3/(a^2*x^2 - 1), x)
```

3.214 $\int \frac{\cosh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

3.214.1 Optimal result

Integrand size = 36, antiderivative size = 58

$$\int \frac{\cosh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Chi}\left(\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a} - \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}$$

output
```
-1/2*Chi(2*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a-1/2*ln((-a*x+1)^(1/2)/(a*x+1)^(
1/2))/a
```

3.214.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int \frac{\cosh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Chi}\left(\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a} - \frac{\log(1-ax)}{4a} + \frac{\log(1+ax)}{4a}$$

input
```
Integrate[Cosh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

output
```
-1/2*CoshIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/a - Log[1 - a*x]/(4*a)
+ Log[1 + a*x]/(4*a)
```

3.214.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7232, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{1-a^2x^2}\,dx$$

$$\Big\downarrow \text{7232}$$

$$-\frac{\int \frac{\sqrt{ax+1}\cosh^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{ax+1}\sin\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\pi}{2}\right)^2}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{\int \left(\frac{\sqrt{ax+1}\cosh\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2\sqrt{1-ax}}+\frac{\sqrt{ax+1}}{2\sqrt{1-ax}}\right)d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{2}\text{Chi}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{1}{2}\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

input
```
Int[Cosh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

output
```
-((CoshIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/2 + Log[Sqrt[1 - a*x]/Sqr
t[1 + a*x]]/2)/a)
```

3.214.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

3.214.4 Maple [F]

$$\int \frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{-a^2x^2+1}dx$$

input
```
int(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

output
```
int(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

3.214.5 Fricas [F]

$$\int \frac{\cosh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx = \int -\frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1} dx$$

input
```
integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="
fricas")
```

output
```
integral(-cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)
```

3.214.6 Sympy [F]

$$\int \frac{\cosh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\cosh^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

input
```
integrate(cosh((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2/(-a**2*x**2+1),x)
```

output
```
-Integral(cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))**2/(a**2*x**2 - 1), x)
```

3.214.7 Maxima [F]

$$\int \frac{\cosh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1}\,dx$$

input
```
integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="
maxima")
```

output
```
1/4*log(a*x + 1)/a - 1/4*log(a*x - 1)/a - 1/4*integrate(e^(2*sqrt(-a*x + 1
)/sqrt(a*x + 1))/(a^2*x^2 - 1), x) - 1/4*integrate(e^(-2*sqrt(-a*x + 1)/sq
rt(a*x + 1))/(a^2*x^2 - 1), x)
```

3.214.8 Giac [F]

$$\int \frac{\cosh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1}\,dx$$

input
```
integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="
giac")
```

output
```
integrate(-cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)
```

3.214.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\cosh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)^2}{a^2\,x^2-1}\,dx$$

input
```
int(-cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2/(a^2*x^2 - 1),x)
```

output
```
-int(cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2/(a^2*x^2 - 1), x)
```

3.215 $\int \frac{\cosh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

3.215.1 Optimal result

Integrand size = 34, antiderivative size = 26

$$\int \frac{\cosh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Chi}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

output
```
-Chi((-a*x+1)^(1/2)/(a*x+1)^(1/2))/a
```

3.215.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Chi}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

input
```
Integrate[Cosh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

output
```
-(CoshIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

3.215.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {7232, 3042, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{1-a^2x^2}dx$$

$$\Big\downarrow \text{7232}$$

$$-\frac{\int \frac{\sqrt{ax+1}\cosh\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{ax+1}\sin\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\pi}{2}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{3782}$$

$$-\frac{\text{Chi}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

input
```
Int[Cosh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

output
```
-(CoshIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

3.215.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

3.215.4 Maple [F]

$$\int \frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2+1}dx$$

input
```
int(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

output
```
int(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

3.215.5 Fricas [F]

$$\int \frac{\cosh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

input
```
integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="fr
icas")
```

output
```
integral(-cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

3.215.6 Sympy [F]

$$\int \frac{\cosh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

input
```
integrate(cosh((-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1),x)
```

output
```
-Integral(cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a**2*x**2 - 1), x)
```

3.215.7 Maxima [F]

$$\int \frac{\cosh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

input
```
integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="ma
xima")
```

output
```
-integrate(cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

3.215.8 Giac [F]

$$\int \frac{\cosh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

input
```
integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="gi
ac")
```

output
```
integrate(-cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

3.215.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\cosh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)}{a^2\,x^2-1}\,dx$$

input
```
int(-cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1),x)
```

output
```
-int(cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1), x)
```

3.216 $\int \frac{\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} \, dx$

3.216.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} \, dx = \operatorname{Int}\left(\frac{\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{(1-ax)(1+ax)}, x\right)$$

output
```
Unintegrable(sech((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a*x+1)/(a*x+1),x)
```

3.216.2 Mathematica [N/A]

Not integrable

Time = 8.83 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} \, dx = \int \frac{\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} \, dx$$

input
```
Integrate[Sech[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

output
```
Integrate[Sech[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2), x]
```

3.216.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7232, 3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{1-a^2x^2}dx$$

$$\Big\downarrow \text{7232}$$

$$-\frac{\int \frac{\sqrt{ax+1}\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{ax+1}\csc\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\pi}{2}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{4680}$$

$$-\frac{\int \frac{\sqrt{ax+1}\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

input
```
Int[Sech[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

output
```
$Aborted
```

3.216.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

3.216.4 Maple [N/A] (verified)

Not integrable

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{1}{(-a^2x^2+1)\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}dx$$

input
```
int(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x)
```

output
```
int(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x)
```

3.216.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{1}{(a^2x^2-1)\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="
fricas")
```

output
```
integral(-1/((a^2*x^2 - 1)*cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

3.216.6 Sympy [N/A]

Not integrable

Time = 7.98 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int \frac{\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{1}{a^2x^2\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right) - \cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

input
```
integrate(1/(-a**2*x**2+1)/cosh((-a*x+1)**(1/2)/(a*x+1)**(1/2)),x)
```

output
```
-Integral(1/(a**2*x**2*cosh(sqrt(-a*x + 1)/sqrt(a*x + 1)) - cosh(sqrt(-a*x
 + 1)/sqrt(a*x + 1))), x)
```

3.216.7 Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{1}{(a^2x^2-1)\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="
maxima")
```

output
```
-integrate(1/((a^2*x^2 - 1)*cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

3.216.8 Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{1}{(a^2x^2-1)\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="
giac")
```

output
```
integrate(-1/((a^2*x^2 - 1)*cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

3.216.9 Mupad [N/A]

Not integrable

Time = 1.76 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{1}{\cosh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)\,\left(a^2\,x^2-1\right)}\,dx$$

input
```
int(-1/(cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))*(a^2*x^2 - 1)),x)
```

output
```
-int(1/(cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))*(a^2*x^2 - 1)), x)
```

3.217 $\int \frac{\operatorname{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

3.217.1 Optimal result

Integrand size = 36, antiderivative size = 36

$$\int \frac{\operatorname{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \operatorname{Int}\left(\frac{\operatorname{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{(1-ax)(1+ax)},x\right)$$

output
```
Unintegrable(sech((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a*x+1)/(a*x+1),x)
```

3.217.2 Mathematica [N/A]

Not integrable

Time = 32.42 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int \frac{\operatorname{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

input
```
Integrate[Sech[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

output
```
Integrate[Sech[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2), x]
```

3.217.3 Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7232, 3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{1-a^2x^2}dx$$

$$\Big\downarrow \text{7232}$$

$$-\frac{\int \frac{\sqrt{ax+1}\text{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{ax+1}\csc\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\pi}{2}\right)^2}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{4680}$$

$$-\frac{\int \frac{\sqrt{ax+1}\text{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

input
```
Int[Sech[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

output
```
$Aborted
```

3.217.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

3.217.4 Maple [N/A] (verified)

Not integrable

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(-a^{2} x^{2}+1\right) \cosh \left(\frac{\sqrt{-a x +1}}{\sqrt{a x +1}}\right)^{2}}d x$$

input
```
int(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x)
```

output
```
int(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x)
```

3.217.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{1}{\left(a^{2} x^{2} - 1\right) \cosh\left(\frac{\sqrt{-a x + 1}}{\sqrt{a x + 1}}\right)^{2}}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm
="fricas")
```

output
```
integral(-1/((a^2*x^2 - 1)*cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))^2), x)
```

3.217.6 Sympy [N/A]

Not integrable

Time = 22.33 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.42

$$\int \frac{\operatorname{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{1}{a^2x^2\cosh^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)-\cosh^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

input
```
integrate(1/(-a**2*x**2+1)/cosh((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2,x)
```

output
```
-Integral(1/(a**2*x**2*cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))**2 - cosh(sqrt(-
a*x + 1)/sqrt(a*x + 1))**2), x)
```

3.217.7 Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.33

$$\int \frac{\operatorname{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{1}{(a^2x^2-1)\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm
="maxima")
```

output
```
2*sqrt(a*x + 1)/(sqrt(-a*x + 1)*a*e^(2*sqrt(-a*x + 1)/sqrt(a*x + 1)) + sqr
t(-a*x + 1)*a) + 2*integrate(sqrt(a*x + 1)/((a^2*x^2 - 1)*sqrt(-a*x + 1)*e
^(2*sqrt(-a*x + 1)/sqrt(a*x + 1)) + (a^2*x^2 - 1)*sqrt(-a*x + 1)), x)
```

3.217.8 Giac [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{1}{(a^2x^2-1)\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm
="giac")
```

output
```
integrate(-1/((a^2*x^2 - 1)*cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))^2), x)
```

3.217.9 Mupad [N/A]

Not integrable

Time = 1.87 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{\operatorname{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{1}{\cosh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)^2\,(a^2\,x^2-1)}\,dx$$

input
```
int(-1/(cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2*(a^2*x^2 - 1)),x)
```

output
```
-int(1/(cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2*(a^2*x^2 - 1)), x)
```

3.218 $\int \frac{x\sinh(x)}{(a+b\cosh(x))^2}\,dx$

3.218.1 Optimal result

Integrand size = 12, antiderivative size = 60

$$\int \frac{x\sinh(x)}{(a+b\cosh(x))^2}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b\sqrt{a+b}} - \frac{x}{b(a+b\cosh(x))}$$

output
```
-x/b/(a+b*cosh(x))+2*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/b/(a-b)^
(1/2)/(a+b)^(1/2)
```

3.218.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98

$$\int \frac{x\sinh(x)}{(a+b\cosh(x))^2}\,dx = -\frac{2\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{b\sqrt{-a^2+b^2}} - \frac{x}{b(a+b\cosh(x))}$$

input
```
Integrate[(x*Sinh[x])/(a + b*Cosh[x])^2,x]
```

output
```
(-2*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(b*Sqrt[-a^2 + b^2]) - x
/(b*(a + b*Cosh[x]))
```

3.218.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5988, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sinh(x)}{(a+b\cosh(x))^2}\,dx$$

$$\Big\downarrow \ 5988$$

$$\frac{\int \frac{1}{a+b\cosh(x)}dx}{b}-\frac{x}{b(a+b\cosh(x))}$$

$$\Big\downarrow \ 3042$$

$$-\frac{x}{b(a+b\cosh(x))}+\frac{\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \ 3138$$

$$\frac{2\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{b}-\frac{x}{b(a+b\cosh(x))}$$

$$\Big\downarrow \ 221$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}-\frac{x}{b(a+b\cosh(x))}$$

input
```
Int[(x*Sinh[x])/(a + b*Cosh[x])^2,x]
```

output
```
(2*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b
]) - x/(b*(a + b*Cosh[x]))
```

3.218.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 5988
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.
)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*((a + b*Cosh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)
^(m - 1)*(a + b*Cosh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

3.218.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(50) = 100$.

Time = 0.39 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.30

method	result	size
risch	$-\frac{2x\,\mathrm{e}^{x}}{b(b\,\mathrm{e}^{2x}+2a\,\mathrm{e}^{x}+b)}+\frac{\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}-b^{2}}-a^{2}+b^{2}}{b\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,b}-\frac{\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}-b^{2}}+a^{2}-b^{2}}{b\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,b}$	138

input
```
int(x*sinh(x)/(a+b*cosh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*x/b*exp(x)/(b*exp(x)^2+2*a*exp(x)+b)+1/(a^2-b^2)^(1/2)/b*ln(exp(x)+(a*(
a^2-b^2)^(1/2)-a^2+b^2)/b/(a^2-b^2)^(1/2))-1/(a^2-b^2)^(1/2)/b*ln(exp(x)+(
a*(a^2-b^2)^(1/2)+a^2-b^2)/b/(a^2-b^2)^(1/2))
```

3.218.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(50) = 100$.

Time = 0.27 (sec) , antiderivative size = 480, normalized size of antiderivative = 8.00

$$\int \frac{x\sinh(x)}{(a+b\cosh(x))^2}\,dx$$
$$= \left[-\frac{2\,(a^2-b^2)x\cosh(x)+2\,(a^2-b^2)x\sinh(x)-\left(b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+\right.}{a^2b^2-b^4+(a^2b^2-b^4)\cosh(x)^2+(a^2b^2-b^4)\sinh(x)^2-}\right.$$
$$-\frac{2\left(\left(a^2-b^2\right)x\cosh(x)+(a^2-b^2)x\sinh(x)+\left(b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+\right.\right.}{a^2b^2-b^4+(a^2b^2-b^4)\cosh(x)^2+(a^2b^2-b^4)\sinh(x)^2+2\,(a^3b-ab^3)\cosh(x)+}$$

input
```
integrate(x*sinh(x)/(a+b*cosh(x))^2,x, algorithm="fricas")
```

output
```
[-(2*(a^2 - b^2)*x*cosh(x) + 2*(a^2 - b^2)*x*sinh(x) - (b*cosh(x)^2 + b*si
nh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)*sqrt(a^2 - b^2)*log
((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cos
h(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*co
sh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)))/(a^
2*b^2 - b^4 + (a^2*b^2 - b^4)*cosh(x)^2 + (a^2*b^2 - b^4)*sinh(x)^2 + 2*(a
^3*b - a*b^3)*cosh(x) + 2*(a^3*b - a*b^3 + (a^2*b^2 - b^4)*cosh(x))*sinh(x
)), -2*((a^2 - b^2)*x*cosh(x) + (a^2 - b^2)*x*sinh(x) + (b*cosh(x)^2 + b*s
inh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)*sqrt(-a^2 + b^2)*a
rctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)))/(a^2*b^2
 - b^4 + (a^2*b^2 - b^4)*cosh(x)^2 + (a^2*b^2 - b^4)*sinh(x)^2 + 2*(a^3*b
- a*b^3)*cosh(x) + 2*(a^3*b - a*b^3 + (a^2*b^2 - b^4)*cosh(x))*sinh(x))]
```

3.218.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1770 vs. $2(46) = 92$.

Time = 152.59 (sec) , antiderivative size = 1770, normalized size of antiderivative = 29.50

$$\int \frac{x\sinh(x)}{(a+b\cosh(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate(x*sinh(x)/(a+b*cosh(x))**2,x)
```

output
```
Piecewise((zoo*(x*tanh(x/2)**2/(tanh(x/2)**2 + 1) - x/(tanh(x/2)**2 + 1) +
 2*tanh(x/2)**2*atan(tanh(x/2))/(tanh(x/2)**2 + 1) + 2*atan(tanh(x/2))/(ta
nh(x/2)**2 + 1)), Eq(a, 0) & Eq(b, 0)), (x*tanh(x/2)**2/(2*b**2) - x/(2*b*
*2) + tanh(x/2)/b**2, Eq(a, b)), (x/(2*b**2) - x/(2*b**2*tanh(x/2)**2) - 1
/(b**2*tanh(x/2)), Eq(a, -b)), ((x*cosh(x) - sinh(x))/a**2, Eq(b, 0)), (-a
*x*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2/(a**2*b*sqrt(a/(a - b) + b/(a
- b))*tanh(x/2)**2 - a**2*b*sqrt(a/(a - b) + b/(a - b)) - 2*a*b**2*sqrt(a/
(a - b) + b/(a - b))*tanh(x/2)**2 + b**3*sqrt(a/(a - b) + b/(a - b))*tanh(
x/2)**2 + b**3*sqrt(a/(a - b) + b/(a - b))) + a*x*sqrt(a/(a - b) + b/(a -
b))/(a**2*b*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 - a**2*b*sqrt(a/(a -
b) + b/(a - b)) - 2*a*b**2*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 + b**3
*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 + b**3*sqrt(a/(a - b) + b/(a - b
))) - a*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))*tanh(x/2)**2/(a**2*b
*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 - a**2*b*sqrt(a/(a - b) + b/(a -
 b)) - 2*a*b**2*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 + b**3*sqrt(a/(a
- b) + b/(a - b))*tanh(x/2)**2 + b**3*sqrt(a/(a - b) + b/(a - b))) + a*log
(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a**2*b*sqrt(a/(a - b) + b/(a -
 b))*tanh(x/2)**2 - a**2*b*sqrt(a/(a - b) + b/(a - b)) - 2*a*b**2*sqrt(a/(
a - b) + b/(a - b))*tanh(x/2)**2 + b**3*sqrt(a/(a - b) + b/(a - b))*tanh(x
/2)**2 + b**3*sqrt(a/(a - b) + b/(a - b))) + a*log(sqrt(a/(a - b) + b/(...
```

3.218.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sinh(x)}{(a+b\cosh(x))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*sinh(x)/(a+b*cosh(x))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.218.8 Giac [F]

$$\int \frac{x\sinh(x)}{(a+b\cosh(x))^2}\,dx = \int \frac{x\sinh(x)}{(b\cosh(x)+a)^2}\,dx$$

input
```
integrate(x*sinh(x)/(a+b*cosh(x))^2,x, algorithm="giac")
```

output
```
integrate(x*sinh(x)/(b*cosh(x) + a)^2, x)
```

3.218.9 Mupad [B] (verification not implemented)

Time = 1.82 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.83

$$\int \frac{x\sinh(x)}{(a+b\cosh(x))^2}\,dx = \frac{2\,\mathrm{atan}\left(\frac{e^x\,(b^4-a^2\,b^2)+a\,b^3+a^2\,b^2\,e^x}{b^2\,\sqrt{b^4-a^2\,b^2}}\right)}{\sqrt{b^4-a^2\,b^2}} - \frac{2\,e^x\,(a^2\,x-b^2\,x)}{(a^2\,b-b^3)\,(b+2\,a\,e^x+b\,e^{2\,x})}$$

input
```
int((x*sinh(x))/(a + b*cosh(x))^2,x)
```

output
```
(2*atan((exp(x)*(b^4 - a^2*b^2) + a*b^3 + a^2*b^2*exp(x))/(b^2*(b^4 - a^2*
b^2)^(1/2))))/(b^4 - a^2*b^2)^(1/2) - (2*exp(x)*(a^2*x - b^2*x))/((a^2*b -
 b^3)*(b + 2*a*exp(x) + b*exp(2*x)))
```

3.219 $\int \frac{x\sinh(x)}{(a+b\cosh(x))^3}\,dx$

3.219.1 Optimal result

Integrand size = 12, antiderivative size = 87

$$\int \frac{x\sinh(x)}{(a+b\cosh(x))^3}\,dx=\frac{a\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b(a+b)^{3/2}}-\frac{x}{2b(a+b\cosh(x))^2}-\frac{\sinh(x)}{2\left(a^2-b^2\right)(a+b\cosh(x))}$$

output
```
a*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(3/2)/b/(a+b)^(3/2)-1
/2*x/b/(a+b*cosh(x))^2-1/2*sinh(x)/(a^2-b^2)/(a+b*cosh(x))
```

3.219.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{x\sinh(x)}{(a+b\cosh(x))^3}\,dx=\frac{1}{2}\left(\frac{\frac{2a\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}}-\frac{x}{(a+b\cosh(x))^2}}{b}-\frac{\sinh(x)}{(a-b)(a+b)(a+b\cosh(x))}\right)$$

input
```
Integrate[(x*Sinh[x])/(a + b*Cosh[x])^3,x]
```

output
```
(((2*a*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) -
x/(a + b*Cosh[x])^2)/b - Sinh[x]/((a - b)*(a + b)*(a + b*Cosh[x])))/2
```

3.219.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5988, 3042, 3143, 25, 27, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sinh(x)}{(a+b\cosh(x))^3}dx$$

$$\Big\downarrow \text{5988}$$

$$\frac{\int \frac{1}{(a+b\cosh(x))^2}dx}{2b}-\frac{x}{2b(a+b\cosh(x))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{x}{2b(a+b\cosh(x))^2}+\frac{\int \frac{1}{\left(a+b\sin\left(ix+\frac{\pi}{2}\right)\right)^2}dx}{2b}$$

$$\Big\downarrow \text{3143}$$

$$\frac{-\frac{\int -\frac{a}{a+b\cosh(x)}dx}{a^2-b^2}-\frac{b\sinh(x)}{(a^2-b^2)(a+b\cosh(x))}}{2b}-\frac{x}{2b(a+b\cosh(x))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{a}{a+b\cosh(x)}dx}{a^2-b^2}-\frac{b\sinh(x)}{(a^2-b^2)(a+b\cosh(x))}}{2b}-\frac{x}{2b(a+b\cosh(x))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{a\int \frac{1}{a+b\cosh(x)}dx}{a^2-b^2}-\frac{b\sinh(x)}{(a^2-b^2)(a+b\cosh(x))}}{2b}-\frac{x}{2b(a+b\cosh(x))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{x}{2b(a+b\cosh(x))^2}+\frac{-\frac{b\sinh(x)}{(a^2-b^2)(a+b\cosh(x))}+\frac{a\int\frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}}{2b}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\frac{2a\int\frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a^2-b^2}-\frac{b\sinh(x)}{(a^2-b^2)(a+b\cosh(x))}}{2b}-\frac{x}{2b(a+b\cosh(x))^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{2a\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}(a^2-b^2)}-\frac{b\sinh(x)}{(a^2-b^2)(a+b\cosh(x))}}{2b}-\frac{x}{2b(a+b\cosh(x))^2}$$

input
```
Int[(x*Sinh[x])/(a + b*Cosh[x])^3,x]
```

output
```
-1/2*x/(b*(a + b*Cosh[x])^2) + ((2*a*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[
a + b]])/(Sqrt[a - b]*Sqrt[a + b]*(a^2 - b^2)) - (b*Sinh[x])/((a^2 - b^2)*
(a + b*Cosh[x])))/(2*b)
```

3.219.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 5988
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.
)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*((a + b*Cosh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)
^(m - 1)*(a + b*Cosh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

3.219.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(73) = 146$.

Time = 2.60 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.66

method	result	size
risch	$-\frac{2a^2x\,e^{2x}-ab\,e^{3x}-2b^2x\,e^{2x}-2a^2e^{2x}-b^2e^{2x}-3b\,e^{x}a-b^2}{b(b\,e^{2x}+2a\,e^{x}+b)^2(a^2-b^2)}+\frac{a\ln\left(e^{x}+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,(a+b)(a-b)b}-\frac{a\ln\left(e^{x}+\frac{a\sqrt{a^2-b^2}+a^2-b^2}{b\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,(a+b)(a-b)b}$	231

input
```
int(x*sinh(x)/(a+b*cosh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/b*(2*a^2*x*exp(x)^2-a*b*exp(x)^3-2*b^2*x*exp(x)^2-2*a^2*exp(x)^2-b^2*ex
p(x)^2-3*b*exp(x)*a-b^2)/(b*exp(x)^2+2*a*exp(x)+b)^2/(a^2-b^2)+1/2/(a^2-b^
2)^(1/2)*a/(a+b)/(a-b)/b*ln(exp(x)+(a*(a^2-b^2)^(1/2)-a^2+b^2)/b/(a^2-b^2)
^(1/2))-1/2/(a^2-b^2)^(1/2)*a/(a+b)/(a-b)/b*ln(exp(x)+(a*(a^2-b^2)^(1/2)+a
^2-b^2)/b/(a^2-b^2)^(1/2))
```

3.219.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 808 vs. 2(73) = 146.

Time = 0.28 (sec) , antiderivative size = 1692, normalized size of antiderivative = 19.45

$$\int \frac{x\sinh(x)}{(a+b\cosh(x))^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*sinh(x)/(a+b*cosh(x))^3,x, algorithm="fricas")
```

output
```
[1/2*(2*a^2*b^2 - 2*b^4 + 2*(a^3*b - a*b^3)*cosh(x)^3 + 2*(a^3*b - a*b^3)*
sinh(x)^3 + 2*(2*a^4 - a^2*b^2 - b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*x)*cosh(x
)^2 + 2*(2*a^4 - a^2*b^2 - b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*x + 3*(a^3*b -
a*b^3)*cosh(x))*sinh(x)^2 - (a*b^2*cosh(x)^4 + a*b^2*sinh(x)^4 + 4*a^2*b*c
osh(x)^3 + 4*a^2*b*cosh(x) + 4*(a*b^2*cosh(x) + a^2*b)*sinh(x)^3 + a*b^2 +
 2*(2*a^3 + a*b^2)*cosh(x)^2 + 2*(3*a*b^2*cosh(x)^2 + 6*a^2*b*cosh(x) + 2*
a^3 + a*b^2)*sinh(x)^2 + 4*(a*b^2*cosh(x)^3 + 3*a^2*b*cosh(x)^2 + a^2*b +
(2*a^3 + a*b^2)*cosh(x))*sinh(x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2
*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) +
 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2
 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)) + 6*(a^3*b - a*b^3)*cosh(
x) + 2*(3*a^3*b - 3*a*b^3 + 3*(a^3*b - a*b^3)*cosh(x)^2 + 2*(2*a^4 - a^2*b
^2 - b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*x)*cosh(x))*sinh(x))/(a^4*b^3 - 2*a^2
*b^5 + b^7 + (a^4*b^3 - 2*a^2*b^5 + b^7)*cosh(x)^4 + (a^4*b^3 - 2*a^2*b^5
+ b^7)*sinh(x)^4 + 4*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cosh(x)^3 + 4*(a^5*b^2
- 2*a^3*b^4 + a*b^6 + (a^4*b^3 - 2*a^2*b^5 + b^7)*cosh(x))*sinh(x)^3 + 2*(
2*a^6*b - 3*a^4*b^3 + b^7)*cosh(x)^2 + 2*(2*a^6*b - 3*a^4*b^3 + b^7 + 3*(a
^4*b^3 - 2*a^2*b^5 + b^7)*cosh(x)^2 + 6*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cosh
(x))*sinh(x)^2 + 4*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cosh(x) + 4*(a^5*b^2 - 2*
a^3*b^4 + a*b^6 + (a^4*b^3 - 2*a^2*b^5 + b^7)*cosh(x)^3 + 3*(a^5*b^2 - ...
```

3.219.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x\sinh(x)}{(a+b\cosh(x))^3}\,dx = \text{Timed out}$$

input
```
integrate(x*sinh(x)/(a+b*cosh(x))**3,x)
```

output
```
Timed out
```

3.219.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sinh(x)}{(a+b\cosh(x))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*sinh(x)/(a+b*cosh(x))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.219.8 Giac [F]

$$\int \frac{x\sinh(x)}{(a+b\cosh(x))^3}\,dx = \int \frac{x\sinh(x)}{(b\cosh(x)+a)^3}\,dx$$

input
```
integrate(x*sinh(x)/(a+b*cosh(x))^3,x, algorithm="giac")
```

output
```
integrate(x*sinh(x)/(b*cosh(x) + a)^3, x)
```

3.219.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sinh(x)}{(a+b\cosh(x))^3}\,dx = \int \frac{x\sinh(x)}{(a+b\cosh(x))^3}\,dx$$

input
```
int((x*sinh(x))/(a + b*cosh(x))^3,x)
```

output
```
int((x*sinh(x))/(a + b*cosh(x))^3, x)
```

3.220 $\int \frac{(2+\cosh^2(a+bx))\sinh(a+bx)}{x}\,dx$

3.220.1 Optimal result

Integrand size = 20, antiderivative size = 47

$$\int \frac{(2+\cosh^2(a+bx))\sinh(a+bx)}{x}\,dx = \frac{9}{4}\text{Chi}(bx)\sinh(a)+\frac{1}{4}\text{Chi}(3bx)\sinh(3a) + \frac{9}{4}\cosh(a)\text{Shi}(bx)+\frac{1}{4}\cosh(3a)\text{Shi}(3bx)$$

output
```
9/4*cosh(a)*Shi(b*x)+1/4*cosh(3*a)*Shi(3*b*x)+9/4*Chi(b*x)*sinh(a)+1/4*Chi
(3*b*x)*sinh(3*a)
```

3.220.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{(2+\cosh^2(a+bx))\sinh(a+bx)}{x}\,dx = \frac{1}{4}(9\text{Chi}(bx)\sinh(a)+\text{Chi}(3bx)\sinh(3a) + 9\cosh(a)\text{Shi}(bx)+\cosh(3a)\text{Shi}(3bx))$$

input
```
Integrate[((2 + Cosh[a + b*x]^2)*Sinh[a + b*x])/x,x]
```

output
```
(9*CoshIntegral[b*x]*Sinh[a] + CoshIntegral[3*b*x]*Sinh[3*a] + 9*Cosh[a]*S
inhIntegral[b*x] + Cosh[3*a]*SinhIntegral[3*b*x])/4
```

3.220.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(a+bx)\left(\cosh^2(a+bx)+2\right)}{x}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{2\sinh(a+bx)}{x}+\frac{\sinh(a+bx)\cosh^2(a+bx)}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{9}{4}\sinh(a)\text{Chi}(bx)+\frac{1}{4}\sinh(3a)\text{Chi}(3bx)+\frac{9}{4}\cosh(a)\text{Shi}(bx)+\frac{1}{4}\cosh(3a)\text{Shi}(3bx)$$

input
```
Int[((2 + Cosh[a + b*x]^2)*Sinh[a + b*x])/x,x]
```

output
```
(9*CoshIntegral[b*x]*Sinh[a])/4 + (CoshIntegral[3*b*x]*Sinh[3*a])/4 + (9*C
osh[a]*SinhIntegral[b*x])/4 + (Cosh[3*a]*SinhIntegral[3*b*x])/4
```

3.220.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.220.4 Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{e^{-3a} \operatorname{Ei}_1(3bx)}{8}+\frac{9 e^{-a} \operatorname{Ei}_1(bx)}{8}-\frac{9 e^{a} \operatorname{Ei}_1(-bx)}{8}-\frac{e^{3a} \operatorname{Ei}_1(-3bx)}{8}$	47

input
```
int((2+cosh(b*x+a)^2)*sinh(b*x+a)/x,x,method=_RETURNVERBOSE)
```

output
```
1/8*exp(-3*a)*Ei(1,3*b*x)+9/8*exp(-a)*Ei(1,b*x)-9/8*exp(a)*Ei(1,-b*x)-1/8*
exp(3*a)*Ei(1,-3*b*x)
```

3.220.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.43

$$\int \frac{(2+\cosh^2(a+bx))\sinh(a+bx)}{x}\,dx = \frac{1}{8}\left(\operatorname{Ei}(3\,bx)-\operatorname{Ei}(-3\,bx)\right)\cosh\left(3\,a\right) + \frac{9}{8}\left(\operatorname{Ei}(bx)-\operatorname{Ei}(-bx)\right)\cosh\left(a\right) + \frac{1}{8}\left(\operatorname{Ei}(3\,bx)+\operatorname{Ei}(-3\,bx)\right)\sinh\left(3\,a\right) + \frac{9}{8}\left(\operatorname{Ei}(bx)+\operatorname{Ei}(-bx)\right)\sinh\left(a\right)$$

input
```
integrate((2+cosh(b*x+a)^2)*sinh(b*x+a)/x,x, algorithm="fricas")
```

output
```
1/8*(Ei(3*b*x) - Ei(-3*b*x))*cosh(3*a) + 9/8*(Ei(b*x) - Ei(-b*x))*cosh(a)
+ 1/8*(Ei(3*b*x) + Ei(-3*b*x))*sinh(3*a) + 9/8*(Ei(b*x) + Ei(-b*x))*sinh(a
)
```

3.220.6 Sympy [F]

$$\int \frac{(2+\cosh^2(a+bx))\sinh(a+bx)}{x}\,dx = \int \frac{(\cosh^2(a+bx)+2)\sinh(a+bx)}{x}\,dx$$

input
```
integrate((2+cosh(b*x+a)**2)*sinh(b*x+a)/x,x)
```

output
```
Integral((cosh(a + b*x)**2 + 2)*sinh(a + b*x)/x, x)
```

3.220.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{(2+\cosh^2(a+bx))\sinh(a+bx)}{x}\,dx = \frac{1}{8}\,\mathrm{Ei}(3\,bx)\,e^{(3\,a)} - \frac{9}{8}\,\mathrm{Ei}(-bx)\,e^{(-a)} - \frac{1}{8}\,\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)} + \frac{9}{8}\,\mathrm{Ei}(bx)\,e^{a}$$

input
```
integrate((2+cosh(b*x+a)^2)*sinh(b*x+a)/x,x, algorithm="maxima")
```

output
```
1/8*Ei(3*b*x)*e^(3*a) - 9/8*Ei(-b*x)*e^(-a) - 1/8*Ei(-3*b*x)*e^(-3*a) + 9/
8*Ei(b*x)*e^a
```

3.220.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{(2+\cosh^2(a+bx))\sinh(a+bx)}{x}\,dx = \frac{1}{8}\,\mathrm{Ei}(3\,bx)\,e^{(3\,a)} - \frac{9}{8}\,\mathrm{Ei}(-bx)\,e^{(-a)} - \frac{1}{8}\,\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)} + \frac{9}{8}\,\mathrm{Ei}(bx)\,e^{a}$$

input
```
integrate((2+cosh(b*x+a)^2)*sinh(b*x+a)/x,x, algorithm="giac")
```

output
```
1/8*Ei(3*b*x)*e^(3*a) - 9/8*Ei(-b*x)*e^(-a) - 1/8*Ei(-3*b*x)*e^(-3*a) + 9/
8*Ei(b*x)*e^a
```

3.220.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(2+\cosh^2(a+bx)\right)\sinh(a+bx)}{x}\,dx=\int \frac{\sinh(a+b\,x)\,\left({\mathrm{cosh}\left(a+b\,x\right)}^2+2\right)}{x}\,dx$$

input
```
int((sinh(a + b*x)*(cosh(a + b*x)^2 + 2))/x,x)
```

output
```
int((sinh(a + b*x)*(cosh(a + b*x)^2 + 2))/x, x)
```

3.221 $\int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$

3.221.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \text{Int}\left(\frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)},x\right)$$

output
```
Unintegrable(x^m*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

3.221.2 Mathematica [N/A]

Not integrable

Time = 2.94 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$$

input
```
Integrate[(x^m*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
```

output
```
Integrate[(x^m*Sinh[c + d*x])/(a + b*Cosh[c + d*x]), x]
```

3.221.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6112}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)}dx$$
$$\Big\downarrow \text{ 6112}$$
$$\int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)}dx$$

input
```
Int[(x^m*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
```

output
```
$Aborted
```

3.221.3.1 Defintions of rubi rules used

rule 6112
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/(Cosh[(c_.)
 + (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Cosh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.221.4 Maple [N/A] (verified)

Not integrable

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m \sinh\left(dx+c\right)}{a+b\cosh\left(dx+c\right)}dx$$

input
```
int(x^m*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

output
```
int(x^m*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

3.221.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh\left(dx+c\right)}{b\cosh\left(dx+c\right)+a}\,dx$$

input
```
integrate(x^m*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(x^m*sinh(d*x + c)/(b*cosh(d*x + c) + a), x)
```

3.221.6 Sympy [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh\left(c+dx\right)}{a+b\cosh\left(c+dx\right)}\,dx$$

input
```
integrate(x**m*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

output
```
Integral(x**m*sinh(c + d*x)/(a + b*cosh(c + d*x)), x)
```

3.221.7 Maxima [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 202, normalized size of antiderivative = 9.18

$$\int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh\left(dx+c\right)}{b\cosh\left(dx+c\right)+a}\,dx$$

input
```
integrate(x^m*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
x*e^(2*d*x + m*log(x) + 2*c)/(b*(m + 1)*e^(2*d*x + 2*c) + 2*a*(m + 1)*e^(d
*x + c) + b*(m + 1)) - 1/2*integrate(2*(2*a*d*x*e^(3*d*x + 3*c) + 2*a*(m +
 1)*e^(d*x + c) + b*(m + 1) + (2*b*d*x*e^(2*c) + b*(m + 1)*e^(2*c))*e^(2*d
*x))*x^m/(b^2*(m + 1)*e^(4*d*x + 4*c) + 4*a*b*(m + 1)*e^(3*d*x + 3*c) + 4*
a*b*(m + 1)*e^(d*x + c) + b^2*(m + 1) + 2*(2*a^2*(m + 1)*e^(2*c) + b^2*(m
+ 1)*e^(2*c))*e^(2*d*x)), x)
```

3.221.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh\left(dx+c\right)}{b\cosh\left(dx+c\right)+a}\,dx$$

input
```
integrate(x^m*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x^m*sinh(d*x + c)/(b*cosh(d*x + c) + a), x)
```

3.221.9 Mupad [N/A]

Not integrable

Time = 1.69 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh(c+d\,x)}{a+b\cosh\left(c+d\,x\right)}\,dx$$

input
```
int((x^m*sinh(c + d*x))/(a + b*cosh(c + d*x)),x)
```

output
```
int((x^m*sinh(c + d*x))/(a + b*cosh(c + d*x)), x)
```

3.222 $\int \frac{x^3 \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$

3.222.1 Optimal result

Integrand size = 22, antiderivative size = 327

$$\begin{aligned}\int \frac{x^3 \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = &-\frac{x^4}{4b}+\frac{x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{x^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd}\\&+\frac{3x^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{3x^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}\\&-\frac{6x\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^3}-\frac{6x\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^3}\\&+\frac{6\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^4}+\frac{6\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^4}\end{aligned}$$

output
```
-1/4*x^4/b+x^3*ln(1+b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b/d+x^3*ln(1+b*exp(d
*x+c)/(a+(a^2-b^2)^(1/2)))/b/d+3*x^2*polylog(2,-b*exp(d*x+c)/(a-(a^2-b^2)^
(1/2)))/b/d^2+3*x^2*polylog(2,-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))/b/d^2-6*x
*polylog(3,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b/d^3-6*x*polylog(3,-b*exp(d
*x+c)/(a+(a^2-b^2)^(1/2)))/b/d^3+6*polylog(4,-b*exp(d*x+c)/(a-(a^2-b^2)^(1
/2)))/b/d^4+6*polylog(4,-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))/b/d^4
```

3.222.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00

$$\int \frac{x^3 \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{x^4}{4b} + \frac{x^3 \log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{x^3 \log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd} + \frac{3x^2 \operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2} + \frac{3x^2 \operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^2} - \frac{6x \operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^3} - \frac{6x \operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^3} + \frac{6 \operatorname{PolyLog}\left(4,\frac{be^{c+dx}}{-a+\sqrt{a^2-b^2}}\right)}{bd^4} + \frac{6 \operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^4}$$

input
```
Integrate[(x^3*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
```

output
```
-1/4*x^4/b + (x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b*d) +
(x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) + (3*x^2*PolyLo
g[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b*d^2) + (3*x^2*PolyLog[2
, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b*d^2) - (6*x*PolyLog[3, -((
b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b*d^3) - (6*x*PolyLog[3, -((b*E^(
c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b*d^3) + (6*PolyLog[4, (b*E^(c + d*x))
/(-a + Sqrt[a^2 - b^2])])/(b*d^4) + (6*PolyLog[4, -((b*E^(c + d*x))/(a + S
qrt[a^2 - b^2]))])/(b*d^4)
```

3.222.3 Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6096, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$$

$$\Big\downarrow \; 6096$$

$$\int \frac{e^{c+dx}x^3}{a+be^{c+dx}-\sqrt{a^2-b^2}}dx + \int \frac{e^{c+dx}x^3}{a+be^{c+dx}+\sqrt{a^2-b^2}}dx - \frac{x^4}{4b}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{3\int x^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2-b^2}}+1\right)dx}{bd}-\frac{3\int x^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2-b^2}}+1\right)dx}{bd}+\frac{x^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^4}{4b}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{3\left(\frac{2\int x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)dx}{d}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}-\frac{3\left(\frac{2\int x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)dx}{d}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}+\frac{x^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^4}{4b}$$

$$\Big\downarrow \text{7163}$$

$$-\frac{3\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{\int \text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)dx}{d}\right)}{d}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}-\frac{3\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{\int \text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)dx}{d}\right)}{d}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}+\frac{x^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^4}{4b}$$

$$\Big\downarrow \text{2720}$$

3.222. $\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$

$$-\frac{3\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{\int e^{-c-dx}\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}-\frac{3\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{\int e^{-c-dx}\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}+\frac{x^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^4}{4b}$$

$$\Big\downarrow \; 7143$$

$$-\frac{3\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d^2}\right)}{d}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}-\frac{3\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{d^2}\right)}{d}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}+\frac{x^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^4}{4b}$$

input
```
Int[(x^3*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
```

output
```
-1/4*x^4/b + (x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b*d) +
(x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) - (3*(-((x^2*Po
lyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/d) + (2*((x*PolyLog[3,
 -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/d - PolyLog[4, -((b*E^(c + d*x
))/(a - Sqrt[a^2 - b^2]))]/d^2))/d))/(b*d) - (3*(-((x^2*PolyLog[2, -((b*E^
(c + d*x))/(a + Sqrt[a^2 - b^2]))])/d) + (2*((x*PolyLog[3, -((b*E^(c + d*x
))/(a + Sqrt[a^2 - b^2]))])/d - PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2
 - b^2]))]/d^2))/d))/(b*d)
```

3.222. $\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$

3.222.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6096
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.222. $\int \frac{x^3 \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$

3.222.4 Maple [F]

$$\int \frac{x^3 \sinh(dx+c)}{a+b\cosh(dx+c)}dx$$

input
```
int(x^3*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

output
```
int(x^3*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

3.222.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(299) = 598$.

Time = 0.27 (sec) , antiderivative size = 624, normalized size of antiderivative = 1.91

$$\int \frac{x^3 \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{d^4x^4 - 12\,d^2x^2\mathrm{Li}_2\left(-\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2-b^2}{b^2}}+b}{b}+1\right) - 12\,d^2x^2\mathrm{Li}_2\left(-\frac{a\cosh(d}{}\right.}{}$$

input
```
integrate(x^3*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/4*(d^4*x^4 - 12*d^2*x^2*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - 12*d^
2*x^2*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sin
h(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + 4*c^3*log(2*b*cosh(d*x + c
) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + 4*c^3*log(2*b*c
osh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + 24*d
*x*polylog(3, -(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*s
inh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) + 24*d*x*polylog(3, -(a*cosh(d*x +
 c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^
2)/b^2))/b) - 4*(d^3*x^3 + c^3)*log((a*cosh(d*x + c) + a*sinh(d*x + c) + (
b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) - 4*(d^3*
x^3 + c^3)*log((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*s
inh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) - 24*polylog(4, -(a*cosh(d*x +
 c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^
2)/b^2))/b) - 24*polylog(4, -(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(
d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b))/(b*d^4)
```

3.222.6 Sympy [F]

$$\int \frac{x^3 \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^3 \sinh{(c+dx)}}{a+b\cosh{(c+dx)}}\,dx$$

input
```
integrate(x**3*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

output
```
Integral(x**3*sinh(c + d*x)/(a + b*cosh(c + d*x)), x)
```

3.222.7 Maxima [F]

$$\int \frac{x^3 \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^3 \sinh{(dx+c)}}{b\cosh{(dx+c)}+a}\,dx$$

input
```
integrate(x^3*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*x^4/b - 1/2*integrate(4*(a*x^3*e^(d*x + c) + b*x^3)/(b^2*e^(2*d*x + 2*
c) + 2*a*b*e^(d*x + c) + b^2), x)
```

3.222.8 Giac [F]

$$\int \frac{x^3 \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^3 \sinh{(dx+c)}}{b\cosh{(dx+c)}+a}\,dx$$

input
```
integrate(x^3*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x^3*sinh(d*x + c)/(b*cosh(d*x + c) + a), x)
```

3.222.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \sinh(c+dx)}{a+b\cosh(c+dx)}\, dx = \int \frac{x^3 \sinh(c+d\,x)}{a+b\cosh\left(c+d\,x\right)}\, dx$$

input
```
int((x^3*sinh(c + d*x))/(a + b*cosh(c + d*x)),x)
```

output
```
int((x^3*sinh(c + d*x))/(a + b*cosh(c + d*x)), x)
```

3.223 $\int \frac{x^2 \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$

3.223.1 Optimal result

Integrand size = 22, antiderivative size = 245

$$\int \frac{x^2\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{x^3}{3b}+\frac{x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{x^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd}+\frac{2x\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{2x\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}-\frac{2\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^3}-\frac{2\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^3}$$

output
```
-1/3*x^3/b+x^2*ln(1+b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b/d+x^2*ln(1+b*exp(d
*x+c)/(a+(a^2-b^2)^(1/2)))/b/d+2*x*polylog(2,-b*exp(d*x+c)/(a-(a^2-b^2)^(1
/2)))/b/d^2+2*x*polylog(2,-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))/b/d^2-2*polyl
og(3,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b/d^3-2*polylog(3,-b*exp(d*x+c)/(a
+(a^2-b^2)^(1/2)))/b/d^3
```

3.223.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.00

$$\int \frac{x^2 \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{x^3}{3b}+\frac{x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{x^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd} +\frac{2x\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{2x\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^2} -\frac{2\,\mathrm{PolyLog}\left(3,\frac{be^{c+dx}}{-a+\sqrt{a^2-b^2}}\right)}{bd^3}-\frac{2\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^3}$$

input
```
Integrate[(x^2*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
```

output
```
-1/3*x^3/b + (x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b*d) +
(x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) + (2*x*PolyLog[
2, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b*d^2) + (2*x*PolyLog[2, -(
(b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b*d^2) - (2*PolyLog[3, (b*E^(c +
 d*x))/(-a + Sqrt[a^2 - b^2])])/(b*d^3) - (2*PolyLog[3, -((b*E^(c + d*x))/
(a + Sqrt[a^2 - b^2]))])/(b*d^3)
```

3.223.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6096, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$$

$$\Big\downarrow \text{6096}$$

$$\int \frac{e^{c+dx}x^2}{a+be^{c+dx}-\sqrt{a^2-b^2}}dx+\int \frac{e^{c+dx}x^2}{a+be^{c+dx}+\sqrt{a^2-b^2}}dx-\frac{x^3}{3b}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{2\int x\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2-b^2}}+1\right)dx}{bd}-\frac{2\int x\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2-b^2}}+1\right)dx}{bd}+\frac{x^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^3}{3b}$$

$\Big\downarrow$ 3011

$$-\frac{2\left(\frac{\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)dx}{d}-\frac{x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}-\frac{2\left(\frac{\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)dx}{d}-\frac{x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}+\frac{x^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^3}{3b}$$

$\Big\downarrow$ 2720

$$-\frac{2\left(\frac{\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)de^{c+dx}}{d^2}-\frac{x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}-\frac{2\left(\frac{\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)de^{c+dx}}{d^2}-\frac{x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}+\frac{x^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^3}{3b}$$

$\Big\downarrow$ 7143

$$-\frac{2\left(\frac{\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d^2}-\frac{x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}-\frac{2\left(\frac{\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{d^2}-\frac{x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}+\frac{x^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^3}{3b}$$

input
```
Int[(x^2*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
```

3.223. $\int \frac{x^2\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$

output
```
-1/3*x^3/b + (x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b*d) +
(x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) - (2*(-((x*Poly
Log[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/d) + PolyLog[3, -((b*E^(
c + d*x))/(a - Sqrt[a^2 - b^2]))]/d^2))/(b*d) - (2*(-((x*PolyLog[2, -((b*E
^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/d) + PolyLog[3, -((b*E^(c + d*x))/(a
+ Sqrt[a^2 - b^2]))]/d^2))/(b*d)
```

3.223.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6096
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.223. $\int \frac{x^2 \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$

3.223.4 Maple [F]

$$\int \frac{x^2 \sinh(dx+c)}{a+b\cosh(dx+c)}dx$$

input
```
int(x^2*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

output
```
int(x^2*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

3.223.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 497 vs. $2(223) = 446$.

Time = 0.27 (sec) , antiderivative size = 497, normalized size of antiderivative = 2.03

$$\int \frac{x^2 \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{d^3x^3 - 6\,dx\mathrm{Li}_2\left(-\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2-b^2}{b^2}}+b}{b}+1\right) - 6\,dx\mathrm{Li}_2\left(-\frac{a\cosh(dx+c)+a}{}\right.}{}$$

input
```
integrate(x^2*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/3*(d^3*x^3 - 6*d*x*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(
d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - 6*d*x*dilo
g(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c)
)*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - 3*c^2*log(2*b*cosh(d*x + c) + 2*b*si
nh(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - 3*c^2*log(2*b*cosh(d*x +
c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - 3*(d^2*x^2 - c
^2)*log((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
 + c))*sqrt((a^2 - b^2)/b^2) + b)/b) - 3*(d^2*x^2 - c^2)*log((a*cosh(d*x +
 c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^
2)/b^2) + b)/b) + 6*polylog(3, -(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*co
sh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) + 6*polylog(3, -(
a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sq
rt((a^2 - b^2)/b^2))/b))/(b*d^3)
```

3.223.6 Sympy [F]

$$\int \frac{x^2 \sinh(c + dx)}{a + b \cosh(c + dx)} dx = \int \frac{x^2 \sinh(c + dx)}{a + b \cosh(c + dx)} dx$$

input
```
integrate(x**2*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

output
```
Integral(x**2*sinh(c + d*x)/(a + b*cosh(c + d*x)), x)
```

3.223.7 Maxima [F]

$$\int \frac{x^2 \sinh(c + dx)}{a + b \cosh(c + dx)} dx = \int \frac{x^2 \sinh(dx + c)}{b \cosh(dx + c) + a} dx$$

input
```
integrate(x^2*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
1/3*x^3/b - 1/2*integrate(4*(a*x^2*e^(d*x + c) + b*x^2)/(b^2*e^(2*d*x + 2*
c) + 2*a*b*e^(d*x + c) + b^2), x)
```

3.223.8 Giac [F]

$$\int \frac{x^2 \sinh(c + dx)}{a + b \cosh(c + dx)} dx = \int \frac{x^2 \sinh(dx + c)}{b \cosh(dx + c) + a} dx$$

input
```
integrate(x^2*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x^2*sinh(d*x + c)/(b*cosh(d*x + c) + a), x)
```

3.223.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^2 \sinh(c+d\,x)}{a+b\cosh\left(c+d\,x\right)}\,dx$$

input
```
int((x^2*sinh(c + d*x))/(a + b*cosh(c + d*x)),x)
```

output
```
int((x^2*sinh(c + d*x))/(a + b*cosh(c + d*x)), x)
```

3.224 $\int \frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$

3.224.1 Optimal result

Integrand size = 20, antiderivative size = 161

$$\int \frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{x^2}{2b}+\frac{x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{x\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd}+\frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}$$

output
```
-1/2*x^2/b+x*ln(1+b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b/d+x*ln(1+b*exp(d*x+c
)/(a+(a^2-b^2)^(1/2)))/b/d+polylog(2,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b/
d^2+polylog(2,-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))/b/d^2
```

3.224.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.99

$$\int \frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{x^2}{2b}+\frac{x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{x\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd}+\frac{\text{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}$$

input
```
Integrate[(x*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
```

output
```
-1/2*x^2/b + (x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b*d) + (x
*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) + PolyLog[2, (b*E^(
c + d*x))/(-a + Sqrt[a^2 - b^2])]/(b*d^2) + PolyLog[2, -((b*E^(c + d*x))/(
a + Sqrt[a^2 - b^2]))]/(b*d^2)
```

3.224.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6096, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}dx$$

$$\Big\downarrow \text{ 6096}$$

$$\int \frac{e^{c+dx}x}{a+be^{c+dx}-\sqrt{a^2-b^2}}dx+\int \frac{e^{c+dx}x}{a+be^{c+dx}+\sqrt{a^2-b^2}}dx-\frac{x^2}{2b}$$

$$\Big\downarrow \text{ 2620}$$

$$-\frac{\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2-b^2}}+1\right)dx}{bd}-\frac{\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2-b^2}}+1\right)dx}{bd}+\frac{x\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^2}{2b}$$

$$\Big\downarrow \text{ 2715}$$

$$-\frac{\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2-b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2-b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{x\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^2}{2b}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{x\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^2}{2b}$$

input
```
Int[(x*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
```

output
```
-1/2*x^2/b + (x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b*d) + (x
*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) + PolyLog[2, -((b*E
^(c + d*x))/(a - Sqrt[a^2 - b^2]))]/(b*d^2) + PolyLog[2, -((b*E^(c + d*x))
/(a + Sqrt[a^2 - b^2]))]/(b*d^2)
```

3.224.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6096
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

3.224.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(147) = 294$.

Time = 0.26 (sec) , antiderivative size = 368, normalized size of antiderivative = 2.29

method	result
risch	$-\frac{x^2}{2b}-\frac{2cx}{db}-\frac{c^2}{d^2b}+\frac{\ln\left(\frac{-e^{dx+c}b+\sqrt{a^2-b^2}-a}{-a+\sqrt{a^2-b^2}}\right)x}{db}+\frac{\ln\left(\frac{-e^{dx+c}b+\sqrt{a^2-b^2}-a}{-a+\sqrt{a^2-b^2}}\right)c}{d^2b}+\frac{\ln\left(\frac{e^{dx+c}b+\sqrt{a^2-b^2}+a}{a+\sqrt{a^2-b^2}}\right)x}{db}+\frac{\ln\left(\frac{e^{dx+c}b}{a+}\right.}{}$

input
```
int(x*sinh(d*x+c)/(a+b*cosh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^2/b-2/d/b*c*x-1/d^2/b*c^2+1/d/b*ln((-exp(d*x+c)*b+(a^2-b^2)^(1/2)-a
)/(-a+(a^2-b^2)^(1/2)))*x+1/d^2/b*ln((-exp(d*x+c)*b+(a^2-b^2)^(1/2)-a)/(-a
+(a^2-b^2)^(1/2)))*c+1/d/b*ln((exp(d*x+c)*b+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2
)^(1/2)))*x+1/d^2/b*ln((exp(d*x+c)*b+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)
))*c+1/d^2/b*dilog((-exp(d*x+c)*b+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))
+1/d^2/b*dilog((exp(d*x+c)*b+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))-1/d^2
/b*c*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)+b)+2/d^2/b*c*ln(exp(d*x+c))
```

3.224.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(145) = 290$.

Time = 0.27 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.20

$$\int \frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{d^2x^2 + 2\,c\log\left(2\,b\cosh\left(dx+c\right) + 2\,b\sinh\left(dx+c\right) + 2\,b\sqrt{\frac{a^2-b^2}{b^2}} + 2\,a\right) + 2\,c\log\left(2\,b\cosh\left(dx+c\right) + 2\right.}{}$$

input
```
integrate(x*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(d^2*x^2 + 2*c*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((
a^2 - b^2)/b^2) + 2*a) + 2*c*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2
*b*sqrt((a^2 - b^2)/b^2) + 2*a) - 2*(d*x + c)*log((a*cosh(d*x + c) + a*sin
h(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b
)/b) - 2*(d*x + c)*log((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x +
c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) - 2*dilog(-(a*cosh(d*x
 + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 -
b^2)/b^2) + b)/b + 1) - 2*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*c
osh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1))/(b*d^2)
```

3.224.6 Sympy [F]

$$\int \frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx=\int \frac{x\sinh{(c+dx)}}{a+b\cosh{(c+dx)}}\,dx$$

input
```
integrate(x*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

output
```
Integral(x*sinh(c + d*x)/(a + b*cosh(c + d*x)), x)
```

3.224.7 Maxima [F]

$$\int \frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx=\int \frac{x\sinh{(dx+c)}}{b\cosh{(dx+c)}+a}\,dx$$

input
```
integrate(x*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*x^2/b - 1/2*integrate(4*(a*x*e^(d*x + c) + b*x)/(b^2*e^(2*d*x + 2*c) +
 2*a*b*e^(d*x + c) + b^2), x)
```

3.224.8 Giac [F]

$$\int \frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx=\int \frac{x\sinh{(dx+c)}}{b\cosh{(dx+c)}+a}\,dx$$

input
```
integrate(x*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x*sinh(d*x + c)/(b*cosh(d*x + c) + a), x)
```

3.224.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x \sinh(c+d\,x)}{a+b\cosh\left(c+d\,x\right)}\,dx$$

input
```
int((x*sinh(c + d*x))/(a + b*cosh(c + d*x)),x)
```

output
```
int((x*sinh(c + d*x))/(a + b*cosh(c + d*x)), x)
```

3.225 $\int \frac{\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$

3.225.1 Optimal result

Integrand size = 19, antiderivative size = 18

$$\int \frac{\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \frac{\log(a+b\cosh(c+dx))}{bd}$$

output
```
ln(a+b*cosh(d*x+c))/b/d
```

3.225.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \frac{\log(a+b\cosh(c+dx))}{bd}$$

input
```
Integrate[Sinh[c + d*x]/(a + b*Cosh[c + d*x]),x]
```

output
```
Log[a + b*Cosh[c + d*x]]/(b*d)
```

3.225.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 26, 3147, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)}{a+b\cosh(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{i\cos\left(ic+idx-\frac{\pi}{2}\right)}{a-b\sin\left(ic+idx-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 26$$

$$-i\int \frac{\cos\left(\frac{1}{2}(2ic-\pi)+idx\right)}{a-b\sin\left(\frac{1}{2}(2ic-\pi)+idx\right)}dx$$

$$\Big\downarrow \; 3147$$

$$\frac{\int \frac{1}{a+b\cosh(c+dx)}d(b\cosh(c+dx))}{bd}$$

$$\Big\downarrow \; 16$$

$$\frac{\log(a+b\cosh(c+dx))}{bd}$$

input
```
Int[Sinh[c + d*x]/(a + b*Cosh[c + d*x]),x]
```

output
```
Log[a + b*Cosh[c + d*x]]/(b*d)
```

3.225.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.225.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\ln(a+b\cosh(dx+c))}{bd}$	19
default	$\frac{\ln(a+b\cosh(dx+c))}{bd}$	19
risch	$-\frac{x}{b}-\frac{2c}{bd}+\frac{\ln\left(\mathrm{e}^{2dx+2c}+\frac{2a\,\mathrm{e}^{dx+c}}{b}+1\right)}{bd}$	48

input
```
int(sinh(d*x+c)/(a+b*cosh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
ln(a+b*cosh(d*x+c))/b/d
```

3.225.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(18) = 36.

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.44

$$\int \frac{\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{dx-\log\left(\frac{2\,(b\cosh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{bd}$$

input
```
integrate(sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
-(d*x - log(2*(b*cosh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))))/(b*d
)
```

3.225.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(14) = 28$.

Time = 0.55 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.28

$$\int \frac{\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \begin{cases} \frac{x\sinh(c)}{a} & \text{for } b=0 \wedge d=0 \\ \frac{\cosh(c+dx)}{ad} & \text{for } b=0 \\ \frac{x\sinh(c)}{a+b\cosh(c)} & \text{for } d=0 \\ \frac{\log\left(\frac{a}{b}+\cosh(c+dx)\right)}{bd} & \text{otherwise} \end{cases}$$

input
```
integrate(sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

output
```
Piecewise((x*sinh(c)/a, Eq(b, 0) & Eq(d, 0)), (cosh(c + d*x)/(a*d), Eq(b,
0)), (x*sinh(c)/(a + b*cosh(c)), Eq(d, 0)), (log(a/b + cosh(c + d*x))/(b*d
), True))
```

3.225.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \frac{\log(b\cosh(dx+c)+a)}{bd}$$

input
```
integrate(sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
log(b*cosh(d*x + c) + a)/(b*d)
```

3.225.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \frac{\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \frac{\log\left(\left|b\left(e^{(dx+c)}+e^{(-dx-c)}\right)+2\,a\right|\right)}{bd}$$

input
```
integrate(sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

output
```
log(abs(b*(e^(d*x + c) + e^(-d*x - c)) + 2*a))/(b*d)
```

3.225.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \frac{\ln\left(a+b\cosh(c+d\,x)\right)}{b\,d}$$

input
```
int(sinh(c + d*x)/(a + b*cosh(c + d*x)),x)
```

output
```
log(a + b*cosh(c + d*x))/(b*d)
```

3.226 $\int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))}\,dx$

3.226.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \text{Int}\left(\frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))},x\right)$$

output
```
Unintegrable(sinh(d*x+c)/x/(a+b*cosh(d*x+c)),x)
```

3.226.2 Mathematica [N/A]

Not integrable

Time = 8.90 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))}\,dx$$

input
```
Integrate[Sinh[c + d*x]/(x*(a + b*Cosh[c + d*x])),x]
```

output
```
Integrate[Sinh[c + d*x]/(x*(a + b*Cosh[c + d*x])), x]
```

3.226.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6112}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))} \, dx$$
$$\Big\downarrow \; 6112$$
$$\int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))}dx$$

input
```
Int[Sinh[c + d*x]/(x*(a + b*Cosh[c + d*x])),x]
```

output
```
$Aborted
```

3.226.3.1 Defintions of rubi rules used

rule 6112
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/(Cosh[(c_.)
 + (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Cosh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.226.4 Maple [N/A] (verified)

Not integrable

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sinh (dx+c)}{x (a+b\cosh (dx+c))}dx$$

input
```
int(sinh(d*x+c)/x/(a+b*cosh(d*x+c)),x)
```

output
```
int(sinh(d*x+c)/x/(a+b*cosh(d*x+c)),x)
```

3.226.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh\left(dx+c\right)}{\left(b\cosh\left(dx+c\right)+a\right)x}\,dx$$

input
```
integrate(sinh(d*x+c)/x/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(sinh(d*x + c)/(b*x*cosh(d*x + c) + a*x), x)
```

3.226.6 Sympy [N/A]

Not integrable

Time = 4.67 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh\left(c+dx\right)}{x\left(a+b\cosh\left(c+dx\right)\right)}\,dx$$

input
```
integrate(sinh(d*x+c)/x/(a+b*cosh(d*x+c)),x)
```

output
```
Integral(sinh(c + d*x)/(x*(a + b*cosh(c + d*x))), x)
```

3.226.7 Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.55

$$\int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh\left(dx+c\right)}{\left(b\cosh\left(dx+c\right)+a\right)x}\,dx$$

input
```
integrate(sinh(d*x+c)/x/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
log(x)/b - 1/2*integrate(4*(a*e^(d*x + c) + b)/(b^2*x*e^(2*d*x + 2*c) + 2*
a*b*x*e^(d*x + c) + b^2*x), x)
```

3.226.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh(dx+c)}{(b\cosh(dx+c)+a)x}\,dx$$

input
```
integrate(sinh(d*x+c)/x/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sinh(d*x + c)/((b*cosh(d*x + c) + a)*x), x)
```

3.226.9 Mupad [N/A]

Not integrable

Time = 1.84 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh(c+d\,x)}{x\,\left(a+b\cosh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(sinh(c + d*x)/(x*(a + b*cosh(c + d*x))),x)
```

output
```
int(sinh(c + d*x)/(x*(a + b*cosh(c + d*x))), x)
```

3.227 $\int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$

3.227.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \text{Int}\left(\frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)},x\right)$$

output
```
Unintegrable(x^m*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

3.227.2 Mathematica [N/A]

Not integrable

Time = 6.92 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$$

input
```
Integrate[(x^m*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]
```

output
```
Integrate[(x^m*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]), x]
```

3.227.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6112}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$$

$$\Big\downarrow \text{ 6112}$$

$$\int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}dx$$

input
```
Int[(x^m*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]
```

output
```
$Aborted
```

3.227.3.1 Defintions of rubi rules used

rule 6112
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/(Cosh[(c_.)
 + (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Cosh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.227.4 Maple [N/A] (verified)

Not integrable

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \sinh\left(dx+c\right)^2}{a+b\cosh\left(dx+c\right)}dx$$

input
```
int(x^m*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

output
```
int(x^m*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

3.227.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh\left(dx+c\right)^2}{b\cosh\left(dx+c\right)+a}\,dx$$

input
```
integrate(x^m*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(x^m*sinh(d*x + c)^2/(b*cosh(d*x + c) + a), x)
```

3.227.6 Sympy [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh^2\left(c+dx\right)}{a+b\cosh\left(c+dx\right)}\,dx$$

input
```
integrate(x**m*sinh(d*x+c)**2/(a+b*cosh(d*x+c)),x)
```

output
```
Integral(x**m*sinh(c + d*x)**2/(a + b*cosh(c + d*x)), x)
```

3.227.7 Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh\left(dx+c\right)^2}{b\cosh\left(dx+c\right)+a}\,dx$$

input
```
integrate(x^m*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(x^m*sinh(d*x + c)^2/(b*cosh(d*x + c) + a), x)
```

3.227.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh\left(dx+c\right)^2}{b\cosh\left(dx+c\right)+a}\,dx$$

input
```
integrate(x^m*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x^m*sinh(d*x + c)^2/(b*cosh(d*x + c) + a), x)
```

3.227.9 Mupad [N/A]

Not integrable

Time = 1.78 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh(c+d\,x)^2}{a+b\cosh\left(c+d\,x\right)}\,dx$$

input
```
int((x^m*sinh(c + d*x)^2)/(a + b*cosh(c + d*x)),x)
```

output
```
int((x^m*sinh(c + d*x)^2)/(a + b*cosh(c + d*x)), x)
```

3.228 $\int \frac{x^3 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$

3.228.1 Optimal result

Integrand size = 24, antiderivative size = 495

$$\int \frac{x^3 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{ax^4}{4b^2}-\frac{6\cosh(c+dx)}{bd^4}-\frac{3x^2\cosh(c+dx)}{bd^2}+\frac{\sqrt{a^2-b^2}\,x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d}-\frac{\sqrt{a^2-b^2}\,x^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d}+\frac{3\sqrt{a^2-b^2}\,x^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2}-\frac{3\sqrt{a^2-b^2}\,x^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d^2}-\frac{6\sqrt{a^2-b^2}\,x\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^3}+\frac{6\sqrt{a^2-b^2}\,x\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d^3}+\frac{6\sqrt{a^2-b^2}\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^4}-\frac{6\sqrt{a^2-b^2}\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d^4}+\frac{6x\sinh(c+dx)}{bd^3}+\frac{x^3\sinh(c+dx)}{bd}$$

output
```
-1/4*a*x^4/b^2-6*cosh(d*x+c)/b/d^4-3*x^2*cosh(d*x+c)/b/d^2+6*x*sinh(d*x+c)
/b/d^3+x^3*sinh(d*x+c)/b/d+x^3*ln(1+b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))*(a^2
-b^2)^(1/2)/b^2/d-x^3*ln(1+b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/
2)/b^2/d+3*x^2*polylog(2,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2
)/b^2/d^2-3*x^2*polylog(2,-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/
2)/b^2/d^2-6*x*polylog(3,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2
)/b^2/d^3+6*x*polylog(3,-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)
/b^2/d^3+6*polylog(4,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^
2/d^4-6*polylog(4,-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d
^4
```

3.228.2 Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 386, normalized size of antiderivative = 0.78

$$\int \frac{x^3\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \frac{-ad^4x^4+4\sqrt{a^2-b^2}\left(d^3x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)-d^3x^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)+3d^2x^2\operatorname{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2-b^2}}\right.\right.}{}$$

input
```
Integrate[(x^3*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]
```

output
```
(-(a*d^4*x^4) + 4*Sqrt[a^2 - b^2]*(d^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sq
rt[a^2 - b^2])] - d^3*x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])] +
 3*d^2*x^2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 - b^2])] - 3*d^2*x^2*
PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))] - 6*d*x*PolyLog[3, (b
*E^(c + d*x))/(-a + Sqrt[a^2 - b^2])] + 6*d*x*PolyLog[3, -((b*E^(c + d*x))
/(a + Sqrt[a^2 - b^2]))] + 6*PolyLog[4, (b*E^(c + d*x))/(-a + Sqrt[a^2 - b
^2])] - 6*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))]) + 4*b*Cosh
[d*x]*(-3*(2 + d^2*x^2)*Cosh[c] + d*x*(6 + d^2*x^2)*Sinh[c]) + 4*b*(d*x*(6
 + d^2*x^2)*Cosh[c] - 3*(2 + d^2*x^2)*Sinh[c])*Sinh[d*x])/(4*b^2*d^4)
```

3.228.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.36 (sec) , antiderivative size = 463, normalized size of antiderivative = 0.94, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {6100, 15, 3042, 3777, 26, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118, 3801, 2694, 27, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \sinh^2(c+dx)}{a+b\cosh(c+dx)} \, dx$$

$$\Big\downarrow \; 6100$$

$$\frac{(a^2-b^2)\int \frac{x^3}{a+b\cosh(c+dx)}dx}{b^2} - \frac{a\int x^3dx}{b^2} + \frac{\int x^3\cosh(c+dx)dx}{b}$$

$$\Big\downarrow \; 15$$

$$\frac{(a^2-b^2)\int \frac{x^3}{a+b\cosh(c+dx)}dx}{b^2} + \frac{\int x^3\cosh(c+dx)dx}{b} - \frac{ax^4}{4b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{(a^2-b^2)\int \frac{x^3}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2} + \frac{\int x^3\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b} - \frac{ax^4}{4b^2}$$

$$\Big\downarrow \; 3777$$

$$\frac{(a^2-b^2)\int \frac{x^3}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2} + \frac{\frac{x^3\sinh(c+dx)}{d} - \frac{3i\int -ix^2\sinh(c+dx)dx}{d}}{b} - \frac{ax^4}{4b^2}$$

$$\Big\downarrow \; 26$$

$$\frac{(a^2-b^2)\int \frac{x^3}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2} + \frac{\frac{x^3\sinh(c+dx)}{d} - \frac{3\int x^2\sinh(c+dx)dx}{d}}{b} - \frac{ax^4}{4b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{(a^2-b^2)\int \frac{x^3}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2} + \frac{\frac{x^3\sinh(c+dx)}{d} - \frac{3\int -ix^2\sin(ic+idx)dx}{d}}{b} - \frac{ax^4}{4b^2}$$

$$\Big\downarrow \; 26$$

$$\frac{(a^2-b^2)\int \frac{x^3}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2} + \frac{\frac{x^3\sinh(c+dx)}{d} + \frac{3i\int x^2\sin(ic+idx)dx}{d}}{b} - \frac{ax^4}{4b^2}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(a^2-b^2)\int \frac{x^3}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{x^3\sinh(c+dx)}{d}+\frac{3i\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\int x\cosh(c+dx)dx}{d}\right)}{d}}{b}-\frac{ax^4}{4b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2-b^2)\int \frac{x^3}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{x^3\sinh(c+dx)}{d}+\frac{3i\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\int x\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}}{b}-\frac{ax^4}{4b^2}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(a^2-b^2)\int \frac{x^3}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{x^3\sinh(c+dx)}{d}+\frac{3i\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{i\int -i\sinh(c+dx)dx}{d}\right)}{d}\right)}{d}}{b}-\frac{ax^4}{4b^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{(a^2-b^2)\int \frac{x^3}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{x^3\sinh(c+dx)}{d}+\frac{3i\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\int \sinh(c+dx)dx}{d}\right)}{d}\right)}{d}}{b}-\frac{ax^4}{4b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2-b^2)\int \frac{x^3}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{x^3\sinh(c+dx)}{d}+\frac{3i\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\int -i\sin(ic+idx)dx}{d}\right)}{d}\right)}{d}}{b}-\frac{ax^4}{4b^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{(a^2-b^2)\int \frac{x^3}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{x^3\sinh(c+dx)}{d}+\frac{3i\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}+\frac{i\int \sin(ic+idx)dx}{d}\right)}{d}\right)}{d}}{b}-\frac{ax^4}{4b^2}$$

$$\Big\downarrow \text{3118}$$

$$\frac{(a^2-b^2)\int \frac{x^3}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}-\frac{ax^4}{4b^2}+\frac{\frac{x^3\sinh(c+dx)}{d}+\frac{3i\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{b}$$

$$\Big\downarrow \text{3801}$$

3.228. $\int \frac{x^3\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$

$$\frac{2\left(a^2-b^2\right)\int \frac{e^{c+dx}x^3}{2e^{c+dx}a+be^{2(c+dx)}+b}dx}{b^2}-\frac{ax^4}{4b^2}+\frac{\frac{x^3\sinh(c+dx)}{d}+\frac{3i\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{b}$$

$$\Big\downarrow \text{2694}$$

$$\frac{2\left(a^2-b^2\right)\left(\frac{b\int \frac{e^{c+dx}x^3}{2\left(a+be^{c+dx}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{b\int \frac{e^{c+dx}x^3}{2\left(a+be^{c+dx}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{b^2}-\frac{ax^4}{4b^2}+\frac{\frac{x^3\sinh(c+dx)}{d}+\frac{3i\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(a^2-b^2\right)\left(\frac{b\int \frac{e^{c+dx}x^3}{a+be^{c+dx}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{b\int \frac{e^{c+dx}x^3}{a+be^{c+dx}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{b^2}-\frac{ax^4}{4b^2}+\frac{\frac{x^3\sinh(c+dx)}{d}+\frac{3i\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{b}$$

$$\Big\downarrow \text{2620}$$

$$\frac{2\left(a^2-b^2\right)\left(\frac{b\left(\frac{x^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}-\frac{3\int x^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2-b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{x^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{3\int x^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2-b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}\right)}{b^2}-\frac{ax^4}{4b^2}+\frac{\frac{x^3\sinh(c+dx)}{d}+\frac{3i\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{b}$$

$$\Big\downarrow \text{3011}$$

3.228. $\int \frac{x^3\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$

$$\frac{2(a^2-b^2)\left(\frac{b\left(\frac{x^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}-\frac{3\left(\frac{2\int x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)dx}{d}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{x^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\ldots\right.}{\ldots}\right.}{b^2}$$

$$\frac{\frac{ax^4}{4b^2}+\frac{x^3\sinh(c+dx)}{d}+\frac{3i\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{b}$$

$\downarrow$ 7163

$$\frac{2(a^2-b^2)\left(\frac{b\left(\frac{x^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}-\frac{3\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{\int\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)dx}{d}\right)}{d}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\ldots\right.}{b^2}$$

$$\frac{\frac{ax^4}{4b^2}+\frac{x^3\sinh(c+dx)}{d}+\frac{3i\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{b}$$

$\downarrow$ 2720

3.228. $\int\frac{x^3\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$

$$2(a^2-b^2)\left(\frac{b\left(\frac{x^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}-\frac{3\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{\int e^{-c-dx}\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{be^{c+d}}{a-\sqrt{a^2}}\right.}{d}\right.}{bd}\right.}{2\sqrt{a^2-b^2}}\right.$$

$$\frac{ax^4}{4b^2}+\frac{\frac{x^3\sinh(c+dx)}{d}+\frac{3i\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{b}$$

$\Big\downarrow$ 7143

3.228. $\int \frac{x^3\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$

$$\frac{2(a^2-b^2)\left(\frac{b\left(\frac{x^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}-\frac{3\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d^2}\right)}{d}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{b\ \cdots}{\cdots}\right)}{b^2}$$

$$\frac{ax^4}{4b^2}+\frac{\frac{x^3\sinh(c+dx)}{d}+\frac{3i\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{b}$$

input

```
Int[(x^3*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]
```

output

```
-1/4*(a*x^4)/b^2 + (2*(a^2 - b^2)*((b*((x^3*Log[1 + (b*E^(c + d*x))/(a - S
qrt[a^2 - b^2])])/(b*d) - (3*(-((x^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqr
t[a^2 - b^2]))])/d) + (2*((x*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 -
b^2]))])/d - PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))]/d^2))/d)
)/(b*d)))/(2*Sqrt[a^2 - b^2]) - (b*((x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt
[a^2 - b^2])])/(b*d) - (3*(-((x^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a
^2 - b^2]))])/d) + (2*((x*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2
]))])/d - PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))]/d^2))/d))/(
b*d)))/(2*Sqrt[a^2 - b^2])))/b^2 + ((x^3*Sinh[c + d*x])/d + ((3*I)*((I*x^2
*Cosh[c + d*x])/d - ((2*I)*(-(Cosh[c + d*x]/d^2) + (x*Sinh[c + d*x])/d))/d
))/d)/b
```

3.228. $\int \frac{x^3\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$

3.228.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

3.228. $\int \frac{x^3 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3801
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[((c + d*x)^m*(E^((-I)*e
+ f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)
*e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c
, d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6100
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_))/(Cosh[(c_.)
+ (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Sin
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Sinh[c + d*x]^(n -
 2)*Cosh[c + d*x], x], x] + Simp[(a^2 - b^2)/b^2   Int[(e + f*x)^m*(Sinh[c
+ d*x]^(n - 2)/(a + b*Cosh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.228. $\int \frac{x^3 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$

3.228.4 Maple [F]

$$\int \frac{x^3 \sinh(dx+c)^2}{a+b\cosh(dx+c)}dx$$

input
```
int(x^3*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

output
```
int(x^3*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

3.228.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1174 vs. $2(451) = 902$.

Time = 0.29 (sec) , antiderivative size = 1174, normalized size of antiderivative = 2.37

$$\int \frac{x^3 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/4*(a*d^4*x^4*cosh(d*x + c) + 2*b*d^3*x^3 + 6*b*d^2*x^2 + 12*b*d*x - 2*(
b*d^3*x^3 - 3*b*d^2*x^2 + 6*b*d*x - 6*b)*cosh(d*x + c)^2 - 2*(b*d^3*x^3 -
3*b*d^2*x^2 + 6*b*d*x - 6*b)*sinh(d*x + c)^2 - 12*(b*d^2*x^2*cosh(d*x + c)
 + b*d^2*x^2*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cosh(d*x + c)
+ a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b
^2) + b)/b + 1) + 12*(b*d^2*x^2*cosh(d*x + c) + b*d^2*x^2*sinh(d*x + c))*s
qrt((a^2 - b^2)/b^2)*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d
*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - 4*(b*c^3*co
sh(d*x + c) + b*c^3*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log(2*b*cosh(d*x
+ c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + 4*(b*c^3*cos
h(d*x + c) + b*c^3*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log(2*b*cosh(d*x +
 c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - 4*((b*d^3*x^3
 + b*c^3)*cosh(d*x + c) + (b*d^3*x^3 + b*c^3)*sinh(d*x + c))*sqrt((a^2 - b
^2)/b^2)*log((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sin
h(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) + 4*((b*d^3*x^3 + b*c^3)*cosh(d*
x + c) + (b*d^3*x^3 + b*c^3)*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log((a*c
osh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt(
(a^2 - b^2)/b^2) + b)/b) - 24*(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 - b^2)/b^2)*polylog(4, -(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x
 + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) + 24*(b*cosh(d*x + c...
```

3.228.6 Sympy [F]

$$\int \frac{x^3 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^3 \sinh^2{(c+dx)}}{a+b\cosh{(c+dx)}}\,dx$$

input
```
integrate(x**3*sinh(d*x+c)**2/(a+b*cosh(d*x+c)),x)
```

output
```
Integral(x**3*sinh(c + d*x)**2/(a + b*cosh(c + d*x)), x)
```

3.228.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a-b>0)', see `assume?` for more
details)Is
```

3.228.8 Giac [F]

$$\int \frac{x^3 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^3 \sinh{(dx+c)}^2}{b\cosh{(dx+c)}+a}\,dx$$

input
```
integrate(x^3*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x^3*sinh(d*x + c)^2/(b*cosh(d*x + c) + a), x)
```

3.228.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^3 \sinh(c+d\,x)^2}{a+b\cosh\left(c+d\,x\right)}\,dx$$

input
```
int((x^3*sinh(c + d*x)^2)/(a + b*cosh(c + d*x)),x)
```

output
```
int((x^3*sinh(c + d*x)^2)/(a + b*cosh(c + d*x)), x)
```

3.229 $\int \frac{x^2 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$

3.229.1 Optimal result

Integrand size = 24, antiderivative size = 370

$$\int \frac{x^2\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{ax^3}{3b^2} - \frac{2x\cosh(c+dx)}{bd^2} + \frac{\sqrt{a^2-b^2}\,x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d} - \frac{\sqrt{a^2-b^2}\,x^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d} + \frac{2\sqrt{a^2-b^2}\,x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2} - \frac{2\sqrt{a^2-b^2}\,x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d^2} - \frac{2\sqrt{a^2-b^2}\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^3} + \frac{2\sqrt{a^2-b^2}\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d^3} + \frac{2\sinh(c+dx)}{bd^3} + \frac{x^2\sinh(c+dx)}{bd}$$

output
```
-1/3*a*x^3/b^2-2*x*cosh(d*x+c)/b/d^2+2*sinh(d*x+c)/b/d^3+x^2*sinh(d*x+c)/b
/d+x^2*ln(1+b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d-x^2*ln
(1+b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d+2*x*polylog(2,-
b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d^2-2*x*polylog(2,-b
*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d^2-2*polylog(3,-b*ex
p(d*x+c)/(a-(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d^3+2*polylog(3,-b*exp(d
*x+c)/(a+(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d^3
```

3.229.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.79

$$\int \frac{x^2 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \frac{-ad^3x^3+3\sqrt{a^2-b^2}\left(d^2x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)-d^2x^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)+2dx\operatorname{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2-b^2}}\right)\right.}{}$$

input
```
Integrate[(x^2*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]
```

output
```
(-(a*d^3*x^3) + 3*Sqrt[a^2 - b^2]*(d^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sq
rt[a^2 - b^2])] - d^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])] +
 2*d*x*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 - b^2])] - 2*d*x*PolyLog[
2, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))] - 2*PolyLog[3, (b*E^(c + d*x)
)/(-a + Sqrt[a^2 - b^2])] + 2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 -
 b^2]))]) + 3*b*Cosh[d*x]*(-2*d*x*Cosh[c] + (2 + d^2*x^2)*Sinh[c]) + 3*b*(
(2 + d^2*x^2)*Cosh[c] - 2*d*x*Sinh[c])*Sinh[d*x])/(3*b^2*d^3)
```

3.229.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.72 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.96, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.708$, Rules used = {6100, 15, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117, 3801, 2694, 27, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \sinh^2(c+dx)}{a+b\cosh(c+dx)}dx$$

$$\Big\downarrow \text{6100}$$

$$\frac{(a^2-b^2)\int \frac{x^2}{a+b\cosh(c+dx)}dx}{b^2}-\frac{a\int x^2dx}{b^2}+\frac{\int x^2\cosh(c+dx)dx}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{(a^2-b^2)\int \frac{x^2}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\int x^2\cosh(c+dx)dx}{b}-\frac{ax^3}{3b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2-b^2)\int \frac{x^2}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\int x^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b}-\frac{ax^3}{3b^2}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(a^2-b^2)\int \frac{x^2}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{x^2\sinh(c+dx)}{d}-\frac{2i\int -ix\sinh(c+dx)dx}{d}}{b}-\frac{ax^3}{3b^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{(a^2-b^2)\int \frac{x^2}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{x^2\sinh(c+dx)}{d}-\frac{2\int x\sinh(c+dx)dx}{d}}{b}-\frac{ax^3}{3b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2-b^2)\int \frac{x^2}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{x^2\sinh(c+dx)}{d}-\frac{2\int -ix\sin(ic+idx)dx}{d}}{b}-\frac{ax^3}{3b^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{(a^2-b^2)\int \frac{x^2}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{x^2\sinh(c+dx)}{d}+\frac{2i\int x\sin(ic+idx)dx}{d}}{b}-\frac{ax^3}{3b^2}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(a^2-b^2)\int \frac{x^2}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\int\cosh(c+dx)dx}{d}\right)}{d}}{b}-\frac{ax^3}{3b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2-b^2)\int \frac{x^2}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}}{b}-\frac{ax^3}{3b^2}$$

$$\Big\downarrow \text{3117}$$

3.229. $\int \frac{x^2\sinh^2(c+dx)}{a+b\cosh(c+dx)}dx$

$$\frac{\left(a^2-b^2\right)\int \frac{x^2}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}-\frac{ax^3}{3b^2}+\frac{\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}}{b}$$

$$\Big\downarrow \text{3801}$$

$$\frac{2\left(a^2-b^2\right)\int \frac{e^{c+dx}x^2}{2e^{c+dx}a+be^{2(c+dx)}+b}dx}{b^2}-\frac{ax^3}{3b^2}+\frac{\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}}{b}$$

$$\Big\downarrow \text{2694}$$

$$\frac{2\left(a^2-b^2\right)\left(\frac{b\int \frac{e^{c+dx}x^2}{2\left(a+be^{c+dx}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{b\int \frac{e^{c+dx}x^2}{2\left(a+be^{c+dx}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{b^2}-\frac{ax^3}{3b^2}+\frac{\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(a^2-b^2\right)\left(\frac{b\int \frac{e^{c+dx}x^2}{a+be^{c+dx}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{b\int \frac{e^{c+dx}x^2}{a+be^{c+dx}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{b^2}-\frac{ax^3}{3b^2}+\frac{\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}}{b}$$

$$\Big\downarrow \text{2620}$$

$$\frac{2\left(a^2-b^2\right)\left(\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}-\frac{2\int x\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2-b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{2\int x\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2-b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}\right)}{b^2}-\frac{ax^3}{3b^2}+\frac{\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}}{b}$$

$$\Big\downarrow \text{3011}$$

3.229. $\int \frac{x^2\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$

$$2(a^2-b^2)\left(\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}-\frac{2\left(\frac{\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)dx}{d}-\frac{x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{2\left(\int \text{P}\right.}{}\right.}{}\right.$$

$$\overline{\qquad b^2\qquad}$$

$$\frac{ax^3}{3b^2}+\frac{\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}}{b}$$

$\downarrow$ 2720

$$2(a^2-b^2)\left(\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}-\frac{2\left(\frac{\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)de^{c+dx}}{d^2}-\frac{x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right.}{bd}\right.}{}\right.$$

$$\overline{\qquad b^2\qquad}$$

$$\frac{ax^3}{3b^2}+\frac{\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}}{b}$$

$\downarrow$ 7143

$$2(a^2-b^2)\left(\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}-\frac{2\left(\frac{\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d^2}-\frac{x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{2\left(\text{PolyLo}\right.}{}\right.}{}\right.$$

$$\overline{\qquad b^2\qquad}$$

$$\frac{ax^3}{3b^2}+\frac{\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}}{b}$$

3.229. $\displaystyle\int \frac{x^2\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$

input
```
Int[(x^2*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]
```

output
```
-1/3*(a*x^3)/b^2 + (2*(a^2 - b^2)*((b*((x^2*Log[1 + (b*E^(c + d*x))/(a - S
qrt[a^2 - b^2])])/(b*d) - (2*(-((x*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[
a^2 - b^2]))])/d) + PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))]/d
^2))/(b*d)))/(2*Sqrt[a^2 - b^2]) - (b*((x^2*Log[1 + (b*E^(c + d*x))/(a + S
qrt[a^2 - b^2])])/(b*d) - (2*(-((x*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[
a^2 - b^2]))])/d) + PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))]/d
^2))/(b*d)))/(2*Sqrt[a^2 - b^2])))/b^2 + ((x^2*Sinh[c + d*x])/d + ((2*I)*(
(I*x*Cosh[c + d*x])/d - (I*Sinh[c + d*x])/d^2))/d)/b
```

3.229.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3801
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[((c + d*x)^m*(E^((-I)*e
+ f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)
*e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c
, d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6100
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_))/(Cosh[(c_.)
+ (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Sin
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Sinh[c + d*x]^(n -
 2)*Cosh[c + d*x], x], x] + Simp[(a^2 - b^2)/b^2   Int[(e + f*x)^m*(Sinh[c
+ d*x]^(n - 2)/(a + b*Cosh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

3.229. $\int \frac{x^2 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.229.4 Maple [F]

$$\int \frac{x^2 \sinh (dx+c)^2}{a+b\cosh (dx+c)}dx$$

input
```
int(x^2*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

output
```
int(x^2*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

3.229.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 937 vs. $2(336) = 672$.

Time = 0.28 (sec) , antiderivative size = 937, normalized size of antiderivative = 2.53

$$\int \frac{x^2 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/6*(2*a*d^3*x^3*cosh(d*x + c) + 3*b*d^2*x^2 + 6*b*d*x - 3*(b*d^2*x^2 - 2
*b*d*x + 2*b)*cosh(d*x + c)^2 - 3*(b*d^2*x^2 - 2*b*d*x + 2*b)*sinh(d*x + c
)^2 - 12*(b*d*x*cosh(d*x + c) + b*d*x*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)
*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
 + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + 12*(b*d*x*cosh(d*x + c) + b*d*x
*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cosh(d*x + c) + a*sinh(d*x
 + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b +
 1) + 6*(b*c^2*cosh(d*x + c) + b*c^2*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*
log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*
a) - 6*(b*c^2*cosh(d*x + c) + b*c^2*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*l
og(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a
) - 6*((b*d^2*x^2 - b*c^2)*cosh(d*x + c) + (b*d^2*x^2 - b*c^2)*sinh(d*x +
c))*sqrt((a^2 - b^2)/b^2)*log((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh
(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) + 6*((b*d^2*x^2
 - b*c^2)*cosh(d*x + c) + (b*d^2*x^2 - b*c^2)*sinh(d*x + c))*sqrt((a^2 - b
^2)/b^2)*log((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sin
h(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) + 12*(b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 - b^2)/b^2)*polylog(3, -(a*cosh(d*x + c) + a*sinh(d*x +
 c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) - 12*(
b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*polylog(3, -(a...
```

3.229.6 Sympy [F]

$$\int \frac{x^2 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^2 \sinh^2\left(c+dx\right)}{a+b\cosh\left(c+dx\right)}\,dx$$

input
```
integrate(x**2*sinh(d*x+c)**2/(a+b*cosh(d*x+c)),x)
```

output
```
Integral(x**2*sinh(c + d*x)**2/(a + b*cosh(c + d*x)), x)
```

3.229.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a-b>0)', see `assume?` for more
details)Is
```

3.229.8 Giac [F]

$$\int \frac{x^2 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^2 \sinh(dx+c)^2}{b\cosh(dx+c)+a}\,dx$$

input
```
integrate(x^2*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x^2*sinh(d*x + c)^2/(b*cosh(d*x + c) + a), x)
```

3.229.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^2 \sinh(c+d\,x)^2}{a+b\cosh(c+d\,x)}\,dx$$

input
```
int((x^2*sinh(c + d*x)^2)/(a + b*cosh(c + d*x)),x)
```

output
```
int((x^2*sinh(c + d*x)^2)/(a + b*cosh(c + d*x)), x)
```

3.230 $\int \frac{x\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$

3.230.1 Optimal result

Integrand size = 22, antiderivative size = 244

$$\int \frac{x\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{ax^2}{2b^2} - \frac{\cosh(c+dx)}{bd^2} + \frac{\sqrt{a^2-b^2}\,x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d} - \frac{\sqrt{a^2-b^2}\,x\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d} + \frac{\sqrt{a^2-b^2}\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2} - \frac{\sqrt{a^2-b^2}\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d^2} + \frac{x\sinh(c+dx)}{bd}$$

output
```
-1/2*a*x^2/b^2-cosh(d*x+c)/b/d^2+x*sinh(d*x+c)/b/d+x*ln(1+b*exp(d*x+c)/(a-
(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d-x*ln(1+b*exp(d*x+c)/(a+(a^2-b^2)^(
1/2)))*(a^2-b^2)^(1/2)/b^2/d+polylog(2,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))*
(a^2-b^2)^(1/2)/b^2/d^2-polylog(2,-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))*(a^2-
b^2)^(1/2)/b^2/d^2
```

3.230.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.77

$$\int \frac{x\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$$
$$= \frac{a(c-dx)(c+dx)-2b\cosh(c+dx)+2\sqrt{a^2-b^2}\left(dx\left(\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)-\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)\right)+\text{Poly}\right.}{2b^2d^2}$$

input
```
Integrate[(x*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]
```

output
```
(a*(c - d*x)*(c + d*x) - 2*b*Cosh[c + d*x] + 2*Sqrt[a^2 - b^2]*(d*x*(Log[1
 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])] - Log[1 + (b*E^(c + d*x))/(a + S
qrt[a^2 - b^2])]) + PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 - b^2])] - P
olyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))]) + 2*b*d*x*Sinh[c + d*
x])/(2*b^2*d^2)
```

3.230.3 Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.99, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {6100, 15, 3042, 3777, 26, 3042, 26, 3118, 3801, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$$

$$\Big\downarrow \text{ 6100}$$

$$\frac{(a^2-b^2)\int \frac{x}{a+b\cosh(c+dx)}dx}{b^2}-\frac{a\int xdx}{b^2}+\frac{\int x\cosh(c+dx)dx}{b}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{(a^2-b^2)\int \frac{x}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\int x\cosh(c+dx)dx}{b}-\frac{ax^2}{2b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(a^2-b^2)\int \frac{x}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\int x\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b}-\frac{ax^2}{2b^2}$$

$$\downarrow 3777$$

$$\frac{(a^2-b^2)\int \frac{x}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{x\sinh(c+dx)}{d}-\frac{i\int -i\sinh(c+dx)dx}{d}}{b}-\frac{ax^2}{2b^2}$$

$$\downarrow 26$$

$$\frac{(a^2-b^2)\int \frac{x}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{x\sinh(c+dx)}{d}-\frac{\int \sinh(c+dx)dx}{d}}{b}-\frac{ax^2}{2b^2}$$

$$\downarrow 3042$$

$$\frac{(a^2-b^2)\int \frac{x}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{x\sinh(c+dx)}{d}-\frac{\int -i\sin(ic+idx)dx}{d}}{b}-\frac{ax^2}{2b^2}$$

$$\downarrow 26$$

$$\frac{(a^2-b^2)\int \frac{x}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{x\sinh(c+dx)}{d}+\frac{i\int \sin(ic+idx)dx}{d}}{b}-\frac{ax^2}{2b^2}$$

$$\downarrow 3118$$

$$\frac{(a^2-b^2)\int \frac{x}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}-\frac{ax^2}{2b^2}+\frac{\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}}{b}$$

$$\downarrow 3801$$

$$\frac{2(a^2-b^2)\int \frac{e^{c+dx}x}{2e^{c+dx}a+be^{2(c+dx)}+b}dx}{b^2}-\frac{ax^2}{2b^2}+\frac{\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}}{b}$$

$$\downarrow 2694$$

$$\frac{2(a^2-b^2)\left(\frac{b\int \frac{e^{c+dx}x}{2\left(a+be^{c+dx}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{b\int \frac{e^{c+dx}x}{2\left(a+be^{c+dx}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{b^2}-\frac{ax^2}{2b^2}+\frac{\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}}{b}$$

$$\downarrow 27$$

$$\frac{2(a^2-b^2)\left(\frac{b\int \frac{e^{c+dx}x}{a+be^{c+dx}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{b\int \frac{e^{c+dx}x}{a+be^{c+dx}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{b^2}-\frac{ax^2}{2b^2}+\frac{\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}}{b}$$

$$\downarrow 2620$$

3.230. $\int \frac{x\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$

$$\frac{2(a^2-b^2)\left(\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}-\frac{\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2-b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2-b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}\right)}{b^2}-\frac{ax^2}{2b^2}+\frac{\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}}{b}$$

$\downarrow$ 2715

$$\frac{2(a^2-b^2)\left(\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}-\frac{\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2-b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2-b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2-b^2}}\right)}{b^2}-\frac{ax^2}{2b^2}+\frac{\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}}{b}$$

$\downarrow$ 2838

$$\frac{2(a^2-b^2)\left(\frac{b\left(\frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{x\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{x\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2-b^2}}\right)}{b^2}-\frac{ax^2}{2b^2}+\frac{\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}}{b}$$

input
```
Int[(x*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]
```

output
```
-1/2*(a*x^2)/b^2 + (2*(a^2 - b^2)*((b*((x*Log[1 + (b*E^(c + d*x))/(a - Sqr
t[a^2 - b^2])])/(b*d) + PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])
)]/(b*d^2)))/(2*Sqrt[a^2 - b^2]) - (b*((x*Log[1 + (b*E^(c + d*x))/(a + Sqr
t[a^2 - b^2])])/(b*d) + PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])
)]/(b*d^2)))/(2*Sqrt[a^2 - b^2])))/b^2 + (-(Cosh[c + d*x]/d^2) + (x*Sinh[c
 + d*x])/d)/b
```

3.230. $\int \frac{x\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$

3.230.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

3.230. $\int \frac{x\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3801
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[((c + d*x)^m*(E^((-I)*e
+ f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)
*e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c
, d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6100
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_))/(Cosh[(c_.)
+ (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Sin
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Sinh[c + d*x]^(n -
 2)*Cosh[c + d*x], x], x] + Simp[(a^2 - b^2)/b^2   Int[(e + f*x)^m*(Sinh[c
+ d*x]^(n - 2)/(a + b*Cosh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

3.230.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 861 vs. $2(222) = 444$.

Time = 1.27 (sec) , antiderivative size = 862, normalized size of antiderivative = 3.53

method	result
risch	$-\frac{a\,x^{2}}{2b^{2}}+\frac{(dx-1)\,\mathrm{e}^{dx+c}}{2b\,d^{2}}-\frac{(dx+1)\,\mathrm{e}^{-dx-c}}{2b\,d^{2}}+\frac{\ln\left(\frac{-\mathrm{e}^{dx+c}b+\sqrt{a^{2}-b^{2}}-a}{-a+\sqrt{a^{2}-b^{2}}}\right)x\,a^{2}}{d\,b^{2}\sqrt{a^{2}-b^{2}}}-\frac{\ln\left(\frac{-\mathrm{e}^{dx+c}b+\sqrt{a^{2}-b^{2}}-a}{-a+\sqrt{a^{2}-b^{2}}}\right)x}{d\sqrt{a^{2}-b^{2}}}-\frac{\ln\left(\frac{\mathrm{e}^{dx+c}b+\sqrt{a}}{a+\sqrt{a^{2}}}\right.}{d\,b^{2}\sqrt{a}}$

input
```
int(x*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*x^2/b^2+1/2*(d*x-1)/b/d^2*exp(d*x+c)-1/2*(d*x+1)/b/d^2*exp(-d*x-c)+
1/d/b^2/(a^2-b^2)^(1/2)*ln((-exp(d*x+c)*b+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)
^(1/2)))*x*a^2-1/d/(a^2-b^2)^(1/2)*ln((-exp(d*x+c)*b+(a^2-b^2)^(1/2)-a)/(-
a+(a^2-b^2)^(1/2)))*x-1/d/b^2/(a^2-b^2)^(1/2)*ln((exp(d*x+c)*b+(a^2-b^2)^(
1/2)+a)/(a+(a^2-b^2)^(1/2)))*x*a^2+1/d/(a^2-b^2)^(1/2)*ln((exp(d*x+c)*b+(a
^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*x+1/d^2/b^2/(a^2-b^2)^(1/2)*ln((-exp
(d*x+c)*b+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))*c*a^2-1/d^2/(a^2-b^2)^(
1/2)*ln((-exp(d*x+c)*b+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))*c-1/d^2/b^
2/(a^2-b^2)^(1/2)*ln((exp(d*x+c)*b+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))
*c*a^2+1/d^2/(a^2-b^2)^(1/2)*ln((exp(d*x+c)*b+(a^2-b^2)^(1/2)+a)/(a+(a^2-b
^2)^(1/2)))*c+1/d^2/b^2/(a^2-b^2)^(1/2)*dilog((-exp(d*x+c)*b+(a^2-b^2)^(1/
2)-a)/(-a+(a^2-b^2)^(1/2)))*a^2-1/d^2/(a^2-b^2)^(1/2)*dilog((-exp(d*x+c)*b
+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))-1/d^2/b^2/(a^2-b^2)^(1/2)*dilog(
(exp(d*x+c)*b+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*a^2+1/d^2/(a^2-b^2)^
(1/2)*dilog((exp(d*x+c)*b+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))-2/d^2/b^
2*c/(-a^2+b^2)^(1/2)*arctan(1/2*(2*exp(d*x+c)*b+2*a)/(-a^2+b^2)^(1/2))*a^2
+2/d^2*c/(-a^2+b^2)^(1/2)*arctan(1/2*(2*exp(d*x+c)*b+2*a)/(-a^2+b^2)^(1/2)
)
```

3.230.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 669 vs. $2(220) = 440$.

Time = 0.29 (sec) , antiderivative size = 669, normalized size of antiderivative = 2.74

$$\int \frac{x \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx =$$

$$-\frac{ad^2x^2\cosh(dx+c) + bdx - (bdx-b)\cosh(dx+c)^2 - (bdx-b)\sinh(dx+c)^2 - 2\left(b\cosh(dx+c) + \right.}{}$$

input
```
integrate(x*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(a*d^2*x^2*cosh(d*x + c) + b*d*x - (b*d*x - b)*cosh(d*x + c)^2 - (b*d
*x - b)*sinh(d*x + c)^2 - 2*(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
- b^2)/b^2)*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) +
 b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + 2*(b*cosh(d*x + c) +
 b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cosh(d*x + c) + a*sinh(d
*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b
 + 1) - 2*(b*c*cosh(d*x + c) + b*c*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*lo
g(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a)
 + 2*(b*c*cosh(d*x + c) + b*c*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log(2*b
*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - 2*
((b*d*x + b*c)*cosh(d*x + c) + (b*d*x + b*c)*sinh(d*x + c))*sqrt((a^2 - b^
2)/b^2)*log((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh
(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) + 2*((b*d*x + b*c)*cosh(d*x + c)
+ (b*d*x + b*c)*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log((a*cosh(d*x + c)
+ a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b
^2) + b)/b) + (a*d^2*x^2 - 2*(b*d*x - b)*cosh(d*x + c))*sinh(d*x + c) + b)
/(b^2*d^2*cosh(d*x + c) + b^2*d^2*sinh(d*x + c))
```

3.230.6 Sympy [F]

$$\int \frac{x\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x\sinh^2{(c+dx)}}{a+b\cosh{(c+dx)}}\,dx$$

input
```
integrate(x*sinh(d*x+c)**2/(a+b*cosh(d*x+c)),x)
```

output
```
Integral(x*sinh(c + d*x)**2/(a + b*cosh(c + d*x)), x)
```

3.230.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a-b>0)', see `assume?` for more
details)Is
```

3.230.8 Giac [F]

$$\int \frac{x\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x\sinh{(dx+c)}^2}{b\cosh{(dx+c)}+a}\,dx$$

input
```
integrate(x*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x*sinh(d*x + c)^2/(b*cosh(d*x + c) + a), x)
```

3.230.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x\,\sinh(c+d\,x)^2}{a+b\cosh{(c+d\,x)}}\,dx$$

input
```
int((x*sinh(c + d*x)^2)/(a + b*cosh(c + d*x)),x)
```

output
```
int((x*sinh(c + d*x)^2)/(a + b*cosh(c + d*x)), x)
```

3.231 $\int \frac{\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$

3.231.1 Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \frac{\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx=-\frac{ax}{b^2}+\frac{2\sqrt{a-b}\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d}+\frac{\sinh(c+dx)}{bd}$$

output
```
-a*x/b^2+sinh(d*x+c)/b/d+2*arctanh((a-b)^(1/2)*tanh(1/2*d*x+1/2*c)/(a+b)^(
1/2))*(a-b)^(1/2)*(a+b)^(1/2)/b^2/d
```

3.231.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int \frac{\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx =\frac{-a(c+dx)+2\sqrt{-a^2+b^2}\arctan\left(\frac{(a-b)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)+b\sinh(c+dx)}{b^2d}$$

input
```
Integrate[Sinh[c + d*x]^2/(a + b*Cosh[c + d*x]),x]
```

output
```
(-(a*(c + d*x)) + 2*Sqrt[-a^2 + b^2]*ArcTan[((a - b)*Tanh[(c + d*x)/2])/Sq
rt[-a^2 + b^2]] + b*Sinh[c + d*x])/(b^2*d)
```

3.231.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 25, 3174, 25, 3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)}{a+b\cosh(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{\cos\left(ic+idx-\frac{\pi}{2}\right)^2}{a-b\sin\left(ic+idx-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2ic-\pi)+idx\right)^2}{a-b\sin\left(\frac{1}{2}(2ic-\pi)+idx\right)}dx$$

$$\Big\downarrow \; 3174$$

$$\frac{\int -\frac{b+a\cosh(c+dx)}{a+b\cosh(c+dx)}dx}{b}+\frac{\sinh(c+dx)}{bd}$$

$$\Big\downarrow \; 25$$

$$\frac{\sinh(c+dx)}{bd}-\frac{\int \frac{b+a\cosh(c+dx)}{a+b\cosh(c+dx)}dx}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sinh(c+dx)}{bd}-\frac{\int \frac{b+a\sin\left(ic+idx+\frac{\pi}{2}\right)}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \; 3214$$

$$\frac{\sinh(c+dx)}{bd}-\frac{\frac{ax}{b}-\frac{(a^2-b^2)\int \frac{1}{a+b\cosh(c+dx)}dx}{b}}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sinh(c+dx)}{bd}-\frac{\frac{ax}{b}-\frac{(a^2-b^2)\int \frac{1}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b}}{b}$$

$$\Big\downarrow \; 3138$$

$$\frac{\sinh(c+dx)}{bd} - \frac{\frac{ax}{b} + \frac{2i(a^2-b^2)\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{1}{2}(c+dx)\right)\right)+a+b}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{bd}}{b}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\sinh(c+dx)}{bd} - \frac{\frac{ax}{b} - \frac{2(a^2-b^2)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}}{b}$$

input
```
Int[Sinh[c + d*x]^2/(a + b*Cosh[c + d*x]),x]
```

output
```
-(((a*x)/b - (2*(a^2 - b^2)*ArcTanh[(Sqrt[a - b]*Tanh[(c + d*x)/2])/Sqrt[a
 + b]])/(Sqrt[a - b]*b*Sqrt[a + b]*d))/b) + Sinh[c + d*x]/(b*d)
```

3.231.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3174
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(b*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
 0] && IntegersQ[2*m, 2*p]
```

3.231. $\int \frac{\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.231.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.77

method	result
derivativedivides	$\frac{-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}-\frac{2\left(-a^2+b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^2\sqrt{(a+b)(a-b)}}-\frac{1}{b\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{a\ln\left(1+\tan\right.}{}}{d}$
default	$\frac{-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}-\frac{2\left(-a^2+b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^2\sqrt{(a+b)(a-b)}}-\frac{1}{b\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{a\ln\left(1+\tan\right.}{}}{d}$
risch	$-\frac{ax}{b^2}+\frac{e^{dx+c}}{2db}-\frac{e^{-dx-c}}{2db}+\frac{\sqrt{a^2-b^2}\,\ln\left(e^{dx+c}-\frac{-a+\sqrt{a^2-b^2}}{b}\right)}{d\,b^2}-\frac{\sqrt{a^2-b^2}\,\ln\left(e^{dx+c}+\frac{a+\sqrt{a^2-b^2}}{b}\right)}{d\,b^2}$

input
```
int(sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b/(tanh(1/2*d*x+1/2*c)-1)+a/b^2*ln(tanh(1/2*d*x+1/2*c)-1)-2/b^2*(-
a^2+b^2)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*d*x+1/2*c)/((a+b)*(a-b
))^(1/2))-1/b/(1+tanh(1/2*d*x+1/2*c))-a/b^2*ln(1+tanh(1/2*d*x+1/2*c)))
```

3.231.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(64) = 128.

Time = 0.27 (sec) , antiderivative size = 415, normalized size of antiderivative = 5.68

$$\int \frac{\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$$

$$= \left[-\frac{2\,adx\cosh(dx+c)-b\cosh(dx+c)^2-b\sinh(dx+c)^2-2\sqrt{a^2-b^2}(\cosh(dx+c)+\sinh(dx+c))\,l}{}\right.$$

$$-\frac{2\,adx\cosh(dx+c)-b\cosh(dx+c)^2-b\sinh(dx+c)^2+4\sqrt{-a^2+b^2}(\cosh(dx+c)+\sinh(dx+c))}{2\left(b^2d\cosh(dx+c)+b^2d\,s\right.}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
[-1/2*(2*a*d*x*cosh(d*x + c) - b*cosh(d*x + c)^2 - b*sinh(d*x + c)^2 - 2*s
qrt(a^2 - b^2)*(cosh(d*x + c) + sinh(d*x + c))*log((b^2*cosh(d*x + c)^2 +
b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 - b^2 + 2*(b^2*cosh(d*x
+ c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 - b^2)*(b*cosh(d*x + c) + b*sinh(d*
x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) +
2*(b*cosh(d*x + c) + a)*sinh(d*x + c) + b)) + 2*(a*d*x - b*cosh(d*x + c))*
sinh(d*x + c) + b)/(b^2*d*cosh(d*x + c) + b^2*d*sinh(d*x + c)), -1/2*(2*a*
d*x*cosh(d*x + c) - b*cosh(d*x + c)^2 - b*sinh(d*x + c)^2 + 4*sqrt(-a^2 +
b^2)*(cosh(d*x + c) + sinh(d*x + c))*arctan(-sqrt(-a^2 + b^2)*(b*cosh(d*x
+ c) + b*sinh(d*x + c) + a)/(a^2 - b^2)) + 2*(a*d*x - b*cosh(d*x + c))*sin
h(d*x + c) + b)/(b^2*d*cosh(d*x + c) + b^2*d*sinh(d*x + c))]
```

3.231.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1122 vs. $2(61) = 122$.

Time = 67.78 (sec) , antiderivative size = 1122, normalized size of antiderivative = 15.37

$$\int \frac{\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)**2/(a+b*cosh(d*x+c)),x)
```

output
```
Piecewise((zoo*x*sinh(c)**2/cosh(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (-d*
x*tanh(c/2 + d*x/2)**2/(b*d*tanh(c/2 + d*x/2)**2 - b*d) + d*x/(b*d*tanh(c/
2 + d*x/2)**2 - b*d) - 2*tanh(c/2 + d*x/2)/(b*d*tanh(c/2 + d*x/2)**2 - b*d
), Eq(a, b)), (d*x*tanh(c/2 + d*x/2)**2/(b*d*tanh(c/2 + d*x/2)**2 - b*d) -
 d*x/(b*d*tanh(c/2 + d*x/2)**2 - b*d) - 2*tanh(c/2 + d*x/2)/(b*d*tanh(c/2
+ d*x/2)**2 - b*d), Eq(a, -b)), ((x*sinh(c + d*x)**2/2 - x*cosh(c + d*x)**
2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d))/a, Eq(b, 0)), (x*sinh(c)**2/(a +
b*cosh(c)), Eq(d, 0)), (-a*d*x*sqrt(a/(a - b) + b/(a - b))*tanh(c/2 + d*x/
2)**2/(b**2*d*sqrt(a/(a - b) + b/(a - b))*tanh(c/2 + d*x/2)**2 - b**2*d*sq
rt(a/(a - b) + b/(a - b))) + a*d*x*sqrt(a/(a - b) + b/(a - b))/(b**2*d*sqr
t(a/(a - b) + b/(a - b))*tanh(c/2 + d*x/2)**2 - b**2*d*sqrt(a/(a - b) + b/
(a - b))) - a*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(c/2 + d*x/2))*tanh(c
/2 + d*x/2)**2/(b**2*d*sqrt(a/(a - b) + b/(a - b))*tanh(c/2 + d*x/2)**2 -
b**2*d*sqrt(a/(a - b) + b/(a - b))) + a*log(-sqrt(a/(a - b) + b/(a - b)) +
 tanh(c/2 + d*x/2))/(b**2*d*sqrt(a/(a - b) + b/(a - b))*tanh(c/2 + d*x/2)*
*2 - b**2*d*sqrt(a/(a - b) + b/(a - b))) + a*log(sqrt(a/(a - b) + b/(a - b
)) + tanh(c/2 + d*x/2))*tanh(c/2 + d*x/2)**2/(b**2*d*sqrt(a/(a - b) + b/(a
 - b))*tanh(c/2 + d*x/2)**2 - b**2*d*sqrt(a/(a - b) + b/(a - b))) - a*log(
sqrt(a/(a - b) + b/(a - b)) + tanh(c/2 + d*x/2))/(b**2*d*sqrt(a/(a - b) +
b/(a - b))*tanh(c/2 + d*x/2)**2 - b**2*d*sqrt(a/(a - b) + b/(a - b))) -...
```

3.231.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.231.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.22

$$\int \frac{\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{\frac{2\,(dx+c)a}{b^2} - \frac{e^{(dx+c)}}{b} + \frac{e^{(-dx-c)}}{b} - \frac{4\,(a^2-b^2)\arctan\left(\frac{be^{(dx+c)}+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}b^2}}{2\,d}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(2*(d*x + c)*a/b^2 - e^(d*x + c)/b + e^(-d*x - c)/b - 4*(a^2 - b^2)*a
rctan((b*e^(d*x + c) + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b^2))/d
```

3.231.9 Mupad [B] (verification not implemented)

Time = 1.90 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.41

$$\int \frac{\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \frac{e^{c+d\,x}}{2\,b\,d} - \frac{e^{-c-d\,x}}{2\,b\,d} - \frac{a\,x}{b^2} + \frac{\ln\left(-\frac{2\,e^{c+d\,x}\,(a^2-b^2)}{b^3} - \frac{2\,\sqrt{a+b}\,\sqrt{a-b}\,(b+a\,e^{c+d\,x})}{b^3}\right)\sqrt{a+b}\,\sqrt{a-b}}{b^2\,d} - \frac{\ln\left(\frac{2\,\sqrt{a+b}\,\sqrt{a-b}\,(b+a\,e^{c+d\,x})}{b^3} - \frac{2\,e^{c+d\,x}\,(a^2-b^2)}{b^3}\right)\sqrt{a+b}\,\sqrt{a-b}}{b^2\,d}$$

input
```
int(sinh(c + d*x)^2/(a + b*cosh(c + d*x)),x)
```

output
```
exp(c + d*x)/(2*b*d) - exp(- c - d*x)/(2*b*d) - (a*x)/b^2 + (log(- (2*exp(
c + d*x)*(a^2 - b^2))/b^3 - (2*(a + b)^(1/2)*(a - b)^(1/2)*(b + a*exp(c +
d*x)))/b^3)*(a + b)^(1/2)*(a - b)^(1/2))/(b^2*d) - (log((2*(a + b)^(1/2)*(
a - b)^(1/2)*(b + a*exp(c + d*x)))/b^3 - (2*exp(c + d*x)*(a^2 - b^2))/b^3)
*(a + b)^(1/2)*(a - b)^(1/2))/(b^2*d)
```

3.232 $\int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))}\,dx$

3.232.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \text{Int}\left(\frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))},x\right)$$

output
```
Unintegrable(sinh(d*x+c)^2/x/(a+b*cosh(d*x+c)),x)
```

3.232.2 Mathematica [N/A]

Not integrable

Time = 19.47 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))}\,dx$$

input
```
Integrate[Sinh[c + d*x]^2/(x*(a + b*Cosh[c + d*x])),x]
```

output
```
Integrate[Sinh[c + d*x]^2/(x*(a + b*Cosh[c + d*x])), x]
```

3.232.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6112}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))} \, dx$$
$$\Big\downarrow \; 6112$$
$$\int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))}dx$$

input
```
Int[Sinh[c + d*x]^2/(x*(a + b*Cosh[c + d*x])),x]
```

output
```
$Aborted
```

3.232.3.1 Defintions of rubi rules used

rule 6112
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/(Cosh[(c_.)
 + (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Cosh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.232.4 Maple [N/A] (verified)

Not integrable

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sinh\left(dx+c\right)^{2}}{x\left(a+b\cosh\left(dx+c\right)\right)}dx$$

input
```
int(sinh(d*x+c)^2/x/(a+b*cosh(d*x+c)),x)
```

output
```
int(sinh(d*x+c)^2/x/(a+b*cosh(d*x+c)),x)
```

3.232.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh\left(dx+c\right)^2}{\left(b\cosh\left(dx+c\right)+a\right)x}\,dx$$

input
```
integrate(sinh(d*x+c)^2/x/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(sinh(d*x + c)^2/(b*x*cosh(d*x + c) + a*x), x)
```

3.232.6 Sympy [N/A]

Not integrable

Time = 43.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh^2\left(c+dx\right)}{x\left(a+b\cosh\left(c+dx\right)\right)}\,dx$$

input
```
integrate(sinh(d*x+c)**2/x/(a+b*cosh(d*x+c)),x)
```

output
```
Integral(sinh(c + d*x)**2/(x*(a + b*cosh(c + d*x))), x)
```

3.232.7 Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 92, normalized size of antiderivative = 3.83

$$\int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh\left(dx+c\right)^2}{\left(b\cosh\left(dx+c\right)+a\right)x}\,dx$$

input
```
integrate(sinh(d*x+c)^2/x/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
2*(a^2*e^c - b^2*e^c)*integrate(e^(d*x)/(b^3*x*e^(2*d*x + 2*c) + 2*a*b^2*x
*e^(d*x + c) + b^3*x), x) + 1/2*Ei(-d*x)*e^(-c)/b + 1/2*Ei(d*x)*e^c/b - a*
log(x)/b^2
```

3.232.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh(dx+c)^2}{(b\cosh(dx+c)+a)x}\,dx$$

input
```
integrate(sinh(d*x+c)^2/x/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sinh(d*x + c)^2/((b*cosh(d*x + c) + a)*x), x)
```

3.232.9 Mupad [N/A]

Not integrable

Time = 1.77 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh(c+d\,x)^2}{x\,\left(a+b\cosh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(sinh(c + d*x)^2/(x*(a + b*cosh(c + d*x))),x)
```

output
```
int(sinh(c + d*x)^2/(x*(a + b*cosh(c + d*x))), x)
```

3.233 $\int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

3.233.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \text{Int}\left(\frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)},x\right)$$

output
```
Unintegrable(x^m*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

3.233.2 Mathematica [N/A]

Not integrable

Time = 10.68 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$$

input
```
Integrate[(x^m*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]
```

output
```
Integrate[(x^m*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]), x]
```

3.233.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6112}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$$
$$\Big\downarrow \; 6112$$
$$\int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}dx$$

input
```
Int[(x^m*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]
```

output
```
$Aborted
```

3.233.3.1 Defintions of rubi rules used

rule 6112
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/(Cosh[(c_.)
 + (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Cosh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.233.4 Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \sinh\left(dx+c\right)^3}{a+b\cosh\left(dx+c\right)}dx$$

input
```
int(x^m*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

output
```
int(x^m*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

3.233.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh\left(dx+c\right)^3}{b\cosh\left(dx+c\right)+a}\,dx$$

input
```
integrate(x^m*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(x^m*sinh(d*x + c)^3/(b*cosh(d*x + c) + a), x)
```

3.233.6 Sympy [N/A]

Not integrable

Time = 2.51 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh^3\left(c+dx\right)}{a+b\cosh\left(c+dx\right)}\,dx$$

input
```
integrate(x**m*sinh(d*x+c)**3/(a+b*cosh(d*x+c)),x)
```

output
```
Integral(x**m*sinh(c + d*x)**3/(a + b*cosh(c + d*x)), x)
```

3.233.7 Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh\left(dx+c\right)^3}{b\cosh\left(dx+c\right)+a}\,dx$$

input
```
integrate(x^m*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(x^m*sinh(d*x + c)^3/(b*cosh(d*x + c) + a), x)
```

3.233.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh(dx+c)^3}{b\cosh(dx+c)+a}\,dx$$

input
```
integrate(x^m*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x^m*sinh(d*x + c)^3/(b*cosh(d*x + c) + a), x)
```

3.233.9 Mupad [N/A]

Not integrable

Time = 1.76 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh(c+d\,x)^3}{a+b\cosh(c+d\,x)}\,dx$$

input
```
int((x^m*sinh(c + d*x)^3)/(a + b*cosh(c + d*x)),x)
```

output
```
int((x^m*sinh(c + d*x)^3)/(a + b*cosh(c + d*x)), x)
```

3.234 $\int \frac{x^3 \sinh^3(c+dx)}{a+b\cosh(c+dx)} \, dx$

3.234.1 Optimal result

Integrand size = 24, antiderivative size = 586

$$\int \frac{x^3\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \frac{3x}{8bd^3}+\frac{x^3}{4bd}-\frac{(a^2-b^2)\,x^4}{4b^3}-\frac{6ax\cosh(c+dx)}{b^2d^3}-\frac{ax^3\cosh(c+dx)}{b^2d}+\frac{(a^2-b^2)\,x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d}+\frac{(a^2-b^2)\,x^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^3d}+\frac{3(a^2-b^2)\,x^2\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^2}+\frac{3(a^2-b^2)\,x^2\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^3d^2}-\frac{6(a^2-b^2)\,x\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^3}-\frac{6(a^2-b^2)\,x\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^3d^3}+\frac{6(a^2-b^2)\,\mathrm{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^4}+\frac{6(a^2-b^2)\,\mathrm{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^3d^4}+\frac{6a\sinh(c+dx)}{b^2d^4}+\frac{3ax^2\sinh(c+dx)}{b^2d^2}-\frac{3\cosh(c+dx)\sinh(c+dx)}{8bd^4}-\frac{3x^2\cosh(c+dx)\sinh(c+dx)}{4bd^2}+\frac{3x\sinh^2(c+dx)}{4bd^3}+\frac{x^3\sinh^2(c+dx)}{2bd}$$

output
```
3/8*x/b/d^3+1/4*x^3/b/d-1/4*(a^2-b^2)*x^4/b^3-6*a*x*cosh(d*x+c)/b^2/d^3-a*
x^3*cosh(d*x+c)/b^2/d+(a^2-b^2)*x^3*ln(1+b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))
/b^3/d+(a^2-b^2)*x^3*ln(1+b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))/b^3/d+3*(a^2-b
^2)*x^2*polylog(2,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b^3/d^2+3*(a^2-b^2)*x
^2*polylog(2,-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))/b^3/d^2-6*(a^2-b^2)*x*poly
log(3,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b^3/d^3-6*(a^2-b^2)*x*polylog(3,-
b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))/b^3/d^3+6*(a^2-b^2)*polylog(4,-b*exp(d*x
+c)/(a-(a^2-b^2)^(1/2)))/b^3/d^4+6*(a^2-b^2)*polylog(4,-b*exp(d*x+c)/(a+(a
^2-b^2)^(1/2)))/b^3/d^4+6*a*sinh(d*x+c)/b^2/d^4+3*a*x^2*sinh(d*x+c)/b^2/d^
2-3/8*cosh(d*x+c)*sinh(d*x+c)/b/d^4-3/4*x^2*cosh(d*x+c)*sinh(d*x+c)/b/d^2+
3/4*x*sinh(d*x+c)^2/b/d^3+1/2*x^3*sinh(d*x+c)^2/b/d
```

3.234.2 Mathematica [A] (verified)

Time = 4.73 (sec) , antiderivative size = 951, normalized size of antiderivative = 1.62

$$\int \frac{x^3 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$$

$$= \frac{8(-a^2+b^2)\left(-x^4+\frac{2b^2(1+e^{2c})\left(d^3x^3\log\left(1+\frac{(a-\sqrt{a^2-b^2})e^{-c-dx}}{b}\right)-3d^2x^2\,\text{PolyLog}\left(2,\frac{(-a+\sqrt{a^2-b^2})e^{-c-dx}}{b}\right)-6dx\,\text{PolyLog}\left(3,\frac{(-a+\sqrt{a^2-b^2})e^{-c-\cdots}}{b}\right.\cdots}{\sqrt{a^2-b^2}(-a+\sqrt{a^2-b^2})d^4}\cdots\right.}{\cdots}$$

input
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Integrate[(x^3*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]
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output
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((8*(-a^2 + b^2)*(-x^4 + (2*b^2*(1 + E^(2*c))*(d^3*x^3*Log[1 + ((a - Sqrt[
a^2 - b^2])*E^(-c - d*x))/b] - 3*d^2*x^2*PolyLog[2, ((-a + Sqrt[a^2 - b^2]
)*E^(-c - d*x))/b] - 6*d*x*PolyLog[3, ((-a + Sqrt[a^2 - b^2])*E^(-c - d*x)
)/b] - 6*PolyLog[4, ((-a + Sqrt[a^2 - b^2])*E^(-c - d*x))/b]))/(Sqrt[a^2 -
 b^2]*(-a + Sqrt[a^2 - b^2])*d^4) + (2*b^2*(1 + E^(2*c))*(d^3*x^3*Log[1 +
((a + Sqrt[a^2 - b^2])*E^(-c - d*x))/b] - 3*d^2*x^2*PolyLog[2, -(((a + Sqr
t[a^2 - b^2])*E^(-c - d*x))/b)] - 6*d*x*PolyLog[3, -(((a + Sqrt[a^2 - b^2]
)*E^(-c - d*x))/b)] - 6*PolyLog[4, -(((a + Sqrt[a^2 - b^2])*E^(-c - d*x))/
b)]))/(Sqrt[a^2 - b^2]*(a + Sqrt[a^2 - b^2])*d^4) + (2*a*(1 + E^(2*c))*(d^
3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])] + 3*d^2*x^2*PolyLog[2
, (b*E^(c + d*x))/(-a + Sqrt[a^2 - b^2])] - 6*d*x*PolyLog[3, (b*E^(c + d*x
))/(-a + Sqrt[a^2 - b^2])] + 6*PolyLog[4, (b*E^(c + d*x))/(-a + Sqrt[a^2 -
 b^2])]))/(Sqrt[a^2 - b^2]*d^4) - (2*a*(1 + E^(2*c))*(d^3*x^3*Log[1 + (b*E
^(c + d*x))/(a + Sqrt[a^2 - b^2])] + 3*d^2*x^2*PolyLog[2, -((b*E^(c + d*x)
)/(a + Sqrt[a^2 - b^2]))] - 6*d*x*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a
^2 - b^2]))] + 6*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))]))/(S
qrt[a^2 - b^2]*d^4)))/(1 + E^(2*c)) - (16*a*b*Cosh[d*x]*(d*x*(6 + d^2*x^2)
*Cosh[c] - 3*(2 + d^2*x^2)*Sinh[c]))/d^4 + (b^2*Cosh[2*d*x]*(2*d*x*(3 + 2*
d^2*x^2)*Cosh[2*c] - 3*(1 + 2*d^2*x^2)*Sinh[2*c]))/d^4 - (16*a*b*(-3*(2 +
d^2*x^2)*Cosh[c] + d*x*(6 + d^2*x^2)*Sinh[c])*Sinh[d*x])/d^4 + (b^2*(-3...
```

3.234.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 3.12 (sec) , antiderivative size = 540, normalized size of antiderivative = 0.92, number of steps used = 29, number of rules used = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.167, Rules used = {6100, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117, 5895, 3042, 25, 3792, 15, 25, 3042, 25, 3115, 24, 6096, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \sinh^3(c+dx)}{a+b\cosh(c+dx)}dx$$

$$\Big\downarrow \text{6100}$$

$$\frac{(a^2-b^2)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}-\frac{a\int x^3\sinh(c+dx)dx}{b^2}+\frac{\int x^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2-b^2)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}-\frac{a\int -ix^3\sin(ic+idx)dx}{b^2}+\frac{\int x^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow 26$$

$$\frac{(a^2-b^2)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\int x^3\sin(ic+idx)dx}{b^2}+\frac{\int x^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow 3777$$

$$\frac{(a^2-b^2)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\int x^2\cosh(c+dx)dx}{d}\right)}{b^2}+\frac{\int x^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow 3042$$

$$\frac{(a^2-b^2)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\int x^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{b^2}+\frac{\int x^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow 3777$$

$$\frac{(a^2-b^2)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}-\frac{2i\int -ix\sinh(c+dx)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int x^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow 26$$

$$\frac{(a^2-b^2)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}-\frac{2\int x\sinh(c+dx)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int x^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow 3042$$

$$\frac{(a^2-b^2)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}-\frac{2\int -ix\sin(ic+idx)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int x^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow 26$$

3.234. $\int \frac{x^3\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

$$\frac{(a^2-b^2)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\int x\sin(ic+idx)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int x^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(a^2-b^2)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\int\cosh(c+dx)dx}{d}\right)}{d}\right)}{d}\right)}{b^2}+\frac{\int x^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2-b^2)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)}{d}\right)}{b^2}+\frac{\int x^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{3117}$$

$$\frac{(a^2-b^2)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\int x^3\cosh(c+dx)\sinh(c+dx)dx}{b}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \text{5895}$$

3.234. $\int \frac{x^3\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

$$\frac{\left(a^2-b^2\right)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\frac{x^3\sinh^2(c+dx)}{2d}-\frac{3\int x^2\sinh^2(c+dx)dx}{2d}}{b}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2-b^2\right)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\frac{x^3\sinh^2(c+dx)}{2d}-\frac{3\int -x^2\sin(ic+idx)^2dx}{2d}}{b}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\left(a^2-b^2\right)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\frac{x^3\sinh^2(c+dx)}{2d}+\frac{3\int x^2\sin(ic+idx)^2dx}{2d}}{b}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \text{3792}$$

$$\frac{\left(a^2-b^2\right)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\frac{3\left(\frac{\int -\sinh^2(c+dx)dx}{2d^2}+\frac{\int x^2dx}{2}+\frac{x\sinh^2(c+dx)}{2d^2}-\frac{x^2\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}+\frac{x^3\sinh^2(c+dx)}{2d}}{b}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \text{15}$$

3.234. $\int \frac{x^3\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

$$\frac{\left(a^2-b^2\right)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\frac{3\left(\frac{\int -\sinh^2(c+dx)dx}{2d^2}+\frac{x\sinh^2(c+dx)}{2d^2}-\frac{x^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x^3}{6}\right)}{2d}+\frac{x^3\sinh^2(c+dx)}{2d}}{b}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}$$

$\downarrow$ 25

$$\frac{\left(a^2-b^2\right)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\frac{3\left(-\frac{\int \sinh^2(c+dx)dx}{2d^2}+\frac{x\sinh^2(c+dx)}{2d^2}-\frac{x^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x^3}{6}\right)}{2d}+\frac{x^3\sinh^2(c+dx)}{2d}}{b}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}$$

$\downarrow$ 3042

$$\frac{\left(a^2-b^2\right)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\frac{x^3\sinh^2(c+dx)}{2d}+\frac{3\left(-\frac{\int -\sin(ic+idx)^2dx}{2d^2}+\frac{x\sinh^2(c+dx)}{2d^2}-\frac{x^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x^3}{6}\right)}{2d}}{b}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}$$

$\downarrow$ 25

$$\frac{\left(a^2-b^2\right)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\frac{x^3\sinh^2(c+dx)}{2d}+\frac{3\left(\frac{\int \sin(ic+idx)^2dx}{2d^2}+\frac{x\sinh^2(c+dx)}{2d^2}-\frac{x^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x^3}{6}\right)}{2d}}{b}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}$$

3.234. $\int \frac{x^3\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

$$\Big\downarrow \ 3115$$

$$\frac{(a^2-b^2)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\frac{3\left(\frac{\frac{\int 1dx}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}}{2d^2}+\frac{x\sinh^2(c+dx)}{2d^2}-\frac{x^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x^3}{6}\right)}{2d}+\frac{x^3\sinh^2(c+dx)}{2d}}{b}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \ 24$$

$$\frac{(a^2-b^2)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}+\frac{\frac{3\left(\frac{x\sinh^2(c+dx)}{2d^2}+\frac{\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}}{2d^2}-\frac{x^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x^3}{6}\right)}{2d}+\frac{x^3\sinh^2(c+dx)}{2d}}{b}$$

$$\Big\downarrow \ 6096$$

$$\frac{(a^2-b^2)\left(\int \frac{e^{c+dx}x^3}{a+be^{c+dx}-\sqrt{a^2-b^2}}dx+\int \frac{e^{c+dx}x^3}{a+be^{c+dx}+\sqrt{a^2-b^2}}dx-\frac{x^4}{4b}\right)}{b^2}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}+\frac{\frac{3\left(\frac{x\sinh^2(c+dx)}{2d^2}+\frac{\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}}{2d^2}-\frac{x^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x^3}{6}\right)}{2d}+\frac{x^3\sinh^2(c+dx)}{2d}}{b}$$

$$\Big\downarrow \ 2620$$

3.234. $\displaystyle\int \frac{x^3\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

$$\frac{(a^2-b^2)\left(-\frac{3\int x^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2-b^2}}+1\right)dx}{bd}-\frac{3\int x^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2-b^2}}+1\right)dx}{bd}+\frac{x^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^4}{4b}\right)}{b^2}$$

$$\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}+$$

$$\frac{\frac{3\left(\frac{x\sinh^2(c+dx)}{2d^2}+\frac{\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}}{2d^2}-\frac{x^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x^3}{6}\right)}{2d}+\frac{x^3\sinh^2(c+dx)}{2d}}{b}$$

$\downarrow$ 3011

$$\frac{(a^2-b^2)\left(-\frac{3\left(\frac{2\int x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)dx}{d}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}-\frac{3\left(\frac{2\int x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)dx}{d}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{be^{c}}{a+\sqrt{}}\right)}{d}\right)}{bd}\right)}{b^2}$$

$$\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}+$$

$$\frac{\frac{3\left(\frac{x\sinh^2(c+dx)}{2d^2}+\frac{\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}}{2d^2}-\frac{x^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x^3}{6}\right)}{2d}+\frac{x^3\sinh^2(c+dx)}{2d}}{b}$$

$\downarrow$ 7163

3.234. $\int \frac{x^3\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

$$\frac{(a^2-b^2)\left(-\frac{3\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{\int \text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)dx}{d}\right)}{d}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}-\frac{3\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-\frac{be^{c+\cdots}}{a+\sqrt{a^{2}\cdots}}\right)}{d}\cdots\right.}{\cdots}\right.}{\cdots}\cdots\right)}{b^2}$$

$$\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}+\frac{\frac{3\left(\frac{x\sinh^2(c+dx)}{2d^2}+\frac{\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}}{2d^2}-\frac{x^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x^3}{6}\right)}{2d}+\frac{x^3\sinh^2(c+dx)}{2d}}{b}$$

$\downarrow$ 2720

$$\frac{(a^2-b^2)\left(-\frac{3\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{\int e^{-c-dx}\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}-\frac{3\left(\frac{2\left(\frac{x\,\text{PolyLo}\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\cdots\right)}{\cdots}$$

$$\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}+\frac{\frac{3\left(\frac{x\sinh^2(c+dx)}{2d^2}+\frac{\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}}{2d^2}-\frac{x^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x^3}{6}\right)}{2d}+\frac{x^3\sinh^2(c+dx)}{2d}}{b}$$

$\downarrow$ 7143

3.234. $\int \frac{x^3\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

$$\frac{(a^2-b^2)\left(-\frac{3\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d^2}\right)}{d}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}-\frac{3\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{d}\right.}{}\right.}{}\right.}{b^2}$$

$$\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}+$$

$$\frac{\frac{3\left(\frac{x\sinh^2(c+dx)}{2d^2}+\frac{\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}}{2d^2}-\frac{x^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x^3}{6}\right)}{2d}+\frac{x^3\sinh^2(c+dx)}{2d}}{b}$$

input
```
Int[(x^3*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]
```

output
```
((a^2 - b^2)*(-1/4*x^4/b + (x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^
2])])/(b*d) + (x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) -
 (3*(-((x^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/d) + (2*
((x*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/d - PolyLog[4, -
((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))]/d^2))/d))/(b*d) - (3*(-((x^2*Poly
Log[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/d) + (2*((x*PolyLog[3, -
((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/d - PolyLog[4, -((b*E^(c + d*x))
/(a + Sqrt[a^2 - b^2]))]/d^2))/d))/(b*d)))/b^2 + (I*a*((I*x^3*Cosh[c + d*x
])/d - ((3*I)*((x^2*Sinh[c + d*x])/d + ((2*I)*((I*x*Cosh[c + d*x])/d - (I*
Sinh[c + d*x])/d^2))/d))/d))/b^2 + ((x^3*Sinh[c + d*x]^2)/(2*d) + (3*(x^3/
6 - (x^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + (x*Sinh[c + d*x]^2)/(2*d^2)
+ (x/2 - (Cosh[c + d*x]*Sinh[c + d*x])/(2*d))/(2*d^2)))/(2*d))/b
```

3.234. $\int \frac{x^3\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

3.234.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.234. $\int \frac{x^3 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

rule 6096
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

rule 6100
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_))/(Cosh[(c_.)
+ (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Sin
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Sinh[c + d*x]^(n -
 2)*Cosh[c + d*x], x], x] + Simp[(a^2 - b^2)/b^2   Int[(e + f*x)^m*(Sinh[c
+ d*x]^(n - 2)/(a + b*Cosh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

3.234. $\int \frac{x^3 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.234.4 Maple [F]

$$\int \frac{x^3 \sinh\left(dx+c\right)^3}{a+b\cosh\left(dx+c\right)}dx$$

input
```
int(x^3*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

output
```
int(x^3*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

3.234.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2025 vs. 2(546) = 1092.

Time = 0.30 (sec) , antiderivative size = 2025, normalized size of antiderivative = 3.46

$$\int \frac{x^3\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
1/32*(4*b^2*d^3*x^3 + 6*b^2*d^2*x^2 + (4*b^2*d^3*x^3 - 6*b^2*d^2*x^2 + 6*b
^2*d*x - 3*b^2)*cosh(d*x + c)^4 + (4*b^2*d^3*x^3 - 6*b^2*d^2*x^2 + 6*b^2*d
*x - 3*b^2)*sinh(d*x + c)^4 + 6*b^2*d*x - 16*(a*b*d^3*x^3 - 3*a*b*d^2*x^2
+ 6*a*b*d*x - 6*a*b)*cosh(d*x + c)^3 - 4*(4*a*b*d^3*x^3 - 12*a*b*d^2*x^2 +
 24*a*b*d*x - 24*a*b - (4*b^2*d^3*x^3 - 6*b^2*d^2*x^2 + 6*b^2*d*x - 3*b^2)
*cosh(d*x + c))*sinh(d*x + c)^3 - 8*((a^2 - b^2)*d^4*x^4 - 2*(a^2 - b^2)*c
^4)*cosh(d*x + c)^2 - 2*(4*(a^2 - b^2)*d^4*x^4 - 8*(a^2 - b^2)*c^4 - 3*(4*
b^2*d^3*x^3 - 6*b^2*d^2*x^2 + 6*b^2*d*x - 3*b^2)*cosh(d*x + c)^2 + 24*(a*b
*d^3*x^3 - 3*a*b*d^2*x^2 + 6*a*b*d*x - 6*a*b)*cosh(d*x + c))*sinh(d*x + c)
^2 + 3*b^2 - 16*(a*b*d^3*x^3 + 3*a*b*d^2*x^2 + 6*a*b*d*x + 6*a*b)*cosh(d*x
 + c) + 96*((a^2 - b^2)*d^2*x^2*cosh(d*x + c)^2 + 2*(a^2 - b^2)*d^2*x^2*co
sh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*d^2*x^2*sinh(d*x + c)^2)*dilog(-(a
*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqr
t((a^2 - b^2)/b^2) + b)/b + 1) + 96*((a^2 - b^2)*d^2*x^2*cosh(d*x + c)^2 +
 2*(a^2 - b^2)*d^2*x^2*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*d^2*x^2*s
inh(d*x + c)^2)*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x +
c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - 32*((a^2 - b^2)*
c^3*cosh(d*x + c)^2 + 2*(a^2 - b^2)*c^3*cosh(d*x + c)*sinh(d*x + c) + (a^2
 - b^2)*c^3*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2
*b*sqrt((a^2 - b^2)/b^2) + 2*a) - 32*((a^2 - b^2)*c^3*cosh(d*x + c)^2 +...
```

3.234.6 Sympy [F]

$$\int \frac{x^3 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^3 \sinh^3{(c+dx)}}{a+b\cosh{(c+dx)}}\,dx$$

input
```
integrate(x**3*sinh(d*x+c)**3/(a+b*cosh(d*x+c)),x)
```

output
```
Integral(x**3*sinh(c + d*x)**3/(a + b*cosh(c + d*x)), x)
```

3.234.7 Maxima [F]

$$\int \frac{x^3 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^3 \sinh(dx+c)^3}{b\cosh(dx+c)+a}\,dx$$

input
```
integrate(x^3*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
1/32*(8*(a^2*d^4*e^(2*c) - b^2*d^4*e^(2*c))*x^4 + (4*b^2*d^3*x^3*e^(4*c) -
 6*b^2*d^2*x^2*e^(4*c) + 6*b^2*d*x*e^(4*c) - 3*b^2*e^(4*c))*e^(2*d*x) - 16
*(a*b*d^3*x^3*e^(3*c) - 3*a*b*d^2*x^2*e^(3*c) + 6*a*b*d*x*e^(3*c) - 6*a*b*
e^(3*c))*e^(d*x) - 16*(a*b*d^3*x^3*e^c + 3*a*b*d^2*x^2*e^c + 6*a*b*d*x*e^c
 + 6*a*b*e^c)*e^(-d*x) + (4*b^2*d^3*x^3 + 6*b^2*d^2*x^2 + 6*b^2*d*x + 3*b^
2)*e^(-2*d*x))*e^(-2*c)/(b^3*d^4) - 1/8*integrate(16*((a^3*e^c - a*b^2*e^c
)*x^3*e^(d*x) + (a^2*b - b^3)*x^3)/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x +
 c) + b^4), x)
```

3.234.8 Giac [F]

$$\int \frac{x^3 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^3 \sinh(dx+c)^3}{b\cosh(dx+c)+a}\,dx$$

input
```
integrate(x^3*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x^3*sinh(d*x + c)^3/(b*cosh(d*x + c) + a), x)
```

3.234.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^3 \sinh(c+d\,x)^3}{a+b\cosh(c+d\,x)}\,dx$$

input
```
int((x^3*sinh(c + d*x)^3)/(a + b*cosh(c + d*x)),x)
```

output
```
int((x^3*sinh(c + d*x)^3)/(a + b*cosh(c + d*x)), x)
```

3.235 $\int \frac{x^2 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

3.235.1 Optimal result

Integrand size = 24, antiderivative size = 432

$$\begin{aligned}\int \frac{x^2\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx &= \frac{x^2}{4bd}-\frac{(a^2-b^2)\,x^3}{3b^3}-\frac{2a\cosh(c+dx)}{b^2d^3} \\ &-\frac{ax^2\cosh(c+dx)}{b^2d}+\frac{(a^2-b^2)\,x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d} \\ &+\frac{(a^2-b^2)\,x^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^3d} \\ &+\frac{2(a^2-b^2)\,x\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^2} \\ &+\frac{2(a^2-b^2)\,x\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^3d^2} \\ &-\frac{2(a^2-b^2)\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^3} \\ &-\frac{2(a^2-b^2)\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^3d^3}+\frac{2ax\sinh(c+dx)}{b^2d^2} \\ &-\frac{x\cosh(c+dx)\sinh(c+dx)}{2bd^2}+\frac{\sinh^2(c+dx)}{4bd^3}+\frac{x^2\sinh^2(c+dx)}{2bd}\end{aligned}$$

output
```
1/4*x^2/b/d-1/3*(a^2-b^2)*x^3/b^3-2*a*cosh(d*x+c)/b^2/d^3-a*x^2*cosh(d*x+c
)/b^2/d+(a^2-b^2)*x^2*ln(1+b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b^3/d+(a^2-b^
2)*x^2*ln(1+b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))/b^3/d+2*(a^2-b^2)*x*polylog(
2,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b^3/d^2+2*(a^2-b^2)*x*polylog(2,-b*ex
p(d*x+c)/(a+(a^2-b^2)^(1/2)))/b^3/d^2-2*(a^2-b^2)*polylog(3,-b*exp(d*x+c)/
(a-(a^2-b^2)^(1/2)))/b^3/d^3-2*(a^2-b^2)*polylog(3,-b*exp(d*x+c)/(a+(a^2-b
^2)^(1/2)))/b^3/d^3+2*a*x*sinh(d*x+c)/b^2/d^2-1/2*x*cosh(d*x+c)*sinh(d*x+c
)/b/d^2+1/4*sinh(d*x+c)^2/b/d^3+1/2*x^2*sinh(d*x+c)^2/b/d
```

3.235.2 Mathematica [A] (verified)

Time = 3.34 (sec) , antiderivative size = 758, normalized size of antiderivative = 1.75

$$\int \frac{x^2\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \frac{8(-a^2+b^2)\left(-2x^3+\frac{3b^2(1+e^{2c})\left(d^2x^2\log\left(1+\frac{(a-\sqrt{a^2-b^2})e^{-c-dx}}{b}\right)-2dx\,\text{PolyLog}\left(2,\frac{(-a+\sqrt{a^2-b^2})e^{-c-dx}}{b}\right)-2\,\text{PolyLog}\left(3,\frac{(-a+\sqrt{a^2-b^2})e^{-c-dx}}{b}\right.\right.}{\sqrt{a^2-b^2}\left(-a+\sqrt{a^2-b^2}\right)d^3}\right.}{}$$

input
```
Integrate[(x^2*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]
```

output
```
((8*(-a^2 + b^2)*(-2*x^3 + (3*b^2*(1 + E^(2*c))*(d^2*x^2*Log[1 + ((a - Sqr
t[a^2 - b^2])*E^(-c - d*x))/b] - 2*d*x*PolyLog[2, ((-a + Sqrt[a^2 - b^2])*
E^(-c - d*x))/b] - 2*PolyLog[3, ((-a + Sqrt[a^2 - b^2])*E^(-c - d*x))/b]))
/(Sqrt[a^2 - b^2]*(-a + Sqrt[a^2 - b^2])*d^3) + (3*b^2*(1 + E^(2*c))*(d^2*
x^2*Log[1 + ((a + Sqrt[a^2 - b^2])*E^(-c - d*x))/b] - 2*d*x*PolyLog[2, -((
(a + Sqrt[a^2 - b^2])*E^(-c - d*x))/b)] - 2*PolyLog[3, -(((a + Sqrt[a^2 -
b^2])*E^(-c - d*x))/b)]))/(Sqrt[a^2 - b^2]*(a + Sqrt[a^2 - b^2])*d^3) + (3
*a*(1 + E^(2*c))*(d^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])] +
 2*d*x*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 - b^2])] - 2*PolyLog[3, (
b*E^(c + d*x))/(-a + Sqrt[a^2 - b^2])]))/(Sqrt[a^2 - b^2]*d^3) - (3*a*(1 +
 E^(2*c))*(d^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])] + 2*d*x*
PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))] - 2*PolyLog[3, -((b*E
^(c + d*x))/(a + Sqrt[a^2 - b^2]))]))/(Sqrt[a^2 - b^2]*d^3)))/(1 + E^(2*c)
) - (24*a*b*Cosh[d*x]*((2 + d^2*x^2)*Cosh[c] - 2*d*x*Sinh[c]))/d^3 + (3*b^
2*Cosh[2*d*x]*((1 + 2*d^2*x^2)*Cosh[2*c] - 2*d*x*Sinh[2*c]))/d^3 - (24*a*b
*(-2*d*x*Cosh[c] + (2 + d^2*x^2)*Sinh[c])*Sinh[d*x])/d^3 + (3*b^2*(-2*d*x*
Cosh[2*c] + (1 + 2*d^2*x^2)*Sinh[2*c])*Sinh[2*d*x])/d^3 + 8*(a^2 - b^2)*x^
3*Tanh[c])/(24*b^3)
```

3.235.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.13 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.90, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {6100, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118, 5895, 3042, 25, 3791, 15, 6096, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \sinh^3(c+dx)}{a+b\cosh(c+dx)}dx$$

$$\Big\downarrow \text{ 6100}$$

$$\frac{(a^2-b^2)\int \frac{x^2\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}-\frac{a\int x^2\sinh(c+dx)dx}{b^2}+\frac{\int x^2\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(a^2-b^2)\int \frac{x^2\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}-\frac{a\int -ix^2\sin(ic+idx)dx}{b^2}+\frac{\int x^2\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{(a^2-b^2)\int \frac{x^2\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\int x^2\sin(ic+idx)dx}{b^2}+\frac{\int x^2\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{(a^2-b^2)\int \frac{x^2\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\int x\cosh(c+dx)dx}{d}\right)}{b^2}+\frac{\int x^2\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(a^2-b^2)\int \frac{x^2\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\int x\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{b^2}+\frac{\int x^2\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{(a^2-b^2)\int \frac{x^2\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{i\int -i\sinh(c+dx)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int x^2\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \ 26$$

$$\frac{(a^2-b^2)\int \frac{x^2\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\int \sinh(c+dx)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int x^2\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \ 3042$$

$$\frac{(a^2-b^2)\int \frac{x^2\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\int -i\sin(ic+idx)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int x^2\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \ 26$$

$$\frac{(a^2-b^2)\int \frac{x^2\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}+\frac{i\int \sin(ic+idx)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int x^2\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \ 3118$$

$$\frac{(a^2-b^2)\int \frac{x^2\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\int x^2\cosh(c+dx)\sinh(c+dx)dx}{b}+\frac{ia\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \ 5895$$

$$\frac{(a^2-b^2)\int \frac{x^2\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\frac{x^2\sinh^2(c+dx)}{2d}-\frac{\int x\sinh^2(c+dx)dx}{d}}{b}+\frac{ia\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \ 3042$$

3.235. $\int \frac{x^2\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

$$\frac{\left(a^2-b^2\right)\int \frac{x^2\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\frac{x^2\sinh^2(c+dx)}{2d}-\frac{\int -x\sin(ic+idx)^2dx}{d}}{b}+\frac{ia\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\left(a^2-b^2\right)\int \frac{x^2\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\frac{x^2\sinh^2(c+dx)}{2d}+\frac{\int x\sin(ic+idx)^2dx}{d}}{b}+\frac{ia\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \ 3791$$

$$\frac{\left(a^2-b^2\right)\int \frac{x^2\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\frac{\frac{\int xdx}{2}+\frac{\sinh^2(c+dx)}{4d^2}-\frac{x\sinh(c+dx)\cosh(c+dx)}{2d}}{d}+\frac{x^2\sinh^2(c+dx)}{2d}}{b}+\frac{ia\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \ 15$$

$$\frac{\left(a^2-b^2\right)\int \frac{x^2\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{b^2}+\frac{\frac{\frac{\sinh^2(c+dx)}{4d^2}-\frac{x\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x^2}{4}}{d}+\frac{x^2\sinh^2(c+dx)}{2d}}{b}$$

$$\Big\downarrow \ 6096$$

$$\frac{\left(a^2-b^2\right)\left(\int \frac{e^{c+dx}x^2}{a+be^{c+dx}-\sqrt{a^2-b^2}}dx+\int \frac{e^{c+dx}x^2}{a+be^{c+dx}+\sqrt{a^2-b^2}}dx-\frac{x^3}{3b}\right)}{b^2}+\frac{ia\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{b^2}+\frac{\frac{\frac{\sinh^2(c+dx)}{4d^2}-\frac{x\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x^2}{4}}{d}+\frac{x^2\sinh^2(c+dx)}{2d}}{b}$$

$$\Big\downarrow \ 2620$$

3.235. $\int \frac{x^2\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

$$\frac{(a^2-b^2)\left(-\frac{2\int x\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2-b^2}}+1\right)dx}{bd}-\frac{2\int x\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2-b^2}}+1\right)dx}{bd}+\frac{x^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^3}{3b}\right)}{b^2}-$$

$$\frac{ia\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{b^2}+\frac{\frac{\frac{\sinh^2(c+dx)}{4d^2}-\frac{x\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x^2}{4}}{d}+\frac{x^2\sinh^2(c+dx)}{2d}}{b}$$

$\downarrow$ 3011

$$\frac{(a^2-b^2)\left(-\frac{2\left(\frac{\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)dx}{d}-\frac{x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}-\frac{2\left(\frac{\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)dx}{d}-\frac{x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}\right.}{b^2}$$

$$\frac{ia\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{b^2}+\frac{\frac{\frac{\sinh^2(c+dx)}{4d^2}-\frac{x\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x^2}{4}}{d}+\frac{x^2\sinh^2(c+dx)}{2d}}{b}$$

$\downarrow$ 2720

$$\frac{(a^2-b^2)\left(-\frac{2\left(\frac{\int e^{-c-dx}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)de^{c+dx}}{d^2}-\frac{x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}-\frac{2\left(\frac{\int e^{-c-dx}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)de^{c+dx}}{d^2}-\right.}{bd}\right.}{b^2}$$

$$\frac{ia\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{b^2}+\frac{\frac{\frac{\sinh^2(c+dx)}{4d^2}-\frac{x\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x^2}{4}}{d}+\frac{x^2\sinh^2(c+dx)}{2d}}{b}$$

$\downarrow$ 7143

$$\frac{(a^2-b^2)\left(-\frac{2\left(\frac{\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d^2}-\frac{x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}-\frac{2\left(\frac{\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{d^2}-\frac{x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}+\right.}{b^2}$$

$$\frac{ia\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{b^2}+\frac{\frac{\frac{\sinh^2(c+dx)}{4d^2}-\frac{x\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x^2}{4}}{d}+\frac{x^2\sinh^2(c+dx)}{2d}}{b}$$

3.235. $\int \frac{x^2\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

input
```
Int[(x^2*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]
```

output
```
((a^2 - b^2)*(-1/3*x^3/b + (x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^
2])])/(b*d) + (x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) -
 (2*(-((x*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/d) + PolyL
og[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))]/d^2))/(b*d) - (2*(-((x*Pol
yLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/d) + PolyLog[3, -((b*E^
(c + d*x))/(a + Sqrt[a^2 - b^2]))]/d^2))/(b*d)))/b^2 + (I*a*((I*x^2*Cosh[c
 + d*x])/d - ((2*I)*(-(Cosh[c + d*x]/d^2) + (x*Sinh[c + d*x])/d))/d))/b^2
+ ((x^2*Sinh[c + d*x]^2)/(2*d) + (x^2/4 - (x*Cosh[c + d*x]*Sinh[c + d*x])/
(2*d) + Sinh[c + d*x]^2/(4*d^2))/d)/b
```

3.235.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

rule 6096
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

3.235. $\int \frac{x^2 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

rule 6100
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_))/(Cosh[(c_.)
+ (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Sin
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Sinh[c + d*x]^(n -
 2)*Cosh[c + d*x], x], x] + Simp[(a^2 - b^2)/b^2   Int[(e + f*x)^m*(Sinh[c
+ d*x]^(n - 2)/(a + b*Cosh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.235.4 Maple [F]

$$\int \frac{x^2 \sinh\left(dx+c\right)^3}{a+b\cosh\left(dx+c\right)}dx$$

input
```
int(x^2*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

output
```
int(x^2*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

3.235.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1622 vs. $2(402) = 804$.

Time = 0.28 (sec) , antiderivative size = 1622, normalized size of antiderivative = 3.75

$$\int \frac{x^2 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
1/48*(6*b^2*d^2*x^2 + 3*(2*b^2*d^2*x^2 - 2*b^2*d*x + b^2)*cosh(d*x + c)^4
+ 3*(2*b^2*d^2*x^2 - 2*b^2*d*x + b^2)*sinh(d*x + c)^4 + 6*b^2*d*x - 24*(a*
b*d^2*x^2 - 2*a*b*d*x + 2*a*b)*cosh(d*x + c)^3 - 12*(2*a*b*d^2*x^2 - 4*a*b
*d*x + 4*a*b - (2*b^2*d^2*x^2 - 2*b^2*d*x + b^2)*cosh(d*x + c))*sinh(d*x +
 c)^3 - 16*((a^2 - b^2)*d^3*x^3 + 2*(a^2 - b^2)*c^3)*cosh(d*x + c)^2 - 2*(
8*(a^2 - b^2)*d^3*x^3 + 16*(a^2 - b^2)*c^3 - 9*(2*b^2*d^2*x^2 - 2*b^2*d*x
+ b^2)*cosh(d*x + c)^2 + 36*(a*b*d^2*x^2 - 2*a*b*d*x + 2*a*b)*cosh(d*x + c
))*sinh(d*x + c)^2 + 3*b^2 - 24*(a*b*d^2*x^2 + 2*a*b*d*x + 2*a*b)*cosh(d*x
 + c) + 96*((a^2 - b^2)*d*x*cosh(d*x + c)^2 + 2*(a^2 - b^2)*d*x*cosh(d*x +
 c)*sinh(d*x + c) + (a^2 - b^2)*d*x*sinh(d*x + c)^2)*dilog(-(a*cosh(d*x +
c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2
)/b^2) + b)/b + 1) + 96*((a^2 - b^2)*d*x*cosh(d*x + c)^2 + 2*(a^2 - b^2)*d
*x*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*d*x*sinh(d*x + c)^2)*dilog(-(
a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sq
rt((a^2 - b^2)/b^2) + b)/b + 1) + 48*((a^2 - b^2)*c^2*cosh(d*x + c)^2 + 2*
(a^2 - b^2)*c^2*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*c^2*sinh(d*x + c
)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2)
 + 2*a) + 48*((a^2 - b^2)*c^2*cosh(d*x + c)^2 + 2*(a^2 - b^2)*c^2*cosh(d*x
 + c)*sinh(d*x + c) + (a^2 - b^2)*c^2*sinh(d*x + c)^2)*log(2*b*cosh(d*x +
c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + 48*(((a^2 -...
```

3.235.6 Sympy [F]

$$\int \frac{x^2 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^2 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$$

input
```
integrate(x**2*sinh(d*x+c)**3/(a+b*cosh(d*x+c)),x)
```

output
```
Integral(x**2*sinh(c + d*x)**3/(a + b*cosh(c + d*x)), x)
```

3.235.7 Maxima [F]

$$\int \frac{x^2 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^2 \sinh(dx+c)^3}{b\cosh(dx+c)+a}\,dx$$

input
```
integrate(x^2*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
1/48*(16*(a^2*d^3*e^(2*c) - b^2*d^3*e^(2*c))*x^3 + 3*(2*b^2*d^2*x^2*e^(4*c
) - 2*b^2*d*x*e^(4*c) + b^2*e^(4*c))*e^(2*d*x) - 24*(a*b*d^2*x^2*e^(3*c) -
 2*a*b*d*x*e^(3*c) + 2*a*b*e^(3*c))*e^(d*x) - 24*(a*b*d^2*x^2*e^c + 2*a*b*
d*x*e^c + 2*a*b*e^c)*e^(-d*x) + 3*(2*b^2*d^2*x^2 + 2*b^2*d*x + b^2)*e^(-2*
d*x))*e^(-2*c)/(b^3*d^3) - 1/8*integrate(16*((a^3*e^c - a*b^2*e^c)*x^2*e^(
d*x) + (a^2*b - b^3)*x^2)/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) + b^4
), x)
```

3.235.8 Giac [F]

$$\int \frac{x^2 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^2 \sinh(dx+c)^3}{b\cosh(dx+c)+a}\,dx$$

input
```
integrate(x^2*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x^2*sinh(d*x + c)^3/(b*cosh(d*x + c) + a), x)
```

3.235.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^2 \sinh(c+d\,x)^3}{a+b\cosh(c+d\,x)}\,dx$$

input
```
int((x^2*sinh(c + d*x)^3)/(a + b*cosh(c + d*x)),x)
```

output
```
int((x^2*sinh(c + d*x)^3)/(a + b*cosh(c + d*x)), x)
```

3.236 $\int \frac{x\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

3.236.1 Optimal result

Integrand size = 22, antiderivative size = 288

$$\begin{aligned}\int \frac{x\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx &= \frac{x}{4bd}-\frac{(a^2-b^2)\,x^2}{2b^3}-\frac{ax\cosh(c+dx)}{b^2d} \\ &+\frac{(a^2-b^2)\,x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d} \\ &+\frac{(a^2-b^2)\,x\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^3d} \\ &+\frac{(a^2-b^2)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^2} \\ &+\frac{(a^2-b^2)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^3d^2}+\frac{a\sinh(c+dx)}{b^2d^2} \\ &-\frac{\cosh(c+dx)\sinh(c+dx)}{4bd^2}+\frac{x\sinh^2(c+dx)}{2bd}\end{aligned}$$

output
```
1/4*x/b/d-1/2*(a^2-b^2)*x^2/b^3-a*x*cosh(d*x+c)/b^2/d+(a^2-b^2)*x*ln(1+b*e
xp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b^3/d+(a^2-b^2)*x*ln(1+b*exp(d*x+c)/(a+(a^2
-b^2)^(1/2)))/b^3/d+(a^2-b^2)*polylog(2,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))
/b^3/d^2+(a^2-b^2)*polylog(2,-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))/b^3/d^2+a*
sinh(d*x+c)/b^2/d^2-1/4*cosh(d*x+c)*sinh(d*x+c)/b/d^2+1/2*x*sinh(d*x+c)^2/
b/d
```

3.236.2 Mathematica [A] (verified)

Time = 1.70 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.30

$$\int \frac{x\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$$
$$= \frac{-8abdx\cosh(c+dx)+2b^2dx\cosh(2(c+dx))+4(a^2-b^2)\left(2c(c+dx)-(c+dx)^2+\frac{4a\sqrt{-(a^2-b^2)^2}c\arctan\left(\right.}{(a^2-b^2)^{3/2}}\right.}{}$$

input
```
Integrate[(x*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]
```

output
```
(-8*a*b*d*x*Cosh[c + d*x] + 2*b^2*d*x*Cosh[2*(c + d*x)] + 4*(a^2 - b^2)*(2
*c*(c + d*x) - (c + d*x)^2 + (4*a*Sqrt[-(a^2 - b^2)^2]*c*ArcTan[(a + b*E^(
c + d*x))/Sqrt[-a^2 + b^2]])/(a^2 - b^2)^(3/2) + (4*a*Sqrt[-(a^2 - b^2)^2]
*c*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 - b^2]])/(-a^2 + b^2)^(3/2) + 2*(c
 + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])] + 2*(c + d*x)*Log[1
 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])] - 2*c*Log[b + 2*a*E^(c + d*x) +
b*E^(2*(c + d*x))] + 2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 - b^2])]
+ 2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))]) + 8*a*b*Sinh[c +
 d*x] - b^2*Sinh[2*(c + d*x)])/(8*b^3*d^2)
```

3.236.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.31 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.92, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {6100, 3042, 26, 3777, 3042, 3117, 5895, 3042, 25, 3115, 24, 6096, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$$
$$\Big\downarrow \text{ 6100}$$
$$\frac{(a^2-b^2)\int \frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}-\frac{a\int x\sinh(c+dx)dx}{b^2}+\frac{\int x\cosh(c+dx)\sinh(c+dx)dx}{b}$$
$$\Big\downarrow \text{ 3042}$$

$$\frac{(a^2-b^2)\int \frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}-\frac{a\int -ix\sin(ic+idx)dx}{b^2}+\frac{\int x\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$\Big\downarrow$ 26

$$\frac{(a^2-b^2)\int \frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\int x\sin(ic+idx)dx}{b^2}+\frac{\int x\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$\Big\downarrow$ 3777

$$\frac{(a^2-b^2)\int \frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\int\cosh(c+dx)dx}{d}\right)}{b^2}+\frac{\int x\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$\Big\downarrow$ 3042

$$\frac{(a^2-b^2)\int \frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{b^2}+\frac{\int x\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$\Big\downarrow$ 3117

$$\frac{(a^2-b^2)\int \frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\int x\cosh(c+dx)\sinh(c+dx)dx}{b}+\frac{ia\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{b^2}$$

$\Big\downarrow$ 5895

$$\frac{(a^2-b^2)\int \frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\frac{x\sinh^2(c+dx)}{2d}-\frac{\int\sinh^2(c+dx)dx}{2d}}{b}+\frac{ia\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{b^2}$$

$\Big\downarrow$ 3042

$$\frac{(a^2-b^2)\int \frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\frac{x\sinh^2(c+dx)}{2d}-\frac{\int-\sin(ic+idx)^2dx}{2d}}{b}+\frac{ia\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{b^2}$$

$\Big\downarrow$ 25

$$\frac{(a^2-b^2)\int \frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\frac{x\sinh^2(c+dx)}{2d}+\frac{\int\sin(ic+idx)^2dx}{2d}}{b}+\frac{ia\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{b^2}$$

$\Big\downarrow$ 3115

$$\frac{(a^2-b^2)\int \frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\frac{\frac{\int 1dx}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}}{2d}+\frac{x\sinh^2(c+dx)}{2d}}{b}+\frac{ia\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{b^2}$$

$\Big\downarrow$ 24

$$\frac{(a^2-b^2)\int \frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{b^2}+\frac{\frac{x\sinh^2(c+dx)}{2d}+\frac{\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}}{2d}}{b}$$

3.236. $\int \frac{x\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

$$\Big\downarrow \text{6096}$$

$$\frac{\left(a^2-b^2\right)\left(\int \frac{e^{c+dx}x}{a+be^{c+dx}-\sqrt{a^2-b^2}}dx+\int \frac{e^{c+dx}x}{a+be^{c+dx}+\sqrt{a^2-b^2}}dx-\frac{x^2}{2b}\right)}{b^2}+\frac{ia\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{b^2}+\frac{\frac{x\sinh^2(c+dx)}{2d}+\frac{\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}}{2d}}{b}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\left(a^2-b^2\right)\left(-\frac{\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2-b^2}}+1\right)dx}{bd}-\frac{\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2-b^2}}+1\right)dx}{bd}+\frac{x\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^2}{2b}\right)}{b^2}+\frac{ia\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{b^2}+\frac{\frac{x\sinh^2(c+dx)}{2d}+\frac{\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}}{2d}}{b}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\left(a^2-b^2\right)\left(-\frac{\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2-b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2-b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{x\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right.}{bd}\right.}{b^2}$$

$$\frac{ia\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{b^2}+\frac{\frac{x\sinh^2(c+dx)}{2d}+\frac{\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}}{2d}}{b}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\left(a^2-b^2\right)\left(\frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{x\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^2}{2b}\right)}{b^2}+\frac{ia\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{b^2}+\frac{\frac{x\sinh^2(c+dx)}{2d}+\frac{\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}}{2d}}{b}$$

input
```
Int[(x*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]
```

output
```
((a^2 - b^2)*(-1/2*x^2/b + (x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]
)])/(b*d) + (x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) + Pol
yLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))]/(b*d^2) + PolyLog[2, -((
b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))]/(b*d^2)))/b^2 + (I*a*((I*x*Cosh[c +
 d*x])/d - (I*Sinh[c + d*x])/d^2))/b^2 + ((x*Sinh[c + d*x]^2)/(2*d) + (x/2
 - (Cosh[c + d*x]*Sinh[c + d*x])/(2*d))/(2*d))/b
```

3.236. $\int \frac{x\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

3.236.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

3.236. $\int \frac{x \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

rule 6096
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

rule 6100
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_))/(Cosh[(c_.)
+ (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Sin
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Sinh[c + d*x]^(n -
 2)*Cosh[c + d*x], x], x] + Simp[(a^2 - b^2)/b^2   Int[(e + f*x)^m*(Sinh[c
+ d*x]^(n - 2)/(a + b*Cosh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

3.236.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 859 vs. $2(268) = 536$.

Time = 6.63 (sec) , antiderivative size = 860, normalized size of antiderivative = 2.99

method	result
risch	$\frac{c^2}{d^2b} - \frac{\operatorname{dilog}\left(\frac{-e^{dx+c}b+\sqrt{a^2-b^2}-a}{-a+\sqrt{a^2-b^2}}\right)}{d^2b} - \frac{\operatorname{dilog}\left(\frac{e^{dx+c}b+\sqrt{a^2-b^2}+a}{a+\sqrt{a^2-b^2}}\right)}{d^2b} + \frac{x^2}{2b} - \frac{\ln\left(\frac{e^{dx+c}b+\sqrt{a^2-b^2}+a}{a+\sqrt{a^2-b^2}}\right)x}{db} - \frac{\ln\left(\frac{e^{dx+c}b+\sqrt{a^2-b}}{a+\sqrt{a^2-b^2}}\right.}{d^2b}$

input
```
int(x*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d^2/b*c^2-1/d^2/b*dilog((-exp(d*x+c)*b+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^
(1/2)))-1/d^2/b*dilog((exp(d*x+c)*b+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2))
)+1/2*x^2/b-1/d/b*ln((exp(d*x+c)*b+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))
*x-1/d^2/b*ln((exp(d*x+c)*b+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*c+1/d^
2/b*c*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)+b)-2/d^2/b*c*ln(exp(d*x+c))-2/d/b
^3*a^2*c*x-1/d/b*ln((-exp(d*x+c)*b+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2))
)*x-1/d^2/b*ln((-exp(d*x+c)*b+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))*c+2
/d/b*c*x-1/2*a*(d*x-1)/d^2/b^2*exp(d*x+c)-1/2*x^2/b^3*a^2-1/d^2/b^3*a^2*c^
2+1/d^2/b^3*a^2*dilog((-exp(d*x+c)*b+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2
)))+1/d^2/b^3*a^2*dilog((exp(d*x+c)*b+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2
)))-1/2*a*(d*x+1)/d^2/b^2*exp(-d*x-c)+1/16*(2*d*x-1)/b/d^2*exp(2*d*x+2*c)+
1/16*(2*d*x+1)/b/d^2*exp(-2*d*x-2*c)+1/d^2/b^3*ln((exp(d*x+c)*b+(a^2-b^2)^
(1/2)+a)/(a+(a^2-b^2)^(1/2)))*a^2*c+1/d/b^3*ln((-exp(d*x+c)*b+(a^2-b^2)^(1
/2)-a)/(-a+(a^2-b^2)^(1/2)))*a^2*x+1/d/b^3*ln((exp(d*x+c)*b+(a^2-b^2)^(1/2
)+a)/(a+(a^2-b^2)^(1/2)))*a^2*x+1/d^2/b^3*ln((-exp(d*x+c)*b+(a^2-b^2)^(1/2
)-a)/(-a+(a^2-b^2)^(1/2)))*a^2*c-1/d^2/b^3*c*a^2*ln(b*exp(2*d*x+2*c)+2*a*e
xp(d*x+c)+b)+2/d^2/b^3*c*a^2*ln(exp(d*x+c))
```

3.236.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1196 vs. $2(266) = 532$.

Time = 0.30 (sec) , antiderivative size = 1196, normalized size of antiderivative = 4.15

$$\int \frac{x\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(x*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
1/16*((2*b^2*d*x - b^2)*cosh(d*x + c)^4 + (2*b^2*d*x - b^2)*sinh(d*x + c)^
4 + 2*b^2*d*x - 8*(a*b*d*x - a*b)*cosh(d*x + c)^3 - 4*(2*a*b*d*x - 2*a*b -
 (2*b^2*d*x - b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 8*((a^2 - b^2)*d^2*x^2
 - 2*(a^2 - b^2)*c^2)*cosh(d*x + c)^2 - 2*(4*(a^2 - b^2)*d^2*x^2 - 8*(a^2
- b^2)*c^2 - 3*(2*b^2*d*x - b^2)*cosh(d*x + c)^2 + 12*(a*b*d*x - a*b)*cosh
(d*x + c))*sinh(d*x + c)^2 + b^2 - 8*(a*b*d*x + a*b)*cosh(d*x + c) + 16*((
a^2 - b^2)*cosh(d*x + c)^2 + 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (
a^2 - b^2)*sinh(d*x + c)^2)*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b
*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + 16*(
(a^2 - b^2)*cosh(d*x + c)^2 + 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) +
(a^2 - b^2)*sinh(d*x + c)^2)*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (
b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - 16*
((a^2 - b^2)*c*cosh(d*x + c)^2 + 2*(a^2 - b^2)*c*cosh(d*x + c)*sinh(d*x +
c) + (a^2 - b^2)*c*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x +
 c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - 16*((a^2 - b^2)*c*cosh(d*x + c)^2
 + 2*(a^2 - b^2)*c*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*c*sinh(d*x +
c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 - b^2)/b^2
) + 2*a) + 16*(((a^2 - b^2)*d*x + (a^2 - b^2)*c)*cosh(d*x + c)^2 + 2*((a^2
 - b^2)*d*x + (a^2 - b^2)*c)*cosh(d*x + c)*sinh(d*x + c) + ((a^2 - b^2)*d*
x + (a^2 - b^2)*c)*sinh(d*x + c)^2)*log((a*cosh(d*x + c) + a*sinh(d*x +...
```

3.236.6 Sympy [F]

$$\int \frac{x \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x \sinh^3{(c+dx)}}{a+b\cosh{(c+dx)}}\,dx$$

input
```
integrate(x*sinh(d*x+c)**3/(a+b*cosh(d*x+c)),x)
```

output
```
Integral(x*sinh(c + d*x)**3/(a + b*cosh(c + d*x)), x)
```

3.236.7 Maxima [F]

$$\int \frac{x\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x\sinh{(dx+c)}^3}{b\cosh{(dx+c)}+a}\,dx$$

input
```
integrate(x*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
1/16*(8*(a^2*d^2*e^(2*c) - b^2*d^2*e^(2*c))*x^2 + (2*b^2*d*x*e^(4*c) - b^2
*e^(4*c))*e^(2*d*x) - 8*(a*b*d*x*e^(3*c) - a*b*e^(3*c))*e^(d*x) - 8*(a*b*d
*x*e^c + a*b*e^c)*e^(-d*x) + (2*b^2*d*x + b^2)*e^(-2*d*x))*e^(-2*c)/(b^3*d
^2) - 1/8*integrate(16*((a^3*e^c - a*b^2*e^c)*x*e^(d*x) + (a^2*b - b^3)*x)
/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) + b^4), x)
```

3.236.8 Giac [F]

$$\int \frac{x\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x\sinh{(dx+c)}^3}{b\cosh{(dx+c)}+a}\,dx$$

input
```
integrate(x*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x*sinh(d*x + c)^3/(b*cosh(d*x + c) + a), x)
```

3.236.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x\sinh(c+d\,x)^3}{a+b\cosh{(c+d\,x)}}\,dx$$

input
```
int((x*sinh(c + d*x)^3)/(a + b*cosh(c + d*x)),x)
```

output
```
int((x*sinh(c + d*x)^3)/(a + b*cosh(c + d*x)), x)
```

3.237 $\int \frac{\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

3.237.1 Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \frac{\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{a\cosh(c+dx)}{b^2d}+\frac{\cosh^2(c+dx)}{2bd}+\frac{(a^2-b^2)\log(a+b\cosh(c+dx))}{b^3d}$$

output
```
-a*cosh(d*x+c)/b^2/d+1/2*cosh(d*x+c)^2/b/d+(a^2-b^2)*ln(a+b*cosh(d*x+c))/b
^3/d
```

3.237.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

$$\int \frac{\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \frac{-4ab\cosh(c+dx)+b^2\cosh(2(c+dx))+4(a^2-b^2)\log(a+b\cosh(c+dx))}{4b^3d}$$

input
```
Integrate[Sinh[c + d*x]^3/(a + b*Cosh[c + d*x]),x]
```

output
```
(-4*a*b*Cosh[c + d*x] + b^2*Cosh[2*(c + d*x)] + 4*(a^2 - b^2)*Log[a + b*Co
sh[c + d*x]])/(4*b^3*d)
```

3.237.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 26, 3147, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)}{a+b\cosh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\cos\left(ic+idx-\frac{\pi}{2}\right)^3}{a-b\sin\left(ic+idx-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cos\left(\frac{1}{2}(2ic-\pi)+idx\right)^3}{a-b\sin\left(\frac{1}{2}(2ic-\pi)+idx\right)}dx$$

$$\Big\downarrow \text{3147}$$

$$-\frac{\int \frac{b^2-b^2\cosh^2(c+dx)}{a+b\cosh(c+dx)}d(b\cosh(c+dx))}{b^3d}$$

$$\Big\downarrow \text{476}$$

$$-\frac{\int \left(a-b\cosh(c+dx)+\frac{b^2-a^2}{a+b\cosh(c+dx)}\right)d(b\cosh(c+dx))}{b^3d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\left(a^2-b^2\right)\log(a+b\cosh(c+dx))+ab\cosh(c+dx)-\frac{1}{2}b^2\cosh^2(c+dx)}{b^3d}$$

input
```
Int[Sinh[c + d*x]^3/(a + b*Cosh[c + d*x]),x]
```

output
```
-((a*b*Cosh[c + d*x] - (b^2*Cosh[c + d*x]^2)/2 - (a^2 - b^2)*Log[a + b*Cos
h[c + d*x]])/(b^3*d))
```

3.237.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.237.4 Maple [A] (verified)

Time = 4.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{-\frac{-\frac{\cosh(dx+c)^2 b}{2}+a\cosh(dx+c)}{b^2}+\frac{(a^2-b^2)\ln(a+b\cosh(dx+c))}{b^3}}{d}$
default	$\frac{-\frac{-\frac{\cosh(dx+c)^2 b}{2}+a\cosh(dx+c)}{b^2}+\frac{(a^2-b^2)\ln(a+b\cosh(dx+c))}{b^3}}{d}$
risch	$-\frac{x\,a^2}{b^3}+\frac{x}{b}+\frac{e^{2dx+2c}}{8db}-\frac{a\,e^{dx+c}}{2d\,b^2}-\frac{e^{-dx-c}a}{2d\,b^2}+\frac{e^{-2dx-2c}}{8db}-\frac{2c\,a^2}{b^3 d}+\frac{2c}{bd}+\frac{\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{b}+1\right)a^2}{b^3 d}-$

input
```
int(sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b^2*(-1/2*cosh(d*x+c)^2*b+a*cosh(d*x+c))+(a^2-b^2)/b^3*ln(a+b*cosh
(d*x+c)))
```

3.237.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. 2(59) = 118.

Time = 0.26 (sec) , antiderivative size = 340, normalized size of antiderivative = 5.57

$$\int \frac{\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$$
$$= \frac{b^2\cosh{(dx+c)}^4 + b^2\sinh{(dx+c)}^4 - 8\,(a^2-b^2)dx\cosh{(dx+c)}^2 - 4\,ab\cosh{(dx+c)}^3 + 4\,(b^2\cosh{(dx-}}{}$$

input
```
integrate(sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
1/8*(b^2*cosh(d*x + c)^4 + b^2*sinh(d*x + c)^4 - 8*(a^2 - b^2)*d*x*cosh(d*
x + c)^2 - 4*a*b*cosh(d*x + c)^3 + 4*(b^2*cosh(d*x + c) - a*b)*sinh(d*x +
c)^3 - 4*a*b*cosh(d*x + c) + 2*(3*b^2*cosh(d*x + c)^2 - 4*(a^2 - b^2)*d*x
- 6*a*b*cosh(d*x + c))*sinh(d*x + c)^2 + b^2 + 8*((a^2 - b^2)*cosh(d*x + c
)^2 + 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*sinh(d*x + c
)^2)*log(2*(b*cosh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(b^2
*cosh(d*x + c)^3 - 4*(a^2 - b^2)*d*x*cosh(d*x + c) - 3*a*b*cosh(d*x + c)^2
 - a*b)*sinh(d*x + c))/(b^3*d*cosh(d*x + c)^2 + 2*b^3*d*cosh(d*x + c)*sinh
(d*x + c) + b^3*d*sinh(d*x + c)^2)
```

3.237.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**3/(a+b*cosh(d*x+c)),x)
```

output
```
Timed out
```

3.237.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(59) = 118$.

Time = 0.19 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.13

$$\int \frac{\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{\left(4\,ae^{(-dx-c)}-b\right)e^{(2\,dx+2\,c)}}{8\,b^2d} + \frac{(a^2-b^2)(dx+c)}{b^3d} - \frac{4\,ae^{(-dx-c)}-be^{(-2\,dx-2\,c)}}{8\,b^2d} + \frac{(a^2-b^2)\log\left(2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}+b\right)}{b^3d}$$

input
```
integrate(sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/8*(4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d) + (a^2 - b^2)*(d*x + c
)/(b^3*d) - 1/8*(4*a*e^(-d*x - c) - b*e^(-2*d*x - 2*c))/(b^2*d) + (a^2 - b
^2)*log(2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) + b)/(b^3*d)
```

3.237.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.44

$$\int \frac{\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \frac{\frac{b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4\,a\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{b^2}+\frac{8\,(a^2-b^2)\log\left(\left|b\left(e^{(dx+c)}+e^{(-dx-c)}\right)+2\,a\right|\right)}{b^3}}{8\,d}$$

input
```
integrate(sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

output
```
1/8*((b*(e^(d*x + c) + e^(-d*x - c))^2 - 4*a*(e^(d*x + c) + e^(-d*x - c)))
/b^2 + 8*(a^2 - b^2)*log(abs(b*(e^(d*x + c) + e^(-d*x - c)) + 2*a))/b^3)/d
```

3.237.9 Mupad [B] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.00

$$\int \frac{\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \frac{e^{-2c-2dx}}{8bd} + \frac{e^{2c+2dx}}{8bd} - \frac{x(a^2-b^2)}{b^3} + \frac{\ln\left(b+2a\,e^{dx}\,e^{c}+b\,e^{2c}\,e^{2dx}\right)(a^2-b^2)}{b^3 d} - \frac{a\,e^{-c-dx}}{2b^2 d} - \frac{a\,e^{c+dx}}{2b^2 d}$$

input
```
int(sinh(c + d*x)^3/(a + b*cosh(c + d*x)),x)
```

output
```
exp(- 2*c - 2*d*x)/(8*b*d) + exp(2*c + 2*d*x)/(8*b*d) - (x*(a^2 - b^2))/b^
3 + (log(b + 2*a*exp(d*x)*exp(c) + b*exp(2*c)*exp(2*d*x))*(a^2 - b^2))/(b^
3*d) - (a*exp(- c - d*x))/(2*b^2*d) - (a*exp(c + d*x))/(2*b^2*d)
```

3.238 $\int \frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))}\,dx$

3.238.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \text{Int}\left(\frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))},x\right)$$

output
```
Unintegrable(sinh(d*x+c)^3/x/(a+b*cosh(d*x+c)),x)
```

3.238.2 Mathematica [N/A]

Not integrable

Time = 36.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))}\,dx$$

input
```
Integrate[Sinh[c + d*x]^3/(x*(a + b*Cosh[c + d*x])),x]
```

output
```
Integrate[Sinh[c + d*x]^3/(x*(a + b*Cosh[c + d*x])), x]
```

3.238.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6112}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))}\,dx$$
$$\Big\downarrow \text{ 6112}$$
$$\int \frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))}dx$$

input
```
Int[Sinh[c + d*x]^3/(x*(a + b*Cosh[c + d*x])),x]
```

output
```
$Aborted
```

3.238.3.1 Defintions of rubi rules used

rule 6112
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.))/(Cosh[(c_.)
 + (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Unintegrable[((e + f*x)^m*F[c +
d*x]^n)/(a + b*Cosh[c + d*x]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
HyperbolicQ[F]
```

3.238.4 Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sinh\left(dx+c\right)^{3}}{x\left(a+b\cosh\left(dx+c\right)\right)}dx$$

input
```
int(sinh(d*x+c)^3/x/(a+b*cosh(d*x+c)),x)
```

output
```
int(sinh(d*x+c)^3/x/(a+b*cosh(d*x+c)),x)
```

3.238.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh(dx+c)^3}{(b\cosh(dx+c)+a)x}\,dx$$

input
```
integrate(sinh(d*x+c)^3/x/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(sinh(d*x + c)^3/(b*x*cosh(d*x + c) + a*x), x)
```

3.238.6 Sympy [N/A]

Not integrable

Time = 22.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh^3(c+dx)}{x\,(a+b\cosh(c+dx))}\,dx$$

input
```
integrate(sinh(d*x+c)**3/x/(a+b*cosh(d*x+c)),x)
```

output
```
Integral(sinh(c + d*x)**3/(x*(a + b*cosh(c + d*x))), x)
```

3.238.7 Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 143, normalized size of antiderivative = 5.96

$$\int \frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh(dx+c)^3}{(b\cosh(dx+c)+a)x}\,dx$$

input
```
integrate(sinh(d*x+c)^3/x/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*Ei(2*d*x)*e^(2*c)/b + 1/2*a*Ei(-d*x)*e^(-c)/b^2 - 1/4*Ei(-2*d*x)*e^(-2
*c)/b - 1/2*a*Ei(d*x)*e^c/b^2 + (a^2 - b^2)*log(x)/b^3 - 1/8*integrate(16*
(a^2*b - b^3 + (a^3*e^c - a*b^2*e^c)*e^(d*x))/(b^4*x*e^(2*d*x + 2*c) + 2*a
*b^3*x*e^(d*x + c) + b^4*x), x)
```

3.238.8 Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh\left(dx+c\right)^3}{\left(b\cosh\left(dx+c\right)+a\right)x}\,dx$$

input
```
integrate(sinh(d*x+c)^3/x/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sinh(d*x + c)^3/((b*cosh(d*x + c) + a)*x), x)
```

3.238.9 Mupad [N/A]

Not integrable

Time = 1.79 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh(c+d\,x)^3}{x\,\left(a+b\cosh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(sinh(c + d*x)^3/(x*(a + b*cosh(c + d*x))),x)
```

output
```
int(sinh(c + d*x)^3/(x*(a + b*cosh(c + d*x))), x)
```

3.239 $\int \cosh (a+b \log (c x^n)) \, dx$

3.239.1 Optimal result

Integrand size = 11, antiderivative size = 54

$$\int \cosh (a+b \log (c x^n)) \, dx = \frac{x \cosh (a+b \log (c x^n))}{1-b^2 n^2} - \frac{b n x \sinh (a+b \log (c x^n))}{1-b^2 n^2}$$

output
```
x*cosh(a+b*ln(c*x^n))/(-b^2*n^2+1)-b*n*x*sinh(a+b*ln(c*x^n))/(-b^2*n^2+1)
```

3.239.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.76

$$\int \cosh (a+b \log (c x^n)) \, dx = \frac{x(-\cosh (a+b \log (c x^n))+b n \sinh (a+b \log (c x^n)))}{-1+b^2 n^2}$$

input
```
Integrate[Cosh[a + b*Log[c*x^n]],x]
```

output
```
(x*(-Cosh[a + b*Log[c*x^n]] + b*n*Sinh[a + b*Log[c*x^n]]))/(-1 + b^2*n^2)
```

3.239.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6044}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a + b\log(cx^n))\,dx$$

$$\Big\downarrow \text{6044}$$

$$\frac{x\cosh(a + b\log(cx^n))}{1 - b^2n^2} - \frac{bnx\sinh(a + b\log(cx^n))}{1 - b^2n^2}$$

input
```
Int[Cosh[a + b*Log[c*x^n]],x]
```

output
```
(x*Cosh[a + b*Log[c*x^n]])/(1 - b^2*n^2) - (b*n*x*Sinh[a + b*Log[c*x^n]])/
(1 - b^2*n^2)
```

3.239.3.1 Defintions of rubi rules used

rule 6044
```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[(-
x)*(Cosh[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 - 1)), x] + Simp[b*d*n*x*(Sinh[
d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 - 1)), x] /; FreeQ[{a, b, c, d, n}, x] &
& NeQ[b^2*d^2*n^2 - 1, 0]
```

3.239.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

method	result	size
parallelrisch	$\frac{x(bn\sinh(a+b\ln(c\,x^n))-\cosh(a+b\ln(c\,x^n)))}{b^2n^2-1}$	42

input
```
int(cosh(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

output
```
x/(b^2*n^2-1)*(b*n*sinh(a+b*ln(c*x^n))-cosh(a+b*ln(c*x^n)))
```

3.239.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \cosh(a+b\log(cx^n))\,dx = \frac{bnx\sinh(bn\log(x)+b\log(c)+a)-x\cosh(bn\log(x)+b\log(c)+a)}{b^2n^2-1}$$

input
```
integrate(cosh(a+b*log(c*x^n)),x, algorithm="fricas")
```

output
```
(b*n*x*sinh(b*n*log(x) + b*log(c) + a) - x*cosh(b*n*log(x) + b*log(c) + a)
)/(b^2*n^2 - 1)
```

3.239.6 Sympy [F]

$$\int \cosh(a+b\log(cx^n))\,dx = \begin{cases} \int \cosh\left(a-\frac{\log(cx^n)}{n}\right)dx & \text{for } b=-\frac{1}{n} \\ \int \cosh\left(a+\frac{\log(cx^n)}{n}\right)dx & \text{for } b=\frac{1}{n} \\ \frac{bnx\sinh(a+b\log(cx^n))}{b^2n^2-1}-\frac{x\cosh(a+b\log(cx^n))}{b^2n^2-1} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(a+b*ln(c*x**n)),x)
```

output
```
Piecewise((Integral(cosh(a - log(c*x**n)/n), x), Eq(b, -1/n)), (Integral(c
osh(a + log(c*x**n)/n), x), Eq(b, 1/n)), (b*n*x*sinh(a + b*log(c*x**n))/(b
**2*n**2 - 1) - x*cosh(a + b*log(c*x**n))/(b**2*n**2 - 1), True))
```

3.239.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \cosh(a+b\log(cx^n))\,dx = \frac{c^bxe^{(b\log(x^n)+a)}}{2(bn+1)}-\frac{xe^{(-a)}}{2(bc^bn-c^b)(x^n)^b}$$

input
```
integrate(cosh(a+b*log(c*x^n)),x, algorithm="maxima")
```

output
```
1/2*c^b*x*e^(b*log(x^n) + a)/(b*n + 1) - 1/2*x*e^(-a)/((b*c^b*n - c^b)*(x^
n)^b)
```

3.239.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \cosh\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{c^bxx^{bn}e^a}{2\left(bn+1\right)}-\frac{xe^{(-a)}}{2\left(bn-1\right)c^bx^{bn}}$$

input
```
integrate(cosh(a+b*log(c*x^n)),x, algorithm="giac")
```

output
```
1/2*c^b*x*x^(b*n)*e^a/(b*n + 1) - 1/2*x*e^(-a)/((b*n - 1)*c^b*x^(b*n))
```

3.239.9 Mupad [B] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \cosh\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{x\,\mathrm{e}^a\,{\left(c\,x^n\right)}^b}{2\,b\,n+2}-\frac{x\,\mathrm{e}^{-a}}{{\left(c\,x^n\right)}^b\,\left(2\,b\,n-2\right)}$$

input
```
int(cosh(a + b*log(c*x^n)),x)
```

output
```
(x*exp(a)*(c*x^n)^b)/(2*b*n + 2) - (x*exp(-a))/((c*x^n)^b*(2*b*n - 2))
```

3.240 $\int \cosh^2(a+b\log(cx^n))\,dx$

3.240.1 Optimal result

Integrand size = 13, antiderivative size = 88

$$\int \cosh^2(a+b\log(cx^n))\,dx = -\frac{2b^2n^2x}{1-4b^2n^2}+\frac{x\cosh^2(a+b\log(cx^n))}{1-4b^2n^2} - \frac{2bnx\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{1-4b^2n^2}$$

output
```
-2*b^2*n^2*x/(-4*b^2*n^2+1)+x*cosh(a+b*ln(c*x^n))^2/(-4*b^2*n^2+1)-2*b*n*x
*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/(-4*b^2*n^2+1)
```

3.240.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

$$\int \cosh^2(a+b\log(cx^n))\,dx = \frac{x(-1+4b^2n^2-\cosh(2(a+b\log(cx^n)))+2bn\sinh(2(a+b\log(cx^n))))}{-2+8b^2n^2}$$

input
```
Integrate[Cosh[a + b*Log[c*x^n]]^2,x]
```

output
```
(x*(-1 + 4*b^2*n^2 - Cosh[2*(a + b*Log[c*x^n])] + 2*b*n*Sinh[2*(a + b*Log[
c*x^n])]))/(-2 + 8*b^2*n^2)
```

3.240.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6046, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{6046}$$

$$-\frac{2b^2n^2\int 1dx}{1-4b^2n^2}+\frac{x\cosh^2(a+b\log(cx^n))}{1-4b^2n^2}-\frac{2bnx\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{1-4b^2n^2}$$

$$\Big\downarrow \text{24}$$

$$\frac{x\cosh^2(a+b\log(cx^n))}{1-4b^2n^2}-\frac{2bnx\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{1-4b^2n^2}-\frac{2b^2n^2x}{1-4b^2n^2}$$

input
```
Int[Cosh[a + b*Log[c*x^n]]^2,x]
```

output
```
(-2*b^2*n^2*x)/(1 - 4*b^2*n^2) + (x*Cosh[a + b*Log[c*x^n]]^2)/(1 - 4*b^2*n
^2) - (2*b*n*x*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/(1 - 4*b^2*n
^2)
```

3.240.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6046
```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> Si
mp[(-x)*(Cosh[d*(a + b*Log[c*x^n])]^p/(b^2*d^2*n^2*p^2 - 1)), x] + (Simp[b*
d*n*p*x*Cosh[d*(a + b*Log[c*x^n])]^(p - 1)*(Sinh[d*(a + b*Log[c*x^n])]/(b^2
*d^2*n^2*p^2 - 1)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 - 1))
   Int[Cosh[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b, c, d, n}
, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - 1, 0]
```

3.240.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$\frac{x\left(4b^{2}n^{2}+2bn\sinh\left(2b\ln\left(c\,x^{n}\right)+2a\right)-\cosh\left(2b\ln\left(c\,x^{n}\right)+2a\right)-1\right)}{8b^{2}n^{2}-2}$	59

input
```
int(cosh(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
```

output
```
x*(4*b^2*n^2+2*b*n*sinh(2*b*ln(c*x^n)+2*a)-cosh(2*b*ln(c*x^n)+2*a)-1)/(8*b
^2*n^2-2)
```

3.240.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.02

$$\int \cosh^2\left(a+b\log\left(cx^n\right)\right)\,dx$$
$$=\frac{4\,bnx\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-x\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2}{2\left(4\,b^2n^2-1\right)}$$

input
```
integrate(cosh(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

output
```
1/2*(4*b*n*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) +
a) - x*cosh(b*n*log(x) + b*log(c) + a)^2 - x*sinh(b*n*log(x) + b*log(c) +
a)^2 + (4*b^2*n^2 - 1)*x)/(4*b^2*n^2 - 1)
```

3.240.6 Sympy [F]

$$\int \cosh^2\left(a+b\log\left(cx^n\right)\right)\,dx$$
$$=\begin{cases}\int\cosh^2\left(a-\frac{\log\left(cx^n\right)}{2n}\right)\,dx\\ \int\cosh^2\left(a+\frac{\log\left(cx^n\right)}{2n}\right)\,dx\\ -\frac{2b^2n^2x\sinh^2\left(a+b\log\left(cx^n\right)\right)}{4b^2n^2-1}+\frac{2b^2n^2x\cosh^2\left(a+b\log\left(cx^n\right)\right)}{4b^2n^2-1}+\frac{2bnx\sinh\left(a+b\log\left(cx^n\right)\right)\cosh\left(a+b\log\left(cx^n\right)\right)}{4b^2n^2-1}-\frac{x\cosh^2\left(a+b\log\left(cx^n\right)\right)}{4b^2n^2-1}\end{cases}$$

input
```
integrate(cosh(a+b*ln(c*x**n))**2,x)
```

output
```
Piecewise((Integral(cosh(a - log(c*x**n)/(2*n))**2, x), Eq(b, -1/(2*n))),
(Integral(cosh(a + log(c*x**n)/(2*n))**2, x), Eq(b, 1/(2*n))), (-2*b**2*n*
*2*x*sinh(a + b*log(c*x**n))**2/(4*b**2*n**2 - 1) + 2*b**2*n**2*x*cosh(a +
 b*log(c*x**n))**2/(4*b**2*n**2 - 1) + 2*b*n*x*sinh(a + b*log(c*x**n))*cos
h(a + b*log(c*x**n))/(4*b**2*n**2 - 1) - x*cosh(a + b*log(c*x**n))**2/(4*b
**2*n**2 - 1), True))
```

3.240.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.76

$$\int \cosh^2(a+b\log(cx^n))\,dx = \frac{c^{2b}xe^{(2b\log(x^n)+2a)}}{4(2bn+1)} + \frac{1}{2}x - \frac{xe^{(-2a)}}{4(2bc^{2b}n - c^{2b})(x^n)^{2b}}$$

input
```
integrate(cosh(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

output
```
1/4*c^(2*b)*x*e^(2*b*log(x^n) + 2*a)/(2*b*n + 1) + 1/2*x - 1/4*x*e^(-2*a)/
((2*b*c^(2*b)*n - c^(2*b))*(x^n)^(2*b))
```

3.240.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.92

$$\int \cosh^2(a+b\log(cx^n))\,dx = \frac{bc^{2b}nxx^{2bn}e^{(2a)}}{2(4b^2n^2-1)} + \frac{2b^2n^2x}{4b^2n^2-1} - \frac{c^{2b}xx^{2bn}e^{(2a)}}{4(4b^2n^2-1)} - \frac{bnxe^{(-2a)}}{2(4b^2n^2-1)c^{2b}x^{2bn}} - \frac{x}{2(4b^2n^2-1)} - \frac{xe^{(-2a)}}{4(4b^2n^2-1)c^{2b}x^{2bn}}$$

input
```
integrate(cosh(a+b*log(c*x^n))^2,x, algorithm="giac")
```

output
```
1/2*b*c^(2*b)*n*x*x^(2*b*n)*e^(2*a)/(4*b^2*n^2 - 1) + 2*b^2*n^2*x/(4*b^2*n
^2 - 1) - 1/4*c^(2*b)*x*x^(2*b*n)*e^(2*a)/(4*b^2*n^2 - 1) - 1/2*b*n*x*e^(-
2*a)/((4*b^2*n^2 - 1)*c^(2*b)*x^(2*b*n)) - 1/2*x/(4*b^2*n^2 - 1) - 1/4*x*e
^(-2*a)/((4*b^2*n^2 - 1)*c^(2*b)*x^(2*b*n))
```

3.240.9 Mupad [B] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.60

$$\int \cosh^2(a+b\log(cx^n))\,dx=\frac{x}{2}-\frac{x\,\mathrm{e}^{-2\,a}}{(c\,x^n)^{2\,b}\,(8\,b\,n-4)}+\frac{x\,\mathrm{e}^{2\,a}\,(c\,x^n)^{2\,b}}{8\,b\,n+4}$$

input
```
int(cosh(a + b*log(c*x^n))^2,x)
```

output
```
x/2 - (x*exp(-2*a))/((c*x^n)^(2*b)*(8*b*n - 4)) + (x*exp(2*a)*(c*x^n)^(2*b
))/(8*b*n + 4)
```

3.241 $\int \cosh^3(a+b\log(cx^n))\,dx$

3.241.1 Optimal result

Integrand size = 13, antiderivative size = 149

$$\int \cosh^3(a+b\log(cx^n))\,dx = -\frac{6b^2n^2x\cosh(a+b\log(cx^n))}{1-10b^2n^2+9b^4n^4}+\frac{x\cosh^3(a+b\log(cx^n))}{1-9b^2n^2} + \frac{6b^3n^3x\sinh(a+b\log(cx^n))}{1-10b^2n^2+9b^4n^4} - \frac{3bnx\cosh^2(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{1-9b^2n^2}$$

output
```
-6*b^2*n^2*x*cosh(a+b*ln(c*x^n))/(9*b^4*n^4-10*b^2*n^2+1)+x*cosh(a+b*ln(c*
x^n))^3/(-9*b^2*n^2+1)+6*b^3*n^3*x*sinh(a+b*ln(c*x^n))/(9*b^4*n^4-10*b^2*n
^2+1)-3*b*n*x*cosh(a+b*ln(c*x^n))^2*sinh(a+b*ln(c*x^n))/(-9*b^2*n^2+1)
```

3.241.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.79

$$\int \cosh^3(a+b\log(cx^n))\,dx = \frac{x((3-27b^2n^2)\cosh(a+b\log(cx^n))+(1-b^2n^2)\cosh(3(a+b\log(cx^n)))+6bn(-1+5b^2n^2+(-1+b^2 ...}{4-40b^2n^2+36b^4n^4}$$

input
```
Integrate[Cosh[a + b*Log[c*x^n]]^3,x]
```

output
```
(x*((3 - 27*b^2*n^2)*Cosh[a + b*Log[c*x^n]] + (1 - b^2*n^2)*Cosh[3*(a + b*
Log[c*x^n])] + 6*b*n*(-1 + 5*b^2*n^2 + (-1 + b^2*n^2)*Cosh[2*(a + b*Log[c*
x^n])])*Sinh[a + b*Log[c*x^n]]))/(4 - 40*b^2*n^2 + 36*b^4*n^4)
```

3.241.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6046, 6044}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{6046}$$

$$-\frac{6b^2n^2\int \cosh(a+b\log(cx^n))\,dx}{1-9b^2n^2}+\frac{x\cosh^3(a+b\log(cx^n))}{1-9b^2n^2}-\frac{3bnx\sinh(a+b\log(cx^n))\cosh^2(a+b\log(cx^n))}{1-9b^2n^2}$$

$$\Big\downarrow \text{6044}$$

$$\frac{x\cosh^3(a+b\log(cx^n))}{1-9b^2n^2}-\frac{3bnx\sinh(a+b\log(cx^n))\cosh^2(a+b\log(cx^n))}{1-9b^2n^2}-\frac{6b^2n^2\left(\frac{x\cosh(a+b\log(cx^n))}{1-b^2n^2}-\frac{bnx\sinh(a+b\log(cx^n))}{1-b^2n^2}\right)}{1-9b^2n^2}$$

input
```
Int[Cosh[a + b*Log[c*x^n]]^3,x]
```

output
```
(x*Cosh[a + b*Log[c*x^n]]^3)/(1 - 9*b^2*n^2) - (3*b*n*x*Cosh[a + b*Log[c*x
^n]]^2*Sinh[a + b*Log[c*x^n]])/(1 - 9*b^2*n^2) - (6*b^2*n^2*((x*Cosh[a + b
*Log[c*x^n]])/(1 - b^2*n^2) - (b*n*x*Sinh[a + b*Log[c*x^n]])/(1 - b^2*n^2)
))/(1 - 9*b^2*n^2)
```

3.241.3.1 Defintions of rubi rules used

rule 6044
```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[(-
x)*(Cosh[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 - 1)), x] + Simp[b*d*n*x*(Sinh[
d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 - 1)), x] /; FreeQ[{a, b, c, d, n}, x] &
& NeQ[b^2*d^2*n^2 - 1, 0]
```

rule 6046
```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> Si
mp[(-x)*(Cosh[d*(a + b*Log[c*x^n])]^p/(b^2*d^2*n^2*p^2 - 1)), x] + (Simp[b*
d*n*p*x*Cosh[d*(a + b*Log[c*x^n])]^(p - 1)*(Sinh[d*(a + b*Log[c*x^n])]/(b^2
*d^2*n^2*p^2 - 1)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 - 1))
   Int[Cosh[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b, c, d, n}
, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - 1, 0]
```

3.241.4 Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.11

method	result
parallelrisch	$\frac{x\left(27b^3n^3\sinh(a+b\ln(c\,x^n))+3b^3n^3\sinh(3b\ln(c\,x^n)+3a)-27b^2n^2\cosh(a+b\ln(c\,x^n))-b^2n^2\cosh(3b\ln(c\,x^n)+3a)-3bn\sinh(a\right.}{36b^4n^4-40b^2n^2+4}$

input
```
int(cosh(a+b*ln(c*x^n))^3,x,method=_RETURNVERBOSE)
```

output
```
x*(27*b^3*n^3*sinh(a+b*ln(c*x^n))+3*b^3*n^3*sinh(3*b*ln(c*x^n)+3*a)-27*b^2
*n^2*cosh(a+b*ln(c*x^n))-b^2*n^2*cosh(3*b*ln(c*x^n)+3*a)-3*b*n*sinh(a+b*ln
(c*x^n))-3*b*n*sinh(3*b*ln(c*x^n)+3*a)+3*cosh(a+b*ln(c*x^n))+cosh(3*b*ln(c
*x^n)+3*a))/(36*b^4*n^4-40*b^2*n^2+4)
```

3.241.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.34

$$\int \cosh^3\left(a+b\log\left(cx^n\right)\right)\,dx =$$
$$-\frac{(b^2n^2-1)x\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^3+3\left(b^2n^2-1\right)x\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\right.}{}$$

input
```
integrate(cosh(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

output
```
-1/4*((b^2*n^2 - 1)*x*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*(b^2*n^2 - 1)*
x*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2 - 3*(b
^3*n^3 - b*n)*x*sinh(b*n*log(x) + b*log(c) + a)^3 + 3*(9*b^2*n^2 - 1)*x*co
sh(b*n*log(x) + b*log(c) + a) - 3*(3*(b^3*n^3 - b*n)*x*cosh(b*n*log(x) + b
*log(c) + a)^2 + (9*b^3*n^3 - b*n)*x)*sinh(b*n*log(x) + b*log(c) + a))/(9*
b^4*n^4 - 10*b^2*n^2 + 1)
```

3.241.6 Sympy [F]

$$\int \cosh^3(a + b\log(cx^n))\,dx$$

$$= \begin{cases} \int \cosh^3\left(a - \frac{\log(cx^n)}{n}\right)dx \\ \int \cosh^3\left(a - \frac{\log(cx^n)}{3n}\right)dx \\ \int \cosh^3\left(a + \frac{\log(cx^n)}{3n}\right)dx \\ \int \cosh^3\left(a + \frac{\log(cx^n)}{n}\right)dx \\ -\frac{6b^3n^3x\sinh^3(a+b\log(cx^n))}{9b^4n^4-10b^2n^2+1} + \frac{9b^3n^3x\sinh(a+b\log(cx^n))\cosh^2(a+b\log(cx^n))}{9b^4n^4-10b^2n^2+1} + \frac{6b^2n^2x\sinh^2(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{9b^4n^4-10b^2n^2+1} - \end{cases}$$

input
```
integrate(cosh(a+b*ln(c*x**n))**3,x)
```

output
```
Piecewise((Integral(cosh(a - log(c*x**n)/n)**3, x), Eq(b, -1/n)), (Integra
l(cosh(a - log(c*x**n)/(3*n))**3, x), Eq(b, -1/(3*n))), (Integral(cosh(a +
 log(c*x**n)/(3*n))**3, x), Eq(b, 1/(3*n))), (Integral(cosh(a + log(c*x**n
)/n)**3, x), Eq(b, 1/n)), (-6*b**3*n**3*x*sinh(a + b*log(c*x**n))**3/(9*b*
*4*n**4 - 10*b**2*n**2 + 1) + 9*b**3*n**3*x*sinh(a + b*log(c*x**n))*cosh(a
 + b*log(c*x**n))**2/(9*b**4*n**4 - 10*b**2*n**2 + 1) + 6*b**2*n**2*x*sinh
(a + b*log(c*x**n))**2*cosh(a + b*log(c*x**n))/(9*b**4*n**4 - 10*b**2*n**2
 + 1) - 7*b**2*n**2*x*cosh(a + b*log(c*x**n))**3/(9*b**4*n**4 - 10*b**2*n*
*2 + 1) - 3*b*n*x*sinh(a + b*log(c*x**n))*cosh(a + b*log(c*x**n))**2/(9*b*
*4*n**4 - 10*b**2*n**2 + 1) + x*cosh(a + b*log(c*x**n))**3/(9*b**4*n**4 -
10*b**2*n**2 + 1), True))
```

3.241.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.77

$$\int \cosh^3(a+b\log(cx^n))\,dx = \frac{c^{3b}xe^{(3b\log(x^n)+3a)}}{8(3bn+1)} + \frac{3c^bxe^{(b\log(x^n)+a)}}{8(bn+1)} - \frac{3xe^{(-b\log(x^n)-a)}}{8(bc^bn-c^b)} - \frac{xe^{(-3a)}}{8(3bc^{3b}n-c^{3b})(x^n)^{3b}}$$

input
```
integrate(cosh(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

output
```
1/8*c^(3*b)*x*e^(3*b*log(x^n) + 3*a)/(3*b*n + 1) + 3/8*c^b*x*e^(b*log(x^n)
 + a)/(b*n + 1) - 3/8*x*e^(-b*log(x^n) - a)/(b*c^b*n - c^b) - 1/8*x*e^(-3*
a)/((3*b*c^(3*b)*n - c^(3*b))*(x^n)^(3*b))
```

3.241.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 665 vs. 2(150) = 300.

Time = 0.29 (sec) , antiderivative size = 665, normalized size of antiderivative = 4.46

$$\begin{aligned}\int \cosh^3(a+b\log(cx^n))\,dx = {} & \frac{3b^3c^{3b}n^3xx^{3bn}e^{(3a)}}{8(9b^4n^4-10b^2n^2+1)} + \frac{27b^3c^bn^3xx^{bn}e^a}{8(9b^4n^4-10b^2n^2+1)} \\ & - \frac{b^2c^{3b}n^2xx^{3bn}e^{(3a)}}{8(9b^4n^4-10b^2n^2+1)} - \frac{27b^2c^bn^2xx^{bn}e^a}{8(9b^4n^4-10b^2n^2+1)} \\ & - \frac{3bc^{3b}nxx^{3bn}e^{(3a)}}{8(9b^4n^4-10b^2n^2+1)} - \frac{27b^3n^3xe^{(-a)}}{8(9b^4n^4-10b^2n^2+1)c^bx^{bn}} \\ & - \frac{3b^3n^3xe^{(-3a)}}{8(9b^4n^4-10b^2n^2+1)c^{3b}x^{3bn}} - \frac{3bc^bnxx^{bn}e^a}{8(9b^4n^4-10b^2n^2+1)} \\ & + \frac{c^{3b}xx^{3bn}e^{(3a)}}{8(9b^4n^4-10b^2n^2+1)} - \frac{27b^2n^2xe^{(-a)}}{8(9b^4n^4-10b^2n^2+1)c^bx^{bn}} \\ & - \frac{b^2n^2xe^{(-3a)}}{8(9b^4n^4-10b^2n^2+1)c^{3b}x^{3bn}} \\ & + \frac{3c^bxx^{bn}e^a}{8(9b^4n^4-10b^2n^2+1)} + \frac{3bnxe^{(-a)}}{8(9b^4n^4-10b^2n^2+1)c^bx^{bn}} \\ & + \frac{3bnxe^{(-3a)}}{8(9b^4n^4-10b^2n^2+1)c^{3b}x^{3bn}} \\ & + \frac{3xe^{(-a)}}{8(9b^4n^4-10b^2n^2+1)c^bx^{bn}} \\ & + \frac{xe^{(-3a)}}{8(9b^4n^4-10b^2n^2+1)c^{3b}x^{3bn}}\end{aligned}$$

input
```
integrate(cosh(a+b*log(c*x^n))^3,x, algorithm="giac")
```

output
```
3/8*b^3*c^(3*b)*n^3*x*x^(3*b*n)*e^(3*a)/(9*b^4*n^4 - 10*b^2*n^2 + 1) + 27/
8*b^3*c^b*n^3*x*x^(b*n)*e^a/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 1/8*b^2*c^(3*b)
*n^2*x*x^(3*b*n)*e^(3*a)/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 27/8*b^2*c^b*n^2*x
*x^(b*n)*e^a/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 3/8*b*c^(3*b)*n*x*x^(3*b*n)*e^
(3*a)/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 27/8*b^3*n^3*x*e^(-a)/((9*b^4*n^4 - 1
0*b^2*n^2 + 1)*c^b*x^(b*n)) - 3/8*b^3*n^3*x*e^(-3*a)/((9*b^4*n^4 - 10*b^2*
n^2 + 1)*c^(3*b)*x^(3*b*n)) - 3/8*b*c^b*n*x*x^(b*n)*e^a/(9*b^4*n^4 - 10*b^
2*n^2 + 1) + 1/8*c^(3*b)*x*x^(3*b*n)*e^(3*a)/(9*b^4*n^4 - 10*b^2*n^2 + 1)
- 27/8*b^2*n^2*x*e^(-a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c^b*x^(b*n)) - 1/8*b
^2*n^2*x*e^(-3*a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c^(3*b)*x^(3*b*n)) + 3/8*c
^b*x*x^(b*n)*e^a/(9*b^4*n^4 - 10*b^2*n^2 + 1) + 3/8*b*n*x*e^(-a)/((9*b^4*n
^4 - 10*b^2*n^2 + 1)*c^b*x^(b*n)) + 3/8*b*n*x*e^(-3*a)/((9*b^4*n^4 - 10*b^
2*n^2 + 1)*c^(3*b)*x^(3*b*n)) + 3/8*x*e^(-a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)
*c^b*x^(b*n)) + 1/8*x*e^(-3*a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c^(3*b)*x^(3*
b*n))
```

3.241.9 Mupad [B] (verification not implemented)

Time = 1.75 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.63

$$\int \cosh^3\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{x\,\mathrm{e}^{3\,a}\,{\left(c\,x^n\right)}^{3\,b}}{24\,b\,n+8}-\frac{x\,\mathrm{e}^{-3\,a}}{{\left(c\,x^n\right)}^{3\,b}\,\left(24\,b\,n-8\right)}-\frac{3\,x\,\mathrm{e}^{-a}}{{\left(c\,x^n\right)}^{b}\,\left(8\,b\,n-8\right)}+\frac{3\,x\,\mathrm{e}^{a}\,{\left(c\,x^n\right)}^{b}}{8\,b\,n+8}$$

input
```
int(cosh(a + b*log(c*x^n))^3,x)
```

output
```
(x*exp(3*a)*(c*x^n)^(3*b))/(24*b*n + 8) - (x*exp(-3*a))/((c*x^n)^(3*b)*(24
*b*n - 8)) - (3*x*exp(-a))/((c*x^n)^b*(8*b*n - 8)) + (3*x*exp(a)*(c*x^n)^b
)/(8*b*n + 8)
```

3.242 $\int \cosh^4(a+b\log(cx^n))\,dx$

3.242.1 Optimal result

Integrand size = 13, antiderivative size = 191

$$\int \cosh^4(a+b\log(cx^n))\,dx=\frac{24b^4n^4x}{1-20b^2n^2+64b^4n^4}-\frac{12b^2n^2x\cosh^2(a+b\log(cx^n))}{1-20b^2n^2+64b^4n^4}+\frac{x\cosh^4(a+b\log(cx^n))}{1-16b^2n^2}+\frac{24b^3n^3x\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{1-20b^2n^2+64b^4n^4}-\frac{4bnx\cosh^3(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{1-16b^2n^2}$$

output
```
24*b^4*n^4*x/(64*b^4*n^4-20*b^2*n^2+1)-12*b^2*n^2*x*cosh(a+b*ln(c*x^n))^2/
(64*b^4*n^4-20*b^2*n^2+1)+x*cosh(a+b*ln(c*x^n))^4/(-16*b^2*n^2+1)+24*b^3*n
^3*x*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/(64*b^4*n^4-20*b^2*n^2+1)-4*b
*n*x*cosh(a+b*ln(c*x^n))^3*sinh(a+b*ln(c*x^n))/(-16*b^2*n^2+1)
```

3.242.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.87

$$\int \cosh^4(a+b\log(cx^n))\,dx$$
$$=\frac{x(3-60b^2n^2+192b^4n^4+(4-64b^2n^2)\cosh(2(a+b\log(cx^n)))+(1-4b^2n^2)\cosh(4(a+b\log(cx^n)))-$$

input
```
Integrate[Cosh[a + b*Log[c*x^n]]^4,x]
```

output
```
(x*(3 - 60*b^2*n^2 + 192*b^4*n^4 + (4 - 64*b^2*n^2)*Cosh[2*(a + b*Log[c*x^
n])] + (1 - 4*b^2*n^2)*Cosh[4*(a + b*Log[c*x^n])] - 8*b*n*Sinh[2*(a + b*Lo
g[c*x^n])] + 128*b^3*n^3*Sinh[2*(a + b*Log[c*x^n])] - 4*b*n*Sinh[4*(a + b*
Log[c*x^n])] + 16*b^3*n^3*Sinh[4*(a + b*Log[c*x^n])]))/(8*(1 - 20*b^2*n^2
+ 64*b^4*n^4))
```

3.242.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6046, 6046, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^4(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{6046}$$

$$-\frac{12b^2n^2\int \cosh^2(a+b\log(cx^n))dx}{1-16b^2n^2}+\frac{x\cosh^4(a+b\log(cx^n))}{1-16b^2n^2}-\frac{4bnx\sinh(a+b\log(cx^n))\cosh^3(a+b\log(cx^n))}{1-16b^2n^2}$$

$$\Big\downarrow \text{6046}$$

$$-\frac{12b^2n^2\left(-\frac{2b^2n^2\int 1dx}{1-4b^2n^2}+\frac{x\cosh^2(a+b\log(cx^n))}{1-4b^2n^2}-\frac{2bnx\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{1-4b^2n^2}\right)}{1-16b^2n^2}+\frac{x\cosh^4(a+b\log(cx^n))}{1-16b^2n^2}-\frac{4bnx\sinh(a+b\log(cx^n))\cosh^3(a+b\log(cx^n))}{1-16b^2n^2}$$

$$\Big\downarrow \text{24}$$

$$\frac{x\cosh^4(a+b\log(cx^n))}{1-16b^2n^2}-\frac{4bnx\sinh(a+b\log(cx^n))\cosh^3(a+b\log(cx^n))}{1-16b^2n^2}-\frac{12b^2n^2\left(\frac{x\cosh^2(a+b\log(cx^n))}{1-4b^2n^2}-\frac{2bnx\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{1-4b^2n^2}-\frac{2b^2n^2x}{1-4b^2n^2}\right)}{1-16b^2n^2}$$

input
```
Int[Cosh[a + b*Log[c*x^n]]^4,x]
```

output
```
(x*Cosh[a + b*Log[c*x^n]]^4)/(1 - 16*b^2*n^2) - (4*b*n*x*Cosh[a + b*Log[c*
x^n]]^3*Sinh[a + b*Log[c*x^n]])/(1 - 16*b^2*n^2) - (12*b^2*n^2*((-2*b^2*n^
2*x)/(1 - 4*b^2*n^2) + (x*Cosh[a + b*Log[c*x^n]]^2)/(1 - 4*b^2*n^2) - (2*b
*n*x*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/(1 - 4*b^2*n^2)))/(1 -
 16*b^2*n^2)
```

3.242.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6046
```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> Si
mp[(-x)*(Cosh[d*(a + b*Log[c*x^n])]^p/(b^2*d^2*n^2*p^2 - 1)), x] + (Simp[b*
d*n*p*x*Cosh[d*(a + b*Log[c*x^n])]^(p - 1)*(Sinh[d*(a + b*Log[c*x^n])]/(b^2
*d^2*n^2*p^2 - 1)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 - 1))
   Int[Cosh[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b, c, d, n}
, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - 1, 0]
```

3.242.4 Maple [A] (verified)

Time = 10.03 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.96

method	result
parallelrisch	$\frac{x(4(-16b^2n^2+1)\cosh(2b\ln(cx^n)+2a)+192b^4n^4+128b^3n^3\sinh(2b\ln(cx^n)+2a)+16b^3n^3\sinh(4b\ln(cx^n)+4a)-4b^2n^2\cosh(4}{512b^4n^4-160b^2n^2+8}$

input
```
int(cosh(a+b*ln(c*x^n))^4,x,method=_RETURNVERBOSE)
```

output
```
1/8*x*(4*(-16*b^2*n^2+1)*cosh(2*b*ln(c*x^n)+2*a)+192*b^4*n^4+128*b^3*n^3*s
inh(2*b*ln(c*x^n)+2*a)+16*b^3*n^3*sinh(4*b*ln(c*x^n)+4*a)-4*b^2*n^2*cosh(4
*b*ln(c*x^n)+4*a)-60*b^2*n^2-8*b*n*sinh(2*b*ln(c*x^n)+2*a)-4*b*n*sinh(4*b*
ln(c*x^n)+4*a)+cosh(4*b*ln(c*x^n)+4*a)+3)/(64*b^4*n^4-20*b^2*n^2+1)
```

3.242.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.53

$$\int \cosh^4(a+b\log(cx^n))\,dx = -\frac{(4\,b^2n^2-1)x\cosh(bn\log(x)+b\log(c)+a)^4-16\,(4\,b^3n^3-bn)x\cosh(bn\log(x)+b\log(c)+a)\sinh(\ldots}{\ldots}$$

input

```
integrate(cosh(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

output

```
-1/8*((4*b^2*n^2 - 1)*x*cosh(b*n*log(x) + b*log(c) + a)^4 - 16*(4*b^3*n^3
- b*n)*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3
 + (4*b^2*n^2 - 1)*x*sinh(b*n*log(x) + b*log(c) + a)^4 + 4*(16*b^2*n^2 - 1
)*x*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*(3*(4*b^2*n^2 - 1)*x*cosh(b*n*lo
g(x) + b*log(c) + a)^2 + 2*(16*b^2*n^2 - 1)*x)*sinh(b*n*log(x) + b*log(c)
+ a)^2 - 3*(64*b^4*n^4 - 20*b^2*n^2 + 1)*x - 16*((4*b^3*n^3 - b*n)*x*cosh(
b*n*log(x) + b*log(c) + a)^3 + (16*b^3*n^3 - b*n)*x*cosh(b*n*log(x) + b*lo
g(c) + a))*sinh(b*n*log(x) + b*log(c) + a))/(64*b^4*n^4 - 20*b^2*n^2 + 1)
```

3.242.6 Sympy [F]

$$\int \cosh^4(a+b\log(cx^n))\,dx$$
$$= \begin{cases} \int \cosh^4\left(a-\frac{\log(cx^n)}{2n}\right)dx \\ \int \cosh^4\left(a-\frac{\log(cx^n)}{4n}\right)dx \\ \int \cosh^4\left(a+\frac{\log(cx^n)}{4n}\right)dx \\ \int \cosh^4\left(a+\frac{\log(cx^n)}{2n}\right)dx \\ \frac{24b^4n^4x\sinh^4(a+b\log(cx^n))}{64b^4n^4-20b^2n^2+1} - \frac{48b^4n^4x\sinh^2(a+b\log(cx^n))\cosh^2(a+b\log(cx^n))}{64b^4n^4-20b^2n^2+1} + \frac{24b^4n^4x\cosh^4(a+b\log(cx^n))}{64b^4n^4-20b^2n^2+1} - \frac{24b^3n^3x\sinh^3(\ldots}{\ldots} \end{cases}$$

input

```
integrate(cosh(a+b*ln(c*x**n))**4,x)
```

output
```
Piecewise((Integral(cosh(a - log(c*x**n)/(2*n))**4, x), Eq(b, -1/(2*n))),
(Integral(cosh(a - log(c*x**n)/(4*n))**4, x), Eq(b, -1/(4*n))), (Integral(
cosh(a + log(c*x**n)/(4*n))**4, x), Eq(b, 1/(4*n))), (Integral(cosh(a + lo
g(c*x**n)/(2*n))**4, x), Eq(b, 1/(2*n))), (24*b**4*n**4*x*sinh(a + b*log(c
*x**n))**4/(64*b**4*n**4 - 20*b**2*n**2 + 1) - 48*b**4*n**4*x*sinh(a + b*l
og(c*x**n))**2*cosh(a + b*log(c*x**n))**2/(64*b**4*n**4 - 20*b**2*n**2 + 1
) + 24*b**4*n**4*x*cosh(a + b*log(c*x**n))**4/(64*b**4*n**4 - 20*b**2*n**2
 + 1) - 24*b**3*n**3*x*sinh(a + b*log(c*x**n))**3*cosh(a + b*log(c*x**n))/
(64*b**4*n**4 - 20*b**2*n**2 + 1) + 40*b**3*n**3*x*sinh(a + b*log(c*x**n))
*cosh(a + b*log(c*x**n))**3/(64*b**4*n**4 - 20*b**2*n**2 + 1) + 12*b**2*n*
*2*x*sinh(a + b*log(c*x**n))**2*cosh(a + b*log(c*x**n))**2/(64*b**4*n**4 -
 20*b**2*n**2 + 1) - 16*b**2*n**2*x*cosh(a + b*log(c*x**n))**4/(64*b**4*n*
*4 - 20*b**2*n**2 + 1) - 4*b*n*x*sinh(a + b*log(c*x**n))*cosh(a + b*log(c*
x**n))**3/(64*b**4*n**4 - 20*b**2*n**2 + 1) + x*cosh(a + b*log(c*x**n))**4
/(64*b**4*n**4 - 20*b**2*n**2 + 1), True))
```

3.242.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.68

$$\int \cosh^4(a+b\log(cx^n))\,dx = \frac{c^{4b}xe^{(4b\log(x^n)+4a)}}{16(4bn+1)} + \frac{c^{2b}xe^{(2b\log(x^n)+2a)}}{4(2bn+1)} + \frac{3}{8}x - \frac{xe^{(-2b\log(x^n)-2a)}}{4(2bc^{2b}n-c^{2b})} - \frac{xe^{(-4a)}}{16(4bc^{4b}n-c^{4b})(x^n)^{4b}}$$

input
```
integrate(cosh(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

output
```
1/16*c^(4*b)*x*e^(4*b*log(x^n) + 4*a)/(4*b*n + 1) + 1/4*c^(2*b)*x*e^(2*b*l
og(x^n) + 2*a)/(2*b*n + 1) + 3/8*x - 1/4*x*e^(-2*b*log(x^n) - 2*a)/(2*b*c^
(2*b)*n - c^(2*b)) - 1/16*x*e^(-4*a)/((4*b*c^(4*b)*n - c^(4*b))*(x^n)^(4*b
))
```

3.242.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 777 vs. $2(192) = 384$.

Time = 0.31 (sec) , antiderivative size = 777, normalized size of antiderivative = 4.07

$$\int \cosh^4(a+b\log(cx^n))\,dx = \frac{b^3c^{4\,b}n^3xx^{4\,bn}e^{(4\,a)}}{64\,b^4n^4-20\,b^2n^2+1}+\frac{8\,b^3c^{2\,b}n^3xx^{2\,bn}e^{(2\,a)}}{64\,b^4n^4-20\,b^2n^2+1}+\frac{24\,b^4n^4x}{64\,b^4n^4-20\,b^2n^2+1}-\frac{b^2c^{4\,b}n^2xx^{4\,bn}e^{(4\,a)}}{4\,(64\,b^4n^4-20\,b^2n^2+1)}-\frac{4\,b^2c^{2\,b}n^2xx^{2\,bn}e^{(2\,a)}}{64\,b^4n^4-20\,b^2n^2+1}-\frac{bc^{4\,b}nxx^{4\,bn}e^{(4\,a)}}{4\,(64\,b^4n^4-20\,b^2n^2+1)}-\frac{bc^{2\,b}nxx^{2\,bn}e^{(2\,a)}}{2\,(64\,b^4n^4-20\,b^2n^2+1)}-\frac{8\,b^3n^3xe^{(-2\,a)}}{(64\,b^4n^4-20\,b^2n^2+1)c^{2\,b}x^{2\,bn}}-\frac{b^3n^3xe^{(-4\,a)}}{(64\,b^4n^4-20\,b^2n^2+1)c^{4\,b}x^{4\,bn}}-\frac{15\,b^2n^2x}{2\,(64\,b^4n^4-20\,b^2n^2+1)}+\frac{c^{4\,b}xx^{4\,bn}e^{(4\,a)}}{16\,(64\,b^4n^4-20\,b^2n^2+1)}+\frac{c^{2\,b}xx^{2\,bn}e^{(2\,a)}}{4\,(64\,b^4n^4-20\,b^2n^2+1)}-\frac{4\,b^2n^2xe^{(-2\,a)}}{(64\,b^4n^4-20\,b^2n^2+1)c^{2\,b}x^{2\,bn}}-\frac{b^2n^2xe^{(-4\,a)}}{4\,(64\,b^4n^4-20\,b^2n^2+1)c^{4\,b}x^{4\,bn}}+\frac{bnxe^{(-2\,a)}}{2\,(64\,b^4n^4-20\,b^2n^2+1)c^{2\,b}x^{2\,bn}}+\frac{bnxe^{(-4\,a)}}{4\,(64\,b^4n^4-20\,b^2n^2+1)c^{4\,b}x^{4\,bn}}+\frac{3\,x}{8\,(64\,b^4n^4-20\,b^2n^2+1)}+\frac{xe^{(-2\,a)}}{4\,(64\,b^4n^4-20\,b^2n^2+1)c^{2\,b}x^{2\,bn}}+\frac{xe^{(-4\,a)}}{16\,(64\,b^4n^4-20\,b^2n^2+1)c^{4\,b}x^{4\,bn}}$$

input
```
integrate(cosh(a+b*log(c*x^n))^4,x, algorithm="giac")
```

output
```
b^3*c^(4*b)*n^3*x*x^(4*b*n)*e^(4*a)/(64*b^4*n^4 - 20*b^2*n^2 + 1) + 8*b^3*
c^(2*b)*n^3*x*x^(2*b*n)*e^(2*a)/(64*b^4*n^4 - 20*b^2*n^2 + 1) + 24*b^4*n^4
*x/(64*b^4*n^4 - 20*b^2*n^2 + 1) - 1/4*b^2*c^(4*b)*n^2*x*x^(4*b*n)*e^(4*a)
/(64*b^4*n^4 - 20*b^2*n^2 + 1) - 4*b^2*c^(2*b)*n^2*x*x^(2*b*n)*e^(2*a)/(64
*b^4*n^4 - 20*b^2*n^2 + 1) - 1/4*b*c^(4*b)*n*x*x^(4*b*n)*e^(4*a)/(64*b^4*n
^4 - 20*b^2*n^2 + 1) - 1/2*b*c^(2*b)*n*x*x^(2*b*n)*e^(2*a)/(64*b^4*n^4 - 2
0*b^2*n^2 + 1) - 8*b^3*n^3*x*e^(-2*a)/((64*b^4*n^4 - 20*b^2*n^2 + 1)*c^(2*
b)*x^(2*b*n)) - b^3*n^3*x*e^(-4*a)/((64*b^4*n^4 - 20*b^2*n^2 + 1)*c^(4*b)*
x^(4*b*n)) - 15/2*b^2*n^2*x/(64*b^4*n^4 - 20*b^2*n^2 + 1) + 1/16*c^(4*b)*x
*x^(4*b*n)*e^(4*a)/(64*b^4*n^4 - 20*b^2*n^2 + 1) + 1/4*c^(2*b)*x*x^(2*b*n)
*e^(2*a)/(64*b^4*n^4 - 20*b^2*n^2 + 1) - 4*b^2*n^2*x*e^(-2*a)/((64*b^4*n^4
 - 20*b^2*n^2 + 1)*c^(2*b)*x^(2*b*n)) - 1/4*b^2*n^2*x*e^(-4*a)/((64*b^4*n^
4 - 20*b^2*n^2 + 1)*c^(4*b)*x^(4*b*n)) + 1/2*b*n*x*e^(-2*a)/((64*b^4*n^4 -
 20*b^2*n^2 + 1)*c^(2*b)*x^(2*b*n)) + 1/4*b*n*x*e^(-4*a)/((64*b^4*n^4 - 20
*b^2*n^2 + 1)*c^(4*b)*x^(4*b*n)) + 3/8*x/(64*b^4*n^4 - 20*b^2*n^2 + 1) + 1
/4*x*e^(-2*a)/((64*b^4*n^4 - 20*b^2*n^2 + 1)*c^(2*b)*x^(2*b*n)) + 1/16*x*e
^(-4*a)/((64*b^4*n^4 - 20*b^2*n^2 + 1)*c^(4*b)*x^(4*b*n))
```

3.242.9 Mupad [B] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.53

$$\int \cosh^4\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{3\,x}{8}-\frac{x\,\mathrm{e}^{-2\,a}}{\left(c\,x^n\right)^{2\,b}\,\left(8\,b\,n-4\right)}+\frac{x\,\mathrm{e}^{2\,a}\,\left(c\,x^n\right)^{2\,b}}{8\,b\,n+4}-\frac{x\,\mathrm{e}^{-4\,a}}{\left(c\,x^n\right)^{4\,b}\,\left(64\,b\,n-16\right)}+\frac{x\,\mathrm{e}^{4\,a}\,\left(c\,x^n\right)^{4\,b}}{64\,b\,n+16}$$

input
```
int(cosh(a + b*log(c*x^n))^4,x)
```

output
```
(3*x)/8 - (x*exp(-2*a))/((c*x^n)^(2*b)*(8*b*n - 4)) + (x*exp(2*a)*(c*x^n)^
(2*b))/(8*b*n + 4) - (x*exp(-4*a))/((c*x^n)^(4*b)*(64*b*n - 16)) + (x*exp(
4*a)*(c*x^n)^(4*b))/(64*b*n + 16)
```

3.243 $\int x^m \cosh(a+b\log(cx^n))\,dx$

3.243.1 Optimal result

Integrand size = 15, antiderivative size = 73

$$\int x^m \cosh(a+b\log(cx^n))\,dx = \frac{(1+m)x^{1+m}\cosh(a+b\log(cx^n))}{(1+m)^2-b^2n^2} - \frac{bnx^{1+m}\sinh(a+b\log(cx^n))}{(1+m)^2-b^2n^2}$$

output
```
(1+m)*x^(1+m)*cosh(a+b*ln(c*x^n))/((1+m)^2-b^2*n^2)-b*n*x^(1+m)*sinh(a+b*l
n(c*x^n))/((1+m)^2-b^2*n^2)
```

3.243.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.74

$$\int x^m \cosh(a+b\log(cx^n))\,dx = \frac{x^{1+m}((1+m)\cosh(a+b\log(cx^n))-bn\sinh(a+b\log(cx^n)))}{(1+m-bn)(1+m+bn)}$$

input
```
Integrate[x^m*Cosh[a + b*Log[c*x^n]],x]
```

output
```
(x^(1 + m)*((1 + m)*Cosh[a + b*Log[c*x^n]] - b*n*Sinh[a + b*Log[c*x^n]]))/
((1 + m - b*n)*(1 + m + b*n))
```

3.243.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {6054}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \cosh(a + b\log(cx^n))\, dx$$

$$\Big\downarrow \text{ 6054}$$

$$\frac{(m+1)x^{m+1}\cosh(a+b\log(cx^n))}{(-bn+m+1)(bn+m+1)} - \frac{bnx^{m+1}\sinh(a+b\log(cx^n))}{(m+1)^2-b^2n^2}$$

input
```
Int[x^m*Cosh[a + b*Log[c*x^n]],x]
```

output
```
((1 + m)*x^(1 + m)*Cosh[a + b*Log[c*x^n]])/((1 + m - b*n)*(1 + m + b*n)) -
 (b*n*x^(1 + m)*Sinh[a + b*Log[c*x^n]])/((1 + m)^2 - b^2*n^2)
```

3.243.3.1 Defintions of rubi rules used

rule 6054
```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-(m + 1))*(e*x)^(m + 1)*(Cosh[d*(a + b*Log[c*x^n])]/(b^2*
d^2*e*n^2 - e*(m + 1)^2)), x] + Simp[b*d*n*(e*x)^(m + 1)*(Sinh[d*(a + b*Log
[c*x^n])]/(b^2*d^2*e*n^2 - e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}
, x] && NeQ[b^2*d^2*n^2 - (m + 1)^2, 0]
```

3.243.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93

method	result	size
parallelrisch	$\frac{x^{1+m}\left(bn\sinh(a+b\ln(c\,x^{n}))-\cosh(a+b\ln(c\,x^{n}))m-\cosh(a+b\ln(c\,x^{n}))\right)}{b^{2}n^{2}-m^{2}-2m-1}$	68

input
```
int(x^m*cosh(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

output
```
x^(1+m)/(b^2*n^2-m^2-2*m-1)*(b*n*sinh(a+b*ln(c*x^n))-cosh(a+b*ln(c*x^n))*m
-cosh(a+b*ln(c*x^n)))
```

3.243.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.36

$$\int x^m \cosh \left(a+b \log \left(c x^n\right)\right) dx =$$
$$-\frac{(m+1)x \cosh \left(bn \log (x)+b \log (c)+a\right) \cosh \left(m \log (x)\right)+(m+1)x \cosh \left(bn \log (x)+b \log (c)+a\right) \text{si}}{b^2n^2-m^2-2\,r}$$

input
```
integrate(x^m*cosh(a+b*log(c*x^n)),x, algorithm="fricas")
```

output
```
-((m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + (m + 1)*x*cos
h(b*n*log(x) + b*log(c) + a)*sinh(m*log(x)) - (b*n*x*cosh(m*log(x)) + b*n*
x*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a))/(b^2*n^2 - m^2 - 2*m -
1)
```

3.243.6 Sympy [F]

$$\int x^m \cosh \left(a+b \log \left(c x^n\right)\right) dx$$
$$= \begin{cases} \log (x) \cosh (a) & \text{for}\ b=0 \wedge m=-1 \\ \int x^m \cosh \left(-a+\frac{m \log (cx^n)}{n}+\frac{\log (cx^n)}{n}\right) dx & \text{for}\ b=-\frac{m+1}{n} \\ \int x^m \cosh \left(a+\frac{m \log (cx^n)}{n}+\frac{\log (cx^n)}{n}\right) dx & \text{for}\ b=\frac{m+1}{n} \\ \frac{bnxx^m \sinh (a+b\log (cx^n))}{b^2n^2-m^2-2m-1}-\frac{mxx^m \cosh (a+b\log (cx^n))}{b^2n^2-m^2-2m-1}-\frac{xx^m \cosh (a+b\log (cx^n))}{b^2n^2-m^2-2m-1} & \text{otherwise} \end{cases}$$

input
```
integrate(x**m*cosh(a+b*ln(c*x**n)),x)
```

output
```
Piecewise((log(x)*cosh(a), Eq(b, 0) & Eq(m, -1)), (Integral(x**m*cosh(-a +
 m*log(c*x**n)/n + log(c*x**n)/n), x), Eq(b, -(m + 1)/n)), (Integral(x**m*
cosh(a + m*log(c*x**n)/n + log(c*x**n)/n), x), Eq(b, (m + 1)/n)), (b*n*x*x
**m*sinh(a + b*log(c*x**n))/(b**2*n**2 - m**2 - 2*m - 1) - m*x*x**m*cosh(a
 + b*log(c*x**n))/(b**2*n**2 - m**2 - 2*m - 1) - x*x**m*cosh(a + b*log(c*x
**n))/(b**2*n**2 - m**2 - 2*m - 1), True))
```

3.243.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int x^m \cosh(a + b\log(cx^n))\, dx = \frac{c^b x e^{(b\log(x^n)+m\log(x)+a)}}{2(bn+m+1)} - \frac{x e^{(-b\log(x^n)+m\log(x)-a)}}{2(bc^b n - c^b(m+1))}$$

input
```
integrate(x^m*cosh(a+b*log(c*x^n)),x, algorithm="maxima")
```

output
```
1/2*c^b*x*e^(b*log(x^n) + m*log(x) + a)/(b*n + m + 1) - 1/2*x*e^(-b*log(x^
n) + m*log(x) - a)/(b*c^b*n - c^b*(m + 1))
```

3.243.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(75) = 150.

Time = 0.27 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.22

$$\begin{aligned}\int x^m \cosh(a + b\log(cx^n))\, dx = {} & \frac{bc^b nx x^{bn} x^m e^a}{2(b^2n^2 - m^2 - 2m - 1)} - \frac{c^b mx x^{bn} x^m e^a}{2(b^2n^2 - m^2 - 2m - 1)} \\ & - \frac{c^b x x^{bn} x^m e^a}{2(b^2n^2 - m^2 - 2m - 1)} - \frac{bnx x^m e^{(-a)}}{2(b^2n^2 - m^2 - 2m - 1)c^b x^{bn}} \\ & - \frac{mx x^m e^{(-a)}}{2(b^2n^2 - m^2 - 2m - 1)c^b x^{bn}} \\ & - \frac{x x^m e^{(-a)}}{2(b^2n^2 - m^2 - 2m - 1)c^b x^{bn}}\end{aligned}$$

input
```
integrate(x^m*cosh(a+b*log(c*x^n)),x, algorithm="giac")
```

output
```
1/2*b*c^b*n*x*x^(b*n)*x^m*e^a/(b^2*n^2 - m^2 - 2*m - 1) - 1/2*c^b*m*x*x^(b
*n)*x^m*e^a/(b^2*n^2 - m^2 - 2*m - 1) - 1/2*c^b*x*x^(b*n)*x^m*e^a/(b^2*n^2
 - m^2 - 2*m - 1) - 1/2*b*n*x*x^m*e^(-a)/((b^2*n^2 - m^2 - 2*m - 1)*c^b*x^
(b*n)) - 1/2*m*x*x^m*e^(-a)/((b^2*n^2 - m^2 - 2*m - 1)*c^b*x^(b*n)) - 1/2*
x*x^m*e^(-a)/((b^2*n^2 - m^2 - 2*m - 1)*c^b*x^(b*n))
```

3.243.9 Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.75

$$\int x^m \cosh(a+b\log(cx^n))\,dx = \frac{x\,x^m\,\mathrm{e}^{-a}}{(c\,x^n)^b\,(2\,m-2\,b\,n+2)}+\frac{x\,x^m\,\mathrm{e}^{a}\,(c\,x^n)^b}{2\,m+2\,b\,n+2}$$

input
```
int(x^m*cosh(a + b*log(c*x^n)),x)
```

output
```
(x*x^m*exp(-a))/((c*x^n)^b*(2*m - 2*b*n + 2)) + (x*x^m*exp(a)*(c*x^n)^b)/(
2*m + 2*b*n + 2)
```

3.244 $\int x^m \cosh^2(a+b\log(cx^n))\,dx$

3.244.1 Optimal result

Integrand size = 17, antiderivative size = 120

$$\int x^m \cosh^2(a+b\log(cx^n))\,dx = -\frac{2b^2n^2x^{1+m}}{(1+m)\left((1+m)^2-4b^2n^2\right)} + \frac{(1+m)x^{1+m}\cosh^2(a+b\log(cx^n))}{(1+m)^2-4b^2n^2} - \frac{2bnx^{1+m}\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{(1+m)^2-4b^2n^2}$$

output
```
-2*b^2*n^2*x^(1+m)/(1+m)/((1+m)^2-4*b^2*n^2)+(1+m)*x^(1+m)*cosh(a+b*ln(c*x
^n))^2/((1+m)^2-4*b^2*n^2)-2*b*n*x^(1+m)*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c
*x^n))/((1+m)^2-4*b^2*n^2)
```

3.244.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.72

$$\int x^m \cosh^2(a+b\log(cx^n))\,dx = \frac{x^{1+m}\left(1+2m+m^2-4b^2n^2+(1+m)^2\cosh(2(a+b\log(cx^n)))-2b(1+m)n\sinh(2(a+b\log(cx^n)))\right)}{2(1+m)(1+m-2bn)(1+m+2bn)}$$

input
```
Integrate[x^m*Cosh[a + b*Log[c*x^n]]^2,x]
```

output
```
(x^(1 + m)*(1 + 2*m + m^2 - 4*b^2*n^2 + (1 + m)^2*Cosh[2*(a + b*Log[c*x^n]
)] - 2*b*(1 + m)*n*Sinh[2*(a + b*Log[c*x^n])]))/(2*(1 + m)*(1 + m - 2*b*n)
*(1 + m + 2*b*n))
```

3.244.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {6056, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \cosh^2(a + b\log(cx^n))\,dx$$

$$\Big\downarrow \text{6056}$$

$$-\frac{2b^2n^2\int x^m dx}{(m+1)^2-4b^2n^2}+\frac{(m+1)x^{m+1}\cosh^2(a+b\log(cx^n))}{-4b^2n^2+m^2+2m+1}-\frac{2bnx^{m+1}\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{(m+1)^2-4b^2n^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{(m+1)x^{m+1}\cosh^2(a+b\log(cx^n))}{-4b^2n^2+m^2+2m+1}-\frac{2bnx^{m+1}\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{(m+1)^2-4b^2n^2}-\frac{2b^2n^2x^{m+1}}{(m+1)\left((m+1)^2-4b^2n^2\right)}$$

input
```
Int[x^m*Cosh[a + b*Log[c*x^n]]^2,x]
```

output
```
(-2*b^2*n^2*x^(1 + m))/((1 + m)*((1 + m)^2 - 4*b^2*n^2)) + ((1 + m)*x^(1 +
 m)*Cosh[a + b*Log[c*x^n]]^2)/(1 + 2*m + m^2 - 4*b^2*n^2) - (2*b*n*x^(1 +
m)*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/((1 + m)^2 - 4*b^2*n^2)
```

3.244.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6056
```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-(m + 1))*(e*x)^(m + 1)*(Cosh[d*(a + b*Log[c*x^n])]^
p/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2)), x] + (Simp[b*d*n*p*(e*x)^(m + 1)*Sinh
[d*(a + b*Log[c*x^n])]*(Cosh[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p
^2 - e*(m + 1)^2)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 - (m
+ 1)^2))   Int[(e*x)^m*Cosh[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ
[{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - (m + 1)^2
, 0]
```

3.244.4 Maple [A] (verified)

Time = 4.94 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.70

method	result	size
parallelrisch	$\frac{\left(2b^2n^2+bn(1+m)\sinh(2b\ln(c\,x^n)+2a)-\frac{(1+m)^2(\cosh(2b\ln(c\,x^n)+2a)+1)}{2}\right)x^{1+m}}{(4b^2n^2-m^2-2m-1)(1+m)}$	84

input
```
int(x^m*cosh(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
```

output
```
(2*b^2*n^2+b*n*(1+m)*sinh(2*b*ln(c*x^n)+2*a)-1/2*(1+m)^2*(cosh(2*b*ln(c*x^
n)+2*a)+1))*x^(1+m)/(4*b^2*n^2-m^2-2*m-1)/(1+m)
```

3.244.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.08

$$\int x^m \cosh^2(a+b\log(cx^n))\,dx$$
$$= \frac{(m^2+2m+1)x\cosh(bn\log(x)+b\log(c)+a)^2\cosh(m\log(x)) - (4b^2n^2-m^2-2m-1)x\cosh(m\log$$

input
```
integrate(x^m*cosh(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

output
```
1/2*((m^2 + 2*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2*cosh(m*log(x)) -
(4*b^2*n^2 - m^2 - 2*m - 1)*x*cosh(m*log(x)) + ((m^2 + 2*m + 1)*x*cosh(m*l
og(x)) + (m^2 + 2*m + 1)*x*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a)
^2 - 4*((b*m + b)*n*x*cosh(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + (b*
m + b)*n*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(m*log(x)))*sinh(b*n*log(x)
 + b*log(c) + a) + ((m^2 + 2*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 -
(4*b^2*n^2 - m^2 - 2*m - 1)*x)*sinh(m*log(x)))/(m^3 - 4*(b^2*m + b^2)*n^2
+ 3*m^2 + 3*m + 1)
```

3.244.6 Sympy [F]

$$\int x^m \cosh^2(a + b\log(cx^n))\,dx$$
$$= \begin{cases} \log(x)\cosh^2(a) \\ \int x^m \cosh^2\left(-a + \frac{m\log(cx^n)}{2n} + \frac{\log(cx^n)}{2n}\right)dx \\ \int x^m \cosh^2\left(a + \frac{m\log(cx^n)}{2n} + \frac{\log(cx^n)}{2n}\right)dx \\ \int \frac{\cosh^2(a+b\log(cx^n))}{x}\,dx \\ -\frac{2b^2n^2xx^m\sinh^2(a+b\log(cx^n))}{4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1} + \frac{2b^2n^2xx^m\cosh^2(a+b\log(cx^n))}{4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1} + \frac{2bmnxx^m\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1} + \frac{2bn}{} \end{cases}$$

input
```
integrate(x**m*cosh(a+b*ln(c*x**n))**2,x)
```

output
```
Piecewise((log(x)*cosh(a)**2, Eq(b, 0) & Eq(m, -1)), (Integral(x**m*cosh(-
a + m*log(c*x**n)/(2*n) + log(c*x**n)/(2*n))**2, x), Eq(b, -(m + 1)/(2*n))
), (Integral(x**m*cosh(a + m*log(c*x**n)/(2*n) + log(c*x**n)/(2*n))**2, x)
, Eq(b, (m + 1)/(2*n))), (Integral(cosh(a + b*log(c*x**n))**2/x, x), Eq(m,
 -1)), (-2*b**2*n**2*x*x**m*sinh(a + b*log(c*x**n))**2/(4*b**2*m*n**2 + 4*
b**2*n**2 - m**3 - 3*m**2 - 3*m - 1) + 2*b**2*n**2*x*x**m*cosh(a + b*log(c
*x**n))**2/(4*b**2*m*n**2 + 4*b**2*n**2 - m**3 - 3*m**2 - 3*m - 1) + 2*b*m
*n*x*x**m*sinh(a + b*log(c*x**n))*cosh(a + b*log(c*x**n))/(4*b**2*m*n**2 +
 4*b**2*n**2 - m**3 - 3*m**2 - 3*m - 1) + 2*b*n*x*x**m*sinh(a + b*log(c*x*
*n))*cosh(a + b*log(c*x**n))/(4*b**2*m*n**2 + 4*b**2*n**2 - m**3 - 3*m**2
- 3*m - 1) - m**2*x*x**m*cosh(a + b*log(c*x**n))**2/(4*b**2*m*n**2 + 4*b**
2*n**2 - m**3 - 3*m**2 - 3*m - 1) - 2*m*x*x**m*cosh(a + b*log(c*x**n))**2/
(4*b**2*m*n**2 + 4*b**2*n**2 - m**3 - 3*m**2 - 3*m - 1) - x*x**m*cosh(a +
b*log(c*x**n))**2/(4*b**2*m*n**2 + 4*b**2*n**2 - m**3 - 3*m**2 - 3*m - 1),
 True))
```

3.244.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.72

$$\int x^m \cosh^2(a + b\log(cx^n)) \, dx = \frac{c^{2b}xe^{(2b\log(x^n)+m\log(x)+2a)}}{4(2bn+m+1)} - \frac{xe^{(-2b\log(x^n)+m\log(x)-2a)}}{4(2bc^{2b}n - c^{2b}(m+1))} + \frac{x^{m+1}}{2(m+1)}$$

input
```
integrate(x^m*cosh(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

output
```
1/4*c^(2*b)*x*e^(2*b*log(x^n) + m*log(x) + 2*a)/(2*b*n + m + 1) - 1/4*x*e^
(-2*b*log(x^n) + m*log(x) - 2*a)/(2*b*c^(2*b)*n - c^(2*b)*(m + 1)) + 1/2*x
^(m + 1)/(m + 1)
```

3.244.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 759 vs. $2(127) = 254$.

Time = 0.30 (sec) , antiderivative size = 759, normalized size of antiderivative = 6.32

$$\int x^m \cosh^2(a + b\log(cx^n))\,dx = \frac{bc^{2\,b}mnxx^{2\,bn}x^m e^{(2\,a)}}{2\,(4\,b^2mn^2 + 4\,b^2n^2 - m^3 - 3\,m^2 - 3\,m - 1)} - \frac{c^{2\,b}m^2xx^{2\,bn}x^m e^{(2\,a)}}{4\,(4\,b^2mn^2 + 4\,b^2n^2 - m^3 - 3\,m^2 - 3\,m - 1)} + \frac{bc^{2\,b}nxx^{2\,bn}x^m e^{(2\,a)}}{2\,(4\,b^2mn^2 + 4\,b^2n^2 - m^3 - 3\,m^2 - 3\,m - 1)} + \frac{2\,b^2n^2xx^m}{4\,b^2mn^2 + 4\,b^2n^2 - m^3 - 3\,m^2 - 3\,m - 1} - \frac{c^{2\,b}mxx^{2\,bn}x^m e^{(2\,a)}}{2\,(4\,b^2mn^2 + 4\,b^2n^2 - m^3 - 3\,m^2 - 3\,m - 1)} - \frac{c^{2\,b}xx^{2\,bn}x^m e^{(2\,a)}}{4\,(4\,b^2mn^2 + 4\,b^2n^2 - m^3 - 3\,m^2 - 3\,m - 1)} - \frac{m^2xx^m}{2\,(4\,b^2mn^2 + 4\,b^2n^2 - m^3 - 3\,m^2 - 3\,m - 1)} - \frac{bmnxx^m e^{(-2\,a)}}{2\,(4\,b^2mn^2 + 4\,b^2n^2 - m^3 - 3\,m^2 - 3\,m - 1)c^{2\,b}x^{2\,bn}} - \frac{mxx^m}{4\,b^2mn^2 + 4\,b^2n^2 - m^3 - 3\,m^2 - 3\,m - 1} - \frac{m^2xx^m e^{(-2\,a)}}{4\,(4\,b^2mn^2 + 4\,b^2n^2 - m^3 - 3\,m^2 - 3\,m - 1)c^{2\,b}x^{2\,bn}} - \frac{bnxx^m e^{(-2\,a)}}{2\,(4\,b^2mn^2 + 4\,b^2n^2 - m^3 - 3\,m^2 - 3\,m - 1)c^{2\,b}x^{2\,bn}} - \frac{xx^m}{2\,(4\,b^2mn^2 + 4\,b^2n^2 - m^3 - 3\,m^2 - 3\,m - 1)} - \frac{mxx^m e^{(-2\,a)}}{2\,(4\,b^2mn^2 + 4\,b^2n^2 - m^3 - 3\,m^2 - 3\,m - 1)c^{2\,b}x^{2\,bn}} - \frac{xx^m e^{(-2\,a)}}{4\,(4\,b^2mn^2 + 4\,b^2n^2 - m^3 - 3\,m^2 - 3\,m - 1)c^{2\,b}x^{2\,bn}}$$

input
```
integrate(x^m*cosh(a+b*log(c*x^n))^2,x, algorithm="giac")
```

3.244. $\int x^m \cosh^2(a + b\log(cx^n))\,dx$

output
```
1/2*b*c^(2*b)*m*n*x*x^(2*b*n)*x^m*e^(2*a)/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 -
 3*m^2 - 3*m - 1) - 1/4*c^(2*b)*m^2*x*x^(2*b*n)*x^m*e^(2*a)/(4*b^2*m*n^2 +
 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) + 1/2*b*c^(2*b)*n*x*x^(2*b*n)*x^m*e^(2
*a)/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) + 2*b^2*n^2*x*x^m/(4
*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/2*c^(2*b)*m*x*x^(2*b*n
)*x^m*e^(2*a)/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/4*c^(2
*b)*x*x^(2*b*n)*x^m*e^(2*a)/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m -
 1) - 1/2*m^2*x*x^m/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/
2*b*m*n*x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1)*
c^(2*b)*x^(2*b*n)) - m*x*x^m/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m
- 1) - 1/4*m^2*x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*
m - 1)*c^(2*b)*x^(2*b*n)) - 1/2*b*n*x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n
^2 - m^3 - 3*m^2 - 3*m - 1)*c^(2*b)*x^(2*b*n)) - 1/2*x*x^m/(4*b^2*m*n^2 +
4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/2*m*x*x^m*e^(-2*a)/((4*b^2*m*n^2 +
4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1)*c^(2*b)*x^(2*b*n)) - 1/4*x*x^m*e^(-2*a)
/((4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1)*c^(2*b)*x^(2*b*n))
```

3.244.9 Mupad [B] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.61

$$\int x^m \cosh^2(a+b\log(cx^n))\,dx = \frac{x\,x^m}{2\,m+2} + \frac{x\,x^m\,\mathrm{e}^{-2\,a}}{(c\,x^n)^{2\,b}\,(4\,m-8\,b\,n+4)} + \frac{x\,x^m\,\mathrm{e}^{2\,a}\,(c\,x^n)^{2\,b}}{4\,m+8\,b\,n+4}$$

input
```
int(x^m*cosh(a + b*log(c*x^n))^2,x)
```

output
```
(x*x^m)/(2*m + 2) + (x*x^m*exp(-2*a))/((c*x^n)^(2*b)*(4*m - 8*b*n + 4)) +
(x*x^m*exp(2*a)*(c*x^n)^(2*b))/(4*m + 8*b*n + 4)
```

3.245 $\int x^m \cosh^3(a+b\log(cx^n))\,dx$

3.245.1 Optimal result

Integrand size = 17, antiderivative size = 203

$$\begin{aligned}\int x^m \cosh^3(a+b\log(cx^n))\,dx = &-\frac{6b^2(1+m)n^2x^{1+m}\cosh(a+b\log(cx^n))}{((1+m)^2-9b^2n^2)((1+m)^2-b^2n^2)} \\ &+\frac{(1+m)x^{1+m}\cosh^3(a+b\log(cx^n))}{(1+m)^2-9b^2n^2} \\ &+\frac{6b^3n^3x^{1+m}\sinh(a+b\log(cx^n))}{((1+m)^2-9b^2n^2)((1+m)^2-b^2n^2)} \\ &-\frac{3bnx^{1+m}\cosh^2(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{(1+m)^2-9b^2n^2}\end{aligned}$$

output
```
-6*b^2*(1+m)*n^2*x^(1+m)*cosh(a+b*ln(c*x^n))/((1+m)^2-9*b^2*n^2)/((1+m)^2-
b^2*n^2)+(1+m)*x^(1+m)*cosh(a+b*ln(c*x^n))^3/((1+m)^2-9*b^2*n^2)+6*b^3*n^3
*x^(1+m)*sinh(a+b*ln(c*x^n))/((1+m)^2-9*b^2*n^2)/((1+m)^2-b^2*n^2)-3*b*n*x
^(1+m)*cosh(a+b*ln(c*x^n))^2*sinh(a+b*ln(c*x^n))/((1+m)^2-9*b^2*n^2)
```

3.245.2 Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.44

$$\int x^m \cosh^3(a+b\log(cx^n))\,dx$$

$$= \frac{1}{4}x^{1+m}\left(\frac{3\sinh(bn\log(x))\left(-bn\cosh(a-bn\log(x)+b\log(cx^n))+(1+m)\sinh(a-bn\log(x)+b\log(cx\right.}{(1+m-bn)(1+m+bn)}\right.$$

$$+\frac{3\cosh(bn\log(x))\left((1+m)\cosh(a-bn\log(x)+b\log(cx^n))-bn\sinh(a-bn\log(x)+b\log(cx^n))\right)}{(1+m-bn)(1+m+bn)}$$

$$+\frac{\sinh(3bn\log(x))\left(-3bn\cosh(3(a-bn\log(x)+b\log(cx^n)))+(1+m)\sinh(3(a-bn\log(x)+b\log(cx\right.}{(1+m-3bn)(1+m+3bn)}$$

$$+\frac{\cosh(3bn\log(x))\left((1+m)\cosh(3(a-bn\log(x)+b\log(cx^n)))-3bn\sinh(3(a-bn\log(x)+b\log(cx^n\right.}{(1+m-3bn)(1+m+3bn)}$$

input
```
Integrate[x^m*Cosh[a + b*Log[c*x^n]]^3,x]
```

output
```
(x^(1 + m)*((3*Sinh[b*n*Log[x]]*(-(b*n*Cosh[a - b*n*Log[x] + b*Log[c*x^n]]
) + (1 + m)*Sinh[a - b*n*Log[x] + b*Log[c*x^n]]))/((1 + m - b*n)*(1 + m +
b*n)) + (3*Cosh[b*n*Log[x]]*((1 + m)*Cosh[a - b*n*Log[x] + b*Log[c*x^n]] -
 b*n*Sinh[a - b*n*Log[x] + b*Log[c*x^n]]))/((1 + m - b*n)*(1 + m + b*n)) +
 (Sinh[3*b*n*Log[x]]*(-3*b*n*Cosh[3*(a - b*n*Log[x] + b*Log[c*x^n])] + (1
+ m)*Sinh[3*(a - b*n*Log[x] + b*Log[c*x^n])]))/((1 + m - 3*b*n)*(1 + m + 3
*b*n)) + (Cosh[3*b*n*Log[x]]*((1 + m)*Cosh[3*(a - b*n*Log[x] + b*Log[c*x^n
])] - 3*b*n*Sinh[3*(a - b*n*Log[x] + b*Log[c*x^n])]))/((1 + m - 3*b*n)*(1
+ m + 3*b*n))))/4
```

3.245.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {6056, 6054}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \cosh^3(a+b\log(cx^n))\,dx$$

$$\downarrow \text{6056}$$

$$-\frac{6b^2n^2\int x^m\cosh(a+b\log(cx^n))\,dx}{(m+1)^2-9b^2n^2}+\frac{(m+1)x^{m+1}\cosh^3(a+b\log(cx^n))}{-9b^2n^2+m^2+2m+1}-\frac{3bnx^{m+1}\sinh(a+b\log(cx^n))\cosh^2(a+b\log(cx^n))}{(m+1)^2-9b^2n^2}$$

$$\Big\downarrow \text{ 6054}$$

$$\frac{(m+1)x^{m+1}\cosh^3(a+b\log(cx^n))}{-9b^2n^2+m^2+2m+1}-\frac{3bnx^{m+1}\sinh(a+b\log(cx^n))\cosh^2(a+b\log(cx^n))}{(m+1)^2-9b^2n^2}-\frac{6b^2n^2\left(\frac{(m+1)x^{m+1}\cosh(a+b\log(cx^n))}{(-bn+m+1)(bn+m+1)}-\frac{bnx^{m+1}\sinh(a+b\log(cx^n))}{(m+1)^2-b^2n^2}\right)}{(m+1)^2-9b^2n^2}$$

input
```
Int[x^m*Cosh[a + b*Log[c*x^n]]^3,x]
```

output
```
((1 + m)*x^(1 + m)*Cosh[a + b*Log[c*x^n]]^3)/(1 + 2*m + m^2 - 9*b^2*n^2) -
 (3*b*n*x^(1 + m)*Cosh[a + b*Log[c*x^n]]^2*Sinh[a + b*Log[c*x^n]])/((1 + m
)^2 - 9*b^2*n^2) - (6*b^2*n^2*(((1 + m)*x^(1 + m)*Cosh[a + b*Log[c*x^n]])/
((1 + m - b*n)*(1 + m + b*n)) - (b*n*x^(1 + m)*Sinh[a + b*Log[c*x^n]])/((1
 + m)^2 - b^2*n^2)))/((1 + m)^2 - 9*b^2*n^2)
```

3.245.3.1 Defintions of rubi rules used

rule 6054
```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-(m + 1))*(e*x)^(m + 1)*(Cosh[d*(a + b*Log[c*x^n])]/(b^2*
d^2*e*n^2 - e*(m + 1)^2)), x] + Simp[b*d*n*(e*x)^(m + 1)*(Sinh[d*(a + b*Log
[c*x^n])]/(b^2*d^2*e*n^2 - e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}
, x] && NeQ[b^2*d^2*n^2 - (m + 1)^2, 0]
```

rule 6056
```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-(m + 1))*(e*x)^(m + 1)*(Cosh[d*(a + b*Log[c*x^n])]^
p/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2)), x] + (Simp[b*d*n*p*(e*x)^(m + 1)*Sinh
[d*(a + b*Log[c*x^n])]*(Cosh[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p
^2 - e*(m + 1)^2)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 - (m
+ 1)^2))   Int[(e*x)^m*Cosh[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ
[{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - (m + 1)^2
, 0]
```

3.245.4 Maple [A] (verified)

Time = 36.28 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.88

method	result
parallelrisch	$-\frac{27\left(\frac{(1+m)(bn+m+1)(bn-m-1)\cosh(3b\ln(c\,x^{n})+3a)}{27}-\frac{bn(bn+m+1)(bn-m-1)\sinh(3b\ln(c\,x^{n})+3a)}{9}+\left(bn-\frac{m}{3}-\frac{1}{3}\right)((1+m)\cosh(a+\right.}{4m^{4}+16m^{3}+(-40b^{2}n^{2}+24)m^{2}+(-80b^{2}n^{2}+16)m+36b^{4}n^{4}-40b^{2}n^{2}+}$

input
```
int(x^m*cosh(a+b*ln(c*x^n))^3,x,method=_RETURNVERBOSE)
```

output
```
-27*(1/27*(1+m)*(b*n+m+1)*(b*n-m-1)*cosh(3*b*ln(c*x^n)+3*a)-1/9*b*n*(b*n+m
+1)*(b*n-m-1)*sinh(3*b*ln(c*x^n)+3*a)+(b*n-1/3*m-1/3)*((1+m)*cosh(a+b*ln(c
*x^n))-b*n*sinh(a+b*ln(c*x^n)))*(b*n+1/3*m+1/3))*x^(1+m)/(4*m^4+16*m^3+(-4
0*b^2*n^2+24)*m^2+(-80*b^2*n^2+16)*m+36*b^4*n^4-40*b^2*n^2+4)
```

3.245.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 584 vs. 2(214) = 428.

Time = 0.28 (sec) , antiderivative size = 584, normalized size of antiderivative = 2.88

$$\int x^{m}\cosh^{3}(a+b\log(cx^{n}))\,dx$$
$$=\frac{(m^{3}-(b^{2}m+b^{2})n^{2}+3\,m^{2}+3\,m+1)x\cosh(bn\log(x)+b\log(c)+a)^{3}\cosh(m\log(x))+3\,(m^{3}-9\,(b^{2}}{}$$

input
```
integrate(x^m*cosh(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

output
```
1/4*((m^3 - (b^2*m + b^2)*n^2 + 3*m^2 + 3*m + 1)*x*cosh(b*n*log(x) + b*log
(c) + a)^3*cosh(m*log(x)) + 3*(m^3 - 9*(b^2*m + b^2)*n^2 + 3*m^2 + 3*m + 1
)*x*cosh(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + 3*((b^3*n^3 - (b*m^2
+ 2*b*m + b)*n)*x*cosh(m*log(x)) + (b^3*n^3 - (b*m^2 + 2*b*m + b)*n)*x*sin
h(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a)^3 + 3*((m^3 - (b^2*m + b^2)*n
^2 + 3*m^2 + 3*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + (
m^3 - (b^2*m + b^2)*n^2 + 3*m^2 + 3*m + 1)*x*cosh(b*n*log(x) + b*log(c) +
a)*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a)^2 + 3*(3*(b^3*n^3 - (b*
m^2 + 2*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)^2*cosh(m*log(x)) + (
9*b^3*n^3 - (b*m^2 + 2*b*m + b)*n)*x*cosh(m*log(x)) + (3*(b^3*n^3 - (b*m^2
 + 2*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)^2 + (9*b^3*n^3 - (b*m^2
 + 2*b*m + b)*n)*x)*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a) + ((m^
3 - (b^2*m + b^2)*n^2 + 3*m^2 + 3*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)
^3 + 3*(m^3 - 9*(b^2*m + b^2)*n^2 + 3*m^2 + 3*m + 1)*x*cosh(b*n*log(x) + b
*log(c) + a))*sinh(m*log(x)))/(9*b^4*n^4 + m^4 + 4*m^3 - 10*(b^2*m^2 + 2*b
^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)
```

3.245.6 Sympy [F]

$$\int x^m \cosh^3(a + b\log(cx^n))\,dx = \text{Too large to display}$$

input
```
integrate(x**m*cosh(a+b*ln(c*x**n))**3,x)
```

output
```
Piecewise((log(x)*cosh(a)**3, Eq(b, 0) & Eq(m, -1)), (Integral(x**m*cosh(-
a + m*log(c*x**n)/(3*n) + log(c*x**n)/(3*n))**3, x), Eq(b, (-m - 1)/(3*n))
), (Integral(x**m*cosh(-a + m*log(c*x**n)/n + log(c*x**n)/n)**3, x), Eq(b,
 (-m - 1)/n)), (Integral(x**m*cosh(a + m*log(c*x**n)/(3*n) + log(c*x**n)/(
3*n))**3, x), Eq(b, (m + 1)/(3*n))), (Integral(x**m*cosh(a + m*log(c*x**n)
/n + log(c*x**n)/n)**3, x), Eq(b, (m + 1)/n)), (-6*b**3*n**3*x*x**m*sinh(a
 + b*log(c*x**n))**3/(9*b**4*n**4 - 10*b**2*m**2*n**2 - 20*b**2*m*n**2 - 1
0*b**2*n**2 + m**4 + 4*m**3 + 6*m**2 + 4*m + 1) + 9*b**3*n**3*x*x**m*sinh(
a + b*log(c*x**n))*cosh(a + b*log(c*x**n))**2/(9*b**4*n**4 - 10*b**2*m**2*
n**2 - 20*b**2*m*n**2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*m**2 + 4*m + 1) +
 6*b**2*m*n**2*x*x**m*sinh(a + b*log(c*x**n))**2*cosh(a + b*log(c*x**n))/(
9*b**4*n**4 - 10*b**2*m**2*n**2 - 20*b**2*m*n**2 - 10*b**2*n**2 + m**4 + 4
*m**3 + 6*m**2 + 4*m + 1) - 7*b**2*m*n**2*x*x**m*cosh(a + b*log(c*x**n))**
3/(9*b**4*n**4 - 10*b**2*m**2*n**2 - 20*b**2*m*n**2 - 10*b**2*n**2 + m**4
+ 4*m**3 + 6*m**2 + 4*m + 1) + 6*b**2*n**2*x*x**m*sinh(a + b*log(c*x**n))*
*2*cosh(a + b*log(c*x**n))/(9*b**4*n**4 - 10*b**2*m**2*n**2 - 20*b**2*m*n*
*2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*m**2 + 4*m + 1) - 7*b**2*n**2*x*x**m
*cosh(a + b*log(c*x**n))**3/(9*b**4*n**4 - 10*b**2*m**2*n**2 - 20*b**2*m*n
**2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*m**2 + 4*m + 1) - 3*b*m**2*n*x*x**m
*sinh(a + b*log(c*x**n))*cosh(a + b*log(c*x**n))**2/(9*b**4*n**4 - 10*b...
```

3.245.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.68

$$\int x^m \cosh^3(a+b\log(cx^n))\,dx = \frac{c^{3b}xe^{(3b\log(x^n)+m\log(x)+3a)}}{8(3bn+m+1)} + \frac{3c^bxe^{(b\log(x^n)+m\log(x)+a)}}{8(bn+m+1)} - \frac{3xe^{(-b\log(x^n)+m\log(x)-a)}}{8(bc^bn-c^b(m+1))} - \frac{xe^{(-3b\log(x^n)+m\log(x)-3a)}}{8(3bc^{3b}n-c^{3b}(m+1))}$$

input
```
integrate(x^m*cosh(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

output
```
1/8*c^(3*b)*x*e^(3*b*log(x^n) + m*log(x) + 3*a)/(3*b*n + m + 1) + 3/8*c^b*
x*e^(b*log(x^n) + m*log(x) + a)/(b*n + m + 1) - 3/8*x*e^(-b*log(x^n) + m*l
og(x) - a)/(b*c^b*n - c^b*(m + 1)) - 1/8*x*e^(-3*b*log(x^n) + m*log(x) - 3
*a)/(3*b*c^(3*b)*n - c^(3*b)*(m + 1))
```

3.245.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3225 vs. $2(214) = 428$.

Time = 0.34 (sec) , antiderivative size = 3225, normalized size of antiderivative = 15.89

$$\int x^m \cosh^3 (a + b \log (cx^n)) \, dx = \text{Too large to display}$$

input
```
integrate(x^m*cosh(a+b*log(c*x^n))^3,x, algorithm="giac")
```

output
```
3/8*b^3*c^(3*b)*n^3*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 -
20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 27/8*b^3*c^b*
n^3*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 1
0*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 1/8*b^2*c^(3*b)*m*n^2*x*x^(3*b*n)*x
^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 +
 4*m^3 + 6*m^2 + 4*m + 1) - 27/8*b^2*c^b*m*n^2*x*x^(b*n)*x^m*e^a/(9*b^4*n^
4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m
 + 1) - 3/8*b*c^(3*b)*m^2*n*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^
2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 1/8*b
^2*c^(3*b)*n^2*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^
2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 3/8*b*c^b*m^2*n*x*
x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*
n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 27/8*b^2*c^b*n^2*x*x^(b*n)*x^m*e^a/(9*b^4
*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 +
4*m + 1) + 1/8*c^(3*b)*m^3*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2
*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 3/4*b*
c^(3*b)*m*n*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m
*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/8*c^b*m^3*x*x^(b*n)
*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4
*m^3 + 6*m^2 + 4*m + 1) - 3/4*b*c^b*m*n*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - ...
```

3.245.9 Mupad [B] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.58

$$\int x^m \cosh^3 (a + b \log (cx^n)) \, dx = \frac{3\,x\,x^m\,\mathrm{e}^{-a}}{\left(c\,x^n\right)^b\,\left(8\,m-8\,b\,n+8\right)} + \frac{x\,x^m\,\mathrm{e}^{-3\,a}}{\left(c\,x^n\right)^{3\,b}\,\left(8\,m-24\,b\,n+8\right)} + \frac{x\,x^m\,\mathrm{e}^{3\,a}\,\left(c\,x^n\right)^{3\,b}}{8\,m+24\,b\,n+8} + \frac{3\,x\,x^m\,\mathrm{e}^{a}\,\left(c\,x^n\right)^{b}}{8\,m+8\,b\,n+8}$$

input
```
int(x^m*cosh(a + b*log(c*x^n))^3,x)
```

output
```
(3*x*x^m*exp(-a))/((c*x^n)^b*(8*m - 8*b*n + 8)) + (x*x^m*exp(-3*a))/((c*x^
n)^(3*b)*(8*m - 24*b*n + 8)) + (x*x^m*exp(3*a)*(c*x^n)^(3*b))/(8*m + 24*b*
n + 8) + (3*x*x^m*exp(a)*(c*x^n)^b)/(8*m + 8*b*n + 8)
```

3.245. $\int x^m \cosh^3(a + b\log(cx^n))\,dx$

3.246 $\int x^m \cosh^4(a+b\log(cx^n))\,dx$

3.246.1 Optimal result

Integrand size = 17, antiderivative size = 266

$$\begin{aligned}\int x^m \cosh^4(a+b\log(cx^n))\,dx &= \frac{24b^4n^4x^{1+m}}{(1+m)\left((1+m)^2-16b^2n^2\right)\left((1+m)^2-4b^2n^2\right)} \\ &- \frac{12b^2(1+m)n^2x^{1+m}\cosh^2(a+b\log(cx^n))}{\left((1+m)^2-16b^2n^2\right)\left((1+m)^2-4b^2n^2\right)} \\ &+ \frac{(1+m)x^{1+m}\cosh^4(a+b\log(cx^n))}{(1+m)^2-16b^2n^2} \\ &+ \frac{24b^3n^3x^{1+m}\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{\left((1+m)^2-16b^2n^2\right)\left((1+m)^2-4b^2n^2\right)} \\ &- \frac{4bnx^{1+m}\cosh^3(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{(1+m)^2-16b^2n^2}\end{aligned}$$

output
```
24*b^4*n^4*x^(1+m)/(1+m)/((1+m)^2-16*b^2*n^2)/((1+m)^2-4*b^2*n^2)-12*b^2*(
1+m)*n^2*x^(1+m)*cosh(a+b*ln(c*x^n))^2/((1+m)^2-16*b^2*n^2)/((1+m)^2-4*b^2
*n^2)+(1+m)*x^(1+m)*cosh(a+b*ln(c*x^n))^4/((1+m)^2-16*b^2*n^2)+24*b^3*n^3*
x^(1+m)*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/((1+m)^2-16*b^2*n^2)/((1+m
)^2-4*b^2*n^2)-4*b*n*x^(1+m)*cosh(a+b*ln(c*x^n))^3*sinh(a+b*ln(c*x^n))/((1
+m)^2-16*b^2*n^2)
```

3.246.2 Mathematica [A] (verified)

Time = 2.52 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.17

$$\int x^m \cosh^4(a+b\log(cx^n))\,dx = \frac{1}{8}x^{1+m}\left(\frac{3}{1+m} + \frac{4\sinh(2bn\log(x))\left(-2bn\cosh(2(a-bn\log(x)+b\log(cx^n))) + (1+m)\sinh(2(a-bn\log(x)+b\log(\right.}{(1+m-2bn)(1+m+2bn)} + \frac{4\cosh(2bn\log(x))\left((1+m)\cosh(2(a-bn\log(x)+b\log(cx^n))) - 2bn\sinh(2(a-bn\log(x)+b\log(cx\right.}{(1+m-2bn)(1+m+2bn)} + \frac{\sinh(4bn\log(x))\left(-4bn\cosh(4(a-bn\log(x)+b\log(cx^n))) + (1+m)\sinh(4(a-bn\log(x)+b\log(cx\right.}{(1+m-4bn)(1+m+4bn)} + \frac{\cosh(4bn\log(x))\left((1+m)\cosh(4(a-bn\log(x)+b\log(cx^n))) - 4bn\sinh(4(a-bn\log(x)+b\log(cx^n\right.}{(1+m-4bn)(1+m+4bn)}\right.$$

input
```
Integrate[x^m*Cosh[a + b*Log[c*x^n]]^4,x]
```

output
```
(x^(1 + m)*(3/(1 + m) + (4*Sinh[2*b*n*Log[x]]*(-2*b*n*Cosh[2*(a - b*n*Log[
x] + b*Log[c*x^n])] + (1 + m)*Sinh[2*(a - b*n*Log[x] + b*Log[c*x^n])]))/((
1 + m - 2*b*n)*(1 + m + 2*b*n)) + (4*Cosh[2*b*n*Log[x]]*((1 + m)*Cosh[2*(a
 - b*n*Log[x] + b*Log[c*x^n])] - 2*b*n*Sinh[2*(a - b*n*Log[x] + b*Log[c*x^
n])]))/((1 + m - 2*b*n)*(1 + m + 2*b*n)) + (Sinh[4*b*n*Log[x]]*(-4*b*n*Cos
h[4*(a - b*n*Log[x] + b*Log[c*x^n])] + (1 + m)*Sinh[4*(a - b*n*Log[x] + b*
Log[c*x^n])]))/((1 + m - 4*b*n)*(1 + m + 4*b*n)) + (Cosh[4*b*n*Log[x]]*((1
 + m)*Cosh[4*(a - b*n*Log[x] + b*Log[c*x^n])] - 4*b*n*Sinh[4*(a - b*n*Log[
x] + b*Log[c*x^n])]))/((1 + m - 4*b*n)*(1 + m + 4*b*n))))/8
```

3.246.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.89, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {6056, 6056, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \cosh^4(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{6056}$$

$$-\frac{12b^2n^2\int x^m\cosh^2(a+b\log(cx^n))\,dx}{(m+1)^2-16b^2n^2}+\frac{(m+1)x^{m+1}\cosh^4(a+b\log(cx^n))}{-16b^2n^2+m^2+2m+1}-\frac{4bnx^{m+1}\sinh(a+b\log(cx^n))\cosh^3(a+b\log(cx^n))}{(m+1)^2-16b^2n^2}$$

$$\Big\downarrow \text{6056}$$

$$-\frac{12b^2n^2\left(-\frac{2b^2n^2\int x^m dx}{(m+1)^2-4b^2n^2}+\frac{(m+1)x^{m+1}\cosh^2(a+b\log(cx^n))}{-4b^2n^2+m^2+2m+1}-\frac{2bnx^{m+1}\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{(m+1)^2-4b^2n^2}\right)}{(m+1)^2-16b^2n^2}+\frac{(m+1)x^{m+1}\cosh^4(a+b\log(cx^n))}{-16b^2n^2+m^2+2m+1}-\frac{4bnx^{m+1}\sinh(a+b\log(cx^n))\cosh^3(a+b\log(cx^n))}{(m+1)^2-16b^2n^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{(m+1)x^{m+1}\cosh^4(a+b\log(cx^n))}{-16b^2n^2+m^2+2m+1}-\frac{12b^2n^2\left(\frac{(m+1)x^{m+1}\cosh^2(a+b\log(cx^n))}{-4b^2n^2+m^2+2m+1}-\frac{2bnx^{m+1}\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{(m+1)^2-4b^2n^2}-\frac{2b^2n^2x^{m+1}}{(m+1)((m+1)^2-4b^2n^2)}\right)}{(m+1)^2-16b^2n^2}-\frac{4bnx^{m+1}\sinh(a+b\log(cx^n))\cosh^3(a+b\log(cx^n))}{(m+1)^2-16b^2n^2}$$

input
```
Int[x^m*Cosh[a + b*Log[c*x^n]]^4,x]
```

output
```
((1 + m)*x^(1 + m)*Cosh[a + b*Log[c*x^n]]^4)/(1 + 2*m + m^2 - 16*b^2*n^2)
- (4*b*n*x^(1 + m)*Cosh[a + b*Log[c*x^n]]^3*Sinh[a + b*Log[c*x^n]])/((1 +
m)^2 - 16*b^2*n^2) - (12*b^2*n^2*((-2*b^2*n^2*x^(1 + m))/((1 + m)*((1 + m)
^2 - 4*b^2*n^2)) + ((1 + m)*x^(1 + m)*Cosh[a + b*Log[c*x^n]]^2)/(1 + 2*m +
 m^2 - 4*b^2*n^2) - (2*b*n*x^(1 + m)*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log
[c*x^n]])/((1 + m)^2 - 4*b^2*n^2)))/((1 + m)^2 - 16*b^2*n^2)
```

3.246.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6056
```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-(m + 1))*(e*x)^(m + 1)*(Cosh[d*(a + b*Log[c*x^n])]^
p/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2)), x] + (Simp[b*d*n*p*(e*x)^(m + 1)*Sinh
[d*(a + b*Log[c*x^n])]*(Cosh[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p
^2 - e*(m + 1)^2)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 - (m
+ 1)^2))   Int[(e*x)^m*Cosh[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ
[{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - (m + 1)^2
, 0]
```

3.246.4 Maple [A] (verified)

Time = 236.93 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.80

method	result
parallelrisch	$-\frac{8x^{1+m}\left(-3b^4n^4-2(1+m)n^3\left(\sinh(2b\ln(c\,x^n)+2a)+\frac{\sinh(4b\ln(c\,x^n)+4a)}{8}\right)b^3+(1+m)^2n^2\left(\cosh(2b\ln(c\,x^n)+2a)+\frac{\cosh(4b\ln(}{1}\right.\right.}{64n^4(1+m)b^4-2}$

input
```
int(x^m*cosh(a+b*ln(c*x^n))^4,x,method=_RETURNVERBOSE)
```

output
```
-8*x^(1+m)*(-3*b^4*n^4-2*(1+m)*n^3*(sinh(2*b*ln(c*x^n)+2*a)+1/8*sinh(4*b*l
n(c*x^n)+4*a))*b^3+(1+m)^2*n^2*(cosh(2*b*ln(c*x^n)+2*a)+1/16*cosh(4*b*ln(c
*x^n)+4*a)+15/16)*b^2+1/8*(1+m)^3*(sinh(2*b*ln(c*x^n)+2*a)+1/2*sinh(4*b*ln
(c*x^n)+4*a))*n*b-1/16*(1+m)^4*(cosh(2*b*ln(c*x^n)+2*a)+1/4*cosh(4*b*ln(c*
x^n)+4*a)+3/4))/(64*n^4*(1+m)*b^4-20*n^2*(1+m)^3*b^2+(1+m)^5)
```

3.246.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1123 vs. 2(283) = 566.

Time = 0.32 (sec) , antiderivative size = 1123, normalized size of antiderivative = 4.22

$$\int x^m \cosh^4(a+b\log(cx^n))\,dx = \text{Too large to display}$$

input
```
integrate(x^m*cosh(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

output
```
1/8*((m^4 + 4*m^3 - 4*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*c
osh(b*n*log(x) + b*log(c) + a)^4*cosh(m*log(x)) + 4*(m^4 + 4*m^3 - 16*(b^2
*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(b*n*log(x) + b*log(c)
+ a)^2*cosh(m*log(x)) + ((m^4 + 4*m^3 - 4*(b^2*m^2 + 2*b^2*m + b^2)*n^2 +
6*m^2 + 4*m + 1)*x*cosh(m*log(x)) + (m^4 + 4*m^3 - 4*(b^2*m^2 + 2*b^2*m +
b^2)*n^2 + 6*m^2 + 4*m + 1)*x*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) +
 a)^4 + 16*((4*(b^3*m + b^3)*n^3 - (b*m^3 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh
(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + (4*(b^3*m + b^3)*n^3 - (b*m^3
 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(m*log(x)
))*sinh(b*n*log(x) + b*log(c) + a)^3 + 3*(64*b^4*n^4 + m^4 + 4*m^3 - 20*(b
^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(m*log(x)) + 2*(3*(m^
4 + 4*m^3 - 4*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(b*n*
log(x) + b*log(c) + a)^2*cosh(m*log(x)) + 2*(m^4 + 4*m^3 - 16*(b^2*m^2 + 2
*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(m*log(x)) + (3*(m^4 + 4*m^3 -
4*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(b*n*log(x) + b*l
og(c) + a)^2 + 2*(m^4 + 4*m^3 - 16*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 +
 4*m + 1)*x)*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a)^2 + 16*((4*(b
^3*m + b^3)*n^3 - (b*m^3 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh(b*n*log(x) + b*l
og(c) + a)^3*cosh(m*log(x)) + (16*(b^3*m + b^3)*n^3 - (b*m^3 + 3*b*m^2 + 3
*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + ((4*(b^...
```

3.246.6 Sympy [F]

$$\int x^m \cosh^4(a + b\log(cx^n))\,dx = \text{Too large to display}$$

input
```
integrate(x**m*cosh(a+b*ln(c*x**n))**4,x)
```

output
```
Piecewise((log(x)*cosh(a)**4, Eq(b, 0) & Eq(m, -1)), (Integral(x**m*cosh(-
a + m*log(c*x**n)/(4*n) + log(c*x**n)/(4*n))**4, x), Eq(b, (-m - 1)/(4*n))
), (Integral(x**m*cosh(-a + m*log(c*x**n)/(2*n) + log(c*x**n)/(2*n))**4, x
), Eq(b, (-m - 1)/(2*n))), (Integral(x**m*cosh(a + m*log(c*x**n)/(4*n) + l
og(c*x**n)/(4*n))**4, x), Eq(b, (m + 1)/(4*n))), (Integral(x**m*cosh(a + m
*log(c*x**n)/(2*n) + log(c*x**n)/(2*n))**4, x), Eq(b, (m + 1)/(2*n))), (In
tegral(cosh(a + b*log(c*x**n))**4/x, x), Eq(m, -1)), (24*b**4*n**4*x*x**m*
sinh(a + b*log(c*x**n))**4/(64*b**4*m*n**4 + 64*b**4*n**4 - 20*b**2*m**3*n
**2 - 60*b**2*m**2*n**2 - 60*b**2*m*n**2 - 20*b**2*n**2 + m**5 + 5*m**4 +
10*m**3 + 10*m**2 + 5*m + 1) - 48*b**4*n**4*x*x**m*sinh(a + b*log(c*x**n))
**2*cosh(a + b*log(c*x**n))**2/(64*b**4*m*n**4 + 64*b**4*n**4 - 20*b**2*m*
*3*n**2 - 60*b**2*m**2*n**2 - 60*b**2*m*n**2 - 20*b**2*n**2 + m**5 + 5*m**
4 + 10*m**3 + 10*m**2 + 5*m + 1) + 24*b**4*n**4*x*x**m*cosh(a + b*log(c*x*
*n))**4/(64*b**4*m*n**4 + 64*b**4*n**4 - 20*b**2*m**3*n**2 - 60*b**2*m**2*
n**2 - 60*b**2*m*n**2 - 20*b**2*n**2 + m**5 + 5*m**4 + 10*m**3 + 10*m**2 +
 5*m + 1) - 24*b**3*m*n**3*x*x**m*sinh(a + b*log(c*x**n))**3*cosh(a + b*lo
g(c*x**n))/(64*b**4*m*n**4 + 64*b**4*n**4 - 20*b**2*m**3*n**2 - 60*b**2*m*
*2*n**2 - 60*b**2*m*n**2 - 20*b**2*n**2 + m**5 + 5*m**4 + 10*m**3 + 10*m**
2 + 5*m + 1) + 40*b**3*m*n**3*x*x**m*sinh(a + b*log(c*x**n))*cosh(a + b*lo
g(c*x**n))**3/(64*b**4*m*n**4 + 64*b**4*n**4 - 20*b**2*m**3*n**2 - 60*b...
```

3.246.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.61

$$\int x^m \cosh^4(a+b\log(cx^n))\,dx = \frac{c^{4b}xe^{(4b\log(x^n)+m\log(x)+4a)}}{16(4bn+m+1)} + \frac{c^{2b}xe^{(2b\log(x^n)+m\log(x)+2a)}}{4(2bn+m+1)} - \frac{xe^{(-2b\log(x^n)+m\log(x)-2a)}}{4(2bc^{2b}n-c^{2b}(m+1))} - \frac{xe^{(-4b\log(x^n)+m\log(x)-4a)}}{16(4bc^{4b}n-c^{4b}(m+1))} + \frac{3x^{m+1}}{8(m+1)}$$

input
```
integrate(x^m*cosh(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

output
```
1/16*c^(4*b)*x*e^(4*b*log(x^n) + m*log(x) + 4*a)/(4*b*n + m + 1) + 1/4*c^(
2*b)*x*e^(2*b*log(x^n) + m*log(x) + 2*a)/(2*b*n + m + 1) - 1/4*x*e^(-2*b*l
og(x^n) + m*log(x) - 2*a)/(2*b*c^(2*b)*n - c^(2*b)*(m + 1)) - 1/16*x*e^(-4
*b*log(x^n) + m*log(x) - 4*a)/(4*b*c^(4*b)*n - c^(4*b)*(m + 1)) + 3/8*x^(m
 + 1)/(m + 1)
```

3.246.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6880 vs. 2(283) = 566.

Time = 0.40 (sec) , antiderivative size = 6880, normalized size of antiderivative = 25.86

$$\int x^m \cosh^4(a + b\log(cx^n))\,dx = \text{Too large to display}$$

input
```
integrate(x^m*cosh(a+b*log(c*x^n))^4,x, algorithm="giac")
```

output
```
b^3*c^(4*b)*m*n^3*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*
b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 1
0*m^3 + 10*m^2 + 5*m + 1) + 8*b^3*c^(2*b)*m*n^3*x*x^(2*b*n)*x^m*e^(2*a)/(6
4*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*
m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/4*b^2*c^(4*b)*
m^2*n^2*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^
2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10
*m^2 + 5*m + 1) + b^3*c^(4*b)*n^3*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 +
64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4
- 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 4*b^2*c^(2*b)*m^2*n^2*x*x^(2*b
*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n
^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)
+ 8*b^3*c^(2*b)*n^3*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 2
0*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 +
 10*m^3 + 10*m^2 + 5*m + 1) + 24*b^4*n^4*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4
- 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^
2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/4*b*c^(4*b)*m^3*n*x*x^(4*b*n)*x^m*e^(4*
a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60
*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/2*b^2*c^(
4*b)*m*n^2*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*...
```

3.246.9 Mupad [B] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.50

$$\int x^m \cosh^4(a + b\log(cx^n))\,dx = \frac{3\,x\,x^m}{8\,m+8} + \frac{x\,x^m\,\mathrm{e}^{-2\,a}}{\left(c\,x^n\right)^{2\,b}\,\left(4\,m-8\,b\,n+4\right)} + \frac{x\,x^m\,\mathrm{e}^{2\,a}\,\left(c\,x^n\right)^{2\,b}}{4\,m+8\,b\,n+4} + \frac{x\,x^m\,\mathrm{e}^{-4\,a}}{\left(c\,x^n\right)^{4\,b}\,\left(16\,m-64\,b\,n+16\right)} + \frac{x\,x^m\,\mathrm{e}^{4\,a}\,\left(c\,x^n\right)^{4\,b}}{16\,m+64\,b\,n+16}$$

input

```
int(x^m*cosh(a + b*log(c*x^n))^4,x)
```

output

```
(3*x*x^m)/(8*m + 8) + (x*x^m*exp(-2*a))/((c*x^n)^(2*b)*(4*m - 8*b*n + 4))
+ (x*x^m*exp(2*a)*(c*x^n)^(2*b))/(4*m + 8*b*n + 4) + (x*x^m*exp(-4*a))/((c
*x^n)^(4*b)*(16*m - 64*b*n + 16)) + (x*x^m*exp(4*a)*(c*x^n)^(4*b))/(16*m +
 64*b*n + 16)
```

3.246. $\int x^m \cosh^4(a + b\log(cx^n))\,dx$

3.247 $\int \frac{\cosh(a+b\log(cx^n))}{x} dx$

3.247.1 Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{\cosh(a+b\log(cx^n))}{x} dx = \frac{\sinh(a+b\log(cx^n))}{bn}$$

output
```
sinh(a+b*ln(c*x^n))/b/n
```

3.247.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(18) = 36.

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.06

$$\int \frac{\cosh(a+b\log(cx^n))}{x} dx = \frac{\cosh(b\log(cx^n))\sinh(a)}{bn} + \frac{\cosh(a)\sinh(b\log(cx^n))}{bn}$$

input
```
Integrate[Cosh[a + b*Log[c*x^n]]/x,x]
```

output
```
(Cosh[b*Log[c*x^n]]*Sinh[a])/(b*n) + (Cosh[a]*Sinh[b*Log[c*x^n]])/(b*n)
```

3.247.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3039, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh (a+b \log (cx^n))}{x} \, dx$$

$$\Big\downarrow \; 3039$$

$$\frac{\int \cosh (a+b \log (cx^n))d\log (cx^n)}{n}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \sin \left(ia+ib\log (cx^n)+\frac{\pi }{2}\right)d\log (cx^n)}{n}$$

$$\Big\downarrow \; 3117$$

$$\frac{\sinh (a+b \log (cx^n))}{bn}$$

input
```
Int[Cosh[a + b*Log[c*x^n]]/x,x]
```

output
```
Sinh[a + b*Log[c*x^n]]/(b*n)
```

3.247.3.1 Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

3.247.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\sinh(a+b\ln(c\,x^n))}{bn}$	19
default	$\frac{\sinh(a+b\ln(c\,x^n))}{bn}$	19
parallelrisch	$\frac{\sinh\left(2b\ln\left(\sqrt{c\,x^n}\right)+a\right)}{bn}$	22

input
```
int(cosh(a+b*ln(c*x^n))/x,x,method=_RETURNVERBOSE)
```

output
```
sinh(a+b*ln(c*x^n))/b/n
```

3.247.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\cosh\left(a+b\log\left(cx^n\right)\right)}{x}\,dx = \frac{\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{bn}$$

input
```
integrate(cosh(a+b*log(c*x^n))/x,x, algorithm="fricas")
```

output
```
sinh(b*n*log(x) + b*log(c) + a)/(b*n)
```

3.247.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(14) = 28.

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{\cosh\left(a+b\log\left(cx^n\right)\right)}{x}\,dx = \begin{cases} \log\left(x\right)\cosh\left(a\right) & \text{for } b=0 \wedge \left(b=0 \vee n=0\right) \\ \log\left(x\right)\cosh\left(a+b\log\left(c\right)\right) & \text{for } n=0 \\ \frac{\sinh\left(a+b\log\left(cx^n\right)\right)}{bn} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(a+b*ln(c*x**n))/x,x)
```

output
```
Piecewise((log(x)*cosh(a), Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*cosh
(a + b*log(c)), Eq(n, 0)), (sinh(a + b*log(c*x**n))/(b*n), True))
```

3.247.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(a+b\log(cx^n))}{x}\,dx = \frac{\sinh(b\log(cx^n)+a)}{bn}$$

input
```
integrate(cosh(a+b*log(c*x^n))/x,x, algorithm="maxima")
```

output
```
sinh(b*log(c*x^n) + a)/(b*n)
```

3.247.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(18) = 36.

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int \frac{\cosh(a+b\log(cx^n))}{x}\,dx = \frac{\left(c^{2b}x^{bn}e^{(2a)} - \frac{1}{x^{bn}}\right)e^{(-a)}}{2\,bc^bn}$$

input
```
integrate(cosh(a+b*log(c*x^n))/x,x, algorithm="giac")
```

output
```
1/2*(c^(2*b)*x^(b*n)*e^(2*a) - 1/x^(b*n))*e^(-a)/(b*c^b*n)
```

3.247.9 Mupad [B] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(a+b\log(cx^n))}{x}\,dx = \frac{\sinh(a+b\,\ln(c\,x^n))}{b\,n}$$

input
```
int(cosh(a + b*log(c*x^n))/x,x)
```

output
```
sinh(a + b*log(c*x^n))/(b*n)
```

3.248 $\int \frac{\cosh^2(a+b\log(cx^n))}{x}\,dx$

3.248.1 Optimal result

Integrand size = 17, antiderivative size = 39

$$\int \frac{\cosh^2\left(a+b\log\left(cx^n\right)\right)}{x}\,dx=\frac{\log(x)}{2}+\frac{\cosh\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{2bn}$$

output
```
1/2*ln(x)+1/2*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/b/n
```

3.248.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{\cosh^2\left(a+b\log\left(cx^n\right)\right)}{x}\,dx=\frac{2(a+b\log\left(cx^n\right))+\sinh\left(2(a+b\log\left(cx^n\right))\right)}{4bn}$$

input
```
Integrate[Cosh[a + b*Log[c*x^n]]^2/x,x]
```

output
```
(2*(a + b*Log[c*x^n]) + Sinh[2*(a + b*Log[c*x^n])])/(4*b*n)
```

3.248.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3039, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(a+b\log(cx^n))}{x}dx$$
$$\Big\downarrow \text{3039}$$
$$\frac{\int \cosh^2(a+b\log(cx^n))d\log(cx^n)}{n}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \sin\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)^2d\log(cx^n)}{n}$$
$$\Big\downarrow \text{3115}$$
$$\frac{\frac{1}{2}\int 1d\log(cx^n)+\frac{\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{2b}}{n}$$
$$\Big\downarrow \text{24}$$
$$\frac{\frac{\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{2b}+\frac{1}{2}\log(cx^n)}{n}$$

input
```
Int[Cosh[a + b*Log[c*x^n]]^2/x,x]
```

output
```
(Log[c*x^n]/2 + (Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/(2*b))/n
```

3.248.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.248.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

method	result	size
parallelrisch	$\frac{2\ln(x)bn+\sinh(2b\ln(c\,x^{n})+2a)}{4bn}$	30
derivativedivides	$\frac{\frac{\cosh(a+b\ln(c\,x^{n}))\sinh(a+b\ln(c\,x^{n}))}{2}+\frac{b\ln(c\,x^{n})}{2}+\frac{a}{2}}{nb}$	45
default	$\frac{\frac{\cosh(a+b\ln(c\,x^{n}))\sinh(a+b\ln(c\,x^{n}))}{2}+\frac{b\ln(c\,x^{n})}{2}+\frac{a}{2}}{nb}$	45

input
```
int(cosh(a+b*ln(c*x^n))^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*ln(x)*b*n+sinh(2*b*ln(c*x^n)+2*a))/b/n
```

3.248.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^2(a+b\log(cx^n))}{x}\,dx = \frac{bn\log(x)+\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c)+a)}{2\,bn}$$

input
```
integrate(cosh(a+b*log(c*x^n))^2/x,x, algorithm="fricas")
```

output
```
1/2*(b*n*log(x) + cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(
c) + a))/(b*n)
```

3.248.6 Sympy [F]

$$\int \frac{\cosh^2(a+b\log(cx^n))}{x}\,dx = \int \frac{\cosh^2(a+b\log(cx^n))}{x}\,dx$$

input
```
integrate(cosh(a+b*ln(c*x**n))**2/x,x)
```

output
```
Integral(cosh(a + b*log(c*x**n))**2/x, x)
```

3.248.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \frac{\cosh^2(a+b\log(cx^n))}{x}\,dx = \frac{e^{(2\,b\log(cx^n)+2\,a)}}{8\,bn} - \frac{e^{(-2\,b\log(cx^n)-2\,a)}}{8\,bn} + \frac{1}{2}\log(x)$$

input
```
integrate(cosh(a+b*log(c*x^n))^2/x,x, algorithm="maxima")
```

output
```
1/8*e^(2*b*log(c*x^n) + 2*a)/(b*n) - 1/8*e^(-2*b*log(c*x^n) - 2*a)/(b*n) +
 1/2*log(x)
```

3.248.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(35) = 70.

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.05

$$\int \frac{\cosh^2(a+b\log(cx^n))}{x}\,dx = \frac{\left(4\,bc^{2\,b}ne^{(2\,a)}\log(x) + c^{4\,b}x^{2\,bn}e^{(4\,a)} - \frac{2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1}{x^{2\,bn}}\right)e^{(-2\,a)}}{8\,bc^{2\,b}n}$$

input
```
integrate(cosh(a+b*log(c*x^n))^2/x,x, algorithm="giac")
```

output
```
1/8*(4*b*c^(2*b)*n*e^(2*a)*log(x) + c^(4*b)*x^(2*b*n)*e^(4*a) - (2*c^(2*b)
*x^(2*b*n)*e^(2*a) + 1)/x^(2*b*n))*e^(-2*a)/(b*c^(2*b)*n)
```

3.248.9 Mupad [B] (verification not implemented)

Time = 1.80 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{\cosh^2(a + b\log(cx^n))}{x}\,dx = \frac{\ln(x^n)}{2\,n} + \frac{\sinh(2\,a + 2\,b\,\ln(c\,x^n))}{4\,b\,n}$$

input
```
int(cosh(a + b*log(c*x^n))^2/x,x)
```

output
```
log(x^n)/(2*n) + sinh(2*a + 2*b*log(c*x^n))/(4*b*n)
```

3.249 $\int \frac{\cosh^3(a+b\log(cx^n))}{x}\,dx$

3.249.1 Optimal result

Integrand size = 17, antiderivative size = 42

$$\int \frac{\cosh^3(a+b\log(cx^n))}{x}\,dx=\frac{\sinh(a+b\log(cx^n))}{bn}+\frac{\sinh^3(a+b\log(cx^n))}{3bn}$$

output
```
sinh(a+b*ln(c*x^n))/b/n+1/3*sinh(a+b*ln(c*x^n))^3/b/n
```

3.249.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^3(a+b\log(cx^n))}{x}\,dx=\frac{\sinh(a+b\log(cx^n))}{bn}+\frac{\sinh^3(a+b\log(cx^n))}{3bn}$$

input
```
Integrate[Cosh[a + b*Log[c*x^n]]^3/x,x]
```

output
```
Sinh[a + b*Log[c*x^n]]/(b*n) + Sinh[a + b*Log[c*x^n]]^3/(3*b*n)
```

3.249.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3039, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(a+b\log(cx^n))}{x} \, dx$$

$$\Big\downarrow \text{ 3039}$$

$$\frac{\int \cosh^3(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \sin\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)^3 d\log(cx^n)}{n}$$

$$\Big\downarrow \text{ 3113}$$

$$\frac{i\int \left(\sinh^2(a+b\log(cx^n))+1\right)d(-i\sinh(a+b\log(cx^n)))}{bn}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{i(-\frac{1}{3}i\sinh^3(a+b\log(cx^n))-i\sinh(a+b\log(cx^n)))}{bn}$$

input
```
Int[Cosh[a + b*Log[c*x^n]]^3/x,x]
```

output
```
(I*((-I)*Sinh[a + b*Log[c*x^n]] - (I/3)*Sinh[a + b*Log[c*x^n]]^3))/(b*n)
```

3.249.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

3.249.4 Maple [A] (verified)

Time = 4.36 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{\left(\frac{2}{3}+\frac{\cosh\left(a+b\ln\left(c\,x^{n}\right)\right)^{2}}{3}\right)\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)}{nb}$	36
default	$\frac{\left(\frac{2}{3}+\frac{\cosh\left(a+b\ln\left(c\,x^{n}\right)\right)^{2}}{3}\right)\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)}{nb}$	36
parallelrisch	$\frac{\sinh\left(3b\ln\left(c\,x^{n}\right)+3a\right)+9\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)}{12bn}$	37

input
```
int(cosh(a+b*ln(c*x^n))^3/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/b*(2/3+1/3*cosh(a+b*ln(c*x^n))^2)*sinh(a+b*ln(c*x^n))
```

3.249.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.26

$$\int \frac{\cosh^3(a+b\log(cx^n))}{x}\,dx$$
$$= \frac{\sinh(bn\log(x)+b\log(c)+a)^3+3\left(\cosh(bn\log(x)+b\log(c)+a)^2+3\right)\sinh(bn\log(x)+b\log(c)+a)}{12\,bn}$$

input
```
integrate(cosh(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

output
```
1/12*(sinh(b*n*log(x) + b*log(c) + a)^3 + 3*(cosh(b*n*log(x) + b*log(c) +
a)^2 + 3)*sinh(b*n*log(x) + b*log(c) + a))/(b*n)
```

3.249.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(32) = 64.

Time = 1.22 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.69

$$\int \frac{\cosh^3(a+b\log(cx^n))}{x}\,dx$$
$$= \begin{cases} \log(x)\cosh^3(a) & \text{for } b=0 \wedge (b=0 \vee n=0) \\ \log(x)\cosh^3(a+b\log(c)) & \text{for } n=0 \\ -\frac{2\sinh^3(a+b\log(cx^n))}{3bn}+\frac{\sinh(a+b\log(cx^n))\cosh^2(a+b\log(cx^n))}{bn} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(a+b*ln(c*x**n))**3/x,x)
```

output
```
Piecewise((log(x)*cosh(a)**3, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*c
osh(a + b*log(c))**3, Eq(n, 0)), (-2*sinh(a + b*log(c*x**n))**3/(3*b*n) +
sinh(a + b*log(c*x**n))*cosh(a + b*log(c*x**n))**2/(b*n), True))
```

3.249.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(40) = 80$.

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.05

$$\int \frac{\cosh^3(a + b\log(cx^n))}{x} dx = \frac{e^{(3b\log(cx^n)+3a)}}{24bn} + \frac{3e^{(b\log(cx^n)+a)}}{8bn} - \frac{3e^{(-b\log(cx^n)-a)}}{8bn} - \frac{e^{(-3b\log(cx^n)-3a)}}{24bn}$$

input
```
integrate(cosh(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

output
```
1/24*e^(3*b*log(c*x^n) + 3*a)/(b*n) + 3/8*e^(b*log(c*x^n) + a)/(b*n) - 3/8
*e^(-b*log(c*x^n) - a)/(b*n) - 1/24*e^(-3*b*log(c*x^n) - 3*a)/(b*n)
```

3.249.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(40) = 80$.

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.93

$$\int \frac{\cosh^3(a + b\log(cx^n))}{x} dx = \frac{\left(c^{6b}x^{3bn}e^{(6a)} + 9c^{4b}x^{bn}e^{(4a)} - \frac{9c^{2b}x^{2bn}e^{(2a)}+1}{x^{3bn}}\right)e^{(-3a)}}{24bc^{3b}n}$$

input
```
integrate(cosh(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

output
```
1/24*(c^(6*b)*x^(3*b*n)*e^(6*a) + 9*c^(4*b)*x^(b*n)*e^(4*a) - (9*c^(2*b)*x
^(2*b*n)*e^(2*a) + 1)/x^(3*b*n))*e^(-3*a)/(b*c^(3*b)*n)
```

3.249.9 Mupad [B] (verification not implemented)

Time = 1.77 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \frac{\cosh^3(a + b\log(cx^n))}{x} dx = \frac{\sinh(a + b\ln(c\,x^n))^3 + 3\sinh(a + b\ln(c\,x^n))}{3\,b\,n}$$

input
```
int(cosh(a + b*log(c*x^n))^3/x,x)
```

output
```
(3*sinh(a + b*log(c*x^n)) + sinh(a + b*log(c*x^n))^3)/(3*b*n)
```

3.250 $\int \frac{\cosh^4(a+b\log(cx^n))}{x}\,dx$

3.250.1 Optimal result

Integrand size = 17, antiderivative size = 73

$$\int \frac{\cosh^4(a+b\log(cx^n))}{x}\,dx = \frac{3\log(x)}{8} + \frac{3\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{8bn} + \frac{\cosh^3(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{4bn}$$

output
```
3/8*ln(x)+3/8*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/b/n+1/4*cosh(a+b*ln(
c*x^n))^3*sinh(a+b*ln(c*x^n))/b/n
```

3.250.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

$$\int \frac{\cosh^4(a+b\log(cx^n))}{x}\,dx = \frac{12(a+b\log(cx^n)) + 8\sinh(2(a+b\log(cx^n))) + \sinh(4(a+b\log(cx^n)))}{32bn}$$

input
```
Integrate[Cosh[a + b*Log[c*x^n]]^4/x,x]
```

output
```
(12*(a + b*Log[c*x^n]) + 8*Sinh[2*(a + b*Log[c*x^n])] + Sinh[4*(a + b*Log[
c*x^n])])/(32*b*n)
```

3.250.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3039, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^4(a+b\log(cx^n))}{x}dx$$
$$\Big\downarrow \text{3039}$$
$$\frac{\int \cosh^4(a+b\log(cx^n))d\log(cx^n)}{n}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \sin\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)^4d\log(cx^n)}{n}$$
$$\Big\downarrow \text{3115}$$
$$\frac{\frac{3}{4}\int \cosh^2(a+b\log(cx^n))d\log(cx^n)+\frac{\sinh(a+b\log(cx^n))\cosh^3(a+b\log(cx^n))}{4b}}{n}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\frac{\sinh(a+b\log(cx^n))\cosh^3(a+b\log(cx^n))}{4b}+\frac{3}{4}\int \sin\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)^2d\log(cx^n)}{n}$$
$$\Big\downarrow \text{3115}$$
$$\frac{\frac{3}{4}\left(\frac{1}{2}\int 1d\log(cx^n)+\frac{\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{2b}\right)+\frac{\sinh(a+b\log(cx^n))\cosh^3(a+b\log(cx^n))}{4b}}{n}$$
$$\Big\downarrow \text{24}$$
$$\frac{\frac{\sinh(a+b\log(cx^n))\cosh^3(a+b\log(cx^n))}{4b}+\frac{3}{4}\left(\frac{\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{2b}+\frac{1}{2}\log(cx^n)\right)}{n}$$

input
```
Int[Cosh[a + b*Log[c*x^n]]^4/x,x]
```

output
```
((Cosh[a + b*Log[c*x^n]]^3*Sinh[a + b*Log[c*x^n]])/(4*b) + (3*(Log[c*x^n]/
2 + (Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/(2*b)))/4)/n
```

3.250.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.250.4 Maple [A] (verified)

Time = 18.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.63

method	result	size
parallelrisch	$\frac{12\ln(x)bn+\sinh(4b\ln(c\,x^n)+4a)+8\sinh(2b\ln(c\,x^n)+2a)}{32bn}$	46
derivativedivides	$\frac{\left(\frac{\cosh(a+b\ln(c\,x^n))^3}{4}+\frac{3\cosh(a+b\ln(c\,x^n))}{8}\right)\sinh(a+b\ln(c\,x^n))+\frac{3b\ln(c\,x^n)}{8}+\frac{3a}{8}}{nb}$	62
default	$\frac{\left(\frac{\cosh(a+b\ln(c\,x^n))^3}{4}+\frac{3\cosh(a+b\ln(c\,x^n))}{8}\right)\sinh(a+b\ln(c\,x^n))+\frac{3b\ln(c\,x^n)}{8}+\frac{3a}{8}}{nb}$	62

input
```
int(cosh(a+b*ln(c*x^n))^4/x,x,method=_RETURNVERBOSE)
```

output
```
1/32*(12*ln(x)*b*n+sinh(4*b*ln(c*x^n)+4*a)+8*sinh(2*b*ln(c*x^n)+2*a))/b/n
```

3.250.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.15

$$\int \frac{\cosh^4(a+b\log(cx^n))}{x}\,dx$$
$$= \frac{\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c)+a)^3+3\,bn\log(x)+\left(\cosh(bn\log(x)+b\log(\ldots\right.}{8\,bn}$$

input
```
integrate(cosh(a+b*log(c*x^n))^4/x,x, algorithm="fricas")
```

output
```
1/8*(cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + 3
*b*n*log(x) + (cosh(b*n*log(x) + b*log(c) + a)^3 + 4*cosh(b*n*log(x) + b*l
og(c) + a))*sinh(b*n*log(x) + b*log(c) + a))/(b*n)
```

3.250.6 Sympy [F]

$$\int \frac{\cosh^4(a+b\log(cx^n))}{x}\,dx = \int \frac{\cosh^4(a+b\log(cx^n))}{x}\,dx$$

input
```
integrate(cosh(a+b*ln(c*x**n))**4/x,x)
```

output
```
Integral(cosh(a + b*log(c*x**n))**4/x, x)
```

3.250.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.27

$$\int \frac{\cosh^4(a+b\log(cx^n))}{x}\,dx = \frac{e^{(4\,b\log(cx^n)+4\,a)}}{64\,bn}+\frac{e^{(2\,b\log(cx^n)+2\,a)}}{8\,bn} - \frac{e^{(-2\,b\log(cx^n)-2\,a)}}{8\,bn}-\frac{e^{(-4\,b\log(cx^n)-4\,a)}}{64\,bn}+\frac{3}{8}\log(x)$$

input
```
integrate(cosh(a+b*log(c*x^n))^4/x,x, algorithm="maxima")
```

output
```
1/64*e^(4*b*log(c*x^n) + 4*a)/(b*n) + 1/8*e^(2*b*log(c*x^n) + 2*a)/(b*n) -
 1/8*e^(-2*b*log(c*x^n) - 2*a)/(b*n) - 1/64*e^(-4*b*log(c*x^n) - 4*a)/(b*n
) + 3/8*log(x)
```

3.250.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.56

$$\int \frac{\cosh^4(a+b\log(cx^n))}{x}\,dx = \frac{\left(24\,bc^{4\,b}ne^{(4\,a)}\log(x)+c^{8\,b}x^{4\,bn}e^{(8\,a)}+8\,c^{6\,b}x^{2\,bn}e^{(6\,a)}-\frac{18\,c^{4\,b}x^{4\,bn}e^{(4\,a)}+8\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1}{x^{4\,bn}}\right)e^{(-4\,a)}}{64\,bc^{4\,b}n}$$

input
```
integrate(cosh(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

output
```
1/64*(24*b*c^(4*b)*n*e^(4*a)*log(x) + c^(8*b)*x^(4*b*n)*e^(8*a) + 8*c^(6*b
)*x^(2*b*n)*e^(6*a) - (18*c^(4*b)*x^(4*b*n)*e^(4*a) + 8*c^(2*b)*x^(2*b*n)*
e^(2*a) + 1)/x^(4*b*n))*e^(-4*a)/(b*c^(4*b)*n)
```

3.250.9 Mupad [B] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68

$$\int \frac{\cosh^4(a+b\log(cx^n))}{x}\,dx = \frac{3\,\ln(x^n)}{8\,n}+\frac{\frac{\sinh(2\,a+2\,b\,\ln(c\,x^n))}{4}+\frac{\sinh(4\,a+4\,b\,\ln(c\,x^n))}{32}}{b\,n}$$

input
```
int(cosh(a + b*log(c*x^n))^4/x,x)
```

output
```
(3*log(x^n))/(8*n) + (sinh(2*a + 2*b*log(c*x^n))/4 + sinh(4*a + 4*b*log(c*
x^n))/32)/(b*n)
```

3.251 $\int \frac{\cosh^5(a+b\log(cx^n))}{x}\,dx$

3.251.1 Optimal result

Integrand size = 17, antiderivative size = 65

$$\int \frac{\cosh^5(a+b\log(cx^n))}{x}\,dx = \frac{\sinh(a+b\log(cx^n))}{bn} + \frac{2\sinh^3(a+b\log(cx^n))}{3bn} + \frac{\sinh^5(a+b\log(cx^n))}{5bn}$$

output
```
sinh(a+b*ln(c*x^n))/b/n+2/3*sinh(a+b*ln(c*x^n))^3/b/n+1/5*sinh(a+b*ln(c*x^
n))^5/b/n
```

3.251.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^5(a+b\log(cx^n))}{x}\,dx = \frac{\sinh(a+b\log(cx^n))}{bn} + \frac{2\sinh^3(a+b\log(cx^n))}{3bn} + \frac{\sinh^5(a+b\log(cx^n))}{5bn}$$

input
```
Integrate[Cosh[a + b*Log[c*x^n]]^5/x,x]
```

output
```
Sinh[a + b*Log[c*x^n]]/(b*n) + (2*Sinh[a + b*Log[c*x^n]]^3)/(3*b*n) + Sinh
[a + b*Log[c*x^n]]^5/(5*b*n)
```

3.251.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3039, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^5(a+b\log(cx^n))}{x}\,dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \cosh^5(a+b\log(cx^n))\,d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)^5 d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3113}$$

$$\frac{i\int \left(\sinh^4(a+b\log(cx^n))+2\sinh^2(a+b\log(cx^n))+1\right) d(-i\sinh(a+b\log(cx^n)))}{bn}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i(-\frac{1}{5}i\sinh^5(a+b\log(cx^n))-\frac{2}{3}i\sinh^3(a+b\log(cx^n))-i\sinh(a+b\log(cx^n)))}{bn}$$

input
```
Int[Cosh[a + b*Log[c*x^n]]^5/x,x]
```

output
```
(I*((-I)*Sinh[a + b*Log[c*x^n]] - ((2*I)/3)*Sinh[a + b*Log[c*x^n]]^3 - (I/
5)*Sinh[a + b*Log[c*x^n]]^5))/(b*n)
```

3.251.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

3.251.4 Maple [A] (verified)

Time = 68.84 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\left(\frac{8}{15}+\frac{\cosh(a+b\ln(c\,x^{n}))^{4}}{5}+\frac{4\cosh(a+b\ln(c\,x^{n}))^{2}}{15}\right)\sinh(a+b\ln(c\,x^{n}))}{nb}$	51
default	$\frac{\left(\frac{8}{15}+\frac{\cosh(a+b\ln(c\,x^{n}))^{4}}{5}+\frac{4\cosh(a+b\ln(c\,x^{n}))^{2}}{15}\right)\sinh(a+b\ln(c\,x^{n}))}{nb}$	51
parallelrisch	$\frac{25\sinh(3b\ln(c\,x^{n})+3a)+150\sinh(a+b\ln(c\,x^{n}))+3\sinh(5b\ln(c\,x^{n})+5a)}{240bn}$	55

input
```
int(cosh(a+b*ln(c*x^n))^5/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/b*(8/15+1/5*cosh(a+b*ln(c*x^n))^4+4/15*cosh(a+b*ln(c*x^n))^2)*sinh(a+b
*ln(c*x^n))
```

3.251.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.62

$$\int \frac{\cosh^5(a+b\log(cx^n))}{x}\,dx$$
$$= \frac{3\sinh(bn\log(x)+b\log(c)+a)^5+5\left(6\cosh(bn\log(x)+b\log(c)+a)^2+5\right)\sinh(bn\log(x)+b\log(c)+a)^3+ \dots}{\dots}$$

input
```
integrate(cosh(a+b*log(c*x^n))^5/x,x, algorithm="fricas")
```

output
```
1/240*(3*sinh(b*n*log(x) + b*log(c) + a)^5 + 5*(6*cosh(b*n*log(x) + b*log(
c) + a)^2 + 5)*sinh(b*n*log(x) + b*log(c) + a)^3 + 15*(cosh(b*n*log(x) + b
*log(c) + a)^4 + 5*cosh(b*n*log(x) + b*log(c) + a)^2 + 10)*sinh(b*n*log(x)
 + b*log(c) + a))/(b*n)
```

3.251.6 Sympy [A] (verification not implemented)

Time = 7.71 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.62

$$\int \frac{\cosh^5(a+b\log(cx^n))}{x}\,dx$$
$$= \begin{cases} \log(x)\cosh^5(a) & \text{for } b=0 \wedge (b= \dots \\ \log(x)\cosh^5(a+b\log(c)) & \text{for } n=0 \\ \frac{8\sinh^5(a+b\log(cx^n))}{15bn} - \frac{4\sinh^3(a+b\log(cx^n))\cosh^2(a+b\log(cx^n))}{3bn} + \frac{\sinh(a+b\log(cx^n))\cosh^4(a+b\log(cx^n))}{bn} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(a+b*ln(c*x**n))**5/x,x)
```

output
```
Piecewise((log(x)*cosh(a)**5, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*c
osh(a + b*log(c))**5, Eq(n, 0)), (8*sinh(a + b*log(c*x**n))**5/(15*b*n) -
4*sinh(a + b*log(c*x**n))**3*cosh(a + b*log(c*x**n))**2/(3*b*n) + sinh(a +
 b*log(c*x**n))*cosh(a + b*log(c*x**n))**4/(b*n), True))
```

3.251.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(61) = 122.

Time = 0.19 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.00

$$\int \frac{\cosh^5(a+b\log(cx^n))}{x}\,dx = \frac{e^{(5\,b\log(cx^n)+5\,a)}}{160\,bn} + \frac{5\,e^{(3\,b\log(cx^n)+3\,a)}}{96\,bn} + \frac{5\,e^{(b\log(cx^n)+a)}}{16\,bn} - \frac{5\,e^{(-b\log(cx^n)-a)}}{16\,bn} - \frac{5\,e^{(-3\,b\log(cx^n)-3\,a)}}{96\,bn} - \frac{e^{(-5\,b\log(cx^n)-5\,a)}}{160\,bn}$$

input
```
integrate(cosh(a+b*log(c*x^n))^5/x,x, algorithm="maxima")
```

output
```
1/160*e^(5*b*log(c*x^n) + 5*a)/(b*n) + 5/96*e^(3*b*log(c*x^n) + 3*a)/(b*n)
 + 5/16*e^(b*log(c*x^n) + a)/(b*n) - 5/16*e^(-b*log(c*x^n) - a)/(b*n) - 5/
96*e^(-3*b*log(c*x^n) - 3*a)/(b*n) - 1/160*e^(-5*b*log(c*x^n) - 5*a)/(b*n)
```

3.251.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.78

$$\int \frac{\cosh^5(a+b\log(cx^n))}{x}\,dx = \frac{\left(3\,c^{10\,b}x^{5\,bn}e^{(10\,a)} + 25\,c^{8\,b}x^{3\,bn}e^{(8\,a)} + 150\,c^{6\,b}x^{bn}e^{(6\,a)} - \frac{150\,c^{4\,b}x^{4\,bn}e^{(4\,a)}+25\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+3}{x^{5\,bn}}\right)e^{(-5\,a)}}{480\,bc^{5\,b}n}$$

input
```
integrate(cosh(a+b*log(c*x^n))^5/x,x, algorithm="giac")
```

output
```
1/480*(3*c^(10*b)*x^(5*b*n)*e^(10*a) + 25*c^(8*b)*x^(3*b*n)*e^(8*a) + 150*
c^(6*b)*x^(b*n)*e^(6*a) - (150*c^(4*b)*x^(4*b*n)*e^(4*a) + 25*c^(2*b)*x^(2
*b*n)*e^(2*a) + 3)/x^(5*b*n))*e^(-5*a)/(b*c^(5*b)*n)
```

3.251.9 Mupad [B] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

$$\int \frac{\cosh^5(a+b\log(cx^n))}{x}\,dx=\frac{\frac{\sinh(a+b\,\ln(c\,x^n))^5}{5}+\frac{2\sinh(a+b\,\ln(c\,x^n))^3}{3}+\sinh(a+b\,\ln(c\,x^n))}{b\,n}$$

input
```
int(cosh(a + b*log(c*x^n))^5/x,x)
```

output
```
(sinh(a + b*log(c*x^n)) + (2*sinh(a + b*log(c*x^n))^3)/3 + sinh(a + b*log(
c*x^n))^5/5)/(b*n)
```

3.252 $\int \frac{\cosh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$

3.252.1 Optimal result

Integrand size = 19, antiderivative size = 67

$$\int \frac{\cosh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{6iE\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{5bn} + \frac{2\cosh^{\frac{3}{2}}(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{5bn}$$

output
```
-6/5*I*(cosh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/cosh(1/2*a+1/2*b*ln(c*x^n))*E
llipticE(I*sinh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/b/n+2/5*cosh(a+b*ln(c*x^n)
)^(3/2)*sinh(a+b*ln(c*x^n))/b/n
```

3.252.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93

$$\int \frac{\cosh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{-6iE\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)+\sqrt{\cosh(a+b\log(cx^n))}\sinh(2(a+b\log(cx^n)))}{5bn}$$

input
```
Integrate[Cosh[a + b*Log[c*x^n]]^(5/2)/x,x]
```

output
```
((-6*I)*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2] + Sqrt[Cosh[a + b*Log[c*x^n
]]]*Sinh[2*(a + b*Log[c*x^n])])/(5*b*n)
```

3.252.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3039, 3042, 3115, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^{\frac{5}{2}}(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \cosh^{\frac{5}{2}}(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)^{5/2}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{3}{5}\int \sqrt{\cosh(a+b\log(cx^n))}d\log(cx^n)+\frac{2\sinh(a+b\log(cx^n))\cosh^{\frac{3}{2}}(a+b\log(cx^n))}{5b}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2\sinh(a+b\log(cx^n))\cosh^{\frac{3}{2}}(a+b\log(cx^n))}{5b}+\frac{3}{5}\int \sqrt{\sin\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{2\sinh(a+b\log(cx^n))\cosh^{\frac{3}{2}}(a+b\log(cx^n))}{5b}-\frac{6iE\left(\frac{1}{2}i(a+b\log(cx^n))|2\right)}{5b}}{n}$$

input
```
Int[Cosh[a + b*Log[c*x^n]]^(5/2)/x,x]
```

output
```
((((-6*I)/5)*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2])/b + (2*Cosh[a + b*Log
[c*x^n]]^(3/2)*Sinh[a + b*Log[c*x^n]])/(5*b))/n
```

3.252.3.1 Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.252.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(97) = 194$.

Time = 1.68 (sec) , antiderivative size = 256, normalized size of antiderivative = 3.82

method	result
derivativedivides	$\frac{2\sqrt{\left(-1+2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\left(8\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^7-16\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^5+10\cosh\left(\frac{a}{2}+\right.\right.}{5n\sqrt{2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^4+\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x}{2}\right.\right.}}$
default	$\frac{2\sqrt{\left(-1+2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\left(8\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^7-16\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^5+10\cosh\left(\frac{a}{2}+\right.\right.}{5n\sqrt{2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^4+\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x}{2}\right.\right.}}$

input
```
int(cosh(a+b*ln(c*x^n))^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2/5/n*((-1+2*cosh(1/2*a+1/2*b*ln(c*x^n))^2)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)
^(1/2)*(8*cosh(1/2*a+1/2*b*ln(c*x^n))^7-16*cosh(1/2*a+1/2*b*ln(c*x^n))^5+1
0*cosh(1/2*a+1/2*b*ln(c*x^n))^3-3*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(
-2*cosh(1/2*a+1/2*b*ln(c*x^n))^2+1)^(1/2)*EllipticE(cosh(1/2*a+1/2*b*ln(c*
x^n)),2^(1/2))-2*cosh(1/2*a+1/2*b*ln(c*x^n)))/(2*sinh(1/2*a+1/2*b*ln(c*x^n
))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/sinh(1/2*a+1/2*b*ln(c*x^n))/(-1+
2*cosh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/b
```

3.252.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 332, normalized size of antiderivative = 4.96

$$\int \frac{\cosh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx =$$
$$-\frac{12\left(\sqrt{2}\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2+2\sqrt{2}\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(\right.\right.\right.}{}$$

input
```
integrate(cosh(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

output
```
-1/10*(12*(sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*sqrt(2)*cosh(b*n*
log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sqrt(2)*sinh(b*n*
log(x) + b*log(c) + a)^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))) - (c
osh(b*n*log(x) + b*log(c) + a)^4 + 4*cosh(b*n*log(x) + b*log(c) + a)*sinh(
b*n*log(x) + b*log(c) + a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 + 6*(cosh
(b*n*log(x) + b*log(c) + a)^2 - 2)*sinh(b*n*log(x) + b*log(c) + a)^2 - 12*
cosh(b*n*log(x) + b*log(c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) + a)^3 -
 6*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) - 1)*s
qrt(cosh(b*n*log(x) + b*log(c) + a)))/(b*n*cosh(b*n*log(x) + b*log(c) + a)
^2 + 2*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)
 + b*n*sinh(b*n*log(x) + b*log(c) + a)^2)
```

3.252.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx=\text{Timed out}$$

input
```
integrate(cosh(a+b*ln(c*x**n))**(5/2)/x,x)
```

output
```
Timed out
```

3.252.7 Maxima [F]

$$\int \frac{\cosh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx=\int \frac{\cosh(b\log(cx^n)+a)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate(cosh(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")
```

output
```
integrate(cosh(b*log(c*x^n) + a)^(5/2)/x, x)
```

3.252.8 Giac [F]

$$\int \frac{\cosh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx=\int \frac{\cosh(b\log(cx^n)+a)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate(cosh(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
```

output
```
integrate(cosh(b*log(c*x^n) + a)^(5/2)/x, x)
```

3.252.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx=\int \frac{{\cosh(a+b\,\ln(c\,x^n))}^{5/2}}{x}\,dx$$

input
```
int(cosh(a + b*log(c*x^n))^(5/2)/x,x)
```

output
```
int(cosh(a + b*log(c*x^n))^(5/2)/x, x)
```

3.253 $\int \frac{\cosh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$

3.253.1 Optimal result

Integrand size = 19, antiderivative size = 67

$$\int \frac{\cosh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{2i\,\text{EllipticF}\left(\frac{1}{2}i(a+b\log(cx^n)),2\right)}{3bn} + \frac{2\sqrt{\cosh(a+b\log(cx^n))}\sinh(a+b\log(cx^n))}{3bn}$$

output
```
-2/3*I*(cosh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/cosh(1/2*a+1/2*b*ln(c*x^n))*E
llipticF(I*sinh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/b/n+2/3*sinh(a+b*ln(c*x^n)
)*cosh(a+b*ln(c*x^n))^(1/2)/b/n
```

3.253.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.10 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.70

$$\int \frac{\cosh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\sinh(2(a+b\log(cx^n))) + 2\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\cosh(2(a+b\log(cx^n))) - \sinh(2(a+b\log(c\ldots}{3bn\sqrt{\cosh(a+b\log(cx^n))}}$$

input
```
Integrate[Cosh[a + b*Log[c*x^n]]^(3/2)/x,x]
```

output
```
(Sinh[2*(a + b*Log[c*x^n])] + 2*Hypergeometric2F1[1/4, 1/2, 5/4, -Cosh[2*(
a + b*Log[c*x^n])] - Sinh[2*(a + b*Log[c*x^n])]]*Sqrt[1 + Cosh[2*(a + b*Lo
g[c*x^n])] + Sinh[2*(a + b*Log[c*x^n])]])/(3*b*n*Sqrt[Cosh[a + b*Log[c*x^n
]]])
```

3.253.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3039, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^{\frac{3}{2}}(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \cosh^{\frac{3}{2}}(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)^{3/2}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{1}{3}\int \frac{1}{\sqrt{\cosh(a+b\log(cx^n))}}d\log(cx^n)+\frac{2\sinh(a+b\log(cx^n))\sqrt{\cosh(a+b\log(cx^n))}}{3b}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2\sinh(a+b\log(cx^n))\sqrt{\cosh(a+b\log(cx^n))}}{3b}+\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{2\sinh(a+b\log(cx^n))\sqrt{\cosh(a+b\log(cx^n))}}{3b}-\frac{2i\,\text{EllipticF}\left(\frac{1}{2}i(a+b\log(cx^n)),2\right)}{3b}}{n}$$

input
```
Int[Cosh[a + b*Log[c*x^n]]^(3/2)/x,x]
```

output
```
((((-2*I)/3)*EllipticF[(I/2)*(a + b*Log[c*x^n]), 2])/b + (2*Sqrt[Cosh[a +
b*Log[c*x^n]]]*Sinh[a + b*Log[c*x^n]])/(3*b))/n
```

3.253.3.1 Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.253.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(97) = 194$.

Time = 0.22 (sec) , antiderivative size = 237, normalized size of antiderivative = 3.54

method	result
derivativedivides	$\frac{2\sqrt{\left(-1+2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\left(4\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^5-6\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^3+\sqrt{-\sinh\left(\frac{a}{2}+\cdots\right.}\right.}{3n\sqrt{2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^4+\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,\sinh\left(\frac{a}{2}\cdots\right.}$
default	$\frac{2\sqrt{\left(-1+2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\left(4\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^5-6\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^3+\sqrt{-\sinh\left(\frac{a}{2}+\cdots\right.}\right.}{3n\sqrt{2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^4+\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,\sinh\left(\frac{a}{2}\cdots\right.}$

input
```
int(cosh(a+b*ln(c*x^n))^(3/2)/x,x,method=_RETURNVERBOSE)
```

3.253. $\int \frac{\cosh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$

output
```
2/3/n*((-1+2*cosh(1/2*a+1/2*b*ln(c*x^n))^2)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)
^(1/2)*(4*cosh(1/2*a+1/2*b*ln(c*x^n))^5-6*cosh(1/2*a+1/2*b*ln(c*x^n))^3+(-
sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cosh(1/2*a+1/2*b*ln(c*x^n))^2+1)^
(1/2)*EllipticF(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))+2*cosh(1/2*a+1/2*b*ln
(c*x^n)))/(2*sinh(1/2*a+1/2*b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^
(1/2)/sinh(1/2*a+1/2*b*ln(c*x^n))/(-1+2*cosh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/
2)/b
```

3.253.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.54

$$\int \frac{\cosh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$$
$$= \frac{2\left(\sqrt{2}\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sqrt{2}\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)\text{weierstrassPInverse}(-4,0,\text{co}}{}$$

input
```
integrate(cosh(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

output
```
1/3*(2*(sqrt(2)*cosh(b*n*log(x) + b*log(c) + a) + sqrt(2)*sinh(b*n*log(x)
+ b*log(c) + a))*weierstrassPInverse(-4, 0, cosh(b*n*log(x) + b*log(c) + a
) + sinh(b*n*log(x) + b*log(c) + a)) + (cosh(b*n*log(x) + b*log(c) + a)^2
+ 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh
(b*n*log(x) + b*log(c) + a)^2 - 1)*sqrt(cosh(b*n*log(x) + b*log(c) + a)))/
(b*n*cosh(b*n*log(x) + b*log(c) + a) + b*n*sinh(b*n*log(x) + b*log(c) + a)
)
```

3.253.6 Sympy [F]

$$\int \frac{\cosh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\cosh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$$

input
```
integrate(cosh(a+b*ln(c*x**n))**(3/2)/x,x)
```

output
```
Integral(cosh(a + b*log(c*x**n))**(3/2)/x, x)
```

3.253.7 Maxima [F]

$$\int \frac{\cosh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\cosh(b\log(cx^n)+a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate(cosh(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate(cosh(b*log(c*x^n) + a)^(3/2)/x, x)
```

3.253.8 Giac [F]

$$\int \frac{\cosh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\cosh(b\log(cx^n)+a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate(cosh(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
```

output
```
integrate(cosh(b*log(c*x^n) + a)^(3/2)/x, x)
```

3.253.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\cosh(a+b\,\ln(c\,x^n))^{3/2}}{x}\,dx$$

input
```
int(cosh(a + b*log(c*x^n))^(3/2)/x,x)
```

output
```
int(cosh(a + b*log(c*x^n))^(3/2)/x, x)
```

3.254 $\int \frac{\sqrt{\cosh(a+b\log(cx^n))}}{x} dx$

3.254.1 Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \frac{\sqrt{\cosh(a+b\log(cx^n))}}{x} dx = -\frac{2iE\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{bn}$$

output
```
-2*I*(cosh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/cosh(1/2*a+1/2*b*ln(c*x^n))*Ell
ipticE(I*sinh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/b/n
```

3.254.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\cosh(a+b\log(cx^n))}}{x} dx = -\frac{2iE\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{bn}$$

input
```
Integrate[Sqrt[Cosh[a + b*Log[c*x^n]]]/x,x]
```

output
```
((-2*I)*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n)
```

3.254.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3039, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cosh(a+b\log(cx^n))}}{x}\,dx$$

$$\Big\downarrow \text{ 3039}$$

$$\frac{\int \sqrt{\cosh(a+b\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \sqrt{\sin\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{ 3119}$$

$$-\frac{2iE\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{bn}$$

input
```
Int[Sqrt[Cosh[a + b*Log[c*x^n]]]/x,x]
```

output
```
((-2*I)*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n)
```

3.254.3.1 Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.254.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(64) = 128.

Time = 0.21 (sec) , antiderivative size = 183, normalized size of antiderivative = 6.54

method	result
derivativedivides	$-\frac{2\sqrt{\left(-1+2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\sqrt{-\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\sqrt{-2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2+1}\,\mathrm{Ellip}}{n\sqrt{2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^4+\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\sqrt{-1+2\cosh\left(\frac{a}{2}+\frac{b\ln(c}{2}\right.}}$
default	$-\frac{2\sqrt{\left(-1+2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\sqrt{-\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\sqrt{-2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2+1}\,\mathrm{Ellip}}{n\sqrt{2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^4+\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\sqrt{-1+2\cosh\left(\frac{a}{2}+\frac{b\ln(c}{2}\right.}}$

input
```
int(cosh(a+b*ln(c*x^n))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-2/n*((-1+2*cosh(1/2*a+1/2*b*ln(c*x^n))^2)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^
(1/2)*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cosh(1/2*a+1/2*b*ln(c*x^n
))^2+1)^(1/2)*EllipticE(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/(2*sinh(1/2*a
+1/2*b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/sinh(1/2*a+1/2*b*
ln(c*x^n))/(-1+2*cosh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/b
```

3.254.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.07

$$\int \frac{\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx = -\frac{2\left(\sqrt{2}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(\right.\right.\right.\right.}{bn}$$

input
```
integrate(cosh(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

output
```
-2*(sqrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(b*n*log
(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))) + sqrt(cosh(b*n*lo
g(x) + b*log(c) + a)))/(b*n)
```

3.254.6 Sympy [F]

$$\int \frac{\sqrt{\cosh\left(a+b\log\left(cx^{n}\right)\right)}}{x}\,dx=\int \frac{\sqrt{\cosh\left(a+b\log\left(cx^{n}\right)\right)}}{x}\,dx$$

input
```
integrate(cosh(a+b*ln(c*x**n))**(1/2)/x,x)
```

output
```
Integral(sqrt(cosh(a + b*log(c*x**n)))/x, x)
```

3.254.7 Maxima [F]

$$\int \frac{\sqrt{\cosh\left(a+b\log\left(cx^{n}\right)\right)}}{x}\,dx=\int \frac{\sqrt{\cosh\left(b\log\left(cx^{n}\right)+a\right)}}{x}\,dx$$

input
```
integrate(cosh(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(cosh(b*log(c*x^n) + a))/x, x)
```

3.254.8 Giac [F]

$$\int \frac{\sqrt{\cosh\left(a+b\log\left(cx^{n}\right)\right)}}{x}\,dx=\int \frac{\sqrt{\cosh\left(b\log\left(cx^{n}\right)+a\right)}}{x}\,dx$$

input
```
integrate(cosh(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(cosh(b*log(c*x^n) + a))/x, x)
```

3.254.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cosh(a+b\log(cx^n))}}{x}\,dx=\int \frac{\sqrt{\cosh\left(a+b\,\ln\left(c\,x^n\right)\right)}}{x}\,dx$$

input
```
int(cosh(a + b*log(c*x^n))^(1/2)/x,x)
```

output
```
int(cosh(a + b*log(c*x^n))^(1/2)/x, x)
```

3.255 $\int \frac{1}{x\sqrt{\cosh(a+b\log(cx^n))}}\,dx$

3.255.1 Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \frac{1}{x\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}}\,dx = -\frac{2i\,\text{EllipticF}\left(\frac{1}{2}i(a+b\log\left(cx^n\right)),2\right)}{bn}$$

output
```
-2*I*(cosh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/cosh(1/2*a+1/2*b*ln(c*x^n))*Ell
ipticF(I*sinh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/b/n
```

3.255.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}}\,dx = -\frac{2i\,\text{EllipticF}\left(\frac{1}{2}i(a+b\log\left(cx^n\right)),2\right)}{bn}$$

input
```
Integrate[1/(x*Sqrt[Cosh[a + b*Log[c*x^n]]]),x]
```

output
```
((-2*I)*EllipticF[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n)
```

3.255.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3039, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\cosh(a+b\log(cx^n))}}dx$$

$$\Big\downarrow \text{ 3039}$$

$$\frac{\int \frac{1}{\sqrt{\cosh(a+b\log(cx^n))}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{1}{\sqrt{\sin\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{ 3120}$$

$$-\frac{2i\,\text{EllipticF}\left(\frac{1}{2}i(a+b\log(cx^n)),2\right)}{bn}$$

input
```
Int[1/(x*Sqrt[Cosh[a + b*Log[c*x^n]]]),x]
```

output
```
((-2*I)*EllipticF[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n)
```

3.255.3.1 Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.255.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(64) = 128.

Time = 0.20 (sec) , antiderivative size = 183, normalized size of antiderivative = 6.54

method	result
derivativedivides	$\frac{2\sqrt{\left(-1+2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}}\,\sqrt{-\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}}\,\sqrt{-2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}+1}\ \mathrm{Elliptic}}{n\sqrt{2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{4}+\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}}\,\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\sqrt{-1+2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right.}}$
default	$\frac{2\sqrt{\left(-1+2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}}\,\sqrt{-\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}}\,\sqrt{-2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}+1}\ \mathrm{Elliptic}}{n\sqrt{2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{4}+\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}}\,\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\sqrt{-1+2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right.}}$

input
```
int(1/x/cosh(a+b*ln(c*x^n))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/n*((-1+2*cosh(1/2*a+1/2*b*ln(c*x^n))^2)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(
1/2)*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cosh(1/2*a+1/2*b*ln(c*x^n)
)^2+1)^(1/2)/(2*sinh(1/2*a+1/2*b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^
2)^(1/2)*EllipticF(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/sinh(1/2*a+1/2*b*l
n(c*x^n))/(-1+2*cosh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/b
```

3.255.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{1}{x\sqrt{\cosh\left(a+b\log\left(cx^{n}\right)\right)}}\,dx$$
$$=\frac{2\sqrt{2}\mathrm{weierstrassPInverse}(-4,0,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right))}{bn}$$

input
```
integrate(1/x/cosh(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(2)*weierstrassPInverse(-4, 0, cosh(b*n*log(x) + b*log(c) + a) + sin
h(b*n*log(x) + b*log(c) + a))/(b*n)
```

3.255.6 Sympy [F]

$$\int \frac{1}{x\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}}\,dx=\int \frac{1}{x\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}}\,dx$$

input
```
integrate(1/x/cosh(a+b*ln(c*x**n))**(1/2),x)
```

output
```
Integral(1/(x*sqrt(cosh(a + b*log(c*x**n)))), x)
```

3.255.7 Maxima [F]

$$\int \frac{1}{x\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}}\,dx=\int \frac{1}{x\sqrt{\cosh\left(b\log\left(cx^n\right)+a\right)}}\,dx$$

input
```
integrate(1/x/cosh(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*sqrt(cosh(b*log(c*x^n) + a))), x)
```

3.255.8 Giac [F]

$$\int \frac{1}{x\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}}\,dx=\int \frac{1}{x\sqrt{\cosh\left(b\log\left(cx^n\right)+a\right)}}\,dx$$

input
```
integrate(1/x/cosh(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(x*sqrt(cosh(b*log(c*x^n) + a))), x)
```

3.255.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{\cosh\left(a+b\log\left(cx^{n}\right)\right)}}\,dx=\int \frac{1}{x\,\sqrt{\cosh\left(a+b\,\ln\left(c\,x^{n}\right)\right)}}\,dx$$

input
```
int(1/(x*cosh(a + b*log(c*x^n))^(1/2)),x)
```

output
```
int(1/(x*cosh(a + b*log(c*x^n))^(1/2)), x)
```

3.256 $\int \frac{1}{x\cosh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

3.256.1 Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \frac{1}{x\cosh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx=\frac{2iE\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{bn}+\frac{2\sinh(a+b\log(cx^n))}{bn\sqrt{\cosh(a+b\log(cx^n))}}$$

output
```
2*I*(cosh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/cosh(1/2*a+1/2*b*ln(c*x^n))*Elli
pticE(I*sinh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/b/n+2*sinh(a+b*ln(c*x^n))/b/n
/cosh(a+b*ln(c*x^n))^(1/2)
```

3.256.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\cosh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx=\frac{2\left(iE\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)+\frac{\sinh(a+b\log(cx^n))}{\sqrt{\cosh(a+b\log(cx^n))}}\right)}{bn}$$

input
```
Integrate[1/(x*Cosh[a + b*Log[c*x^n]]^(3/2)),x]
```

output
```
(2*(I*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2] + Sinh[a + b*Log[c*x^n]]/Sqrt
[Cosh[a + b*Log[c*x^n]]]))/(b*n)
```

3.256.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3039, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\cosh^{\frac{3}{2}}(a+b\log(cx^n))}dx$$
$$\Big\downarrow \text{3039}$$
$$\frac{\int \frac{1}{\cosh^{\frac{3}{2}}(a+b\log(cx^n))}d\log(cx^n)}{n}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \frac{1}{\sin\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)^{3/2}}d\log(cx^n)}{n}$$
$$\Big\downarrow \text{3116}$$
$$\frac{\frac{2\sinh(a+b\log(cx^n))}{b\sqrt{\cosh(a+b\log(cx^n))}}-\int \sqrt{\cosh(a+b\log(cx^n))}d\log(cx^n)}{n}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\frac{2\sinh(a+b\log(cx^n))}{b\sqrt{\cosh(a+b\log(cx^n))}}-\int \sqrt{\sin\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)}d\log(cx^n)}{n}$$
$$\Big\downarrow \text{3119}$$
$$\frac{\frac{2\sinh(a+b\log(cx^n))}{b\sqrt{\cosh(a+b\log(cx^n))}}+\frac{2iE\left(\frac{1}{2}i(a+b\log(cx^n))|2\right)}{b}}{n}$$

input
```
Int[1/(x*Cosh[a + b*Log[c*x^n]]^(3/2)),x]
```

output
```
(((2*I)*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2])/b + (2*Sinh[a + b*Log[c*x^
n]])/(b*Sqrt[Cosh[a + b*Log[c*x^n]]]))/n
```

3.256.3.1 Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.256.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.24

method	result
derivativedivides	$\frac{4\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}+2\sqrt{-2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}-1}\,\sqrt{-\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}}\ \text{EllipticE}\left(\cosh\left(\frac{a}{2}\right.\right.}{n\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\sqrt{-1+2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}}\,b}$
default	$\frac{4\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}+2\sqrt{-2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}-1}\,\sqrt{-\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}}\ \text{EllipticE}\left(\cosh\left(\frac{a}{2}\right.\right.}{n\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\sqrt{-1+2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}}\,b}$

input
```
int(1/x/cosh(a+b*ln(c*x^n))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/n*(2*cosh(1/2*a+1/2*b*ln(c*x^n))*sinh(1/2*a+1/2*b*ln(c*x^n))^2+(-2*sinh(
1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*E
llipticE(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2)))/sinh(1/2*a+1/2*b*ln(c*x^n))
/(-1+2*cosh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/b
```

3.256.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.86

$$\int \frac{1}{x\cosh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{2\left(\left(\sqrt{2}\cosh(bn\log(x)+b\log(c)+a)^2+2\sqrt{2}\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c)\right.\right.}{}$$

input
```
integrate(1/x/cosh(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

output
```
2*((sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*sqrt(2)*cosh(b*n*log(x)
+ b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sqrt(2)*sinh(b*n*log(x)
+ b*log(c) + a)^2 + sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))) +
 2*(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*
sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2)*sqrt(
cosh(b*n*log(x) + b*log(c) + a)))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^2 +
 2*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + b
*n*sinh(b*n*log(x) + b*log(c) + a)^2 + b*n)
```

3.256.6 Sympy [F]

$$\int \frac{1}{x\cosh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\cosh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$$

input
```
integrate(1/x/cosh(a+b*ln(c*x**n))**(3/2),x)
```

output
```
Integral(1/(x*cosh(a + b*log(c*x**n))**(3/2)), x)
```

3.256.7 Maxima [F]

$$\int \frac{1}{x\cosh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\cosh(b\log(cx^n)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/cosh(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*cosh(b*log(c*x^n) + a)^(3/2)), x)
```

3.256.8 Giac [F]

$$\int \frac{1}{x\cosh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\cosh(b\log(cx^n)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/cosh(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/(x*cosh(b*log(c*x^n) + a)^(3/2)), x)
```

3.256.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\cosh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\cosh\left(a+b\,\ln\left(c\,x^n\right)\right)^{3/2}}\,dx$$

input
```
int(1/(x*cosh(a + b*log(c*x^n))^(3/2)),x)
```

output
```
int(1/(x*cosh(a + b*log(c*x^n))^(3/2)), x)
```

3.257 $\int \frac{1}{x \cosh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

3.257.1 Optimal result

Integrand size = 19, antiderivative size = 67

$$\int \frac{1}{x \cosh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = -\frac{2i\,\text{EllipticF}\left(\frac{1}{2}i(a+b\log(cx^n)),2\right)}{3bn} + \frac{2\sinh(a+b\log(cx^n))}{3bn\cosh^{\frac{3}{2}}(a+b\log(cx^n))}$$

output
```
-2/3*I*(cosh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/cosh(1/2*a+1/2*b*ln(c*x^n))*E
llipticF(I*sinh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/b/n+2/3*sinh(a+b*ln(c*x^n)
)/b/n/cosh(a+b*ln(c*x^n))^(3/2)
```

3.257.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.82

$$\int \frac{1}{x \cosh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{2\Big(\sinh(a+b\log(cx^n)) + \cosh(a+b\log(cx^n))\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\cosh(2(a+b\log(cx^n)))-\right.}{3bn\cosh^{\frac{3}{2}}(a+b\log(c}$$

input
```
Integrate[1/(x*Cosh[a + b*Log[c*x^n]]^(5/2)),x]
```

output
```
(2*(Sinh[a + b*Log[c*x^n]] + Cosh[a + b*Log[c*x^n]]*Hypergeometric2F1[1/4,
 1/2, 5/4, -Cosh[2*(a + b*Log[c*x^n])] - Sinh[2*(a + b*Log[c*x^n])]]*Sqrt[
1 + Cosh[2*(a + b*Log[c*x^n])] + Sinh[2*(a + b*Log[c*x^n])]]))/(3*b*n*Cosh
[a + b*Log[c*x^n]]^(3/2))
```

3.257.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3039, 3042, 3116, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\cosh^{\frac{5}{2}}(a+b\log(cx^n))}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \frac{1}{\cosh^{\frac{5}{2}}(a+b\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sin\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)^{5/2}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\frac{1}{3}\int \frac{1}{\sqrt{\cosh(a+b\log(cx^n))}}d\log(cx^n)+\frac{2\sinh(a+b\log(cx^n))}{3b\cosh^{\frac{3}{2}}(a+b\log(cx^n))}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2\sinh(a+b\log(cx^n))}{3b\cosh^{\frac{3}{2}}(a+b\log(cx^n))}+\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{2\sinh(a+b\log(cx^n))}{3b\cosh^{\frac{3}{2}}(a+b\log(cx^n))}-\frac{2i\,\text{EllipticF}\left(\frac{1}{2}i(a+b\log(cx^n)),2\right)}{3b}}{n}$$

input
```
Int[1/(x*Cosh[a + b*Log[c*x^n]]^(5/2)),x]
```

output
```
((((-2*I)/3)*EllipticF[(I/2)*(a + b*Log[c*x^n]), 2])/b + (2*Sinh[a + b*Log
[c*x^n]])/(3*b*Cosh[a + b*Log[c*x^n]]^(3/2)))/n
```

3.257.3.1 Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.257.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(97) = 194$.

Time = 0.24 (sec) , antiderivative size = 295, normalized size of antiderivative = 4.40

method	result
derivativedivides	$\frac{2\left(2\sqrt{-\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\sqrt{-2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2-1}\ \text{EllipticF}\left(\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right),\sqrt{2}\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2+2}{3n\sqrt{2\sinh\left(\frac{a}{2}+\right.}}$
default	$\frac{2\left(2\sqrt{-\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\sqrt{-2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2-1}\ \text{EllipticF}\left(\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right),\sqrt{2}\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2+2}{3n\sqrt{2\sinh\left(\frac{a}{2}+\right.}}$

input
```
int(1/x/cosh(a+b*ln(c*x^n))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/n*(2*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*sinh(1/2*a+1/2*b*ln(c*
x^n))^2-1)^(1/2)*EllipticF(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))*sinh(1/2*a
+1/2*b*ln(c*x^n))^2+2*cosh(1/2*a+1/2*b*ln(c*x^n))*sinh(1/2*a+1/2*b*ln(c*x^
n))^2+(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*sinh(1/2*a+1/2*b*ln(c*x^n
))^2-1)^(1/2)*EllipticF(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2)))*((-1+2*cosh(
1/2*a+1/2*b*ln(c*x^n))^2)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/(2*sinh(1/2
*a+1/2*b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/(-1+2*cosh(1/2*
a+1/2*b*ln(c*x^n))^2)^(3/2)/sinh(1/2*a+1/2*b*ln(c*x^n))/b
```

3.257.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 501, normalized size of antiderivative = 7.48

$$\int \frac{1}{x\cosh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$$
$$= \frac{2\left(\left(\sqrt{2}\cosh(bn\log(x)+b\log(c)+a)^4+4\sqrt{2}\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c)\right.\right.}{}$$

input
```
integrate(1/x/cosh(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

output
```
2/3*((sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^4 + 4*sqrt(2)*cosh(b*n*log(x
) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + sqrt(2)*sinh(b*n*log
(x) + b*log(c) + a)^4 + 2*(3*sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^2 + s
qrt(2))*sinh(b*n*log(x) + b*log(c) + a)^2 + 2*sqrt(2)*cosh(b*n*log(x) + b*
log(c) + a)^2 + 4*(sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^3 + sqrt(2)*cos
h(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) + sqrt(2))*w
eierstrassPInverse(-4, 0, cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x
) + b*log(c) + a)) + 2*(cosh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*log
(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2 + sinh(b*n*log(x) +
b*log(c) + a)^3 + (3*cosh(b*n*log(x) + b*log(c) + a)^2 - 1)*sinh(b*n*log(x
) + b*log(c) + a) - cosh(b*n*log(x) + b*log(c) + a))*sqrt(cosh(b*n*log(x)
+ b*log(c) + a)))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^4 + 4*b*n*cosh(b*n*
log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + b*n*sinh(b*n*lo
g(x) + b*log(c) + a)^4 + 2*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*(3*b*
n*cosh(b*n*log(x) + b*log(c) + a)^2 + b*n)*sinh(b*n*log(x) + b*log(c) + a)
^2 + b*n + 4*(b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + b*n*cosh(b*n*log(x)
+ b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a))
```

3.257.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\cosh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \text{Timed out}$$

input
```
integrate(1/x/cosh(a+b*ln(c*x**n))**(5/2),x)
```

output
```
Timed out
```

3.257.7 Maxima [F]

$$\int \frac{1}{x\cosh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\cosh(b\log(cx^n)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x/cosh(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*cosh(b*log(c*x^n) + a)^(5/2)), x)
```

3.257.8 Giac [F]

$$\int \frac{1}{x\cosh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\cosh(b\log(cx^n)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x/cosh(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/(x*cosh(b*log(c*x^n) + a)^(5/2)), x)
```

3.257.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\cosh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\cosh(a+b\,\ln(c\,x^n))^{5/2}}\,dx$$

input
```
int(1/(x*cosh(a + b*log(c*x^n))^(5/2)),x)
```

output
```
int(1/(x*cosh(a + b*log(c*x^n))^(5/2)), x)
```

3.258 $\int \cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\,dx$

3.258.1 Optimal result

Integrand size = 18, antiderivative size = 206

$$\int \cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)dx$$
$$=-\frac{1}{4}x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)+\frac{5e^{-2a}x(cx^n)^{-4/n}\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(1+e^{-2a}(cx^n)^{-4/n}\right)^2}$$
$$+\frac{5x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(1+e^{-2a}(cx^n)^{-4/n}\right)}-\frac{5e^{-3a}x(cx^n)^{-6/n}\operatorname{csch}^{-1}\left(e^a(cx^n)^{2/n}\right)\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(1+e^{-2a}(cx^n)^{-4/n}\right)^{5/2}}$$

output
```
-1/4*x*cosh(a+2*ln(c*x^n)/n)^(5/2)+5/4*x*cosh(a+2*ln(c*x^n)/n)^(5/2)/exp(2
*a)/((c*x^n)^(4/n))/(1+1/exp(2*a)/((c*x^n)^(4/n)))^2+5/12*x*cosh(a+2*ln(c*
x^n)/n)^(5/2)/(1+1/exp(2*a)/((c*x^n)^(4/n)))-5/4*x*arccsch(exp(a)*(c*x^n)^
(2/n))*cosh(a+2*ln(c*x^n)/n)^(5/2)/exp(3*a)/((c*x^n)^(6/n))/(1+1/exp(2*a)/
((c*x^n)^(4/n)))^(5/2)
```

3.258.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.41

$$\int \cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\,dx = \frac{1}{14}e^{2a}x(cx^n)^{4/n}\left(1 + e^{2a}(cx^n)^{4/n}\right)\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\text{Hypergeometric2F1}\left(2,\frac{7}{2},\frac{9}{2},1+e^{2a}(cx^n)^{4/n}\right)$$

input
```
Integrate[Cosh[a + (2*Log[c*x^n])/n]^(5/2),x]
```

output
```
(E^(2*a)*x*(c*x^n)^(4/n)*(1 + E^(2*a)*(c*x^n)^(4/n))*Cosh[a + (2*Log[c*x^n
])/n]^(5/2)*Hypergeometric2F1[2, 7/2, 9/2, 1 + E^(2*a)*(c*x^n)^(4/n)])/14
```

3.258.3 Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {6052, 6060, 876, 872, 868, 773, 247, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\,dx$$

$$\Big\downarrow \text{6052}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)d(cx^n)}{n}$$

$$\Big\downarrow \text{6060}$$

$$\frac{x(cx^n)^{-6/n}\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\int (cx^n)^{\frac{6}{n}-1}\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{5/2}d(cx^n)}{n\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{5/2}}$$

$$\Big\downarrow \text{876}$$

$$\frac{x(cx^n)^{-6/n}\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\left(\frac{5}{2}\int(cx^n)^{\frac{6}{n}-1}\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{3/2}d(cx^n)-\frac{1}{4}n(cx^n)^{6/n}\left(e^{-2a}(cx^n)^{-4/n}+\right.\right.}{n\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{5/2}}$$

$\downarrow$ 872

$$\frac{x(cx^n)^{-6/n}\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\left(\frac{5}{2}\left(e^{-2a}\int(cx^n)^{\frac{2}{n}-1}\sqrt{e^{-2a}(cx^n)^{-4/n}+1}d(cx^n)+\frac{1}{6}n(cx^n)^{6/n}\left(e^{-2a}(cx^n)^{-4/n}\right.\right.\right.}{n\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{5/2}}$$

$\downarrow$ 868

$$\frac{x(cx^n)^{-6/n}\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\left(\frac{5}{2}\left(\frac{1}{2}e^{-2a}n\int\sqrt{\frac{e^{-2a}x^{-2n}}{c^2}+1}d(cx^n)^{2/n}+\frac{1}{6}n(cx^n)^{6/n}\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{3/2}\right)\right.}{n\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{5/2}}$$

$\downarrow$ 773

$$\frac{x(cx^n)^{-6/n}\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\left(\frac{5}{2}\left(\frac{1}{6}n(cx^n)^{6/n}\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{3/2}-\frac{1}{2}e^{-2a}n\int\frac{x^{-2n}\sqrt{c^2e^{-2a}x^{2n}+1}}{c^2}d\frac{x^{-n}}{c}\right)-\right.}{n\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{5/2}}$$

$\downarrow$ 247

$$\frac{x(cx^n)^{-6/n}\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\left(\frac{5}{2}\left(\frac{1}{6}n(cx^n)^{6/n}\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{3/2}-\frac{1}{2}e^{-2a}n\left(e^{-2a}\int\frac{1}{\sqrt{c^2e^{-2a}x^{2n}+1}}d\frac{x^{-n}}{c}-\right.\right.\right.}{n\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{5/2}}$$

$\downarrow$ 222

$$\frac{x(cx^n)^{-6/n}\left(\frac{5}{2}\left(\frac{1}{6}n(cx^n)^{6/n}\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{3/2}-\frac{1}{2}e^{-2a}n\left(e^{-a}\text{arcsinh}\left(\frac{e^{-a}x^{-n}}{c}\right)-\frac{x^{-n}\sqrt{e^{-2a}c^2x^{2n}+1}}{c}\right)\right)-\frac{1}{4}n(\right.}{n\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{5/2}}$$

input

```
Int[Cosh[a + (2*Log[c*x^n])/n]^(5/2),x]
```

3.258. $\int\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)dx$

output
```
(x*(-1/4*(n*(c*x^n)^(6/n)*(1 + 1/(E^(2*a)*(c*x^n)^(4/n)))^(5/2)) + (5*((n*
(c*x^n)^(6/n)*(1 + 1/(E^(2*a)*(c*x^n)^(4/n)))^(3/2))/6 - (n*(-(Sqrt[1 + (c
^2*x^(2*n))/E^(2*a)]/(c*x^n)) + ArcSinh[1/(c*E^a*x^n)]/E^a))/(2*E^(2*a))))
/2)*Cosh[a + (2*Log[c*x^n])/n]^(5/2))/(n*(c*x^n)^(6/n)*(1 + 1/(E^(2*a)*(c*
x^n)^(4/n)))^(5/2))
```

3.258.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

rule 872
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*(
(a + b*x^n)^p/(m + 1)), x] - Simp[b*n*(p/(m + 1))   Int[x^(m + n)*(a + b*x^
n)^(p - 1), x], x] /; FreeQ[{a, b, m, n}, x] && EqQ[(m + 1)/n + p, 0] && Gt
Q[p, 0]
```

rule 876
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m, n}, x] &
& IntegerQ[p + Simplify[(m + 1)/n]] && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

3.258. $\int \cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\,dx$

rule 6052
```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> S
imp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Cosh[d*(a + b*Log[x])]^p, x
], x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1]
)
```

rule 6060
```
Int[Cosh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[Cosh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p
)   Int[(e*x)^m*x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[
{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

3.258.4 Maple [F]

$$\int \cosh\left(a+\frac{2\ln\left(c\,x^{n}\right)}{n}\right)^{\frac{5}{2}}dx$$

input
```
int(cosh(a+2*ln(c*x^n)/n)^(5/2),x)
```

output
```
int(cosh(a+2*ln(c*x^n)/n)^(5/2),x)
```

3.258.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.91

$$\int \cosh^{\frac{5}{2}}\left(a+\frac{2\log\left(cx^{n}\right)}{n}\right)\,dx = \frac{\left(15\sqrt{2}x^{3}e^{\left(\frac{3\,(an+2\,\log(c))}{2\,n}\right)}\log\left(\frac{x^{4}e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}-2\sqrt{2}\sqrt{\frac{1}{2}}x\sqrt{\frac{x^{4}e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}+1}{x^{2}}}+2}{x^{4}}\right)+4\sqrt{\frac{1}{2}}\left(2\,x^{8}e^{\left(\frac{4\,(an+2\,\log(c))}{n}\right)}\right.\right.}{192\,x^{3}}$$

input
```
integrate(cosh(a+2*log(c*x^n)/n)^(5/2),x, algorithm="fricas")
```

output
```
1/192*(15*sqrt(2)*x^3*e^(3/2*(a*n + 2*log(c))/n)*log((x^4*e^(2*(a*n + 2*lo
g(c))/n) - 2*sqrt(2)*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n + 2*log(c))/n) + 1)/x
^2) + 2)/x^4) + 4*sqrt(1/2)*(2*x^8*e^(4*(a*n + 2*log(c))/n) + 14*x^4*e^(2*
(a*n + 2*log(c))/n) - 3)*sqrt((x^4*e^(2*(a*n + 2*log(c))/n) + 1)/x^2)*e^(-
1/2*(a*n + 2*log(c))/n))*e^(-2*(a*n + 2*log(c))/n)/x^3
```

3.258.6 Sympy [F(-1)]

Timed out.

$$\int \cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\,dx=\text{Timed out}$$

input
```
integrate(cosh(a+2*ln(c*x**n)/n)**(5/2),x)
```

output
```
Timed out
```

3.258.7 Maxima [F]

$$\int \cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\,dx=\int \cosh\left(a+\frac{2\,\log(cx^n)}{n}\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(cosh(a+2*log(c*x^n)/n)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cosh(a + 2*log(c*x^n)/n)^(5/2), x)
```

3.258.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3088 vs. $2(192)=384$.

Time = 135.03 (sec) , antiderivative size = 3088, normalized size of antiderivative = 14.99

$$\int \cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\,dx=\text{Too large to display}$$

input
```
integrate(cosh(a+2*log(c*x^n)/n)^(5/2),x, algorithm="giac")
```

output
```
1/48*sqrt(2)*sqrt(c^(6/n)*x^6*e^(3*a) + c^(2/n)*x^2*e^a)*c^(2/n)*x^3*e^a +
 1/192*sqrt(2)*(15*c^(8/n)*e^(9/2*a)*log(sqrt(-sqrt(x^8*abs(c)^(8/n)*e^(4*
a) + 2*x^4*abs(c)^(4/n)*cos(2*pi*sgn(c)/n - 2*pi/n)*e^(2*a) + 1)*cos(1/2*p
i*sgn(x^4*abs(c)^(4/n)*cos(-1/2*pi*sgn(c)/n + 1/2*pi/n)^4*e^(3*a) - 6*x^4*
abs(c)^(4/n)*cos(-1/2*pi*sgn(c)/n + 1/2*pi/n)^2*e^(3*a)*sin(-1/2*pi*sgn(c)
/n + 1/2*pi/n)^2 + x^4*abs(c)^(4/n)*e^(3*a)*sin(-1/2*pi*sgn(c)/n + 1/2*pi/
n)^4 + e^a)*sgn(4*x^4*abs(c)^(4/n)*cos(-1/2*pi*sgn(c)/n + 1/2*pi/n)^3*e^(3
*a)*sin(-1/2*pi*sgn(c)/n + 1/2*pi/n) - 4*x^4*abs(c)^(4/n)*cos(-1/2*pi*sgn(
c)/n + 1/2*pi/n)*e^(3*a)*sin(-1/2*pi*sgn(c)/n + 1/2*pi/n)^3) - 1/2*pi*sgn(
4*x^4*abs(c)^(4/n)*cos(-1/2*pi*sgn(c)/n + 1/2*pi/n)^3*e^(3*a)*sin(-1/2*pi*
sgn(c)/n + 1/2*pi/n) - 4*x^4*abs(c)^(4/n)*cos(-1/2*pi*sgn(c)/n + 1/2*pi/n)
*e^(3*a)*sin(-1/2*pi*sgn(c)/n + 1/2*pi/n)^3) - arctan(-4*x^4*abs(c)^(4/n)*
e^(2*a)*tan(-1/2*pi*sgn(c)/n + 1/2*pi/n)^3/(x^4*abs(c)^(4/n)*e^(2*a)*tan(-
1/2*pi*sgn(c)/n + 1/2*pi/n)^4 - 6*x^4*abs(c)^(4/n)*e^(2*a)*tan(-1/2*pi*sgn
(c)/n + 1/2*pi/n)^2 + x^4*abs(c)^(4/n)*e^(2*a) + tan(-1/2*pi*sgn(c)/n + 1/
2*pi/n)^4 + 2*tan(-1/2*pi*sgn(c)/n + 1/2*pi/n)^2 + 1) + 4*x^4*abs(c)^(4/n)
*e^(2*a)*tan(-1/2*pi*sgn(c)/n + 1/2*pi/n)/(x^4*abs(c)^(4/n)*e^(2*a)*tan(-1
/2*pi*sgn(c)/n + 1/2*pi/n)^4 - 6*x^4*abs(c)^(4/n)*e^(2*a)*tan(-1/2*pi*sgn(
c)/n + 1/2*pi/n)^2 + x^4*abs(c)^(4/n)*e^(2*a) + tan(-1/2*pi*sgn(c)/n + 1/2
*pi/n)^4 + 2*tan(-1/2*pi*sgn(c)/n + 1/2*pi/n)^2 + 1))*e^a - 2*(x^8*abs...
```

3.258.9 Mupad [F(-1)]

Timed out.

$$\int \cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\,dx=\int \cosh\left(a+\frac{2\,\ln(c\,x^n)}{n}\right)^{5/2}\,dx$$

input
```
int(cosh(a + (2*log(c*x^n))/n)^(5/2),x)
```

output
```
int(cosh(a + (2*log(c*x^n))/n)^(5/2), x)
```

3.259 $\int \sqrt{\cosh \left(a+\frac{2 \log(cx^n)}{n}\right)} \, dx$

3.259.1 Optimal result

Integrand size = 18, antiderivative size = 102

$$\int \sqrt{\cosh \left(a+\frac{2 \log (cx^n)}{n}\right)} \, dx = \frac{1}{2} x \sqrt{\cosh \left(a+\frac{2 \log (cx^n)}{n}\right)} - \frac{e^{-a} x (cx^n)^{-2/n} \operatorname{csch}^{-1}\left(e^a (cx^n)^{2/n}\right) \sqrt{\cosh \left(a+\frac{2 \log(cx^n)}{n}\right)}}{2 \sqrt{1+e^{-2a} (cx^n)^{-4/n}}}$$

output
```
1/2*x*cosh(a+2*ln(c*x^n)/n)^(1/2)-1/2*x*arccsch(exp(a)*(c*x^n)^(2/n))*cosh
(a+2*ln(c*x^n)/n)^(1/2)/exp(a)/((c*x^n)^(2/n))/(1+1/exp(2*a)/((c*x^n)^(4/n
)))^(1/2)
```

3.259.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.73

$$\int \sqrt{\cosh \left(a+\frac{2 \log (cx^n)}{n}\right)} \, dx = \frac{1}{2} x \left(1 - \frac{\operatorname{arctanh}\left(\sqrt{1+e^{2a} (cx^n)^{4/n}}\right)}{\sqrt{1+e^{2a} (cx^n)^{4/n}}}\right) \sqrt{\cosh \left(a+\frac{2 \log (cx^n)}{n}\right)}$$

input
```
Integrate[Sqrt[Cosh[a + (2*Log[c*x^n])/n]],x]
```

output
```
(x*(1 - ArcTanh[Sqrt[1 + E^(2*a)*(c*x^n)^(4/n)]]/Sqrt[1 + E^(2*a)*(c*x^n)^
(4/n)])*Sqrt[Cosh[a + (2*Log[c*x^n])/n]])/2
```

3.259.3 Rubi [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6052, 6060, 868, 773, 247, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cosh\left(a+\frac{2\log\left(cx^n\right)}{n}\right)}dx$$

$$\Big\downarrow \text{ 6052}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\sqrt{\cosh\left(a+\frac{2\log(cx^n)}{n}\right)}d(cx^n)}{n}$$

$$\Big\downarrow \text{ 6060}$$

$$\frac{x(cx^n)^{-2/n}\sqrt{\cosh\left(a+\frac{2\log(cx^n)}{n}\right)}\int (cx^n)^{\frac{2}{n}-1}\sqrt{e^{-2a}\left(cx^n\right)^{-4/n}+1}d(cx^n)}{n\sqrt{e^{-2a}\left(cx^n\right)^{-4/n}+1}}$$

$$\Big\downarrow \text{ 868}$$

$$\frac{x(cx^n)^{-2/n}\sqrt{\cosh\left(a+\frac{2\log(cx^n)}{n}\right)}\int \sqrt{\frac{e^{-2a}x^{-2n}}{c^2}+1}d(cx^n)^{2/n}}{2\sqrt{e^{-2a}\left(cx^n\right)^{-4/n}+1}}$$

$$\Big\downarrow \text{ 773}$$

$$-\frac{x(cx^n)^{-2/n}\sqrt{\cosh\left(a+\frac{2\log(cx^n)}{n}\right)}\int \frac{x^{-2n}\sqrt{c^2e^{-2a}x^{2n}+1}}{c^2}d\frac{x^{-n}}{c}}{2\sqrt{e^{-2a}\left(cx^n\right)^{-4/n}+1}}$$

$$\Big\downarrow \text{ 247}$$

$$-\frac{x(cx^n)^{-2/n}\sqrt{\cosh\left(a+\frac{2\log(cx^n)}{n}\right)}\left(e^{-2a}\int\frac{1}{\sqrt{c^2e^{-2a}x^{2n}+1}}d\frac{x^{-n}}{c}-\frac{x^{-n}\sqrt{e^{-2a}c^2x^{2n}+1}}{c}\right)}{2\sqrt{e^{-2a}\left(cx^n\right)^{-4/n}+1}}$$

$$\Big\downarrow \text{222}$$

$$-\frac{x(cx^n)^{-2/n}\left(e^{-a}\text{arcsinh}\left(\frac{e^{-a}x^{-n}}{c}\right)-\frac{x^{-n}\sqrt{e^{-2a}c^2x^{2n}+1}}{c}\right)\sqrt{\cosh\left(a+\frac{2\log(cx^n)}{n}\right)}}{2\sqrt{e^{-2a}\left(cx^n\right)^{-4/n}+1}}$$

input
```
Int[Sqrt[Cosh[a + (2*Log[c*x^n])/n]],x]
```

output
```
-1/2*(x*(-(Sqrt[1 + (c^2*x^(2*n))/E^(2*a)]/(c*x^n)) + ArcSinh[1/(c*E^a*x^n
)]/E^a)*Sqrt[Cosh[a + (2*Log[c*x^n])/n]])/((c*x^n)^(2/n)*Sqrt[1 + 1/(E^(2*
a)*(c*x^n)^(4/n))])
```

3.259.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

3.259. $\int\sqrt{\cosh\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx$

rule 6052
```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> S
imp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Cosh[d*(a + b*Log[x])]^p, x
], x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1]
)
```

rule 6060
```
Int[Cosh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[Cosh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p
)   Int[(e*x)^m*x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[
{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

3.259.4 Maple [F]

$$\int \sqrt{\cosh \left(a+\frac{2 \ln \left(c\, x^{n}\right)}{n}\right)}dx$$

input
```
int(cosh(a+2*ln(c*x^n)/n)^(1/2),x)
```

output
```
int(cosh(a+2*ln(c*x^n)/n)^(1/2),x)
```

3.259.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.38

$$\int \sqrt{\cosh \left(a+\frac{2 \log \left(cx^{n}\right)}{n}\right)}\, dx = \frac{1}{8}\left(4\sqrt{\frac{1}{2}}x\sqrt{\frac{x^4e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}+1}{x^2}}e^{\left(\frac{an+2\,\log(c)}{2\,n}\right)}+\sqrt{2}e^{\left(\frac{an+2\,\log(c)}{2\,n}\right)}\log\left(\frac{x^4e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}-2\sqrt{2}\sqrt{\frac{1}{2}}x\sqrt{\frac{x^4e^{\left(\right.}}{}}}{x^4}\right.\right.$$

input
```
integrate(cosh(a+2*log(c*x^n)/n)^(1/2),x, algorithm="fricas")
```

output
```
1/8*(4*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n + 2*log(c))/n) + 1)/x^2)*e^(1/2*(a*
n + 2*log(c))/n) + sqrt(2)*e^(1/2*(a*n + 2*log(c))/n)*log((x^4*e^(2*(a*n +
 2*log(c))/n) - 2*sqrt(2)*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n + 2*log(c))/n) +
 1)/x^2) + 2)/x^4))*e^(-(a*n + 2*log(c))/n)
```

3.259.6 Sympy [F]

$$\int \sqrt{\cosh \left(a+\frac{2 \log (c x^n)}{n}\right)}\, dx=\int \sqrt{\cosh \left(a+\frac{2 \log (c x^n)}{n}\right)}\, dx$$

input
```
integrate(cosh(a+2*ln(c*x**n)/n)**(1/2),x)
```

output
```
Integral(sqrt(cosh(a + 2*log(c*x**n)/n)), x)
```

3.259.7 Maxima [F]

$$\int \sqrt{\cosh \left(a+\frac{2 \log (c x^n)}{n}\right)}\, dx=\int \sqrt{\cosh \left(a+\frac{2 \log (c x^n)}{n}\right)}\, dx$$

input
```
integrate(cosh(a+2*log(c*x^n)/n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(cosh(a + 2*log(c*x^n)/n)), x)
```

3.259.8 Giac [F(-1)]

Timed out.

$$\int \sqrt{\cosh \left(a+\frac{2 \log (c x^n)}{n}\right)}\, dx=\text{Timed out}$$

input
```
integrate(cosh(a+2*log(c*x^n)/n)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.259.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cosh\left(a+\frac{2\log\left(cx^n\right)}{n}\right)}\,dx=\int \sqrt{\cosh\left(a+\frac{2\,\ln\left(c\,x^n\right)}{n}\right)}\,dx$$

input
```
int(cosh(a + (2*log(c*x^n))/n)^(1/2),x)
```

output
```
int(cosh(a + (2*log(c*x^n))/n)^(1/2), x)
```

3.260 $\int \frac{1}{\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx$

3.260.1 Optimal result

Integrand size = 18, antiderivative size = 42

$$\int \frac{1}{\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = -\frac{x\left(1+e^{-2a}(cx^n)^{-4/n}\right)}{2\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

output
```
-1/2*x*(1+1/exp(2*a)/((c*x^n)^(4/n)))/cosh(a+2*ln(c*x^n)/n)^(3/2)
```

3.260.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.45

$$\int \frac{1}{\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = \frac{-\cosh\left(a-2\log(x)+\frac{2\log(cx^n)}{n}\right)+\sinh\left(a-2\log(x)+\frac{2\log(cx^n)}{n}\right)}{x\sqrt{\cosh\left(a+\frac{2\log(cx^n)}{n}\right)}}$$

input
```
Integrate[Cosh[a + (2*Log[c*x^n])/n]^(-3/2),x]
```

output
```
(-Cosh[a - 2*Log[x] + (2*Log[c*x^n])/n] + Sinh[a - 2*Log[x] + (2*Log[c*x^n
])/n])/(x*Sqrt[Cosh[a + (2*Log[c*x^n])/n]])
```

3.260.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6052, 6060, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}dx$$

$$\Big\downarrow \text{6052}$$

$$\frac{x(cx^n)^{-1/n}\int \frac{(cx^n)^{\frac{1}{n}-1}}{\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}d(cx^n)}{n}$$

$$\Big\downarrow \text{6060}$$

$$\frac{x(cx^n)^{2/n}\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{3/2}\int \frac{(cx^n)^{-1-\frac{2}{n}}}{\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{3/2}}d(cx^n)}{n\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

$$\Big\downarrow \text{796}$$

$$-\frac{x\left(e^{-2a}(cx^n)^{-4/n}+1\right)}{2\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

input
```
Int[Cosh[a + (2*Log[c*x^n])/n]^(-3/2),x]
```

output
```
-1/2*(x*(1 + 1/(E^(2*a)*(c*x^n)^(4/n))))/Cosh[a + (2*Log[c*x^n])/n]^(3/2)
```

3.260.3.1 Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 6052
```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> S
imp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Cosh[d*(a + b*Log[x])]^p, x
], x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1]
)
```

rule 6060
```
Int[Cosh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[Cosh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p
)   Int[(e*x)^m*x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[
{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

3.260.4 Maple [F]

$$\int \frac{1}{\cosh\left(a+\frac{2\ln(c\,x^{n})}{n}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/cosh(a+2*ln(c*x^n)/n)^(3/2),x)
```

output
```
int(1/cosh(a+2*ln(c*x^n)/n)^(3/2),x)
```

3.260.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.62

$$\int \frac{1}{\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = -\frac{2\sqrt{\frac{1}{2}}x\sqrt{\frac{x^4e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}+1}{x^2}}e^{\left(-\frac{an+2\,\log(c)}{2\,n}\right)}}{x^4e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}+1}$$

input
```
integrate(1/cosh(a+2*log(c*x^n)/n)^(3/2),x, algorithm="fricas")
```

output
```
-2*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n + 2*log(c))/n) + 1)/x^2)*e^(-1/2*(a*n +
 2*log(c))/n)/(x^4*e^(2*(a*n + 2*log(c))/n) + 1)
```

3.260.6 Sympy [F]

$$\int \frac{1}{\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx=\int \frac{1}{\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx$$

input
```
integrate(1/cosh(a+2*ln(c*x**n)/n)**(3/2),x)
```

output
```
Integral(cosh(a + 2*log(c*x**n)/n)**(-3/2), x)
```

3.260.7 Maxima [F]

$$\int \frac{1}{\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx=\int \frac{1}{\cosh\left(a+\frac{2\log(cx^n)}{n}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cosh(a+2*log(c*x^n)/n)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cosh(a + 2*log(c*x^n)/n)^(-3/2), x)
```

3.260.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/cosh(a+2*log(c*x^n)/n)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.260.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx=\int \frac{1}{\cosh\left(a+\frac{2\,\ln(c\,x^n)}{n}\right)^{3/2}}\,dx$$

input
```
int(1/cosh(a + (2*log(c*x^n))/n)^(3/2),x)
```

output
```
int(1/cosh(a + (2*log(c*x^n))/n)^(3/2), x)
```

3.261 $\int \frac{1}{\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx$

3.261.1 Optimal result

Integrand size = 18, antiderivative size = 101

$$\int \frac{1}{\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = -\frac{x\left(1+e^{-2a}(cx^n)^{-4/n}\right)}{6\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}-\frac{e^{-2a}x(cx^n)^{-4/n}\left(1+e^{-2a}(cx^n)^{-4/n}\right)}{15\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

output
```
-1/6*x*(1+1/exp(2*a)/((c*x^n)^(4/n)))/cosh(a+2*ln(c*x^n)/n)^(7/2)-1/15*x*(
1+1/exp(2*a)/((c*x^n)^(4/n)))/exp(2*a)/((c*x^n)^(4/n))/cosh(a+2*ln(c*x^n)/
n)^(7/2)
```

3.261.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.20

$$\int \frac{1}{\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = \frac{\left((2+5x^4)\cosh\left(a-2\log(x)+\frac{2\log(cx^n)}{n}\right)+(-2+5x^4)\sinh\left(a-2\log(x)+\frac{2\log(cx^n)}{n}\right)\right)\left(-\cosh\left(2a-4\right.\right.}{15x^5\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

input
```
Integrate[Cosh[a + (2*Log[c*x^n])/n]^(-7/2),x]
```

output
```
(((2 + 5*x^4)*Cosh[a - 2*Log[x] + (2*Log[c*x^n])/n] + (-2 + 5*x^4)*Sinh[a
- 2*Log[x] + (2*Log[c*x^n])/n])*(-Cosh[2*a - 4*Log[x] + (4*Log[c*x^n])/n]
+ Sinh[2*a - 4*Log[x] + (4*Log[c*x^n])/n]))/(15*x^5*Cosh[a + (2*Log[c*x^n]
)/n]^(5/2))
```

3.261.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.40, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6052, 6060, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}dx$$

$$\Big\downarrow \text{6052}$$

$$\frac{x(cx^n)^{-1/n}\int \frac{(cx^n)^{\frac{1}{n}-1}}{\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}d(cx^n)}{n}$$

$$\Big\downarrow \text{6060}$$

$$\frac{x(cx^n)^{6/n}\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{7/2}\int \frac{(cx^n)^{-1-\frac{6}{n}}}{\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{7/2}}d(cx^n)}{n\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

$$\Big\downarrow \text{803}$$

$$\frac{x(cx^n)^{6/n}\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{7/2}\left(\frac{2}{3}e^{-2a}\int \frac{(cx^n)^{-1-\frac{10}{n}}}{\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{7/2}}d(cx^n)-\frac{n(cx^n)^{-6/n}}{6\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{5/2}}\right)}{n\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

$$\Big\downarrow \text{796}$$

$$\frac{x(cx^n)^{6/n}\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{7/2}\left(-\frac{e^{-2a}n(cx^n)^{-10/n}}{15\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{5/2}}-\frac{n(cx^n)^{-6/n}}{6\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{5/2}}\right)}{n\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

input
```
Int[Cosh[a + (2*Log[c*x^n])/n]^(-7/2),x]
```

output
```
(x*(c*x^n)^(6/n)*(1 + 1/(E^(2*a)*(c*x^n)^(4/n)))^(7/2)*(-1/15*n/(E^(2*a)*(
c*x^n)^(10/n)*(1 + 1/(E^(2*a)*(c*x^n)^(4/n)))^(5/2)) - n/(6*(c*x^n)^(6/n)*
(1 + 1/(E^(2*a)*(c*x^n)^(4/n)))^(5/2))))/(n*Cosh[a + (2*Log[c*x^n])/n]^(7/
2))
```

3.261.3.1 Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 6052
```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> S
imp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Cosh[d*(a + b*Log[x])]^p, x
], x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1]
)
```

rule 6060
```
Int[Cosh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[Cosh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p
)   Int[(e*x)^m*x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[
{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

3.261.4 Maple [F]

$$\int \frac{1}{\cosh\left(a+\frac{2\ln(c x^n)}{n}\right)^{\frac{7}{2}}}dx$$

input
```
int(1/cosh(a+2*ln(c*x^n)/n)^(7/2),x)
```

output
```
int(1/cosh(a+2*ln(c*x^n)/n)^(7/2),x)
```

3.261.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.27

$$\int \frac{1}{\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = -\frac{8\sqrt{\frac{1}{2}}\left(5\,x^5e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}+2\,x\right)\sqrt{\frac{x^4e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}+1}{x^2}}e^{\left(-\frac{an+2\,\log(c)}{2\,n}\right)}}{15\left(x^{12}e^{\left(\frac{6\,(an+2\,\log(c))}{n}\right)}+3\,x^8e^{\left(\frac{4\,(an+2\,\log(c))}{n}\right)}+3\,x^4e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}+1\right)}$$

input
```
integrate(1/cosh(a+2*log(c*x^n)/n)^(7/2),x, algorithm="fricas")
```

output
```
-8/15*sqrt(1/2)*(5*x^5*e^(2*(a*n + 2*log(c))/n) + 2*x)*sqrt((x^4*e^(2*(a*n
 + 2*log(c))/n) + 1)/x^2)*e^(-1/2*(a*n + 2*log(c))/n)/(x^12*e^(6*(a*n + 2*
log(c))/n) + 3*x^8*e^(4*(a*n + 2*log(c))/n) + 3*x^4*e^(2*(a*n + 2*log(c))/
n) + 1)
```

3.261.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/cosh(a+2*ln(c*x**n)/n)**(7/2),x)
```

output
```
Timed out
```

3.261.7 Maxima [F]

$$\int \frac{1}{\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = \int \frac{1}{\cosh\left(a+\frac{2\log(cx^n)}{n}\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cosh(a+2*log(c*x^n)/n)^(7/2),x, algorithm="maxima")
```

output
```
integrate(cosh(a + 2*log(c*x^n)/n)^(-7/2), x)
```

3.261.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/cosh(a+2*log(c*x^n)/n)^(7/2),x, algorithm="giac")
```

output
```
Timed out
```

3.261.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = \int \frac{1}{\cosh\left(a+\frac{2\ln(c\,x^n)}{n}\right)^{7/2}}\,dx$$

input
```
int(1/cosh(a + (2*log(c*x^n))/n)^(7/2),x)
```

output
```
int(1/cosh(a + (2*log(c*x^n))/n)^(7/2), x)
```

3.262 $\int \cosh\left(\frac{a+bx}{c+dx}\right)\,dx$

3.262.1 Optimal result

Integrand size = 14, antiderivative size = 101

$$\int \cosh\left(\frac{a+bx}{c+dx}\right)\,dx=\frac{(c+dx)\cosh\left(\frac{a+bx}{c+dx}\right)}{d}+\frac{(bc-ad)\text{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)\sinh\left(\frac{b}{d}\right)}{d^2}-\frac{(bc-ad)\cosh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}$$

output
```
(d*x+c)*cosh((b*x+a)/(d*x+c))/d-(-a*d+b*c)*cosh(b/d)*Shi((-a*d+b*c)/d/(d*x
+c))/d^2+(-a*d+b*c)*Chi((-a*d+b*c)/d/(d*x+c))*sinh(b/d)/d^2
```

3.262.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 305 vs. 2(101) = 202.

Time = 1.40 (sec) , antiderivative size = 305, normalized size of antiderivative = 3.02

$$\int \cosh\left(\frac{a+bx}{c+dx}\right)\,dx$$
$$=\frac{cde^{-\frac{a+bx}{c+dx}}+cde^{\frac{a+bx}{c+dx}}+2d^2x\cosh\left(\frac{b}{d}\right)\cosh\left(\frac{-bc+ad}{d(c+dx)}\right)+2d^2x\sinh\left(\frac{b}{d}\right)\sinh\left(\frac{-bc+ad}{d(c+dx)}\right)-(bc-ad)\left(\text{Chi}\left(\frac{bc-ad}{cd+d^2x}\right)\right.}{}$$

input
```
Integrate[Cosh[(a + b*x)/(c + d*x)],x]
```

output
```
((c*d)/E^((a + b*x)/(c + d*x)) + c*d*E^((a + b*x)/(c + d*x)) + 2*d^2*x*Cos
h[b/d]*Cosh[(-(b*c) + a*d)/(d*(c + d*x))] + 2*d^2*x*Sinh[b/d]*Sinh[(-(b*c)
 + a*d)/(d*(c + d*x))] - (b*c - a*d)*(CoshIntegral[(b*c - a*d)/(c*d + d^2*
x)]*(Cosh[b/d] - Sinh[b/d]) - CoshIntegral[(-(b*c) + a*d)/(d*(c + d*x))]*(
Cosh[b/d] + Sinh[b/d]) - Cosh[b/d]*SinhIntegral[(-(b*c) + a*d)/(d*(c + d*x
))] - Sinh[b/d]*SinhIntegral[(-(b*c) + a*d)/(d*(c + d*x))] + Cosh[b/d]*Sin
hIntegral[(b*c - a*d)/(c*d + d^2*x)] - Sinh[b/d]*SinhIntegral[(b*c - a*d)/
(c*d + d^2*x)]))/(2*d^2)
```

3.262.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.15, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {6142, 3042, 3778, 26, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh\left(\frac{a+bx}{c+dx}\right) dx$$

$$\Big\downarrow \text{6142}$$

$$-\frac{\int (c+dx)^2 \cosh\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right) d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int (c+dx)^2 \sin\left(\frac{ib}{d}-\frac{i(bc-ad)}{d(c+dx)}+\frac{\pi}{2}\right) d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{-\left((c+dx)\cosh\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)-\frac{i(bc-ad)\int -i(c+dx)\sinh\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}}{d}$$

$$\Big\downarrow \text{26}$$

$$-\frac{-\frac{(bc-ad)\int (c+dx)\sinh\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}-\left((c+dx)\cosh\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\left((c+dx)\cosh\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)-\frac{(bc-ad)\int -i(c+dx)\sin\left(\frac{ib}{d}-\frac{i(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}}{d}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{-(c+dx)\cosh\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)+\frac{i(bc-ad)\int(c+dx)\sin\left(\frac{ib}{d}-\frac{i(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}}{d}$$

$$\Big\downarrow \text{ 3784}$$

$$-\frac{-(c+dx)\cosh\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)+\frac{i(bc-ad)\left(i\sinh\left(\frac{b}{d}\right)\int(c+dx)\cosh\left(\frac{bc-ad}{d(c+dx)}\right)d\frac{1}{c+dx}+\cosh\left(\frac{b}{d}\right)\int -i(c+dx)\sinh\left(\frac{bc-ad}{d(c+dx)}\right)d\frac{1}{c+dx}\right)}{d}}{d}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{-(c+dx)\cosh\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)+\frac{i(bc-ad)\left(i\sinh\left(\frac{b}{d}\right)\int(c+dx)\cosh\left(\frac{bc-ad}{d(c+dx)}\right)d\frac{1}{c+dx}-i\cosh\left(\frac{b}{d}\right)\int(c+dx)\sinh\left(\frac{bc-ad}{d(c+dx)}\right)d\frac{1}{c+dx}\right)}{d}}{d}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{-(c+dx)\cosh\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)+\frac{i(bc-ad)\left(i\sinh\left(\frac{b}{d}\right)\int(c+dx)\sin\left(\frac{i(bc-ad)}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}-i\cosh\left(\frac{b}{d}\right)\int -i(c+dx)\sin\left(\frac{i(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}\right)}{d}}{d}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{-(c+dx)\cosh\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)+\frac{i(bc-ad)\left(i\sinh\left(\frac{b}{d}\right)\int(c+dx)\sin\left(\frac{i(bc-ad)}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}-\cosh\left(\frac{b}{d}\right)\int(c+dx)\sin\left(\frac{i(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}\right)}{d}}{d}$$

$$\Big\downarrow \text{ 3779}$$

$$-\frac{-(c+dx)\cosh\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)+\frac{i(bc-ad)\left(i\sinh\left(\frac{b}{d}\right)\int(c+dx)\sin\left(\frac{i(bc-ad)}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}-i\cosh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc-ad}{d(c+dx)}\right)\right)}{d}}{d}$$

$$\Big\downarrow \text{ 3782}$$

$$-\frac{-(c+dx)\cosh\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)+\frac{i(bc-ad)\left(i\sinh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)-i\cosh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc-ad}{d(c+dx)}\right)\right)}{d}}{d}$$

input
```
Int[Cosh[(a + b*x)/(c + d*x)],x]
```

3.262. $\int \cosh\left(\frac{a+bx}{c+dx}\right)\,dx$

output
```
-((-((c + d*x)*Cosh[b/d - (b*c - a*d)/(d*(c + d*x))]) + (I*(b*c - a*d)*(I*
CoshIntegral[(b*c - a*d)/(d*(c + d*x))]*Sinh[b/d] - I*Cosh[b/d]*SinhIntegr
al[(b*c - a*d)/(d*(c + d*x))]))/d)/d)
```

3.262.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6142
```
Int[Cosh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> Simp[-d^(-1)   Subst[Int[Cosh[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x]
, x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c -
a*d, 0]
```

3.262.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(101) = 202$.

Time = 0.21 (sec) , antiderivative size = 347, normalized size of antiderivative = 3.44

method	result
risch	$\frac{e^{-\frac{bx+a}{dx+c}} a}{\frac{2da}{dx+c} - \frac{2cb}{dx+c}} - \frac{e^{-\frac{bx+a}{dx+c}} cb}{2d\left(\frac{da}{dx+c} - \frac{cb}{dx+c}\right)} - \frac{e^{-\frac{b}{d}} \operatorname{Ei}_1\left(\frac{da-cb}{d(dx+c)}\right) a}{2d} + \frac{e^{-\frac{b}{d}} \operatorname{Ei}_1\left(\frac{da-cb}{d(dx+c)}\right) bc}{2d^2} + \frac{d\, e^{\frac{bx+a}{dx+c}} xa}{2da-2cb} - \frac{e^{\frac{bx+a}{dx+c}} xcb}{2(da-cb)} + \frac{e^{\frac{bx+a}{dx+c}} ca}{2da-2cb}$

input
```
int(cosh((b*x+a)/(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(-(b*x+a)/(d*x+c))/(d/(d*x+c)*a-1/(d*x+c)*c*b)*a-1/2/d*exp(-(b*x+a)
/(d*x+c))/(d/(d*x+c)*a-1/(d*x+c)*c*b)*c*b-1/2/d*exp(-1/d*b)*Ei(1,(a*d-b*c)
/d/(d*x+c))*a+1/2/d^2*exp(-1/d*b)*Ei(1,(a*d-b*c)/d/(d*x+c))*b*c+1/2*d*exp(
(b*x+a)/(d*x+c))/(a*d-b*c)*x*a-1/2*exp((b*x+a)/(d*x+c))/(a*d-b*c)*x*c*b+1/
2*exp((b*x+a)/(d*x+c))/(a*d-b*c)*c*a-1/2/d*exp((b*x+a)/(d*x+c))/(a*d-b*c)*
c^2*b+1/2/d*exp(1/d*b)*Ei(1,-(a*d-b*c)/d/(d*x+c))*a-1/2/d^2*exp(1/d*b)*Ei(
1,-(a*d-b*c)/d/(d*x+c))*b*c
```

3.262.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.69

$$\int \cosh\left(\frac{a+bx}{c+dx}\right)\, dx$$
$$= \frac{2\left(d^2x+cd\right)\cosh\left(\frac{bx+a}{dx+c}\right) - \left((bc-ad)\operatorname{Ei}\left(\frac{bc-ad}{d^2x+cd}\right) - (bc-ad)\operatorname{Ei}\left(-\frac{bc-ad}{d^2x+cd}\right)\right)\cosh\left(\frac{b}{d}\right) + \left((bc-ad)\operatorname{Ei}\left(\frac{bc-ad}{d^2x+cd}\right.\right.}{2\,d^2}$$

input
```
integrate(cosh((b*x+a)/(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(2*(d^2*x + c*d)*cosh((b*x + a)/(d*x + c)) - ((b*c - a*d)*Ei((b*c - a*
d)/(d^2*x + c*d)) - (b*c - a*d)*Ei(-(b*c - a*d)/(d^2*x + c*d)))*cosh(b/d)
+ ((b*c - a*d)*Ei((b*c - a*d)/(d^2*x + c*d)) + (b*c - a*d)*Ei(-(b*c - a*d)
/(d^2*x + c*d)))*sinh(b/d))/d^2
```

3.262.6 Sympy [F]

$$\int \cosh\left(\frac{a+bx}{c+dx}\right)\,dx = \int \cosh\left(\frac{a+bx}{c+dx}\right)\,dx$$

input
```
integrate(cosh((b*x+a)/(d*x+c)),x)
```

output
```
Integral(cosh((a + b*x)/(c + d*x)), x)
```

3.262.7 Maxima [F]

$$\int \cosh\left(\frac{a+bx}{c+dx}\right)\,dx = \int \cosh\left(\frac{bx+a}{dx+c}\right)\,dx$$

input
```
integrate(cosh((b*x+a)/(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(cosh((b*x + a)/(d*x + c)), x)
```

3.262.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 764 vs. $2(101) = 202$.

Time = 1.96 (sec) , antiderivative size = 764, normalized size of antiderivative = 7.56

$$\int \cosh\left(\frac{a+bx}{c+dx}\right)\,dx$$
$$= \frac{\left(b^3c^2\mathrm{Ei}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\frac{b}{d}} - 2\,ab^2cd\mathrm{Ei}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\frac{b}{d}} - \frac{(bx+a)b^2c^2d\mathrm{Ei}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\frac{b}{d}}}{dx+c} + a^2bd^2\mathrm{Ei}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right.\right.}{}$$
$$- \frac{\left(b^3c^2\mathrm{Ei}\left(\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\left(-\frac{b}{d}\right)} - 2\,ab^2cd\mathrm{Ei}\left(\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\left(-\frac{b}{d}\right)} - \frac{(bx+a)b^2c^2d\mathrm{Ei}\left(\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\left(-\frac{b}{d}\right)}}{dx+c} + a^2bd^2\mathrm{Ei}\left(\frac{b-(\right.\right.}{}$$

input
```
integrate(cosh((b*x+a)/(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(b^3*c^2*Ei(-(b - (b*x + a)*d/(d*x + c))/d)*e^(b/d) - 2*a*b^2*c*d*Ei(-
(b - (b*x + a)*d/(d*x + c))/d)*e^(b/d) - (b*x + a)*b^2*c^2*d*Ei(-(b - (b*x
 + a)*d/(d*x + c))/d)*e^(b/d)/(d*x + c) + a^2*b*d^2*Ei(-(b - (b*x + a)*d/(
d*x + c))/d)*e^(b/d) + 2*(b*x + a)*a*b*c*d^2*Ei(-(b - (b*x + a)*d/(d*x + c
))/d)*e^(b/d)/(d*x + c) - (b*x + a)*a^2*d^3*Ei(-(b - (b*x + a)*d/(d*x + c)
)/d)*e^(b/d)/(d*x + c) + b^2*c^2*d*e^((b*x + a)/(d*x + c)) - 2*a*b*c*d^2*e
^((b*x + a)/(d*x + c)) + a^2*d^3*e^((b*x + a)/(d*x + c)))*(b*c/(b*c - a*d)
^2 - a*d/(b*c - a*d)^2)/(b*d^2 - (b*x + a)*d^3/(d*x + c)) - 1/2*(b^3*c^2*E
i((b - (b*x + a)*d/(d*x + c))/d)*e^(-b/d) - 2*a*b^2*c*d*Ei((b - (b*x + a)*
d/(d*x + c))/d)*e^(-b/d) - (b*x + a)*b^2*c^2*d*Ei((b - (b*x + a)*d/(d*x +
c))/d)*e^(-b/d)/(d*x + c) + a^2*b*d^2*Ei((b - (b*x + a)*d/(d*x + c))/d)*e^
(-b/d) + 2*(b*x + a)*a*b*c*d^2*Ei((b - (b*x + a)*d/(d*x + c))/d)*e^(-b/d)/
(d*x + c) - (b*x + a)*a^2*d^3*Ei((b - (b*x + a)*d/(d*x + c))/d)*e^(-b/d)/(
d*x + c) - b^2*c^2*d*e^(-(b*x + a)/(d*x + c)) + 2*a*b*c*d^2*e^(-(b*x + a)/
(d*x + c)) - a^2*d^3*e^(-(b*x + a)/(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b
*c - a*d)^2)/(b*d^2 - (b*x + a)*d^3/(d*x + c))
```

3.262.9 Mupad [F(-1)]

Timed out.

$$\int \cosh\left(\frac{a+bx}{c+dx}\right)\,dx = \int \cosh\left(\frac{a+b\,x}{c+d\,x}\right)\,dx$$

input
```
int(cosh((a + b*x)/(c + d*x)),x)
```

output
```
int(cosh((a + b*x)/(c + d*x)), x)
```

3.263 $\int \cosh^2\left(\frac{a+bx}{c+dx}\right) dx$

3.263.1 Optimal result

Integrand size = 16, antiderivative size = 107

$$\int \cosh^2\left(\frac{a+bx}{c+dx}\right) dx = \frac{(c+dx)\cosh^2\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(bc-ad)\text{Chi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)\sinh\left(\frac{2b}{d}\right)}{d^2} - \frac{(bc-ad)\cosh\left(\frac{2b}{d}\right)\text{Shi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}$$

output
```
(d*x+c)*cosh((b*x+a)/(d*x+c))^2/d-(-a*d+b*c)*cosh(2*b/d)*Shi(2*(-a*d+b*c)/
d/(d*x+c))/d^2+(-a*d+b*c)*Chi(2*(-a*d+b*c)/d/(d*x+c))*sinh(2*b/d)/d^2
```

3.263.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 475 vs. 2(107) = 214.

Time = 3.17 (sec) , antiderivative size = 475, normalized size of antiderivative = 4.44

$$\int \cosh^2\left(\frac{a+bx}{c+dx}\right) dx = \frac{cde^{-\frac{2(a+bx)}{c+dx}} + cde^{\frac{2(a+bx)}{c+dx}} + 2d^2x + 2d^2x\cosh\left(\frac{2b}{d}\right)\cosh\left(\frac{2(-bc+ad)}{d(c+dx)}\right) - 2(bc-ad)\text{Chi}\left(\frac{2bc-2ad}{cd+d^2x}\right)\left(\cosh\left(\frac{2b}{d}\right) - \text{s}\right.}{}$$

input
```
Integrate[Cosh[(a + b*x)/(c + d*x)]^2,x]
```

output
```
((c*d)/E^((2*(a + b*x))/(c + d*x)) + c*d*E^((2*(a + b*x))/(c + d*x)) + 2*d
^2*x + 2*d^2*x*Cosh[(2*b)/d]*Cosh[(2*(-(b*c) + a*d))/(d*(c + d*x))] - 2*(b
*c - a*d)*CoshIntegral[(2*b*c - 2*a*d)/(c*d + d^2*x)]*(Cosh[(2*b)/d] - Sin
h[(2*b)/d]) + 2*(b*c - a*d)*CoshIntegral[(2*(-(b*c) + a*d))/(d*(c + d*x))]
*(Cosh[(2*b)/d] + Sinh[(2*b)/d]) + 2*d^2*x*Sinh[(2*b)/d]*Sinh[(2*(-(b*c) +
 a*d))/(d*(c + d*x))] + 2*b*c*Cosh[(2*b)/d]*SinhIntegral[(2*(-(b*c) + a*d)
)/(d*(c + d*x))] - 2*a*d*Cosh[(2*b)/d]*SinhIntegral[(2*(-(b*c) + a*d))/(d*
(c + d*x))] + 2*b*c*Sinh[(2*b)/d]*SinhIntegral[(2*(-(b*c) + a*d))/(d*(c +
d*x))] - 2*a*d*Sinh[(2*b)/d]*SinhIntegral[(2*(-(b*c) + a*d))/(d*(c + d*x))
] - 2*b*c*Cosh[(2*b)/d]*SinhIntegral[(2*b*c - 2*a*d)/(c*d + d^2*x)] + 2*a*
d*Cosh[(2*b)/d]*SinhIntegral[(2*b*c - 2*a*d)/(c*d + d^2*x)] + 2*b*c*Sinh[(
2*b)/d]*SinhIntegral[(2*b*c - 2*a*d)/(c*d + d^2*x)] - 2*a*d*Sinh[(2*b)/d]*
SinhIntegral[(2*b*c - 2*a*d)/(c*d + d^2*x)])/(4*d^2)
```

3.263.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.14, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6142, 3042, 3794, 27, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2\left(\frac{a+bx}{c+dx}\right)dx$$

$$\Big\downarrow \text{6142}$$

$$-\frac{\int (c+dx)^2\cosh^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int (c+dx)^2\sin\left(\frac{ib}{d}-\frac{i(bc-ad)}{d(c+dx)}+\frac{\pi}{2}\right)^2d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{3794}$$

$$-\frac{-\left((c+dx)\cosh^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)-\frac{2i(bc-ad)\int -\frac{1}{2}i(c+dx)\sinh\left(\frac{2b}{d}-\frac{2(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}}{d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{(bc-ad)\int (c+dx)\sinh\left(\frac{2b}{d}-\frac{2(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}-\left((c+dx)\cosh^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)}{d}$$

$\big\downarrow$ 3042

$$-\frac{-\left((c+dx)\cosh^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)-\frac{(bc-ad)\int -i(c+dx)\sin\left(\frac{2ib}{d}-\frac{2i(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}}{d}$$

$\big\downarrow$ 26

$$-\frac{-(c+dx)\cosh^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)+\frac{i(bc-ad)\int (c+dx)\sin\left(\frac{2ib}{d}-\frac{2i(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}}{d}$$

$\big\downarrow$ 3784

$$-\frac{-(c+dx)\cosh^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)+\frac{i(bc-ad)\left(i\sinh\left(\frac{2b}{d}\right)\int (c+dx)\cosh\left(\frac{2(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}+\cosh\left(\frac{2b}{d}\right)\int -i(c+dx)\sinh\left(\frac{2(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}\right)}{d}}{d}$$

$\big\downarrow$ 26

$$-\frac{-(c+dx)\cosh^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)+\frac{i(bc-ad)\left(i\sinh\left(\frac{2b}{d}\right)\int (c+dx)\cosh\left(\frac{2(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}-i\cosh\left(\frac{2b}{d}\right)\int (c+dx)\sinh\left(\frac{2(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}\right)}{d}}{d}$$

$\big\downarrow$ 3042

$$-\frac{-(c+dx)\cosh^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)+\frac{i(bc-ad)\left(i\sinh\left(\frac{2b}{d}\right)\int (c+dx)\sin\left(\frac{2i(bc-ad)}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}-i\cosh\left(\frac{2b}{d}\right)\int -i(c+dx)\sin\left(\frac{2i(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}\right)}{d}}{d}$$

$\big\downarrow$ 26

$$-\frac{-(c+dx)\cosh^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)+\frac{i(bc-ad)\left(i\sinh\left(\frac{2b}{d}\right)\int (c+dx)\sin\left(\frac{2i(bc-ad)}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}-\cosh\left(\frac{2b}{d}\right)\int (c+dx)\sin\left(\frac{2i(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}\right)}{d}}{d}$$

$\big\downarrow$ 3779

$$-\frac{-(c+dx)\cosh^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)+\frac{i(bc-ad)\left(i\sinh\left(\frac{2b}{d}\right)\int (c+dx)\sin\left(\frac{2i(bc-ad)}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}-i\cosh\left(\frac{2b}{d}\right)\text{Shi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)\right)}{d}}{d}$$

$\big\downarrow$ 3782

$$-\frac{-(c+dx)\cosh^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)+\frac{i(bc-ad)\left(i\sinh\left(\frac{2b}{d}\right)\text{Chi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)-i\cosh\left(\frac{2b}{d}\right)\text{Shi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)\right)}{d}}{d}$$

3.263. $\int \cosh^2\left(\frac{a+bx}{c+dx}\right)\,dx$

input
```
Int[Cosh[(a + b*x)/(c + d*x)]^2,x]
```

output
```
-((-((c + d*x)*Cosh[b/d - (b*c - a*d)/(d*(c + d*x))]^2) + (I*(b*c - a*d)*(
I*CoshIntegral[(2*(b*c - a*d))/(d*(c + d*x))]*Sinh[(2*b)/d] - I*Cosh[(2*b)
/d]*SinhIntegral[(2*(b*c - a*d))/(d*(c + d*x))]))/d)/d)
```

3.263.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 6142
```
Int[Cosh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> Simp[-d^(-1)   Subst[Int[Cosh[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x]
, x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c -
a*d, 0]
```

3.263.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(107) = 214$.

Time = 0.85 (sec) , antiderivative size = 358, normalized size of antiderivative = 3.35

method	result
risch	$\frac{x}{2}+\frac{\mathrm{e}^{-\frac{2(bx+a)}{dx+c}}a}{\frac{4da}{dx+c}-\frac{4cb}{dx+c}}-\frac{\mathrm{e}^{-\frac{2(bx+a)}{dx+c}}cb}{4d\left(\frac{da}{dx+c}-\frac{cb}{dx+c}\right)}-\frac{\mathrm{e}^{-\frac{2b}{d}}\operatorname{Ei}_1\left(\frac{2da-2cb}{(dx+c)d}\right)a}{2d}+\frac{\mathrm{e}^{-\frac{2b}{d}}\operatorname{Ei}_1\left(\frac{2da-2cb}{(dx+c)d}\right)bc}{2d^2}+\frac{d\,\mathrm{e}^{\frac{2bx+2a}{dx+c}}xa}{4da-4cb}-\frac{\mathrm{e}^{\frac{2bx+2a}{dx+c}}xcb}{4(da-cb)}+$

input
```
int(cosh((b*x+a)/(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x+1/4*exp(-2*(b*x+a)/(d*x+c))/(d/(d*x+c)*a-1/(d*x+c)*c*b)*a-1/4/d*exp(
-2*(b*x+a)/(d*x+c))/(d/(d*x+c)*a-1/(d*x+c)*c*b)*c*b-1/2/d*exp(-2/d*b)*Ei(1
,2*(a*d-b*c)/d/(d*x+c))*a+1/2/d^2*exp(-2/d*b)*Ei(1,2*(a*d-b*c)/d/(d*x+c))*
b*c+1/4*d*exp(2*(b*x+a)/(d*x+c))/(a*d-b*c)*x*a-1/4*exp(2*(b*x+a)/(d*x+c))/
(a*d-b*c)*x*c*b+1/4*exp(2*(b*x+a)/(d*x+c))/(a*d-b*c)*c*a-1/4/d*exp(2*(b*x+
a)/(d*x+c))/(a*d-b*c)*c^2*b+1/2/d*exp(2/d*b)*Ei(1,-2*(a*d-b*c)/d/(d*x+c))*
a-1/2/d^2*exp(2/d*b)*Ei(1,-2*(a*d-b*c)/d/(d*x+c))*b*c
```

3.263.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(107) = 214$.

Time = 0.30 (sec) , antiderivative size = 366, normalized size of antiderivative = 3.42

$$\int \cosh^2\left(\frac{a+bx}{c+dx}\right)\,dx$$
$$= \frac{d^2x + (d^2x+cd)\cosh\left(\frac{bx+a}{dx+c}\right)^2 + \left(d^2x - (bc-ad)\text{Ei}\left(-\frac{2\,(bc-ad)}{d^2x+cd}\right)\cosh\left(\frac{2\,b}{d}\right) + cd\right)\sinh\left(\frac{bx+a}{dx+c}\right)^2 + \left((bc-a\right.}{}$$

input
```
integrate(cosh((b*x+a)/(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*(d^2*x + (d^2*x + c*d)*cosh((b*x + a)/(d*x + c))^2 + (d^2*x - (b*c - a
*d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*cosh(2*b/d) + c*d)*sinh((b*x + a)/(d*
x + c))^2 + ((b*c - a*d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*cosh((b*x + a)/(
d*x + c))^2 - (b*c - a*d)*Ei(2*(b*c - a*d)/(d^2*x + c*d)))*cosh(2*b/d) + (
(b*c - a*d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*cosh((b*x + a)/(d*x + c))^2 -
 (b*c - a*d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*sinh((b*x + a)/(d*x + c))^2
+ (b*c - a*d)*Ei(2*(b*c - a*d)/(d^2*x + c*d)))*sinh(2*b/d))/(d^2*cosh((b*x
 + a)/(d*x + c))^2 - d^2*sinh((b*x + a)/(d*x + c))^2)
```

3.263.6 Sympy [F]

$$\int \cosh^2\left(\frac{a+bx}{c+dx}\right)\,dx = \int \cosh^2\left(\frac{a+bx}{c+dx}\right)\,dx$$

input
```
integrate(cosh((b*x+a)/(d*x+c))**2,x)
```

output
```
Integral(cosh((a + b*x)/(c + d*x))**2, x)
```

3.263.7 Maxima [F]

$$\int \cosh^2\left(\frac{a+bx}{c+dx}\right) dx = \int \cosh\left(\frac{bx+a}{dx+c}\right)^2 dx$$

input
```
integrate(cosh((b*x+a)/(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/2*x + 1/4*integrate(e^(2*b*c/(d^2*x + c*d) - 2*a/(d*x + c) - 2*b/d), x)
+ 1/4*integrate(e^(-2*b*c/(d^2*x + c*d) + 2*a/(d*x + c) + 2*b/d), x)
```

3.263.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 749 vs. $2(107) = 214$.

Time = 6.84 (sec) , antiderivative size = 749, normalized size of antiderivative = 7.00

$$\int \cosh^2\left(\frac{a+bx}{c+dx}\right) dx = \frac{\left(2\,b^3c^2\text{Ei}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)e^{\left(\frac{2\,b}{d}\right)} - 4\,ab^2cd\text{Ei}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)e^{\left(\frac{2\,b}{d}\right)} - \frac{2\,(bx+a)b^2c^2d\text{Ei}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)e^{\left(\frac{2\,b}{d}\right)}}{dx+c} +$$

input
```
integrate(cosh((b*x+a)/(d*x+c))^2,x, algorithm="giac")
```

output
```
1/4*(2*b^3*c^2*Ei(-2*(b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d) - 4*a*b^2*c*
d*Ei(-2*(b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d) - 2*(b*x + a)*b^2*c^2*d*E
i(-2*(b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d)/(d*x + c) + 2*a^2*b*d^2*Ei(-
2*(b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d) + 4*(b*x + a)*a*b*c*d^2*Ei(-2*(
b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d)/(d*x + c) - 2*(b*x + a)*a^2*d^3*Ei
(-2*(b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d)/(d*x + c) - 2*b^3*c^2*Ei(2*(b
 - (b*x + a)*d/(d*x + c))/d)*e^(-2*b/d) + 4*a*b^2*c*d*Ei(2*(b - (b*x + a)*
d/(d*x + c))/d)*e^(-2*b/d) + 2*(b*x + a)*b^2*c^2*d*Ei(2*(b - (b*x + a)*d/(
d*x + c))/d)*e^(-2*b/d)/(d*x + c) - 2*a^2*b*d^2*Ei(2*(b - (b*x + a)*d/(d*x
 + c))/d)*e^(-2*b/d) - 4*(b*x + a)*a*b*c*d^2*Ei(2*(b - (b*x + a)*d/(d*x +
c))/d)*e^(-2*b/d)/(d*x + c) + 2*(b*x + a)*a^2*d^3*Ei(2*(b - (b*x + a)*d/(d
*x + c))/d)*e^(-2*b/d)/(d*x + c) + b^2*c^2*d*e^(2*(b*x + a)/(d*x + c)) - 2
*a*b*c*d^2*e^(2*(b*x + a)/(d*x + c)) + a^2*d^3*e^(2*(b*x + a)/(d*x + c)) +
 b^2*c^2*d*e^(-2*(b*x + a)/(d*x + c)) - 2*a*b*c*d^2*e^(-2*(b*x + a)/(d*x +
 c)) + a^2*d^3*e^(-2*(b*x + a)/(d*x + c)) + 2*b^2*c^2*d - 4*a*b*c*d^2 + 2*
a^2*d^3)*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)/(b*d^2 - (b*x + a)*d^3/(d
*x + c))
```

3.263.9 Mupad [F(-1)]

Timed out.

$$\int \cosh^2\left(\frac{a+bx}{c+dx}\right)\,dx = \int \cosh\left(\frac{a+b\,x}{c+d\,x}\right)^2\,dx$$

input
```
int(cosh((a + b*x)/(c + d*x))^2,x)
```

output
```
int(cosh((a + b*x)/(c + d*x))^2, x)
```

3.264 $\int e^{a+bx} \cosh^4(a+bx) \, dx$

3.264.1 Optimal result

Integrand size = 16, antiderivative size = 83

$$\int e^{a+bx} \cosh^4(a+bx) \, dx = -\frac{e^{-3a-3bx}}{48b} - \frac{e^{-a-bx}}{4b} + \frac{3e^{a+bx}}{8b} + \frac{e^{3a+3bx}}{12b} + \frac{e^{5a+5bx}}{80b}$$

output
```
-1/48*exp(-3*b*x-3*a)/b-1/4*exp(-b*x-a)/b+3/8*exp(b*x+a)/b+1/12*exp(3*b*x+
3*a)/b+1/80*exp(5*b*x+5*a)/b
```

3.264.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.75

$$\int e^{a+bx} \cosh^4(a+bx) \, dx = \frac{e^{-3(a+bx)} \left(-5 - 60e^{2(a+bx)} + 90e^{4(a+bx)} + 20e^{6(a+bx)} + 3e^{8(a+bx)}\right)}{240b}$$

input
```
Integrate[E^(a + b*x)*Cosh[a + b*x]^4,x]
```

output
```
(-5 - 60*E^(2*(a + b*x)) + 90*E^(4*(a + b*x)) + 20*E^(6*(a + b*x)) + 3*E^(
8*(a + b*x)))/(240*b*E^(3*(a + b*x)))
```

3.264.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2720, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\cosh^4(a+bx)\,dx$$

$$\Big\downarrow \; 2720$$

$$\frac{\int \frac{1}{16}e^{-4a-4bx}\left(1+e^{2a+2bx}\right)^4de^{a+bx}}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{\int e^{-4a-4bx}\left(1+e^{2a+2bx}\right)^4de^{a+bx}}{16b}$$

$$\Big\downarrow \; 244$$

$$\frac{\int \left(6+e^{-4a-4bx}+4e^{-2a-2bx}+4e^{2a+2bx}+e^{4a+4bx}\right)de^{a+bx}}{16b}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{1}{3}e^{-3a-3bx}-4e^{-a-bx}+6e^{a+bx}+\frac{4}{3}e^{3a+3bx}+\frac{1}{5}e^{5a+5bx}}{16b}$$

input
```
Int[E^(a + b*x)*Cosh[a + b*x]^4,x]
```

output
```
(-1/3*E^(-3*a - 3*b*x) - 4*E^(-a - b*x) + 6*E^(a + b*x) + (4*E^(3*a + 3*b*
x))/3 + E^(5*a + 5*b*x)/5)/(16*b)
```

3.264.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.264.4 Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.54

method	result
derivativedivides	$\frac{\frac{\cosh(bx+a)^5}{5}+\left(\frac{8}{15}+\frac{\cosh(bx+a)^4}{5}+\frac{4\cosh(bx+a)^2}{15}\right)\sinh(bx+a)}{b}$
default	$\frac{\frac{\cosh(bx+a)^5}{5}+\left(\frac{8}{15}+\frac{\cosh(bx+a)^4}{5}+\frac{4\cosh(bx+a)^2}{15}\right)\sinh(bx+a)}{b}$
risch	$-\frac{e^{-3bx-3a}}{48b}-\frac{e^{-bx-a}}{4b}+\frac{3\,e^{bx+a}}{8b}+\frac{e^{3bx+3a}}{12b}+\frac{e^{5bx+5a}}{80b}$
parallelrisch	$-\frac{e^{bx+a}(20\cosh(2bx+2a)-24\cosh(bx+a)+\cosh(4bx+4a)-4\sinh(4bx+4a)-40\sinh(2bx+2a)+24\sinh(bx+a)-45)}{120b}$

input
```
int(exp(b*x+a)*cosh(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/5*cosh(b*x+a)^5+(8/15+1/5*cosh(b*x+a)^4+4/15*cosh(b*x+a)^2)*sinh(b*
x+a))
```

3.264.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.36

$$\int e^{a+bx}\cosh^4(a+bx)\,dx =$$
$$-\frac{\cosh(bx+a)^4 - 16\cosh(bx+a)\sinh(bx+a)^3 + \sinh(bx+a)^4 + 2\left(3\cosh(bx+a)^2 + 10\right)\sinh(bx+}{120\left(b\cosh(bx+a) - b\sinh\right.}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^4,x, algorithm="fricas")
```

output
```
-1/120*(cosh(b*x + a)^4 - 16*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)
^4 + 2*(3*cosh(b*x + a)^2 + 10)*sinh(b*x + a)^2 + 20*cosh(b*x + a)^2 - 16*
(cosh(b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a) - 45)/(b*cosh(b*x + a) -
 b*sinh(b*x + a))
```

3.264.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(65) = 130.

Time = 2.14 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.67

$$\int e^{a+bx}\cosh^4(a+bx)\,dx$$
$$= \begin{cases} \frac{8e^a e^{bx}\sinh^4(a+bx)}{15b} - \frac{8e^a e^{bx}\sinh^3(a+bx)\cosh(a+bx)}{15b} - \frac{4e^a e^{bx}\sinh^2(a+bx)\cosh^2(a+bx)}{5b} + \frac{4e^a e^{bx}\sinh(a+bx)\cosh^3(a+bx)}{5b} + \frac{e^a e}{} \\ xe^a\cosh^4(a) \end{cases}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)**4,x)
```

output
```
Piecewise((8*exp(a)*exp(b*x)*sinh(a + b*x)**4/(15*b) - 8*exp(a)*exp(b*x)*s
inh(a + b*x)**3*cosh(a + b*x)/(15*b) - 4*exp(a)*exp(b*x)*sinh(a + b*x)**2*
cosh(a + b*x)**2/(5*b) + 4*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**3/
(5*b) + exp(a)*exp(b*x)*cosh(a + b*x)**4/(5*b), Ne(b, 0)), (x*exp(a)*cosh(
a)**4, True))
```

3.264.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.82

$$\int e^{a+bx}\cosh^4(a+bx)\,dx = \frac{e^{(5\,bx+5\,a)}}{80\,b} + \frac{e^{(3\,bx+3\,a)}}{12\,b} + \frac{3\,e^{(bx+a)}}{8\,b} - \frac{e^{(-bx-a)}}{4\,b} - \frac{e^{(-3\,bx-3\,a)}}{48\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/80*e^(5*b*x + 5*a)/b + 1/12*e^(3*b*x + 3*a)/b + 3/8*e^(b*x + a)/b - 1/4*
e^(-b*x - a)/b - 1/48*e^(-3*b*x - 3*a)/b
```

3.264.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

$$\int e^{a+bx}\cosh^4(a+bx)\,dx = -\frac{5\left(12\,e^{(2\,bx+2\,a)}+1\right)e^{(-3\,bx-3\,a)} - 3\,e^{(5\,bx+5\,a)} - 20\,e^{(3\,bx+3\,a)} - 90\,e^{(bx+a)}}{240\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^4,x, algorithm="giac")
```

output
```
-1/240*(5*(12*e^(2*b*x + 2*a) + 1)*e^(-3*b*x - 3*a) - 3*e^(5*b*x + 5*a) -
20*e^(3*b*x + 3*a) - 90*e^(b*x + a))/b
```

3.264.9 Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.70

$$\int e^{a+bx}\cosh^4(a+bx)\,dx = \frac{90\,\mathrm{e}^{a+b\,x} - 60\,\mathrm{e}^{-a-b\,x} - 5\,\mathrm{e}^{-3\,a-3\,b\,x} + 20\,\mathrm{e}^{3\,a+3\,b\,x} + 3\,\mathrm{e}^{5\,a+5\,b\,x}}{240\,b}$$

input
```
int(cosh(a + b*x)^4*exp(a + b*x),x)
```

output
```
(90*exp(a + b*x) - 60*exp(- a - b*x) - 5*exp(- 3*a - 3*b*x) + 20*exp(3*a +
 3*b*x) + 3*exp(5*a + 5*b*x))/(240*b)
```

3.265 $\int e^{a+bx}\cosh^3(a+bx)\,dx$

3.265.1 Optimal result

Integrand size = 16, antiderivative size = 57

$$\int e^{a+bx}\cosh^3(a+bx)\,dx=-\frac{e^{-2a-2bx}}{16b}+\frac{3e^{2a+2bx}}{16b}+\frac{e^{4a+4bx}}{32b}+\frac{3x}{8}$$

output
```
-1/16*exp(-2*b*x-2*a)/b+3/16*exp(2*b*x+2*a)/b+1/32*exp(4*b*x+4*a)/b+3/8*x
```

3.265.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int e^{a+bx}\cosh^3(a+bx)\,dx=\frac{-\frac{1}{16}e^{-2a-2bx}+\frac{3}{16}e^{2a+2bx}+\frac{1}{32}e^{4a+4bx}+\frac{3bx}{8}}{b}$$

input
```
Integrate[E^(a + b*x)*Cosh[a + b*x]^3,x]
```

output
```
(-1/16*E^(-2*a - 2*b*x) + (3*E^(2*a + 2*b*x))/16 + E^(4*a + 4*b*x)/32 + (3
*b*x)/8)/b
```

3.265.3 Rubi [A] (warning: unable to verify)

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2720, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\cosh^3(a+bx)\,dx$$

$$\Big\downarrow \; 2720$$

$$\frac{\int \frac{1}{8}e^{-3a-3bx}\left(1+e^{2a+2bx}\right)^3de^{a+bx}}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{\int e^{-3a-3bx}\left(1+e^{2a+2bx}\right)^3de^{a+bx}}{8b}$$

$$\Big\downarrow \; 243$$

$$\frac{\int e^{-2a-2bx}\left(1+e^{2a+2bx}\right)^3de^{2a+2bx}}{16b}$$

$$\Big\downarrow \; 49$$

$$\frac{\int \left(3+e^{-2a-2bx}+3e^{-a-bx}+e^{2a+2bx}\right)de^{2a+2bx}}{16b}$$

$$\Big\downarrow \; 2009$$

$$\frac{-e^{-a-bx}+\frac{7}{2}e^{2a+2bx}+3\log\left(e^{2a+2bx}\right)}{16b}$$

input
```
Int[E^(a + b*x)*Cosh[a + b*x]^3,x]
```

output
```
(-E^(-a - b*x) + (7*E^(2*a + 2*b*x))/2 + 3*Log[E^(2*a + 2*b*x)])/(16*b)
```

3.265.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.265.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

method	result	size
risch	$-\frac{\mathrm{e}^{-2bx-2a}}{16b}+\frac{3\,\mathrm{e}^{2bx+2a}}{16b}+\frac{\mathrm{e}^{4bx+4a}}{32b}+\frac{3x}{8}$	47
derivativedivides	$\frac{\frac{\cosh(bx+a)^4}{4}+\left(\frac{\cosh(bx+a)^3}{4}+\frac{3\cosh(bx+a)}{8}\right)\sinh(bx+a)+\frac{3bx}{8}+\frac{3a}{8}}{b}$	49
default	$\frac{\frac{\cosh(bx+a)^4}{4}+\left(\frac{\cosh(bx+a)^3}{4}+\frac{3\cosh(bx+a)}{8}\right)\sinh(bx+a)+\frac{3bx}{8}+\frac{3a}{8}}{b}$	49
parallelrisch	$\frac{\mathrm{e}^{bx+a}(12bx\cosh(bx+a)-12bx\sinh(bx+a)+\cosh(bx+a)+11\sinh(bx+a)-\cosh(3bx+3a)+3\sinh(3bx+3a))}{32b}$	69

input
```
int(exp(b*x+a)*cosh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*exp(-2*b*x-2*a)/b+3/16*exp(2*b*x+2*a)/b+1/32*exp(4*b*x+4*a)/b+3/8*x
```

3.265.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(46) = 92$.

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.67

$$\int e^{a+bx}\cosh^3(a+bx)\,dx =$$
$$-\frac{\cosh(bx+a)^3 + 3\cosh(bx+a)\sinh(bx+a)^2 - 3\sinh(bx+a)^3 - 6(2bx+1)\cosh(bx+a) + 3(4bx}{32(b\cosh(bx+a) - b\sinh(bx+a))}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/32*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 - 3*sinh(b*x + a)
^3 - 6*(2*b*x + 1)*cosh(b*x + a) + 3*(4*b*x - 3*cosh(b*x + a)^2 - 2)*sinh(
b*x + a))/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

3.265.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(48) = 96$.

Time = 0.86 (sec) , antiderivative size = 182, normalized size of antiderivative = 3.19

$$\int e^{a+bx}\cosh^3(a+bx)\,dx$$
$$= \begin{cases} \frac{3xe^ae^{bx}\sinh^3(a+bx)}{8} - \frac{3xe^ae^{bx}\sinh^2(a+bx)\cosh(a+bx)}{8} - \frac{3xe^ae^{bx}\sinh(a+bx)\cosh^2(a+bx)}{8} + \frac{3xe^ae^{bx}\cosh^3(a+bx)}{8} - \frac{3e^ae^{bx}\sinh}{8b} \\ xe^a\cosh^3(a) \end{cases}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)**3,x)
```

output
```
Piecewise((3*x*exp(a)*exp(b*x)*sinh(a + b*x)**3/8 - 3*x*exp(a)*exp(b*x)*si
nh(a + b*x)**2*cosh(a + b*x)/8 - 3*x*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a
+ b*x)**2/8 + 3*x*exp(a)*exp(b*x)*cosh(a + b*x)**3/8 - 3*exp(a)*exp(b*x)*s
inh(a + b*x)**3/(8*b) + 3*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**2/(
4*b) - exp(a)*exp(b*x)*cosh(a + b*x)**3/(8*b), Ne(b, 0)), (x*exp(a)*cosh(a
)**3, True))
```

3.265.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int e^{a+bx}\cosh^3(a+bx)\,dx = \frac{3\,(bx+a)}{8\,b}+\frac{e^{(4\,bx+4\,a)}}{32\,b}+\frac{3\,e^{(2\,bx+2\,a)}}{16\,b}-\frac{e^{(-2\,bx-2\,a)}}{16\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^3,x, algorithm="maxima")
```

output
```
3/8*(b*x + a)/b + 1/32*e^(4*b*x + 4*a)/b + 3/16*e^(2*b*x + 2*a)/b - 1/16*e
^(-2*b*x - 2*a)/b
```

3.265.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int e^{a+bx}\cosh^3(a+bx)\,dx = \frac{12\,bx-2\left(3\,e^{(2\,bx+2\,a)}+1\right)e^{(-2\,bx-2\,a)}+12\,a+e^{(4\,bx+4\,a)}+6\,e^{(2\,bx+2\,a)}}{32\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/32*(12*b*x - 2*(3*e^(2*b*x + 2*a) + 1)*e^(-2*b*x - 2*a) + 12*a + e^(4*b*
x + 4*a) + 6*e^(2*b*x + 2*a))/b
```

3.265.9 Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int e^{a+bx}\cosh^3(a+bx)\,dx = \frac{3\,x}{8}+\frac{\frac{3\,\mathrm{e}^{2\,a+2\,b\,x}}{16}-\frac{\mathrm{e}^{-2\,a-2\,b\,x}}{16}+\frac{\mathrm{e}^{4\,a+4\,b\,x}}{32}}{b}$$

input
```
int(cosh(a + b*x)^3*exp(a + b*x),x)
```

output
```
(3*x)/8 + ((3*exp(2*a + 2*b*x))/16 - exp(- 2*a - 2*b*x)/16 + exp(4*a + 4*b
*x)/32)/b
```

3.266 $\int e^{a+bx}\cosh^2(a+bx)\,dx$

3.266.1 Optimal result

Integrand size = 16, antiderivative size = 49

$$\int e^{a+bx}\cosh^2(a+bx)\,dx=-\frac{e^{-a-bx}}{4b}+\frac{e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{12b}$$

output
```
-1/4*exp(-b*x-a)/b+1/2*exp(b*x+a)/b+1/12*exp(3*b*x+3*a)/b
```

3.266.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int e^{a+bx}\cosh^2(a+bx)\,dx=\frac{e^{-a-bx}\left(-3+6e^{2(a+bx)}+e^{4(a+bx)}\right)}{12b}$$

input
```
Integrate[E^(a + b*x)*Cosh[a + b*x]^2,x]
```

output
```
(E^(-a - b*x)*(-3 + 6*E^(2*(a + b*x)) + E^(4*(a + b*x))))/(12*b)
```

3.266.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2720, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{1}{4}e^{-2a-2bx}\left(1+e^{2a+2bx}\right)^2de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int e^{-2a-2bx}\left(1+e^{2a+2bx}\right)^2de^{a+bx}}{4b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(2+e^{-2a-2bx}+e^{2a+2bx}\right)de^{a+bx}}{4b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-e^{-a-bx}+2e^{a+bx}+\frac{1}{3}e^{3a+3bx}}{4b}$$

input
```
Int[E^(a + b*x)*Cosh[a + b*x]^2,x]
```

output
```
(-E^(-a - b*x) + 2*E^(a + b*x) + E^(3*a + 3*b*x)/3)/(4*b)
```

3.266.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.266.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^3}{3}+\left(\frac{2}{3}+\frac{\cosh(bx+a)^2}{3}\right)\sinh(bx+a)}{b}$	35
default	$\frac{\frac{\cosh(bx+a)^3}{3}+\left(\frac{2}{3}+\frac{\cosh(bx+a)^2}{3}\right)\sinh(bx+a)}{b}$	35
risch	$-\frac{e^{-bx-a}}{4b}+\frac{e^{bx+a}}{2b}+\frac{e^{3bx+3a}}{12b}$	41
parallelrisch	$\frac{e^{bx+a}(-2\sinh(bx+a)+2\cosh(bx+a)+3+2\sinh(2bx+2a)-\cosh(2bx+2a))}{6b}$	52

input
```
int(exp(b*x+a)*cosh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/3*cosh(b*x+a)^3+(2/3+1/3*cosh(b*x+a)^2)*sinh(b*x+a))
```

3.266.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10

$$\int e^{a+bx}\cosh^2(a+bx)\,dx = -\frac{\cosh(bx+a)^2-4\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2-3}{6\,(b\cosh(bx+a)-b\sinh(bx+a))}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/6*(cosh(b*x + a)^2 - 4*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 -
3)/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

3.266.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(34) = 68$.

Time = 0.40 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.59

$$\int e^{a+bx}\cosh^2(a+bx)\,dx = \begin{cases} -\frac{2e^ae^{bx}\sinh^2(a+bx)}{3b} + \frac{2e^ae^{bx}\sinh(a+bx)\cosh(a+bx)}{3b} + \frac{e^ae^{bx}\cosh^2(a+bx)}{3b} & \text{for } b \neq 0 \\ xe^a\cosh^2(a) & \text{otherwise} \end{cases}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)**2,x)
```

output
```
Piecewise((-2*exp(a)*exp(b*x)*sinh(a + b*x)**2/(3*b) + 2*exp(a)*exp(b*x)*s
inh(a + b*x)*cosh(a + b*x)/(3*b) + exp(a)*exp(b*x)*cosh(a + b*x)**2/(3*b),
 Ne(b, 0)), (x*exp(a)*cosh(a)**2, True))
```

3.266.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int e^{a+bx}\cosh^2(a+bx)\,dx = \frac{e^{(3\,bx+3\,a)}}{12\,b} + \frac{e^{(bx+a)}}{2\,b} - \frac{e^{(-bx-a)}}{4\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/12*e^(3*b*x + 3*a)/b + 1/2*e^(b*x + a)/b - 1/4*e^(-b*x - a)/b
```

3.266.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.69

$$\int e^{a+bx}\cosh^2(a+bx)\,dx = \frac{e^{(3\,bx+3\,a)} + 6\,e^{(bx+a)} - 3\,e^{(-bx-a)}}{12\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^2,x, algorithm="giac")
```

output
```
1/12*(e^(3*b*x + 3*a) + 6*e^(b*x + a) - 3*e^(-b*x - a))/b
```

3.266.9 Mupad [B] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.69

$$\int e^{a+bx}\cosh^2(a+bx)\,dx = \frac{6\,\mathrm{e}^{a+b\,x} - 3\,\mathrm{e}^{-a-b\,x} + \mathrm{e}^{3\,a+3\,b\,x}}{12\,b}$$

input
```
int(cosh(a + b*x)^2*exp(a + b*x),x)
```

output
```
(6*exp(a + b*x) - 3*exp(- a - b*x) + exp(3*a + 3*b*x))/(12*b)
```

3.267 $\int e^{a+bx} \cosh(a+bx)\,dx$

3.267.1 Optimal result

Integrand size = 14, antiderivative size = 23

$$\int e^{a+bx} \cosh(a+bx)\,dx = \frac{e^{2a+2bx}}{4b} + \frac{x}{2}$$

output
```
1/4*exp(2*b*x+2*a)/b+1/2*x
```

3.267.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int e^{a+bx} \cosh(a+bx)\,dx = \frac{e^{2a+2bx}}{4b} + \frac{x}{2}$$

input
```
Integrate[E^(a + b*x)*Cosh[a + b*x],x]
```

output
```
E^(2*a + 2*b*x)/(4*b) + x/2
```

3.267.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2720, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{1}{2}e^{-a-bx}\left(1+e^{2a+2bx}\right)de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int e^{-a-bx}\left(1+e^{2a+2bx}\right)de^{a+bx}}{2b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(e^{-a-bx}+e^{a+bx}\right)de^{a+bx}}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}e^{2a+2bx}+\log\left(e^{a+bx}\right)}{2b}$$

input
```
Int[E^(a + b*x)*Cosh[a + b*x],x]
```

output
```
(E^(2*a + 2*b*x)/2 + Log[E^(a + b*x)])/(2*b)
```

3.267.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.267.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{e^{2bx+2a}}{4b}+\frac{x}{2}$	19
derivativedivides	$\frac{\frac{\cosh(bx+a)^2}{2}+\frac{\cosh(bx+a)\sinh(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}}{b}$	37
default	$\frac{\frac{\cosh(bx+a)^2}{2}+\frac{\cosh(bx+a)\sinh(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}}{b}$	37
parallelrisch	$\frac{e^{bx+a}(bx\cosh(bx+a)-bx\sinh(bx+a)+\sinh(bx+a))}{2b}$	38

input
```
int(exp(b*x+a)*cosh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*exp(2*b*x+2*a)/b+1/2*x
```

3.267.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(18) = 36.

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.17

$$\int e^{a+bx}\cosh(a+bx)\,dx=\frac{(2\,bx+1)\cosh(bx+a)-(2\,bx-1)\sinh(bx+a)}{4\,(b\cosh(bx+a)-b\sinh(bx+a))}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a),x, algorithm="fricas")
```

output
```
1/4*((2*b*x + 1)*cosh(b*x + a) - (2*b*x - 1)*sinh(b*x + a))/(b*cosh(b*x +
a) - b*sinh(b*x + a))
```

3.267.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(15) = 30$.

Time = 0.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.48

$$\int e^{a+bx}\cosh(a+bx)\,dx = \begin{cases} -\frac{xe^{a}e^{bx}\sinh{(a+bx)}}{2} + \frac{xe^{a}e^{bx}\cosh{(a+bx)}}{2} + \frac{e^{a}e^{bx}\sinh{(a+bx)}}{b} - \frac{e^{a}e^{bx}\cosh{(a+bx)}}{2b} & \text{for } b \neq 0 \\ xe^{a}\cosh{(a)} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a),x)
```

output
```
Piecewise((-x*exp(a)*exp(b*x)*sinh(a + b*x)/2 + x*exp(a)*exp(b*x)*cosh(a +
 b*x)/2 + exp(a)*exp(b*x)*sinh(a + b*x)/b - exp(a)*exp(b*x)*cosh(a + b*x)/
(2*b), Ne(b, 0)), (x*exp(a)*cosh(a), True))
```

3.267.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int e^{a+bx}\cosh(a+bx)\,dx = \frac{1}{2}\,x + \frac{a}{2\,b} + \frac{e^{(2\,bx+2\,a)}}{4\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*x + 1/2*a/b + 1/4*e^(2*b*x + 2*a)/b
```

3.267.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int e^{a+bx}\cosh(a+bx)\,dx = \frac{2\,bx + 2\,a + e^{(2\,bx+2\,a)}}{4\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a),x, algorithm="giac")
```

output
```
1/4*(2*b*x + 2*a + e^(2*b*x + 2*a))/b
```

3.267.9 Mupad [B] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int e^{a+bx}\cosh(a+bx)\,dx=\frac{x}{2}+\frac{\mathrm{e}^{2\,a+2\,b\,x}}{4\,b}$$

input
```
int(cosh(a + b*x)*exp(a + b*x),x)
```

output
```
x/2 + exp(2*a + 2*b*x)/(4*b)
```

3.268 $\int e^{a+bx}\text{sech}(a+bx)\,dx$

3.268.1 Optimal result

Integrand size = 14, antiderivative size = 17

$$\int e^{a+bx}\text{sech}(a+bx)\,dx=\frac{\log\left(1+e^{2a+2bx}\right)}{b}$$

output
```
ln(1+exp(2*b*x+2*a))/b
```

3.268.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int e^{a+bx}\text{sech}(a+bx)\,dx=\frac{\log\left(1+e^{2a+2bx}\right)}{b}$$

input
```
Integrate[E^(a + b*x)*Sech[a + b*x],x]
```

output
```
Log[1 + E^(2*a + 2*b*x)]/b
```

3.268.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2720, 27, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\text{sech}(a+bx)\,dx$$

$$\Big\downarrow \ 2720$$

$$\frac{\int \frac{2e^{a+bx}}{1+e^{2a+2bx}}de^{a+bx}}{b}$$

$$\Big\downarrow \ 27$$

$$\frac{2\int \frac{e^{a+bx}}{1+e^{2a+2bx}}de^{a+bx}}{b}$$

$$\Big\downarrow \ 240$$

$$\frac{\log\left(e^{2a+2bx}+1\right)}{b}$$

input
```
Int[E^(a + b*x)*Sech[a + b*x],x]
```

output
```
Log[1 + E^(2*a + 2*b*x)]/b
```

3.268.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.268.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\ln(\cosh(bx+a))+bx+a}{b}$	17
default	$\frac{\ln(\cosh(bx+a))+bx+a}{b}$	17
risch	$-\frac{2a}{b}+\frac{\ln\left(1+e^{2bx+2a}\right)}{b}$	24

input
```
int(exp(b*x+a)*sech(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(ln(cosh(b*x+a))+b*x+a)
```

3.268.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.76

$$\int e^{a+bx}\text{sech}(a+bx)\,dx=\frac{\log\left(\frac{2\,\cosh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b}$$

input
```
integrate(exp(b*x+a)*sech(b*x+a),x, algorithm="fricas")
```

output
```
log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a)))/b
```

3.268.6 Sympy [F]

$$\int e^{a+bx}\text{sech}(a+bx)\,dx = e^{a}\int e^{bx}\,\text{sech}\,(a+bx)\,dx$$

input
```
integrate(exp(b*x+a)*sech(b*x+a),x)
```

output
```
exp(a)*Integral(exp(b*x)*sech(a + b*x), x)
```

3.268.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int e^{a+bx}\text{sech}(a+bx)\,dx = \frac{\log\left(e^{(2\,bx+2\,a)}+1\right)}{b}$$

input
```
integrate(exp(b*x+a)*sech(b*x+a),x, algorithm="maxima")
```

output
```
log(e^(2*b*x + 2*a) + 1)/b
```

3.268.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int e^{a+bx}\text{sech}(a+bx)\,dx = \frac{\log\left(e^{(2\,bx+2\,a)}+1\right)}{b}$$

input
```
integrate(exp(b*x+a)*sech(b*x+a),x, algorithm="giac")
```

output
```
log(e^(2*b*x + 2*a) + 1)/b
```

3.268.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int e^{a+bx}\operatorname{sech}(a+bx)\,dx=\frac{\ln\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}{b}$$

input
```
int(exp(a + b*x)/cosh(a + b*x),x)
```

output
```
log(exp(2*a + 2*b*x) + 1)/b
```

3.269 $\int e^{a+bx}\mathbf{sech}^2(a+bx)\,dx$

3.269.1 Optimal result

Integrand size = 16, antiderivative size = 40

$$\int e^{a+bx}\text{sech}^2(a+bx)\,dx=-\frac{2e^{a+bx}}{b\left(1+e^{2a+2bx}\right)}+\frac{2\arctan\left(e^{a+bx}\right)}{b}$$

output
```
-2*exp(b*x+a)/b/(1+exp(2*b*x+2*a))+2*arctan(exp(b*x+a))/b
```

3.269.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int e^{a+bx}\text{sech}^2(a+bx)\,dx=\frac{-\frac{2e^{a+bx}}{1+e^{2a+2bx}}+2\arctan\left(e^{a+bx}\right)}{b}$$

input
```
Integrate[E^(a + b*x)*Sech[a + b*x]^2,x]
```

output
```
((-2*E^(a + b*x))/(1 + E^(2*a + 2*b*x)) + 2*ArcTan[E^(a + b*x)])/b
```

3.269.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2720, 27, 252, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\text{sech}^2(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{4e^{2a+2bx}}{(1+e^{2a+2bx})^2}de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\int \frac{e^{2a+2bx}}{(1+e^{2a+2bx})^2}de^{a+bx}}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{4\left(\frac{1}{2}\int \frac{1}{1+e^{2a+2bx}}de^{a+bx}-\frac{e^{a+bx}}{2(e^{2a+2bx}+1)}\right)}{b}$$

$$\Big\downarrow \text{216}$$

$$\frac{4\left(\frac{1}{2}\arctan\left(e^{a+bx}\right)-\frac{e^{a+bx}}{2(e^{2a+2bx}+1)}\right)}{b}$$

input
```
Int[E^(a + b*x)*Sech[a + b*x]^2,x]
```

output
```
(4*(-1/2*E^(a + b*x)/(1 + E^(2*a + 2*b*x)) + ArcTan[E^(a + b*x)]/2))/b
```

3.269.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.269.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.62

method	result	size
derivativedivides	$\frac{-\frac{1}{\cosh(bx+a)}+2\arctan\left(e^{bx+a}\right)}{b}$	25
default	$\frac{-\frac{1}{\cosh(bx+a)}+2\arctan\left(e^{bx+a}\right)}{b}$	25
risch	$-\frac{2\,e^{bx+a}}{b\left(1+e^{2bx+2a}\right)}+\frac{i\ln\left(e^{bx+a}+i\right)}{b}-\frac{i\ln\left(e^{bx+a}-i\right)}{b}$	58

input
```
int(exp(b*x+a)*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/cosh(b*x+a)+2*arctan(exp(b*x+a)))
```

3.269.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(37) = 74.

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.62

$$\int e^{a+bx}\operatorname{sech}^2(a+bx)\,dx = \frac{2\left(\left(\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2+1\right)\arctan(\cosh(bx+a)+\sinh(bx-$$
$$b\cosh(bx+a)^2+2\,b\cosh(bx+a)\sinh(bx+a)+b\sinh(bx+a)^2+$$

input
```
integrate(exp(b*x+a)*sech(b*x+a)^2,x, algorithm="fricas")
```

output
```
2*((cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)
*arctan(cosh(b*x + a) + sinh(b*x + a)) - cosh(b*x + a) - sinh(b*x + a))/(b
*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 + b
)
```

3.269.6 Sympy [F]

$$\int e^{a+bx}\operatorname{sech}^2(a+bx)\,dx = e^a\int e^{bx}\operatorname{sech}^2(a+bx)\,dx$$

input
```
integrate(exp(b*x+a)*sech(b*x+a)**2,x)
```

output
```
exp(a)*Integral(exp(b*x)*sech(a + b*x)**2, x)
```

3.269.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int e^{a+bx}\operatorname{sech}^2(a+bx)\,dx = \frac{2\arctan\left(e^{(bx+a)}\right)}{b} - \frac{2\,e^{(bx+a)}}{b(e^{(2\,bx+2\,a)}+1)}$$

input
```
integrate(exp(b*x+a)*sech(b*x+a)^2,x, algorithm="maxima")
```

output
```
2*arctan(e^(b*x + a))/b - 2*e^(b*x + a)/(b*(e^(2*b*x + 2*a) + 1))
```

3.269.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int e^{a+bx}\operatorname{sech}^2(a+bx)\,dx = -\frac{2\left(\frac{e^{(bx+a)}}{e^{(2bx+2a)}+1} - \arctan\left(e^{(bx+a)}\right)\right)}{b}$$

input
```
integrate(exp(b*x+a)*sech(b*x+a)^2,x, algorithm="giac")
```

output
```
-2*(e^(b*x + a)/(e^(2*b*x + 2*a) + 1) - arctan(e^(b*x + a)))/b
```

3.269.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int e^{a+bx}\operatorname{sech}^2(a+bx)\,dx = \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} - \frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(exp(a + b*x)/cosh(a + b*x)^2,x)
```

output
```
(2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2) - (2*exp(a + b*x))/(
b*(exp(2*a + 2*b*x) + 1))
```

3.270 $\int e^{a+bx}\operatorname{sech}^3(a+bx)\,dx$

3.270.1 Optimal result

Integrand size = 16, antiderivative size = 29

$$\int e^{a+bx}\operatorname{sech}^3(a+bx)\,dx=\frac{2e^{4a+4bx}}{b\left(1+e^{2a+2bx}\right)^2}$$

output
```
2*exp(4*b*x+4*a)/b/(1+exp(2*b*x+2*a))^2
```

3.270.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int e^{a+bx}\operatorname{sech}^3(a+bx)\,dx=\frac{2e^{4a+4bx}}{b\left(1+e^{2a+2bx}\right)^2}$$

input
```
Integrate[E^(a + b*x)*Sech[a + b*x]^3,x]
```

output
```
(2*E^(4*a + 4*b*x))/(b*(1 + E^(2*a + 2*b*x))^2)
```

3.270.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2720, 27, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\text{sech}^3(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{8e^{3a+3bx}}{(1+e^{2a+2bx})^3}de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{8\int \frac{e^{3a+3bx}}{(1+e^{2a+2bx})^3}de^{a+bx}}{b}$$

$$\Big\downarrow \text{242}$$

$$\frac{2e^{4a+4bx}}{b\left(e^{2a+2bx}+1\right)^2}$$

input
```
Int[E^(a + b*x)*Sech[a + b*x]^3,x]
```

output
```
(2*E^(4*a + 4*b*x))/(b*(1 + E^(2*a + 2*b*x))^2)
```

3.270.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.270.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{-\frac{1}{2\cosh(bx+a)^2}+\tanh(bx+a)}{b}$	22
default	$\frac{-\frac{1}{2\cosh(bx+a)^2}+\tanh(bx+a)}{b}$	22
risch	$-\frac{2(2\,\mathrm{e}^{2bx+2a}+1)}{b(1+\mathrm{e}^{2bx+2a})^2}$	32
parallelrisch	$\frac{\mathrm{e}^{bx+a}(\cosh(bx+a)+\sinh(bx+a))}{b(1+\cosh(2bx+2a))}$	37

input
```
int(exp(b*x+a)*sech(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2/cosh(b*x+a)^2+tanh(b*x+a))
```

3.270.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(27) = 54.

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.97

$$\int e^{a+bx}\mathrm{sech}^3(a+bx)\,dx =$$
$$-\frac{2\,(3\,\cosh\,(bx+a)+\sinh\,(bx+a))}{b\cosh\,(bx+a)^3+3\,b\cosh\,(bx+a)\sinh\,(bx+a)^2+b\sinh\,(bx+a)^3+3\,b\cosh\,(bx+a)+(3\,b\cosh\,(bx-$$

input
```
integrate(exp(b*x+a)*sech(b*x+a)^3,x, algorithm="fricas")
```

output
```
-2*(3*cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a
)*sinh(b*x + a)^2 + b*sinh(b*x + a)^3 + 3*b*cosh(b*x + a) + (3*b*cosh(b*x
+ a)^2 + b)*sinh(b*x + a))
```

3.270.6 Sympy [F]

$$\int e^{a+bx}\text{sech}^3(a+bx)\,dx = e^a\int e^{bx}\,\text{sech}^3\,(a+bx)\,dx$$

input
```
integrate(exp(b*x+a)*sech(b*x+a)**3,x)
```

output
```
exp(a)*Integral(exp(b*x)*sech(a + b*x)**3, x)
```

3.270.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(27) = 54.

Time = 0.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.34

$$\int e^{a+bx}\text{sech}^3(a+bx)\,dx = -\frac{4\,e^{(2\,bx+2\,a)}}{b(e^{(4\,bx+4\,a)}+2\,e^{(2\,bx+2\,a)}+1)} - \frac{2}{b(e^{(4\,bx+4\,a)}+2\,e^{(2\,bx+2\,a)}+1)}$$

input
```
integrate(exp(b*x+a)*sech(b*x+a)^3,x, algorithm="maxima")
```

output
```
-4*e^(2*b*x + 2*a)/(b*(e^(4*b*x + 4*a) + 2*e^(2*b*x + 2*a) + 1)) - 2/(b*(e
^(4*b*x + 4*a) + 2*e^(2*b*x + 2*a) + 1))
```

3.270.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int e^{a+bx}\text{sech}^3(a+bx)\,dx = -\frac{2\,\left(2\,e^{(2\,bx+2\,a)}+1\right)}{b(e^{(2\,bx+2\,a)}+1)^2}$$

input
```
integrate(exp(b*x+a)*sech(b*x+a)^3,x, algorithm="giac")
```

output
```
-2*(2*e^(2*b*x + 2*a) + 1)/(b*(e^(2*b*x + 2*a) + 1)^2)
```

3.270.9 Mupad [B] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int e^{a+bx}\text{sech}^3(a+bx)\,dx = -\frac{2\,\left(2\,{\mathrm{e}}^{2\,a+2\,b\,x}+1\right)}{b\,{\left({\mathrm{e}}^{2\,a+2\,b\,x}+1\right)}^2}$$

input
```
int(exp(a + b*x)/cosh(a + b*x)^3,x)
```

output
```
-(2*(2*exp(2*a + 2*b*x) + 1))/(b*(exp(2*a + 2*b*x) + 1)^2)
```

3.271 $\int e^{a+bx}\text{sech}^4(a+bx)\,dx$

3.271.1 Optimal result

Integrand size = 16, antiderivative size = 95

$$\int e^{a+bx}\text{sech}^4(a+bx)\,dx = -\frac{8e^{3a+3bx}}{3b\left(1+e^{2a+2bx}\right)^3}-\frac{2e^{a+bx}}{b\left(1+e^{2a+2bx}\right)^2}+\frac{e^{a+bx}}{b\left(1+e^{2a+2bx}\right)}+\frac{\arctan\left(e^{a+bx}\right)}{b}$$

output
```
-8/3*exp(3*b*x+3*a)/b/(1+exp(2*b*x+2*a))^3-2*exp(b*x+a)/b/(1+exp(2*b*x+2*a
))^2+exp(b*x+a)/b/(1+exp(2*b*x+2*a))+arctan(exp(b*x+a))/b
```

3.271.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.67

$$\int e^{a+bx}\text{sech}^4(a+bx)\,dx = \frac{e^{a+bx}\left(-3-8e^{2(a+bx)}+3e^{4(a+bx)}\right)}{3b\left(1+e^{2(a+bx)}\right)^3}+\frac{\arctan\left(e^{a+bx}\right)}{b}$$

input
```
Integrate[E^(a + b*x)*Sech[a + b*x]^4,x]
```

output
```
(E^(a + b*x)*(-3 - 8*E^(2*(a + b*x)) + 3*E^(4*(a + b*x))))/(3*b*(1 + E^(2*
(a + b*x)))^3) + ArcTan[E^(a + b*x)]/b
```

3.271.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2720, 27, 252, 252, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\text{sech}^4(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{16e^{4a+4bx}}{(1+e^{2a+2bx})^4}de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{16\int \frac{e^{4a+4bx}}{(1+e^{2a+2bx})^4}de^{a+bx}}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{16\left(\frac{1}{2}\int \frac{e^{2a+2bx}}{(1+e^{2a+2bx})^3}de^{a+bx}-\frac{e^{3a+3bx}}{6(e^{2a+2bx}+1)^3}\right)}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{16\left(\frac{1}{2}\left(\frac{1}{4}\int \frac{1}{(1+e^{2a+2bx})^2}de^{a+bx}-\frac{e^{a+bx}}{4(e^{2a+2bx}+1)^2}\right)-\frac{e^{3a+3bx}}{6(e^{2a+2bx}+1)^3}\right)}{b}$$

$$\Big\downarrow \text{215}$$

$$\frac{16\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{2}\int \frac{1}{1+e^{2a+2bx}}de^{a+bx}+\frac{e^{a+bx}}{2(e^{2a+2bx}+1)}\right)-\frac{e^{a+bx}}{4(e^{2a+2bx}+1)^2}\right)-\frac{e^{3a+3bx}}{6(e^{2a+2bx}+1)^3}\right)}{b}$$

$$\Big\downarrow \text{216}$$

$$\frac{16\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{2}\arctan\left(e^{a+bx}\right)+\frac{e^{a+bx}}{2(e^{2a+2bx}+1)}\right)-\frac{e^{a+bx}}{4(e^{2a+2bx}+1)^2}\right)-\frac{e^{3a+3bx}}{6(e^{2a+2bx}+1)^3}\right)}{b}$$

input
```
Int[E^(a + b*x)*Sech[a + b*x]^4,x]
```

output
```
(16*(-1/6*E^(3*a + 3*b*x)/(1 + E^(2*a + 2*b*x))^3 + (-1/4*E^(a + b*x)/(1 +
 E^(2*a + 2*b*x))^2 + (E^(a + b*x)/(2*(1 + E^(2*a + 2*b*x))) + ArcTan[E^(a
 + b*x)]/2)/4)/2))/b
```

3.271.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.271.4 Maple [A] (verified)

Time = 2.34 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.39

method	result	size
derivativedivides	$\frac{-\frac{1}{3\cosh(bx+a)^3}+\frac{\operatorname{sech}(bx+a)\tanh(bx+a)}{2}+\arctan\left(e^{bx+a}\right)}{b}$	37
default	$\frac{-\frac{1}{3\cosh(bx+a)^3}+\frac{\operatorname{sech}(bx+a)\tanh(bx+a)}{2}+\arctan\left(e^{bx+a}\right)}{b}$	37
risch	$\frac{e^{bx+a}\left(3\,e^{4bx+4a}-8\,e^{2bx+2a}-3\right)}{3b\left(1+e^{2bx+2a}\right)^3}+\frac{i\ln\left(e^{bx+a}+i\right)}{2b}-\frac{i\ln\left(e^{bx+a}-i\right)}{2b}$	82

input
```
int(exp(b*x+a)*sech(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/3/cosh(b*x+a)^3+1/2*sech(b*x+a)*tanh(b*x+a)+arctan(exp(b*x+a)))
```

3.271.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 513 vs. 2(86) = 172.

Time = 0.26 (sec) , antiderivative size = 513, normalized size of antiderivative = 5.40

$$\int e^{a+bx}\operatorname{sech}^4(a+bx)\,dx$$
$$=\frac{3\cosh(bx+a)^5+15\cosh(bx+a)\sinh(bx+a)^4+3\sinh(bx+a)^5+2\left(15\cosh(bx+a)^2-4\right)\sinh(bx$$

input
```
integrate(exp(b*x+a)*sech(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/3*(3*cosh(b*x + a)^5 + 15*cosh(b*x + a)*sinh(b*x + a)^4 + 3*sinh(b*x + a
)^5 + 2*(15*cosh(b*x + a)^2 - 4)*sinh(b*x + a)^3 - 8*cosh(b*x + a)^3 + 6*(
5*cosh(b*x + a)^3 - 4*cosh(b*x + a))*sinh(b*x + a)^2 + 3*(cosh(b*x + a)^6
+ 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2
 + 1)*sinh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + 3*cosh(
b*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 + 1)*
sinh(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 + 2*cosh(b*x + a)
^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a
)) + 3*(5*cosh(b*x + a)^4 - 8*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - 3*cosh(
b*x + a))/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(
b*x + a)^6 + 3*b*cosh(b*x + a)^4 + 3*(5*b*cosh(b*x + a)^2 + b)*sinh(b*x +
a)^4 + 4*(5*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^3 + 3*b*c
osh(b*x + a)^2 + 3*(5*b*cosh(b*x + a)^4 + 6*b*cosh(b*x + a)^2 + b)*sinh(b*
x + a)^2 + 6*(b*cosh(b*x + a)^5 + 2*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*s
inh(b*x + a) + b)
```

3.271.6 Sympy [F]

$$\int e^{a+bx}\operatorname{sech}^4(a+bx)\,dx = e^a\int e^{bx}\operatorname{sech}^4(a+bx)\,dx$$

input
```
integrate(exp(b*x+a)*sech(b*x+a)**4,x)
```

output
```
exp(a)*Integral(exp(b*x)*sech(a + b*x)**4, x)
```

3.271.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.87

$$\int e^{a+bx}\operatorname{sech}^4(a+bx)\,dx = \frac{\arctan\left(e^{(bx+a)}\right)}{b} + \frac{3\,e^{(5\,bx+5\,a)} - 8\,e^{(3\,bx+3\,a)} - 3\,e^{(bx+a)}}{3\,b(e^{(6\,bx+6\,a)} + 3\,e^{(4\,bx+4\,a)} + 3\,e^{(2\,bx+2\,a)} + 1)}$$

input
```
integrate(exp(b*x+a)*sech(b*x+a)^4,x, algorithm="maxima")
```

output
```
arctan(e^(b*x + a))/b + 1/3*(3*e^(5*b*x + 5*a) - 8*e^(3*b*x + 3*a) - 3*e^(
b*x + a))/(b*(e^(6*b*x + 6*a) + 3*e^(4*b*x + 4*a) + 3*e^(2*b*x + 2*a) + 1)
)
```

3.271.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.63

$$\int e^{a+bx}\operatorname{sech}^4(a+bx)\,dx = \frac{\frac{3\,e^{(5\,bx+5\,a)}-8\,e^{(3\,bx+3\,a)}-3\,e^{(bx+a)}}{\left(e^{(2\,bx+2\,a)}+1\right)^3}+3\,\arctan\left(e^{(bx+a)}\right)}{3\,b}$$

input
```
integrate(exp(b*x+a)*sech(b*x+a)^4,x, algorithm="giac")
```

output
```
1/3*((3*e^(5*b*x + 5*a) - 8*e^(3*b*x + 3*a) - 3*e^(b*x + a))/(e^(2*b*x + 2
*a) + 1)^3 + 3*arctan(e^(b*x + a)))/b
```

3.271.9 Mupad [B] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.37

$$\int e^{a+bx}\operatorname{sech}^4(a+bx)\,dx = \frac{\operatorname{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^a\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}}-\frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(2\,\mathrm{e}^{2\,a+2\,b\,x}+\mathrm{e}^{4\,a+4\,b\,x}+1\right)} - \frac{8\,\mathrm{e}^{3\,a+3\,b\,x}}{3\,b\,\left(3\,\mathrm{e}^{2\,a+2\,b\,x}+3\,\mathrm{e}^{4\,a+4\,b\,x}+\mathrm{e}^{6\,a+6\,b\,x}+1\right)}+\frac{\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(exp(a + b*x)/cosh(a + b*x)^4,x)
```

output
```
atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b)/(b^2)^(1/2) - (2*exp(a + b*x))/(b*(2
*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (8*exp(3*a + 3*b*x))/(3*b*(3*
exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1)) + exp(a + b
*x)/(b*(exp(2*a + 2*b*x) + 1))
```

3.272 $\int e^{a+bx}\textbf{sech}^5(a+bx)\,dx$

3.272.1 Optimal result

Integrand size = 16, antiderivative size = 60

$$\int e^{a+bx}\text{sech}^5(a+bx)\,dx=-\frac{4}{b(1+e^{2a+2bx})^4}+\frac{32}{3b(1+e^{2a+2bx})^3}-\frac{8}{b(1+e^{2a+2bx})^2}$$

output
```
-4/b/(1+exp(2*b*x+2*a))^4+32/3/b/(1+exp(2*b*x+2*a))^3-8/b/(1+exp(2*b*x+2*a
))^2
```

3.272.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

$$\int e^{a+bx}\text{sech}^5(a+bx)\,dx=-\frac{4(1+4e^{2(a+bx)}+6e^{4(a+bx)})}{3b(1+e^{2(a+bx)})^4}$$

input
```
Integrate[E^(a + b*x)*Sech[a + b*x]^5,x]
```

output
```
(-4*(1 + 4*E^(2*(a + b*x)) + 6*E^(4*(a + b*x))))/(3*b*(1 + E^(2*(a + b*x))
)^4)
```

3.272.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2720, 27, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\text{sech}^5(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{32e^{5a+5bx}}{(1+e^{2a+2bx})^5}de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{32\int \frac{e^{5a+5bx}}{(1+e^{2a+2bx})^5}de^{a+bx}}{b}$$

$$\Big\downarrow \text{243}$$

$$\frac{16\int \frac{e^{2a+2bx}}{(1+e^{2a+2bx})^5}de^{2a+2bx}}{b}$$

$$\Big\downarrow \text{53}$$

$$\frac{16\int \left(\frac{1}{(1+e^{2a+2bx})^3}-\frac{2}{(1+e^{2a+2bx})^4}+\frac{1}{(1+e^{2a+2bx})^5}\right)de^{2a+2bx}}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{16\left(-\frac{1}{2(e^{2a+2bx}+1)^2}+\frac{2}{3(e^{2a+2bx}+1)^3}-\frac{1}{4(e^{2a+2bx}+1)^4}\right)}{b}$$

input
```
Int[E^(a + b*x)*Sech[a + b*x]^5,x]
```

output
```
(16*(-1/4*1/(1 + E^(2*a + 2*b*x))^4 + 2/(3*(1 + E^(2*a + 2*b*x))^3) - 1/(2
*(1 + E^(2*a + 2*b*x))^2)))/b
```

3.272.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.272.4 Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.58

method	result	size
derivativedivides	$\frac{-\frac{1}{4\cosh(bx+a)^4}+\left(\frac{2}{3}+\frac{\operatorname{sech}(bx+a)^2}{3}\right)\tanh(bx+a)}{b}$	35
default	$\frac{-\frac{1}{4\cosh(bx+a)^4}+\left(\frac{2}{3}+\frac{\operatorname{sech}(bx+a)^2}{3}\right)\tanh(bx+a)}{b}$	35
risch	$-\frac{4(6\,e^{4bx+4a}+4\,e^{2bx+2a}+1)}{3b(1+e^{2bx+2a})^4}$	43
parallelrisch	$\frac{2\,e^{bx+a}(\cosh(3bx+3a)+\sinh(3bx+3a)+4\cosh(bx+a)+4\sinh(bx+a))}{3b(3+\cosh(4bx+4a)+4\cosh(2bx+2a))}$	71

input
```
int(exp(b*x+a)*sech(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/4/cosh(b*x+a)^4+(2/3+1/3*sech(b*x+a)^2)*tanh(b*x+a))
```

3.272.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(55) = 110.

Time = 0.25 (sec) , antiderivative size = 233, normalized size of antiderivative = 3.88

$$\int e^{a+bx}\text{sech}^5(a+bx)\,dx =$$

$$-\frac{}{3\left(b\cosh\left(bx+a\right)^6+6\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^5+b\sinh\left(bx+a\right)^6+4\,b\cosh\left(bx+a\right)^4+\left(15\,b\cosh\right.\right.}$$

input
```
integrate(exp(b*x+a)*sech(b*x+a)^5,x, algorithm="fricas")
```

output
```
-4/3*(7*cosh(b*x + a)^2 + 10*cosh(b*x + a)*sinh(b*x + a) + 7*sinh(b*x + a)
^2 + 4)/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*
x + a)^6 + 4*b*cosh(b*x + a)^4 + (15*b*cosh(b*x + a)^2 + 4*b)*sinh(b*x + a
)^4 + 4*(5*b*cosh(b*x + a)^3 + 4*b*cosh(b*x + a))*sinh(b*x + a)^3 + 7*b*co
sh(b*x + a)^2 + (15*b*cosh(b*x + a)^4 + 24*b*cosh(b*x + a)^2 + 7*b)*sinh(b
*x + a)^2 + 2*(3*b*cosh(b*x + a)^5 + 8*b*cosh(b*x + a)^3 + 5*b*cosh(b*x +
a))*sinh(b*x + a) + 4*b)
```

3.272.6 Sympy [F]

$$\int e^{a+bx}\text{sech}^5(a+bx)\,dx = e^a\int e^{bx}\,\text{sech}^5\,(a+bx)\,dx$$

input
```
integrate(exp(b*x+a)*sech(b*x+a)**5,x)
```

output
```
exp(a)*Integral(exp(b*x)*sech(a + b*x)**5, x)
```

3.272.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(55) = 110$.

Time = 0.19 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.87

$$\int e^{a+bx}\operatorname{sech}^5(a+bx)\,dx = -\frac{8\,e^{(4\,bx+4\,a)}}{b(e^{(8\,bx+8\,a)}+4\,e^{(6\,bx+6\,a)}+6\,e^{(4\,bx+4\,a)}+4\,e^{(2\,bx+2\,a)}+1)} - \frac{16\,e^{(2\,bx+2\,a)}}{3\,b(e^{(8\,bx+8\,a)}+4\,e^{(6\,bx+6\,a)}+6\,e^{(4\,bx+4\,a)}+4\,e^{(2\,bx+2\,a)}+1)} - \frac{4}{3\,b(e^{(8\,bx+8\,a)}+4\,e^{(6\,bx+6\,a)}+6\,e^{(4\,bx+4\,a)}+4\,e^{(2\,bx+2\,a)}+1)}$$

input
```
integrate(exp(b*x+a)*sech(b*x+a)^5,x, algorithm="maxima")
```

output
```
-8*e^(4*b*x + 4*a)/(b*(e^(8*b*x + 8*a) + 4*e^(6*b*x + 6*a) + 6*e^(4*b*x +
4*a) + 4*e^(2*b*x + 2*a) + 1)) - 16/3*e^(2*b*x + 2*a)/(b*(e^(8*b*x + 8*a)
+ 4*e^(6*b*x + 6*a) + 6*e^(4*b*x + 4*a) + 4*e^(2*b*x + 2*a) + 1)) - 4/3/(b
*(e^(8*b*x + 8*a) + 4*e^(6*b*x + 6*a) + 6*e^(4*b*x + 4*a) + 4*e^(2*b*x + 2
*a) + 1))
```

3.272.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.70

$$\int e^{a+bx}\operatorname{sech}^5(a+bx)\,dx = -\frac{4\left(6\,e^{(4\,bx+4\,a)}+4\,e^{(2\,bx+2\,a)}+1\right)}{3\,b(e^{(2\,bx+2\,a)}+1)^4}$$

input
```
integrate(exp(b*x+a)*sech(b*x+a)^5,x, algorithm="giac")
```

output
```
-4/3*(6*e^(4*b*x + 4*a) + 4*e^(2*b*x + 2*a) + 1)/(b*(e^(2*b*x + 2*a) + 1)^
4)
```

3.272.9 Mupad [B] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.70

$$\int e^{a+bx}\text{sech}^5(a+bx)\,dx=-\frac{4\,\left(4\,{\mathrm{e}}^{2\,a+2\,b\,x}+6\,{\mathrm{e}}^{4\,a+4\,b\,x}+1\right)}{3\,b\,{\left({\mathrm{e}}^{2\,a+2\,b\,x}+1\right)}^4}$$

input
```
int(exp(a + b*x)/cosh(a + b*x)^5,x)
```

output
```
-(4*(4*exp(2*a + 2*b*x) + 6*exp(4*a + 4*b*x) + 1))/(3*b*(exp(2*a + 2*b*x)
+ 1)^4)
```

3.273 $\int e^x \cosh^2(2x)\,dx$

3.273.1 Optimal result

Integrand size = 10, antiderivative size = 26

$$\int e^x \cosh^2(2x)\,dx = -\frac{1}{12}e^{-3x} + \frac{e^x}{2} + \frac{e^{5x}}{20}$$

output
```
-1/12/exp(3*x)+1/2*exp(x)+1/20*exp(5*x)
```

3.273.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int e^x \cosh^2(2x)\,dx = -\frac{1}{12}e^{-3x} + \frac{e^x}{2} + \frac{e^{5x}}{20}$$

input
```
Integrate[E^x*Cosh[2*x]^2,x]
```

output
```
-1/12*1/E^(3*x) + E^x/2 + E^(5*x)/20
```

3.273.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2720, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \cosh^2(2x) \, dx$$

$$\Big\downarrow \text{2720}$$

$$\int \frac{1}{4} e^{-4x} \left(e^{4x}+1\right)^2 de^x$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4} \int e^{-4x} \left(1+e^{4x}\right)^2 de^x$$

$$\Big\downarrow \text{802}$$

$$\frac{1}{4} \int \left(2+e^{-4x}+e^{4x}\right) de^x$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4} \left(-\frac{1}{3} e^{-3x}+2 e^x+\frac{e^{5x}}{5}\right)$$

input
```
Int[E^x*Cosh[2*x]^2,x]
```

output
```
(-1/3*1/E^(3*x) + 2*E^x + E^(5*x)/5)/4
```

3.273.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.273.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

method	result	size
parallelrisch	$-\frac{e^{x} \left(-15+\cosh \left(4 x \right)-4 \sinh \left(4 x \right)\right)}{30}$	17
risch	$\frac{{e}^{5 x}}{20}+\frac{{e}^{x}}{2}-\frac{{e}^{-3 x}}{12}$	18
default	$\frac{\sinh \left(x \right)}{2}+\frac{\sinh \left(3 x \right)}{12}+\frac{\sinh \left(5 x \right)}{20}+\frac{\cosh \left(x \right)}{2}-\frac{\cosh \left(3 x \right)}{12}+\frac{\cosh \left(5 x \right)}{20}$	34

input
```
int(exp(x)*cosh(2*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/30*exp(x)*(-15+cosh(4*x)-4*sinh(4*x))
```

3.273.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(17) = 34.

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.81

$$\int e^x \cosh^2(2x)\,dx = -\frac{\cosh(x)^4 - 16\cosh(x)^3\sinh(x) + 6\cosh(x)^2\sinh(x)^2 - 16\cosh(x)\sinh(x)^3 + \sinh(x)^4 - 15}{30\left(\cosh(x) - \sinh(x)\right)}$$

input
```
integrate(exp(x)*cosh(2*x)^2,x, algorithm="fricas")
```

output
```
-1/30*(cosh(x)^4 - 16*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 16*cosh(
x)*sinh(x)^3 + sinh(x)^4 - 15)/(cosh(x) - sinh(x))
```

3.273.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(19) = 38.

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int e^x \cosh^2(2x)\,dx = -\frac{8e^x \sinh^2(2x)}{15} + \frac{4e^x \sinh(2x)\cosh(2x)}{15} + \frac{7e^x \cosh^2(2x)}{15}$$

input
```
integrate(exp(x)*cosh(2*x)**2,x)
```

output
```
-8*exp(x)*sinh(2*x)**2/15 + 4*exp(x)*sinh(2*x)*cosh(2*x)/15 + 7*exp(x)*cos
h(2*x)**2/15
```

3.273.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int e^x \cosh^2(2x)\,dx = \frac{1}{20}\,e^{(5\,x)} - \frac{1}{12}\,e^{(-3\,x)} + \frac{1}{2}\,e^x$$

input
```
integrate(exp(x)*cosh(2*x)^2,x, algorithm="maxima")
```

output
```
1/20*e^(5*x) - 1/12*e^(-3*x) + 1/2*e^x
```

3.273.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int e^x \cosh^2(2x)\,dx = \frac{1}{20}\,e^{(5\,x)} - \frac{1}{12}\,e^{(-3\,x)} + \frac{1}{2}\,e^x$$

input
```
integrate(exp(x)*cosh(2*x)^2,x, algorithm="giac")
```

output
```
1/20*e^(5*x) - 1/12*e^(-3*x) + 1/2*e^x
```

3.273.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int e^x \cosh^2(2x)\, dx = \frac{e^{5x}}{20} - \frac{e^{-3x}}{12} + \frac{e^x}{2}$$

input
```
int(cosh(2*x)^2*exp(x),x)
```

output
```
exp(5*x)/20 - exp(-3*x)/12 + exp(x)/2
```

3.274 $\int e^x \cosh(2x)\,dx$

3.274.1 Optimal result

Integrand size = 8, antiderivative size = 19

$$\int e^x \cosh(2x)\,dx = -\frac{e^{-x}}{2} + \frac{e^{3x}}{6}$$

output
```
-1/2/exp(x)+1/6*exp(3*x)
```

3.274.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int e^x \cosh(2x)\,dx = \frac{1}{6}e^{-x}\left(-3 + e^{4x}\right)$$

input
```
Integrate[E^x*Cosh[2*x],x]
```

output
```
(-3 + E^(4*x))/(6*E^x)
```

3.274.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2720, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \cosh(2x)\,dx$$

$$\Big\downarrow \; 2720$$

$$\int \frac{1}{2}e^{-2x}\left(e^{4x}+1\right)de^x$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\int e^{-2x}\left(1+e^{4x}\right)de^x$$

$$\Big\downarrow \; 802$$

$$\frac{1}{2}\int \left(e^{-2x}+e^{2x}\right)de^x$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{e^{3x}}{3}-e^{-x}\right)$$

input
```
Int[E^x*Cosh[2*x],x]
```

output
```
(-E^(-x) + E^(3*x)/3)/2
```

3.274.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.274.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
risch	$\frac{e^{3x}}{6}-\frac{e^{-x}}{2}$	14
parallelrisch	$-\frac{e^{x}(\cosh(2x)-2\sinh(2x))}{3}$	16
default	$\frac{\sinh(x)}{2}+\frac{\sinh(3x)}{6}-\frac{\cosh(x)}{2}+\frac{\cosh(3x)}{6}$	22

input
```
int(exp(x)*cosh(2*x),x,method=_RETURNVERBOSE)
```

output
```
1/6*exp(3*x)-1/2*exp(-x)
```

3.274.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int e^x \cosh(2x)\,dx=-\frac{\cosh\left(x\right)^2-4\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^2}{3\left(\cosh\left(x\right)-\sinh\left(x\right)\right)}$$

input
```
integrate(exp(x)*cosh(2*x),x, algorithm="fricas")
```

output
```
-1/3*(cosh(x)^2 - 4*cosh(x)*sinh(x) + sinh(x)^2)/(cosh(x) - sinh(x))
```

3.274.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^x \cosh(2x)\,dx = \frac{2e^x \sinh(2x)}{3} - \frac{e^x \cosh(2x)}{3}$$

input
```
integrate(exp(x)*cosh(2*x),x)
```

output
```
2*exp(x)*sinh(2*x)/3 - exp(x)*cosh(2*x)/3
```

3.274.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int e^x \cosh(2x)\,dx = \frac{1}{6}\,e^{(3\,x)} - \frac{1}{2}\,e^{(-x)}$$

input
```
integrate(exp(x)*cosh(2*x),x, algorithm="maxima")
```

output
```
1/6*e^(3*x) - 1/2*e^(-x)
```

3.274.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int e^x \cosh(2x)\,dx = \frac{1}{6}\,e^{(3\,x)} - \frac{1}{2}\,e^{(-x)}$$

input
```
integrate(exp(x)*cosh(2*x),x, algorithm="giac")
```

output
```
1/6*e^(3*x) - 1/2*e^(-x)
```

3.274.9 Mupad [B] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int e^x \cosh(2x)\, dx = \frac{e^{-x} \left(e^{4x} - 3\right)}{6}$$

input
```
int(cosh(2*x)*exp(x),x)
```

output
```
(exp(-x)*(exp(4*x) - 3))/6
```

3.275 $\int e^x \mathbf{sech}(2x)\,dx$

3.275.1 Optimal result

Integrand size = 8, antiderivative size = 92

$$\int e^x \operatorname{sech}(2x)\,dx = -\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}} + \frac{\arctan\left(1+\sqrt{2}e^x\right)}{\sqrt{2}} + \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{2\sqrt{2}}$$

output
```
1/2*arctan(-1+exp(x)*2^(1/2))*2^(1/2)+1/2*arctan(1+exp(x)*2^(1/2))*2^(1/2)
+1/4*ln(1+exp(2*x)-exp(x)*2^(1/2))*2^(1/2)-1/4*ln(1+exp(2*x)+exp(x)*2^(1/2
))*2^(1/2)
```

3.275.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.26

$$\int e^x \operatorname{sech}(2x)\,dx = \frac{2}{3}e^{3x}\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-e^{4x}\right)$$

input
```
Integrate[E^x*Sech[2*x],x]
```

output
```
(2*E^(3*x)*Hypergeometric2F1[3/4, 1, 7/4, -E^(4*x)])/3
```

3.275.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.250$, Rules used = {2720, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \operatorname{sech}(2x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int \frac{2e^{2x}}{e^{4x}+1}de^x$$

$$\Big\downarrow \text{27}$$

$$2\int \frac{e^{2x}}{1+e^{4x}}de^x$$

$$\Big\downarrow \text{826}$$

$$2\left(\frac{1}{2}\int \frac{1+e^{2x}}{1+e^{4x}}de^x-\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x\right)$$

$$\Big\downarrow \text{1476}$$

$$2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{1-\sqrt{2}e^x+e^{2x}}de^x+\frac{1}{2}\int \frac{1}{1+\sqrt{2}e^x+e^{2x}}de^x\right)-\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x\right)$$

$$\Big\downarrow \text{1082}$$

$$2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-1-e^{2x}}d\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\int \frac{1}{-1-e^{2x}}d\left(1+\sqrt{2}e^x\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x\right)$$

$$\Big\downarrow \text{217}$$

$$2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x\right)$$

$$\Big\downarrow \text{1479}$$

$$2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(1+\sqrt{2}e^x\right)}{1+\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(1+\sqrt{2}e^x\right)}{1+\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)\right)$$

$$\Big\downarrow 27$$

$$2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}-\frac{1}{2}\int \frac{1+\sqrt{2}e^x}{1+\sqrt{2}e^x+e^{2x}}de^x\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)\right)$$

$$\Big\downarrow 1103$$

$$2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}}\right)\right)$$

input
```
Int[E^x*Sech[2*x],x]
```

output
```
2*((-(ArcTan[1 - Sqrt[2]*E^x]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*E^x]/Sqrt[2])/
2 + (Log[1 - Sqrt[2]*E^x + E^(2*x)]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*E^x + E^
(2*x)]/(2*Sqrt[2]))/2)
```

3.275.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.275. $\int e^x \text{sech}(2x) \, dx$

3.275.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.27

method	result	size
risch	$2\left(\sum_{\_R=\mathrm{RootOf}\left(256\_Z^4+1\right)} \_R\ln\left(64\_R^3+\mathrm{e}^x\right)\right)$	25

input
```
int(exp(x)*sech(2*x),x,method=_RETURNVERBOSE)
```

output
```
2*sum(_R*ln(64*_R^3+exp(x)),_R=RootOf(256*_Z^4+1))
```

3.275.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.88

$$\begin{aligned}\int e^x \operatorname{sech}(2x)\,dx =& \left(\frac{1}{4}i-\frac{1}{4}\right)\sqrt{2}\log\left((i+1)\sqrt{2}+2\cosh(x)+2\sinh(x)\right)\\ &-\left(\frac{1}{4}i+\frac{1}{4}\right)\sqrt{2}\log\left(-(i-1)\sqrt{2}+2\cosh(x)+2\sinh(x)\right)\\ &+\left(\frac{1}{4}i+\frac{1}{4}\right)\sqrt{2}\log\left((i-1)\sqrt{2}+2\cosh(x)+2\sinh(x)\right)\\ &-\left(\frac{1}{4}i-\frac{1}{4}\right)\sqrt{2}\log\left(-(i+1)\sqrt{2}+2\cosh(x)+2\sinh(x)\right)\end{aligned}$$

input
```
integrate(exp(x)*sech(2*x),x, algorithm="fricas")
```

output
```
(1/4*I - 1/4)*sqrt(2)*log((I + 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) - (1/4*
I + 1/4)*sqrt(2)*log(-(I - 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) + (1/4*I +
1/4)*sqrt(2)*log((I - 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) - (1/4*I - 1/4)*
sqrt(2)*log(-(I + 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x))
```

3.275.6 Sympy [F]

$$\int e^x \text{sech}(2x)\,dx = \int e^x \operatorname{sech}(2x)\,dx$$

input
```
integrate(exp(x)*sech(2*x),x)
```

output
```
Integral(exp(x)*sech(2*x), x)
```

3.275.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\int e^x \text{sech}(2x)\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right) + \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right) - \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}e^x + e^{(2x)} + 1\right) + \frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}e^x + e^{(2x)} + 1\right)$$

input
```
integrate(exp(x)*sech(2*x),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/2*sqrt(2)*arctan(-1/
2*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/4*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1)
+ 1/4*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1)
```

3.275.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\int e^x \text{sech}(2x)\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right) + \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right) - \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}e^x + e^{(2x)} + 1\right) + \frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}e^x + e^{(2x)} + 1\right)$$

input
```
integrate(exp(x)*sech(2*x),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/2*sqrt(2)*arctan(-1/
2*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/4*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1)
+ 1/4*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1)
```

3.275.9 Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.84

$$\int e^x \operatorname{sech}(2x)\,dx = \sqrt{2}\,\ln\left(4+\sqrt{2}\,e^x\,(-2-2i)\right)\,\left(\frac{1}{4}+\frac{1}{4}i\right) + \sqrt{2}\,\ln\left(4+\sqrt{2}\,e^x\,(-2+2i)\right)\,\left(\frac{1}{4}-\frac{1}{4}i\right) + \sqrt{2}\,\ln\left(4+\sqrt{2}\,e^x\,(2-2i)\right)\,\left(-\frac{1}{4}+\frac{1}{4}i\right) + \sqrt{2}\,\ln\left(4+\sqrt{2}\,e^x\,(2+2i)\right)\,\left(-\frac{1}{4}-\frac{1}{4}i\right)$$

input
```
int(exp(x)/cosh(2*x),x)
```

output
```
2^(1/2)*log(4 - 2^(1/2)*exp(x)*(2 + 2i))*(1/4 + 1i/4) + 2^(1/2)*log(4 - 2^
(1/2)*exp(x)*(2 - 2i))*(1/4 - 1i/4) - 2^(1/2)*log(2^(1/2)*exp(x)*(2 - 2i)
+ 4)*(1/4 - 1i/4) - 2^(1/2)*log(2^(1/2)*exp(x)*(2 + 2i) + 4)*(1/4 + 1i/4)
```

3.276 $\int e^x \text{sech}^2(2x)\,dx$

3.276.1 Optimal result

Integrand size = 10, antiderivative size = 111

$$\int e^x \text{sech}^2(2x)\,dx = -\frac{e^x}{1+e^{4x}} - \frac{\arctan\left(1-\sqrt{2}e^x\right)}{2\sqrt{2}} + \frac{\arctan\left(1+\sqrt{2}e^x\right)}{2\sqrt{2}} - \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}} + \frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}}$$

output
```
-exp(x)/(1+exp(4*x))+1/4*arctan(-1+exp(x)*2^(1/2))*2^(1/2)+1/4*arctan(1+ex
p(x)*2^(1/2))*2^(1/2)-1/8*ln(1+exp(2*x)-exp(x)*2^(1/2))*2^(1/2)+1/8*ln(1+e
xp(2*x)+exp(x)*2^(1/2))*2^(1/2)
```

3.276.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95

$$\int e^x \text{sech}^2(2x)\,dx = \frac{1}{8}\left(-\frac{8e^x}{1+e^{4x}} - 2\sqrt{2}\arctan\left(1-\sqrt{2}e^x\right) + 2\sqrt{2}\arctan\left(1+\sqrt{2}e^x\right) - \sqrt{2}\log\left(1-\sqrt{2}e^x+e^{2x}\right) + \sqrt{2}\log\left(1+\sqrt{2}e^x+e^{2x}\right)\right)$$

input
```
Integrate[E^x*Sech[2*x]^2,x]
```

output
```
((-8*E^x)/(1 + E^(4*x)) - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*E^x] + 2*Sqrt[2]*Ar
cTan[1 + Sqrt[2]*E^x] - Sqrt[2]*Log[1 - Sqrt[2]*E^x + E^(2*x)] + Sqrt[2]*L
og[1 + Sqrt[2]*E^x + E^(2*x)])/8
```

3.276.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.13, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules used = {2720, 27, 817, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \text{sech}^2(2x) \, dx$$

$$\Big\downarrow \text{2720}$$

$$\int \frac{4e^{4x}}{(e^{4x}+1)^2}de^x$$

$$\Big\downarrow \text{27}$$

$$4\int \frac{e^{4x}}{(1+e^{4x})^2}de^x$$

$$\Big\downarrow \text{817}$$

$$4\left(\frac{1}{4}\int \frac{1}{1+e^{4x}}de^x-\frac{e^x}{4\,(e^{4x}+1)}\right)$$

$$\Big\downarrow \text{755}$$

$$4\left(\frac{1}{4}\left(\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x+\frac{1}{2}\int \frac{1+e^{2x}}{1+e^{4x}}de^x\right)-\frac{e^x}{4\,(e^{4x}+1)}\right)$$

$$\Big\downarrow \text{1476}$$

$$4\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{1-\sqrt{2}e^x+e^{2x}}de^x+\frac{1}{2}\int \frac{1}{1+\sqrt{2}e^x+e^{2x}}de^x\right)+\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x\right)-\frac{e^x}{4\,(e^{4x}+1)}\right)$$

$$\Big\downarrow \text{1082}$$

$$4\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{\int \frac{1}{-1-e^{2x}}d(1-\sqrt{2}e^x)}{\sqrt{2}}-\frac{\int \frac{1}{-1-e^{2x}}d(1+\sqrt{2}e^x)}{\sqrt{2}}\right)+\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x\right)-\frac{e^x}{4\,(e^{4x}+1)}\right)$$

$$\Big\downarrow \text{217}$$

$$4\left(\frac{1}{4}\left(\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)\right)-\frac{e^x}{4\,(e^{4x}+1)}\right)$$

$$\Big\downarrow \text{1479}$$

$$4\left(\frac{1}{4}\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(1+\sqrt{2}e^x\right)}{1+\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)\right)-\frac{}{4(e}\right.$$

$$\Big\downarrow \text{25}$$

$$4\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(1+\sqrt{2}e^x\right)}{1+\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)\right)-\frac{e^x}{4(e^{4x}+}\right.$$

$$\Big\downarrow \text{27}$$

$$4\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}+\frac{1}{2}\int \frac{1+\sqrt{2}e^x}{1+\sqrt{2}e^x+e^{2x}}de^x\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)\right)-\frac{}{4(}\right.$$

$$\Big\downarrow \text{1103}$$

$$4\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}}-\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}}\right)\right)-\right.$$

input
```
Int[E^x*Sech[2*x]^2,x]
```

output
```
4*(-1/4*E^x/(1 + E^(4*x)) + ((-(ArcTan[1 - Sqrt[2]*E^x]/Sqrt[2]) + ArcTan[
1 + Sqrt[2]*E^x]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*E^x + E^(2*x)]/Sqrt[2]
 + Log[1 + Sqrt[2]*E^x + E^(2*x)]/(2*Sqrt[2]))/2)/4)
```

3.276.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.276. $\int e^x \operatorname{sech}^2(2x)\,dx$

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.276.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.35 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.32

method	result	size
risch	$-\frac{e^x}{1+e^{4x}}+4\left(\sum_{\_R=\mathrm{RootOf}(65536\_Z^4+1)}\_R\ln\left(e^x+16\_R\right)\right)$	36

input
```
int(exp(x)*sech(2*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-exp(x)/(1+exp(4*x))+4*sum(_R*ln(exp(x)+16*_R),_R=RootOf(65536*_Z^4+1))
```

3.276.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 349, normalized size of antiderivative = 3.14

$$\int e^x\mathrm{sech}^2(2x)\,dx$$
$$= \frac{\left((i+1)\sqrt{2}\cosh(x)^4+(4i+4)\sqrt{2}\cosh(x)^3\sinh(x)+(6i+6)\sqrt{2}\cosh(x)^2\sinh(x)^2+(4i+4)\sqrt{2}\cos\right.}{}$$

input
```
integrate(exp(x)*sech(2*x)^2,x, algorithm="fricas")
```

output
```
1/8*(((I + 1)*sqrt(2)*cosh(x)^4 + (4*I + 4)*sqrt(2)*cosh(x)^3*sinh(x) + (6
*I + 6)*sqrt(2)*cosh(x)^2*sinh(x)^2 + (4*I + 4)*sqrt(2)*cosh(x)*sinh(x)^3
+ (I + 1)*sqrt(2)*sinh(x)^4 + (I + 1)*sqrt(2))*log((I + 1)*sqrt(2) + 2*cos
h(x) + 2*sinh(x)) + (-(I - 1)*sqrt(2)*cosh(x)^4 - (4*I - 4)*sqrt(2)*cosh(x
)^3*sinh(x) - (6*I - 6)*sqrt(2)*cosh(x)^2*sinh(x)^2 - (4*I - 4)*sqrt(2)*co
sh(x)*sinh(x)^3 - (I - 1)*sqrt(2)*sinh(x)^4 - (I - 1)*sqrt(2))*log(-(I - 1
)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) + ((I - 1)*sqrt(2)*cosh(x)^4 + (4*I - 4
)*sqrt(2)*cosh(x)^3*sinh(x) + (6*I - 6)*sqrt(2)*cosh(x)^2*sinh(x)^2 + (4*I
 - 4)*sqrt(2)*cosh(x)*sinh(x)^3 + (I - 1)*sqrt(2)*sinh(x)^4 + (I - 1)*sqrt
(2))*log((I - 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) + (-(I + 1)*sqrt(2)*cosh
(x)^4 - (4*I + 4)*sqrt(2)*cosh(x)^3*sinh(x) - (6*I + 6)*sqrt(2)*cosh(x)^2*
sinh(x)^2 - (4*I + 4)*sqrt(2)*cosh(x)*sinh(x)^3 - (I + 1)*sqrt(2)*sinh(x)^
4 - (I + 1)*sqrt(2))*log(-(I + 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) - 8*cos
h(x) - 8*sinh(x))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2
 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 1)
```

3.276.6 Sympy [F]

$$\int e^x \text{sech}^2(2x)\,dx = \int e^x \operatorname{sech}^2(2x)\,dx$$

input
```
integrate(exp(x)*sech(2*x)**2,x)
```

output
```
Integral(exp(x)*sech(2*x)**2, x)
```

3.276.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.79

$$\int e^x \text{sech}^2(2x)\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right)+\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right)-\frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}e^x+e^{(2x)}+1\right)-\frac{e^x}{e^{(4x)}+1}$$

input
```
integrate(exp(x)*sech(2*x)^2,x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/4*sqrt(2)*arctan(-1/
2*sqrt(2)*(sqrt(2) - 2*e^x)) + 1/8*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1)
- 1/8*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - e^x/(e^(4*x) + 1)
```

3.276.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.79

$$\int e^x \operatorname{sech}^2(2x)\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right) + \frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right)-\frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}e^x+e^{(2x)}+1\right)-\frac{e^x}{e^{(4x)}+1}$$

input
```
integrate(exp(x)*sech(2*x)^2,x, algorithm="giac")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/4*sqrt(2)*arctan(-1/
2*sqrt(2)*(sqrt(2) - 2*e^x)) + 1/8*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1)
- 1/8*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - e^x/(e^(4*x) + 1)
```

3.276.9 Mupad [B] (verification not implemented)

Time = 1.80 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.77

$$\int e^x \operatorname{sech}^2(2x)\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\mathrm{e}^x-\frac{\sqrt{2}}{2}\right)\right)}{4}-\frac{\mathrm{e}^x}{\mathrm{e}^{4x}+1}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\mathrm{e}^x+\frac{\sqrt{2}}{2}\right)\right)}{4} - \frac{\sqrt{2}\,\ln\left(\left(\mathrm{e}^x-\frac{\sqrt{2}}{2}\right)^2+\frac{1}{2}\right)}{8}+\frac{\sqrt{2}\,\ln\left(\left(\mathrm{e}^x+\frac{\sqrt{2}}{2}\right)^2+\frac{1}{2}\right)}{8}$$

input
```
int(exp(x)/cosh(2*x)^2,x)
```

output
```
(2^(1/2)*atan(2^(1/2)*(exp(x) - 2^(1/2)/2)))/4 - exp(x)/(exp(4*x) + 1) + (
2^(1/2)*atan(2^(1/2)*(exp(x) + 2^(1/2)/2)))/4 - (2^(1/2)*log((exp(x) - 2^(
1/2)/2)^2 + 1/2))/8 + (2^(1/2)*log((exp(x) + 2^(1/2)/2)^2 + 1/2))/8
```

3.277 $\int e^x \cosh^2(3x)\,dx$

3.277.1 Optimal result

Integrand size = 10, antiderivative size = 26

$$\int e^x \cosh^2(3x)\,dx = -\frac{1}{20}e^{-5x} + \frac{e^x}{2} + \frac{e^{7x}}{28}$$

output
```
-1/20/exp(5*x)+1/2*exp(x)+1/28*exp(7*x)
```

3.277.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int e^x \cosh^2(3x)\,dx = -\frac{1}{20}e^{-5x} + \frac{e^x}{2} + \frac{e^{7x}}{28}$$

input
```
Integrate[E^x*Cosh[3*x]^2,x]
```

output
```
-1/20*1/E^(5*x) + E^x/2 + E^(7*x)/28
```

3.277.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2720, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \cosh^2(3x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int \frac{1}{4}e^{-6x}\left(e^{6x}+1\right)^2de^x$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int e^{-6x}\left(1+e^{6x}\right)^2de^x$$

$$\Big\downarrow \text{802}$$

$$\frac{1}{4}\int \left(2+e^{-6x}+e^{6x}\right)de^x$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{1}{5}e^{-5x}+2e^x+\frac{e^{7x}}{7}\right)$$

input
```
Int[E^x*Cosh[3*x]^2,x]
```

output
```
(-1/5*1/E^(5*x) + 2*E^x + E^(7*x)/7)/4
```

3.277.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.277.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

method	result	size
parallelrisch	$-\frac{e^{x}\left(-35+\cosh\left(6x\right)-6\sinh\left(6x\right)\right)}{70}$	17
risch	$\frac{e^{7x}}{28}+\frac{e^{x}}{2}-\frac{e^{-5x}}{20}$	18
default	$\frac{\sinh\left(x\right)}{2}+\frac{\sinh\left(5x\right)}{20}+\frac{\sinh\left(7x\right)}{28}+\frac{\cosh\left(x\right)}{2}-\frac{\cosh\left(5x\right)}{20}+\frac{\cosh\left(7x\right)}{28}$	34

input
```
int(exp(x)*cosh(3*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/70*exp(x)*(-35+cosh(6*x)-6*sinh(6*x))
```

3.277.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(17) = 34$.

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.58

$$\int e^{x}\cosh^{2}(3x)\,dx =$$
$$-\frac{\cosh\left(x\right)^{6}-36\cosh\left(x\right)^{5}\sinh\left(x\right)+15\cosh\left(x\right)^{4}\sinh\left(x\right)^{2}-120\cosh\left(x\right)^{3}\sinh\left(x\right)^{3}+15\cosh\left(x\right)^{2}\sinh\left(\right.}{70\left(\cosh\left(x\right)-\sinh\left(x\right)\right)}$$

input
```
integrate(exp(x)*cosh(3*x)^2,x, algorithm="fricas")
```

output
```
-1/70*(cosh(x)^6 - 36*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 - 120*cos
h(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 - 36*cosh(x)*sinh(x)^5 + sinh(x)
^6 - 35)/(cosh(x) - sinh(x))
```

3.277.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(19) = 38$.

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int e^x \cosh^2(3x)\,dx = -\frac{18e^x \sinh^2{(3x)}}{35} + \frac{6e^x \sinh{(3x)}\cosh{(3x)}}{35} + \frac{17e^x \cosh^2{(3x)}}{35}$$

input
```
integrate(exp(x)*cosh(3*x)**2,x)
```

output
```
-18*exp(x)*sinh(3*x)**2/35 + 6*exp(x)*sinh(3*x)*cosh(3*x)/35 + 17*exp(x)*c
osh(3*x)**2/35
```

3.277.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int e^x \cosh^2(3x)\,dx = \frac{1}{28}\,e^{(7\,x)} - \frac{1}{20}\,e^{(-5\,x)} + \frac{1}{2}\,e^x$$

input
```
integrate(exp(x)*cosh(3*x)^2,x, algorithm="maxima")
```

output
```
1/28*e^(7*x) - 1/20*e^(-5*x) + 1/2*e^x
```

3.277.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int e^x \cosh^2(3x)\,dx = \frac{1}{28}\,e^{(7\,x)} - \frac{1}{20}\,e^{(-5\,x)} + \frac{1}{2}\,e^x$$

input
```
integrate(exp(x)*cosh(3*x)^2,x, algorithm="giac")
```

output
```
1/28*e^(7*x) - 1/20*e^(-5*x) + 1/2*e^x
```

3.277.9 Mupad [B] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int e^x \cosh^2(3x)\,dx = \frac{e^{7x}}{28} - \frac{e^{-5x}}{20} + \frac{e^x}{2}$$

input
```
int(cosh(3*x)^2*exp(x),x)
```

output
```
exp(7*x)/28 - exp(-5*x)/20 + exp(x)/2
```

3.278 $\int e^x \cosh(3x)\,dx$

3.278.1 Optimal result

Integrand size = 8, antiderivative size = 19

$$\int e^x \cosh(3x)\,dx = -\frac{1}{4}e^{-2x} + \frac{e^{4x}}{8}$$

output
```
-1/4/exp(2*x)+1/8*exp(4*x)
```

3.278.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int e^x \cosh(3x)\,dx = \frac{1}{8}e^{-2x}\left(-2 + e^{6x}\right)$$

input
```
Integrate[E^x*Cosh[3*x],x]
```

output
```
(-2 + E^(6*x))/(8*E^(2*x))
```

3.278.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2720, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \cosh(3x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int \frac{1}{2}e^{-3x}\left(e^{6x}+1\right)de^x$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int e^{-3x}\left(1+e^{6x}\right)de^x$$

$$\Big\downarrow \text{802}$$

$$\frac{1}{2}\int \left(e^{-3x}+e^{3x}\right)de^x$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{e^{4x}}{4}-\frac{1}{2}e^{-2x}\right)$$

input
```
Int[E^x*Cosh[3*x],x]
```

output
```
(-1/2*1/E^(2*x) + E^(4*x)/4)/2
```

3.278.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.278.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
risch	$\frac{{\rm e}^{4x}}{8}-\frac{{\rm e}^{-2x}}{4}$	14
parallelrisch	$-\frac{{\rm e}^{x}\left(\cosh\left(3x\right)-3\sinh\left(3x\right)\right)}{8}$	16
default	$\frac{\sinh\left(4x\right)}{8}+\frac{\sinh\left(2x\right)}{4}-\frac{\cosh\left(2x\right)}{4}+\frac{\cosh\left(4x\right)}{8}$	26

input
```
int(exp(x)*cosh(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/8*exp(4*x)-1/4*exp(-2*x)
```

3.278.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(13) = 26.

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.00

$$\int e^{x}\cosh(3x)\,dx=-\frac{\cosh\left(x\right)^{3}-9\cosh\left(x\right)^{2}\sinh\left(x\right)+3\cosh\left(x\right)\sinh\left(x\right)^{2}-3\sinh\left(x\right)^{3}}{8\left(\cosh\left(x\right)-\sinh\left(x\right)\right)}$$

input
```
integrate(exp(x)*cosh(3*x),x, algorithm="fricas")
```

output
```
-1/8*(cosh(x)^3 - 9*cosh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 - 3*sinh(x)^3)
/(cosh(x) - sinh(x))
```

3.278.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^x \cosh(3x)\,dx = \frac{3e^x \sinh(3x)}{8} - \frac{e^x \cosh(3x)}{8}$$

input
```
integrate(exp(x)*cosh(3*x),x)
```

output
```
3*exp(x)*sinh(3*x)/8 - exp(x)*cosh(3*x)/8
```

3.278.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int e^x \cosh(3x)\,dx = \frac{1}{8}\,e^{(4\,x)} - \frac{1}{4}\,e^{(-2\,x)}$$

input
```
integrate(exp(x)*cosh(3*x),x, algorithm="maxima")
```

output
```
1/8*e^(4*x) - 1/4*e^(-2*x)
```

3.278.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int e^x \cosh(3x)\,dx = \frac{1}{8}\,e^{(4\,x)} - \frac{1}{4}\,e^{(-2\,x)}$$

input
```
integrate(exp(x)*cosh(3*x),x, algorithm="giac")
```

output
```
1/8*e^(4*x) - 1/4*e^(-2*x)
```

3.278.9 Mupad [B] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int e^x \cosh(3x)\,dx = \frac{e^{-2x}\,(e^{6x} - 2)}{8}$$

input
```
int(cosh(3*x)*exp(x),x)
```

output
```
(exp(-2*x)*(exp(6*x) - 2))/8
```

3.279 $\int e^x \mathbf{sech}(3x)\,dx$

3.279.1 Optimal result

Integrand size = 8, antiderivative size = 55

$$\int e^x \operatorname{sech}(3x)\,dx = -\frac{\arctan\left(\frac{1-2e^{2x}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{3}\log\left(1+e^{2x}\right) + \frac{1}{6}\log\left(1-e^{2x}+e^{4x}\right)$$

output
```
-1/3*ln(1+exp(2*x))+1/6*ln(1-exp(2*x)+exp(4*x))-1/3*arctan(1/3*(1-2*exp(2*
x))*3^(1/2))*3^(1/2)
```

3.279.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.44

$$\int e^x \operatorname{sech}(3x)\,dx = \frac{1}{2}e^{4x}\,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},-e^{6x}\right)$$

input
```
Integrate[E^x*Sech[3*x],x]
```

output
```
(E^(4*x)*Hypergeometric2F1[2/3, 1, 5/3, -E^(6*x)])/2
```

3.279.3 Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.64, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {2720, 27, 807, 821, 16, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \text{sech}(3x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int \frac{2e^{3x}}{e^{6x}+1}de^x$$

$$\Big\downarrow \text{27}$$

$$2\int \frac{e^{3x}}{1+e^{6x}}de^x$$

$$\Big\downarrow \text{807}$$

$$\int \frac{e^{2x}}{e^{3x}+1}de^{2x}$$

$$\Big\downarrow \text{821}$$

$$\frac{1}{3}\int \left(1+e^{2x}\right)de^{2x}-\frac{1}{3}\int \frac{1}{1+e^{2x}}de^{2x}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\int \left(1+e^{2x}\right)de^{2x}-\frac{1}{3}\log\left(e^{2x}+1\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{3}\left(\frac{3\int 1de^{2x}}{2}+\frac{1}{2}\int \left(-1+2e^{2x}\right)de^{2x}\right)-\frac{1}{3}\log\left(e^{2x}+1\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(\frac{3\int 1de^{2x}}{2}-\frac{1}{2}\int \left(1-2e^{2x}\right)de^{2x}\right)-\frac{1}{3}\log\left(e^{2x}+1\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{3}\left(-3\int \frac{1}{-2-2e^{2x}}d\left(-1+2e^{2x}\right)-\frac{1}{2}\int \left(1-2e^{2x}\right)de^{2x}\right)-\frac{1}{3}\log\left(e^{2x}+1\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2e^{2x}-1}{\sqrt{3}}\right)-\frac{1}{2}\int\left(1-2e^{2x}\right)de^{2x}\right)-\frac{1}{3}\log\left(e^{2x}+1\right)$$
$$\Big\downarrow \text{1103}$$
$$\frac{\arctan\left(\frac{2e^{2x}-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}\log\left(e^{2x}+1\right)$$

input
```
Int[E^x*Sech[3*x],x]
```

output
```
ArcTan[(-1 + 2*E^(2*x))/Sqrt[3]]/Sqrt[3] - Log[1 + E^(2*x)]/3
```

3.279.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.279.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.44

method	result	size
risch	$\frac{\ln\left(\mathrm{e}^{2x}-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{6}+\frac{i\ln\left(\mathrm{e}^{2x}-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{6}+\frac{\ln\left(\mathrm{e}^{2x}-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{6}-\frac{i\ln\left(\mathrm{e}^{2x}-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{6}-\frac{\ln\left(1+\mathrm{e}^{2x}\right)}{3}$	79

input
```
int(exp(x)*sech(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/6*ln(exp(2*x)-1/2+1/2*I*3^(1/2))+1/6*I*ln(exp(2*x)-1/2+1/2*I*3^(1/2))*3^
(1/2)+1/6*ln(exp(2*x)-1/2-1/2*I*3^(1/2))-1/6*I*ln(exp(2*x)-1/2-1/2*I*3^(1/
2))*3^(1/2)-1/3*ln(1+exp(2*x))
```

3.279.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.51

$$\int e^x\text{sech}(3x)\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}\cosh\left(x\right)+3\sqrt{3}\sinh\left(x\right)}{3\left(\cosh\left(x\right)-\sinh\left(x\right)\right)}\right) + \frac{1}{6}\log\left(\frac{2\cosh\left(x\right)^2+2\sinh\left(x\right)^2-1}{\cosh\left(x\right)^2-2\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^2}\right) - \frac{1}{3}\log\left(\frac{2\cosh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}\right)$$

input
```
integrate(exp(x)*sech(3*x),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(-1/3*(sqrt(3)*cosh(x) + 3*sqrt(3)*sinh(x))/(cosh(x) -
sinh(x))) + 1/6*log((2*cosh(x)^2 + 2*sinh(x)^2 - 1)/(cosh(x)^2 - 2*cosh(x)
*sinh(x) + sinh(x)^2)) - 1/3*log(2*cosh(x)/(cosh(x) - sinh(x)))
```

3.279.6 Sympy [F]

$$\int e^x\text{sech}(3x)\,dx = \int e^x\,\text{sech}\,(3x)\,dx$$

input
```
integrate(exp(x)*sech(3*x),x)
```

output
```
Integral(exp(x)*sech(3*x), x)
```

3.279.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.29

$$\int e^x\text{sech}(3x)\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\sqrt{3}+2\,e^x\right) + \frac{1}{3}\sqrt{3}\arctan\left(-\sqrt{3}+2\,e^x\right) + \frac{1}{6}\log\left(\sqrt{3}e^x+e^{(2\,x)}+1\right) + \frac{1}{6}\log\left(-\sqrt{3}e^x+e^{(2\,x)}+1\right) - \frac{1}{3}\log\left(e^{(2\,x)}+1\right)$$

input
```
integrate(exp(x)*sech(3*x),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(sqrt(3) + 2*e^x) + 1/3*sqrt(3)*arctan(-sqrt(3) + 2*e^x
) + 1/6*log(sqrt(3)*e^x + e^(2*x) + 1) + 1/6*log(-sqrt(3)*e^x + e^(2*x) +
1) - 1/3*log(e^(2*x) + 1)
```

3.279.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

$$\int e^x \mathrm{sech}(3x)\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,e^{(2\,x)}-1\right)\right) + \frac{1}{6}\log\left(e^{(4\,x)}-e^{(2\,x)}+1\right) - \frac{1}{3}\log\left(e^{(2\,x)}+1\right)$$

input
```
integrate(exp(x)*sech(3*x),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(2*x) - 1)) + 1/6*log(e^(4*x) - e^(2*x
) + 1) - 1/3*log(e^(2*x) + 1)
```

3.279.9 Mupad [B] (verification not implemented)

Time = 1.82 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.18

$$\int e^x \mathrm{sech}(3x)\,dx = -\frac{\ln\left(8\,\mathrm{e}^{2\,x}+8\right)}{3} - \ln\left(24\,\mathrm{e}^{2\,x}\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+8\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) + \ln\left(8-24\,\mathrm{e}^{2\,x}\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

input
```
int(exp(x)/cosh(3*x),x)
```

output
```
log(8 - 24*exp(2*x)*((3^(1/2)*1i)/6 + 1/6))*((3^(1/2)*1i)/6 + 1/6) - log(2
4*exp(2*x)*((3^(1/2)*1i)/6 - 1/6) + 8)*((3^(1/2)*1i)/6 - 1/6) - log(8*exp(
2*x) + 8)/3
```

3.280 $\int e^x \mathbf{sech}^2(3x)\,dx$

3.280.1 Optimal result

Integrand size = 10, antiderivative size = 110

$$\int e^x \text{sech}^2(3x)\,dx = -\frac{2e^x}{3(1+e^{6x})} + \frac{2\arctan(e^x)}{9} - \frac{1}{9}\arctan\left(\sqrt{3}-2e^x\right) + \frac{1}{9}\arctan\left(\sqrt{3}+2e^x\right) - \frac{\log\left(1-\sqrt{3}e^x+e^{2x}\right)}{6\sqrt{3}} + \frac{\log\left(1+\sqrt{3}e^x+e^{2x}\right)}{6\sqrt{3}}$$

output
```
-2/3*exp(x)/(1+exp(6*x))+2/9*arctan(exp(x))+1/9*arctan(2*exp(x)-3^(1/2))+1
/9*arctan(2*exp(x)+3^(1/2))-1/18*ln(1+exp(2*x)-exp(x)*3^(1/2))*3^(1/2)+1/1
8*ln(1+exp(2*x)+exp(x)*3^(1/2))*3^(1/2)
```

3.280.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.31

$$\int e^x \text{sech}^2(3x)\,dx = \frac{2}{3}e^x\left(-\frac{1}{1+e^{6x}} + \text{Hypergeometric2F1}\left(\frac{1}{6},1,\frac{7}{6},-e^{6x}\right)\right)$$

input
```
Integrate[E^x*Sech[3*x]^2,x]
```

output
```
(2*E^x*(-(1 + E^(6*x))^(-1) + Hypergeometric2F1[1/6, 1, 7/6, -E^(6*x)]))/3
```

3.280.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules used = {2720, 27, 817, 753, 27, 216, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \text{sech}^2(3x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int \frac{4e^{6x}}{(e^{6x}+1)^2}de^x$$

$$\Big\downarrow \text{27}$$

$$4\int \frac{e^{6x}}{(1+e^{6x})^2}de^x$$

$$\Big\downarrow \text{817}$$

$$4\left(\frac{1}{6}\int \frac{1}{1+e^{6x}}de^x-\frac{e^x}{6(e^{6x}+1)}\right)$$

$$\Big\downarrow \text{753}$$

$$4\left(\frac{1}{6}\left(\frac{1}{3}\int \frac{1}{1+e^{2x}}de^x+\frac{1}{3}\int \frac{2-\sqrt{3}e^x}{2\left(1-\sqrt{3}e^x+e^{2x}\right)}de^x+\frac{1}{3}\int \frac{2+\sqrt{3}e^x}{2\left(1+\sqrt{3}e^x+e^{2x}\right)}de^x\right)-\frac{e^x}{6(e^{6x}+1)}\right)$$

$$\Big\downarrow \text{27}$$

$$4\left(\frac{1}{6}\left(\frac{1}{3}\int \frac{1}{1+e^{2x}}de^x+\frac{1}{6}\int \frac{2-\sqrt{3}e^x}{1-\sqrt{3}e^x+e^{2x}}de^x+\frac{1}{6}\int \frac{2+\sqrt{3}e^x}{1+\sqrt{3}e^x+e^{2x}}de^x\right)-\frac{e^x}{6(e^{6x}+1)}\right)$$

$$\Big\downarrow \text{216}$$

$$4\left(\frac{1}{6}\left(\frac{1}{6}\int \frac{2-\sqrt{3}e^x}{1-\sqrt{3}e^x+e^{2x}}de^x+\frac{1}{6}\int \frac{2+\sqrt{3}e^x}{1+\sqrt{3}e^x+e^{2x}}de^x+\frac{\arctan(e^x)}{3}\right)-\frac{e^x}{6(e^{6x}+1)}\right)$$

$$\Big\downarrow \text{1142}$$

$$4\left(\frac{1}{6}\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{1-\sqrt{3}e^x+e^{2x}}de^x-\frac{1}{2}\sqrt{3}\int -\frac{\sqrt{3}-2e^x}{1-\sqrt{3}e^x+e^{2x}}de^x\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{1+\sqrt{3}e^x+e^{2x}}de^x+\frac{1}{2}\sqrt{3}\int \frac{}{1+}\right.\right.\right.$$

$$\Big\downarrow \text{25}$$

$$4\left(\frac{1}{6}\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1}{1-\sqrt{3}e^x+e^{2x}}de^x+\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-2e^x}{1-\sqrt{3}e^x+e^{2x}}de^x\right)+\frac{1}{6}\left(\frac{1}{2}\int\frac{1}{1+\sqrt{3}e^x+e^{2x}}de^x+\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}}{1+\sqrt{}}\right.\right.\right.$$

$$\Big\downarrow \text{1083}$$

$$4\left(\frac{1}{6}\left(\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-2e^x}{1-\sqrt{3}e^x+e^{2x}}de^x-\int\frac{1}{-1-e^{2x}}d\left(-\sqrt{3}+2e^x\right)\right)+\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}+2e^x}{1+\sqrt{3}e^x+e^{2x}}de^x-\int\frac{}{-1-}\right.\right.\right.$$

$$\Big\downarrow \text{217}$$

$$4\left(\frac{1}{6}\left(\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-2e^x}{1-\sqrt{3}e^x+e^{2x}}de^x-\arctan\left(\sqrt{3}-2e^x\right)\right)+\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}+2e^x}{1+\sqrt{3}e^x+e^{2x}}de^x+\arctan\left(2e^x+\sqrt{}\right.\right.\right.\right.$$

$$\Big\downarrow \text{1103}$$

$$4\left(\frac{1}{6}\left(\frac{\arctan(e^x)}{3}+\frac{1}{6}\left(-\arctan\left(\sqrt{3}-2e^x\right)-\frac{1}{2}\sqrt{3}\log\left(-\sqrt{3}e^x+e^{2x}+1\right)\right)+\frac{1}{6}\left(\arctan\left(2e^x+\sqrt{3}\right)+\frac{1}{2}\sqrt{3}\,\mathrm{lc}\right.\right.\right.$$

input
```
Int[E^x*Sech[3*x]^2,x]
```

output
```
4*(-1/6*E^x/(1 + E^(6*x)) + (ArcTan[E^x]/3 + (-ArcTan[Sqrt[3] - 2*E^x] - (
Sqrt[3]*Log[1 - Sqrt[3]*E^x + E^(2*x)])/2)/6 + (ArcTan[Sqrt[3] + 2*E^x] +
(Sqrt[3]*Log[1 + Sqrt[3]*E^x + E^(2*x)])/2)/6)/6)
```

3.280.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 753
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))   Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n))   Sum[u,
{k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.280. $\int e^x \operatorname{sech}^2(3x)\,dx$

3.280.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.36 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.54

method	result	size
risch	$-\frac{2\,\mathrm{e}^{x}}{3(1+\mathrm{e}^{6x})}+4\left(\sum_{\_R=\mathrm{RootOf}\left(1679616\_Z^{4}-1296\_Z^{2}+1\right)}\_R\ln\left(\mathrm{e}^{x}+36\_R\right)\right)+\frac{i\ln(\mathrm{e}^{x}+i)}{9}-\frac{i\ln(\mathrm{e}^{x}-i)}{9}$	59

input
```
int(exp(x)*sech(3*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*exp(x)/(1+exp(6*x))+4*sum(_R*ln(exp(x)+36*_R),_R=RootOf(1679616*_Z^4-
1296*_Z^2+1))+1/9*I*ln(exp(x)+I)-1/9*I*ln(exp(x)-I)
```

3.280.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 479, normalized size of antiderivative = 4.35

$$\int e^{x}\mathrm{sech}^{2}(3x)\,dx$$
$$=\frac{\left(\cosh(x)^{6}+6\cosh(x)^{5}\sinh(x)+15\cosh(x)^{4}\sinh(x)^{2}+20\cosh(x)^{3}\sinh(x)^{3}+15\cosh(x)^{2}\sinh(x)^{4}\right.}{}$$

input
```
integrate(exp(x)*sech(3*x)^2,x, algorithm="fricas")
```

output
```
1/18*((cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(
x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6
+ 1)*sqrt(2*I*sqrt(3) + 2)*log(sqrt(2*I*sqrt(3) + 2) + 2*cosh(x) + 2*sinh(
x)) - (cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(
x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6
+ 1)*sqrt(2*I*sqrt(3) + 2)*log(-sqrt(2*I*sqrt(3) + 2) + 2*cosh(x) + 2*sinh
(x)) + (cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh
(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6
 + 1)*sqrt(-2*I*sqrt(3) + 2)*log(sqrt(-2*I*sqrt(3) + 2) + 2*cosh(x) + 2*si
nh(x)) - (cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*co
sh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)
^6 + 1)*sqrt(-2*I*sqrt(3) + 2)*log(-sqrt(-2*I*sqrt(3) + 2) + 2*cosh(x) + 2
*sinh(x)) + 4*(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 +
20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + si
nh(x)^6 + 1)*arctan(cosh(x) + sinh(x)) - 12*cosh(x) - 12*sinh(x))/(cosh(x)
^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3
 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 1)
```

3.280.6 Sympy [F]

$$\int e^x \text{sech}^2(3x)\,dx = \int e^x \operatorname{sech}^2(3x)\,dx$$

input
```
integrate(exp(x)*sech(3*x)**2,x)
```

output
```
Integral(exp(x)*sech(3*x)**2, x)
```

3.280.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.72

$$\int e^x \text{sech}^2(3x)\,dx = \frac{1}{18}\sqrt{3}\log\left(\sqrt{3}e^x + e^{(2x)} + 1\right) - \frac{1}{18}\sqrt{3}\log\left(-\sqrt{3}e^x + e^{(2x)} + 1\right) - \frac{2e^x}{3\left(e^{(6x)} + 1\right)} + \frac{1}{9}\arctan\left(\sqrt{3} + 2e^x\right) + \frac{1}{9}\arctan\left(-\sqrt{3} + 2e^x\right) + \frac{2}{9}\arctan\left(e^x\right)$$

input
```
integrate(exp(x)*sech(3*x)^2,x, algorithm="maxima")
```

output
```
1/18*sqrt(3)*log(sqrt(3)*e^x + e^(2*x) + 1) - 1/18*sqrt(3)*log(-sqrt(3)*e^
x + e^(2*x) + 1) - 2/3*e^x/(e^(6*x) + 1) + 1/9*arctan(sqrt(3) + 2*e^x) + 1
/9*arctan(-sqrt(3) + 2*e^x) + 2/9*arctan(e^x)
```

3.280.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.72

$$\int e^x \operatorname{sech}^2(3x)\,dx = \frac{1}{18}\sqrt{3}\log\left(\sqrt{3}e^x + e^{(2x)} + 1\right) - \frac{1}{18}\sqrt{3}\log\left(-\sqrt{3}e^x + e^{(2x)} + 1\right) - \frac{2e^x}{3\left(e^{(6x)} + 1\right)} + \frac{1}{9}\arctan\left(\sqrt{3} + 2e^x\right) + \frac{1}{9}\arctan\left(-\sqrt{3} + 2e^x\right) + \frac{2}{9}\arctan\left(e^x\right)$$

input
```
integrate(exp(x)*sech(3*x)^2,x, algorithm="giac")
```

output
```
1/18*sqrt(3)*log(sqrt(3)*e^x + e^(2*x) + 1) - 1/18*sqrt(3)*log(-sqrt(3)*e^
x + e^(2*x) + 1) - 2/3*e^x/(e^(6*x) + 1) + 1/9*arctan(sqrt(3) + 2*e^x) + 1
/9*arctan(-sqrt(3) + 2*e^x) + 2/9*arctan(e^x)
```

3.280.9 Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.76

$$\int e^x \operatorname{sech}^2(3x)\,dx = \frac{2\,\mathrm{atan}(e^x)}{9} + \frac{\mathrm{atan}\left(2\,e^x + \sqrt{3}\right)}{9} + \frac{\mathrm{atan}\left(2\,e^x - \sqrt{3}\right)}{9} - \frac{2\,e^x}{3\,\left(e^{6\,x} + 1\right)} - \frac{\sqrt{3}\,\ln\left(\left(\frac{2\,e^x}{3} - \frac{\sqrt{3}}{3}\right)^2 + \frac{1}{9}\right)}{18} + \frac{\sqrt{3}\,\ln\left(\left(\frac{2\,e^x}{3} + \frac{\sqrt{3}}{3}\right)^2 + \frac{1}{9}\right)}{18}$$

input
```
int(exp(x)/cosh(3*x)^2,x)
```

output
```
(2*atan(exp(x)))/9 + atan(2*exp(x) + 3^(1/2))/9 + atan(2*exp(x) - 3^(1/2))
/9 - (2*exp(x))/(3*(exp(6*x) + 1)) - (3^(1/2)*log(((2*exp(x))/3 - 3^(1/2)/
3)^2 + 1/9))/18 + (3^(1/2)*log(((2*exp(x))/3 + 3^(1/2)/3)^2 + 1/9))/18
```

3.281 $\int e^x \cosh^2(4x) \, dx$

3.281.1 Optimal result

Integrand size = 10, antiderivative size = 26

$$\int e^x \cosh^2(4x) \, dx = -\frac{1}{28} e^{-7x} + \frac{e^x}{2} + \frac{e^{9x}}{36}$$

output
```
-1/28/exp(7*x)+1/2*exp(x)+1/36*exp(9*x)
```

3.281.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int e^x \cosh^2(4x) \, dx = -\frac{1}{28} e^{-7x} + \frac{e^x}{2} + \frac{e^{9x}}{36}$$

input
```
Integrate[E^x*Cosh[4*x]^2,x]
```

output
```
-1/28*1/E^(7*x) + E^x/2 + E^(9*x)/36
```

3.281.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2720, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \cosh^2(4x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int \frac{1}{4}e^{-8x}\left(e^{8x}+1\right)^2de^x$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int e^{-8x}\left(1+e^{8x}\right)^2de^x$$

$$\Big\downarrow \text{802}$$

$$\frac{1}{4}\int \left(2+e^{-8x}+e^{8x}\right)de^x$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{1}{7}e^{-7x}+2e^x+\frac{e^{9x}}{9}\right)$$

input
```
Int[E^x*Cosh[4*x]^2,x]
```

output
```
(-1/7*1/E^(7*x) + 2*E^x + E^(9*x)/9)/4
```

3.281.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.281.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

method	result	size
parallelrisch	$-\frac{e^{x}(-63+\cosh(8x)-8\sinh(8x))}{126}$	17
risch	$\frac{e^{9x}}{36}+\frac{e^{x}}{2}-\frac{e^{-7x}}{28}$	18
default	$\frac{\sinh(x)}{2}+\frac{\sinh(7x)}{28}+\frac{\sinh(9x)}{36}+\frac{\cosh(x)}{2}-\frac{\cosh(7x)}{28}+\frac{\cosh(9x)}{36}$	34

input
```
int(exp(x)*cosh(4*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/126*exp(x)*(-63+cosh(8*x)-8*sinh(8*x))
```

3.281.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(17) = 34.

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.35

$$\int e^x \cosh^2(4x)\,dx =$$
$$-\frac{\cosh(x)^8-64\cosh(x)^7\sinh(x)+28\cosh(x)^6\sinh(x)^2-448\cosh(x)^5\sinh(x)^3+70\cosh(x)^4\sinh(}{126(\cosh(x)-s}$$

input
```
integrate(exp(x)*cosh(4*x)^2,x, algorithm="fricas")
```

output
```
-1/126*(cosh(x)^8 - 64*cosh(x)^7*sinh(x) + 28*cosh(x)^6*sinh(x)^2 - 448*co
sh(x)^5*sinh(x)^3 + 70*cosh(x)^4*sinh(x)^4 - 448*cosh(x)^3*sinh(x)^5 + 28*
cosh(x)^2*sinh(x)^6 - 64*cosh(x)*sinh(x)^7 + sinh(x)^8 - 63)/(cosh(x) - si
nh(x))
```

3.281.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(19) = 38$.

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int e^x \cosh^2(4x)\,dx = -\frac{32e^x\sinh^2(4x)}{63} + \frac{8e^x\sinh(4x)\cosh(4x)}{63} + \frac{31e^x\cosh^2(4x)}{63}$$

input
```
integrate(exp(x)*cosh(4*x)**2,x)
```

output
```
-32*exp(x)*sinh(4*x)**2/63 + 8*exp(x)*sinh(4*x)*cosh(4*x)/63 + 31*exp(x)*c
osh(4*x)**2/63
```

3.281.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int e^x \cosh^2(4x)\,dx = \frac{1}{36}\,e^{(9\,x)} - \frac{1}{28}\,e^{(-7\,x)} + \frac{1}{2}\,e^x$$

input
```
integrate(exp(x)*cosh(4*x)^2,x, algorithm="maxima")
```

output
```
1/36*e^(9*x) - 1/28*e^(-7*x) + 1/2*e^x
```

3.281.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int e^x \cosh^2(4x)\, dx = \frac{1}{36}\, e^{(9\,x)} - \frac{1}{28}\, e^{(-7\,x)} + \frac{1}{2}\, e^x$$

input
```
integrate(exp(x)*cosh(4*x)^2,x, algorithm="giac")
```

output
```
1/36*e^(9*x) - 1/28*e^(-7*x) + 1/2*e^x
```

3.281.9 Mupad [B] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int e^x \cosh^2(4x)\, dx = \frac{\mathrm{e}^{9\,x}}{36} - \frac{\mathrm{e}^{-7\,x}}{28} + \frac{\mathrm{e}^{x}}{2}$$

input
```
int(cosh(4*x)^2*exp(x),x)
```

output
```
exp(9*x)/36 - exp(-7*x)/28 + exp(x)/2
```

3.282 $\int e^x \cosh(4x)\,dx$

3.282.1 Optimal result

Integrand size = 8, antiderivative size = 19

$$\int e^x \cosh(4x)\,dx = -\frac{1}{6}e^{-3x} + \frac{e^{5x}}{10}$$

output
```
-1/6/exp(3*x)+1/10*exp(5*x)
```

3.282.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int e^x \cosh(4x)\,dx = -\frac{1}{6}e^{-3x} + \frac{e^{5x}}{10}$$

input
```
Integrate[E^x*Cosh[4*x],x]
```

output
```
-1/6*1/E^(3*x) + E^(5*x)/10
```

3.282.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2720, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \cosh(4x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int \frac{1}{2}e^{-4x}\left(e^{8x}+1\right)de^x$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int e^{-4x}\left(1+e^{8x}\right)de^x$$

$$\Big\downarrow \text{802}$$

$$\frac{1}{2}\int \left(e^{-4x}+e^{4x}\right)de^x$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{e^{5x}}{5}-\frac{1}{3}e^{-3x}\right)$$

input
```
Int[E^x*Cosh[4*x],x]
```

output
```
(-1/3*1/E^(3*x) + E^(5*x)/5)/2
```

3.282.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.282.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
risch	$\frac{e^{5x}}{10}-\frac{e^{-3x}}{6}$	14
parallelrisch	$-\frac{e^{x}(\cosh(4x)-4\sinh(4x))}{15}$	16
default	$\frac{\sinh(3x)}{6}+\frac{\sinh(5x)}{10}-\frac{\cosh(3x)}{6}+\frac{\cosh(5x)}{10}$	26

input
```
int(exp(x)*cosh(4*x),x,method=_RETURNVERBOSE)
```

output
```
1/10*exp(5*x)-1/6*exp(-3*x)
```

3.282.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(13) = 26.

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.42

$$\int e^{x}\cosh(4x)\,dx = -\frac{\cosh(x)^{4}-16\cosh(x)^{3}\sinh(x)+6\cosh(x)^{2}\sinh(x)^{2}-16\cosh(x)\sinh(x)^{3}+\sinh(x)^{4}}{15(\cosh(x)-\sinh(x))}$$

input
```
integrate(exp(x)*cosh(4*x),x, algorithm="fricas")
```

output
```
-1/15*(cosh(x)^4 - 16*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 16*cosh(
x)*sinh(x)^3 + sinh(x)^4)/(cosh(x) - sinh(x))
```

3.282.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^x \cosh(4x)\,dx = \frac{4e^x \sinh(4x)}{15} - \frac{e^x \cosh(4x)}{15}$$

input
```
integrate(exp(x)*cosh(4*x),x)
```

output
```
4*exp(x)*sinh(4*x)/15 - exp(x)*cosh(4*x)/15
```

3.282.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int e^x \cosh(4x)\,dx = \frac{1}{10}\,e^{(5\,x)} - \frac{1}{6}\,e^{(-3\,x)}$$

input
```
integrate(exp(x)*cosh(4*x),x, algorithm="maxima")
```

output
```
1/10*e^(5*x) - 1/6*e^(-3*x)
```

3.282.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int e^x \cosh(4x)\,dx = \frac{1}{10}\,e^{(5\,x)} - \frac{1}{6}\,e^{(-3\,x)}$$

input
```
integrate(exp(x)*cosh(4*x),x, algorithm="giac")
```

output
```
1/10*e^(5*x) - 1/6*e^(-3*x)
```

3.282.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int e^x \cosh(4x)\,dx = \frac{e^{-3x}\,(3\,e^{8x}-5)}{30}$$

input
```
int(cosh(4*x)*exp(x),x)
```

output
```
(exp(-3*x)*(3*exp(8*x) - 5))/30
```

3.283 $\int e^x \operatorname{sech}(4x)\,dx$

3.283.1 Optimal result

Integrand size = 8, antiderivative size = 371

$$\begin{aligned}\int e^x \operatorname{sech}(4x)\,dx = {} & \frac{\arctan\left(\frac{\sqrt{2-\sqrt{2}}-2e^x}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}} - \frac{\arctan\left(\frac{\sqrt{2+\sqrt{2}}-2e^x}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}} \\ & - \frac{\arctan\left(\frac{\sqrt{2-\sqrt{2}}+2e^x}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}} + \frac{\arctan\left(\frac{\sqrt{2+\sqrt{2}}+2e^x}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}} \\ & - \frac{\log\left(1-\sqrt{2-\sqrt{2}}e^x+e^{2x}\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}} + \frac{\log\left(1+\sqrt{2-\sqrt{2}}e^x+e^{2x}\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}} \\ & + \frac{\log\left(1-\sqrt{2+\sqrt{2}}e^x+e^{2x}\right)}{4\sqrt{2\left(2+\sqrt{2}\right)}} - \frac{\log\left(1+\sqrt{2+\sqrt{2}}e^x+e^{2x}\right)}{4\sqrt{2\left(2+\sqrt{2}\right)}}\end{aligned}$$

output
```
-1/2*arctan((-2*exp(x)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(4-2*2^(1/2))
^(1/2)+1/2*arctan((2*exp(x)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(4-2*2^(
1/2))^(1/2)-1/4*ln(1+exp(2*x)-exp(x)*(2-2^(1/2))^(1/2))/(4-2*2^(1/2))^(1/2
)+1/4*ln(1+exp(2*x)+exp(x)*(2-2^(1/2))^(1/2))/(4-2*2^(1/2))^(1/2)+1/2*arct
an((-2*exp(x)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)-1/
2*arctan((2*exp(x)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/
2)+1/4*ln(1+exp(2*x)-exp(x)*(2+2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)-1/4*ln(
1+exp(2*x)+exp(x)*(2+2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)
```

3.283.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.06

$$\int e^x \text{sech}(4x)\,dx = \frac{2}{5}e^{5x}\,\text{Hypergeometric2F1}\left(\frac{5}{8},1,\frac{13}{8},-e^{8x}\right)$$

input
```
Integrate[E^x*Sech[4*x],x]
```

output
```
(2*E^(5*x)*Hypergeometric2F1[5/8, 1, 13/8, -E^(8*x)])/5
```

3.283.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {2720, 27, 828, 1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \text{sech}(4x)\,dx$$

$$\Big\downarrow \text{ 2720}$$

$$\int \frac{2e^{4x}}{e^{8x}+1}de^x$$

$$\Big\downarrow \text{ 27}$$

$$2\int \frac{e^{4x}}{1+e^{8x}}de^x$$

$$\Big\downarrow \text{ 828}$$

$$2\left(\frac{\int \frac{e^{2x}}{1-\sqrt{2}e^{2x}+e^{4x}}de^x}{2\sqrt{2}}-\frac{\int \frac{e^{2x}}{1+\sqrt{2}e^{2x}+e^{4x}}de^x}{2\sqrt{2}}\right)$$

$$\Big\downarrow \text{ 1447}$$

$$2\left(\frac{\frac{1}{2}\int \frac{1+e^{2x}}{1-\sqrt{2}e^{2x}+e^{4x}}de^x-\frac{1}{2}\int \frac{1-e^{2x}}{1-\sqrt{2}e^{2x}+e^{4x}}de^x}{2\sqrt{2}}-\frac{\frac{1}{2}\int \frac{1+e^{2x}}{1+\sqrt{2}e^{2x}+e^{4x}}de^x-\frac{1}{2}\int \frac{1-e^{2x}}{1+\sqrt{2}e^{2x}+e^{4x}}de^x}{2\sqrt{2}}\right)$$

$$\Big\downarrow 1475$$

$$2\left(\frac{\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{1-\sqrt{2+\sqrt{2}}e^x+e^{2x}}de^x+\frac{1}{2}\int \frac{1}{1+\sqrt{2+\sqrt{2}}e^x+e^{2x}}de^x\right)-\frac{1}{2}\int \frac{1-e^{2x}}{1-\sqrt{2}e^{2x}+e^{4x}}de^x}{2\sqrt{2}}-\frac{\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{1-\sqrt{2-\sqrt{2}}e^x+e^{2x}}de^x+\frac{1}{2}\int \right.}{}\right.$$

$$\Big\downarrow 1083$$

$$2\left(\frac{\frac{1}{2}\left(-\int \frac{1}{-2+\sqrt{2}-e^{2x}}d\left(-\sqrt{2+\sqrt{2}}+2e^x\right)-\int \frac{1}{-2+\sqrt{2}-e^{2x}}d\left(\sqrt{2+\sqrt{2}}+2e^x\right)\right)-\frac{1}{2}\int \frac{1-e^{2x}}{1-\sqrt{2}e^{2x}+e^{4x}}de^x}{2\sqrt{2}}-\frac{\frac{1}{2}\left(-\int \right.}{}\right.$$

$$\Big\downarrow 217$$

$$2\left(\frac{\frac{1}{2}\left(\frac{\arctan\left(\frac{2e^x-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{\sqrt{2-\sqrt{2}}}+\frac{\arctan\left(\frac{2e^x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{\sqrt{2-\sqrt{2}}}\right)-\frac{1}{2}\int \frac{1-e^{2x}}{1-\sqrt{2}e^{2x}+e^{4x}}de^x}{2\sqrt{2}}-\frac{\frac{1}{2}\left(\frac{\arctan\left(\frac{2e^x-\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{\sqrt{2+\sqrt{2}}}+\frac{\arctan\left(\frac{2e^x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{\sqrt{2+\sqrt{2}}}\right.}{2\sqrt{2}}\right.$$

$$\Big\downarrow 1478$$

$$2\left(\frac{\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2+\sqrt{2}}-2e^x}{1-\sqrt{2+\sqrt{2}}e^x+e^{2x}}de^x}{2\sqrt{2+\sqrt{2}}}+\frac{\int -\frac{\sqrt{2+\sqrt{2}}+2e^x}{1+\sqrt{2+\sqrt{2}}e^x+e^{2x}}de^x}{2\sqrt{2+\sqrt{2}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2e^x-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{\sqrt{2-\sqrt{2}}}+\frac{\arctan\left(\frac{2e^x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2}}-\frac{\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2-}}{1-\sqrt{2-}}}{2\sqrt{}}\right.}{}\right.$$

$$\Big\downarrow 25$$

$$2\left(\frac{\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2+\sqrt{2}}-2e^x}{1-\sqrt{2+\sqrt{2}}e^x+e^{2x}}de^x}{2\sqrt{2+\sqrt{2}}}-\frac{\int \frac{\sqrt{2+\sqrt{2}}+2e^x}{1+\sqrt{2+\sqrt{2}}e^x+e^{2x}}de^x}{2\sqrt{2+\sqrt{2}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2e^x-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{\sqrt{2-\sqrt{2}}}+\frac{\arctan\left(\frac{2e^x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2}}-\frac{\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2-}}{1-\sqrt{2-}}}{2\sqrt{}}\right.}{}\right.$$

$$\Big\downarrow 1103$$

3.283. $\int e^x \text{sech}(4x)\, dx$

$$2\left(\frac{\frac{1}{2}\left(\frac{\arctan\left(\frac{2e^x-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{\sqrt{2-\sqrt{2}}}+\frac{\arctan\left(\frac{2e^x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{\sqrt{2-\sqrt{2}}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2+\sqrt{2}}e^x+e^{2x}+1\right)}{2\sqrt{2+\sqrt{2}}}-\frac{\log\left(\sqrt{2+\sqrt{2}}e^x+e^{2x}+1\right)}{2\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2}}-\frac{\frac{1}{2}\left(\frac{\arctan\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.$$

input
```
Int[E^x*Sech[4*x],x]
```

output
```
2*(-1/2*((ArcTan[(-Sqrt[2 - Sqrt[2]] + 2*E^x)/Sqrt[2 + Sqrt[2]]]/Sqrt[2 +
Sqrt[2]] + ArcTan[(Sqrt[2 - Sqrt[2]] + 2*E^x)/Sqrt[2 + Sqrt[2]]]/Sqrt[2 +
Sqrt[2]])/2 + (Log[1 - Sqrt[2 - Sqrt[2]]*E^x + E^(2*x)]/(2*Sqrt[2 - Sqrt[2
]]) - Log[1 + Sqrt[2 - Sqrt[2]]*E^x + E^(2*x)]/(2*Sqrt[2 - Sqrt[2]]))/2)/S
qrt[2] + ((ArcTan[(-Sqrt[2 + Sqrt[2]] + 2*E^x)/Sqrt[2 - Sqrt[2]]]/Sqrt[2 -
 Sqrt[2]] + ArcTan[(Sqrt[2 + Sqrt[2]] + 2*E^x)/Sqrt[2 - Sqrt[2]]]/Sqrt[2 -
 Sqrt[2]])/2 + (Log[1 - Sqrt[2 + Sqrt[2]]*E^x + E^(2*x)]/(2*Sqrt[2 + Sqrt[
2]]) - Log[1 + Sqrt[2 + Sqrt[2]]*E^x + E^(2*x)]/(2*Sqrt[2 + Sqrt[2]]))/2)/
(2*Sqrt[2]))
```

3.283.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 828
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Simp[s^3/(2*Sqrt[2]*b*r)   Int[x^
(m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Simp[s^3/(2*S
qrt[2]*b*r)   Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x]
, x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] &&
 GtQ[a/b, 0]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.283. $\int e^x \text{sech}(4x)\, dx$

3.283.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.07

method	result	size
risch	$2\left(\sum_{\_R=\mathrm{RootOf}\left(16777216\_Z^{8}+1\right)}\_R\ln\left(-32768\_R^{5}+\mathrm{e}^{x}\right)\right)$	25

input
```
int(exp(x)*sech(4*x),x,method=_RETURNVERBOSE)
```

output
```
2*sum(_R*ln(-32768*_R^5+exp(x)),_R=RootOf(16777216*_Z^8+1))
```

3.283.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.45

$$\begin{aligned}\int e^{x}\mathrm{sech}(4x)\,dx =& \left(\frac{1}{8}i+\frac{1}{8}\right)\sqrt{2}(-1)^{\frac{1}{8}}\log\left((i+1)\sqrt{2}(-1)^{\frac{5}{8}}+2\cosh\left(x\right)+2\sinh\left(x\right)\right)\\&-\left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}(-1)^{\frac{1}{8}}\log\left(-(i-1)\sqrt{2}(-1)^{\frac{5}{8}}+2\cosh\left(x\right)+2\sinh\left(x\right)\right)\\&+\left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}(-1)^{\frac{1}{8}}\log\left((i-1)\sqrt{2}(-1)^{\frac{5}{8}}+2\cosh\left(x\right)+2\sinh\left(x\right)\right)\\&-\left(\frac{1}{8}i+\frac{1}{8}\right)\sqrt{2}(-1)^{\frac{1}{8}}\log\left(-(i+1)\sqrt{2}(-1)^{\frac{5}{8}}+2\cosh\left(x\right)+2\sinh\left(x\right)\right)\\&-\frac{1}{4}(-1)^{\frac{1}{8}}\log\left((-1)^{\frac{5}{8}}+\cosh\left(x\right)+\sinh\left(x\right)\right)\\&-\frac{1}{4}i(-1)^{\frac{1}{8}}\log\left(i(-1)^{\frac{5}{8}}+\cosh\left(x\right)+\sinh\left(x\right)\right)\\&+\frac{1}{4}i(-1)^{\frac{1}{8}}\log\left(-i(-1)^{\frac{5}{8}}+\cosh\left(x\right)+\sinh\left(x\right)\right)\\&+\frac{1}{4}(-1)^{\frac{1}{8}}\log\left(-(-1)^{\frac{5}{8}}+\cosh\left(x\right)+\sinh\left(x\right)\right)\end{aligned}$$

input
```
integrate(exp(x)*sech(4*x),x, algorithm="fricas")
```

output
```
(1/8*I + 1/8)*sqrt(2)*(-1)^(1/8)*log((I + 1)*sqrt(2)*(-1)^(5/8) + 2*cosh(x
) + 2*sinh(x)) - (1/8*I - 1/8)*sqrt(2)*(-1)^(1/8)*log(-(I - 1)*sqrt(2)*(-1
)^(5/8) + 2*cosh(x) + 2*sinh(x)) + (1/8*I - 1/8)*sqrt(2)*(-1)^(1/8)*log((I
 - 1)*sqrt(2)*(-1)^(5/8) + 2*cosh(x) + 2*sinh(x)) - (1/8*I + 1/8)*sqrt(2)*
(-1)^(1/8)*log(-(I + 1)*sqrt(2)*(-1)^(5/8) + 2*cosh(x) + 2*sinh(x)) - 1/4*
(-1)^(1/8)*log((-1)^(5/8) + cosh(x) + sinh(x)) - 1/4*I*(-1)^(1/8)*log(I*(-
1)^(5/8) + cosh(x) + sinh(x)) + 1/4*I*(-1)^(1/8)*log(-I*(-1)^(5/8) + cosh(
x) + sinh(x)) + 1/4*(-1)^(1/8)*log(-(-1)^(5/8) + cosh(x) + sinh(x))
```

3.283.6 Sympy [F]

$$\int e^x \operatorname{sech}(4x)\,dx = \int e^x \operatorname{sech}(4x)\,dx$$

input
```
integrate(exp(x)*sech(4*x),x)
```

output
```
Integral(exp(x)*sech(4*x), x)
```

3.283.7 Maxima [F]

$$\int e^x \operatorname{sech}(4x)\,dx = \int e^x \operatorname{sech}(4x)\,dx$$

input
```
integrate(exp(x)*sech(4*x),x, algorithm="maxima")
```

output
```
integrate(e^x*sech(4*x), x)
```

3.283.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.67

$$\int e^x \operatorname{sech}(4x)\,dx = \frac{1}{4}\sqrt{\sqrt{2}+2}\arctan\left(\frac{\sqrt{\sqrt{2}+2}+2\,e^x}{\sqrt{-\sqrt{2}+2}}\right) + \frac{1}{4}\sqrt{\sqrt{2}+2}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}-2\,e^x}{\sqrt{-\sqrt{2}+2}}\right) - \frac{1}{4}\sqrt{-\sqrt{2}+2}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}+2\,e^x}{\sqrt{\sqrt{2}+2}}\right) - \frac{1}{4}\sqrt{-\sqrt{2}+2}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}-2\,e^x}{\sqrt{\sqrt{2}+2}}\right) - \frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}e^x+e^{(2\,x)}+1\right) + \frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(-\sqrt{\sqrt{2}+2}e^x+e^{(2\,x)}+1\right) + \frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{-\sqrt{2}+2}e^x+e^{(2\,x)}+1\right) - \frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(-\sqrt{-\sqrt{2}+2}e^x+e^{(2\,x)}+1\right)$$

input
```
integrate(exp(x)*sech(4*x),x, algorithm="giac")
```

output
```
1/4*sqrt(sqrt(2) + 2)*arctan((sqrt(sqrt(2) + 2) + 2*e^x)/sqrt(-sqrt(2) + 2
)) + 1/4*sqrt(sqrt(2) + 2)*arctan(-(sqrt(sqrt(2) + 2) - 2*e^x)/sqrt(-sqrt(
2) + 2)) - 1/4*sqrt(-sqrt(2) + 2)*arctan((sqrt(-sqrt(2) + 2) + 2*e^x)/sqrt
(sqrt(2) + 2)) - 1/4*sqrt(-sqrt(2) + 2)*arctan(-(sqrt(-sqrt(2) + 2) - 2*e^
x)/sqrt(sqrt(2) + 2)) - 1/8*sqrt(-sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*e^x +
 e^(2*x) + 1) + 1/8*sqrt(-sqrt(2) + 2)*log(-sqrt(sqrt(2) + 2)*e^x + e^(2*x
) + 1) + 1/8*sqrt(sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) -
 1/8*sqrt(sqrt(2) + 2)*log(-sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1)
```

3.283.9 Mupad [B] (verification not implemented)

Time = 5.65 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.29

$$\int e^x \mathrm{sech}(4x)\,dx = -\ln\left(32768\,\mathrm{e}^x\left(\frac{\sqrt{\sqrt{2}+2}}{8}+\frac{\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{8}\right)^3-512\right)\left(\frac{\sqrt{\sqrt{2}+2}}{8}+\frac{\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{8}\right)+\ln\left(32768\,\mathrm{e}^x\left(\frac{\sqrt{\sqrt{2}+2}}{8}+\frac{\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{8}\right)^3+512\right)\left(\frac{\sqrt{\sqrt{2}+2}}{8}+\frac{\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{8}\right)-\ln\left(32768\,\mathrm{e}^x\left(-\frac{\sqrt{2-\sqrt{2}}}{8}+\frac{\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{8}\right)^3-512\right)\left(-\frac{\sqrt{2-\sqrt{2}}}{8}+\frac{\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{8}\right)+\ln\left(32768\,\mathrm{e}^x\left(-\frac{\sqrt{2-\sqrt{2}}}{8}+\frac{\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{8}\right)^3+512\right)\left(-\frac{\sqrt{2-\sqrt{2}}}{8}+\frac{\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{8}\right)+\sqrt{2}\,\ln\left(-512+\sqrt{2}\,\mathrm{e}^x\left(\frac{\sqrt{\sqrt{2}+2}}{8}+\frac{\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{8}\right)^3(16384-16384\mathrm{i})\right)\left(\frac{\sqrt{\sqrt{2}+2}}{8}+\frac{\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{8}\right)\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\,\ln\left(512+\sqrt{2}\,\mathrm{e}^x\left(\frac{\sqrt{\sqrt{2}+2}}{8}+\frac{\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{8}\right)^3(16384-16384\mathrm{i})\right)\left(\frac{\sqrt{\sqrt{2}+2}}{8}+\frac{\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{8}\right)\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\,\ln\left(-512+\sqrt{2}\,\mathrm{e}^x\left(\frac{\sqrt{\sqrt{2}+2}}{8}+\frac{\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{8}\right)^3(16384+16384\mathrm{i})\right)\left(\frac{\sqrt{\sqrt{2}+2}}{8}+\frac{\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{8}\right)\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\,\ln\left(512+\sqrt{2}\,\mathrm{e}^x\left(\frac{\sqrt{\sqrt{2}+2}}{8}+\frac{\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{8}\right)^3(16384+16384\mathrm{i})\right)\left(\frac{\sqrt{\sqrt{2}+2}}{8}+\frac{\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{8}\right)\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)$$

input
```
int(exp(x)/cosh(4*x),x)
```

output
```
log(32768*exp(x)*((2^(1/2) + 2)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*1i)/8)^3 +
512)*((2^(1/2) + 2)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*1i)/8) - log(32768*exp(
x)*((2^(1/2) + 2)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*1i)/8)^3 - 512)*((2^(1/2)
 + 2)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*1i)/8) - log(32768*exp(x)*(((2^(1/2)
+ 2)^(1/2)*1i)/8 - (2 - 2^(1/2))^(1/2)/8)^3 - 512)*(((2^(1/2) + 2)^(1/2)*1
i)/8 - (2 - 2^(1/2))^(1/2)/8) + log(32768*exp(x)*(((2^(1/2) + 2)^(1/2)*1i)
/8 - (2 - 2^(1/2))^(1/2)/8)^3 + 512)*(((2^(1/2) + 2)^(1/2)*1i)/8 - (2 - 2^
(1/2))^(1/2)/8) + 2^(1/2)*log(2^(1/2)*exp(x)*((2^(1/2) + 2)^(1/2)/8 + ((2
- 2^(1/2))^(1/2)*1i)/8)^3*(16384 - 16384i) - 512)*((2^(1/2) + 2)^(1/2)/8 +
 ((2 - 2^(1/2))^(1/2)*1i)/8)*(1/2 + 1i/2) - 2^(1/2)*log(2^(1/2)*exp(x)*((2
^(1/2) + 2)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*1i)/8)^3*(16384 - 16384i) + 512
)*((2^(1/2) + 2)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*1i)/8)*(1/2 + 1i/2) + 2^(1
/2)*log(2^(1/2)*exp(x)*((2^(1/2) + 2)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*1i)/8
)^3*(16384 + 16384i) - 512)*((2^(1/2) + 2)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*
1i)/8)*(1/2 - 1i/2) - 2^(1/2)*log(2^(1/2)*exp(x)*((2^(1/2) + 2)^(1/2)/8 +
((2 - 2^(1/2))^(1/2)*1i)/8)^3*(16384 + 16384i) + 512)*((2^(1/2) + 2)^(1/2)
/8 + ((2 - 2^(1/2))^(1/2)*1i)/8)*(1/2 - 1i/2)
```

3.283. $\int e^x \operatorname{sech}(4x)\,dx$

3.284 $\int e^x \mathrm{sech}^2(4x)\,dx$

3.284.1 Optimal result

Integrand size = 10, antiderivative size = 379

$$\begin{aligned}\int e^x \mathrm{sech}^2(4x)\,dx = &-\frac{e^x}{2(1+e^{8x})}-\frac{\arctan\left(\frac{\sqrt{2-\sqrt{2}}-2e^x}{\sqrt{2+\sqrt{2}}}\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\arctan\left(\frac{\sqrt{2+\sqrt{2}}-2e^x}{\sqrt{2-\sqrt{2}}}\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}}\\
&+\frac{\arctan\left(\frac{\sqrt{2-\sqrt{2}}+2e^x}{\sqrt{2+\sqrt{2}}}\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}}+\frac{\arctan\left(\frac{\sqrt{2+\sqrt{2}}+2e^x}{\sqrt{2-\sqrt{2}}}\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}}\\
&-\frac{1}{32}\sqrt{2-\sqrt{2}}\log\left(1-\sqrt{2-\sqrt{2}}e^x+e^{2x}\right)\\
&+\frac{1}{32}\sqrt{2-\sqrt{2}}\log\left(1+\sqrt{2-\sqrt{2}}e^x+e^{2x}\right)\\
&-\frac{1}{32}\sqrt{2+\sqrt{2}}\log\left(1-\sqrt{2+\sqrt{2}}e^x+e^{2x}\right)\\
&+\frac{1}{32}\sqrt{2+\sqrt{2}}\log\left(1+\sqrt{2+\sqrt{2}}e^x+e^{2x}\right)\end{aligned}$$

output
```
-1/2*exp(x)/(1+exp(8*x))-1/32*ln(1+exp(2*x)-exp(x)*(2-2^(1/2))^(1/2))*(2-2
^(1/2))^(1/2)+1/32*ln(1+exp(2*x)+exp(x)*(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/
2)-1/8*arctan((-2*exp(x)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))/(4-2*2^(1/2
))^(1/2)+1/8*arctan((2*exp(x)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))/(4-2*2
^(1/2))^(1/2)-1/32*ln(1+exp(2*x)-exp(x)*(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/
2)+1/32*ln(1+exp(2*x)+exp(x)*(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)-1/8*arct
an((-2*exp(x)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)+1/
8*arctan((2*exp(x)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/
2)
```

3.284.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.09

$$\int e^x \operatorname{sech}^2(4x)\,dx = \frac{1}{2}e^x\left(-\frac{1}{1+e^{8x}} + \text{Hypergeometric2F1}\left(\frac{1}{8},1,\frac{9}{8},-e^{8x}\right)\right)$$

input
```
Integrate[E^x*Sech[4*x]^2,x]
```

output
```
(E^x*(-(1 + E^(8*x))^(-1) + Hypergeometric2F1[1/8, 1, 9/8, -E^(8*x)]))/2
```

3.284.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {2720, 27, 817, 757, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \operatorname{sech}^2(4x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int \frac{4e^{8x}}{(e^{8x}+1)^2}de^x$$

$$\Big\downarrow \text{27}$$

$$4\int \frac{e^{8x}}{(1+e^{8x})^2}de^x$$

$$\Big\downarrow \text{817}$$

$$4\left(\frac{1}{8}\int \frac{1}{1+e^{8x}}de^x-\frac{e^x}{8\left(e^{8x}+1\right)}\right)$$

$$\Big\downarrow \text{757}$$

$$4\left(\frac{1}{8}\left(\frac{\int \frac{\sqrt{2}-e^{2x}}{1-\sqrt{2}e^{2x}+e^{4x}}de^x}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}+e^{2x}}{1+\sqrt{2}e^{2x}+e^{4x}}de^x}{2\sqrt{2}}\right)-\frac{e^x}{8\left(e^{8x}+1\right)}\right)$$

$$\Big\downarrow \text{1483}$$

$$4\left(\frac{1}{8}\left(\frac{\frac{\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}+\left(1-\sqrt{2}\right)e^x}{1-\sqrt{2-\sqrt{2}}e^x+e^{2x}}de^x}{2\sqrt{2-\sqrt{2}}}+\frac{\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}-\left(1-\sqrt{2}\right)e^x}{1+\sqrt{2-\sqrt{2}}e^x+e^{2x}}de^x}{2\sqrt{2-\sqrt{2}}}}{2\sqrt{2}}+\frac{\frac{\int \frac{\sqrt{2\left(2+\sqrt{2}\right)}-\left(1+\sqrt{2}\right)e^x}{1-\sqrt{2+\sqrt{2}}e^x+e^{2x}}de^x}{2\sqrt{2+\sqrt{2}}}+\frac{\int \frac{\sqrt{2\left(2+\sqrt{2}\right)}+\left(1+\sqrt{2}\right)e^x}{1+\sqrt{2+\sqrt{2}}e^x+e^{2x}}de^x}{2\sqrt{2+\sqrt{2}}}}{2\sqrt{2}}\right)-\frac{\quad}{8\left(\right.}$$

$$\Big\downarrow \text{1142}$$

$$4\left(\frac{1}{8}\left(\frac{\frac{\frac{1}{2}\sqrt{2+\sqrt{2}}\int \frac{1}{1-\sqrt{2-\sqrt{2}}e^x+e^{2x}}de^x+\frac{1}{2}\left(1-\sqrt{2}\right)\int -\frac{\sqrt{2-\sqrt{2}}-2e^x}{1-\sqrt{2-\sqrt{2}}e^x+e^{2x}}de^x}{2\sqrt{2-\sqrt{2}}}+\frac{\frac{1}{2}\sqrt{2+\sqrt{2}}\int \frac{1}{1+\sqrt{2-\sqrt{2}}e^x+e^{2x}}de^x-\frac{1}{2}\left(1-\sqrt{2}\right)\int \frac{\sqrt{2-\sqrt{2}}+2}{1+\sqrt{2-\sqrt{2}}e^x}}{2\sqrt{2-\sqrt{2}}}}{2\sqrt{2}}\right.\right.$$

$$\Big\downarrow \text{25}$$

$$4\left(\frac{1}{8}\left(\frac{\frac{\frac{1}{2}\sqrt{2+\sqrt{2}}\int \frac{1}{1-\sqrt{2-\sqrt{2}}e^x+e^{2x}}de^x-\frac{1}{2}\left(1-\sqrt{2}\right)\int \frac{\sqrt{2-\sqrt{2}}-2e^x}{1-\sqrt{2-\sqrt{2}}e^x+e^{2x}}de^x}{2\sqrt{2-\sqrt{2}}}+\frac{\frac{1}{2}\sqrt{2+\sqrt{2}}\int \frac{1}{1+\sqrt{2-\sqrt{2}}e^x+e^{2x}}de^x-\frac{1}{2}\left(1-\sqrt{2}\right)\int \frac{\sqrt{2-\sqrt{2}}+2e^x}{1+\sqrt{2-\sqrt{2}}e^x+e}}{2\sqrt{2-\sqrt{2}}}}{2\sqrt{2}}\right.\right.$$

$$\Big\downarrow \text{1083}$$

$$4\left(\frac{1}{8}\left(\frac{\frac{-\sqrt{2+\sqrt{2}}\int \frac{1}{-2-\sqrt{2}-e^{2x}}d\left(-\sqrt{2-\sqrt{2}}+2e^x\right)-\frac{1}{2}\left(1-\sqrt{2}\right)\int \frac{\sqrt{2-\sqrt{2}}-2e^x}{1-\sqrt{2-\sqrt{2}}e^x+e^{2x}}de^x}{2\sqrt{2-\sqrt{2}}}+\frac{-\sqrt{2+\sqrt{2}}\int \frac{1}{-2-\sqrt{2}-e^{2x}}d\left(\sqrt{2-\sqrt{2}}+2e^x\right)-\frac{1}{2}\left(1-\right.}{2\sqrt{2-\sqrt{2}}}}{2\sqrt{2}}\right.\right.$$

$$\Big\downarrow \text{217}$$

3.284. $\int e^x\text{sech}^2(4x)\,dx$

$$4\left(\frac{1}{8}\left(\frac{\frac{\arctan\left(\frac{2e^x-\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)-\frac{1}{2}\left(1-\sqrt{2}\right)\int\frac{\sqrt{2-\sqrt{2}}-2e^x}{1-\sqrt{2-\sqrt{2}}e^x+e^{2x}}de^x}{2\sqrt{2-\sqrt{2}}}+\frac{\arctan\left(\frac{2e^x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)-\frac{1}{2}\left(1-\sqrt{2}\right)\int\frac{\sqrt{2-\sqrt{2}}+2e^x}{1+\sqrt{2-\sqrt{2}}e^x+e^{2x}}de^x}{2\sqrt{2-\sqrt{2}}}}{2\sqrt{2}}+\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\cdots}{\cdots}\right.\right.$$

$\Big\downarrow$ 1103

$$4\left(\frac{1}{8}\left(\frac{\frac{\arctan\left(\frac{2e^x-\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)+\frac{1}{2}\left(1-\sqrt{2}\right)\log\left(-\sqrt{2-\sqrt{2}}e^x+e^{2x}+1\right)}{2\sqrt{2-\sqrt{2}}}+\frac{\arctan\left(\frac{2e^x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)-\frac{1}{2}\left(1-\sqrt{2}\right)\log\left(\sqrt{2-\sqrt{2}}e^x+e^{2x}+1\right)}{2\sqrt{2-\sqrt{2}}}}{2\sqrt{2}}+\frac{\mathrm{arcta}\cdots}{\cdots}\right.\right.$$

input
```
Int[E^x*Sech[4*x]^2,x]
```

output
```
4*(-1/8*E^x/(1 + E^(8*x)) + (((ArcTan[(-Sqrt[2 - Sqrt[2]] + 2*E^x)/Sqrt[2
+ Sqrt[2]]] + ((1 - Sqrt[2])*Log[1 - Sqrt[2 - Sqrt[2]]*E^x + E^(2*x)])/2)/
(2*Sqrt[2 - Sqrt[2]]) + (ArcTan[(Sqrt[2 - Sqrt[2]] + 2*E^x)/Sqrt[2 + Sqrt[
2]]] - ((1 - Sqrt[2])*Log[1 + Sqrt[2 - Sqrt[2]]*E^x + E^(2*x)])/2)/(2*Sqrt
[2 - Sqrt[2]]))/(2*Sqrt[2]) + ((ArcTan[(-Sqrt[2 + Sqrt[2]] + 2*E^x)/Sqrt[2
 - Sqrt[2]]] - ((1 + Sqrt[2])*Log[1 - Sqrt[2 + Sqrt[2]]*E^x + E^(2*x)])/2)
/(2*Sqrt[2 + Sqrt[2]]) + (ArcTan[(Sqrt[2 + Sqrt[2]] + 2*E^x)/Sqrt[2 - Sqrt
[2]]] + ((1 + Sqrt[2])*Log[1 + Sqrt[2 + Sqrt[2]]*E^x + E^(2*x)])/2)/(2*Sqr
t[2 + Sqrt[2]]))/(2*Sqrt[2]))/8)
```

3.284.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 757
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b,
 4]], s = Denominator[Rt[a/b, 4]]}, Simp[r/(2*Sqrt[2]*a)   Int[(Sqrt[2]*r -
 s*x^(n/4))/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] + Simp[r/(2*S
qrt[2]*a)   Int[(Sqrt[2]*r + s*x^(n/4))/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^
(n/2)), x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 1] && GtQ[a/b, 0]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.284. $\int e^x \operatorname{sech}^2(4x)\, dx$

3.284.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.35 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.09

method	result	size
risch	$-\frac{e^{x}}{2(1+e^{8x})}+4\left(\sum_{\_R=\mathrm{RootOf}(281474976710656\_Z^{8}+1)}\_R\ln\left(e^{x}+64\_R\right)\right)$	36

input
```
int(exp(x)*sech(4*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(x)/(1+exp(8*x))+4*sum(_R*ln(exp(x)+64*_R),_R=RootOf(2814749767106
56*_Z^8+1))
```

3.284.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 1215, normalized size of antiderivative = 3.21

$$\int e^{x}\mathrm{sech}^{2}(4x)\,dx=\text{Too large to display}$$

input
```
integrate(exp(x)*sech(4*x)^2,x, algorithm="fricas")
```

output
```
1/32*(((I + 1)*sqrt(2)*(-1)^(1/8)*cosh(x)^8 + (8*I + 8)*sqrt(2)*(-1)^(1/8)
*cosh(x)^7*sinh(x) + (28*I + 28)*sqrt(2)*(-1)^(1/8)*cosh(x)^6*sinh(x)^2 +
(56*I + 56)*sqrt(2)*(-1)^(1/8)*cosh(x)^5*sinh(x)^3 + (70*I + 70)*sqrt(2)*(
-1)^(1/8)*cosh(x)^4*sinh(x)^4 + (56*I + 56)*sqrt(2)*(-1)^(1/8)*cosh(x)^3*s
inh(x)^5 + (28*I + 28)*sqrt(2)*(-1)^(1/8)*cosh(x)^2*sinh(x)^6 + (8*I + 8)*
sqrt(2)*(-1)^(1/8)*cosh(x)*sinh(x)^7 + (I + 1)*sqrt(2)*(-1)^(1/8)*sinh(x)^
8 + (I + 1)*sqrt(2)*(-1)^(1/8))*log((I + 1)*sqrt(2)*(-1)^(1/8) + 2*cosh(x)
 + 2*sinh(x)) + (-(I - 1)*sqrt(2)*(-1)^(1/8)*cosh(x)^8 - (8*I - 8)*sqrt(2)
*(-1)^(1/8)*cosh(x)^7*sinh(x) - (28*I - 28)*sqrt(2)*(-1)^(1/8)*cosh(x)^6*s
inh(x)^2 - (56*I - 56)*sqrt(2)*(-1)^(1/8)*cosh(x)^5*sinh(x)^3 - (70*I - 70
)*sqrt(2)*(-1)^(1/8)*cosh(x)^4*sinh(x)^4 - (56*I - 56)*sqrt(2)*(-1)^(1/8)*
cosh(x)^3*sinh(x)^5 - (28*I - 28)*sqrt(2)*(-1)^(1/8)*cosh(x)^2*sinh(x)^6 -
 (8*I - 8)*sqrt(2)*(-1)^(1/8)*cosh(x)*sinh(x)^7 - (I - 1)*sqrt(2)*(-1)^(1/
8)*sinh(x)^8 - (I - 1)*sqrt(2)*(-1)^(1/8))*log(-(I - 1)*sqrt(2)*(-1)^(1/8)
 + 2*cosh(x) + 2*sinh(x)) + ((I - 1)*sqrt(2)*(-1)^(1/8)*cosh(x)^8 + (8*I -
 8)*sqrt(2)*(-1)^(1/8)*cosh(x)^7*sinh(x) + (28*I - 28)*sqrt(2)*(-1)^(1/8)*
cosh(x)^6*sinh(x)^2 + (56*I - 56)*sqrt(2)*(-1)^(1/8)*cosh(x)^5*sinh(x)^3 +
 (70*I - 70)*sqrt(2)*(-1)^(1/8)*cosh(x)^4*sinh(x)^4 + (56*I - 56)*sqrt(2)*
(-1)^(1/8)*cosh(x)^3*sinh(x)^5 + (28*I - 28)*sqrt(2)*(-1)^(1/8)*cosh(x)^2*
sinh(x)^6 + (8*I - 8)*sqrt(2)*(-1)^(1/8)*cosh(x)*sinh(x)^7 + (I - 1)*sq...
```

3.284.6 Sympy [F]

$$\int e^x \operatorname{sech}^2(4x) \, dx = \int e^x \operatorname{sech}^2(4x) \, dx$$

input
```
integrate(exp(x)*sech(4*x)**2,x)
```

output
```
Integral(exp(x)*sech(4*x)**2, x)
```

3.284.7 Maxima [F]

$$\int e^x \operatorname{sech}^2(4x)\,dx = \int e^x\operatorname{sech}(4x)^2\,dx$$

input
```
integrate(exp(x)*sech(4*x)^2,x, algorithm="maxima")
```

output
```
-1/2*e^x/(e^(8*x) + 1) + 4*integrate(1/8*e^x/(e^(8*x) + 1), x)
```

3.284.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.69

$$\begin{aligned}\int e^x \operatorname{sech}^2(4x)\,dx &= \frac{1}{16}\sqrt{-\sqrt{2}+2}\arctan\left(\frac{\sqrt{\sqrt{2}+2}+2\,e^x}{\sqrt{-\sqrt{2}+2}}\right)\\
&+\frac{1}{16}\sqrt{-\sqrt{2}+2}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}-2\,e^x}{\sqrt{-\sqrt{2}+2}}\right)\\
&+\frac{1}{16}\sqrt{\sqrt{2}+2}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}+2\,e^x}{\sqrt{\sqrt{2}+2}}\right)\\
&+\frac{1}{16}\sqrt{\sqrt{2}+2}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}-2\,e^x}{\sqrt{\sqrt{2}+2}}\right)\\
&+\frac{1}{32}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}e^x+e^{(2\,x)}+1\right)\\
&-\frac{1}{32}\sqrt{\sqrt{2}+2}\log\left(-\sqrt{\sqrt{2}+2}e^x+e^{(2\,x)}+1\right)\\
&+\frac{1}{32}\sqrt{-\sqrt{2}+2}\log\left(\sqrt{-\sqrt{2}+2}e^x+e^{(2\,x)}+1\right)\\
&-\frac{1}{32}\sqrt{-\sqrt{2}+2}\log\left(-\sqrt{-\sqrt{2}+2}e^x+e^{(2\,x)}+1\right)-\frac{e^x}{2\,(e^{(8\,x)}+1)}\end{aligned}$$

input
```
integrate(exp(x)*sech(4*x)^2,x, algorithm="giac")
```

output
```
1/16*sqrt(-sqrt(2) + 2)*arctan((sqrt(sqrt(2) + 2) + 2*e^x)/sqrt(-sqrt(2) +
 2)) + 1/16*sqrt(-sqrt(2) + 2)*arctan(-(sqrt(sqrt(2) + 2) - 2*e^x)/sqrt(-s
qrt(2) + 2)) + 1/16*sqrt(sqrt(2) + 2)*arctan((sqrt(-sqrt(2) + 2) + 2*e^x)/
sqrt(sqrt(2) + 2)) + 1/16*sqrt(sqrt(2) + 2)*arctan(-(sqrt(-sqrt(2) + 2) -
2*e^x)/sqrt(sqrt(2) + 2)) + 1/32*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*e
^x + e^(2*x) + 1) - 1/32*sqrt(sqrt(2) + 2)*log(-sqrt(sqrt(2) + 2)*e^x + e^
(2*x) + 1) + 1/32*sqrt(-sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2)*e^x + e^(2*x)
+ 1) - 1/32*sqrt(-sqrt(2) + 2)*log(-sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1)
- 1/2*e^x/(e^(8*x) + 1)
```

3.284. $\int e^x \operatorname{sech}^2(4x)\,dx$

3.284.9 Mupad [B] (verification not implemented)

Time = 4.50 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.25

$$\int e^x \operatorname{sech}^2(4x)\,dx = -\frac{e^x}{2\,(e^{8\,x}+1)} + \ln\left(-\frac{e^x}{2}-\frac{\sqrt{\sqrt{2}+2}}{4}-\frac{\sqrt{2-\sqrt{2}}\,1i}{4}\right)\left(\frac{\sqrt{\sqrt{2}+2}}{32}+\frac{\sqrt{2-\sqrt{2}}\,1i}{32}\right) - \ln\left(\frac{\sqrt{\sqrt{2}+2}}{4}-\frac{e^x}{2}+\frac{\sqrt{2-\sqrt{2}}\,1i}{4}\right)\left(\frac{\sqrt{\sqrt{2}+2}}{32}+\frac{\sqrt{2-\sqrt{2}}\,1i}{32}\right) + \ln\left(\frac{\sqrt{2-\sqrt{2}}}{4}-\frac{e^x}{2}-\frac{\sqrt{\sqrt{2}+2}\,1i}{4}\right)\left(-\frac{\sqrt{2-\sqrt{2}}}{32}+\frac{\sqrt{\sqrt{2}+2}\,1i}{32}\right) - \ln\left(-\frac{e^x}{2}-\frac{\sqrt{2-\sqrt{2}}}{4}+\frac{\sqrt{\sqrt{2}+2}\,1i}{4}\right)\left(-\frac{\sqrt{2-\sqrt{2}}}{32}+\frac{\sqrt{\sqrt{2}+2}\,1i}{32}\right) + \sqrt{2}\,\ln\left(-\frac{e^x}{2}+\sqrt{2}\left(\frac{\sqrt{\sqrt{2}+2}}{32}+\frac{\sqrt{2-\sqrt{2}}\,1i}{32}\right)(-4-4i)\right)\left(\frac{\sqrt{\sqrt{2}+2}}{32}+\frac{\sqrt{2-\sqrt{2}}\,1i}{32}\right)\left(\frac{1}{2}+\frac{1}{2}i\right) + \sqrt{2}\,\ln\left(-\frac{e^x}{2}+\sqrt{2}\left(\frac{\sqrt{\sqrt{2}+2}}{32}+\frac{\sqrt{2-\sqrt{2}}\,1i}{32}\right)(-4+4i)\right)\left(\frac{\sqrt{\sqrt{2}+2}}{32}+\frac{\sqrt{2-\sqrt{2}}\,1i}{32}\right)\left(\frac{1}{2}-\frac{1}{2}i\right) + \sqrt{2}\,\ln\left(-\frac{e^x}{2}+\sqrt{2}\left(\frac{\sqrt{\sqrt{2}+2}}{32}+\frac{\sqrt{2-\sqrt{2}}\,1i}{32}\right)(4-4i)\right)\left(\frac{\sqrt{\sqrt{2}+2}}{32}+\frac{\sqrt{2-\sqrt{2}}\,1i}{32}\right)\left(-\frac{1}{2}+\frac{1}{2}i\right) + \sqrt{2}\,\ln\left(-\frac{e^x}{2}+\sqrt{2}\left(\frac{\sqrt{\sqrt{2}+2}}{32}+\frac{\sqrt{2-\sqrt{2}}\,1i}{32}\right)(4+4i)\right)\left(\frac{\sqrt{\sqrt{2}+2}}{32}+\frac{\sqrt{2-\sqrt{2}}\,1i}{32}\right)\left(-\frac{1}{2}-\frac{1}{2}i\right)$$

input
```
int(exp(x)/cosh(4*x)^2,x)
```

output
```
log(- exp(x)/2 - (2^(1/2) + 2)^(1/2)/4 - ((2 - 2^(1/2))^(1/2)*1i)/4)*((2^(
1/2) + 2)^(1/2)/32 + ((2 - 2^(1/2))^(1/2)*1i)/32) - exp(x)/(2*(exp(8*x) +
1)) - log((2^(1/2) + 2)^(1/2)/4 - exp(x)/2 + ((2 - 2^(1/2))^(1/2)*1i)/4)*(
(2^(1/2) + 2)^(1/2)/32 + ((2 - 2^(1/2))^(1/2)*1i)/32) + log((2 - 2^(1/2))^
(1/2)/4 - ((2^(1/2) + 2)^(1/2)*1i)/4 - exp(x)/2)*(((2^(1/2) + 2)^(1/2)*1i)
/32 - (2 - 2^(1/2))^(1/2)/32) - log(((2^(1/2) + 2)^(1/2)*1i)/4 - exp(x)/2
- (2 - 2^(1/2))^(1/2)/4)*(((2^(1/2) + 2)^(1/2)*1i)/32 - (2 - 2^(1/2))^(1/2
)/32) + 2^(1/2)*log(- exp(x)/2 - 2^(1/2)*((2^(1/2) + 2)^(1/2)/32 + ((2 - 2
^(1/2))^(1/2)*1i)/32)*(4 + 4i))*((2^(1/2) + 2)^(1/2)/32 + ((2 - 2^(1/2))^(
1/2)*1i)/32)*(1/2 + 1i/2) + 2^(1/2)*log(- exp(x)/2 - 2^(1/2)*((2^(1/2) + 2
)^(1/2)/32 + ((2 - 2^(1/2))^(1/2)*1i)/32)*(4 - 4i))*((2^(1/2) + 2)^(1/2)/3
2 + ((2 - 2^(1/2))^(1/2)*1i)/32)*(1/2 - 1i/2) - 2^(1/2)*log(2^(1/2)*((2^(1
/2) + 2)^(1/2)/32 + ((2 - 2^(1/2))^(1/2)*1i)/32)*(4 - 4i) - exp(x)/2)*((2^
(1/2) + 2)^(1/2)/32 + ((2 - 2^(1/2))^(1/2)*1i)/32)*(1/2 - 1i/2) - 2^(1/2)*
log(2^(1/2)*((2^(1/2) + 2)^(1/2)/32 + ((2 - 2^(1/2))^(1/2)*1i)/32)*(4 + 4i
) - exp(x)/2)*((2^(1/2) + 2)^(1/2)/32 + ((2 - 2^(1/2))^(1/2)*1i)/32)*(1/2
+ 1i/2)
```

3.284. $\int e^x \text{sech}^2(4x)\,dx$

3.285 $\int F^{c(a+bx)} \cosh^3(d+ex) \, dx$

3.285.1 Optimal result

Integrand size = 18, antiderivative size = 202

$$\int F^{c(a+bx)} \cosh^3(d+ex) \, dx = -\frac{bcF^{c(a+bx)} \cosh^3(d+ex) \log(F)}{9e^2 - b^2c^2 \log^2(F)} - \frac{6bce^2 F^{c(a+bx)} \cosh(d+ex) \log(F)}{9e^4 - 10b^2c^2e^2 \log^2(F) + b^4c^4 \log^4(F)} + \frac{3eF^{c(a+bx)} \cosh^2(d+ex) \sinh(d+ex)}{9e^2 - b^2c^2 \log^2(F)} + \frac{6e^3 F^{c(a+bx)} \sinh(d+ex)}{9e^4 - 10b^2c^2e^2 \log^2(F) + b^4c^4 \log^4(F)}$$

output
```
-b*c*F^(c*(b*x+a))*cosh(e*x+d)^3*ln(F)/(9*e^2-b^2*c^2*ln(F)^2)-6*b*c*e^2*F
^(c*(b*x+a))*cosh(e*x+d)*ln(F)/(9*e^4-10*b^2*c^2*e^2*ln(F)^2+b^4*c^4*ln(F)
^4)+3*e*F^(c*(b*x+a))*cosh(e*x+d)^2*sinh(e*x+d)/(9*e^2-b^2*c^2*ln(F)^2)+6*
e^3*F^(c*(b*x+a))*sinh(e*x+d)/(9*e^4-10*b^2*c^2*e^2*ln(F)^2+b^4*c^4*ln(F)^
4)
```

3.285.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.79

$$\int F^{c(a+bx)}\cosh^3(d+ex)\,dx$$
$$=\frac{F^{c(a+bx)}\left(3\cosh(d+ex)\left(-9bce^2\log(F)+b^3c^3\log^3(F)\right)+\cosh(3(d+ex))\left(-bce^2\log(F)+b^3c^3\log^3(F)\right)\right.}{4\left(9e^4-10b^2c^2e^2\log^2(F)+b^4c^4\log\right.}$$

input
```
Integrate[F^(c*(a + b*x))*Cosh[d + e*x]^3,x]
```

output
```
(F^(c*(a + b*x))*(3*Cosh[d + e*x]*(-9*b*c*e^2*Log[F] + b^3*c^3*Log[F]^3) +
 Cosh[3*(d + e*x)]*(-(b*c*e^2*Log[F]) + b^3*c^3*Log[F]^3) + 6*e*(5*e^2 - b
^2*c^2*Log[F]^2 + Cosh[2*(d + e*x)]*(e^2 - b^2*c^2*Log[F]^2))*Sinh[d + e*x
]))/(4*(9*e^4 - 10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4))
```

3.285.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6000, 5998}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3(d+ex)F^{c(a+bx)}\,dx$$
$$\Big\downarrow \text{6000}$$
$$\frac{6e^2\int F^{c(a+bx)}\cosh(d+ex)dx}{9e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\cosh^3(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)}+\frac{3e\sinh(d+ex)\cosh^2(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)}$$
$$\Big\downarrow \text{5998}$$
$$-\frac{bc\log(F)\cosh^3(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)}+\frac{3e\sinh(d+ex)\cosh^2(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)}+\frac{6e^2\left(\frac{e\sinh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\cosh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}\right)}{9e^2-b^2c^2\log^2(F)}$$

input
```
Int[F^(c*(a + b*x))*Cosh[d + e*x]^3,x]
```

output
```
-((b*c*F^(c*(a + b*x))*Cosh[d + e*x]^3*Log[F])/(9*e^2 - b^2*c^2*Log[F]^2))
 + (3*e*F^(c*(a + b*x))*Cosh[d + e*x]^2*Sinh[d + e*x])/(9*e^2 - b^2*c^2*Lo
g[F]^2) + (6*e^2*(-((b*c*F^(c*(a + b*x))*Cosh[d + e*x]*Log[F])/(e^2 - b^2*
c^2*Log[F]^2)) + (e*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2
)))/(9*e^2 - b^2*c^2*Log[F]^2)
```

3.285.3.1 Defintions of rubi rules used

rule 5998
```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

rule 6000
```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]^n/(e^2*n^2 - b^2*c
^2*Log[F]^2)), x] + (Simp[e*n*F^(c*(a + b*x))*Sinh[d + e*x]*(Cosh[d + e*x]^
(n - 1)/(e^2*n^2 - b^2*c^2*Log[F]^2)), x] + Simp[n*(n - 1)*(e^2/(e^2*n^2 -
b^2*c^2*Log[F]^2))   Int[F^(c*(a + b*x))*Cosh[d + e*x]^(n - 2), x], x]) /;
FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n
, 1]
```

3.285.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.73

method	result
parallelrisch	$\frac{F^{c(bx+a)}\left(\left(\ln(F)^3 b^3 c^3-\ln(F) bc\, e^2\right)\cosh(3ex+3d)+\left(-3\ln(F)^2 b^2 c^2 e+3e^3\right)\sinh(3ex+3d)-3(bc\ln(F)-3e)(bc\ln(F)+3e)(-\ln}{4b^4c^4\ln(F)^4-40b^2c^2e^2\ln(F)^2+36e^4}$
risch	$\frac{\left(\ln(F)^3 b^3 c^3 e^{6ex+6d}+3\ln(F)^3 b^3 c^3 e^{4ex+4d}-3\ln(F)^2 b^2 c^2 e\, e^{6ex+6d}+3\ln(F)^3 b^3 c^3 e^{2ex+2d}-3\ln(F)^2 b^2 c^2 e\, e^{4ex+4d}-\ln(F) bc\, e^2\right.}{}$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
F^(c*(b*x+a))*((ln(F)^3*b^3*c^3-ln(F)*b*c*e^2)*cosh(3*e*x+3*d)+(-3*ln(F)^2
*b^2*c^2*e+3*e^3)*sinh(3*e*x+3*d)-3*(b*c*ln(F)-3*e)*(b*c*ln(F)+3*e)*(-ln(F
)*cosh(e*x+d)*b*c+e*sinh(e*x+d)))/(4*b^4*c^4*ln(F)^4-40*b^2*c^2*e^2*ln(F)^
2+36*e^4)
```

3.285.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2218 vs. $2(199) = 398$.

Time = 0.34 (sec) , antiderivative size = 2218, normalized size of antiderivative = 10.98

$$\int F^{c(a+bx)} \cosh^3(d+ex)\, dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^3,x, algorithm="fricas")
```

output
```
1/8*((3*e^3*cosh(e*x + d)^6 + 27*e^3*cosh(e*x + d)^4 + (b^3*c^3*log(F)^3 -
 3*b^2*c^2*e*log(F)^2 - b*c*e^2*log(F) + 3*e^3)*sinh(e*x + d)^6 + 6*(b^3*c
^3*cosh(e*x + d)*log(F)^3 - 3*b^2*c^2*e*cosh(e*x + d)*log(F)^2 - b*c*e^2*c
osh(e*x + d)*log(F) + 3*e^3*cosh(e*x + d))*sinh(e*x + d)^5 - 27*e^3*cosh(e
*x + d)^2 + 3*(15*e^3*cosh(e*x + d)^2 + (5*b^3*c^3*cosh(e*x + d)^2 + b^3*c
^3)*log(F)^3 + 9*e^3 - (15*b^2*c^2*e*cosh(e*x + d)^2 + b^2*c^2*e)*log(F)^2
 - (5*b*c*e^2*cosh(e*x + d)^2 + 9*b*c*e^2)*log(F))*sinh(e*x + d)^4 + (b^3*
c^3*cosh(e*x + d)^6 + 3*b^3*c^3*cosh(e*x + d)^4 + 3*b^3*c^3*cosh(e*x + d)^
2 + b^3*c^3)*log(F)^3 + 4*(15*e^3*cosh(e*x + d)^3 + 27*e^3*cosh(e*x + d) +
 (5*b^3*c^3*cosh(e*x + d)^3 + 3*b^3*c^3*cosh(e*x + d))*log(F)^3 - 3*(5*b^2
*c^2*e*cosh(e*x + d)^3 + b^2*c^2*e*cosh(e*x + d))*log(F)^2 - (5*b*c*e^2*co
sh(e*x + d)^3 + 27*b*c*e^2*cosh(e*x + d))*log(F))*sinh(e*x + d)^3 - 3*e^3
- 3*(b^2*c^2*e*cosh(e*x + d)^6 + b^2*c^2*e*cosh(e*x + d)^4 - b^2*c^2*e*cos
h(e*x + d)^2 - b^2*c^2*e)*log(F)^2 + 3*(15*e^3*cosh(e*x + d)^4 + 54*e^3*co
sh(e*x + d)^2 + (5*b^3*c^3*cosh(e*x + d)^4 + 6*b^3*c^3*cosh(e*x + d)^2 + b
^3*c^3)*log(F)^3 - 9*e^3 - (15*b^2*c^2*e*cosh(e*x + d)^4 + 6*b^2*c^2*e*cos
h(e*x + d)^2 - b^2*c^2*e)*log(F)^2 - (5*b*c*e^2*cosh(e*x + d)^4 + 54*b*c*e
^2*cosh(e*x + d)^2 + 9*b*c*e^2)*log(F))*sinh(e*x + d)^2 - (b*c*e^2*cosh(e*
x + d)^6 + 27*b*c*e^2*cosh(e*x + d)^4 + 27*b*c*e^2*cosh(e*x + d)^2 + b*c*e
^2)*log(F) + 6*(3*e^3*cosh(e*x + d)^5 + 18*e^3*cosh(e*x + d)^3 - 9*e^3*...
```

3.285.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1658 vs. $2(199) = 398$.

Time = 15.18 (sec) , antiderivative size = 1658, normalized size of antiderivative = 8.21

$$\int F^{c(a+bx)}\cosh^3(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F**(c*(b*x+a))*cosh(e*x+d)**3,x)
```

output
```
Piecewise((x*cosh(d)**3, Eq(F, 1) & Eq(e, 0)), (F**(a*c)*x*cosh(d)**3, Eq(
b, 0) & Eq(e, 0)), (x*cosh(d)**3, Eq(c, 0) & Eq(e, 0)), (3*F**(a*c + b*c*x
)*x*sinh(b*c*x*log(F) - d)**3/8 - 3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F) -
 d)**2*cosh(b*c*x*log(F) - d)/8 - 3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F) -
 d)*cosh(b*c*x*log(F) - d)**2/8 + 3*F**(a*c + b*c*x)*x*cosh(b*c*x*log(F) -
 d)**3/8 - 5*F**(a*c + b*c*x)*sinh(b*c*x*log(F) - d)**3/(8*b*c*log(F)) + F
**(a*c + b*c*x)*sinh(b*c*x*log(F) - d)**2*cosh(b*c*x*log(F) - d)/(4*b*c*lo
g(F)) + F**(a*c + b*c*x)*sinh(b*c*x*log(F) - d)*cosh(b*c*x*log(F) - d)**2/
(b*c*log(F)) - 3*F**(a*c + b*c*x)*cosh(b*c*x*log(F) - d)**3/(8*b*c*log(F))
, Eq(e, -b*c*log(F))), (-F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 - d)**3/8
+ 3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 - d)**2*cosh(b*c*x*log(F)/3 - d
)/8 - 3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 - d)*cosh(b*c*x*log(F)/3 -
d)**2/8 + F**(a*c + b*c*x)*x*cosh(b*c*x*log(F)/3 - d)**3/8 + 11*F**(a*c +
b*c*x)*sinh(b*c*x*log(F)/3 - d)**3/(8*b*c*log(F)) - 15*F**(a*c + b*c*x)*si
nh(b*c*x*log(F)/3 - d)**2*cosh(b*c*x*log(F)/3 - d)/(4*b*c*log(F)) + 3*F**(
a*c + b*c*x)*sinh(b*c*x*log(F)/3 - d)*cosh(b*c*x*log(F)/3 - d)**2/(b*c*log
(F)) - F**(a*c + b*c*x)*cosh(b*c*x*log(F)/3 - d)**3/(8*b*c*log(F)), Eq(e,
-b*c*log(F)/3)), (-F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 + d)**3/8 + 3*F*
*(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 + d)**2*cosh(b*c*x*log(F)/3 + d)/8 -
3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 + d)*cosh(b*c*x*log(F)/3 + d)*...
```

3.285.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.66

$$\int F^{c(a+bx)}\cosh^3(d+ex)\,dx = \frac{F^{ac}e^{(bcx\log(F)+3\,ex+3\,d)}}{8\,(bc\log(F)+3\,e)} + \frac{3\,F^{ac}e^{(bcx\log(F)+ex+d)}}{8\,(bc\log(F)+e)} + \frac{3\,F^{ac}e^{(bcx\log(F)-ex)}}{8\,(bce^{d}\log(F)-ee^{d})} + \frac{F^{ac}e^{(bcx\log(F)-3\,ex)}}{8\,(bce^{(3\,d)}\log(F)-3\,ee^{(3\,d)})}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^3,x, algorithm="maxima")
```

output
```
1/8*F^(a*c)*e^(b*c*x*log(F) + 3*e*x + 3*d)/(b*c*log(F) + 3*e) + 3/8*F^(a*c
)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + e) + 3/8*F^(a*c)*e^(b*c*x*log(F
) - e*x)/(b*c*e^d*log(F) - e*e^d) + 1/8*F^(a*c)*e^(b*c*x*log(F) - 3*e*x)/(
b*c*e^(3*d)*log(F) - 3*e*e^(3*d))
```

3.285.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 1211, normalized size of antiderivative = 6.00

$$\int F^{c(a+bx)} \cosh^3(d+ex) \, dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^3,x, algorithm="giac")
```

output
```
1/4*(2*(b*c*log(abs(F)) + 3*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1
/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(ab
s(F)) + 3*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*
pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4
*(b*c*log(abs(F)) + 3*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 3*e)*
x + 3*d) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*s
gn(F) - 1/2*I*pi*a*c)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F))
 + 48*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn
(F) + 1/2*I*pi*a*c)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F))
+ 48*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 3*e)*x + 3*d) + 3/4*(2*(b
*c*log(abs(F)) + e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*s
gn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) + e)^
2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/
2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs
(F)) + e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + 3*I*(I*e
^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*
a*c)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) + 16*e) - I*e^(-
1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*
c)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F)) + 16*e))*e^(a*c*l
og(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + 3/4*(2*(b*c*log(abs(F)) - e...
```

3.285.9 Mupad [B] (verification not implemented)

Time = 2.66 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.76

$$\int F^{c(a+bx)}\cosh^3(d+ex)\,dx$$
$$=\frac{F^{a\,c+b\,c\,x}\,\left(6\,e^3\,\sinh(d+e\,x)+3\,e^3\,\cosh(d+e\,x)^2\,\sinh(d+e\,x)+b^3\,c^3\,\cosh(d+e\,x)^3\,\ln\left(F\right)^3-b\,c\,e^2\,\cosh(d\right.}{b^4\,c^4\,\ln\left(F\right)^4-10\,b^2\,c^2\,e^2\,\mathrm{l}}$$

input
```
int(F^(c*(a + b*x))*cosh(d + e*x)^3,x)
```

output
```
(F^(a*c + b*c*x)*(6*e^3*sinh(d + e*x) + 3*e^3*cosh(d + e*x)^2*sinh(d + e*x
) + b^3*c^3*cosh(d + e*x)^3*log(F)^3 - b*c*e^2*cosh(d + e*x)^3*log(F) - 6*
b*c*e^2*cosh(d + e*x)*log(F) - 3*b^2*c^2*e*cosh(d + e*x)^2*sinh(d + e*x)*l
og(F)^2))/(9*e^4 + b^4*c^4*log(F)^4 - 10*b^2*c^2*e^2*log(F)^2)
```

3.286 $\int F^{c(a+bx)} \cosh^2(d+ex)\,dx$

3.286.1 Optimal result

Integrand size = 18, antiderivative size = 132

$$\int F^{c(a+bx)} \cosh^2(d+ex)\,dx = \frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)} - \frac{bcF^{c(a+bx)}\cosh^2(d+ex)\log(F)}{4e^2-b^2c^2\log^2(F)} + \frac{2eF^{c(a+bx)}\cosh(d+ex)\sinh(d+ex)}{4e^2-b^2c^2\log^2(F)}$$

output
```
2*e^2*F^(c*(b*x+a))/b/c/ln(F)/(4*e^2-b^2*c^2*ln(F)^2)-b*c*F^(c*(b*x+a))*co
sh(e*x+d)^2*ln(F)/(4*e^2-b^2*c^2*ln(F)^2)+2*e*F^(c*(b*x+a))*cosh(e*x+d)*si
nh(e*x+d)/(4*e^2-b^2*c^2*ln(F)^2)
```

3.286.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.64

$$\int F^{c(a+bx)} \cosh^2(d+ex)\,dx = \frac{F^{c(a+bx)}\left(-4e^2+b^2c^2\log^2(F)+b^2c^2\cosh(2(d+ex))\log^2(F)-2bce\log(F)\sinh(2(d+ex))\right)}{-8bce^2\log(F)+2b^3c^3\log^3(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Cosh[d + e*x]^2,x]
```

output
```
(F^(c*(a + b*x))*(-4*e^2 + b^2*c^2*Log[F]^2 + b^2*c^2*Cosh[2*(d + e*x)]*Lo
g[F]^2 - 2*b*c*e*Log[F]*Sinh[2*(d + e*x)]))/(-8*b*c*e^2*Log[F] + 2*b^3*c^3
*Log[F]^3)
```

3.286.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6000, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6000}$$

$$\frac{2e^2\int F^{c(a+bx)}dx}{4e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\cosh^2(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}+\frac{2e\sinh(d+ex)\cosh(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}$$

$$\Big\downarrow \text{2624}$$

$$-\frac{bc\log(F)\cosh^2(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}+\frac{2e\sinh(d+ex)\cosh(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}+\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)}$$

input
```
Int[F^(c*(a + b*x))*Cosh[d + e*x]^2,x]
```

output
```
(2*e^2*F^(c*(a + b*x)))/(b*c*Log[F]*(4*e^2 - b^2*c^2*Log[F]^2)) - (b*c*F^(
c*(a + b*x))*Cosh[d + e*x]^2*Log[F])/(4*e^2 - b^2*c^2*Log[F]^2) + (2*e*F^(
c*(a + b*x))*Cosh[d + e*x]*Sinh[d + e*x])/(4*e^2 - b^2*c^2*Log[F]^2)
```

3.286.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 6000
```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]^n/(e^2*n^2 - b^2*c
^2*Log[F]^2)), x] + (Simp[e*n*F^(c*(a + b*x))*Sinh[d + e*x]*(Cosh[d + e*x]^
(n - 1)/(e^2*n^2 - b^2*c^2*Log[F]^2)), x] + Simp[n*(n - 1)*(e^2/(e^2*n^2 -
b^2*c^2*Log[F]^2))   Int[F^(c*(a + b*x))*Cosh[d + e*x]^(n - 2), x], x]) /;
FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n
, 1]
```

3.286.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.68

method	result
parallelrisch	$-\frac{2F^{c(bx+a)}\left(-\frac{c^2b^2\ln(F)^2\cosh(2ex+2d)}{2}-\frac{b^2c^2\ln(F)^2}{2}+\ln(F)bce\sinh(2ex+2d)+2e^2\right)}{2\ln(F)^3b^3c^3-8\ln(F)bce^2}$
risch	$\frac{\left(\ln(F)^2b^2c^2\mathrm{e}^{4ex+4d}+2\ln(F)^2b^2c^2\mathrm{e}^{2ex+2d}-2\ln(F)bce\,\mathrm{e}^{4ex+4d}+b^2c^2\ln(F)^2+2bc\ln(F)e-8e^2\mathrm{e}^{2ex+2d}\right)\mathrm{e}^{-2ex-2d}F^{c(bx+a)}}{4bc\ln(F)(bc\ln(F)-2e)(bc\ln(F)+2e)}$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*F^(c*(b*x+a))*(-1/2*c^2*b^2*ln(F)^2*cosh(2*e*x+2*d)-1/2*b^2*c^2*ln(F)^2
+ln(F)*b*c*e*sinh(2*e*x+2*d)+2*e^2)/(2*ln(F)^3*b^3*c^3-8*ln(F)*b*c*e^2)
```

3.286.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 699 vs. 2(128) = 256.

Time = 0.27 (sec) , antiderivative size = 699, normalized size of antiderivative = 5.30

$$\int F^{c(a+bx)}\cosh^2(d+ex)\,dx$$
$$= \frac{\left(\left(b^2c^2\log\left(F\right)^2-2\,bce\log\left(F\right)\right)\sinh\left(ex+d\right)^4-8\,e^2\cosh\left(ex+d\right)^2+4\left(b^2c^2\cosh\left(ex+d\right)\log\left(F\right)^2-2\,bc\right.}{}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^2,x, algorithm="fricas")
```

output
```
1/4*(((b^2*c^2*log(F)^2 - 2*b*c*e*log(F))*sinh(e*x + d)^4 - 8*e^2*cosh(e*x
 + d)^2 + 4*(b^2*c^2*cosh(e*x + d)*log(F)^2 - 2*b*c*e*cosh(e*x + d)*log(F)
)*sinh(e*x + d)^3 + (b^2*c^2*cosh(e*x + d)^4 + 2*b^2*c^2*cosh(e*x + d)^2 +
 b^2*c^2)*log(F)^2 - 2*(6*b*c*e*cosh(e*x + d)^2*log(F) - (3*b^2*c^2*cosh(e
*x + d)^2 + b^2*c^2)*log(F)^2 + 4*e^2)*sinh(e*x + d)^2 - 2*(b*c*e*cosh(e*x
 + d)^4 - b*c*e)*log(F) - 4*(2*b*c*e*cosh(e*x + d)^3*log(F) + 4*e^2*cosh(e
*x + d) - (b^2*c^2*cosh(e*x + d)^3 + b^2*c^2*cosh(e*x + d))*log(F)^2)*sinh
(e*x + d))*cosh((b*c*x + a*c)*log(F)) + ((b^2*c^2*log(F)^2 - 2*b*c*e*log(F
))*sinh(e*x + d)^4 - 8*e^2*cosh(e*x + d)^2 + 4*(b^2*c^2*cosh(e*x + d)*log(
F)^2 - 2*b*c*e*cosh(e*x + d)*log(F))*sinh(e*x + d)^3 + (b^2*c^2*cosh(e*x +
 d)^4 + 2*b^2*c^2*cosh(e*x + d)^2 + b^2*c^2)*log(F)^2 - 2*(6*b*c*e*cosh(e*
x + d)^2*log(F) - (3*b^2*c^2*cosh(e*x + d)^2 + b^2*c^2)*log(F)^2 + 4*e^2)*
sinh(e*x + d)^2 - 2*(b*c*e*cosh(e*x + d)^4 - b*c*e)*log(F) - 4*(2*b*c*e*co
sh(e*x + d)^3*log(F) + 4*e^2*cosh(e*x + d) - (b^2*c^2*cosh(e*x + d)^3 + b^
2*c^2*cosh(e*x + d))*log(F)^2)*sinh(e*x + d))*sinh((b*c*x + a*c)*log(F)))/
(b^3*c^3*cosh(e*x + d)^2*log(F)^3 - 4*b*c*e^2*cosh(e*x + d)^2*log(F) + (b^
3*c^3*log(F)^3 - 4*b*c*e^2*log(F))*sinh(e*x + d)^2 + 2*(b^3*c^3*cosh(e*x +
 d)*log(F)^3 - 4*b*c*e^2*cosh(e*x + d)*log(F))*sinh(e*x + d))
```

3.286.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 706 vs. $2(119) = 238$.

Time = 1.13 (sec) , antiderivative size = 706, normalized size of antiderivative = 5.35

$$\int F^{c(a+bx)}\cosh^2(d+ex)\,dx$$

$$= \begin{cases} x\cosh^2(d) \\ -\frac{x\sinh^2(d+ex)}{2} + \frac{x\cosh^2(d+ex)}{2} + \frac{\sinh(d+ex)\cosh(d+ex)}{2e} \\ F^{ac}\left(-\frac{x\sinh^2(d+ex)}{2} + \frac{x\cosh^2(d+ex)}{2} + \frac{\sinh(d+ex)\cosh(d+ex)}{2e}\right) \\ -\frac{x\sinh^2(d+ex)}{2} + \frac{x\cosh^2(d+ex)}{2} + \frac{\sinh(d+ex)\cosh(d+ex)}{2e} \\ \frac{F^{ac+bcx}x\sinh^2\left(\frac{bcx\log(F)}{2}-d\right)}{4} - \frac{F^{ac+bcx}x\sinh\left(\frac{bcx\log(F)}{2}-d\right)\cosh\left(\frac{bcx\log(F)}{2}-d\right)}{2} + \frac{F^{ac+bcx}x\cosh^2\left(\frac{bcx\log(F)}{2}-d\right)}{4} - \frac{F^{ac+bcx}\sin}{} \\ \frac{F^{ac+bcx}x\sinh^2\left(\frac{bcx\log(F)}{2}+d\right)}{4} - \frac{F^{ac+bcx}x\sinh\left(\frac{bcx\log(F)}{2}+d\right)\cosh\left(\frac{bcx\log(F)}{2}+d\right)}{2} + \frac{F^{ac+bcx}x\cosh^2\left(\frac{bcx\log(F)}{2}+d\right)}{4} - \frac{F^{ac+bcx}\sin}{} \\ \frac{F^{ac+bcx}b^2c^2\log(F)^2\cosh^2(d+ex)}{b^3c^3\log(F)^3-4bce^2\log(F)} - \frac{2F^{ac+bcx}bce\log(F)\sinh(d+ex)\cosh(d+ex)}{b^3c^3\log(F)^3-4bce^2\log(F)} + \frac{2F^{ac+bcx}e^2\sinh^2(d+ex)}{b^3c^3\log(F)^3-4bce^2\log(F)} - \frac{2F^{ac+bcx}e^2\cosh^2}{b^3c^3\log(F)^3-4bce^2} \end{cases}$$

input
```
integrate(F**(c*(b*x+a))*cosh(e*x+d)**2,x)
```

output
```
Piecewise((x*cosh(d)**2, Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(e, 0)), (-x*s
inh(d + e*x)**2/2 + x*cosh(d + e*x)**2/2 + sinh(d + e*x)*cosh(d + e*x)/(2*
e), Eq(F, 1)), (F**(a*c)*(-x*sinh(d + e*x)**2/2 + x*cosh(d + e*x)**2/2 + s
inh(d + e*x)*cosh(d + e*x)/(2*e)), Eq(b, 0)), (-x*sinh(d + e*x)**2/2 + x*c
osh(d + e*x)**2/2 + sinh(d + e*x)*cosh(d + e*x)/(2*e), Eq(c, 0)), (F**(a*c
 + b*c*x)*x*sinh(b*c*x*log(F)/2 - d)**2/4 - F**(a*c + b*c*x)*x*sinh(b*c*x*
log(F)/2 - d)*cosh(b*c*x*log(F)/2 - d)/2 + F**(a*c + b*c*x)*x*cosh(b*c*x*l
og(F)/2 - d)**2/4 - F**(a*c + b*c*x)*sinh(b*c*x*log(F)/2 - d)*cosh(b*c*x*l
og(F)/2 - d)/(2*b*c*log(F)) + F**(a*c + b*c*x)*cosh(b*c*x*log(F)/2 - d)**2
/(b*c*log(F)), Eq(e, -b*c*log(F)/2)), (F**(a*c + b*c*x)*x*sinh(b*c*x*log(F
)/2 + d)**2/4 - F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/2 + d)*cosh(b*c*x*log
(F)/2 + d)/2 + F**(a*c + b*c*x)*x*cosh(b*c*x*log(F)/2 + d)**2/4 - F**(a*c
+ b*c*x)*sinh(b*c*x*log(F)/2 + d)*cosh(b*c*x*log(F)/2 + d)/(2*b*c*log(F))
+ F**(a*c + b*c*x)*cosh(b*c*x*log(F)/2 + d)**2/(b*c*log(F)), Eq(e, b*c*log
(F)/2)), (F**(a*c + b*c*x)*b**2*c**2*log(F)**2*cosh(d + e*x)**2/(b**3*c**3
*log(F)**3 - 4*b*c*e**2*log(F)) - 2*F**(a*c + b*c*x)*b*c*e*log(F)*sinh(d +
 e*x)*cosh(d + e*x)/(b**3*c**3*log(F)**3 - 4*b*c*e**2*log(F)) + 2*F**(a*c
+ b*c*x)*e**2*sinh(d + e*x)**2/(b**3*c**3*log(F)**3 - 4*b*c*e**2*log(F)) -
 2*F**(a*c + b*c*x)*e**2*cosh(d + e*x)**2/(b**3*c**3*log(F)**3 - 4*b*c*e**
2*log(F)), True))
```

3.286.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.71

$$\int F^{c(a+bx)} \cosh^2(d+ex)\,dx = \frac{F^{ac}e^{(bcx\log(F)+2\,ex+2\,d)}}{4\,(bc\log{(F)}+2\,e)} + \frac{F^{ac}e^{(bcx\log(F)-2\,ex)}}{4\,(bce^{(2\,d)}\log{(F)}-2\,ee^{(2\,d)})} + \frac{F^{bcx+ac}}{2\,bc\log{(F)}}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^2,x, algorithm="maxima")
```

output
```
1/4*F^(a*c)*e^(b*c*x*log(F) + 2*e*x + 2*d)/(b*c*log(F) + 2*e) + 1/4*F^(a*c
)*e^(b*c*x*log(F) - 2*e*x)/(b*c*e^(2*d)*log(F) - 2*e*e^(2*d)) + 1/2*F^(b*c
*x + a*c)/(b*c*log(F))
```

3.286.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 889, normalized size of antiderivative = 6.73

$$\int F^{c(a+bx)} \cosh^2(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^2,x, algorithm="giac")
```

output
```
(2*b*c*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*p
i*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2)
- (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*p
i*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*
b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*(I*e^(1/2*I*pi*b*c*x*
sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(2*I*pi*b*c*
sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F))) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1
/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-2*I*pi*b*c*sgn(F) +
2*I*pi*b*c + 4*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) +
 1/2*(2*(b*c*log(abs(F)) + 2*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x -
1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(a
bs(F)) + 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2
*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 +
4*(b*c*log(abs(F)) + 2*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 2*e)
*x + 2*d) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*
sgn(F) - 1/2*I*pi*a*c)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*b*c*log(abs(F))
 + 16*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn
(F) + 1/2*I*pi*a*c)/(-4*I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)) +
 16*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 2*e)*x + 2*d) + 1/2*(2*(b*
c*log(abs(F)) - 2*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a...
```

3.286.9 Mupad [B] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.76

$$\int F^{c(a+bx)} \cosh^2(d+ex)\,dx =$$
$$-\frac{2\,F^{a\,c+b\,c\,x}\,e^2-F^{a\,c+b\,c\,x}\,b^2\,c^2\,\cosh(d+e\,x)^2\,\ln\left(F\right)^2+2\,F^{a\,c+b\,c\,x}\,b\,c\,e\cosh(d+e\,x)\,\sinh(d+e\,x)\,\ln\left(F\right)}{b^3\,c^3\ln\left(F\right)^3-4\,b\,c\,e^2\,\ln\left(F\right)}$$

input
```
int(F^(c*(a + b*x))*cosh(d + e*x)^2,x)
```

output
```
-(2*F^(a*c + b*c*x)*e^2 - F^(a*c + b*c*x)*b^2*c^2*cosh(d + e*x)^2*log(F)^2
 + 2*F^(a*c + b*c*x)*b*c*e*cosh(d + e*x)*sinh(d + e*x)*log(F))/(b^3*c^3*lo
g(F)^3 - 4*b*c*e^2*log(F))
```

3.286. $\int F^{c(a+bx)} \cosh^2(d+ex)\,dx$

3.287 $\int F^{c(a+bx)} \cosh(d+ex)\,dx$

3.287.1 Optimal result

Integrand size = 16, antiderivative size = 75

$$\int F^{c(a+bx)} \cosh(d+ex)\,dx = -\frac{bcF^{c(a+bx)}\cosh(d+ex)\log(F)}{e^2-b^2c^2\log^2(F)}+\frac{eF^{c(a+bx)}\sinh(d+ex)}{e^2-b^2c^2\log^2(F)}$$

output
```
-b*c*F^(c*(b*x+a))*cosh(e*x+d)*ln(F)/(e^2-b^2*c^2*ln(F)^2)+e*F^(c*(b*x+a))
*sinh(e*x+d)/(e^2-b^2*c^2*ln(F)^2)
```

3.287.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.67

$$\int F^{c(a+bx)} \cosh(d+ex)\,dx = \frac{F^{c(a+bx)}(-bc\cosh(d+ex)\log(F)+e\sinh(d+ex))}{(e-bc\log(F))(e+bc\log(F))}$$

input
```
Integrate[F^(c*(a + b*x))*Cosh[d + e*x],x]
```

output
```
(F^(c*(a + b*x))*(-(b*c*Cosh[d + e*x]*Log[F]) + e*Sinh[d + e*x]))/((e - b*
c*Log[F])*(e + b*c*Log[F]))
```

3.287.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {5998}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{ 5998}$$

$$\frac{e\sinh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\cosh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}$$

input
```
Int[F^(c*(a + b*x))*Cosh[d + e*x],x]
```

output
```
-((b*c*F^(c*(a + b*x))*Cosh[d + e*x]*Log[F])/(e^2 - b^2*c^2*Log[F]^2)) + (
e*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
```

3.287.3.1 Defintions of rubi rules used

rule 5998
```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

3.287.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.69

method	result	size
parallelrisch	$-\frac{F^{c(bx+a)}(-\ln(F)\cosh(ex+d)bc+e\sinh(ex+d))}{b^2c^2\ln(F)^2-e^2}$	52
risch	$\frac{(\ln(F)bc\,{\rm e}^{2ex+2d}+bc\ln(F)-e\,{\rm e}^{2ex+2d}+e){\rm e}^{-ex-d}F^{c(bx+a)}}{2(bc\ln(F)-e)(e+bc\ln(F))}$	74

input
```
int(F^(c*(b*x+a))*cosh(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-F^(c*(b*x+a))*(-ln(F)*cosh(e*x+d)*b*c+e*sinh(e*x+d))/(b^2*c^2*ln(F)^2-e^2
)
```

3.287.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(77) = 154$.

Time = 0.28 (sec) , antiderivative size = 246, normalized size of antiderivative = 3.28

$$\int F^{c(a+bx)}\cosh(d+ex)\,dx = -\frac{\left(e\cosh\left(ex+d\right)^2-\left(bc\log\left(F\right)-e\right)\sinh\left(ex+d\right)^2-\left(bc\cosh\left(ex+d\right)^2+bc\right)\log\left(F\right)-2\left(bc\cosh\left(ex+\right.\right.}{}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d),x, algorithm="fricas")
```

output
```
-1/2*((e*cosh(e*x + d)^2 - (b*c*log(F) - e)*sinh(e*x + d)^2 - (b*c*cosh(e*
x + d)^2 + b*c)*log(F) - 2*(b*c*cosh(e*x + d)*log(F) - e*cosh(e*x + d))*si
nh(e*x + d) - e)*cosh((b*c*x + a*c)*log(F)) + (e*cosh(e*x + d)^2 - (b*c*lo
g(F) - e)*sinh(e*x + d)^2 - (b*c*cosh(e*x + d)^2 + b*c)*log(F) - 2*(b*c*co
sh(e*x + d)*log(F) - e*cosh(e*x + d))*sinh(e*x + d) - e)*sinh((b*c*x + a*c
)*log(F)))/(b^2*c^2*cosh(e*x + d)*log(F)^2 - e^2*cosh(e*x + d) + (b^2*c^2*
log(F)^2 - e^2)*sinh(e*x + d))
```

3.287.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(68) = 136$.

Time = 0.62 (sec) , antiderivative size = 265, normalized size of antiderivative = 3.53

$$\int F^{c(a+bx)}\cosh(d+ex)\,dx = \begin{cases} x\cosh\left(d\right) & \text{for}\ F=1\wedge e=0\\ F^{ac}x\cosh\left(d\right) & \text{for}\ b=0\wedge e=0\\ x\cosh\left(d\right) & \text{for}\ c=0\wedge e=0\\ -\frac{F^{ac+bcx}x\sinh\left(bcx\log\left(F\right)-d\right)}{2}+\frac{F^{ac+bcx}x\cosh\left(bcx\log\left(F\right)-d\right)}{2}+\frac{F^{ac+bcx}\sinh\left(bcx\log\left(F\right)-d\right)}{2bc\log\left(F\right)} & \text{for}\ e=-bc\log\left(F\right)\\ -\frac{F^{ac+bcx}x\sinh\left(bcx\log\left(F\right)+d\right)}{2}+\frac{F^{ac+bcx}x\cosh\left(bcx\log\left(F\right)+d\right)}{2}+\frac{F^{ac+bcx}\sinh\left(bcx\log\left(F\right)+d\right)}{2bc\log\left(F\right)} & \text{for}\ e=bc\log\left(F\right)\\ \frac{F^{ac+bcx}bc\log\left(F\right)\cosh\left(d+ex\right)}{b^2c^2\log\left(F\right)^2-e^2}-\frac{F^{ac+bcx}e\sinh\left(d+ex\right)}{b^2c^2\log\left(F\right)^2-e^2} & \text{otherwise} \end{cases}$$

input
```
integrate(F**(c*(b*x+a))*cosh(e*x+d),x)
```

output
```
Piecewise((x*cosh(d), Eq(F, 1) & Eq(e, 0)), (F**(a*c)*x*cosh(d), Eq(b, 0)
& Eq(e, 0)), (x*cosh(d), Eq(c, 0) & Eq(e, 0)), (-F**(a*c + b*c*x)*x*sinh(b
*c*x*log(F) - d)/2 + F**(a*c + b*c*x)*x*cosh(b*c*x*log(F) - d)/2 + F**(a*c
 + b*c*x)*sinh(b*c*x*log(F) - d)/(2*b*c*log(F)), Eq(e, -b*c*log(F))), (-F*
*(a*c + b*c*x)*x*sinh(b*c*x*log(F) + d)/2 + F**(a*c + b*c*x)*x*cosh(b*c*x*
log(F) + d)/2 + F**(a*c + b*c*x)*sinh(b*c*x*log(F) + d)/(2*b*c*log(F)), Eq
(e, b*c*log(F))), (F**(a*c + b*c*x)*b*c*log(F)*cosh(d + e*x)/(b**2*c**2*lo
g(F)**2 - e**2) - F**(a*c + b*c*x)*e*sinh(d + e*x)/(b**2*c**2*log(F)**2 -
e**2), True))
```

3.287.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.84

$$\int F^{c(a+bx)} \cosh(d+ex)\, dx = \frac{F^{ac}e^{(bcx\log(F)+ex+d)}}{2\,(bc\log{(F)}+e)} + \frac{F^{ac}e^{(bcx\log(F)-ex)}}{2\,(bce^{d}\log{(F)}-ee^{d})}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d),x, algorithm="maxima")
```

output
```
1/2*F^(a*c)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + e) + 1/2*F^(a*c)*e^(b
*c*x*log(F) - e*x)/(b*c*e^d*log(F) - e*e^d)
```

3.287.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 597, normalized size of antiderivative = 7.96

$$\int F^{c(a+bx)} \cosh(d+ex)\, dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d),x, algorithm="giac")
```

output
```
(2*(b*c*log(abs(F)) + e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*
a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F))
+ e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x
 - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*lo
g(abs(F)) + e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + 1/2
*I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/
2*I*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F)) + 2*e) - I*e^(
-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a
*c)/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log(abs(F)) + 2*e))*e^(a*c*log(ab
s(F)) + (b*c*log(abs(F)) + e)*x + d) + (2*(b*c*log(abs(F)) - e)*cos(-1/2*p
i*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*s
gn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2) - (pi*b*c*sgn(F) - pi*b*c)*
sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/
((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2))*e^(a*c*log(abs(F
)) + (b*c*log(abs(F)) - e)*x - d) + 1/2*I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/
2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi
*b*c + 2*b*c*log(abs(F)) - 2*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b
*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(F) + I*pi*b*c +
2*b*c*log(abs(F)) - 2*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - e)*x - d
)
```

3.287.9 Mupad [B] (verification not implemented)

Time = 1.78 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

$$\int F^{c(a+bx)} \cosh(d+ex)\,dx = -\frac{F^{a\,c+b\,c\,x}\,\mathrm{e}^{-d-e\,x}\,\left(e-e\,\mathrm{e}^{2\,d+2\,e\,x}+b\,c\,\ln\left(F\right)+b\,c\,\mathrm{e}^{2\,d+2\,e\,x}\,\ln\left(F\right)\right)}{2\,\left(e^2-b^2\,c^2\,{\ln\left(F\right)}^2\right)}$$

input
```
int(F^(c*(a + b*x))*cosh(d + e*x),x)
```

output
```
-(F^(a*c + b*c*x)*exp(- d - e*x)*(e - e*exp(2*d + 2*e*x) + b*c*log(F) + b*
c*exp(2*d + 2*e*x)*log(F)))/(2*(e^2 - b^2*c^2*log(F)^2))
```

3.288 $\int F^{c(a+bx)}\text{sech}(d+ex)\,dx$

3.288.1 Optimal result

Integrand size = 16, antiderivative size = 68

$$\int F^{c(a+bx)}\text{sech}(d+ex)\,dx = \frac{2e^{d+ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right),-e^{2(d+ex)}\right)}{e+bc\log(F)}$$

output
```
2*exp(e*x+d)*F^(c*(b*x+a))*hypergeom([1, 1/2*(e+b*c*ln(F))/e],[3/2+1/2*b*c
*ln(F)/e],-exp(2*e*x+2*d))/(e+b*c*ln(F))
```

3.288.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.03

$$\int F^{c(a+bx)}\text{sech}(d+ex)\,dx = \frac{2e^{d+ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+\frac{bc\log(F)}{2e},\frac{3}{2}+\frac{bc\log(F)}{2e},-e^{2(d+ex)}\right)}{e+bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Sech[d + e*x],x]
```

output
```
(2*E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[1, 1/2 + (b*c*Log[F])/(2*
e), 3/2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(e + b*c*Log[F])
```

3.288.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {6015}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6015}$$

$$\frac{2e^{d+ex}F^{c(a+bx)}\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right),-e^{2(d+ex)}\right)}{bc\log(F)+e}$$

input
```
Int[F^(c*(a + b*x))*Sech[d + e*x],x]
```

output
```
(2*E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*e)
, (3 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))])/(e + b*c*Log[F])
```

3.288.3.1 Defintions of rubi rules used

rule 6015
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[2^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hyper
geometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)), -E^
(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

3.288.4 Maple [F]

$$\int F^{c(bx+a)}\operatorname{sech}(ex+d)\,dx$$

input
```
int(F^(c*(b*x+a))*sech(e*x+d),x)
```

output
```
int(F^(c*(b*x+a))*sech(e*x+d),x)
```

3.288.5 Fricas [F]

$$\int F^{c(a+bx)}\text{sech}(d+ex)\,dx = \int F^{(bx+a)c}\,\text{sech}\,(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d),x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*sech(e*x + d), x)
```

3.288.6 Sympy [F]

$$\int F^{c(a+bx)}\text{sech}(d+ex)\,dx = \int F^{c(a+bx)}\,\text{sech}\,(d+ex)\,dx$$

input
```
integrate(F**(c*(b*x+a))*sech(e*x+d),x)
```

output
```
Integral(F**(c*(a + b*x))*sech(d + e*x), x)
```

3.288.7 Maxima [F]

$$\int F^{c(a+bx)}\text{sech}(d+ex)\,dx = \int F^{(bx+a)c}\,\text{sech}\,(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d),x, algorithm="maxima")
```

output
```
-4*F^(a*c)*e*integrate(e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + (b*c*e^(4*
d)*log(F) - e*e^(4*d))*e^(4*e*x) + 2*(b*c*e^(2*d)*log(F) - e*e^(2*d))*e^(2
*e*x) - e), x) + 2*F^(a*c)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + (b*c*e
^(2*d)*log(F) - e*e^(2*d))*e^(2*e*x) - e)
```

3.288.8 Giac [F]

$$\int F^{c(a+bx)}\operatorname{sech}(d+ex)\,dx = \int F^{(bx+a)c}\operatorname{sech}(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d),x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*sech(e*x + d), x)
```

3.288.9 Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\operatorname{sech}(d+ex)\,dx = \int \frac{F^{c\,(a+b\,x)}}{\cosh\left(d+e\,x\right)}\,dx$$

input
```
int(F^(c*(a + b*x))/cosh(d + e*x),x)
```

output
```
int(F^(c*(a + b*x))/cosh(d + e*x), x)
```

3.289 $\int F^{c(a+bx)}\text{sech}^2(d+ex)\,dx$

3.289.1 Optimal result

Integrand size = 18, antiderivative size = 70

$$\int F^{c(a+bx)}\text{sech}^2(d+ex)\,dx = \frac{4e^{2(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{bc\log(F)}{2e},2+\frac{bc\log(F)}{2e},-e^{2(d+ex)}\right)}{2e+bc\log(F)}$$

output
```
4*exp(2*e*x+2*d)*F^(c*(b*x+a))*hypergeom([2, 1+1/2*b*c*ln(F)/e],[2+1/2*b*c
*ln(F)/e],-exp(2*e*x+2*d))/(b*c*ln(F)+2*e)
```

3.289.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}\text{sech}^2(d+ex)\,dx = \frac{4e^{2(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{bc\log(F)}{2e},2+\frac{bc\log(F)}{2e},-e^{2(d+ex)}\right)}{2e+bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Sech[d + e*x]^2,x]
```

output
```
(4*E^(2*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/(
2*e), 2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(2*e + b*c*Log[F])
```

3.289.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {6015}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^2(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \; 6015$$

$$\frac{4e^{2(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,\frac{bc\log(F)}{2e}+1,\frac{bc\log(F)}{2e}+2,-e^{2(d+ex)}\right)}{bc\log(F)+2e}$$

input
```
Int[F^(c*(a + b*x))*Sech[d + e*x]^2,x]
```

output
```
(4*E^(2*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/(
2*e), 2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(2*e + b*c*Log[F])
```

3.289.3.1 Defintions of rubi rules used

rule 6015
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[2^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hyper
geometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)), -E^
(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

3.289.4 Maple [F]

$$\int F^{c(bx+a)} \operatorname{sech}(ex+d)^2\,dx$$

input
```
int(F^(c*(b*x+a))*sech(e*x+d)^2,x)
```

output
```
int(F^(c*(b*x+a))*sech(e*x+d)^2,x)
```

3.289.5 Fricas [F]

$$\int F^{c(a+bx)}\operatorname{sech}^2(d+ex)\,dx = \int F^{(bx+a)c}\operatorname{sech}(ex+d)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*sech(e*x + d)^2, x)
```

3.289.6 Sympy [F]

$$\int F^{c(a+bx)}\operatorname{sech}^2(d+ex)\,dx = \int F^{c(a+bx)}\operatorname{sech}^2(d+ex)\,dx$$

input
```
integrate(F**(c*(b*x+a))*sech(e*x+d)**2,x)
```

output
```
Integral(F**(c*(a + b*x))*sech(d + e*x)**2, x)
```

3.289.7 Maxima [F]

$$\int F^{c(a+bx)}\operatorname{sech}^2(d+ex)\,dx = \int F^{(bx+a)c}\operatorname{sech}(ex+d)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)^2,x, algorithm="maxima")
```

output
```
16*F^(a*c)*b*c*e*integrate(F^(b*c*x)/(b^2*c^2*log(F)^2 - 6*b*c*e*log(F) +
8*e^2 + (b^2*c^2*e^(6*d)*log(F)^2 - 6*b*c*e*e^(6*d)*log(F) + 8*e^2*e^(6*d)
)*e^(6*e*x) + 3*(b^2*c^2*e^(4*d)*log(F)^2 - 6*b*c*e*e^(4*d)*log(F) + 8*e^2
*e^(4*d))*e^(4*e*x) + 3*(b^2*c^2*e^(2*d)*log(F)^2 - 6*b*c*e*e^(2*d)*log(F)
 + 8*e^2*e^(2*d))*e^(2*e*x)), x)*log(F) - 4*(4*F^(a*c)*e - (F^(a*c)*b*c*e^
(2*d)*log(F) - 4*F^(a*c)*e*e^(2*d))*e^(2*e*x))*F^(b*c*x)/(b^2*c^2*log(F)^2
 - 6*b*c*e*log(F) + 8*e^2 + (b^2*c^2*e^(4*d)*log(F)^2 - 6*b*c*e*e^(4*d)*lo
g(F) + 8*e^2*e^(4*d))*e^(4*e*x) + 2*(b^2*c^2*e^(2*d)*log(F)^2 - 6*b*c*e*e^
(2*d)*log(F) + 8*e^2*e^(2*d))*e^(2*e*x))
```

3.289.8 Giac [F]

$$\int F^{c(a+bx)}\text{sech}^2(d+ex)\,dx = \int F^{(bx+a)c}\,\text{sech}\,(ex+d)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*sech(e*x + d)^2, x)
```

3.289.9 Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\text{sech}^2(d+ex)\,dx = \int \frac{F^{c\,(a+b\,x)}}{\cosh\,(d+e\,x)^2}\,dx$$

input
```
int(F^(c*(a + b*x))/cosh(d + e*x)^2,x)
```

output
```
int(F^(c*(a + b*x))/cosh(d + e*x)^2, x)
```

3.290 $\int F^{c(a+bx)}\text{sech}^3(d+ex)\,dx$

3.290.1 Optimal result

Integrand size = 18, antiderivative size = 124

$$\int F^{c(a+bx)}\text{sech}^3(d+ex)\,dx = \frac{e^{d+ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right),-e^{2(d+ex)}\right)(e-bc\log(F))}{e^2} + \frac{bcF^{c(a+bx)}\log(F)\text{sech}(d+ex)}{2e^2} + \frac{F^{c(a+bx)}\text{sech}(d+ex)\tanh(d+ex)}{2e}$$

output
```
exp(e*x+d)*F^(c*(b*x+a))*hypergeom([1, 1/2*(e+b*c*ln(F))/e],[3/2+1/2*b*c*l
n(F)/e],-exp(2*e*x+2*d))*(e-b*c*ln(F))/e^2+1/2*b*c*F^(c*(b*x+a))*ln(F)*sec
h(e*x+d)/e^2+1/2*F^(c*(b*x+a))*sech(e*x+d)*tanh(e*x+d)/e
```

3.290.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.77

$$\int F^{c(a+bx)}\text{sech}^3(d+ex)\,dx = \frac{F^{c(a+bx)}\left(2e^{d+ex}\,\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right),-e^{2(d+ex)}\right)(e-bc\log(F))+\text{sech}(d+\right.}{2e^2}$$

input
```
Integrate[F^(c*(a + b*x))*Sech[d + e*x]^3,x]
```

output
```
(F^(c*(a + b*x))*(2*E^(d + e*x)*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*e
), (3 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))]*(e - b*c*Log[F]) + Sech[d + e
*x]*(b*c*Log[F] + e*Tanh[d + e*x])))/(2*e^2)
```

3.290.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6013, 6015}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^3(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6013}$$

$$\frac{1}{2}\left(1-\frac{b^2c^2\log^2(F)}{e^2}\right)\int F^{c(a+bx)}\operatorname{sech}(d+ex)dx+\frac{bc\log(F)\operatorname{sech}(d+ex)F^{c(a+bx)}}{2e^2}+\frac{\tanh(d+ex)\operatorname{sech}(d+ex)F^{c(a+bx)}}{2e}$$

$$\Big\downarrow \text{6015}$$

$$\frac{e^{d+ex}F^{c(a+bx)}\left(1-\frac{b^2c^2\log^2(F)}{e^2}\right)\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right),-e^{2(d+ex)}\right)}{bc\log(F)+e}+\frac{bc\log(F)\operatorname{sech}(d+ex)F^{c(a+bx)}}{2e^2}+\frac{\tanh(d+ex)\operatorname{sech}(d+ex)F^{c(a+bx)}}{2e}$$

input
```
Int[F^(c*(a + b*x))*Sech[d + e*x]^3,x]
```

output
```
(E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*e),
(3 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))]*(1 - (b^2*c^2*Log[F]^2)/e^2))/(e
 + b*c*Log[F]) + (b*c*F^(c*(a + b*x))*Log[F]*Sech[d + e*x])/(2*e^2) + (F^(
c*(a + b*x))*Sech[d + e*x]*Tanh[d + e*x])/(2*e)
```

3.290.3.1 Defintions of rubi rules used

rule 6013
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sech[d + e*x]^(n - 2)/(e^2*(n - 1)*
(n - 2))), x] + (Simp[F^(c*(a + b*x))*Sech[d + e*x]^(n - 1)*(Sinh[d + e*x]/
(e*(n - 1))), x] + Simp[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n
- 2))   Int[F^(c*(a + b*x))*Sech[d + e*x]^(n - 2), x], x]) /; FreeQ[{F, a,
b, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1] &&
 NeQ[n, 2]
```

rule 6015
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[2^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hyper
geometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)), -E^
(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

3.290.4 Maple [F]

$$\int F^{c(bx+a)} \operatorname{sech}(ex+d)^3 \, dx$$

input
```
int(F^(c*(b*x+a))*sech(e*x+d)^3,x)
```

output
```
int(F^(c*(b*x+a))*sech(e*x+d)^3,x)
```

3.290.5 Fricas [F]

$$\int F^{c(a+bx)}\operatorname{sech}^3(d+ex)\,dx = \int F^{(bx+a)c} \operatorname{sech}(ex+d)^3 \, dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*sech(e*x + d)^3, x)
```

3.290.6 Sympy [F]

$$\int F^{c(a+bx)}\text{sech}^3(d+ex)\,dx = \int F^{c(a+bx)}\,\text{sech}^3\,(d+ex)\,dx$$

input
```
integrate(F**(c*(b*x+a))*sech(e*x+d)**3,x)
```

output
```
Integral(F**(c*(a + b*x))*sech(d + e*x)**3, x)
```

3.290.7 Maxima [F]

$$\int F^{c(a+bx)}\text{sech}^3(d+ex)\,dx = \int F^{(bx+a)c}\,\text{sech}\,(ex+d)^3\;dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)^3,x, algorithm="maxima")
```

output
```
48*(F^(a*c)*b*c*e*e^d*log(F) + F^(a*c)*e^2*e^d)*integrate(e^(b*c*x*log(F)
+ e*x)/(b^2*c^2*log(F)^2 - 8*b*c*e*log(F) + 15*e^2 + (b^2*c^2*e^(8*d)*log(
F)^2 - 8*b*c*e*e^(8*d)*log(F) + 15*e^2*e^(8*d))*e^(8*e*x) + 4*(b^2*c^2*e^(
6*d)*log(F)^2 - 8*b*c*e*e^(6*d)*log(F) + 15*e^2*e^(6*d))*e^(6*e*x) + 6*(b^
2*c^2*e^(4*d)*log(F)^2 - 8*b*c*e*e^(4*d)*log(F) + 15*e^2*e^(4*d))*e^(4*e*x
) + 4*(b^2*c^2*e^(2*d)*log(F)^2 - 8*b*c*e*e^(2*d)*log(F) + 15*e^2*e^(2*d))
*e^(2*e*x)), x) - 8*(6*F^(a*c)*e*e^(e*x + d) - (F^(a*c)*b*c*e^(3*d)*log(F)
 - 5*F^(a*c)*e*e^(3*d))*e^(3*e*x))*F^(b*c*x)/(b^2*c^2*log(F)^2 - 8*b*c*e*l
og(F) + 15*e^2 + (b^2*c^2*e^(6*d)*log(F)^2 - 8*b*c*e*e^(6*d)*log(F) + 15*e
^2*e^(6*d))*e^(6*e*x) + 3*(b^2*c^2*e^(4*d)*log(F)^2 - 8*b*c*e*e^(4*d)*log(
F) + 15*e^2*e^(4*d))*e^(4*e*x) + 3*(b^2*c^2*e^(2*d)*log(F)^2 - 8*b*c*e*e^(
2*d)*log(F) + 15*e^2*e^(2*d))*e^(2*e*x))
```

3.290.8 Giac [F]

$$\int F^{c(a+bx)}\text{sech}^3(d+ex)\,dx=\int F^{(bx+a)c}\,\text{sech}\,(ex+d)^3\;dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*sech(e*x + d)^3, x)
```

3.290.9 Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\text{sech}^3(d+ex)\,dx=\int \frac{F^{c\,(a+b\,x)}}{\cosh\,(d+e\,x)^3}\,dx$$

input
```
int(F^(c*(a + b*x))/cosh(d + e*x)^3,x)
```

output
```
int(F^(c*(a + b*x))/cosh(d + e*x)^3, x)
```

3.291 $\int F^{c(a+bx)}\text{sech}^4(d+ex)\,dx$

3.291.1 Optimal result

Integrand size = 18, antiderivative size = 133

$$\int F^{c(a+bx)}\text{sech}^4(d+ex)\,dx = \frac{2e^{2(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{bc\log(F)}{2e},2+\frac{bc\log(F)}{2e},-e^{2(d+ex)}\right)(2e-bc\log(F))}{3e^2} + \frac{bcF^{c(a+bx)}\log(F)\text{sech}^2(d+ex)}{6e^2} + \frac{F^{c(a+bx)}\text{sech}^2(d+ex)\tanh(d+ex)}{3e}$$

output
```
2/3*exp(2*e*x+2*d)*F^(c*(b*x+a))*hypergeom([2, 1+1/2*b*c*ln(F)/e],[2+1/2*b
*c*ln(F)/e],-exp(2*e*x+2*d))*(2*e-b*c*ln(F))/e^2+1/6*b*c*F^(c*(b*x+a))*ln(
F)*sech(e*x+d)^2/e^2+1/3*F^(c*(b*x+a))*sech(e*x+d)^2*tanh(e*x+d)/e
```

3.291.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.76

$$\int F^{c(a+bx)}\text{sech}^4(d+ex)\,dx = \frac{F^{c(a+bx)}\left(4e^{2(d+ex)}\,\text{Hypergeometric2F1}\left(2,1+\frac{bc\log(F)}{2e},2+\frac{bc\log(F)}{2e},-e^{2(d+ex)}\right)(2e-bc\log(F))+\text{sech}^2(d\right.}{6e^2}$$

input
```
Integrate[F^(c*(a + b*x))*Sech[d + e*x]^4,x]
```

output
```
(F^(c*(a + b*x))*(4*E^(2*(d + e*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/
(2*e), 2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))]*(2*e - b*c*Log[F]) + Sech
[d + e*x]^2*(b*c*Log[F] + 2*e*Tanh[d + e*x])))/(6*e^2)
```

3.291.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6013, 6015}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{sech}^4(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6013}$$

$$\frac{1}{6}\left(4-\frac{b^2c^2\log^2(F)}{e^2}\right)\int F^{c(a+bx)}\text{sech}^2(d+ex)dx+\frac{bc\log(F)\text{sech}^2(d+ex)F^{c(a+bx)}}{6e^2}+\frac{\tanh(d+ex)\text{sech}^2(d+ex)F^{c(a+bx)}}{3e}$$

$$\Big\downarrow \text{6015}$$

$$\frac{2e^{2(d+ex)}F^{c(a+bx)}\left(4-\frac{b^2c^2\log^2(F)}{e^2}\right)\text{Hypergeometric2F1}\left(2,\frac{bc\log(F)}{2e}+1,\frac{bc\log(F)}{2e}+2,-e^{2(d+ex)}\right)}{3(bc\log(F)+2e)}+\frac{bc\log(F)\text{sech}^2(d+ex)F^{c(a+bx)}}{6e^2}+\frac{\tanh(d+ex)\text{sech}^2(d+ex)F^{c(a+bx)}}{3e}$$

input
```
Int[F^(c*(a + b*x))*Sech[d + e*x]^4,x]
```

output
```
(2*E^(2*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/(
2*e), 2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))]*(4 - (b^2*c^2*Log[F]^2)/e^
2))/(3*(2*e + b*c*Log[F])) + (b*c*F^(c*(a + b*x))*Log[F]*Sech[d + e*x]^2)/
(6*e^2) + (F^(c*(a + b*x))*Sech[d + e*x]^2*Tanh[d + e*x])/(3*e)
```

3.291.3.1 Defintions of rubi rules used

rule 6013
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sech[d + e*x]^(n - 2)/(e^2*(n - 1)*
(n - 2))), x] + (Simp[F^(c*(a + b*x))*Sech[d + e*x]^(n - 1)*(Sinh[d + e*x]/
(e*(n - 1))), x] + Simp[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n
- 2))   Int[F^(c*(a + b*x))*Sech[d + e*x]^(n - 2), x], x]) /; FreeQ[{F, a,
b, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1] &&
 NeQ[n, 2]
```

rule 6015
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[2^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hyper
geometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)), -E^
(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

3.291.4 Maple [F]

$$\int F^{c(bx+a)} \operatorname{sech}(ex+d)^4 \, dx$$

input
```
int(F^(c*(b*x+a))*sech(e*x+d)^4,x)
```

output
```
int(F^(c*(b*x+a))*sech(e*x+d)^4,x)
```

3.291.5 Fricas [F]

$$\int F^{c(a+bx)}\operatorname{sech}^4(d+ex)\,dx = \int F^{(bx+a)c} \operatorname{sech}(ex+d)^4 \, dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)^4,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*sech(e*x + d)^4, x)
```

3.291.6 Sympy [F]

$$\int F^{c(a+bx)}\operatorname{sech}^4(d+ex)\,dx=\int F^{c(a+bx)}\operatorname{sech}^4(d+ex)\,dx$$

input
```
integrate(F**(c*(b*x+a))*sech(e*x+d)**4,x)
```

output
```
Integral(F**(c*(a + b*x))*sech(d + e*x)**4, x)
```

3.291.7 Maxima [F]

$$\int F^{c(a+bx)}\operatorname{sech}^4(d+ex)\,dx=\int F^{(bx+a)c}\operatorname{sech}(ex+d)^4\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)^4,x, algorithm="maxima")
```

output
```
-128*(F^(a*c)*b^2*c^2*e*log(F)^2 + 2*F^(a*c)*b*c*e^2*log(F))*integrate(F^(
b*c*x)/(b^3*c^3*log(F)^3 - 18*b^2*c^2*e*log(F)^2 + 104*b*c*e^2*log(F) - 19
2*e^3 + (b^3*c^3*e^(10*d)*log(F)^3 - 18*b^2*c^2*e*e^(10*d)*log(F)^2 + 104*
b*c*e^2*e^(10*d)*log(F) - 192*e^3*e^(10*d))*e^(10*e*x) + 5*(b^3*c^3*e^(8*d
)*log(F)^3 - 18*b^2*c^2*e*e^(8*d)*log(F)^2 + 104*b*c*e^2*e^(8*d)*log(F) -
192*e^3*e^(8*d))*e^(8*e*x) + 10*(b^3*c^3*e^(6*d)*log(F)^3 - 18*b^2*c^2*e*e
^(6*d)*log(F)^2 + 104*b*c*e^2*e^(6*d)*log(F) - 192*e^3*e^(6*d))*e^(6*e*x)
+ 10*(b^3*c^3*e^(4*d)*log(F)^3 - 18*b^2*c^2*e*e^(4*d)*log(F)^2 + 104*b*c*e
^2*e^(4*d)*log(F) - 192*e^3*e^(4*d))*e^(4*e*x) + 5*(b^3*c^3*e^(2*d)*log(F)
^3 - 18*b^2*c^2*e*e^(2*d)*log(F)^2 + 104*b*c*e^2*e^(2*d)*log(F) - 192*e^3*
e^(2*d))*e^(2*e*x)), x) + 16*(8*F^(a*c)*b*c*e*log(F) + 16*F^(a*c)*e^2 + (F
^(a*c)*b^2*c^2*e^(4*d)*log(F)^2 - 14*F^(a*c)*b*c*e*e^(4*d)*log(F) + 48*F^(
a*c)*e^2*e^(4*d))*e^(4*e*x) - 8*(F^(a*c)*b*c*e*e^(2*d)*log(F) - 8*F^(a*c)*
e^2*e^(2*d))*e^(2*e*x))*F^(b*c*x)/(b^3*c^3*log(F)^3 - 18*b^2*c^2*e*log(F)^
2 + 104*b*c*e^2*log(F) - 192*e^3 + (b^3*c^3*e^(8*d)*log(F)^3 - 18*b^2*c^2*
e*e^(8*d)*log(F)^2 + 104*b*c*e^2*e^(8*d)*log(F) - 192*e^3*e^(8*d))*e^(8*e*
x) + 4*(b^3*c^3*e^(6*d)*log(F)^3 - 18*b^2*c^2*e*e^(6*d)*log(F)^2 + 104*b*c
*e^2*e^(6*d)*log(F) - 192*e^3*e^(6*d))*e^(6*e*x) + 6*(b^3*c^3*e^(4*d)*log(
F)^3 - 18*b^2*c^2*e*e^(4*d)*log(F)^2 + 104*b*c*e^2*e^(4*d)*log(F) - 192*e^
3*e^(4*d))*e^(4*e*x) + 4*(b^3*c^3*e^(2*d)*log(F)^3 - 18*b^2*c^2*e*e^(2*...
```

3.291.8 Giac [F]

$$\int F^{c(a+bx)}\text{sech}^4(d+ex)\,dx = \int F^{(bx+a)c}\,\text{sech}\,(ex+d)^4\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)^4,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*sech(e*x + d)^4, x)
```

3.291.9 Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\text{sech}^4(d+ex)\,dx = \int \frac{F^{c\,(a+b\,x)}}{\cosh\left(d+e\,x\right)^4}\,dx$$

input
```
int(F^(c*(a + b*x))/cosh(d + e*x)^4,x)
```

output
```
int(F^(c*(a + b*x))/cosh(d + e*x)^4, x)
```

3.292 $\int e^{c(a+bx)} \cosh^2(ac+bcx)^{5/2}\,dx$

3.292.1 Optimal result

Integrand size = 25, antiderivative size = 250

$$\begin{aligned}\int e^{c(a+bx)}\cosh^2(ac+bcx)^{5/2}\,dx &= -\frac{e^{-4c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\operatorname{sech}(ac+bcx)}{128bc}\\ &-\frac{5e^{-2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\operatorname{sech}(ac+bcx)}{64bc}\\ &+\frac{5e^{2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\operatorname{sech}(ac+bcx)}{32bc}\\ &+\frac{5e^{4c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\operatorname{sech}(ac+bcx)}{128bc}\\ &+\frac{e^{6c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\operatorname{sech}(ac+bcx)}{192bc}+\frac{5}{16}x\sqrt{\cosh^2(ac+bcx)}\operatorname{sech}(ac+bcx)\end{aligned}$$

output
```
-1/128*sech(b*c*x+a*c)*(cosh(b*c*x+a*c)^2)^(1/2)/b/c/exp(4*c*(b*x+a))-5/64
*sech(b*c*x+a*c)*(cosh(b*c*x+a*c)^2)^(1/2)/b/c/exp(2*c*(b*x+a))+5/32*exp(2
*c*(b*x+a))*sech(b*c*x+a*c)*(cosh(b*c*x+a*c)^2)^(1/2)/b/c+5/128*exp(4*c*(b
*x+a))*sech(b*c*x+a*c)*(cosh(b*c*x+a*c)^2)^(1/2)/b/c+1/192*exp(6*c*(b*x+a)
)*sech(b*c*x+a*c)*(cosh(b*c*x+a*c)^2)^(1/2)/b/c+5/16*x*sech(b*c*x+a*c)*(co
sh(b*c*x+a*c)^2)^(1/2)
```

3.292.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.44

$$\int e^{c(a+bx)} \cosh^2(ac+bcx)^{5/2} dx = \frac{\left(-\frac{1}{128}e^{-4c(a+bx)} - \frac{5}{64}e^{-2c(a+bx)} + \frac{5}{32}e^{2c(a+bx)} + \frac{5}{128}e^{4c(a+bx)} + \frac{1}{192}e^{6c(a+bx)} + \frac{5bcx}{16}\right)\cosh^2(c(a+b$$

input
```
Integrate[E^(c*(a + b*x))*(Cosh[a*c + b*c*x]^2)^(5/2),x]
```

output
```
((-1/128*1/E^(4*c*(a + b*x)) - 5/(64*E^(2*c*(a + b*x))) + (5*E^(2*c*(a + b
*x)))/32 + (5*E^(4*c*(a + b*x)))/128 + E^(6*c*(a + b*x))/192 + (5*b*c*x)/1
6)*(Cosh[c*(a + b*x)]^2)^(5/2)*Sech[c*(a + b*x)]^5)/(b*c)
```

3.292.3 Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.41, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {7271, 2720, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)} \cosh^2(ac+bcx)^{5/2} dx$$

$$\Big\downarrow \text{7271}$$

$$\sqrt{\cosh^2(ac+bcx)}\text{sech}(ac+bcx) \int e^{c(a+bx)} \cosh^5(ac+bxc)dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\sqrt{\cosh^2(ac+bcx)}\text{sech}(ac+bcx) \int \frac{1}{32}e^{-5c(a+bx)}\left(1+e^{2c(a+bx)}\right)^5 de^{c(a+bx)}}{bc}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{\cosh^2(ac+bcx)}\text{sech}(ac+bcx) \int e^{-5c(a+bx)}\left(1+e^{2c(a+bx)}\right)^5 de^{c(a+bx)}}{32bc}$$

$$\Big\downarrow \text{243}$$

$$\frac{\sqrt{\cosh^2(ac+bcx)}\text{sech}(ac+bcx)\int e^{-3c(a+bx)}\left(1+e^{2c(a+bx)}\right)^5de^{2c(a+bx)}}{64bc}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\sqrt{\cosh^2(ac+bcx)}\text{sech}(ac+bcx)\int \left(10+e^{-3c(a+bx)}+5e^{-2c(a+bx)}+10e^{-c(a+bx)}+6e^{2c(a+bx)}\right)de^{2c(a+bx)}}{64bc}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(-\frac{1}{2}e^{-2c(a+bx)}-5e^{-c(a+bx)}+\frac{25}{2}e^{2c(a+bx)}+\frac{1}{3}e^{3c(a+bx)}+10\log\left(e^{2c(a+bx)}\right)\right)\sqrt{\cosh^2(ac+bcx)}\text{sech}(ac+bcx)}{64bc}$$

input
```
Int[E^(c*(a + b*x))*(Cosh[a*c + b*c*x]^2)^(5/2),x]
```

output
```
(Sqrt[Cosh[a*c + b*c*x]^2]*(-1/2*1/E^(2*c*(a + b*x)) - 5/E^(c*(a + b*x)) +
 (25*E^(2*c*(a + b*x)))/2 + E^(3*c*(a + b*x))/3 + 10*Log[E^(2*c*(a + b*x))
])*Sech[a*c + b*c*x])/(64*b*c)
```

3.292.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

3.292.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.35

method	result
default	$\frac{\operatorname{csgn}(\cosh(c(bx+a)))\left(\frac{\cosh(bcx+ac)^6}{6}+\left(\frac{\cosh(bcx+ac)^5}{6}+\frac{5\cosh(bcx+ac)^3}{24}+\frac{5\cosh(bcx+ac)}{16}\right)\sinh(bcx+ac)+\frac{5bcx}{16}+\frac{5ac}{16}\right)}{cb}$
risch	$\frac{5x\sqrt{\left(1+e^{2c(bx+a)}\right)^2 e^{-2c(bx+a)}}\, e^{c(bx+a)}}{16\left(1+e^{2c(bx+a)}\right)}+\frac{\sqrt{\left(1+e^{2c(bx+a)}\right)^2 e^{-2c(bx+a)}}\, e^{7c(bx+a)}}{192bc\left(1+e^{2c(bx+a)}\right)}+\frac{5\sqrt{\left(1+e^{2c(bx+a)}\right)^2 e^{-2c(bx+a)}}\, e^{5c(bx+a)}}{128bc\left(1+e^{2c(bx+a)}\right)}+$

input
```
int(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
csgn(cosh(c*(b*x+a)))/c/b*(1/6*cosh(b*c*x+a*c)^6+(1/6*cosh(b*c*x+a*c)^5+5/
24*cosh(b*c*x+a*c)^3+5/16*cosh(b*c*x+a*c))*sinh(b*c*x+a*c)+5/16*b*c*x+5/16
*a*c)
```

3.292.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.87

$$\int e^{c(a+bx)}\cosh^2(ac+bcx)^{5/2}\,dx =$$

$$-\frac{\cosh(bcx+ac)^5+5\cosh(bcx+ac)\sinh(bcx+ac)^4-5\sinh(bcx+ac)^5-5\left(10\cosh(bcx+ac)^2+9\right)\mathrm{si}}{}$$

input
```
integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/384*(cosh(b*c*x + a*c)^5 + 5*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^4 - 5*
sinh(b*c*x + a*c)^5 - 5*(10*cosh(b*c*x + a*c)^2 + 9)*sinh(b*c*x + a*c)^3 +
 15*cosh(b*c*x + a*c)^3 + 5*(2*cosh(b*c*x + a*c)^3 + 9*cosh(b*c*x + a*c))*
sinh(b*c*x + a*c)^2 - 60*(2*b*c*x + 1)*cosh(b*c*x + a*c) - 5*(5*cosh(b*c*x
 + a*c)^4 - 24*b*c*x + 27*cosh(b*c*x + a*c)^2 + 12)*sinh(b*c*x + a*c))/(b*
c*cosh(b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

3.292.6 Sympy [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\cosh^2(ac+bcx)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)**2)**(5/2),x)
```

output
```
Timed out
```

3.292.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.45

$$\int e^{c(a+bx)}\cosh^2(ac+bcx)^{5/2}\,dx=\frac{5\,(bcx+ac)}{16\,bc}+\frac{e^{(6\,bcx+6\,ac)}}{192\,bc}+\frac{5\,e^{(4\,bcx+4\,ac)}}{128\,bc}+\frac{5\,e^{(2\,bcx+2\,ac)}}{32\,bc}-\frac{5\,e^{(-2\,bcx-2\,ac)}}{64\,bc}-\frac{e^{(-4\,bcx-4\,ac)}}{128\,bc}$$

input
```
integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

output
```
5/16*(b*c*x + a*c)/(b*c) + 1/192*e^(6*b*c*x + 6*a*c)/(b*c) + 5/128*e^(4*b*
c*x + 4*a*c)/(b*c) + 5/32*e^(2*b*c*x + 2*a*c)/(b*c) - 5/64*e^(-2*b*c*x - 2
*a*c)/(b*c) - 1/128*e^(-4*b*c*x - 4*a*c)/(b*c)
```

3.292.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.40

$$\int e^{c(a+bx)}\cosh^2(ac+bcx)^{5/2}\,dx = \frac{120\,bcx - 3\left(30\,e^{(4\,bcx+4\,ac)} + 10\,e^{(2\,bcx+2\,ac)} + 1\right)e^{(-4\,bcx-4\,ac)} + \left(2\,e^{(6\,bcx+18\,ac)} + 15\,e^{(4\,bcx+16\,ac)} -}{384\,bc}$$

input
```
integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

output
```
1/384*(120*b*c*x - 3*(30*e^(4*b*c*x + 4*a*c) + 10*e^(2*b*c*x + 2*a*c) + 1)
*e^(-4*b*c*x - 4*a*c) + (2*e^(6*b*c*x + 18*a*c) + 15*e^(4*b*c*x + 16*a*c)
+ 60*e^(2*b*c*x + 14*a*c))*e^(-12*a*c))/(b*c)
```

3.292.9 Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\cosh^2(ac+bcx)^{5/2}\,dx = \int e^{c\,(a+b\,x)}\,\left(\cosh(a\,c+b\,c\,x)^2\right)^{5/2}\,dx$$

input
```
int(exp(c*(a + b*x))*(cosh(a*c + b*c*x)^2)^(5/2),x)
```

output
```
int(exp(c*(a + b*x))*(cosh(a*c + b*c*x)^2)^(5/2), x)
```

3.293 $\int e^{c(a+bx)} \cosh^2(ac+bcx)^{3/2} dx$

3.293.1 Optimal result

Integrand size = 25, antiderivative size = 162

$$\int e^{c(a+bx)} \cosh^2(ac+bcx)^{3/2} dx = -\frac{e^{-2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\text{sech}(ac+bcx)}{16bc} + \frac{3e^{2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\text{sech}(ac+bcx)}{16bc} + \frac{e^{4c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\text{sech}(ac+bcx)}{32bc} + \frac{3}{8}x\sqrt{\cosh^2(ac+bcx)}\text{sech}(ac+bcx)$$

output
```
-1/16*sech(b*c*x+a*c)*(cosh(b*c*x+a*c)^2)^(1/2)/b/c/exp(2*c*(b*x+a))+3/16*
exp(2*c*(b*x+a))*sech(b*c*x+a*c)*(cosh(b*c*x+a*c)^2)^(1/2)/b/c+1/32*exp(4*
c*(b*x+a))*sech(b*c*x+a*c)*(cosh(b*c*x+a*c)^2)^(1/2)/b/c+3/8*x*sech(b*c*x+
a*c)*(cosh(b*c*x+a*c)^2)^(1/2)
```

3.293.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.50

$$\int e^{c(a+bx)} \cosh^2(ac+bcx)^{3/2} dx = \frac{\left(-\frac{1}{16}e^{-2c(a+bx)} + \frac{3}{16}e^{2c(a+bx)} + \frac{1}{32}e^{4c(a+bx)} + \frac{3bcx}{8}\right)\cosh^2(c(a+bx))^{3/2}\text{sech}^3(c(a+bx))}{bc}$$

input
```
Integrate[E^(c*(a + b*x))*(Cosh[a*c + b*c*x]^2)^(3/2),x]
```

output
```
((-1/16*1/E^(2*c*(a + b*x)) + (3*E^(2*c*(a + b*x)))/16 + E^(4*c*(a + b*x))
/32 + (3*b*c*x)/8)*(Cosh[c*(a + b*x)]^2)^(3/2)*Sech[c*(a + b*x)]^3)/(b*c)
```

3.293.3 Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.46, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {7271, 2720, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)} \cosh^2(ac+bcx)^{3/2} \, dx$$

$$\Big\downarrow \text{7271}$$

$$\sqrt{\cosh^2(ac+bcx)}\text{sech}(ac+bcx) \int e^{c(a+bx)} \cosh^3(ac+bxc)dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\sqrt{\cosh^2(ac+bcx)}\text{sech}(ac+bcx) \int \frac{1}{8}e^{-3c(a+bx)} \left(1+e^{2c(a+bx)}\right)^3 de^{c(a+bx)}}{bc}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{\cosh^2(ac+bcx)}\text{sech}(ac+bcx) \int e^{-3c(a+bx)} \left(1+e^{2c(a+bx)}\right)^3 de^{c(a+bx)}}{8bc}$$

$$\Big\downarrow \text{243}$$

$$\frac{\sqrt{\cosh^2(ac+bcx)}\text{sech}(ac+bcx) \int e^{-2c(a+bx)} \left(1+e^{2c(a+bx)}\right)^3 de^{2c(a+bx)}}{16bc}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{\cosh^2(ac+bcx)}\text{sech}(ac+bcx) \int \left(3+e^{-2c(a+bx)}+3e^{-c(a+bx)}+e^{2c(a+bx)}\right) de^{2c(a+bx)}}{16bc}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-e^{-c(a+bx)}+\frac{7}{2}e^{2c(a+bx)}+3\log\left(e^{2c(a+bx)}\right)\right)\sqrt{\cosh^2(ac+bcx)}\text{sech}(ac+bcx)}{16bc}$$

input
```
Int[E^(c*(a + b*x))*(Cosh[a*c + b*c*x]^2)^(3/2),x]
```

output
```
(Sqrt[Cosh[a*c + b*c*x]^2]*(-E^(-(c*(a + b*x))) + (7*E^(2*c*(a + b*x)))/2
+ 3*Log[E^(2*c*(a + b*x))])*Sech[a*c + b*c*x])/(16*b*c)
```

3.293.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

3.293.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.46

method	result
default	$\frac{\operatorname{csgn}(\cosh(c(bx+a)))\left(\frac{\cosh(bcx+ac)^4}{4}+\left(\frac{\cosh(bcx+ac)^3}{4}+\frac{3\cosh(bcx+ac)}{8}\right)\sinh(bcx+ac)+\frac{3bcx}{8}+\frac{3ac}{8}\right)}{cb}$
risch	$\frac{3x\sqrt{\left(1+\mathrm{e}^{2c(bx+a)}\right)^2\mathrm{e}^{-2c(bx+a)}}\,\mathrm{e}^{c(bx+a)}}{8\left(1+\mathrm{e}^{2c(bx+a)}\right)}+\frac{\sqrt{\left(1+\mathrm{e}^{2c(bx+a)}\right)^2\mathrm{e}^{-2c(bx+a)}}\,\mathrm{e}^{5c(bx+a)}}{32bc\left(1+\mathrm{e}^{2c(bx+a)}\right)}+\frac{3\sqrt{\left(1+\mathrm{e}^{2c(bx+a)}\right)^2\mathrm{e}^{-2c(bx+a)}}\,\mathrm{e}^{3c(bx+a)}}{16bc\left(1+\mathrm{e}^{2c(bx+a)}\right)}-$

input
```
int(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
csgn(cosh(c*(b*x+a)))/c/b*(1/4*cosh(b*c*x+a*c)^4+(1/4*cosh(b*c*x+a*c)^3+3/
8*cosh(b*c*x+a*c))*sinh(b*c*x+a*c)+3/8*b*c*x+3/8*a*c)
```

3.293.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.78

$$\int e^{c(a+bx)}\cosh^2(ac+bcx)^{3/2}\,dx =$$
$$-\frac{\cosh(bcx+ac)^3+3\cosh(bcx+ac)\sinh(bcx+ac)^2-3\sinh(bcx+ac)^3-6(2bcx+1)\cosh(bcx+ac)}{32(bc\cosh(bcx+ac)-bc\sinh(bcx+ac))}$$

input
```
integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/32*(cosh(b*c*x + a*c)^3 + 3*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 - 3*s
inh(b*c*x + a*c)^3 - 6*(2*b*c*x + 1)*cosh(b*c*x + a*c) + 3*(4*b*c*x - 3*co
sh(b*c*x + a*c)^2 - 2)*sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c) - b*c*sin
h(b*c*x + a*c))
```

3.293.6 Sympy [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\cosh^2(ac+bcx)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)**2)**(3/2),x)
```

output
```
Timed out
```

3.293.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.46

$$\int e^{c(a+bx)}\cosh^2(ac+bcx)^{3/2}\,dx = \frac{3\,(bcx+ac)}{8\,bc} + \frac{e^{(4\,bcx+4\,ac)}}{32\,bc} + \frac{3\,e^{(2\,bcx+2\,ac)}}{16\,bc} - \frac{e^{(-2\,bcx-2\,ac)}}{16\,bc}$$

input
```
integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

output
```
3/8*(b*c*x + a*c)/(b*c) + 1/32*e^(4*b*c*x + 4*a*c)/(b*c) + 3/16*e^(2*b*c*x
 + 2*a*c)/(b*c) - 1/16*e^(-2*b*c*x - 2*a*c)/(b*c)
```

3.293.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.45

$$\int e^{c(a+bx)}\cosh^2(ac+bcx)^{3/2}\,dx = \frac{12\,bcx - 2\left(3\,e^{(2\,bcx+2\,ac)}+1\right)e^{(-2\,bcx-2\,ac)} + \left(e^{(4\,bcx+8\,ac)}+6\,e^{(2\,bcx+6\,ac)}\right)e^{(-4\,ac)}}{32\,bc}$$

input
```
integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

output
```
1/32*(12*b*c*x - 2*(3*e^(2*b*c*x + 2*a*c) + 1)*e^(-2*b*c*x - 2*a*c) + (e^(
4*b*c*x + 8*a*c) + 6*e^(2*b*c*x + 6*a*c))*e^(-4*a*c))/(b*c)
```

3.293.9 Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)} \cosh^2(ac+bcx)^{3/2} \, dx = \int e^{c\,(a+b\,x)} \left(\cosh(a\,c+b\,c\,x)^2\right)^{3/2} dx$$

input
```
int(exp(c*(a + b*x))*(cosh(a*c + b*c*x)^2)^(3/2),x)
```

output
```
int(exp(c*(a + b*x))*(cosh(a*c + b*c*x)^2)^(3/2), x)
```

3.294 $\int e^{c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,dx$

3.294.1 Optimal result

Integrand size = 25, antiderivative size = 74

$$\int e^{c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,dx=\frac{e^{2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\operatorname{sech}(ac+bcx)}{4bc}+\frac{1}{2}x\sqrt{\cosh^2(ac+bcx)}\operatorname{sech}(ac+bcx)$$

output
```
1/4*exp(2*c*(b*x+a))*sech(b*c*x+a*c)*(cosh(b*c*x+a*c)^2)^(1/2)/b/c+1/2*x*s
ech(b*c*x+a*c)*(cosh(b*c*x+a*c)^2)^(1/2)
```

3.294.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.65

$$\int e^{c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,dx=\frac{\left(e^{2c(a+bx)}+2bcx\right)\sqrt{\cosh^2(c(a+bx))}\operatorname{sech}(c(a+bx))}{4bc}$$

input
```
Integrate[E^(c*(a + b*x))*Sqrt[Cosh[a*c + b*c*x]^2],x]
```

output
```
((E^(2*c*(a + b*x)) + 2*b*c*x)*Sqrt[Cosh[c*(a + b*x)]^2]*Sech[c*(a + b*x)]
)/(4*b*c)
```

3.294.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7271, 2720, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,dx$$

$$\Big\downarrow \text{7271}$$

$$\sqrt{\cosh^2(ac+bcx)}\text{sech}(ac+bcx)\int e^{c(a+bx)}\cosh(ac+bxc)dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\sqrt{\cosh^2(ac+bcx)}\text{sech}(ac+bcx)\int \frac{1}{2}e^{-c(a+bx)}\left(1+e^{2c(a+bx)}\right)de^{c(a+bx)}}{bc}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{\cosh^2(ac+bcx)}\text{sech}(ac+bcx)\int e^{-c(a+bx)}\left(1+e^{2c(a+bx)}\right)de^{c(a+bx)}}{2bc}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{\cosh^2(ac+bcx)}\text{sech}(ac+bcx)\int \left(e^{-c(a+bx)}+e^{c(a+bx)}\right)de^{c(a+bx)}}{2bc}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{1}{2}e^{2c(a+bx)}+\log\left(e^{c(a+bx)}\right)\right)\sqrt{\cosh^2(ac+bcx)}\text{sech}(ac+bcx)}{2bc}$$

input
```
Int[E^(c*(a + b*x))*Sqrt[Cosh[a*c + b*c*x]^2],x]
```

output
```
(Sqrt[Cosh[a*c + b*c*x]^2]*(E^(2*c*(a + b*x))/2 + Log[E^(c*(a + b*x))])*Se
ch[a*c + b*c*x])/(2*b*c)
```

3.294.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

3.294.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{\operatorname{csgn}(\cosh(c(bx+a)))\left(\frac{\cosh(bcx+ac)^2}{2}+\frac{\sinh(bcx+ac)\cosh(bcx+ac)}{2}+\frac{bcx}{2}+\frac{ac}{2}\right)}{cb}$	60
risch	$\frac{x\sqrt{\left(1+e^{2c(bx+a)}\right)^2 e^{-2c(bx+a)}}\, e^{c(bx+a)}}{2+2\, e^{2c(bx+a)}}+\frac{\sqrt{\left(1+e^{2c(bx+a)}\right)^2 e^{-2c(bx+a)}}\, e^{3c(bx+a)}}{4bc\left(1+e^{2c(bx+a)}\right)}$	106

input
```
int(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
csgn(cosh(c*(b*x+a)))/c/b*(1/2*cosh(b*c*x+a*c)^2+1/2*sinh(b*c*x+a*c)*cosh(
b*c*x+a*c)+1/2*b*c*x+1/2*a*c)
```

3.294.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.89

$$\int e^{c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,dx = \frac{(2\,bcx+1)\cosh\left(bcx+ac\right)-(2\,bcx-1)\sinh\left(bcx+ac\right)}{4\left(bc\cosh\left(bcx+ac\right)-bc\sinh\left(bcx+ac\right)\right)}$$

input
```
integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/4*((2*b*c*x + 1)*cosh(b*c*x + a*c) - (2*b*c*x - 1)*sinh(b*c*x + a*c))/(b
*c*cosh(b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

3.294.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.94 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.20

$$\int e^{c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,dx$$
$$= \begin{cases} x\sqrt{\cosh^2(ac)}e^{ac} & \text{for } b=0 \\ x & \text{for } c=0 \\ 0 & \text{for } a=-\frac{2bcx-}{2c} \\ -\frac{x\sqrt{\cosh^2(ac+bcx)}e^{ac}e^{bcx}\sinh(ac+bcx)}{2\cosh(ac+bcx)}+\frac{x\sqrt{\cosh^2(ac+bcx)}e^{ac}e^{bcx}}{2}+\frac{\sqrt{\cosh^2(ac+bcx)}e^{ac}e^{bcx}\sinh(ac+bcx)}{2bc\cosh(ac+bcx)} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)**2)**(1/2),x)
```

output
```
Piecewise((x*sqrt(cosh(a*c)**2)*exp(a*c), Eq(b, 0)), (x, Eq(c, 0)), (0, Eq
(a, -(2*b*c*x - I*pi)/(2*c))), (-x*sqrt(cosh(a*c + b*c*x)**2)*exp(a*c)*exp
(b*c*x)*sinh(a*c + b*c*x)/(2*cosh(a*c + b*c*x)) + x*sqrt(cosh(a*c + b*c*x)
**2)*exp(a*c)*exp(b*c*x)/2 + sqrt(cosh(a*c + b*c*x)**2)*exp(a*c)*exp(b*c*x
)*sinh(a*c + b*c*x)/(2*b*c*cosh(a*c + b*c*x)), True))
```

3.294.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.39

$$\int e^{c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,dx = \frac{1}{2}\,x + \frac{a}{2\,b} + \frac{e^{(2\,bcx+2\,ac)}}{4\,bc}$$

input
```
integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*x + 1/2*a/b + 1/4*e^(2*b*c*x + 2*a*c)/(b*c)
```

3.294.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.31

$$\int e^{c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,dx = \frac{1}{2}\,x + \frac{e^{(2\,bcx+2\,ac)}}{4\,bc}$$

input
```
integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*x + 1/4*e^(2*b*c*x + 2*a*c)/(b*c)
```

3.294.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.03

$$\int e^{c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,dx = \frac{\left(x\,\mathrm{e}^{a\,c+b\,c\,x}+\frac{\mathrm{e}^{3\,a\,c+3\,b\,c\,x}}{2\,b\,c}\right)\,\sqrt{{\left(\frac{\mathrm{e}^{a\,c+b\,c\,x}}{2}+\frac{\mathrm{e}^{-a\,c-b\,c\,x}}{2}\right)}^2}}{\mathrm{e}^{2\,a\,c+2\,b\,c\,x}+1}$$

input
```
int(exp(c*(a + b*x))*(cosh(a*c + b*c*x)^2)^(1/2),x)
```

output
```
((x*exp(a*c + b*c*x) + exp(3*a*c + 3*b*c*x)/(2*b*c))*((exp(a*c + b*c*x)/2
+ exp(- a*c - b*c*x)/2)^2)^(1/2))/(exp(2*a*c + 2*b*c*x) + 1)
```

3.295 $\int \frac{e^{c(a+bx)}}{\sqrt{\cosh^2(ac+bcx)}}\,dx$

3.295.1 Optimal result

Integrand size = 25, antiderivative size = 44

$$\int \frac{e^{c(a+bx)}}{\sqrt{\cosh^2(ac+bcx)}}\,dx=\frac{\cosh(ac+bcx)\log\left(1+e^{2c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}$$

output
```
cosh(b*c*x+a*c)*ln(1+exp(2*c*(b*x+a)))/b/c/(cosh(b*c*x+a*c)^2)^(1/2)
```

3.295.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{e^{c(a+bx)}}{\sqrt{\cosh^2(ac+bcx)}}\,dx=\frac{\cosh(c(a+bx))\log\left(1+e^{2c(a+bx)}\right)}{bc\sqrt{\cosh^2(c(a+bx))}}$$

input
```
Integrate[E^(c*(a + b*x))/Sqrt[Cosh[a*c + b*c*x]^2],x]
```

output
```
(Cosh[c*(a + b*x)]*Log[1 + E^(2*c*(a + b*x))])/(b*c*Sqrt[Cosh[c*(a + b*x)]
^2])
```

3.295.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {7271, 2720, 27, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c(a+bx)}}{\sqrt{\cosh^2(ac+bcx)}}dx$$

$$\Big\downarrow \text{7271}$$

$$\frac{\cosh(ac+bcx)\int e^{c(a+bx)}\text{sech}(ac+bxc)dx}{\sqrt{\cosh^2(ac+bcx)}}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\cosh(ac+bcx)\int \frac{2e^{c(a+bx)}}{1+e^{2c(a+bx)}}de^{c(a+bx)}}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\cosh(ac+bcx)\int \frac{e^{c(a+bx)}}{1+e^{2c(a+bx)}}de^{c(a+bx)}}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$\Big\downarrow \text{240}$$

$$\frac{\log\left(e^{2c(a+bx)}+1\right)\cosh(ac+bcx)}{bc\sqrt{\cosh^2(ac+bcx)}}$$

input
```
Int[E^(c*(a + b*x))/Sqrt[Cosh[a*c + b*c*x]^2],x]
```

output
```
(Cosh[a*c + b*c*x]*Log[1 + E^(2*c*(a + b*x))])/(b*c*Sqrt[Cosh[a*c + b*c*x]
^2])
```

3.295.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

3.295.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.66

method	result	size
default	$\operatorname{csgn}\left(\cosh\left(c(bx+a)\right)\right)\left(x+\frac{\ln\left(\cosh\left(c(bx+a)\right)\right)}{cb}\right)$	29
risch	$\frac{\ln\left(\mathrm{e}^{2bcx}+\mathrm{e}^{-2ac}\right)\left(1+\mathrm{e}^{2c(bx+a)}\right)\mathrm{e}^{-c(bx+a)}}{bc\sqrt{\left(1+\mathrm{e}^{2c(bx+a)}\right)^{2}\mathrm{e}^{-2c(bx+a)}}}$	66

input
```
int(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
csgn(cosh(c*(b*x+a)))*(x+1/c/b*ln(cosh(c*(b*x+a))))
```

3.295.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{e^{c(a+bx)}}{\sqrt{\cosh^2(ac+bcx)}}\,dx = \frac{\log\left(\frac{2\cosh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{bc}$$

input
```
integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

output
```
log(2*cosh(b*c*x + a*c)/(cosh(b*c*x + a*c) - sinh(b*c*x + a*c)))/(b*c)
```

3.295.6 Sympy [F]

$$\int \frac{e^{c(a+bx)}}{\sqrt{\cosh^2(ac+bcx)}}\,dx = e^{ac}\int \frac{e^{bcx}}{\sqrt{\cosh^2(ac+bcx)}}\,dx$$

input
```
integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)**2)**(1/2),x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)/sqrt(cosh(a*c + b*c*x)**2), x)
```

3.295.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.48

$$\int \frac{e^{c(a+bx)}}{\sqrt{\cosh^2(ac+bcx)}}\,dx = \frac{\log\left(e^{(2\,bcx+2\,ac)}+1\right)}{bc}$$

input
```
integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

output
```
log(e^(2*b*c*x + 2*a*c) + 1)/(b*c)
```

3.295.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.45

$$\int \frac{e^{c(a+bx)}}{\sqrt{\cosh^2(ac+bcx)}}\,dx = \frac{\log\left(e^{(2\,bcx)} + e^{(-2\,ac)}\right)}{bc}$$

input
```
integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

output
```
log(e^(2*b*c*x) + e^(-2*a*c))/(b*c)
```

3.295.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{c(a+bx)}}{\sqrt{\cosh^2(ac+bcx)}}\,dx = \int \frac{e^{c\,(a+b\,x)}}{\sqrt{\cosh\left(a\,c+b\,c\,x\right)^2}}\,dx$$

input
```
int(exp(c*(a + b*x))/(cosh(a*c + b*c*x)^2)^(1/2),x)
```

output
```
int(exp(c*(a + b*x))/(cosh(a*c + b*c*x)^2)^(1/2), x)
```

3.296 $\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{3/2}}\,dx$

3.296.1 Optimal result

Integrand size = 25, antiderivative size = 56

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{3/2}}\,dx = \frac{2e^{4c(a+bx)}\cosh(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^2\sqrt{\cosh^2(ac+bcx)}}$$

output
```
2*exp(4*c*(b*x+a))*cosh(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^2/(cosh(b*c*x+
a*c)^2)^(1/2)
```

3.296.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{3/2}}\,dx = \frac{4e^{5c(a+bx)}\sqrt{\cosh^2(c(a+bx))}}{bc\left(1+e^{2c(a+bx)}\right)^3}$$

input
```
Integrate[E^(c*(a + b*x))/(Cosh[a*c + b*c*x]^2)^(3/2),x]
```

output
```
(4*E^(5*c*(a + b*x))*Sqrt[Cosh[c*(a + b*x)]^2])/(b*c*(1 + E^(2*c*(a + b*x)
))^3)
```

3.296.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {7271, 2720, 27, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{3/2}}\,dx$$

$$\Big\downarrow \text{7271}$$

$$\frac{\cosh(ac+bcx)\int e^{c(a+bx)}\text{sech}^3(ac+bxc)dx}{\sqrt{\cosh^2(ac+bcx)}}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\cosh(ac+bcx)\int \frac{8e^{3c(a+bx)}}{\left(1+e^{2c(a+bx)}\right)^3}de^{c(a+bx)}}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{8\cosh(ac+bcx)\int \frac{e^{3c(a+bx)}}{\left(1+e^{2c(a+bx)}\right)^3}de^{c(a+bx)}}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$\Big\downarrow \text{242}$$

$$\frac{2e^{4c(a+bx)}\cosh(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)^2\sqrt{\cosh^2(ac+bcx)}}$$

input
```
Int[E^(c*(a + b*x))/(Cosh[a*c + b*c*x]^2)^(3/2),x]
```

output
```
(2*E^(4*c*(a + b*x))*Cosh[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a + b*x)))^2*Sqr
t[Cosh[a*c + b*c*x]^2])
```

3.296.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

3.296.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{\operatorname{csgn}(\cosh(c(bx+a)))\left(\frac{\tanh(c(bx+a))^2}{2}+\tanh(c(bx+a))\right)}{cb}$	38
risch	$-\frac{2\left(2\,\mathrm{e}^{2c(bx+a)}+1\right)\mathrm{e}^{-c(bx+a)}}{bc\sqrt{\left(1+\mathrm{e}^{2c(bx+a)}\right)^2\mathrm{e}^{-2c(bx+a)}}\,\left(1+\mathrm{e}^{2c(bx+a)}\right)}$	69

input
```
int(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
csgn(cosh(c*(b*x+a)))/c/b*(1/2*tanh(c*(b*x+a))^2+tanh(c*(b*x+a)))
```

3.296.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(52) = 104.

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.14

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{3/2}}\,dx = -\frac{2\,(3\,\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right))}{bc\cosh\left(bcx+ac\right)^3+3\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^2+bc\sinh\left(bcx+ac\right)^3+3\,bc\cosh\left(bcx+ac\right)+(3$$

input
```
integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

output
```
-2*(3*cosh(b*c*x + a*c) + sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^3 + 3*
b*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 + b*c*sinh(b*c*x + a*c)^3 + 3*b*
c*cosh(b*c*x + a*c) + (3*b*c*cosh(b*c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c))
```

3.296.6 Sympy [F]

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{3/2}}\,dx = e^{ac}\int \frac{e^{bcx}}{\left(\cosh^2\left(ac+bcx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)**2)**(3/2),x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)/(cosh(a*c + b*c*x)**2)**(3/2), x)
```

3.296.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.50

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{3/2}}\,dx = -\frac{4\,e^{(2\,bcx+2\,ac)}}{bc(e^{(4\,bcx+4\,ac)}+2\,e^{(2\,bcx+2\,ac)}+1)} - \frac{2}{bc(e^{(4\,bcx+4\,ac)}+2\,e^{(2\,bcx+2\,ac)}+1)}$$

input
```
integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

output
```
-4*e^(2*b*c*x + 2*a*c)/(b*c*(e^(4*b*c*x + 4*a*c) + 2*e^(2*b*c*x + 2*a*c) +
 1)) - 2/(b*c*(e^(4*b*c*x + 4*a*c) + 2*e^(2*b*c*x + 2*a*c) + 1))
```

3.296.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.68

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{3/2}}\,dx = -\frac{2\left(2\,e^{(2\,bcx+2\,ac)}+1\right)}{bc\left(e^{(2\,bcx+2\,ac)}+1\right)^2}$$

input
```
integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

output
```
-2*(2*e^(2*b*c*x + 2*a*c) + 1)/(b*c*(e^(2*b*c*x + 2*a*c) + 1)^2)
```

3.296.9 Mupad [B] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.36

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{3/2}}\,dx = -\frac{4\,\mathrm{e}^{a\,c+b\,c\,x}\,\left(2\,\mathrm{e}^{2\,a\,c+2\,b\,c\,x}+1\right)\,\sqrt{{\left(\frac{\mathrm{e}^{a\,c+b\,c\,x}}{2}+\frac{\mathrm{e}^{-a\,c-b\,c\,x}}{2}\right)}^2}}{b\,c\,{\left(\mathrm{e}^{2\,a\,c+2\,b\,c\,x}+1\right)}^3}$$

input
```
int(exp(c*(a + b*x))/(cosh(a*c + b*c*x)^2)^(3/2),x)
```

output
```
-(4*exp(a*c + b*c*x)*(2*exp(2*a*c + 2*b*c*x) + 1)*((exp(a*c + b*c*x)/2 + e
xp(- a*c - b*c*x)/2)^2)^(1/2))/(b*c*(exp(2*a*c + 2*b*c*x) + 1)^3)
```

3.297 $\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{5/2}}\,dx$

3.297.1 Optimal result

Integrand size = 25, antiderivative size = 141

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{5/2}}\,dx = -\frac{4\cosh(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^4\sqrt{\cosh^2(ac+bcx)}} + \frac{32\cosh(ac+bcx)}{3bc\left(1+e^{2c(a+bx)}\right)^3\sqrt{\cosh^2(ac+bcx)}} - \frac{8\cosh(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^2\sqrt{\cosh^2(ac+bcx)}}$$

output
```
-4*cosh(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^4/(cosh(b*c*x+a*c)^2)^(1/2)+32
/3*cosh(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^3/(cosh(b*c*x+a*c)^2)^(1/2)-8*
cosh(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^2/(cosh(b*c*x+a*c)^2)^(1/2)
```

3.297.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.51

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{5/2}}\,dx = -\frac{4\left(1+4e^{2c(a+bx)}+6e^{4c(a+bx)}\right)\cosh(c(a+bx))}{3bc\left(1+e^{2c(a+bx)}\right)^4\sqrt{\cosh^2(c(a+bx))}}$$

input
```
Integrate[E^(c*(a + b*x))/(Cosh[a*c + b*c*x]^2)^(5/2),x]
```

output
```
(-4*(1 + 4*E^(2*c*(a + b*x)) + 6*E^(4*c*(a + b*x)))*Cosh[c*(a + b*x)])/(3*
b*c*(1 + E^(2*c*(a + b*x)))^4*Sqrt[Cosh[c*(a + b*x)]^2])
```

3.297.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.62, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {7271, 2720, 27, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{5/2}}dx$$

$$\Big\downarrow \text{7271}$$

$$\frac{\cosh(ac+bcx)\int e^{c(a+bx)}\text{sech}^5(ac+bxc)dx}{\sqrt{\cosh^2(ac+bcx)}}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\cosh(ac+bcx)\int \frac{32e^{5c(a+bx)}}{\left(1+e^{2c(a+bx)}\right)^5}de^{c(a+bx)}}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{32\cosh(ac+bcx)\int \frac{e^{5c(a+bx)}}{\left(1+e^{2c(a+bx)}\right)^5}de^{c(a+bx)}}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$\Big\downarrow \text{243}$$

$$\frac{16\cosh(ac+bcx)\int \frac{e^{2c(a+bx)}}{\left(1+e^{2c(a+bx)}\right)^5}de^{2c(a+bx)}}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$\Big\downarrow \text{53}$$

$$\frac{16\cosh(ac+bcx)\int \left(\frac{1}{\left(1+e^{2c(a+bx)}\right)^3}-\frac{2}{\left(1+e^{2c(a+bx)}\right)^4}+\frac{1}{\left(1+e^{2c(a+bx)}\right)^5}\right)de^{2c(a+bx)}}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{16\left(-\frac{1}{2\left(e^{2c(a+bx)}+1\right)^2}+\frac{2}{3\left(e^{2c(a+bx)}+1\right)^3}-\frac{1}{4\left(e^{2c(a+bx)}+1\right)^4}\right)\cosh(ac+bcx)}{bc\sqrt{\cosh^2(ac+bcx)}}$$

input
```
Int[E^(c*(a + b*x))/(Cosh[a*c + b*c*x]^2)^(5/2),x]
```

output
```
(16*(-1/4*1/(1 + E^(2*c*(a + b*x)))^4 + 2/(3*(1 + E^(2*c*(a + b*x)))^3) -
1/(2*(1 + E^(2*c*(a + b*x)))^2))*Cosh[a*c + b*c*x])/(b*c*Sqrt[Cosh[a*c + b
*c*x]^2])
```

3.297.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

3.297. $\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{5/2}}\,dx$

3.297.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.46

method	result	size
default	$-\frac{\operatorname{csgn}(\cosh(c(bx+a)))\left(\frac{\tanh(c(bx+a))^4}{4}+\frac{\tanh(c(bx+a))^3}{3}-\frac{\tanh(c(bx+a))^2}{2}-\tanh(c(bx+a))\right)}{cb}$	65
risch	$-\frac{4(6\,\mathrm{e}^{4c(bx+a)}+4\,\mathrm{e}^{2c(bx+a)}+1)\mathrm{e}^{-c(bx+a)}}{3bc\sqrt{(1+\mathrm{e}^{2c(bx+a)})^2\mathrm{e}^{-2c(bx+a)}}\,(1+\mathrm{e}^{2c(bx+a)})^3}$	80

input
```
int(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-csgn(cosh(c*(b*x+a)))/c/b*(1/4*tanh(c*(b*x+a))^4+1/3*tanh(c*(b*x+a))^3-1/
2*tanh(c*(b*x+a))^2-tanh(c*(b*x+a)))
```

3.297.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. 2(130) = 260.

Time = 0.25 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.23

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{5/2}}\,dx =$$

$$-\frac{}{3\left(bc\cosh\left(bcx+ac\right)^6+6\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^5+bc\sinh\left(bcx+ac\right)^6+4\,bc\cosh\left(bcx+ac\right)^4\right.}$$

input
```
integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

output
```
-4/3*(7*cosh(b*c*x + a*c)^2 + 10*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 7*s
inh(b*c*x + a*c)^2 + 4)/(b*c*cosh(b*c*x + a*c)^6 + 6*b*c*cosh(b*c*x + a*c)
*sinh(b*c*x + a*c)^5 + b*c*sinh(b*c*x + a*c)^6 + 4*b*c*cosh(b*c*x + a*c)^4
 + (15*b*c*cosh(b*c*x + a*c)^2 + 4*b*c)*sinh(b*c*x + a*c)^4 + 7*b*c*cosh(b
*c*x + a*c)^2 + 4*(5*b*c*cosh(b*c*x + a*c)^3 + 4*b*c*cosh(b*c*x + a*c))*si
nh(b*c*x + a*c)^3 + (15*b*c*cosh(b*c*x + a*c)^4 + 24*b*c*cosh(b*c*x + a*c)
^2 + 7*b*c)*sinh(b*c*x + a*c)^2 + 4*b*c + 2*(3*b*c*cosh(b*c*x + a*c)^5 + 8
*b*c*cosh(b*c*x + a*c)^3 + 5*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c))
```

3.297.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)**2)**(5/2),x)
```

output
```
Timed out
```

3.297.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.48

$$\begin{aligned}\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{5/2}}\,dx = &-\frac{8\,e^{(4\,bcx+4\,ac)}}{bc(e^{(8\,bcx+8\,ac)}+4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,bcx+2\,ac)}+1)}\\&-\frac{16\,e^{(2\,bcx+2\,ac)}}{3\,bc(e^{(8\,bcx+8\,ac)}+4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,bcx+2\,ac)}+1)}\\&-\frac{4}{3\,bc(e^{(8\,bcx+8\,ac)}+4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,bcx+2\,ac)}+1)}\end{aligned}$$

input
```
integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

output
```
-8*e^(4*b*c*x + 4*a*c)/(b*c*(e^(8*b*c*x + 8*a*c) + 4*e^(6*b*c*x + 6*a*c) +
 6*e^(4*b*c*x + 4*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1)) - 16/3*e^(2*b*c*x + 2
*a*c)/(b*c*(e^(8*b*c*x + 8*a*c) + 4*e^(6*b*c*x + 6*a*c) + 6*e^(4*b*c*x + 4
*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1)) - 4/3/(b*c*(e^(8*b*c*x + 8*a*c) + 4*e^
(6*b*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1))
```

3.297.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.36

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{5/2}}\,dx = -\frac{4\left(6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,bcx+2\,ac)}+1\right)}{3\,bc(e^{(2\,bcx+2\,ac)}+1)^4}$$

input
```
integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

output
```
-4/3*(6*e^(4*b*c*x + 4*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1)/(b*c*(e^(2*b*c*x
+ 2*a*c) + 1)^4)
```

3.297.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.63

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{5/2}}\,dx = -\frac{8\,e^{a\,c+b\,c\,x}\,\sqrt{\left(\frac{e^{a\,c+b\,c\,x}}{2}+\frac{e^{-a\,c-b\,c\,x}}{2}\right)^2}\,\left(4\,e^{2\,a\,c+2\,b\,c\,x}+6\,e^{4\,a\,c+4\,b\,c\,x}+1\right)}{3\,b\,c\,\left(e^{2\,a\,c+2\,b\,c\,x}+1\right)^5}$$

input
```
int(exp(c*(a + b*x))/(cosh(a*c + b*c*x)^2)^(5/2),x)
```

output
```
-(8*exp(a*c + b*c*x)*((exp(a*c + b*c*x)/2 + exp(- a*c - b*c*x)/2)^2)^(1/2)
*(4*exp(2*a*c + 2*b*c*x) + 6*exp(4*a*c + 4*b*c*x) + 1))/(3*b*c*(exp(2*a*c
+ 2*b*c*x) + 1)^5)
```

3.298 $\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{7/2}}\,dx$

3.298.1 Optimal result

Integrand size = 25, antiderivative size = 191

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{7/2}}\,dx = \frac{32\cosh(ac+bcx)}{3bc\left(1+e^{2c(a+bx)}\right)^6\sqrt{\cosh^2(ac+bcx)}} - \frac{192\cosh(ac+bcx)}{5bc\left(1+e^{2c(a+bx)}\right)^5\sqrt{\cosh^2(ac+bcx)}} + \frac{48\cosh(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^4\sqrt{\cosh^2(ac+bcx)}} - \frac{64\cosh(ac+bcx)}{3bc\left(1+e^{2c(a+bx)}\right)^3\sqrt{\cosh^2(ac+bcx)}}$$

output
```
32/3*cosh(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^6/(cosh(b*c*x+a*c)^2)^(1/2)-
192/5*cosh(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^5/(cosh(b*c*x+a*c)^2)^(1/2)
+48*cosh(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^4/(cosh(b*c*x+a*c)^2)^(1/2)-6
4/3*cosh(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^3/(cosh(b*c*x+a*c)^2)^(1/2)
```

3.298.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.44

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{7/2}}\,dx = -\frac{16\left(1+6e^{2c(a+bx)}+15e^{4c(a+bx)}+20e^{6c(a+bx)}\right)\cosh(c(a+bx))}{15bc\left(1+e^{2c(a+bx)}\right)^6\sqrt{\cosh^2(c(a+bx))}}$$

input
```
Integrate[E^(c*(a + b*x))/(Cosh[a*c + b*c*x]^2)^(7/2),x]
```

output
```
(-16*(1 + 6*E^(2*c*(a + b*x)) + 15*E^(4*c*(a + b*x)) + 20*E^(6*c*(a + b*x)
))*Cosh[c*(a + b*x)])/(15*b*c*(1 + E^(2*c*(a + b*x)))^6*Sqrt[Cosh[c*(a + b
*x)]^2])
```

3.298.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.55, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {7271, 2720, 27, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{7/2}}dx$$

$$\Big\downarrow \text{7271}$$

$$\frac{\cosh(ac+bcx)\int e^{c(a+bx)}\text{sech}^7(ac+bxc)dx}{\sqrt{\cosh^2(ac+bcx)}}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\cosh(ac+bcx)\int \frac{128e^{7c(a+bx)}}{\left(1+e^{2c(a+bx)}\right)^7}de^{c(a+bx)}}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{128\cosh(ac+bcx)\int \frac{e^{7c(a+bx)}}{\left(1+e^{2c(a+bx)}\right)^7}de^{c(a+bx)}}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$\Big\downarrow \text{243}$$

$$\frac{64\cosh(ac+bcx)\int\frac{e^{3c(a+bx)}}{\left(1+e^{2c(a+bx)}\right)^7}de^{2c(a+bx)}}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$\Big\downarrow \text{53}$$

$$\frac{64\cosh(ac+bcx)\int\left(\frac{1}{\left(1+e^{2c(a+bx)}\right)^4}-\frac{3}{\left(1+e^{2c(a+bx)}\right)^5}+\frac{3}{\left(1+e^{2c(a+bx)}\right)^6}-\frac{1}{\left(1+e^{2c(a+bx)}\right)^7}\right)de^{2c(a+bx)}}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{64\left(-\frac{1}{3\left(e^{2c(a+bx)}+1\right)^3}+\frac{3}{4\left(e^{2c(a+bx)}+1\right)^4}-\frac{3}{5\left(e^{2c(a+bx)}+1\right)^5}+\frac{1}{6\left(e^{2c(a+bx)}+1\right)^6}\right)\cosh(ac+bcx)}{bc\sqrt{\cosh^2(ac+bcx)}}$$

input
```
Int[E^(c*(a + b*x))/(Cosh[a*c + b*c*x]^2)^(7/2),x]
```

output
```
(64*(1/(6*(1 + E^(2*c*(a + b*x)))^6) - 3/(5*(1 + E^(2*c*(a + b*x)))^5) + 3
/(4*(1 + E^(2*c*(a + b*x)))^4) - 1/(3*(1 + E^(2*c*(a + b*x)))^3))*Cosh[a*c
 + b*c*x])/(b*c*Sqrt[Cosh[a*c + b*c*x]^2])
```

3.298.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.298. $\int\frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{7/2}}\,dx$

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

3.298.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.45

method	result	size
default	$\frac{\operatorname{csgn}(\cosh(c(bx+a)))\left(\frac{\tanh(c(bx+a))^6}{6}+\frac{\tanh(c(bx+a))^5}{5}-\frac{\tanh(c(bx+a))^4}{2}-\frac{2\tanh(c(bx+a))^3}{3}+\frac{\tanh(c(bx+a))^2}{2}+\tanh(c(bx+a))\right)}{cb}$	86
risch	$-\frac{16\left(20\,\mathrm{e}^{6c(bx+a)}+15\,\mathrm{e}^{4c(bx+a)}+6\,\mathrm{e}^{2c(bx+a)}+1\right)\mathrm{e}^{-c(bx+a)}}{15bc\sqrt{\left(1+\mathrm{e}^{2c(bx+a)}\right)^2\mathrm{e}^{-2c(bx+a)}}\left(1+\mathrm{e}^{2c(bx+a)}\right)^5}$	91

input
```
int(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
csgn(cosh(c*(b*x+a)))/c/b*(1/6*tanh(c*(b*x+a))^6+1/5*tanh(c*(b*x+a))^5-1/2
*tanh(c*(b*x+a))^4-2/3*tanh(c*(b*x+a))^3+1/2*tanh(c*(b*x+a))^2+tanh(c*(b*x
+a)))
```

3.298.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 589 vs. 2(173) = 346.

Time = 0.26 (sec) , antiderivative size = 589, normalized size of antiderivative = 3.08

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{7/2}}\,dx =$$

$$-\frac{}{15\left(bc\cosh\left(bcx+ac\right)^9+9\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^8+bc\sinh\left(bcx+ac\right)^9+6\,bc\cosh\left(bcx+ac\right)^7\right.}$$

input
```
integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(7/2),x, algorithm="fricas")
```

output
```
-16/15*(21*cosh(b*c*x + a*c)^3 + 63*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2
+ 19*sinh(b*c*x + a*c)^3 + 3*(19*cosh(b*c*x + a*c)^2 + 3)*sinh(b*c*x + a*c
) + 21*cosh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^9 + 9*b*c*cosh(b*c*x + a*
c)*sinh(b*c*x + a*c)^8 + b*c*sinh(b*c*x + a*c)^9 + 6*b*c*cosh(b*c*x + a*c)
^7 + 6*(6*b*c*cosh(b*c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c)^7 + 15*b*c*cosh
(b*c*x + a*c)^5 + 42*(2*b*c*cosh(b*c*x + a*c)^3 + b*c*cosh(b*c*x + a*c))*s
inh(b*c*x + a*c)^6 + 3*(42*b*c*cosh(b*c*x + a*c)^4 + 42*b*c*cosh(b*c*x + a
*c)^2 + 5*b*c)*sinh(b*c*x + a*c)^5 + 21*b*c*cosh(b*c*x + a*c)^3 + 3*(42*b*
c*cosh(b*c*x + a*c)^5 + 70*b*c*cosh(b*c*x + a*c)^3 + 25*b*c*cosh(b*c*x + a
*c))*sinh(b*c*x + a*c)^4 + (84*b*c*cosh(b*c*x + a*c)^6 + 210*b*c*cosh(b*c*
x + a*c)^4 + 150*b*c*cosh(b*c*x + a*c)^2 + 19*b*c)*sinh(b*c*x + a*c)^3 + 2
1*b*c*cosh(b*c*x + a*c) + 3*(12*b*c*cosh(b*c*x + a*c)^7 + 42*b*c*cosh(b*c*
x + a*c)^5 + 50*b*c*cosh(b*c*x + a*c)^3 + 21*b*c*cosh(b*c*x + a*c))*sinh(b
*c*x + a*c)^2 + 3*(3*b*c*cosh(b*c*x + a*c)^8 + 14*b*c*cosh(b*c*x + a*c)^6
+ 25*b*c*cosh(b*c*x + a*c)^4 + 19*b*c*cosh(b*c*x + a*c)^2 + 3*b*c)*sinh(b*
c*x + a*c))
```

3.298.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)**2)**(7/2),x)
```

output
```
Timed out
```

3.298.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(173) = 346$.

Time = 0.21 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.02

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{7/2}}\,dx = -\frac{64\,e^{(6\,bcx+6\,ac)}}{3\,bc(e^{(12\,bcx+12\,ac)}+6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}+20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1)} - \frac{16\,e^{(4\,bcx+4\,ac)}}{bc(e^{(12\,bcx+12\,ac)}+6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}+20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1)} - \frac{32\,e^{(2\,bcx+2\,ac)}}{5\,bc(e^{(12\,bcx+12\,ac)}+6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}+20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1)} - \frac{16}{15\,bc(e^{(12\,bcx+12\,ac)}+6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}+20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1)}$$

input
```
integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(7/2),x, algorithm="maxima")
```

output
```
-64/3*e^(6*b*c*x + 6*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) + 6*e^(10*b*c*x + 10
*a*c) + 15*e^(8*b*c*x + 8*a*c) + 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x +
4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1)) - 16*e^(4*b*c*x + 4*a*c)/(b*c*(e^(12*
b*c*x + 12*a*c) + 6*e^(10*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) + 20*e^
(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1)) -
 32/5*e^(2*b*c*x + 2*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) + 6*e^(10*b*c*x + 10
*a*c) + 15*e^(8*b*c*x + 8*a*c) + 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x +
4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1)) - 16/15/(b*c*(e^(12*b*c*x + 12*a*c) +
 6*e^(10*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) + 20*e^(6*b*c*x + 6*a*c)
 + 15*e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1))
```

3.298.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.34

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{7/2}}\,dx = -\frac{16\,(20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1)}{15\,bc(e^{(2\,bcx+2\,ac)}+1)^6}$$

input
```
integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(7/2),x, algorithm="giac")
```

output
```
-16/15*(20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2
*a*c) + 1)/(b*c*(e^(2*b*c*x + 2*a*c) + 1)^6)
```

3.298.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.81

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{7/2}}\,dx = \frac{96\,\mathrm{e}^{2\,a\,c+2\,b\,c\,x}\,\sqrt{{\left(\frac{\mathrm{e}^{a\,c+b\,c\,x}}{2}+\frac{\mathrm{e}^{-a\,c-b\,c\,x}}{2}\right)}^2}}{b\,c\,\left(\mathrm{e}^{a\,c+b\,c\,x}+\mathrm{e}^{3\,a\,c+3\,b\,c\,x}\right)\,{\left(\mathrm{e}^{2\,a\,c+2\,b\,c\,x}+1\right)}^4} - \frac{128\,\mathrm{e}^{2\,a\,c+2\,b\,c\,x}\,\sqrt{{\left(\frac{\mathrm{e}^{a\,c+b\,c\,x}}{2}+\frac{\mathrm{e}^{-a\,c-b\,c\,x}}{2}\right)}^2}}{3\,b\,c\,\left(\mathrm{e}^{a\,c+b\,c\,x}+\mathrm{e}^{3\,a\,c+3\,b\,c\,x}\right)\,{\left(\mathrm{e}^{2\,a\,c+2\,b\,c\,x}+1\right)}^3} - \frac{384\,\mathrm{e}^{2\,a\,c+2\,b\,c\,x}\,\sqrt{{\left(\frac{\mathrm{e}^{a\,c+b\,c\,x}}{2}+\frac{\mathrm{e}^{-a\,c-b\,c\,x}}{2}\right)}^2}}{5\,b\,c\,\left(\mathrm{e}^{a\,c+b\,c\,x}+\mathrm{e}^{3\,a\,c+3\,b\,c\,x}\right)\,{\left(\mathrm{e}^{2\,a\,c+2\,b\,c\,x}+1\right)}^5} + \frac{64\,\mathrm{e}^{2\,a\,c+2\,b\,c\,x}\,\sqrt{{\left(\frac{\mathrm{e}^{a\,c+b\,c\,x}}{2}+\frac{\mathrm{e}^{-a\,c-b\,c\,x}}{2}\right)}^2}}{3\,b\,c\,\left(\mathrm{e}^{a\,c+b\,c\,x}+\mathrm{e}^{3\,a\,c+3\,b\,c\,x}\right)\,{\left(\mathrm{e}^{2\,a\,c+2\,b\,c\,x}+1\right)}^6}$$

input
```
int(exp(c*(a + b*x))/(cosh(a*c + b*c*x)^2)^(7/2),x)
```

output
```
(96*exp(2*a*c + 2*b*c*x)*((exp(a*c + b*c*x)/2 + exp(- a*c - b*c*x)/2)^2)^(
1/2))/(b*c*(exp(a*c + b*c*x) + exp(3*a*c + 3*b*c*x))*(exp(2*a*c + 2*b*c*x)
 + 1)^4) - (128*exp(2*a*c + 2*b*c*x)*((exp(a*c + b*c*x)/2 + exp(- a*c - b*
c*x)/2)^2)^(1/2))/(3*b*c*(exp(a*c + b*c*x) + exp(3*a*c + 3*b*c*x))*(exp(2*
a*c + 2*b*c*x) + 1)^3) - (384*exp(2*a*c + 2*b*c*x)*((exp(a*c + b*c*x)/2 +
exp(- a*c - b*c*x)/2)^2)^(1/2))/(5*b*c*(exp(a*c + b*c*x) + exp(3*a*c + 3*b
*c*x))*(exp(2*a*c + 2*b*c*x) + 1)^5) + (64*exp(2*a*c + 2*b*c*x)*((exp(a*c
+ b*c*x)/2 + exp(- a*c - b*c*x)/2)^2)^(1/2))/(3*b*c*(exp(a*c + b*c*x) + ex
p(3*a*c + 3*b*c*x))*(exp(2*a*c + 2*b*c*x) + 1)^6)
```

3.299 $\int e^x \cosh(a+bx)\,dx$

3.299.1 Optimal result

Integrand size = 10, antiderivative size = 41

$$\int e^x \cosh(a+bx)\,dx = \frac{e^x \cosh(a+bx)}{1-b^2} - \frac{be^x \sinh(a+bx)}{1-b^2}$$

output
```
exp(x)*cosh(b*x+a)/(-b^2+1)-b*exp(x)*sinh(b*x+a)/(-b^2+1)
```

3.299.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int e^x \cosh(a+bx)\,dx = \frac{e^x(-\cosh(a+bx)+b\sinh(a+bx))}{-1+b^2}$$

input
```
Integrate[E^x*Cosh[a + b*x],x]
```

output
```
(E^x*(-Cosh[a + b*x] + b*Sinh[a + b*x]))/(-1 + b^2)
```

3.299.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5998}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \cosh(a+bx)\, dx$$
$$\Big\downarrow \text{5998}$$
$$\frac{e^x \cosh(a+bx)}{1-b^2} - \frac{be^x \sinh(a+bx)}{1-b^2}$$

input
```
Int[E^x*Cosh[a + b*x],x]
```

output
```
(E^x*Cosh[a + b*x])/(1 - b^2) - (b*E^x*Sinh[a + b*x])/(1 - b^2)
```

3.299.3.1 Defintions of rubi rules used

rule 5998
```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

3.299.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

method	result	size
parallelrisch	$\frac{{\mathrm e}^{x} \left(b \sinh \left(b x +a \right)-\cosh \left(b x +a \right)\right)}{b^{2}-1}$	28
risch	$\frac{{\mathrm e}^{b x +a +x}}{2+2 b}-\frac{{\mathrm e}^{-b x -a +x}}{2 \left(b -1\right)}$	33
default	$\frac{\sinh \left(x \left(b -1\right)+a \right)}{2 b -2}+\frac{\sinh \left(\left(1+b \right) x +a \right)}{2+2 b}-\frac{\cosh \left(x \left(b -1\right)+a \right)}{2 \left(b -1\right)}+\frac{\cosh \left(\left(1+b \right) x +a \right)}{2+2 b}$	62

input
```
int(exp(x)*cosh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
exp(x)/(b^2-1)*(b*sinh(b*x+a)-cosh(b*x+a))
```

3.299.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10

$$\int e^x \cosh(a+bx)\,dx = -\frac{\cosh(bx+a)\cosh(x) - (b\cosh(x)+b\sinh(x))\sinh(bx+a) + \cosh(bx+a)\sinh(x)}{b^2-1}$$

input
```
integrate(exp(x)*cosh(b*x+a),x, algorithm="fricas")
```

output
```
-(cosh(b*x + a)*cosh(x) - (b*cosh(x) + b*sinh(x))*sinh(b*x + a) + cosh(b*x
 + a)*sinh(x))/(b^2 - 1)
```

3.299.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(31) = 62$.

Time = 0.23 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.41

$$\int e^x \cosh(a+bx)\,dx = \begin{cases} \frac{xe^x\sinh(a-x)}{2} + \frac{xe^x\cosh(a-x)}{2} - \frac{e^x\sinh(a-x)}{2} & \text{for } b=-1 \\ -\frac{xe^x\sinh(a+x)}{2} + \frac{xe^x\cosh(a+x)}{2} + \frac{e^x\cosh(a+x)}{2} & \text{for } b=1 \\ \frac{be^x\sinh(a+bx)}{b^2-1} - \frac{e^x\cosh(a+bx)}{b^2-1} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(x)*cosh(b*x+a),x)
```

output
```
Piecewise((x*exp(x)*sinh(a - x)/2 + x*exp(x)*cosh(a - x)/2 - exp(x)*sinh(a
 - x)/2, Eq(b, -1)), (-x*exp(x)*sinh(a + x)/2 + x*exp(x)*cosh(a + x)/2 + e
xp(x)*cosh(a + x)/2, Eq(b, 1)), (b*exp(x)*sinh(a + b*x)/(b**2 - 1) - exp(x
)*cosh(a + b*x)/(b**2 - 1), True))
```

3.299.7 Maxima [F(-2)]

Exception generated.

$$\int e^x \cosh(a+bx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(exp(x)*cosh(b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-b>0)', see `assume?` for more d
etails)Is
```

3.299.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int e^x \cosh(a+bx)\,dx = \frac{e^{(bx+a+x)}}{2\,(b+1)} - \frac{e^{(-bx-a+x)}}{2\,(b-1)}$$

input
```
integrate(exp(x)*cosh(b*x+a),x, algorithm="giac")
```

output
```
1/2*e^(b*x + a + x)/(b + 1) - 1/2*e^(-b*x - a + x)/(b - 1)
```

3.299.9 Mupad [B] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10

$$\int e^x \cosh(a+bx)\,dx = -\frac{e^{x-a-b\,x}\,\left(b+e^{2\,a+2\,b\,x}-b\,e^{2\,a+2\,b\,x}+1\right)}{2\,\left(b^2-1\right)}$$

input
```
int(cosh(a + b*x)*exp(x),x)
```

output
```
-(exp(x - a - b*x)*(b + exp(2*a + 2*b*x) - b*exp(2*a + 2*b*x) + 1))/(2*(b^
2 - 1))
```

3.300 $\int e^x \cosh(a + cx^2)\, dx$

3.300.1 Optimal result

Integrand size = 12, antiderivative size = 85

$$\int e^x \cosh(a + cx^2)\, dx = -\frac{e^{-a+\frac{1}{4c}}\sqrt{\pi}\operatorname{erf}\left(\frac{1-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}} + \frac{e^{a-\frac{1}{4c}}\sqrt{\pi}\operatorname{erfi}\left(\frac{1+2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

output
```
-1/4*exp(-a+1/4/c)*erf(1/2*(-2*c*x+1)/c^(1/2))*Pi^(1/2)/c^(1/2)+1/4*exp(a-
1/4/c)*erfi(1/2*(2*c*x+1)/c^(1/2))*Pi^(1/2)/c^(1/2)
```

3.300.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.93

$$\int e^x \cosh(a + cx^2)\, dx = \frac{e^{-\frac{1}{4}/c}\sqrt{\pi}\left(e^{\frac{1}{2}/c}\operatorname{erf}\left(\frac{-1+2cx}{2\sqrt{c}}\right)(\cosh(a) - \sinh(a)) + \operatorname{erfi}\left(\frac{1+2cx}{2\sqrt{c}}\right)(\cosh(a) + \sinh(a))\right)}{4\sqrt{c}}$$

input
```
Integrate[E^x*Cosh[a + c*x^2],x]
```

output
```
(Sqrt[Pi]*(E^(1/(2*c))*Erf[(-1 + 2*c*x)/(2*Sqrt[c])]*(Cosh[a] - Sinh[a]) +
 Erfi[(1 + 2*c*x)/(2*Sqrt[c])]*(Cosh[a] + Sinh[a])))/(4*Sqrt[c]*E^(1/(4*c)
))
```

3.300.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \cosh\left(a+cx^2\right) \, dx$$

$$\Big\downarrow \text{ 6039}$$

$$\int \left(\frac{1}{2}e^{-a-cx^2+x}+\frac{1}{2}e^{a+cx^2+x}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{\pi}e^{a-\frac{1}{4c}}\text{erfi}\left(\frac{2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\sqrt{\pi}e^{\frac{1}{4c}-a}\text{erf}\left(\frac{1-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

input
```
Int[E^x*Cosh[a + c*x^2],x]
```

output
```
-1/4*(E^(-a + 1/(4*c))*Sqrt[Pi]*Erf[(1 - 2*c*x)/(2*Sqrt[c])])/Sqrt[c] + (E
^(a - 1/(4*c))*Sqrt[Pi]*Erfi[(1 + 2*c*x)/(2*Sqrt[c])])/(4*Sqrt[c])
```

3.300.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.300.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85

method	result	size
risch	$\frac{\sqrt{\pi}\,\mathrm{e}^{-\frac{4ac-1}{4c}}\operatorname{erf}\left(\sqrt{c}\,x-\frac{1}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{4ac-1}{4c}}\operatorname{erf}\left(\sqrt{-c}\,x-\frac{1}{2\sqrt{-c}}\right)}{4\sqrt{-c}}$	72

input
```
int(exp(x)*cosh(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*Pi^(1/2)*exp(-1/4*(4*a*c-1)/c)/c^(1/2)*erf(c^(1/2)*x-1/2/c^(1/2))+1/4*
Pi^(1/2)*exp(1/4*(4*a*c-1)/c)/(-c)^(1/2)*erf((-c)^(1/2)*x-1/2/(-c)^(1/2))
```

3.300.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.22

$$\int e^x \cosh\left(a+cx^2\right)\,dx =$$
$$-\frac{\sqrt{\pi}\sqrt{-c}\left(\cosh\left(\frac{4\,ac-1}{4\,c}\right)+\sinh\left(\frac{4\,ac-1}{4\,c}\right)\right)\operatorname{erf}\left(\frac{(2\,cx+1)\sqrt{-c}}{2\,c}\right)-\sqrt{\pi}\sqrt{c}\left(\cosh\left(\frac{4\,ac-1}{4\,c}\right)-\sinh\left(\frac{4\,ac-1}{4\,c}\right)\right)\operatorname{erf}\left(\frac{2\,cx\cdots}{2\sqrt{\cdots}}\right.}{4\,c}$$

input
```
integrate(exp(x)*cosh(c*x^2+a),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(pi)*sqrt(-c)*(cosh(1/4*(4*a*c - 1)/c) + sinh(1/4*(4*a*c - 1)/c)
)*erf(1/2*(2*c*x + 1)*sqrt(-c)/c) - sqrt(pi)*sqrt(c)*(cosh(1/4*(4*a*c - 1)
/c) - sinh(1/4*(4*a*c - 1)/c))*erf(1/2*(2*c*x - 1)/sqrt(c)))/c
```

3.300.6 Sympy [F]

$$\int e^x \cosh\left(a+cx^2\right)\,dx=\int e^x \cosh\left(a+cx^2\right)dx$$

input
```
integrate(exp(x)*cosh(c*x**2+a),x)
```

output
```
Integral(exp(x)*cosh(a + c*x**2), x)
```

3.300.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

$$\int e^x \cosh\left(a+cx^2\right)\,dx = \frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-c}x-\frac{1}{2\sqrt{-c}}\right)e^{\left(a-\frac{1}{4c}\right)}}{4\sqrt{-c}}+\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{c}x-\frac{1}{2\sqrt{c}}\right)e^{\left(-a+\frac{1}{4c}\right)}}{4\sqrt{c}}$$

input
```
integrate(exp(x)*cosh(c*x^2+a),x, algorithm="maxima")
```

output
```
1/4*sqrt(pi)*erf(sqrt(-c)*x - 1/2/sqrt(-c))*e^(a - 1/4/c)/sqrt(-c) + 1/4*s
qrt(pi)*erf(sqrt(c)*x - 1/2/sqrt(c))*e^(-a + 1/4/c)/sqrt(c)
```

3.300.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int e^x \cosh\left(a+cx^2\right)\,dx = -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c}\left(2x+\frac{1}{c}\right)\right)e^{\left(\frac{4ac-1}{4c}\right)}}{4\sqrt{-c}} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{c}\left(2x-\frac{1}{c}\right)\right)e^{\left(-\frac{4ac-1}{4c}\right)}}{4\sqrt{c}}$$

input
```
integrate(exp(x)*cosh(c*x^2+a),x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*erf(-1/2*sqrt(-c)*(2*x + 1/c))*e^(1/4*(4*a*c - 1)/c)/sqrt(-c
) - 1/4*sqrt(pi)*erf(-1/2*sqrt(c)*(2*x - 1/c))*e^(-1/4*(4*a*c - 1)/c)/sqrt
(c)
```

3.300.9 Mupad [F(-1)]

Timed out.

$$\int e^x \cosh\left(a+cx^2\right)\,dx = \int \mathrm{e}^x \cosh\left(c\,x^2+a\right)\,dx$$

input
```
int(exp(x)*cosh(a + c*x^2),x)
```

output
```
int(exp(x)*cosh(a + c*x^2), x)
```

3.301 $\int e^x \cosh(a+bx+cx^2)\,dx$

3.301.1 Optimal result

Integrand size = 15, antiderivative size = 101

$$\int e^x \cosh\left(a+bx+cx^2\right)\,dx=-\frac{e^{-a+\frac{(1-b)^2}{4c}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{1-b-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{e^{a-\frac{(1+b)^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1+b+2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

output
```
-1/4*exp(-a+1/4*(1-b)^2/c)*erf(1/2*(-2*c*x-b+1)/c^(1/2))*Pi^(1/2)/c^(1/2)+
1/4*exp(a-1/4*(1+b)^2/c)*erfi(1/2*(2*c*x+b+1)/c^(1/2))*Pi^(1/2)/c^(1/2)
```

3.301.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.90

$$\int e^x \cosh\left(a+bx+cx^2\right)\,dx =\frac{e^{-\frac{(1+b)^2}{4c}}\sqrt{\pi}\left(e^{\frac{1+b^2}{2c}}\mathrm{erf}\left(\frac{-1+b+2cx}{2\sqrt{c}}\right)(\cosh(a)-\sinh(a))+\mathrm{erfi}\left(\frac{1+b+2cx}{2\sqrt{c}}\right)(\cosh(a)+\sinh(a))\right)}{4\sqrt{c}}$$

input
```
Integrate[E^x*Cosh[a + b*x + c*x^2],x]
```

output
```
(Sqrt[Pi]*(E^((1 + b^2)/(2*c))*Erf[(-1 + b + 2*c*x)/(2*Sqrt[c])]*(Cosh[a]
- Sinh[a]) + Erfi[(1 + b + 2*c*x)/(2*Sqrt[c])]*(Cosh[a] + Sinh[a])))/(4*Sq
rt[c]*E^((1 + b)^2/(4*c)))
```

3.301.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \cosh\left(a+bx+cx^2\right) dx$$

$$\Big\downarrow \ 6039$$

$$\int \left(\frac{1}{2}e^{-a+(1-b)x-cx^2}+\frac{1}{2}e^{a+(b+1)x+cx^2}\right) dx$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{\pi}e^{a-\frac{(b+1)^2}{4c}}\text{erfi}\left(\frac{b+2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\sqrt{\pi}e^{\frac{(1-b)^2}{4c}-a}\text{erf}\left(\frac{-b-2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

input
```
Int[E^x*Cosh[a + b*x + c*x^2],x]
```

output
```
-1/4*(E^(-a + (1 - b)^2/(4*c))*Sqrt[Pi]*Erf[(1 - b - 2*c*x)/(2*Sqrt[c])])/
Sqrt[c] + (E^(a - (1 + b)^2/(4*c))*Sqrt[Pi]*Erfi[(1 + b + 2*c*x)/(2*Sqrt[c
])])/(4*Sqrt[c])
```

3.301.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.301.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96

method	result	size
risch	$\frac{\sqrt{\pi}\,\mathrm{e}^{-\frac{4ac-b^{2}+2b-1}{4c}}\operatorname{erf}\left(\sqrt{c}\,x-\frac{1-b}{2\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{4ac-b^{2}-2b-1}{4c}}\operatorname{erf}\left(-\sqrt{-c}\,x+\frac{1+b}{2\sqrt{-c}}\right)}{4\sqrt{-c}}$	97

input
```
int(exp(x)*cosh(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*Pi^(1/2)*exp(-1/4*(4*a*c-b^2+2*b-1)/c)/c^(1/2)*erf(c^(1/2)*x-1/2*(1-b)
/c^(1/2))-1/4*Pi^(1/2)*exp(1/4*(4*a*c-b^2-2*b-1)/c)/(-c)^(1/2)*erf(-(-c)^(
1/2)*x+1/2*(1+b)/(-c)^(1/2))
```

3.301.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.29

$$\int e^{x}\cosh\left(a+bx+cx^{2}\right)\,dx =$$

$$-\frac{\sqrt{\pi}\sqrt{-c}\left(\cosh\left(-\frac{b^{2}-4\,ac+2\,b+1}{4\,c}\right)+\sinh\left(-\frac{b^{2}-4\,ac+2\,b+1}{4\,c}\right)\right)\operatorname{erf}\left(\frac{(2\,cx+b+1)\sqrt{-c}}{2\,c}\right)-\sqrt{\pi}\sqrt{c}\left(\cosh\left(-\frac{b^{2}-4\,ac-2}{4\,c}\right.\right.}{4\,c}$$

input
```
integrate(exp(x)*cosh(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(pi)*sqrt(-c)*(cosh(-1/4*(b^2 - 4*a*c + 2*b + 1)/c) + sinh(-1/4*
(b^2 - 4*a*c + 2*b + 1)/c))*erf(1/2*(2*c*x + b + 1)*sqrt(-c)/c) - sqrt(pi)
*sqrt(c)*(cosh(-1/4*(b^2 - 4*a*c - 2*b + 1)/c) - sinh(-1/4*(b^2 - 4*a*c -
2*b + 1)/c))*erf(1/2*(2*c*x + b - 1)/sqrt(c)))/c
```

3.301.6 Sympy [F]

$$\int e^x \cosh \left(a+bx+cx^2\right) \, dx = \int e^x \cosh \left(a+bx+cx^2\right) dx$$

input
```
integrate(exp(x)*cosh(c*x**2+b*x+a),x)
```

output
```
Integral(exp(x)*cosh(a + b*x + c*x**2), x)
```

3.301.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.80

$$\int e^x \cosh \left(a+bx+cx^2\right) \, dx = \frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-c}x - \frac{b+1}{2\sqrt{-c}}\right) e^{\left(a-\frac{(b+1)^2}{4c}\right)}}{4\sqrt{-c}} + \frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{c}x + \frac{b-1}{2\sqrt{c}}\right) e^{\left(-a+\frac{(b-1)^2}{4c}\right)}}{4\sqrt{c}}$$

input
```
integrate(exp(x)*cosh(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
1/4*sqrt(pi)*erf(sqrt(-c)*x - 1/2*(b + 1)/sqrt(-c))*e^(a - 1/4*(b + 1)^2/c
)/sqrt(-c) + 1/4*sqrt(pi)*erf(sqrt(c)*x + 1/2*(b - 1)/sqrt(c))*e^(-a + 1/4
*(b - 1)^2/c)/sqrt(c)
```

3.301.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.90

$$\int e^x \cosh \left(a+bx+cx^2\right) \, dx = -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c}\left(2x+\frac{b+1}{c}\right)\right) e^{\left(-\frac{b^2-4ac+2b+1}{4c}\right)}}{4\sqrt{-c}} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{c}\left(2x+\frac{b-1}{c}\right)\right) e^{\left(\frac{b^2-4ac-2b+1}{4c}\right)}}{4\sqrt{c}}$$

input
```
integrate(exp(x)*cosh(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*erf(-1/2*sqrt(-c)*(2*x + (b + 1)/c))*e^(-1/4*(b^2 - 4*a*c +
2*b + 1)/c)/sqrt(-c) - 1/4*sqrt(pi)*erf(-1/2*sqrt(c)*(2*x + (b - 1)/c))*e^
(1/4*(b^2 - 4*a*c - 2*b + 1)/c)/sqrt(c)
```

3.301.9 Mupad [F(-1)]

Timed out.

$$\int e^x \cosh \left(a+bx+cx^2\right) \, dx=\int e^x \cosh\left(c\,x^2+b\,x+a\right) \,dx$$

input
```
int(exp(x)*cosh(a + b*x + c*x^2),x)
```

output
```
int(exp(x)*cosh(a + b*x + c*x^2), x)
```

3.302 $\int e^{x^2} \cosh(a+bx)\,dx$

3.302.1 Optimal result

Integrand size = 12, antiderivative size = 65

$$\int e^{x^2} \cosh(a+bx)\,dx = \frac{1}{4}e^{-a-\frac{b^2}{4}}\sqrt{\pi}\text{erfi}\left(\frac{1}{2}(-b+2x)\right)+\frac{1}{4}e^{a-\frac{b^2}{4}}\sqrt{\pi}\text{erfi}\left(\frac{1}{2}(b+2x)\right)$$

output
```
-1/4*exp(-a-1/4*b^2)*erfi(1/2*b-x)*Pi^(1/2)+1/4*exp(a-1/4*b^2)*erfi(1/2*b+
x)*Pi^(1/2)
```

3.302.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int e^{x^2} \cosh(a+bx)\,dx = \frac{1}{4}e^{-\frac{b^2}{4}}\sqrt{\pi}\left(\text{erfi}\left(\frac{b}{2}-x\right)(-\cosh(a)+\sinh(a)) + \text{erfi}\left(\frac{b}{2}+x\right)(\cosh(a)+\sinh(a))\right)$$

input
```
Integrate[E^x^2*Cosh[a + b*x],x]
```

output
```
(Sqrt[Pi]*(Erfi[b/2 - x]*(-Cosh[a] + Sinh[a]) + Erfi[b/2 + x]*(Cosh[a] + S
inh[a])))/(4*E^(b^2/4))
```

3.302.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^2} \cosh(a+bx)\,dx$$

$$\Big\downarrow \text{ 6039}$$

$$\int \left(\frac{1}{2}e^{-a-bx+x^2}+\frac{1}{2}e^{a+bx+x^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}\sqrt{\pi}e^{-a-\frac{b^2}{4}}\text{erfi}\left(\frac{1}{2}(2x-b)\right)+\frac{1}{4}\sqrt{\pi}e^{a-\frac{b^2}{4}}\text{erfi}\left(\frac{1}{2}(b+2x)\right)$$

input
```
Int[E^x^2*Cosh[a + b*x],x]
```

output
```
(E^(-a - b^2/4)*Sqrt[Pi]*Erfi[(-b + 2*x)/2])/4 + (E^(a - b^2/4)*Sqrt[Pi]*E
rfi[(b + 2*x)/2])/4
```

3.302.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.302.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

method	result	size
risch	$\frac{i\sqrt{\pi}\,\mathrm{e}^{-a-\frac{b^{2}}{4}}\operatorname{erf}\left(-ix+\frac{1}{2}ib\right)}{4}-\frac{i\sqrt{\pi}\,\mathrm{e}^{a-\frac{b^{2}}{4}}\operatorname{erf}\left(ix+\frac{1}{2}ib\right)}{4}$	52

input
```
int(exp(x^2)*cosh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*I*Pi^(1/2)*exp(-a-1/4*b^2)*erf(-I*x+1/2*I*b)-1/4*I*Pi^(1/2)*exp(a-1/4*
b^2)*erf(I*x+1/2*I*b)
```

3.302.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.11

$$\int e^{x^2}\cosh(a+bx)\,dx$$
$$=\frac{1}{4}\sqrt{\pi}\Bigg(\cosh\left(\frac{1}{4}b^2-a\right)\operatorname{erfi}\left(\frac{1}{2}b+x\right)+\cosh\left(\frac{1}{4}b^2+a\right)\operatorname{erfi}\left(-\frac{1}{2}b+x\right)-\operatorname{erfi}\left(-\frac{1}{2}b+x\right)\sinh\Bigg(\frac{1}{4}b^2$$

input
```
integrate(exp(x^2)*cosh(b*x+a),x, algorithm="fricas")
```

output
```
1/4*sqrt(pi)*(cosh(1/4*b^2 - a)*erfi(1/2*b + x) + cosh(1/4*b^2 + a)*erfi(-
1/2*b + x) - erfi(-1/2*b + x)*sinh(1/4*b^2 + a) - erfi(1/2*b + x)*sinh(1/4
*b^2 - a))
```

3.302.6 Sympy [F]

$$\int e^{x^2}\cosh(a+bx)\,dx=\int e^{x^2}\cosh\left(a+bx\right)dx$$

input
```
integrate(exp(x**2)*cosh(b*x+a),x)
```

output
```
Integral(exp(x**2)*cosh(a + b*x), x)
```

3.302.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int e^{x^2}\cosh(a+bx)\,dx = -\frac{1}{4}i\sqrt{\pi}\,\mathrm{erf}\left(\frac{1}{2}i\,b+i\,x\right)e^{\left(-\frac{1}{4}\,b^2+a\right)} - \frac{1}{4}i\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}i\,b+i\,x\right)e^{\left(-\frac{1}{4}\,b^2-a\right)}$$

input
```
integrate(exp(x^2)*cosh(b*x+a),x, algorithm="maxima")
```

output
```
-1/4*I*sqrt(pi)*erf(1/2*I*b + I*x)*e^(-1/4*b^2 + a) - 1/4*I*sqrt(pi)*erf(-
1/2*I*b + I*x)*e^(-1/4*b^2 - a)
```

3.302.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int e^{x^2}\cosh(a+bx)\,dx = \frac{1}{4}i\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}i\,b-i\,x\right)e^{\left(-\frac{1}{4}\,b^2+a\right)} + \frac{1}{4}i\sqrt{\pi}\,\mathrm{erf}\left(\frac{1}{2}i\,b-i\,x\right)e^{\left(-\frac{1}{4}\,b^2-a\right)}$$

input
```
integrate(exp(x^2)*cosh(b*x+a),x, algorithm="giac")
```

output
```
1/4*I*sqrt(pi)*erf(-1/2*I*b - I*x)*e^(-1/4*b^2 + a) + 1/4*I*sqrt(pi)*erf(1
/2*I*b - I*x)*e^(-1/4*b^2 - a)
```

3.302.9 Mupad [F(-1)]

Timed out.

$$\int e^{x^2}\cosh(a+bx)\,dx = \int \cosh(a+b\,x)\,\mathrm{e}^{x^2}\,dx$$

input
```
int(cosh(a + b*x)*exp(x^2),x)
```

output
```
int(cosh(a + b*x)*exp(x^2), x)
```

3.303 $\int e^{x^2}\cosh\left(a+cx^2\right)\,dx$

3.303.1 Optimal result

Integrand size = 14, antiderivative size = 65

$$\int e^{x^2}\cosh\left(a+cx^2\right)\,dx=\frac{e^{-a}\sqrt{\pi}\,\text{erfi}\left(\sqrt{1-c}\,x\right)}{4\sqrt{1-c}}+\frac{e^{a}\sqrt{\pi}\,\text{erfi}\left(\sqrt{1+c}\,x\right)}{4\sqrt{1+c}}$$

output
```
1/4*erfi(x*(1-c)^(1/2))*Pi^(1/2)/exp(a)/(1-c)^(1/2)+1/4*exp(a)*erfi(x*(1+c
)^(1/2))*Pi^(1/2)/(1+c)^(1/2)
```

3.303.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.09

$$\int e^{x^2}\cosh\left(a+cx^2\right)\,dx$$
$$=\frac{\sqrt{\pi}\left(\sqrt{-1+c}(1+c)\text{erf}\left(\sqrt{-1+c}\,x\right)(\cosh(a)-\sinh(a))+(-1+c)\sqrt{1+c}\,\text{erfi}\left(\sqrt{1+c}\,x\right)(\cosh(a)+\sinh(\right.}{4\left(-1+c^2\right)}$$

input
```
Integrate[E^x^2*Cosh[a + c*x^2],x]
```

output
```
(Sqrt[Pi]*(Sqrt[-1 + c]*(1 + c)*Erf[Sqrt[-1 + c]*x]*(Cosh[a] - Sinh[a]) +
(-1 + c)*Sqrt[1 + c]*Erfi[Sqrt[1 + c]*x]*(Cosh[a] + Sinh[a])))/(4*(-1 + c^
2))
```

3.303.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^2}\cosh\left(a+cx^2\right)\,dx$$
$$\Big\downarrow \text{6039}$$
$$\int \left(\frac{1}{2}e^{(1-c)x^2-a}+\frac{1}{2}e^{a+(c+1)x^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{\sqrt{\pi}e^{-a}\text{erfi}\left(\sqrt{1-c}x\right)}{4\sqrt{1-c}}+\frac{\sqrt{\pi}e^{a}\text{erfi}\left(\sqrt{c+1}x\right)}{4\sqrt{c+1}}$$

input
```
Int[E^x^2*Cosh[a + c*x^2],x]
```

output
```
(Sqrt[Pi]*Erfi[Sqrt[1 - c]*x])/(4*Sqrt[1 - c]*E^a) + (E^a*Sqrt[Pi]*Erfi[Sq
rt[1 + c]*x])/(4*Sqrt[1 + c])
```

3.303.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.303.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

method	result	size
risch	$\frac{\sqrt{\pi}\,e^{-a}\,\mathrm{erf}\left(\sqrt{c-1}\,x\right)}{4\sqrt{c-1}}+\frac{\sqrt{\pi}\,e^{a}\,\mathrm{erf}\left(\sqrt{-c-1}\,x\right)}{4\sqrt{-c-1}}$	48

input
```
int(exp(x^2)*cosh(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*Pi^(1/2)*exp(-a)/(c-1)^(1/2)*erf((c-1)^(1/2)*x)+1/4*Pi^(1/2)*exp(a)/(-
c-1)^(1/2)*erf((-c-1)^(1/2)*x)
```

3.303.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.17

$$\int e^{x^2}\cosh\left(a+cx^2\right)\,dx$$
$$=\frac{\sqrt{\pi}((c+1)\cosh(a)-(c+1)\sinh(a))\sqrt{c-1}\,\mathrm{erf}\left(\sqrt{c-1}x\right)-\sqrt{\pi}((c-1)\cosh(a)+(c-1)\sinh(a))\sqrt{-}}{4\left(c^2-1\right)}$$

input
```
integrate(exp(x^2)*cosh(c*x^2+a),x, algorithm="fricas")
```

output
```
1/4*(sqrt(pi)*((c + 1)*cosh(a) - (c + 1)*sinh(a))*sqrt(c - 1)*erf(sqrt(c -
 1)*x) - sqrt(pi)*((c - 1)*cosh(a) + (c - 1)*sinh(a))*sqrt(-c - 1)*erf(sqr
t(-c - 1)*x))/(c^2 - 1)
```

3.303.6 Sympy [F]

$$\int e^{x^2}\cosh\left(a+cx^2\right)\,dx=\int e^{x^2}\cosh\left(a+cx^2\right)dx$$

input
```
integrate(exp(x**2)*cosh(c*x**2+a),x)
```

output
```
Integral(exp(x**2)*cosh(a + c*x**2), x)
```

3.303.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

$$\int e^{x^2}\cosh\left(a+cx^2\right)\,dx=\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{c-1}x\right)e^{(-a)}}{4\sqrt{c-1}}+\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-c-1}x\right)e^{a}}{4\sqrt{-c-1}}$$

input
```
integrate(exp(x^2)*cosh(c*x^2+a),x, algorithm="maxima")
```

output
```
1/4*sqrt(pi)*erf(sqrt(c - 1)*x)*e^(-a)/sqrt(c - 1) + 1/4*sqrt(pi)*erf(sqrt
(-c - 1)*x)*e^a/sqrt(-c - 1)
```

3.303.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

$$\int e^{x^2}\cosh\left(a+cx^2\right)\,dx=-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{c-1}x\right)e^{(-a)}}{4\sqrt{c-1}}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-c-1}x\right)e^{a}}{4\sqrt{-c-1}}$$

input
```
integrate(exp(x^2)*cosh(c*x^2+a),x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*erf(-sqrt(c - 1)*x)*e^(-a)/sqrt(c - 1) - 1/4*sqrt(pi)*erf(-s
qrt(-c - 1)*x)*e^a/sqrt(-c - 1)
```

3.303.9 Mupad [F(-1)]

Timed out.

$$\int e^{x^2}\cosh\left(a+cx^2\right)\,dx=\int \mathrm{e}^{x^2}\cosh\left(c\,x^2+a\right)\,dx$$

input
```
int(exp(x^2)*cosh(a + c*x^2),x)
```

output
```
int(exp(x^2)*cosh(a + c*x^2), x)
```

3.304 $\int e^{x^2} \cosh \left(a+b x+c x^2\right) \, dx$

3.304.1 Optimal result

Integrand size = 17, antiderivative size = 115

$$\int e^{x^2} \cosh \left(a+b x+c x^2\right) \, dx=-\frac{e^{-a-\frac{b^2}{4(1-c)}} \sqrt{\pi } \text{erfi}\left(\frac{b-2(1-c)x}{2\sqrt{1-c}}\right)}{4\sqrt{1-c}}+\frac{e^{a-\frac{b^2}{4(1+c)}} \sqrt{\pi } \text{erfi}\left(\frac{b+2(1+c)x}{2\sqrt{1+c}}\right)}{4\sqrt{1+c}}$$

output
```
-1/4*exp(-a-1/4*b^2/(1-c))*erfi(1/2*(b-2*(1-c)*x)/(1-c)^(1/2))*Pi^(1/2)/(1
-c)^(1/2)+1/4*exp(a-1/4*b^2/(1+c))*erfi(1/2*(b+2*(1+c)*x)/(1+c)^(1/2))*Pi^
(1/2)/(1+c)^(1/2)
```

3.304.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.06

$$\int e^{x^2} \cosh \left(a+b x+c x^2\right) \, dx$$
$$=\frac{e^{-\frac{b^2}{4+4c}} \sqrt{\pi } \left(\sqrt{-1+c}(1+c)e^{\frac{b^2 c}{2\left(-1+c^2\right)}} \text{erf}\left(\frac{b+2(-1+c)x}{2\sqrt{-1+c}}\right) (\cosh (a)-\sinh (a))+(-1+c)\sqrt{1+c}\text{erfi}\left(\frac{b+2(1+c)x}{2\sqrt{1+c}}\right)\right.}{4\left(-1+c^2\right)}$$

input
```
Integrate[E^x^2*Cosh[a + b*x + c*x^2],x]
```

output
```
(Sqrt[Pi]*(Sqrt[-1 + c]*(1 + c)*E^((b^2*c)/(2*(-1 + c^2)))*Erf[(b + 2*(-1
+ c)*x)/(2*Sqrt[-1 + c])]*(Cosh[a] - Sinh[a]) + (-1 + c)*Sqrt[1 + c]*Erfi[
(b + 2*(1 + c)*x)/(2*Sqrt[1 + c])]*(Cosh[a] + Sinh[a])))/(4*(-1 + c^2)*E^(
b^2/(4 + 4*c)))
```

3.304.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^2} \cosh\left(a + bx + cx^2\right) dx$$

$$\Big\downarrow \text{6039}$$

$$\int \left(\frac{1}{2} e^{-a-bx+(1-c)x^2} + \frac{1}{2} e^{a+bx+(c+1)x^2}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi} e^{a-\frac{b^2}{4(c+1)}} \text{erfi}\left(\frac{b+2(c+1)x}{2\sqrt{c+1}}\right)}{4\sqrt{c+1}} - \frac{\sqrt{\pi} e^{-a-\frac{b^2}{4(1-c)}} \text{erfi}\left(\frac{b-2(1-c)x}{2\sqrt{1-c}}\right)}{4\sqrt{1-c}}$$

input
```
Int[E^x^2*Cosh[a + b*x + c*x^2],x]
```

output
```
-1/4*(E^(-a - b^2/(4*(1 - c)))*Sqrt[Pi]*Erfi[(b - 2*(1 - c)*x)/(2*Sqrt[1 -
 c])])/Sqrt[1 - c] + (E^(a - b^2/(4*(1 + c)))*Sqrt[Pi]*Erfi[(b + 2*(1 + c)
*x)/(2*Sqrt[1 + c])])/(4*Sqrt[1 + c])
```

3.304.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.304.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.91

method	result	size
risch	$\frac{\sqrt{\pi}\, \mathrm{e}^{-\frac{4 a c - b^{2}-4 a}{4 (c-1)}} \operatorname{erf}\left(\sqrt{c-1}\, x+\frac{b}{2 \sqrt{c-1}}\right)}{4 \sqrt{c-1}}-\frac{\sqrt{\pi}\, \mathrm{e}^{\frac{4 a c - b^{2}+4 a}{4 c+4}} \operatorname{erf}\left(-\sqrt{-c-1}\, x+\frac{b}{2 \sqrt{-c-1}}\right)}{4 \sqrt{-c-1}}$	105

input
```
int(exp(x^2)*cosh(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*Pi^(1/2)*exp(-1/4*(4*a*c-b^2-4*a)/(c-1))/(c-1)^(1/2)*erf((c-1)^(1/2)*x
+1/2*b/(c-1)^(1/2))-1/4*Pi^(1/2)*exp(1/4*(4*a*c-b^2+4*a)/(c+1))/(-c-1)^(1/
2)*erf(-(-c-1)^(1/2)*x+1/2*b/(-c-1)^(1/2))
```

3.304.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.43

$$\int e^{x^2} \cosh\left(a+bx+cx^2\right)\, dx$$
$$= \frac{\sqrt{\pi}\left((c+1)\cosh\left(-\frac{b^2-4\,ac+4\,a}{4\,(c-1)}\right)-(c+1)\sinh\left(-\frac{b^2-4\,ac+4\,a}{4\,(c-1)}\right)\right)\sqrt{c-1}\operatorname{erf}\left(\frac{2\,(c-1)x+b}{2\,\sqrt{c-1}}\right)-\sqrt{\pi}\Big((c-1)\cosh}{4\,(c^2-1)}$$

input
```
integrate(exp(x^2)*cosh(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
1/4*(sqrt(pi)*((c + 1)*cosh(-1/4*(b^2 - 4*a*c + 4*a)/(c - 1)) - (c + 1)*si
nh(-1/4*(b^2 - 4*a*c + 4*a)/(c - 1)))*sqrt(c - 1)*erf(1/2*(2*(c - 1)*x + b
)/sqrt(c - 1)) - sqrt(pi)*((c - 1)*cosh(-1/4*(b^2 - 4*a*c - 4*a)/(c + 1))
+ (c - 1)*sinh(-1/4*(b^2 - 4*a*c - 4*a)/(c + 1)))*sqrt(-c - 1)*erf(1/2*(2*
(c + 1)*x + b)*sqrt(-c - 1)/(c + 1)))/(c^2 - 1)
```

3.304.6 Sympy [F]

$$\int e^{x^2}\cosh\left(a+bx+cx^2\right)\,dx=\int e^{x^2}\cosh\left(a+bx+cx^2\right)dx$$

input
```
integrate(exp(x**2)*cosh(c*x**2+b*x+a),x)
```

output
```
Integral(exp(x**2)*cosh(a + b*x + c*x**2), x)
```

3.304.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.77

$$\int e^{x^2}\cosh\left(a+bx+cx^2\right)\,dx=\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-c-1}x-\frac{b}{2\sqrt{-c-1}}\right)e^{\left(a-\frac{b^2}{4\,(c+1)}\right)}}{4\sqrt{-c-1}}+\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{c-1}x+\frac{b}{2\sqrt{c-1}}\right)e^{\left(-a+\frac{b^2}{4\,(c-1)}\right)}}{4\sqrt{c-1}}$$

input
```
integrate(exp(x^2)*cosh(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
1/4*sqrt(pi)*erf(sqrt(-c - 1)*x - 1/2*b/sqrt(-c - 1))*e^(a - 1/4*b^2/(c +
1))/sqrt(-c - 1) + 1/4*sqrt(pi)*erf(sqrt(c - 1)*x + 1/2*b/sqrt(c - 1))*e^(
-a + 1/4*b^2/(c - 1))/sqrt(c - 1)
```

3.304.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.88

$$\int e^{x^2}\cosh\left(a+bx+cx^2\right)\,dx = -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c-1}\left(2\,x+\frac{b}{c+1}\right)\right)e^{\left(-\frac{b^2-4\,ac-4\,a}{4\,(c+1)}\right)}}{4\,\sqrt{-c-1}} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{c-1}\left(2\,x+\frac{b}{c-1}\right)\right)e^{\left(\frac{b^2-4\,ac+4\,a}{4\,(c-1)}\right)}}{4\,\sqrt{c-1}}$$

input
```
integrate(exp(x^2)*cosh(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*erf(-1/2*sqrt(-c - 1)*(2*x + b/(c + 1)))*e^(-1/4*(b^2 - 4*a*
c - 4*a)/(c + 1))/sqrt(-c - 1) - 1/4*sqrt(pi)*erf(-1/2*sqrt(c - 1)*(2*x +
b/(c - 1)))*e^(1/4*(b^2 - 4*a*c + 4*a)/(c - 1))/sqrt(c - 1)
```

3.304.9 Mupad [F(-1)]

Timed out.

$$\int e^{x^2}\cosh\left(a+bx+cx^2\right)\,dx = \int \mathrm{e}^{x^2}\cosh\left(c\,x^2+b\,x+a\right)\,dx$$

input
```
int(exp(x^2)*cosh(a + b*x + c*x^2),x)
```

output
```
int(exp(x^2)*cosh(a + b*x + c*x^2), x)
```

3.305 $\int f^{a+bx}\cosh(d+fx^2)\,dx$

3.305.1 Optimal result

Integrand size = 16, antiderivative size = 110

$$\int f^{a+bx}\cosh(d+fx^2)\,dx = \frac{1}{4}e^{-d+\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right) + \frac{1}{4}e^{d-\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{2fx+b\log(f)}{2\sqrt{f}}\right)$$

output
```
1/4*exp(-d+1/4*b^2*ln(f)^2/f)*f^(-1/2+a)*erf(1/2*(2*f*x-b*ln(f))/f^(1/2))*
Pi^(1/2)+1/4*exp(d-1/4*b^2*ln(f)^2/f)*f^(-1/2+a)*erfi(1/2*(2*f*x+b*ln(f))/
f^(1/2))*Pi^(1/2)
```

3.305.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.93

$$\int f^{a+bx}\cosh(d+fx^2)\,dx = \frac{1}{4}e^{-\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\left(e^{\frac{b^2\log^2(f)}{2f}}\mathrm{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)(\cosh(d) - \sinh(d)) + \mathrm{erfi}\left(\frac{2fx+b\log(f)}{2\sqrt{f}}\right)(\cosh(d)+\sinh(d))\right)$$

input
```
Integrate[f^(a + b*x)*Cosh[d + f*x^2],x]
```

output
```
(f^(-1/2 + a)*Sqrt[Pi]*(E^((b^2*Log[f]^2)/(2*f))*Erf[(2*f*x - b*Log[f])/(2
*Sqrt[f])]*(Cosh[d] - Sinh[d]) + Erfi[(2*f*x + b*Log[f])/(2*Sqrt[f])]*(Cos
h[d] + Sinh[d])))/(4*E^((b^2*Log[f]^2)/(4*f)))
```

3.305.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx} \cosh\left(d+fx^2\right) dx$$

$$\Big\downarrow \text{6039}$$

$$\int \left(\frac{1}{2}e^{-d-fx^2}f^{a+bx}+\frac{1}{2}e^{d+fx^2}f^{a+bx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{4f}-d}\text{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{d-\frac{b^2\log^2(f)}{4f}}\text{erfi}\left(\frac{b\log(f)+2fx}{2\sqrt{f}}\right)$$

input
```
Int[f^(a + b*x)*Cosh[d + f*x^2],x]
```

output
```
(E^(-d + (b^2*Log[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(2*f*x - b*Log[f]
)/(2*Sqrt[f])])/4 + (E^(d - (b^2*Log[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Er
fi[(2*f*x + b*Log[f])/(2*Sqrt[f])])/4
```

3.305.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.305.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.91

method	result	size
risch	$-\frac{\operatorname{erf}\left(-\sqrt{f}\, x+\frac{\ln(f) b}{2\sqrt{f}}\right)\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{b^{2} \ln(f)^{2}-4 d f}{4 f}}}{4\sqrt{f}}-\frac{\operatorname{erf}\left(-\sqrt{-f}\, x+\frac{\ln(f) b}{2\sqrt{-f}}\right)\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}-4 d f}{4 f}}}{4\sqrt{-f}}$	100

input
```
int(f^(b*x+a)*cosh(f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4*erf(-f^(1/2)*x+1/2*ln(f)*b/f^(1/2))/f^(1/2)*Pi^(1/2)*f^a*exp(1/4*(b^2
*ln(f)^2-4*d*f)/f)-1/4*erf(-(-f)^(1/2)*x+1/2*ln(f)*b/(-f)^(1/2))/(-f)^(1/2
)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-4*d*f)/f)
```

3.305.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(84) = 168.

Time = 0.28 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.92

$$\int f^{a+bx}\cosh\left(d+fx^2\right)\,dx =$$
$$-\frac{\sqrt{\pi}\sqrt{-f}\cosh\left(\frac{b^2\log(f)^2-4\,af\log(f)-4\,df}{4\,f}\right)\operatorname{erf}\left(\frac{(2\,fx+b\log(f))\sqrt{-f}}{2\,f}\right)+\sqrt{\pi}\sqrt{f}\cosh\left(\frac{b^2\log(f)^2+4\,af\log(f)-4\,df}{4\,f}\right)\operatorname{erf}}{}$$

input
```
integrate(f^(b*x+a)*cosh(f*x^2+d),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(pi)*sqrt(-f)*cosh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f)*
erf(1/2*(2*f*x + b*log(f))*sqrt(-f)/f) + sqrt(pi)*sqrt(f)*cosh(1/4*(b^2*lo
g(f)^2 + 4*a*f*log(f) - 4*d*f)/f)*erf(-1/2*(2*f*x - b*log(f))/sqrt(f)) + s
qrt(pi)*sqrt(f)*erf(-1/2*(2*f*x - b*log(f))/sqrt(f))*sinh(1/4*(b^2*log(f)^
2 + 4*a*f*log(f) - 4*d*f)/f) - sqrt(pi)*sqrt(-f)*erf(1/2*(2*f*x + b*log(f)
)*sqrt(-f)/f)*sinh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f))/f
```

3.305.6 Sympy [F]

$$\int f^{a+bx}\cosh\left(d+fx^2\right)\,dx=\int f^{a+bx}\cosh\left(d+fx^2\right)dx$$

input
```
integrate(f**(b*x+a)*cosh(f*x**2+d),x)
```

output
```
Integral(f**(a + b*x)*cosh(d + f*x**2), x)
```

3.305.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.82

$$\int f^{a+bx}\cosh\left(d+fx^2\right)\,dx=\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}\operatorname{erf}\left(\sqrt{f}x-\frac{b\log\left(f\right)}{2\sqrt{f}}\right)e^{\left(\frac{b^2\log(f)^2}{4f}-d\right)}+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-f}x-\frac{b\log(f)}{2\sqrt{-f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4f}+d\right)}}{4\sqrt{-f}}$$

input
```
integrate(f^(b*x+a)*cosh(f*x^2+d),x, algorithm="maxima")
```

output
```
1/4*sqrt(pi)*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*b*log(f)/sqrt(f))*e^(1/4*b^2*
log(f)^2/f - d) + 1/4*sqrt(pi)*f^a*erf(sqrt(-f)*x - 1/2*b*log(f)/sqrt(-f))
*e^(-1/4*b^2*log(f)^2/f + d)/sqrt(-f)
```

3.305.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.96

$$\int f^{a+bx}\cosh\left(d+fx^2\right)\,dx=-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{f}\left(2x-\frac{b\log(f)}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+4af\log(f)-4df}{4f}\right)}}{4\sqrt{f}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-f}\left(2x+\frac{b\log(f)}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4af\log(f)-4df}{4f}\right)}}{4\sqrt{-f}}$$

input
```
integrate(f^(b*x+a)*cosh(f*x^2+d),x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*erf(-1/2*sqrt(f)*(2*x - b*log(f)/f))*e^(1/4*(b^2*log(f)^2 +
4*a*f*log(f) - 4*d*f)/f)/sqrt(f) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-f)*(2*x + b
*log(f)/f))*e^(-1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f)/sqrt(-f)
```

3.305.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\cosh\left(d+fx^2\right)\,dx=\int f^{a+b\,x}\cosh\left(f\,x^2+d\right)\,dx$$

input
```
int(f^(a + b*x)*cosh(d + f*x^2),x)
```

output
```
int(f^(a + b*x)*cosh(d + f*x^2), x)
```

3.306 $\int f^{a+bx} \cosh^2(d+fx^2)\,dx$

3.306.1 Optimal result

Integrand size = 18, antiderivative size = 148

$$\int f^{a+bx} \cosh^2(d+fx^2)\,dx = \frac{1}{8}e^{-2d+\frac{b^2\log^2(f)}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right) + \frac{1}{8}e^{2d-\frac{b^2\log^2(f)}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{4fx+b\log(f)}{2\sqrt{2}\sqrt{f}}\right) + \frac{f^{a+bx}}{2b\log(f)}$$

output
```
1/2*f^(b*x+a)/b/ln(f)+1/16*exp(-2*d+1/8*b^2*ln(f)^2/f)*f^(-1/2+a)*erf(1/4*
(4*f*x-b*ln(f))*2^(1/2)/f^(1/2))*2^(1/2)*Pi^(1/2)+1/16*exp(2*d-1/8*b^2*ln(
f)^2/f)*f^(-1/2+a)*erfi(1/4*(4*f*x+b*ln(f))*2^(1/2)/f^(1/2))*2^(1/2)*Pi^(1
/2)
```

3.306.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.01

$$\int f^{a+bx} \cosh^2(d+fx^2)\,dx = \frac{1}{16}f^a\left(\frac{8f^{bx}}{b\log(f)} + \frac{e^{\frac{b^2\log^2(f)}{8f}}\sqrt{2\pi}\operatorname{erf}\left(\frac{4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)(\cosh(2d)-\sinh(2d))}{\sqrt{f}} + \frac{e^{-\frac{b^2\log^2(f)}{8f}}\sqrt{2\pi}\operatorname{erfi}\left(\frac{4fx+b\log(f)}{2\sqrt{2}\sqrt{f}}\right)(\cosh(2d)+\sinh(2d))}{\sqrt{f}}\right)$$

input
```
Integrate[f^(a + b*x)*Cosh[d + f*x^2]^2,x]
```

output
```
(f^a*((8*f^(b*x))/(b*Log[f]) + (E^((b^2*Log[f]^2)/(8*f))*Sqrt[2*Pi]*Erf[(4
*f*x - b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*(Cosh[2*d] - Sinh[2*d]))/Sqrt[f] + (
Sqrt[2*Pi]*Erfi[(4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*(Cosh[2*d] + Sinh[
2*d]))/(E^((b^2*Log[f]^2)/(8*f))*Sqrt[f])))/16
```

3.306.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx}\cosh^2\left(d+fx^2\right)\,dx$$

$$\Big\downarrow \text{6039}$$

$$\int \left(\frac{1}{4}e^{-2d-2fx^2}f^{a+bx}+\frac{1}{4}e^{2d+2fx^2}f^{a+bx}+\frac{1}{2}f^{a+bx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{8f}-2d}\text{erf}\left(\frac{4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{2d-\frac{b^2\log^2(f)}{8f}}\text{erfi}\left(\frac{b\log(f)+4fx}{2\sqrt{2}\sqrt{f}}\right)+\frac{f^{a+bx}}{2b\log(f)}$$

input
```
Int[f^(a + b*x)*Cosh[d + f*x^2]^2,x]
```

output
```
(E^(-2*d + (b^2*Log[f]^2)/(8*f))*f^(-1/2 + a)*Sqrt[Pi/2]*Erf[(4*f*x - b*Lo
g[f])/(2*Sqrt[2]*Sqrt[f])])/8 + (E^(2*d - (b^2*Log[f]^2)/(8*f))*f^(-1/2 +
a)*Sqrt[Pi/2]*Erfi[(4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])])/8 + f^(a + b*x
)/(2*b*Log[f])
```

3.306.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.306.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.85

method	result	size
risch	$-\frac{\operatorname{erf}\left(-\sqrt{2}\,\sqrt{f}\,x+\frac{b\ln(f)\sqrt{2}}{4\sqrt{f}}\right)\sqrt{2}\,\sqrt{\pi}\,f^{a}\mathrm{e}^{\frac{b^{2}\ln(f)^{2}-16df}{8f}}}{16\sqrt{f}}-\frac{\operatorname{erf}\left(-\sqrt{-2f}\,x+\frac{b\ln(f)}{2\sqrt{-2f}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{b^{2}\ln(f)^{2}-16df}{8f}}}{8\sqrt{-2f}}+\frac{f^{a}f^{bx}}{2b\ln(f)}$	126

input
```
int(f^(b*x+a)*cosh(f*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/16*erf(-2^(1/2)*f^(1/2)*x+1/4*b*ln(f)*2^(1/2)/f^(1/2))/f^(1/2)*2^(1/2)*
Pi^(1/2)*f^a*exp(1/8*(b^2*ln(f)^2-16*d*f)/f)-1/8*erf(-(-2*f)^(1/2)*x+1/2*b
*ln(f)/(-2*f)^(1/2))/(-2*f)^(1/2)*Pi^(1/2)*f^a*exp(-1/8*(b^2*ln(f)^2-16*d*
f)/f)+1/2*f^a*f^(b*x)/b/ln(f)
```

3.306.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(114) = 228.

Time = 0.26 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.88

$$\int f^{a+bx}\cosh^2\left(d+fx^2\right)\,dx =$$

$$-\frac{\sqrt{2}\sqrt{\pi}b\sqrt{-f}\cosh\left(\frac{b^2\log(f)^2-8\,af\log(f)-16\,df}{8\,f}\right)\operatorname{erf}\left(\frac{\sqrt{2}(4\,fx+b\log(f))\sqrt{-f}}{4\,f}\right)\log(f)+\sqrt{2}\sqrt{\pi}b\sqrt{f}\cosh\left(\frac{b^2\log(f)^2}{\cdots}\right.}{\cdots}$$

input
```
integrate(f^(b*x+a)*cosh(f*x^2+d)^2,x, algorithm="fricas")
```

output
```
-1/16*(sqrt(2)*sqrt(pi)*b*sqrt(-f)*cosh(1/8*(b^2*log(f)^2 - 8*a*f*log(f) -
 16*d*f)/f)*erf(1/4*sqrt(2)*(4*f*x + b*log(f))*sqrt(-f)/f)*log(f) + sqrt(2
)*sqrt(pi)*b*sqrt(f)*cosh(1/8*(b^2*log(f)^2 + 8*a*f*log(f) - 16*d*f)/f)*er
f(-1/4*sqrt(2)*(4*f*x - b*log(f))/sqrt(f))*log(f) + sqrt(2)*sqrt(pi)*b*sqr
t(f)*erf(-1/4*sqrt(2)*(4*f*x - b*log(f))/sqrt(f))*log(f)*sinh(1/8*(b^2*log
(f)^2 + 8*a*f*log(f) - 16*d*f)/f) - sqrt(2)*sqrt(pi)*b*sqrt(-f)*erf(1/4*sq
rt(2)*(4*f*x + b*log(f))*sqrt(-f)/f)*log(f)*sinh(1/8*(b^2*log(f)^2 - 8*a*f
*log(f) - 16*d*f)/f) - 8*f*cosh((b*x + a)*log(f)) - 8*f*sinh((b*x + a)*log
(f)))/(b*f*log(f))
```

3.306.6 Sympy [F]

$$\int f^{a+bx}\cosh^2\left(d+fx^2\right)\,dx=\int f^{a+bx}\cosh^2\left(d+fx^2\right)dx$$

input
```
integrate(f**(b*x+a)*cosh(f*x**2+d)**2,x)
```

output
```
Integral(f**(a + b*x)*cosh(d + f*x**2)**2, x)
```

3.306.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.86

$$\int f^{a+bx}\cosh^2\left(d+fx^2\right)\,dx=\frac{\sqrt{2}\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{2}\sqrt{f}x-\frac{\sqrt{2}b\log(f)}{4\sqrt{f}}\right)e^{\left(\frac{b^2\log(f)^2}{8f}-2d\right)}}{16\sqrt{f}}+\frac{\sqrt{2}\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{2}\sqrt{-f}x-\frac{\sqrt{2}b\log(f)}{4\sqrt{-f}}\right)e^{\left(-\frac{b^2\log(f)^2}{8f}+2d\right)}}{16\sqrt{-f}}+\frac{f^{bx+a}}{2b\log(f)}$$

input
```
integrate(f^(b*x+a)*cosh(f*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/16*sqrt(2)*sqrt(pi)*f^a*erf(sqrt(2)*sqrt(f)*x - 1/4*sqrt(2)*b*log(f)/sqr
t(f))*e^(1/8*b^2*log(f)^2/f - 2*d)/sqrt(f) + 1/16*sqrt(2)*sqrt(pi)*f^a*erf
(sqrt(2)*sqrt(-f)*x - 1/4*sqrt(2)*b*log(f)/sqrt(-f))*e^(-1/8*b^2*log(f)^2/
f + 2*d)/sqrt(-f) + 1/2*f^(b*x + a)/(b*log(f))
```

3.306.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.40

$$\int f^{a+bx}\cosh^2\left(d+fx^2\right)\,dx =$$

$$-\frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{4}\sqrt{2}\sqrt{f}\left(4x-\frac{b\log(f)}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+8af\log(f)-16df}{8f}\right)}}{16\sqrt{f}}$$
$$-\frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{4}\sqrt{2}\sqrt{-f}\left(4x+\frac{b\log(f)}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-8af\log(f)-16df}{8f}\right)}}{16\sqrt{-f}}$$
$$+\left(\frac{2b\cos\left(-\frac{1}{2}\pi bx\mathrm{sgn}(f)+\frac{1}{2}\pi bx-\frac{1}{2}\pi a\mathrm{sgn}(f)+\frac{1}{2}\pi a\right)\log(|f|)}{4b^2\log(|f|)^2+(\pi b\mathrm{sgn}(f)-\pi b)^2}-\frac{(\pi b\mathrm{sgn}(f)-\pi b)\sin\left(-\frac{1}{2}\pi bx\mathrm{sgn}(f)\right.}{4b^2\log(|f|)^2+(\pi b\mathrm{s}}\right.$$
$$+i\left(\frac{ie^{\left(\frac{1}{2}i\pi bx\mathrm{sgn}(f)-\frac{1}{2}i\pi bx+\frac{1}{2}i\pi a\mathrm{sgn}(f)-\frac{1}{2}i\pi a\right)}}{2i\pi b\mathrm{sgn}(f)-2i\pi b+4b\log(|f|)}-\frac{ie^{\left(-\frac{1}{2}i\pi bx\mathrm{sgn}(f)+\frac{1}{2}i\pi bx-\frac{1}{2}i\pi a\mathrm{sgn}(f)+\frac{1}{2}i\pi a\right)}}{-2i\pi b\mathrm{sgn}(f)+2i\pi b+4b\log(|f|)}\right)e^{(bx\log(|f|)+a\log(|f|))}$$

input
```
integrate(f^(b*x+a)*cosh(f*x^2+d)^2,x, algorithm="giac")
```

output
```
-1/16*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*sqrt(f)*(4*x - b*log(f)/f))*e^(1/8
*(b^2*log(f)^2 + 8*a*f*log(f) - 16*d*f)/f)/sqrt(f) - 1/16*sqrt(2)*sqrt(pi)
*erf(-1/4*sqrt(2)*sqrt(-f)*(4*x + b*log(f)/f))*e^(-1/8*(b^2*log(f)^2 - 8*a
*f*log(f) - 16*d*f)/f)/sqrt(-f) + (2*b*cos(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x
 - 1/2*pi*a*sgn(f) + 1/2*pi*a)*log(abs(f))/(4*b^2*log(abs(f))^2 + (pi*b*sg
n(f) - pi*b)^2) - (pi*b*sgn(f) - pi*b)*sin(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x
 - 1/2*pi*a*sgn(f) + 1/2*pi*a)/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)
^2))*e^(b*x*log(abs(f)) + a*log(abs(f))) + I*(I*e^(1/2*I*pi*b*x*sgn(f) - 1
/2*I*pi*b*x + 1/2*I*pi*a*sgn(f) - 1/2*I*pi*a)/(2*I*pi*b*sgn(f) - 2*I*pi*b
+ 4*b*log(abs(f))) - I*e^(-1/2*I*pi*b*x*sgn(f) + 1/2*I*pi*b*x - 1/2*I*pi*a
*sgn(f) + 1/2*I*pi*a)/(-2*I*pi*b*sgn(f) + 2*I*pi*b + 4*b*log(abs(f))))*e^(
b*x*log(abs(f)) + a*log(abs(f)))
```

3.306.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\cosh^2\left(d+fx^2\right)\,dx=\int f^{a+b\,x}\,{\mathrm{cosh}\left(f\,x^2+d\right)}^2\,dx$$

input
```
int(f^(a + b*x)*cosh(d + f*x^2)^2,x)
```

output
```
int(f^(a + b*x)*cosh(d + f*x^2)^2, x)
```

3.307 $\int f^{a+bx} \cosh^3(d+fx^2)\,dx$

3.307.1 Optimal result

Integrand size = 18, antiderivative size = 239

$$\int f^{a+bx}\cosh^3\left(d+fx^2\right)\,dx = \frac{3}{16}e^{-d+\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right) + \frac{1}{16}e^{-3d+\frac{b^2\log^2(f)}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{3}}\,\mathrm{erf}\left(\frac{6fx-b\log(f)}{2\sqrt{3}\sqrt{f}}\right) + \frac{3}{16}e^{d-\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{2fx+b\log(f)}{2\sqrt{f}}\right) + \frac{1}{16}e^{3d-\frac{b^2\log^2(f)}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{3}}\,\mathrm{erfi}\left(\frac{6fx+b\log(f)}{2\sqrt{3}\sqrt{f}}\right)$$

output
```
1/48*exp(-3*d+1/12*b^2*ln(f)^2/f)*f^(-1/2+a)*erf(1/6*(6*f*x-b*ln(f))*3^(1/
2)/f^(1/2))*3^(1/2)*Pi^(1/2)+1/48*exp(3*d-1/12*b^2*ln(f)^2/f)*f^(-1/2+a)*e
rfi(1/6*(6*f*x+b*ln(f))*3^(1/2)/f^(1/2))*3^(1/2)*Pi^(1/2)+3/16*exp(-d+1/4*
b^2*ln(f)^2/f)*f^(-1/2+a)*erf(1/2*(2*f*x-b*ln(f))/f^(1/2))*Pi^(1/2)+3/16*e
xp(d-1/4*b^2*ln(f)^2/f)*f^(-1/2+a)*erfi(1/2*(2*f*x+b*ln(f))/f^(1/2))*Pi^(1
/2)
```

3.307.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.20

$$\int f^{a+bx}\cosh^3\left(d+fx^2\right)\,dx=\frac{1}{16}e^{-\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{3}}\left(3\sqrt{3}\cosh(d)\text{erfi}\left(\frac{2fx+b\log(f)}{2\sqrt{f}}\right)+e^{\frac{b^2\log^2(f)}{6f}}\cosh(3d)\text{erfi}\left(\frac{6fx+b\log(f)}{2\sqrt{3}\sqrt{f}}\right)+3\sqrt{3}e^{\frac{b^2\log^2(f)}{2f}}\text{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)(\cosh(d)-\sinh(d))+3\sqrt{3}\text{erfi}\left(\frac{2fx+b\log(f)}{2\sqrt{f}}\right)\sinh(d)+e^{\frac{b^2\log^2(f)}{3f}}\text{erf}\left(\frac{6fx-b\log(f)}{2\sqrt{3}\sqrt{f}}\right)(\cosh(3d)-\sinh(3d))+e^{\frac{b^2\log^2(f)}{6f}}\text{erfi}\left(\frac{6fx+b\log(f)}{2\sqrt{3}\sqrt{f}}\right)\sinh(3d)\right)$$

input
```
Integrate[f^(a + b*x)*Cosh[d + f*x^2]^3,x]
```

output
```
(f^(-1/2 + a)*Sqrt[Pi/3]*(3*Sqrt[3]*Cosh[d]*Erfi[(2*f*x + b*Log[f])/(2*Sqr
t[f])] + E^((b^2*Log[f]^2)/(6*f))*Cosh[3*d]*Erfi[(6*f*x + b*Log[f])/(2*Sqr
t[3]*Sqrt[f])] + 3*Sqrt[3]*E^((b^2*Log[f]^2)/(2*f))*Erf[(2*f*x - b*Log[f])
/(2*Sqrt[f])]*(Cosh[d] - Sinh[d]) + 3*Sqrt[3]*Erfi[(2*f*x + b*Log[f])/(2*S
qrt[f])]*Sinh[d] + E^((b^2*Log[f]^2)/(3*f))*Erf[(6*f*x - b*Log[f])/(2*Sqrt
[3]*Sqrt[f])]*(Cosh[3*d] - Sinh[3*d]) + E^((b^2*Log[f]^2)/(6*f))*Erfi[(6*f
*x + b*Log[f])/(2*Sqrt[3]*Sqrt[f])]*Sinh[3*d]))/(16*E^((b^2*Log[f]^2)/(4*f
)))
```

3.307.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx}\cosh^3\left(d+fx^2\right)\,dx$$

$$\Big\downarrow \text{6039}$$

$$\int \left(\frac{1}{8}e^{-3d-3fx^2}f^{a+bx}+\frac{3}{8}e^{-d-fx^2}f^{a+bx}+\frac{3}{8}e^{d+fx^2}f^{a+bx}+\frac{1}{8}e^{3d+3fx^2}f^{a+bx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{4f}-d}\text{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{12f}-3d}\text{erf}\left(\frac{6fx-b\log(f)}{2\sqrt{3}\sqrt{f}}\right)+\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}e^{d-\frac{b^2\log^2(f)}{4f}}\text{erfi}\left(\frac{b\log(f)+2fx}{2\sqrt{f}}\right)+\frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{1}{2}}e^{3d-\frac{b^2\log^2(f)}{12f}}\text{erfi}\left(\frac{b\log(f)+6fx}{2\sqrt{3}\sqrt{f}}\right)$$

input
```
Int[f^(a + b*x)*Cosh[d + f*x^2]^3,x]
```

output
```
(3*E^(-d + (b^2*Log[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(2*f*x - b*Log[
f])/(2*Sqrt[f])])/16 + (E^(-3*d + (b^2*Log[f]^2)/(12*f))*f^(-1/2 + a)*Sqrt
[Pi/3]*Erf[(6*f*x - b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16 + (3*E^(d - (b^2*Lo
g[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erfi[(2*f*x + b*Log[f])/(2*Sqrt[f])])
/16 + (E^(3*d - (b^2*Log[f]^2)/(12*f))*f^(-1/2 + a)*Sqrt[Pi/3]*Erfi[(6*f*x
 + b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16
```

3.307.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.307.4 Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{\text{erf}\left(-\sqrt{3}\sqrt{f}\,x+\frac{\ln(f)b\sqrt{3}}{6\sqrt{f}}\right)\sqrt{3}\sqrt{\pi}\,f^a\text{e}^{\frac{b^2\ln(f)^2-36df}{12f}}}{48\sqrt{f}}-\frac{\text{erf}\left(-\sqrt{-3f}\,x+\frac{\ln(f)b}{2\sqrt{-3f}}\right)\sqrt{\pi}\,f^a\text{e}^{-\frac{b^2\ln(f)^2-36df}{12f}}}{16\sqrt{-3f}}-\frac{3\,\text{erf}\left(-\sqrt{f}\,x+\frac{\ln(f)}{2\sqrt{f}}\right.}{1}$

input
```
int(f^(b*x+a)*cosh(f*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/48*erf(-3^(1/2)*f^(1/2)*x+1/6*ln(f)*b*3^(1/2)/f^(1/2))/f^(1/2)*3^(1/2)*
Pi^(1/2)*f^a*exp(1/12*(b^2*ln(f)^2-36*d*f)/f)-1/16*erf(-(-3*f)^(1/2)*x+1/2
*ln(f)*b/(-3*f)^(1/2))/(-3*f)^(1/2)*Pi^(1/2)*f^a*exp(-1/12*(b^2*ln(f)^2-36
*d*f)/f)-3/16*erf(-f^(1/2)*x+1/2*ln(f)*b/f^(1/2))/f^(1/2)*Pi^(1/2)*f^a*exp
(1/4*(b^2*ln(f)^2-4*d*f)/f)-3/16*erf(-(-f)^(1/2)*x+1/2*ln(f)*b/(-f)^(1/2))
/(-f)^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-4*d*f)/f)
```

3.307.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(181) = 362$.

Time = 0.29 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.85

$$\int f^{a+bx}\cosh^3\left(d+fx^2\right)\,dx =$$
$$-\frac{\sqrt{3}\sqrt{\pi}\sqrt{-f}\cosh\left(\frac{b^2\log(f)^2-12\,af\log(f)-36\,df}{12\,f}\right)\operatorname{erf}\left(\frac{\sqrt{3}(6\,fx+b\log(f))\sqrt{-f}}{6\,f}\right)+\sqrt{3}\sqrt{\pi}\sqrt{f}\cosh\left(\frac{b^2\log(f)^2+12\,af\log(}{12\,f}\right.}{}$$

input
```
integrate(f^(b*x+a)*cosh(f*x^2+d)^3,x, algorithm="fricas")
```

output
```
-1/48*(sqrt(3)*sqrt(pi)*sqrt(-f)*cosh(1/12*(b^2*log(f)^2 - 12*a*f*log(f) -
 36*d*f)/f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f))*sqrt(-f)/f) + sqrt(3)*sqrt(
pi)*sqrt(f)*cosh(1/12*(b^2*log(f)^2 + 12*a*f*log(f) - 36*d*f)/f)*erf(-1/6*
sqrt(3)*(6*f*x - b*log(f))/sqrt(f)) + sqrt(3)*sqrt(pi)*sqrt(f)*erf(-1/6*sq
rt(3)*(6*f*x - b*log(f))/sqrt(f))*sinh(1/12*(b^2*log(f)^2 + 12*a*f*log(f)
- 36*d*f)/f) - sqrt(3)*sqrt(pi)*sqrt(-f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f)
)*sqrt(-f)/f)*sinh(1/12*(b^2*log(f)^2 - 12*a*f*log(f) - 36*d*f)/f) + 9*sqr
t(pi)*sqrt(-f)*cosh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f)*erf(1/2*(
2*f*x + b*log(f))*sqrt(-f)/f) + 9*sqrt(pi)*sqrt(f)*cosh(1/4*(b^2*log(f)^2
+ 4*a*f*log(f) - 4*d*f)/f)*erf(-1/2*(2*f*x - b*log(f))/sqrt(f)) + 9*sqrt(p
i)*sqrt(f)*erf(-1/2*(2*f*x - b*log(f))/sqrt(f))*sinh(1/4*(b^2*log(f)^2 + 4
*a*f*log(f) - 4*d*f)/f) - 9*sqrt(pi)*sqrt(-f)*erf(1/2*(2*f*x + b*log(f))*s
qrt(-f)/f)*sinh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f))/f
```

3.307.6 Sympy [F]

$$\int f^{a+bx}\cosh^3\left(d+fx^2\right)\,dx=\int f^{a+bx}\cosh^3\left(d+fx^2\right)dx$$

input
```
integrate(f**(b*x+a)*cosh(f*x**2+d)**3,x)
```

output
```
Integral(f**(a + b*x)*cosh(d + f*x**2)**3, x)
```

3.307.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.84

$$\int f^{a+bx}\cosh^3\left(d+fx^2\right)\,dx=\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}\operatorname{erf}\left(\sqrt{f}x-\frac{b\log(f)}{2\sqrt{f}}\right)e^{\left(\frac{b^2\log(f)^2}{4f}-d\right)}+\frac{\sqrt{3}\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{3}\sqrt{f}x-\frac{\sqrt{3}b\log(f)}{6\sqrt{f}}\right)e^{\left(\frac{b^2\log(f)^2}{12f}-3d\right)}}{48\sqrt{f}}+\frac{\sqrt{3}\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{3}\sqrt{-f}x-\frac{\sqrt{3}b\log(f)}{6\sqrt{-f}}\right)e^{\left(-\frac{b^2\log(f)^2}{12f}+3d\right)}}{48\sqrt{-f}}+\frac{3\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-f}x-\frac{b\log(f)}{2\sqrt{-f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4f}+d\right)}}{16\sqrt{-f}}$$

input
```
integrate(f^(b*x+a)*cosh(f*x^2+d)^3,x, algorithm="maxima")
```

output
```
3/16*sqrt(pi)*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*b*log(f)/sqrt(f))*e^(1/4*b^2
*log(f)^2/f - d) + 1/48*sqrt(3)*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(f)*x - 1/6*s
qrt(3)*b*log(f)/sqrt(f))*e^(1/12*b^2*log(f)^2/f - 3*d)/sqrt(f) + 1/48*sqrt
(3)*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(-f)*x - 1/6*sqrt(3)*b*log(f)/sqrt(-f))*e
^(-1/12*b^2*log(f)^2/f + 3*d)/sqrt(-f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-f)*x
- 1/2*b*log(f)/sqrt(-f))*e^(-1/4*b^2*log(f)^2/f + d)/sqrt(-f)
```

3.307.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.93

$$\int f^{a+bx}\cosh^3\left(d+fx^2\right)\,dx$$
$$= -\frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{6}\sqrt{3}\sqrt{f}\left(6x-\frac{b\log(f)}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+12\,af\log(f)-36\,df}{12\,f}\right)}}{48\sqrt{f}}$$
$$-\frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{6}\sqrt{3}\sqrt{-f}\left(6x+\frac{b\log(f)}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-12\,af\log(f)-36\,df}{12\,f}\right)}}{48\sqrt{-f}}$$
$$-\frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{f}\left(2x-\frac{b\log(f)}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+4\,af\log(f)-4\,df}{4\,f}\right)}}{16\sqrt{f}}$$
$$-\frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-f}\left(2x+\frac{b\log(f)}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,af\log(f)-4\,df}{4\,f}\right)}}{16\sqrt{-f}}$$

input
```
integrate(f^(b*x+a)*cosh(f*x^2+d)^3,x, algorithm="giac")
```

output
```
-1/48*sqrt(3)*sqrt(pi)*erf(-1/6*sqrt(3)*sqrt(f)*(6*x - b*log(f)/f))*e^(1/1
2*(b^2*log(f)^2 + 12*a*f*log(f) - 36*d*f)/f)/sqrt(f) - 1/48*sqrt(3)*sqrt(p
i)*erf(-1/6*sqrt(3)*sqrt(-f)*(6*x + b*log(f)/f))*e^(-1/12*(b^2*log(f)^2 -
12*a*f*log(f) - 36*d*f)/f)/sqrt(-f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(f)*(2*x
- b*log(f)/f))*e^(1/4*(b^2*log(f)^2 + 4*a*f*log(f) - 4*d*f)/f)/sqrt(f) - 3
/16*sqrt(pi)*erf(-1/2*sqrt(-f)*(2*x + b*log(f)/f))*e^(-1/4*(b^2*log(f)^2 -
 4*a*f*log(f) - 4*d*f)/f)/sqrt(-f)
```

3.307.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\cosh^3\left(d+fx^2\right)\,dx = \int f^{a+b\,x}\cosh\left(f\,x^2+d\right)^3\,dx$$

input
```
int(f^(a + b*x)*cosh(d + f*x^2)^3,x)
```

output
```
int(f^(a + b*x)*cosh(d + f*x^2)^3, x)
```

3.308 $\int f^{a+bx}\cosh\left(d+ex+fx^2\right)\,dx$

3.308.1 Optimal result

Integrand size = 19, antiderivative size = 115

$$\int f^{a+bx}\cosh\left(d+ex+fx^2\right)\,dx=\frac{1}{4}e^{-d+\frac{(e-b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{e+2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{4}e^{d-\frac{(e+b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e+2fx+b\log(f)}{2\sqrt{f}}\right)$$

output
```
1/4*exp(-d+1/4*(e-b*ln(f))^2/f)*f^(-1/2+a)*erf(1/2*(e+2*f*x-b*ln(f))/f^(1/
2))*Pi^(1/2)+1/4*exp(d-1/4*(e+b*ln(f))^2/f)*f^(-1/2+a)*erfi(1/2*(e+2*f*x+b
*ln(f))/f^(1/2))*Pi^(1/2)
```

3.308.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.07

$$\int f^{a+bx}\cosh\left(d+ex+fx^2\right)\,dx=\frac{1}{4}e^{-\frac{e^2+b^2\log^2(f)}{4f}}f^{a-\frac{be+f}{2f}}\sqrt{\pi}\left(e^{\frac{e^2+b^2\log^2(f)}{2f}}\mathrm{erf}\left(\frac{e+2fx-b\log(f)}{2\sqrt{f}}\right)(\cosh(d)-\sinh(d))+\mathrm{erfi}\left(\frac{e+2fx+b\log(f)}{2\sqrt{f}}\right)(\cosh(d)+\sinh(d))\right)$$

input
```
Integrate[f^(a + b*x)*Cosh[d + e*x + f*x^2],x]
```

output
```
(f^(a - (b*e + f)/(2*f))*Sqrt[Pi]*(E^((e^2 + b^2*Log[f]^2)/(2*f))*Erf[(e +
 2*f*x - b*Log[f])/(2*Sqrt[f])]*(Cosh[d] - Sinh[d]) + Erfi[(e + 2*f*x + b*
Log[f])/(2*Sqrt[f])]*(Cosh[d] + Sinh[d])))/(4*E^((e^2 + b^2*Log[f]^2)/(4*f
)))
```

3.308.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx}\cosh\left(d+ex+fx^2\right)\,dx$$

$$\Big\downarrow \text{6039}$$

$$\int \left(\frac{1}{2}f^{a+bx}e^{-d-ex-fx^2}+\frac{1}{2}f^{a+bx}e^{d+ex+fx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{(e-b\log(f))^2}{4f}-d}\text{erf}\left(\frac{-b\log(f)+e+2fx}{2\sqrt{f}}\right)+\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{d-\frac{(b\log(f)+e)^2}{4f}}\text{erfi}\left(\frac{b\log(f)+e+2fx}{2\sqrt{f}}\right)$$

input
```
Int[f^(a + b*x)*Cosh[d + e*x + f*x^2],x]
```

output
```
(E^(-d + (e - b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(e + 2*f*x - b*
Log[f])/(2*Sqrt[f])])/4 + (E^(d - (e + b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqr
t[Pi]*Erfi[(e + 2*f*x + b*Log[f])/(2*Sqrt[f])])/4
```

3.308.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.308.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.10

method	result	siz
risch	$-\frac{\operatorname{erf}\left(-\sqrt{f}\, x+\frac{b \ln(f)-e}{2 \sqrt{f}}\right) \sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{b^{2} \ln(f)^{2}-2 \ln(f) b e-4 d f+e^{2}}{4 f}}}{4 \sqrt{f}}-\frac{\operatorname{erf}\left(-\sqrt{-f}\, x+\frac{e+b \ln(f)}{2 \sqrt{-f}}\right) \sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}+2 \ln(f) b e-4 d f+e^{2}}{4 f}}}{4 \sqrt{-f}}$	12

input
```
int(f^(b*x+a)*cosh(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4*erf(-f^(1/2)*x+1/2*(b*ln(f)-e)/f^(1/2))/f^(1/2)*Pi^(1/2)*f^a*exp(1/4*
(b^2*ln(f)^2-2*ln(f)*b*e-4*d*f+e^2)/f)-1/4*erf(-(-f)^(1/2)*x+1/2*(e+b*ln(f
))/(-f)^(1/2))/(-f)^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+2*ln(f)*b*e-4
*d*f+e^2)/f)
```

3.308.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. 2(90) = 180.

Time = 0.27 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.18

$$\int f^{a+bx} \cosh \left(d+ex+fx^2\right) \, dx =$$
$$-\frac{\sqrt{\pi}\sqrt{-f}\cosh\left(\frac{b^2\log(f)^2+e^2-4\,df+2\,(be-2\,af)\log(f)}{4\,f}\right)\operatorname{erf}\left(\frac{(2\,fx+b\log(f)+e)\sqrt{-f}}{2\,f}\right)+\sqrt{\pi}\sqrt{f}\cosh\left(\frac{b^2\log(f)^2+e^2-4\,df-}{4\,f}\right.}{}$$

input
```
integrate(f^(b*x+a)*cosh(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(pi)*sqrt(-f)*cosh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f + 2*(b*e - 2*
a*f)*log(f))/f)*erf(1/2*(2*f*x + b*log(f) + e)*sqrt(-f)/f) + sqrt(pi)*sqrt
(f)*cosh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f - 2*(b*e - 2*a*f)*log(f))/f)*erf(
-1/2*(2*f*x - b*log(f) + e)/sqrt(f)) - sqrt(pi)*sqrt(-f)*erf(1/2*(2*f*x +
b*log(f) + e)*sqrt(-f)/f)*sinh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f + 2*(b*e -
2*a*f)*log(f))/f) + sqrt(pi)*sqrt(f)*erf(-1/2*(2*f*x - b*log(f) + e)/sqrt(
f))*sinh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f - 2*(b*e - 2*a*f)*log(f))/f))/f
```

3.308.6 Sympy [F]

$$\int f^{a+bx}\cosh\left(d+ex+fx^2\right)\,dx=\int f^{a+bx}\cosh\left(d+ex+fx^2\right)dx$$

input
```
integrate(f**(b*x+a)*cosh(f*x**2+e*x+d),x)
```

output
```
Integral(f**(a + b*x)*cosh(d + e*x + f*x**2), x)
```

3.308.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89

$$\int f^{a+bx}\cosh\left(d+ex+fx^2\right)\,dx=\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}\operatorname{erf}\left(\sqrt{f}x-\frac{b\log(f)-e}{2\sqrt{f}}\right)e^{\left(-d+\frac{(b\log(f)-e)^2}{4f}\right)}+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-f}x-\frac{b\log(f)+e}{2\sqrt{-f}}\right)e^{\left(d-\frac{(b\log(f)+e)^2}{4f}\right)}}{4\sqrt{-f}}$$

input
```
integrate(f^(b*x+a)*cosh(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
1/4*sqrt(pi)*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*(b*log(f) - e)/sqrt(f))*e^(-d
 + 1/4*(b*log(f) - e)^2/f) + 1/4*sqrt(pi)*f^a*erf(sqrt(-f)*x - 1/2*(b*log(
f) + e)/sqrt(-f))*e^(d - 1/4*(b*log(f) + e)^2/f)/sqrt(-f)
```

3.308.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.15

$$\int f^{a+bx}\cosh\left(d+ex+fx^2\right)\,dx$$
$$=-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-f}\left(2x+\frac{b\log(f)+e}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2+2be\log(f)-4af\log(f)+e^2-4df}{4f}\right)}}{4\sqrt{-f}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{f}\left(2x-\frac{b\log(f)-e}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2-2be\log(f)+4af\log(f)+e^2-4df}{4f}\right)}}{4\sqrt{f}}$$

input
```
integrate(f^(b*x+a)*cosh(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*erf(-1/2*sqrt(-f)*(2*x + (b*log(f) + e)/f))*e^(-1/4*(b^2*log
(f)^2 + 2*b*e*log(f) - 4*a*f*log(f) + e^2 - 4*d*f)/f)/sqrt(-f) - 1/4*sqrt(
pi)*erf(-1/2*sqrt(f)*(2*x - (b*log(f) - e)/f))*e^(1/4*(b^2*log(f)^2 - 2*b*
e*log(f) + 4*a*f*log(f) + e^2 - 4*d*f)/f)/sqrt(f)
```

3.308.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\cosh\left(d+ex+fx^2\right)\,dx=\int f^{a+b\,x}\cosh\left(f\,x^2+e\,x+d\right)\,dx$$

input
```
int(f^(a + b*x)*cosh(d + e*x + f*x^2),x)
```

output
```
int(f^(a + b*x)*cosh(d + e*x + f*x^2), x)
```

3.309 $\int f^{a+bx}\cosh^2(d+ex+fx^2)\,dx$

3.309.1 Optimal result

Integrand size = 21, antiderivative size = 161

$$\int f^{a+bx}\cosh^2(d+ex+fx^2)\,dx=\frac{1}{8}e^{-2d+\frac{(2e-b\log(f))^2}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{2e+4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)+\frac{1}{8}e^{2d-\frac{(2e+b\log(f))^2}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{2e+4fx+b\log(f)}{2\sqrt{2}\sqrt{f}}\right)+\frac{f^{a+bx}}{2b\log(f)}$$

output
```
1/2*f^(b*x+a)/b/ln(f)+1/16*exp(-2*d+1/8*(2*e-b*ln(f))^2/f)*f^(-1/2+a)*erf(
1/4*(2*e+4*f*x-b*ln(f))*2^(1/2)/f^(1/2))*2^(1/2)*Pi^(1/2)+1/16*exp(2*d-1/8
*(2*e+b*ln(f))^2/f)*f^(-1/2+a)*erfi(1/4*(2*e+4*f*x+b*ln(f))*2^(1/2)/f^(1/2
))*2^(1/2)*Pi^(1/2)
```

3.309.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.37

$$\int f^{a+bx}\cosh^2(d+ex+fx^2)\,dx$$
$$=\frac{e^{-\frac{4e^2+b^2\log^2(f)}{8f}}f^{a-\frac{be+f}{2f}}\left(4\sqrt{2}e^{\frac{4e^2+b^2\log^2(f)}{8f}}f^{\frac{1}{2}+b\left(\frac{e}{2f}+x\right)}+be^{\frac{4e^2+b^2\log^2(f)}{4f}}\sqrt{\pi}\text{erf}\left(\frac{2e+4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)\log(f)(\cosh(2d)-\right.}{8\sqrt{2}b\log(f)}$$

input
```
Integrate[f^(a + b*x)*Cosh[d + e*x + f*x^2]^2,x]
```

output
```
(f^(a - (b*e + f)/(2*f))*(4*Sqrt[2]*E^((4*e^2 + b^2*Log[f]^2)/(8*f))*f^(1/
2 + b*(e/(2*f) + x)) + b*E^((4*e^2 + b^2*Log[f]^2)/(4*f))*Sqrt[Pi]*Erf[(2*
e + 4*f*x - b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*Log[f]*(Cosh[2*d] - Sinh[2*d])
+ b*Sqrt[Pi]*Erfi[(2*e + 4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*Log[f]*(Co
sh[2*d] + Sinh[2*d])))/(8*Sqrt[2]*b*E^((4*e^2 + b^2*Log[f]^2)/(8*f))*Log[f
])
```

3.309.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx} \cosh^2\left(d+ex+fx^2\right) dx$$

$$\Big\downarrow \text{ 6039}$$

$$\int \left(\frac{1}{4} f^{a+bx} e^{-2d-2ex-2fx^2}+\frac{1}{4} f^{a+bx} e^{2d+2ex+2fx^2}+\frac{1}{2} f^{a+bx}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{8}\sqrt{\frac{\pi}{2}} f^{a-\frac{1}{2}} e^{\frac{(2e-b\log(f))^2}{8f}-2d} \text{erf}\left(\frac{-b\log(f)+2e+4fx}{2\sqrt{2}\sqrt{f}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}} f^{a-\frac{1}{2}} e^{2d-\frac{(b\log(f)+2e)^2}{8f}} \text{erfi}\left(\frac{b\log(f)+2e+4fx}{2\sqrt{2}\sqrt{f}}\right)+\frac{f^{a+bx}}{2b\log(f)}$$

input
```
Int[f^(a + b*x)*Cosh[d + e*x + f*x^2]^2,x]
```

output
```
(E^(-2*d + (2*e - b*Log[f])^2/(8*f))*f^(-1/2 + a)*Sqrt[Pi/2]*Erf[(2*e + 4*
f*x - b*Log[f])/(2*Sqrt[2]*Sqrt[f])])/8 + (E^(2*d - (2*e + b*Log[f])^2/(8*
f))*f^(-1/2 + a)*Sqrt[Pi/2]*Erfi[(2*e + 4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[
f])])/8 + f^(a + b*x)/(2*b*Log[f])
```

3.309.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.309.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.98

method	result
risch	$-\frac{\operatorname{erf}\left(-\sqrt{2}\,\sqrt{f}\,x+\frac{(b\ln(f)-2e)\sqrt{2}}{4\sqrt{f}}\right)\sqrt{2}\,\sqrt{\pi}\,f^{a}\mathrm{e}^{\frac{b^{2}\ln(f)^{2}-4\ln(f)be-16df+4e^{2}}{8f}}}{16\sqrt{f}}-\frac{\operatorname{erf}\left(-\sqrt{-2f}\,x+\frac{2e+b\ln(f)}{2\sqrt{-2f}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{b^{2}\ln(f)^{2}+4\ln(f)b}{8f}}}{8\sqrt{-2f}}$

input
```
int(f^(b*x+a)*cosh(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/16*erf(-2^(1/2)*f^(1/2)*x+1/4*(b*ln(f)-2*e)*2^(1/2)/f^(1/2))/f^(1/2)*2^
(1/2)*Pi^(1/2)*f^a*exp(1/8*(b^2*ln(f)^2-4*ln(f)*b*e-16*d*f+4*e^2)/f)-1/8*e
rf(-(-2*f)^(1/2)*x+1/2*(2*e+b*ln(f))/(-2*f)^(1/2))/(-2*f)^(1/2)*Pi^(1/2)*f
^a*exp(-1/8*(b^2*ln(f)^2+4*ln(f)*b*e-16*d*f+4*e^2)/f)+1/2*f^a*f^(b*x)/b/ln
(f)
```

3.309.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(126) = 252.

Time = 0.26 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.07

$$\int f^{a+bx}\cosh^{2}\left(d+ex+fx^{2}\right)\,dx =$$

$$-\frac{\sqrt{2}\sqrt{\pi}b\sqrt{-f}\cosh\left(\frac{b^{2}\log(f)^{2}+4e^{2}-16df+4(be-2af)\log(f)}{8f}\right)\operatorname{erf}\left(\frac{\sqrt{2}(4fx+b\log(f)+2e)\sqrt{-f}}{4f}\right)\log(f)+\sqrt{2}\sqrt{\pi}b\sqrt{f}\,\mathrm{c}}{}$$

input
```
integrate(f^(b*x+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

output
```
-1/16*(sqrt(2)*sqrt(pi)*b*sqrt(-f)*cosh(1/8*(b^2*log(f)^2 + 4*e^2 - 16*d*f
 + 4*(b*e - 2*a*f)*log(f))/f)*erf(1/4*sqrt(2)*(4*f*x + b*log(f) + 2*e)*sqr
t(-f)/f)*log(f) + sqrt(2)*sqrt(pi)*b*sqrt(f)*cosh(1/8*(b^2*log(f)^2 + 4*e^
2 - 16*d*f - 4*(b*e - 2*a*f)*log(f))/f)*erf(-1/4*sqrt(2)*(4*f*x - b*log(f)
 + 2*e)/sqrt(f))*log(f) - sqrt(2)*sqrt(pi)*b*sqrt(-f)*erf(1/4*sqrt(2)*(4*f
*x + b*log(f) + 2*e)*sqrt(-f)/f)*log(f)*sinh(1/8*(b^2*log(f)^2 + 4*e^2 - 1
6*d*f + 4*(b*e - 2*a*f)*log(f))/f) + sqrt(2)*sqrt(pi)*b*sqrt(f)*erf(-1/4*s
qrt(2)*(4*f*x - b*log(f) + 2*e)/sqrt(f))*log(f)*sinh(1/8*(b^2*log(f)^2 + 4
*e^2 - 16*d*f - 4*(b*e - 2*a*f)*log(f))/f) - 8*f*cosh((b*x + a)*log(f)) -
8*f*sinh((b*x + a)*log(f)))/(b*f*log(f))
```

3.309.6 Sympy [F]

$$\int f^{a+bx}\cosh^2\left(d+ex+fx^2\right)\,dx=\int f^{a+bx}\cosh^2\left(d+ex+fx^2\right)dx$$

input
```
integrate(f**(b*x+a)*cosh(f*x**2+e*x+d)**2,x)
```

output
```
Integral(f**(a + b*x)*cosh(d + e*x + f*x**2)**2, x)
```

3.309.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.89

$$\int f^{a+bx}\cosh^2\left(d+ex+fx^2\right)\,dx$$
$$=\frac{\sqrt{2}\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{2}\sqrt{-f}x-\frac{\sqrt{2}(b\log(f)+2e)}{4\sqrt{-f}}\right)e^{\left(2d-\frac{(b\log(f)+2e)^2}{8f}\right)}}{16\sqrt{-f}}$$
$$+\frac{\sqrt{2}\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{2}\sqrt{f}x-\frac{\sqrt{2}(b\log(f)-2e)}{4\sqrt{f}}\right)e^{\left(-2d+\frac{(b\log(f)-2e)^2}{8f}\right)}}{16\sqrt{f}}+\frac{f^{bx+a}}{2\,b\log(f)}$$

input
```
integrate(f^(b*x+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

output
```
1/16*sqrt(2)*sqrt(pi)*f^a*erf(sqrt(2)*sqrt(-f)*x - 1/4*sqrt(2)*(b*log(f) +
 2*e)/sqrt(-f))*e^(2*d - 1/8*(b*log(f) + 2*e)^2/f)/sqrt(-f) + 1/16*sqrt(2)
*sqrt(pi)*f^a*erf(sqrt(2)*sqrt(f)*x - 1/4*sqrt(2)*(b*log(f) - 2*e)/sqrt(f)
)*e^(-2*d + 1/8*(b*log(f) - 2*e)^2/f)/sqrt(f) + 1/2*f^(b*x + a)/(b*log(f))
```

3.309.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 387, normalized size of antiderivative = 2.40

$$\int f^{a+bx}\cosh^2\left(d+ex+fx^2\right)\,dx$$
$$= -\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{4}\sqrt{2}\sqrt{-f}\left(4x+\frac{b\log(f)+2e}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2+4be\log(f)-8af\log(f)+4e^2-16df}{8f}\right)}}{16\sqrt{-f}}$$
$$-\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{4}\sqrt{2}\sqrt{f}\left(4x-\frac{b\log(f)-2e}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2-4be\log(f)+8af\log(f)+4e^2-16df}{8f}\right)}}{16\sqrt{f}}$$
$$+\left(\frac{2b\cos\left(-\frac{1}{2}\pi bx\text{sgn}(f)+\frac{1}{2}\pi bx-\frac{1}{2}\pi a\text{sgn}(f)+\frac{1}{2}\pi a\right)\log\left(|f|\right)}{4b^2\log\left(|f|\right)^2+\left(\pi b\text{sgn}\left(f\right)-\pi b\right)^2}-\frac{\left(\pi b\text{sgn}(f)-\pi b\right)\sin\left(-\frac{1}{2}\pi bx\text{sgn}(f)+\right.}{4b^2\log\left(|f|\right)^2+\left(\pi b\text{s}\right.}\right.$$
$$+i\left(\frac{ie^{\left(\frac{1}{2}i\pi bx\text{sgn}(f)-\frac{1}{2}i\pi bx+\frac{1}{2}i\pi a\text{sgn}(f)-\frac{1}{2}i\pi a\right)}}{2i\pi b\text{sgn}\left(f\right)-2i\pi b+4b\log\left(|f|\right)}-\frac{ie^{\left(-\frac{1}{2}i\pi bx\text{sgn}(f)+\frac{1}{2}i\pi bx-\frac{1}{2}i\pi a\text{sgn}(f)+\frac{1}{2}i\pi a\right)}}{-2i\pi b\text{sgn}\left(f\right)+2i\pi b+4b\log\left(|f|\right)}\right)e^{\left(bx\log(|f|)+a\log(|f|)\right)}$$

input
```
integrate(f^(b*x+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="giac")
```

output
```
-1/16*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*sqrt(-f)*(4*x + (b*log(f) + 2*e)/f
))*e^(-1/8*(b^2*log(f)^2 + 4*b*e*log(f) - 8*a*f*log(f) + 4*e^2 - 16*d*f)/f
)/sqrt(-f) - 1/16*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*sqrt(f)*(4*x - (b*log(
f) - 2*e)/f))*e^(1/8*(b^2*log(f)^2 - 4*b*e*log(f) + 8*a*f*log(f) + 4*e^2 -
 16*d*f)/f)/sqrt(f) + (2*b*cos(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*
sgn(f) + 1/2*pi*a)*log(abs(f))/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)
^2) - (pi*b*sgn(f) - pi*b)*sin(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*
sgn(f) + 1/2*pi*a)/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2))*e^(b*x*
log(abs(f)) + a*log(abs(f))) + I*(I*e^(1/2*I*pi*b*x*sgn(f) - 1/2*I*pi*b*x
+ 1/2*I*pi*a*sgn(f) - 1/2*I*pi*a)/(2*I*pi*b*sgn(f) - 2*I*pi*b + 4*b*log(ab
s(f))) - I*e^(-1/2*I*pi*b*x*sgn(f) + 1/2*I*pi*b*x - 1/2*I*pi*a*sgn(f) + 1/
2*I*pi*a)/(-2*I*pi*b*sgn(f) + 2*I*pi*b + 4*b*log(abs(f))))*e^(b*x*log(abs(
f)) + a*log(abs(f)))
```

3.309.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\cosh^2\left(d+ex+fx^2\right)\,dx=\int f^{a+b\,x}\,\cosh\left(f\,x^2+e\,x+d\right)^2\,dx$$

input
```
int(f^(a + b*x)*cosh(d + e*x + f*x^2)^2,x)
```

output
```
int(f^(a + b*x)*cosh(d + e*x + f*x^2)^2, x)
```

3.310 $\int f^{a+bx}\cosh^3(d+ex+fx^2)\,dx$

3.310.1 Optimal result

Integrand size = 21, antiderivative size = 257

$$\begin{aligned}\int f^{a+bx}\cosh^3(d+ex+fx^2)\,dx &= \frac{3}{16}e^{-d+\frac{(e-b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\text{erf}\left(\frac{e+2fx-b\log(f)}{2\sqrt{f}}\right)\\ &+\frac{1}{16}e^{-3d+\frac{(3e-b\log(f))^2}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{3}}\text{erf}\left(\frac{3e+6fx-b\log(f)}{2\sqrt{3}\sqrt{f}}\right)\\ &+\frac{3}{16}e^{d-\frac{(e+b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\text{erfi}\left(\frac{e+2fx+b\log(f)}{2\sqrt{f}}\right)\\ &+\frac{1}{16}e^{3d-\frac{(3e+b\log(f))^2}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{3}}\text{erfi}\left(\frac{3e+6fx+b\log(f)}{2\sqrt{3}\sqrt{f}}\right)\end{aligned}$$

output
```
1/48*exp(-3*d+1/12*(3*e-b*ln(f))^2/f)*f^(-1/2+a)*erf(1/6*(3*e+6*f*x-b*ln(f
))*3^(1/2)/f^(1/2))*3^(1/2)*Pi^(1/2)+1/48*exp(3*d-1/12*(3*e+b*ln(f))^2/f)*
f^(-1/2+a)*erfi(1/6*(3*e+6*f*x+b*ln(f))*3^(1/2)/f^(1/2))*3^(1/2)*Pi^(1/2)+
3/16*exp(-d+1/4*(e-b*ln(f))^2/f)*f^(-1/2+a)*erf(1/2*(e+2*f*x-b*ln(f))/f^(1
/2))*Pi^(1/2)+3/16*exp(d-1/4*(e+b*ln(f))^2/f)*f^(-1/2+a)*erfi(1/2*(e+2*f*x
+b*ln(f))/f^(1/2))*Pi^(1/2)
```

3.310.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.37

$$\int f^{a+bx}\cosh^3\left(d+ex+fx^2\right)\,dx$$
$$= \frac{1}{16}e^{-\frac{3e^2+b^2\log^2(f)}{4f}}f^{a-\frac{be+f}{2f}}\sqrt{\frac{\pi}{3}}\left(3\sqrt{3}e^{\frac{e^2}{2f}}\cosh(d)\mathrm{erfi}\left(\frac{e+2fx+b\log(f)}{2\sqrt{f}}\right)\right.$$
$$+e^{\frac{b^2\log^2(f)}{6f}}\cosh(3d)\mathrm{erfi}\left(\frac{3e+6fx+b\log(f)}{2\sqrt{3}\sqrt{f}}\right)$$
$$+3\sqrt{3}e^{\frac{2e^2+b^2\log^2(f)}{2f}}\mathrm{erf}\left(\frac{e+2fx-b\log(f)}{2\sqrt{f}}\right)(\cosh(d)-\sinh(d))$$
$$+3\sqrt{3}e^{\frac{e^2}{2f}}\mathrm{erfi}\left(\frac{e+2fx+b\log(f)}{2\sqrt{f}}\right)\sinh(d)$$
$$+e^{\frac{9e^2+2b^2\log^2(f)}{6f}}\mathrm{erf}\left(\frac{3e+6fx-b\log(f)}{2\sqrt{3}\sqrt{f}}\right)(\cosh(3d)-\sinh(3d))$$
$$\left.+e^{\frac{b^2\log^2(f)}{6f}}\mathrm{erfi}\left(\frac{3e+6fx+b\log(f)}{2\sqrt{3}\sqrt{f}}\right)\sinh(3d)\right)$$

input
```
Integrate[f^(a + b*x)*Cosh[d + e*x + f*x^2]^3,x]
```

output
```
(f^(a - (b*e + f)/(2*f))*Sqrt[Pi/3]*(3*Sqrt[3]*E^(e^2/(2*f))*Cosh[d]*Erfi[
(e + 2*f*x + b*Log[f])/(2*Sqrt[f])] + E^((b^2*Log[f]^2)/(6*f))*Cosh[3*d]*E
rfi[(3*e + 6*f*x + b*Log[f])/(2*Sqrt[3]*Sqrt[f])] + 3*Sqrt[3]*E^((2*e^2 +
b^2*Log[f]^2)/(2*f))*Erf[(e + 2*f*x - b*Log[f])/(2*Sqrt[f])]*(Cosh[d] - Si
nh[d]) + 3*Sqrt[3]*E^(e^2/(2*f))*Erfi[(e + 2*f*x + b*Log[f])/(2*Sqrt[f])]*
Sinh[d] + E^((9*e^2 + 2*b^2*Log[f]^2)/(6*f))*Erf[(3*e + 6*f*x - b*Log[f])/
(2*Sqrt[3]*Sqrt[f])]*(Cosh[3*d] - Sinh[3*d]) + E^((b^2*Log[f]^2)/(6*f))*Er
fi[(3*e + 6*f*x + b*Log[f])/(2*Sqrt[3]*Sqrt[f])]*Sinh[3*d]))/(16*E^((3*e^2
 + b^2*Log[f]^2)/(4*f)))
```

3.310.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx}\cosh^3\left(d+ex+fx^2\right)\,dx$$

$$\Big\downarrow \text{6039}$$

$$\int \left(\frac{3}{8}f^{a+bx}\exp\left(-3(d+ex+fx^2)+2d+2ex+2fx^2\right)+\frac{3}{8}f^{a+bx}\exp\left(-3(d+ex+fx^2)+4d+4ex+4fx^2\right)+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{(e-b\log(f))^2}{4f}-d}\operatorname{erf}\left(\frac{-b\log(f)+e+2fx}{2\sqrt{f}}\right)+\frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{1}{2}}e^{\frac{(3e-b\log(f))^2}{12f}-3d}\operatorname{erf}\left(\frac{-b\log(f)+3e+6fx}{2\sqrt{3}\sqrt{f}}\right)+\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}e^{d-\frac{(b\log(f)+e)^2}{4f}}\operatorname{erfi}\left(\frac{b\log(f)+e+2fx}{2\sqrt{f}}\right)+\frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{1}{2}}e^{3d-\frac{(b\log(f)+3e)^2}{12f}}\operatorname{erfi}\left(\frac{b\log(f)+3e+6fx}{2\sqrt{3}\sqrt{f}}\right)$$

input
```
Int[f^(a + b*x)*Cosh[d + e*x + f*x^2]^3,x]
```

output
```
(3*E^(-d + (e - b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(e + 2*f*x -
b*Log[f])/(2*Sqrt[f])])/16 + (E^(-3*d + (3*e - b*Log[f])^2/(12*f))*f^(-1/2
 + a)*Sqrt[Pi/3]*Erf[(3*e + 6*f*x - b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16 + (
3*E^(d - (e + b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erfi[(e + 2*f*x + b
*Log[f])/(2*Sqrt[f])])/16 + (E^(3*d - (3*e + b*Log[f])^2/(12*f))*f^(-1/2 +
 a)*Sqrt[Pi/3]*Erfi[(3*e + 6*f*x + b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16
```

3.310.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.310.4 Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{\operatorname{erf}\left(-\sqrt{3}\,\sqrt{f}\,x+\frac{(b\ln(f)-3e)\sqrt{3}}{6\sqrt{f}}\right)\sqrt{3}\,\sqrt{\pi}\,f^{a}\mathrm{e}^{\frac{b^{2}\ln(f)^{2}-6\ln(f)be-36df+9e^{2}}{12f}}}{48\sqrt{f}}-\frac{\operatorname{erf}\left(-\sqrt{-3f}\,x+\frac{3e+b\ln(f)}{2\sqrt{-3f}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{b^{2}\ln(f)^{2}+6\ln(f)b}{12f}}}{16\sqrt{-3f}}$

input
```
int(f^(b*x+a)*cosh(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/48*erf(-3^(1/2)*f^(1/2)*x+1/6*(b*ln(f)-3*e)*3^(1/2)/f^(1/2))/f^(1/2)*3^
(1/2)*Pi^(1/2)*f^a*exp(1/12*(b^2*ln(f)^2-6*ln(f)*b*e-36*d*f+9*e^2)/f)-1/16
*erf(-(-3*f)^(1/2)*x+1/2*(3*e+b*ln(f))/(-3*f)^(1/2))/(-3*f)^(1/2)*Pi^(1/2)
*f^a*exp(-1/12*(b^2*ln(f)^2+6*ln(f)*b*e-36*d*f+9*e^2)/f)-3/16*erf(-f^(1/2)
*x+1/2*(b*ln(f)-e)/f^(1/2))/f^(1/2)*Pi^(1/2)*f^a*exp(1/4*(b^2*ln(f)^2-2*ln
(f)*b*e-4*d*f+e^2)/f)-3/16*erf(-(-f)^(1/2)*x+1/2*(e+b*ln(f))/(-f)^(1/2))/(
-f)^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+2*ln(f)*b*e-4*d*f+e^2)/f)
```

3.310.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(199) = 398$.

Time = 0.26 (sec) , antiderivative size = 539, normalized size of antiderivative = 2.10

$$\int f^{a+bx}\cosh^{3}\left(d+ex+fx^{2}\right)\,dx=$$
$$-\frac{\sqrt{3}\sqrt{\pi}\sqrt{-f}\cosh\left(\frac{b^{2}\log(f)^{2}+9e^{2}-36df+6(be-2af)\log(f)}{12f}\right)\operatorname{erf}\left(\frac{\sqrt{3}(6fx+b\log(f)+3e)\sqrt{-f}}{6f}\right)+\sqrt{3}\sqrt{\pi}\sqrt{f}\cosh\left(\frac{b^{2}\log}{}\right.}{}$$

input
```
integrate(f^(b*x+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

output
```
-1/48*(sqrt(3)*sqrt(pi)*sqrt(-f)*cosh(1/12*(b^2*log(f)^2 + 9*e^2 - 36*d*f
+ 6*(b*e - 2*a*f)*log(f))/f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f) + 3*e)*sqrt
(-f)/f) + sqrt(3)*sqrt(pi)*sqrt(f)*cosh(1/12*(b^2*log(f)^2 + 9*e^2 - 36*d*
f - 6*(b*e - 2*a*f)*log(f))/f)*erf(-1/6*sqrt(3)*(6*f*x - b*log(f) + 3*e)/s
qrt(f)) - sqrt(3)*sqrt(pi)*sqrt(-f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f) + 3*
e)*sqrt(-f)/f)*sinh(1/12*(b^2*log(f)^2 + 9*e^2 - 36*d*f + 6*(b*e - 2*a*f)*
log(f))/f) + sqrt(3)*sqrt(pi)*sqrt(f)*erf(-1/6*sqrt(3)*(6*f*x - b*log(f) +
 3*e)/sqrt(f))*sinh(1/12*(b^2*log(f)^2 + 9*e^2 - 36*d*f - 6*(b*e - 2*a*f)*
log(f))/f) + 9*sqrt(pi)*sqrt(-f)*cosh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f + 2*
(b*e - 2*a*f)*log(f))/f)*erf(1/2*(2*f*x + b*log(f) + e)*sqrt(-f)/f) + 9*sq
rt(pi)*sqrt(f)*cosh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f - 2*(b*e - 2*a*f)*log(
f))/f)*erf(-1/2*(2*f*x - b*log(f) + e)/sqrt(f)) - 9*sqrt(pi)*sqrt(-f)*erf(
1/2*(2*f*x + b*log(f) + e)*sqrt(-f)/f)*sinh(1/4*(b^2*log(f)^2 + e^2 - 4*d*
f + 2*(b*e - 2*a*f)*log(f))/f) + 9*sqrt(pi)*sqrt(f)*erf(-1/2*(2*f*x - b*lo
g(f) + e)/sqrt(f))*sinh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f - 2*(b*e - 2*a*f)*
log(f))/f))/f
```

3.310.6 Sympy [F]

$$\int f^{a+bx}\cosh^3\left(d+ex+fx^2\right)\,dx=\int f^{a+bx}\cosh^3\left(d+ex+fx^2\right)dx$$

input
```
integrate(f**(b*x+a)*cosh(f*x**2+e*x+d)**3,x)
```

output
```
Integral(f**(a + b*x)*cosh(d + e*x + f*x**2)**3, x)
```

3.310.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.89

$$\int f^{a+bx}\cosh^3\left(d+ex+fx^2\right)\,dx$$
$$=\frac{\sqrt{3}\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{3}\sqrt{-f}x-\frac{\sqrt{3}(b\log(f)+3e)}{6\sqrt{-f}}\right)e^{\left(3d-\frac{(b\log(f)+3e)^2}{12f}\right)}}{48\sqrt{-f}}$$
$$+\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}\operatorname{erf}\left(\sqrt{f}x-\frac{b\log(f)-e}{2\sqrt{f}}\right)e^{\left(-d+\frac{(b\log(f)-e)^2}{4f}\right)}$$
$$+\frac{\sqrt{3}\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{3}\sqrt{f}x-\frac{\sqrt{3}(b\log(f)-3e)}{6\sqrt{f}}\right)e^{\left(-3d+\frac{(b\log(f)-3e)^2}{12f}\right)}}{48\sqrt{f}}$$
$$+\frac{3\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-f}x-\frac{b\log(f)+e}{2\sqrt{-f}}\right)e^{\left(d-\frac{(b\log(f)+e)^2}{4f}\right)}}{16\sqrt{-f}}$$

input
```
integrate(f^(b*x+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

output
```
1/48*sqrt(3)*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(-f)*x - 1/6*sqrt(3)*(b*log(f) +
 3*e)/sqrt(-f))*e^(3*d - 1/12*(b*log(f) + 3*e)^2/f)/sqrt(-f) + 3/16*sqrt(p
i)*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*(b*log(f) - e)/sqrt(f))*e^(-d + 1/4*(b*
log(f) - e)^2/f) + 1/48*sqrt(3)*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(f)*x - 1/6*s
qrt(3)*(b*log(f) - 3*e)/sqrt(f))*e^(-3*d + 1/12*(b*log(f) - 3*e)^2/f)/sqrt
(f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-f)*x - 1/2*(b*log(f) + e)/sqrt(-f))*e^(d
 - 1/4*(b*log(f) + e)^2/f)/sqrt(-f)
```

3.310.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.09

$$\int f^{a+bx}\cosh^3\left(d+ex+fx^2\right)\,dx$$
$$= -\frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{6}\sqrt{3}\sqrt{-f}\left(6x+\frac{b\log(f)+3e}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2+6be\log(f)-12af\log(f)+9e^2-36df}{12f}\right)}}{48\sqrt{-f}} - \frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{6}\sqrt{3}\sqrt{f}\left(6x-\frac{b\log(f)-3e}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2-6be\log(f)+12af\log(f)+9e^2-36df}{12f}\right)}}{48\sqrt{f}} - \frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-f}\left(2x+\frac{b\log(f)+e}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2+2be\log(f)-4af\log(f)+e^2-4df}{4f}\right)}}{16\sqrt{-f}} - \frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{f}\left(2x-\frac{b\log(f)-e}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2-2be\log(f)+4af\log(f)+e^2-4df}{4f}\right)}}{16\sqrt{f}}$$

input
```
integrate(f^(b*x+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="giac")
```

output
```
-1/48*sqrt(3)*sqrt(pi)*erf(-1/6*sqrt(3)*sqrt(-f)*(6*x + (b*log(f) + 3*e)/f
))*e^(-1/12*(b^2*log(f)^2 + 6*b*e*log(f) - 12*a*f*log(f) + 9*e^2 - 36*d*f)
/f)/sqrt(-f) - 1/48*sqrt(3)*sqrt(pi)*erf(-1/6*sqrt(3)*sqrt(f)*(6*x - (b*lo
g(f) - 3*e)/f))*e^(1/12*(b^2*log(f)^2 - 6*b*e*log(f) + 12*a*f*log(f) + 9*e
^2 - 36*d*f)/f)/sqrt(f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-f)*(2*x + (b*log(f)
 + e)/f))*e^(-1/4*(b^2*log(f)^2 + 2*b*e*log(f) - 4*a*f*log(f) + e^2 - 4*d*
f)/f)/sqrt(-f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(f)*(2*x - (b*log(f) - e)/f))*
e^(1/4*(b^2*log(f)^2 - 2*b*e*log(f) + 4*a*f*log(f) + e^2 - 4*d*f)/f)/sqrt(
f)
```

3.310.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\cosh^3\left(d+ex+fx^2\right)\,dx = \int f^{a+b\,x}\,{\cosh\left(f\,x^2+e\,x+d\right)}^3\,dx$$

input
```
int(f^(a + b*x)*cosh(d + e*x + f*x^2)^3,x)
```

output
```
int(f^(a + b*x)*cosh(d + e*x + f*x^2)^3, x)
```

3.311 $\int f^{a+cx^2} \cosh(d+ex)\,dx$

3.311.1 Optimal result

Integrand size = 16, antiderivative size = 133

$$\int f^{a+cx^2} \cosh(d+ex)\,dx = -\frac{e^{-d-\frac{e^2}{4c\log(f)}} f^a \sqrt{\pi}\,\text{erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}} + \frac{e^{d-\frac{e^2}{4c\log(f)}} f^a \sqrt{\pi}\,\text{erfi}\left(\frac{e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

output
```
1/4*exp(-d-1/4*e^2/c/ln(f))*f^a*erfi(1/2*(-e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1
/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/4*exp(d-1/4*e^2/c/ln(f))*f^a*erfi(1/2*
(e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

3.311.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.78

$$\int f^{a+cx^2} \cosh(d+ex)\,dx = \frac{e^{-\frac{e^2}{4c\log(f)}} f^a \sqrt{\pi}\left(\text{erfi}\left(\frac{-e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(d)-\sinh(d)) + \text{erfi}\left(\frac{e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(d)+\sinh(d))\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Integrate[f^(a + c*x^2)*Cosh[d + e*x],x]
```

output
```
(f^a*Sqrt[Pi]*(Erfi[(-e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[d]
 - Sinh[d]) + Erfi[(e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[d] +
 Sinh[d])))/(4*Sqrt[c]*E^(e^2/(4*c*Log[f]))*Sqrt[Log[f]])
```

3.311.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2}\cosh(d+ex)\,dx$$

$$\Big\downarrow \text{6039}$$

$$\int \left(\frac{1}{2}e^{-d-ex}f^{a+cx^2}+\frac{1}{2}e^{d+ex}f^{a+cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}f^a e^{d-\frac{e^2}{4c\log(f)}}\text{erfi}\left(\frac{2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a e^{-\frac{e^2}{4c\log(f)}-d}\text{erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + c*x^2)*Cosh[d + e*x],x]
```

output
```
-1/4*(E^(-d - e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - 2*c*x*Log[f])/(2*Sq
rt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log[f]]) + (E^(d - e^2/(4*c*Log[f]))*f
^a*Sqrt[Pi]*Erfi[(e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*
Sqrt[Log[f]])
```

3.311.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.311.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.88

method	result	size
risch	$\frac{\operatorname{erf}\left(\sqrt{-c\ln\left(f\right)}\, x+\frac{e}{2\sqrt{-c\ln\left(f\right)}}\right)\sqrt{\pi}\, f^{a} {\mathrm e}^{-\frac{4 d\ln\left(f\right) c+e^{2}}{4\ln\left(f\right) c}}}{4\sqrt{-c\ln\left(f\right)}}-\frac{\operatorname{erf}\left(-\sqrt{-c\ln\left(f\right)}\, x+\frac{e}{2\sqrt{-c\ln\left(f\right)}}\right)\sqrt{\pi}\, f^{a} {\mathrm e}^{\frac{4 d\ln\left(f\right) c-e^{2}}{4\ln\left(f\right) c}}}{4\sqrt{-c\ln\left(f\right)}}$	117

input
```
int(f^(c*x^2+a)*cosh(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*erf((-c*ln(f))^(1/2)*x+1/2*e/(-c*ln(f))^(1/2))/(-c*ln(f))^(1/2)*Pi^(1/
2)*f^a*exp(-1/4*(4*d*ln(f)*c+e^2)/ln(f)/c)-1/4*erf(-(-c*ln(f))^(1/2)*x+1/2
*e/(-c*ln(f))^(1/2))/(-c*ln(f))^(1/2)*Pi^(1/2)*f^a*exp(1/4*(4*d*ln(f)*c-e^
2)/ln(f)/c)
```

3.311.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(101) = 202.

Time = 0.28 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.62

$$\int f^{a+cx^2}\cosh(d+ex)\,dx =$$
$$-\frac{\sqrt{-c\log\left(f\right)}\left(\sqrt{\pi}\cosh\left(\frac{4\,ac\log\left(f\right)^{2}+4\,cd\log\left(f\right)-e^{2}}{4\,c\log\left(f\right)}\right)+\sqrt{\pi}\sinh\left(\frac{4\,ac\log\left(f\right)^{2}+4\,cd\log\left(f\right)-e^{2}}{4\,c\log\left(f\right)}\right)\right)\operatorname{erf}\left(\frac{(2\,cx\log\left(f\right)+e)\sqrt{-c}}{2\,c\log\left(f\right)}\right.}{}$$

input
```
integrate(f^(c*x^2+a)*cosh(e*x+d),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) -
e^2)/(c*log(f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) - e^2)
/(c*log(f))))*erf(1/2*(2*c*x*log(f) + e)*sqrt(-c*log(f))/(c*log(f))) + sqr
t(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - e^2)/(c*l
og(f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - e^2)/(c*log(f
))))*erf(1/2*(2*c*x*log(f) - e)*sqrt(-c*log(f))/(c*log(f))))/(c*log(f))
```

3.311.6 Sympy [F]

$$\int f^{a+cx^2}\cosh(d+ex)\,dx=\int f^{a+cx^2}\cosh\left(d+ex\right)dx$$

input
```
integrate(f**(c*x**2+a)*cosh(e*x+d),x)
```

output
```
Integral(f**(a + c*x**2)*cosh(d + e*x), x)
```

3.311.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.79

$$\int f^{a+cx^2}\cosh(d+ex)\,dx=\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)}x-\frac{e}{2\sqrt{-c\log(f)}}\right)e^{\left(d-\frac{e^2}{4\,c\log(f)}\right)}}{4\sqrt{-c\log\left(f\right)}}+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)}x+\frac{e}{2\sqrt{-c\log(f)}}\right)e^{\left(-d-\frac{e^2}{4\,c\log(f)}\right)}}{4\sqrt{-c\log\left(f\right)}}$$

input
```
integrate(f^(c*x^2+a)*cosh(e*x+d),x, algorithm="maxima")
```

output
```
1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*e/sqrt(-c*log(f)))*e^(d - 1/4
*e^2/(c*log(f)))/sqrt(-c*log(f)) + 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x
+ 1/2*e/sqrt(-c*log(f)))*e^(-d - 1/4*e^2/(c*log(f)))/sqrt(-c*log(f))
```

3.311.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.99

$$\int f^{a+cx^2}\cosh(d+ex)\,dx$$
$$= -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{4\,ac\log(f)^2+4\,cd\log(f)-e^2}{4\,c\log(f)}\right)}}{4\sqrt{-c\log(f)}} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x-\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{4\,ac\log(f)^2-4\,cd\log(f)-e^2}{4\,c\log(f)}\right)}}{4\sqrt{-c\log(f)}}$$

input
```
integrate(f^(c*x^2+a)*cosh(e*x+d),x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + e/(c*log(f))))*e^(1/4*(4*a*c
*log(f)^2 + 4*c*d*log(f) - e^2)/(c*log(f)))/sqrt(-c*log(f)) - 1/4*sqrt(pi)
*erf(-1/2*sqrt(-c*log(f))*(2*x - e/(c*log(f))))*e^(1/4*(4*a*c*log(f)^2 - 4
*c*d*log(f) - e^2)/(c*log(f)))/sqrt(-c*log(f))
```

3.311.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\cosh(d+ex)\,dx = \int f^{c\,x^2+a}\cosh(d+e\,x)\,dx$$

input
```
int(f^(a + c*x^2)*cosh(d + e*x),x)
```

output
```
int(f^(a + c*x^2)*cosh(d + e*x), x)
```

3.312 $\int f^{a+cx^2} \cosh^2(d+ex)\,dx$

3.312.1 Optimal result

Integrand size = 18, antiderivative size = 161

$$\int f^{a+cx^2} \cosh^2(d+ex)\,dx = \frac{f^a\sqrt{\pi}\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}} - \frac{e^{-2d-\frac{e^2}{c\log(f)}} f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e-cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}} + \frac{e^{2d-\frac{e^2}{c\log(f)}} f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e+cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

output
```
1/8*exp(-2*d-e^2/c/ln(f))*f^a*erfi((-e+c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^
(1/2)/c^(1/2)/ln(f)^(1/2)+1/8*exp(2*d-e^2/c/ln(f))*f^a*erfi((e+c*x*ln(f))/
c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/4*f^a*erfi(x*c^(1/2)*l
n(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

3.312.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.81

$$\int f^{a+cx^2} \cosh^2(d+ex)\,dx = \frac{e^{-\frac{e^2}{c\log(f)}} f^a\sqrt{\pi}\left(2e^{\frac{e^2}{c\log(f)}}\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right) + \operatorname{erfi}\left(\frac{-e+cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(2d)-\sinh(2d)) + \operatorname{erfi}\left(\frac{e+cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)(\mathrm{c}\right.}{8\sqrt{c}\sqrt{\log(f)}}$$

input
```
Integrate[f^(a + c*x^2)*Cosh[d + e*x]^2,x]
```

output
```
(f^a*Sqrt[Pi]*(2*E^(e^2/(c*Log[f]))*Erfi[Sqrt[c]*x*Sqrt[Log[f]]] + Erfi[(-
e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(e
+ c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])))/(8*Sqrt[c]
*E^(e^2/(c*Log[f]))*Sqrt[Log[f]])
```

3.312.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2}\cosh^2(d+ex)\,dx$$

$$\Big\downarrow \text{6039}$$

$$\int \left(\frac{1}{4}e^{-2d-2ex}f^{a+cx^2}+\frac{1}{4}e^{2d+2ex}f^{a+cx^2}+\frac{1}{2}f^{a+cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{\pi}f^a e^{-\frac{e^2}{c\log(f)}-2d}\operatorname{erfi}\left(\frac{e-cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a e^{2d-\frac{e^2}{c\log(f)}}\operatorname{erfi}\left(\frac{cx\log(f)+e}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + c*x^2)*Cosh[d + e*x]^2,x]
```

output
```
(f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) - (E^
(-2*d - e^2/(c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - c*x*Log[f])/(Sqrt[c]*Sqrt[L
og[f]])])/(8*Sqrt[c]*Sqrt[Log[f]]) + (E^(2*d - e^2/(c*Log[f]))*f^a*Sqrt[Pi
]*Erfi[(e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])])/(8*Sqrt[c]*Sqrt[Log[f]])
```

3.312.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.312.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.86

method	result
risch	$\frac{\operatorname{erf}\left(\sqrt{-c\ln(f)}\,x+\frac{e}{\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{2d\ln(f)c+e^{2}}{\ln(f)c}}}{8\sqrt{-c\ln(f)}}-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{e}{\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{\frac{2d\ln(f)c-e^{2}}{\ln(f)c}}}{8\sqrt{-c\ln(f)}}+\frac{f^{a}\sqrt{\pi}\operatorname{erf}\left(\sqrt{-c\ln\right.}}{4\sqrt{-c\ln(f)}}$

input
```
int(f^(c*x^2+a)*cosh(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*erf((-c*ln(f))^(1/2)*x+e/(-c*ln(f))^(1/2))/(-c*ln(f))^(1/2)*Pi^(1/2)*f
^a*exp(-(2*d*ln(f)*c+e^2)/ln(f)/c)-1/8*erf(-(-c*ln(f))^(1/2)*x+e/(-c*ln(f)
)^(1/2))/(-c*ln(f))^(1/2)*Pi^(1/2)*f^a*exp((2*d*ln(f)*c-e^2)/ln(f)/c)+1/4*
f^a*Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

3.312.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.50

$$\int f^{a+cx^2}\cosh^2(d+ex)\,dx =$$
$$-\frac{2\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh(a\log(f))+\sqrt{\pi}\sinh(a\log(f))\right)\operatorname{erf}\left(\sqrt{-c\log(f)}x\right)+\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\right.}{}$$

input
```
integrate(f^(c*x^2+a)*cosh(e*x+d)^2,x, algorithm="fricas")
```

output
```
-1/8*(2*sqrt(-c*log(f))*(sqrt(pi)*cosh(a*log(f)) + sqrt(pi)*sinh(a*log(f))
)*erf(sqrt(-c*log(f))*x) + sqrt(-c*log(f))*(sqrt(pi)*cosh((a*c*log(f)^2 +
2*c*d*log(f) - e^2)/(c*log(f))) + sqrt(pi)*sinh((a*c*log(f)^2 + 2*c*d*log(
f) - e^2)/(c*log(f))))*erf((c*x*log(f) + e)*sqrt(-c*log(f))/(c*log(f))) +
sqrt(-c*log(f))*(sqrt(pi)*cosh((a*c*log(f)^2 - 2*c*d*log(f) - e^2)/(c*log(
f))) + sqrt(pi)*sinh((a*c*log(f)^2 - 2*c*d*log(f) - e^2)/(c*log(f))))*erf(
(c*x*log(f) - e)*sqrt(-c*log(f))/(c*log(f))))/(c*log(f))
```

3.312.6 Sympy [F]

$$\int f^{a+cx^2}\cosh^2(d+ex)\,dx=\int f^{a+cx^2}\cosh^2{(d+ex)}\,dx$$

input
```
integrate(f**(c*x**2+a)*cosh(e*x+d)**2,x)
```

output
```
Integral(f**(a + c*x**2)*cosh(d + e*x)**2, x)
```

3.312.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.81

$$\int f^{a+cx^2}\cosh^2(d+ex)\,dx=\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x-\frac{e}{\sqrt{-c\log(f)}}\right)e^{\left(2d-\frac{e^2}{c\log(f)}\right)}}{8\sqrt{-c\log(f)}}+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x+\frac{e}{\sqrt{-c\log(f)}}\right)e^{\left(-2d-\frac{e^2}{c\log(f)}\right)}}{8\sqrt{-c\log(f)}}+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x\right)}{4\sqrt{-c\log(f)}}$$

input
```
integrate(f^(c*x^2+a)*cosh(e*x+d)^2,x, algorithm="maxima")
```

output
```
1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - e/sqrt(-c*log(f)))*e^(2*d - e^2/(
c*log(f)))/sqrt(-c*log(f)) + 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x + e/sq
rt(-c*log(f)))*e^(-2*d - e^2/(c*log(f)))/sqrt(-c*log(f)) + 1/4*sqrt(pi)*f^
a*erf(sqrt(-c*log(f))*x)/sqrt(-c*log(f))
```

3.312.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.93

$$\int f^{a+cx^2}\cosh^2(d+ex)\,dx = -\frac{\sqrt{\pi}f^a \operatorname{erf}\left(-\sqrt{-c\log(f)}x\right)}{4\sqrt{-c\log(f)}} - \frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log(f)}\left(x+\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{ac\log(f)^2+2\,cd\log(f)-e^2}{c\log(f)}\right)}}{8\sqrt{-c\log(f)}} - \frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log(f)}\left(x-\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{ac\log(f)^2-2\,cd\log(f)-e^2}{c\log(f)}\right)}}{8\sqrt{-c\log(f)}}$$

input
```
integrate(f^(c*x^2+a)*cosh(e*x+d)^2,x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*f^a*erf(-sqrt(-c*log(f))*x)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*e
rf(-sqrt(-c*log(f))*(x + e/(c*log(f))))*e^((a*c*log(f)^2 + 2*c*d*log(f) -
e^2)/(c*log(f)))/sqrt(-c*log(f)) - 1/8*sqrt(pi)*erf(-sqrt(-c*log(f))*(x -
e/(c*log(f))))*e^((a*c*log(f)^2 - 2*c*d*log(f) - e^2)/(c*log(f)))/sqrt(-c*
log(f))
```

3.312.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\cosh^2(d+ex)\,dx = \int f^{c\,x^2+a}\cosh(d+e\,x)^2\,dx$$

input
```
int(f^(a + c*x^2)*cosh(d + e*x)^2,x)
```

output
```
int(f^(a + c*x^2)*cosh(d + e*x)^2, x)
```

3.313 $\int f^{a+cx^2}\cosh^3(d+ex)\,dx$

3.313.1 Optimal result

Integrand size = 18, antiderivative size = 271

$$\int f^{a+cx^2}\cosh^3(d+ex)\,dx = -\frac{3e^{-d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} - \frac{e^{-3d-\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{3e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{3e^{d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{e^{3d-\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{3e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

output
```
3/16*exp(-d-1/4*e^2/c/ln(f))*f^a*erfi(1/2*(-e+2*c*x*ln(f))/c^(1/2)/ln(f)^(
1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/16*exp(-3*d-9/4*e^2/c/ln(f))*f^a*erfi
(1/2*(-3*e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+
3/16*exp(d-1/4*e^2/c/ln(f))*f^a*erfi(1/2*(e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/
2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/16*exp(3*d-9/4*e^2/c/ln(f))*f^a*erfi(1/
2*(3*e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

3.313.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.79

$$\int f^{a+cx^2}\cosh^3(d+ex)\,dx$$
$$= \frac{e^{-\frac{9e^2}{4c\log(f)}} f^a\sqrt{\pi}\left((\cosh(d)+\sinh(d))\left(3e^{\frac{2e^2}{c\log(f)}}\text{erfi}\left(\frac{e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)+3e^{\frac{2e^2}{c\log(f)}}\text{erfi}\left(\frac{-e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(2d)-\sin\right.\right.}{16\sqrt{c}\sqrt{\log(}}$$

input
```
Integrate[f^(a + c*x^2)*Cosh[d + e*x]^3,x]
```

output
```
(f^a*Sqrt[Pi]*((Cosh[d] + Sinh[d])*(3*E^((2*e^2)/(c*Log[f]))*Erfi[(e + 2*c
*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])] + 3*E^((2*e^2)/(c*Log[f]))*Erfi[(-e +
 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(3
*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])) + Er
fi[(-3*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[3*d] - Sinh[3*d])
))/(16*Sqrt[c]*E^((9*e^2)/(4*c*Log[f]))*Sqrt[Log[f]])
```

3.313.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2}\cosh^3(d+ex)\,dx$$
$$\Big\downarrow \text{6039}$$
$$\int \left(\frac{1}{8}e^{-3d-3ex}f^{a+cx^2}+\frac{3}{8}e^{-d-ex}f^{a+cx^2}+\frac{3}{8}e^{d+ex}f^{a+cx^2}+\frac{1}{8}e^{3d+3ex}f^{a+cx^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{3\sqrt{\pi}f^a e^{-\frac{e^2}{4c\log(f)}-d}\text{erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a e^{-\frac{9e^2}{4c\log(f)}-3d}\text{erfi}\left(\frac{3e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{3\sqrt{\pi}f^a e^{d-\frac{e^2}{4c\log(f)}}\text{erfi}\left(\frac{2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a e^{3d-\frac{9e^2}{4c\log(f)}}\text{erfi}\left(\frac{2cx\log(f)+3e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + c*x^2)*Cosh[d + e*x]^3,x]
```

output
```
(-3*E^(-d - e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - 2*c*x*Log[f])/(2*Sqrt
[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) - (E^(-3*d - (9*e^2)/(4*c*Lo
g[f]))*f^a*Sqrt[Pi]*Erfi[(3*e - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(
16*Sqrt[c]*Sqrt[Log[f]]) + (3*E^(d - e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(
e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (
E^(3*d - (9*e^2)/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(3*e + 2*c*x*Log[f])/(2*S
qrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]])
```

3.313.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.313.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.86

method	result
risch	$\frac{\operatorname{erf}\left(\sqrt{-c\ln(f)}\,x+\frac{3e}{2\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{3\left(4d\ln(f)c+3e^{2}\right)}{4\ln(f)c}}}{16\sqrt{-c\ln(f)}}-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{3e}{2\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{\frac{3d\ln(f)c-\frac{9e^{2}}{4}}{c\ln(f)}}}{16\sqrt{-c\ln(f)}}+\frac{3\operatorname{erf}\left(\sqrt{-\right.}$

input
```
int(f^(c*x^2+a)*cosh(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*erf((-c*ln(f))^(1/2)*x+3/2*e/(-c*ln(f))^(1/2))/(-c*ln(f))^(1/2)*Pi^(1
/2)*f^a*exp(-3/4*(4*d*ln(f)*c+3*e^2)/ln(f)/c)-1/16*erf(-(-c*ln(f))^(1/2)*x
+3/2*e/(-c*ln(f))^(1/2))/(-c*ln(f))^(1/2)*Pi^(1/2)*f^a*exp(3/4*(4*d*ln(f)*
c-3*e^2)/ln(f)/c)+3/16*erf((-c*ln(f))^(1/2)*x+1/2*e/(-c*ln(f))^(1/2))/(-c*
ln(f))^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(4*d*ln(f)*c+e^2)/ln(f)/c)-3/16*erf(-(-
c*ln(f))^(1/2)*x+1/2*e/(-c*ln(f))^(1/2))/(-c*ln(f))^(1/2)*Pi^(1/2)*f^a*exp
(1/4*(4*d*ln(f)*c-e^2)/ln(f)/c)
```

3.313.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(205) = 410$.

Time = 0.28 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.57

$$\int f^{a+cx^2}\cosh^3(d+ex)\,dx =$$
$$-\frac{\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\left(\frac{4ac\log(f)^2+12cd\log(f)-9e^2}{4c\log(f)}\right)+\sqrt{\pi}\sinh\left(\frac{4ac\log(f)^2+12cd\log(f)-9e^2}{4c\log(f)}\right)\right)\operatorname{erf}\left(\frac{(2cx\log(f)+3}{2c\log}\right.}{}$$

input
```
integrate(f^(c*x^2+a)*cosh(e*x+d)^3,x, algorithm="fricas")
```

output
```
-1/16*(sqrt(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 + 12*c*d*log(f)
- 9*e^2)/(c*log(f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 + 12*c*d*log(f) -
 9*e^2)/(c*log(f))))*erf(1/2*(2*c*x*log(f) + 3*e)*sqrt(-c*log(f))/(c*log(f
))) + 3*sqrt(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f)
- e^2)/(c*log(f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) - e^
2)/(c*log(f))))*erf(1/2*(2*c*x*log(f) + e)*sqrt(-c*log(f))/(c*log(f))) + 3
*sqrt(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - e^2)/
(c*log(f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - e^2)/(c*l
og(f))))*erf(1/2*(2*c*x*log(f) - e)*sqrt(-c*log(f))/(c*log(f))) + sqrt(-c*
log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 - 12*c*d*log(f) - 9*e^2)/(c*log
(f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 - 12*c*d*log(f) - 9*e^2)/(c*log(
f))))*erf(1/2*(2*c*x*log(f) - 3*e)*sqrt(-c*log(f))/(c*log(f))))/(c*log(f))
```

3.313.6 Sympy [F]

$$\int f^{a+cx^2}\cosh^3(d+ex)\,dx = \int f^{a+cx^2}\cosh^3(d+ex)\,dx$$

input
```
integrate(f**(c*x**2+a)*cosh(e*x+d)**3,x)
```

output
```
Integral(f**(a + c*x**2)*cosh(d + e*x)**3, x)
```

3.313.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.78

$$\int f^{a+cx^2}\cosh^3(d+ex)\,dx = \frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x-\frac{3e}{2\sqrt{-c\log(f)}}\right)e^{\left(3d-\frac{9e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}} + \frac{3\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x-\frac{e}{2\sqrt{-c\log(f)}}\right)e^{\left(d-\frac{e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}} + \frac{3\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x+\frac{e}{2\sqrt{-c\log(f)}}\right)e^{\left(-d-\frac{e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}} + \frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x+\frac{3e}{2\sqrt{-c\log(f)}}\right)e^{\left(-3d-\frac{9e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}$$

input
```
integrate(f^(c*x^2+a)*cosh(e*x+d)^3,x, algorithm="maxima")
```

output
```
1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 3/2*e/sqrt(-c*log(f)))*e^(3*d -
9/4*e^2/(c*log(f)))/sqrt(-c*log(f)) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f)
)*x - 1/2*e/sqrt(-c*log(f)))*e^(d - 1/4*e^2/(c*log(f)))/sqrt(-c*log(f)) +
3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x + 1/2*e/sqrt(-c*log(f)))*e^(-d - 1
/4*e^2/(c*log(f)))/sqrt(-c*log(f)) + 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))
*x + 3/2*e/sqrt(-c*log(f)))*e^(-3*d - 9/4*e^2/(c*log(f)))/sqrt(-c*log(f))
```

3.313.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.97

$$\int f^{a+cx^2}\cosh^3(d+ex)\,dx$$
$$= -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{3e}{c\log(f)}\right)\right)e^{\left(\frac{4ac\log(f)^2+12cd\log(f)-9e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}$$
$$-\frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{4ac\log(f)^2+4cd\log(f)-e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}$$
$$-\frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x-\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{4ac\log(f)^2-4cd\log(f)-e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}$$
$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x-\frac{3e}{c\log(f)}\right)\right)e^{\left(\frac{4ac\log(f)^2-12cd\log(f)-9e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}$$

input
```
integrate(f^(c*x^2+a)*cosh(e*x+d)^3,x, algorithm="giac")
```

output
```
-1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + 3*e/(c*log(f))))*e^(1/4*(4*
a*c*log(f)^2 + 12*c*d*log(f) - 9*e^2)/(c*log(f)))/sqrt(-c*log(f)) - 3/16*s
qrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + e/(c*log(f))))*e^(1/4*(4*a*c*log(f
)^2 + 4*c*d*log(f) - e^2)/(c*log(f)))/sqrt(-c*log(f)) - 3/16*sqrt(pi)*erf(
-1/2*sqrt(-c*log(f))*(2*x - e/(c*log(f))))*e^(1/4*(4*a*c*log(f)^2 - 4*c*d*
log(f) - e^2)/(c*log(f)))/sqrt(-c*log(f)) - 1/16*sqrt(pi)*erf(-1/2*sqrt(-c
*log(f))*(2*x - 3*e/(c*log(f))))*e^(1/4*(4*a*c*log(f)^2 - 12*c*d*log(f) -
9*e^2)/(c*log(f)))/sqrt(-c*log(f))
```

3.313.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\cosh^3(d+ex)\,dx = \int f^{c\,x^2+a}\cosh(d+e\,x)^3\,dx$$

input
```
int(f^(a + c*x^2)*cosh(d + e*x)^3,x)
```

output
```
int(f^(a + c*x^2)*cosh(d + e*x)^3, x)
```

3.314 $\int f^{a+cx^2} \cosh(d+fx^2)\,dx$

3.314.1 Optimal result

Integrand size = 18, antiderivative size = 81

$$\int f^{a+cx^2}\cosh\left(d+fx^2\right)\,dx=\frac{e^{-d}f^a\sqrt{\pi}\operatorname{erf}\left(x\sqrt{f-c\log(f)}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^d f^a\sqrt{\pi}\operatorname{erfi}\left(x\sqrt{f+c\log(f)}\right)}{4\sqrt{f+c\log(f)}}$$

output
```
1/4*f^a*erf(x*(f-c*ln(f))^(1/2))*Pi^(1/2)/exp(d)/(f-c*ln(f))^(1/2)+1/4*exp
(d)*f^a*erfi(x*(f+c*ln(f))^(1/2))*Pi^(1/2)/(f+c*ln(f))^(1/2)
```

3.314.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int f^{a+cx^2}\cosh\left(d+fx^2\right)\,dx=\frac{1}{4}f^a\sqrt{\pi}\left(\frac{\operatorname{erf}\left(x\sqrt{f-c\log(f)}\right)(\cosh(d)-\sinh(d))}{\sqrt{f-c\log(f)}}+\frac{\operatorname{erfi}\left(x\sqrt{f+c\log(f)}\right)(\cosh(d)+\sinh(d))}{\sqrt{f+c\log(f)}}\right)$$

input
```
Integrate[f^(a + c*x^2)*Cosh[d + f*x^2],x]
```

output
```
(f^a*Sqrt[Pi]*((Erf[x*Sqrt[f - c*Log[f]]]*(Cosh[d] - Sinh[d]))/Sqrt[f - c*
Log[f]] + (Erfi[x*Sqrt[f + c*Log[f]]]*(Cosh[d] + Sinh[d]))/Sqrt[f + c*Log[
f]]))/4
```

3.314.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2} \cosh\left(d+fx^2\right) dx$$

$$\Big\downarrow \text{6039}$$

$$\int \left(\frac{1}{2}e^{-d-fx^2}f^{a+cx^2}+\frac{1}{2}e^{d+fx^2}f^{a+cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}e^{-d}f^a\text{erf}\left(x\sqrt{f-c\log(f)}\right)}{4\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}e^d f^a\text{erfi}\left(x\sqrt{c\log(f)+f}\right)}{4\sqrt{c\log(f)+f}}$$

input
```
Int[f^(a + c*x^2)*Cosh[d + f*x^2],x]
```

output
```
(f^a*Sqrt[Pi]*Erf[x*Sqrt[f - c*Log[f]]])/(4*E^d*Sqrt[f - c*Log[f]]) + (E^d
*f^a*Sqrt[Pi]*Erfi[x*Sqrt[f + c*Log[f]]])/(4*Sqrt[f + c*Log[f]])
```

3.314.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.314.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

method	result	size
risch	$\frac{f^{a} {\mathrm e}^{-d} \sqrt{\pi}\, \operatorname{erf}\left(x \sqrt{f -c \ln \left(f \right)}\right)}{4 \sqrt{f -c \ln \left(f \right)}}+\frac{f^{a} {\mathrm e}^{d} \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{-c \ln \left(f \right)-f}\, x \right)}{4 \sqrt{-c \ln \left(f \right)-f}}$	70

input
```
int(f^(c*x^2+a)*cosh(f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*f^a*exp(-d)*Pi^(1/2)/(f-c*ln(f))^(1/2)*erf(x*(f-c*ln(f))^(1/2))+1/4*f^
a*exp(d)*Pi^(1/2)/(-c*ln(f)-f)^(1/2)*erf((-c*ln(f)-f)^(1/2)*x)
```

3.314.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(63) = 126.

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.79

$$\int f^{a+cx^2} \cosh \left(d+fx^2\right) \, dx = -\frac{\left(\sqrt{\pi}(c\log\left(f\right)+f)\cosh\left(a\log\left(f\right)-d\right)+\sqrt{\pi}(c\log\left(f\right)+f)\sinh\left(a\log\left(f\right)-d\right)\right)\sqrt{-c\log\left(f\right)+f}\,\operatorname{erf}\left(\sqrt{}\right.}{}$$

input
```
integrate(f^(c*x^2+a)*cosh(f*x^2+d),x, algorithm="fricas")
```

output
```
-1/4*((sqrt(pi)*(c*log(f) + f)*cosh(a*log(f) - d) + sqrt(pi)*(c*log(f) + f
)*sinh(a*log(f) - d))*sqrt(-c*log(f) + f)*erf(sqrt(-c*log(f) + f)*x) + (sq
rt(pi)*(c*log(f) - f)*cosh(a*log(f) + d) + sqrt(pi)*(c*log(f) - f)*sinh(a*
log(f) + d))*sqrt(-c*log(f) - f)*erf(sqrt(-c*log(f) - f)*x))/(c^2*log(f)^2
 - f^2)
```

3.314.6 Sympy [F]

$$\int f^{a+cx^2}\cosh\left(d+fx^2\right)\,dx=\int f^{a+cx^2}\cosh\left(d+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+a)*cosh(f*x**2+d),x)
```

output
```
Integral(f**(a + c*x**2)*cosh(d + f*x**2), x)
```

3.314.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85

$$\int f^{a+cx^2}\cosh\left(d+fx^2\right)\,dx=\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+f}x\right)e^{(-d)}}{4\sqrt{-c\log\left(f\right)+f}}+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-f}x\right)e^{d}}{4\sqrt{-c\log\left(f\right)-f}}$$

input
```
integrate(f^(c*x^2+a)*cosh(f*x^2+d),x, algorithm="maxima")
```

output
```
1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x)*e^(-d)/sqrt(-c*log(f) + f) + 1
/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x)*e^d/sqrt(-c*log(f) - f)
```

3.314.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int f^{a+cx^2}\cosh\left(d+fx^2\right)\,dx=-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)-f}x\right)e^{(a\log(f)+d)}}{4\sqrt{-c\log\left(f\right)-f}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)+f}x\right)e^{(a\log(f)-d)}}{4\sqrt{-c\log\left(f\right)+f}}$$

input
```
integrate(f^(c*x^2+a)*cosh(f*x^2+d),x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*erf(-sqrt(-c*log(f) - f)*x)*e^(a*log(f) + d)/sqrt(-c*log(f)
- f) - 1/4*sqrt(pi)*erf(-sqrt(-c*log(f) + f)*x)*e^(a*log(f) - d)/sqrt(-c*l
og(f) + f)
```

3.314.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\cosh\left(d+fx^2\right)\,dx=\int f^{c\,x^2+a}\cosh\left(f\,x^2+d\right)\,dx$$

input
```
int(f^(a + c*x^2)*cosh(d + f*x^2),x)
```

output
```
int(f^(a + c*x^2)*cosh(d + f*x^2), x)
```

3.315 $\int f^{a+cx^2}\cosh^2(d+fx^2)\,dx$

3.315.1 Optimal result

Integrand size = 20, antiderivative size = 128

$$\int f^{a+cx^2}\cosh^2\left(d+fx^2\right)\,dx=\frac{f^a\sqrt{\pi}\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-2d}f^a\sqrt{\pi}\text{erf}\left(x\sqrt{2f-c\log(f)}\right)}{8\sqrt{2f-c\log(f)}}+\frac{e^{2d}f^a\sqrt{\pi}\text{erfi}\left(x\sqrt{2f+c\log(f)}\right)}{8\sqrt{2f+c\log(f)}}$$

output
```
1/4*f^a*erfi(x*c^(1/2)*ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/8*f^a*e
rf(x*(2*f-c*ln(f))^(1/2))*Pi^(1/2)/exp(2*d)/(2*f-c*ln(f))^(1/2)+1/8*exp(2*
d)*f^a*erfi(x*(2*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*f+c*ln(f))^(1/2)
```

3.315.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.40

$$\int f^{a+cx^2}\cosh^2\left(d+fx^2\right)\,dx=\frac{f^a\sqrt{\pi}\left(\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)\left(-8f^2+2c^2\log^2(f)\right)+\sqrt{c}\sqrt{\log(f)}\left(\text{erf}\left(x\sqrt{2f-c\log(f)}\right)\sqrt{2f-c\log(f)}(2\right.\right.}{8\sqrt{c}\sqrt{\log}}$$

input
```
Integrate[f^(a + c*x^2)*Cosh[d + f*x^2]^2,x]
```

output
```
(f^a*Sqrt[Pi]*(Erfi[Sqrt[c]*x*Sqrt[Log[f]]]*(-8*f^2 + 2*c^2*Log[f]^2) + Sq
rt[c]*Sqrt[Log[f]]*(Erf[x*Sqrt[2*f - c*Log[f]]]*Sqrt[2*f - c*Log[f]]*(2*f
+ c*Log[f])*(-Cosh[2*d] + Sinh[2*d]) - Erfi[x*Sqrt[2*f + c*Log[f]]]*(2*f -
 c*Log[f])*Sqrt[2*f + c*Log[f]]*(Cosh[2*d] + Sinh[2*d]))))/(8*Sqrt[c]*Sqrt
[Log[f]]*(-4*f^2 + c^2*Log[f]^2))
```

3.315.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2} \cosh^2\left(d+fx^2\right) dx$$

$$\Big\downarrow \text{6039}$$

$$\int \left(\frac{1}{4}e^{-2d-2fx^2} f^{a+cx^2} + \frac{1}{4}e^{2d+2fx^2} f^{a+cx^2} + \frac{1}{2}f^{a+cx^2}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}e^{-2d} f^a \text{erf}\left(x\sqrt{2f-c\log(f)}\right)}{8\sqrt{2f-c\log(f)}} + \frac{\sqrt{\pi}e^{2d} f^a \text{erfi}\left(x\sqrt{c\log(f)+2f}\right)}{8\sqrt{c\log(f)+2f}} + \frac{\sqrt{\pi} f^a \text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + c*x^2)*Cosh[d + f*x^2]^2,x]
```

output
```
(f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) + (f^
a*Sqrt[Pi]*Erf[x*Sqrt[2*f - c*Log[f]]])/(8*E^(2*d)*Sqrt[2*f - c*Log[f]]) +
 (E^(2*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[2*f + c*Log[f]]])/(8*Sqrt[2*f + c*Log[f
]])
```

3.315.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.315.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{f^{a} e^{-2 d} \sqrt{\pi}\, \operatorname{erf}\left(x \sqrt{2 f - c \ln\left(f\right)}\right)}{8 \sqrt{2 f - c \ln\left(f\right)}}+\frac{f^{a} e^{2 d} \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{-c \ln\left(f\right)-2 f}\, x\right)}{8 \sqrt{-c \ln\left(f\right)-2 f}}+\frac{f^{a} \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{-c \ln\left(f\right)}\, x\right)}{4 \sqrt{-c \ln\left(f\right)}}$	101

input
```
int(f^(c*x^2+a)*cosh(f*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*f^a*exp(-2*d)*Pi^(1/2)/(2*f-c*ln(f))^(1/2)*erf(x*(2*f-c*ln(f))^(1/2))+
1/8*f^a*exp(2*d)*Pi^(1/2)/(-c*ln(f)-2*f)^(1/2)*erf((-c*ln(f)-2*f)^(1/2)*x)
+1/4*f^a*Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

3.315.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(98) = 196.

Time = 0.27 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.98

$$\int f^{a+cx^2} \cosh^2\left(d+fx^2\right)\, dx =$$

$$-\frac{\left(\sqrt{\pi}\left(c^2\log\left(f\right)^2+2\,cf\log\left(f\right)\right)\cosh\left(a\log\left(f\right)-2\,d\right)+\sqrt{\pi}\left(c^2\log\left(f\right)^2+2\,cf\log\left(f\right)\right)\sinh\left(a\log\left(f\right)-2\right.}{}$$

input
```
integrate(f^(c*x^2+a)*cosh(f*x^2+d)^2,x, algorithm="fricas")
```

output
```
-1/8*((sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh(a*log(f) - 2*d) + sqrt(
pi)*(c^2*log(f)^2 + 2*c*f*log(f))*sinh(a*log(f) - 2*d))*sqrt(-c*log(f) + 2
*f)*erf(sqrt(-c*log(f) + 2*f)*x) + (sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))
*cosh(a*log(f) + 2*d) + sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*sinh(a*log(
f) + 2*d))*sqrt(-c*log(f) - 2*f)*erf(sqrt(-c*log(f) - 2*f)*x) + 2*(sqrt(pi
)*(c^2*log(f)^2 - 4*f^2)*cosh(a*log(f)) + sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*
sinh(a*log(f)))*sqrt(-c*log(f))*erf(sqrt(-c*log(f))*x))/(c^3*log(f)^3 - 4*
c*f^2*log(f))
```

3.315.6 Sympy [F]

$$\int f^{a+cx^2}\cosh^2\left(d+fx^2\right)\,dx=\int f^{a+cx^2}\cosh^2\left(d+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+a)*cosh(f*x**2+d)**2,x)
```

output
```
Integral(f**(a + c*x**2)*cosh(d + f*x**2)**2, x)
```

3.315.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.78

$$\int f^{a+cx^2}\cosh^2\left(d+fx^2\right)\,dx=\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-2\,f}x\right)e^{(2\,d)}}{8\,\sqrt{-c\log\left(f\right)-2\,f}}+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+2\,f}x\right)e^{(-2\,d)}}{8\,\sqrt{-c\log\left(f\right)+2\,f}}+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)}x\right)}{4\,\sqrt{-c\log\left(f\right)}}$$

input
```
integrate(f^(c*x^2+a)*cosh(f*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x)*e^(2*d)/sqrt(-c*log(f) - 2*f
) + 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 2*f)*x)*e^(-2*d)/sqrt(-c*log(f)
+ 2*f) + 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x)/sqrt(-c*log(f))
```

3.315.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.84

$$\int f^{a+cx^2}\cosh^2\left(d+fx^2\right)\,dx = -\frac{\sqrt{\pi}f^a\operatorname{erf}\left(-\sqrt{-c\log(f)}x\right)}{4\sqrt{-c\log(f)}} - \frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log(f)-2f}x\right)e^{(a\log(f)+2d)}}{8\sqrt{-c\log(f)-2f}} - \frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log(f)+2f}x\right)e^{(a\log(f)-2d)}}{8\sqrt{-c\log(f)+2f}}$$

input
```
integrate(f^(c*x^2+a)*cosh(f*x^2+d)^2,x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*f^a*erf(-sqrt(-c*log(f))*x)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*e
rf(-sqrt(-c*log(f) - 2*f)*x)*e^(a*log(f) + 2*d)/sqrt(-c*log(f) - 2*f) - 1/
8*sqrt(pi)*erf(-sqrt(-c*log(f) + 2*f)*x)*e^(a*log(f) - 2*d)/sqrt(-c*log(f)
 + 2*f)
```

3.315.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\cosh^2\left(d+fx^2\right)\,dx = \int f^{c\,x^2+a}\cosh\left(f\,x^2+d\right)^2\,dx$$

input
```
int(f^(a + c*x^2)*cosh(d + f*x^2)^2,x)
```

output
```
int(f^(a + c*x^2)*cosh(d + f*x^2)^2, x)
```

3.316 $\int f^{a+cx^2} \cosh^3(d+fx^2)\,dx$

3.316.1 Optimal result

Integrand size = 20, antiderivative size = 171

$$\int f^{a+cx^2}\cosh^3(d+fx^2)\,dx=\frac{3e^{-d}f^a\sqrt{\pi}\operatorname{erf}\left(x\sqrt{f-c\log(f)}\right)}{16\sqrt{f-c\log(f)}}+\frac{e^{-3d}f^a\sqrt{\pi}\operatorname{erf}\left(x\sqrt{3f-c\log(f)}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3e^{d}f^a\sqrt{\pi}\operatorname{erfi}\left(x\sqrt{f+c\log(f)}\right)}{16\sqrt{f+c\log(f)}}+\frac{e^{3d}f^a\sqrt{\pi}\operatorname{erfi}\left(x\sqrt{3f+c\log(f)}\right)}{16\sqrt{3f+c\log(f)}}$$

output
```
3/16*f^a*erf(x*(f-c*ln(f))^(1/2))*Pi^(1/2)/exp(d)/(f-c*ln(f))^(1/2)+1/16*f
^a*erf(x*(3*f-c*ln(f))^(1/2))*Pi^(1/2)/exp(3*d)/(3*f-c*ln(f))^(1/2)+3/16*e
xp(d)*f^a*erfi(x*(f+c*ln(f))^(1/2))*Pi^(1/2)/(f+c*ln(f))^(1/2)+1/16*exp(3*
d)*f^a*erfi(x*(3*f+c*ln(f))^(1/2))*Pi^(1/2)/(3*f+c*ln(f))^(1/2)
```

3.316.2 Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.58

$$\int f^{a+cx^2}\cosh^3\left(d+fx^2\right)\,dx$$
$$=\frac{f^a\sqrt{\pi}\Big(3\text{erf}\Big(x\sqrt{f-c\log(f)}\Big)\sqrt{f-c\log(f)}\left(9f^3+9cf^2\log(f)-c^2f\log^2(f)-c^3\log^3(f)\right)(\cosh(d)-\text{si}}{}$$

input
```
Integrate[f^(a + c*x^2)*Cosh[d + f*x^2]^3,x]
```

output
```
(f^a*Sqrt[Pi]*(3*Erf[x*Sqrt[f - c*Log[f]]]*Sqrt[f - c*Log[f]]*(9*f^3 + 9*c
*f^2*Log[f] - c^2*f*Log[f]^2 - c^3*Log[f]^3)*(Cosh[d] - Sinh[d]) + (f - c*
Log[f])*(Erf[x*Sqrt[3*f - c*Log[f]]]*Sqrt[3*f - c*Log[f]]*(3*f^2 + 4*c*f*L
og[f] + c^2*Log[f]^2)*(Cosh[3*d] - Sinh[3*d]) + (3*f - c*Log[f])*(3*Erfi[x
*Sqrt[f + c*Log[f]]]*Sqrt[f + c*Log[f]]*(3*f + c*Log[f])*(Cosh[d] + Sinh[d
]) + Erfi[x*Sqrt[3*f + c*Log[f]]]*(f + c*Log[f])*Sqrt[3*f + c*Log[f]]*(Cos
h[3*d] + Sinh[3*d])))))/(16*(9*f^4 - 10*c^2*f^2*Log[f]^2 + c^4*Log[f]^4))
```

3.316.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2}\cosh^3\left(d+fx^2\right)\,dx$$
$$\Big\downarrow \text{6039}$$
$$\int \left(\frac{1}{8}e^{-3d-3fx^2}f^{a+cx^2}+\frac{3}{8}e^{-d-fx^2}f^{a+cx^2}+\frac{3}{8}e^{d+fx^2}f^{a+cx^2}+\frac{1}{8}e^{3d+3fx^2}f^{a+cx^2}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{3\sqrt{\pi}e^{-d}f^a\operatorname{erf}\left(x\sqrt{f-c\log(f)}\right)}{16\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}e^{-3d}f^a\operatorname{erf}\left(x\sqrt{3f-c\log(f)}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3\sqrt{\pi}e^{d}f^a\operatorname{erfi}\left(x\sqrt{c\log(f)+f}\right)}{16\sqrt{c\log(f)+f}}+\frac{\sqrt{\pi}e^{3d}f^a\operatorname{erfi}\left(x\sqrt{c\log(f)+3f}\right)}{16\sqrt{c\log(f)+3f}}$$

input
```
Int[f^(a + c*x^2)*Cosh[d + f*x^2]^3,x]
```

output
```
(3*f^a*Sqrt[Pi]*Erf[x*Sqrt[f - c*Log[f]]])/(16*E^d*Sqrt[f - c*Log[f]]) + (
f^a*Sqrt[Pi]*Erf[x*Sqrt[3*f - c*Log[f]]])/(16*E^(3*d)*Sqrt[3*f - c*Log[f]]
) + (3*E^d*f^a*Sqrt[Pi]*Erfi[x*Sqrt[f + c*Log[f]]])/(16*Sqrt[f + c*Log[f]]
) + (E^(3*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[3*f + c*Log[f]]])/(16*Sqrt[3*f + c*L
og[f]])
```

3.316.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.316.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.84

method	result
risch	$\frac{f^{a}\mathrm{e}^{-3d}\sqrt{\pi}\,\operatorname{erf}\left(x\sqrt{3f-c\ln(f)}\right)}{16\sqrt{3f-c\ln(f)}}+\frac{f^{a}\mathrm{e}^{3d}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-c\ln(f)-3f}\,x\right)}{16\sqrt{-c\ln(f)-3f}}+\frac{3f^{a}\mathrm{e}^{-d}\sqrt{\pi}\,\operatorname{erf}\left(x\sqrt{f-c\ln(f)}\right)}{16\sqrt{f-c\ln(f)}}+\frac{3f^{a}\mathrm{e}^{d}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-c\ln\right.}{16\sqrt{-c\ln(f)-}}$

input
```
int(f^(c*x^2+a)*cosh(f*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*f^a*exp(-3*d)*Pi^(1/2)/(3*f-c*ln(f))^(1/2)*erf(x*(3*f-c*ln(f))^(1/2))
+1/16*f^a*exp(3*d)*Pi^(1/2)/(-c*ln(f)-3*f)^(1/2)*erf((-c*ln(f)-3*f)^(1/2)*
x)+3/16*f^a*exp(-d)*Pi^(1/2)/(f-c*ln(f))^(1/2)*erf(x*(f-c*ln(f))^(1/2))+3/
16*f^a*exp(d)*Pi^(1/2)/(-c*ln(f)-f)^(1/2)*erf((-c*ln(f)-f)^(1/2)*x)
```

3.316.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(135) = 270$.

Time = 0.28 (sec) , antiderivative size = 491, normalized size of antiderivative = 2.87

$$\int f^{a+cx^2} \cosh^3\left(d+fx^2\right) dx = -\frac{\left(\sqrt{\pi}\left(c^3\log\left(f\right)^3+3\,c^2f\log\left(f\right)^2-cf^2\log\left(f\right)-3\,f^3\right)\cosh\left(a\log\left(f\right)-3\,d\right)+\sqrt{\pi}\left(c^3\log\left(f\right)^3+3\,c^2f\log\right.\right.}{}$$

input
```
integrate(f^(c*x^2+a)*cosh(f*x^2+d)^3,x, algorithm="fricas")
```

output
```
-1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*
cosh(a*log(f) - 3*d) + sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*l
og(f) - 3*f^3)*sinh(a*log(f) - 3*d))*sqrt(-c*log(f) + 3*f)*erf(sqrt(-c*log
(f) + 3*f)*x) + 3*(sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2 - 9*c*f^2*log(f
) - 9*f^3)*cosh(a*log(f) - d) + sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2 -
9*c*f^2*log(f) - 9*f^3)*sinh(a*log(f) - d))*sqrt(-c*log(f) + f)*erf(sqrt(-
c*log(f) + f)*x) + 3*(sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*lo
g(f) + 9*f^3)*cosh(a*log(f) + d) + sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2
 - 9*c*f^2*log(f) + 9*f^3)*sinh(a*log(f) + d))*sqrt(-c*log(f) - f)*erf(sqr
t(-c*log(f) - f)*x) + (sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*l
og(f) + 3*f^3)*cosh(a*log(f) + 3*d) + sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log
(f)^2 - c*f^2*log(f) + 3*f^3)*sinh(a*log(f) + 3*d))*sqrt(-c*log(f) - 3*f)*
erf(sqrt(-c*log(f) - 3*f)*x))/(c^4*log(f)^4 - 10*c^2*f^2*log(f)^2 + 9*f^4)
```

3.316.6 Sympy [F]

$$\int f^{a+cx^2} \cosh^3\left(d+fx^2\right) dx = \int f^{a+cx^2} \cosh^3\left(d+fx^2\right) dx$$

input
```
integrate(f**(c*x**2+a)*cosh(f*x**2+d)**3,x)
```

output
```
Integral(f**(a + c*x**2)*cosh(d + f*x**2)**3, x)
```

3.316.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.84

$$\int f^{a+cx^2}\cosh^3\left(d+fx^2\right)\,dx = \frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log\left(f\right)-3\,f}x\right)e^{(3\,d)}}{16\,\sqrt{-c\log\left(f\right)-3\,f}} + \frac{3\,\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log\left(f\right)+f}x\right)e^{(-d)}}{16\,\sqrt{-c\log\left(f\right)+f}} + \frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log\left(f\right)+3\,f}x\right)e^{(-3\,d)}}{16\,\sqrt{-c\log\left(f\right)+3\,f}} + \frac{3\,\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log\left(f\right)-f}x\right)e^{d}}{16\,\sqrt{-c\log\left(f\right)-f}}$$

input
```
integrate(f^(c*x^2+a)*cosh(f*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x)*e^(3*d)/sqrt(-c*log(f) - 3*
f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x)*e^(-d)/sqrt(-c*log(f) +
f) + 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 3*f)*x)*e^(-3*d)/sqrt(-c*log(f
) + 3*f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x)*e^d/sqrt(-c*log(f)
 - f)
```

3.316.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.91

$$\int f^{a+cx^2}\cosh^3\left(d+fx^2\right)\,dx = -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-c\log\left(f\right)-3\,f}x\right)e^{(a\log(f)+3\,d)}}{16\,\sqrt{-c\log\left(f\right)-3\,f}} - \frac{3\,\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-c\log\left(f\right)-f}x\right)e^{(a\log(f)+d)}}{16\,\sqrt{-c\log\left(f\right)-f}} - \frac{3\,\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-c\log\left(f\right)+f}x\right)e^{(a\log(f)-d)}}{16\,\sqrt{-c\log\left(f\right)+f}} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-c\log\left(f\right)+3\,f}x\right)e^{(a\log(f)-3\,d)}}{16\,\sqrt{-c\log\left(f\right)+3\,f}}$$

input
```
integrate(f^(c*x^2+a)*cosh(f*x^2+d)^3,x, algorithm="giac")
```

output
```
-1/16*sqrt(pi)*erf(-sqrt(-c*log(f) - 3*f)*x)*e^(a*log(f) + 3*d)/sqrt(-c*lo
g(f) - 3*f) - 3/16*sqrt(pi)*erf(-sqrt(-c*log(f) - f)*x)*e^(a*log(f) + d)/s
qrt(-c*log(f) - f) - 3/16*sqrt(pi)*erf(-sqrt(-c*log(f) + f)*x)*e^(a*log(f)
 - d)/sqrt(-c*log(f) + f) - 1/16*sqrt(pi)*erf(-sqrt(-c*log(f) + 3*f)*x)*e^
(a*log(f) - 3*d)/sqrt(-c*log(f) + 3*f)
```

3.316.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\cosh^3\left(d+fx^2\right)\,dx=\int f^{c\,x^2+a}\cosh\left(f\,x^2+d\right)^3\,dx$$

input
```
int(f^(a + c*x^2)*cosh(d + f*x^2)^3,x)
```

output
```
int(f^(a + c*x^2)*cosh(d + f*x^2)^3, x)
```

3.317 $\int f^{a+cx^2} \cosh\left(d+ex+fx^2\right) dx$

3.317.1 Optimal result

Integrand size = 21, antiderivative size = 140

$$\int f^{a+cx^2}\cosh\left(d+ex+fx^2\right)dx=\frac{e^{-d+\frac{e^2}{4f-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{e+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^{d-\frac{e^2}{4(f+c\log(f))}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{4\sqrt{f+c\log(f)}}$$

output
```
1/4*exp(-d+e^2/(4*f-4*c*ln(f)))*f^a*erf(1/2*(e+2*x*(f-c*ln(f)))/(f-c*ln(f)
)^(1/2))*Pi^(1/2)/(f-c*ln(f))^(1/2)+1/4*exp(d-1/4*e^2/(f+c*ln(f)))*f^a*erf
i(1/2*(e+2*x*(f+c*ln(f)))/(f+c*ln(f))^(1/2))*Pi^(1/2)/(f+c*ln(f))^(1/2)
```

3.317.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.18

$$\int f^{a+cx^2}\cosh\left(d+ex+fx^2\right)dx$$
$$=\frac{e^{-\frac{e^2}{4(f+c\log(f))}}f^a\sqrt{\pi}\left(e^{\frac{e^2f}{2f^2-2c^2\log^2(f)}}\operatorname{erf}\left(\frac{e+2fx-2cx\log(f)}{2\sqrt{f-c\log(f)}}\right)\sqrt{f+c\log(f)}(\cosh(d)-\sinh(d))+\operatorname{erfi}\left(\frac{e+2fx+2cx\log}{2\sqrt{f+c\log(f}}\right.\right.}{4\sqrt{f-c\log(f)}\sqrt{f+c\log(f)}}$$

input
```
Integrate[f^(a + c*x^2)*Cosh[d + e*x + f*x^2],x]
```

output
```
(f^a*Sqrt[Pi]*(E^((e^2*f)/(2*f^2 - 2*c^2*Log[f]^2))*Erf[(e + 2*f*x - 2*c*x
*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f + c*Log[f]]*(Cosh[d] - Sinh[d]) +
Erfi[(e + 2*f*x + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[f - c*Log[f]]
*(Cosh[d] + Sinh[d])))/(4*E^(e^2/(4*(f + c*Log[f])))*Sqrt[f - c*Log[f]]*Sq
rt[f + c*Log[f]])
```

3.317.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2}\cosh\left(d+ex+fx^2\right)\,dx$$

$$\Big\downarrow \text{6039}$$

$$\int \left(\frac{1}{2}f^{a+cx^2}e^{-d-ex-fx^2}+\frac{1}{2}f^{a+cx^2}e^{d+ex+fx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}f^a e^{\frac{e^2}{4f-4c\log(f)}-d}\text{erf}\left(\frac{2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}f^a e^{d-\frac{e^2}{4(c\log(f)+f)}}\text{erfi}\left(\frac{2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}}$$

input
```
Int[f^(a + c*x^2)*Cosh[d + e*x + f*x^2],x]
```

output
```
(E^(-d + e^2/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(e + 2*x*(f - c*Log[f]))
/(2*Sqrt[f - c*Log[f]])])/(4*Sqrt[f - c*Log[f]]) + (E^(d - e^2/(4*(f + c*L
og[f])))*f^a*Sqrt[Pi]*Erfi[(e + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])
])/(4*Sqrt[f + c*Log[f]])
```

3.317.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.317.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.05

method	result
risch	$\frac{\operatorname{erf}\left(x\sqrt{f-c\ln(f)}+\frac{e}{2\sqrt{f-c\ln(f)}}\right)\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{4d\ln(f)c-4df+e^{2}}{4(c\ln(f)-f)}}}{4\sqrt{f-c\ln(f)}}-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)-f}\,x+\frac{e}{2\sqrt{-c\ln(f)-f}}\right)\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{4d\ln(f)c+4df-e^{2}}{4f+4c\ln(f)}}}{4\sqrt{-c\ln(f)-f}}$

input
```
int(f^(c*x^2+a)*cosh(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*erf(x*(f-c*ln(f))^(1/2)+1/2*e/(f-c*ln(f))^(1/2))/(f-c*ln(f))^(1/2)*Pi^
(1/2)*f^a*exp(-1/4*(4*d*ln(f)*c-4*d*f+e^2)/(c*ln(f)-f))-1/4*erf(-(-c*ln(f)
-f)^(1/2)*x+1/2*e/(-c*ln(f)-f)^(1/2))/(-c*ln(f)-f)^(1/2)*Pi^(1/2)*f^a*exp(
1/4*(4*d*ln(f)*c+4*d*f-e^2)/(f+c*ln(f)))
```

3.317.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(119) = 238.

Time = 0.27 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.29

$$\int f^{a+cx^2}\cosh\left(d+ex+fx^2\right)\,dx =$$
$$-\frac{\left(\sqrt{\pi}(c\log(f)+f)\cosh\left(\frac{4\,ac\log(f)^2-e^2+4\,df-4\,(cd+af)\log(f)}{4\,(c\log(f)-f)}\right)+\sqrt{\pi}(c\log(f)+f)\sinh\left(\frac{4\,ac\log(f)^2-e^2+4\,df-4\,(\right.\right.}{4\,(c\log(f)-}}$$

input
```
integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
-1/4*((sqrt(pi)*(c*log(f) + f)*cosh(1/4*(4*a*c*log(f)^2 - e^2 + 4*d*f - 4*
(c*d + a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c*log(f) + f)*sinh(1/4*(4*
a*c*log(f)^2 - e^2 + 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f) - f)))*sqrt(-
c*log(f) + f)*erf(1/2*(2*c*x*log(f) - 2*f*x - e)*sqrt(-c*log(f) + f)/(c*lo
g(f) - f)) + (sqrt(pi)*(c*log(f) - f)*cosh(1/4*(4*a*c*log(f)^2 - e^2 + 4*d
*f + 4*(c*d + a*f)*log(f))/(c*log(f) + f)) + sqrt(pi)*(c*log(f) - f)*sinh(
1/4*(4*a*c*log(f)^2 - e^2 + 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) + f)))
*sqrt(-c*log(f) - f)*erf(1/2*(2*c*x*log(f) + 2*f*x + e)*sqrt(-c*log(f) - f
)/(c*log(f) + f)))/(c^2*log(f)^2 - f^2)
```

3.317.6 Sympy [F]

$$\int f^{a+cx^2}\cosh\left(d+ex+fx^2\right)\,dx=\int f^{a+cx^2}\cosh\left(d+ex+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+a)*cosh(f*x**2+e*x+d),x)
```

output
```
Integral(f**(a + c*x**2)*cosh(d + e*x + f*x**2), x)
```

3.317.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91

$$\int f^{a+cx^2}\cosh\left(d+ex+fx^2\right)\,dx$$
$$=\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)-f}\,x-\frac{e}{2\sqrt{-c\log(f)-f}}\right)e^{\left(d-\frac{e^2}{4(c\log(f)+f)}\right)}}{4\sqrt{-c\log(f)-f}}+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)+f}\,x+\frac{e}{2\sqrt{-c\log(f)+f}}\right)e^{\left(-d-\frac{e^2}{4(c\log(f)-f)}\right)}}{4\sqrt{-c\log(f)+f}}$$

input
```
integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*e/sqrt(-c*log(f) - f))*e^
(d - 1/4*e^2/(c*log(f) + f))/sqrt(-c*log(f) - f) + 1/4*sqrt(pi)*f^a*erf(sq
rt(-c*log(f) + f)*x + 1/2*e/sqrt(-c*log(f) + f))*e^(-d - 1/4*e^2/(c*log(f)
 - f))/sqrt(-c*log(f) + f)
```

3.317.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.23

$$\int f^{a+cx^2}\cosh\left(d+ex+fx^2\right)\,dx$$
$$= -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-f}\left(2\,x+\frac{e}{c\log(f)+f}\right)\right)e^{\left(\frac{4\,ac\log(f)^2+4\,cd\log(f)+4\,af\log(f)-e^2+4\,df}{4\,(c\log(f)+f)}\right)}}{4\,\sqrt{-c\log\left(f\right)-f}}$$
$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)+f}\left(2\,x-\frac{e}{c\log(f)-f}\right)\right)e^{\left(\frac{4\,ac\log(f)^2-4\,cd\log(f)-4\,af\log(f)-e^2+4\,df}{4\,(c\log(f)-f)}\right)}}{4\,\sqrt{-c\log\left(f\right)+f}}$$

input
```
integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + e/(c*log(f) + f)))*e^(1/
4*(4*a*c*log(f)^2 + 4*c*d*log(f) + 4*a*f*log(f) - e^2 + 4*d*f)/(c*log(f) +
 f))/sqrt(-c*log(f) - f) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x
- e/(c*log(f) - f)))*e^(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - 4*a*f*log(f)
- e^2 + 4*d*f)/(c*log(f) - f))/sqrt(-c*log(f) + f)
```

3.317.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\cosh\left(d+ex+fx^2\right)\,dx=\int f^{c\,x^2+a}\cosh\left(f\,x^2+e\,x+d\right)\,dx$$

input
```
int(f^(a + c*x^2)*cosh(d + e*x + f*x^2),x)
```

output
```
int(f^(a + c*x^2)*cosh(d + e*x + f*x^2), x)
```

3.318 $\int f^{a+cx^2}\cosh^2\left(d+ex+fx^2\right)\,dx$

3.318.1 Optimal result

Integrand size = 23, antiderivative size = 183

$$\int f^{a+cx^2}\cosh^2\left(d+ex+fx^2\right)\,dx=\frac{f^a\sqrt{\pi}\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-2d+\frac{e^2}{2f-c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{e+x(2f-c\log(f))}{\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{e^{2d-\frac{e^2}{2f+c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e+x(2f+c\log(f))}{\sqrt{2f+c\log(f)}}\right)}{8\sqrt{2f+c\log(f)}}$$

output
```
1/4*f^a*erfi(x*c^(1/2)*ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/8*exp(-
2*d+e^2/(2*f-c*ln(f)))*f^a*erf((e+x*(2*f-c*ln(f)))/(2*f-c*ln(f))^(1/2))*Pi
^(1/2)/(2*f-c*ln(f))^(1/2)+1/8*exp(2*d-e^2/(2*f+c*ln(f)))*f^a*erfi((e+x*(2
*f+c*ln(f)))/(2*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*f+c*ln(f))^(1/2)
```

3.318.2 Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.41

$$\int f^{a+cx^2}\cosh^2\left(d+ex+fx^2\right)\,dx = \frac{e^{\frac{e^2}{2f-c\log(f)}}f^a\sqrt{\pi}\left(-2e^{\frac{e^2}{-2f+c\log(f)}}\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)\left(4f^2-c^2\log^2(f)\right)-\sqrt{c}\sqrt{\log(f)}\left(\operatorname{erf}\left(\frac{e+2fx-cx\log(f)}{\sqrt{2f-c\log(f)}}\right)\right.\right.}{}$$

input
```
Integrate[f^(a + c*x^2)*Cosh[d + e*x + f*x^2]^2,x]
```

output
```
(E^(e^2/(2*f - c*Log[f]))*f^a*Sqrt[Pi]*(-2*E^(e^2/(-2*f + c*Log[f]))*Erfi[
Sqrt[c]*x*Sqrt[Log[f]]]*(4*f^2 - c^2*Log[f]^2) - Sqrt[c]*Sqrt[Log[f]]*(Erf
[(e + 2*f*x - c*x*Log[f])/Sqrt[2*f - c*Log[f]]]*Sqrt[2*f - c*Log[f]]*(2*f
+ c*Log[f])*(Cosh[2*d] - Sinh[2*d]) + E^((4*e^2*f)/(-4*f^2 + c^2*Log[f]^2)
)*Erfi[(e + 2*f*x + c*x*Log[f])/Sqrt[2*f + c*Log[f]]]*(2*f - c*Log[f])*Sqr
t[2*f + c*Log[f]]*(Cosh[2*d] + Sinh[2*d]))))/(8*Sqrt[c]*Sqrt[Log[f]]*(-4*f
^2 + c^2*Log[f]^2))
```

3.318.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2}\cosh^2\left(d+ex+fx^2\right)\,dx$$

$$\Big\downarrow \text{ 6039}$$

$$\int \left(\frac{1}{4}f^{a+cx^2}e^{-2d-2ex-2fx^2}+\frac{1}{4}f^{a+cx^2}e^{2d+2ex+2fx^2}+\frac{1}{2}f^{a+cx^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{\pi}f^a e^{\frac{e^2}{2f-c\log(f)}-2d}\operatorname{erf}\left(\frac{x(2f-c\log(f))+e}{\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi}f^a e^{2d-\frac{e^2}{c\log(f)+2f}}\operatorname{erfi}\left(\frac{x(c\log(f)+2f)+e}{\sqrt{c\log(f)+2f}}\right)}{8\sqrt{c\log(f)+2f}}+\frac{\sqrt{\pi}f^a\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + c*x^2)*Cosh[d + e*x + f*x^2]^2,x]
```

output
```
(f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^
(-2*d + e^2/(2*f - c*Log[f]))*f^a*Sqrt[Pi]*Erf[(e + x*(2*f - c*Log[f]))/Sq
rt[2*f - c*Log[f]]])/(8*Sqrt[2*f - c*Log[f]]) + (E^(2*d - e^2/(2*f + c*Log
[f]))*f^a*Sqrt[Pi]*Erfi[(e + x*(2*f + c*Log[f]))/Sqrt[2*f + c*Log[f]]])/(8
*Sqrt[2*f + c*Log[f]])
```

3.318.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.318.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.97

method	result
risch	$\frac{\operatorname{erf}\left(x\sqrt{2f-c\ln(f)}+\frac{e}{\sqrt{2f-c\ln(f)}}\right)\sqrt{\pi}\,f^a\mathrm{e}^{-\frac{2d\ln(f)c-4df+e^2}{c\ln(f)-2f}}}{8\sqrt{2f-c\ln(f)}}-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)-2f}\,x+\frac{e}{\sqrt{-c\ln(f)-2f}}\right)\sqrt{\pi}\,f^a\mathrm{e}^{\frac{2d\ln(f)c+4df-e^2}{2f+c\ln(f)}}}{8\sqrt{-c\ln(f)-2f}}$

input
```
int(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*erf(x*(2*f-c*ln(f))^(1/2)+e/(2*f-c*ln(f))^(1/2))/(2*f-c*ln(f))^(1/2)*P
i^(1/2)*f^a*exp(-(2*d*ln(f)*c-4*d*f+e^2)/(c*ln(f)-2*f))-1/8*erf(-(-c*ln(f)
-2*f)^(1/2)*x+e/(-c*ln(f)-2*f)^(1/2))/(-c*ln(f)-2*f)^(1/2)*Pi^(1/2)*f^a*ex
p((2*d*ln(f)*c+4*d*f-e^2)/(2*f+c*ln(f)))+1/4*f^a*Pi^(1/2)/(-c*ln(f))^(1/2)
*erf((-c*ln(f))^(1/2)*x)
```

3.318.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(155) = 310$.

Time = 0.29 (sec) , antiderivative size = 420, normalized size of antiderivative = 2.30

$$\int f^{a+cx^2}\cosh^2\left(d+ex+fx^2\right)\,dx =$$

$$-\frac{2\left(\sqrt{\pi}\left(c^2\log\left(f\right)^2-4\,f^2\right)\cosh\left(a\log\left(f\right)\right)+\sqrt{\pi}\left(c^2\log\left(f\right)^2-4\,f^2\right)\sinh\left(a\log\left(f\right)\right)\right)\sqrt{-c\log\left(f\right)}\operatorname{erf}\left(\sqrt{\right.}$$

input
```
integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

output
```
-1/8*(2*(sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*cosh(a*log(f)) + sqrt(pi)*(c^2*lo
g(f)^2 - 4*f^2)*sinh(a*log(f)))*sqrt(-c*log(f))*erf(sqrt(-c*log(f))*x) + (
sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh((a*c*log(f)^2 - e^2 + 4*d*f -
2*(c*d + a*f)*log(f))/(c*log(f) - 2*f)) + sqrt(pi)*(c^2*log(f)^2 + 2*c*f*l
og(f))*sinh((a*c*log(f)^2 - e^2 + 4*d*f - 2*(c*d + a*f)*log(f))/(c*log(f)
- 2*f)))*sqrt(-c*log(f) + 2*f)*erf((c*x*log(f) - 2*f*x - e)*sqrt(-c*log(f)
 + 2*f)/(c*log(f) - 2*f)) + (sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*cosh((
a*c*log(f)^2 - e^2 + 4*d*f + 2*(c*d + a*f)*log(f))/(c*log(f) + 2*f)) + sqr
t(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*sinh((a*c*log(f)^2 - e^2 + 4*d*f + 2*(
c*d + a*f)*log(f))/(c*log(f) + 2*f)))*sqrt(-c*log(f) - 2*f)*erf((c*x*log(f
) + 2*f*x + e)*sqrt(-c*log(f) - 2*f)/(c*log(f) + 2*f)))/(c^3*log(f)^3 - 4*
c*f^2*log(f))
```

3.318.6 Sympy [F]

$$\int f^{a+cx^2}\cosh^2\left(d+ex+fx^2\right)\,dx=\int f^{a+cx^2}\cosh^2\left(d+ex+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+a)*cosh(f*x**2+e*x+d)**2,x)
```

output
```
Integral(f**(a + c*x**2)*cosh(d + e*x + f*x**2)**2, x)
```

3.318.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.88

$$\int f^{a+cx^2}\cosh^2\left(d+ex+fx^2\right)\,dx$$
$$=\frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)-2\,f}x-\frac{e}{\sqrt{-c\log(f)-2\,f}}\right)e^{\left(2\,d-\frac{e^2}{c\log(f)+2\,f}\right)}}{8\,\sqrt{-c\log(f)-2\,f}}+\frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)+2\,f}x+\frac{e}{\sqrt{-c\log(f)+2\,f}}\right)e^{\left(-2\,d-\frac{e^2}{c\log(f)-2\,f}\right)}}{8\,\sqrt{-c\log(f)+2\,f}}+\frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)}x\right)}{4\,\sqrt{-c\log(f)}}$$

input
```
integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

output
```
1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x - e/sqrt(-c*log(f) - 2*f))*e^
(2*d - e^2/(c*log(f) + 2*f))/sqrt(-c*log(f) - 2*f) + 1/8*sqrt(pi)*f^a*erf(
sqrt(-c*log(f) + 2*f)*x + e/sqrt(-c*log(f) + 2*f))*e^(-2*d - e^2/(c*log(f)
 - 2*f))/sqrt(-c*log(f) + 2*f) + 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x)/s
qrt(-c*log(f))
```

3.318.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.08

$$\int f^{a+cx^2}\cosh^2\left(d+ex+fx^2\right)\,dx$$
$$= -\frac{\sqrt{\pi}f^a \operatorname{erf}\left(-\sqrt{-c\log(f)}x\right)}{4\sqrt{-c\log(f)}}$$
$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log(f)-2f}\left(x+\frac{e}{c\log(f)+2f}\right)\right)e^{\left(\frac{ac\log(f)^2+2cd\log(f)+2af\log(f)-e^2+4df}{c\log(f)+2f}\right)}}{8\sqrt{-c\log(f)-2f}}$$
$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log(f)+2f}\left(x-\frac{e}{c\log(f)-2f}\right)\right)e^{\left(\frac{ac\log(f)^2-2cd\log(f)-2af\log(f)-e^2+4df}{c\log(f)-2f}\right)}}{8\sqrt{-c\log(f)+2f}}$$

input
```
integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*f^a*erf(-sqrt(-c*log(f))*x)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*e
rf(-sqrt(-c*log(f) - 2*f)*(x + e/(c*log(f) + 2*f)))*e^((a*c*log(f)^2 + 2*c
*d*log(f) + 2*a*f*log(f) - e^2 + 4*d*f)/(c*log(f) + 2*f))/sqrt(-c*log(f) -
 2*f) - 1/8*sqrt(pi)*erf(-sqrt(-c*log(f) + 2*f)*(x - e/(c*log(f) - 2*f)))*
e^((a*c*log(f)^2 - 2*c*d*log(f) - 2*a*f*log(f) - e^2 + 4*d*f)/(c*log(f) -
2*f))/sqrt(-c*log(f) + 2*f)
```

3.318.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\cosh^2\left(d+ex+fx^2\right)\,dx = \int f^{c\,x^2+a}\cosh\left(f\,x^2+e\,x+d\right)^2\,dx$$

input
```
int(f^(a + c*x^2)*cosh(d + e*x + f*x^2)^2,x)
```

output
```
int(f^(a + c*x^2)*cosh(d + e*x + f*x^2)^2, x)
```

3.319 $\int f^{a+cx^2} \cosh^3(d+ex+fx^2)\,dx$

3.319.1 Optimal result

Integrand size = 23, antiderivative size = 300

$$\int f^{a+cx^2}\cosh^3(d+ex+fx^2)\,dx = \frac{3e^{-d+\frac{e^2}{4f-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{e+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}} + \frac{e^{-3d+\frac{9e^2}{12f-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{3e+2x(3f-c\log(f))}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}} + \frac{3e^{d-\frac{e^2}{4(f+c\log(f))}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{16\sqrt{f+c\log(f)}} + \frac{e^{3d-\frac{9e^2}{4(3f+c\log(f))}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{3e+2x(3f+c\log(f))}{2\sqrt{3f+c\log(f)}}\right)}{16\sqrt{3f+c\log(f)}}$$

output
```
3/16*exp(-d+e^2/(4*f-4*c*ln(f)))*f^a*erf(1/2*(e+2*x*(f-c*ln(f)))/(f-c*ln(f
))^(1/2))*Pi^(1/2)/(f-c*ln(f))^(1/2)+1/16*exp(-3*d+9*e^2/(12*f-4*c*ln(f)))
*f^a*erf(1/2*(3*e+2*x*(3*f-c*ln(f)))/(3*f-c*ln(f))^(1/2))*Pi^(1/2)/(3*f-c*
ln(f))^(1/2)+3/16*exp(d-1/4*e^2/(f+c*ln(f)))*f^a*erfi(1/2*(e+2*x*(f+c*ln(f
)))/(f+c*ln(f))^(1/2))*Pi^(1/2)/(f+c*ln(f))^(1/2)+1/16*exp(3*d-9/4*e^2/(3*
f+c*ln(f)))*f^a*erfi(1/2*(3*e+2*x*(3*f+c*ln(f)))/(3*f+c*ln(f))^(1/2))*Pi^(
1/2)/(3*f+c*ln(f))^(1/2)
```

3.319.2 Mathematica [A] (verified)

Time = 4.44 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.59

$$\int f^{a+cx^2}\cosh^3\left(d+ex+fx^2\right)\,dx$$
$$= \frac{e^{-\frac{1}{4}e^2\left(\frac{1}{f+c\log(f)}+\frac{9}{3f+c\log(f)}\right)}f^a\sqrt{\pi}\left(3e^{\frac{1}{4}e^2\left(\frac{1}{f-c\log(f)}+\frac{1}{f+c\log(f)}+\frac{9}{3f+c\log(f)}\right)}\operatorname{erf}\left(\frac{e+2fx-2cx\log(f)}{2\sqrt{f-c\log(f)}}\right)\sqrt{f-c\log(f)}\,(9f^3 -}{}$$

input
```
Integrate[f^(a + c*x^2)*Cosh[d + e*x + f*x^2]^3,x]
```

output
```
(f^a*Sqrt[Pi]*(3*E^((e^2*((f - c*Log[f])^(-1) + (f + c*Log[f])^(-1) + 9/(3
*f + c*Log[f])))/4)*Erf[(e + 2*f*x - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]
*Sqrt[f - c*Log[f]]*(9*f^3 + 9*c*f^2*Log[f] - c^2*f*Log[f]^2 - c^3*Log[f]^
3)*(Cosh[d] - Sinh[d]) + (f - c*Log[f])*(E^((e^2*(9/(3*f - c*Log[f]) + (f
+ c*Log[f])^(-1) + 9/(3*f + c*Log[f])))/4)*Erf[(3*e + 6*f*x - 2*c*x*Log[f]
)/(2*Sqrt[3*f - c*Log[f]])]*Sqrt[3*f - c*Log[f]]*(3*f^2 + 4*c*f*Log[f] + c
^2*Log[f]^2)*(Cosh[3*d] - Sinh[3*d]) + (3*f - c*Log[f])*(3*E^((9*e^2)/(4*(
3*f + c*Log[f])))*Erfi[(e + 2*f*x + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*
Sqrt[f + c*Log[f]]*(3*f + c*Log[f])*(Cosh[d] + Sinh[d]) + E^(e^2/(4*(f + c
*Log[f])))*Erfi[(3*e + 6*f*x + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*(f
+ c*Log[f])*Sqrt[3*f + c*Log[f]]*(Cosh[3*d] + Sinh[3*d])))))/(16*E^((e^2*(
(f + c*Log[f])^(-1) + 9/(3*f + c*Log[f])))/4)*(9*f^4 - 10*c^2*f^2*Log[f]^2
 + c^4*Log[f]^4))
```

3.319.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2}\cosh^3\left(d+ex+fx^2\right)\,dx$$

$$\Big\downarrow \; 6039$$

$$\int \left(\frac{3}{8}f^{a+cx^2}\exp\left(-3(d+ex+fx^2)+2d+2ex+2fx^2\right)+\frac{3}{8}f^{a+cx^2}\exp\left(-3(d+ex+fx^2)+4d+4ex+4fx^2\right)\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\sqrt{\pi}f^a e^{\frac{e^2}{4f-4c\log(f)}-d}\text{erf}\left(\frac{2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}f^a e^{\frac{9e^2}{12f-4c\log(f)}-3d}\text{erf}\left(\frac{2x(3f-c\log(f))+3e}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3\sqrt{\pi}f^a e^{d-\frac{e^2}{4(c\log(f)+f)}}\text{erfi}\left(\frac{2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{16\sqrt{c\log(f)+f}}+\frac{\sqrt{\pi}f^a e^{3d-\frac{9e^2}{4(c\log(f)+3f)}}\text{erfi}\left(\frac{2x(c\log(f)+3f)+3e}{2\sqrt{c\log(f)+3f}}\right)}{16\sqrt{c\log(f)+3f}}$$

input
```
Int[f^(a + c*x^2)*Cosh[d + e*x + f*x^2]^3,x]
```

output
```
(3*E^(-d + e^2/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(e + 2*x*(f - c*Log[f]
))/(2*Sqrt[f - c*Log[f]])])/(16*Sqrt[f - c*Log[f]]) + (E^(-3*d + (9*e^2)/(
12*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(3*e + 2*x*(3*f - c*Log[f]))/(2*Sqrt[
3*f - c*Log[f]])])/(16*Sqrt[3*f - c*Log[f]]) + (3*E^(d - e^2/(4*(f + c*Log
[f])))*f^a*Sqrt[Pi]*Erfi[(e + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])
/(16*Sqrt[f + c*Log[f]]) + (E^(3*d - (9*e^2)/(4*(3*f + c*Log[f])))*f^a*Sqr
t[Pi]*Erfi[(3*e + 2*x*(3*f + c*Log[f]))/(2*Sqrt[3*f + c*Log[f]])])/(16*Sqr
t[3*f + c*Log[f]])
```

3.319.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.319.4 Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.01

method	result
risch	$\frac{\operatorname{erf}\left(x\sqrt{3f-c\ln(f)}+\frac{3e}{2\sqrt{3f-c\ln(f)}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{3\left(4d\ln(f)c-12df+3e^{2}\right)}{4(c\ln(f)-3f)}}}{16\sqrt{3f-c\ln(f)}}-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)-3f}\,x+\frac{3e}{2\sqrt{-c\ln(f)-3f}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{\frac{3d\ln(f)c+9}{3f+c\ln}}}{16\sqrt{-c\ln(f)-3f}}$

input
```
int(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*erf(x*(3*f-c*ln(f))^(1/2)+3/2*e/(3*f-c*ln(f))^(1/2))/(3*f-c*ln(f))^(1
/2)*Pi^(1/2)*f^a*exp(-3/4*(4*d*ln(f)*c-12*d*f+3*e^2)/(c*ln(f)-3*f))-1/16*e
rf(-(-c*ln(f)-3*f)^(1/2)*x+3/2*e/(-c*ln(f)-3*f)^(1/2))/(-c*ln(f)-3*f)^(1/2
)*Pi^(1/2)*f^a*exp(3/4*(4*d*ln(f)*c+12*d*f-3*e^2)/(3*f+c*ln(f)))+3/16*erf(
x*(f-c*ln(f))^(1/2)+1/2*e/(f-c*ln(f))^(1/2))/(f-c*ln(f))^(1/2)*Pi^(1/2)*f^
a*exp(-1/4*(4*d*ln(f)*c-4*d*f+e^2)/(c*ln(f)-f))-3/16*erf(-(-c*ln(f)-f)^(1/
2)*x+1/2*e/(-c*ln(f)-f)^(1/2))/(-c*ln(f)-f)^(1/2)*Pi^(1/2)*f^a*exp(1/4*(4*
d*ln(f)*c+4*d*f-e^2)/(f+c*ln(f)))
```

3.319.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 847 vs. $2(253) = 506$.

Time = 0.29 (sec) , antiderivative size = 847, normalized size of antiderivative = 2.82

$$\int f^{a+cx^2}\cosh^3\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

output
```
-1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*
cosh(1/4*(4*a*c*log(f)^2 - 9*e^2 + 36*d*f - 12*(c*d + a*f)*log(f))/(c*log(
f) - 3*f)) + sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*
f^3)*sinh(1/4*(4*a*c*log(f)^2 - 9*e^2 + 36*d*f - 12*(c*d + a*f)*log(f))/(c
*log(f) - 3*f)))*sqrt(-c*log(f) + 3*f)*erf(1/2*(2*c*x*log(f) - 6*f*x - 3*e
)*sqrt(-c*log(f) + 3*f)/(c*log(f) - 3*f)) + 3*(sqrt(pi)*(c^3*log(f)^3 + c^
2*f*log(f)^2 - 9*c*f^2*log(f) - 9*f^3)*cosh(1/4*(4*a*c*log(f)^2 - e^2 + 4*
d*f - 4*(c*d + a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c^3*log(f)^3 + c^2
*f*log(f)^2 - 9*c*f^2*log(f) - 9*f^3)*sinh(1/4*(4*a*c*log(f)^2 - e^2 + 4*d
*f - 4*(c*d + a*f)*log(f))/(c*log(f) - f)))*sqrt(-c*log(f) + f)*erf(1/2*(2
*c*x*log(f) - 2*f*x - e)*sqrt(-c*log(f) + f)/(c*log(f) - f)) + 3*(sqrt(pi)
*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*cosh(1/4*(4*a*c*
log(f)^2 - e^2 + 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) + f)) + sqrt(pi)*
(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*sinh(1/4*(4*a*c*l
og(f)^2 - e^2 + 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) + f)))*sqrt(-c*log
(f) - f)*erf(1/2*(2*c*x*log(f) + 2*f*x + e)*sqrt(-c*log(f) - f)/(c*log(f)
+ f)) + (sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f^3)
*cosh(1/4*(4*a*c*log(f)^2 - 9*e^2 + 36*d*f + 12*(c*d + a*f)*log(f))/(c*log
(f) + 3*f)) + sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3
*f^3)*sinh(1/4*(4*a*c*log(f)^2 - 9*e^2 + 36*d*f + 12*(c*d + a*f)*log(f)...
```

3.319.6 Sympy [F]

$$\int f^{a+cx^2} \cosh^3\left(d+ex+fx^2\right) \, dx = \int f^{a+cx^2} \cosh^3\left(d+ex+fx^2\right) dx$$

input
```
integrate(f**(c*x**2+a)*cosh(f*x**2+e*x+d)**3,x)
```

output
```
Integral(f**(a + c*x**2)*cosh(d + e*x + f*x**2)**3, x)
```

3.319.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.88

$$\int f^{a+cx^2}\cosh^3\left(d+ex+fx^2\right)\,dx$$
$$=\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)-3f}x-\frac{3e}{2\sqrt{-c\log(f)-3f}}\right)e^{\left(3d-\frac{9e^2}{4(c\log(f)+3f)}\right)}}{16\sqrt{-c\log(f)-3f}}$$
$$+\frac{3\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)-f}x-\frac{e}{2\sqrt{-c\log(f)-f}}\right)e^{\left(d-\frac{e^2}{4(c\log(f)+f)}\right)}}{16\sqrt{-c\log(f)-f}}$$
$$+\frac{3\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)+f}x+\frac{e}{2\sqrt{-c\log(f)+f}}\right)e^{\left(-d-\frac{e^2}{4(c\log(f)-f)}\right)}}{16\sqrt{-c\log(f)+f}}$$
$$+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)+3f}x+\frac{3e}{2\sqrt{-c\log(f)+3f}}\right)e^{\left(-3d-\frac{9e^2}{4(c\log(f)-3f)}\right)}}{16\sqrt{-c\log(f)+3f}}$$

input
```
integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

output
```
1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x - 3/2*e/sqrt(-c*log(f) - 3*f
))*e^(3*d - 9/4*e^2/(c*log(f) + 3*f))/sqrt(-c*log(f) - 3*f) + 3/16*sqrt(pi
)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*e/sqrt(-c*log(f) - f))*e^(d - 1/4*e^
2/(c*log(f) + f))/sqrt(-c*log(f) - f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(
f) + f)*x + 1/2*e/sqrt(-c*log(f) + f))*e^(-d - 1/4*e^2/(c*log(f) - f))/sqr
t(-c*log(f) + f) + 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 3*f)*x + 3/2*e/s
qrt(-c*log(f) + 3*f))*e^(-3*d - 9/4*e^2/(c*log(f) - 3*f))/sqrt(-c*log(f) +
 3*f)
```

3.319.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.17

$$\int f^{a+cx^2}\cosh^3\left(d+ex+fx^2\right)\,dx$$
$$= -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)-3f}\left(2x+\frac{3e}{c\log(f)+3f}\right)\right)e^{\left(\frac{4ac\log(f)^2+12cd\log(f)+12af\log(f)-9e^2+36df}{4(c\log(f)+3f)}\right)}}{16\sqrt{-c\log(f)-3f}}$$
$$-\frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)-f}\left(2x+\frac{e}{c\log(f)+f}\right)\right)e^{\left(\frac{4ac\log(f)^2+4cd\log(f)+4af\log(f)-e^2+4df}{4(c\log(f)+f)}\right)}}{16\sqrt{-c\log(f)-f}}$$
$$-\frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)+f}\left(2x-\frac{e}{c\log(f)-f}\right)\right)e^{\left(\frac{4ac\log(f)^2-4cd\log(f)-4af\log(f)-e^2+4df}{4(c\log(f)-f)}\right)}}{16\sqrt{-c\log(f)+f}}$$
$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)+3f}\left(2x-\frac{3e}{c\log(f)-3f}\right)\right)e^{\left(\frac{4ac\log(f)^2-12cd\log(f)-12af\log(f)-9e^2+36df}{4(c\log(f)-3f)}\right)}}{16\sqrt{-c\log(f)+3f}}$$

input
```
integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="giac")
```

output
```
-1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 3*f)*(2*x + 3*e/(c*log(f) + 3*f))
)*e^(1/4*(4*a*c*log(f)^2 + 12*c*d*log(f) + 12*a*f*log(f) - 9*e^2 + 36*d*f)
/(c*log(f) + 3*f))/sqrt(-c*log(f) - 3*f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*
log(f) - f)*(2*x + e/(c*log(f) + f)))*e^(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f
) + 4*a*f*log(f) - e^2 + 4*d*f)/(c*log(f) + f))/sqrt(-c*log(f) - f) - 3/16
*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x - e/(c*log(f) - f)))*e^(1/4*(4
*a*c*log(f)^2 - 4*c*d*log(f) - 4*a*f*log(f) - e^2 + 4*d*f)/(c*log(f) - f))
/sqrt(-c*log(f) + f) - 1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + 3*f)*(2*x -
 3*e/(c*log(f) - 3*f)))*e^(1/4*(4*a*c*log(f)^2 - 12*c*d*log(f) - 12*a*f*lo
g(f) - 9*e^2 + 36*d*f)/(c*log(f) - 3*f))/sqrt(-c*log(f) + 3*f)
```

3.319.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2} \cosh^3\left(d+ex+fx^2\right) dx = \int f^{c x^2+a} \cosh\left(f x^2+e x+d\right)^3 dx$$

input
```
int(f^(a + c*x^2)*cosh(d + e*x + f*x^2)^3,x)
```

output
```
int(f^(a + c*x^2)*cosh(d + e*x + f*x^2)^3, x)
```

3.320 $\int f^{a+bx+cx^2}\cosh(d+ex)\,dx$

3.320.1 Optimal result

Integrand size = 19, antiderivative size = 153

$$\int f^{a+bx+cx^2}\cosh(d+ex)\,dx=-\frac{e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{d-\frac{(e+b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

output
```
1/4*exp(-d-1/4*(e-b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-e+b*ln(f)+2*c*x*ln(f)
)/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/4*exp(d-1/4*(e+b*ln(
f))^2/c/ln(f))*f^a*erfi(1/2*(e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*P
i^(1/2)/c^(1/2)/ln(f)^(1/2)
```

3.320.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.88

$$\int f^{a+bx+cx^2}\cosh(d+ex)\,dx$$
$$=\frac{e^{-\frac{e(e+2b\log(f))}{4c\log(f)}}f^{a-\frac{b^2}{4c}}\sqrt{\pi}\left(e^{\frac{be}{c}}\mathrm{erfi}\left(\frac{-e+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(d)-\sinh(d))+\mathrm{erfi}\left(\frac{e+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(d)+\sinh\right.}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Cosh[d + e*x],x]
```

output
```
(f^(a - b^2/(4*c))*Sqrt[Pi]*(E^((b*e)/c)*Erfi[(-e + (b + 2*c*x)*Log[f])/(2
*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[d] - Sinh[d]) + Erfi[(e + (b + 2*c*x)*Log[f]
)/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[d] + Sinh[d])))/(4*Sqrt[c]*E^((e*(e + 2*
b*Log[f]))/(4*c*Log[f]))*Sqrt[Log[f]])
```

3.320.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(d+ex)f^{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{6039}$$

$$\int \left(\frac{1}{2}e^{-d-ex}f^{a+bx+cx^2}+\frac{1}{2}e^{d+ex}f^{a+bx+cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}f^a e^{d-\frac{(b\log(f)+e)^2}{4c\log(f)}}\text{erfi}\left(\frac{b\log(f)+2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a e^{-\frac{(e-b\log(f))^2}{4c\log(f)}-d}\text{erfi}\left(\frac{-b\log(f)-2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + b*x + c*x^2)*Cosh[d + e*x],x]
```

output
```
-1/4*(E^(-d - (e - b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - b*Log[
f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log[f]]) + (E^
(d - (e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e + b*Log[f] + 2*c*
x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]])
```

3.320.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.320.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.06

method	result
risch	$-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{b\ln(f)-e}{2\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^{a}f^{-\frac{b^2}{4c}}\,\mathrm{e}^{\frac{2\ln(f)be-4d\ln(f)c-e^2}{4\ln(f)c}}}{4\sqrt{-c\ln(f)}}-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{e+b\ln(f)}{2\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^{a}f^{-\frac{b^2}{4c}}\,\mathrm{e}^{-\frac{2\ln(f)be}{}}}{4\sqrt{-c\ln(f)}}$

input
```
int(f^(c*x^2+b*x+a)*cosh(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-e)/(-c*ln(f))^(1/2))/(-c*ln(f))^
(1/2)*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(1/4*(2*ln(f)*b*e-4*d*ln(f)*c-e^2)/ln
(f)/c)-1/4*erf(-(-c*ln(f))^(1/2)*x+1/2*(e+b*ln(f))/(-c*ln(f))^(1/2))/(-c*l
n(f))^(1/2)*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(-1/4*(2*ln(f)*b*e-4*d*ln(f)*c+
e^2)/ln(f)/c)
```

3.320.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(121) = 242.

Time = 0.28 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.71

$$\int f^{a+bx+cx^2}\cosh(d+ex)\,dx = -\frac{\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\left(-\frac{(b^2-4ac)\log(f)^2+e^2-2(2cd-be)\log(f)}{4c\log(f)}\right)+\sqrt{\pi}\sinh\left(-\frac{(b^2-4ac)\log(f)^2+e^2-2(2cd-be)\log(f)}{4c\log(f)}\right.\right.}{}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(e*x+d),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 -
2*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*lo
g(f)^2 + e^2 - 2*(2*c*d - b*e)*log(f))/(c*log(f))))*erf(1/2*((2*c*x + b)*l
og(f) + e)*sqrt(-c*log(f))/(c*log(f))) + sqrt(-c*log(f))*(sqrt(pi)*cosh(-1
/4*((b^2 - 4*a*c)*log(f)^2 + e^2 + 2*(2*c*d - b*e)*log(f))/(c*log(f))) + s
qrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 + 2*(2*c*d - b*e)*log(f))/
(c*log(f))))*erf(1/2*((2*c*x + b)*log(f) - e)*sqrt(-c*log(f))/(c*log(f))))
/(c*log(f))
```

3.320.6 Sympy [F]

$$\int f^{a+bx+cx^2}\cosh(d+ex)\,dx = \int f^{a+bx+cx^2}\cosh{(d+ex)}\,dx$$

input
```
integrate(f**(c*x**2+b*x+a)*cosh(e*x+d),x)
```

output
```
Integral(f**(a + b*x + c*x**2)*cosh(d + e*x), x)
```

3.320.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.84

$$\int f^{a+bx+cx^2}\cosh(d+ex)\,dx = \frac{\sqrt{\pi}f^a \operatorname{erf}\left(\sqrt{-c\log{(f)}}x - \frac{b\log(f)+e}{2\sqrt{-c\log(f)}}\right)e^{\left(d-\frac{(b\log(f)+e)^2}{4\,c\log(f)}\right)}}{4\sqrt{-c\log{(f)}}} + \frac{\sqrt{\pi}f^a \operatorname{erf}\left(\sqrt{-c\log{(f)}}x - \frac{b\log(f)-e}{2\sqrt{-c\log(f)}}\right)e^{\left(-d-\frac{(b\log(f)-e)^2}{4\,c\log(f)}\right)}}{4\sqrt{-c\log{(f)}}}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(e*x+d),x, algorithm="maxima")
```

output
```
1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + e)/sqrt(-c*log(f)
))*e^(d - 1/4*(b*log(f) + e)^2/(c*log(f)))/sqrt(-c*log(f)) + 1/4*sqrt(pi)*
f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) - e)/sqrt(-c*log(f)))*e^(-d - 1/
4*(b*log(f) - e)^2/(c*log(f)))/sqrt(-c*log(f))
```

3.320.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.09

$$\int f^{a+bx+cx^2}\cosh(d+ex)\,dx$$
$$= -\frac{\sqrt{\pi}\,\text{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)}\left(2\,x+\frac{b\log(f)-e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2+4\,cd\log(f)-2\,be\log(f)+e^2}{4\,c\log(f)}\right)}}{4\,\sqrt{-c\log\left(f\right)}} - \frac{\sqrt{\pi}\,\text{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)}\left(2\,x+\frac{b\log(f)+e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2-4\,cd\log(f)+2\,be\log(f)+e^2}{4\,c\log(f)}\right)}}{4\,\sqrt{-c\log\left(f\right)}}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(e*x+d),x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) - e)/(c*log(f))))*
e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d*log(f) - 2*b*e*log(f) + e^2
)/(c*log(f)))/sqrt(-c*log(f)) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x
 + (b*log(f) + e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*
c*d*log(f) + 2*b*e*log(f) + e^2)/(c*log(f)))/sqrt(-c*log(f))
```

3.320.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\cosh(d+ex)\,dx = \int f^{c\,x^2+b\,x+a}\cosh\left(d+e\,x\right)\,dx$$

input
```
int(f^(a + b*x + c*x^2)*cosh(d + e*x),x)
```

output
```
int(f^(a + b*x + c*x^2)*cosh(d + e*x), x)
```

3.321 $\int f^{a+bx+cx^2} \cosh^2(d+ex)\,dx$

3.321.1 Optimal result

Integrand size = 21, antiderivative size = 219

$$\int f^{a+bx+cx^2} \cosh^2(d+ex)\,dx = \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}} - \frac{e^{-2d-\frac{(2e-b\log(f))^2}{4c\log(f)}} f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{2e-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}} + \frac{e^{2d-\frac{(2e+b\log(f))^2}{4c\log(f)}} f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{2e+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

output
```
1/8*exp(-2*d-1/4*(2*e-b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-2*e+b*ln(f)+2*c*x
*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/8*exp(2*d-1/4*
(2*e+b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(2*e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln
(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/4*f^(a-1/4*b^2/c)*erfi(1/2*(2*c*
x+b)*ln(f)^(1/2)/c^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

3.321.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.84

$$\int f^{a+bx+cx^2}\cosh^2(d+ex)\,dx$$
$$=\frac{e^{-\frac{e(e+b\log(f))}{c\log(f)}}f^{a-\frac{b^2}{4c}}\sqrt{\pi}\left(2e^{\frac{e(e+b\log(f))}{c\log(f)}}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)+e^{\frac{2be}{c}}\operatorname{erfi}\left(\frac{-2e+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(2d)-\sinh(2d))+\right.}{8\sqrt{c}\sqrt{\log(f)}}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Cosh[d + e*x]^2,x]
```

output
```
(f^(a - b^2/(4*c))*Sqrt[Pi]*(2*E^((e*(e + b*Log[f]))/(c*Log[f]))*Erfi[((b
+ 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])] + E^((2*b*e)/c)*Erfi[(-2*e + (b + 2*c*
x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(2*e +
 (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])))/(
8*Sqrt[c]*E^((e*(e + b*Log[f]))/(c*Log[f]))*Sqrt[Log[f]])
```

3.321.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2(d+ex)f^{a+bx+cx^2}\,dx$$
$$\Big\downarrow \text{6039}$$
$$\int\left(\frac{1}{4}e^{-2d-2ex}f^{a+bx+cx^2}+\frac{1}{4}e^{2d+2ex}f^{a+bx+cx^2}+\frac{1}{2}f^{a+bx+cx^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\operatorname{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a e^{-\frac{(2e-b\log(f))^2}{4c\log(f)}-2d}\operatorname{erfi}\left(\frac{-b\log(f)-2cx\log(f)+2e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a e^{2d-\frac{(b\log(f)+2e)^2}{4c\log(f)}}\operatorname{erfi}\left(\frac{b\log(f)+2cx\log(f)+2e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + b*x + c*x^2)*Cosh[d + e*x]^2,x]
```

output
```
(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/
(4*Sqrt[c]*Sqrt[Log[f]]) - (E^(-2*d - (2*e - b*Log[f])^2/(4*c*Log[f]))*f^a
*Sqrt[Pi]*Erfi[(2*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/
(8*Sqrt[c]*Sqrt[Log[f]]) + (E^(2*d - (2*e + b*Log[f])^2/(4*c*Log[f]))*f^a*
Sqrt[Pi]*Erfi[(2*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(
8*Sqrt[c]*Sqrt[Log[f]])
```

3.321.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.321.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{b\ln(f)-2e}{2\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^{a}f^{-\frac{b^{2}}{4c}}\,\mathrm{e}^{\frac{\ln(f)be-2d\ln(f)c-e^{2}}{\ln(f)c}}}{8\sqrt{-c\ln(f)}}-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{2e+b\ln(f)}{2\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^{a}f^{-\frac{b^{2}}{4c}}\,\mathrm{e}^{-\frac{\ln(f)be-2}{\ln}}}{8\sqrt{-c\ln(f)}}$

input
```
int(f^(c*x^2+b*x+a)*cosh(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-2*e)/(-c*ln(f))^(1/2))/(-c*ln(f)
)^(1/2)*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp((ln(f)*b*e-2*d*ln(f)*c-e^2)/ln(f)/
c)-1/8*erf(-(-c*ln(f))^(1/2)*x+1/2*(2*e+b*ln(f))/(-c*ln(f))^(1/2))/(-c*ln(
f))^(1/2)*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(-(ln(f)*b*e-2*d*ln(f)*c+e^2)/ln(
f)/c)-1/4*f^a*Pi^(1/2)*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/
2)*x+1/2*ln(f)*b/(-c*ln(f))^(1/2))
```

3.321.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(167) = 334.

Time = 0.27 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.56

$$\int f^{a+bx+cx^2}\cosh^2(d+ex)\,dx =$$
$$-\frac{2\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\left(-\frac{(b^2-4ac)\log(f)}{4c}\right)+\sqrt{\pi}\sinh\left(-\frac{(b^2-4ac)\log(f)}{4c}\right)\right)\operatorname{erf}\left(\frac{(2cx+b)\sqrt{-c\log(f)}}{2c}\right)+\sqrt{-c\,l}}{}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(e*x+d)^2,x, algorithm="fricas")
```

output
```
-1/8*(2*sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*(b^2 - 4*a*c)*log(f)/c) + sqrt
(pi)*sinh(-1/4*(b^2 - 4*a*c)*log(f)/c))*erf(1/2*(2*c*x + b)*sqrt(-c*log(f)
)/c) + sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 4*e^2
 - 4*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)
*log(f)^2 + 4*e^2 - 4*(2*c*d - b*e)*log(f))/(c*log(f))))*erf(1/2*((2*c*x +
 b)*log(f) + 2*e)*sqrt(-c*log(f))/(c*log(f))) + sqrt(-c*log(f))*(sqrt(pi)*
cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 4*e^2 + 4*(2*c*d - b*e)*log(f))/(c*log
(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 4*e^2 + 4*(2*c*d - b*
e)*log(f))/(c*log(f))))*erf(1/2*((2*c*x + b)*log(f) - 2*e)*sqrt(-c*log(f))
/(c*log(f))))/(c*log(f))
```

3.321.6 Sympy [F]

$$\int f^{a+bx+cx^2}\cosh^2(d+ex)\,dx = \int f^{a+bx+cx^2}\cosh^2(d+ex)\,dx$$

input
```
integrate(f**(c*x**2+b*x+a)*cosh(e*x+d)**2,x)
```

output
```
Integral(f**(a + b*x + c*x**2)*cosh(d + e*x)**2, x)
```

3.321.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.84

$$\int f^{a+bx+cx^2}\cosh^2(d+ex)\,dx = \frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x-\frac{b\log(f)+2e}{2\sqrt{-c\log(f)}}\right)e^{\left(2d-\frac{(b\log(f)+2e)^2}{4c\log(f)}\right)}}{8\sqrt{-c\log(f)}} + \frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x-\frac{b\log(f)-2e}{2\sqrt{-c\log(f)}}\right)e^{\left(-2d-\frac{(b\log(f)-2e)^2}{4c\log(f)}\right)}}{8\sqrt{-c\log(f)}} + \frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x-\frac{b\log(f)}{2\sqrt{-c\log(f)}}\right)}{4\sqrt{-c\log(f)}f^{\frac{b^2}{4c}}}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(e*x+d)^2,x, algorithm="maxima")
```

output
```
1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + 2*e)/sqrt(-c*log(
f)))*e^(2*d - 1/4*(b*log(f) + 2*e)^2/(c*log(f)))/sqrt(-c*log(f)) + 1/8*sqr
t(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) - 2*e)/sqrt(-c*log(f)))*e^
(-2*d - 1/4*(b*log(f) - 2*e)^2/(c*log(f)))/sqrt(-c*log(f)) + 1/4*sqrt(pi)*
f^a*erf(sqrt(-c*log(f))*x - 1/2*b*log(f)/sqrt(-c*log(f)))/(sqrt(-c*log(f))
*f^(1/4*b^2/c))
```

3.321.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.02

$$\int f^{a+bx+cx^2}\cosh^2(d+ex)\,dx = -\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b}{c}\right)\right)e^{\left(-\frac{b^2\log(f)-4ac\log(f)}{4c}\right)}}{4\sqrt{-c\log(f)}} - \frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b\log(f)-2e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+8cd\log(f)-4be\log(f)+4e^2}{4c\log(f)}\right)}}{8\sqrt{-c\log(f)}} - \frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b\log(f)+2e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-8cd\log(f)+4be\log(f)+4e^2}{4c\log(f)}\right)}}{8\sqrt{-c\log(f)}}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(e*x+d)^2,x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + b/c))*e^(-1/4*(b^2*log(f) -
4*a*c*log(f))/c)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(
2*x + (b*log(f) - 2*e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2
 + 8*c*d*log(f) - 4*b*e*log(f) + 4*e^2)/(c*log(f)))/sqrt(-c*log(f)) - 1/8*
sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) + 2*e)/(c*log(f))))*e^(
-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 8*c*d*log(f) + 4*b*e*log(f) + 4*e^2)
/(c*log(f)))/sqrt(-c*log(f))
```

3.321.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\cosh^2(d+ex)\,dx=\int f^{c\,x^2+b\,x+a}\cosh(d+e\,x)^2\,dx$$

input
```
int(f^(a + b*x + c*x^2)*cosh(d + e*x)^2,x)
```

output
```
int(f^(a + b*x + c*x^2)*cosh(d + e*x)^2, x)
```

3.322 $\int f^{a+bx+cx^2} \cosh^3(d+ex)\,dx$

3.322.1 Optimal result

Integrand size = 21, antiderivative size = 315

$$\int f^{a+bx+cx^2}\cosh^3(d+ex)\,dx = -\frac{3e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} - \frac{e^{-3d-\frac{(3e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{3e-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{3e^{d-\frac{(e+b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{e^{3d-\frac{(3e+b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{3e+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

output
```
3/16*exp(-d-1/4*(e-b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-e+b*ln(f)+2*c*x*ln(f
))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/16*exp(-3*d-1/4*(3*
e-b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-3*e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f
)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+3/16*exp(d-1/4*(e+b*ln(f))^2/c/ln(f)
)*f^a*erfi(1/2*(e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/
2)/ln(f)^(1/2)+1/16*exp(3*d-1/4*(3*e+b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(3*e
+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

3.322.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.83

$$\int f^{a+bx+cx^2}\cosh^3(d+ex)\,dx$$
$$= \frac{e^{-\frac{3e(3e+2b\log(f))}{4c\log(f)}}f^{a-\frac{b^2}{4c}}\sqrt{\pi}\left((\cosh(d)+\sinh(d))\left(3e^{\frac{e(2e+b\log(f))}{c\log(f)}}\operatorname{erfi}\left(\frac{e+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)+3e^{\frac{2e(e+b\log(f))}{c\log(f)}}\operatorname{erfi}\left(\frac{-e+(b+2c}{2\sqrt{c}\sqrt{l}}\right.\right.\right.}{}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Cosh[d + e*x]^3,x]
```

output
```
(f^(a - b^2/(4*c))*Sqrt[Pi]*((Cosh[d] + Sinh[d])*(3*E^((e*(2*e + b*Log[f])
)/(c*Log[f]))*Erfi[(e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])] + 3*
E^((2*e*(e + b*Log[f]))/(c*Log[f]))*Erfi[(-e + (b + 2*c*x)*Log[f])/(2*Sqrt
[c]*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(3*e + (b + 2*c*x)*Log[f
])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])) + E^((3*b*e)/c)*Erfi
[(-3*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[3*d] - Sinh[3
*d])))/(16*Sqrt[c]*E^((3*e*(3*e + 2*b*Log[f]))/(4*c*Log[f]))*Sqrt[Log[f]])
```

3.322.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3(d+ex)f^{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{ 6039}$$

$$\int \left(\frac{1}{8}e^{-3d-3ex}f^{a+bx+cx^2}+\frac{3}{8}e^{-d-ex}f^{a+bx+cx^2}+\frac{3}{8}e^{d+ex}f^{a+bx+cx^2}+\frac{1}{8}e^{3d+3ex}f^{a+bx+cx^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{3\sqrt{\pi}f^ae^{-\frac{(e-b\log(f))^2}{4c\log(f)}-d}\operatorname{erfi}\left(\frac{-b\log(f)-2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^ae^{-\frac{(3e-b\log(f))^2}{4c\log(f)}-3d}\operatorname{erfi}\left(\frac{-b\log(f)-2cx\log(f)+3e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{3\sqrt{\pi}f^ae^{d-\frac{(b\log(f)+e)^2}{4c\log(f)}}\operatorname{erfi}\left(\frac{b\log(f)+2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^ae^{3d-\frac{(b\log(f)+3e)^2}{4c\log(f)}}\operatorname{erfi}\left(\frac{b\log(f)+2cx\log(f)+3e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + b*x + c*x^2)*Cosh[d + e*x]^3,x]
```

output
```
(-3*E^(-d - (e - b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - b*Log[f]
 - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) - (E
^(-3*d - (3*e - b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(3*e - b*Log[f
] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (
3*E^(d - (e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e + b*Log[f] +
2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (E^(3
*d - (3*e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(3*e + b*Log[f] +
2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]])
```

3.322.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.322.4 Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{b\ln(f)-3e}{2\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^a f^{-\frac{b^2}{4c}}\,\mathrm{e}^{\frac{\frac{3\ln(f)be}{2}-3d\ln(f)c-\frac{9e^2}{4}}{c\ln(f)}}}{16\sqrt{-c\ln(f)}}-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{3e+b\ln(f)}{2\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^a f^{-\frac{b^2}{4c}}\,\mathrm{e}^{-\frac{3(2\ln\cdots}{\cdots}}}{16\sqrt{-c\ln(f)}}$

input
```
int(f^(c*x^2+b*x+a)*cosh(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-3*e)/(-c*ln(f))^(1/2))/(-c*ln(f
))^(1/2)*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(3/4*(2*ln(f)*b*e-4*d*ln(f)*c-3*e^
2)/ln(f)/c)-1/16*erf(-(-c*ln(f))^(1/2)*x+1/2*(3*e+b*ln(f))/(-c*ln(f))^(1/2
))/(-c*ln(f))^(1/2)*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(-3/4*(2*ln(f)*b*e-4*d*
ln(f)*c+3*e^2)/ln(f)/c)-3/16*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-e)/(-c*l
n(f))^(1/2))/(-c*ln(f))^(1/2)*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(1/4*(2*ln(f)
*b*e-4*d*ln(f)*c-e^2)/ln(f)/c)-3/16*erf(-(-c*ln(f))^(1/2)*x+1/2*(e+b*ln(f)
)/(-c*ln(f))^(1/2))/(-c*ln(f))^(1/2)*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(-1/4*
(2*ln(f)*b*e-4*d*ln(f)*c+e^2)/ln(f)/c)
```

3.322.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(247) = 494$.

Time = 0.26 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.67

$$\int f^{a+bx+cx^2}\cosh^3(d+ex)\,dx =$$
$$-\frac{\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\left(-\frac{(b^2-4ac)\log(f)^2+9e^2-6(2cd-be)\log(f)}{4c\log(f)}\right)+\sqrt{\pi}\sinh\left(-\frac{(b^2-4ac)\log(f)^2+9e^2-6(2cd-be)\log}{4c\log(f)}\right.\right.}{}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(e*x+d)^3,x, algorithm="fricas")
```

output
```
-1/16*(sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*e^2
 - 6*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)
*log(f)^2 + 9*e^2 - 6*(2*c*d - b*e)*log(f))/(c*log(f))))*erf(1/2*((2*c*x +
 b)*log(f) + 3*e)*sqrt(-c*log(f))/(c*log(f))) + 3*sqrt(-c*log(f))*(sqrt(pi
)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 2*(2*c*d - b*e)*log(f))/(c*log
(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 2*(2*c*d - b*e)
*log(f))/(c*log(f))))*erf(1/2*((2*c*x + b)*log(f) + e)*sqrt(-c*log(f))/(c*
log(f))) + 3*sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 +
 e^2 + 2*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*
a*c)*log(f)^2 + e^2 + 2*(2*c*d - b*e)*log(f))/(c*log(f))))*erf(1/2*((2*c*x
 + b)*log(f) - e)*sqrt(-c*log(f))/(c*log(f))) + sqrt(-c*log(f))*(sqrt(pi)*
cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*e^2 + 6*(2*c*d - b*e)*log(f))/(c*log
(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*e^2 + 6*(2*c*d - b*
e)*log(f))/(c*log(f))))*erf(1/2*((2*c*x + b)*log(f) - 3*e)*sqrt(-c*log(f))
/(c*log(f))))/(c*log(f))
```

3.322.6 Sympy [F]

$$\int f^{a+bx+cx^2}\cosh^3(d+ex)\,dx=\int f^{a+bx+cx^2}\cosh^3(d+ex)\,dx$$

input
```
integrate(f**(c*x**2+b*x+a)*cosh(e*x+d)**3,x)
```

output
```
Integral(f**(a + b*x + c*x**2)*cosh(d + e*x)**3, x)
```

3.322.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.83

$$\int f^{a+bx+cx^2}\cosh^3(d+ex)\,dx=\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x-\frac{b\log(f)+3e}{2\sqrt{-c\log(f)}}\right)e^{\left(3d-\frac{(b\log(f)+3e)^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}+\frac{3\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x-\frac{b\log(f)+e}{2\sqrt{-c\log(f)}}\right)e^{\left(d-\frac{(b\log(f)+e)^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}+\frac{3\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x-\frac{b\log(f)-e}{2\sqrt{-c\log(f)}}\right)e^{\left(-d-\frac{(b\log(f)-e)^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x-\frac{b\log(f)-3e}{2\sqrt{-c\log(f)}}\right)e^{\left(-3d-\frac{(b\log(f)-3e)^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(e*x+d)^3,x, algorithm="maxima")
```

output
```
1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + 3*e)/sqrt(-c*log
(f)))*e^(3*d - 1/4*(b*log(f) + 3*e)^2/(c*log(f)))/sqrt(-c*log(f)) + 3/16*s
qrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + e)/sqrt(-c*log(f)))*e^
(d - 1/4*(b*log(f) + e)^2/(c*log(f)))/sqrt(-c*log(f)) + 3/16*sqrt(pi)*f^a*
erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) - e)/sqrt(-c*log(f)))*e^(-d - 1/4*(b
*log(f) - e)^2/(c*log(f)))/sqrt(-c*log(f)) + 1/16*sqrt(pi)*f^a*erf(sqrt(-c
*log(f))*x - 1/2*(b*log(f) - 3*e)/sqrt(-c*log(f)))*e^(-3*d - 1/4*(b*log(f)
 - 3*e)^2/(c*log(f)))/sqrt(-c*log(f))
```

3.322.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.08

$$\int f^{a+bx+cx^2}\cosh^3(d+ex)\,dx$$
$$= -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b\log(f)-3e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+12cd\log(f)-6be\log(f)+9e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}$$
$$-\frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b\log(f)-e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+4cd\log(f)-2be\log(f)+e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}$$
$$-\frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b\log(f)+e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-4cd\log(f)+2be\log(f)+e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}$$
$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b\log(f)+3e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-12cd\log(f)+6be\log(f)+9e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(e*x+d)^3,x, algorithm="giac")
```

output
```
-1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) - 3*e)/(c*log(f))
))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 12*c*d*log(f) - 6*b*e*log(f) +
 9*e^2)/(c*log(f)))/sqrt(-c*log(f)) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f
))*(2*x + (b*log(f) - e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)
^2 + 4*c*d*log(f) - 2*b*e*log(f) + e^2)/(c*log(f)))/sqrt(-c*log(f)) - 3/16
*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) + e)/(c*log(f))))*e^(-
1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) + 2*b*e*log(f) + e^2)/(c
*log(f)))/sqrt(-c*log(f)) - 1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x +
(b*log(f) + 3*e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 12*
c*d*log(f) + 6*b*e*log(f) + 9*e^2)/(c*log(f)))/sqrt(-c*log(f))
```

3.322.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\cosh^3(d+ex)\,dx = \int f^{c\,x^2+b\,x+a}\cosh(d+e\,x)^3\,dx$$

input
```
int(f^(a + b*x + c*x^2)*cosh(d + e*x)^3,x)
```

output
```
int(f^(a + b*x + c*x^2)*cosh(d + e*x)^3, x)
```

3.323 $\int f^{a+bx+cx^2} \cosh(d+fx^2)\,dx$

3.323.1 Optimal result

Integrand size = 21, antiderivative size = 154

$$\int f^{a+bx+cx^2}\cosh(d+fx^2)\,dx = -\frac{e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}} + \frac{e^{d-\frac{b^2\log^2(f)}{4(f+c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\log(f)+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{4\sqrt{f+c\log(f)}}$$

output
```
-1/4*exp(-d+b^2*ln(f)^2/(4*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(f)-2*x*(f-c*ln(
f)))/(f-c*ln(f))^(1/2))*Pi^(1/2)/(f-c*ln(f))^(1/2)+1/4*exp(d-1/4*b^2*ln(f)
^2/(f+c*ln(f)))*f^a*erfi(1/2*(b*ln(f)+2*x*(f+c*ln(f)))/(f+c*ln(f))^(1/2))*
Pi^(1/2)/(f+c*ln(f))^(1/2)
```

3.323.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.20

$$\int f^{a+bx+cx^2}\cosh(d+fx^2)\,dx = \frac{e^{-\frac{b^2\log^2(f)}{4(f+c\log(f))}}f^a\sqrt{\pi}\left(e^{\frac{b^2f\log^2(f)}{2f^2-2c^2\log^2(f)}}\mathrm{erf}\left(\frac{2fx-(b+2cx)\log(f)}{2\sqrt{f-c\log(f)}}\right)\sqrt{f-c\log(f)}(f+c\log(f))(\cosh(d)-\sinh(d))+e}{4\left(f^2-c^2\log^2(f)\right)}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Cosh[d + f*x^2],x]
```

output
```
(f^a*Sqrt[Pi]*(E^((b^2*f*Log[f]^2)/(2*f^2 - 2*c^2*Log[f]^2))*Erf[(2*f*x -
(b + 2*c*x)*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f - c*Log[f]]*(f + c*Log[
f])*(Cosh[d] - Sinh[d]) + Erfi[(2*f*x + (b + 2*c*x)*Log[f])/(2*Sqrt[f + c*
Log[f]])]*(f - c*Log[f])*Sqrt[f + c*Log[f]]*(Cosh[d] + Sinh[d])))/(4*E^((b
^2*Log[f]^2)/(4*(f + c*Log[f])))*(f^2 - c^2*Log[f]^2))
```

3.323.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh\left(d+fx^2\right) f^{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{6039}$$

$$\int \left(\frac{1}{2}e^{-d-fx^2}f^{a+bx+cx^2}+\frac{1}{2}e^{d+fx^2}f^{a+bx+cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}f^a e^{d-\frac{b^2\log^2(f)}{4(c\log(f)+f)}}\operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+f)}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}}-\frac{\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{4f-4c\log(f)}-d}\operatorname{erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}$$

input
```
Int[f^(a + b*x + c*x^2)*Cosh[d + f*x^2],x]
```

output
```
-1/4*(E^(-d + (b^2*Log[f]^2)/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f
] - 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/Sqrt[f - c*Log[f]] + (E^(
d - (b^2*Log[f]^2)/(4*(f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*(
f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(4*Sqrt[f + c*Log[f]])
```

3.323.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.323.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.04

method	result
risch	$-\frac{\operatorname{erf}\left(-x\sqrt{f-c\ln(f)}+\frac{\ln(f)b}{2\sqrt{f-c\ln(f)}}\right)\sqrt{\pi}\, f^{a}\mathrm{e}^{-\frac{b^{2}\ln(f)^{2}+4d\ln(f)c-4df}{4(c\ln(f)-f)}}}{4\sqrt{f-c\ln(f)}}-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)-f}\,x+\frac{\ln(f)b}{2\sqrt{-c\ln(f)-f}}\right)\sqrt{\pi}\, f^{a}\mathrm{e}^{-\frac{b^{2}\ln(f)^{2}}{4(f}}}{4\sqrt{-c\ln(f)-f}}$

input
```
int(f^(c*x^2+b*x+a)*cosh(f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4*erf(-x*(f-c*ln(f))^(1/2)+1/2*ln(f)*b/(f-c*ln(f))^(1/2))/(f-c*ln(f))^(
1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+4*d*ln(f)*c-4*d*f)/(c*ln(f)-f))-1/
4*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)-f)^(1/2))/(-c*ln(f)-f)^(
1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-4*d*ln(f)*c-4*d*f)/(f+c*ln(f)))
```

3.323.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. 2(131) = 262.

Time = 0.27 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.10

$$\int f^{a+bx+cx^2}\cosh\left(d+fx^2\right)\,dx =$$
$$-\frac{\left(\sqrt{\pi}(c\log(f)+f)\cosh\left(-\frac{(b^2-4\,ac)\log(f)^2-4\,df+4\,(cd+af)\log(f)}{4\,(c\log(f)-f)}\right)+\sqrt{\pi}(c\log(f)+f)\sinh\left(-\frac{(b^2-4\,ac)\log(f)^2}{4\,(c}\right.\right.}{}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d),x, algorithm="fricas")
```

output
```
-1/4*((sqrt(pi)*(c*log(f) + f)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*f +
 4*(c*d + a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c*log(f) + f)*sinh(-1/4
*((b^2 - 4*a*c)*log(f)^2 - 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) - f)))*
sqrt(-c*log(f) + f)*erf(-1/2*(2*f*x - (2*c*x + b)*log(f))*sqrt(-c*log(f) +
 f)/(c*log(f) - f)) + (sqrt(pi)*(c*log(f) - f)*cosh(-1/4*((b^2 - 4*a*c)*lo
g(f)^2 - 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f) + f)) + sqrt(pi)*(c*log(f
) - f)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*f - 4*(c*d + a*f)*log(f))/(
c*log(f) + f)))*sqrt(-c*log(f) - f)*erf(1/2*(2*f*x + (2*c*x + b)*log(f))*s
qrt(-c*log(f) - f)/(c*log(f) + f)))/(c^2*log(f)^2 - f^2)
```

3.323.6 Sympy [F]

$$\int f^{a+bx+cx^2} \cosh \left(d+fx^2\right) \, dx = \int f^{a+bx+cx^2} \cosh \left(d+fx^2\right) dx$$

input
```
integrate(f**(c*x**2+b*x+a)*cosh(f*x**2+d),x)
```

output
```
Integral(f**(a + b*x + c*x**2)*cosh(d + f*x**2), x)
```

3.323.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.90

$$\int f^{a+bx+cx^2} \cosh \left(d+fx^2\right) \, dx$$
$$= \frac{\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log(f)-f}\,x-\frac{b\log(f)}{2\sqrt{-c\log(f)-f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4\,(c\log(f)+f)}+d\right)}}{4\sqrt{-c\log(f)-f}} + \frac{\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log(f)+f}\,x-\frac{b\log(f)}{2\sqrt{-c\log(f)+f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4\,(c\log(f)-f)}-d\right)}}{4\sqrt{-c\log(f)+f}}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d),x, algorithm="maxima")
```

output
```
1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*b*log(f)/sqrt(-c*log(f) -
 f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + f) + d)/sqrt(-c*log(f) - f) + 1/4*sq
rt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*b*log(f)/sqrt(-c*log(f) + f))*e
^(-1/4*b^2*log(f)^2/(c*log(f) - f) - d)/sqrt(-c*log(f) + f)
```

3.323.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.18

$$\int f^{a+bx+cx^2}\cosh\left(d+fx^2\right)\,dx$$
$$= -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-f}\left(2x+\frac{b\log(f)}{c\log(f)+f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-4cd\log(f)-4af\log(f)-4df}{4(c\log(f)+f)}\right)}}{4\sqrt{-c\log\left(f\right)-f}}$$
$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)+f}\left(2x+\frac{b\log(f)}{c\log(f)-f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+4cd\log(f)+4af\log(f)-4df}{4(c\log(f)-f)}\right)}}{4\sqrt{-c\log\left(f\right)+f}}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d),x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + b*log(f)/(c*log(f) + f))
)*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) - 4*a*f*log(f) - 4
*d*f)/(c*log(f) + f))/sqrt(-c*log(f) - f) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*
log(f) + f)*(2*x + b*log(f)/(c*log(f) - f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c
*log(f)^2 + 4*c*d*log(f) + 4*a*f*log(f) - 4*d*f)/(c*log(f) - f))/sqrt(-c*l
og(f) + f)
```

3.323.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\cosh\left(d+fx^2\right)\,dx=\int f^{c\,x^2+b\,x+a}\cosh\left(f\,x^2+d\right)\,dx$$

input
```
int(f^(a + b*x + c*x^2)*cosh(d + f*x^2),x)
```

output
```
int(f^(a + b*x + c*x^2)*cosh(d + f*x^2), x)
```

3.324 $\int f^{a+bx+cx^2}\cosh^2(d+fx^2)\,dx$

3.324.1 Optimal result

Integrand size = 23, antiderivative size = 225

$$\int f^{a+bx+cx^2}\cosh^2(d+fx^2)\,dx=\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2d+\frac{b^2\log^2(f)}{8f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-2x(2f-c\log(f))}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{e^{2d-\frac{b^2\log^2(f)}{8f+4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\log(f)+2x(2f+c\log(f))}{2\sqrt{2f+c\log(f)}}\right)}{8\sqrt{2f+c\log(f)}}$$

output
```
1/4*f^(a-1/4*b^2/c)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))*Pi^(1/2)/c^(1/
2)/ln(f)^(1/2)-1/8*exp(-2*d+b^2*ln(f)^2/(8*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln
(f)-2*x*(2*f-c*ln(f)))/(2*f-c*ln(f))^(1/2))*Pi^(1/2)/(2*f-c*ln(f))^(1/2)+1
/8*exp(2*d-b^2*ln(f)^2/(8*f+4*c*ln(f)))*f^a*erfi(1/2*(b*ln(f)+2*x*(2*f+c*l
n(f)))/(2*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*f+c*ln(f))^(1/2)
```

3.324.2 Mathematica [A] (verified)

Time = 1.58 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.14

$$\int f^{a+bx+cx^2}\cosh^2\left(d+fx^2\right)\,dx = \frac{1}{8}f^a\sqrt{\pi}\left(\frac{2f^{-\frac{b^2}{4c}}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{\sqrt{c}\sqrt{\log(f)}} - \frac{e^{-\frac{b^2\log^2(f)}{8f+4c\log(f)}}\left(e^{\frac{b^2f\log^2(f)}{4f^2-c^2\log^2(f)}}\operatorname{erf}\left(\frac{4fx-(b+2cx)\log(f)}{2\sqrt{2f-c\log(f)}}\right)\sqrt{2f-c\log(f)}(2f+c\log(f))(\cosh(2d)-\sinh(2d))+e\right.}{-4f^2+c^2\log^2(f)}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Cosh[d + f*x^2]^2,x]
```

output
```
(f^a*Sqrt[Pi]*((2*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(Sqrt[c]*f
^(b^2/(4*c))*Sqrt[Log[f]]) - (E^((b^2*f*Log[f]^2)/(4*f^2 - c^2*Log[f]^2))*
Erf[(4*f*x - (b + 2*c*x)*Log[f])/(2*Sqrt[2*f - c*Log[f]])]*Sqrt[2*f - c*Lo
g[f]]*(2*f + c*Log[f])*(Cosh[2*d] - Sinh[2*d]) + Erfi[(4*f*x + (b + 2*c*x)
*Log[f])/(2*Sqrt[2*f + c*Log[f]])]*(2*f - c*Log[f])*Sqrt[2*f + c*Log[f]]*(
Cosh[2*d] + Sinh[2*d]))/(E^((b^2*Log[f]^2)/(8*f + 4*c*Log[f]))*(-4*f^2 + c
^2*Log[f]^2))))/8
```

3.324.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2\left(d+fx^2\right)f^{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{6039}$$

$$\int \left(\frac{1}{4}e^{-2d-2fx^2}f^{a+bx+cx^2}+\frac{1}{4}e^{2d+2fx^2}f^{a+bx+cx^2}+\frac{1}{2}f^{a+bx+cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{8f-4c\log(f)}-2d}\operatorname{erf}\left(\frac{b\log(f)-2x(2f-c\log(f))}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi}f^a e^{2d-\frac{b^2\log^2(f)}{4c\log(f)+8f}}\operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+2f)}{2\sqrt{c\log(f)+2f}}\right)}{8\sqrt{c\log(f)+2f}}+\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\operatorname{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + b*x + c*x^2)*Cosh[d + f*x^2]^2,x]
```

output
```
(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/
(4*Sqrt[c]*Sqrt[Log[f]]) - (E^(-2*d + (b^2*Log[f]^2)/(8*f - 4*c*Log[f]))*f
^a*Sqrt[Pi]*Erf[(b*Log[f] - 2*x*(2*f - c*Log[f]))/(2*Sqrt[2*f - c*Log[f]])
])/(8*Sqrt[2*f - c*Log[f]]) + (E^(2*d - (b^2*Log[f]^2)/(8*f + 4*c*Log[f]))
*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*(2*f + c*Log[f]))/(2*Sqrt[2*f + c*Log[f
]])])/(8*Sqrt[2*f + c*Log[f]])
```

3.324.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.324.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{\operatorname{erf}\left(-x\sqrt{2f-c\ln(f)}+\frac{\ln(f)b}{2\sqrt{2f-c\ln(f)}}\right)\sqrt{\pi}\,f^a\mathrm{e}^{-\frac{b^2\ln(f)^2+8d\ln(f)c-16df}{4(c\ln(f)-2f)}}}{8\sqrt{2f-c\ln(f)}}-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)-2f}\,x+\frac{\ln(f)b}{2\sqrt{-c\ln(f)-2f}}\right)\sqrt{\pi}\,f^a\mathrm{e}^{-\frac{b^2\ln}{}}}{8\sqrt{-c\ln(f)-2f}}$

input
```
int(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*erf(-x*(2*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(2*f-c*ln(f))^(1/2))/(2*f-c*ln
(f))^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+8*d*ln(f)*c-16*d*f)/(c*ln(f)
-2*f))-1/8*erf(-(-c*ln(f)-2*f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)-2*f)^(1/2))/(
-c*ln(f)-2*f)^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-8*d*ln(f)*c-16*d*f)
/(2*f+c*ln(f)))-1/4*f^a*Pi^(1/2)*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*
ln(f))^(1/2)*x+1/2*ln(f)*b/(-c*ln(f))^(1/2))
```

3.324.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(185) = 370$.

Time = 0.27 (sec) , antiderivative size = 466, normalized size of antiderivative = 2.07

$$\int f^{a+bx+cx^2}\cosh^2\left(d+fx^2\right)\,dx = -\frac{\left(\sqrt{\pi}\left(c^2\log\left(f\right)^2+2\,cf\log\left(f\right)\right)\cosh\left(-\frac{\left(b^2-4\,ac\right)\log\left(f\right)^2-16\,df+8\,\left(cd+af\right)\log\left(f\right)}{4\,\left(c\log\left(f\right)-2\,f\right)}\right)+\sqrt{\pi}\left(c^2\log\left(f\right)^2+2\,cf\log\left(\right.\right.$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^2,x, algorithm="fricas")
```

output
```
-1/8*((sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh(-1/4*((b^2 - 4*a*c)*log
(f)^2 - 16*d*f + 8*(c*d + a*f)*log(f))/(c*log(f) - 2*f)) + sqrt(pi)*(c^2*l
og(f)^2 + 2*c*f*log(f))*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 16*d*f + 8*(c*
d + a*f)*log(f))/(c*log(f) - 2*f)))*sqrt(-c*log(f) + 2*f)*erf(-1/2*(4*f*x
- (2*c*x + b)*log(f))*sqrt(-c*log(f) + 2*f)/(c*log(f) - 2*f)) + (sqrt(pi)*
(c^2*log(f)^2 - 2*c*f*log(f))*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 16*d*f -
 8*(c*d + a*f)*log(f))/(c*log(f) + 2*f)) + sqrt(pi)*(c^2*log(f)^2 - 2*c*f*
log(f))*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 16*d*f - 8*(c*d + a*f)*log(f))
/(c*log(f) + 2*f)))*sqrt(-c*log(f) - 2*f)*erf(1/2*(4*f*x + (2*c*x + b)*log
(f))*sqrt(-c*log(f) - 2*f)/(c*log(f) + 2*f)) + 2*(sqrt(pi)*(c^2*log(f)^2 -
 4*f^2)*cosh(-1/4*(b^2 - 4*a*c)*log(f)/c) + sqrt(pi)*(c^2*log(f)^2 - 4*f^2
)*sinh(-1/4*(b^2 - 4*a*c)*log(f)/c))*sqrt(-c*log(f))*erf(1/2*(2*c*x + b)*s
qrt(-c*log(f))/c))/(c^3*log(f)^3 - 4*c*f^2*log(f))
```

3.324.6 Sympy [F]

$$\int f^{a+bx+cx^2}\cosh^2\left(d+fx^2\right)\,dx=\int f^{a+bx+cx^2}\cosh^2\left(d+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+b*x+a)*cosh(f*x**2+d)**2,x)
```

output
```
Integral(f**(a + b*x + c*x**2)*cosh(d + f*x**2)**2, x)
```

3.324.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.88

$$\begin{aligned}\int f^{a+bx+cx^2}\cosh^2\left(d+fx^2\right)\,dx\\ =\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-2\,f}x-\frac{b\log(f)}{2\sqrt{-c\log(f)-2\,f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4\,(c\log(f)+2\,f)}+2\,d\right)}}{8\sqrt{-c\log\left(f\right)-2\,f}}\\ +\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+2\,f}x-\frac{b\log(f)}{2\sqrt{-c\log(f)+2\,f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4\,(c\log(f)-2\,f)}-2\,d\right)}}{8\sqrt{-c\log\left(f\right)+2\,f}}\\ +\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)}x-\frac{b\log(f)}{2\sqrt{-c\log(f)}}\right)}{4\sqrt{-c\log\left(f\right)}f^{\frac{b^2}{4\,c}}}\end{aligned}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x - 1/2*b*log(f)/sqrt(-c*log(f)
 - 2*f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + 2*f) + 2*d)/sqrt(-c*log(f) - 2*f
) + 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 2*f)*x - 1/2*b*log(f)/sqrt(-c*lo
g(f) + 2*f))*e^(-1/4*b^2*log(f)^2/(c*log(f) - 2*f) - 2*d)/sqrt(-c*log(f) +
 2*f) + 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*b*log(f)/sqrt(-c*log(
f)))/(sqrt(-c*log(f))*f^(1/4*b^2/c))
```

3.324.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.06

$$\int f^{a+bx+cx^2}\cosh^2\left(d+fx^2\right)\,dx = -\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)-2\,f}\left(2\,x+\frac{b\log(f)}{c\log(f)+2\,f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2-8\,cd\log(f)-8\,af\log(f)-16\,df}{4\,(c\log(f)+2\,f)}\right)}}{8\,\sqrt{-c\log\left(f\right)-2\,f}} - \frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)+2\,f}\left(2\,x+\frac{b\log(f)}{c\log(f)-2\,f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2+8\,cd\log(f)+8\,af\log(f)-16\,df}{4\,(c\log(f)-2\,f)}\right)}}{8\,\sqrt{-c\log\left(f\right)+2\,f}} - \frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)}\left(2\,x+\frac{b}{c}\right)\right)e^{\left(-\frac{b^2\log(f)-4\,ac\log(f)}{4\,c}\right)}}{4\,\sqrt{-c\log\left(f\right)}}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^2,x, algorithm="giac")
```

output
```
-1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 2*f)*(2*x + b*log(f)/(c*log(f) + 2
*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 8*c*d*log(f) - 8*a*f*log(f)
 - 16*d*f)/(c*log(f) + 2*f))/sqrt(-c*log(f) - 2*f) - 1/8*sqrt(pi)*erf(-1/2
*sqrt(-c*log(f) + 2*f)*(2*x + b*log(f)/(c*log(f) - 2*f)))*e^(-1/4*(b^2*log
(f)^2 - 4*a*c*log(f)^2 + 8*c*d*log(f) + 8*a*f*log(f) - 16*d*f)/(c*log(f) -
 2*f))/sqrt(-c*log(f) + 2*f) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x
+ b/c))*e^(-1/4*(b^2*log(f) - 4*a*c*log(f))/c)/sqrt(-c*log(f))
```

3.324.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\cosh^2\left(d+fx^2\right)\,dx = \int f^{c\,x^2+b\,x+a}\cosh\left(f\,x^2+d\right)^2\,dx$$

input
```
int(f^(a + b*x + c*x^2)*cosh(d + f*x^2)^2,x)
```

output
```
int(f^(a + b*x + c*x^2)*cosh(d + f*x^2)^2, x)
```

3.325 $\int f^{a+bx+cx^2}\cosh^3\left(d+fx^2\right)\,dx$

3.325.1 Optimal result

Integrand size = 23, antiderivative size = 323

$$\int f^{a+bx+cx^2}\cosh^3\left(d+fx^2\right)\,dx=-\frac{3e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\sqrt{\pi}\,\text{erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}-\frac{e^{-3d+\frac{b^2\log^2(f)}{12f-4c\log(f)}}f^a\sqrt{\pi}\,\text{erf}\left(\frac{b\log(f)-2x(3f-c\log(f))}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3e^{d-\frac{b^2\log^2(f)}{4(f+c\log(f))}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{b\log(f)+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{16\sqrt{f+c\log(f)}}+\frac{e^{3d-\frac{b^2\log^2(f)}{4(3f+c\log(f))}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{b\log(f)+2x(3f+c\log(f))}{2\sqrt{3f+c\log(f)}}\right)}{16\sqrt{3f+c\log(f)}}$$

output
```
-3/16*exp(-d+b^2*ln(f)^2/(4*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(f)-2*x*(f-c*ln
(f)))/(f-c*ln(f))^(1/2))*Pi^(1/2)/(f-c*ln(f))^(1/2)-1/16*exp(-3*d+b^2*ln(f
)^2/(12*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(f)-2*x*(3*f-c*ln(f)))/(3*f-c*ln(f)
)^(1/2))*Pi^(1/2)/(3*f-c*ln(f))^(1/2)+3/16*exp(d-1/4*b^2*ln(f)^2/(f+c*ln(f
)))*f^a*erfi(1/2*(b*ln(f)+2*x*(f+c*ln(f)))/(f+c*ln(f))^(1/2))*Pi^(1/2)/(f+
c*ln(f))^(1/2)+1/16*exp(3*d-1/4*b^2*ln(f)^2/(3*f+c*ln(f)))*f^a*erfi(1/2*(b
*ln(f)+2*x*(3*f+c*ln(f)))/(3*f+c*ln(f))^(1/2))*Pi^(1/2)/(3*f+c*ln(f))^(1/2
)
```

3.325.2 Mathematica [A] (verified)

Time = 4.94 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.55

$$\int f^{a+bx+cx^2}\cosh^3\left(d+fx^2\right)\,dx$$
$$= \frac{e^{-\frac{b^2\log^2(f)(2f+c\log(f))}{2(f+c\log(f))(3f+c\log(f))}}f^a\sqrt{\pi}\left(3e^{\frac{1}{4}b^2\log^2(f)\left(\frac{1}{f-c\log(f)}+\frac{1}{f+c\log(f)}+\frac{1}{3f+c\log(f)}\right)}\operatorname{erf}\left(\frac{2fx-(b+2cx)\log(f)}{2\sqrt{f-c\log(f)}}\right)\sqrt{f-c\log(f)}(9\ldots}{}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Cosh[d + f*x^2]^3,x]
```

output
```
(f^a*Sqrt[Pi]*(3*E^((b^2*Log[f]^2*((f - c*Log[f])^(-1) + (f + c*Log[f])^(-
1) + (3*f + c*Log[f])^(-1)))/4)*Erf[(2*f*x - (b + 2*c*x)*Log[f])/(2*Sqrt[f
 - c*Log[f]])]*Sqrt[f - c*Log[f]]*(9*f^3 + 9*c*f^2*Log[f] - c^2*f*Log[f]^2
 - c^3*Log[f]^3)*(Cosh[d] - Sinh[d]) + (f - c*Log[f])*(E^((b^2*Log[f]^2*((
3*f - c*Log[f])^(-1) + (f + c*Log[f])^(-1) + (3*f + c*Log[f])^(-1)))/4)*Er
f[(6*f*x - (b + 2*c*x)*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Sqrt[3*f - c*Log[
f]]*(3*f^2 + 4*c*f*Log[f] + c^2*Log[f]^2)*(Cosh[3*d] - Sinh[3*d]) + (3*f -
 c*Log[f])*(3*E^((b^2*Log[f]^2)/(12*f + 4*c*Log[f]))*Erfi[(2*f*x + (b + 2*
c*x)*Log[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[f + c*Log[f]]*(3*f + c*Log[f])*(
Cosh[d] + Sinh[d]) + E^((b^2*Log[f]^2)/(4*(f + c*Log[f])))*Erfi[(6*f*x + (
b + 2*c*x)*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*(f + c*Log[f])*Sqrt[3*f + c*L
og[f]]*(Cosh[3*d] + Sinh[3*d])))))/(16*E^((b^2*Log[f]^2*(2*f + c*Log[f]))/
(2*(f + c*Log[f])*(3*f + c*Log[f])))*(9*f^4 - 10*c^2*f^2*Log[f]^2 + c^4*Lo
g[f]^4))
```

3.325.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3\left(d+fx^2\right)f^{a+bx+cx^2}\,dx$$
$$\Big\downarrow \; 6039$$

$$\int \left(\frac{1}{8}e^{-3d-3fx^2}f^{a+bx+cx^2} + \frac{3}{8}e^{-d-fx^2}f^{a+bx+cx^2} + \frac{3}{8}e^{d+fx^2}f^{a+bx+cx^2} + \frac{1}{8}e^{3d+3fx^2}f^{a+bx+cx^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{3\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{4f-4c\log(f)}-d}\operatorname{erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}} - \frac{\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{12f-4c\log(f)}-3d}\operatorname{erf}\left(\frac{b\log(f)-2x(3f-c\log(f))}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}} + \frac{3\sqrt{\pi}f^a e^{d-\frac{b^2\log^2(f)}{4(c\log(f)+f)}}\operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+f)}{2\sqrt{c\log(f)+f}}\right)}{16\sqrt{c\log(f)+f}} + \frac{\sqrt{\pi}f^a e^{3d-\frac{b^2\log^2(f)}{4(c\log(f)+3f)}}\operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+3f)}{2\sqrt{c\log(f)+3f}}\right)}{16\sqrt{c\log(f)+3f}}$$

input
```
Int[f^(a + b*x + c*x^2)*Cosh[d + f*x^2]^3,x]
```

output
```
(-3*E^(-d + (b^2*Log[f]^2)/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f]
- 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(16*Sqrt[f - c*Log[f]]) - (
E^(-3*d + (b^2*Log[f]^2)/(12*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] -
 2*x*(3*f - c*Log[f]))/(2*Sqrt[3*f - c*Log[f]])])/(16*Sqrt[3*f - c*Log[f]]
) + (3*E^(d - (b^2*Log[f]^2)/(4*(f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(b*Log[
f] + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(16*Sqrt[f + c*Log[f]])
+ (E^(3*d - (b^2*Log[f]^2)/(4*(3*f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(b*Log[
f] + 2*x*(3*f + c*Log[f]))/(2*Sqrt[3*f + c*Log[f]])])/(16*Sqrt[3*f + c*Log
[f]])
```

3.325.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.325.4 Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.01

method	result
risch	$-\frac{\operatorname{erf}\left(-x\sqrt{3f-c\ln(f)}+\frac{\ln(f)b}{2\sqrt{3f-c\ln(f)}}\right)\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2}\ln(f)^{2}+12d\ln(f)c-36df}{4(c\ln(f)-3f)}}}{16\sqrt{3f-c\ln(f)}}-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)-3f}\, x+\frac{\ln(f)b}{2\sqrt{-c\ln(f)-3f}}\right)\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2}}{}}}{16\sqrt{-c\ln(f)-3f}}$

input
```
int(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*erf(-x*(3*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(3*f-c*ln(f))^(1/2))/(3*f-c*l
n(f))^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+12*d*ln(f)*c-36*d*f)/(c*ln(
f)-3*f))-1/16*erf(-(-c*ln(f)-3*f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)-3*f)^(1/2)
)/(-c*ln(f)-3*f)^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-12*d*ln(f)*c-36*
d*f)/(3*f+c*ln(f)))-3/16*erf(-x*(f-c*ln(f))^(1/2)+1/2*ln(f)*b/(f-c*ln(f))^
(1/2))/(f-c*ln(f))^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+4*d*ln(f)*c-4*
d*f)/(c*ln(f)-f))-3/16*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)-f)^
(1/2))/(-c*ln(f)-f)^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-4*d*ln(f)*c-4
*d*f)/(f+c*ln(f)))
```

3.325.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 851 vs. $2(275) = 550$.

Time = 0.28 (sec) , antiderivative size = 851, normalized size of antiderivative = 2.63

$$\int f^{a+bx+cx^2}\cosh^3\left(d+fx^2\right)\, dx = \text{Too large to display}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^3,x, algorithm="fricas")
```

output
```
-1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*
cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f + 12*(c*d + a*f)*log(f))/(c*log
(f) - 3*f)) + sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3
*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f + 12*(c*d + a*f)*log(f))/
(c*log(f) - 3*f)))*sqrt(-c*log(f) + 3*f)*erf(-1/2*(6*f*x - (2*c*x + b)*log
(f))*sqrt(-c*log(f) + 3*f)/(c*log(f) - 3*f)) + 3*(sqrt(pi)*(c^3*log(f)^3 +
 c^2*f*log(f)^2 - 9*c*f^2*log(f) - 9*f^3)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^
2 - 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c^3*log(f)^3
 + c^2*f*log(f)^2 - 9*c*f^2*log(f) - 9*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f
)^2 - 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) - f)))*sqrt(-c*log(f) + f)*e
rf(-1/2*(2*f*x - (2*c*x + b)*log(f))*sqrt(-c*log(f) + f)/(c*log(f) - f)) +
 3*(sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*cosh
(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f) +
f)) + sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*si
nh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f)
+ f)))*sqrt(-c*log(f) - f)*erf(1/2*(2*f*x + (2*c*x + b)*log(f))*sqrt(-c*lo
g(f) - f)/(c*log(f) + f)) + (sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c
*f^2*log(f) + 3*f^3)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f - 12*(c*d
+ a*f)*log(f))/(c*log(f) + 3*f)) + sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)
^2 - c*f^2*log(f) + 3*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f -...
```

3.325.6 Sympy [F]

$$\int f^{a+bx+cx^2}\cosh^3\left(d+fx^2\right)\,dx=\int f^{a+bx+cx^2}\cosh^3\left(d+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+b*x+a)*cosh(f*x**2+d)**3,x)
```

output
```
Integral(f**(a + b*x + c*x**2)*cosh(d + f*x**2)**3, x)
```

3.325.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.89

$$\int f^{a+bx+cx^2}\cosh^3\left(d+fx^2\right)\,dx$$
$$=\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-3\,f}x-\frac{b\log(f)}{2\sqrt{-c\log(f)-3\,f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4\,(c\log(f)+3\,f)}+3\,d\right)}}{16\sqrt{-c\log\left(f\right)-3\,f}}$$
$$+\frac{3\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-f}x-\frac{b\log(f)}{2\sqrt{-c\log(f)-f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4\,(c\log(f)+f)}+d\right)}}{16\sqrt{-c\log\left(f\right)-f}}$$
$$+\frac{3\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+f}x-\frac{b\log(f)}{2\sqrt{-c\log(f)+f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4\,(c\log(f)-f)}-d\right)}}{16\sqrt{-c\log\left(f\right)+f}}$$
$$+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+3\,f}x-\frac{b\log(f)}{2\sqrt{-c\log(f)+3\,f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4\,(c\log(f)-3\,f)}-3\,d\right)}}{16\sqrt{-c\log\left(f\right)+3\,f}}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x - 1/2*b*log(f)/sqrt(-c*log(f
) - 3*f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + 3*f) + 3*d)/sqrt(-c*log(f) - 3*
f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*b*log(f)/sqrt(-c*lo
g(f) - f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + f) + d)/sqrt(-c*log(f) - f) +
3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*b*log(f)/sqrt(-c*log(f)
+ f))*e^(-1/4*b^2*log(f)^2/(c*log(f) - f) - d)/sqrt(-c*log(f) + f) + 1/16*
sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 3*f)*x - 1/2*b*log(f)/sqrt(-c*log(f) + 3
*f))*e^(-1/4*b^2*log(f)^2/(c*log(f) - 3*f) - 3*d)/sqrt(-c*log(f) + 3*f)
```

3.325.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.14

$$\int f^{a+bx+cx^2}\cosh^3\left(d+fx^2\right)\,dx =$$
$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)-3f}\left(2x+\frac{b\log(f)}{c\log(f)+3f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-12cd\log(f)-12af\log(f)-36df}{4(c\log(f)+3f)}\right)}}{16\sqrt{-c\log(f)-3f}}$$
$$-\frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)-f}\left(2x+\frac{b\log(f)}{c\log(f)+f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-4cd\log(f)-4af\log(f)-4df}{4(c\log(f)+f)}\right)}}{16\sqrt{-c\log(f)-f}}$$
$$-\frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)+f}\left(2x+\frac{b\log(f)}{c\log(f)-f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+4cd\log(f)+4af\log(f)-4df}{4(c\log(f)-f)}\right)}}{16\sqrt{-c\log(f)+f}}$$
$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)+3f}\left(2x+\frac{b\log(f)}{c\log(f)-3f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+12cd\log(f)+12af\log(f)-36df}{4(c\log(f)-3f)}\right)}}{16\sqrt{-c\log(f)+3f}}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^3,x, algorithm="giac")
```

output
```
-1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 3*f)*(2*x + b*log(f)/(c*log(f) +
3*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 12*c*d*log(f) - 12*a*f*log
(f) - 36*d*f)/(c*log(f) + 3*f))/sqrt(-c*log(f) - 3*f) - 3/16*sqrt(pi)*erf(
-1/2*sqrt(-c*log(f) - f)*(2*x + b*log(f)/(c*log(f) + f)))*e^(-1/4*(b^2*log
(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) - 4*a*f*log(f) - 4*d*f)/(c*log(f) +
f))/sqrt(-c*log(f) - f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x
+ b*log(f)/(c*log(f) - f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d
*log(f) + 4*a*f*log(f) - 4*d*f)/(c*log(f) - f))/sqrt(-c*log(f) + f) - 1/16
*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + 3*f)*(2*x + b*log(f)/(c*log(f) - 3*f))
)*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 12*c*d*log(f) + 12*a*f*log(f) -
 36*d*f)/(c*log(f) - 3*f))/sqrt(-c*log(f) + 3*f)
```

3.325.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\cosh^3\left(d+fx^2\right)\,dx=\int f^{c\,x^2+b\,x+a}\cosh\left(f\,x^2+d\right)^3\,dx$$

input
```
int(f^(a + b*x + c*x^2)*cosh(d + f*x^2)^3,x)
```

output
```
int(f^(a + b*x + c*x^2)*cosh(d + f*x^2)^3, x)
```

3.326 $\int f^{a+bx+cx^2}\cosh\left(d+ex+fx^2\right)\,dx$

3.326.1 Optimal result

Integrand size = 24, antiderivative size = 161

$$\int f^{a+bx+cx^2}\cosh\left(d+ex+fx^2\right)\,dx=\frac{e^{-d+\frac{(e-b\log(f))^2}{4(f-c\log(f))}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{e-b\log(f)+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^{d-\frac{(e+b\log(f))^2}{4(f+c\log(f))}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e+b\log(f)+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{4\sqrt{f+c\log(f)}}$$

output
```
1/4*exp(-d+1/4*(e-b*ln(f))^2/(f-c*ln(f)))*f^a*erf(1/2*(e-b*ln(f)+2*x*(f-c*
ln(f)))/(f-c*ln(f))^(1/2))*Pi^(1/2)/(f-c*ln(f))^(1/2)+1/4*exp(d-1/4*(e+b*l
n(f))^2/(f+c*ln(f)))*f^a*erfi(1/2*(e+b*ln(f)+2*x*(f+c*ln(f)))/(f+c*ln(f))^
(1/2))*Pi^(1/2)/(f+c*ln(f))^(1/2)
```

3.326.2 Mathematica [A] (warning: unable to verify)

Time = 1.07 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.56

$$\int f^{a+bx+cx^2}\cosh\left(d+ex+fx^2\right)\,dx$$

$$=\frac{e^{-\frac{e^2+b^2\log^2(f)}{4(f+c\log(f))}}f^{a+\frac{bef}{-f^2+c^2\log^2(f)}}\sqrt{\pi}\left(e^{\frac{f\left(e^2+b^2\log^2(f)\right)}{2\left(f^2-c^2\log^2(f)\right)}}f^{\frac{be}{2(f+c\log(f))}}\operatorname{erf}\left(\frac{e+2fx-(b+2cx)\log(f)}{2\sqrt{f-c\log(f)}}\right)\sqrt{f-c\log(f)}(f+c\log(\right.}{4\left(f^2-\right.}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Cosh[d + e*x + f*x^2],x]
```

output
```
(f^(a + (b*e*f)/(-f^2 + c^2*Log[f]^2))*Sqrt[Pi]*(E^((f*(e^2 + b^2*Log[f]^2
))/(2*(f^2 - c^2*Log[f]^2)))*f^((b*e)/(2*(f + c*Log[f])))*Erf[(e + 2*f*x -
 (b + 2*c*x)*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f - c*Log[f]]*(f + c*Log
[f])*(Cosh[d] - Sinh[d]) + f^((b*e)/(2*f - 2*c*Log[f]))*Erfi[(e + 2*f*x +
(b + 2*c*x)*Log[f])/(2*Sqrt[f + c*Log[f]])]*(f - c*Log[f])*Sqrt[f + c*Log[
f]]*(Cosh[d] + Sinh[d])))/(4*E^((e^2 + b^2*Log[f]^2)/(4*(f + c*Log[f])))*(
f^2 - c^2*Log[f]^2))
```

3.326.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx+cx^2}\cosh\left(d+ex+fx^2\right)dx$$

$$\Big\downarrow \text{6039}$$

$$\int \left(\frac{1}{2}e^{-d-ex-fx^2}f^{a+bx+cx^2}+\frac{1}{2}e^{d+ex+fx^2}f^{a+bx+cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}f^a e^{\frac{(e-b\log(f))^2}{4(f-c\log(f))}-d}\text{erf}\left(\frac{-b\log(f)+2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}f^a e^{d-\frac{(b\log(f)+e)^2}{4(c\log(f)+f)}}\text{erfi}\left(\frac{b\log(f)+2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}}$$

input
```
Int[f^(a + b*x + c*x^2)*Cosh[d + e*x + f*x^2],x]
```

output
```
(E^(-d + (e - b*Log[f])^2/(4*(f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[(e - b*Log[
f] + 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(4*Sqrt[f - c*Log[f]]) +
 (E^(d - (e + b*Log[f])^2/(4*(f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(e + b*Log
[f] + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(4*Sqrt[f + c*Log[f]])
```

3.326.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.326.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.16

method	result
risch	$-\frac{\operatorname{erf}\left(-x\sqrt{f-c\ln(f)}+\frac{b\ln(f)-e}{2\sqrt{f-c\ln(f)}}\right)\sqrt{\pi}\, f^{a}\, \mathrm{e}^{-\frac{b^{2}\ln(f)^{2}-2\ln(f)be+4d\ln(f)c-4df+e^{2}}{4(c\ln(f)-f)}}}{4\sqrt{f-c\ln(f)}}-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)-f}\,x+\frac{e+b\ln(f)}{2\sqrt{-c\ln(f)-f}}\right)\sqrt{\pi}\, f}{4\sqrt{-c\ln(}}$

input
```
int(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4*erf(-x*(f-c*ln(f))^(1/2)+1/2*(b*ln(f)-e)/(f-c*ln(f))^(1/2))/(f-c*ln(f
))^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-2*ln(f)*b*e+4*d*ln(f)*c-4*d*f+
e^2)/(c*ln(f)-f))-1/4*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*(e+b*ln(f))/(-c*ln(f)-
f)^(1/2))/(-c*ln(f)-f)^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+2*ln(f)*b*
e-4*d*ln(f)*c-4*d*f+e^2)/(f+c*ln(f)))
```

3.326.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. 2(139) = 278.

Time = 0.27 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.25

$$\int f^{a+bx+cx^2}\cosh\left(d+ex+fx^2\right)\,dx =$$

$$-\frac{\left(\sqrt{\pi}(c\log\left(f\right)+f)\cosh\left(-\frac{(b^2-4\,ac)\log(f)^2+e^2-4\,df+2\,(2\,cd-be+2\,af)\log(f)}{4\,(c\log(f)-f)}\right)+\sqrt{\pi}(c\log\left(f\right)+f)\sinh\left(-\frac{(b^2-4\,a}{}\right.\right.}{}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
-1/4*((sqrt(pi)*(c*log(f) + f)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4
*d*f + 2*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c*log(f
) + f)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f + 2*(2*c*d - b*e +
2*a*f)*log(f))/(c*log(f) - f)))*sqrt(-c*log(f) + f)*erf(-1/2*(2*f*x - (2*c
*x + b)*log(f) + e)*sqrt(-c*log(f) + f)/(c*log(f) - f)) + (sqrt(pi)*(c*log
(f) - f)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f - 2*(2*c*d - b*e
+ 2*a*f)*log(f))/(c*log(f) + f)) + sqrt(pi)*(c*log(f) - f)*sinh(-1/4*((b^2
 - 4*a*c)*log(f)^2 + e^2 - 4*d*f - 2*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(
f) + f)))*sqrt(-c*log(f) - f)*erf(1/2*(2*f*x + (2*c*x + b)*log(f) + e)*sqr
t(-c*log(f) - f)/(c*log(f) + f)))/(c^2*log(f)^2 - f^2)
```

3.326.6 Sympy [F]

$$\int f^{a+bx+cx^2}\cosh\left(d+ex+fx^2\right)\,dx=\int f^{a+bx+cx^2}\cosh\left(d+ex+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+b*x+a)*cosh(f*x**2+e*x+d),x)
```

output
```
Integral(f**(a + b*x + c*x**2)*cosh(d + e*x + f*x**2), x)
```

3.326.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.94

$$\int f^{a+bx+cx^2}\cosh\left(d+ex+fx^2\right)\,dx$$
$$=\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)-f}\,x-\frac{b\log(f)+e}{2\sqrt{-c\log(f)-f}}\right)e^{\left(-\frac{(b\log(f)+e)^2}{4(c\log(f)+f)}+d\right)}}{4\sqrt{-c\log(f)-f}}+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)+f}\,x-\frac{b\log(f)-e}{2\sqrt{-c\log(f)+f}}\right)e^{\left(-\frac{(b\log(f)-e)^2}{4(c\log(f)-f)}-d\right)}}{4\sqrt{-c\log(f)+f}}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*(b*log(f) + e)/sqrt(-c*lo
g(f) - f))*e^(-1/4*(b*log(f) + e)^2/(c*log(f) + f) + d)/sqrt(-c*log(f) - f
) + 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*(b*log(f) - e)/sqrt(-
c*log(f) + f))*e^(-1/4*(b*log(f) - e)^2/(c*log(f) - f) - d)/sqrt(-c*log(f)
 + f)
```

3.326.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.29

$$\int f^{a+bx+cx^2}\cosh\left(d+ex+fx^2\right)\,dx=$$

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)-f}\left(2x+\frac{b\log(f)+e}{c\log(f)+f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-4cd\log(f)+2be\log(f)-4af\log(f)+e^2-4df}{4(c\log(f)+f)}\right)}}{4\sqrt{-c\log(f)-f}}$$
$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)+f}\left(2x+\frac{b\log(f)-e}{c\log(f)-f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+4cd\log(f)-2be\log(f)+4af\log(f)+e^2-4df}{4(c\log(f)-f)}\right)}}{4\sqrt{-c\log(f)+f}}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + (b*log(f) + e)/(c*log(f)
 + f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) + 2*b*e*log(
f) - 4*a*f*log(f) + e^2 - 4*d*f)/(c*log(f) + f))/sqrt(-c*log(f) - f) - 1/4
*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x + (b*log(f) - e)/(c*log(f) - f
)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d*log(f) - 2*b*e*log(f) +
 4*a*f*log(f) + e^2 - 4*d*f)/(c*log(f) - f))/sqrt(-c*log(f) + f)
```

3.326.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\cosh\left(d+ex+fx^2\right)\,dx=\int f^{c\,x^2+b\,x+a}\cosh\left(f\,x^2+e\,x+d\right)\,dx$$

input
```
int(f^(a + b*x + c*x^2)*cosh(d + e*x + f*x^2),x)
```

output
```
int(f^(a + b*x + c*x^2)*cosh(d + e*x + f*x^2), x)
```

3.327 $\int f^{a+bx+cx^2}\cosh^2\left(d+ex+fx^2\right)\,dx$

3.327.1 Optimal result

Integrand size = 26, antiderivative size = 239

$$\int f^{a+bx+cx^2}\cosh^2\left(d+ex+fx^2\right)\,dx=\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-2d+\frac{(2e-b\log(f))^2}{8f-4c\log(f)}}f^a\sqrt{\pi}\,\text{erf}\left(\frac{2e-b\log(f)+2x(2f-c\log(f))}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{e^{2d-\frac{(2e+b\log(f))^2}{8f+4c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{2e+b\log(f)+2x(2f+c\log(f))}{2\sqrt{2f+c\log(f)}}\right)}{8\sqrt{2f+c\log(f)}}$$

output
```
1/4*f^(a-1/4*b^2/c)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))*Pi^(1/2)/c^(1/
2)/ln(f)^(1/2)+1/8*exp(-2*d+(2*e-b*ln(f))^2/(8*f-4*c*ln(f)))*f^a*erf(1/2*(
2*e-b*ln(f)+2*x*(2*f-c*ln(f)))/(2*f-c*ln(f))^(1/2))*Pi^(1/2)/(2*f-c*ln(f))
^(1/2)+1/8*exp(2*d-(2*e+b*ln(f))^2/(8*f+4*c*ln(f)))*f^a*erfi(1/2*(2*e+b*ln
(f)+2*x*(2*f+c*ln(f)))/(2*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*f+c*ln(f))^(1/2)
```

3.327.2 Mathematica [A] (warning: unable to verify)

Time = 4.37 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.42

$$\int f^{a+bx+cx^2}\cosh^2\left(d+ex+fx^2\right)\,dx=\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$
$$-\frac{e^{-\frac{4e^2+b^2\log^2(f)}{8f+4c\log(f)}}f^{a+\frac{4bef}{-4f^2+c^2\log^2(f)}}\sqrt{\pi}\left(e^{\frac{f\left(4e^2+b^2\log^2(f)\right)}{4f^2-c^2\log^2(f)}}f^{\frac{be}{2f+c\log(f)}}\text{erf}\left(\frac{2(e+2fx)-(b+2cx)\log(f)}{2\sqrt{2f-c\log(f)}}\right)\sqrt{2f-c\log(f)}(2f}{}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Cosh[d + e*x + f*x^2]^2,x]
```

output
```
(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/
(4*Sqrt[c]*Sqrt[Log[f]]) - (f^(a + (4*b*e*f)/(-4*f^2 + c^2*Log[f]^2))*Sqrt
[Pi]*(E^((f*(4*e^2 + b^2*Log[f]^2))/(4*f^2 - c^2*Log[f]^2))*f^((b*e)/(2*f
+ c*Log[f]))*Erf[(2*(e + 2*f*x) - (b + 2*c*x)*Log[f])/(2*Sqrt[2*f - c*Log[
f]])]*Sqrt[2*f - c*Log[f]]*(2*f + c*Log[f])*(Cosh[2*d] - Sinh[2*d]) + f^((
b*e)/(2*f - c*Log[f]))*Erfi[(2*(e + 2*f*x) + (b + 2*c*x)*Log[f])/(2*Sqrt[2
*f + c*Log[f]])]*(2*f - c*Log[f])*Sqrt[2*f + c*Log[f]]*(Cosh[2*d] + Sinh[2
*d])))/(8*E^((4*e^2 + b^2*Log[f]^2)/(8*f + 4*c*Log[f]))*(-4*f^2 + c^2*Log[
f]^2))
```

3.327.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx+cx^2}\cosh^2\left(d+ex+fx^2\right)\,dx$$

$$\Big\downarrow \text{6039}$$

$$\int\left(\frac{1}{4}e^{-2d-2ex-2fx^2}f^{a+bx+cx^2}+\frac{1}{4}e^{2d+2ex+2fx^2}f^{a+bx+cx^2}+\frac{1}{2}f^{a+bx+cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\operatorname{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a\exp\left(\frac{(2e-b\log(f))^2}{8f-4c\log(f)}-2d\right)\operatorname{erf}\left(\frac{-b\log(f)+2x(2f-c\log(f))+2e}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi}f^a\exp\left(2d-\frac{(b\log(f)+2e)^2}{4c\log(f)+8f}\right)\operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+2f)+2e}{2\sqrt{c\log(f)+2f}}\right)}{8\sqrt{c\log(f)+2f}}$$

input
```
Int[f^(a + b*x + c*x^2)*Cosh[d + e*x + f*x^2]^2,x]
```

output
```
(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/
(4*Sqrt[c]*Sqrt[Log[f]]) + (E^(-2*d + (2*e - b*Log[f])^2/(8*f - 4*c*Log[f]
))*f^a*Sqrt[Pi]*Erf[(2*e - b*Log[f] + 2*x*(2*f - c*Log[f]))/(2*Sqrt[2*f -
c*Log[f]])])/(8*Sqrt[2*f - c*Log[f]]) + (E^(2*d - (2*e + b*Log[f])^2/(8*f
+ 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(2*e + b*Log[f] + 2*x*(2*f + c*Log[f]))/(
2*Sqrt[2*f + c*Log[f]])])/(8*Sqrt[2*f + c*Log[f]])
```

3.327.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.327.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.04

method	result
risch	$-\frac{\operatorname{erf}\left(-x\sqrt{2f-c\ln(f)}+\frac{b\ln(f)-2e}{2\sqrt{2f-c\ln(f)}}\right)\sqrt{\pi}\,f^a\,\mathrm{e}^{-\frac{b^2\ln(f)^2-4\ln(f)be+8d\ln(f)c-16df+4e^2}{4(c\ln(f)-2f)}}}{8\sqrt{2f-c\ln(f)}}-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)-2f}\,x+\frac{2e+b\ln(f)}{2\sqrt{-c\ln(f)-2f}}\right.}{8\sqrt{}}$

input
```
int(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*erf(-x*(2*f-c*ln(f))^(1/2)+1/2*(b*ln(f)-2*e)/(2*f-c*ln(f))^(1/2))/(2*
f-c*ln(f))^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-4*ln(f)*b*e+8*d*ln(f)*
c-16*d*f+4*e^2)/(c*ln(f)-2*f))-1/8*erf(-(-c*ln(f)-2*f)^(1/2)*x+1/2*(2*e+b*
ln(f))/(-c*ln(f)-2*f)^(1/2))/(-c*ln(f)-2*f)^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b
^2*ln(f)^2+4*ln(f)*b*e-8*d*ln(f)*c-16*d*f+4*e^2)/(2*f+c*ln(f)))-1/4*f^a*Pi
^(1/2)*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*ln(f)*b
/(-c*ln(f))^(1/2))
```

3.327.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 516 vs. $2(197) = 394$.

Time = 0.27 (sec) , antiderivative size = 516, normalized size of antiderivative = 2.16

$$\int f^{a+bx+cx^2}\cosh^2\left(d+ex+fx^2\right)\,dx = -\frac{\left(\sqrt{\pi}\left(c^2\log(f)^2+2\,cf\log(f)\right)\cosh\left(-\frac{(b^2-4\,ac)\log(f)^2+4\,e^2-16\,df+4\,(2\,cd-be+2\,af)\log(f)}{4\,(c\log(f)-2\,f)}\right)+\sqrt{\pi}\left(c^2\log(f)^2+\right.\right.}{}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

output
```
-1/8*((sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh(-1/4*((b^2 - 4*a*c)*log
(f)^2 + 4*e^2 - 16*d*f + 4*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) - 2*f))
 + sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*sinh(-1/4*((b^2 - 4*a*c)*log(f)^
2 + 4*e^2 - 16*d*f + 4*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) - 2*f)))*sq
rt(-c*log(f) + 2*f)*erf(-1/2*(4*f*x - (2*c*x + b)*log(f) + 2*e)*sqrt(-c*lo
g(f) + 2*f)/(c*log(f) - 2*f)) + (sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*co
sh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 4*e^2 - 16*d*f - 4*(2*c*d - b*e + 2*a*f)
*log(f))/(c*log(f) + 2*f)) + sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*sinh(-
1/4*((b^2 - 4*a*c)*log(f)^2 + 4*e^2 - 16*d*f - 4*(2*c*d - b*e + 2*a*f)*log
(f))/(c*log(f) + 2*f)))*sqrt(-c*log(f) - 2*f)*erf(1/2*(4*f*x + (2*c*x + b)
*log(f) + 2*e)*sqrt(-c*log(f) - 2*f)/(c*log(f) + 2*f)) + 2*(sqrt(pi)*(c^2*
log(f)^2 - 4*f^2)*cosh(-1/4*(b^2 - 4*a*c)*log(f)/c) + sqrt(pi)*(c^2*log(f)
^2 - 4*f^2)*sinh(-1/4*(b^2 - 4*a*c)*log(f)/c))*sqrt(-c*log(f))*erf(1/2*(2*
c*x + b)*sqrt(-c*log(f))/c))/(c^3*log(f)^3 - 4*c*f^2*log(f))
```

3.327.6 Sympy [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\cosh^2\left(d+ex+fx^2\right)\,dx=\text{Timed out}$$

input
```
integrate(f**(c*x**2+b*x+a)*cosh(f*x**2+e*x+d)**2,x)
```

output
```
Timed out
```

3.327.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.90

$$\int f^{a+bx+cx^2}\cosh^2\left(d+ex+fx^2\right)\,dx$$
$$=\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-2\,f}x-\frac{b\log(f)+2\,e}{2\sqrt{-c\log(f)-2\,f}}\right)e^{\left(-\frac{(b\log(f)+2\,e)^2}{4\,(c\log(f)+2\,f)}+2\,d\right)}}{8\sqrt{-c\log\left(f\right)-2\,f}}+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+2\,f}x-\frac{b\log(f)-2\,e}{2\sqrt{-c\log(f)+2\,f}}\right)e^{\left(-\frac{(b\log(f)-2\,e)^2}{4\,(c\log(f)-2\,f)}-2\,d\right)}}{8\sqrt{-c\log\left(f\right)+2\,f}}+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)}x-\frac{b\log(f)}{2\sqrt{-c\log(f)}}\right)}{4\sqrt{-c\log\left(f\right)}f^{\frac{b^2}{4\,c}}}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

output
```
1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x - 1/2*(b*log(f) + 2*e)/sqrt(-
c*log(f) - 2*f))*e^(-1/4*(b*log(f) + 2*e)^2/(c*log(f) + 2*f) + 2*d)/sqrt(-
c*log(f) - 2*f) + 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 2*f)*x - 1/2*(b*lo
g(f) - 2*e)/sqrt(-c*log(f) + 2*f))*e^(-1/4*(b*log(f) - 2*e)^2/(c*log(f) -
2*f) - 2*d)/sqrt(-c*log(f) + 2*f) + 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x
 - 1/2*b*log(f)/sqrt(-c*log(f)))/(sqrt(-c*log(f))*f^(1/4*b^2/c))
```

3.327.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.13

$$\int f^{a+bx+cx^2}\cosh^2\left(d+ex+fx^2\right)\,dx=$$

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-2f}\left(2x+\frac{b\log(f)+2e}{c\log(f)+2f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-8cd\log(f)+4be\log(f)-8af\log(f)+4e^2-16df}{4(c\log(f)+2f)}\right)}}{8\sqrt{-c\log\left(f\right)-2f}}$$
$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)+2f}\left(2x+\frac{b\log(f)-2e}{c\log(f)-2f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+8cd\log(f)-4be\log(f)+8af\log(f)+4e^2-16df}{4(c\log(f)-2f)}\right)}}{8\sqrt{-c\log\left(f\right)+2f}}$$
$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)}\left(2x+\frac{b}{c}\right)\right)e^{\left(-\frac{b^2\log(f)-4ac\log(f)}{4c}\right)}}{4\sqrt{-c\log\left(f\right)}}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="giac")
```

output
```
-1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 2*f)*(2*x + (b*log(f) + 2*e)/(c*lo
g(f) + 2*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 8*c*d*log(f) + 4*b*
e*log(f) - 8*a*f*log(f) + 4*e^2 - 16*d*f)/(c*log(f) + 2*f))/sqrt(-c*log(f)
 - 2*f) - 1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + 2*f)*(2*x + (b*log(f) - 2
*e)/(c*log(f) - 2*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 8*c*d*log(
f) - 4*b*e*log(f) + 8*a*f*log(f) + 4*e^2 - 16*d*f)/(c*log(f) - 2*f))/sqrt(
-c*log(f) + 2*f) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + b/c))*e^(-
1/4*(b^2*log(f) - 4*a*c*log(f))/c)/sqrt(-c*log(f))
```

3.327.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\cosh^2\left(d+ex+fx^2\right)\,dx=\int f^{c\,x^2+b\,x+a}\,{\cosh\left(f\,x^2+e\,x+d\right)}^2\,dx$$

input
```
int(f^(a + b*x + c*x^2)*cosh(d + e*x + f*x^2)^2,x)
```

output
```
int(f^(a + b*x + c*x^2)*cosh(d + e*x + f*x^2)^2, x)
```

3.328 $\int f^{a+bx+cx^2} \cosh^3(d+ex+fx^2)\,dx$

3.328.1 Optimal result

Integrand size = 26, antiderivative size = 344

$$\int f^{a+bx+cx^2}\cosh^3\left(d+ex+fx^2\right)\,dx=\frac{3e^{-d+\frac{(e-b\log(f))^2}{4(f-c\log(f))}}f^a\sqrt{\pi}\text{erf}\left(\frac{e-b\log(f)+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\frac{e^{-3d+\frac{(3e-b\log(f))^2}{12f-4c\log(f)}}f^a\sqrt{\pi}\text{erf}\left(\frac{3e-b\log(f)+2x(3f-c\log(f))}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3e^{d-\frac{(e+b\log(f))^2}{4(f+c\log(f))}}f^a\sqrt{\pi}\text{erfi}\left(\frac{e+b\log(f)+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{16\sqrt{f+c\log(f)}}+\frac{e^{3d-\frac{(3e+b\log(f))^2}{4(3f+c\log(f))}}f^a\sqrt{\pi}\text{erfi}\left(\frac{3e+b\log(f)+2x(3f+c\log(f))}{2\sqrt{3f+c\log(f)}}\right)}{16\sqrt{3f+c\log(f)}}$$

output
```
3/16*exp(-d+1/4*(e-b*ln(f))^2/(f-c*ln(f)))*f^a*erf(1/2*(e-b*ln(f)+2*x*(f-c
*ln(f)))/(f-c*ln(f))^(1/2))*Pi^(1/2)/(f-c*ln(f))^(1/2)+1/16*exp(-3*d+(3*e-
b*ln(f))^2/(12*f-4*c*ln(f)))*f^a*erf(1/2*(3*e-b*ln(f)+2*x*(3*f-c*ln(f)))/(
3*f-c*ln(f))^(1/2))*Pi^(1/2)/(3*f-c*ln(f))^(1/2)+3/16*exp(d-1/4*(e+b*ln(f)
)^2/(f+c*ln(f)))*f^a*erfi(1/2*(e+b*ln(f)+2*x*(f+c*ln(f)))/(f+c*ln(f))^(1/2
))*Pi^(1/2)/(f+c*ln(f))^(1/2)+1/16*exp(3*d-1/4*(3*e+b*ln(f))^2/(3*f+c*ln(f
)))*f^a*erfi(1/2*(3*e+b*ln(f)+2*x*(3*f+c*ln(f)))/(3*f+c*ln(f))^(1/2))*Pi^(
1/2)/(3*f+c*ln(f))^(1/2)
```

3.328.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2991 vs. $2(344) = 688$.

Time = 6.46 (sec) , antiderivative size = 2991, normalized size of antiderivative = 8.69

$$\int f^{a+bx+cx^2} \cosh^3\left(d+ex+fx^2\right) dx = \text{Result too large to show}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Cosh[d + e*x + f*x^2]^3,x]
```

output
```
(f^a*Sqrt[Pi]*((27*f^3*Cosh[d]*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(
2*Sqrt[f - c*Log[f]])]*Sqrt[f - c*Log[f]])/E^((-e^2 + 2*b*e*Log[f] - b^2*L
og[f]^2)/(4*(f - c*Log[f]))) + (27*c*f^2*Cosh[d]*Erf[(e + 2*f*x - b*Log[f]
 - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]*Sqrt[f - c*Log[f]])/E^((-e
^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) - (3*c^2*f*Cosh[d]*E
rf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]^2*
Sqrt[f - c*Log[f]])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[
f]))) - (3*c^3*Cosh[d]*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f
 - c*Log[f]])]*Log[f]^3*Sqrt[f - c*Log[f]])/E^((-e^2 + 2*b*e*Log[f] - b^2*
Log[f]^2)/(4*(f - c*Log[f]))) + (3*f^3*Cosh[3*d]*Erf[(3*e + 6*f*x - b*Log[
f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Sqrt[3*f - c*Log[f]])/E^((-9*
e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) + (c*f^2*Cosh[3*d
]*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Lo
g[f]*Sqrt[3*f - c*Log[f]])/E^((-9*e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3
*f - c*Log[f]))) - (3*c^2*f*Cosh[3*d]*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*
Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]^2*Sqrt[3*f - c*Log[f]])/E^((-9*e^
2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) - (c^3*Cosh[3*d]*Er
f[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]
^3*Sqrt[3*f - c*Log[f]])/E^((-9*e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f
 - c*Log[f]))) + (27*f^3*Cosh[d]*Erfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log...
```

3.328.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx+cx^2} \cosh^3\left(d+ex+fx^2\right) dx$$

$\Big\downarrow$ 6039

$$\int \left(\frac{3}{8} f^{a+bx+cx^2} \exp\left(-3\left(d+ex+fx^2\right)+2d+2ex+2fx^2\right)+\frac{3}{8} f^{a+bx+cx^2} \exp\left(-3\left(d+ex+fx^2\right)+4d+4ex+\right.\right.$$

$\Big\downarrow$ 2009

$$\frac{\sqrt{\pi} f^a \exp\left(\frac{(3e-b\log(f))^2}{12f-4c\log(f)}-3d\right) \operatorname{erf}\left(\frac{-b\log(f)+2x(3f-c\log(f))+3e}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3\sqrt{\pi} f^a e^{\frac{(e-b\log(f))^2}{4(f-c\log(f))}-d} \operatorname{erf}\left(\frac{-b\log(f)+2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi} f^a \exp\left(3d-\frac{(b\log(f)+3e)^2}{4(c\log(f)+3f)}\right) \operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+3f)+3e}{2\sqrt{c\log(f)+3f}}\right)}{16\sqrt{c\log(f)+3f}}+\frac{3\sqrt{\pi} f^a e^{d-\frac{(b\log(f)+e)^2}{4(c\log(f)+f)}} \operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{16\sqrt{c\log(f)+f}}$$

input
```
Int[f^(a + b*x + c*x^2)*Cosh[d + e*x + f*x^2]^3,x]
```

output
```
(3*E^(-d + (e - b*Log[f])^2/(4*(f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[(e - b*Lo
g[f] + 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(16*Sqrt[f - c*Log[f]]
) + (E^(-3*d + (3*e - b*Log[f])^2/(12*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(3
*e - b*Log[f] + 2*x*(3*f - c*Log[f]))/(2*Sqrt[3*f - c*Log[f]])])/(16*Sqrt[
3*f - c*Log[f]]) + (3*E^(d - (e + b*Log[f])^2/(4*(f + c*Log[f])))*f^a*Sqrt
[Pi]*Erfi[(e + b*Log[f] + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(16
*Sqrt[f + c*Log[f]]) + (E^(3*d - (3*e + b*Log[f])^2/(4*(3*f + c*Log[f])))*
f^a*Sqrt[Pi]*Erfi[(3*e + b*Log[f] + 2*x*(3*f + c*Log[f]))/(2*Sqrt[3*f + c*
Log[f]])])/(16*Sqrt[3*f + c*Log[f]])
```

3.328. $\int f^{a+bx+cx^2} \cosh^3\left(d+ex+fx^2\right) dx$

3.328.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

3.328.4 Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.12

method	result
risch	$-\frac{\operatorname{erf}\left(-x\sqrt{3f-c\ln(f)}+\frac{b\ln(f)-3e}{2\sqrt{3f-c\ln(f)}}\right)\sqrt{\pi}\,f^{a}\,\mathrm{e}^{-\frac{b^2\ln(f)^2-6\ln(f)be+12d\ln(f)c-36df+9e^2}{4(c\ln(f)-3f)}}}{16\sqrt{3f-c\ln(f)}}-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)-3f}\,x+\frac{3e+b\ln(f)}{2\sqrt{-c\ln(f)-3\ldots}}$

input
```
int(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*erf(-x*(3*f-c*ln(f))^(1/2)+1/2*(b*ln(f)-3*e)/(3*f-c*ln(f))^(1/2))/(3
*f-c*ln(f))^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-6*ln(f)*b*e+12*d*ln(f
)*c-36*d*f+9*e^2)/(c*ln(f)-3*f))-1/16*erf(-(-c*ln(f)-3*f)^(1/2)*x+1/2*(3*e
+b*ln(f))/(-c*ln(f)-3*f)^(1/2))/(-c*ln(f)-3*f)^(1/2)*Pi^(1/2)*f^a*exp(-1/4
*(b^2*ln(f)^2+6*ln(f)*b*e-12*d*ln(f)*c-36*d*f+9*e^2)/(3*f+c*ln(f)))-3/16*e
rf(-x*(f-c*ln(f))^(1/2)+1/2*(b*ln(f)-e)/(f-c*ln(f))^(1/2))/(f-c*ln(f))^(1/
2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-2*ln(f)*b*e+4*d*ln(f)*c-4*d*f+e^2)/(
c*ln(f)-f))-3/16*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*(e+b*ln(f))/(-c*ln(f)-f)^(1
/2))/(-c*ln(f)-f)^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+2*ln(f)*b*e-4*d
*ln(f)*c-4*d*f+e^2)/(f+c*ln(f)))
```

3.328.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 939 vs. 2(295) = 590.

Time = 0.28 (sec) , antiderivative size = 939, normalized size of antiderivative = 2.73

$$\int f^{a+bx+cx^2}\cosh^3\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

output
```
-1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*
cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*e^2 - 36*d*f + 6*(2*c*d - b*e + 2*a*
f)*log(f))/(c*log(f) - 3*f)) + sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 -
 c*f^2*log(f) - 3*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*e^2 - 36*d*f
+ 6*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) - 3*f)))*sqrt(-c*log(f) + 3*f)
*erf(-1/2*(6*f*x - (2*c*x + b)*log(f) + 3*e)*sqrt(-c*log(f) + 3*f)/(c*log(
f) - 3*f)) + 3*(sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2 - 9*c*f^2*log(f) -
 9*f^3)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f + 2*(2*c*d - b*e +
 2*a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2
- 9*c*f^2*log(f) - 9*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f
+ 2*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) - f)))*sqrt(-c*log(f) + f)*erf
(-1/2*(2*f*x - (2*c*x + b)*log(f) + e)*sqrt(-c*log(f) + f)/(c*log(f) - f))
 + 3*(sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*co
sh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f - 2*(2*c*d - b*e + 2*a*f)*lo
g(f))/(c*log(f) + f)) + sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*
log(f) + 9*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f - 2*(2*c*d
 - b*e + 2*a*f)*log(f))/(c*log(f) + f)))*sqrt(-c*log(f) - f)*erf(1/2*(2*f*
x + (2*c*x + b)*log(f) + e)*sqrt(-c*log(f) - f)/(c*log(f) + f)) + (sqrt(pi
)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f^3)*cosh(-1/4*((b^2
 - 4*a*c)*log(f)^2 + 9*e^2 - 36*d*f - 6*(2*c*d - b*e + 2*a*f)*log(f))/(...
```

3.328.6 Sympy [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\cosh^3\left(d+ex+fx^2\right)\,dx=\text{Timed out}$$

input
```
integrate(f**(c*x**2+b*x+a)*cosh(f*x**2+e*x+d)**3,x)
```

output
```
Timed out
```

3.328.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.92

$$\int f^{a+bx+cx^2} \cosh^3\left(d+ex+fx^2\right) dx$$
$$= \frac{\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log\left(f\right)-3f}x - \frac{b\log(f)+3e}{2\sqrt{-c\log(f)-3f}}\right) e^{\left(-\frac{(b\log(f)+3e)^2}{4(c\log(f)+3f)}+3d\right)}}{16\sqrt{-c\log\left(f\right)-3f}}$$
$$+ \frac{3\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log\left(f\right)-f}x - \frac{b\log(f)+e}{2\sqrt{-c\log(f)-f}}\right) e^{\left(-\frac{(b\log(f)+e)^2}{4(c\log(f)+f)}+d\right)}}{16\sqrt{-c\log\left(f\right)-f}}$$
$$+ \frac{3\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log\left(f\right)+f}x - \frac{b\log(f)-e}{2\sqrt{-c\log(f)+f}}\right) e^{\left(-\frac{(b\log(f)-e)^2}{4(c\log(f)-f)}-d\right)}}{16\sqrt{-c\log\left(f\right)+f}}$$
$$+ \frac{\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log\left(f\right)+3f}x - \frac{b\log(f)-3e}{2\sqrt{-c\log(f)+3f}}\right) e^{\left(-\frac{(b\log(f)-3e)^2}{4(c\log(f)-3f)}-3d\right)}}{16\sqrt{-c\log\left(f\right)+3f}}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

output
```
1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x - 1/2*(b*log(f) + 3*e)/sqrt(
-c*log(f) - 3*f))*e^(-1/4*(b*log(f) + 3*e)^2/(c*log(f) + 3*f) + 3*d)/sqrt(
-c*log(f) - 3*f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*(b*lo
g(f) + e)/sqrt(-c*log(f) - f))*e^(-1/4*(b*log(f) + e)^2/(c*log(f) + f) + d
)/sqrt(-c*log(f) - f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*
(b*log(f) - e)/sqrt(-c*log(f) + f))*e^(-1/4*(b*log(f) - e)^2/(c*log(f) - f
) - d)/sqrt(-c*log(f) + f) + 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 3*f)*x
 - 1/2*(b*log(f) - 3*e)/sqrt(-c*log(f) + 3*f))*e^(-1/4*(b*log(f) - 3*e)^2/
(c*log(f) - 3*f) - 3*d)/sqrt(-c*log(f) + 3*f)
```

3.328.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.24

$$\int f^{a+bx+cx^2}\cosh^3\left(d+ex+fx^2\right)\,dx =$$

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)-3\,f}\left(2\,x+\frac{b\log(f)+3\,e}{c\log(f)+3\,f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2-12\,cd\log(f)+6\,be\log(f)-12\,af\log(f)+9\,e^2-36\,df}{4\,(c\log(f)+3\,f)}\right)}}{16\,\sqrt{-c\log\left(f\right)-3\,f}}$$
$$-\frac{3\,\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)-f}\left(2\,x+\frac{b\log(f)+e}{c\log(f)+f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2-4\,cd\log(f)+2\,be\log(f)-4\,af\log(f)+e^2-4\,df}{4\,(c\log(f)+f)}\right)}}{16\,\sqrt{-c\log\left(f\right)-f}}$$
$$-\frac{3\,\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)+f}\left(2\,x+\frac{b\log(f)-e}{c\log(f)-f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2+4\,cd\log(f)-2\,be\log(f)+4\,af\log(f)+e^2-4\,df}{4\,(c\log(f)-f)}\right)}}{16\,\sqrt{-c\log\left(f\right)+f}}$$
$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)+3\,f}\left(2\,x+\frac{b\log(f)-3\,e}{c\log(f)-3\,f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2+12\,cd\log(f)-6\,be\log(f)+12\,af\log(f)+9\,e^2-36\,df}{4\,(c\log(f)-3\,f)}\right)}}{16\,\sqrt{-c\log\left(f\right)+3\,f}}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="giac")
```

output
```
-1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 3*f)*(2*x + (b*log(f) + 3*e)/(c*l
og(f) + 3*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 12*c*d*log(f) + 6*
b*e*log(f) - 12*a*f*log(f) + 9*e^2 - 36*d*f)/(c*log(f) + 3*f))/sqrt(-c*log
(f) - 3*f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + (b*log(f) +
 e)/(c*log(f) + f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f)
 + 2*b*e*log(f) - 4*a*f*log(f) + e^2 - 4*d*f)/(c*log(f) + f))/sqrt(-c*log(
f) - f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x + (b*log(f) - e)
/(c*log(f) - f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d*log(f) -
2*b*e*log(f) + 4*a*f*log(f) + e^2 - 4*d*f)/(c*log(f) - f))/sqrt(-c*log(f)
+ f) - 1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + 3*f)*(2*x + (b*log(f) - 3*e
)/(c*log(f) - 3*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 12*c*d*log(f
) - 6*b*e*log(f) + 12*a*f*log(f) + 9*e^2 - 36*d*f)/(c*log(f) - 3*f))/sqrt(
-c*log(f) + 3*f)
```

3.328.9 Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\cosh^3\left(d+ex+fx^2\right)\,dx=\int f^{c\,x^2+b\,x+a}\cosh\left(f\,x^2+e\,x+d\right)^3\,dx$$

input
```
int(f^(a + b*x + c*x^2)*cosh(d + e*x + f*x^2)^3,x)
```

output
```
int(f^(a + b*x + c*x^2)*cosh(d + e*x + f*x^2)^3, x)
```

3.329 $\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)}+x\sqrt{\cosh(x)}\right)\,dx$

3.329.1 Optimal result

Integrand size = 17, antiderivative size = 20

$$\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)}+x\sqrt{\cosh(x)}\right)\,dx=-4\sqrt{\cosh(x)}+\frac{2x\sinh(x)}{\sqrt{\cosh(x)}}$$

output
```
2*x*sinh(x)/cosh(x)^(1/2)-4*cosh(x)^(1/2)
```

3.329.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 46 vs. 2(20) = 40.

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.30

$$\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)}+x\sqrt{\cosh(x)}\right)\,dx=\frac{2\sinh(x)\left(x-\frac{2\cosh(x)\sinh(x)\sqrt{\tanh^2\left(\frac{x}{2}\right)}}{(-1+\cosh(x))^{3/2}\sqrt{1+\cosh(x)}}\right)}{\sqrt{\cosh(x)}}$$

input
```
Integrate[x/Cosh[x]^(3/2) + x*Sqrt[Cosh[x]],x]
```

output
```
(2*Sinh[x]*(x - (2*Cosh[x]*Sinh[x]*Sqrt[Tanh[x/2]^2])/((-1 + Cosh[x])^(3/2
)*Sqrt[1 + Cosh[x]])))/Sqrt[Cosh[x]]
```

3.329.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)}+x\sqrt{\cosh(x)}\right) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2x\sinh(x)}{\sqrt{\cosh(x)}}-4\sqrt{\cosh(x)}$$

input
```
Int[x/Cosh[x]^(3/2) + x*Sqrt[Cosh[x]],x]
```

output
```
-4*Sqrt[Cosh[x]] + (2*x*Sinh[x])/Sqrt[Cosh[x]]
```

3.329.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.329.4 Maple [F]

$$\int \left(\frac{x}{\cosh\left(x\right)^{\frac{3}{2}}}+x\sqrt{\cosh\left(x\right)}\right)dx$$

input
```
int(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x)
```

output
```
int(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x)
```

3.329.5 Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)} + x\sqrt{\cosh(x)}\right) dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.329.6 Sympy [F]

$$\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)} + x\sqrt{\cosh(x)}\right) dx = \int \frac{x(\cosh^2(x) + 1)}{\cosh^{\frac{3}{2}}(x)}\, dx$$

input
```
integrate(x/cosh(x)**(3/2)+x*cosh(x)**(1/2),x)
```

output
```
Integral(x*(cosh(x)**2 + 1)/cosh(x)**(3/2), x)
```

3.329.7 Maxima [F]

$$\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)} + x\sqrt{\cosh(x)}\right) dx = \int x\sqrt{\cosh(x)} + \frac{x}{\cosh(x)^{\frac{3}{2}}}\, dx$$

input
```
integrate(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*sqrt(cosh(x)) + x/cosh(x)^(3/2), x)
```

3.329.8 Giac [F]

$$\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)}+x\sqrt{\cosh(x)}\right)\,dx=\int x\sqrt{\cosh\left(x\right)}+\frac{x}{\cosh\left(x\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*sqrt(cosh(x)) + x/cosh(x)^(3/2), x)
```

3.329.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)}+x\sqrt{\cosh(x)}\right)\,dx=-\frac{2\sqrt{\frac{\mathrm{e}^{-x}}{2}+\frac{\mathrm{e}^{x}}{2}}\,\left(x+2\,\mathrm{e}^{2\,x}-x\,\mathrm{e}^{2\,x}+2\right)}{\mathrm{e}^{2\,x}+1}$$

input
```
int(x*cosh(x)^(1/2) + x/cosh(x)^(3/2),x)
```

output
```
-(2*(exp(-x)/2 + exp(x)/2)^(1/2)*(x + 2*exp(2*x) - x*exp(2*x) + 2))/(exp(2
*x) + 1)
```

3.330 $\int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)}-\frac{x}{3\sqrt{\cosh(x)}}\right)\,dx$

3.330.1 Optimal result

Integrand size = 20, antiderivative size = 24

$$\int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)}-\frac{x}{3\sqrt{\cosh(x)}}\right)\,dx=\frac{4}{3\sqrt{\cosh(x)}}+\frac{2x\sinh(x)}{3\cosh^{\frac{3}{2}}(x)}$$

output
```
2/3*x*sinh(x)/cosh(x)^(3/2)+4/3/cosh(x)^(1/2)
```

3.330.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)}-\frac{x}{3\sqrt{\cosh(x)}}\right)\,dx=\frac{2(2+x\tanh(x))}{3\sqrt{\cosh(x)}}$$

input
```
Integrate[x/Cosh[x]^(5/2) - x/(3*Sqrt[Cosh[x]]),x]
```

output
```
(2*(2 + x*Tanh[x]))/(3*Sqrt[Cosh[x]])
```

3.330.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)}-\frac{x}{3\sqrt{\cosh(x)}}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{4}{3\sqrt{\cosh(x)}}+\frac{2x\sinh(x)}{3\cosh^{\frac{3}{2}}(x)}$$

input
```
Int[x/Cosh[x]^(5/2) - x/(3*Sqrt[Cosh[x]]),x]
```

output
```
4/(3*Sqrt[Cosh[x]]) + (2*x*Sinh[x])/(3*Cosh[x]^(3/2))
```

3.330.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.330.4 Maple [F]

$$\int \left(\frac{x}{\cosh\left(x\right)^{\frac{5}{2}}}-\frac{x}{3\sqrt{\cosh\left(x\right)}}\right)dx$$

input
```
int(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x)
```

output
```
int(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x)
```

3.330.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(16) = 32.

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 4.54

$$\int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cosh(x)}}\right) dx$$
$$= \frac{4\left((x+2)\cosh(x)^3 + 3(x+2)\cosh(x)\sinh(x)^2 + (x+2)\sinh(x)^3 - (x-2)\cosh(x) + (3(x+2)\cosh\right.}{3\left(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\left(3\cosh(x)^2+1\right)\sinh(x)^2 + 2\cosh(x)^2 + 4(\cosh\right.}$$

input
```
integrate(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x, algorithm="fricas")
```

output
```
4/3*((x + 2)*cosh(x)^3 + 3*(x + 2)*cosh(x)*sinh(x)^2 + (x + 2)*sinh(x)^3 -
 (x - 2)*cosh(x) + (3*(x + 2)*cosh(x)^2 - x + 2)*sinh(x))*sqrt(cosh(x))/(c
osh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2
 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

3.330.6 Sympy [F]

$$\int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cosh(x)}}\right) dx = -\frac{\int \left(-\frac{3x}{\cosh^{\frac{5}{2}}(x)}\right) dx + \int \frac{x}{\sqrt{\cosh(x)}}\, dx}{3}$$

input
```
integrate(x/cosh(x)**(5/2)-1/3*x/cosh(x)**(1/2),x)
```

output
```
-(Integral(-3*x/cosh(x)**(5/2), x) + Integral(x/sqrt(cosh(x)), x))/3
```

3.330.7 Maxima [F]

$$\int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cosh(x)}}\right) dx = \int -\frac{x}{3\sqrt{\cosh(x)}} + \frac{x}{\cosh(x)^{\frac{5}{2}}}\, dx$$

input
```
integrate(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(-1/3*x/sqrt(cosh(x)) + x/cosh(x)^(5/2), x)
```

3.330.8 Giac [F]

$$\int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cosh(x)}}\right) dx = \int -\frac{x}{3\sqrt{\cosh(x)}} + \frac{x}{\cosh(x)^{\frac{5}{2}}} dx$$

input
```
integrate(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(-1/3*x/sqrt(cosh(x)) + x/cosh(x)^(5/2), x)
```

3.330.9 Mupad [B] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.75

$$\int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cosh(x)}}\right) dx = \frac{4\,e^x\,\sqrt{\frac{e^{-x}}{2}+\frac{e^x}{2}}\,\left(2\,e^{2\,x}-x+x\,e^{2\,x}+2\right)}{3\,\left(e^{2\,x}+1\right)^2}$$

input
```
int(x/cosh(x)^(5/2) - x/(3*cosh(x)^(1/2)),x)
```

output
```
(4*exp(x)*(exp(-x)/2 + exp(x)/2)^(1/2)*(2*exp(2*x) - x + x*exp(2*x) + 2))/
(3*(exp(2*x) + 1)^2)
```

3.331 $\int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cosh(x)}\right)\,dx$

3.331.1 Optimal result

Integrand size = 20, antiderivative size = 47

$$\int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cosh(x)}\right)\,dx=\frac{4}{15\cosh^{\frac{3}{2}}(x)}-\frac{12\sqrt{\cosh(x)}}{5}+\frac{2x\sinh(x)}{5\cosh^{\frac{5}{2}}(x)}+\frac{6x\sinh(x)}{5\sqrt{\cosh(x)}}$$

output
```
4/15/cosh(x)^(3/2)+2/5*x*sinh(x)/cosh(x)^(5/2)+6/5*x*sinh(x)/cosh(x)^(1/2)
-12/5*cosh(x)^(1/2)
```

3.331.2 Mathematica [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.36

$$\int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cosh(x)}\right)\,dx =\frac{1}{5}\sqrt{\cosh(x)}\left(-\frac{12\sinh^2(x)}{\sqrt{-1+\cosh(x)}(1+\cosh(x))^{3/2}\sqrt{\tanh^2\left(\frac{x}{2}\right)}}+6x\tanh(x)+\operatorname{sech}^2(x)\left(\frac{4}{3}+2x\tanh(x)\right)\right)$$

input
```
Integrate[x/Cosh[x]^(7/2) + (3*x*Sqrt[Cosh[x]])/5,x]
```

output
```
(Sqrt[Cosh[x]]*((-12*Sinh[x]^2)/(Sqrt[-1 + Cosh[x]]*(1 + Cosh[x])^(3/2)*Sq
rt[Tanh[x/2]^2]) + 6*x*Tanh[x] + Sech[x]^2*(4/3 + 2*x*Tanh[x])))/5
```

3.331.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cosh(x)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4}{15\cosh^{\frac{3}{2}}(x)}-\frac{12\sqrt{\cosh(x)}}{5}+\frac{2x\sinh(x)}{5\cosh^{\frac{5}{2}}(x)}+\frac{6x\sinh(x)}{5\sqrt{\cosh(x)}}$$

input
```
Int[x/Cosh[x]^(7/2) + (3*x*Sqrt[Cosh[x]])/5,x]
```

output
```
4/(15*Cosh[x]^(3/2)) - (12*Sqrt[Cosh[x]])/5 + (2*x*Sinh[x])/(5*Cosh[x]^(5/
2)) + (6*x*Sinh[x])/(5*Sqrt[Cosh[x]])
```

3.331.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.331.4 Maple [F]

$$\int \left(\frac{x}{\cosh\left(x\right)^{\frac{7}{2}}}+\frac{3x\sqrt{\cosh\left(x\right)}}{5}\right)dx$$

input
```
int(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x)
```

output
```
int(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x)
```

3.331.5 Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cosh(x)}\right)dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.331.6 Sympy [F(-1)]

Timed out.

$$\int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cosh(x)}\right)dx = \text{Timed out}$$

input
```
integrate(x/cosh(x)**(7/2)+3/5*x*cosh(x)**(1/2),x)
```

output
```
Timed out
```

3.331.7 Maxima [F]

$$\int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cosh(x)}\right)\,dx=\int \frac{3}{5}\,x\sqrt{\cosh\left(x\right)}+\frac{x}{\cosh\left(x\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(3/5*x*sqrt(cosh(x)) + x/cosh(x)^(7/2), x)
```

3.331.8 Giac [F]

$$\int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cosh(x)}\right)\,dx=\int \frac{3}{5}\,x\sqrt{\cosh\left(x\right)}+\frac{x}{\cosh\left(x\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(3/5*x*sqrt(cosh(x)) + x/cosh(x)^(7/2), x)
```

3.331.9 Mupad [B] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.34

$$\int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cosh(x)}\right)\,dx=\frac{e^{2\,x}\,\left(\frac{8\,x}{5}+\frac{16}{15}\right)\,\sqrt{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}}{\left(e^{2\,x}+1\right)^2}-\left(\frac{6\,x}{5}+\frac{12}{5}\right)\,\sqrt{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}+\frac{12\,x\,e^{2\,x}\,\sqrt{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}}{5\,\left(e^{2\,x}+1\right)}-\frac{16\,x\,e^{2\,x}\,\sqrt{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}}{5\,\left(e^{2\,x}+1\right)^3}$$

input
```
int((3*x*cosh(x)^(1/2))/5 + x/cosh(x)^(7/2),x)
```

output
```
(exp(2*x)*((8*x)/5 + 16/15)*(exp(-x)/2 + exp(x)/2)^(1/2))/(exp(2*x) + 1)^2
 - ((6*x)/5 + 12/5)*(exp(-x)/2 + exp(x)/2)^(1/2) + (12*x*exp(2*x)*(exp(-x)
/2 + exp(x)/2)^(1/2))/(5*(exp(2*x) + 1)) - (16*x*exp(2*x)*(exp(-x)/2 + exp
(x)/2)^(1/2))/(5*(exp(2*x) + 1)^3)
```

3.332 $\int \left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)}+x^2\sqrt{\cosh(x)}\right) dx$

3.332.1 Optimal result

Integrand size = 21, antiderivative size = 36

$$\int \left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)}+x^2\sqrt{\cosh(x)}\right) dx = -8x\sqrt{\cosh(x)}-16iE\left(\frac{ix}{2}\middle|2\right)+\frac{2x^2\sinh(x)}{\sqrt{\cosh(x)}}$$

output
```
-16*I*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(1/2*x),2^(1/2))+2
*x^2*sinh(x)/cosh(x)^(1/2)-8*x*cosh(x)^(1/2)
```

3.332.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.11

$$\int \left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)}+x^2\sqrt{\cosh(x)}\right) dx$$
$$= \frac{4\sqrt{\cosh(x)}(\cosh(x)+\sinh(x))\left(-4(-2+x)\cosh(x)+x^2\sinh(x)+8\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2}\right.\right.}{1+e^{2x}}$$

input
```
Integrate[x^2/Cosh[x]^(3/2) + x^2*Sqrt[Cosh[x]],x]
```

output
```
(4*Sqrt[Cosh[x]]*(Cosh[x] + Sinh[x])*(-4*(-2 + x)*Cosh[x] + x^2*Sinh[x] +
8*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^(2*x)]*(-Cosh[x] + Sinh[x])*Sqrt[1
+ Cosh[2*x] + Sinh[2*x]]))/(1 + E^(2*x))
```

3.332.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)}+x^2\sqrt{\cosh(x)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2x^2\sinh(x)}{\sqrt{\cosh(x)}}-8x\sqrt{\cosh(x)}-16iE\left(\frac{ix}{2}\middle|2\right)$$

input
```
Int[x^2/Cosh[x]^(3/2) + x^2*Sqrt[Cosh[x]],x]
```

output
```
-8*x*Sqrt[Cosh[x]] - (16*I)*EllipticE[(I/2)*x, 2] + (2*x^2*Sinh[x])/Sqrt[C
osh[x]]
```

3.332.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.332.4 Maple [F]

$$\int \left(\frac{x^2}{\cosh(x)^{\frac{3}{2}}}+x^2\sqrt{\cosh(x)}\right)dx$$

input
```
int(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x)
```

output
```
int(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x)
```

3.332.5 Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)} + x^2\sqrt{\cosh(x)} \right) dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.332.6 Sympy [F]

$$\int \left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)} + x^2\sqrt{\cosh(x)} \right) dx = \int \frac{x^2(\cosh^2(x) + 1)}{\cosh^{\frac{3}{2}}(x)}\, dx$$

input
```
integrate(x**2/cosh(x)**(3/2)+x**2*cosh(x)**(1/2),x)
```

output
```
Integral(x**2*(cosh(x)**2 + 1)/cosh(x)**(3/2), x)
```

3.332.7 Maxima [F]

$$\int \left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)} + x^2\sqrt{\cosh(x)} \right) dx = \int x^2\sqrt{\cosh(x)} + \frac{x^2}{\cosh(x)^{\frac{3}{2}}}\, dx$$

input
```
integrate(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2*sqrt(cosh(x)) + x^2/cosh(x)^(3/2), x)
```

3.332.8 Giac [F]

$$\int \left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)} + x^2\sqrt{\cosh(x)}\right) dx = \int x^2\sqrt{\cosh(x)} + \frac{x^2}{\cosh(x)^{\frac{3}{2}}} \, dx$$

input
```
integrate(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2*sqrt(cosh(x)) + x^2/cosh(x)^(3/2), x)
```

3.332.9 Mupad [F(-1)]

Timed out.

$$\int \left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)} + x^2\sqrt{\cosh(x)}\right) dx = \int x^2\sqrt{\cosh(x)} + \frac{x^2}{\cosh(x)^{3/2}} \, dx$$

input
```
int(x^2*cosh(x)^(1/2) + x^2/cosh(x)^(3/2),x)
```

output
```
int(x^2*cosh(x)^(1/2) + x^2/cosh(x)^(3/2), x)
```

3.333 $\int (x+\cosh(x))^2\,dx$

3.333.1 Optimal result

Integrand size = 6, antiderivative size = 30

$$\int (x+\cosh(x))^2\,dx=\frac{x}{2}+\frac{x^3}{3}-2\cosh(x)+2x\sinh(x)+\frac{1}{2}\cosh(x)\sinh(x)$$

output
```
1/2*x+1/3*x^3-2*cosh(x)+2*x*sinh(x)+1/2*cosh(x)*sinh(x)
```

3.333.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int (x+\cosh(x))^2\,dx=\frac{1}{6}\left(3\cosh(x)(-4+\sinh(x))+x\left(3+2x^2+12\sinh(x)\right)\right)$$

input
```
Integrate[(x + Cosh[x])^2,x]
```

output
```
(3*Cosh[x]*(-4 + Sinh[x]) + x*(3 + 2*x^2 + 12*Sinh[x]))/6
```

3.333.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x+\cosh(x))^2\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(x^2+\cosh^2(x)+2x\cosh(x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^3}{3}+\frac{x}{2}+2x\sinh(x)-2\cosh(x)+\frac{1}{2}\sinh(x)\cosh(x)$$

input
```
Int[(x + Cosh[x])^2,x]
```

output
```
x/2 + x^3/3 - 2*Cosh[x] + 2*x*Sinh[x] + (Cosh[x]*Sinh[x])/2
```

3.333.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.333.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{x}{2}+\frac{x^3}{3}-2\cosh(x)+2x\sinh(x)+\frac{\cosh(x)\sinh(x)}{2}$	25
parts	$\frac{x}{2}+\frac{x^3}{3}-2\cosh(x)+2x\sinh(x)+\frac{\cosh(x)\sinh(x)}{2}$	25
parallelrisch	$\frac{x^3}{3}+\frac{x}{2}-2+2x\sinh(x)+\frac{\sinh(2x)}{4}-2\cosh(x)$	26
risch	$\frac{x^3}{3}+\frac{x}{2}+\frac{e^{2x}}{8}+(-1+x)e^{x}+(-1-x)e^{-x}-\frac{e^{-2x}}{8}$	38

input
```
int((x+cosh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x+1/3*x^3-2*cosh(x)+2*x*sinh(x)+1/2*cosh(x)*sinh(x)
```

3.333.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int (x+\cosh(x))^2\,dx = \frac{1}{3}\,x^3+\frac{1}{2}\,(4\,x+\cosh(x))\sinh(x)+\frac{1}{2}\,x-2\,\cosh(x)$$

input
```
integrate((x+cosh(x))^2,x, algorithm="fricas")
```

output
```
1/3*x^3 + 1/2*(4*x + cosh(x))*sinh(x) + 1/2*x - 2*cosh(x)
```

3.333.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.37

$$\int (x+\cosh(x))^2\,dx = \frac{x^3}{3}-\frac{x\sinh^2(x)}{2}+2x\sinh(x)+\frac{x\cosh^2(x)}{2}+\frac{\sinh(x)\cosh(x)}{2}-2\cosh(x)$$

input
```
integrate((x+cosh(x))**2,x)
```

output
```
x**3/3 - x*sinh(x)**2/2 + 2*x*sinh(x) + x*cosh(x)**2/2 + sinh(x)*cosh(x)/2
 - 2*cosh(x)
```

3.333.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int (x+\cosh(x))^2\,dx = \frac{1}{3}\,x^3 - (x+1)e^{(-x)} + (x-1)e^{x} + \frac{1}{2}\,x + \frac{1}{8}\,e^{(2\,x)} - \frac{1}{8}\,e^{(-2\,x)}$$

input
```
integrate((x+cosh(x))^2,x, algorithm="maxima")
```

output
```
1/3*x^3 - (x + 1)*e^(-x) + (x - 1)*e^x + 1/2*x + 1/8*e^(2*x) - 1/8*e^(-2*x
)
```

3.333.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int (x+\cosh(x))^2\,dx = \frac{1}{3}\,x^3 - (x+1)e^{(-x)} + (x-1)e^{x} + \frac{1}{2}\,x + \frac{1}{8}\,e^{(2\,x)} - \frac{1}{8}\,e^{(-2\,x)}$$

input
```
integrate((x+cosh(x))^2,x, algorithm="giac")
```

output
```
1/3*x^3 - (x + 1)*e^(-x) + (x - 1)*e^x + 1/2*x + 1/8*e^(2*x) - 1/8*e^(-2*x
)
```

3.333.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int (x+\cosh(x))^2\,dx = \frac{x}{2} - 2\cosh(x) + \frac{\cosh(x)\,\sinh(x)}{2} + 2\,x\sinh(x) + \frac{x^3}{3}$$

input
```
int((x + cosh(x))^2,x)
```

output
```
x/2 - 2*cosh(x) + (cosh(x)*sinh(x))/2 + 2*x*sinh(x) + x^3/3
```

3.334 $\int (x+\cosh(x))^3\,dx$

3.334.1 Optimal result

Integrand size = 6, antiderivative size = 56

$$\int (x+\cosh(x))^3\,dx = \frac{3x^2}{4}+\frac{x^4}{4}-6x\cosh(x)-\frac{3\cosh^2(x)}{4}+7\sinh(x)+3x^2\sinh(x)+\frac{3}{2}x\cosh(x)\sinh(x)+\frac{\sinh^3(x)}{3}$$

output
```
3/4*x^2+1/4*x^4-6*x*cosh(x)-3/4*cosh(x)^2+7*sinh(x)+3*x^2*sinh(x)+3/2*x*co
sh(x)*sinh(x)+1/3*sinh(x)^3
```

3.334.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int (x+\cosh(x))^3\,dx = -6x\cosh(x)-\frac{3}{8}\cosh(2x)+\frac{1}{12}\left(9x^2+3x^4+9\left(9+4x^2\right)\sinh(x)+9x\sinh(2x)+\sinh(3x)\right)$$

input
```
Integrate[(x + Cosh[x])^3,x]
```

output
```
-6*x*Cosh[x] - (3*Cosh[2*x])/8 + (9*x^2 + 3*x^4 + 9*(9 + 4*x^2)*Sinh[x] +
9*x*Sinh[2*x] + Sinh[3*x])/12
```

3.334.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x+\cosh(x))^3 \, dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(x^3+3x^2\cosh(x)+\cosh^3(x)+3x\cosh^2(x)\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^4}{4}+\frac{3x^2}{4}+3x^2\sinh(x)+\frac{\sinh^3(x)}{3}+7\sinh(x)-\frac{3\cosh^2(x)}{4}-6x\cosh(x)+\frac{3}{2}x\sinh(x)\cosh(x)$$

input
```
Int[(x + Cosh[x])^3,x]
```

output
```
(3*x^2)/4 + x^4/4 - 6*x*Cosh[x] - (3*Cosh[x]^2)/4 + 7*Sinh[x] + 3*x^2*Sinh
[x] + (3*x*Cosh[x]*Sinh[x])/2 + Sinh[x]^3/3
```

3.334.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.334.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

method	result
parallelrisch	$-\frac{5}{8}+\frac{x^{4}}{4}+\frac{3x^{2}}{4}+3x^{2}\sinh\left(x\right)-6x\cosh\left(x\right)+\frac{3x\sinh(2x)}{4}+\frac{\sinh(3x)}{12}+\frac{27\sinh(x)}{4}-\frac{3\cosh(2x)}{8}$
default	$\left(\frac{2}{3}+\frac{\cosh(x)^{2}}{3}\right)\sinh\left(x\right)+\frac{3x\cosh(x)\sinh(x)}{2}+\frac{3x^{2}}{4}-\frac{3\cosh(x)^{2}}{4}+3x^{2}\sinh\left(x\right)-6x\cosh\left(x\right)+6\sin$
parts	$\left(\frac{2}{3}+\frac{\cosh(x)^{2}}{3}\right)\sinh\left(x\right)+\frac{3x\cosh(x)\sinh(x)}{2}+\frac{3x^{2}}{4}-\frac{3\cosh(x)^{2}}{4}+3x^{2}\sinh\left(x\right)-6x\cosh\left(x\right)+6\sin$
risch	$\frac{x^{4}}{4}+\frac{3x^{2}}{4}+\frac{9}{16}+\frac{\mathrm{e}^{3x}}{24}+\left(-\frac{3}{16}+\frac{3x}{8}\right)\mathrm{e}^{2x}+\left(\frac{27}{8}-3x+\frac{3}{2}x^{2}\right)\mathrm{e}^{x}+\left(-\frac{27}{8}-3x-\frac{3}{2}x^{2}\right)\mathrm{e}^{-x}+\left(-\frac{3}{16}\right.$

input
```
int((x+cosh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-5/8+1/4*x^4+3/4*x^2+3*x^2*sinh(x)-6*x*cosh(x)+3/4*x*sinh(2*x)+1/12*sinh(3
*x)+27/4*sinh(x)-3/8*cosh(2*x)
```

3.334.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\begin{aligned}\int(x+\cosh(x))^{3}\,dx=&\ \frac{1}{4}\,x^{4}+\frac{1}{12}\,\sinh\left(x\right)^{3}+\frac{3}{4}\,x^{2}-6\,x\cosh\left(x\right)-\frac{3}{8}\,\cosh\left(x\right)^{2}\\&+\frac{1}{4}\left(12\,x^{2}+6\,x\cosh\left(x\right)+\cosh\left(x\right)^{2}+27\right)\sinh\left(x\right)-\frac{3}{8}\,\sinh\left(x\right)^{2}\end{aligned}$$

input
```
integrate((x+cosh(x))^3,x, algorithm="fricas")
```

output
```
1/4*x^4 + 1/12*sinh(x)^3 + 3/4*x^2 - 6*x*cosh(x) - 3/8*cosh(x)^2 + 1/4*(12
*x^2 + 6*x*cosh(x) + cosh(x)^2 + 27)*sinh(x) - 3/8*sinh(x)^2
```

3.334.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.52

$$\int (x+\cosh(x))^3\,dx = \frac{x^4}{4} - \frac{3x^2\sinh^2(x)}{4} + 3x^2\sinh(x) + \frac{3x^2\cosh^2(x)}{4} + \frac{3x\sinh(x)\cosh(x)}{2} - 6x\cosh(x) - \frac{2\sinh^3(x)}{3} + \sinh(x)\cosh^2(x) + 6\sinh(x) - \frac{3\cosh^2(x)}{4}$$

input
```
integrate((x+cosh(x))**3,x)
```

output
```
x**4/4 - 3*x**2*sinh(x)**2/4 + 3*x**2*sinh(x) + 3*x**2*cosh(x)**2/4 + 3*x*
sinh(x)*cosh(x)/2 - 6*x*cosh(x) - 2*sinh(x)**3/3 + sinh(x)*cosh(x)**2 + 6*
sinh(x) - 3*cosh(x)**2/4
```

3.334.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.45

$$\int (x+\cosh(x))^3\,dx = \frac{1}{4}x^4 + \frac{3}{4}x^2 + \frac{3}{16}(2x-1)e^{(2x)} - \frac{3}{2}(x^2+2x+2)e^{(-x)} - \frac{3}{16}(2x+1)e^{(-2x)} + \frac{3}{2}(x^2-2x+2)e^x + \frac{1}{24}e^{(3x)} - \frac{3}{8}e^{(-x)} - \frac{1}{24}e^{(-3x)} + \frac{3}{8}e^x$$

input
```
integrate((x+cosh(x))^3,x, algorithm="maxima")
```

output
```
1/4*x^4 + 3/4*x^2 + 3/16*(2*x - 1)*e^(2*x) - 3/2*(x^2 + 2*x + 2)*e^(-x) -
3/16*(2*x + 1)*e^(-2*x) + 3/2*(x^2 - 2*x + 2)*e^x + 1/24*e^(3*x) - 3/8*e^(
-x) - 1/24*e^(-3*x) + 3/8*e^x
```

3.334.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.34

$$\int (x+\cosh(x))^3\,dx = \frac{1}{4}\,x^4 + \frac{3}{4}\,x^2 + \frac{3}{16}\,(2\,x-1)e^{(2\,x)} - \frac{3}{8}\,\left(4\,x^2+8\,x+9\right)e^{(-x)} - \frac{3}{16}\,(2\,x+1)e^{(-2\,x)} + \frac{3}{8}\,\left(4\,x^2-8\,x+9\right)e^{x} + \frac{1}{24}\,e^{(3\,x)} - \frac{1}{24}\,e^{(-3\,x)}$$

input
```
integrate((x+cosh(x))^3,x, algorithm="giac")
```

output
```
1/4*x^4 + 3/4*x^2 + 3/16*(2*x - 1)*e^(2*x) - 3/8*(4*x^2 + 8*x + 9)*e^(-x)
- 3/16*(2*x + 1)*e^(-2*x) + 3/8*(4*x^2 - 8*x + 9)*e^x + 1/24*e^(3*x) - 1/2
4*e^(-3*x)
```

3.334.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int (x+\cosh(x))^3\,dx = \frac{20\sinh(x)}{3} + 3\,x^2\sinh(x) - \frac{3\cosh(x)^2}{4} + \frac{\cosh(x)^2\sinh(x)}{3} - 6\,x\cosh(x) + \frac{3\,x^2}{4} + \frac{x^4}{4} + \frac{3\,x\cosh(x)\,\sinh(x)}{2}$$

input
```
int((x + cosh(x))^3,x)
```

output
```
(20*sinh(x))/3 + 3*x^2*sinh(x) - (3*cosh(x)^2)/4 + (cosh(x)^2*sinh(x))/3 -
 6*x*cosh(x) + (3*x^2)/4 + x^4/4 + (3*x*cosh(x)*sinh(x))/2
```

3.335 $\int \frac{\cosh(a+bx)}{c+dx^2} dx$

3.335.1 Optimal result

Integrand size = 16, antiderivative size = 213

$$\int \frac{\cosh(a+bx)}{c+dx^2} dx = \frac{\cosh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}} - \frac{\cosh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{2\sqrt{-c}\sqrt{d}} - \frac{\sinh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Shi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}} - \frac{\sinh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Shi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{2\sqrt{-c}\sqrt{d}}$$

output
```
-1/2*Chi(b*x+b*(-c)^(1/2)/d^(1/2))*cosh(a-b*(-c)^(1/2)/d^(1/2))/(-c)^(1/2)
/d^(1/2)+1/2*Chi(-b*x+b*(-c)^(1/2)/d^(1/2))*cosh(a+b*(-c)^(1/2)/d^(1/2))/(
-c)^(1/2)/d^(1/2)-1/2*Shi(b*x+b*(-c)^(1/2)/d^(1/2))*sinh(a-b*(-c)^(1/2)/d^
(1/2))/(-c)^(1/2)/d^(1/2)+1/2*Shi(b*x-b*(-c)^(1/2)/d^(1/2))*sinh(a+b*(-c)^
(1/2)/d^(1/2))/(-c)^(1/2)/d^(1/2)
```

3.335.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.78

$$\int \frac{\cosh(a+bx)}{c+dx^2} dx = -\frac{ie^{-a-\frac{ib\sqrt{c}}{\sqrt{d}}}\left(e^{2a+\frac{2ib\sqrt{c}}{\sqrt{d}}}\text{ExpIntegralEi}\left(b\left(-\frac{i\sqrt{c}}{\sqrt{d}}+x\right)\right)-e^{2a}\text{ExpIntegralEi}\left(b\left(\frac{i\sqrt{c}}{\sqrt{d}}+x\right)\right)-e^{\frac{2ib\sqrt{c}}{\sqrt{d}}}\text{ExpInte}\right.}{4\sqrt{c}\sqrt{d}}$$

input
```
Integrate[Cosh[a + b*x]/(c + d*x^2),x]
```

output
```
((-1/4*I)*E^(-a - (I*b*Sqrt[c])/Sqrt[d])*(E^(2*a + ((2*I)*b*Sqrt[c])/Sqrt[
d])*ExpIntegralEi[b*(((-I)*Sqrt[c])/Sqrt[d] + x)] - E^(2*a)*ExpIntegralEi[
b*((I*Sqrt[c])/Sqrt[d] + x)] - E^(((2*I)*b*Sqrt[c])/Sqrt[d])*ExpIntegralEi
[((-I)*b*Sqrt[c])/Sqrt[d] - b*x] + ExpIntegralEi[(I*b*Sqrt[c])/Sqrt[d] - b
*x]))/(Sqrt[c]*Sqrt[d])
```

3.335.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(a+bx)}{c+dx^2} \, dx$$

$$\Big\downarrow \ 5804$$

$$\int \left(\frac{\sqrt{-c}\cosh(a+bx)}{2c\left(\sqrt{-c}-\sqrt{d}x\right)}+\frac{\sqrt{-c}\cosh(a+bx)}{2c\left(\sqrt{-c}+\sqrt{d}x\right)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{\cosh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\cosh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Chi}\left(xb+\frac{\sqrt{-c}b}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\sinh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Shi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\sinh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Shi}\left(xb+\frac{\sqrt{-c}b}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}$$

input
```
Int[Cosh[a + b*x]/(c + d*x^2),x]
```

output
```
(Cosh[a + (b*Sqrt[-c])/Sqrt[d]]*CoshIntegral[(b*Sqrt[-c])/Sqrt[d] - b*x])/
(2*Sqrt[-c]*Sqrt[d]) - (Cosh[a - (b*Sqrt[-c])/Sqrt[d]]*CoshIntegral[(b*Sqr
t[-c])/Sqrt[d] + b*x])/(2*Sqrt[-c]*Sqrt[d]) - (Sinh[a + (b*Sqrt[-c])/Sqrt[
d]]*SinhIntegral[(b*Sqrt[-c])/Sqrt[d] - b*x])/(2*Sqrt[-c]*Sqrt[d]) - (Sinh
[a - (b*Sqrt[-c])/Sqrt[d]]*SinhIntegral[(b*Sqrt[-c])/Sqrt[d] + b*x])/(2*Sq
rt[-c]*Sqrt[d])
```

3.335.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5804
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

3.335.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{\mathrm{e}^{\frac{b\sqrt{-cd}+da}{d}}\mathrm{Ei}_1\left(\frac{b\sqrt{-cd}-(bx+a)d+da}{d}\right)}{4\sqrt{-cd}}+\frac{\mathrm{e}^{\frac{-b\sqrt{-cd}+da}{d}}\mathrm{Ei}_1\left(-\frac{b\sqrt{-cd}+(bx+a)d-da}{d}\right)}{4\sqrt{-cd}}-\frac{\mathrm{e}^{-\frac{b\sqrt{-cd}+da}{d}}\mathrm{Ei}_1\left(-\frac{b\sqrt{-cd}-(bx+a)d+da}{d}\right)}{4\sqrt{-cd}}$

input
```
int(cosh(b*x+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/4/(-c*d)^(1/2)*exp((b*(-c*d)^(1/2)+d*a)/d)*Ei(1,(b*(-c*d)^(1/2)-(b*x+a)
*d+d*a)/d)+1/4/(-c*d)^(1/2)*exp((-b*(-c*d)^(1/2)+d*a)/d)*Ei(1,-(b*(-c*d)^(
1/2)+(b*x+a)*d-d*a)/d)-1/4/(-c*d)^(1/2)*exp(-(b*(-c*d)^(1/2)+d*a)/d)*Ei(1,
-(b*(-c*d)^(1/2)-(b*x+a)*d+d*a)/d)+1/4/(-c*d)^(1/2)*exp(-(-b*(-c*d)^(1/2)+
d*a)/d)*Ei(1,(b*(-c*d)^(1/2)+(b*x+a)*d-d*a)/d)
```

3.335.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(157) = 314.

Time = 0.26 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.48

$$\int \frac{\cosh(a+bx)}{c+dx^2}\,dx =$$
$$-\frac{\left(\sqrt{-\frac{b^2c}{d}}\mathrm{Ei}\left(bx-\sqrt{-\frac{b^2c}{d}}\right)+\sqrt{-\frac{b^2c}{d}}\mathrm{Ei}\left(-bx+\sqrt{-\frac{b^2c}{d}}\right)\right)\cosh\left(a+\sqrt{-\frac{b^2c}{d}}\right)-\left(\sqrt{-\frac{b^2c}{d}}\mathrm{Ei}\left(bx+\sqrt{\right.\right.}{}$$

input
```
integrate(cosh(b*x+a)/(d*x^2+c),x, algorithm="fricas")
```

output
```
-1/4*((sqrt(-b^2*c/d)*Ei(b*x - sqrt(-b^2*c/d)) + sqrt(-b^2*c/d)*Ei(-b*x +
sqrt(-b^2*c/d)))*cosh(a + sqrt(-b^2*c/d)) - (sqrt(-b^2*c/d)*Ei(b*x + sqrt(
-b^2*c/d)) + sqrt(-b^2*c/d)*Ei(-b*x - sqrt(-b^2*c/d)))*cosh(-a + sqrt(-b^2
*c/d)) + (sqrt(-b^2*c/d)*Ei(b*x - sqrt(-b^2*c/d)) - sqrt(-b^2*c/d)*Ei(-b*x
 + sqrt(-b^2*c/d)))*sinh(a + sqrt(-b^2*c/d)) + (sqrt(-b^2*c/d)*Ei(b*x + sq
rt(-b^2*c/d)) - sqrt(-b^2*c/d)*Ei(-b*x - sqrt(-b^2*c/d)))*sinh(-a + sqrt(-
b^2*c/d)))/(b*c)
```

3.335.6 Sympy [F]

$$\int \frac{\cosh(a+bx)}{c+dx^2}\,dx=\int \frac{\cosh(a+bx)}{c+dx^2}\,dx$$

input
```
integrate(cosh(b*x+a)/(d*x**2+c),x)
```

output
```
Integral(cosh(a + b*x)/(c + d*x**2), x)
```

3.335.7 Maxima [F]

$$\int \frac{\cosh(a+bx)}{c+dx^2}\,dx=\int \frac{\cosh(bx+a)}{dx^2+c}\,dx$$

input
```
integrate(cosh(b*x+a)/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate(cosh(b*x + a)/(d*x^2 + c), x)
```

3.335.8 Giac [F]

$$\int \frac{\cosh(a+bx)}{c+dx^2}\,dx=\int \frac{\cosh(bx+a)}{dx^2+c}\,dx$$

input
```
integrate(cosh(b*x+a)/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)/(d*x^2 + c), x)
```

3.335.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)}{c+dx^2}\,dx = \int \frac{\cosh(a+b\,x)}{d\,x^2+c}\,dx$$

input
```
int(cosh(a + b*x)/(c + d*x^2),x)
```

output
```
int(cosh(a + b*x)/(c + d*x^2), x)
```

3.336 $\int \frac{\cosh(a+bx)}{c+dx+ex^2}\,dx$

3.336.1 Optimal result

Integrand size = 19, antiderivative size = 271

$$\int \frac{\cosh(a+bx)}{c+dx+ex^2}\,dx = \frac{\cosh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\mathrm{Chi}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}} - \frac{\cosh\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)\mathrm{Chi}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}} + \frac{\sinh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\mathrm{Shi}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}} - \frac{\sinh\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)\mathrm{Shi}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}$$

output
```
Chi(b*x+1/2*b*(d-(-4*c*e+d^2)^(1/2))/e)*cosh(a-1/2*b*(d-(-4*c*e+d^2)^(1/2)
)/e)/(-4*c*e+d^2)^(1/2)-Chi(b*x+1/2*b*(d+(-4*c*e+d^2)^(1/2))/e)*cosh(a-1/2
*b*(d+(-4*c*e+d^2)^(1/2))/e)/(-4*c*e+d^2)^(1/2)+Shi(b*x+1/2*b*(d-(-4*c*e+d
^2)^(1/2))/e)*sinh(a-1/2*b*(d-(-4*c*e+d^2)^(1/2))/e)/(-4*c*e+d^2)^(1/2)-Sh
i(b*x+1/2*b*(d+(-4*c*e+d^2)^(1/2))/e)*sinh(a-1/2*b*(d+(-4*c*e+d^2)^(1/2))/
e)/(-4*c*e+d^2)^(1/2)
```

3.336.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.81

$$\int \frac{\cosh(a+bx)}{c+dx+ex^2}\,dx = \frac{e^{-a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}}\left(e^{\frac{bd}{e}}\,\text{ExpIntegralEi}\left(-\frac{b\left(d-\sqrt{d^2-4ce}+2ex\right)}{2e}\right)+e^{2a+\frac{b\sqrt{d^2-4ce}}{e}}\,\text{ExpIntegralEi}\left(\frac{b\left(d-\sqrt{d^2-4ce}+2ex\right)}{2e}\right.\right.}{2\sqrt{d^2-4c}}$$

input
```
Integrate[Cosh[a + b*x]/(c + d*x + e*x^2),x]
```

output
```
(E^(-a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e))*(E^((b*d)/e)*ExpIntegralEi[-1/
2*(b*(d - Sqrt[d^2 - 4*c*e] + 2*e*x))/e] + E^(2*a + (b*Sqrt[d^2 - 4*c*e])/
e)*ExpIntegralEi[(b*(d - Sqrt[d^2 - 4*c*e] + 2*e*x))/(2*e)] - E^((b*(d + S
qrt[d^2 - 4*c*e]))/e)*ExpIntegralEi[-1/2*(b*(d + Sqrt[d^2 - 4*c*e] + 2*e*x
))/e] - E^(2*a)*ExpIntegralEi[(b*(d + Sqrt[d^2 - 4*c*e] + 2*e*x))/(2*e)]))
/(2*Sqrt[d^2 - 4*c*e])
```

3.336.3 Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(a+bx)}{c+dx+ex^2}\,dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{2e\cosh(a+bx)}{\sqrt{d^2-4ce}\left(-\sqrt{d^2-4ce}+d+2ex\right)}-\frac{2e\cosh(a+bx)}{\sqrt{d^2-4ce}\left(\sqrt{d^2-4ce}+d+2ex\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\cosh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\operatorname{Chi}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\cosh\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\operatorname{Chi}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}+\frac{\sinh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\operatorname{Shi}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\sinh\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\operatorname{Shi}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}$$

input
```
Int[Cosh[a + b*x]/(c + d*x + e*x^2),x]
```

output
```
(Cosh[a - (b*(d - Sqrt[d^2 - 4*c*e]))/(2*e)]*CoshIntegral[(b*(d - Sqrt[d^2
 - 4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e] - (Cosh[a - (b*(d + Sqrt[d^2 -
 4*c*e]))/(2*e)]*CoshIntegral[(b*(d + Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sq
rt[d^2 - 4*c*e] + (Sinh[a - (b*(d - Sqrt[d^2 - 4*c*e]))/(2*e)]*SinhIntegra
l[(b*(d - Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e] - (Sinh[a -
(b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)]*SinhIntegral[(b*(d + Sqrt[d^2 - 4*c*e])
)/(2*e) + b*x])/Sqrt[d^2 - 4*c*e]
```

3.336.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.336.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.39

method	result
risch	$-\frac{b\,\mathrm{e}^{\frac{2ea-bd+\sqrt{-4b^2ce+b^2d^2}}{2e}}\operatorname{Ei}_1\left(\frac{-2e(bx+a)+2ea-bd+\sqrt{-4b^2ce+b^2d^2}}{2e}\right)}{2\sqrt{-4b^2ce+b^2d^2}}+\frac{b\,\mathrm{e}^{\frac{2ea-bd-\sqrt{-4b^2ce+b^2d^2}}{2e}}\operatorname{Ei}_1\left(-\frac{2e(bx+a)-2ea+bd+\sqrt{-4b}}{2e}\right.}{2\sqrt{-4b^2ce+b^2d^2}}$

input
```
int(cosh(b*x+a)/(e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2*b/(-4*b^2*c*e+b^2*d^2)^(1/2)*exp(1/2/e*(2*e*a-b*d+(-4*b^2*c*e+b^2*d^2
)^(1/2)))*Ei(1,1/2*(-2*e*(b*x+a)+2*e*a-b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)+
1/2*b/(-4*b^2*c*e+b^2*d^2)^(1/2)*exp(1/2/e*(2*e*a-b*d-(-4*b^2*c*e+b^2*d^2)
^(1/2)))*Ei(1,-1/2*(2*e*(b*x+a)-2*e*a+b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)-1
/2*b/(-4*b^2*c*e+b^2*d^2)^(1/2)*exp(-1/2/e*(2*e*a-b*d+(-4*b^2*c*e+b^2*d^2)
^(1/2)))*Ei(1,-1/2*(-2*e*(b*x+a)+2*e*a-b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)+
1/2*b/(-4*b^2*c*e+b^2*d^2)^(1/2)*exp(-1/2/e*(2*e*a-b*d-(-4*b^2*c*e+b^2*d^2
)^(1/2)))*Ei(1,1/2*(2*e*(b*x+a)-2*e*a+b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)
```

3.336.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 671 vs. $2(231) = 462$.

Time = 0.27 (sec) , antiderivative size = 671, normalized size of antiderivative = 2.48

$$\int \frac{\cosh(a+bx)}{c+dx+ex^2}\,dx =$$
$$-\frac{\left(e\sqrt{\frac{b^2d^2-4\,b^2ce}{e^2}}\operatorname{Ei}\left(\frac{2\,bex+bd+e\sqrt{\frac{b^2d^2-4\,b^2ce}{e^2}}}{2\,e}\right)+e\sqrt{\frac{b^2d^2-4\,b^2ce}{e^2}}\operatorname{Ei}\left(-\frac{2\,bex+bd+e\sqrt{\frac{b^2d^2-4\,b^2ce}{e^2}}}{2\,e}\right)\right)\cosh\left(\frac{bd-2\,ae+e\sqrt{b}}{2\,e}\right.}{}$$

input
```
integrate(cosh(b*x+a)/(e*x^2+d*x+c),x, algorithm="fricas")
```

output
```
-1/2*((e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(2*b*e*x + b*d + e*sqrt((b
^2*d^2 - 4*b^2*c*e)/e^2))/e) + e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(-1/2*(
2*b*e*x + b*d + e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e))*cosh(1/2*(b*d - 2*a
*e + e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e) - (e*sqrt((b^2*d^2 - 4*b^2*c*e)
/e^2)*Ei(1/2*(2*b*e*x + b*d - e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e) + e*sq
rt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(-1/2*(2*b*e*x + b*d - e*sqrt((b^2*d^2 - 4
*b^2*c*e)/e^2))/e))*cosh(-1/2*(b*d - 2*a*e - e*sqrt((b^2*d^2 - 4*b^2*c*e)/
e^2))/e) - (e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(2*b*e*x + b*d + e*sq
rt((b^2*d^2 - 4*b^2*c*e)/e^2))/e) - e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(-
1/2*(2*b*e*x + b*d + e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e))*sinh(1/2*(b*d
- 2*a*e + e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e) - (e*sqrt((b^2*d^2 - 4*b^2
*c*e)/e^2)*Ei(1/2*(2*b*e*x + b*d - e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e) -
 e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(-1/2*(2*b*e*x + b*d - e*sqrt((b^2*d^
2 - 4*b^2*c*e)/e^2))/e))*sinh(-1/2*(b*d - 2*a*e - e*sqrt((b^2*d^2 - 4*b^2*
c*e)/e^2))/e))/(b*d^2 - 4*b*c*e)
```

3.336.6 Sympy [F]

$$\int \frac{\cosh(a+bx)}{c+dx+ex^2}\,dx = \int \frac{\cosh{(a+bx)}}{c+dx+ex^2}\,dx$$

input
```
integrate(cosh(b*x+a)/(e*x**2+d*x+c),x)
```

output
```
Integral(cosh(a + b*x)/(c + d*x + e*x**2), x)
```

3.336.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh(a+bx)}{c+dx+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(b*x+a)/(e*x^2+d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c*e-d^2>0)', see `assume?` for
 more deta
```

3.336.8 Giac [F]

$$\int \frac{\cosh(a+bx)}{c+dx+ex^2}\,dx = \int \frac{\cosh\left(bx+a\right)}{ex^2+dx+c}\,dx$$

input
```
integrate(cosh(b*x+a)/(e*x^2+d*x+c),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)/(e*x^2 + d*x + c), x)
```

3.336.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)}{c+dx+ex^2}\,dx = \int \frac{\cosh(a+b\,x)}{e\,x^2+d\,x+c}\,dx$$

input
```
int(cosh(a + b*x)/(c + d*x + e*x^2),x)
```

output
```
int(cosh(a + b*x)/(c + d*x + e*x^2), x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                           convert(leaf_count_result,string),"$ vs. $2(",
                           convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
            return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =" "
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


    print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```